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ABSTRACT

This paper compares different solution methods for computing the equilibrium of dynamic stochastic general equilibrium (DSGE) models with rare disasters along the line of those proposed by Rietz (1988), Barro (2006}, Gabaix (2012), and Gourio (2012). DSGE models with rare disasters require solution methods that can handle the large non-linearities triggered by low-probability, high-impact events with sufficient accuracy and speed. We solve a standard New Keynesian model with Epstein-Zin preferences and time-varying disaster risk with perturbation, Taylor projection, and Smolyak collocation. Our main finding is that Taylor projection delivers the best accuracy/speed tradeoff among the tested solutions. We also document that even third-order perturbations may generate solutions that suffer from accuracy problems and that Smolyak collocation can be costly in terms of run time and memory requirements.

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1 Introduction

Rietz (1988), Barro (2006), and Gabaix (2012) have popularized the idea that low-probability events with a large negative impact on consumption ("rare disasters") can account for many asset pricing puzzles, such as the equity premium puzzle of Mehra and Prescott (1985).[1] Barro (2006), in particular, argues that a rare disaster model calibrated to match data from 35 countries can reproduce the observed high equity premium, the low risk-free rate, and the stock market volatility. Barro assumed disaster probabilities of 1.7 percent a year and declines in output/consumption in a range of 15 to 64 percent.

Many other researchers have followed Barro's lead and formulated, calibrated/estimated, and solved models with disaster probabilities and declines in consumption that are roughly in agreement with Barro's original proposal.[2] Furthermore, the approach has been extended to analyze business cycles (Gourio, 2012), credit risk (Gourio, 2013), and foreign exchange markets (Farhi and Gabaix, 2008 and Gourio, Siemer, and Verdelhan, 2013). These calibrations/estimations share a common feature: they induce large non-linearities in the solution. This is not a surprise. The mechanism that makes rare disasters work is the large precautionary behavior responses induced in normal times by the probability of tail events.

Dealing with these non-linearities is not too challenging when we work with endowment economies. A judicious choice of functional forms and parameterization allow a researcher to either derive closed-form solutions or formulae that can be easily evaluated.

The situation changes, however, when we move to production models, such as those of Gourio (2012, 2013), Andreasen (2012), Isoré and Szczerbowicz (2013, 2015), and Petrosky-Nadeau, Zhang, and Kuehn (2015). Suddenly, having an accurate solution is of foremost importance. For example, rare disaster models have the promise of helping to design policies to prevent disasters (with measures such as financial stability policy) and to mitigate them once they have occurred (with measures such as bailouts and unconventional monetary policy). The considerable welfare losses associated with rare disasters reported by Barro (2009) suggest that any progress along the lines of having *accurate* quantitative models to design counter-disaster policies is a highly rewarding endeavor.

But we do not care only about accuracy. We also care about speed. Models that can be useful for policy analysis usually require estimation of parameter values, which involves the repeated solution of the model, and that the models be as rich in terms of detail as the

[1] See also Barro (2009), who, with the help of Epstein and Zin (1989) preferences, can fix some counterfactual implications of models with power utility and high-risk aversion regarding the responses of the price/dividend ratio to increases in uncertainty.

[2] Among many others, Barro and Ursúa (2012), Barro and Jin (2011), Nakamura, Steinsson, Barro, and Ursúa (2013), Wachter (2013), and Tsai and Wachter (2015).

most recent generation of dynamic stochastic general equilibrium (DSGE) models, which are characterized by many state variables.

Gourio (2012, 2013) and Petrosky-Nadeau, Zhang, and Kuehn (2015) solve their models with standard projection methods (Judd, 1992). Projection methods are highly accurate (Aruoba, Fernández-Villaverde, and Rubio-Ramírez, 2006), but they suffer from an acute curse of dimensionality. Thus, the previous papers concentrate in analyzing relatively small models. Andreasen (2012) and Isoré and Szczerbowicz (2013, 2015) solve more fully-fledged models with perturbation solutions. Perturbation solutions are fast to compute and can handle many state variables. However, Isoré and Szczerbowicz (2013) only undertake first- and second-order perturbations and Andreasen (2012) and Isoré and Szczerbowicz (2015) a third-order perturbation. We will argue below that there are reasons to be cautious about the properties of these perturbation solutions (see also Levintal, 2015). Perturbations are inherently local solution methods and rare disasters often trigger equilibrium dynamics that travel far away from the approximation point of the perturbation. Moreover, perturbations may fail to approximate accurately asset prices and risk premia due to the strong volatility embedded in these models.

To get around the limitations of existing algorithms, we apply a new solution method, Taylor projection, to compute DSGE models with rare disasters. This method, recently proposed by Levintal (2016), is a hybrid of Taylor-based perturbations and projections (and hence its name). Like standard projection methods, Taylor projection starts from a residual function created by plugging the unknown decision rules of the agents into the equilibrium conditions of the model and searching for coefficients that make that residual function as close to zero as possible. The novelty of the approach is that, instead of "projecting" the residual function according to an inner product, we approximate the residual function around the steady state of the model using a Taylor series, and find the solution that zeros the Taylor series. We show that Taylor projection is sufficiently accurate and fast so as to open the door to the solution and estimation of rich models with rare disasters, including New Keynesian models such as those in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007).

To do so, we first propose in section 2 a standard New Keynesian model augmented with Epstein-Zin preferences and time-varying rare disaster risk. We also present seven simpler versions of the model. In what we will call version 1, we start with a benchmark real business cycle model, also with Epstein-Zin preferences and time-varying rare disaster risk. This model only has four state variables (capital, the classical technology shock, and two additional state variables associated with the time-varying rare disaster risk). Then, we start adding more shocks and price rigidities, until we get to version 8, our complete New Keynesian model with twelve state variables. Our layer-by-layer analysis allows us to gauge how accuracy and run time change as new mechanisms are added to the model and as the dimensionality of the state space grows.

In section 3, we calibrate the model with a baseline parameterization, which captures rare disasters, and with a non-disaster parameterization, where we shut down rare disasters. The latter calibration will help us in measuring the effect of disasters on the accuracy and speed of our solution methods.

In section 4, we describe how we solve each of the eight versions of the model, with the two calibrations, using perturbation, Taylor projection, and Smolyak collocation. We implement different levels of each of the three solution methods: perturbations from order 1 to 5, Taylor projections from order 1 to 3, and Smolyak collocation from level 1 to 3. Therefore, we generate eleven solutions per each of the eight versions of the model and each of the two calibrations, for a total of 176 possible solutions (although we did not find a few of the Smolyak solutions because of convergence/memory constraints).

In section 5, we present our main results. Our first finding is that first-, second-, and third-order perturbations fail to provide a satisfactory accuracy. This is particularly true for the risk-free interest rate and several impulse response functions. Our second finding is that fifth-order perturbations are much more accurate, but they become cumbersome to compute and require a non-trivial runtime and some skill at memory management. Our third finding is that second- and third-order Taylor projections offer an outstanding compromise between accuracy and speed. Second-order Taylor projections can be as accurate as Smolyak collocations and, yet, be solved in a fraction of the time. Third-order Taylor projection takes longer to run, but their accuracy can be quite high, even in testbed as challenging as the New Keynesian model with rare disasters.[3]

We postulate, therefore, that a new generation of solution methods, such as Taylor projection (but also, potentially, others such as those in Maliar and Maliar, 2014) can be an important tool in fulfilling the promises of production models with rare disasters. We are ready now to start our analysis by moving into the description of the model.

2 A DSGE Model with Rare Disasters

We build a standard New Keynesian model along the lines of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). In the model, there is a representative

[3] We provide `MATLAB` codes that implement the Taylor projection method for a general class of DSGE models. Given these codes, the implementation of our method in larger, more complex models should be relatively easy and straightforward.

household that consumes and saves, a final good producer, a continuum of intermediate good producers subject to Calvo pricing, and a monetary authority that sets up the nominal interest rate following a Taylor rule. Given the goals of this paper and to avoid excessive complexity in the model, we avoid wage rigidities.

