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NOMINALLY DENOMINATED SOVEREIGN
DEBT, RISK SHIFTING, AND REPUTATION

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ABSTRACT

This paper analyzes a reputational equilibrium in a model in which nominally denominated sovereign debt serves to shift risk associated with the unpredictability of tax revenues from the sovereign to its lenders. The analysis answers the following set of related questions: Why would a sovereign refrain from inflating when faced with servicing a large quantity of nominal debt? If a sovereign does not plan to use inflation to repudiate its nominal debts, why would it want to issue nominal debt in the first place? What are the distinguishing features of those sovereigns who are willing and able to issue nominal debts?

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Over the course of history, certain sovereigns--the USA and UK are prominent examples--have regularly issued debts that are nominally denominated in the units of the sovereign's own currency. Although this practice is far from universal, it is nevertheless a puzzling phenomenon, because such debt involves promises to pay a fixed number of units of an asset whose real value the sovereign itself controls through its power to issue fiat money. Presumably, a sovereign can issue nominally denominated debt only if lenders believe that the sovereign will not use its power to inflate in order to repudiate its debts. In practice, sovereign issuers of nominally denominated debt regularly validate this belief, even to the extent of deflating in periods, typically postwar periods, following the issuance of large quantities of debt.

These observations suggest the following set of related questions:

1. Why would a sovereign refrain from inflating, or even deflate, when faced with servicing a large quantity of nominally denominated debt?
2. If a sovereign does not plan to use inflation to repudiate its nominally denominated debts, why would it want to issue such debt in the first place?
3. What are the distinguishing features of those sovereigns who are willing and able to issue nominally denominated debt?

To answer these questions, this paper develops a theory that focuses on the unpredictability of tax revenues, net of nondiscretionary expenditures, and the desirability of shifting some of the associated risk from the sovereign to its lenders. If contracts contingent on net tax revenues are sufficiently costly to write, verify, and enforce, shifting risk by making debt servicing directly contingent on net tax revenues is not feasible. In

this situation, if the price level and real output are inversely related and real output and net tax revenues are positively related, then nominally denominated debt can help to achieve the desired risk shifting. This advantage of nominally denominated debt, however, requires that lenders expect the sovereign to refrain from attempting to produce unexpected inflation in order to repudiate its debts.

The analysis assumes that, because monetary policy is verifiable, lenders can differentiate unexpected inflation that is associated with negative shocks to aggregate output from unexpected inflation that would result from a sovereign's efforts to repudiate its nominally denominated debts. In this theory, the choice by the sovereign to validate lender expectations about monetary policy--by, for example, maintaining or resuming convertibility to an external standard like gold--depends on the sovereign's concern for its trustworthy reputation. A trustworthy reputation is valuable because it serves as collateral that provides continued ability to issue nominally denominated debt.

The analysis derives a reputational equilibrium in which the amount of nominally denominated debt is such that the short-run benefits to the sovereign from deliberately creating unexpected inflation are smaller than the long-run costs from the loss of a trustworthy reputation. Any unexpected inflation that occurs in the reputational equilibrium is the result of negative shocks to aggregate output and serves to shift risk from the sovereign to its lenders. The analysis shows that, depending on such factors as the probability of the sovereign's surviving in power and the probability of the lenders' forgetting a repudiation, the equilibrium amount of nominally denominated debt can be adequate to permit efficient risk

shifting or can be constrained to limit risk shifting. In extreme cases, reputational forces can fail to support anything other than a trivial equilibrium with zero nominal debt.

In the existing literature on nominally denominated sovereign debt, Lucas and Stokey (1983) develop a normative model of debt as a device to shift risk. In their model, efficiency requires that the sovereign commit itself to service its debts according to a contingent nominal servicing schedule and also commit monetary policy to achieve a specific price path. Such commitments would effectively convert nominally denominated debts into real contingent claims. Neither Lucas and Stokey nor the further discussion in Lucas (1986) explores the implications of the unenforceability of such commitments.

