NBER WORKING PAPER SERIES

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Working Paper 22330
http://www.nber.org/papers/w22330

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 2016, Revised March 2021

This paper previously circulated under the title ”Debt Crises: For Whom the Bell Tolls.” We thank Vladimir Asriyan, Marco Bassetto, Anmol Bhandari, Patrick Kehoe, Monika Piazzesi, Alex Vardoulakis, Laura Veldkamp, Venky Venkateswaran and seminar participants at Arizona State, Banco Central de Chile, Cambridge, Carlson School at Minnesota, Cornell, Harvard, Johns Hopkins, McGill, NYU, Penn State, Richmond Fed, Stanford, UCL, Wharton, Yale, the 2015 NBER Conference on “MacroeconomicsWithin and Across Borders”, the 2015 Barcelona GSE Summer Forum, the 2015 Conference for Junior Macroeconomists at the EIEF in Rome, the 2015 QED Frontier of Macroeconomics Conference, and the 2019 UVA Symposium on Financial Economics for useful comments. Cole and Ordonez received support from the NSF through grant 1851707. The usual waiver of liability applies. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 22330
June 2016, Revised March 2021
JEL No. F34,F42,G15,H63

ABSTRACT

We develop a theory of information spillovers in primary sovereign bond markets where governments raise funds from a common pool of competitive investors who may acquire information about default risk and later trade in secondary markets. Strategic complementarities in information acquisition lead to the co-existence of an informed regime with high yields and high volatility, and a Pareto-dominant uninformed regime with low yields and low volatility. Small shocks to default risk in a single country may trigger information acquisition, retrenchment of capital flows, and sharp yield increases within and across countries. Competitive secondary markets strengthen information acquisition incentives, raise primary market yields, and amplify spillovers.

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1 Introduction

Two empirical regularities in sovereign bond markets have received widespread attention. The first is that increases in sovereign yields (particularly during sovereign debt crises) often spill over to other seemingly unrelated countries. Examples include the Russian crisis of 1998, the Mexican crisis of 1994, the Latin American crises of the 1980s, and the recent Eurozone crisis. The second is that these movements typically lead to a retrenchment of capital flows and increased market segmentation that further raises yields by reducing cross-country diversification (see, for example, Milesi-Ferretti et al. (2011) and Lane (2012)). We develop a new heterogeneous information model of sovereign debt markets that is consistent with this evidence.

We differ from the existing macroeconomic literature in three ways. First, since government revenues are determined when selling new bonds, we focus on primary rather than secondary market prices.[1] Second, we study the role of asymmetric information in determining bond yields and yield volatility. This allows us to establish a new information-based channel of yield shocks and spillovers that leads to the existence of multiple equilibria within a country, but is unrelated to rollover crises. Third, we show how the interaction between primary and secondary markets reinforces the link between information and bond yields. Perhaps contrary to conventional wisdom, secondary markets raise the value of acquiring information in primary markets, increasing yields and yield volatility.

We study a model in which two countries run simultaneous auctions in primary markets to raise a given amount of revenue by selling bonds to ex-ante identical risk-averse investors, who may participate in both countries' auctions and later trade in secondary markets.[2] The only other asset available to investors is a risk-free investment with zero net return. To focus on demand determinants of bond yields, we model defaults as mechanically determined by an exogenous realization of a country-specific state. The state can be *good* (low default probability) or *bad* (high default probability). There are no fundamental links between countries; default risk is inde-

[1] Many models link country fundamentals to secondary market spreads. See for example Reinhart, Rogoff, and Savastano (2003), Tomz and Wright (2007), Broner, Martin, and Ventura (2010), Tomz and Wright (2013) and Aguiar and Amador (2014)). For a quantitative literature that accounts for the effect of default on sovereign spreads see Aguiar and Gopinath (2006), Arellano (2008), Chatterjee and Eyigungor (2012), Hatchondo and Martinez (2009). Aguiar et al. (2016) surveys this literature.

[2] Lizarazo (2013) and Broner, Lorenzoni, and Schmukler (2013) discuss the importance of risk aversion for explaining the behavior of sovereign spreads.

pendently distributed across countries.

Prior to participating in primary markets, investors can exert costly effort to learn about the state of the world in one or both countries. This decision determines an investor's type as either *informed* or *uninformed* about the probability of default, such that informed investors can adjust their bid upon learning this information. While information acquisition could pertain to learning about macroeconomic performance or financial indicators, we view it primarily as relating to soft information such as internal negotiations about government policy, the formation of political coalitions, debt renegotiation strategies with large external creditors or the outcomes of pertinent court cases.[3] Given this interpretation, our analysis applies primarily to volatile emerging market economies and the Eurozone periphery.[4]

We model primary markets as multi-unit discriminatory-price auctions, the predominant protocol used by these economies to sell bonds.[5] Under this format, investors submit multiple sealed bids consisting of a price and a commitment to buy a certain number of bonds at that price. The government orders bids in descending order of prices and executes bids at the bid price until it raises the required revenue. This leads to a lowest-accepted *marginal price*, with all bids at prices above the marginal price also accepted. Since there are many bidders, we assume individual investors take the set of marginal prices as given. This price-taking assumption leads to a tractable setting for studying endogenous information acquisition in primary sovereign debt markets.

For any possible marginal price, informed investors bid more aggressively upon good news and more conservatively after bad news. Hence the presence of informed investors leads to price dispersion that creates a form of the winner's curse for the remaining uninformed investors: any bid at the high price associated with the good state is also accepted when the state is bad. This leads to a tradeoff for the uninformed between capturing infra-marginal rents in the good state and overpaying in the bad state. The value of information, measured as the difference in expected utility

[3]The complex debt restructuring process of Argentina's defaulted bonds in 2001, which included a 2005 restructuring, repayment of obligations to the IMF, a second debt swap in 2010, a 2014 "selective default" with holdouts, etc., provide a vivid illustration of the intricacies of information we model and the implications for new debt issuance.

[4]In Cole, Neuhann, and Ordoñez (2020) we provide evidence on the relevance of information frictions and the nature of information using Mexican Cetes auctions.

[5]Brenner, Galai, and Sade (2009) find that the majority of their sample of 83 countries, including 83% of OECD countries and many countries that have experienced sovereign default episodes in the past, sell bonds using discriminatory price (pay your bid) auctions.

between informed and uninformed investors, lies in avoiding this tradeoff.

We find that the value of information is non-monotonic with respect to the fraction of investors that are informed. When there are few informed investors, the value of information is increasing in the fraction of informed investors. This is because an increase in informed bidding increases cross-state price dispersion and increases the cost of overpaying for uninformed investors. Uninformed investors respond by submitting fewer bids at higher prices. Once the fraction of informed investors is large enough such that the uninformed have retreated from participating at high prices, a further increase in the fraction of informed reduces the value of information, as there are now no uninformed investors to exploit and more informed investors to compete with.

The result of this non-monotonicity is the co-existence of two information regimes for appropriate information costs. One is the *uninformed regime* in which no investor acquires information. Yields are then determined by the unconditional required risk premium, and volatility is muted because prices do not respond to the realized state. The other is the *informed regime* in which some investors do acquire information and prices are volatile because they vary with the state. Importantly, since information acquisition amounts to rent-seeking at the expense of other investors that is fully offset by the cost of information acquisition, investors strictly prefer the uninformed regime, while the government faces higher price volatility and possibly lower average prices when there is information acquisition. In this sense, information can lead to sudden change in yields and precipitate crises. The co-existence of information regimes depends on fundamentals. When there is little risk there is little value in learning and so safe countries are likely to raise funds in an uninformed regime. On the other hand, information is valuable when fundamentals are volatile and so risky countries are likely to suffer from amplification through information acquisition. Moreover, small shocks to default risk may be sufficient to trigger a sudden switch to the informed regime, with concomitant increases in yields and volatility. We view this as an attractive feature of a theory of spillovers and yield shocks.

Information acquisition also leads to cross-country spillovers. We establish three distinct channels, all of which contribute to retrenchment of capital and market segmentation after bad shocks. The first channel, *risk appetite*, does not rely directly on asymmetric information but amplifies its effects. Whenever investor preferences satisfy decreasing absolute risk aversion, an increase in default risk in one country raises investors risk aversion when investing in the other country. This tends to raise the required risk premium and lowers bond prices in bond countries. Notably, we find that these spillovers are particularly strong when global debt burdens are high. The second channel, *segmentation*, is information-based and relates to imperfect diversification. Informed investors allocate a larger fraction of their risky investments to the country in which they are informed in order to exploit their information advantage. Uninformed investors, on the other hand, shift their risky investments to the country with fewer informed investors to escape the winner's curse. Both investors types thus hold less diversified portfolios, raising risk premia in both countries. Importantly, this is the case even if investors only acquire information in one country. The third, *information intensity*, channel relates to information regimes. An investor has stronger incentives to acquire information about a country when buying a lot of that country's bonds. An uninformed investor who shifts his portfolio towards a second country with fewer informed becomes at the same time more exposed to that country, increasing his incentives to acquire information in the second country. Since information acquisition lowers prices, such *information regime contagion* also increases yields.

Our last contribution is to analyze the impact of secondary market trading on primary market outcomes and information acquisition. This is pertinent from a positive and normative perspective: most government bonds can be traded in secondary markets, and the establishment of liquid secondary markets was the explicit goal of various market liberalization initiatives. Perhaps contrary to conventional wisdom, we find that secondary markets have a deleterious impact on primary market prices. We develop these results under the assumption that marginal auction prices are common knowledge in the secondary market so that trading takes place under symmetric information. The only remaining motive for trade in secondary markets is then the sharing of differential default risk after primary markets.

The equilibrium with secondary markets works as follows. Informed investors buy a large number of bonds in the primary market to sell a fraction in the secondary market at pure arbitrage profit. Since there is asymmetric information only in the primary market, uninformed investors wait for the secondary market to avoid the winner's curse. Secondary markets are thus costly to the government because fewer investors participate in the primary market, depressing the price at which the government can sell its bonds. Secondary market trading also raises information acquisition incentives because the option to resell allows informed investors to aggressively exploit their information advantage without being excessively exposed to the country in which they are informed. This novel adverse feedback effect to primary market prices should be weighed against other potential benefits of secondary markets.

Related Literature. Previous work has explored spillovers in sovereign debt markets, but not from the perspective of endogenous heterogenous information and the interplay between primary and secondary markets. The most common view of spillovers relies on real linkages, such as trade in goods or correlated shocks, that may transmit negative shocks from one country to the next. Nevertheless, it is often difficult to empirically identify linkages that are powerful enough to induce the observed degree of spillovers. This led to a new set of explanations that rely on self-fulfilling debt crises either through feedback effects as in Calvo (1988) and Lorenzoni and Werning (2013) or rollover problems, as in Cole and Kehoe (2000), Aguiar et al. (2015), and Bocola and Dovis (2015).

We explore here a different form of spillovers, which stem not from country fundamentals (the supply side) but rather from the investment and information acquisition decisions of common investors (the demand side). Previous work has explored spillovers generated by a global pool of investors, based on changes in wealth as in Kyle and Xiong (2001) or Goldstein and Pauzner (2004), borrowing constraints as in Yuan (2005), short-selling constraints as in Calvo and Mendoza (1999), and exogenous private information in Walrasian markets as in Kodres and Pritsker (2002). Broner, Gelos, and Reinhart (2004) provide empirical evidence of the importance of portfolio effects for spillovers. Our innovation is combining a common pool of investors with endogenous information heterogeneity and a rich dual market structure.

Closer to our insight, Van Nieuwerburgh and Veldkamp (2009) also use a model of information acquisition to study home bias and segmentation in financial markets. They consider information acquisition in competitive secondary markets, showing it is a strategic substitute. Our model features a strategic complementarity in primary markets that leads to equilibrium multiplicity and contagion of information regimes. Ahnert and Bertsch (2020) study a global-games model of sequential regime change in which there is information-based contagion. There is no portfolio choice or prices in their model, so their main focus is on contagion of default itself. Our focus is on price spillovers upon raising funds in primary markets. Bukchandani and Huang (1989) consider the interaction of primary and secondary markets when primary market bidders have an incentive to signal private information. They consider risk-neutral agent in single-unit unit auctions and show overbidding at auction compared to the case without secondary markets. We consider multi-unit auctions with risk-averse bidders and endogenous information acquisition and find that primary market prices decline. Broner, Martin, and Ventura (2010) argue that secondary markets support sovereign borrowing capacity by providing commitment against default on foreign creditors. Our work complements this view, as we show that sovereign markets may induce harmful information acquisition and reduce primary market prices for given borrowing capacity.

The paper proceeds as follows. The next section describes our model of primary and secondary sovereign debt markets in two countries with a common pool of investors. Section 3 characterizes the equilibrium without secondary markets and describes the sources of information multiplicity in each country and the effects on informational spillovers. Section 4 studies the role of secondary markets on bond yields, information acqusition, and spillovers. Section 5 concludes.

2 Model

2.1 Environment

We study a two-period economy with a single numeraire good, a measure one of ex-ante identical risk-averse investors with fixed per-capita wealth W and two countries, indexed by $j \in \{1, 2\}$. The government of country j needs to raise a fixed amount $D_j \geq 0$ by auctioning sovereign bonds in the primary market. Thereafter, bonds may also be traded among investors in a centralized competitive secondary market.

Investors care only about consumption at the final date. Their preferences are represented by a common flow utility function u that is strictly increasing and concave and twice continuously differentiable. Furthermore, preferences satisfy the Inada conditions and feature weakly decreasing absolute risk aversion (standard CRRA preferences fulfill these properties). Investors can invest in government bonds or a risk-free asset whose net return is normalized to zero. There is no borrowing: investors can spend no more than W at either the primary and secondary markets. There is also no short-selling: investors cannot submit negative bids at auction, and, in the secondary market, can sell at most all bonds acquired at auction.

Without loss of generality, a bond auctioned at date 1 promises one real unit of consumption at date 2. Bonds are risky because they deliver a unit of the numeraire only if the issuing government does not default. In a default, the recovery rate is zero. Default is summarized by $\delta_j \in \{0, 1\}$, where $\delta_j = 1$ denotes default and $\delta_j = 0$ denotes repayment, and $\vec{\delta} = [\delta_1, \delta_2]$. To focus on demand determinants of bond yields, we assume that governments behave mechanically. Specifically, country j's default probability $\kappa_j(\theta_j) = \Pr\{\delta_j = 1|\theta_j\}$ is a random variable that depends only on the realization of a country-specific fundamental $\theta_j \in \{b, g\}$. Without loss of generality, $\kappa_j(g) < \kappa_j(b)$. The probability of state θ_j is $f_j(\theta_j)$, and the unconditional default probability is

$$\bar{\kappa}_j = f_j(b)\kappa_j(b) + f_j(g)\kappa_j(g).$$

To focus on information-based contagion rather than real linkages, we assume that θ_j is independently distributed across countries and we define $\vec{\theta} \equiv [\theta_1, \theta_2]$.

2.2 Information Structure

Prior to bidding for bonds in primary markets, investors can acquire information (learn the realization of θ_1 and/or θ_2) by paying a utility cost. We denote the decision to acquire information in country j by $a_j \in \{0, 1\}$. The associated cost is $C(a_1, a_2) \geq 0$ and is weakly increasing in each argument. The information acquisition defines the investor's *type*, which we index by $i \in \{a_1 a_2\}$. We use $\mathcal{F}^i$ to denote type i information set and $n^i \in [0, 1]$ its mass, with $\sum_i n^i = 1$. Since investors are identical conditional on their information set, we study a representative investor of each type. We denote the set of types informed in j by $\mathcal{I}_j \equiv \{i : a^i_j = 1\}$ and the set of types uninformed in j by $\mathcal{U}_j \equiv \{i : a^i_j = 0\}$. [6] The mass of investors who acquire information in j is $\bar{n}_j = \sum_{i \in \mathcal{I}_j} n^i$.

To transparently characterize portfolios and spillovers, we assume that asset markets are partially segmented. Specifically, each investor splits up into two traders at time zero, with each trader tasked with trading and possibly acquiring information in one specific country. Traders cannot share information. This ensures that bids in

[6]Notice that there are four possible types (a^i_1, a^i_2) in terms of information (this is (0,0), (1,0), (0,1) and (1,1)). In the first passages of the paper, in which we focus on the effects of asymmetric information in one country (say Country 1) we will assume no information in the other (Country 2) and then we will just have two types, (0,0) and (1,0). We get back to four types when discussing contagion of information regimes across countries.

country j are not contingent on the *realization* of θ_{-j}. However, they will be contingent on the information acquisition *strategy* in $-j$.[7]

2.3 Primary Market

Governments sell bonds using discriminatory multi-unit auctions. Investors can submit multiple bids, each of which represent a commitment to purchase a non-negative number of bonds at a particular price should the government decide to execute the bid. The government treats each bid independently, sorts all bids from the highest to the lowest bid price, and executes all bids at the bid price in descending order of prices until it generates revenue D_j. Since there is a fixed revenue target, the total number of bonds sold is an equilibrium object. A *marginal price* is the lowest accepted price for a given θ_j, and we denote it by $P_j(\theta_j)$.