As we described in the introduction, we augment the standard New Keynesian model along two dimensions. First, we introduce Epstein-Zin preferences. Beyond being extremely popular in macroeconomics and asset pricing, these preferences have been studied in the context of New Keynesian models by Andreasen (2012), Rudebusch and Swanson (2012), and Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2013), among several others. Second, we add a time-varying rare disaster risk. Rare disasters impose two permanent shocks on the real economy: a productivity shock and a capital depreciation shock. When a disaster occurs, technology and capital fall immediately. This specification should be viewed as a reduced form that captures severe disruptions in production, such as those caused by a war or a large natural catastrophe, and failures of firms and financial institutions, such as those caused by massive labor unrest or a financial panic.

We present first the full New Keynesian model and some of its asset pricing implications. Then, in subsection 2.7, we briefly describe the simpler versions of the model mentioned in the introduction.

2.1 The household

A representative household's preferences are representable by an Epstein-Zin aggregator between the period utility U_t and the continuation utility V_{t+1}:

$$V_t^{1-\psi} = U_t^{1-\psi} + \beta \mathbb{E}_t \left(V_{t+1}^{1-\gamma} \right)^{\frac{1-\psi}{1-\gamma}} \tag{1}$$

where the period utility over consumption c_t and labor l_t is given by $U_t = e^{\xi_t} c_t \left(1 - l_t\right)^{\nu}$ and $\mathbb{E}_t$ is the conditional expectation operator. The parameter γ controls risk aversion (Swanson, 2012) and the intertemporal elasticity of substitution (IES) is given by $1/\widehat{\psi}$, where $\widehat{\psi} = 1 - (1+\nu)(1-\psi)$ (Gourio, 2012). The intertemporal preference shock ξ_t follows:

$$\xi_t = \rho_\xi \xi_{t-1} + \sigma_\xi \epsilon_{\xi,t}, \;\; \epsilon_{\xi,t} \sim \mathcal{N}(0,1).$$

The household's budget constraint is given by:

$$c_t + x_t + \frac{b_{t+1}}{p_t} = w_t l_t + r_t k_t + R_{t-1} \frac{b_t}{p_t} + F_t + T_t, \tag{2}$$

where x_t is investment in physical capital, w_t denotes the real wage, r_t is the real rental price of capital, F_t are the real profits of the firms in the economy, and T_t is a real lump-sum transfer from the government. The household trades a nominal bond b_t that pays a nominal gross interest rate of R_t. We transform the nominal bond into real quantities by dividing by the price p_t of the final good. There is, as well, a full set of Arrow securities. With complete markets and a zero net supply condition for those securities, we can omit them from the budget constraint without further consequences.

Investment x_t induces the law of motion for capital:

$$k_t^* = (1-\delta)\, k_t + \mu_t \left(1 - S\left[\frac{x_t}{x_{t-1}}\right]\right) x_t \tag{3}$$

where

$$\log k_t = \log k_{t-1}^* - d_t \theta_t \tag{4}$$

and

$$S\left[\frac{x_t}{x_{t-1}}\right] = \frac{\kappa}{2}\left(\frac{x_t}{x_{t-1}} - \Lambda_x\right)^2.$$

Here, k_{t-1}^* is the capital decision taken by the household in period $t-1$. Actual capital k_t, however, depends on the disaster shock. Define an indicator function d_t that takes values 0 or 1. If a disaster occurs in t (i.e., $d_t = 1$), k_t falls by θ_t. Gourio (2012) interprets θ_t as the permanent capital depreciation caused by the disaster.

We want, in addition, to capture the idea that the disaster risk can be time-varying. To do so, we add an AR structure to the log of θ_t:

$$\log \theta_t = (1-\rho_\theta) \log \bar{\theta} + \rho_\theta \log \theta_{t-1} + \sigma_\theta \epsilon_{\theta,t}, \;\; \epsilon_{\theta,t} \sim \mathcal{N}(0,1)$$

which resembles those in models with stochastic volatility (Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez, 2015; see also a similar specification in Gabaix, 2012). Note, nevertheless, that d_t is non-Gaussian, a fact that will have a material effect on the dynamics of the model. We specify the evolution of θ_t in logs to ensure positive values of this variable.

The second term on the right-hand side of equation (3):

$$\mu_t \left(1 - S\left[\frac{x_t}{x_{t-1}}\right]\right) x_t$$

includes two parts: First, an investment-specific technological shock μ_t that follows:

$$\log \mu_t = \log \mu_{t-1} + \Lambda_\mu + \sigma_\mu \epsilon_{\mu,t}, \;\; \epsilon_{\mu,t} \sim \mathcal{N}(0,1).$$

Second, a quadratic capital adjustment cost function that depends on investment growth (Christiano, Eichenbaum, and Evans, 2005).

The household maximizes its preferences (1) subject to the budget constraint (2) and the law of motion for capital (3). The optimality conditions for this problem are:

$$\mathbb{E}_t \left(M_{t+1} \exp\left(-d_{t+1}\theta_{t+1}\right) \left[r_{t+1} + q_{t+1}\left(1-\delta\right)\right]\right) = q_t \tag{5}$$

$$\begin{aligned} 1 = q_t \mu_t & \left[\left(1 - S\left[\frac{x_t}{x_{t-1}}\right]\right) - S'\left[\frac{x_t}{x_{t-1}}\right]\frac{x_t}{x_{t-1}}\right] \\ & + \mathbb{E}_t \left(M_{t+1}\left[q_{t+1}\mu_{t+1} S'\left[\frac{x_{t+1}}{x_t}\right]\left(\frac{x_{t+1}}{x_t}\right)^2\right]\right), \end{aligned} \tag{6}$$

$$\nu \frac{c_t}{1-l_t} = w_t, \tag{7}$$

where λ_t is the Lagrange multiplier associated with the budget constraint, q_t is the Lagrange multiplier associated with the evolution law of capital (as a ratio of λ_t), and M_{t+1} is the stochastic discount factor:

$$M_{t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{V_{t+1}^{\psi-\gamma}}{\mathbb{E}_t \left(V_{t+1}^{1-\gamma}\right)^{\frac{\psi-\gamma}{1-\gamma}}}.$$

A non-arbitrage condition also determines the nominal gross return on bonds:

$$1 = \mathbb{E}_t M_{t+1} \frac{R_t}{\Pi_{t+1}}.$$

See the appendix for more details.

2.2 The final good producer

The final good y_t is produced by a perfectly competitive firm that bundles a continuum of intermediate goods y_{it} using the production function:

$$y_t = \left(\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{8}$$

where ε is the elasticity of substitution. The final good producer maximizes profits subject to the production function (8) and taking as given the price of the final good, p_t, and all intermediate goods prices p_{it}. Well-known results tell us that:

$$p_t = \left(\int_0^1 p_{it}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}.$$

2.3 Intermediate good producers

There is a continuum of differentiated intermediate good producers that combine capital and labor with the production function:

$$y_{i.t} = \max\left\{A_t k_{i,t}^{\alpha} l_{i,t}^{1-\alpha} - \phi z_t, 0\right\}. \tag{9}$$

The common neutral technological level A_t follows a random walk with a drift in logs:

$$\log A_t = \log A_{t-1} + \Lambda_A + \sigma_A \epsilon_{A,t} - (1-\alpha)\, d_t \theta_t,\ \ \epsilon_{A,t} \sim \mathcal{N}(0,1),$$

subject to a Gaussian shock $\epsilon_{A,t}$ and a rare disaster shock d_t with a time-varying impact θ_t. Following Gourio (2012), disasters reduce physical capital and total output by the same factor. This can be easily generalized at the cost of heavier notation and, possibly, additional state variables. Note the presence of a common fixed cost ϕz_t, which we index by a measure of technology,

$$z_t = A_t^{\frac{1}{1-\alpha}} \mu_t^{\frac{\alpha}{1-\alpha}}$$

to ensure that such fixed cost remains relevant along the equilibrium dynamics of the model.