In contrast, Bohn's (1985) analysis of nominally denominated debt as a device to shift risk emphasizes the potential time inconsistency of monetary policy commitments. To rationalize a finite inflation rate, Bohn assumes that sovereigns are averse to inflation, whether expected or not, and derives a myopic equilibrium in which the expected inflation rate is sufficiently high and the amount of debt sufficiently small that the marginal cost of unexpected additional inflation equals the marginal benefit from reduced real debt servicing. Bohn's analysis does not consider the possible role of reputational forces in supporting a larger amount of debt.

In another related paper, Grossman and Van Huyck (1987), we develop the idea of sovereign debt as a contingent claim that allows excusable default in verifiably bad states of the world, and we derive a reputational equilibrium in which the sovereign always chooses to meet its contingent real servicing obligations rather than to repudiate its debts. The present

paper considers a complementary problem in which claims that are directly contingent on net tax revenues are assumed not to be feasible. This infeasibility motivates the issuance of nominally denominated debt and the associated use of inflation that is unexpected, but not deliberate, to enforce the contingent claim.

I. Analytical Framework

To simplify the analysis, assume that the sovereign's lenders are risk neutral and that all of the sovereign's debts are nominally denominated claims that mature in one period. The use of loans as a device to facilitate risk shifting allows the prepayment of indemnities and permits a large agent like a sovereign to draw on the resources of many small and anonymous insurers, with whom it would be costly to write and to enforce contracts requiring the payment of an indemnity after a bad realization. In the present case, the prepayment of indemnities as loans with nominal servicing commitments allows the execution of the insurance contract without requiring the lenders to verify the state of the world.

The sovereign's objective in period τ is to maximize the expectation of the present value of current and future utility over an horizon of h periods, which corresponds to the prospective longevity of the sovereign's survival in power. This expectation, U_τ , is given by

$$(1) \quad U_\tau = u(c_\tau) + E_\tau \sum_{t=\tau+1}^{\tau+h} \beta^{t-\tau} u(c_t),$$

with $u'(c_t) > 0$, $u''(c_t) < 0$, $u'(0) = \infty$, and $0 < \beta \leq 1$, where E_τ is an operator that denotes an expectation conditional on information available in

period τ , c_t is a measure of discretionary public expenditures in period t , $c_t \geq 0$, and β reflects the sovereign's constant rate of pure time preference.

The analysis assumes that h is a random variable defined over the positive integers, and, for simplicity, assumes that in any period the probability that the current sovereignty will terminate is $1-\gamma$, where $0 \leq \gamma \leq 1$. Thus, h has the geometric probability distribution $[h, q(h)]$, where $q(h) = \gamma^{h-1}(1-\gamma)$. The probability that h will turn out to be less than $s+1$ periods is $1-\gamma^s$. Because h is unbounded, evaluating U_τ requires a calculation of expected utility over an infinite horizon, with utility at date t discounted to reflect the probability that h would turn out to be less than t . Specifically, equation (1) and the geometric distribution for h imply

$$(2) \quad U_\tau = u(c_\tau) + \sum_{t=\tau+1}^{\infty} (\beta\gamma)^{t-\tau} E_\tau [u(c_t)].$$

The analysis assumes that lenders know β , γ , and the function $u(c)$, as well as all other aspects of the structure of the model. [Note that, if the probability of termination, $1-\gamma$, is positive, then U_τ is defined even if β equals unity.]

Current discretionary public expenditure equals current net real tax revenue plus the net real revenue from borrowing contracted in the previous period--that is,

$$(3) \quad c_t = n_t + (1+\rho - \frac{1+i_{t-1}}{1+\pi_t}) b_{t-1},$$

where n_t is real net tax revenue in period t , ρ is a constant risk-free one-period rate of return, i_{t-1} is the nominal interest rate on borrowing contracted in period $t-1$, π_t is the rate of change in the price index for real national product from period $t-1$ to period t , and b_{t-1} is the real value of the amount borrowed in period $t-1$. [Note that equation (3) does not consider the possibility of financing current expenditures out of savings or out of current borrowing. This abstraction allows us to avoid the difficult and yet unsolved technical problem of formulating a tractable model of reputation with multiple state variables. Backus and Driffill (1986) consider the general properties of such a model. The important point is that, given that feasible limits on savings and accumulated debt are finite, these devices alone could not facilitate complete smoothing of expenditures. As an analytical device, the randomness of n_t can represent the net variability of net tax revenues after allowing for net current saving.]