Since it is a weakly dominant strategy to bid only at prices that are marginal in at least one state of the world, we take as given that bids at all other prices are zero. Excess demand at the marginal price is rationed pro-rata, but rationing does not occur in equilibrium.[8] Let $B^i_j(\theta_j) \geq 0$ denotes trader i's bid in country j at the marginal price $P_j(\theta_j)$. The set of states in which this bid is accepted is

$$\mathcal{A}_j(\theta_j) = \{\theta'_j : P_j(\theta'_j) \geq P_j(\theta_j)\}.$$

This set always includes θ_j, but it also includes $\theta'_j \neq \theta_j$ if $P_j(\theta'_j) \geq P_j(\theta_j)$. Let $\mathcal{B}^i_j(\theta_j)$ denote the *realized* quantity of country-j bonds acquired by investor i in state θ_j. Because only informed investors can submit state-contingent bids, we have

$$\mathcal{B}^i_j(\theta_j) = \begin{cases} B^i_j(\theta_j) & \text{if } i \text{ is informed in } j \\ \sum_{\theta'_j \in \mathcal{A}_j(\theta_j)} B^i_j(\theta'_j) & \text{if } i \text{ is uninformed in } j. \end{cases}$$

We need to distinguish between the bids that an investor makes, $B^i_j(\theta_j)$, and the bonds that he acquires, $\mathcal{B}^i_j(\theta_j)$. For the informed investor who bids at the correct marginal

[7]This reduces the number of equilibrium prices from 16 to 8 without affecting the basic mechanisms.

[8]An investor can avoid rationing by offering an infinitesimally higher price, something the uninformed investors would strictly prefer when bidding at the higher price. Even if this were not an issue, for any equilibrium with rationing there is an equivalent equilibrium in which bidders scale down their bids by the rationing factor so long as the marginal prices are distinct, which they are here.

price, these two are the same; for the uninformed investor they may not be because some bids may have been submitted at prices above the realized marginal price.

Investor i's total expenditure on bonds in country j and state θ_j thus is

$$X_j^i(\theta_j) = \begin{cases} P_j(\theta_j)B_j^i(\theta_j) & \text{if } i \text{ is informed in } j \\ \sum_{\theta_j' \in \mathcal{A}_j(\theta_j)} P_j(\theta')B_j^i(\theta') & \text{if } i \text{ is uninformed in } j. \end{cases}$$

The market-clearing condition in country j and state θ_j is

$$\sum_i n^i X_j^i(\theta_j) = D_j. \tag{1}$$

2.4 Secondary Market

The secondary market opens once the primary market closes, and auction marginal prices are public knowledge prior to secondary market trading. If there are informed investors participating in the primary market, auction prices are fully revealing of the state ex-post. Otherwise, no investor is informed. In either case, the secondary market operates under symmetric information.

We denote with hats secondary market figures of primary market counterparts. For instance, we denote purchases by $\widehat{B}_j^i(\theta_j)$, and market-clearing prices by $\widehat{P}_j(\theta_j)$. Negative quantities indicate sales, and investors can sell no more than the total quantity of bonds acquired at auction, $\widehat{B}_j^i(\theta_j) \geq -\mathcal{B}_j^i(\theta_j)$. Secondary market expenditures are $\widehat{X}_j^i(\theta_j) = \widehat{P}_j(\theta_j)\widehat{B}_j^i(\theta_j)$ and then secondary market clearing requires

$$\sum_i n^i \widehat{B}_j^i(\theta_j) = 0. \tag{2}$$

2.5 Investors' Decision Problems and Equilibrium Definition

Investors face two sequential decision problems. The first is the choice of an information acquisition strategy $\{f_1(g), a_2\}$. The second is a portfolio choice problem whereby each type chooses a bidding strategy $\mathcal{S}^i$ to maximize expected utility derived from second-period consumption. The bidding strategy is a tuple of primary and secondary market bids for each j and θ_j,

$$\mathcal{S}^i \equiv \left\{ \left\{ B_j^i(\theta_j), \widehat{B}_j^i(\theta_j) \right\}_{\theta_j \in \{g,b\}} \right\}_{j \in \{1,2\}}$$

Bids determine the final number of bonds held by the investor for each j and θ_j as

$$\widehat{\mathcal{B}}^i_j(\theta_j) = \mathcal{B}^i_j(\theta_j) + \widehat{B}^i_j(\theta_j)$$

This implies that investment in the risk-free asset after the auction satisfies

$$w^i(\vec{\theta}) = W - \sum_j X^i_j(\theta_j) \quad \text{for all } \vec{\theta}.$$

while total holdings of the risk-free asset at secondary market close are given by

$$\widehat{w}^i(\vec{\theta}) = w^i(\vec{\theta}) - \sum_j \widehat{X}^i_j(\theta_j) \quad \text{for all } \vec{\theta}.$$

The resulting consumption profile is

$$c^i(\vec{\theta}, \vec{\delta}, \mathcal{S}^i) = \widehat{w}^i(\vec{\theta}) + (1-\delta_1)\widehat{\mathcal{B}}^i_1(\theta_1) + (1-\delta_2)\widehat{\mathcal{B}}^i_2(\theta_2) \quad \text{for all } \vec{\theta} \text{ and } \vec{\delta}.$$

We can now define investors' decision problems and the equilibrium concept.

Definition 1 (Portfolio choice problem)**.** *Type i's portfolio choice problem is*

$$\begin{aligned} V^i = \max_{\mathcal{S}^i} \quad & \mathbb{E}\left[u(c^i(\vec{\theta}, \vec{\delta}, \mathcal{S}^i)) \middle| \mathcal{F}^i\right] \\ \textit{s.t.} \quad & B^i_j(\theta_j) \geq 0 \textit{ and } \widehat{B}^i_j(\theta_j) \geq -\mathcal{B}^i_j(\theta_j) \quad \textit{for all } j \textit{ and } \theta_j \\ & w^i(\vec{\theta}) \geq 0 \textit{ and } \widehat{w}^i(\vec{\theta}) \geq 0 \textit{ for all } \vec{\theta}. \end{aligned}$$

The first pair of constraints ensures that bids are non-negative at auction and that there is no short-selling in the secondary market. The second pair of constraints ensures that investors do not borrow at any date.

Given a solution to the portfolio choice problem for every investor type, we can define the preceding information acquisition problem. The solution to this problem determines an investor's type going forward.

Definition 2 (Information acquisition problem)**.** *Let $\iota(a_1, a_2)$ denote the type induced by $\{a_1, a_2\}$. Then the information acquisition problem is*

$$\max_{\{a_1,a_2\}} V^{\iota(a_1,a_2)} - C(a_1, a_2).$$

An equilibrium combines market clearing at auction and in the secondary market with solutions to investors' decision problems.

Definition 3 (Equilibrium)**.** *An equilibrium consists of pricing functions $P_j : \{b, g\} \rightarrow [0, 1]$ and $\widehat{P}_j : \{b, g\} \rightarrow [0, 1]$ for each j, an information acquisition strategy $\{a_1, a_2\}$ for each investor, and bidding strategies $\mathcal{S}^{\iota(a_1,a_2)}$ for all $\{a_i, a_2\}$ on the path of play such that: (i) $\mathcal{S}^{\iota(a_i,a_2)}$ solves type $\iota(a_1, a_2)$'s portfolio choice problem, (ii) $\{a_1, a_2\}$ solves the information acquisition problem for each investor, and (iii) market-clearing conditions* (1) *and* (2) *hold.*

Throughout the paper we use numerical examples to illustrate the key economic mechanisms. Unless stated otherwise, we will use the following parameters.

Definition 4 (Baseline Parameters for Numerical Examples)**.** *Utility is $U(\cdot) = \log(\cdot)$. Countries are ex-ante symmetric. Wealth is $W = 800$ and outstanding debt is $D_j = 300$. Default probabilities satisfy $\kappa_j(g) = 0.1$, $\kappa_j(b) = 0.35$, and $f_j(g) = 0.6$. Hence $\bar{\kappa}_j = 0.2$.*

3 Auction Equilibrium

We first characterize equilibrium without secondary markets. This allows us to precisely characterize optimal bids at auction, and it provides a benchmark to evaluate the effects of secondary market trading. The equilibrium definition is Definition 3, augmented with the requirement that all secondary market bids are zero.

When deciding on the number of bids to submit at marginal price $P_j(\theta_j)$, investors form expectations with respect to the states in which a given bid will be accepted. For investor i, the set of feasible states is determined by the information set $\mathcal{F}^i$. The set of states in which a bid at price $P_j(\theta_j)$ is accepted is $\mathcal{A}_j(\theta_j)$. This in turn depends on the ordering of prices across states, which is as follows.

Lemma 1. *If no investor learns θ_j, marginal prices are the same in all states, $P_j(g) = P_j(b)$. If some investors learn θ_j, the marginal price is strictly higher in the good state, $P_j(g) > P_j(b)$.*

The intersection $\mathcal{F}^i \cap \mathcal{A}_j(\theta_j)$ captures the relevant set of states when submitting bids at $P_j(\theta_j)$. If no investor acquires information, the relevant set is the same for all investors, $\mathcal{F}^i_j \cap \mathcal{A}_j(\theta_j) = \{g, b\}$ for all θ_j. If some investors are informed, the inference problem is more complicated. For an informed investor, the relevant set always contains the true state only, $\mathcal{F}^i_j \cap \mathcal{A}_j(\theta_j) = \theta_j$ for all θ_j if $a^i_j = 1$. For uninformed investors, the ordering of state-specific prices implies that bids at the high marginal price are also accepted in the bad state. Since these investors cannot directly distinguish states based on their information, the relevant set for bids at $P_j(g)$ contains all states, $\mathcal{F}_j^i \cap \mathcal{A}_j(g) = \{g, b\}$. The same ordering of prices also implies that bids at $P_j(b)$ will *not* be accepted in the good state. Hence $\mathcal{F}_j^i \cap \mathcal{A}_j(b) = b$ if i is uninformed even though the investor cannot directly distinguish states. Thus, uninformed face adverse selection (the winner's curse) only at the high price.

Optimal bidding strategies trade off the expected marginal utility loss from default against the expected marginal benefit of the yield earned after repayment in all relevant states. Since bids are associated with specific prices, it is helpful to summarize investor i's expected marginal utility for bids in country j given state θ_j and a hypothetical default decision δ_j by

$$m_j^i(\theta_j, \delta_j) = \mathbb{E}\Big[u'(c^i(\vec{\theta}, \vec{\delta}))\Big|\mathcal{F}^i, \theta_j, \delta_j\Big].$$

Here the expectation is taken over states of the world and default decisions in country $-j$. Taking ratios of marginal utility given, default in j and repayment in j yields the relevant *marginal rate of substitution* (MRS) for evaluating bids at $P_j(\theta_j)$, which is

$$M_j^i(\theta_j) = \frac{\sum_{\theta_j' \in \mathcal{F}^i \cap \mathcal{A}_j(\theta_j)} f_j(\theta_j') \kappa_j(\theta_j') m_j^i(\theta_j', 1)}{\sum_{\theta_j' \in \mathcal{F}^i \cap \mathcal{A}_j(\theta_j)} f_j(\theta_j') \left(1 - \kappa_j(\theta_j')\right) m_j^i(\theta_j', 0)}.$$

Proposition 1 below shows that first-order conditions for marginal investors equalize this marginal rate of substitution with bond yields in a given country. The MRS differs across investors through variation in $\mathcal{F}^i \cap \mathcal{A}_j(\theta_j)$ and portfolios in the other country.

Proposition 1 (Marginal Investor and Prices)**.** *Fix any share of informed investors in Country j. Let $M_j^*(\theta_j)$ denote the marginal rate of subsitution for the marginal investor in country j and state θ_j. Bond prices satisfy the marginal investor's first-order condition*

$$\frac{1 - P_j(\theta_j)}{P_j(\theta_j)} = M_j^*(\theta_j).$$

If there are no informed investors in j, then uninformed investors are marginal in every state and there exists a single marginal price $\bar{P}_j$ such that:

$$\frac{1 - \bar{P}_j}{\bar{P}_j} = M_j^i(g) = M_j^i(b) \qquad \textit{for all uninformed types } i \in \mathcal{U}_j.$$

If there are informed investors, then informed investors are marginal in every state and

$$\frac{1 - P_j(\theta_j)}{P_j(\theta_j)} = M_j^i(\theta_j) \qquad \textit{for all informed types } i \in \mathcal{I}_j.$$

while uninformed investors are not marginal and may not bid in the good state. That is, uinformed investor optimality conditions satisfy

$$M_j^U(b) = \frac{1 - P_j(b)}{P_j(b)} \qquad \textit{and} \qquad M_j^U(g) \geq \frac{1 - P_j(g)}{P_j(g)} \qquad \textit{if } i \in \mathcal{U}_j,$$

where the inequality is strict if and only if the short-sale constraint binds for $B_j^U(g)$.

Optimal portfolios give rise to standard asset pricing relationships: marginal investors price bonds such that bond yields are equal to state-contingent marginal rates of substitution. If no investor acquires information, marginal rates of substitution are independent of the state and this relationship holds for all investors in every state. If some investors acquire information, only informed investors are marginal in every state, while uninformed investors instead may cease to bid at the high price in order to escape the winner's curse.

The following analytical example illustrates the proposition by considering the special case where investors hold no bonds in Country 2. This assumption allows us to write down tractable versions of the relevant marginal rates of substitution. Asymmetric information introduces portfolio differences in *all* states even though the winner's curse only applies to bids at the high price. This is because such bids are accepted in all states, thereby altering marginal incentives to bid at the low price even when such bids are effectively state-contingent.

Example 1. *Let* $D_2 = 0$*. For informed investors,* $i \in \mathcal{I}_1$*, the relevant MRS in state* θ_1 *is*

$$M_1^i(\theta_1) = \frac{\kappa_1(\theta_1)u'\Big(W - P_1(\theta_1)B_1^i(\theta_1)\Big)}{(1 - \kappa_1(\theta_1))u'\Big(W + (1 - P_1(\theta_1))B_1^i(\theta_1)\Big)}.$$

and is state-separable, i.e. it does not depend on bids at the other marginal price.

For uninformed investors, $i \in \mathcal{U}_1$, the relevant MRS for bids at $P_1(g)$ is

$$M_1^i(g) = \frac{\begin{array}{l} f_1(g)\kappa_1(g)u'\Big(W - P_1(g)B_1^i(g)\Big) \\ \quad + f_1(b)\kappa_1(b)u'\Big(W - P_1(g)B_1^i(g)) - P_1(b)B_1^i(b)\Big) \end{array}}{\begin{array}{l} f_1(g)(1-\kappa_1(g))u'\Big(W + (1-P_1(g))B_1^i(g)\Big) \\ \quad + f_1(b)(1-\kappa_1(b))u'\Big(W + (1-P_1(g))B_1^i(g) + (1-P_1(b))B_1^i(b)\Big) \end{array}}$$

and is not separable across states, while the relevant MRS for bids at $P_1(b)$ is

$$M_1^i(b) = \frac{\kappa_1(b)u'\Big(W - P_1(g)B_1^i(g)) - P_1(b)B_1^i(b)\Big)}{(1-\kappa_1(b))u'\Big(W + (1-P_1(g))B_1^i(g) + (1-P_1(b))B_1^i(b)\Big)}$$

and takes into account that uninformed bids at $P_1(g)$ are also accepted in the bad state.

3.1 Within-Country Effects of Asymmetric Information

We now characterize how asymmetric information affects portfolios and prices *within* a specific country (say Country 1). To isolate within-country effects, we assume that all investors are uninformed and hold a fixed portfolio of bonds in the other country (Country 2). We relax this assumption in the next section, where we study optimal global portfolios.

To simplify notation, we use superscripts I and U to denote informed and uninformed investors in Country 1, respectively, and define $\bar{P}_1$ to be the equilibrium price that obtains in Country 1 when there are no informed investors. In a slight abuse of notation, we will index equilibrium outcomes by n_1, the share of informed investors in Country 1. The case with $n_1 = 0$ is the *uninformed regime* and the case with $n_1 > 0$ is the *informed regime*.

We first study the effects of exogenous variation in the share of informed investors n_1 on optimal portfolios and prices. When there are informed investors there is price dispersion and uninformed investors shy away from bidding at the high price because these bids are also accepted in the bad state, with high default probabilities.

Proposition 2 (Portfolios and Price Dispersion)**.** *Assume there are n_1 informed investors in Country 1, and let all investors hold the same portfolio in country 2. Then in Country 1:*

1. *Informed investors spend more in the good state that uninformed investors and less in the bad state,* $X_1^I(g) > X_1^U(g)$ *and* $X_1^I(b) \leq X_1^U(b)$*. The second inequality is strict if and only if uninformed investors submit bids at the high marginal price,* $B_1^U(g) > 0$*.*

2. *The high-state marginal price* $P_1(g)$ *is strictly increasing in the share of informed investors in Country 1 and converges to the uninformed equilibrium price as* $n_1 \to 0$*.*

3. *The bad-state marginal price* $P_1(b)$ *is strictly lower than the uninformed equilibrium price* $\bar{P}_1$ *for all* $n_1 > 0$ *and* $\lim_{n_1 \to 0} P_1(b) < \bar{P}_1$*.*

Uninformed investors submit fewer bids at the high marginal price due to the winner's curse, and thus spend less than informed investors in the good state. By the market-clearing condition, the high-state marginal price is thus strictly increasing in n_1. Because uninformed bids at the high price are also accepted in the bad state and uninformed investors can purchase bonds at $P_1(b)$ without being adversely selected, their total *expenditures* on bonds in the bad state are higher than for informed investors. The comparative statics of the low marginal price with respect to n_1 are more involved. There are two competing effects. First, informed investors spend less in the bad state which contributes to a decline in $P_1(b)$. Second, holding bids fixed, uninformed expenditures are *increasing* in n_1. This is because $P_1(g)$ is increasing in n_1 and uninformed bids at $P_1(g)$ are also executed in the bad state. This effect thus pushes the price up. The total effect depends on number of uninformed bids submitted at the high price, which in turn responds endogenously to the extent of the winner's curse. In sum, $P_1(b)$ may be non-monotonic in n_1. We will return to this issue when discussing *expected average bond prices* below. Importantly, $P_1(b)$ lies below the uninformed price everywhere, and there is strict marginal price dispersion even when n_1 is vanishingly small. This feature of the model is an important driver of equilibrium multiplicity.

It is possible to derive closed-form solutions for equilibrium prices in our analytical example with $D_2 = 0$. The example show that bonds offer a risk premium that depends on the level of debt relative to investor wealth. Moreover, price differences in the limit $n_1 \to 0$ depend on the variance of default probabilities through $\kappa_1(b) - \bar{k}_1$.

Example 1 (Continued). *Let* $D_2 = 0$ *and* $u(\cdot) = \log(\cdot)$*.In the uninformed regime with a unique marginal price, uninformed demand is* $\bar{B}_1^U = \frac{(1-\bar{\kappa}_1-\bar{P}_1)W}{\bar{P}_1(1-\bar{P}_1)}$ *and the marginal price is* *such that $\bar{P}_1 \bar{B}_1^U = D$. Hence the uninformed equilibrium price is*

$$\bar{P}_1 = 1 - \frac{\bar{\kappa}_1 W}{W - D}.$$

In the informed regime, informed investor demand is $B_1^I(\theta_1) = \frac{(1-\kappa_1(\theta_1) - P_1(\theta_1))W}{P_1(\theta_1)(1-P_1(\theta_1))}$ and, by market-clearing, prices in the limit with no information are given by

$$\lim_{n_1 \to 0} P_1(g) = \bar{P}_1 \qquad \lim_{n_1 \to 0} P_1(b) = 1 - \frac{\kappa_1(b)W}{W - D + \frac{\kappa_1(b) - \bar{\kappa}_1}{1 - \bar{\kappa}_1} D}.$$

In the full-information limit where $n_1 \to 1$, informed regime prices satisfy

$$\lim_{n_1 \to 1} P_1(g) = 1 - \frac{\kappa_1(g)W}{W - D} \qquad \lim_{n_1 \to 1} P_1(b) = 1 - \frac{\kappa_1(g)W}{W - D}.$$

Figure 1 further illustrates the proposition for the entire range of n_1 using the numerical example introduced in Definition 4 where $D_2 > 0$. We hold prices and bids in Country 2 fixed at the level that would obtain in an equilibrium where there are no informed investors. In addition to the marginal prices in each state, of relevance to government finances is the *expected average price* $\mathbb{E}[P_1]$ the government receives. This depends on the mix of bids submitted in both states and the share of informed investors. In the good state, all accepted bids are executed at $P_1(g)$. In the bad state, some uninformed bids are executed at $P_1(g)$ and the remainder is executed at $P_1(b)$,

$$\mathbb{E}[P_1] \equiv f_1(g) P_1(g) + f_1(b) \left(\frac{(1 - n_1) B_1^U(g) P_1(g) + \left((1 - n) B_1^U(b) + n_1 B_1^I(b) \right) P_1(b)}{(1 - n) \left(B_1^U(g) + B_1^U(b) \right) + n_1 B_1^I(b)} \right)$$

The horizontal line shows the uninformed equilibrium price $\bar{P}_1$. The marginal price $P_1(g)$ is monotonically increasing in n_1, and converges to $\bar{P}_1$ as the share of informed investors approaches zero. In the given example, $P_1(b)$ is strictly decreasing and expected average prices lies strictly below the uninformed equilibrium price unless the share of informed investors is very close to one. This is because the discount the government must offer to risk-averse investors in the bad state is greater than the premium it can charge in the good state. Moreover, price differences between states are sufficiently large enough that uninformed investors withdraw from bidding at the high price very quickly.

The fact that the average price is below the uninformed price is not a general re-

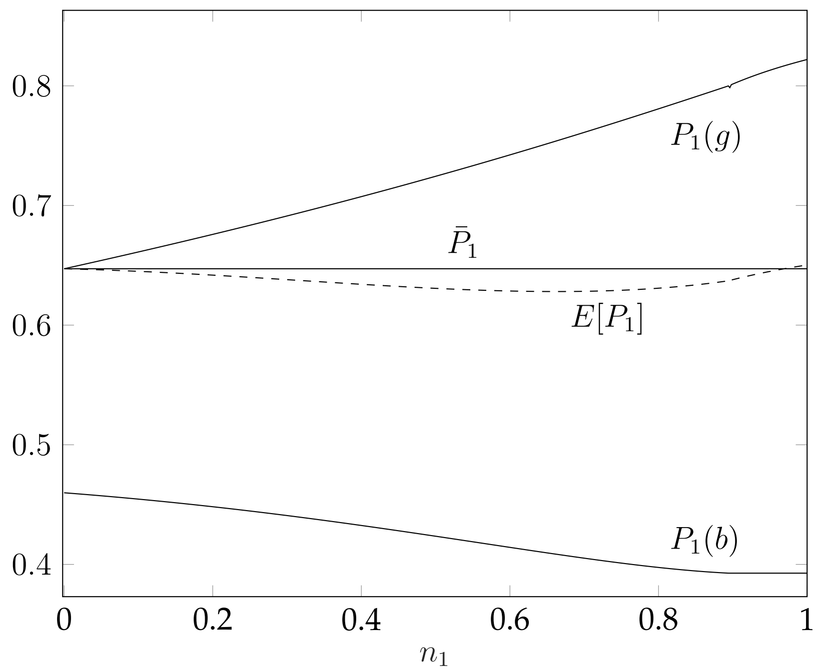

Figure 1: Prices in Country 1 as a function of n_1 given a fixed bond portfolion in Country 2.

sult. If price differences are relatively small (for example because default probabilities do not vary much by state), uninformed investors continue to submit relatively many bids at the high price even when n_1 is relatively large. In this case, the government can capture a part of the winner's curse by executing high-price bids even in the bad state, and the average price may be above the uninformed price. However, the government always faces more price volatility when investors acquire information. This is relevant for a theory of crises where the focus is on bad states. Additionally, the costs of price volatility are larger in a more general model where default probabilities increase with the government's need to issue debt at lower prices.

3.1.1 Endogenous Information Acquisition and the Value of Information

So far we have taken the share of informed investors as given. We now study how it is determined in equilibrium. Since all investors are uninformed in Country 2, let $K \equiv C(1,0)$ denote the marginal cost of acquiring information in Country 1. Fixing Country 2 portfolios, the value of information in Country 1 is

$$\Delta V(n_1) = V^I(n_1) - V^U(n_1).$$

In the informed regime, $\Delta V(n_1)$ is the *equilibrium* difference in expected utility obtained by informed and uninformed investors. In the uninformed regime, $\Delta V(0)$ is the *counterfactual* expected utility gain achieved by a single deviating investor who becomes informed but does not alter equilibrium prices. We refer to this value as $\bar{\Delta}V$. This leads to the following self-evident result.

Proposition 3. *(Information Acquisition) It is strictly optimal to learn θ_1 if $\Delta V(n_1) > K$. There exists an equilibrium without information acquisition if and only if $\bar{\Delta}V \leq K$, and there exists an equilibrium with information acquisition if and only if $\Delta V(n_1) \geq K$ for some $n_1 > 0$. Any equilibrium with an interior share of informed investors, $n_1^* \in (0,1)$, must satisfy $\Delta V(n_1^*) = K$.*

The difficulty lies in computing the value of information, since it depends on the share of informed investors through its effect on prices. The next result shows that that information acquisition is a strategic complement if the share of informed investors is sufficiently small. Furthermore, the discontinuous change in marginal price schedules at $n_1 = 0$ (see Proposition 2 for the derivation) generates a discontinuous change in the value of information at $n_1 = 0$. This feature of the auction protocol allows for the co-existence of the informed and uninformed regime for appropriate information costs.

Proposition 4 (Complementarity and Multiplicity)**.** *There exists a threshold share of informed investors $\bar{n}_1 > 0$ such that the value of information is strictly higher if $n_1 \in (0, \bar{n}_1]$ than if $n_1 = 0$. The informed and uninformed regime co-exist if and only if $K \in [\bar{\Delta}V, \max_{n_1} \Delta V(n_1)]$. The maximal share of informed investors is decreasing in K.*

Our example allows us compute the value of information in closed form.

Example 1 (Continued)**.** *In the uninformed regime, uninformed investors' consumption is $(1-\bar{\kappa}_1)W/\bar{P}_1$ after repayment and $\bar{\kappa}_1 W/(1-\bar{P}_1)$ after default. The counterfactual informed investor's consumption is $(1-\kappa_1(\theta_1))W/\bar{P}_1$ after repayment and $\kappa_1(\theta_1)W/(1-\bar{P}_1)$ after default. Hence the value of information is*

$$\bar{\Delta}V = \sum_{\theta_1} f_1(\theta_1)\left[\log(\kappa_1(\theta_1)^{\kappa_1(\theta_1)}(1-\kappa_1(\theta_1))^{1-\kappa_1(\theta_1)}\right] - \log(\bar{\kappa_1}^{\bar{\kappa_1}}(1-\bar{\kappa_1})^{1-\bar{\kappa_1}}),$$

and is strictly positive and strictly increasing in a mean-preserving spread of default probabilities around $\bar{\kappa}_1$ by the the strict convexity of $\log(\kappa^{\kappa}(1-\kappa)^{1-\kappa})$ on $(0,1)$.

Next consider the limit of the informed regime as $n_1 \to 0$. Market clearing requires that uninformed investors continue to purchase essentially all bonds in all states. Hence, in this limit, they achieve the same utility as in the uniformed regime. This is not true for informed investors, who may submit bids at two distinct marginal prices. The resulting consumption profile in state θ_1 is $(1-\kappa_1(\theta_1))W/P_1(\theta_1)$ after repayment and $\kappa_1(\theta_1)W/(1-P_1(\theta_1)$ after default. Hence the value of information is

$$\lim_{n_1 \to 0} \Delta V(n_1) = \Delta V(0) + f_1(b) \lim_{n_1 \to 0} \log \left(\frac{\bar{P}_1}{P_1(b)} \right)^{1-\kappa_1(b)} \left(\frac{1-\bar{P}_1}{1-P_1(b)} \right)^{\kappa_1(b)} .$$

It is easy to verify that the second term is strictly positive because $\lim_{n_1 \to 0} P_1(b) < \bar{P}_1$.

The example highlights that fundamental volatility raises the value of information. This is because fundamental volatility creates volatility in optimal state-contingent bidding strategies. Since only informed investors can submit state-contingent bids, this raises the benefit of being informed. Below we use this observation to argue that (small) fundamental shocks can trigger switches in the information regime.

Figure 2 illustrates the proposition for the whole range of n_1 using our baseline numerical example. We plot the value of information in the uninformed and informed regime, and parameters are as in Definition 4. The value of information jumps at $n_1 = 0$ as the information regime switches from uninformed to informed.[9] Within the informed regime, it is non-monotonic due to the interaction of two forces. On the one hand, an increase in n_1 raises the price spread $P_1(g) - P_1(b)$ and, thus, the severity of the winner's curse for the uninformed investor. This raises the value of information and leads to a strategic complementarity in information acquisition. On the other hand, an increase in n_1 strengthens competition for good bonds among informed investors, dissipating rents on infra-marginal bond purchases. The first force dominates if n_1 is small, and the second force dominates if n_1 is large. This is due to a composition effect: the share of uninformed bids at the high price declines as $P_1(g)$ increases with n_1. The slope of this decline determines the comparative statics of the value of information.

Which type of shocks can induce information acquisition in a country? One trivial possibility is that the cost of information falls. A more interesting one is that the

[9] In Cole, Neuhann, and Ordoñez (2020) we augment the one-country auction model with a demand shock similar to Grossman and Stiglitz (1980), and show this smoothes the discontinuity in the value of information at $n = 0$ while preserving the strategic complementarity in information acquisition as well as the scope for equilibrium multiplicity.

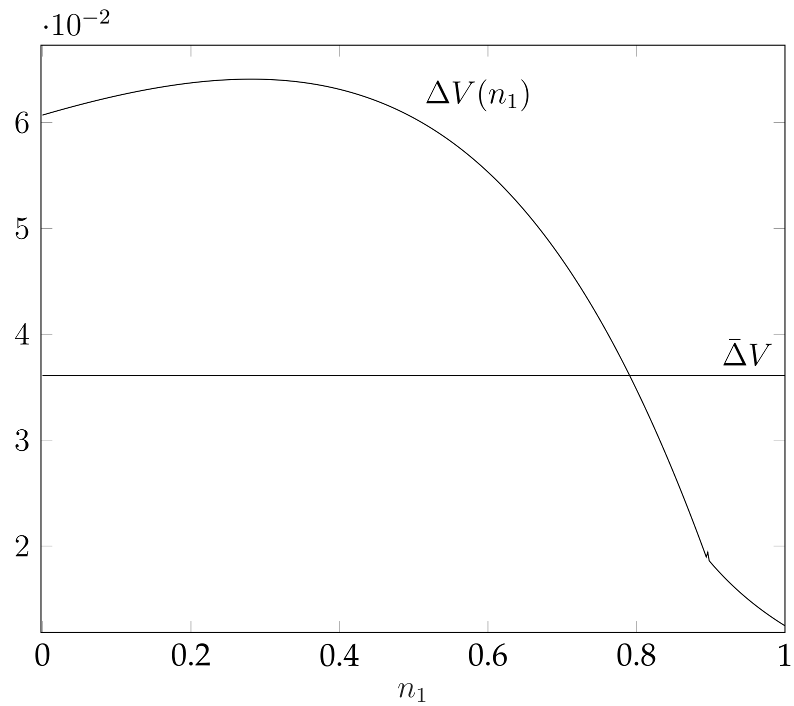

Figure 2: The value of information in Country 1 as a function of n_1.

value of information increases because default risk rises. Figure 3 plots the value of information in the uninformed regime and in the informed regime in the limit $n_1 \to 0$ as a function of the bad-state default probability $\kappa_1(b)$. An increase in $\kappa_1(b)$ raises default risk and increases the variance of default risk across states. An equilibrium with information exists if the value of information exceeds its cost K for some value of n_1. The solid black lines show the value of information in both the informed and uninformed regimes. The regions in which an informed equilibrium exists thus expands as default risk rises. (The argument extends analogously to plotting the value of information at $n_1 >> 0$).

3.2 Cross-Country Spillovers through Risk and Information

We now study optimally chosen portfolios in both countries and characterize three distinct mechanisms of cross-country spillovers, by which we mean the notion that shocks to one country affect prices in the other country.

The first channel is independent of information effects and instead relies only on changes in *risk appetite* due to decreasing absolute risk aversion. We establish this

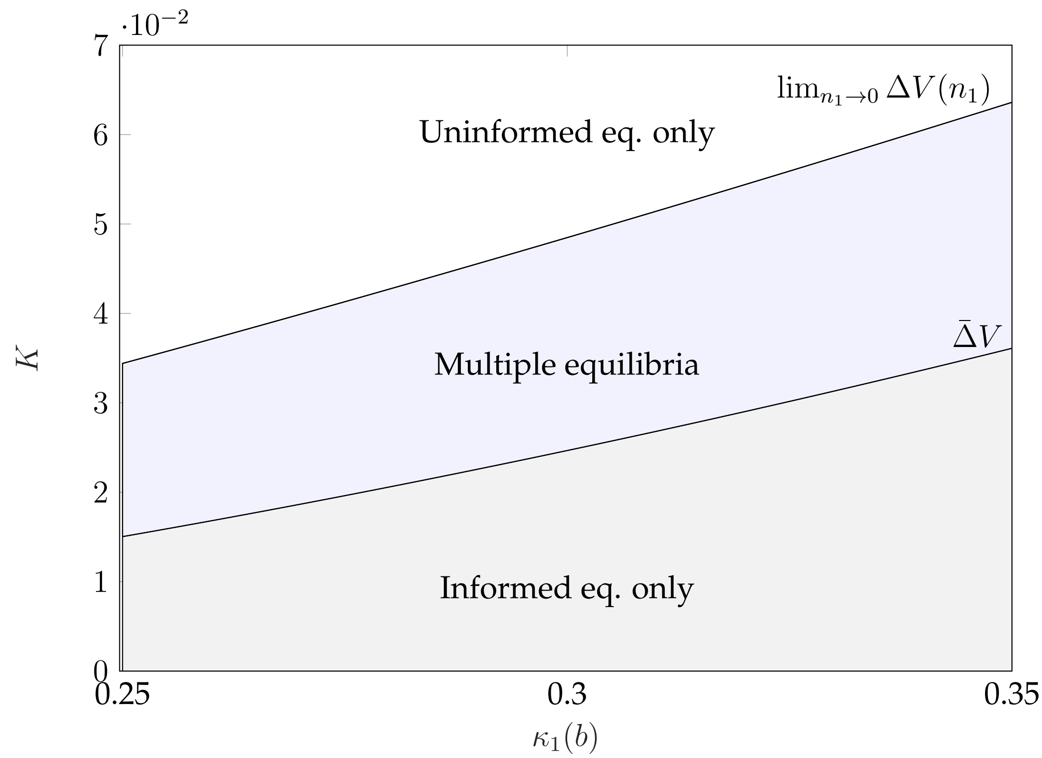

Figure 3: Information regimes in Country 1 given $\kappa_1(b)$.

channel by showing that changes in default risk in one country may affect prices in the other country even when no investor is informed. The second channel is *information-based* and operates within a fixed information regime. We show that increasing the share of informed investors in one country affects asset prices in the other country even when there is no change in the informed share in that country. This channel also leads to retrenchment of capital and hampers cross-country diversification, and it is fully independent of the risk-appetite channel. We show this using a second-order approximation to optimal portfolios that mechanically shuts down the effects of decreasing absolute risk aversion. The third channel relies on spillovers of *information regimes*: an increase in the share of informed investors in one country can result in a switch to the informed regime in the other country.

3.2.1 Spillovers through Risk Appetite

We now establish that *endogenous changes in risk appetite* can lead to simultaneous movements in all countries' prices in response to fundamental shocks in a single country. Specifically, we study the effects of a shock to the default probability in one country (say Country 1) on the price of bonds in the other country (Country 2). To show that this mechanism is independent of asymmetric information, we assume that no investor is informed in any country and we simplify notation by dropping superscripts indicating investors types.

Figure 4 illustrates price comovement by plotting equilibrium prices as a function of Country 1's unconditional default probability $\bar{\kappa}_1$, maintaining fixed Country 2's unconditional default probability $\bar{\kappa}_2$. We use the baseline parameters from Definition 4 and log utility. Prices decline in both countries, but fall more steeply in Country 1. The strength of this correlation intensifies at larger coefficients of risk aversion.

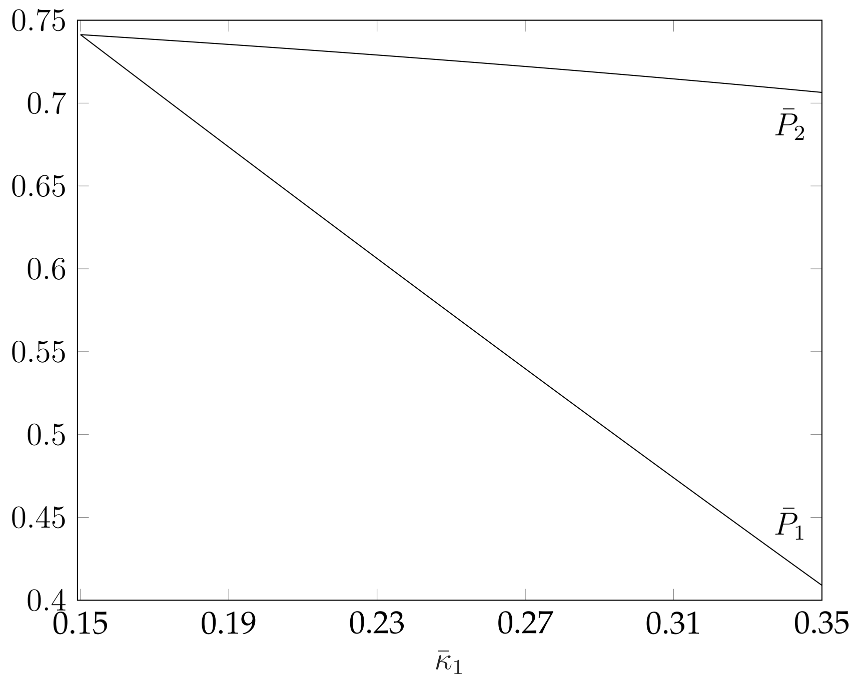

Figure 4: Prices in Uninformed Equilibrium as a function of $\bar{\kappa}_1$.

We formalize next the conditions under which risk appetite spillovers occur, and discuss the central role for decreasing absolute risk aversion in mediating this channel. By Proposition 1, define the marginal *net benefit* of investing in country j as

$$F_j = \frac{1 - P_j}{P_j} - M_j, \tag{3}$$

and recall that equilibrium is such that $F_1 = F_2 = 0$. Notice that the first term in

F_j is simply the yield, and that, when all investors are uninformed, we can further decompose $M_j = m_j(1)\kappa_j/m_j(0)(1-\kappa_j)$. The response of prices *in both countries* to a marginal change in Country 1's unconditional default probability $\bar{\kappa}_1$ is given by

$$\begin{bmatrix} \frac{\partial P_1}{\partial \bar{\kappa}_1} \\ \frac{\partial P_2}{\partial \bar{\kappa}_1} \end{bmatrix} = \frac{1}{\frac{\partial F_1}{\partial P_1}\frac{\partial F_2}{\partial P_2} - \frac{\partial F_1}{\partial P_2}\frac{\partial F_2}{\partial P_1}} \begin{bmatrix} -\frac{\partial F_2}{\partial P_2}\frac{\partial F_1}{\partial \bar{\kappa}_1} + \frac{\partial F_1}{\partial P_2}\frac{\partial F_2}{\partial \bar{\kappa}_1} \\ -\frac{\partial F_1}{\partial P_1}\frac{\partial F_2}{\partial \bar{\kappa}_1} + \frac{\partial F_2}{\partial P_1}\frac{\partial F_1}{\partial \bar{\kappa}_1} \end{bmatrix}.$$

Price effects can be decomposed into four components. Two operate within-country. The first is the change in the net benefit of investing in j given a change in j's default probability, $\partial F_j/\partial \kappa_j$. The second is the change in the net benefit of investing in j given a change in country j's price, $\partial F_j/\partial P_j$. Both of these effects are naturally negative if investors are risk averse. That is, $\partial F_j/\partial \kappa_j < 0$ and $\partial F_j/\partial P_j < 0$ for any strictly concave utility function. The other two are subtle cross-country effects. Importantly, their sign and magnitude depends on the third derivative of the utility function. The first is the *default risk contagion channel*, defined as the change in the net benefit of investing in j given a change in the other country's default probability, $\partial F_j/\partial \kappa_{-j}$. There is contagion of this sort when an increase in κ_{-j} lowers the net benefit of investing in j, i.e. $\partial F_j/\partial \kappa_{-j} < 0$. The second is the *pure price contagion channel*, defined as the change in the net benefit of investing in j given a change in the other country's bond price, $\partial F_j/\partial P_{-j}$. This sort of contagion happens if a decrease in P_{-j} decreases the benefit of investing in country j, i.e. $\partial F_j/\partial P_{-j} > 0$. The next Proposition formalizes the conditions for risk-based spillovers.

Proposition 5 (Risk-based contagion)**.** *Assume there are no informed investors in either country. Then the following statements hold:*

(i) An increase in κ_1 simultaneously decreases prices in both countries if and only if

$$\left[\frac{\partial F_1}{\partial P_1}\frac{\partial F_2}{\partial P_2} - \frac{\partial F_1}{\partial P_2}\frac{\partial F_2}{\partial P_1}\right]\left[-\frac{\partial F_{-j}}{\partial P_{-j}}\frac{\partial F_j}{\partial \bar{\kappa}_1} + \frac{\partial F_j}{\partial P_{-j}}\frac{\partial F_{-j}}{\partial \bar{\kappa}_1}\right] < 0 \quad \textit{for all } j.$$

(ii) There is contagion through the default risk channel ($\partial F_j/\partial \kappa_{-j} < 0$) if and only if there is decreasing absolute risk aversion. There is no cross-country contagion ($\partial F_j/\partial \kappa_{-j} = \partial F_j/\partial P_{-j} = 0$) if and only if there is constant absolute risk aversion.

(iii) Under decreasing absolute risk aversion, a shock to $\bar{\kappa}_1$ that lowers P_1 also lowers P_2 if

$$\frac{\partial F_j}{\partial P_{-j}} > \frac{\frac{\partial F_{-j}}{\partial P_{-j}}\frac{\partial F_j}{\partial \bar{\kappa}_1}}{\frac{\partial F_{-j}}{\partial \bar{\kappa}_1}} \quad \textit{where} \quad \frac{\frac{\partial F_{-j}}{\partial P_{-j}}\frac{\partial F_j}{\partial \bar{\kappa}_1}}{\frac{\partial F_{-j}}{\partial \bar{\kappa}_1}} < 0.$$

The proposition first states a general necessary and sufficient condition for contagion. The first term on the left-hand side compares the magnitude of within-country price effects with cross-country price effects, while second term determines the magnitude of contagion due to default risk. The second statement in the proposition shows that the sign of the default risk contagion channel is determined by the properties of investors' absolute risk aversion: there is contagion through default risk if and only if there is decreasing absolute risk aversion (DARA). This is because an increase in default risk places more weight on states with low consumption. Under DARA, this leads to an increase in average risk aversion and a higher required risk premium. With constant absolute risk aversion (CARA) instead, a level change in consumption does not change the required risk premium and prices are perfectly insulated from fundamental shocks in the other country.

The sign of the pure price contagion channel is $\partial F_j / \partial P_{-j}$ is ambiguous under DARA. To account for this, the third statement provides a sufficient condition for simultaneous price decreases which ensures that any positive spillovers from pure price contagion do not not outweigh the negative spillovers from default risk contagion. (We provide sufficient conditions for negative price spillovers below.)

The intuition for the ambiguous sign is as follows. By market-clearing, total expenditures in each country are fixed at D_j. Thus, a price decrease in country j does not alter consumption after default but raises consumption after repayment. This creates two forces. First, it makes the investor wealthier in some states of the world, raising the willingness to buy more bonds in country j. Second, it creates dispersion in consumption that raises risk aversion on average. The first *wealth force* tends to encourage buying bonds in country j. The second *risk aversion force* tends to discourage buying bonds in country j. We find that the first effect dominates when debt levels are small and there is little dispersion in marginal utility, while the second effect dominates when debt levels are high and marginal utility is very steep after a default.

Specifically, the next corollary shows that the sign of the pure price contagion channel is determined by a novel twisted definition of absolute risk aversion that takes into account the dispersion of consumption induced by the other country's default decision. It also provides conditions such that that the risk-aversion force dominates and there is indeed pure price contagion.

Corollary 1. *Let $w = W - D_1 - D_2$ denote consumption after a default in both countries. There is pure price contagion, $\partial F_j / \partial P_{-j} > 0$, if and only if*

$$\frac{-u''(w + B_{-j})}{(1 - \bar{\kappa}_{-j})u'(w + B_{-j}) + \bar{\kappa}_{-j}u'(w)} < \frac{-u''(w + B_j + B_{-j})}{(1 - \bar{\kappa}_{-j})u'(w + B_j + B_{-j}) + \bar{\kappa}_{-j}u'(w + B_j)}.$$

This condition is violated if D_{-j} is sufficiently close to zero, and it is satisfied if $D_1 + D_2$ is sufficiently close to W.

The corollary provides a sufficient condition for prices to co-move through the pure price contagion channel by ensuring that the third statement of Proposition 5 holds. The condition is similar to the standard definition of absolute risk aversion, but the denominator accounts for contagion by taking a weighted average over marginal utility after default and repayment in the other country. Importantly, it is more likely to be satisfied when agents place a higher weight on the low-consumption state where both countries default (high $\bar{\kappa}_{-j}$) or if wealth after default is very low after a simultaneous default (low w). Hence our model predicts particularly strong spillovers when global debt burdens are high and fundamentals are poor.

3.2.2 Spillovers through Asymmetric Information

Next we show that variation in the share of informed investors in one country affects prices in both countries. Information regimes are fixed. For simplicity, no investor is informed in Country 2 (uninformed regime), but a fraction n_1 is informed in Country 1 (informed regime). To highlight that this channel is independent of the portfolio-risk effect discussed in the previous proposition, we study a second-order approximation of the optimal portfolio problem. Specifically, we assume that the utility function satisfies constant relative risk aversion (CRRA) with risk-aversion coefficient γ, and approximate around zero bond holdings. We recover optimal portfolios that are functions of the mean return and return volatility of bonds at a given marginal price only. We use I to index investors with information in Country 1, and U to index investors without any information. An important aspect of the mechanism is that asymmetric information leads to market segmentation which strengthens as default risk increases.

The realized rate of a return on a country-j bond bought in state θ_j at price $P_j(\theta_j)$ given default decision δ_j is $R_j(\theta_j, \delta_j) = \frac{1-\delta_j - P_j(\theta_j)}{P_j(\theta_j)}$. We define $\widehat{R}^i_j(\theta_j) \equiv \mathbb{E}[R_j(\theta_j, \delta_j)|\mathcal{F}^i]$ and $\widehat{\sigma}^i_j(\theta_j) \equiv \sqrt{\mathbb{V}[R_j(\theta_j, \delta_j)|\mathcal{F}^i]}$ to be the expected return and standard deviation of a Country-j bond purchased at marginal price $P_j(\theta_j)$ given information set $\mathcal{F}^i$. These may differ across differentially informed investors. The associated Sharpe ratio is

$$S^i_j(\theta_j) = \frac{\widehat{R}^i_j(\theta_j)}{\widehat{\sigma}^i_j(\theta_j)}.$$

It is immediate that uninformed investors expect a lower Sharpe ratio when bidding at the high price as long as expected default probabilities are below 50%.

Lemma 2. *Let $\bar{\kappa}_1 < \frac{1}{2}$. For $\theta_1 = g$, $S^I_1(\theta_1) > S^U_1(\theta_1)$ and $\frac{\partial(S^I_1(\theta_1) - S^U_1(\theta_1))}{\partial P_1(g)} < 0$.*

Uninformed investors face a unfavorable risk-return trade-off in the high state because bids at the high price are also accepted in the bad state. This raises expected default probabilities on bonds purchased at $P_1(g)$. We restrict attention to $\bar{\kappa}_1 < \frac{1}{2}$ because increasing default risk would lower volatility otherwise, and clutter the underlying forces. The next result characterizes optimal portfolios given the approximation. Denote portfolio shares scaled by the coefficient of risk aversion by

$$\omega^i_j(\theta_j) \equiv \frac{\gamma P_j(\theta_j) B^i_j(\theta_j)}{W}.$$

To simplify notation, let $s^i_j(\theta_j) \equiv \frac{S^i_j(\theta_j)^2}{1+S^i_j(\theta_j)^2}$ denote a scaled version of the state-contingent Sharpe ratio and $s^i_j \equiv \sum_{\theta_j} f_j(\theta_j) s^i_j(\theta_j)$ its expectation over states for country j.