Intermediate good producers rent labor and capital in perfectly competitive markets with flexible wages and rental rates of capital. However, intermediate good producers set prices à la Calvo. In each period, a fraction $1-\theta_p$ of intermediate good producers reoptimize their prices to $p_t^* = p_{it}$ (the reset price is common across all firms that update their prices). All other firms keep their old prices. This pricing structure yields the standard Calvo block (see derivation in the appendix):

$$\frac{k_t}{l_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t} \tag{10}$$

$$g_t^1 = mc_t y_t + \theta_p \mathbb{E} M_{t+1} \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{-\varepsilon} g_{t+1}^1 \tag{11}$$

$$g_t^2 = \Pi_t^* y_t + \theta_p \mathbb{E} M_{t+1} \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{1-\varepsilon} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right) g_{t+1}^2 \tag{12}$$

$$\varepsilon g_t^1 = (\varepsilon - 1)\, g_t^2 \tag{13}$$

$$mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{w_t^{1-\alpha} r_t^{\alpha}}{A_t}. \tag{14}$$

Here, $\Pi_t \equiv \frac{p_t}{p_{t-1}}$ is the inflation rate in terms of the final good, $\Pi_t^* \equiv \frac{p_t^*}{p_t}$ is the ratio between the reset price and the price of the final good, mc_t is the marginal cost of the intermediate good producer, and g_t^1 and g_t^2 are auxiliary variables that allow us to write this block recursively.

2.4 The monetary authority

The monetary authority sets the nominal interest rate according to the Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left(\left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi} \left(\frac{\frac{y_t}{y_{t-1}}}{\exp\left(\Lambda_y\right)}\right)^{\gamma_y}\right)^{1-\gamma_R} e^{\sigma_m \epsilon_{m,t}} \tag{15}$$

where $\epsilon_{m,t} \sim \mathcal{N}(0,1)$ is a monetary shock, the variable Π is the target level of inflation, and R the implicit target for the nominal gross return of bonds (which depends on Π, β, and the growth rate along the balanced growth path of the model). The proceedings from monetary policy are distributed lump-sum to the representative household.

2.5 Aggregation

The aggregate resource constraint is given by:

$$c_t + x_t = \frac{1}{v_t^p}\left(A_t k_t^\alpha l_t^{1-\alpha} - \phi z_t\right) \tag{16}$$

where

$$v_t^p = \int_0^1 \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} di$$

is a measure of price dispersion with law of motion:

$$v_t^p = \theta_p \left(\frac{\Pi_{t-1}^\chi}{\Pi_t}\right)^{-\varepsilon} v_{t-1}^p + \left(1-\theta_p\right)\left(\Pi_t^*\right)^{-\varepsilon}.$$

2.6 Asset prices

Rare disasters have a large impact on asset prices. In fact, this is the reason they have become so popular. Thus, it is worthwhile to spend some space reviewing the asset pricing implications of the model. First, we have that the price of a one-period risk-free real bond, q_t^f, is determined by the Euler condition:

$$q_t^f = \mathbb{E}_t\left(M_{t+1}\right).$$

Second, the price of a claim to the stream of dividends $div_t = y_t - w_t l_t - x_t$ (all income minus labor income and investment), which we can call equity, is equal to:

$$q_t^e = \mathbb{E}_t\left(M_{t+1}\left(div_{t+1} + q_{t+1}^e\right)\right).$$

In our budget constraint, we specified that the household owns the physical capital and rents it to the firm. Given our complete markets assumption, this is equivalent to the firm owning the physical capital and the household owning these claims to dividends. Our specification makes deriving optimality conditions slightly easier.

Third, we can define the price-earnings ratio:

$$\frac{q_t^e}{div_t} = \mathbb{E}_t \left(M_{t+1} \frac{div_{t+1}}{div_t} \left(1 + \frac{q_{t+1}^e}{div_{t+1}} \right) \right).$$

All these prices can be solved indirectly, once we have obtained the solution of M_{t+1} and other endogenous variables, or simultaneously. In this paper, we will solve for q_t^f and q_t^e simultaneously with the other endogenous variables. This will show the flexibility of our approach. In general, it is not a good numerical strategy to solve simultaneously for volatile asset prices. For example, the price of a consol fluctuates wildly, especially if the expected return is low or even negative. This happens when disaster risk suddenly rises. The perturbation solution for the price of this asset would display large Taylor coefficients that converge very slowly. The Taylor projection method may even fail to give a solution, because it builds on the assumption that variables fluctuate within the convergence domain of their Taylor series. Note that in models with financial frictions, it is necessary to solve simultaneously for real variables and asset prices, as the latter influences the values of the former.

2.7 Stripping down the full model

In our analysis below, we will solve eight versions of the model in order to examine the computational properties of the solution for models of different size and complexity.

Version 1 of the model is a benchmark real business cycle model with Epstein-Zin preferences and time-varying disaster risk. Prices are fully flexible, the intermediate good producers do not have market power (i.e., ε goes to infinity), and there are no adjustment costs in investment. Hence, instead of the the Calvo block (10)-(14), factor prices are determined by their marginal products:

$$r_t = \alpha A_t k_t^{\alpha-1} l_t^{1-\alpha} \tag{17}$$

$$w_t = (1-\alpha) A_t k_t^{\alpha} l_t^{-\alpha}. \tag{18}$$

The benchmark version consists of four state variables: planned capital k_{t-1}^*, disaster shock d_t, disaster risk θ_t, and technology innovations $\sigma_A \epsilon_{A,t}$. Also, since the model satisfies the classical dichotomy, we can ignore the Taylor rule.

Version 2 of the model introduces investment adjustment costs to version 1, but not the investment-specific technological shock. This adds past investment x_{t-1} as another state variable. We still ignore the monetary part of the model.

Version 3 of the model reintroduces price rigidity. Since we start using the Calvo block (10)-(14), we need two additional state variables: past inflation Π_{t-1} and price dispersion v_{t-1}^{p}. However, in this version 3, we employ a simple Taylor rule that responds only to inflation. Versions 4 and 5 extend the Taylor rule so it responds to output growth and the past interest rate. These two versions introduce past output and interest rate as additional state variables. But, in all three versions, there are no monetary shocks to the Taylor rule.

Finally, versions 6, 7, and 8 of the model introduce the investment-specific technological shock, the monetary shocks, and the preference shocks. These shocks are added to the vector of state variables one by one. Therefore, the full model that we described in detail in this section (version 8) contains 12 state variables.

3 Calibration

Before we compute the model, we normalize all relevant variables to obtain stationarity. We follow the normalization scheme in Fernández-Villaverde and Rubio-Ramírez (2006).

The model is calibrated at a quarterly frequency. When needed, Gaussian shocks are discretized by monomial rules with $2n_\epsilon$ nodes (for n_ϵ shocks). Parameter values are listed in Table 1. Most parameters are taken from Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2015), who perform a structural estimation of a very similar DSGE model (hereafter FQR). There are three exceptions. The first exception is Epstein-Zin parameters and standard deviation of TFP shocks, which we take from Gourio (2012).

The second exception is the three parameters in the Taylor rule, which we calibrate somewhat more conservatively than those in FQR. Specifically, we pick the inflation target to be 2 percent annually, the inflation parameter γ_Π to be 1.3, which satisfies the Taylor principle, and the interest smoothing parameter γ_R to be 0.5. The estimated values of γ_R and γ_Π in FQR are less common in the literature. Furthermore, when combined with rare disasters, they generate too strong, and empirically implausible, nonlinearities.

The third exception is the parameters related to disasters. In the baseline calibration, we calibrate the mean disaster impact $\bar{\theta}$ such that output loss in a disaster is 40 percent. This is broadly in line with the figures presented by Barro (2006), who indicates an average contraction of 35 percent (compared to trend). We do not account for partial recoveries, so the impact of disaster risk may be overstated. For our purposes, this type of bias makes the model more difficult to solve because the nonlinearity is stronger. The persistence of disaster risk is set at $\rho_\theta = 0.9$, which is close to Gourio (2012) and Gabaix (2012), although these researchers use specifications that are slightly different from ours. The standard deviation of the disaster risk is calibrated at $\sigma_\theta = .025$. The four disaster parameters - disaster probability, mean impact, persistence, and standard deviation - have a strong effect on the precautionary saving motive and, hence, on asset prices and equilibrium dynamics. Ideally, these parameters should be jointly estimated, but, to keep our focus, we do not pursue this route in the present paper. Instead, we choose parameter values that generate realistic risk premia and that are broadly consistent with the previous literature.

We also consider an alternative no-disaster calibration. In this calibration, we set the mean and standard deviation of the disaster impact very close to zero, while keeping all the other parameter values as in the baseline calibration in Table 1. We do so in order to benchmark our results without disasters and gauge the role of large risks in terms of accuracy and computational time.

4 Solution Methods

Given that we will deal with models with up to 12 state variables, we can only investigate solution methods that scale well in terms of the dimensionality of the state space. This eliminates, for example, value function iteration or projection methods based on tensors. The three methods left on the table are perturbation (a particular case of which is linearization), Taylor projection, and Smolyak collocation.[4] The methods are implemented for different polynomial orders. More concretely, we will aim at computing 176 solutions, with 11 solutions per each of the eight versions of the model -perturbations from order 1 to 5, Taylor projections from order 1 to 3, and Smolyak collocation from level 1 to 3- and the two calibrations described above, the baseline calibration and the no-disaster calibration. As we will describe below, we could not find a few of the Smolyak collocation solutions.

Perturbation and Smolyak collocation are well-known methodologies. They are described in detail in Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016). In comparison, Taylor projection is a new method recently proposed by Levintal (2016). We discuss the three methods briefly in the next pages. But, before doing so, we need to introduce some notation.

[4] Judd, Maliar, and Maliar (2011) is a possible alternative solution, based on a simulation method. Maliar and Maliar (2014) survey the recent developments in simulation methods. We abstract from simulation methods, because the Smolyak collocation method is already satisfactory in terms of computational costs. Possibly, for larger models simulation methods may be more efficient than Smolyak collocation, although we will later introduce some comments on why we conjecture that, for this class of models, simulation methods may be difficult to implement.

Following Schmitt-Grohé and Uribe (2004), we cast the model in the following form:

$$\mathbb{E}_t f\left(y_{t+1}, y_t, x_{t+1}, x_t\right) = 0 \tag{19}$$

$$y_t = g\left(x_t\right) \tag{20}$$

$$x_{t+1} = h\left(x_t\right) + \eta\epsilon_{t+1}, \tag{21}$$

where x_t is a vector of n_x state variables, y_t is a vector of n_y control variables, $f : \mathbb{R}^{2n_x+2n_y} \to \mathbb{R}^{n_x+n_y}$, $g : \mathbb{R}^{n_x} \to \mathbb{R}^{n_y}$, $h : \mathbb{R}^{n_x} \to \mathbb{R}^{n_x}$, η is a known matrix of dimensions $n_x \times n_\epsilon$, and ϵ is a $n_\epsilon \times 1$ vector of zero mean shocks. The first equation gathers all expectational conditions, the second one maps states into controls, and the last one gives us the law of motion for states. Equations (19)-(21) constitute a system of $n_y + n_x$ functional equations in the unknown policy functions g and h. In practical applications, some of the elements of h are known (e.g. the evolution of the exogenous state variables), so the number of unknown functions and equations is smaller.

4.1 Perturbation

Perturbation introduces a perturbation parameter σ that controls the volatility of the model. Specifically, equation (21) is replaced with:

$$x_{t+1} = h\left(x_t\right) + \sigma\eta\epsilon_{t+1}.$$

At $\sigma = 0$, the model boils down to a deterministic model. The steady state of the deterministic model, denoted $\bar{x}$, is calculated (assuming it exists). Then, by applying the implicit function theorem, we recover the derivatives of the policy functions g and h with respect to x and σ. Having these derivatives, the policy functions are approximated by a Taylor series around $\bar{x}$. To capture risk effects, the Taylor series must include at least second-order terms (Schmitt-Grohé and Uribe, 2004).

High-order perturbation solutions have been developed and explored by Judd (1998), Gaspar and Judd (1997), Jin and Judd (2002), Schmitt-Grohé and Uribe (2004), and Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006), among others. Obtaining perturbation solutions is easy for low orders, but the problem becomes cumbersome at high orders, especially for large models. In this paper, we use the perturbation algorithm presented in Levintal (2015), which allows to solve models with non-Gaussian shocks up to the fifth order.

4.2 Smolyak collocation

Collocation is one of the projection methods introduced by Judd (1992). The policy functions $g(x)$ and $h(x)$ are approximated by polynomial functions $\widehat{g}(x, \Theta_g)$ and $\widehat{h}(x, \Theta_h)$, where Θ_g and Θ_h are the polynomial coefficients of $\widehat{g}$ and $\widehat{h}$, respectively. Let $\Theta = (\Theta_g, \Theta_h)$ denote a vector of size n_Θ of all polynomial coefficients. Substituting in equation (19) yields a residual function $R(x_t, \Theta)$:

$$R(x_t, \Theta) = \mathbb{E}_t f\left(\widehat{g}\left(\hat{h}(x_t, \Theta_h) + \eta\epsilon_{t+1}, \Theta_g\right), \widehat{g}(x_t, \Theta_g), \widehat{h}(x_t, \Theta_h) + \eta\epsilon_{t+1}, x_t\right).$$

Collocation methods evaluate the residual function $R(x, \Theta)$ at N points $\{x_1, \ldots, x_N\}$, and find the vector Θ for which the residual function is zero at all points. This requires solving a nonlinear system for Θ:

$$R(x_i, \Theta) = 0, \;\; \forall i = 1, \ldots, N. \tag{22}$$

The number of grid points N is chosen such that the number of conditions is equal to the number of coefficients to be solved (n_Θ).

Since DSGE models are multidimensional, the choice of the basis function is crucial for computational costs. We follow Krüger and Kubler (2004) by using Smolyak polynomials as the basis function. Smolyak polynomials are products of Chebyshev polynomials, but unlike tensor products, which grow exponentially, the number of terms in Smolyak polynomials grows polynomially with the number of state variables. We implement Smolyak polynomials of levels 1, 2, and 3. These approximation levels vary in the size of the basis function. The level 1 approximation contains $1 + 2n_x$ terms, the level 2 contains $1 + 4n_x + (4n_x(n_x - 1))/2$ terms, and the level 3 contains $1 + 8n_x + 12n_x(n_x - 1)/2 + 8n_x(n_x - 1)(n_x - 2)/6$ terms (see Krüger and Kubler, 2004, for details). The Smolyak approximation level is different from the polynomial order, as it contains higher order terms. For instance, an approximation of level 1 contains quadratic terms. Hence, the number of terms in a Smolyak basis of level k is larger than the number of terms in a k order complete polynomial.

The first step of this approach is to construct the grid $\{x_1, \ldots, x_N\}$. The bounds of the grid affect the accuracy of the solution. For a given basis function, a wider grid reduces accuracy, because the same approximating function has to fit a larger domain of the state space. Generally, we would like to have a good fit at points that the model is more likely to visit, at the expense of other less likely points.

Disaster models pose a special challenge for grid-based methods, because the disaster periods are points of low likelihood, but with a large impact. Hence, methods that build a grid over a high probability region are not appropriate for disaster models (see a recent summary in Maliar and Maliar, 2014). For this reason, we choose a more conservative approach and construct the grid by a hypercube. Specifically, we obtain a third-order perturbation solution, which is computationally cheap, and use it to simulate the model. Then, we take the smallest hypercube that contains all the simulation points (including the disaster periods) and build a Smolyak grid over the hypercube. In the level-3 Smolyak approximations, we had to increase the size of the hypercube by up to 60 percent; otherwise, the Jacobian would be severely ill-conditioned (we use the Newton method; see below). Our grid method is extremely fast, so we ignore its computational costs in our run time comparisons.[5]

The final, and most demanding, step is to solve the nonlinear system (22). Previous studies have used time iteration, e.g., Krüger and Kubler (2004), Malin, Krüger, and Kubler (2011), and Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015), but this method can be slow. More recently, Maliar and Maliar (2014) have advocated the use of fixed-point iteration as a faster algorithm. For the size of our models (up to 12 state variables), we find that a Newton method with analytic Jacobian performs surprisingly well. The run time of the Newton method is faster than that of the fixed-point methods reported in the literature for models of similar size, e.g. see Judd, Maliar, Maliar, and Valero (2014). Moreover, the Newton method ensures convergence if the initial guess is sufficiently good, whereas fixed-point iteration does not guarantee convergence even if it starts near the solution. Our initial guess is a third-order perturbation solution, which proves to be sufficiently accurate for the models we study. Thus, the Newton method converges in just a few iterations. The main cost we encounter is the memory constraint, which becomes binding at a level-3 approximation for the largest model (12 state variables).[6]

The algorithm employed in solving the nonlinear system dictates the type of basis function and grid that should be used. Since we apply the Newton method, we must use a basis function and a grid that yield a numerically stable system. Our implementation of Smolyak collocation has this property. By comparison, methods that use derivative-free solvers (e.g., Maliar and Maliar, 2015) gain more flexibility in the choice of basis functions and grids, but lose the convergence property of Newton-type solvers, which are particularly convenient in our case because we have access to a good initial guess.

[5]Judd, Maliar, Maliar, and Valero (2014) propose to replace the hypercube with a parallelotope that encloses the ergodic set. This technique may increase accuracy if the state variables are highly correlated. In our case, the correlation between the state variables is very low (piecewise correlation is 0.14 on average), so the potential gain from this method is small, while computational costs are higher. More recently, Maliar and Maliar (2014, 2015) have proposed new types of grids. We skip the implementation of these more advanced techniques because our collocation method already performs well and the new ideas, which carry computational costs of their own, may be more useful in other classes of models.

[6]We work on a Dell computer with a Intel(R) Core(TM) i7-5600U Processor and 16GB RAM, and our codes are written in `MATLAB/MEX`.

4.3 Taylor projection

Taylor projection is a new type of projection method proposed by Levintal (2016). As with standard projection methods, the goal is to find Θ for which the residual function $R(x,\Theta)$, defined by equation (22), is approximately zero over a certain domain of the state space that is of interest. Taylor projection builds on the Taylor theorem, which states that $R(x,\Theta)$ can be approximated in the neighborhood of x_0 by a kth-order Taylor series about x_0. If the kth-order Taylor series is exactly zero (i.e., all the Taylor coefficients up to the kth-order are zero), then $R(x,\Theta)\approx 0$ in the neighborhood of x_0. Thus, Taylor projection finds Θ for which the kth-order Taylor series of the residual function about x_0 is exactly zero. This amounts to finding Θ that solves:

$$
\begin{aligned}
&R(x_0,\Theta)=0\\
&\left.\frac{\partial R(x,\Theta)}{\partial x_i}\right|_{x_0}=0,\ \forall i=1,\dots,n_x\\
&\left.\frac{\partial^2 R(x,\Theta)}{\partial x_{i_1}\partial x_{i_2}}\right|_{x_0}=0,\ \forall i_1,i_2=1,\dots,n_x\\
&\vdots\\
&\left.\frac{\partial^k R(x,\Theta)}{\partial x_{i_1}\cdots\partial x_{i_k}}\right|_{x_0}=0,\ \forall i_1,\dots,i_k=1,\dots,n_x.
\end{aligned} \tag{23}
$$

Namely, the residual function and all its derivatives up to the kth-order should be zero at x_0. When this holds, all the terms of the kth-order Taylor series of $R(x,\Theta)$ about x_0 are zero.

System (23) is solved for Θ using the Newton method with the analytic Jacobian. For comparability with Smolyak collocation, we use the same initial guess, which is a third-order perturbation solution, and the same stopping rule for the Newton method.

Taylor projection offers several computational advantages over standard projection methods. First, a grid is not required. The polynomial coefficients are identified by information that comes from the model derivatives, rather than a grid of points. Second, the basis function is a complete polynomial. This gives additional flexibility over Smolyak polynomials. For instance, interaction terms can be captured by a second-order solution, which has $1+n_x+n_x(n_x+1)/2$ terms in the basis function. In Smolyak polynomials, interactions show up only at the level-2 approximation with $1+4n_x+(4n_x(n_x-1))/2$ terms in the basis function (asymptotically 4 times larger). More terms in the basis function translate into a larger Jacobian, which is the main computational bottleneck of the Newton method. Finally, the Jacobian of Taylor projection is much sparser than the one from collocation. Hence, the computation of the Jacobian and the Newton step is cheaper.

The main cost of Taylor projection is the computation of all the derivatives. Note that the Jacobian requires differentiation of the nonlinear system (23) with respect to Θ. These derivatives can be computed efficiently by the chain rule method developed by Levintal (2015). This method expresses high-order chain rules in compact matrix notation that exploits symmetry, permutations, and repeated partial derivatives. The chain rules can also take advantage of sparse matrix (or tensor) operations. For more details, see Levintal (2016).

5 Results

We are finally ready to discuss our results. In three subsections, we will describe our findings in terms of accuracy, simulations, and computational costs.

5.1 Accuracy

Following the literature, we assess the accuracy of the various solution methods in two ways. As proposed by Judd (1992), we compare the mean and maximum unit-free Euler errors across the ergodic set of the model. We approximate this ergodic set by simulating the model with the solution that was found to be the most accurate (third-order Taylor projection).

We first report accuracy measures for the no-disasters calibration model to benchmark our results. Tables 2-3 report the mean and maximum error for this calibration. As expected, all 11 solutions are reasonably accurate for each of the 8 versions of the model. The mean Euler errors (in log10 units) range from around -2.7 (for a first-order perturbation) to -10.2 (for a level-3 Smolyak). The max Euler errors range from -1.3 (for a first-order perturbation) to -9.3 (for a level-3 Smolyak). These results replicate the well-understood notion that models with weak volatility can be approximated well by linearization. See, for a similar result, Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006).[7]

Tables 4-5 report the accuracy measures for the baseline calibration.[8] The accuracy measures change significantly when disasters are introduced into the model. The mean and maximum errors are now, across all solutions, 2-3 orders of magnitude larger than before.

[7] All through this section, we approximate the same set of variables by all methods and use the model equations to solve for the remaining variables. While applying perturbation methods, researchers usually employ instead the perturbation solution for all variables. We avoid that practice because we want to be consistent across all solution methods.

[8] The results for the level-1 Smolyak collocation are partial, because the Newton solver did not converge in all cases. The level-3 Smolyak could not be solved for version 8 of the model due to insufficient memory. Also, for the level-3 Smolyak and to avoid ill-conditioned Jacobians, the size of the grid was increased by 30 percent for version 3 of the model and by 60 percent for versions 4-7.

First-order perturbation and Taylor projection solutions are severely inaccurate, with max Euler errors as high as -0.4. Higher-order perturbation solutions are more accurate, but errors are still relatively large. In particular, we find that a third-order perturbation solution is unlikely to be accurate enough, with mean Euler errors between -1.8 and -2.4 and max Euler errors between -1.7 and -1.9. Even a fifth-order perturbation can generate a disappointing mean Euler error of between -1.9 and -3.5.

In comparison, second- and third-order Taylor projections deliver a much more solid accuracy, with mean Euler errors between -3.6 and -6.9. Interestingly, the max Euler errors are about two orders of magnitude larger, suggesting that in a few rare cases these solutions are significantly less accurate. We will later explore whether the differences between mean and max Euler errors are economically significant.

The Smolyak solution improves over the fifth-order perturbation solution, but it is less accurate than a Taylor projection of comparable order. How can this happen given the higher-order terms in the polynomials forming the Smolyak solution? Because of the strong nonlinearity generated by rare disasters. The Smolyak method has to extrapolate outside the grid. Since the grid already contains extreme points (rare disasters), extrapolating outside these extreme points introduces even more extreme points (e.g., a disaster period that occurs right after a disaster period). By comparison, Taylor projection evaluates the residual function and its derivatives at one point, which is a normal period. Thus, it has to extrapolate only for next-period likely outcomes, which can be either normal or disaster periods. This reduces the approximation errors that contaminate the solution.

To dig deeper, we plot in Figure 1 the model residuals across the ergodic set for the four most accurate solutions (second- and third-order Taylor projection and Smolyak collocation). We use version 7 of the model, for which we have all four solutions. These plots reveal the different pattern of the errors of Taylor projection compared to Smolyak collocation. Taylor projection exhibits very small errors throughout most of the sample, except for two peaks of high errors, which occur around disaster periods. Since Taylor projection zeros the Taylor series of the residual function, the residuals are small as long as the model stays around the center of the Taylor series (the steady state in our case). Namely, Taylor projection yields a locally accurate solution, which deteriorates at points distant from the center. Fortunately, these points are relatively unlikely, even considering the effects of disaster risk.

In principle, it is possible to improve the accuracy of Taylor projection by solving the model at multiple points, as done in Levintal (2016). For instance, we could solve the model also at a disaster period and use this solution when the model visits that point. However, for these solutions to be accurate an important condition must hold: the state variables must not change dramatically (in probability) from the current period to the future period. This condition holds when the model is in a normal state, because it is highly likely that it stays at a normal state the next period as well. However, if the model is in a disaster state, it is very likely that it will change to a normal state the next period. Hence, solving the model in a disaster state is prone to higher approximation errors. Nevertheless, it is important to understand this property of Taylor projection, because one can build the model in such a way that the future state of the economy is likely to be similar to the current state (for instance, by increasing the frequency of the calibration or the persistence of the exogenous shocks).

The Smolyak errors depicted in Figure 1 are more evenly distributed than the Taylor projection errors. This is not surprising, because the collocation algorithm minimizes residuals across the collocation points, which represent the ergodic set. This also reflects the uniform convergence of projection methods (Judd, 1998). The problem is that the disaster periods tilt the solution towards these rare episodes at the expense of the more likely normal states. As a result, the errors in normal states get larger, because the curvature of the basis function is limited. The solution to the problem is to increase the Smolyak order, but as shown below, the computational costs are too high.

5.2 Simulations

Our second step is to compare the equilibrium dynamics generated by the different solutions. In particular, we look at two standard outputs from DSGE models: moments from simulations and impulse response functions.

Rare disasters generate a strong impact on asset prices and risk premia. The solution methods should be able to approximate these effects. Hence, we examine how the different solutions approximate the prices of the two assets in our model: equity and risk-free bonds. Tables 6-7 present the mean risk-free rate and the mean return on equity across simulations generated by the different methods. We focus on version 7 of the model, for which we have all the solutions. By the previous accuracy measures, the most accurate solutions are Taylor projection of orders 2 and 3, and Smolyak collocation of orders 2 and 3. The mean risk-free rate in these four solutions is 1.7-1.8 percent. Note that despite the differences in mean and maximum Euler errors, from an economic viewpoint, these four solutions yield roughly the same result. The differences across the four solutions are smaller than 0.06.

By comparison, perturbation solutions, which have been found to be less accurate, generate a much higher risk-free rate, ranging from 4.8 percent at the first order to 2.3 percent at the fifth order. At the third order (a popular choice when solving models with stochastic volatility), the risk-free rate is 2.9 percent. Therefore, our evidence suggests that perturbation methods fail to approximate accurately the risk-free rate, unless one goes for very high orders. At the fifth order, the approximation errors are relatively small, which is consistent with the results obtained by Levintal (2015). Nevertheless, researchers that seek higher accuracy should use different methods. The approximation of the mean return on equity is more volatile across the different perturbation solutions, but fairly close to the 5.8-5.9 percent obtained by the four accurate solutions.

We next examine impulse response functions. We focus on the disaster variables, which generate the main nonlinearity in our model. Figure 2 presents the response of the model to a disaster shock. The initial point is the fixed point to which the model converges in the absence of shocks. The figure plots the response of output, investment, and consumption. In the left panels we plot three perturbation solutions and a third-order Taylor projection. In the right panels we plot the three Taylor projections and Smolyak levels 2 and 3 (the mnemonics in the figure should be easy to read). Although the scale of the shock is large and, therefore, it tends to cluster all impulse response functions, we can see some non-trivial differences in the impulse response functions from low-order perturbations with respect to all the other impulse response functions (furthermore, the model is solved for the detrended variables, which are much less volatile).

Figure 3 plots the impulse response functions of a disaster risk shock (θ_t). We assume that the disaster impact θ_t rises from a contraction of 40 percent to a contraction of 45 percent, which under our calibration is a 3.5 standard deviation event. As explained in section 3, this relatively small change has a large impact on the model, because the model is highly sensitive to the disaster parameters. All solutions generate in response a decline in detrended output, investment, and consumption, but the magnitudes differ considerably. As before, the left panels of the figure compare the perturbation solutions to a third-order Taylor projection. Low-order perturbation solutions fail to approximate well the model dynamics, although the fifth-order perturbation is relatively accurate. The right panel of Figure 3 shows a similarity of the four most accurate solutions (second- and third-order Taylor projection and Smolyak levels 2 and 3). We read this figure, as well as the results from Tables 6-7, as suggesting that the solutions generated by a second- and third-order Taylor projection are economically indistinguishable from the solutions from a Smolyak collocation.

Figure 4 shows similar impulse response functions, but only for the four most accurate solutions. The left panel depicts the same impulse response as in Figure 3 with some zooming in. The right panel shows impulse response functions for a larger shock, which increases the anticipated disaster impact from 40 percent to 50 percent. Given the standard deviation of θ_t, which is calibrated at 2.5 percent, the shock we consider is a 7 standard deviation event. Barro (2006) points out that, while rare, this is a shock that is sometimes observed in the data. Note that differences among the solutions are economically small (the scale is log).

Nevertheless, there seem to be two clusters of solutions: second-order Taylor projection and Smolyak level-2 and third-order Taylor projection and Smolyak level-3.

We conclude from this analysis that second- and third-order Taylor projections and Smolyak solutions are economically similar. We could not find a significant difference between these solutions. The other solutions are relatively poor approximations, except for the fifth-order perturbation solution, which is reasonably good.

5.3 Computational costs

Our previous findings suggest that the second- and third-order Taylor projections and Smolyak solutions are similar. However, when it comes to computational costs, there are more than considerable differences among the solutions. Table 8 reports total run time (in seconds) for each solution. The second-order Taylor projection is the fastest method among the four accurate solutions by a large difference. It takes less than 3 seconds to solve the seventh version of the model with second-order Taylor projection, 100 seconds with third-order Taylor projection, 31 seconds with second-order Smolyak and 4,067 seconds with third-order Smolyak. Given that these solutions are roughly equivalent, this is a remarkable result. Taylor projection allows us to solve large and highly nonlinear models in a few seconds, and potentially to nest the solution within an estimation algorithm, where the model needs to be solved hundreds of times for different parameter values. Note also that a second-order Taylor projection takes considerably less time than a fifth-order perturbation (3.5 seconds versus 60.3 seconds for the full model), even if its mean Euler errors are smaller (-3.6 versus -2.2).

The marginal costs of the different methods are extremely heterogeneous. Moving from version 7 to version 8 of the model adds only one exogenous state variable. This change increases the run time of a second-order Taylor projection by 0.8 second. By comparison, a third-order Taylor projection takes about 61 more seconds, Smolyak level-2 takes roughly 31 more seconds, and Smolyak level-3 could not be computed due to insufficient memory. Extrapolating these trends forward implies that the differences in computational costs across solutions would increase rapidly with the size of the model.

We conclude that the second-order Taylor projection solution delivers the best accuracy/speed tradeoff among the tested solutions. The run time of this method is sufficiently fast to enable estimation of the model, which would be much more difficult with the other methods tested. For researchers interested in higher accuracy at the expense of higher costs, we recommend the third-order Taylor projection solution, which is faster than a Smolyak solution of comparable order.

Finally, we provide `MATLAB` codes that perform the Taylor projection method on the class of models defined in section 4. Given these codes, Taylor projection is as straightforward and easy to implement as standard perturbation methods. In comparison, coding a Smolyak collocation requires some degree of skill and care.

6 Conclusions

Models with rare disasters have become a popular line of research in macroeconomics and finance. However, rare disasters, by inducing significant non-linearities, present non-trivial computational challenges that have been largely ignored in the literature or dealt with only in a non-systematic fashion. To fill this gap, in this paper, we formulated and solved a New Keynesian model with time-varying disaster risk (including several simpler versions of it). Our findings were as follows. First, low-order perturbation solutions (first, second, and third) do not offer enough accuracy as measured by the Euler errors, computed statistics, or impulse response functions. A fifth-order perturbation fixes part of the problem, but it is still not entirely satisfactory regarding accuracy and it imposes some serious computational costs. Second, a second-order Taylor projection seems an excellent choice, with a satisfactory balance of accuracy and run time. A third-order Taylor projection can handle a medium size model with even better accuracy, but at a higher cost. Finally, Smolyak collocation methods were accurate, but they were hard to implement (we failed to find a solution on several occasions and faced memory limitations) and suffered from long run times.

This paper should be read only as a preliminary progress report. There is much more to be learned about the properties of models with rare disasters than we can cover in one paper. However, we hope that our results will stimulate further investigation on the topic.

7 Appendix

7.1 Euler Conditions

Define the household's maximization problem as follows:

$$\max_{c_t, k_t^*, x_t, l_t} \left\{ U_t^{1-\psi} + \beta \mathbb{E}_t \left(V_{t+1}^{1-\gamma} \right)^{\frac{1-\psi}{1-\gamma}} \right\}$$
$$\text{s.t. } c_t + x_t - w_t l_t - r_t k_t - F_t - T_t = 0$$
$$k_t^* - (1-\delta) k_t - \mu_t \left(1 - S \left[\frac{x_t}{x_{t-1}} \right] \right) x_t = 0$$
$$k_{t+1} = k_t^* \exp(-d_{t+1} \theta_{t+1}) .$$

Note that the value function V_t depends on the household's actual stock of capital k_t and on past investment x_{t-1}, as well as on aggregate variables and shocks that the household takes as given. Thus, let us use $V_{k,t}$ and $V_{x,t}$ to denote the derivatives of V_t with respect to capital k_t and past investment x_{t-1} (assuming differentiability). These derivatives are obtained by the envelope theorem:

$$(1-\psi) V_t^{-\psi} V_{k,t} = \lambda_t r_t + Q_t (1-\delta) \tag{24}$$
$$(1-\psi) V_t^{-\psi} V_{x,t} = Q_t \mu_t S' \left[\frac{x_t}{x_{t-1}} \right] \left(\frac{x_t}{x_{t-1}} \right)^2 , \tag{25}$$

where λ_t and Q_t are the Lagrange multipliers associated with the budget constraint and the evolution law of capital (they enter the Lagrangian in negative sign). We exclude the third constraint from the Lagrangian and substitute it directly in the value function or the other constraints, whenever necessary.

Differentiating the Lagrangian with respect to c_t, k_t^*, x_t, and l_t yields the first-order conditions:

$$(1-\psi) U_t^{-\psi} U_{c,t} = \lambda_t \tag{26}$$
$$(1-\psi) \beta E_t \left(V_{t+1}^{1-\gamma} \right)^{\frac{\gamma-\psi}{1-\gamma}} E_t \left(V_{t+1}^{-\gamma} V_{k,t+1} \exp(-d_{t+1}\theta_{t+1}) \right) = Q_t \tag{27}$$
$$\lambda_t = Q_t \mu_t \left[\left(1 - S \left[\frac{x_t}{x_{t-1}} \right] \right) - S' \left[\frac{x_t}{x_{t-1}} \right] \frac{x_t}{x_{t-1}} \right] + \tag{28}$$
$$+ (1-\psi) \beta E_t \left(V_{t+1}^{1-\gamma} \right)^{\frac{\gamma-\psi}{1-\gamma}} E_t \left(V_{t+1}^{-\gamma} V_{x,t+1} \right)$$
$$(1-\psi) U_t^{-\psi} U_{l,t} = -\lambda_t w_t \tag{29}$$

Substituting the envelope conditions (24)-(25) and defining:

$$q_t = \frac{Q_t}{\lambda_t}$$

yields equations (5)-(7) in the main text.

7.2 The Calvo Block

The intermediate good producer that is allowed to adjust prices maximizes the discounted value of its profits. Fernández-Villaverde and Rubio-Ramírez (2006, pp. 12-13) derive the first-order conditions of this problem for expected utility preferences, which yield the following recursion:

$$g_t^1 = \lambda_t mc_t y_t + \beta\theta_p E_t \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{-\epsilon} g_{t+1}^1$$

$$g_t^2 = \lambda_t \Pi_t^* y_t + \beta\theta_p E_t \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{1-\epsilon} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right) g_{t+1}^2.$$

To adjust these conditions to Epstein-Zin preferences, divide by λ_t to have:

$$\frac{g_t^1}{\lambda_t} = mc_t y_t + \beta\theta_p E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{-\epsilon} \frac{g_{t+1}^1}{\lambda_{t+1}} \tag{30}$$

$$\frac{g_t^2}{\lambda_t} = \Pi_t^* y_t + \beta\theta_p E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{1-\epsilon} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right) \frac{g_{t+1}^2}{\lambda_{t+1}} \tag{31}$$

Note that $\beta\frac{\lambda_{t+1}}{\lambda_t}$ is the stochastic discount factor in expected utility preferences. In Epstein-Zin preferences the stochastic discount factor is given instead by (2.1). Substituting and defining $\bar{g}_t^1 = \frac{g_t^1}{\lambda_t}$, $\bar{g}_t^2 = \frac{g_t^2}{\lambda_t}$ yields (10)-(14). The other conditions in the Calvo block follow directly from Fernández-Villaverde and Rubio-Ramírez (2006, pp. 12-13).

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Table 1: Baseline Calibration

Parameter	Value	Source
Leisure preference (ν)	2.33	Gourio (2012)
Risk aversion (γ)	3.8	Gourio (2012)
Inverse IES ($\widehat{\psi}$)	0.5	Gourio (2012)
Trend growth of TFP (Λ_A)	.0028	FQR (2015)
Std of TFP shocks (σ_A)	.01	Gourio (2012)
Trend growth of investment shock (Λ_μ)	0	
Std of investment shock (σ_μ)	.0024	FQR (2015)
Discount factor (β)	.99	FQR (2015)
Cobb-Douglas parameter (α)	.21	FQR (2015)
Depreciation (δ)	.025	FQR (2015)
Fixed production costs (ϕ)	0	FQR (2015)
Disaster probability	.0043	Gourio (2012)
Mean Disaster size ($\bar{\theta}$)	.5108	
Persistence of disaster risk shock (ρ_θ)	.9	
Std of disaster risk shock (σ_θ)	.025	
Adjustment cost parameter (κ)	9.5	FQR (2015)
Calvo parameter (θ_p)	.8139	FQR (2015)
Automatic price adjustment (χ)	.6186	FQR (2015)
Elasticity of substitution (ϵ)	10	FQR (2015)
Inflation target (Π)	1.0050	
Inflation parameter in Taylor rule* (γ_Π)	1.3	
Output growth parameter in Taylor rule (γ_y)	.2458	FQR (2015)
Interest smoothness in Taylor rule* (γ_R)	.5	
Std of monetary shock ($\sigma_{m,t}$)	.0025	FQR (2015)
Persistence of intertemporal shock (ρ_ξ)	.1182	FQR (2015)
Std of intertemporal shock (σ_ξ)	.1376	FQR (2015)

Table 2: No disasters: Mean Euler Errors (log10) across the ergodic set

Model	state vars	Perturbation					Taylor Projection			Smolyak collocation		
		1st	2nd	3rd	4th	5th	1st	2nd	3rd	1st	2nd	3rd
1. Benchmark with EZ and disasters	4	-2.8	-3.4	-5.3	-6.4	-7.1	-3.3	-6.2	-8.7	-3.3	-7.4	-10.2
2. + capital adjustment costs	5	-2.7	-3.9	-5.0	-6.7	-7.1	-3.1	-4.9	-6.8	-2.7	-5.1	-7.1
3. + Calvo	7	-2.8	-4.0	-4.4	-5.9	-6.5	-3.3	-5.2	-7.0	-3.2	-5.3	-7.0
4. + Taylor rule depends on output growth	8	-2.8	-4.1	-4.7	-6.2	-6.9	-3.3	-5.2	-7.0	-3.2	-5.3	-6.5
5. + Taylor rule is smoothed	9	-2.8	-4.1	-4.6	-6.1	-6.7	-3.3	-5.2	-7.0	-3.2	-5.2	-6.5
6. + investment shock	10	-2.8	-4.1	-4.6	-6.1	-6.7	-3.1	-4.9	-6.8	-3.2	-5.3	-6.7
7. + monetary shock	11	-2.7	-3.9	-4.6	-5.9	-6.6	-2.9	-4.7	-6.4	-3.0	-5.1	-6.4
8. + intertemporal preference shock	12	-2.8	-4.0	-4.6	-5.7	-6.6	-3.0	-4.8	-6.5	-3.2	-5.2	-

Table 3: No disasters: Max Euler Errors (log10) across the ergodic set

Model	state vars	Perturbation					Taylor Projection			Smolyak collocation		
		1st	2nd	3rd	4th	5th	1st	2nd	3rd	1st	2nd	3rd
1. Benchmark with EZ and disasters	4	-1.3	-2.3	-4.2	-5.4	-7.1	-1.3	-3.4	-5.5	-1.9	-6.2	-9.3
2. + capital adjustment costs	5	-1.8	-2.9	-4.2	-5.4	-6.6	-1.8	-2.9	-4.2	-1.4	-3.2	-5.5
3. + Calvo	7	-1.6	-2.7	-4.1	-5.1	-6.0	-1.6	-2.8	-4.1	-1.9	-4.2	-5.4
4. + Taylor rule depends on output growth	8	-1.6	-2.8	-4.1	-5.4	-6.4	-1.6	-2.8	-4.1	-1.9	-4.2	-5.0
5. + Taylor rule is smoothed	9	-1.6	-2.8	-4.2	-5.4	-6.5	-1.6	-2.8	-4.1	-1.9	-4.1	-4.9
6. + investment shock	10	-1.8	-3.0	-4.5	-5.5	-6.5	-1.8	-3.1	-4.5	-2.1	-4.1	-4.9
7. + monetary shock	11	-1.8	-3.0	-4.4	-5.4	-6.4	-1.8	-3.1	-4.4	-2.0	-3.9	-4.9
8. + intertemporal preference shock	12	-1.7	-3.0	-4.0	-5.1	-6.1	-1.7	-3.1	-4.4	-2.1	-4.1	-

Table 4: Disaster Models - Mean Euler Errors (log10) across the ergodic set

Model	state vars	Perturbation					Taylor Projection			Smolyak collocation		
		1st	2nd	3rd	4th	5th	1st	2nd	3rd	1st	2nd	3rd
1. Benchmark with EZ and disasters	4	-1.7	-2.0	-2.4	-2.9	-3.5	-3.1	-5.3	-6.9	-3.2	-6.0	-8.5
2. + capital adjustment costs	5	-1.6	-2.0	-2.4	-2.8	-3.2	-2.5	-4.1	-5.4	-	-1.6	-3.6
3. + Calvo	7	-1.7	-2.0	-1.8	-1.7	-1.9	-2.4	-3.8	-4.8	-1.0	-2.6	-3.6
4. + Taylor rule depends on output growth	8	-1.8	-2.1	-2.1	-2.0	-2.2	-2.5	-4.0	-5.1	-1.0	-2.7	-3.5
5. + Taylor rule is smoothed	9	-1.7	-2.1	-2.1	-2.1	-2.2	-2.2	-3.7	-4.5	-	-2.7	-3.7
6. + investment shock	10	-1.8	-2.1	-2.1	-2.1	-2.2	-2.3	-3.7	-4.5	-	-2.6	-3.6
7. + monetary shock	11	-1.8	-2.2	-2.1	-2.1	-2.2	-2.2	-3.6	-4.5	-	-2.6	-3.8
8. + intertemporal preference shock	12	-1.7	-2.1	-2.1	-2.1	-2.2	-2.2	-3.6	-4.4	-	-2.6	-

Table 5: Disaster Models - Max Euler Errors (log10) across the ergodic set

Model	state vars	Perturbation					Taylor Projection			Smolyak collocation		
		1st	2nd	3rd	4th	5th	1st	2nd	3rd	1st	2nd	3rd
1. Benchmark with EZ and disasters	4	-1.5	-1.7	-1.9	-2.1	-2.4	-1.5	-3.1	-3.8	-1.8	-4.9	-7.3
2. + capital adjustment costs	5	-0.8	-1.2	-1.8	-2.1	-2.4	-0.8	-1.1	-1.8	-	-0.7	-2.4
3. + Calvo	7	-0.5	-1.6	-1.7	-1.6	-1.6	-0.6	-1.7	-2.5	-0.1	-1.7	-2.6
4. + Taylor rule depends on output growth	8	-0.5	-1.7	-1.8	-1.8	-1.9	-0.5	-1.7	-2.3	-0.1	-1.7	-2.6
5. + Taylor rule is smoothed	9	-0.5	-1.6	-1.8	-1.9	-1.9	-0.5	-1.6	-2.2	-	-1.7	-2.6
6. + investment shock	10	-0.8	-1.7	-1.8	-1.9	-1.9	-0.8	-1.7	-2.3	-	-1.7	-2.7
7. + monetary shock	11	-0.7	-1.8	-1.9	-1.9	-2.0	-0.9	-2.0	-2.4	-	-1.9	-2.8
8. + intertemporal preference shock	12	-0.4	-1.5	-1.8	-1.8	-1.9	-0.5	-1.5	-2.1	-	-1.9	-

Table 6: Disaster Models - Risk-free rate (% annualized) - simulation average

Model	state vars	Perturbation					Taylor Projection			Smolyak collocation		
		1st	2nd	3rd	4th	5th	1st	2nd	3rd	1st	2nd	3rd
1. Benchmark with EZ and disasters	4	4.6	3.1	1.8	1.0	0.7	0.6	0.5	0.5	0.5	0.5	0.5
2. + capital adjustment costs	5	4.6	2.9	1.6	0.8	0.4	0.3	0.2	0.2	-	0.1	0.2
3. + