NBER WORKING PAPER SERIES

CAPITAL SHARE DYNAMICS WHEN FIRMS INSURE MANAGERS

Barney Hartman-Glaser
Hanno Lustig
Mindy X. Zhang

Working Paper 22651
http://www.nber.org/papers/w22651

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 2016

Previously circulated as "National Income Accounting When Firms Insure Managers: Understanding Firm Size and Compensation Inequality." We received detailed feedback from Hengjie Ai, Andy Atkeson, Alti Aydogan, Frederico Belo, Jonathan Berk, Peter DeMarzo, Darrell Duffie, Bernard Dumas, Andres Donangelo, Mike Elsby (discussant), Bob Hall, Lars Hansen, Ben Hebert, Chad Jones, Pat Kehoe, Arvind Krishnamurthy, Pablo Kurlat, Ed Lazear, Monika Piazzesi, Chris Tonetti, Sebastian DiTella, Martin Schneider, and Andy Skrypacz. The authors acknowledge comments received from seminar participants at Insead, the Stanford GSB, UT Austin, Universite de Lausanne and EPFL, the Carlson School at the University of Minnesota, the Federal Reserve Bank of Atlanta, the Federal Reserve Bank of New York, the Capital Markets group at the NBER Summer Institute, SITE, and the 2016 Labor and Finance group meeting. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

Capital Share Dynamics When Firms Insure Managers
Barney Hartman-Glaser, Hanno Lustig, and Mindy X. Zhang
NBER Working Paper No. 22651
September 2016, Revised December 2016
JEL No. E25,G30

ABSTRACT

The share of the average firm's value added that accrues to its owners has declined, even though the aggregate capital share has increased. These changes in factor shares partly reflect a larger firm-level risk insurance premium paid by workers to owners. The largest firms in the right tail account for a larger share of output, but the compensation of workers at these firms has not kept up. We develop a model in which firms provide managers with insurance against firm-specific shocks. Larger, more productive firms return a larger share of rents to shareholders, while less productive firms endogenously exit. An increase in firm-level risk lowers the threshold at which firms exit and increases the measure of firms in the right tail of the size distribution, pushing up the aggregate capital share in the economy, but lowering the average firm's capital share. As predicted by the model, the increase in firm size inequality is not matched by an increase in inter-firm labor compensation inequality.

Barney Hartman-Glaser
University of California at Los Angeles
110 Westwood Plaza
Suite C421
Los Angeles, CA 90095
barney.hartman-glaser@anderson.ucla.edu

Hanno Lustig
Stanford Graduate School of Business
655 Knight Way
Stanford, CA 94305
and NBER
hlustig@stanford.edu

Mindy X. Zhang
University of Texas at Austin
2110 Speedway B6600
Austin, TX 78703
xiaolan.zhang@mccombs.utexas.edu

1 Introduction

Over the last decades, publicly traded U.S. firms have experienced a large increase in firm-specific volatility of both firm-level cash flow as well as returns (see, e.g., Campbell, Lettau, Malkiel, and Xu, 2001; Comin and Philippon, 2005; Zhang, 2014; Bloom, 2014; Herskovic, Kelly, Lustig, and Van Nieuwerburgh, 2015). At the same time, the share of total value added that accrues to the owners of these firms, the aggregate capital share, has also increased, while the average firm's capital share has decreased. We develop an equilibrium model which links these two facts and provides novel implications for national income accounting. Our model demonstrates that when shareholder's insure manager's against idiosyncratic risk, level capital shares vary substantially over the size distribution of firms, with the largest and most productive firms having the highest capital share. As such, compositional changes in firm-level capital shares have implications for aggregates. In a statistical decomposition, we find that the increase in firm size inequality induced by the increase in risk accounts for most of the increase in the aggregate capital share for publicly traded firms. The capital share at the average firm, however, has decreased. Inter-firm compensation inequality has increased (see Song, Price, Guvenen, Bloom, and von Wachter, 2015, for recent evidence), but not enough to offset the effect of the increase the firm size inequality on the capital share.

Since shareholders of publicly traded firms can diversify idiosyncratic firm-specific risk away, while risk-averse workers cannot, it is efficient for firms to provide managers with insurance against firm-specific risk. We analyze a simple compensation contract in an equilibrium model of industry dynamics (see, e.g., Hopenhayn, 1992) that pays managers a fixed wage while allocating the remainder of profits to shareholders. The level of managerial compensation is set in equilibrium to capture the value of ex-ante identical firms, as in Atkeson and Kehoe (2005). Ex-post these firms are subject to permanent idiosyncratic shocks that lead some firms to increase in size and productivity while others decrease and potentially exit. We use this model as a laboratory to analyze the impact of changes in firm-level risk on the distribution of rents.

Standard national income accounting applied inside this model yields a new perspective on capital share dynamics. The manager's compensation is set such that the net present value of starting a new firm, computed by integrating over all paths using the density for a new firm, is zero, but the national income accounts only integrate over all firms that are currently active using the stationary size distribution, without discounting. As a result, the aggregate capital share calculation puts more probability mass on the right tail than the NPV calculation. As firm-level risk increases and the right tail of the firm size distribution grows, managers capture a smaller fraction of aggregate rents ex post, even though they capture all of the ex ante rents. This effect is partly offset by a larger mass of unprofitable firms in the left tail of the stationary size distribution, but, in our model, an increase in firm-level risk invariably increases the capital share. Only when managers receive equity-only compensation is the capital share invariant to changes in firm-level volatility.

At the heart of this mechanism is the selection effect that arrises by measuring the distribution of rents excluding firms that have endogenously exited.[1] This effect is closely related to Hopenhayn (2002)'s observation that selection biases average Tobin's Q estimates for industries above one. In a similar manner, the capital share computed in national income accounts also produces a biased estimate of the ex ante profitability of new firms. Moreover, an increase in selection increases the size of the bias. This effect explains the measured divergence between aggregate compensation and profits: Compensation is tied to ex ante profitability, not the ex post realized one. This result also has a natural insurance interpretation. When idiosyncratic risk increases, managers effectively pay a larger idiosyncratic

[1] Jovanovic (1982) is the first study of selection in an equilibrium model of industry dynamics. Selection has also been found to be quantitatively important. Luttmer (2007) attributes about 50 percent of output growth to selection in a model with firm-specific productivity improvements, selection of successful firms and imitation by entrants.

insurance premium to shareholders ex post. The increase in the ex post premium leads to an increase in the aggregate capital share, even though the shareholders are risk-neutral and receive zero rents ex ante. This mechanism has interesting cross-sectional implications. Only the capital share of the largest firms in the right tail increases as risk increases, but they determine aggregate capital share dynamics, echoing Gabaix (2011)'s observation that we need to study the behavior of large firms to understand macroeconomic aggregates. The capital share of the smallest firms will actually decrease. As a result, the average capital share across all firms will tend to decrease.

Between 1960 and 2010, the U.S. labor share of total output in the non-farm business sector of the U.S. economy has shrunk by 15 percent. This phenomenon does not seem limited to the U.S. (see, e.g., Piketty and Zucman, 2014). In the universe of U.S. publicly traded firms, we find that the capital share, measured as total operating income divided by total value added and plotted in Figure 1, has increased from 40% to 60% since 1980, while the labor share has experienced a similar decline. We show that the shrinkage in the labor (capital) share is concentrated among the largest (smallest) firms in the U.S. In fact, the equal-weighted average labor (capital) share of publicly traded companies has increased (decreased), starting in the 80s. This new cross-sectional evidence is consistent with the selection mechanism: The divergence between the average and the aggregate labor share is a key prediction of selection. In a calibrated version of our model, we find that an increase in the size of economic rents (see, e.g., Furman and Orszag, 2015; Barkai, 2016) together with an increase in volatility are needed to quantitatively match the increase in the aggregate capital share and the decrease in the average capital share. The main competing hypothesis is that firms are increasingly substituting capital for labor (see, e.g., Karabarbounis and Neiman, 2013). As far as we know, this mechanism does not predict a divergence between the average and aggregate labor share that we document in the data.

Firm-level risk and the firm size inequality that results plays a key role in U.S. factor

Figure 1. Capital Share.

The figure presents the aggregate capital share for all firms in the Compustat public firms database. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014). Aggregate capital share = $\sum_i$ Operating Income$_i$ divided by $\sum_i$ VA$_i$ for each year.

share dynamics. Consistent with the selection mechanism, we find that the decline in the aggregate U.S. labor share for publicly traded firms cannot be attributed to the averages of log firm-level output and log compensation, but is entirely due to differential changes in the higher-order moments of the cross-sectional firm size and firm compensation distribution, as predicted by our model. In particular, starting in the late seventies, the increases in the variance and kurtosis of the log output distribution are not matched by similar risk increases for log compensation. An increase in firm size inequality that is unmatched by a commensurate increase in inter-firm compensation inequality mechanically lowers the aggregate labor income share. Even though inter-firm wage inequality has increased, as was recently pointed out by Song et al. (2015), the increase was too small to offset the increase in firm size inequality.

In a series of papers, Luttmer (2007, 2012) characterizes the stationary size distribution of firms when firm-specific productivity is subject to permanent shocks.[2] Firms incur a

[2]Other work on characterizing the firm size distribution includes Miao (2005); Gourio and Roys (2014);

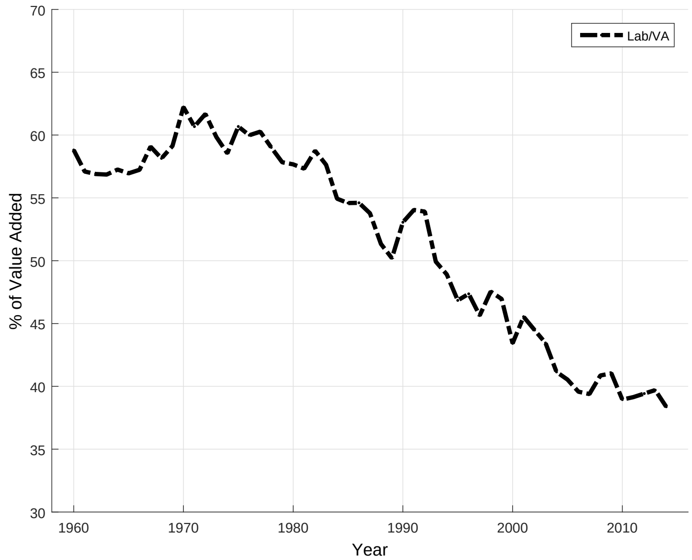


Figure 2. Labor Share.
The figure presents the aggregate labor share for all firms in the Compustat public firms database. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014). Aggregate labor share $= \sum_i$ Labor Cost$_i$ divided by $\sum_i$ VA$_i$ for each year.

fixed cost of operating a firm. The selection effect of exit at the bottom of the distribution informs the shape of the stationary size distribution, which is Pareto with an endogenous tail index. Our work explores the impact of changes in the stationary size distribution on the distribution of rents in our laboratory economy.

There is a large literature on optimal risk sharing contracts between workers and firms (see Thomas and Worall, 1988; Holmstrom and Milgrom, 1991; Kocherlakota, 1996; Krueger and Uhlig, 2006; Lustig, Syverson, and Nieuwerburgh, 2011; Lagakos and Ordoñez, 2011; Berk and Walden, 2013; Zhang, 2014). This literature has analyzed the trade-off between insurance and incentives. We analyze the case of two-sided limited commitment on the part of the firm and the manager, similar to Ai and Li (2015); Ai, Kiku, and Li (2013).

There is strong evidence that firms insure workers. Guiso, Pistaferri, and Schivardi (2005) were the first to study insurance within the firm using U.S. micro data, and they find that

Moll (2016). Perla, Tonetti, Benhabib, et al. (2014) examine the endogenous productivity distribution in an environment where firms choose to innovate, adopt new technology or keep producing with the old technology.

firms fully insure workers against transitory shocks, but not against permanent shocks (see also Rute Cardoso and Portela, 2009; Fuss and Wintr, 2009; Lagakos and Ordoñez, 2011; Friedrich, Laun, Meghir, and Pistaferri, 2014; Fagereng, Guiso, and Pistaferri, 2016, for foreign evidence). Lagakos and Ordoñez (2011) find that wages of low-skilled workers are more responsive to shocks than those of high-skilled workers. In our model, unskilled labor does not benefit from insurance.

When we introduce moral hazard and other frictions that hamper risk sharing, our mechanism will be mitigated. However, we show that when we allow managers to have some exposure to firm performance, our primary results remain unchanged. The intuition is that so long as a firm's owners are providing some insurance to its managers, and can exit when productivity declines, the selection problem still applies. Gabaix and Landier (2008); Edmans, Gabaix, and Landier (2009) find that equilibrium CEO compensation in a competitive market for CEO talent will be comprised of a cash component and an equity component. We analyze the implications of this class of contracts for our key results.

The rest of this paper is organized as follows. Section 2 describes the benchmark model that we use as a laboratory. Section 3 considers a simple endowment version of this economy in which managers are completely insured. We derive the stationary firm size distribution in the benchmark model, and it describes the implications for the aggregate capital share. Section 4 considers a large class of compensation contracts that allow for performance sensitivity. Finally, section 5 analyzes the capital share in the full version of our economy with unskilled labor and physical capital. Section 6 present empirical evidence on U.S. capital share dynamics. Section 7 uses a calibrated version of our economy as a laboratory to explore the quantitative effect of changes in volatility on factor shares. Finally, section 8 concludes by showing that compensation inequality has not kept up with size inequality.

2 A Dynamic Model of Industry Equilibrium with Entry and Exit

In this section we present a model to rationalize the facts we present above. The model is very similar to that analyzed by Atkeson and Kehoe (2005). In the model, firms produce cash flows according to a simple production function. Importantly, the shareholders of a given firm hold an option to cease operations when productivity falls. This is the classic abandonment option that has been well studied in the real options literature. As is standard in that literature, increasing the volatility of the firms cash flows increases the value of the option to wait to abandon, and thus decreases the threshold in productivity at which the firm ceases operations.

Given the solution to the optimal abandonment problem, we characterize the stationary distribution of firms. Increasing (idiosyncratic) cash flow volatility leads more firms to delay abandonment and survive long enough to become very productive. As such, the average across firms of the capital share of profits can be increasing in volatility.

2.1 Environment

The economy is populated by a measure of ex ante identical firms each operating a standard production technology. A given firm i with productivity X_{it} has a single manager, rents physical capital K_{it} and employs unskilled labor L_{it}. The total output produced by this firm is given by

$$Y_{it} = X_{it}^{\nu} F(K_{it}, L_{it})^{1-\nu},$$

where F is homogeneous of degree one and $0 < \nu < 1$. ν governs the decreasing returns to scale at the firm level. Lucas refers to ν as the span of control parameter of the firm's manager. Firm productivity evolves according to:

$$dX_{it} = \mu X_{it} dt + \sigma X_{it} dZ_t^i - X_{it} dN_{it}; \quad \text{for } X_{it} > X_{\min} \tag{1}$$

where Z_{it} is a standard Brownian motion independent across firms, N_{it} is a Poisson process with intensity λ, and $X_{\min} > 0$ is some minimum level of productivity. If $dN_{it} = 1$, or of X_{it} reaches $X_{\min}$, X_i jumps to zero, and the firm exits. The process N_{it} gives rise to what is often referred to as an exogenous death rate of firms and is necessary to guarantee the existence of a stationary distribution of firms for all parameterizations of the model. Since all firms are identical up to their current level of productivity, we omit the subscript i for the remainder of the discussion.

Each firm is owned by a shareholder and requires a skilled manager to operate. We assume shareholders are risk-neutral and discount cash flows at the risk free rate $r > \mu$ while managers value a stream of payment $\{c_t\}_{t \geq 0}$ according to the following utility function

$$U(\{c_t\}_{t \geq 0}) = E\left[\int_0^\infty e^{-rt} u(c_t) dt\right],$$

where $u'(c) \geq 0$ and $u''(c) > 0$. We normalize the measure of managers in the economy to one.