Proposition 6 (Segmentation). *Up to second order, investor i's optimal portfolio satisfies*

$$\omega^i_1(g) = \frac{s^i_1(g)}{\widehat{R}^i_1(g)}\left(\frac{1-s^i_2}{1-s^i_1 s^i_2}\right), \qquad \omega^i_1(b) = \frac{s^i_1(b)}{\widehat{R}^i_1(b)}\left(\frac{1-s^i_2}{1-s^i_1 s^i_2}\right), \quad \textit{and} \quad \omega^i_2 = \frac{s^i_2}{\widehat{R}^i_2}\left(\frac{1-s^i_1}{1-s^i_1 s^i_2}\right).$$

If $\bar{\kappa}_1 < \frac{1}{2}$, then portfolios display segmentation: $\omega^U_1(g) < \omega^I_1(g)$, $\omega^U_2 > \omega^I_2$ and $\frac{\partial(\omega^U_2 - \omega^I_2)}{\partial P_1(g)} < 0$.

Optimal portfolios address standard risk and return trade-offs: bond purchases are increasing in own Sharpe ratios, and portfolio weights are determined by relative

Sharpe ratios. There is segmentation because uninformed investors face a worse risk-return trade-off when buying bonds at $P_1(g)$ and respond by allocating more funds to Country 2. Segmentation (differences in funds allocated to Country 2) decreases in $P_1(g)$ as it reduces the Sharpe ratios for informed faster given that the uniformed reduce participation in Country 1. Since $P_1(g)$ is increasing in the share of informed investors, more information in Country 1 induces more retrenchment to Country 2 by both informed and uninformed.

Figure 5 illustrates this result using the baseline numerical example from Definition 4. We plot the portfolio shares across the two countries, defined as the ratio of expenditures in each country over wealth W. Since portfolios are stochastic in Country 1, we plot expected portfolio shares for that country. By market-clearing, solid lines depict benchmark expenditure shares in either the uninformed regime ($n_1 = 0$) or the informed regime where all investors are informed in Country 1($n_1 = 1$).

As the share of informed investors in Country 1, n_1, increases, the uninformed faces more adverse selection and invest less in Country 1 and more in Country 2 (dotted lines diverge as n_1 increases), while informed investors face more competition from other informed and move in the opposite direction (solid lines converge as n_1 increases).

Importantly, when there are informed investors in Country 1, the informed pull back from Country 2 and invest more in Country 1 in order to exploit their information advantage. Uninformed investors, instead, pull back from Country 1 due to adverse selection and invest more in Country 2. This segmentation leads to less diversification and lower risk appetite globally. Notice also that informed investor expenditures in Country 1 are decreasing in n_1 because there is more competition for information rents as the share of informed investors increases. This reduces the profitability of investing in Country 1 relative to investing in the risk-free asset or in Country 2.

Figure 6 shows that segmentation interacts with decreasing absolute risk aversion to lower prices in both countries. As a benchmark, the horizontal lines in both panels show bond prices in the uninformed regime. Country 1's average bond price lies strictly below the uninformed equilibrium for the informational reasons laid out in the previous section. Importantly, Country 2's bond price also lies strictly below the uninformed price even though no investors acquires information in that country. This is due to lower diversification and lower risk appetite globally. Hence a

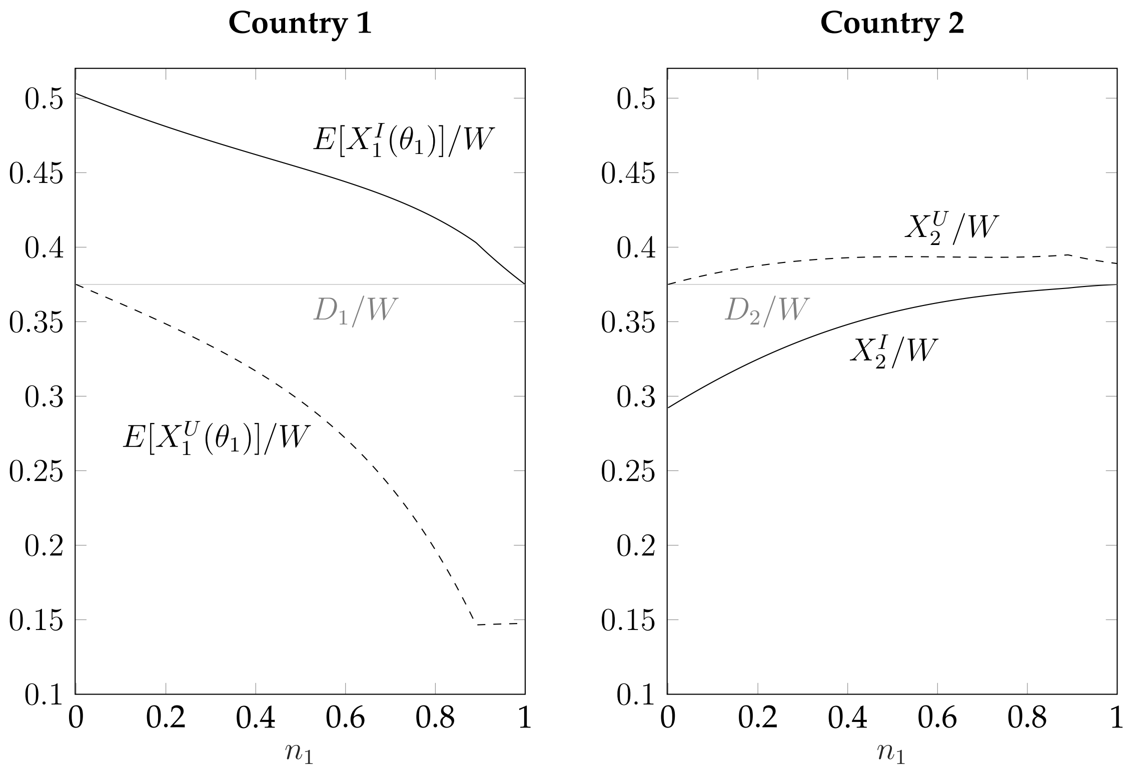

Figure 5: Effects of n_1 on portfolio shares across countries and investors.

country's bonds price is highest when no investor is informed in either country. This effect of information in one country on the price in another is what we refer to as *informational spillovers*.

3.2.3 Spillover of Information Regimes

We now show that changes in the share of informed investors in one country affect incentives to acquire information in the other country. We begin with our baseline model where there are n_1 informed investors in Country 1 and no informed investors in Country 2. We then compute incentives to become informed in Country 2. Since there are no other investors who are in informed in that country, we measure the value of information for a deviating investor whose individual information acquisiton decision does not alter asset prices. Given that there is asymmetric information in Country 1, we compute this value both for an investor who is informed in Country 1 (denoted by $\hat{\Delta}V^{\{1,1\}}(n_1)$) and one who is uninformed in Country 1 (denoted by $\hat{\Delta}V^{\{0,1\}}(n_1)$).Figure 7 plots these two functions in black. The gray lines show the

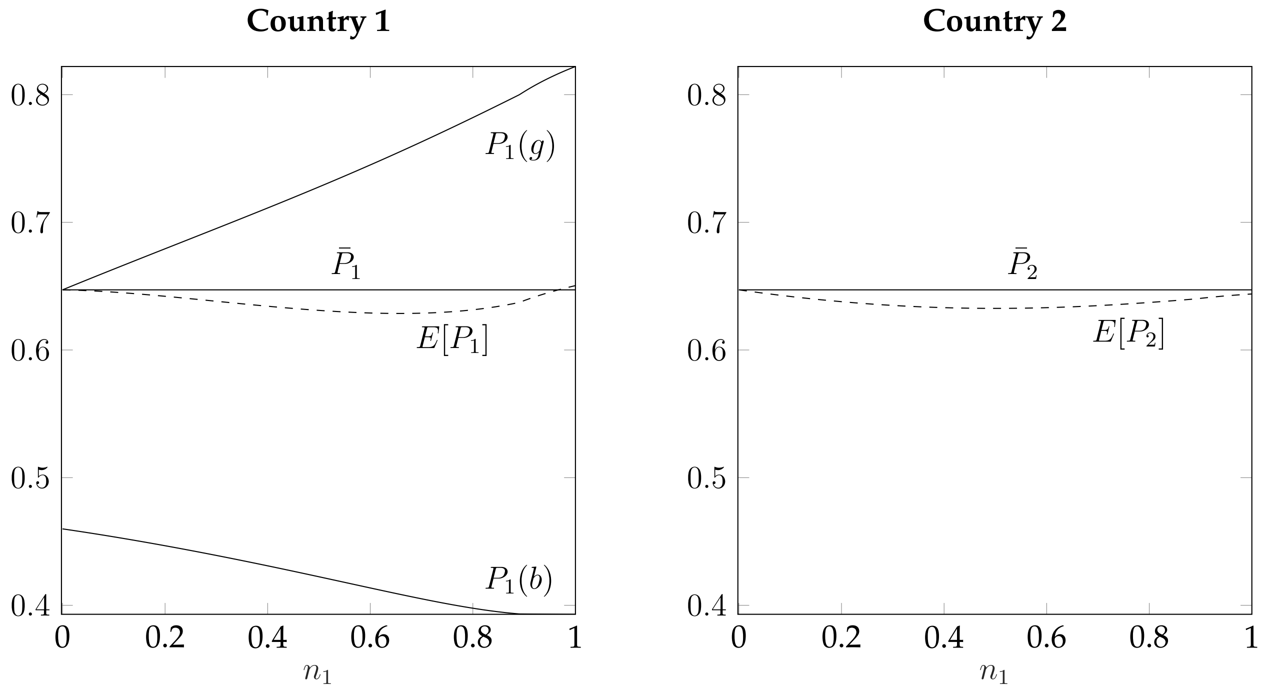

Figure 6: Prices in informed equilibrium as a function of n_1.

value of information in Country 1 that are familiar from Figure 2.

The incentive to acquire information in Country 2 is always strictly higher when there is some information in Country 1, and the additional incentive to become informed in a second country is smaller than the incentive to become informed in a first country. The intuition is that a country without informed investors becomes a "safe haven" where uninformed investors do not face adverse selection. Thus information acquisition in Country 1 leads to a migration of uninformed capital to Country 2. Since Country 2 now represents a higher share of uninformed investors' portfolio, they have a stronger incentives to acquire information in the "safe haven". The existence of informed investors thus begets further information acquisition, creating a novel channel of contagion through spillovers in the informational regime.

4 Equilibrium with Secondary Markets

We now consider the effects of secondary market trading on primary market prices and incentives to acquire information. Auction prices are public knowledge prior

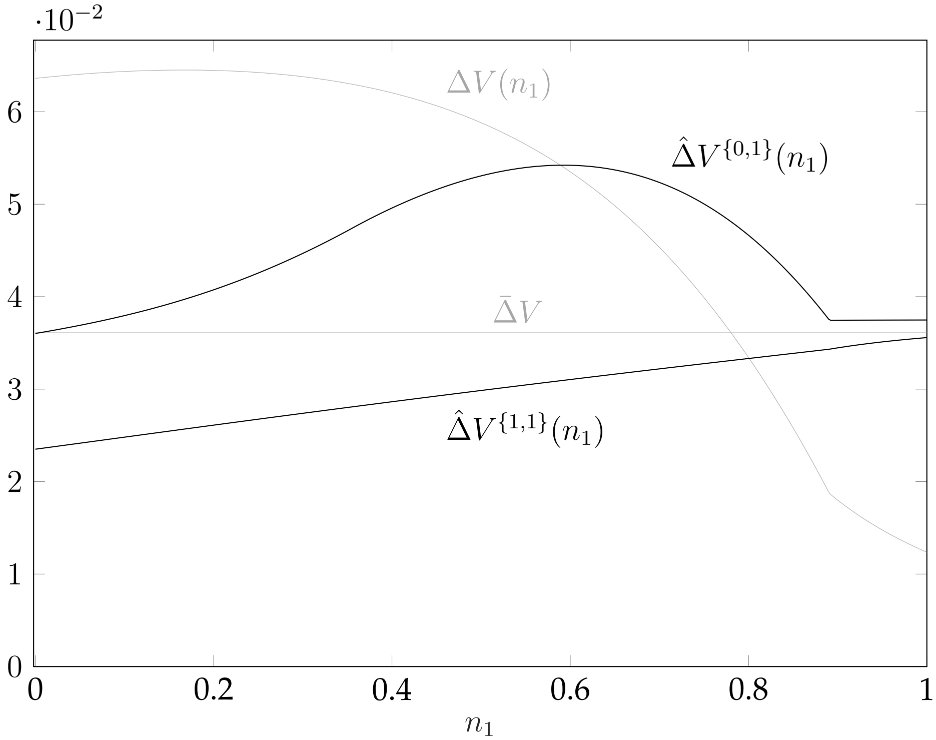

Figure 7: Value of Information as a function of n_1.

to the opening of the secondary market. Since marginal auction prices differ across states if some investors acquire information, there can be no asymmetric information in the secondary market, and the only motive for trade is reallocating differential risk exposure acquired at auction.

Lemma 3. *In every country, secondary markets operate under symmetric information.*

The fact that information is revealed prior to the secondary market might be interpreted to mean that information is worthless in the auction. In fact, the opposite is true. Because uninformed investors have the option to wait for the secondary market, they are less willing to participate in the auction but *are* willing to pay a mark-up to trade under symmetric information. This mark-up is earned by informed investors who buy bonds at the auction in order to sell them at a riskless arbitrage profit in the secondary market. We demonstrate these pricing patterns in the next proposition, where we denote equilibrium outcomes in the auction equilibrium by superscript A, i.e. $P_j^A(\theta)$ denotes auction prices in country j when there is no secondary market.

Proposition 7 (Equilibrium with Secondary Markets)**.** *With secondary markets, equilibrium prices satisfy:*

(i) If no investor acquires information in country j, then $P_j(\theta_j) = \hat{P}_j(\theta_j)$ for all θ_j and the equilibrium with secondary markets delivers the same prices and allocations as the auction equilibrium.

(ii) If some investors acquire information in country j, then informed investors earn arbitrage profits in the high state by buying low at the auction and selling high in the secondary market, but there are no arbitrage opportunities in the bad state. Prices satisfy $P_j(b) = \hat{P}_j(b)$ and $P_j(g) < \hat{P}_j(g)$.

(iii) As the share of informed investors in country j approaches zero, $n_j \to 0$, the limiting behavior of auction prices is the same as in the auction-only equilibrium, $\lim_{n_j \to 0} P_j(\theta_j) = \lim_{n_1 \to 0} P_j^A(\theta_j)$. Moreover, the good state features a strict arbitrage opportunity between primary and secondary markets in this limit, $\lim_{n_1 \to 0} P_j(g) < \lim_{n_1 \to 0} \hat{P}_j(\theta_j)$.

The first statement shows that the secondary market is irrelevant when there is no asymmetric information at the auction. This is because all investors are symmetric and so there is no trading motive in secondary markets. The second statement shows that there are arbitrage profits only in the high state. This is because uninformed investors face the winner's curse only when bidding at $P_1(g)$. Conversely, they are unwilling to pay a premium to escape adverse selection in the bad state.

Importantly, the third statement shows that the arbitrage persists even as the share of informed investors approaches zero. The intuition is simple: if primary and secondary market prices were to converge to each other, all uninformed investors would strictly prefer to wait for the secondary market rather than buy at the auction. This is inconsistent with market clearing when almost all investors are uninformed.

The next result maps these pricing patterns into implications for the value of information. We find that the effects of secondary markets are non-monotonic in n_1. When the share of informed investors n_1 is small, the value of information is strictly higher with secondary markets than in their absence. Hence informed equilibrium exists for a wider range of information costs, and secondary markets *amplify* the complementarity in information acquisition.

If sufficiently many investors have already acquired information and n_1 is large, information is impounded into prices more efficiently than in the absence of secondary markets, and the value of information is lower. As a point of comparison, we define the *full information auction equilibrium* to be the equilibrium that obtains when there no secondary markets and all investors are informed in Country 1.

Proposition 8 (Value of Information)**.** *When secondary markets open after the auction:*

(i) As $n_1 \to 0$, the value of information is strictly higher than without secondary markets.

(ii) The range of information costs for which an informed equilibrium exists is strictly larger.

(iii) If and only if $n_1 \geq \hat{n}_1 \equiv \frac{D_1}{W-D_2}$, the value of information is zero, the equilibrium with secondary markets delivers the same allocations and prices as the full information auction equilibrium, and there is no cross-market arbitrage, $P_j(\theta_j) = \hat{P}_j(\theta_j)$ for all θ_j.

(iv) Any equilibrium with endogenous information acquisition satisfies $n_1 < \hat{n}_1$.

Statements (i) and (ii) consider the value of information when the share of informed investors is small. In the absence of secondary markets, exploiting an information advantage requires taking a large position in a risky bond. When there is a secondary market, informed investors can purchase the same bond at a similar price in the primary market, *and* offload risk exposure in the secondary market while earning arbitrage profits. The range of information costs that can rationalize an informed equilibrium is thus necessarily greater and there may exist an informed equilibrium in the presence of secondary markets but not in their absence. Since information raises yields, the presence of liquid secondary markets may thus raise government's financing costs. This is contrary to conventional wisdom and common policy advice.

Statements (iii) and (iv) consider the case where the share of informed investors is relatively large, and shows that limits to arbitrage are endogenous: if there are sufficiently many informed investors willing to buy at auction and sell in the secondary market, price differences shrink and the arbitrage is eliminated. Informed investors then buy the entire primary market in the good state, and sell to uninformed investors in the secondary market at zero markup. The threshold is such that the wealth of informed investors is enough to purchase both countries' stock of debt outright. When there is no arbitrage, uninformed investors can trade as if they are informed, and the value of information is zero. Equilibria with endogenous information acquisition thus necessarily entail arbitrage, and costly information entails large arbitrage profits.

Figure 8 illustrates the proposition by showing that arbitrage profits harm the government by lowering auction prices. The left panel shows prices in Country 1, the right panel shows prices in Country 2. We show both the prices at the auction ($\hat{P}_1(\theta)$) and in the secondary markets ($P_1(\theta)$) for Country 1, and how they vary with n_1. We also show the corresponding prices in the model without secondary markets ($P_1^A(\theta)$) in grey, along with a horizontal line showing the uninformed equilibrium prices for comparison purposes. In Country 2, everyone is uninformed, so there is a single price schedule in which primary and secondary market prices coincide.

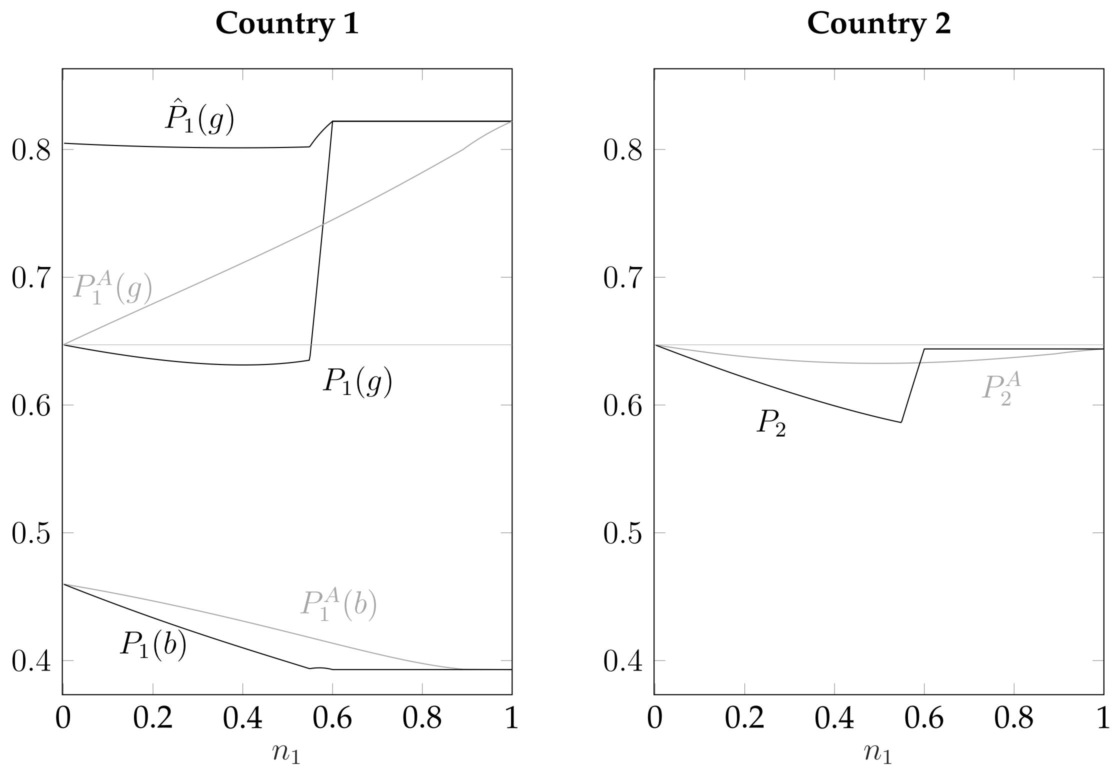

Figure 8: Effects of n_1 on prices and the value of information.

The most striking observation is that prices in primary markets are strictly lower *in all states* compared to both the uninformed equilibrium and the auction equilibrium without secondary markets as long as the share of informed investors is sufficiently small. Note that this is the relevant region when information acquisition is endogenous and the cost of information is not trivial. The intuition is as follows. In the presence of secondary markets, uninformed investors always have the option to trade under symmetric information by waiting out the auction. But when there are relatively few informed investors, the auction can clear only if some uninformed investors can be persuaded to participate in the auction. Given the benefit to waiting for the secondary market, this requires a sizable price discount at auction.

Since this mechanism primarily affects the good state where uninformed investors face adverse selection at auction, it can explain why even the good-state auction price is lower than in the uninformed equilibrium price. Nevertheless, this is a striking departure from standard models of information revelation where good news tends to raise prices while bad news tends to lower. Moreover, due to the auction protocol there are consequences for the bad state as well. Since $P_1(g)$ falls, uninformed bids at the high marginal price that are executed in the bad state now. Hence $P_1(b)$ must fall further to clear the market. This effect is exacerbated by the fact that uninformed investors also delay some high-price bids to the secondary market.

The introduction of secondary markets also magnifies spillover effects and adversely affects prices in Country 2. This is the case even though there is no motive to retrade bonds in a country where there is no asymmetric information. The spillover operates through capital reallocation. Informed investors earn arbitrage profits in Country 1. Hence it is optimal for them to reallocate more funds from Country 2 to Country 1. This mechanism is reminiscent of Proposition 6 where we showed that the informed spend less in Country 2 in order to take advantage of a more favorable risk-return trade-off in Country 1. With secondary markets, this effect is amplified because arbitrage profits are risk-free for the informed.

The arbitrage narrows as more investors become informed. Hence $\hat{P}_1(g)$ is initially declining in n_1, and uninformed investors respond by postponing more of their investments to the secondary market. At a certain point (around $n_1 = 0.55$ in our example), uninformed investors no longer submit bids at $P_1(g)$ in the auction. At this point, the gains from information decline dramatically as arbitrage opportunities are competed away. This generates the kink in the price schedules, as informed investors respond by shifting a share of their portfolio back to Country 2 because it is less attractive to forego diversification benefits to capture arbitrage rents. In contrast to the case without secondary markets, uninformed investors now *benefit* from more informed investors because it allows them to avoid adverse selection at lower cost. This lowers their overall portfolio risk and generates a relative increase in their demand for Country 2 bonds. Both effects combine to generate a reversal in the comparative statics of the price in Country 2.

Taken together, the impact of asymmetric information on primary market prices in the presence of secondary markets changes sharply around intermediate levels of n_1. When there are not many informed investors, secondary markets generate arbitrage opportunities for informed investors that magnify their reallocation of funds towards the informed country *and* allow uninformed uninformed to avoid the winner's curse. Both effects depress prices in both countries. On the other hand, uninformed investors benefit from secondary markets because they can buy bad bonds in primary and good bonds in secondary markets, as if they were informed. This allows the uninformed to take on more risk exposure overall, and leads to a better risk allocation. The latter effect dominates when n_1 is high and arbitrage spreads are low.

With endogenous costly information acquisition, any equilibrium satisfies $n_1^* < \hat{n}_1$. As long as the cost of information is not too low, the presence of secondary markets thus leads to strictly *lower* prices at auction *in all states and all countries*. Since government revenues are determined by the price in primary markets, our model provides a channel by which liquid aftermarkets can depress government revenues. One way to interpret this result is that secondary markets force a transfer of resources from the government to informed investors. Since these adverse affects are more pronounced when the share of investors is small, they are a particularly relevant concern in emerging market economies with more uncertainty and higher costs of information acquisition.

5 Conclusion

This paper constructs a simple model of portfolio choice with information acquisition by a global pool of risk-averse investors who can buy sovereign debt issued by a number of different countries in primary markets, and traded later in secondary markets. There are three novelties in our approach. First, we allows for endogenous asymmetric information about fundamental default risk. Second, we focus on primary markets and the role of commonly-used discriminatory price protocols in determining the equilibrium degree of information asymmetry and its impact on yields and spillovers. Third, we explore the implications of secondary markets, and their interaction with primary markets and asymmetric information.

In this setting we uncover three important sources of spillovers in sovereign bond spreads: First, spillovers do not require fundamental linkages or common factors, just a common pool of prudent investors who re-balance portfolios in response to country-specific default risk shocks. Second, asymmetric information generates spillovers through endogenous market segmentation: informed investors tend to invest more in the country in which they are informed, which generates price risk that increases background risk and affects bond prices globally. In this regard, we also show that endogenous price risk leads to complementarities in information acquisition. Finally, there are also spillovers on the incentives to acquire information: investors acquiring information about fundamentals in one country increases the likelihood that investors also want to become informed about the fundamentals in other countries, even without economies of scale in information acquisition. As information asymmetries lead to lower prices and higher volatility, all these novel sources of spillovers reinforce each other.

By introducing secondary markets and analyzing their interaction with primary markets in the presence of endogenous asymmetric information, we have shown that aftermarkets introduce risk-free arbitrage opportunities for informed investors, thereby encouraging information acquisition and discouraging the participation of uninformed investors in primary markets. Both effects combine to reduce prices in primary markets and government revenues in all states and in all countries. Our results highlight that it is not straightforward to interpret changes in sovereign debt prices as informative about changes in country fundamentals, as they depend not only on publicly observable information but also on privately acquired information. Moreover, they depend not only on the particular country's informational regime, but also on the information regime in other countries.

We purposefully made several assumptions to isolate the effects of asymmetric information on bond prices and spillovers. Relaxing some of these assumptions would likely magnify the effects we uncover. Examples include allowing default probabilities to respond endogenously to bond prices, introducing fundamental linkages across countries , time-varying risk aversion, allowing for exogenous market segmentation, or assuming economies of scale in the production of information. Relaxing other assumptions, such allowing information to affect real choices and allocations, would likely introduce countervailing benefits of information acquisition that are absent in our setting.

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A Appendix: Proofs

A.1 Proof of Lemma 1

If all investors are uninformed in j, the set of submitted bids cannot differ by state. By the auction clearing condition, marginal prices thus also do not differ by state. Now assume there is a strictly positive mass of informed investors in j. Given the ranking of default probabilities, it is clear that we must have $P_j(g) \geq P_j(b)$. Assume for a contradiction that $P_j(g) = P_j(b)$. Then any uninformed bid at the supposed marginal price is accepted in all states and purchased bonds default with probability $\bar{\kappa}_j$. Conversely, any informed bid in state g is defaulted on with probability $\kappa_j(g) < \bar{\kappa}_j$ while any informed bid in state b is defaulted on with probability$\kappa_j(b) > \bar{\kappa}_j$. By private optimality, informed investors thus submit more bids in the good state and fewer bids in the bad state than uninformed investors. This is a contradiction with the auction-clearing condition.

A.2 Proof of Proposition 1

Investors' risk-aversion implies that we must have $P_j(\theta_j) < 1 - \kappa_j(\theta_j)$ whenever there are informed investors in j, and $P_j(g) = P_j(b) < 1 - \bar{\kappa}_j$ if there no informed investors. Hence bonds offer a strictly positive risk premium (excess return over storage), and each country's default decision is uncorrelated with that of the other country. Given a twice continously differentiable utility function, a risk-averse investor must purchase a strictly positive quantity of any perfectly divisible risky gamble if it offers a strictly positive expected return. When there are no informed investors, uninformed investors face such a gamble and thus their first-order condition for optimal bids must hold with equality. When there are informed investors, informed investors also face such a gamble in every state, and thus their first-order condition holds with equality. Lemma 1 shows that the presence of informed investors leads to price dispersion. As a result, uninformed investors' bids at $P_j(g)$ are also accepted if $\theta_j = b$, and the expected default probability on a bond acquired at $P_j(g)$ is $\bar{\kappa}_1$. If $P_j(g) < 1 - \bar{\kappa}_1$, uninformed investors face a gamble with negative expected returns and the short-sale constraint on bids at $P_j(g)$ binds. Hence uninformed investors are marginal if there are no informed investors, and otherwise informed investors are marginal investors in every state. The stated optimality conditions are the first-order conditions for optimality derived from differentiating the objective function with respect to bids. Given the convexity of constraints and the strict concavity of the objective function, first-order conditions are necessary and sufficient for portfolio optimality.

A.3 Proof of Proposition 2

First statement. Let B_2 denote investors' bids in Country 2 given marginal price P_2. Assume that uninformed investors submit bids in all states, so that all first-order conditions for optimal bids hold with equality. We will first show that informed investors spend less than uninformed investors in the bad state, $P_1(b)B_1^I(b) < P_1(g)B_1^U(g) + P_1(b)B_1^U(b)$. For a contradiction, suppose not. Then for any $\tilde{W} \in \{W - P_2B_2, W + (1 - P_2B_2\}$, marginal utility after default satisfies

$$P_1(b)\kappa_1(b)u'(\tilde{W} - P_1(b)B_1^I(b)) \geq P_1(b)\kappa_1(b)u'(\tilde{W} - P_1(g)B_1^I(g) - P_1(b)B_1^U(b)).$$

First-order conditions for bids at $P_1(b)$ (as stated in Proposition 2) then imply that, for any $\tilde{W} \in \{W - P_2B_2, W + (1 - P_2B_2\}$, marginal utility after repayment satisfies

$$u'\Big(\tilde{W} + (1 - P_1(b))B_1^I(b)\Big) \geq u'\Big(\tilde{W} + (1 - P_1(g))B_1^U(g) + (1 - P_1(b))B_1^U(b)\Big).$$

By the concavity of $u(\cdot)$, we have

$$B_1^I(b) - \Big(B_1^U(g) + B_1^U(b)\Big) \leq P_1(b)B_1^I(b) - \Big(P_1(g)B_1^U(g) + P_1(b)B_1^U(b)\Big).$$

We have assumed for a contradiction that $P_1(b)B_1^I(b) \geq P_1(g)B_1^U(g) + P_1(b)B_1^U(b)$. Moreover, $P_1(b) < 1$ by investors' risk aversion. Hence the right-hand side of the preceding inequality satisfies

$$P_1(b)B_1^I(b) - \Big(P_1(g)B_1^U(g) + P_1(b)B_1^U(b)\Big) < B_1^I(b) - \Big(\frac{P_1(g)}{P_1(b)}B_1^U(g) + B_1^U(b)\Big).$$

Since $P_1(g) \geq P_1(b)$, the contradiction obtains.

Next, we show that informed investors spend more than uninformed investors in the good state, $P_1(g)B_1^I(g) > P_1(g)B_1^U(g)$. For any fixed repayment or default decision in Country 2 and associated risk-free holdings $\tilde{W} \in \{W - P_2B_2, W + (1 - P_2B_2\}$, uninformed investors' first-order condition for bids at $P_1(g)$ can be written as

$$\begin{aligned}
&f_1(b)\Big[P_1(g)\kappa_1(b)u'(\tilde{W} - P_1(g)B_1^U(g) - P_1(b)B_1^U(b))\ldots \\
&-(1 - P_1(g))(1 - \kappa_1(b))u'(\tilde{W} + (1 - P_1(g))B_1^U(g) + (1 - P_1(b))B_1^U(b))\Big] \\
&=f_1(g)\Big[(1 - P_1(g))(1 - \kappa_1(g))u'(\tilde{W}(1 - P_1(g))B_1^U(g)) - P_1(g)\kappa_1(g)u'(\tilde{W} - P_1(g)B_1^U(g))\Big].
\end{aligned}$$

Since $P_1(g) \geq P_1(b)$, the first-order condition for bids at $P_1(b)$ implies that the left-hand side is positive. This implies

$$\frac{(1 - \kappa_1(g))u'(\tilde{W} + (1 - P_1(g))\,B_1^U(g))}{\kappa_1(g)u'(\tilde{W} - P_1(g)B_1^U(g))} > \frac{P_1(g)}{(1 - P_1(g))}.$$

Comparing with informed investors' FOC for bids at $P_1(g)$ implies the result.