Current real net tax revenues, the first source of financing for current discretionary expenditure, depend positively on current real national product, y_t . Specifically,

$$(4) \quad n_t = f(y_t),$$

with $f'(y_t)$ strictly positive and continuous. Real national product is a continuous stationary exogenous random variable whose realizations are bounded according to $0 < y_{\min} < y_t < y_{\max}$ and whose probability distribution is $p(y)$. Denote the mean value of n_t as \bar{n} .

One possible interpretation of this setup is that n_t represents tax revenues net of the minimal level of public expenditures essential for law, order, and national security, that c_t represents additional expenditures on public services, that U_r represents the utility of a representative individual, and that $f(\cdot)$ represents an optimal tax policy. Another possibility, which may be more realistic, is to regard the sovereign as a proprietor and to interpret c_t as the proprietor's consumption and U_r as the proprietor's utility. In this case, n_t would represent the proprietor's maximum revenue from taxes and other sources net of expenditures that is consistent with its survival as sovereign with probability γ . [A possible extension of the analysis would be to model n_t and the implied value of γ as choice variables.] On either interpretation, the concavity of $u(c_t)$ motivates the sovereign to want to issue and to service debt in such a way that it can stabilize c_t in the face of variations in n_t . As stressed above, however, the analysis assumes that shifting risk by making real debt servicing directly contingent on net tax revenue is not feasible.

Given the nominal interest rate contracted in period $t-1$, the net borrowing proceeds, which are the second source of financing for current discretionary expenditure, depend positively on the current inflation rate. The current inflation rate depends, in turn, on current monetary policy and on the current realization of real national product. Assume, for simplicity, that the instrument of monetary policy is the exchange rate, denoted by x_t , $x_t \geq 0$, and defined as the value of a nominal monetary unit in units of a monetary standard. For example, x_t could be the gold value or the yen value of a dollar. A critical element of the information structure is that lenders can readily observe x_t and thereby verify the sovereign's

monetary policy.

The decision rule governing monetary policy has the general form,

$$(5) \quad x_t = M(y_t; A_{t-1}),$$

where A_{t-1} is a vector of all relevant predetermined variables, which include x_{t-1} , i_{t-1} , and b_{t-1} . The function $M(\cdot)$ allows the possibility that the choice of x_t depends on the realization of real national product, and hence of real net tax revenue, as well as on predetermined variables, which include the nominal debt servicing obligation contracted in period $t-1$.

For a given ratio of x_t to x_{t-1} , the inflation rate depends on the movement in the terms of trade. Specifically, let $1/v_t$ represent an index of the value of a unit of national product in units of a monetary standard, for example, gold. Given these definitions, we have

$$(6) \quad 1 + \pi_t = \frac{x_{t-1}}{x_t} \frac{v_{t-1}}{v_t}.$$

Assume that the interaction of demand and supply in world markets causes the value of a unit of national product, $1/v_t$, to vary inversely with the level of real national product. Specifically,

$$(7) \quad v_t = g(y_t),$$

where $g'(y_t)$ is strictly positive and continuous. Denote the mean realization of v_t as \bar{v} .

The supply of loans is a critical constraint on the sovereign's choices. Given competition among the sovereign's lenders, who are risk neutral, market clearing implies that if b_{t-1} is positive, the expected real interest rate on sovereign borrowing equals the alternative risk-free real rate of return, ρ . Accordingly, for $b_{t-1} > 0$ the nominal interest rate, i_{t-1} , must satisfy

$$(8) \quad E_{t-1} \left(\frac{1+i_{t-1}}{1+\pi_t} \right) = 1+\rho,$$

where $E_{t-1}(\cdot) = E(\cdot | A_{t-1})$. To see the implications of equation (8) for the nominal interest rate, observe that equation (6) implies

$$(9) \quad E_{t-1} [(1+\pi_t)^{-1}] = \frac{E_{t-1}(x_t v_t)}{x_{t-1} v_{t-1}}$$

and that, using equations (5) and (7), we can write

$$(10) \quad E_{t-1}(x_t v_t) = \int M^e(y; A_{t-1}) g(y) p(y) dy,$$

where the function $M^e(\cdot)$ describes the decision rule that lenders believe governs the sovereign's monetary policy--that is, the sovereign's choice of x_t . The final component needed to complete the model is the determination of $M^e(\cdot)$.