Firms can enter and exit the economy at the discretion of their shareholder's. When a firm exits, its shareholder receives the liquidation value of the firm, which we normalize to zero. The manager of a firm which exits immediately enters the managerial labor market. There is a competitive fringe of shareholders waiting to create new firms. When an shareholder creates a new firm, she matches with a manager then pays a cost P for the technology blueprint to begin production. After creating a new firm, the firm's initial productivity is drawn from a Pareto distribution with density

$$f(X) = \frac{\rho}{X^{1+\rho}}; \quad X \in [X_{\min}, \infty).$$

This distribution implies that the log-productivity of an entering firm is exponentially distributed with parameter $\rho > 1$ and simplifies the characterization of equilibrium that follows. We denote the rate at which new firms are created by ψ_t. Note that this implies that the entry rate at a given point X is $\psi_t f(X)$.

2.2 Compensation Contract

Upon matching with a manager, a shareholder in a new firm offers a long term contract to the manager prior to the realization of the firm's productivity and payment of the cost P. The manager can reject the contract at which point she is instantaneously matched with a new shareholder. Formally, this contract can be denoted by a process $\{c_t\}_{t\geq 0}$ determining payment to the manager of c_t at time t. We assume that the investor cannot commit to continue operations or to pay the manager once the firm has ceased operations. We also assume that the manager can choose to exit the contract and match with a new firm at any time and that she does not have access to a savings technology. When the firm ceases operations, the manager is matched with a entering firm and signs a new contract. This contracting environment features a two sided limited commitment problem similar to Ai and Li (2015); Ai et al. (2013). Importantly the outside option of the manager will depend on the value of starting a new firm, which is endogenously determined in equilibrium.

2.3 The Investors' Problem

We denote the utility the manager receives upon entering this market by U_0, which is also the manager's reservation utility. At the inception of the contract, the investor and manager takes U_0 as exogenously given, although it will be determined in equilibrium by the market for managers. The investor will continue operations as long as doing so yields a positive present value. This means that the investor operating for the firm is the solution to a standard *abondonment option* common in the real options literate. Specifically, the investor operates the firm until a stopping time denoted by τ. The investor's problem is thus

$$\max_{K,L,\tau,c} E\left[\int_0^\tau e^{-rt}(Y_t - c_t)dt\right] \tag{2}$$

such that

$$U_0 \leq E\left[\int_t^\tau e^{-r(s-t)}u(c_s)ds + e^{-r(\tau-t)}U_0\right] \text{ for all } t > 0. \tag{3}$$

Intuitively, the manager's limited commitment constraint given in equation (3) must bind as delivering more continuation utility to the manager can only ever reduce the investor's value for the firm. As a result, the manager value for the contract is constant over time and it is without loss of generality to restrict attention to contracts that offer the manager a fixed wage c until the firm exits, at which point the manager reenters the market and receives her outside option.

2.4 Equilibrium

We will focus our analysis on equilibria in which the measure of firms at any given level of productivity is stationary. We denote by $\phi(x)$ the stationary distribution of log-productivity, where $x = \log(X)$ throughout.

Definition 1. A stationary equilibrium consists of a rental rate κ for physical capital, a demand for physical capital as a function of productivity $K(X)$, a wage rate w for unskilled labor, demand for unskilled labor $L(X)$ as a function of X, a compensation c^* for the managers, an entry rate of new firms ψ^*, an exit policy for the shareholder $\bar{X}$, and a stationary distribution $\phi(x)$ such that

1. The exit policy $\bar{X}$ solves the investors problem given by (4).

2. The stationary distribution $\phi(x)$ is consistent with the entry rate of new firms ψ and the exit policy $\bar{X}$.

3. Creating a new firm leaves the investor with zero expected NPV

$$\int_{X_{\min}}^{\infty} V(X;c)f(X)dX = P.$$

4. The markets for physical capital, unskilled labor, and managers clear

$$\int_{X_{\min}}^{\infty} K(x)\phi(x)dx = \int_{X_{\min}}^{\infty} L(x)\phi(x)dx \int_{X_{\min}}^{\infty} \phi(x)dx = 1.$$

Conditions 3 of the above definition merits some discussion. Condition 3 derives from the existence of the competitive fringe of investors waiting to create new firms. If an investor in a new firm offers a contract that leaves her with positive ex ante expected NPV, then the manager will reject since she can simply reenter the market and instantaneously match with a new firm. Thus, we allocate all of the ex ante bargaining power to the manager. This condition determines the level of compensation to the managers in the economy. An alternative definition would be to allocate some bargaining power to the investor, however, doing so will not drastically change the results.

3 An Endowment Economy

To demonstrate the main forces behind our results, we start by analyzing an endowment version of this economy in which we abstract from physical capital and unskilled labor. In this endowment economy, firm-level output is simply determined by firm-level productivity. In this simple version of the economy, $Y_t = X_t$. Thus, we can simplify the investors problem to

$$V(X;c) = \max_{\tau} E\left[\int_0^{\tau} e^{-rt}(X_t - c)dt | X_0 = X\right], \tag{4}$$

where $V(X;c)$ is the value of operating a firm with current productivity X given a manager contract c. The payment c to the manager then acts as a fixed cost or operating leverage. As such, the investor in a given firm will choose to exit if productivity X is low enough as in the classic problems of optimal abandonment considered in the real options literature or optimal default as in Leland (1994). It is without loss of generality to restrict attention to firm exit times that are given by threshold rules of the form

$$\tau = \inf\{t | X_t \leq \bar{X} \text{ or } dN_t = 1\}$$

for some $\bar{X} \geq 0$.

3.1 Equilibrium Analysis

In this section, we characterize the stationary equilibrium of the model and study its implications for national income accounting. To solve for the firm value function and exit policy of the investor, we use standard techniques from the real options literature. An application of Ito's formula and the dynamic programing principal imply that $V(X;c)$ must satisfy the following ordinary differential equation

$$(r+\lambda)V(X;c) = X - c + \mu X \frac{\partial}{\partial X}V(X;c) + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2}{\partial X^2}V(X;c), \tag{5}$$

with the boundary conditions

$$V(\bar{X}(c);c) = 0, \tag{6}$$

$$\frac{\partial}{\partial X}V(\bar{X}(c);c) = 0, \tag{7}$$

$$\lim_{X\to\infty}\left|V(X;c) - \left(\frac{X}{r+\lambda-\mu} - \frac{c}{r+\lambda}\right)\right| = 0. \tag{8}$$

Conditions (6) and (7) are the standard value matching and smooth pasting conditions pinning down the optimal exercise boundary for the abandonment option. Condition (8) arises because as X_t tends to infinity, abandonment occurs with zero probability and the value of the firm must tend to the present value of a growing cash flow less a fixed cost.

The solution to equations (5)-(8) is given by

$$\bar{X}(c) = \frac{\eta}{\eta+1}\frac{c(r+\lambda-\mu)}{r+\lambda} \tag{9}$$

$$V(X;c) = \frac{X}{r+\lambda-\mu} - \frac{c}{r+\lambda} - \left(\frac{\bar{X}(c)}{r+\lambda-\mu} - \frac{c}{r+\lambda}\right)\left(\frac{X}{\bar{X}(c)}\right)^{-\eta} \tag{10}$$

where $\eta = \frac{\mu-\frac{1}{2}\sigma^2+\sqrt{(\mu-\frac{1}{2}\sigma^2)^2+2(r+\lambda)\sigma^2}}{\sigma^2}$. Note that an increase in firm-level volatility σ invariably lowers the abandonment threshold, simply because an increase in volatility raises the option value of keeping the firm alive. This feature of the abandonment option will play key role in our analysis as will become apparent when we discuss the stationary distribution of firm size. Specifically, an increase in firm-level volatility will lead to a increase mass of firm's that delay exit, increasing the mass of firms that have low productivity as well the mass of firms that survive long enough to achieve high productivity.

Given the solution for firm value conditional on a manager wage c as well as our assumption on the distribution of productivity of new firms, we can solve for the equilibrium compensation in closed form. We have

$$c^* = \left(\frac{P(r+\lambda)(\rho-1)(\rho-\eta)}{\eta} \left(\frac{\eta(r+\lambda-\mu)}{(\eta+1)(r+\lambda)} \right)^{\rho} \right)^{-\frac{1}{\rho-1}}. \tag{11}$$

The derivation of c^* is given in section of the Appendix.

To remain stationary, the expected change via inflow and outflow in the measure of firms at a given level of x must be equal to the measure of firms that exogenously die at the rate λ less the measure of firms that endogenously enter at the rate $\psi g(x)$ (see p. 273 in Dixit and Pindyck, 1994). This leads to the following Kolmogorov forward equation for $\phi(x)$

$$\frac{1}{2}\sigma^2 \phi''(x) - \left(\mu - \frac{1}{2}\sigma^2 \right) \phi'(x) - \lambda \phi(x) + \psi g(x) = 0. \tag{12}$$

where $g(x) = \rho e^{-\rho x}$. is the density of initial log productivity x for entering firms. A similarly argument gives a boundary condition for $\phi(x)$ at the exit barrier $\bar{x} = \log \bar{X}$

$$\phi(\bar{x}) = 0. \tag{13}$$

The final equation that determines the stationary distribution of firm size is given by the market clearing condition for managers

$$\int_{\bar{x}}^{\infty} \phi(x) dx = 1. \tag{14}$$

The solution to equations (12)-(14) is given by

$$\phi(x) = \frac{\rho\gamma}{\rho-\gamma} \left(e^{-\gamma(x-\bar{x})} - e^{-\rho(x-\bar{x})} \right) \tag{15}$$

for $x \in [\bar{x}, \infty)$, where $\gamma = \frac{-(\mu - \frac{1}{2}\sigma^2) + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2\lambda}}{\sigma^2}$. The solution also allows us to characterize the aggregate entry rate of new firms

$$\psi = \frac{\gamma(\rho(\mu - \frac{1}{2}\sigma^2) + \frac{1}{2}\rho^2\sigma^2 - \lambda)}{\rho - \gamma} e^{\rho\bar{x}}. \tag{16}$$

We note the our assumption on the density of productivity of entering firms allows for the simple closed form solutions above. The general solution to the ODE given in equation (12) is exponential. By assuming that $g(x)$ is exponential as well, we are left with a solution to equation (12) for which it is possible to solve the boundary condition given in equation (13).

Figure 3 plots the stationary distribution of firm productivity for different levels of σ. The other parameters are calibrated at $r = 5\%, \mu = 2\%, \lambda = .05, \rho = 3, P = 1$. As σ increases, the stationary distribution shifts to the left and becomes more diffuse, with a fatter right tail. The shift to the left is due to the fact that as firm-level volatility increase, the value of the option to wait to exit increases, and the optimal point at which the investor chooses to exit necessarily decreases.

The effect of firm-level volatility on the shape of $\phi(x)$ visible in figure 3 is born out by examining the higher-ordered moments of $\phi(x)$. Table 1 reports the standard deviation, the skewness and the kurtosis of the log size distribution as we increase σ. As σ increases, the right skewness increases from 0.12 to 2.74 and the excess kurtosis of the log size distribution increases from 0.15 to 7.31. This overall widening of the distribution with a fattening of the left tail comes from two effects. First there is a direct effect of σ on the dispersion of the distribution of firm size. When firm-level productitivity is more volatile, the stationary distribution of firms must be more dispersed. This is evident by examining the dependence of γ on σ. The second effect operates through the abandonment option. When the option to wait to exit becomes more valuable, more firms delay exit, and as a result more firms survive long enough to become very productive. As a result the right tail of the distribution

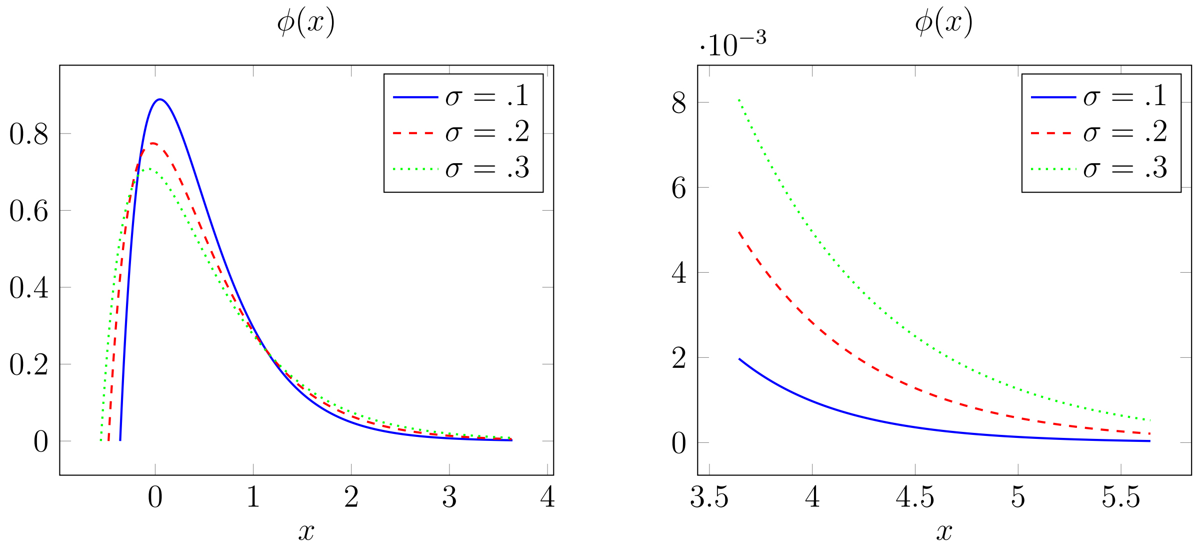


Figure 3. The stationary distribution of log-productivity for $\sigma = .1, .2$, and $.3$. Parameter values: $r = 5\%, \mu = 2\%, \lambda = .05, \rho = 3, p = 1$.

Table 1. Higher order moments of the log-size distribution implied by the model

σ	Mean	Std. Dev.	Skew.	Kurt.
.1	1.879	0.700	0.120	0.151
.2	1.493	0.696	2.186	5.631
.3	1.181	0.789	2.742	7.310

Moments of the stationary distribution of log-productivity for $\sigma = .1, .2$, and $.3$. Parameter values: $r = 5\%, \mu = 2\%, \lambda = .05, \rho = 3, p = 1$.

becomes fatter. In the next section, we show that this effect has important implications for national income accounting.

3.2 National Income Accounting

Armed with this stationary distribution, we can do national income accounting inside our model. We use this stationary distribution to calculate the total and average profit share for a range of σ. Specifically, we use the stationary distribution to calculate the aggregate capital share and the average firm's capital share, respectively:

$$\text{Capital Share of Profits} = \Pi = \frac{\int_{\bar{x}}^{\infty}(e^x - c)\phi(x)dx}{\int_{\bar{x}}^{\infty} e^x\phi(x)dx},$$
$$= 1 - \frac{c}{\int_{\bar{x}}^{\infty} e^x\phi(x)dx},$$

$$\text{Average Capital Share of Profits} = \int_{\bar{x}}^{\infty}\left(\frac{e^x - c}{e^x}\right)\phi(x)dx.$$
$$= 1 - \int_{\bar{x}}^{\infty}\frac{c}{e^x}\phi(x)dx,$$

We note that our expressions for both aggregate and average capital share are gross of the costs of starting new firms. Including these costs leads to a less transparent expressions and does not change the results of the analysis below.

To gain an intuition for the effect of a comparative static change in idiosyncratic volatility on the aggregate capital share, it is useful to decompose the expression into its constituent parts. The denominator of the second term Π is the total profits to all firms in the economy, and is given by $\int_{\bar{x}}^{\infty} e^x\phi(x)dx = \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\rho}{\rho-1}\right)\bar{X}$, while the numerator is the total compensation paid to managers, and is given by $c = \left(\frac{(r+\lambda)(\eta+1)}{\eta}\right)\left(\frac{1}{r+\lambda-\mu}\right)\bar{X}$. It suffices to normalize these terms by $\bar{X}$ since it is a common factor in both. As σ increases, the total profits in the economy, normalized by the minimum productivity of active firms $\bar{X}$, increases because the right tail of the stationary distribution of x gets fatter.

Now consider the numerator. By the value matching condition pinning down $\bar{X}$, the present value of compensation c to a given manager must be equal to the present value of all future gross cash flows to the firm assuming that it will exit at $\bar{X}$. Thus, the expression for c given above states that total compensation to managers is the present value of all gross cash flows forgone by an exiting firm. This present value, normalized again by $\bar{X}$, also increases in σ for the same reason that total profits increase–there is a greater measure of future paths of the firm that result in high productivity. However, these high future draws of productivity are discounted at the rate r and thus have a smaller effect on total compensation paid to managers than on total profits. This intuition implies that the capital share of profits should be increasing in σ.

To show this intuition is in fact correct, we can combine the terms above to get the following simple closed form expression for Π

$$\Pi = 1 - \left(\frac{r+\lambda}{r+\lambda-\mu}\right)\left(\frac{\rho-1}{\rho}\right)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\eta+1}{\eta}\right). \tag{17}$$

We can then calculate the derivative of Π with respect to the volatility parameter σ:

$$\frac{\partial\Pi}{\partial\sigma} = -\left(\frac{r+\lambda}{r+\lambda-\mu}\right)\left(\frac{\rho-1}{\rho}\right)\left[\left(\frac{\eta+1}{\eta}\right)\frac{1}{\gamma^2}\frac{\partial\gamma}{\partial\sigma} - \left(\frac{\gamma-1}{\gamma}\right)\frac{1}{\eta^2}\frac{\partial\eta}{\partial\sigma}\right]. \tag{18}$$

So $\partial\Pi/\partial\sigma$ is positive if and only if

$$\eta(\eta+1)\frac{\partial\gamma}{\partial\sigma} \leq \gamma(\gamma-1)\frac{\partial\eta}{\partial\sigma}. \tag{19}$$

It is straightforward to show that $\eta(\eta+1)\frac{\partial\eta}{\partial\sigma} \leq 0$ and $\gamma(\gamma-1)\frac{\partial\gamma}{\partial\sigma} \leq 0$, so to verify (19), it is equivalent to verify

$$\frac{\eta(\eta+1)\frac{\partial\gamma}{\partial\sigma}}{\gamma(\gamma-1)\frac{\partial\eta}{\partial\sigma}} \geq 1. \tag{20}$$

One can show that

$$\frac{\eta(\eta+1)\frac{\partial\gamma}{\partial\sigma}}{\gamma(\gamma-1)\frac{\partial\eta}{\partial\sigma}} = \frac{\sqrt{(\mu-\frac{1}{2}\sigma^2)^2 + 2(r+\lambda)\sigma^2}}{\sqrt{(\mu-\frac{1}{2}\sigma^2)^2 + 2\lambda\sigma^2}} > 1 \tag{21}$$

which verifies that $\partial\Pi/\partial\sigma > 0$. Hence, in our model, the aggregate capital share always increases as volatility increases, as long as $r > 0$.

The expression given in equation (21) validates our intuition about the effect of discounting on the relative sensitivity of firm value and total stationary profits to changes in idiosyncratic volatility. Indeed, it shows that the strict positive sign of the comparative static requires that the discount rate r be positive. To understand this effect, it is helpful to consider the limiting case of no discounting. As r approaches zero, the ex ante average value of a firm (normalized by r) approaches the aggregate value of all payments to investors:

$$\begin{aligned}\lim_{r\to 0}\int_{X_{\min}}^{\infty} rV(X)f(X)dX &= \lim_{r\to 0}\int_{X_{\min}}^{\infty} E\left[\int_0^{\tau} re^{rt}(X_t - c)dt|X_0 = X\right] f(X)dX \\ &= \lim_{r\to 0}\int_0^{\infty}\int_{\bar{x}}^{\infty} re^{rt}(e^x - c)\phi_t(x)dxdt \\ &= \int_{\bar{x}}^{\infty}(e^x - c)\phi(x)dx.\end{aligned}$$

where

$$\phi_t(x) = \frac{\partial}{\partial x}\int_{\bar{x}}^{\infty} E[\mathbb{1}(x_t > x)\mathbb{1}(t \le \tau)|x_t = y]g(y)dy$$

is the distribution of log productivity x_t for a firm given an initial value drawn from $g(\cdot)$ conditional on the firm not having yet exited. Intuitively, as r goes to zero, the present value of all future cash flows is just given by the expectation of cash flow in the limit as t goes to infinity, i.e., in the stationary distribution. Returning to our intuition, the total compensation paid to managers is then proportional to total profits, and σ has no effect on the capital share.

Figure 4 plots a calibrated example. The figure plots the total and average capital share of profit as a functions of σ. We use the following parameter values: $r = 5\%, \mu = 2\%, \lambda = .05, \rho = 3, p = 1$. We can see that the total capital share of profits is increasing in σ while the average capital share of profits is decreasing. The intuition is as follows. As σ increases, the value of the option to delay abandonment increases, and hence the optimal threshold at which firms exit decreases. Holding the total measure of firms fixed, this means that the distribution of profits becomes more dispersed. The increase in mass of firms in the right tail of the firm size distribution increases the total profit share, because the profit share measures the ex post profitability of existing firms. This is effectively a selection bias. The profit share of entering firms is set by setting the NPV of the investor's stake in the firm to zero. This NPV calculation integrates over all possible future paths for firm-level productivity, including those that lead the investor to choose to exit. In contrast, the stationary distribution of existing firms only consider firms that have survived. Surviving firms necessarily have a higher capital share of profits, otherwise the investor would have chosen to exit.

Our model also makes a novel prediction about the capital share at the average firm. The increase in mass of firms that delay exist means that there will be more firms with a low capital share. Thus an increase in firm-level volatility can decrease the average profit share. This is in contrast to the effect one would expect to see if the increase in total capital share of profits is due to a greater growth rate in the value of capital relative to wages that may follow the substitution of capital for labor. In that case, one would expect both the total and average capital share to increase.

We can also examine the capital share of firm value, with similar results as for profits. Figure 5 plots the total and average capital share of firm value derived from the model.

Finally, we examine the comparative statics of the capital share with respect to the entry parameter. First we have

$$\frac{\partial \Pi}{\partial \rho} = -\left(\frac{r+\lambda}{r+\lambda-\mu}\right)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\eta+1}{\eta}\right)\frac{1}{\rho^2} < 0.$$

To understand this comparative static, note that an increase in ρ means that the right tail of the entry distribution becomes thinner and entering firms are on average smaller. This in

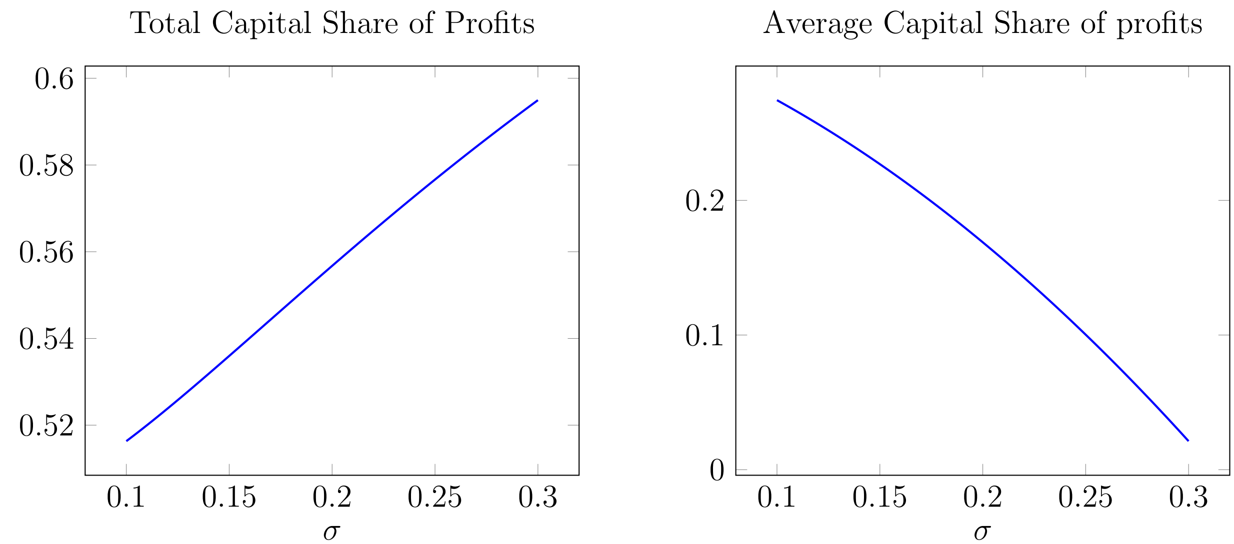


Figure 4. The total and average capital share of profit as a functions of σ. Parameter values: $r = 5\%, \mu = 2\%, \lambda = .05, \rho = 3, p = 1$.

turn implies that the capital share decreases because smaller firms have lower capital shares.

4 Pay for Performance

In this section we allow the manager some exposure firm performance. Edmans et al. (2009) derive CEO compensation in a competitive equilibrium with a talent assignment and moral hazard problem. This exposure could arise for a variety of reasons. For example, there could be a firm-level agency conflict between the manager and investors or the investor could be risk averse. In either case, the optimal contract will call for the manger to bear some exposure to firm performance, either for incentive purposes or to improve risk sharing. The precise form of the optimal contract will depend on the nature of the agency problem or the exact preferences of the managers and investors. A concern with our results thus far might be that such a sensitivity could mitigate the insurance nature of the relationship between firms' owners and their managers, thus decreasing or reversing the effect of firm level

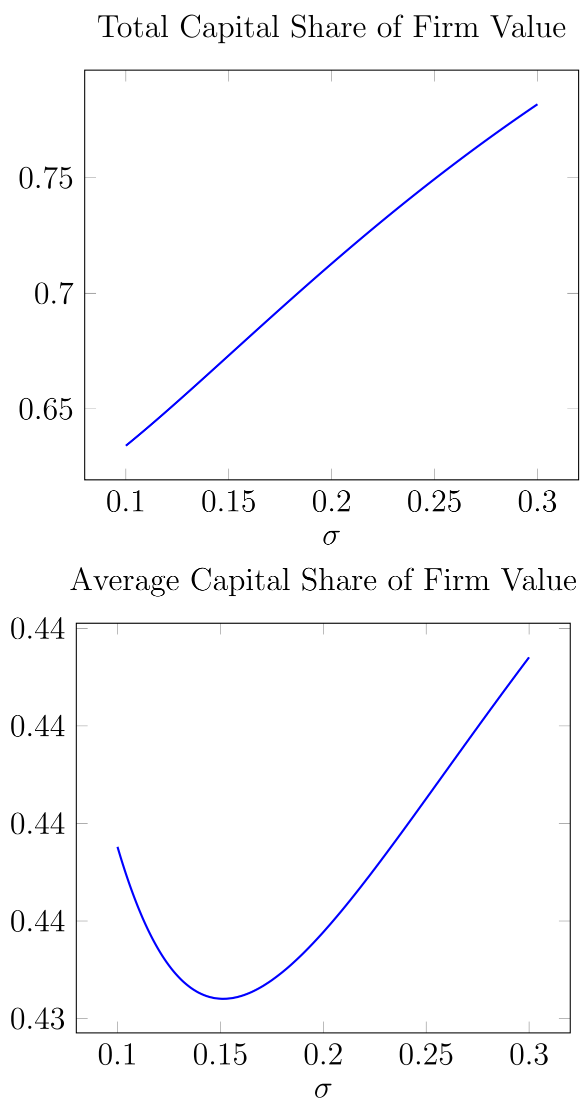


Figure 5. The total and average capital share of firm value as a functions of σ. Parameter values: $r = 5\%, \mu = 2\%, \lambda = .05, \rho = 3, p = 1$.

volatility on the capital share of profits. Rather than solve directly for an optimal contract for a particular problem, we assume that the manager's contract takes the following simple affine form

$$c_t = \beta X_t + w. \tag{22}$$

The sensitivity β of the managers payment c_t to the level of productvity is determined by either the severity of the agency problem or the nature of the risk-sharing problem, and is exogenous from the standpoint of our model. The fixed wage w is set in equilibrium in the same manner as total wages are set above. This contract has the advantage of being particularly tractable to analyze in the context of our model of equilibrium.

For a given fixed wage w, the investors problem is

$$\max_{\tau} \left[\int_0^{\tau} e^{-rt}((1-\beta)X_t - w)dt \right]. \tag{23}$$

Again, standard arguments imply that the investor's value function $V(X)$ must statisfy the following ODE

$$(r+\lambda)V = (1-\beta)X - w + \mu X V' + \frac{1}{2}\sigma^2 X^2 V'', \tag{24}$$

with the boundary conditions

$$V(\bar{X}) = 0, \tag{25}$$

$$V'(\bar{X}) = 0, \tag{26}$$

$$\lim_{X\to\infty} \left| V(X) - \left(\frac{(1-\beta)X}{r+\lambda-\mu} - \frac{w}{r+\lambda} \right) \right| = 0. \tag{27}$$

This problem is essentially the same as one given in equations (5)-(8), up to a scaling of the leading term by a factor of $(1-\beta)$. Thus, the solution to equation (24)-(27) is

$$\bar{X} = \left(\frac{1}{1-\beta}\right)\left(\frac{\eta}{\eta+1}\right)\frac{w(r+\lambda-\mu)}{r+\lambda}$$
$$V(X) = \frac{(1-\beta)X}{r+\lambda-\mu} - \frac{c}{r+\lambda} - \left(\frac{(1-\beta)\bar{X}}{r+\lambda-\mu} - \frac{c}{r+\lambda}\right)\left(\frac{X}{\bar{X}}\right)^{-\eta}$$

where η is defined as above.

Given the solution for the investor's value, we can apply the investor's zero ex-ante profit condition to determine the fixed component of the manager's equilibrium contract. Doing this calculation yields

$$w^* = \left(\frac{P(r+\lambda)(\rho-1)(\rho-\eta)}{\eta}\left(\frac{\eta(r+\lambda-\mu)}{(1-\beta)(\eta+1)(r+\lambda)}\right)^{\rho}\right)^{-\frac{1}{\rho-1}}. \tag{28}$$

Comparing equations (11) and (28) reveals that the fixed component of the equilibrium affine contract is just the equilibrium wage under full insurance scaled by a function of β. The intuition here is that we can essentially view the investor's problem under the affine contract has identically to the problem under full insurance when the firm's productivity is scaled by a factor of $1-\beta$.

Now note that the stationary distribution of firm productivity is unaffected by our assumption of affine contracts, up to a shifting of the optimal abandonment threshold, i.e. the left support of the stationary distribution. Thus we can again calculate the total capital share of profits in the stationary distribution to get

$$\Pi = 1 - (1-\beta)\left(\frac{r+\lambda}{r+\lambda-\mu}\right)\left(\frac{\rho-1}{\rho}\right)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\eta+1}{\eta}\right). \tag{29}$$

Comparing equations (17) and (29) shows that the total capital share profits under the affine contract depends on γ and η, and hence on σ, in the same manner as the total capital share of profits under full insurance. In other words, allowing the manager to share in success of the successful firms does not change our main results.

5 The Full Production Economy

In this section we return to the production economy analyzed by Atkeson and Kehoe (2005). We maintain our assumptions on the preferences of investors and managers as well as the structure of entry and exit in economy from our basic model. Given these assumptions, it is still optimal to offer managers a fixed wage. Also, investors face the same basic exit decision as in the endowment economy up to an adjustment to net profits for the payments to physical capital and labor. Investor will thus choose to exit when productivity falls bellow some threshold $\bar{X}$. Note that since firms rent physical capital and unskilled labor in spot markets, a given firm's demand for these inputs will be a function of it's current productivity.

5.1 Equilibrium Analysis

To characterize equilibrium, we begin by considering the allocation of physical capital and unskilled labor across active firms. Given spot rates for physical capital and unskilled labor and some current level of productivity, a given firm chooses capital and labor to maximize profits net of the rental costs of physical capital and unskilled labor:

$$(K_t, L_t) = \arg\max_{K,L} \left\{ X_t^{\nu} F(K, L)^{1-\nu} - wL - \kappa K \right\}.$$

The homogeneity of the production function F implies that the solution (K_t, L_t) of the maximization above is linear in X_t. Market clearing then implies that physical capital and unskilled labor are allocated across firms according to the following linear allocation rule:

$$K_t = \frac{k}{\widehat{X}} X_t,$$
$$L_t = \frac{l}{\widehat{X}} X_t,$$

where

$$\widehat{X}_t = \int_{\bar{x}}^{\infty} e^x \phi(x) dx$$

is the average productivity in the economy given the distribution of log productivity $\phi(x)$. This allocation rule implies that the output of any given firm is a linear function of aggregate output:

$$Y_t = \frac{y}{\widehat{X}} X_t$$

where $y = \widehat{X}^{\nu} F(k, l)^{1-\nu}$ is aggregate output. As a result, a firm's gross earnings (operating profit) are proportional to X_t:

$$Y_t - wL_t - \kappa K_t = \frac{\nu y}{\widehat{X}} X_t.$$

For convenience, we let $\widehat{F} = \frac{\nu y}{\widehat{X}}$. We refer to $\widehat{F}$ as equilibrium rents normalized by productivity X.

Having determined the allocation of physical capital and unskilled labor, we can now analyze the investors optimal abandonment decision. Thus, we can simplify the investors problem to

$$V(X; c, \hat{F}) = \max_{\tau} E\left[\int_0^{\tau} e^{-rt} \left(\widehat{F} X_t - c\right) dt | X_0 = X\right], \tag{30}$$

where $V(X; c, \hat{F})$ is the value of operating a firm with current productivity X given a manager contract c and rents $\widehat{F}$. The solution technique for the investor's problem is essentially the same as in the case with constant physical capital and labor up to a change in the coefficients in the ODE and determination of the optimal abandonment threshold. Given c and $\widehat{F}$, $V(X;c,\widehat{F})$ must satisfy the following ordinary differential equation

$$(r+\lambda)V(X;c,\widehat{F}) = \widehat{F}X - c + \mu X\frac{\partial}{\partial X}V(X;c,\widehat{F}) + \frac{1}{2}\sigma^2X^2\frac{\partial^2}{\partial X^2}V(X;c,\widehat{F}), \tag{31}$$

with the boundary conditions

$$V(\bar{X}(c,\widehat{F}));c,\widehat{F})) = 0, \tag{32}$$

$$\frac{\partial}{\partial X}V(\bar{X}(c,\widehat{F}));c,\widehat{F})) = 0, \tag{33}$$

$$\lim_{X\to\infty}\left|V(X;c,\widehat{F}) - \left(\frac{\widehat{F}X}{r+\lambda-\mu} - \frac{c}{r+\lambda}\right)\right| = 0. \tag{34}$$

Where $\bar{X}(c,\widehat{F})$ is the abandoment threshold given c and $\widehat{F}$. Conditions (32) and (33) are the standard value matching and smooth pasting conditions, while condition (34) arises because as X_t tends to infinity, abandonment occurs with zero probability as in the simple model we analyzed above. The smooth pasting condition (33) need only hold if $\bar{X}(c,\widehat{F}) > X_{\min}$.

The solution to equations (31)-(34) is given by

$$\bar{X}(c,\widehat{F}) = \left(\frac{\eta}{\eta+1}\right)\left(\frac{r+\lambda-\mu}{r+\lambda}\right)\left(\frac{c}{\widehat{F}}\right) \tag{35}$$

$$V(X;c,\widehat{F}) = \frac{\widehat{F}X}{r+\lambda-\mu} - \frac{c}{r+\lambda} - \left(\frac{\widehat{F}\bar{X}(c,\widehat{F})}{r+\lambda-\mu} - \frac{c}{r+\lambda}\right)\left(\frac{X}{\bar{X}(c,\widehat{F})}\right)^{-\eta} \tag{36}$$

where η is as defined above.

Next consider equilibrium managerial compensation. As in the endowment economy, c is set so as to give zero profits to the investors for starting a new firm. Given the solution to the investors value function and the Pareto entry distribution of new firms, it is straightforward to solve the investor's ex ante zero profit condition for the equilibrium c. We have

$$c^* = \left(\frac{P(r+\lambda)(\rho-1)(\rho-\eta)}{\eta} \left(\frac{\eta(r+\lambda-\mu)}{(\eta+1)(r+\lambda)\hat{F}} \right)^{\rho} \right)^{-\frac{1}{\rho-1}}. \tag{37}$$

Comparing the equilibrium managerial wage in the production economy vs the endowment economy, we see that the two are identical up to an adjustment for equilibrium rents $\widehat{F}$.

5.2 Stationary Size Distribution

Finally, we consider the equilibrium distribution of productivity $\phi(x)$ as well as rents $\widehat{F}(\bar{X})$ given an exit threshold $\bar{X}$. Note that since X_t has the same dynamics as in the endowment economy model, the form of the stationary distribution for productivity is unchanged. The equilibrium average productivity is then

$$\widehat{X}(\bar{X}) = \frac{\bar{X}\gamma\rho}{(\gamma-1)(\rho-1)}. \tag{38}$$

This in turn implies that equilibrium rents are

$$\widehat{F}(\bar{X}) = \nu \left(\frac{\bar{X}\gamma\rho}{(\gamma-1)(\rho-1)} \right)^{\nu-1} F(k,l)^{1-\nu}. \tag{39}$$

An equilibrium is then characterized by a solution $(\bar{X}, \widehat{F})$ to equations (35), (37), and (39). One can show that such a solution exists and is unique.

5.3 National Income Accounting

As in the endowment model, we can do national income accounting within our model. Specifically, we can calculate the aggregate capital share of output:

$$\begin{aligned}\text{Capital Share of Output} = \Pi &= \frac{y - wl - c}{y} \\ &= 1 - (1-\nu)(1-\alpha(k,l)) - \frac{c}{y},\end{aligned}$$

where $1 - \alpha(k,l) = \frac{1}{F(k,l)^{-1}} \frac{\partial F(k,l)}{\partial l}$ is the elasticity of the production function F with respect to unskilled labor. In words, the capital share of output is one minus the total labor share where the labor share is the sum the share of output that accrues to unskilled and skilled labor. Using the definition of $\widehat{F}$ and equations (35) and (38), we can write total output as

$$y = \frac{\widehat{F}\widehat{X}}{\nu} = \left(\frac{\gamma\rho}{(\gamma-1)(\rho-1)}\right)\left(\frac{\eta}{\eta+1}\right)\left(\frac{r+\lambda-\mu}{r+\lambda}\right)\left(\frac{c}{\nu}\right), \tag{40}$$

so that total output y is linear in c. Thus the total capital share of output simplifies to

$$\Pi = 1 - (1-\nu)(1-\alpha(k,l)) - \nu\left(\frac{r+\lambda}{r+\lambda-\mu}\right)\left(\frac{\rho-1}{\rho}\right)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\eta+1}{\eta}\right), \tag{41}$$

which is essentially the same as in the endowment economy less the unskilled labor share of output and an adjustment to the managerial share of output for the elasticity of output with respect to productivity. Importantly, the comparative static of total capital share with respect to idiosyncratic volatility σ will have the same positive sign in both the endowment economy and the production economy. Intuitively, the share of output that goes to unskilled labor is given by the shape of the production function and does not depend on aggregate rents or production other than through the aggregate quantity of physical capital and unskilled labor. At the same time the *share* of output that goes to the managers is determined by the equilibrium exit policy of firms and also does not directly depend on aggregate production. Thus, the capital share of output does not directly depend on aggregate output other than through aggregate quantity of physical capital and unskilled labor.

6 Understanding U.S. Capital Share Dynamics

In this section, we present empirical evidence on the joint dynamics of compensation, firm size and the implied capital share dynamics. We show the findings are consistent with the findings of our model.

6.1 Data

Our baseline results are from analyzing two sources of data. To obtain cross-sectional evidence, we use widely available accounting data from Compustat/CRSP Merged Fundamentals Annual, which includes all publicly-traded firms. The sample is from 1960 to 2014. We exclude financial firms with SIC codes in the interval 6000-6799, and also exclude firms whose sales, employee numbers and total asset values are negative. We examine the distribution of factor share of output in the publicly traded firm sample.

The capital income of the firm is measured by operating income before depreciation (OIBDP). OIBDP equals sales minus operating expenses including the cost of goods sold, labor cost and other administrative expenses. The aggregate capital share of output is the ratio of total OIBDP aggregated across al firms to total value added. We use the ratio of the cost of labor to value added as the measure of labor share of output. The cost of labor is the staff expenses – total (XLR) in Compustat. Value added is computed as the sum of OIBDP and XLR. However, the limitation of XLR is that XLR in Compustat is sparse with only roughly 13% firm-year observations in the sample. We follow Donangelo, Gourio, and Palacios (2015) to construct the extended labor cost. We first estimate the average labor cost per employee (XLR/EMP) within industry for each year using the available XLR observations, and then labor cost of a firm with missing XLR equals the number of employees times the average labor cost per employee of the same industry[3] during that year.

6.2 Time Series Dynamics in Capital Share

Figure 1 plots the aggregate capital share of value added, which has increased from 41 to 62 %. Figure 2 plots the aggregate labor share, which has decreased from 60% to less than 40% in the Compustat sample. These trends in the aggregate factor shares are not operative for a typical U.S. firm. Figure 6 plots the average and aggregate capital share as a fraction of sales in the publicly traded firm sample. We use sales instead of value added, because the latter can be negative at the firm level. The average capital income/sales ratio is the cross-sectional mean of the capital income /sales ratio for a given year. The large declines in the average capital income/sales ratio are driven mostly by small firms with negative operating margins. This decline is especially pronounced in R&D intensive industries such as biotech, but we observe this decline across-the-board. As the firm-level volatility increases, the aggregate and the average capital income/sales ratios share diverge. The aggregate ratio equals the sum of capital income (OIBDP) across all the firms divided by aggregate sales. The average ratio drops from 13% in 1960 to -40% in 2014, while the aggregate ratio increases with a less dramatic scale, from 14% in 1960 to 17% in 2014. The initial decline in the aggregate capital income/sales ratio disappears when we adjust the capital income measure for the expensing of R&D by adding it back to operating income.

We find similar patterns in the labor share dynamics. The aggregate labor income/sales ratio in the non-farm business sector has declined by 15%. However, the the trend of the average labor share of output in the publicly traded firm sample did not decline. Figure 7

[3]We follow Donangelo et al. (2015) and use Fama-French 17 industry classifications. The result is robust using 2-digit SIC code.

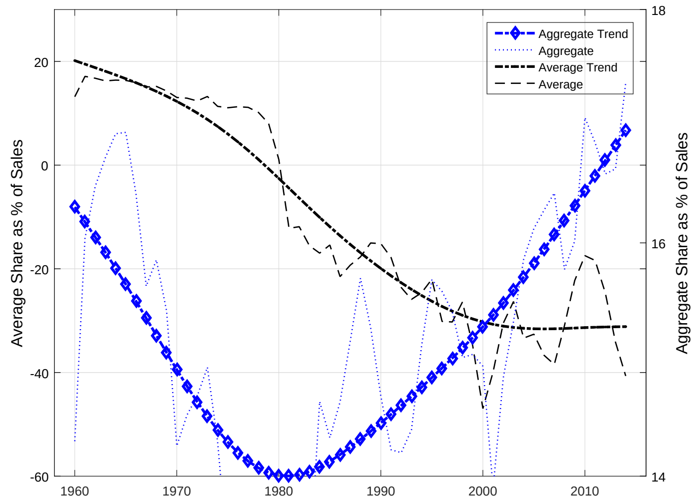


Figure 6. Average and Aggregate Capital Income/Sales Ratio

Capital Income/Sales ratio equals the ratio of operating income to sales. Aggregate Capital Income/Sales ratio = $\sum_i$ Operating Income$_i$ divided by $\sum_i$ Sales$_i$ for each year. Average Capital Income/Sales ratio = mean(Operating Income divided by Sales) for each year. The dash lines are the HP-filtered trends. Source: Compustat/CRSP Merged Fundamentals Annual (1960–2014).

shows the time series of both the average and the aggregate Labor Income/Sales ratio in our sample using the estimated labor cost. The average labor share rises from 32% in 1960 to 40% in 2014, while the aggregate labor share drops from 25% to 19% during the same period of time.

The divergence in the moments of the firm-level capital and labor share distribution are consistent with the mechanism we highlight in our model. Specifically, the trends we observe in the data a consistent with changes in firm-level volatility causing a shift in the distribution of firm size that favors the owners of capital.

6.3 Cross Sectional Variation in Capital Share Dynamics

Figure 8 presents the average and aggregate capital share for difference size groups. All firms are sorted into five groups based on their total assets. Within each group, we compute the average capital share. The average capital share tends to decline more in the smaller size

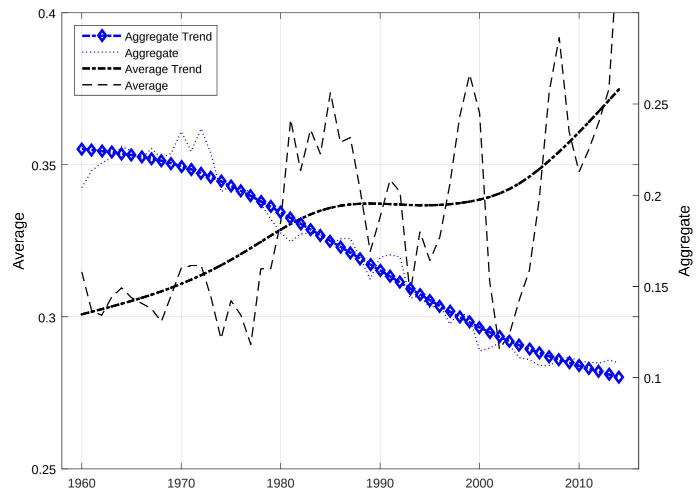


Figure 7. Aggregate and Average Labor Income/Sales Ratio

The figure presents the average and aggregate Labor Income/Sales ratio in Compustat public firms database. Labor Income/Sales ratio is measured by the ratio of estimated staff expenses (XLR) to sales. The dashed lines are the HP-filtered trends. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

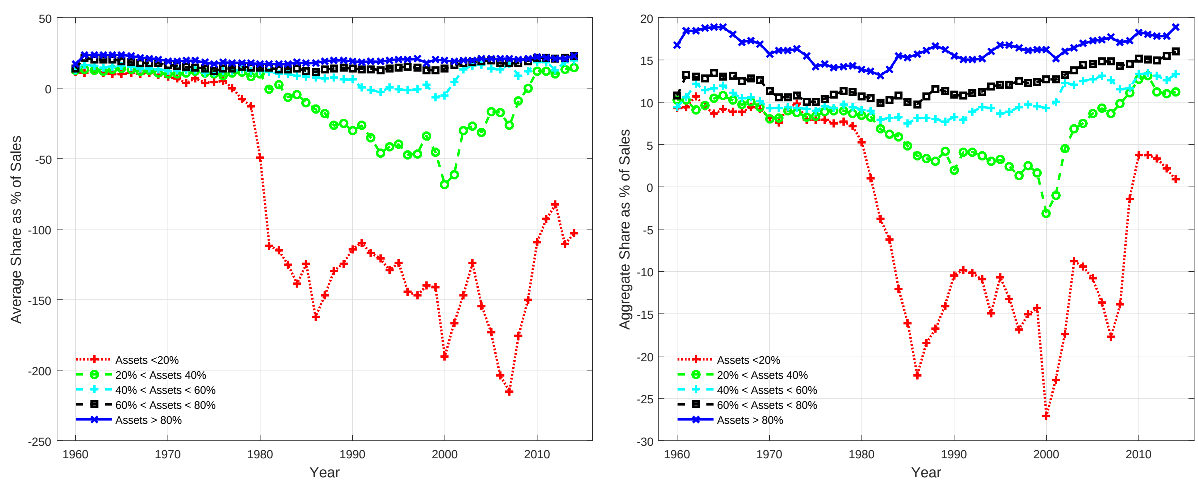


Figure 8. Capital Share of Output by Firm Size

The plot shows the average and aggregate capital share in each size quintile. Size is measured by total assets, and the capital share is measured as operating income (OIBDP) divided by sales. The sample includes all Compustat firms (both **active and inactive**), 1960-2014. The sample is winsorized at 1%.

quintiles. Aggregate capital share increases in total, but the increase in profitability mainly happens in the larger firms. Over the period from 1960 to 2014, firm-level volatility has gone up, and hence small firms with low profitability stay in the industry as they find the abandon options are more valuable. During the same period, we find that the capital share of the large firms (last quintile) remains stable. The dispersion of capital share across size groups increases over time as the volatility increases. Changes in the capital share of firms on the right tail of size distribution imply the change in the selection process of the public firms.

