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OPTIMAL MONETARY POLICY IN A COLLATERALIZED ECONOMY

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ABSTRACT

In the last forty or so years the U.S. financial system has morphed from a mostly insured retail deposit-based system into a system with significant amounts of wholesale short-term debt that relies on collateral, and in particular Treasuries, which have a convenience yield. In the new economy the quality of collateral matters: when Treasuries are scarce, the private sector produces (imperfect) substitutes, mortgage-backed and asset-backed securities (MBS). When the ratio of MBS to Treasuries is high, a financial crisis is more likely. The central bank's open market operations affect the quality of collateral because the bank exchanges cash for Treasuries (one kind of money for another). We analyze optimal central bank policy in this context as a dynamic game between the central bank and private agents. In equilibrium, the central bank sometimes optimally triggers recessions to reduce systemic fragility.

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1. **Introduction**

The Financial Crisis of 2007-2008 was not the result of an unfortunate convergence of a number of bad factors, but was the result of a long-term, permanent, transformation of the U.S. financial system into one involving significant amounts of wholesale funding, which is vulnerable to runs. This vulnerability has not been solved. In this paper argue that this transformation has important implications for monetary policy.

In the last forty years the U.S. banking system transformed from a system that mostly produced retail (insured) demand deposits to a system that produces significant amounts of short-term debt for the wholesale market. The new system produces short-term debt to a large extent with backing collateral made from the very loans that the traditional banking system originated, a process called "securitization." In the past, since demand deposits are insured, bank regulators/examiners monitored the on-balance sheet loan collateral backing the deposits and monetary policy could be conducted without reference to these backing loan portfolios. The new system, however, is different because the short-term debt is not insured, and so the quality of the backing assets –the collateral–matters for financial fragility. The backing assets are asset-backed and mortgage-backed securities and U.S. Treasuries. In the new system monetary policy *cannot* be separated from macroprudential policy because open market operations affect the quality of collateral in the economy. In this paper we show how collateral quality and monetary policy are intertwined and examine optimal monetary policy in this context.

The transformation of the financial system is dramatic. Figure 1, reproduced from Gorton, Lewellen and Metrick (2012), displays the transformation of the U.S. banking system over the period 1952Q1 through 2009Q1. The figure shows the components of privately-produced safe debt as a percentage of the total amount of privately produced safe debt.[1] The darkest part of the figure, at the bottom, is demand deposits. As a percentage of the total, demand deposits have fallen from about 80% to 31%. Moving upwards, the next component is money-like debt (e.g., repo, commercial paper, money market funds). This component rose from 11% to 21%. Upwards next are private-label AAA asset-backed and mortgage-backed securities. This category rose from zero to 18%. The "shadow banking system" is the sum of the ABS/MBS and money-like debt components, which grew from 11% to 38%, larger than demand deposits. This transformation seems to have accelerated in the late 1980s, but in any case it is apparent that the change is not a temporary change.

[1] Privately-produced debt cannot be "safe" in the same sense that Treasuries are safe. Nevertheless we will refer to the privately-produced substitutes as safe debt (which is the norm in the literature).

Gorton and Muir (2015) describe this change in the composition of money as corresponding to a transition from a system of immobile collateral – bank loans staying on bank balance sheets until maturity – to a system of mobile collateral, which is created by securitizing bank loans, that is producing bonds from the loans. Asset-backed and mortgage-backed securities (ABS/MBS) are the mobile collateral; they can be used as collateral for repo or for derivative positions, etc. The issue we address here arises in a world where collateral is mobile and, in particular, privately-produced collateral consists of mortgage-backed securities.

When Treasuries are scarce, the private sector produces substitutes to use as collateral and as a store of value, asset-backed and mortgage-backed securities (ABS/MBS) prior to the recent crisis (see Krishnamurthy and Vissing-Jørgensen (2012, 2015)). When the ratio of privately produced "safe debt" ABS/MBS to Treasuries is high, i.e., there is a shortage of government-produced safe debt, a financial crisis is more likely because the privately-produced safe debt, used to back short-term bank debt (repo and asset-backed commercial paper), is not riskless in every state of the world. We study a setting where the central bank cares about the quality of collateral in the economy. Macroprudential policy aims to keep the MBS-to-Treasury ratio low to reduce financial fragility. This introduces a conflict between maximizing output and minimizing financial fragility.

We take the transformation of the financial system as permanent and fundamental. In addition, we state the following stylized facts:

1. There is a convenience yield associated with U.S. Treasury debt. See, e.g., Duffee (1996), Krishnamurthy and Vissing-Jørgensen (2012, 2015), Nagel (2014), Gorton and Muir (2015),Carlson, Duygan-Bump, Natalucci, Nelson, Ochoa, Stein, and Van den Heuvel (2014), and Greenwood, Hanson, and Stein (2015).

2. When Treasury debt is scarce, the convenience yield rises and the private sector produces substitutes, mortgage-backed securities in the recent crisis. See Krishnamurthy and Vissing-Jørgensen (2015), Gorton, Lewellen and Metrick (2012), Xie (2012), and Sunderam (2012).

3. Credit booms—high growth in private debt—typically precede financial crises. See, e.g., Schularick and Taylor (2012), Jorda, Schularick and Taylor (2011), Laevan and Valencia (2012), Desmirguc-Kunt and Detragiache (1998) and Gorton and Ordoñez (2015).[2]

4. When the ratio of Treasury debt to GDP is low, a financial crisis is more likely. Alternatively, when the ratio of MBS-to-Treasuries is high, crisis is more likely. This is implied by points (2) and (3).

[2]The literature on credit booms and crises is large and we have only cited a few of the many papers.

In this mobile collateral world there is a fundamental friction: *cash cannot be securitized and Treasuries cannot be used to satisfy cash-in-advance constraints.*[3] In other words, cash and Treasuries are not perfect substitutes because non-banks cannot hold reserves at the central bank and there is (currently) no way to directly intermediate to create a marketable bond backed by reserves at the Fed, a bond that anyone could hold, trade, or pledge as collateral. If anyone could open an account at the central bank, hold interest-bearing reserves directly, and issue a tradable and pledgeable bond against those reserves, then reserves and T-bills would become very close substitutes. But, this is not currently institutionally feasible.[4] On the other hand, Treasuries are only issued in large denominations and cannot be a hand-to-hand currency. This means that cash and Treasuries are fundamentally different. This difference and a cash-in-advance constraint are the frictions in the model.

Since large agents are the end-users for Treasuries and ABS/MBS, to back short-term debt or to store value, they are one clientele demanding one kind of money, Treasuries and ABS/MBS. Households are largely the source of the demand for traditional money, M0. These clienteles are distinct because of the fundamental friction, which means that Treasuries and ABS/MBS are not substitutes for cash or demand deposits. Taking account of both types of money has important implications for monetary policy since there are now two clienteles demanding two different types of "money." Since each clientele has a demand for its type of money, optimal monetary policy has a difficult trade-off in terms of welfare. On the one hand, each clientele needs a form of "money", but on the other hand, inflation is a function of cash and financial fragility is a function of the quality of collateral.

In this paper, we introduce a macroprudential policy for the central bank to pursue to manage financial fragility in the context described above. We show that this policy interferes with what would otherwise be the optimal monetary policy (in an economy which never has crises), but that this interference is optimal to reduce financial fragility. The model is a dynamic game between the central bank and private agents. The equilibrium concept we use considerably simplifies the analysis of this type of game. In equilibrium, the economy will experience deflation during recessions, and a boom-bust pattern is the equilibrium outcome under the optimal policy.

Here "macroprudential policy" is taken to mean optimal management of the quality of the collateral in the economy, the ratio of ABS/MBS to Treasuries. (We simply refer to MBS hereafter.) Financial fragility—the likelihood of a crisis-- is increasing in this ratio, and this reduces welfare. More generally, the ratio would be credit to the private sector divided by

[3]ABS/MBS can be transformed into cash via the discount window, but this faces the problem of stigma and, in any case, is not permanent.

[4]Garratt, Martin, McAndrews and Nosal (2015) discuss segregated balance accounts at the Fed, but not securitization of the deposits in such accounts.

Treasuries. MBS and Treasuries are used to back repo, and for money market funds and mortgage-backed commercial paper, i.e. short-term debt which is the root of crises. Here, rather than regulate the quantity of short-term debt directly, the central bank regulates indirectly, via collateral quality. This is very natural since open market operations already trade cash for Treasuries, and vice versa.[5] So, whether or not the central bank recognizes this, it is in fact affecting the quality of collateral in the economy. Here, the central bank explicitly recognizes this. In the mobile collateral world macroprudential policy cannot be separated from monetary policy.

We model the money demands of the two clienteles in the following way. We employ a cash-in-advance model, in which households demand cash to pay for cash-goods consumption. Firms demand Treasuries and MBS, implicitly to use as collateral to borrow. Financial intermediaries will briefly appear in the model, but the use of Treasuries and MBS to back repo, for example, is in the background. The demand for this second kind of money is modelled by specifying production to be a function of real Treasury securities and MBS, recognizing that privately-created MBS cannot supply as much "liquidity" as Treasuries. Entering Treasuries and MBS into the production function is a reduced form for the combined financial and non-financial sectors. As we will explain, securitization plays a role in the model. The creation of MBS is endogenous in the model and occurs via securitization of mortgages.

The dynamics in this economy are quite different from the standard model (without macroprudential policy). Here, the central bank trades one form of money for another to balance between the safe public collateral, Treasuries, and the fragile private collateral, MBS, as well as worrying about inflation. As discussed above, if the central bank does not inject enough Treasuries into the economy, the private sector generates more MBS. A change in Treasuries is negatively correlated with the production of MBS. At the same time, the central bank's money supply affects the price level, which affects the real value of the collateral (Treasuries plus MBS). A high initial money supply drives up the price level today, which decreases the real value of the collateral for production; to balance that, the central bank needs to conduct an expansionary monetary policy which drives up the amount of MBS and, consequently, more collateral in nominal term. However, if the ratio of MBS to Treasuries is too high, this cannot be optimal due to the prohibitively high risk of financial fragility. Therefore, with a high initial money supply, it may be optimal for the central bank to reduce the amount of cash in nominal terms by selling Treasuries through open market operations, which reduces the amount of MBS, the money supply and the price level. In this way, the real value of liquidity and output may drop, but not as much as the welfare loss of financial fragility avoided.

The "macroprudential policy" we discuss here is different from what is usually meant by this term. Usually it refers to the supervision and regulation of "banks," including capital and liquidity requirements, stress tests, as well as examinations, stress tests, and required

[5] There are, of course, other ways to deal with the vulnerability of short-term wholesale debt. But, these have not been adopted, so we focus on the current environment.

reporting. The authorities engaged in these activities include parts of the government that are separate from the central bank. Usually it is argued that monetary policy and macroprudential policy are distinct, as a practical matter, but moreover monetary policy should be kept separate. Svensson (2015), for example, argues that monetary policy cannot achieve financial stability in any case, but the concept of a "financial crisis" is not specified. Bernanke (2015) argues that as a practical matter the two policies should be separate because with respect to financial stability monetary policy "is a blunt tool." Also see Williams (2015) and the references therein.

The debate about the role of financial stability in setting monetary policy has generally focused on whether the central bank should address asset price bubbles.[6] In our setting an asset price bubble may be viewed as a high level of housing prices, which occurs precisely because there is an insufficient amount of Treasuries, causing the endogenous creation of MBS. But, in our model there is no "bubble", though a high level of MBS appears as one. In our setting if the growth of MBS makes MBS large relative Treasuries then the central bank must act to decrease this ratio. The transformation of the financial system means that open market operations have an effect now that was not present before.[7] Consequently, in this paper we take the view, as does Stein (2014), that the risks of a financial crisis cannot be completely mitigated with conventional non-monetary tools.

The model we analyze is an infinitely-repeated game between one large player—the central bank---and many small players, agents in the private economy. It is a Ramsey problem in which the central bank cannot commit to its optimal policy. Such settings have been the subject of a large amount of research because of the results of Kydland and Prescott (1977), showing that dynamic programming cannot be used as a solution method because of the dynamic inconsistency. A recursive characterization of Perfect Public Equilibrium (PPE), which was first defined in Fudenberg, Levine, and Maskin (1994), for a dynamic game was proposed by Abreu, Pearce, and Stacchetti (1986) (APS). APS sheds light on this issue. In any PPE, the strategy of the large player is dynamically consistent although there is no commitment. Moreover, APS, in a game with a finite number of players, show that past histories can be summarized by

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[6] There is a very large literature on this topic, see the IMF Staff Report (2015) and Conlon (2015) for references.