II. An Irrevocably Committed Monetary Policy

Suppose that in each period $t-1$ the monetary authority acting as an

agent of the sovereign could irrevocably commit itself to set a specific announced positive exchange rate in period t . Denote this announced exchange rate as ${}_{t-1}x_t$. This announcement could be either explicit or implicit. For example, it could take the form of a generally understood commitment to return to a historical monetary standard, like gold, at the historical parity.

Given that it is irrevocable, such a commitment would determine the expectation that lenders form in period $t-1$ about x_t . Specifically, such a commitment would imply that the function $M(\cdot)$ governing monetary policy, given in general by equation (5), has the special properties,

$$(11) \quad x_t = M(y_t; A_{t-1}) = {}_{t-1}x_t \quad \text{for all } y_t \text{ and } A_{t-1},$$

that the function $M^e(\cdot)$ corresponds to the function $M(\cdot)$, and that, from equation (10),

$$(12) \quad E_{t-1}(x_t v_t) = {}_{t-1}x_t \bar{v}.$$

Substituting equation (12) into equation (9), and substituting equations (9), (8), and (6) into equation (3) yields

$$(13) \quad c_t = n_t + (1+\rho)(1-v_t/\bar{v})b_{t-1}$$

The essential property of equation (13) is the dependence of current discretionary expenditure, through unexpected inflation, on the deviation in the terms of trade from its expected value. For example, if because of a

low realization of national product, the gold value of units of national product, $1/v_t$, turns out to be higher than expected, then the inflation rate is also unexpectedly high, the real value of the sovereign's nominal servicing obligation is reduced, and net revenue from borrowing is larger than expected. The important observation is that this windfall would serve to indemnify the sovereign for the unexpectedly poor realization of net real tax revenue resulting from the low realization of national product. For a given realization of y_t , and hence of v_t and n_t , the size of this indemnity depends positively on the real value of the amount borrowed, b_{t-1} .

Note that equation (13) does not involve the actual exchange rate. Given that x_t is predetermined and positive, its actual value is not important because the nominal interest rate incorporates any expected inflation. [Note also that this simple model lacks sufficient structure to pin down the inflation rate. Although the sovereign's utility depends on unexpected inflation, it does not depend separately on either expected inflation or actual inflation. In order to determine the inflation rate, the model would have to be extended to include separate effects of expected or actual inflation either directly on the sovereign's utility or on the sovereign's revenue, as, for example, in the model of seigniorage and reputation analyzed in Grossman and Van Huyck (1986) or in the generic model of monetary policy and reputation analyzed in Grossman 1987).]

With the exchange rate predetermined and positive, given that y_t is stationary and that debt matures in one period, the problem of maximizing U_t reduces to the choice in each period $t-1$ of the value of b_{t-1} that maximizes $E_{t-1}u(c_t)$. Substituting equations (4) and (7) into equation (13) and applying the operator E_{t-1} yields

$$(14) \quad E_{t-1} u(c_t) = \int u\{f(y) + (1+\rho)[1-g(y)/\bar{v}]b_{t-1}\}p(y)dy.$$

The time-invariant critical value for this problem, denoted by \hat{b} , satisfies the first-order condition,

$$(15) \quad \int u'\{f(y) + (1+\rho)[1-g(y)/\bar{v}]\hat{b}\}[1-g(y)/\bar{v}]p(y)dy = 0.$$

The assumptions that $u'(\cdot)$, $f'(\cdot)$, and $g'(\cdot)$ are strictly positive and continuous and that $u''(\cdot)$ is strictly negative imply that a positive value of \hat{b} exists. Moreover, the second-order condition,

$$\int u''(\cdot)[1-g(y)/\bar{v}]^2 p(y)dy < 0,$$

implies that this critical value is unique and yields maximum expected utility. In this sense, the real value of borrowing given by \hat{b} implies an efficient amount of risk shifting from the sovereign to its lenders.

Equation (15) is satisfied if c_t and, hence, $u'(\cdot)$, the marginal utility of discretionary expenditure, is independent of y_t . To obtain a closed-form solution for \hat{b} , assume that n_t and v_t have a correlation coefficient equal to one or, equivalently, that $f(y)$ is an affine transformation of $g(y)$: $f(y) = \kappa + \mu g(y)$. Given this assumption, the real amount of borrowing given by

$$(16) \quad \hat{b} = \frac{\bar{n} - \kappa}{1 + \rho}.$$

makes $u'(\cdot)$ independent of y_t and, hence, satisfies equation (15).

Equation (16) says that, under appropriate simplifying assumptions, the sovereign would borrow in each period a real amount in the form of nominally denominated debt equal to the discounted expected value of its real net tax revenue less a constant. In this case, efficient risk shifting would be complete. In other words, c_t would be independent of y_t and its value, denoted \hat{c} , would be equal to \bar{n} , the mean value of real net tax revenues. [For a correlation coefficient between n_t and v_t less than one, efficient risk shifting with a predetermined exchange rate would not be complete.]

III. An Opportunistic Monetary Policy

In reality, monetary policy, including exchange rate policy, is not subject to irrevocable commitments. Indeed, the power to abrogate commitments without having to answer to a higher authority is an essential property of sovereignty and, hence, of economic policy. Accordingly, although providing a useful benchmark, the preceding case in which a committed monetary policy supports efficient and complete risk shifting does not provide an empirically relevant analysis of nominally denominated sovereign debt.

To consider another useful benchmark case, suppose that, in addition to being incapable of making irrevocable policy commitments, the sovereign sets the current exchange rate without regard either for its own exchange rate announcements or for any effect that its actual current monetary policy has on expectations about its future monetary policy. Instead, in period τ the sovereign opportunistically chooses the current exchange rate, x_τ , to maximize U_τ , taking the lenders' expectation about future exchange rates as given. Because a decrease in x_τ , by equations (3) and (6), increases c_τ and U_τ , the solution to this problem would be to set x_τ equal to zero--that

is, to repudiate any outstanding nominally denominated debt through devaluation and inflation.

Assuming that lenders correctly perceive that the sovereign would face the same problem and would obtain the same solution in the next period, lenders would anticipate repudiation through inflation of any nominally denominated debt in the future. Accordingly, the condition for a positive supply of loans, given by equation (8), could not be satisfied for any finite nominal interest rate and, hence, b_{t-1} could not be positive. In other words, an opportunistic monetary policy would imply an autarkic equilibrium. The sovereign would be unable to issue any nominally denominated debt and it would be unable to shift any risk.

IV. A Reputational Equilibrium

The analysis in the preceding sections assumed either that the sovereign irrevocably commits its monetary policy, in which case the quantity of nominally denominated debt would be sufficient for efficient risk shifting, or that the sovereign makes monetary policy opportunistically, in which case it could not issue any nominally denominated debt. To develop a more general analysis, suppose that, although the sovereign cannot irrevocably commit its monetary policy, it can influence lenders' expectations about future monetary policy by its choice of current monetary policy.

The linkage between current monetary policy and lenders' beliefs about the decision rule that governs monetary policy is the sovereign's reputation for trustworthiness. Given this linkage, a rational sovereign would consider how its current monetary policy is likely to affect its reputation and how its reputation affects its ability to issue nominally denominated

debt now and in the future. Only a sovereign that irrationally ignored its reputation would behave opportunistically. [Assuming that the process by which it appoints and removes the individual policymakers who constitute the monetary authority permits the sovereign to translate its objectives into policy, reputation resides with the sovereign and not with individual policymakers.]

The sovereign's reputation depends on the announcement in each period $t-1$ of a specific positive exchange rate for period t and the subsequent validation, or repudiation, of this announcement. Denote the announced exchange rate again as ${}_{t-1}x_t$, but in this section this announcement, whether explicit or implicit, is not an irrevocable commitment.

In a reputational equilibrium, the amount of nominally denominated debt is such that for any realization of y_t the expected value of the sovereign's utility is at least as large if the sovereign validates its announced exchange rate by setting x_t equal to ${}_{t-1}x_t$ as it would be if the sovereign were to set x_t equal to zero. Using their knowledge of the sovereign's objectives and constraints, lenders are able to derive the set of amounts of debt that satisfy this condition. Given the stationary structure of the model, this set is time invariant. The maximal element in this set, denoted b^m , is the largest amount of nominally denominated debt for which the condition for a positive supply of loans, given by equation (8), could be satisfied with a finite nominal interest rate.

To analyze the determination of b^m , assume that all sovereigns always behave rationally, except for an infinitesimal fraction, ϵ , of sovereigns who inexplicably lose the rational ability to resist the temptation to behave opportunistically. Such a loss of rational restraint could result

either from idiosyncratic irrationality or from a breakdown in the process by which the individuals who compose the sovereignty reach their decisions. Either infirmity, however uncommonly it occurs, is intrinsic and irreversible.

Knowing this pattern of sovereign behavior, lenders, when dealing with a specific sovereign, attach probability $1-\epsilon$, which equals approximately unity, to rational and, hence, trustworthy behavior as long as this sovereign has not behaved opportunistically within the memory of these lenders. In this case, the sovereign could issue nominally denominated debt up to the amount b^m with a contracted nominal interest rate that implies an expected real rate of return equal to ρ .

If, alternatively, this sovereign has used devaluation and inflation to repudiate its nominally denominated debts within the memory of current lenders, these lenders would expect such opportunistic behavior in the future. In this case, the supply function of loans would imply that this sovereign would be unable to issue any nominally denominated debt. Note that this outcome would depend only on the expectations of atomistic lenders and would not require or involve collusive strategic behavior by lenders.

Denote the length of time over which current lenders would remember a repudiation to be k periods, where $0 \leq k \leq \infty$. The analysis assumes that k is a random variable and, for simplicity, assumes that in any period the probability that lenders would permanently forget a past repudiation is a constant $1 - \delta$, where $0 \leq \delta \leq 1$. Thus, k has the geometric probability distribution $[k, q(k)]$, where $q(k)$ equals $\delta^{k-1}(1-\delta)$. The probability that k would turn out to be less than $s + 1$ periods is $1 - \delta^s$. If δ equals unity, then k is infinite and a sovereign would never recover a trustworthy

reputation once it has been lost.

Given that the sovereign has a trustworthy reputation in period $\tau - 1$, these assumptions about the sovereign's reputation and lenders' expectations imply that

$$(17) \quad \begin{aligned} &\text{for } t = \tau, M^e(y_t; A_{t-1}) = {}_{t-1}x_t \text{ and } b_{t-1} \leq b^m, \text{ and} \\ &\text{for } \tau + h \geq t > \tau, \text{ either } M^e(y_t; A_{t-1}) = {}_{t-1}x_t \text{ and } b_{t-1} \leq b^m \\ &\quad \text{if } x_{t-j} \geq {}_{t-j-1}x_{t-j} \quad \text{for all } j = 1 \dots k, \\ &\quad \text{or } b_{t-1} = 0 \quad \text{otherwise.} \end{aligned}$$

Taking account of reputation, the rational sovereign's problem in period τ is to choose a program $\{b_t, x_t, {}_t x_{t+1}\}_{t=\tau}^{\infty}$ to maximize U_τ , as given by equation (2), subject to equation (3) and conditions (8) and (17). The solution to this problem describes a reputational equilibrium. The important feature of this equilibrium is the program for b_t . Given the stationary structure of the model, this program, denoted by b^* , is time invariant.