We then compare the average and aggregate factor share of output for different volatility sectors. Figure 13 in the separate appendix presents the cross-section of factor shares for low volatility and high volatility sectors. The decline of capital share in the small firms group (first quintile) is much more sizable in the high volatility sector than in the low volatility sector. Finally, we also note that the average capital share of firms that were liquidated is declining over time (see Figure 14 in separate appendix), consistent with an increase in the option value of waiting.

To address the concern of changing composition of public firm sample, we examine the distribution of capital share controlling for industries. We examine four main industries in this paper: consumer goods, manufacturing, health products and information, computer and technology (high tech) industry. The definition of consumer goods, manufacturing and health products are taken from Fama-French 5 industry classification. The high tech industry definition is from BEA Industry Economic Accounts[4]. We fix the definitions of industries over time, and sort firms into five different size groups within each industry. We find similar cross-sectional patterns within each industry: the dispersion of capital share across size groups increases over the last five decades, while the more significant decline in capital share

[4]The high tech industry is classified using NAICS, consisting of computer and electronic products, publishing and software, information and data processing, and computer system design and related services.

happens in the smaller size quintiles (see Figure 15 in the separate appendix). We also see stronger selection effects in the high tech industry and in the health products industry which have relatively high firm-level volatility, and smaller effects in manufacturing and consumer goods. For example, we observe highly negative operating margins for small biotech firms that employ researchers but have low revenues from sales. Biotech also is an industry characterized by high volatility.

7 Quantitative Experiments in Calibrated Model

In this section, we explore the quantitative implications of our model. We calibrate the economy to match the empirical moments of the U.S. Compustat sample, and then we consider the effects of changes in the underlying parameters to quantify the effect of our selection mechanism on the moments of the capital share.[5]

We first calibrate the model to match the aggregate moments from the U.S. publicly-traded firm sample over the period from 1960 to 1970. Panel A in Table 2 reports the moments we set out to match. Our model fails to match the cross-sectional standard deviation in firm-level capital share, partly because the model cannot match the cross-sectional dispersion in the size distribution. Panel B reports the parameters we chose to match these moments. Panel C reports some pre-set parameters. We set $\sigma = 0.2$ to match the average idiosyncratic sales volatility. μ is chosen to match the average TFP growth rate 2%. The choice of μ set the lower bound of parameter λ which governs the exogenous death rate of firms. We calibrate λ and ρ to match the cross-sectional standard deviation of the firm-level capital share and the capital share at the exit threshold $\bar{X}$. Our calibration of λ and ρ also

[5] Firm-level value added VA_i is OIBDP plus Extended XLR. To deal with negative values, we identify the minimum of operating income (OIBDP) for each year, and we increase the value added of all firms by the absolute value of the minimum OIBDP$\times$(1+1%). The average capital share is computed using OIBDP divided by the adjusted value added. The standard deviation and skewness of capital share is also estimated using the adjusted value added measure. The aggregate capital share is calculated using the unadjusted value added.

Table 2. Benchmark Calibration

Benchmark calibration. Panel A reports target moments. Panel B reports calibrated parameters. Panel C reports pre-set parameters. The sample covers 1960-1970. Compustat universe excluding SIC code from 6000 to 6799.

Panel A: Capital Share Moments 1960-1970		
	Data	Model
Average Capital Share	0.208	0.246
Aggregate Capital Share	0.419	0.318
Standard Deviation	0.152	0.082
Skewness Capital Share	0.710	-0.021
Exit Threshold CS	0.076	0.064
Entry Rate	-	0.013
Panel B: Calibrated Parameters		
σ	0.2	Idiosyncratic Vol
μ	0.02	Firm Growth
λ	0.055	Exogenous Exit Rate
ρ	3.5	Entrants Firm Size Distribution
α	0.27	Aggregate Physical Capital Share of output
Panel C: Pre-set Parameters		
r	0.05	Discount Rate
k/l	1	Capital/Labor Ratio
p	1	Sunk Cost
ν	0.2	Share of Rents in GDP (AK, 2005)

produces a reasonable exit/entry rate of the public firms at 1.34%; the average IPO rate is 3.4% in the 1980-2015 sample.[6] ν is not easily identifiable in the data. We choose $\nu = 0.2$, following Atkeson and Kehoe (2005).

Next, we use the benchmark calibration of the model to conduct a series of experiments in Table . In Panel A, we increase in turn σ, ρ and ν. σ is 20% per annum in the benchmark calibration. Firm level volatility plotted in Figure 9 has increased dramatically over the past five decades (see Comin and Philippon, 2005; Zhang, 2014; Herskovic et al., 2015). The measure of cash flow volatility and stock return volatility have doubled over the period 1960-2010. When we double the volatility, the model predicts a decline in the average capital share of 6.6 pps. and an increase the aggregate capital share of 1.4 pps. In the benchmark

[6]IPO rates prior to 1980 are not available. Fama and French 2005 suggested that the IPO before 1979 is much lower.

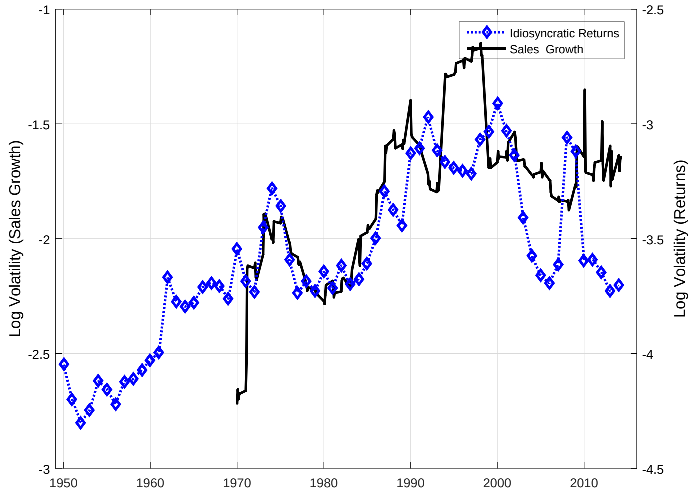


Figure 9. Firm Level Volatility of U.S. Public Firms

The dashed line is annualized idiosyncratic firm-level stock return volatility. Idiosyncratic returns are constructed within each calendar year by estimating a Fama French 3-factor model using all observations within the year. Idiosyncratic volatility is then calculated as the standard deviation of residuals of the factor model within the calendar year. We obtain the time series of idiosyncratic volatility by average across firms at each year. The solid line is the firm-level cash flow volatility, estimated for all CRSP/Compustat firms using the 20 quarterly year-on-year sales growth observations for calendar years. The idiosyncratic sales growth is the standard deviation of residuals of a factor specification. The factors for sales growth are the first 3 major principal components. Source: CRSP 1960-2014 and Compustat/CRSP Merged Fundamentals Annual 1950-2014.

calibration of our model, the owners only collect 13.9% of total rents at the average firm, but they collect 49.7% of aggregate rents. Doubling vol increases the aggregate share of rents collected by owners by 7.2 pps., while decreasing the average share of rents by 33 pps. This 7.2 pps increase translates into a 1.4 pps. increase in the aggregate capital share. To visualize the joint effect of changes in volatility and the rents, Figure 10 plots the aggregate capital share (panel on the left) and the average capital share (panel on the right) against σ and ν. Increases in ν have a minor effect on the aggregate capital share, except when these are ausgmented by increases in σ. Finally, only increases in σ lower the average capital share.

We also consider the effects of increasing ρ, the parameter governing the Pareto-entry distribution, as well as the size of rents in the economy ν. Interestingly, doubling ν, the

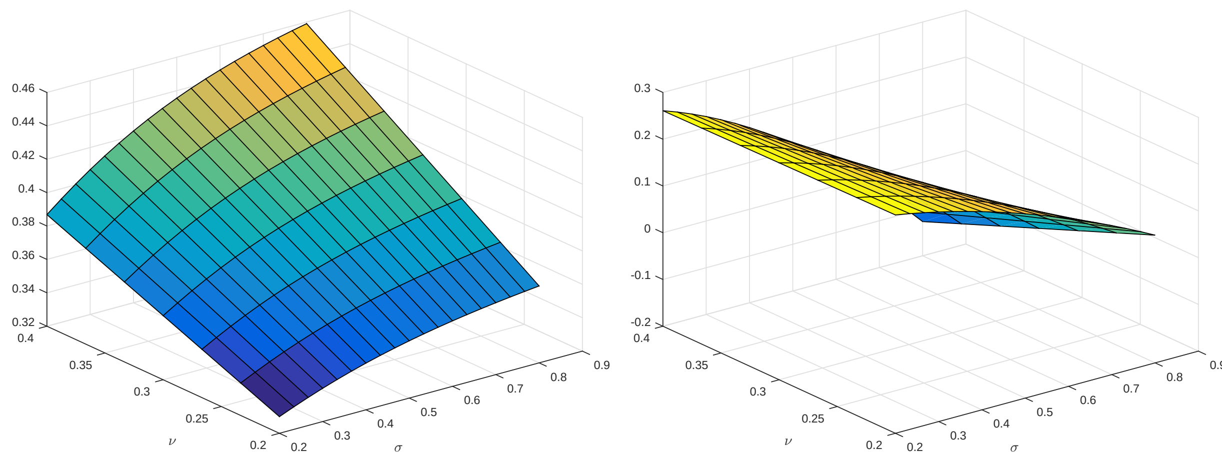


Figure 10. Aggregate and Average Capital Share

Figure plots the aggregate (left panel) and average (right panel) capital share against ν and σ. $r = 0.10, \mu = 0.02, \lambda = 0.055, \rho = 3.5, \alpha = 0.27$.

share of GDP due to rents, only increases the aggregate capital share by 4.5 pps., simply because the owners only get 13.9% of rents. ν has no bearing on the actual distribution of rents. That explains why we need to consider a joint increase in risk and the size of rents to match the moments in the data. We also report moments when the discount rate is 10%. Interestingly, the increase in the aggregate share is larger when the discount rate is higher. Higher discount rates imply a larger gap between the ex ante and ex post calculations.

Panel B of Table reports the effect of a joint increase in σ and ν. In this case, the model predicts an increase of 7.4 pps. in the aggregate capital share , and a decrease of 15.9 pps. in the average capital share. When we increase the discount rate to 10%, the increase in the aggregate capital share is 9.3 pps. while the decrease in the average capital share is 9.2 pps. Finally, Table 4 considers the joint distribution of firm-level capital shares and size in the data and the model. Our model does an adequate job matching the size pattern in average capital shares: negative capital shares for the smallest firms and large capital shares for the largest firms.

Table 3. Changes in Capital Share Moments from 1960-1970 to 1990-2014.

Changes in Moments of Firm-level Capital Share Distribution and Changes in Aggregate Capital Share. The 'Data' column reports the difference between moments in 1990-2014 and 1960-1970. The 'Model' column reports the differences in the moments of 2 stationary size distributions in response to changes in parameters starting from the benchmark calibration. In Panel A, we sequentially double σ, ρ and ν, and compute the moments of the new stationary size distribution. In Panel B, we double σ and ν jointly, and we compute the moments of the new stationary size distribution.

	Data	Model		
Panel A: Univariate Experiment				
		$\sigma \to 2\sigma$	$\rho \to 3\rho$	$\nu \to 2\nu$
		Discount Rate $r = 0.05$		
Average Capital Share	-0.118	-0.066	-0.029	-0.027
Aggregate Capital Share	0.138	0.014	-0.027	0.045
Capital Share at $\bar{X}$	-0.574	-0.190	0.000	-0.210
		Discount Rate $r = 0.10$		
Average Capital Share	-0.118	-0.043	-0.026	-0.006
Aggregate Capital Share	0.138	0.018	-0.024	0.057
Capital Share at $\bar{X}$	-0.574	-0.133	0.000	-0.167
Panel B: Multivariate Experiment				
		$\sigma \to 2\sigma$	$\sigma \to 2\sigma$	
		$\nu \to 2\nu$	$\nu \to 2\nu$	
		DR $r = 0.05$	DR $r = 0.10$	
Average Capital Share	-0.118	-0.159	-0.092	
Aggregate Capital Share	0.138	0.074	0.093	
Capital Share at $\bar{X}$	-0.574	-0.589	-0.432	

8 Firm Size and Firm Compensation Inequality

Finally, we return to the data to provide more evidence in support of our mechanism. Firm-level risk and firm size inequality are the key drivers of U.S. labor share and capital share dynamics. We can use the cumulant generating function to decompose capital share dynamics. The aggregate labor share is the ratio of two cross-sectional moments: $1 - \Pi = \frac{\mathbb{E}(lab)}{\mathbb{E}(sales)}$ where Π denotes the aggregate capital share. We can decompose these cross-sectional moments using higher-order moments of the log sales and log labor income distribution. In particular, we can expand the log of the aggregate labor share as the difference in the

Table 4. Firm-level Capital Share and Size Distribution in Multivariate Experiment

Moments of Firm-level Capital Share and Size Distribution. In Panel A, the 'Data' row reports the average capital share for size quintiles in 1960-1970. The 'Model' row reports the average capital share for size quintiles, computed from the benchmark calibration. In Panel B, the 'Data' row reports the average capital share for size quintiles in 1990-2014. The 'Model' row reports the average capital share for size quintiles, computed after doubling ν and σ. The discount rate r is 0.05.