Lastly, assume that the short-sale constraint binds for uninformed bids at $P_1(g)$. Then uninformed investors' decision problem for bids at $P_1(b)$ is identical to that of informed investors (else the only difference is that the uninformed know bids at $P_1(g)$ are also going to be accepted). Hence they choose the same bidding strategy at $P_1(b)$.

Second statement. The first part follows directly from the first statement. Since informed investors spend relatively more in the good state for any marginal price, and increase in their mass must lead to a price increase. (The analogous statement does not necessarily hold for the bad state because market-clearing condition in state $\theta_1 = b$ is a function of $P_1(g)$ also.) In the limit as $n_1 \to 0$, uninformed investors must clear the market, $\lim_{n1 \to 0} P_1(g) B_1^U(g) = D_1$. Since all high-price bids are also accepted in the bad state, uninformed investor's first-order condition then implies the result.

Third statement. Since $P_1(b) < 1 - \kappa_1(b)$, all investors face a gamble with strictly positive expected returns at $P_1(b)$. Hence by Proposition 1 the first-order condition for bids at $P_1(b)$ binds with equality for all investors and $B_1^i(b) > 0$ for all i if $n_1 > 0$. Since $\lim_{n_1 \to 0} P_1(g) = \bar{P}_1$ and $P_1(g) > P_1(b)$ for all $n_1 > 0$ by Lemma 1, we have ruled out $\lim_{n_1 \to 0} P_1(b) > \bar{P}_1$. Now suppose for a contradiction that $\lim_{n_1 \to 0} P_1(b) = \bar{P}_1$. If $B_1^U(b) = 0$, uninformed investors' consumption is invariant to the state. By the first-order condition for bids at $P_1(g)$, these investors are indifferent on the margin between buying and selling a bond that defaults with probability $\bar{\kappa}_1$. Since $\lim_{n_1 \to 0} P_1(b) = \bar{P}_1$, the continutity of marginal utility implies that there exists $\bar{n}_1 > 0$ sufficiently close to zero such that it is strictly optimal to submit negative bids at $P_1(b)$ because the associated bonds default with probability $\kappa_1(b) > \bar{\kappa}_1$. Contradiction.

Next, we show that $P_1(b) < \bar{P}_1$ for all $n_1 > 0$. Suppose for a contradition that $P_1(b) \geq \bar{P}_1$. By definition, $\bar{P}_1$ is the price at which uninformed investors are willing to spend D_1 on bonds given that the acquired bonds default with probability $\bar{\kappa}_1$. Recall also that $P_1(g) \geq P_1(b)$ by Lemma 1. Hence if $P_1(b) \geq \bar{P}_1$, first-order conditions for bid optimality imply that $X_1^U(b) = P_1(g) B_1^U(g) + P_1(b) B_1^U(b) < D_1$. The first statement of this proposition showed that $X_1^U(g) \leq X_1^U(g)$. Hence $n_1 X_1^I(b) + (1 - n_1) X_1^U(b) < D_1$, a contradiction with the market-clearing condition.

Q.E.D.

A.4 Proof of Proposition 3

The optimal information acquisition decision follows immediately from observing that the marginal cost of information is fixed and each individual investor takes the value of information as given. The remainder is a direct implication.

Q.E.D.

A.5 Proof of Proposition 4

In the uninformed equilibrium, prices are invariant to the state, $P_1(g) = P_1(b) = \bar{P}_1$. Let $\bar{B}_1 = D_1/\bar{P}_1$ denote the equilibrium bids of uninformed investors in the uninformed equilibrium. Proposition 2 and its proof show that the informed equilibrium satisfies $\lim_{n_1 \to 0} P_1(g) = \bar{P}_1$, $\lim_{n_1 \to 0} P_1(b) < \bar{P}_1$, $\lim_{n_1 \to 0} P_1(b) < \bar{P}_1$, $\lim_{n_1 \to 0} B_1^U(g) = \bar{B}_1$ and $\lim_{n_1 \to 0} B_1^U(b) = 0$. Hence in the limit as $n_1 \to 0$, uninformed investors purchase bonds only at $P_1(g)$ and obtain the same utility as in the uninformed equilibrium. Hence we must show that informed investors do strictly better in the limit of the informed equilibrium as $n_1 \to 0$. By the fact that $\lim_{n_1 \to 0} P_1(g) = \bar{P}_1$, informed investors face the same decision problem (and obtain the same utility advantage over uninformed investors) in the good state. In the bad state, informed investors face a strictly lower marginal price in the limit of the uninformed equilibrium than in the uninformed equilibrium. Hence they are strictly better in the informed equilibrium if and only if the short-sale constraint does not bind at $P_1^0(b) \equiv \lim_{n_1 \to 0} P_1(b)$. We now show that this constraint does not bind. Recall that $P_1^0(b)$ is such that uninformed investors are willing to purchase a vanishingly small number of bonds in a neighborhood around $n_1 = 0$. This requires $P_1(b) < 1 - \kappa_1(b)$. Since informed investors can make state-contingent bids and hold only uncorrelated risks in Country 2, it is strictly optimal to purchase bonds at $P_1^0(b)$.

The previous arguments have shown that $\Delta\bar{V} < \lim_{n_1 \to 0} \Delta V(n_1)$, and we can find a cost of information such that it is strictly suboptimal to acquire information if no other investor does so, but strictly optimal to acquire information if some other investors do so as well. Since K is the cost of acquiring information, it is trivial that the share of informed investors in any equilibrium with endogenous information acquisition is weakly increasing in K.

Q.E.D.

A.6 Proof of Proposition 5

First Statement. Write the system of first-order conditions in vector form as

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Differentiating with respect to $\bar{\kappa}_1$ and applying the implicit function theorem gives

$$\begin{bmatrix} \frac{\partial F_1}{\partial P_1} & \frac{\partial F_1}{\partial P_2} \\ \frac{\partial F_2}{\partial P_1} & \frac{\partial F_2}{\partial P_2} \end{bmatrix} \begin{bmatrix} \frac{\partial P_1}{\partial \bar{\kappa}_1} \\ \frac{\partial P_2}{\partial \bar{\kappa}_1} \end{bmatrix} = \begin{bmatrix} -\frac{\partial F_1}{\partial \bar{\kappa}_1} \\ -\frac{\partial F_2}{\partial \bar{\kappa}_1} \end{bmatrix}$$

Define the determinant of the square matrix as

$$\det = \frac{\partial F_1}{\partial P_1}\frac{\partial F_2}{\partial P_2} - \frac{\partial F_1}{\partial P_2}\frac{\partial F_2}{\partial P_1}$$

Then

$$\begin{bmatrix} \frac{\partial P_1}{\partial \bar{\kappa}_1} \\ \frac{\partial P_2}{\partial \bar{\kappa}_1} \end{bmatrix} = \frac{1}{\det} \begin{bmatrix} \frac{\partial F_2}{\partial P_2} & -\frac{\partial F_1}{\partial P_2} \\ -\frac{\partial F_2}{\partial P_1} & \frac{\partial F_1}{\partial P_1} \end{bmatrix} \begin{bmatrix} -\frac{\partial F_1}{\partial \bar{\kappa}_1} \\ -\frac{\partial F_2}{\partial \bar{\kappa}_1} \end{bmatrix}$$

The first statement follows from this expression.

Second Statement. We will first show that there is no contagion with CARA preferences. Without loss of generality, consider a representative uninformed investor and drop superscripts indicating types. By market-clearing, $B_j = \frac{D_j}{P_j}$ for j, and risk-free holdings satisfy $w = W - D_1 - D_2$. The resulting consumption profile depends only on the default decisions δ_1 and δ_2,

$$c(\delta_1, \delta_2) = w + (1 - \delta_1)B_1 + (1 - \delta_2)B_2.$$

where $\delta_j = 1$ if j defaults and $\delta_j = 0$ otherwise. Expected marginal utility conditional on δ_j is

$$m_j(\delta_j) = \bar{\kappa}_{-j}u'\left(w + (1 - \delta_j)B_j\right) + (1 - \bar{\kappa}_{-j})u'\left(w + (1 - \delta_j)B_j + B_{-j}\right)$$

First-order conditions for bids in Country 1 and Country 2 are, respectively,

$$(1 - \bar{\kappa}_1)(1 - P_1)m_1(0) - \bar{\kappa}_1 P_1 m_1(1) = 0 \tag{4}$$

$$(1 - \bar{\kappa}_2)(1 - P_2)m_2(0) - \bar{\kappa}_2 P_2 m_2(1) = 0. \tag{5}$$

Let $y_j \equiv (1 - P_j)/P_j$ denote j's yield and redefine the appropriate ratio of marginal utilities (or ratio of state prices) as $M_j = \frac{\kappa_j}{1-\kappa_j}\widetilde{M}_j$, with

$$\widetilde{M}_j \equiv \frac{m_j(1)}{m_j(0)} = \frac{\bar{\kappa}_{-j}u'\left(w\right) + (1 - \bar{\kappa}_{-j})u'\left(w + B_{-j}\right)}{\bar{\kappa}_{-j}u'\left(w + B_j\right) + (1 - \bar{\kappa}_{-j})u'\left(w + B_j + B_{-j}\right)}$$

$$= \frac{u'\left(w + B_{-j}\right)}{u'\left(w + B_j + B_{-j}\right)} \frac{1 + \bar{\kappa}_{-j}\left[\frac{u'(w)}{u'(w+B_{-j})} - 1\right]}{1 + \bar{\kappa}_{-j}\left[\frac{u'(w+B_j)}{u'(w+B_j+B_{-j})} - 1\right]}$$

If preferences satisfy CARA, then $u'(c) = \gamma e^{-\gamma c}$ and the second term of the previous line is equal to one for any default probabilities and debt levels. Hence

$$\widetilde{M}_j = e^{\gamma B_j}$$

Given this result, we can express the pricing equation for each country as

$$\frac{1 - P_j}{P_j} = \frac{\bar{\kappa}_j}{1 - \bar{\kappa}_j} e^{\gamma \frac{D_j}{P_j}}$$

which is independent of any variables indexed by $-j$.

Based on this new notation, $F_j = y_j - \frac{\kappa_j}{1-\bar{\kappa}_j}\widetilde{M}_j$. Hence

$$\frac{\partial F_j}{\partial \bar{\kappa}_{-j}} = -\frac{\bar{\kappa}_j}{1-\bar{\kappa}_j}\frac{\partial \widetilde{M}_j}{\partial \bar{\kappa}_{-j}}.$$

Hence the sign is the opposite of the sign of $\frac{\partial \widetilde{M}_j}{\partial \bar{\kappa}_{-j}}$. We will show that that the latter is strictly positive if and only if preferences satisfy DARA (as we discussed this is zero with CARA). Hence $\frac{\partial F_j}{\partial \bar{\kappa}_{-j}} < 0$. Differentiating $\widetilde{M}_j$ with respect to $\bar{\kappa}_{-j}$ yields

$$\frac{\partial \widetilde{M}_j}{\partial \bar{\kappa}_{-j}} = \frac{\Big(u'(w) - u'(w + B_{-j})\Big) - \Big(u'(w + B_j) - u'(w + B_j + B_{-j})\Big)\widetilde{M}_j}{m_j(0)}$$

Observe that

$$\frac{\partial \widetilde{M}_j}{\partial \bar{\kappa}_{-j}} > 0 \Leftrightarrow \frac{\Big(u'(w) - u'(w + B_{-j})\Big)}{\Big(u'(w + B_j) - u'(w + B_j + B_{-j})\Big)} > \widetilde{M}_j.$$

After some algebra, this condition can be rewritten as

$$\frac{\partial \widetilde{M}_j}{\partial \bar{\kappa}_{-j}} > 0 \Leftrightarrow \frac{u'(w) - u'(w + B_{-j})}{u'(w + B_{-j})} > \frac{u'(w + B_j) - u'(w + B_j + B_{-j})}{u'(w + B_j + B_{-j})}$$

We now show this holds if u satisfies decreasing absolute risk aversion (DARA). Let

$$\Omega = \frac{u'(\tilde{W}) - u'(\tilde{W} + B)}{u'(\tilde{W} + B)}.$$

Then the claim is equivalent to Ω strictly decreasing in $\tilde{W}$ for any $B, \tilde{W} > 0$. This holds by definition of DARA since

$$\frac{\partial \Omega}{\partial \tilde{W}} < 0 \Leftrightarrow \frac{-u''(\tilde{W})}{u'(\tilde{W})} > \frac{-u''(\tilde{W} + B)}{u'(\tilde{W} + B)}.$$

Third Statement. By the second statement, we have that $\frac{\partial F_{-j}}{\partial P_{-j}}$, $\frac{\partial F_j}{\partial \bar{\kappa}_1}$, $\frac{\partial F_{-j}}{\partial \bar{\kappa}_1}$ are all strictly negative. Under the stated condition, we therefore have the that the second term of the condition in Statement (i) is strictly negative for each j. If prices are to decline in Country 1, we must have $\frac{\partial F_1}{\partial P_1}\frac{\partial F_2}{\partial P_2} - \frac{\partial F_1}{\partial P_2}\frac{\partial F_2}{\partial P_1} > 0$, and Country 2 prices also decline.

Q.E.D.

A.7 Proof of Corollary 1

There is pure price contagion if $\partial F_j/\partial P_{-j} > 0$. From the third statement of Proposition 5 this is the case if and only if,

$$\frac{\partial \widetilde{M}_j}{\partial B_{-j}} > 0 \Leftrightarrow \frac{-u''(w+B_{-j})}{u'(w+B_{-j})} < \frac{-u''(w+B_j+B_{-j})}{u'(w+B_j+B_{-j})}\left[\frac{1-\bar{\kappa}_{-j}+\bar{\kappa}_{-j}\frac{u'(w)}{u'(w+B_{-j})}}{1-\bar{\kappa}_{-j}+\bar{\kappa}_{-j}\frac{u'(w+B_j)}{u'(w+B_j+B_{-j})}}\right].$$

We can rewrite this condition as

$$\frac{-u''(w+B_{-j})}{u'(w+B_{-j})} < \frac{-u''(w+B_j+B_{-j})}{u'(w+B_j+B_{-j})}\left[\frac{\frac{(1-\bar{\kappa}_{-j})u'(w+B_{-j})+\bar{\kappa}_{-j}u'(w)}{u'(w+B_{-j})}}{\frac{(1-\bar{\kappa}_{-j})u'(w+B_j+B_{-j})+\bar{\kappa}_{-j}u'(w+B_j)}{u'(w+B_j+B_{-j})}}\right].$$

Cancelling $u'\left(w+B_j+B_{-j}\right)$ in the denominator

$$\frac{-u''(w+B_{-j})}{u'(w+B_{-j})} < \frac{-u''(w+B_j+B_{-j})}{u'(w+B_j)}\left[\frac{(1-\bar{\kappa}_{-j})u'\left(w+B_{-j}\right)+\bar{\kappa}_{-j}u'\left(w\right)}{(1-\bar{\kappa}_{-j})u'\left(w+B_j+B_{-j}\right)+\bar{\kappa}_{-j}u'\left(w+B_j\right)}\right].$$

Cancelling $u'\left(w+B_j\right)$ in both sides and rearranging

$$\frac{-u''(w+B_{-j})}{(1-\bar{\kappa}_{-j})u'\left(w+B_{-j}\right)+\bar{\kappa}_{-j}u'\left(w\right)} < \frac{-u''(w+B_j+B_{-j})}{(1-\bar{\kappa}_{-j})u'\left(w+B_j+B_{-j}\right)+\bar{\kappa}_{-j}u'\left(w+B_j\right)}.$$

In the limit as $D_{-j} \to 0$, we have that $B_{-j} \to 0$ and the condition is not fulfilled by DARA. In the limit as $w \to 0$, the left hand side goes to 0 and the condition is fulfilled.