To derive b^* , define V_τ to be the expected value of the sovereign's utility over an horizon that corresponds to either the prospective longevity of the sovereign's survival in power or the prospective longevity of lenders' memories of a repudiation, whichever is shorter--that is

$$(18) \quad V_\tau = u(c_\tau) + E_\tau \sum_{t=\tau+1}^{\tau+\min(h,k)} \beta^{t-\tau} u(c_t).$$

Only at most the next k periods are relevant, because the sovereign's

utility after period $\tau + k$ is independent of the decision regarding repudiation in period τ . Given the geometric probability distributions for h and k , equation (18) implies

$$(19) \quad V_{\tau} = u(c_{\tau}) + \sum_{t=\tau+1}^{\infty} (\beta\gamma\delta)^{t-\tau} E_{\tau}[u(c_t)].$$

According to equation (19), the contribution of expected utility in period t to V_{τ} is smaller the larger is t , the larger is the sovereign's rate of pure time preference, the larger is probability that in any period the current sovereignty will terminate, and the larger is the probability that in any period lenders would forget a past repudiation.

The definition of b^* implies that b^* is less than or equal to b^m and, hence, is a member of the set of amounts of debt that satisfy

$$(20) \quad V_{\tau}^* \geq V_{\tau}^0 \text{ for all possible realizations of } y_{\tau},$$

where V_{τ}^* is the expected utility associated with borrowing b^* and setting x_t equal to ${}_{t-1}x_t$ for all $t \geq \tau$ and V_{τ}^0 is the expected utility associated with borrowing b^* and setting x_{τ} equal to zero. Condition (20) says that b^* is such that for all possible realizations of y_{τ} a plan that for all $t \geq \tau$ involves validating lenders' expectations that x_t will equal ${}_{t-1}x_t$ would generate in period τ higher expected utility for a rational sovereign than would a decision to repudiate.

Given that, if the sovereign set x_{τ} equal to ${}_{\tau-1}x_{\tau}$, lenders do not change their expectations, any value of b^* that satisfies condition (20)

also satisfies the analogous condition for period $\tau + 1$ and, by extension, for every subsequent period. Therefore, the sovereign's plan to keep its trustworthy reputation in the future is time consistent.

In the reputational equilibrium, as in the hypothetical case of an irrevocably committed monetary policy, the expected and actual exchange rates are equal. Accordingly, equation (13) gives the equilibrium relation between discretionary expenditures, denoted by c_t^* , and the real value of borrowing, b^* . Thus, to calculate V_τ^* , set b_{t-1} equal to b^* and substitute equation (13) for c_t into equation (20). Given the probability distribution $p(y)$, this calculation yields

$$(21) \quad V_\tau^* = u[n_\tau + (1 + \rho)(1 - v_r/\bar{v})b^*] \\ + \frac{\alpha}{1-\alpha} \int u[n + (1 + \rho)(1 - v/\bar{v})b^*]p(y)dy,$$

where $\alpha = \beta\gamma\delta$. To calculate V_τ^0 , observe that by repudiating its debts in period τ , the sovereign would obtain discretionary expenditure in period τ equal to $n_\tau + (1+\rho)b^*$. At the same time, by condition (17) repudiation would cause the sovereign to lose its trustworthy reputation. Consequently, in the next h or k periods, the equilibrium under anticipated repudiation, analyzed in Section III, would obtain. In this equilibrium, the sovereign would be unable to borrow and, accordingly, would obtain discretionary expenditure equal to n_t . Substituting these terms into equation (19) yields

$$(22) \quad V_\tau^0 = u[n_\tau + (1+\rho)b^*] + \frac{\alpha}{1-\alpha} \int u(n)p(y)dy.$$

V. Does Reputation Support Efficient Risk Shifting?

The analysis in Section II presented an example in which risk shifting is efficient and complete, the implied time-invariant real amount of borrowing, \hat{b} , is equal to $(\bar{n}-\kappa)/(1+\rho)$, and the associated time-invariant discretionary expenditure, \hat{c} , is equal to \bar{n} . The combination of \hat{b} and \hat{c} yields the highest value of U_r subject to x_t equals $_{t-1}x_t$ for all $t \geq r$. Thus, if condition (20) is satisfied for b^* equal to \hat{b} --that is, if \hat{b} is not larger than b^m --then the sovereign would borrow \hat{b} , and \hat{b} and \hat{c} are the reputational equilibrium. If, alternatively, condition (20) is not satisfied for b^* equal to \hat{b} , lenders would not permit the sovereign to borrow \hat{b} . Such a constraint would prevent the sovereign from achieving efficient and complete risk shifting.

To analyze the conditions that determine whether \hat{b} and \hat{c} are the reputational equilibrium, it is necessary to evaluate condition (20) for b^* equal to \hat{b} . Making the appropriate substitutions in equations (21) and (22) gives

$$(23) \quad v_r^* \Big|_{b^*=b} = u(\bar{n}) + \frac{\alpha}{1-\alpha} u(\bar{n}) \quad \text{and}$$

$$(24) \quad v_r^0 \Big|_{b^*=b} = u(n_r + \bar{n} - \kappa) + \frac{\alpha}{1-\alpha} \int u(n)p(y)dy.$$

Comparison of equations (23) and (24) shows that current considerations favor repudiation because \bar{n} is smaller than $n_r + \bar{n} - \kappa$, whereas future considerations discourage repudiation because $\frac{\alpha}{1-\alpha} u(\bar{n})$ is larger than $\frac{\alpha}{1-\alpha} \int u(n)p(y)dy$. Looking more closely at future consider-

ations, the factors that weigh against repudiation are a larger value of α , which implies less discounting of future events, and greater concavity (risk aversion) of the function $u(\cdot)$ and a larger variance of the stochastic variable y_t , both of which make risk shifting more important. Looking more closely at current considerations, the incentive to repudiate is larger the larger the realization of y_r and, hence, of n_r and v_r . The effect arises because, for a given exchange rate, the larger is real national product, the lower is the price level, and larger is the real cost of servicing the existing nominally denominated debt.

Comparison of equations (23) and (24) reveals that if condition (20) is satisfied for y_{\max} , then condition (20) will be satisfied for all possible realizations of y_t . Suppose, however, that \hat{b} does not satisfy condition (20) for large realizations of y_r . In other words, suppose that \hat{b} is so big that for a large enough realization of y_r a sovereign that had borrowed \hat{b} would choose to repudiate its debt through devaluation and inflation rather than to keep its trustworthy reputation. In that case, b^* would be less than \hat{b} .

To confirm this result, observe that for large values of y_r equations (21) and (22) imply

$$(25) \quad \frac{\partial(v_r^* - v_r^0)}{\partial b^*} \Big|_{b^* = \hat{b}} = (1+\rho) [u'(\bar{n})(1-v_r/\bar{v}) - u'(n_r + \bar{n} - \kappa)] < 0.$$

Equation (25) says that, if \hat{b} is not the reputational equilibrium, then, with lenders forming expectations according to condition (17), the supply constraint given by equation (8) might be consistent with a smaller amount

of borrowing. Of course, this smaller amount of borrowing implies less than efficient and complete risk shifting. In extreme, but perhaps frequently realistic, cases—for example, cases in which the sovereign has a low value of α —the equilibrium amount of nominally denominated debt can be zero.

VI. Summary

This paper has analyzed a reputational equilibrium in a model in which, because net real tax revenue and unexpected inflation are inversely related, nominally denominated sovereign debt serves to shift the risk associated with net real tax revenue from the sovereign to its lenders. The model assumes that lenders can differentiate unexpected inflation that is associated with negative shocks to aggregate output from unexpected inflation that would result if a sovereign set its exchange rate opportunistically, without regard for its announced exchange rate policy, in an attempt to repudiate its debts. In the reputational equilibrium, the amount of nominally denominated debt is such that the short-run gains from repudiation of the debt through unexpected devaluation and inflation are smaller than the long-run costs from the loss of a trustworthy reputation. Accordingly, the sovereign always resists the temptation to repudiate.

The equilibrium amount of debt can be sufficient to permit efficient and complete risk shifting or it can be constrained in a way that limits risk shifting. The factors that tend to produce such a debt constraint include a high rate of pure time preference for a sovereign, a high probability per period that the sovereign will not survive in power, a high probability per period that lenders would forget a past repudiation,^{and} a low risk aversion for the sovereign.

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