[7] Adrian and Shin (2008, 2009) identify other channels that are important. They argue that monetary policy and financial stability polies "are closely linked" because short-term interest rates influence the size and leverage of financial intermediaries. Also see Gorio and Zhu (2008). Stein (2012) identifies still another channel in which banks create too much short-term debt, an issue that monetary policy could address.

promised future utilities, continuation values, and the values of agents can be described recursively. This approach has been widely used in macroeconomics.[8]

Our solution method is most closely related to Phelan and Stacchetti (2001) (PS), an extension of APS for a strategic game between a large player and a continuum of small players, where there is a public state variable.[9] PS do not augment payoffs using continuation values directly as APS do, but rather write the continuation values as a product of the choice variables and marginal values of these variables. By augmenting the payoffs in this way, agents are enticed to stay on the equilibrium path, without a need to characterize the payoffs for agents off the equilibrium path. This captures the key feature of small players, that is, as a price-taker, an individual small player's action does not affect the other small players' payoffs.

We closely follow PS with one important difference. The difference concerns the equilibrium concept. PS use Perfect Public Equilibrium and determine the *set* of equilibria by determining the appropriate correspondences. Because functions cannot be used, it is very difficult to analytically solve the game or to compute the equilibrium numerically.[10] Computation of the equilibrium is also an issue in closely-related models, such as Chang (1998) and Chari and Kehoe (1990). We strengthen the equilibrium concept, following Gorton, He and Huang (2014), so that we can work with functions. This may be of independent interest.

Papers closest to ours include Stein (2012) who analyzes a world of unregulated private money production, which has an externality in that too much is produced, increasing fragility because of fire sales. Stein (2012) argues that this externality can be addressed by a central bank that uses reserve requirements to implement a cap-and-trade system for the private money. In our setting the central bank uses open market operations to address financial fragility, so there is an effect on inflation as well.

Other papers consider monetary policy in the context of a shortage of safe assets due to a crisis. As in our model, Caballero and Farhi (2014), and Andolfatto and Williamson (2015) analyze environments in which cash and government bonds are not substitutes and this is the essential friction. Caballero and Farhi (2014) study the implications of a safe debt shortage following a crisis onset. One of their main points is that the shortage of safe debt constrains the effectiveness of monetary policy when the economy is up against the zero lower bound. The usual mechanisms for equilibration and policy effectiveness weaken or do not work at all, a situation they describe as a "safety trap." Andolfatto and Williams (2015) specify a general equilibrium model in which there are demands for both cash and government bonds to be used in transactions. But cash and government bonds are not substitutes. There are two goods.

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[8]See Ljungqvist and Sargent (2004), chapter 22, for a summary. An alternative, closely related, approach introduces Lagrange multipliers as co-sate variables. See Kydland and Prescott (1980) and Marcet and Marimon (2011).

[9]Atkeson (1991) showed that APS can be used when there is a publicly observed state variable.

[10] On the computation issues see Judd, Yeltekin and Conklin (2003) and Sleet and Yeltekin (2002).

One (good 1) requires cash for purchases and the other (good 2) requires some cash and some bonds. If the two constraints do not bind, households will be indifferent about producing and selling each of the two goods. “Scarce safe debt” means that the cash-and-bonds-in-advance constraint binds. In this world, a lower nominal interest rate (higher bond price) reduces consumption in the market for good 2 and increases consumption in the market for good 1. Essentially, an increase in the bond price is an increase in the relative price of consumption of good 2, from the household's point of view. But, reducing the demand for good 2 reduces aggregate output and welfare. In this context they analyze a number of monetary policies. None of these papers, however, consider pro-active monetary policies for avoiding financial crises.

Benigno and Nisticò (2016) also look at a setting where there is a shortage of safe assets. They study a model with multiple assets with different liquidity properties, modelled essentially as exogenous haircuts. Households only find out the haircut at the time they try to use the asset to purchase goods. They are interested in what happens when an exogenous “liquidity shock” worsens the quality of some of the assets for use in exchange. The shocked assets—bonds, say—suffer a fall in price and a rise in their yield because there is a shortage of safe debt now. The shortage of safe debt means that the demand for consumption goes down. If there are no nominal rigidities so the price level falls and if there are nominal rigidities, there would be a recession. How should the central bank respond? Again, an important role of the central bank is to supply safe assets.

In Section 2 we start the analysis by specifying and solving a two period model. We then assume specific functional forms and examine monetary policy. Section 3 presents and solves the infinite horizon model and also presents results with specific functional forms. The conclusion is Section 4.

2. The Two-Period Model

We begin with a two-period model in order to convey the basic setting and intuition. In the two-period model there is no commitment problem for the central bank, as will be seen. We first explain the model and then briefly discuss the assumptions.

2.1 Model Setup

There are two types of goods available each period, perishable cash goods, which are subject to a cash-in-advance constraint (CIAC), and credit goods, which are housing services, at prices p and q, respectively. There is a continuum (of measure 1) of infinitely–lived identical agents. They consume cash goods and housing services, and they also own the proceeds from production and home rental. Agents are arranged on a circle. Each agent is renting a house from the agent on the left and renting out a house he owns to an agent on the right. Each agent will purchase housing services and receive payments from the rental of housing services. Think of goods as being of different colors. Agents prefer the color of the goods produced by the agents to their left. For markets to be competitive there must be a double continuum. But, because agents are otherwise identical we will speak of a representative agent. The central bank conducts monetary policy through trading in open market operations, adjusting the quantity of cash (M) and Treasuries (TB) in the economy.

The supply of houses, H, is constant, and each period one unit of house generates one unit of housing services. Units of houses can be rented out at price, q per unit. Define Q to be the market value of one unit of a house. In the background, the agent will obtain a variable rate mortgage to buy h units of house; the agent makes rQ in mortgage payments for each unit of house he buys, with a total mortgage payment of rQh each period. In equilibrium, renting one unit of housing services at price q is equivalent to buying one unit of house with a mortgage payment rQ, that is $q = rQ$ (i.e., the house price is the annuity value of the mortgage payments).

Mortgage-backed-securities (MBS) are generated from housing sales, and in equilibrium the amount of MBS is proportional to the total amount of credit generated to finance housing sales, that is, $MBS = QH$ in equilibrium.

Each agent owns a firm which produces cash goods using real Treasuries and real MBS as inputs. (This is discussed below.) The liquidity of MBS is not as good as Treasuries, since MBS are privately-produced; with respect to liquidity services MBS are only worth δMBS, where $\delta < 1$. The value of δ can be thought of as a haircut. It is a measure of market liquidity; the higher the value of δ, the better the market liquidity of MBS (and the more that can be borrowed). We assume that the production function takes the following form, $y = f((TB + \delta MBS)/p, \varepsilon)$. We interpret $(TB + \delta MBS)/p$ as the amount of liquidity (in real terms) provided through using Treasuries and mortgage-backed securities as collateral. Therefore, we define $L \equiv (TB + \delta MBS)/p$. Implicitly, the firm is borrowing to buy capital.

We now describe how the MBS are created (via securitization) and used in four steps (we only show the net changes to the balance sheets). For clarity we omit cash balances and Treasury holdings.

Step 1: An agent/firm borrows Qh from a bank to buy h units of a house. At the same time the agent/firm receives QH from the agent buying his house. He deposits this in the bank. In equilibrium, market clearing will require that $h = H$.

Bank		**Special Purpose Vehicle**		**Agent/Firm**	
Asset	Liability	Asset	Liability	Asset	Liability
Qh	QH	0	0	QH	Qh

Step 2: The bank securitizes the loan *Qh* through a special purpose vehicle, and the agent/firm uses its deposits to buy the *MBS* issued by the special purpose vehicle.

Bank		**Special Purpose Vehicle**		**Agent/Firm**	
Asset	Liability	Asset	Liability	Asset	Liability
0	0	*Qh*	*MBS*	*MBS*	*Qh*

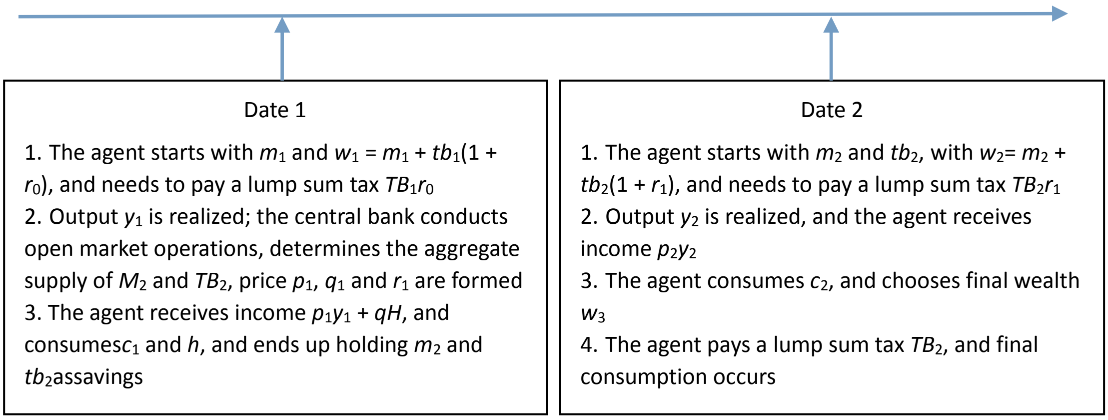
Step 3: The agent/firm uses *MBS* as collateral to borrow δMBS from the bank for production.

Bank		**Special Purpose Vehicle**		**Agent/Firm**	
Asset	Liability	Asset	Liability	Asset	Liability
δMBS	δMBS	*Qh*	*MBS*	$MBS+\delta MBS$	$Qh+\delta MBS$

Step 4: The agent/firm produces and pays off all its debt (including the mortgage payment and the principal of debt); the special purpose vehicle passes the debt payment *Qh* to the agent/firm, who is the *MBS* holder, and the *MBS* are cleared; the bank receives δMBS loan repayments and pays off its own debt.

Bank		**Special Purpose Vehicle**		**Agent/Firm**	
Asset	Liability	Asset	Liability	Asset	Liability
0	0	0	0	0	0

The time line is as follows:



Note:

(i) The representative agent has wealth w_t at the beginning of period t, and pays a lump sum tax $TB_t r_{t-1}$. Define net worth as $nw_t \equiv m_t + tb_t(1 + r_{t-1}) - TB_t r_{t-1}$.

(ii) At the aggregate level, we assume that the sum of cash and Treasuries, net of the tax payment, is a constant, and we define $NW \equiv M_1 + TB_1 = M_2 + TB_2$.

(iii) At the end of date 1, each unit of house has a resale value of $Q' = q/r_1$ Therefore, we have the value of the house at the beginning of date 1, $Q = (q + Q')/(1 + r_1) = q/r_1$. This assumption makes the model consistent with the infinite horizon model.

(iv) Open market operations occur at date 1 and affect output at date 2. The quantity of Treasuries is taxed away and is irrelevant for the final consumption, by assumption.

(v) Final consumption is the real value of final wealth minus the tax payment, normalized by the date 2 cash goods price. The composition of final wealth does not affect utility, by assumption. Specifying the final consumption in this way closes the model.

At date 1, given the cash goods price p_1, the housing services price q, and the interest rate r_1, the representative agent is choosing cash goods consumption, c_1, housing services consumption, h, a cash amount, m_2, and Treasuries, tb_2, subject to the budget constraint and the cash-in-advance constraint.

$$\begin{aligned} &\max_{c_1, h_1, m_2, tb_2} u(c_1, h) + \beta U(m_2, tb_2) - \psi(MBS/TB_2) \qquad (1) \\ &s.t. \quad p_1 c_1 + qh + m_2 + tb_2 \leq p_1 y_1 + qH + nw_1 \\ &\qquad\; p_1 c_1 \leq m_1 . \end{aligned}$$

In the above optimization problem, $U(m_2, tb_2)$ is the utility generated from date 2 consumption, as a function of the individual state variables, m_2 and tb_2.The representative agent also suffers a welfare loss due to a possible financial crisis, namely $\psi(MBS/TB)$, which—as discussed in the Introduction (and further below) —is a function of the ratio of MBS/Treasuries at date 1, and which is also a function of the aggregate state variable, M_2. We refer to the likelihood of a financial crisis as “financial fragility” henceforth. The individual agent cannot internalize the cost of financial fragility as it is a function of the aggregate state variable only, and he behaves as if the cost does not exist.

At $t = 2$, the representative agent starts with wealth $w_2 = m_2 + tb_2(1 + r_1)$, pays a lump sum tax $TB_2 r_1$, and receives income from cash goods output y_2 and housing services supplied, h. With $MBS = QH$(in equilibrium), we have $y_2 = f((TB_2 + \delta MBS)/p_1, \varepsilon)$. The agent also pays a lump sum tax TB_2 from his final wealth w_3. Given the cash goods price, p_2, the representative agent is choosing cash goods consumption, c_2, and final wealth, w_3, subject to the budget constraint and the cash-in-advance constraint.

$$
\begin{aligned}
U(m_2, tb_2) = E\Big[\max_{c_2, w_3}[v(c_2) + \alpha v(c_3)]\Big] \\
s.t. \quad & p_2 c_2 + w_3 \le p_2 y_2 + n w_2 \\
& p_2 c_2 \le m_2 \\
& c_3 = n w_3 / p_2 = \left(w_3 - TB_2\right) / p_2 \qquad (2) \\
& n w_2 = m_2 + tb_2(1 + r_1) \text{ - } TB_2 r \\
& y_2 = f((TB_2 + \delta MBS) / p_1, \varepsilon).
\end{aligned}
$$

Note that since housing consumption at date 2 plays no role (there is simply a residual value to the house, *Q'*), we have omitted it from (2).