A.8 Proof of Lemma 2

The return of a Country-1 bond bought at the high price (in state g) in case of default is -1 (with expected probability $\kappa_1^i(g)$) and in case of repayment $\frac{1-P_1(g)}{P_1(g)}$ (with expected probability $1-\kappa_1^i(g)$). This implies that the expected return of such bond is $\widehat{R}_1^i(g) = \frac{1-\kappa_1^i(g)-P_1(g)}{P_1(g)}$ and the standard deviation is $\widehat{\sigma}_1^i = \frac{\sqrt{\kappa_1^i(g)(1-\kappa_1^i(g))}}{P_1(g)}$. Since $\kappa_1^I(g) = \kappa_1(g)$ and $\kappa_1^U(g) = \bar{\kappa}_1$, the difference in Sharpe ratios can be written as

$$S_1^I(g) - S_1^U(g) = \frac{1-\kappa_1(g)}{\sqrt{\kappa_1(g)(1-\kappa_1(g))}} - \frac{1-\bar{\kappa}_1}{\sqrt{\bar{\kappa}_1(1-\bar{\kappa}_1)}} - P_1(g)\left(\frac{1}{\sqrt{\kappa_1(g)(1-\kappa_1(g))}} - \frac{1}{\sqrt{\bar{\kappa}_1(1-\bar{\kappa}_1)}}\right)$$

If $\bar{\kappa}_1 < \frac{1}{2}$, then $S_1^I(g) - S_1^U(g) > 0$ and strictly decreasing in $P_1(g)$. **Q.E.D.**

A.9 Proof of Proposition 6

Let $n_1 \in (0,1)$. There are 8 possible states: for each $\theta_j \in \{g, b\}$, each country may default (d) or repay (r). Since there is no information in Country 2, we can proceed as if there were only one state with default probability $\bar{\kappa}_2$. Simplify notation by writing state-contingent consumption as $\{c^i_{rr}(\theta), c^i_{rd}(\theta), c^i_{dr}(\theta), c^i_{dd}(\theta)\}$. Then i's objective function can be written as

$$V^i = f_1(g)\left\{ \begin{array}{c} \kappa_1(g)\Big[\bar{\kappa}_2 U(c^i_{dd}(g)) + (1-\bar{\kappa}_2)U(c^i_{dr}(g))\Big] \\ +(1-\kappa_1(g))\Big[\bar{\kappa}_2 U(c^i_{rd}(g)) + (1-\bar{\kappa}_2)U(c^i_{rr}(g))\Big] \end{array} \right\}$$
$$+ f_1(b)\left\{ \begin{array}{c} \kappa_1(b)\Big[\bar{\kappa}_2 U(c^b_{dd}) + (1-\bar{\kappa}_2)U(c^b_{dr})\Big] \\ +(1-\kappa_1(b))\Big[\bar{\kappa}_2 U(c^b_{rd}) + (1-\bar{\kappa}_2)U(c^b_{rr})\Big] \end{array} \right\}$$

We compute a second-order Taylor approximation of the objective function around $B^i_j(\theta_j) = 0$ for all i, all j, and all θ_j. For informed investors, the associated first-order conditions with respect to $B^i_1(g)$, $B^i_1(b)$ and B^i_2 are, respectively,

$$\begin{aligned} 0 = & f_1(g)(1-\kappa_1(g)-P_1(g))U'(W) \\ & + f_1(g)\Big[\kappa_1(g)(-P_1(g))^2 + (1-\kappa_1(g))(1-P_1(g))^2\Big]U''(W)B^I_1(g) \\ & + f_1(g)(1-\kappa_1(g)-P_1(g))(1-\bar{\kappa}_2-P_2)U''(W)B^I_2 \end{aligned} \tag{6}$$

$$\begin{aligned} 0 = & f_1(b)(1-\kappa_1(b)-P_1(b))U'(W) \\ & + f_1(b)\Big[\kappa_1(b)(-P_1(b))^2 + (1-\kappa_1(b))(1-P_1(b))^2\Big]U''(W)B^I_1(b) \\ & + f_1(b)(1-\kappa_1(b)-P_1(b))(1-\bar{\kappa}_2-P_2)U''(W)B^I_2 \end{aligned} \tag{7}$$

$$\begin{aligned} 0 = & (1-\bar{\kappa}_2-P_2)U'(W) \\ & + \Big[\bar{\kappa}_2(-P_2)^2 + (1-\bar{\kappa}_2)(1-P_2)^2\Big]U''(W)B^I_2 \\ & + f_1(g)(1-\bar{\kappa}_2-P_2)(1-\kappa_1(g)-P_1(g))U''(W)B^I_1(g) \\ & + f_1(b)(1-\bar{\kappa}_2-P_2)(1-\kappa_1(b)-P_1(b))U''(W)B^I_1(b) \end{aligned} \tag{8}$$

Define informed expected rates of return by $\tilde{r}^I_1(g) = \frac{1-\kappa_1(g)-P_1(g)}{P_1(g)}$, $\tilde{r}^I_1(b) = \frac{1-\kappa_1(b)-P_1(b)}{P_1(b)}$ and $\tilde{r}^I_2 = \frac{1-\bar{\kappa}_2-P_2}{P_2}$ and let $\sigma^I_1(g)$, $\sigma^I_1(b)$, and σ^I_2 denote the associated standard deviations. The first term of the RHS of (6) can be rewritten in terms of returns as

$$f_1(g)(1-\kappa_1(g)-P_1(g))U'(W) = f_1(g)\tilde{r}_1(g)P_1(g)U'(W)$$

and the second term as

$$f_1(g)\Big[\kappa_1(g)(-P_1(g))^2+(1-\kappa_1(g))(1-P_1(g))^2\Big]U''(W)B_1^I(g) = f_1(g)\mathbb{E}\left[\left(r_1^I(g)\right)^2\right]P_1(g)^2U''(W)B_1^I(g)$$

All other terms in equations (6)-(8) can be analogously rewritten. Let $U(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$, and define the state-contingent portfolio weights $\omega_1^I(g) = \frac{P_1(g)B_1^I(g)}{W}$, $\omega_1^I(b) = \frac{P_1(b)B_1^I(b)}{W}$, and $\omega_2^I = \frac{P_2B_2^I}{W}$. Since $Var(x) = \mathbb{E}[x^2] - (\mathbb{E}[x])^2$, the system of equations is

$$\tilde{r}_1^I(g) = \gamma\omega_1^I(g)\left(\left(\sigma_1^I(g)\right)^2 + \left(\tilde{r}_1^I(g)\right)^2\right) + \gamma\omega_2^I\left(\tilde{r}_1^I(g)\tilde{r}_2^I\right) \tag{9}$$

$$\tilde{r}_1^I(b) = \gamma\omega_1^I(b)\left(\left(\sigma_1^I(b)\right)^2 + \left(\tilde{r}_1^I(b)\right)^2\right) + \gamma\omega_2^I\left(\tilde{r}_1^I(b)\tilde{r}_2^I\right) \tag{10}$$

$$\tilde{r}_2^I = \gamma\omega_2^I\left(\left(\sigma_2^I\right)^2 + \left(\tilde{r}_2^I\right)^2\right) + f_1(g)\gamma\omega_1^I(g)\tilde{r}_1^I(g)\tilde{r}_2^I + f_1(b)\gamma\omega_1^I(b)\tilde{r}_1^I(b)\tilde{r}_2^I \tag{11}$$

Optimality conditions for uninformed investors are analogous, modulo adjusting expected returns and standard deviations to take into account that bids $P_1(g)$ are also accepted in the bad state. To facilitate comparisons of optimal portfolios, going forward we denote expected returns for a given information set simply by R_g, R_b and R_2. Let σ_g, σ_b, and σ_2 denote the associated standard deviations, and S_g, S_b and S_2 the Sharpe ratios. Optimal portfolios then satisfy the following system of equations, with the only differences across types accounted for by differences in expected returns and volatilities:

$$\omega_g = \left(\frac{R_g}{\sigma_g^2 + R_g^2}\right)(1 - \omega_2 R_2)$$

$$\omega_b = \left(\frac{R_b}{\sigma_b^2 + R_b^2}\right)(1 - \omega_2 R_2)$$

$$\omega_2 = \left(\frac{R_2}{\sigma_2^2 + R_2^2}\right)(1 - f_1(g)\omega_g R_g - f_1(b)\omega_b R_b)$$

Multiplying by $R_i(1/\sigma_i^2)$, dividing by $(1/\sigma_i^2)$ and defining $s = \frac{S^2}{1+S^2}$, which is strictly increasing in S, we can rewrite these expressions as

$$R_g\omega_g = s_g(1 - R_2\omega_2)$$

$$R_b\omega_b = s_b(1 - R_2\omega_2)$$

$$R_2\omega_2 = s_2(1 - f_1(g)R_g\omega_g - f_1(b)R_b\omega_b)$$

Then plug in the first two equations into the third to give:

$$R_2\omega_2 = s_2\Bigg(1 - f_1(g)s_g(1 - R_2\omega_2) - f_1(b)s_b(1 - R_2\omega_2)\Bigg)$$

It follows that

$$\omega_2 = \frac{1}{R_2}\left(\frac{1 - f_1(g)s_g - f_1(b)s_b}{\frac{1}{s_2} - f_1(g)s_g - f_1(b)s_b}\right)$$
$$\omega_g = \frac{s_g}{R_g}\left(\frac{\frac{1}{s_2} - 1}{\frac{1}{s_2} - f_1(g)s_g - f_1(b)s_b}\right)$$
$$\omega_b = \frac{s_b}{R_b}\left(\frac{\frac{1}{s_2} - 1}{\frac{1}{s_2} - f_1(g)s_g - f_1(b)s_b}\right)$$

Since $\frac{\partial \omega_g}{\partial S_g} > 0$, then from Lemma 2, $\omega_1^I(g) > \omega_1^U(g)$. Since $\frac{\partial \omega_2}{\partial S_g} < 0$, then from Lemma 2, $\omega_2^I < \omega_2^U$ and $\frac{\partial(\omega_2^U - \omega_2^I)}{\partial P_1(g)} < 0$.

Q.E.D.

A.10 Proof of Proposition 7

First Statement. Let all investors be uninformed. Since all investors are symmetric, there is no arbitrage across markets. If they are no price differences across markets, all investors must hold the same portfolio ex-post. Since no information is revealed at any stage, any equilibrium must feature the same allocation as the auction equilibrium.

Second Statement. Consider the bad state. If $P_j(b) > \hat{P}_j(b)$, it is strictly optimal to submit zero bids at auction, which is a contradiction with auction market clearing. Next, suppose $P_j(b) < \hat{P}_j(b)$. Recall that all investors' bids at $P_j(b)$ are executed if and only if $\theta_1 = b$. Hence is strictly optimal for all investors to sell bonds in the secondary market. Hence the secondary market cannot clear. Now consider the good state. By auction market-clearing, we cannot have $P_j(g) > \hat{P}_j(g)$ because all investors would then strictly prefer to trade in the secondary market. Next, we show that we must have $P_j(g) < \hat{P}_j(g)$. Suppose not, $P_j(g) = \hat{P}_j(g)$. Then uninformed investors can trade under perfect information in the secondary market but receive the same price as in the auction. Hence the value of information is zero, and there is no incentive to become informed. $P_j(g) < \hat{P}_j(g)$ is sustainable in equilibrium because uninformed investors are adversely selected if buying at auction. Hence as long as $\hat{P}_j(g) - P_j(g)$ is sufficiently small, uninformed investors strictly prefer to buy in the secondary market.

Suppose $\theta_j = b$, both informed and uninformed investors would trade to arbitrage price differences (recall that uninformed bids at the low price are accepted if and only if $\theta_1 = b$). But if all investors take the same side of the arbitrage in the primary market, then the secondary market cannot clear. Now turn to the good state. If

$P_j(g) > q_j(g)$, then all investors find it strictly optimal to wait, and the primary market does not clear. Hence the informed cannot do better than the uninformed, and there are no incentives to acquire information. Hence, when there is information in the auction it must be that $P_j(g) < q_j(g)$.

Now, turning to bids, informed investors fully exploit the arbitrage opportunity using all wealth allocated to country j to buy bonds in the good state and by selling a fraction to uninformed investors in the secondary market. Uninformed investors cannot exploit the arbitrage in the same manner ecause they run the risk of overpaying in the bad state. To see why arbitrage can persist, note that the supply of assets in the secondary market is bounded above by $\sum_{i:a^i_j=1} \frac{n^i \tilde{W}}{P_j(g)}$, while the demand for bonds in the primary market is decreasing in the fraction of informed investors. All else equal, reducing the number of informed investors thus widens the gap between primary and secondary market prices in the high state.

Third statement. In the limit $n_1 \to 0$, almost all investors are ex-ante identical. By market-clearing, it follows trivially that auction prices must converge to the limiting prices of the auction-only equilibrium. Now consider the limit of secondary market prices. Since $\hat{P}_1(b) = P_1(b)$ for all $n_1 > 0$, we have $\lim_{n_1 \to 0} \hat{P}_1(b) = \lim_{n_1 \to 0} P^A_1(b)$. Next consider the high state. The case $\lim_{n_1 \to 0} \hat{P}_1(g) < \lim_{n_1 \to 0} P_1(g)$ can be immediately ruled out by the second statement. Suppose for a contradiction that $\lim_{n_1 \to 0} \hat{P}_1(g) = \lim_{n_1 \to 0} P_1(g)$. Since $\lim_{n_1 \to 0} P_1(b) < \lim_{n_1 \to 0} P_1(g)$, for n_1 sufficiently small it is strictly optimal for any uninformed investor to submit zero bids at $P_1(g)$ and purchase bonds only in the secondary market. Since $n_1 W < D_1$ for n_1 sufficiently small, we have a contradiction with market clearing. **Q.E.D.**

Proposition 9 (Value of Information)**.** *When there are secondary markets after the auction:*

(i) As $n_1 \to 0$, the value of information is strictly higher than without secondary markets.

(ii) The range of information costs for which an informed equilibrium exists is strictly larger.

(iii) If and only if $n_1 \geq \hat{n}_1 \equiv \frac{D_1}{W - D_2}$, the value of information is zero, the equilibrium with secondary markets delivers the same allocations and prices as the full information auction equilibrium, and there is no cross-market arbitrage, $P_j(\theta_j) = \hat{P}_j(\theta_j)$ for all θ_j.

(iv) Any equilibrium with endogenous information acquisition satisfies $n_1 < \hat{n}_1$.

A.11 Proof of Proposition 9

First Statement. By Proposition 7, $\lim_{n_1 \to 0} \hat{P}_j(\theta_j) = \lim_{n_1 \to 0} P^A_j(\theta_j)$ and $\lim_{n_1 \to 0} \hat{P}_1(g) > \lim_{n_1 \to 0} P_1(g)$. By the Inada condition, it is always strictly optimal to invest a strictly positive amount of wealth into the risk-free asset in the auction equilibrium say $\tilde{W}$. The following is a feasible portfolio that generates strictly higher utitlity than the optimal auction-only portfolio: (i) buy the same portfolio at auction, (ii) in addition spend

$\tilde{W}$ on bonds in state g in Country 1, and (iii) sell the additional bonds purchased with $\tilde{W}$ in the secondary market at a strict profit. This portfolio has higher average returns and less volatility, and so it is strictly preferred. Since uninformed investors obtain the same utility as in the auction equilibrium in the limit $n_1 \to 0$, the result follows.

Second Statement. Follows immediately from the first statement.

Third Statement. There is enough informed capital to fully arbitrage prices if and only if $n \geq \hat{n}_1$. Let $\widehat{P}_1(g)d$, $\widehat{B}^I(g)$, and B_2^I denote the equilibrium good-state price and informed bids in the equilibrium in which all investors are informed and there are no secondary markets. In this equilibrium, informed investors spend $\widehat{P}_2\widehat{B}_2^I$ in Country 2. By auction-clearing, $\widehat{P}_2\widehat{B}_2^I = D_2$. By the budget constraint, informed investors have $W - D_2$ in capital to invest in Country 1. In order for informed buy the entire supply of bonds in Country 1 at price $\widehat{P}_1$ if $\theta_1 = g$, we require that $n_1(W - D_2) \geq \widehat{P}_1 B_1^I(g) = D_1$, where the last equality follows from auction clearing. This holds iff $n_1 \geq \hat{n}_1$. Hence iff $n \geq \hat{n}_1$, we can construct an equilibrium in which informed investors buy the entire supply of bonds in the primary market when $\theta_1 = g$, and then sell some of these bonds to uninformed investors in the secondary market at the same price. This implies that uninformed investors can buy bonds as if they were informed and choose not to participate in primary markets. Hence the equilibrium must be such that all prices are identical to the fully informed equilibrium.

Fourth statement. By the third statement, uninformed investors choose the same ex-post portfolio as informed investors if and only if $n_1 \geq \hat{n}_1$. Hence the value of information is positive if and only if $n_1 < \hat{n}_1$.