Panel A: 1960-1970 Sample					
	-20%	20%-40%	40%-60%	60%-80%	80%-
Data	0.042	0.094	0.157	0.237	0.357
$\sigma = 0.2, \nu = 0.2$	0.118	0.196	0.247	0.301	0.393
Panel B: 1990-2014 Sample					
	-20%	20%-40%	40%-60%	60%-80%	80%-
Data	-0.119	-0.137	0.059	0.264	0.497
$\sigma = 0.4$	-0.031	0.102	0.188	0.272	0.388
$\sigma = 0.4, \nu = 0.4$	-0.334	-0.070	0.103	0.271	0.497

cumulant-generating functions of log output and log capital at the firm level:

$$\log(1 - \Pi) = \log \mathbb{E}(lab) - \log \mathbb{E}(sales) = \sum_{j=1}^{\infty} \frac{\kappa_j(\log lab)}{j!} - \sum_{j=1}^{\infty} \frac{\kappa_j(\log sales)}{j!}, \tag{42}$$

where the cumulants are defined by: κ_1 (mean), κ_2 (variance), $\kappa_3/\kappa_2^{3/2}$ (skewness), and κ_4/κ_2^2 (kurtosis). Similarly, note that the average labor income share can be decomposed as:

$$\log(\textit{Average Labor Share}) = \log \mathbb{E}(lab/sales) = \sum_{j=1}^{\infty} \frac{\kappa_j(\log lab - \log sales)}{j!}. \tag{43}$$

The first cumulant has the same effect on the average and the aggregate labor share, but the higher-order terms do not. For example, an increase in the variance of size not matched by an increase in variance of log compensation will increase the aggregate labor share, but decrease the average labor share. We decompose one minus the aggregate capital share $\log(1-\Pi) \approx \log\left((Sales - OIBDP)/Sales\right)$, instead of $\log\left(Labor/Sales\right)$, because the labor data is sparse in Compustat. In the appendix, we also use compensation date directly. We refer to $(Sales - OIBDP)$ as labor income, even though it includes other items. Table

5 provides a decade-by-decade overview of all four cumulants for sales and labor income. The top panel considers all sectors, including financials. The bottom panel considers only non-financials. The sum of all these weighted differences in cumulants adds up the log labor income share. The means of the log sales and log labor income distribution do not contribute much to the decline in the labor income share. All of the time series variation in the aggregate labor share is induced by higher order moments. Note that a common measure of risk is the entropy of a random variable $\mathbb{L}(x) = \log(\mathbb{E}(x)) - \mathbb{E}\log(x)$. Hence, much of the change in the aggregate labor share is attributable to difference in entropy between firm size (sales) and firm labor income.

$$\Delta \log(1 - \Pi) \approx \mathbb{L}(lab) - \mathbb{L}(sales) = \sum_{j=2}^{\infty} \frac{\kappa_j(\log lab)}{j!} - \sum_{j=2}^{\infty} \frac{\kappa_j(\log sales)}{j!}$$

We can interpret $\mathbb{L}(lab)$ as a measure of inter-firm compensation inequality, and $\mathbb{L}(sales)$ as a measure of firm size inequality. The log of aggregate labor income share in this sample declines because the overall cross-sectional inequality in labor compensation increases far less than the overall inequality in the size distribution. Across-the-board, both for sales and compensation, we see large increase in variance and kurtosis starting in the seventies, together with large increases in negative skewness. However, these increases are much larger for sales than for labor income. Between 1960 and 2014, we record a 279 log point increase in the cumulant sum for firm sales, but only 208 log points for compensation, which implies a 71 log point increase in the difference between the size and compensation weighted sum of cumulants. Table 6 reports the differences between the moments of the compensation and size distribution. The last column shows that 69 log points, almost the entire change, are due to the difference in firm size and firm compensation inequality ($\approx \mathbb{L}(lab) - \mathbb{L}(sales)$). Inter-firm compensation inequality has increased (see Song et al., 2015, for recent evidence), but not enough to offset the effect of the increase the firm size inequality on the capital share. In this sample, the changes in the means have no bearing on the aggregate labor share. When we exclude financials, reported in the bottom panel, the results are even stronger. Changes in the means do not account for any of the labor share decline.

Basically, the increase in the cross-sectional variance and kurtosis of the log size distribution that started in the late 70s in our universe of firms is not matched by similar changes in the log labor compensation distribution. These forces are only partly mitigated by an increase in negative skewness of the log size distribution, because of a growing mass of small, unprofitable firms in the left tail of the size distribution, that is not offset a similar increase in the log labor compensation distribution. As documented by Fama and French (2004), the wave of new listings that started in 1980 gave rise to a large mass of unprofitable firms in the left tail of the size distribution. The increasing left skewness of profitability and right skewness of growth after 1979 are not due to younger firms seeking a public listing. Loughran and Ritter (1995) conclude that during 1980-1990 there is no downtrend in the average age of firms going public. Starting in 1996, this trend in new listings reversed itself, and there was a sharp decline in the number of listed firms (Doidge, Karolyi, and Stulz (2015)). The decline is concentrated mostly in smaller firms.

9 Conclusion

We propose a mechanism whereby an increase in firm-level volatility can have important effects on national income accounting. A firm's owner insures is managers against firm-level productivity shocks. As a result, that owner may choose to exit if productivity becomes too low. The level of the managers' compensation is set based on expected firm value–which necessarily integrates over paths that end in exit. In contrast, when accounting for income, one typically integrates over surviving firms which necessarily feature lower capital shares of profit. This leads to an difference between the aggregate capital share of income, which is

Table 5. Firm Size and Compensation Inequality

The table shows the first four cumulants of the log labor compensation minus the same cumulants for the log sales distribution. Labor compensation is computed at the firm level as Sales minus operating income before depreciation (OIBDP). The sample includes all Compustat firms (both active and inactive), 1960-2014. The sample is winsorized at 1%.

	κ_1	$\frac{\kappa_2}{2}$	$\frac{\kappa_3}{3!}$	$\frac{\kappa_4}{4!}$	$\sum_{j=1}^{4} \frac{\kappa_j}{j!}$	$\sum_{j=2}^{4} \frac{\kappa_j}{j!}$
				Sales		
1960 - 1969	4.560	1.208	0.027	0.019	5.814	1.255
1970 - 1979	4.367	1.610	-0.014	0.243	6.205	1.839
1980 - 1989	4.068	3.138	-0.841	0.549	6.915	2.847
1990 - 1999	4.541	2.704	-0.334	0.571	7.482	2.941
2000 - 2009	5.445	2.665	-0.470	0.566	8.206	2.761
2010 - present	5.968	2.693	-0.932	0.876	8.605	2.637
2010's-1960's	1.408	1.485	-0.959	0.857	2.791	1.382
				Labor Compensation		
1960 - 1969	4.384	1.242	0.008	0.032	5.666	1.282
1970 - 1979	4.192	1.647	0.006	0.110	5.954	1.763
1980 - 1989	3.954	2.793	-0.041	-0.385	6.320	2.366
1990 - 1999	4.421	2.338	0.332	-0.122	6.969	2.548
2000 - 2009	5.332	2.265	0.287	-0.210	7.674	2.343
2010 - present	5.774	2.358	0.086	-0.472	7.746	1.972
2010's-1960's	1.390	1.116	0.077	-0.504	2.080	0.689

Table 6. Time Series of Cumulants

The table shows the differences in the first four cumulants of the log labor compensation minus the same cumulants for the log sales distribution. Labor compensation is computed at the firm level as Sales minus operating income before depreciation (OIBDP). The sample includes all Compustat firms (both active and inactive), 1960-2014. The sample is winsorized at 1%.

	$\kappa_1(\log lab) - \kappa_1(\log sales)$	$\frac{\kappa_2(\log lab)}{2} - \frac{\kappa_2(\log sales)}{2}$	$\frac{\kappa_3(\log lab)}{3!} - \frac{\kappa_3(\log sales)}{3!}$	$\frac{\kappa_4(\log lab)}{4!} - \frac{\kappa_4(\log sales)}{4!}$	$\sum_{j=1}^4 \frac{\kappa_j(\log lab)}{j!} - \sum_{j=1}^4 \frac{\kappa_j(\log sales)}{j!}$	$\sum_{j=2}^4 \frac{\kappa_j(\log lab)}{j!} - \sum_{j=2}^4 \frac{\kappa_j(\log sales)}{j!}$
1960 - 1969	-0.176	0.034	-0.019	0.013	-0.148	0.028
1970 - 1979	-0.175	0.037	0.020	-0.133	-0.251	-0.076
1980 - 1989	-0.114	-0.346	0.800	-0.935	-0.594	-0.480
1990 - 1999	-0.120	-0.366	0.666	-0.693	-0.513	-0.393
2000 - 2009	-0.114	-0.400	0.757	-0.776	-0.532	-0.418
2010 - present	-0.194	-0.335	1.018	-1.348	-0.859	-0.665
2010's-1960's	-0.018	-0.369	1.037	-1.360	-0.711	-0.693

calculated ex post, and the capital share of value at the origination of firm, which is calculated ex ante. When firm-level volatility increases, the difference can increase, increasing the aggregate capital share and decreasing the average capital share. We also present time series and cross sectional evidence for Compustat firms consistent with our proposed mechanism.

References

Ai, H., Kiku, D., Li, R., 2013. A mechanism design model of firm dynamics: The case of limited commitment. Available at SSRN 2347821.

Ai, H., Li, R., 2015. Investment and CEO compensation under limited commitment. Journal of Financial Economics 116 (3), 452 – 472.
URL http://www.sciencedirect.com/science/article/pii/S0304405X15000458

Atkeson, A., Kehoe, P., 2005. Modeling and measuring organizational capital. Journal of Political Economy 113, 1026–1053.

Barkai, S., 2016. Declining labor and capital shares, Working Paper, University of Chicago.

Berk, J. B., Walden, J., 2013. Limited capital market participation and human capital risk. Review of Asset Pricing Studies 3 (1), 1–37.
URL http://raps.oxfordjournals.org/content/3/1/1.abstract

Bloom, N., May 2014. Fluctuations in uncertainty. Journal of Economic Perspectives 28 (2), 153–76.
URL http://www.aeaweb.org/articles?id=10.1257/jep.28.2.153

Campbell, J. Y., Lettau, M., Malkiel, B. G., Xu, Y., 2001. Have individual stocks become more volatile? an empirical exploration of idiosyncratic risk. The Journal of Finance 56 (1), 1–43.
URL http://dx.doi.org/10.1111/0022-1082.00318

Comin, D., Philippon, T., 2005. The rise in firm-level volatility: Causes and consequences. NBER Macroeconomics Annual.

Dixit, A. K., Pindyck, R. S., 1994. Investment under uncertainty. Princeton university press.

Doidge, C., Karolyi, G. A., Stulz, R. M., May 2015. The u.s. listing gap. Working Paper 21181, National Bureau of Economic Research.
URL http://www.nber.org/papers/w21181

Donangelo, A., Gourio, F., Palacios, M., 2015. Labor share and the value premium. Tech. rep.

Edmans, A., Gabaix, X., Landier, A., 2009. A multiplicative model of optimal ceo incentives in market equilibrium. Review of Financial Studies 22 (12), 4881–4917.
URL http://rfs.oxfordjournals.org/content/22/12/4881.abstract

Fagereng, A., Guiso, L., Pistaferri, L., 2016. Back to background risk?

Fama, E. F., French, K. R., 2004. New lists: Fundamentals and survival rates. Journal of Financial Economics 73 (2), 229 – 269.
URL http://www.sciencedirect.com/science/article/pii/S0304405X04000315

Friedrich, B., Laun, L., Meghir, C., Pistaferri, L., 2014. Earnings dynamics and firm-level shocks. Tech. rep., Discussion paper.

Furman, J., Orszag, P., 2015. A firm-level perspective on the role of rents in the rise in inequality. Presentation at "A Just Society" Centennial Event in Honor of Joseph Stiglitz Columbia University.

Fuss, C., Wintr, L., 2009. Rigid labour compensation and flexible employment? firm-level evidence with regard to productivity for belgium. Firm-Level Evidence with Regard to Productivity for Belgium (March 11, 2009). National Bank of Belgium Working Paper (159).

Gabaix, X., 2011. The granular origins of aggregate fluctuations. Econometrica 79 (3), 733–772.
URL http://dx.doi.org/10.3982/ECTA8769

Gabaix, X., Landier, A., 2008. Why has ceo pay increased so much? The Quarterly Journal of Economics 123 (1), 49–100.
URL http://qje.oxfordjournals.org/content/123/1/49.abstract

Gourio, F., Roys, N., 2014. Size-dependent regulations, firm size distribution, and reallocation. Quantitative Economics 5, 1759–7331.

Guiso, L., Pistaferri, L., Schivardi, F., 2005. Insurance within the firm. Journal of Political Economy 113 (5), 1054–1087.
URL http://www.jstor.org/stable/10.1086/432136

Herskovic, B., Kelly, B. T., Lustig, H. N., Van Nieuwerburgh, S., 2015. The common factor in idiosyncratic volatility: Quantitative asset pricing implications. Journal of Financial Economics.

Holmstrom, B., Milgrom, P., 1991. Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design. Journal of Law, Economics, & Organization 7, 24–52.
URL http://www.jstor.org/stable/764957

Hopenhayn, H., 2002. Exit, selection, and the value of firms. Journal of Economic Dynamics and Control 16(3-4), Pages 621–653.

Hopenhayn, H. A., 1992. Entry, exit, and firm dynamics in long run equilibrium. Econometrica 60 (5), 1127–1150.
URL http://www.jstor.org/stable/2951541

Jovanovic, B., 1982. Selection and the evolution of industry. Econometrica 50 (3), 649–670.
URL http://www.jstor.org/stable/1912606

Karabarbounis, L., Neiman, B., 2013. The global decline of the labor share. Tech. rep., National Bureau of Economic Research.

Kocherlakota, N., 1996. Implications of efficient risk sharing without commitment. Review of Economic Studies 63, 595–610.

Krueger, D., Uhlig, H., 2006. Competitive risk sharing contracts with one-sided commitment. Journal of Monetary Economics 53 (7), 1661 – 1691.
URL http://www.sciencedirect.com/science/article/pii/S0304393206000663

Lagakos, D., Ordoñez, G. L., 2011. Which workers get insurance within the firm? Journal of Monetary Economics 58 (6–8), 632 – 645.
URL http://www.sciencedirect.com/science/article/pii/S0304393211001176

Leland, H. E., 1994. Corporate debt value, bond covenants, and optimal capital structure. The Journal of Finance 49 (4), 1213–1252.
URL http://dx.doi.org/10.1111/j.1540-6261.1994.tb02452.x

Loughran, T., Ritter, J. R., 1995. The new issues puzzle. The Journal of Finance 50 (1), 23–51.
URL http://dx.doi.org/10.1111/j.1540-6261.1995.tb05166.x

Lustig, H., Syverson, C., Nieuwerburgh, S. V., 2011. Technological change and the growing inequality in managerial compensation. Journal of Financial Economics 99 (3), 601 – 627.
URL http://www.sciencedirect.com/science/article/pii/S0304405X10002199

Luttmer, E. G., 2012. Technology diffusion and growth. Journal of Economic Theory 147 (2), 602 – 622, issue in honor of David Cass.
URL http://www.sciencedirect.com/science/article/pii/S0022053111000196

Luttmer, E. G. J., 2007. Selection, growth, and the size distribution of firms. The Quarterly Journal of Economics 122 (3), 1103–1144.
URL http://www.jstor.org/stable/25098869

Miao, J., 2005. Optimal capital structure and industry dynamics. Journal of Finance 60, 2621–2659.

Moll, B., 2016. Productivity losses from financial frictions: Can self-financing undo capital misallocation? American Economic Review (forthcoming).

Perla, J., Tonetti, C., Benhabib, J., et al., 2014. The growth dynamics of innovation, diffusion, and the technology frontier. In: 2014 Meeting Papers. No. 818. Society for Economic Dynamics.

Piketty, T., Zucman, G., 2014. Capital is back: Wealth-income ratios in rich countries, 1700-2010. The Quarterly Journal of Economics.
URL http://qje.oxfordjournals.org/content/early/2014/05/21/qje.qju018.abstract

Rute Cardoso, A., Portela, M., 2009. Micro foundations for wage flexibility: Wage insurance at the firm level*. Scandinavian Journal of Economics 111 (1), 29–50.
URL http://dx.doi.org/10.1111/j.1467-9442.2008.01553.x

Song, J., Price, D. J., Guvenen, F., Bloom, N., von Wachter, T., May 2015. Firming up inequality. Working Paper 21199, National Bureau of Economic Research.
URL http://www.nber.org/papers/w21199

Thomas, J., Worall, T., 1988. Self-enforcing wage contracts. Review of Economic Studies 15, 541–554.

Zhang, M. X., 2014. Who bears firm-level risk? implications for cash flow volatility, working Paper U.T. Austin.

A Proofs

A.1 Derivation of equilibrium compensation

The equilibrium wage is given by the solution to the following equation

$$\int_{\bar{X}(c)}^{\infty} V(X;c)f(X)dX = P$$

where $\bar{X}(c)$ is given by equation (9) and $V(X;c)$ is given by equation (10). Note that this equation is equivalent to condition 3 of Definition 1. First evaluate the integral on the left hand side we have

$$\begin{aligned}
&\int_{\bar{X}(c)}^{\infty} \left(\frac{X}{r+\lambda-\mu} - \frac{c}{r+\lambda} - \left(\frac{\bar{X}(c)}{r+\lambda-\mu} - \frac{c}{r+\lambda} \right) \left(\frac{X}{\bar{X}(c)} \right)^{-\eta} \right) \frac{\rho}{X^{1+\rho}} dX \\
=&(\rho \bar{X}(c))^{\rho} \left(\frac{\bar{X}(c)}{(r+\lambda-\mu)(\rho-1)} + \frac{c}{(r+\lambda)\rho} + \frac{\bar{X}(c)(r+\lambda) - c(r+\lambda-\mu)}{(r+\lambda-\mu)(r+\lambda)(\eta+\rho)} \right) \\
=&\left(\frac{\eta(r+\lambda-\mu)}{(1+\eta)(r+\lambda)} \right)^{-\rho} \left(\frac{\eta}{(r+\lambda)(\rho-1)(\eta+\rho)} \right) c^{-(\rho-1)}.
\end{aligned}$$

Note that our assumption on the Pareto form $f(X)$ facilitates that computation of the integral above because both $V(X;c)$ and $f(X)$ are power functions. This integral represent the expected value of the firm to the shareholder after paying the fixed cost but before realizing the initial productivity of the firm. Since $\rho > 1$, it is monotonically increasing in c, and we can solve to get the expression for equilibrium compensation given in (11).

A.2 Derivation of stationary distribution

The ODE for $\phi(x)$ has the following general solution

$$\phi(x) = A_1 e^{\gamma_1 x} + A_2 e^{-\gamma_2 x} + A_3 e^{-\rho x} \tag{44}$$

where γ_1 and γ_2 are given by:

$$\gamma_1 = \frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2\lambda}}{\sigma^2} \tag{45}$$

$$\gamma_2 = \frac{-(\mu - \frac{1}{2}\sigma^2) + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2\lambda}}{\sigma^2} \tag{46}$$

First note that $\gamma_1 > 0$ implies $A_1 = 0$. To ease notation we drop the subscript on γ_2. Next note that an application of the ODE gives

$$A_3 = -\frac{\rho\psi}{\frac{1}{2}\rho^2\sigma^2 + \rho(\mu - \frac{1}{2}\sigma^2) - \lambda}. \tag{47}$$

Finally, the boundary condition implies that

$$A_2 e^{-\gamma\bar{x}} + A_3 e^{-\rho\bar{x}} = 0$$

so

$$A_2 = -A_3 e^{(\gamma-\rho)\bar{x}}. \tag{48}$$

The result in equation (15) directly follows from the above and an application of the market clearing condition for managers.

A.3 Derivation of Total and Average Capital Share of Profits.

We have

$$\begin{aligned}\Pi = \frac{\int_{\bar{x}}^{\infty}(e^x - c)\phi(x)dx}{\int_{\bar{x}}^{\infty} e^x\phi(x)dx} &= 1 - \frac{c\int_{\bar{x}}^{\infty}\phi(x)dx}{\int_{\bar{x}}^{\infty} e^x\phi(x)dx} \\ &= 1 - \frac{c}{\int_{\bar{x}}^{\infty} e^x\phi(x)dx}\end{aligned}$$

where the second step follows from the market clearing condition given in equation (14). To continue the calculation we have

$$\begin{aligned}
\int_{\bar{x}}^{\infty} e^x \phi(x) dx &= \int_{\bar{x}}^{\infty} \frac{\rho\gamma}{\rho-\gamma} \left(e^{-(\gamma-1)x+\gamma\bar{x}} - e^{-(\rho-1)x+\rho\bar{x}}\right) dx \\
&= \bar{X} \left(\frac{\rho}{\rho-1}\right) \left(\frac{\gamma}{\gamma-1}\right) \\
&= \left(\frac{\rho}{\rho-1}\right) \left(\frac{\gamma}{\gamma-1}\right) \left(\frac{\eta}{\eta+1}\right) \left(\frac{r+\lambda-\mu}{r+\lambda}\right) c
\end{aligned}$$

Substituting this expression into the expression for Π given above yields the desired result. The derivation of the average capital share is similar.

B Data Appendix

B.1 Data Construction

The Sample: Compustat/CRSP Merged Fundamental Annual contains widely available accounting data and stock return data for all publicly-traded firms. The sample is from 1960 to 2014, including all Compustat firms (both active and inactive). We exclude financial firms with SIC codes 6000-6799 for our main analysis, and exclude firms whose sales, employee numbers and total asset values are negative. Last, we also exclude firms with currency code as Canadian dollars to focus on the sample of U.S. firms only. The sample is winsorized at 1%.

Construction of Main Variables We measure firm's capital income using operating income before depreciation (OIBDP). The capital share of output is defined as OIBDP/Sales.

Labor income is measured using labor cost reported by the public firms. Since Staff Expenses (XLR) is not required to file for public firms, we get sparse observations of labor cost directly from the database. Following Donangelo et al. (2015), we construct the *extended labor cost* (XLR). First, we estimate the average labor cost per employee (XLR/EMP) within the industry-size group for each year. Industries are classified using 17 Fama-French industry definition, and firms are sorted into 20 size groups based on their total assets, so we obtain total 340 industry-size groups. Then, labor cost of a firm with missing XLR equals the number of employees times the average labor cost per employee of the same industry-size group during that year. We winsorize the extended XLR at 5% to exclude outliers from the approximation. We measure labor share of output as extended labor cost (XLR)/Sales.

Value added (VA) is defined as OIBDP + extended XLR. We winsorize the VA at 5% to exclude outliers from the approximation of extended XLR. We also calculate capital share as OIBDP/VA, and labor share as extended XLR/VA.

We measure firm-level volatility using both idiosyncratic cash flow volatility and idiosyncratic stock return volatility. Idiosyncratic stock returns are constructed within each year by obtaining the residual of a Fama-French 3-factor model using all observations within that year. Idiosyncratic stock return volatility is calculated as the standard deviation of residuals within that year. To obtain idiosyncratic cash flow volatility, we first estimate the first 3 major principal components of quarterly sales growth. The idiosyncratic cash flow volatility is the standard deviation of residuals of a sales growth factor specification using 20 quarterly year-on-year observations.

Industry Classification We classify firms into five industries: *consumer goods, manufacturing, health products, high tech, and others.* The classification of consumer goods, manufacturing, and health products industries are taken from Fama-French 5-industry classification. The high-tech industry category is defined following the definition of the information, computer, and technology industry classification from the BEA Industry Economic Accounts, which consists of computer and electronic products, publishing industries (including software), information and data processing services, as well as computer systems design and related services. We classified all the remaining firms (including the finance industry) into other industries. The traditional industries in our paper combines the consumer goods and manufacturing industries. To categorize the new economy industries, or highly intangible-intensive industry, we use our definition of high-tech (ICT) industries.

B.2 Figures and Tables

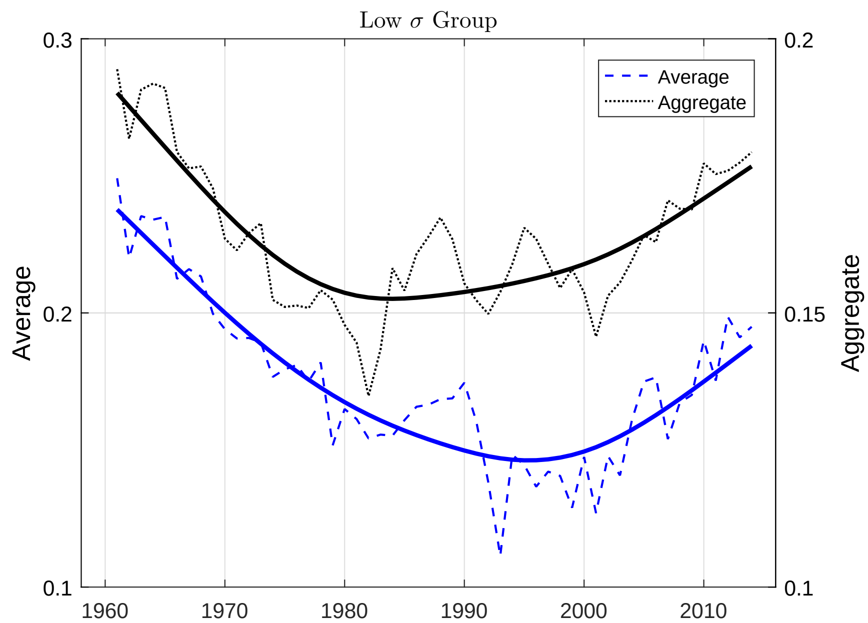
In this session, we provide more details of the figures and tables in the paper.

Average and Aggregate Factor Share Using Compustat sample, we construct the average and aggregate factor shares in the following way. For each year, aggregate factor share $= \frac{\Sigma_i \text{Factor Income}}{\Sigma_i \text{Output}}$, and the average factor share $= \Sigma_i \left(\frac{\text{Factor Income}}{\text{Output}}\right)/N$.

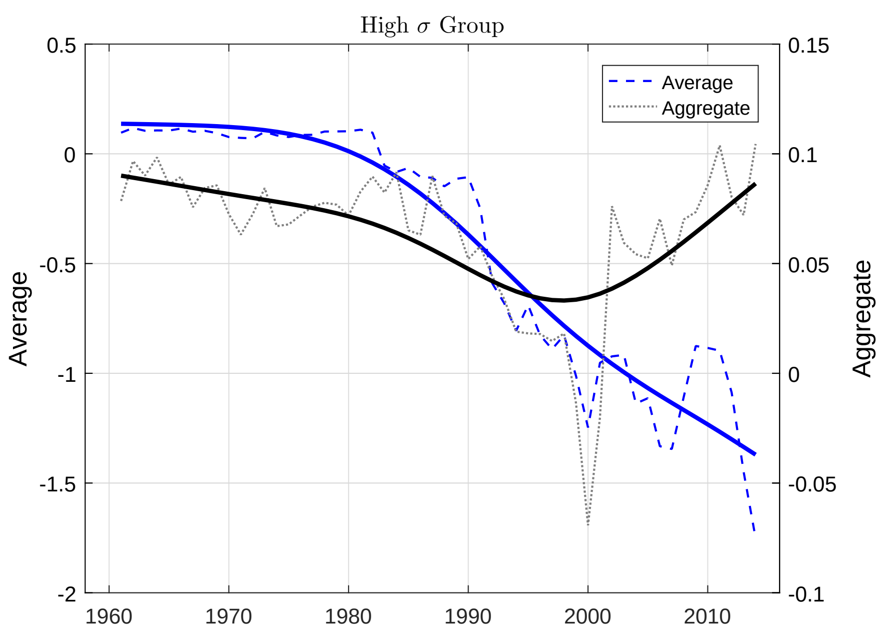
Size Groups For each year, all firms are sorted into five groups based on their total assets. Within each group, we compute the average and aggregate labor share and capital share. Figure 8 and Figure 15 show the time series of the average and aggregate capital share of each size group.

Volatility Groups For each year, all firms are sorted into three groups (low, median, and high) based on the idiosyncratic stock return volatility. Within each group, we compute the average and aggregate labor share and capital share. Figure 11 and Figure 12 show the time series of factor shares of the high volatility and the low volatility groups.

Delisting Threshold We obtain the security delisting information from CRSP U.S. Stock Events database. Delisting Code is a 3-digit integer code. It either indicates that a security is still trading or provides a specific reason for delisting. For our interest, we consider delisting for being liquidated (delist code from 400 to 490), or delisting by the current exchange for various reasons due to the poor company fundamentals, e.g, insolvency, bankruptcy, insufficient capital (delist code from 550 to 591). Then, we calculate the average capital shares prior (3 years or 5 years) to delisting. Figure 14 shows the time series of the pre-delisting performance from 1970 to 2014.



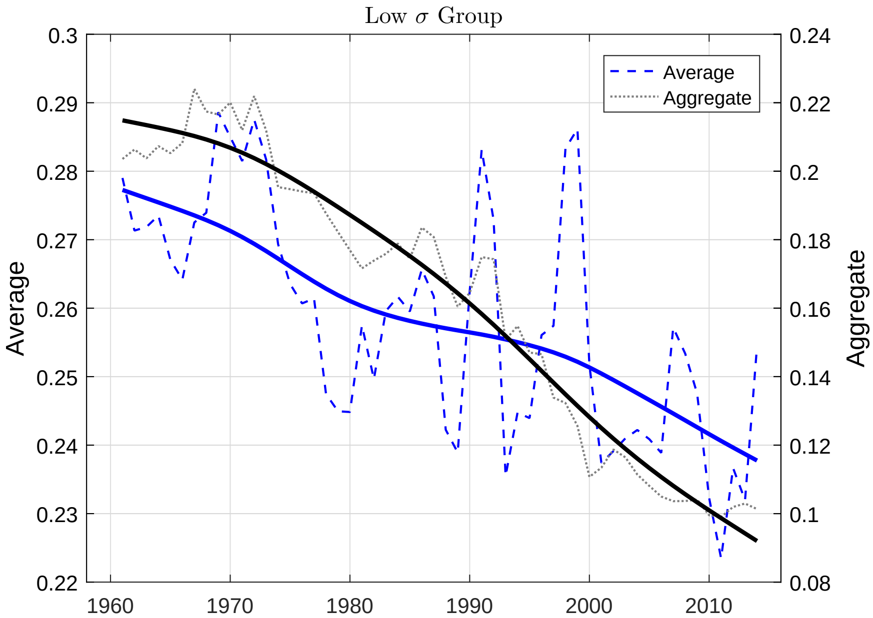
(a)



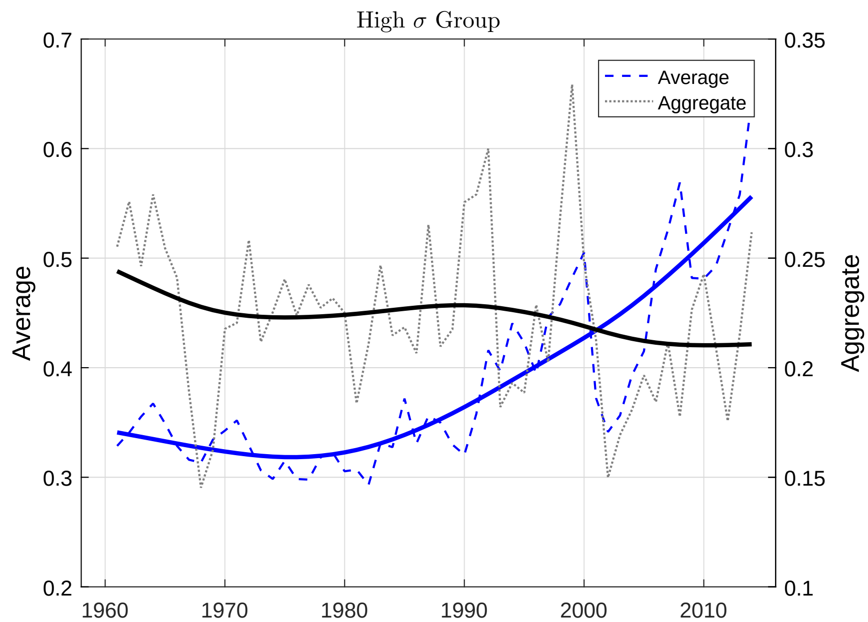
(b)

Figure 11. Average and Aggregate Capital Share of Output – Volatility Sectors

The plots present the time series of capital share of output for low and high volatility groups. For both (a) and (b), we classify firms into three groups (low, median, high) based on the idiosyncratic return volatility. Panel (a) shows the average and aggregate time series of operating income (OIBDP) to sales ratio in the group of firms with low idiosyncratic volatility. Panel (b) shows the average and aggregate time series of operating income (OIBDP) to sales ratio in the group of firms with high idiosyncratic volatility. The plots show data moments of the lowest and highest volatility groups. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).



(a)



(b)

Figure 12. Average and Aggregate Labor Share of Output – Volatility Sectors

The plots present the time series of labor share of output for low and high volatility groups. For both (a) and (b), we classify firms into three groups (low, median, high) based on the idiosyncratic return volatility. Panel (a) shows the average and aggregate time series of extended labor cost (XLR) to sales ratio in the group of firms with low idiosyncratic volatility. Panel (b) shows the average and aggregate time series of extended labor cost (XLR) to sales ratio in the group of firms with high idiosyncratic volatility. The plots show data moments of the lowest and highest volatility groups. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

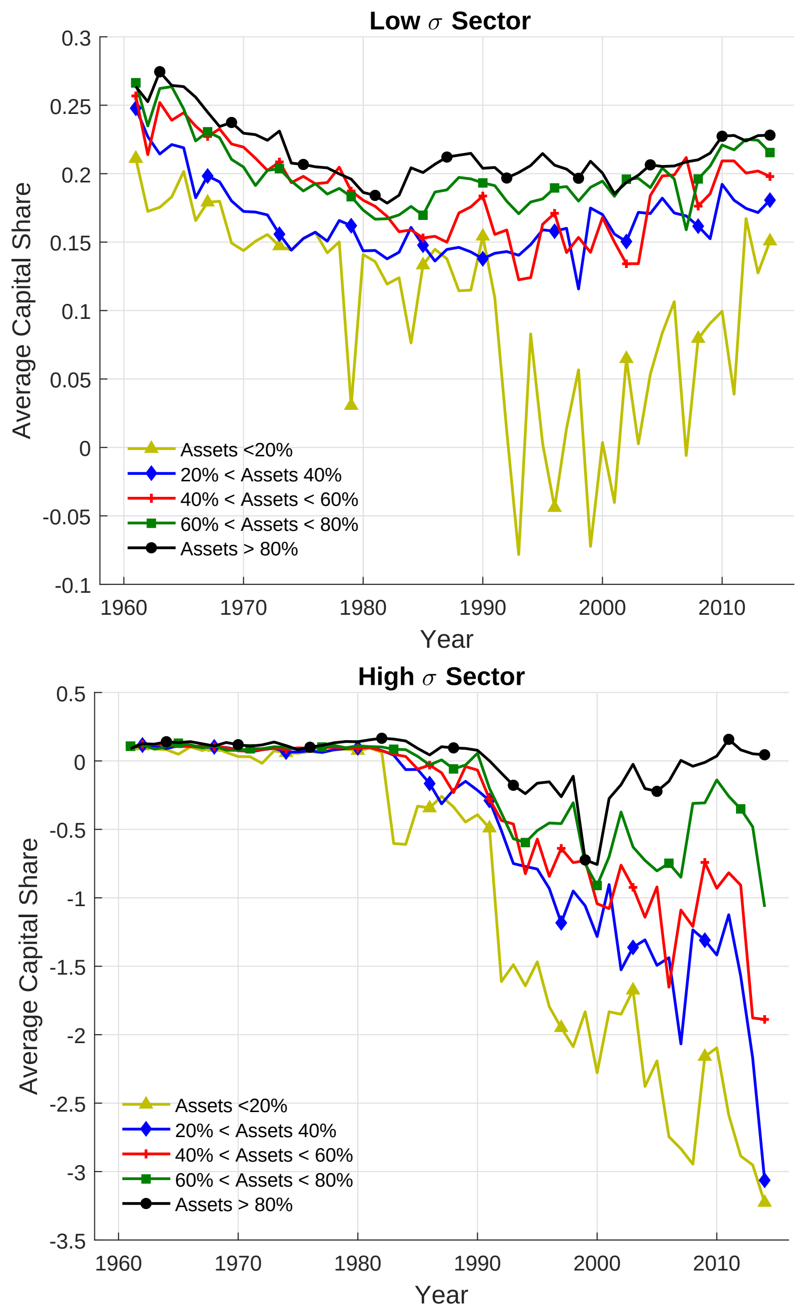


Figure 13. Average Capital Share of Output – Volatility Sectors

The plots present the average capital share of output within each size quintile for both high and low volatility sectors. Size is measured by total assets, and the capital share is measured as operating income (OIBDP) divided by sales. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

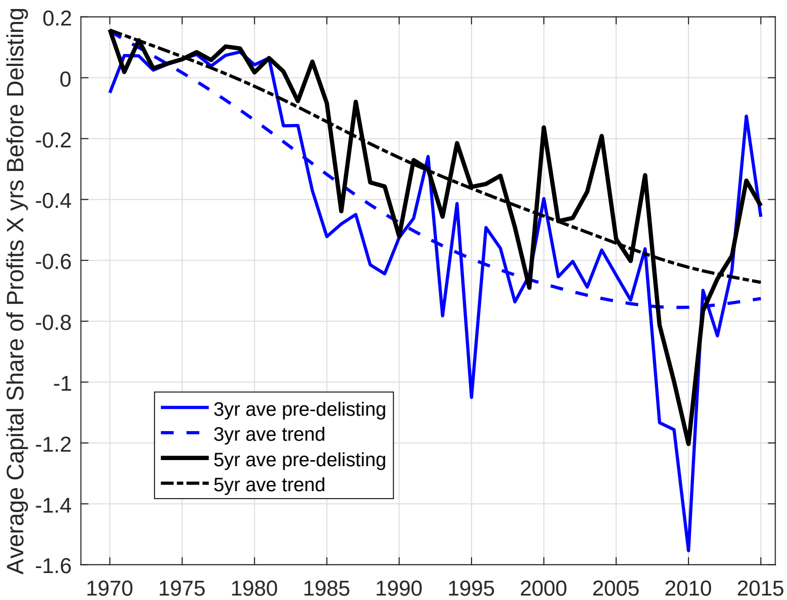


Figure 14. Exit Threshold: Average Capital Share X-years before Delisting

The plots present the average capital share of output 3-yr or 5-yr before delisting. We define firms' exiting the public firm domain by delisting code from 400 to 490 and from 550 to 591. The dashed line is the HP filtered trend. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014) and CRSP delisting code.

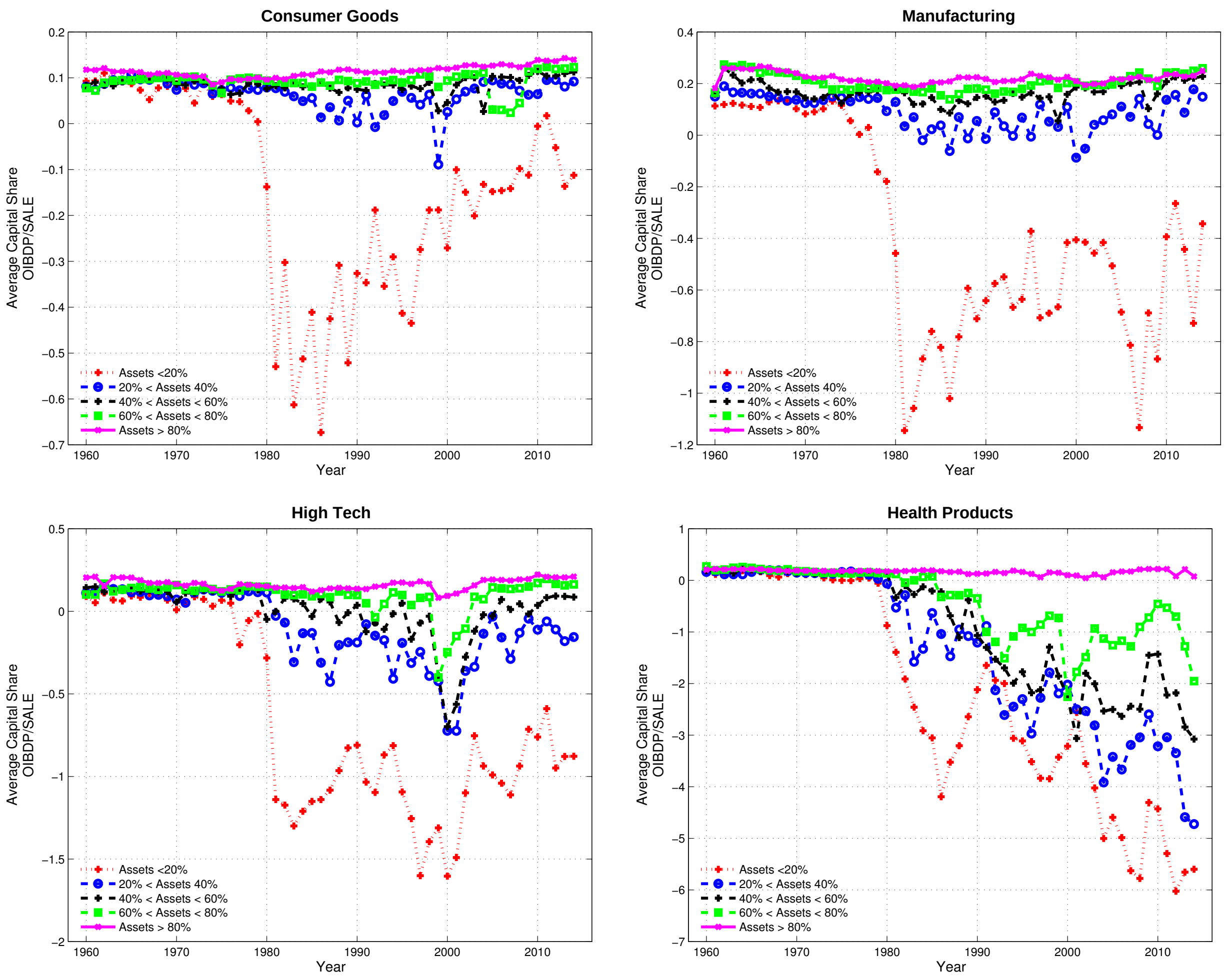


Figure 15. Average Capital Share of Output – Industries

We use revised Fama-French 5 industry classification. Within each industry, we sort firms into five groups based on their total assets. The plot shows the average capital share within each size group for four different industries. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

Table 7. Industry Cumulants

The table shows the sum of the cumulants of the log aggregate labor share distribution. The jth cumulant κ_j of the aggregate labor share is the jth cumulant of log labor compensation minus the jth cumulant of log sales. For each industry, we report the sum of the first four cumulants and the sum from the 2nd to the 4th cumulant. Labor compensation is computed at the firm level as Sales minus operating income before depreciation (OIBDP). The sample includes all Compustat firms (both active and inactive), 1960-2014. The sample is winsorized at 1%.

Periods	Consumer Goods		Manufacturing		Health Product		High Tech	
	$\sum_{j=1}^{4} \frac{\kappa_j}{j!}$	$\sum_{j=2}^{4} \frac{\kappa_j}{j!}$	$\sum_{j=1}^{4} \frac{\kappa_j}{j!}$	$\sum_{j=2}^{4} \frac{\kappa_j}{j!}$	$\sum_{j=1}^{4} \frac{\kappa_j}{j!}$	$\sum_{j=2}^{4} \frac{\kappa_j}{j!}$	$\sum_{j=1}^{4} \frac{\kappa_j}{j!}$	$\sum_{j=2}^{4} \frac{\kappa_j}{j!}$
1960 - 1969	-0.101	-0.003	-0.16	0.049	-0.339	-0.148	-0.237	-0.093
1970 - 1979	-0.083	0.009	-0.396	-0.197	-0.215	-0.066	-0.238	-0.11
1980 - 1989	-0.487	-0.422	-0.544	-0.416	-0.159	-0.287	-0.345	-0.328
1990 - 1999	-0.293	-0.217	-0.538	-0.363	-0.554	-0.89	-0.432	-0.437
2000 - 2009	-0.153	-0.065	-0.627	-0.439	-0.214	-0.736	-0.529	-0.565
2010 - present	-0.150	-0.034	-0.474	-0.224	0.329	-0.159	-0.434	-0.356
2010s- 1960s	-0.049	-0.031	-0.314	-0.273	0.668	-0.011	-0.197	-0.263

Table 8. Cumulants of Average Labor Share

The table shows the first four cumulants of the log labor share distribution. Labor compensation is computed at the firm level as Sales minus operating income before depreciation (OIBDP), and labor share is proxied using labor compensation scaled by Sales. The sample includes all Compustat firms (both active and inactive), 1960-2014. The sample is winsorized at 1%.

Periods	Panel A: Including Financials					
	$\kappa_1(\log \frac{lab}{sales})$	$\frac{\kappa_2(\log \frac{lab}{sales})}{2}$	$\frac{\kappa_3(\log \frac{lab}{sales})}{3!}$	$\frac{\kappa_4(\log \frac{lab}{sales})}{4!}$	$\sum_{j=1}^{4} \frac{\kappa_j(\log \frac{lab}{sales})}{j!}$	$\sum_{j=2}^{4} \frac{\kappa_j(\log \frac{lab}{sales})}{j!}$
1960 - 1969	-0.193	0.022	-0.002	0.000	-0.173	0.020
1970 - 1979	-0.172	0.040	-0.002	0.003	-0.131	0.042
1980 - 1989	-0.075	0.157	0.025	0.031	0.139	0.213
1990 - 1999	-0.066	0.207	0.048	0.056	0.245	0.311
2000 - 2009	-0.064	0.253	0.072	0.079	0.340	0.405
2010 - present	-0.130	0.281	0.092	0.100	0.342	0.472
2010s- 1960s	0.063	0.259	0.093	0.099	0.515	0.452

Periods	Panel B: Excluding Financials					
	$\kappa_1(\log \frac{lab}{sales})$	$\frac{\kappa_2(\log \frac{lab}{sales})}{2}$	$\frac{\kappa_3(\log \frac{lab}{sales})}{3!}$	$\frac{\kappa_4(\log \frac{lab}{sales})}{4!}$	$\sum_{j=1}^{4} \frac{\kappa_j(\log \frac{lab}{sales})}{j!}$	$\sum_{j=2}^{4} \frac{\kappa_j(\log \frac{lab}{sales})}{j!}$
1970 - 1979	-0.151	0.026	0.001	0.002	-0.122	0.029
1980 - 1989	-0.039	0.128	0.034	0.026	0.149	0.188
1990 - 1999	-0.003	0.192	0.065	0.053	0.307	0.309
2000 - 2009	0.017	0.255	0.096	0.081	0.449	0.432
2010 - present	-0.039	0.275	0.114	0.103	0.453	0.492
2010s- 1960s	0.148	0.256	0.115	0.103	0.622	0.475