We now turn to the central bank's objective function. Subject to a boundary condition on expected deflation (discussed below), the central bank is choosing M_2 to maximize social welfare, which is the representative agent's utility function consisting of two parts: (i) The expected utility of the representative agent from consumption; (ii) The welfare loss due to financial fragility. Unlike the representative agent, the central bank can internalize the cost of financial fragility by choosing the money supply (and hence the amount of Treasuries in the economy).

The optimization problem of the central bank is:

$$
\begin{aligned}
& Max_{M_2} \left\{u(c_1, h_1) + \beta U(M_2, TB_2) - \psi(MBS / TB_2)\right\} \\
& s.t. \quad E[p_2 / p_1] \ge \underline{\pi}.
\end{aligned} \qquad (3)
$$

The central bank's objective function is concerned with the effects of money (monetary policy), but it is also concerned about managing financial fragility (the likelihood of a financial crisis). The constraint on deflation means that the central bank does not want to have a deflation in the economy, which increases the real value of debt, and which may aggravate recessions and lead to a deflationary spiral. We discuss this constraint further below.

For a given amount of Treasuries, production of MBS constitutes a credit boom and has an externality that private agents do not take into account; it raises the likelihood of a crisis. The welfare loss, ψ, is a function of the ratio of MBS to TB_2. When relatively more mortgage-backed securities are used as collateral for production (a credit boom), the probability of financial fragility increases, and we use the reduced form function ψ to capture the expected welfare loss from the likelihood of financial fragility ($\psi'>0$ and $\psi''>0$). In equilibrium there will never be an explicit financial crisis (though the strategy space admits this), but the objective function of the central bank specifies that welfare changes are smoothly changing with the ψ function. In other words, it is painful for the central bank to have a higher likelihood of crisis, since it does not know exactly what ratio of MBS to Treasuries will result in a crisis.

The central bank's choice of the aggregate variable, M_2, affects social welfare through several channels: (i) Supply channel: M_2 affects the level of p_1 (if the cash-in-advance constraint is not binding at date 1), TB_2 and *MBS* (through *q*), and consequently affects output at date 2 (because TB_2, *MBS* and p_1 affect real liquidity); (ii) Demand channel (price effect): M_2 affects the price level p_1 (if the cash-in-advance constraint is not binding at date 1), q and p_2 (if the cash-in-advance constraint is binding at date 2), as well as the interest rate level r_1, and therefore affects the consumption and saving behavior of the agent; (iii) Demand channel (wealth effect): output is also one source of income for an agent; (iv) Externality: the ratio of MBS/Treasuries at date 1 determines the welfare loss due to its effect on financial fragility.

The representative agent, being small, does not take the possibility of financial fragility into account. Only the central bank can internalize the welfare loss due to the likelihood of financial fragility by choosing the aggregate level of the money supply. The representative agent cannot affect the aggregate money supply, and he behaves as if the welfare loss due to financial fragility does not exist, though he suffers if there is a higher likelihood of a financial crisis. In other words, the household worries when the economy becomes more fragile, because the household can observe the ratio MBS/TB. Here, "worries" reduce utility.

We will focus on symmetric equilibrium in which all agents start with the same endowment and behave the same way. In general, there can be many equilibria for a strategic game between the central bank and the continuum of small agents. We focus on the sequential equilibrium in which after observing every value of M_2, all agents will behave competitively and rationally. We define the sequential equilibrium below.

Definition 1 (Sequential Equilibrium): A sequential equilibrium satisfies the following two conditions:

(i) Given the central bank's choice of M_2, and the realization of cash goods output at date 2, $\{p_1, q, r_1, p_2\}$ and $\{c_1, h, m_2, tb_2, c_2, w_3\}$ are the competitive equilibrium of this economy, that is: Given $\{p_1, q, r_1, p_2\}$, $\{c_1, h, m_2, tb_2, c_2, w_3\}$ solves (1) and (2), and $\{p_1, q, r_1, p_2\}$ are such that markets are cleared, with $c_1 = y_1$, $h = H$, $m_2 = M_2$, $tb_2 = TB_2 = NW - M_2$, $c_2 = y_2$, and $w_3 = M_2 + TB_2$.

(ii) The central bank's choice of M_2 maximizes social welfare.

2.2 Discussion of the Model

Housing plays a central role in the model because the amount of privately-produced collateral, mortgage-backed securities, is endogenously determined. This is consistent with the importance of housing for the macroeconomy, see, e.g., Jorda, Schularick and Taylor (2014, 2015), Mian and Sufi (2014), Mian, Sufi, and Verner (2016), and Leamer (2007), and it is consistent with housing often being at the center of financial crises.

A key ingredient of the model is the demand for Treasuries and MBS. Putting Treasuries and MBS in the production function is a reduced form for the use of Treasuries and MBS as collateral in the economy. It is simpler than using a "collateral in advance" constraint.[11] Privately-produced collateral, MBS, is not as liquid as Treasuries, hence the $\delta<1$ parameter.

As discussed in the Introduction, financial crises tend to occur when there is insufficient government debt in the economy, resulting in the private sector creating a substitute, but inferior form of collateral, MBS—a credit boom. The credit boom is an externality because it creates financial fragility. Private agents cannot affect the quantities of Treasuries and MBS in the economy. This becomes the job of the central bank, as represented here by the ψ function.

Note the timing of the open market operations. Agents enter the first period with $m_1$and tb_1, but before transacting in the first period, the central bank conducts open market operations. This affects first period agent decisions via prices. Agents then enter period 2 with m_2 and tb_2. So, in the two-period model, there is no commitment problem, as there will be in the infinite horizon model. Still, in the two-period model, the central bank acts strategically, as a large player.

Note that the MBS implicitly pay interest. In the background, every agent is borrowing money to buy a house, paying a variable interest rate, while depositing all the money from selling his own house and receiving a variable interest rate via the MBS. So interest payments on the mortgages and the MBS cancel out.

We assume a constant housing supply for simplicity, but note that the price of houses can change. Also, housing services are proportional to the housing stock (with proportionality one) for simplicity. Note that in order to pay off his mortgage, an agent sells the house. But, the new housing price may be such that he defaults on his mortgage. For simplicity this is costless.

2.3. Equilibrium Characterization

2.3.1 Individual Agent Optimization in a Competitive Equilibrium

To simplify the analysis below (for the two period model), we assume there is no uncertainty in production, that is, $y_2 = f((TB_2 + \delta MBS)/p_1)$,and we also assume:

$$u(c_1, h_1) = \ln c_1 + \ln h_1$$
$$v(c_2) = \ln c_2, v(c_3) = \ln c_3.$$

[11]There is an older literature on money in the production function. The issue first arose in monetary growth models, e.g., Levhari and Patinkin (1968) and the debate evolved from that point. Examples include Friedman (1969), Fischer (1974), and Saving (1972). Examples of the empirical literature include Sinai and Stokes (1972) and You (1981). Nguyen (1986) reviews a lot of the literature. Benchimol (2010) is a recent example.

We will start by solving the model backwards starting from t =2. In the second period there are no central bank actions.

Proposition 1 (Equilibrium Prices): In a sequential equilibrium, at date 1, if the central bank chooses M_2 such that $M_2<M_1\beta$, then the cash-in-advance constraint is *not binding* for the representative agent, and we have:

$$p_1 = \frac{M_2}{\beta y_1}, q = \frac{M_2}{\beta H}, 1 + r_1 = \frac{1}{\alpha}.$$

If the central bank chooses M_2 such that $M_2>M_1\beta$, then the cash-in-advance constraint *is binding* for the representative agent, and we have:

$$p_1 = \frac{M_1}{y_1}, q = \frac{M_2}{\beta H}, 1 + r_1 = \frac{1}{a}.$$

At date 2 the cash-in-advance constraint (CIAC) is always binding for the representative agent, and we have:

$$p_2 = M_2 / y_2.$$

Proof: See Appendix.■

We solve the equilibrium prices using backward induction. Notice that, the money supply chosen by the central bank, M_2, at date 1, is the key for equilibrium prices, i.e., the central bank is NOT a price taker.

In the proof of Proposition 1, one of the key steps was to characterize $U(m_2, tb_2)$ defined in (2) and use the derivatives with respect to m_2 and tb_2 to calculate the marginal value of money and the marginal value of Treasuries for the representative agent. $U(m_2, tb_2)$ should be interpreted as the expected utility of an agent with m_2 and tb_2 while everyone else is holding M_2 and TB_2, and this is the key feature of a small agent. Notice that, $U_m(m_2 = M_2, tb_2 = TB_2)$ and $U_{tb}(m_2 = M_2, tb_2 = TB_2)$ are in general not the same as $U_M(m_2 = M_2, tb_2 = TB_2)$ and $U_{TB}(m_2 = M_2, tb_2 = TB_2)$. Intuitively, an individual agent is a price-taker, and he can only change his own choices of m_2 and tb_2 without affecting the aggregate level of M_2 and TB_2, while the central bank can change the aggregate economic variables. In the infinite horizon model, in general, the functional form $U(m_2, tb_2)$ usually cannot be derived analytically, however, we can show that the value of its derivatives with respect to m_2 and tb_2 at $m_2 = M_2$ and $tb_2 = TB_2$ are sufficient to solve for the symmetric equilibrium in which all agents hold the same amounts of m_2 and tb_2. So, instead of knowing the functional form of $U(m_2, tb_2)$, we only need to know two values, which reduces the complexity of the problem substantially as will be seen.

2.3.2 Optimal Monetary Policy

The central bank trades through open market operations to maximize social welfare defined in (3):

$$\begin{aligned} V &= u(y_1, H) + \beta U(M_2, TB_2) - \psi(MBS / TB_2) \\ &= \ln(y_1) + \ln(H) + \beta(1+\alpha)\ln(f(L)) - \psi(MBS / TB_2), \end{aligned}$$

subject to the deflation bound constraint with $MBS = QH = qH / r_1 = \alpha M_2 / \beta(1-\alpha)$ and $L = \left(TB_2 + \delta MBS\right) / p_1$.

In the above optimization problem, the money supply affects social welfare through the real value of collateral and the cost of financial fragility. The decision problem of the central bank depends on whether the cash-in-advance constraint is binding or not at date 1. From Proposition 1, we know that, when the central bank chooses a low money supply, M_2, the value of money is high at date 2 (because the price level is low), agents are hoarding cash and the price level at date 1 is low, and the cash-in-advance constraint is not binding. In this case we will say that the economy is in recession (the CIAC is not binding). When the central bank chooses a high money supply, M_2, the value of money is low at date 2, agents spend all their cash on hand and the price level is high at date 1, and the cash-in-advance constraint is binding. In this case we say that the economy is booming (the CIAC is binding).

Assuming the production function has the simple linear form with $f(L) = AL$, we have the following:

Lemma 1 (Money Supply and Output): With $\delta\alpha/(1-\alpha)>\beta$, the amounts of real collateral and output at date 2 are decreasing with M_2 during recession (CIAC not binding) and increasing with M_2 when the economy is booming (CIAC binding).

Proof: See Appendix.■

The above lemma shows the key link between the money supply and output; it works through the real value of collateral. And, this depends on whether the cash-in-advance constraint is binding or not. Under the parametric restriction in the proposition, our use of the terms recession and boom is consistent with output movements.

From Proposition 1, we can see that inflation between date 1 and date 2, π, is increasing with the money supply chosen by the central bank, M_2. For future use, we define M_π as the money supply such that $\pi(M_\pi) \equiv \underline{\pi}$, and we have:

$$M_\pi \equiv \frac{A\underline{\pi}\beta NW}{\beta - A\underline{\pi}(\delta\alpha / (1-\alpha) - \beta)}.$$

Notice that, both the inflation rate π and M_π are independent of date 1 output, y_1, and the initial money supply, M_1 (this is driven by the model assumptions). Recall that $NW=TB_2+M_2$ is constant.

The welfare loss from financial fragility, ψ, is increasing with the ratio of MBS to Treasuries, and, suppose it takes the following form for tractability, with $\gamma > 0$ being some constant:

$$\psi\left(\frac{\alpha M_2}{\beta(1-\alpha)(NW-M_2)}\right)=\gamma\left[\ln NW-\ln(NW-M_2)\right].$$

Recall that $TB_2=NW-M_2$. Let:

$$M_\psi \equiv \frac{\beta(1+\alpha)(\delta\alpha/(1-\alpha)-\beta)-\gamma\beta}{\beta(1+\alpha)(\delta\alpha/(1-\alpha)-\beta)+\gamma(\delta\alpha/(1-\alpha)-\beta)}NW.$$

Assume $M_\psi > M_\pi$. We can show that M_ψ is the optimal money supply that maximizes the social welfare in (3) if the cash-in-advance constraint is binding at date 1. Notice that M_ψ is also a constant independent of date 1 output, y_1, and initial money supply, M_1. It is easy to see that M_ψ decreases as the risk of financial fragility, as measured by γ, rises. Intuitively, if an expansionary monetary policy (M_ψ) is optimal, with a higher marginal cost of financial fragility, the central bank wants to lower the money supply (sell Treasuries) and this increases the real value of collateral and increases the marginal utility from cash-goods consumption.

With the results in Proposition1 and Lemmas 1, we can fully characterize the optimal money supply, which is stated in the following Proposition.

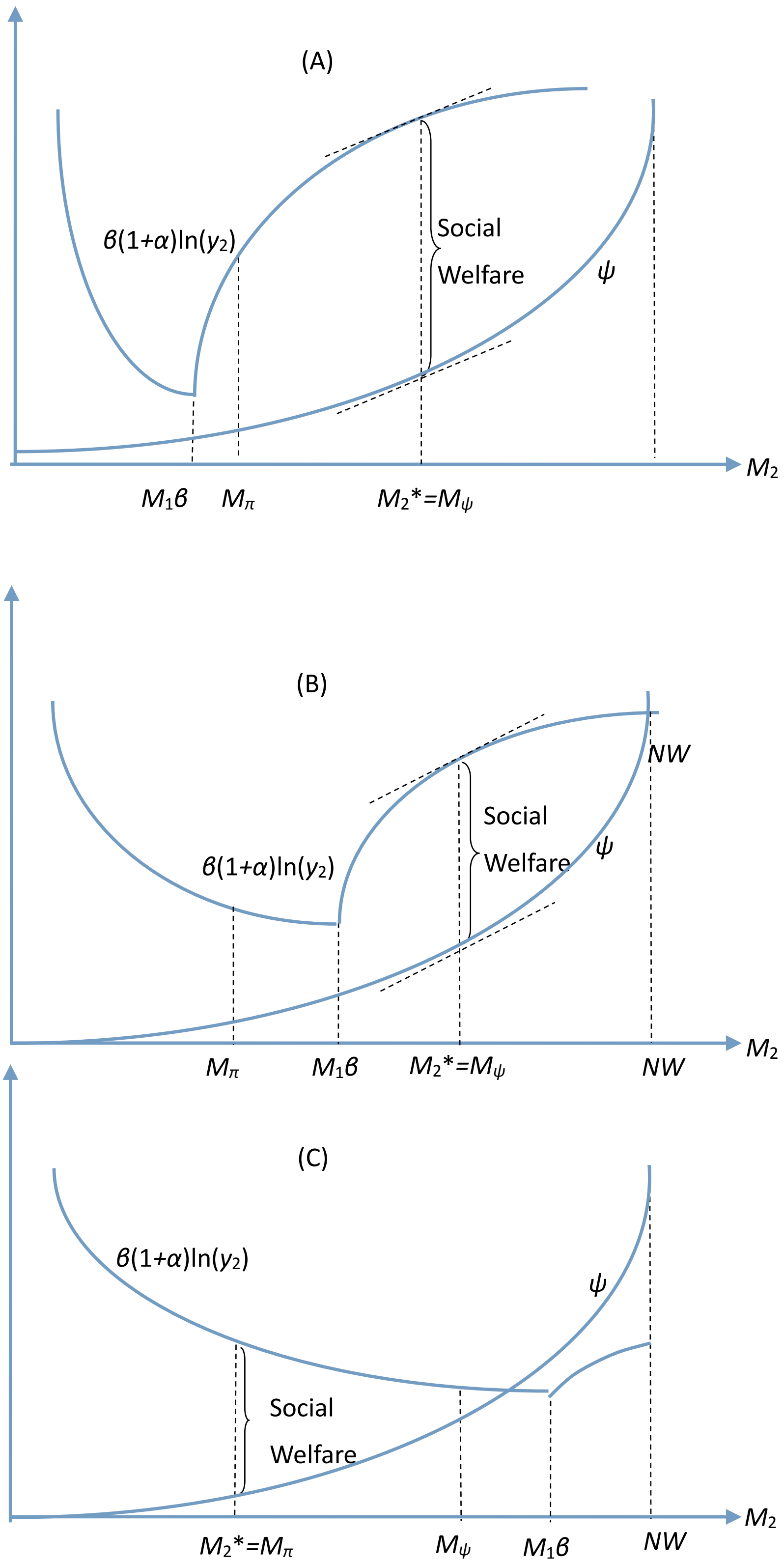
Proposition 2 (Optimal Monetary Policy): There exists a cutoff value of the initial money supply $M_1^*\in(M_\pi/\beta,\ M_\psi/\beta)$, above which the economy is in recession, and a contractionary monetary policy (M_π) is optimal, and below which the economy is booming, and an expansionary monetary policy (M_ψ) is optimal.

Proof: See Appendix. ■

When the economy is in recession, output is decreasing with the money supply, and the welfare loss from financial fragility is increasing with the money supply. Therefore, the optimal money supply is at the lower bound. When the economy is booming, output is increasing with the money supply, but the welfare loss from financial fragility is also increasing with the money supply. Therefore, the optimal money supply balances the gain from the output increase with the welfare loss due to financial fragility.

The above proposition tells us that when the initial money supply is high, it is more likely that the economy is going into a recession with deflation and low welfare loss due to financial fragility. This boom-bust pattern is the equilibrium outcome under the optimal monetary policy.

We plot the three cases of the optimal monetary policy in the figures below. In the figures the x-axis is M_2 and the y-axis is social welfare. Figure (A) is for the case where $M_\psi > M_\pi > M_1\beta$; Figure (B) is for the case $M_\psi > M_1\beta > M_\pi$; and Figure (C) is for the case $M_1\beta > M_\psi > M_\pi$. In the figures the upper curve is the utility from output, and the bottom curve is the disutility from financial fragility. The dashed lines are the marginal utilities.



From case (C), we can see that imposing a lower bound on inflation/deflation prevents the corner solution (M_2=0), and this is the only reason for the constraint.

2.4 Summary

The two-period model shows optimal monetary policy in an economy where collateral is central and the central bank seeks to mitigate the likelihood of a financial crisis. Monetary policy works through the real value of collateral. The two-period model, however, is simplified because the cash-in-advance constraint is always binding in the second period and there is no commitment issue for the central bank. In the second period there is no further action by the central bank and there are no expectations about the future path of monetary policy.

3. Infinite Horizon Model

We now turn to the infinite horizon model, which admits dynamics. In this setting, there is a commitment problem; agents do not know the future path of monetary policy. Also, the cash-advance-constraint in the future may or may not be binding.

3.1. Model Set-Up

The model is a straightforward extension of the two-period model to an infinite horizon problem.

The time line for period *t* is as follows:

1. At the beginning of period *t* the agent starts with m_t and $tb_t(1+r_{t-1})$, but needs to pay a lump sum tax of $TB_t r_{t-1}$;
2. Output y_t is realized, and then the central bank conducts open market operations, which determines the aggregate supplies of M_{t+1} and TB_{t+1}; prices p_t, q_t and r_t are formed;
3. The agent receives income $p_t y_t + q_t H$ and chooses the consumption of cash goods, c_t, and housing services, h_t, subject to the budget constraint and the cash-in-advance constraint, generating utility $u(c_t, h_t)$, while holding m_{t+1} and tb_{t+1} as savings.

Remarks: As before, the lump sum tax is used for interest payments on outstanding Treasuries. We assume the central bank/government will tax the exact amount to cover the interest payments. After paying the lump sum tax, the representative agent carries a net wealth of $nw_t \equiv m_t + tb_t(1+r_{t-1}) - TB_t r_{t-1}$ into the beginning of period *t*, and he also receives income $p_t y_t + q_t H$. On-the-equilibrium path $nw_t = M_t + TB_t = NW$, which is a constant as we assume the only monetary policy allowed in the model is the open market operations, which involve a one-for-one exchange of cash and Treasuries.

The representative agent's lifetime utility from consumption net of the welfare loss from financial fragility can be written as $E\left[\sum_{t=1}^{\infty}\beta^{t}\left(u(c_t,h_t)-\psi(MBS_t/TB_{t+1})\right)\right]$, where $u(.)$ is the utility function and $\psi(.)$ is a convex function increasing in MBS_t/TB_{t+1}, and $\beta\in(0, 1)$ is the discount factor. We assume that $u(.)$ is increasing and concave in both c and h, that is, $u_c > 0$, $u_{cc} \leq 0$, $u_h > 0$, and $u_{hh} \leq 0$. The term $\psi(MBS_t/TB_{t+1})$ affects the welfare of each agent in equilibrium, but it does not affect their optimization problems as it is a function of aggregate variables only. The central bank is maximizing social welfare, which is the representative agent's lifetime utility from consumption net of the cost of financial fragility.

In period t, after output y_t is realized, the central bank chooses the amount of Treasuries to trade, ΔTB_{t+1}, which leads to $TB_{t+1} = TB_t + \Delta TB_{t+1}$ and $M_{t+1} = M_t - \Delta TB_{t+1}$, and the representative agent chooses consumption of cash goods, c_t, consumption of housing services, h_t, cash holdings m_{t+1}, and Treasury bill holdings, tb_{t+1}, as functions of the prices in the economy, p_t, q_t and r_t. The equilibrium prices p_t, q_t and r_t are such that all markets clear, that is, $c_t = y_t$, $h_t = H$, $m_{t+1} = M_{t+1}$, and $tb_{t+1} = TB_{t+1}$.

Similar to the two-period case, we will study sequential equilibria with public strategies in which the strategies of both the central bank and the representative agent depend only on public information. This type of equilibrium is called Perfect Public Equilibrium (PPE) (see Fudenberg, Levine, and Maskin (1994)). The public history at time t is denoted as η^t:

$$\eta^t = \begin{cases} \{M_1, TB_1, y_1, r_0\} & for\ t=1 \\ \{M_1, TB_1, y_1, r_0\} \cup \{M_{\tau+1}, TB_{\tau+1}, y_{\tau+1}, p_\tau, q_\tau, r_\tau\}_{\tau=1}^{t-1} & for\ t>1. \end{cases}$$

The representative agent's strategy can be written as:

$$\sigma_a = \{c_t(\eta^t, p_t, q_t, r_t), h_t(\eta^t, p_t, q_t, r_t), m_t(\eta^t, p_t, q_t, r_t), tb_t(\eta^t, p_t, q_t, r_t)\}_{t=1}^{\infty}.$$

The central bank's strategy can be written as:

$$\sigma_b = \{TB_t(\eta^t)\}_{t=1}^{\infty}.$$

We can see that the only state variable of the economy is the money supply in the economy, and we denote the economy that started with M_1 as $\Phi(M_1)$. A strategy profile for the economy $\Phi(M_1)$ is denoted as $\sigma = (\sigma_a, \sigma_b)$. We will use the Strong Markov Perfect Public Equilibrium concept to define our equilibrium, as in Gorton, He and Huang (2014), which is defined below. We will first construct an auxiliary competitive equilibrium by assuming that the central bank adopts an exogenous strategy σ_b. This auxiliary equilibrium will be useful subsequently because along the equilibrium path private agents behave as if the central bank has an exogenous strategy. So, subsequently determined equilibria must be in the set of equilibria for this auxiliary problem.

3.2. An Auxiliary Competitive Equilibrium

Given the central bank's exogenous strategy σ_b, the representative agent is solving the following optimization problem:

$$\begin{aligned} &\max E[\textstyle\sum_{t=1}^{\infty}\beta^t\left(u(c_t,h_t)-\psi(MBS_t/TB_{t+1})\right)] \\ &s.t. \quad \begin{aligned} &p_t c_t + q_t h_t + m_{t+1} + tb_{t+1} \le p_t y_t + q_t H + w_t - TB_t r_{t-1} \\ &p_t c_t \le m_t \end{aligned} \end{aligned} \tag{4}$$

Given the realization of cash goods output, the competitive equilibrium of this economy (conditional on the central bank's strategy), denoted as $\Phi(M_1|\sigma_b)$, is a sequence of cash goods prices, housing services prices and interest rates, $\{p_t,q_t,r_t\}_{t=1}^{\infty}$ and a sequence of cash goods consumption, housing consumption, cash holding and Treasuries holding $\{c_t,h_t,m_{t+1},tb_{t+1}\}_{t=1}^{\infty}$, such that:

1. Given $\{p_t,q_t,r_t\}_{t=1}^{\infty}$, $\{c_t,h_t,m_{t+1},tb_{t+1}\}_{t=1}^{\infty}$ maximizes $E\left[\sum_{t=1}^{\infty}\beta^t\left(u(c_t,h_t)-\psi(MBS_t/TB_{t+1})\right)\right]$ subject to the budget constraints and cash-in-advance constraints.
2. Markets are cleared, that is, $c_t = y_t$, $h_t = H$, $m_{t+1} = M_{t+1}$, and $tb_{t+1} = TB_{t+1}$, for any t.

In equilibrium, the representative agent consumes today and saves in the form of cash and Treasuries. To construct the auxiliary equilibrium, we need to define the marginal value of additional cash holding, X, and the marginal value of additional Treasuries holding, Z. Under the usual regularity conditions, the budget constraint is binding, but not necessarily the cash-in-advance constraint. We will define X and Z separately in the case when the cash-in-advance constraint is binding and in the case when it is not binding.

Lemma 2 (Marginal Values of Cash and Treasuries): When cash-in-advance constraint is binding (B), the marginal value of cash holding and Treasury holdings at time t+1 are:

$$\begin{aligned} X_{t+1}^B &= u_c(y_{t+1},H)/p_{t+1} \\ Z_{t+1}^B &= u_h(y_{t+1},H)(1+r_t)/q_{t+1}, \end{aligned}$$

When cash-in-advance constraint is binding (NB), the marginal value of cash holding and Treasury holdings at time t+1 are:

$$X_{t+1}^{NB} = u_c(y_{t+1}, H) / p_{t+1}$$
$$Z_{t+1}^{NB} = u_c(y_{t+1}, H)(1 + r_t) / p_{t+1}$$
$$= u_h(y_{t+1}, H)(1 + r_t) / q_{t+1}.$$

Proof: See Appendix. ■

Combining the two cases in Lemma 2, we have:

$$X_{t+1} = E_B[X_{t+1}^B]\Pr_{t+1}(B) + E_{NB}[X_{t+1}^{NB}]\Pr_{t+1}(NB)$$
$$Z_{t+1} = E_B[Z_{t+1}^B]\Pr_{t+1}(B) + E_{NB}[Z_{t+1}^{NB}]\Pr_{t+1}(NB)$$

The variables X_{t+1} and Z_{t+1} represent the representative agent's marginal expected lifetime utility for an additional amount of cash and Treasuries, respectively, that he holds at the beginning of period t+1. These two variables summarize all the information the representative agent needs to make the consumption and saving decision at time t. In our two-period model, X_{t+1} and Z_{t+1} correspond to the derivatives of $U(m_2, tb_2)$ with respect to m_2 and tb_2, respectively, at $m_2 = M_2$ and $tb_2 = TB_2$, which is the only information we need from date 2 when we characterize the date 1 problem for our two-period model in Section 2.2.2 (see the Lagrange conditions).

Suppose we have solved the auxiliary competitive equilibrium for the economy $\Phi(M_1|\sigma_b)$. We can then calculate the values of X_{t+1} and Z_{t+1} for every period. Let us construct a one-period economy where the representative agent owns initial cash holdings and Treasuries of m_t and tb_t, and X_{t+1} and Z_{t+1} are taken as exogenously given. Since the central bank's trading strategy is now exogenously given, the representative agent in this one-period economy chooses cash goods consumption, c_t, housing services consumption, h_t, cash holdings, m_{t+1}, and Treasuries, tb_{t+1}, with an augmented utility function over consumption in cash goods and housing and end-of-period cash holding and Treasuries holding, subject to the budget constraint and cash-in-advance constraint:

$$\max\left[u(c_t, h_t) + \beta X_{t+1} m_{t+1} + \beta Z_{t+1} tb_{t+1}\right]$$
$$s.t. \quad \begin{array}{l} p_t c_t + q_t h_t + m_{t+1} + tb_{t+1} \le p_t y_t + q_t H + w_t - TB_t r_{t-1} \\ p_t c_t \le m_t. \end{array} \quad (5)$$

The competitive equilibrium of this static one-period economy consists of prices $\{p_t, q_t, r_t\}$ and consumptions plus savings $\{c_t, h_t, m_t, tb_t\}$ such that the following two conditions are satisfied:

1. Given $\{p_t, q_t, r_t\}$, $\{c_t, h_t, m_{t+1}, tb_{t+1}\}$ maximizes $u(c_t, h_t) + \beta X_{t+1} m_{t+1} + \beta Z_{t+1} tb_{t+1}$;

2. $\{p_t, q_t, r_t\}$ are such that $c_t = y_t$, $h_t = H$, $m_{t+1} = M_{t+1}$, and $tb_{t+1} = TB_{t+1}$.

Let CE(m_t, tb_t, X_{t+1}, Z_{t+1}) denote the set of competitive equilibrium allocations (c_t, h_t, m_{t+1}, tb_{t+1}) of this static one-period economy. We can show that the auxiliary competitive equilibrium of this economy is equivalent to a corresponding one-period economy constructed below with a transversality condition. As shown in Phelan and Stacchetti (2001), the transversality condition holds if we impose boundary conditions on the representative agent's marginal utility, for which we assume that, the money supply is bounded from below away from zero, that is $0 < \underline{M} < M < NW$, output is bounded from above and below away from zero, that is $0 < \underline{y} < y < \bar{y} < \infty$, and $u(c,h)$ is concave with $0 < \underline{u}_c < u_c(c,h) < \bar{u}_c < \infty$ and $0 < \underline{u}_h < u_h(c,h) < \bar{u}_h < \infty$.

With the above equivalence result, we can represent the infinite-horizon dynamic problem in recursive form, as shown below.

3.3. Strong Markov Perfect Public Equilibria

We now proceed to characterize the dynamic equilibrium with the central bank's decisions endogenized. For a given strategy profile $\sigma = (\sigma_a, \sigma_b)$, the representative agent has the following expected lifetime utility:

$$V[\sigma] = E\left[\sum_{t=1}^{\infty} \beta^t \left(u(c_t, h_t) - \psi(MBS_t / TB_{t+1})\right)\right].$$

Definition 2 (Perfect Public Equilibrium) A strategy profile $\sigma = (\sigma_a, \sigma_b)$ is a Perfect Public Equilibrium (PPE) for the economy $\Phi(M_1)$ if for any $\tau \geq 1$, and history η^τ, the following conditions are satisfied:

1. Given the representative agent's strategy σ_a, the central bank has no incentive to deviate, that is, $V[(\sigma_{a\tau}, \sigma_{b\tau})] > V[(\sigma_{a\tau}, \sigma'_{b\tau})]$ for any $\sigma'_{b\tau} \neq \sigma_{b\tau}$, where $(\sigma_{a\tau}, \sigma_{b\tau})$ is the truncated equilibrium strategy profile $\sigma = (\sigma_a, \sigma_b)$ starting from $\tau \geq 1$;

2. $\{c_t, h_t, m_{t+1}, tb_{t+1}\}_{t=\tau}^{\infty}$ resulting from the representative agent's strategy $\sigma_{a\tau}$ is an auxiliary competitive equilibrium outcome of the economy $\Phi(M_\tau | \sigma_{b\tau})$.

The definition of PPE imposes two conditions. The first condition requires sequential optimality, that is, the central bank's continuation strategies must be best responses to the representative continuation strategies after any history η^τ. The second condition states the optimality of the representative agent's strategy in an auxiliary competitive equilibrium we analyzed in Section 3.1.

From the definition of PPE, we know that the continuation payoffs of a PPE after any history have to correspond to PPE profiles, so the lifetime expected payoffs can be factored into current payoffs and continuation PPE payoffs. As in Phelan and Stacchetti (2001), the recursive formalization involves not only the payoffs to the central bank and the representative agent, but also the marginal values of cash and Treasuries for the representative agent, which are the key features for the auxiliary competitive equilibrium. For any strategy profile $\sigma = (\sigma_a, \sigma_b)$, we define the marginal value of cash and Treasuries at the beginning of the game as

$$X[\sigma] = E[u_c(c_1, h_1) / p_1]$$
$$Z[\sigma] = E[u_h(c_1, h_1)(1 + r_0) / q_1].$$

Following Phelan and Stacchetti (2001), we can show the recursive factorization of the defined PPE in terms of $V[\sigma]$, $X[\sigma]$ and $Z[\sigma]$, which can be replaced with simplified state-dependent value correspondences due to the existence of multiple PPEs. In our model, the state variable is the distribution of money holdings across agents, however, when all dispersed agents hold the same amount of money, the state variable degenerates to a single variable—the aggregate money supply in the economy, M. We define

$$(V(M), X(M), Z(M)) = \{(V[\sigma], X[\sigma], Z[\sigma]) \mid \sigma \text{ is a PPE for the economy } \Phi(M)\}.$$

The recursive formalization of the PPE only delivers value correspondences that depend on M, but the strategies of the central bank and the representative agent still depend on the history. For tractability, we restrict attention to strategies where the central bank's and the representative agent's strategies only depend on the state variable, M. These strategies are known as Markovian strategies. A Markov Perfect Public Equilibrium (MPPE) is a Perfect Public Equilibrium in which the central bank and the representative agent play time-invariant Markovian strategies. As in Gorton, He and Huang (2014), we impose a further restriction on the off-equilibrium-path strategies, and require off-equilibrium strategies to be the same as on-equilibrium strategies when the state variables are the same. Markovian strategies satisfying this consistency conditions are named as Strong Markov Perfect Public Equilibrium (SMPPE), which is formally defined below.

Definition 3 (Strong Markov Perfect Public Equilibrium) A Strong Markov Perfect Public Equilibrium (SMPPE) is a Markov Perfect Public Equilibrium (MPPE) that yields the same MPPE in every truncated continuation game regardless of on- or off-the-equilibrium path beliefs.

By imposing this restriction on off-equilibrium threats, we can study functions instead of correspondences. We can write an SMPPE as a set of functions, $\{V(M, y), X(M, y), Z(M, y), M'(M, y), p(M, M', y), q(M, M', y), r(M, M', y), c(M, M', y), h(M, M', y), m'(M, M, y), tb'(M, M', y)\}$ that are derived from solving the optimization problems of the representative agent and the central bank.

$$
\begin{aligned}
&\max_{c,h,m',tb'} \left\{ u(c,h) - \psi(MBS/TB') + \beta E_{y'}[X(M',y')]m' + \beta E_{y'}[Z(M',y')]tb' \right\} \\
&\qquad pc + qh + m' + tb' \le py + qH + nw \\
&\qquad pc \le m \\
&s.t. \quad nw = m + tb(1+r_0) - TBr_0 \\
&\qquad y' = f\left(\frac{TB' + \delta MBS}{p}, \varepsilon' \right),
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
&\max_{M'} \left\{ u(y,H) - \psi(MBS/TB') + \beta E_{y'}\left[V(M',y') \right] \right\} \\
&s.t. \quad y' = f\left(\frac{TB' + \delta MBS}{p}, \varepsilon' \right),
\end{aligned} \tag{7}
$$

where *nw* is equal to the initial wealth, *w*, net of the lump sum tax payment for the representative agent, *MBS* is equal to *QH* with $Q = q/r$, and r_0 is the interest rate from last period

Remarks: here we use {*V*(*M*, *y*), *X*(*M*, *y*), *Z*(*M*, *y*)}, values after *y* is realized, instead of the expected values, {*V*(*M*), *X*(*M*), *Z*(*M*)}. It is more convenient to state the results in terms of the realized values.

Substituting in the market clearing conditions, we have the following conditions that need to be satisfied:

1. *V*(*M*, *y*) is the value function of the central bank:

$$
\begin{aligned}
&V(M,y) = u(y,H) - \psi(MBS/TB') + \beta E_{y'}\left[V(M',y') \right] \\
&s.t. \quad y' = f\left(\frac{TB' + \delta MBS}{p}, \varepsilon' \right).
\end{aligned} \tag{C1}
$$

2. *X*(*M*, *y*) is the marginal value of cash (*m*) for the representative agent:

$$
X(M,y) = \begin{cases} u_c(y,H)/p & \text{if} \quad u_c(y,H)/p > u_h(y,H)/q \\ u_c(y,H)c_w^{NB} + u_h(y,H)h_w^{NB} & \text{if} \quad py < M. \end{cases} \tag{C2}
$$

3. *Z*(*M*, *y*) is the marginal value of Treasuries (*tb*) for the representative agent:

$$Z(M,y)=\begin{cases} u_h(y,H)(1+r_0)/q & \text{if} \quad u_c(y,H)/p>u_h(y,H)/q \\ u_c(y,H)(1+r_0)/p=u_h(y,H)(1+r_0)/q & \text{if} \quad py<M. \end{cases} \tag{C3}$$

4. Optimal cash holding, *m'*, for the representative agent:

$$\begin{aligned} & u_h(y,H)/q=\beta E_{y'}[X(M',y')] \\ & s.t. \qquad y'=f\left(\frac{TB'+\delta MBS}{p},\varepsilon'\right). \end{aligned} \tag{C4}$$

5. Optimal Treasuries holding, *tb'*, for the representative agent:

$$\begin{aligned} & u_h(y,H)/q=\beta E_{y'}[Z(M',y')] \\ & s.t. \qquad y'=f\left(\frac{TB'+\delta MBS}{p},\varepsilon'\right). \end{aligned} \tag{C5}$$

6. Optimal consumption of cash goods:

$$p=\begin{cases} M/y=(NW-TB)/y & \text{if} \quad u_c(y,H)/p>u_h(y,H)/q \\ qu_c(y,H)/u_h(y,H) & \text{if} \quad py<M, \end{cases} \tag{C6}$$

where *NW* is a constant that is equal to the sum of aggregate cash, *M*, and aggregate Treasuries, *TB*.

7. Optimal open market operations by the central bank:

$$\begin{aligned} & M'(M,y)=\text{argmax}_{M'}\left\{u(y,H)-\psi(MBS/TB')+\beta E_{y'}\left[V(M',y')\right]\right\} \\ & s.t. \qquad y'=f\left(\frac{TB'+\delta MBS}{p},\varepsilon'\right). \end{aligned} \tag{C7}$$

Remarks:

(i) With the binding budget constraint, we only have three first order conditions (C4-C6), and the optimal housing consumption is implied from the binding budget constraint. If the cash-in-advance constraint is also binding, then the first order condition in (C6) reduces to the binding cash-in-advance constraint.

(ii) The continuation values (or marginal values), *V*(*M'*, *y'*), *X*(*M'*, *y'*) and *Z*(*M'*, *y'*), in the above conditions reflect the consistency of the continuation game no matter whether it is on- or of-the-equilibrium path, and this is the key feature of SMPPE.

Proposition 3 (Existence of SMPPE) If $f(L,\varepsilon)$ is continuous and $u(c,h)$ is continuous and have continuous differentials, then there exists a Strong Markov Perfect Public Equilibrium.

Proof: See Appendix. ■

3.4. An Example with Policy Analysis

We assume $u(c,h) = \ln(c) + \ln(h)$, and the production, $y' = f(L,\varepsilon)$, has common support $Y=[\underline{y},\bar{y}]$.

We also assume that the cost of financial fragility $\psi(\rho)$ satisfies $\psi'(\rho) > 0$ ($\rho \equiv MBS/TB$) and the Inada conditions, that is, $\lim_{\rho\to 0}\psi'(\rho) = 0$ and $\lim_{\rho\to\infty}\psi'(\rho) = \infty$.

Next, we are going to characterize some features of an SMPPE.

Proposition 4 (The Taylor Rule with Housing Rental Prices): Let g_q be the expected appreciation in home rental prices. The interest rate is approximately equal to the expected home rental price appreciation rate plus one minus the time discounting rate i.e., $r \approx g_q + (1-\beta)$.

Proof: See Appendix.■

ThisTaylor rule is derived from (C5) above with the assumed functional form, i.e., the FOC for optimal Treasury holdings. To get an additional unit of Treasury bonds today, the agent needs to give up some housing services (credit goods) consumption today, while his gain is the principal plus interest rate tomorrow, which can be transformed into housing services consumption tomorrow. So, the trade-off is between waiting to consume housing services tomorrow and paying the house (rental) price appreciation versus the interest received. Note that this does not apply to cash goods consumption, which is constrained by the amount of cash.

In our cash-in-advance economy, housing, i.e., the credit good, expenditure is closely linked to the money supply chosen by the central bank while the cash good spending is sometimes disconnected. The price level of the cash good is constrained by the initial money supply, and is linked to the money supply chosen by the central bank as well as the housing rental price, but only during a recession (i.e., when the CIAC is not binding). When the economy is booming (the CIAC is binding), the money supply chosen by the central bank does not have an impact on the price level today while it does affect the interest rate and the housing rental price. The housing price is highly correlated with the money supply, and our theory provides an explanation for that and suggests that the optimal monetary policy should pay close attention to asset markets, in particular, the housing market.

Proposition 5 (Optimal Expansionary Policy): In any period, there exists *some output level*with non-zero measure, whichafter being observed by the central bank,causes the central bankto optimally choose a money supply such that the economy booms (i.e., CIAC is binding) regardless of the initial money supply.

Proof: See Appendix. ■

The above proposition says that, regardless of the initial money supply, the central bank will optimally conduct expansionary monetary policy, creating a boom, but only for certain output levels. Intuitively, ex-ante money and Treasuries have the same expected value for the representative agent in equilibrium (otherwise only one of them will be held by the agent in equilibrium). However, we know that if the economy is in recession (CIAC is not binding), Treasuries are more valuable as they pay interest, and on the other hand, if the economy is booming (CIAC is binding), cash is more valuable as there is a shortage of cash. If the economy is almost surely in recession, Treasuries will always be more valuable unless the interest rate is zero, but zero interest rate cannot be optimal as there will be a very large amount of MBS in the economy and cost of financial fragility is prohibitively high.

However, when the initial money supply is very high, the chance of getting into recession becomes substantial, as we describe in the following proposition.

Proposition 6 (Boom-Bust): Assume that output is smoothly distributed for any amount of collateral, and in particular, we assume $\underline{y}E[1/y'] \leq \underline{\pi}$ (i.e., the variation in output is large enough). When the initial money supply is high enough, then there exists some low output level with non-zero measure, which after being observed by the central bank, causes the central bank to optimally choose a money supply that triggers a recession (i.e., CIAC is not binding).

Proof: See Appendix. ■

Intuitively, with a large initial money supply, M, when the output is low, the central bank has to choose a relatively low level of money supply M' to drive down the price this period with CIAC not binding. Otherwise, the price level this period would be too high, and the central bank has to choose a very high M' to keep the economy away from deflation, but this cannot be optimal as the risk of financial fragility is too high. Proposition 6 is similar to Stein (2014): " . . . Monetary policy should be less accommodative—by which I mean that it should be willing to tolerate a larger forecast shortfall of the path of the unemployment rate from its full-employment level . . . " (p. 2).

From Proposition 6, we can see that imposing a lower bound on inflation/deflation is a key assumption for boom-bust pattern. Intuitively, when the output is very low this period causing the price level to surge up, the central bank has to choose an expansionary monetary policy to avoid deflation, but this will increase the probability of financial crisis. Therefore, a recession will be triggered when an expansionary monetary policy cannot be sustained when the initial money supply is already very high (and there is a negative shock on output).

3.5. Discussion of the Value of Central Bank Intervention

While doing nothing (allowing a crisis to occur) is always in its strategy space, central bank intervention can improve social welfare, which is measured by the representative agent's lifetime utility from consumption net of the cost of financial fragility. In the two-period model, the optimal money supply M_2 is either M_π or M_ψ, which are independent of output given the assumptions in Section 2. But, as long as M_1 is not at either M_π or M_ψ then there is an M_2 which can improve welfare, which is the utility from consumption net of the cost of financial fragility.

For the infinite horizon case, the channels through which central bank intervention improves social welfare are more complicated. Intuitively, a low output this period would drive up the price level this period (if the CIAC is binding) and drive down the real value of collateral and next period's output. The central bank can conduct an expansionary monetary policy to increase the nominal amount of collateral, keeping the economy booming. However, if the initial money supply is already very high, an expansionary monetary policy after a low output realization might not be optimal as the cost of financial fragility is too high, and the optimal monetary policy could be contractionary, which drives down the price level (if the CIAC is not binding) and the economy goes into recession. In a recession, the nominal amount of the collateral goes down, but the real value of the collateral does not go down as much because the price level also goes down.

Similarly, a high output realization this period would drive down the price level this period (if the CIAC is binding) and drive up the real value of collateral and next period's output. The central bank can conduct a contractionary monetary policy to reduce the nominal amount of collateral to balance the marginal gain from the output increase with the marginal cost of financial fragility, while keeping the economy booming. However, if the initial money supply is very low, a contractionary monetary policy after a high output might not be optimal as the cost of financial fragility is low, and the optimal monetary policy could be expansionary, which drives up the price level and the economy could get into booming.

To illustrate some of the intuition above, we can check the welfare improvement of a one-shot deviation from no-intervention. To simplify the illustration, besides the assumptions in Section 3.4., we further assume the realization of output can be only one of two values, y_G or y_B, and that the real value of the collateral affects the probability distribution of output, i.e., prob(y_G)

= $f(L)$, with $f(L)$ increasing with the real value of the collateral, L. The representative agent's value functions can be expressed as:

$$\begin{aligned} &V_G = u(y_G, H) - \psi(\rho_G) + \beta\left[f(L_G)V_G + (1 - f(L_G))V_B\right] \\ &> V_B = u(y_B, H) - \psi(\rho_B) + \beta\left[f(L_B)V_G + (1 - f(L_B))V_B\right] \end{aligned}.$$

Now consider a one-shot marginal money supply shock to the economy, in which $M' = M \pm \Delta M$, and we have the following:

Proposition 7: With $\delta/(1\text{-}\beta)$ greater than 1, an increase in the money supply M' will increase both ρ and L, and we have three cases for the optimal choice of money supply:

(i) M is very high; the optimal monetary policy is contractionary for both output levels, i.e., $M'(G) = M'(B) = M - \Delta M$.

(ii) M is at an intermediate level; the optimal monetary policy is contractionary for the high output level, i.e., $M'(G) = M - \Delta M$, and expansionary for the low output level, i.e., $M'(B) = M + \Delta M$.

(iii) M is very low; the optimal monetary policy is expansionary for both output levels, i.e., $M'(G) = M'(B) = M - \Delta M$.

Proof: See Appendix. ■

From the above example, we can see that case (ii) is very similar to the conventional countercyclical type monetary policy, but it is due to a completely different non-interest rate channel. Cases (i) and (iii) are slightly different types of central bank intervention. If the economy experienced a long-lasting monetary expansion, a contractionary monetary policy is almost surely optimal. If the economy experienced a long lasting monetary contraction, an expansionary monetary policy is almost surely optimal.

In our dynamic game with frequent central bank intervention, the mechanism is more complicated as the central bank intervention might drive the economy into recession, which could be optimal.

4. Conclusion

Monetary policy and macroprudential policy are linked because of the transformation of the financial system, which occurred over the last thirty or so years, dramatically increasing the role of collateral. Bank loans, which were previously passively held on bank balance sheets as immobile collateral became mobile via securitization. In the credit boom preceding the crisis, it was mortgage-backed securities that grew enormously. A large portion of Treasuries are held abroad, so in the United States, the ratio of MBS to Treasuries rose dramatically, ending in a financial crisis. Privately-produced collateral is not riskless, so when the ratio of MBS to Treasuries increases, financial fragility increases.

Cash and Treasuries are not substitutes because cash cannot be securitized and Treasuries cannot be used to satisfy cash-in-advance constraints. Open market operations exchange cash for Treasuries affecting the quality of collateral in the economy. If insured demand deposits are the dominate form of privately-produced safe debt, then the quality of collateral, bank loans, can be monitored by bank examiners and the central bank can focus on inflation targeting.

One possible solution to the issue of the quality of private collateral is for the government to oversee, monitor, its production (see Gorton and Metrick (2010)). But, this is not likely to eliminate the problem that privately-produced collateral is not riskless. So, in a world where privately-produced collateral, MBS, is important, the central bank needs to respond to a decline in collateral quality to reduce financial fragility. The central bank undertakes this macroprudential role by incorporating the costs of financial fragility into its policy. In the setting here, it is the real value of collateral which is central to monetary policy. But, trading one kind of money for another complicates monetary policy.

As a practical matter how should the central bank behave? Financial crises are not predictable, but they are related to the quality of collateral in the economy. This suggests that measures of the scarcity of Treasuries would be an indicator of fragility. There are signs of a scarcity of good collateral. One is an increase the convenience yield such as the GC repo minus Treasury rate spread. Stein (2014) focuses on bond risk premia, which when very low may also be signaling an excess demand for collateral. This does not exclude other possible indicators such as financial firm leverage, but that requires knowing where the "banks" are. The financial crisis illustrated that this is not always easy.

Appendix: Proofs

Proof of Proposition 1: We solve for equilibrium prices using backward induction. At date 2, with the multipliers $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$, the Lagrange conditions for (2) are:

$$v_c(c_2) - \lambda_1 p_2 - \lambda_2 p_2 = 0$$
$$\alpha v_c(nw_3 / p_2) / p_2 - \lambda_1 = 0$$
$$p_2 c_2 + w_3 \leq p_2 y_2 + nw_2$$
$$p_2 c_2 \leq m_2.$$

It is easy to show that the budget constraint is binding, which implies $\lambda_1 > 0$. Eliminating λ_1 from the Lagrangean and substituting in the market clearing conditions, $c_2 = y_2$, $c_3 = nw_3 / p_2 = (w_3 - TB_2)/p_2 = M_2/p_2$, we have:

$$v_c(y_2) - \alpha v_c(M_2 / p_2) - \lambda_2 p_2 = 0$$
$$p_2 y_2 \leq M_2.$$

If the cash-in-advance condition is not binding, that is, $p_2 < M_2/y_2$, then $\lambda_2 = 0$, or, $v_c(y_2) = \alpha v_c(M_2 / p_2)$. With the assumptions on the utility function, we have

$$v_c(y_2) = 1 / y_2 = \alpha v_c(M_2 / p_2) = \alpha p_2 / M_2$$
$$\Rightarrow p_2 = M_2 / \alpha y_2 > M_2 / y_2$$

which constitutes a contradiction. Therefore, we must have the cash-in-advance constraint binding at date 2 with $\lambda_2 > 0$, and we have $p_2 = M_2 / y_2$.

For the date 1 price, we need to characterize the functional form of $U(m_2, tb_2)$ for an agent with holdings m_2 and tb_2, whether on-the-equilibrium path or off-the-equilibrium path at date 2. Using $p_2 = M_2 / y_2$, when the cash-in-advance constraint is binding, we have:

$$c_2 = m_2 / p_2 = m_2 y_2 / M_2$$
$$nw_3 = M_2 + \left(tb_2 - TB_2\right)(1 + r_1)$$

If the following condition holds, then the cash-in-advance constraint is binding:

$$m_2 < nw_3 / \alpha \text{ with } nw_3 = M_2 + (tb_2 - TB_2)(1 + r_1).$$

In which case:

$$U(m_2, tb_2) = \ln(c_2) + \alpha \ln(nw_3 / p_2)$$
$$= \ln(m_2 f / M_2) + \alpha \ln(nw_3 f / M_2)$$
$$= (1+\alpha)\ln(f) + \ln(m_2 / M_2) + \alpha \ln(nw_3 / M_2).$$

We can also characterize the form of $U(m_2, tb_2)$ when the cash-in-advance constraint is not binding, and we have $\lambda_2 = 0$ and

$$c_2 = nw_3 / \alpha p_2 = nw_3 y_2 / \alpha M_2$$
$$nw_3 = \frac{\alpha}{1+\alpha}\left[M_2 + m_2 + (tb_2 - TB_2)(1+r_1)\right]$$

Using the condition in the lemma, we have:

$$\begin{aligned}
&U(m_2, tb_2) = \ln(c_2) + \alpha \ln(nw_3 / p_2) \\
&= \ln(nw_3 f / \alpha M_2) + \alpha \ln(nw_3 f / M_2) \\
&= (1+\alpha)\ln(f) + (1+\alpha)\ln(nw_3 / M_2) - \ln(\alpha).
\end{aligned}$$

At date 1, with the multipliers $\mu_1 \geq 0$ and$\mu_2 \geq 0$, the Lagrange conditions for (1) are:

$$\begin{aligned}
&u_c(c_1, h) - \mu_1 p_1 - \mu_2 p_1 = 0 \\
&u_h(c_1, h) - \mu_1 q = 0 \\
&\beta U_m - \mu_1 = 0 \\
&\beta U_{tb} - \mu_1 = 0 \\
&p_1 c_1 + qh + m_2 + tb_2 \leq p_1 y_1 + qH + nw_1 \\
&p_1 c_1 \leq m_1.
\end{aligned}$$

Substituting in the market clearing conditions $m_2 = M_2$ and $tb_2 = TB_2$ into $U(m_2, tb_2)$, we have:

$$\begin{aligned}
&U_m(m_2 = M_2, tb_2 = TB_2) = 1 / M_2 \\
&U_{tb}(m_2 = M_2, tb_2 = TB_2) = \alpha(1+r_1) / M_2.
\end{aligned}$$

For the Lagrange conditions, substitute in the market clearing conditions, $c_1 = y_1$, $h = H$, and we have:

$$\begin{aligned}
&\mu_1 = \beta / M_2, 1 + r_1 = 1/\alpha, q = M_2 / \beta H \\
&1 / y_1 - \mu_1 p_1 - \mu_2 p_1 = 0 \\
&p_1 y_1 \leq M_1.
\end{aligned}$$

When the cash-in-advance constraint *is* binding, we have:

$$\begin{aligned}
&p_1 = M_1 / y_1 \\
&1 / y_1 - \mu_1 p_1 = \mu_2 p_1 > 0 \Rightarrow M_2 > M_1 \beta.
\end{aligned}$$

When the cash-in-advance constraint is *not* binding, we have:

$$\mu_2 = 0$$
$$p_1 = 1/y_1\mu_1 = M_2/\beta y_1$$
$$p_1 y_1 = M_2/\beta < M_1 \Rightarrow M_2 < M_1\beta. \quad \blacksquare$$

Proof of Lemma 1: The output can be written as:

$$y_2 = \begin{cases} \dfrac{A[\beta NW + (\delta\alpha/(1-\alpha) - \beta)M_2]y_1}{M_2} & \text{if } M_2 < M_1\beta \\ \dfrac{A[\beta NW + (\delta\alpha/(1-\alpha) - \beta)M_2]y_1}{\beta M_1} & \text{if } M_2 > M_1\beta. \end{cases}$$

The result is immediate. ■

Proof of Proposition 2: The optimal monetary policy solves the following optimization problem:

$$\max_{M_2} V(M_2) \equiv \beta(1+\alpha)\ln(f(L)) - \psi(MBS/TB_2) + C$$
$$= \begin{cases} \beta(1+\alpha)\ln f\left(\dfrac{[\beta NW + (\delta\alpha/(1-\alpha) - \beta)M_2]y_1}{M_2}\right) - \psi\left(\dfrac{\alpha M_2}{\beta(1-\alpha)(NW - M_2)}\right) + C \text{ if } M_2 < M_1\beta \\ \beta(1+\alpha)\ln f\left(\dfrac{[\beta NW + (\delta\alpha/(1-\alpha) - \beta)M_2]y_1}{\beta M_1}\right) - \psi\left(\dfrac{\alpha M_2}{\beta(1-\alpha)(NW - M_2)}\right) + C \text{ if } M_2 > M_1\beta. \end{cases}$$

With the assumption that $M_\psi > M_\pi$, we know that the lower bound on the money supply is M_π. When $M_\psi > M_\pi > M_1\beta$, the optimal money supply is M_ψ as M_ψ yields the highest welfare for any $M_2 > M_1\beta$. When $M_\psi > M_1\beta > M_\pi$, we know that in the region with $M_2 < M_1\beta$, the optimal money supply is M_π, as both the output and the risk of financial fragility increase as M_2 decreases, while in the region $M_2 > M_1\beta$, the optimal money supply is M_ψ, by Proposition 2, therefore the optimal money supply is M_π or M_ψ, whichever gives the highest value of social welfare. When $M_1\beta > M_\psi > M_\pi$, we know in the region $M_2 > M_1\beta$, social welfare is decreasing with M_2, while in the region $M_2 < M_1\beta$, social welfare is decreasing with M_2; therefore, the lower bound on money supply, M_π, yields the highest welfare, and is the optimal money supply. We can see that the proposition statement is true when $M_\psi > M_\pi > M_1\beta$ or $M_1\beta > M_\psi > M_\pi$, so we only need to show the case with $M_\psi > M_1\beta > M_\pi$. We first prove that when M_1 satisfies $M_\psi > M_1\beta > M_\pi$, we have $V(M_\pi|M_1)/V(M_\psi|M_1)$ increasing with M_1. To see that, we have:

$$V(M_\pi | M_1) = \ln y_1 + \ln H + \beta(1+\alpha)\ln\frac{A\left[\beta NW + (\delta\alpha/(1-\alpha) - \beta)M_\pi\right]y_1}{M_\pi} - \gamma\ln\frac{NW}{NW - M_\pi},$$

$$V(M_{\psi}\,|\,M_1)=\ln y_1+\ln H+\beta(1+\alpha)\ln\frac{A\left[\beta NW+(\delta\alpha/(1-\alpha)-\beta)M_{\psi}\right]y_1}{\beta M_1}-\gamma\ln\frac{NW}{NW-M_{\psi}}.$$

The result is immediate as $V(M_\pi|M_1)$ is independent of M_1 while $V(M_\psi|M_1)$ is decreasing with M_1. We can check that, when $M_1=M_\pi/\beta$, we have $V(M_\pi|M_1)<V(M_\psi|M_1)$; when $M_1=M_\psi/\beta$, we have $V(M_\pi|M_1)>V(M_\psi|M_1)$. Therefore, there exists some $M_1^*\in(M_\pi/\beta, M_\psi/\beta)$ such that $V(M_\pi|M_1^*) = V(M_\psi|M_1^*)$. ■

Proof of Lemma 2: Cash-in-advance constraint is binding (B): If on-the-equilibrium path, the cash-in-advance constraint is binding, the additional cash holding today will increase cash goods consumption tomorrow, while the additional Treasuries holding today will increase housing consumption tomorrow, given everything else the same. We can write a one-period optimization problem as follows

$$\begin{aligned}&\max_{c_{t+1},h_{t+1}} u(c_{t+1},h_{t+1})\\&\qquad p_{t+1}c_{t+1}+q_{t+1}h_{t+1}+m_{t+2}+tb_{t+2}=p_{t+1}y_{t+1}+q_{t+1}H+w_{t+1}-TB_{t+1}r_t\\&s.t.\quad w_{t+1}=m_{t+1}+tb_{t+1}(1+r_t)\\&\qquad p_{t+1}c_{t+1}=m_{t+1}\end{aligned}$$

The binding cash-in-advance constraint and budget constraint give:

$$\begin{aligned}c_{t+1}^B&=m_{t+1}/p_{t+1}\\h_{t+1}^B&=\left(p_{t+1}y_{t+1}+q_{t+1}H+w_{t+1}-TB_{t+1}r_t-p_{t+1}c_{t+1}-m_{t+2}-tb_{t+2}\right)/q_{t+1}\\&=\left(p_{t+1}y_{t+1}+q_{t+1}H+tb_{t+1}(1+r_t)-TB_{t+1}r_t-m_{t+2}-tb_{t+2}\right)/q_{t+1}\end{aligned}$$

Define the marginal value of cash holdings at time t+1 as follows

$$X_{t+1}^B=u_c(y_{t+1},H)/p_{t+1},$$

which is just the first order condition of expected t+1 utility with respect to m_{t+1} at $c_{t+1}^B(w_{t+1})=y_{t+1}$ and $h_{t+1}^B(w_{t+1})=H$.

Similarly, define the expected marginal value of Treasury holdings at time t+1 as follows:

$$Z_{t+1}^B=u_h(y_{t+1},H)(1+r_t)/q_{t+1},$$

which is just the first order condition of expected t+1 utility with respect to tb_{t+1} at $c_{t+1}^B(w_{t+1})=y_{t+1}$ and $h_{t+1}^B(w_{t+1})=H$.

For the cash-in-advance constraint to be binding, we need:

$$u_c(y_{t+1},H)/p_{t+1} > u_h(y_{t+1},H)/q_{t+1}$$
$$\Rightarrow X^B_{t+1} > Z^B_{t+1}/(1+r_t)$$

Cash-in-advance constraint is *not* binding (NB): If on-the-equilibrium path, the cash-in-advance constraint is *not* binding, that is $p_{t+1}y_{t+1} < m_t$, then to calculate the additional utility from extra cash holdings and Treasury holdings, we write a one-period optimization problem as follows:

$$\max_{c_{t+1},h_{t+1}} u(c_{t+1},h_{t+1})$$
$$s.t.\quad p_{t+1}c_{t+1} + q_{t+1}h_{t+1} + m_{t+2} + tb_{t+2} = p_{t+1}y_{t+1} + q_{t+1}H + w_{t+1} - TB_{t+1}r_t$$
$$w_{t+1} = m_{t+1} + tb_{t+1}(1+r_t)$$

in which c_{t+1} and h_{t+1} are the only choice variables, and m_{t+2} and tb_{t+2} can be deemed to be fixed. Denote the solution to the above problem as $c^{NB}_{t+1}(w_{t+1})$ and $h^{NB}_{t+1}(w_{t+1})$, which are functions of the initial wealth, w_{t+1}.

Now we can define the marginal value of cash holdings at time t+1 as follows

$$X^{NB}_{t+1} = u_c(c^{NB}_{t+1},h^{NB}_{t+1})\frac{dc^{NB}_{t+1}}{dw_{t+1}}\frac{\partial w_{t+1}}{\partial m_{t+1}} + u_h(c^{NB}_{t+1},h^{NB}_{t+1})\frac{dh^{NB}_{t+1}}{dw_{t+1}}\frac{\partial w_{t+1}}{\partial m_{t+1}},$$
$$= u_c(y_{t+1},H)c^{NB}_{w,t+1} + u_h(y_{t+1},H)h^{NB}_{w,t+1}$$

which is again the first order condition of expected t+1 utility with respect to m_t at $c^{NB}_{t+1}(w_{t+1}) = y_{t+1}$ and $h^{NB}_{t+1}(w_{t+1}) = H$.

Similarly, we can define the expected marginal value of Treasury holdings at time t+1 as follows

$$Z^{NB}_{t+1} = u_c(c^{NB}_{t+1},h^{NB}_{t+1})\frac{dc^{NB}_{t+1}}{dw_{t+1}}\frac{\partial w_{t+1}}{\partial tb_{t+1}} + u_h(c^{NB}_{t+1},h^{NB}_{t+1})\frac{dh^{NB}_{t+1}}{dw_{t+1}}\frac{\partial w_{t+1}}{\partial tb_{t+1}}$$
$$= u_c(y_{t+1},H)c^{NB}_{w,t+1}(1+r_t) + u_h(y_{t+1},H)h^{NB}_{w,t+1}(1+r_t)$$
$$= X^{NB}_{t+1}(1+r_t)$$

which is again the first order condition of expected t+1 utility with respect to tb_{t+1} at $c^{NB}_{t+1}(w_{t+1}) = y_{t+1}$ and $h^{NB}_{t+1}(w_{t+1}) = H$.

Furthermore, given the binding budget constraint, it is easy to check that

$$u_c(y_{t+1},H)/p_{t+1}=u_h(y_{t+1},H)/q_{t+1}$$
$$p_{t+1}c_{w,t+1}^{NB}+q_{t+1}h_{w,t+1}^{NB}=1$$

which implies

$$u_c(y_{t+1},H)c_{w,t+1}^{NB}+u_h(y_{t+1},H)h_{w,t+1}^{NB}=u_c(y_{t+1},H)/p_{t+1}=u_h(y_{t+1},H)/q_{t+1}.$$

Therefore, we can further simplify the expressions of X_{t+1}^{NB} and Z_{t+1}^{NB} as follows:

$$X_{t+1}^{NB}=u_c(y_{t+1},H)/p_{t+1}$$
$$Z_{t+1}^{NB}=u_c(y_{t+1},H)(1+r_t)/p_{t+1}=u_h(y_{t+1},H)(1+r_t)/q_{t+1}. \blacksquare$$

Proof of Proposition 3: Given the pricing functions $p(M, M', y)$ and $q(M, M', y)$, for any value function, $U(M, y)$, the central bank chooses next period's money supply, M', to maximize the representative agent's utility.

Let C_U be the set of continuous functions from $[0,NW]\times[\underline{y},\overline{y}]$ to R, and let ρ be the sup-norm defined on C_U, that is $\rho(V_1,V_2) = \sup\{V_1(M, y) - V_2(M, y)\}$. Given continuous $p(M, M', y)$, $q(M, M', y)$ and $V(M, y)$, define a mapping T_U: $C_U \rightarrow C_U$ as follows:

$$T_1(V)(M,y)=\max_{M'}\left\{u(y,H)-\psi(MBS/TB')+\beta E_{y'}\left[V(TB',y')\right]\right\}$$
$$s.t. \qquad y'=f\left(\frac{TB'+\delta MBS}{p},\varepsilon'\right)$$

We can show that $T_1(V)$ is a contraction mapping. According to Berge's Maximum Theorem, we know that there exists a fixed point, V, which is continuous and satisfies $T_1(V) = V$. Moreover, the corresponding solution $M'(M, y)$ is compact-valued upper hemi-continuous correspondences ($\underline{M'}(M, y)$ is not empty-valued as the objective function is continuous and the choice sets are compact by the Extreme Value Theorem. We know that a compact-valued upper hemi-continuous correspondence contains a continuous function,[12] and we can pick

[12]To see this, we know that a compact-valued upper hemi-continuous correspondence (Ω: $X \rightarrow Y$) has the following property: for every sequence $\{x_n\} \rightarrow x$ and every sequence $\{y_n\}$ such that $y_n \in \Omega(x_n)$ for all n, there exists a convergent subsequence of $\{y_n\}$ whose limit point y is in $\Omega(x)$. See, for example, Stokey et al. (1989).

such a continuous function from $\underline{M}'(M, y)$, defined as $M'(M, y)$. Therefore, we have a map from continuous pricing functions ($p(M, M', y)$, $q(M, M', y)$) to $M'(M, y)$, which is also continuous.

Given continuous $M'(M, y)$, with $M'' = M'(M', y')$, define:

$$T_q(p) = u_h(y,H) / \tilde{q}(TB,TB',y) = \beta E[u_c(y',H) / p(TB',TB'',y')]$$
$$T_p(p) = u_c(y,H) / \tilde{p}(TB,TB',y)$$
$$= \begin{cases} u_c(y/H)y/(NW-TB) & \text{if } u_c(y/H)y/(NW-TB) > u_h(y,H)/\tilde{q}(TB,TB',y) \\ T_q(p) = u_h(y,H)/\tilde{q}(TB,TB',y) & \text{otherwise} \end{cases}$$

Basically, T_p maps p to $\tilde{p}$ through T_q (and $\tilde{q}$) using the first order condition with respect to m' for the representative agent's optimization problem.

Let C_p be the set of continuous function from $[0,NW]\times[\underline{y},\overline{y}]$ to R, and let ρ be the sup-norm defined on C_p. We can show that $T_p(.)$ is a contraction mapping, and there exists a continuous function $p(M, M', y)$ such that, $u_c(y, H)/ p(M, M', y) = T_p(p)$. Thus, we establish a mapping from $M'(M, y)$ to $p(M, M', y)$, from which we can derive $q(M, M', y)$ from T_q.

With C being the set of continuous pricing functions from $[0,NW]^2\times[\underline{y},\overline{y}]$ to R^2, so far we have established a mapping from $(p(M, M', y), q(M, M', y)) \in C$ to $(p(M, M', y), q(M, M', y)) \in C$, which we call T. We can show that C is a non-empty weakly compact convex subset of a Banach space (because the set of continuous functions defined on a compact set with the sup-norm is compact, convex, and complete), and T is a continuous mapping as it is the product of two continuous mappings. By the Brouwer-Schauder-Tychonoff Fixed Point Theorem, we know there exists a fixed point $\{p(M, M', y), q(M, M', y)\} \in C([0,NW]^2\times[\underline{y},\overline{y}])$ such that $(p, q) \in T(p, q)$.[13]■

Proof of Proposition 4: (C3) and (C5) imply: $1/qH = \beta E\left[Z(M',y')\right] = \beta E[(1+r)/q'H]$, which can be written as:

$$\frac{1}{\beta(1+r)} = E\left[q/q'\right] = 1 - E[(q'-q)/q'] \approx 1 - E[(q'-q)/q] = 1 - g_q.$$

13 See, for example, Aliprantis and Border (1999).

Taking logs we have $r \approx g_q + (1-\beta)$.■

Proof of Proposition 5: Suppose that for almost all realizations of output *y'*, the economy is in recession next period, i.e. the cash-in-advance constraint is not binding Then we have:

$$
\begin{aligned}
1/qH &= \beta E\left[X(M',y')\right] = \beta E\left[1/q'H\right] \\
1/qH &= \beta E\left[Z(M',y')\right] = \beta E\left[(1+r)/q'H\right],
\end{aligned}
$$

which implies $r = 0$. However, $r = 0$ implies *MBS* = infinity, which yields an infinitely high cost of financial fragility and cannot be an equilibrium outcome. This is never optimal, so the central bank adopts an expansionary monetary policy. ■

Proof of Proposition 6: Given that the initial money supply *M* large enough (close to *NW*), assume for any output *y*, the optimal monetary policy is such that the economy is booming (CIAC is binding) this period. For the lower bound of output level, $\underline{y}$, the price level this period will be $p = M/\underline{y}$. We know that the price level for next period must satisfy $p' \leq M'/y'$ by CIAC, and this implies $M'>M$ as we know $E[p']/p \geq \underline{\pi}$ but $\underline{y}E[1/y'] \leq \underline{\pi}$. When *M* approaches the upper limit, *NW*, we must have M' approaching *NW*, which implies $\rho = MBS/TB'$ is approaching infinity and the cost of financial fragility goes to infinity. Therefore, the money supply *M'* such that $M'>M$ and the economy is always booming cannot be the optimal monetary policy for the central bank when the output is low. ■

Proof of Proposition 7:We can show that the CIAC is binding in equilibrium for both output levels without central bank intervention. Consider a one-shot marginal money supply shock to the economy, in which $M'=M\pm\Delta M$; we only need consider the marginal benefit of the central bank operation with CIAC always binding (i.e., the economy is always booming). We have the following results:

$$
\begin{aligned}
& p_G y_G = p_B y_B = M,\;\; q_G H = q_B H = M'/\beta \\
& \beta(1+r) = 1,\;\; X_G = X_B = 1/M \\
& Z_G = (1+r)/q_G H = Z_B = (1+r)/q_B H = 1/M' \\
& \rho_G = MBS_G / TB' = \rho_B = MBS_B / TB' = M'/(1-\beta)(NW - M') \\
& L_G = (TB' + \delta MBS_G)/p_G = \frac{NW - M' + \delta M'/(1-\beta)}{M/y_G} \\
& > L_B = (TB' + \delta MBS_B)/p_G = \frac{NW - M' + \delta M'/(1-\beta)}{M/y_B}
\end{aligned}
$$

When M is very high, both ρ and L are very high, with convexity of $\psi()$ and concavity of $f()$, we know that $\psi_\rho \rho_{M'}(G) = \psi_\rho \rho_{M'}(B) > f_L L_{M'}(B)(V_G – V_B) > f_L L_{M'}(G)(V_G – V_B)$, and so the optimal monetary policy is contractionary for both output levels, i.e., $M'(G) = M'(B) = M – \Delta M$.

When M is at an intermediate level, we have $f_L L_{M'}(B)(V_G – V_B) > \psi_\rho \rho_{M'}(G) = \psi_\rho \rho_{M'}(B) > f_L L_{M'}(G)(V_G – V_B)$, and the optimal monetary policy is contractionary for the high output level, i.e., $M'(G) = M – \Delta M$, and expansionary for the low output level, i.e., $M'(B) = M + \Delta M$.

When M is very low, we have $f_L L_{M'}(B)(V_G – V_B) > f_L L_{M'}(G)(V_G – V_B) > \psi_\rho \rho_{M'}(G) = \psi_\rho \rho_{M'}(B)$, and the optimal monetary policy is expansionary for both output levels, i.e., $M'(G) = M'(B) = M – \Delta M$.

■

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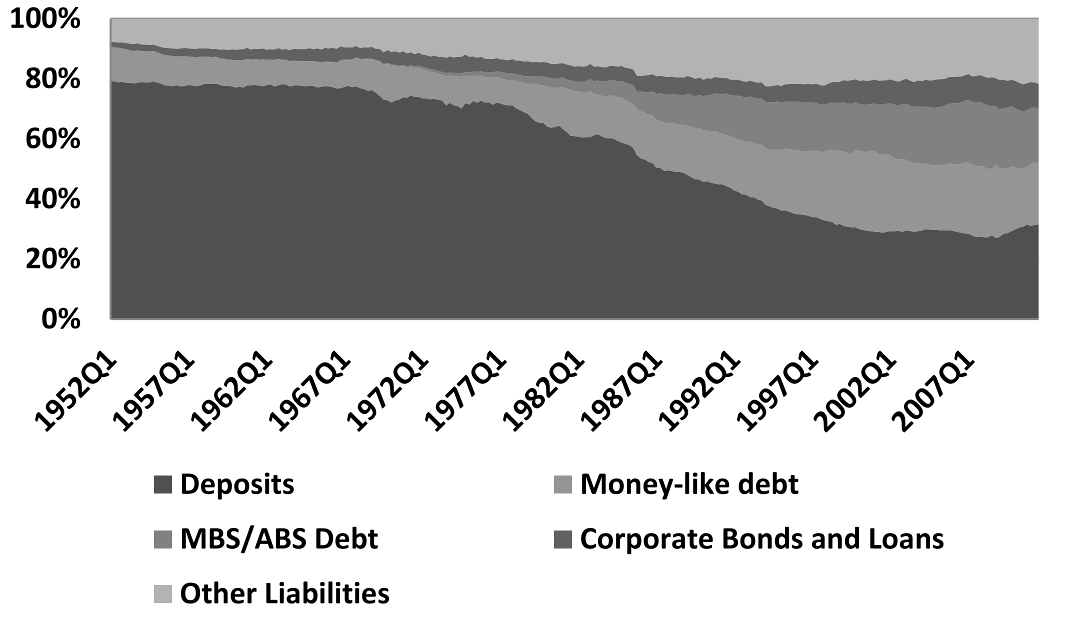
Figure 1: Components of Privately-Produced Safe Debt as a Fraction of Total Privately-Produced Safe Debt (U.S.)



Source: Gorton, Lewellen, and Metrick (2012), based on Flow of Funds data.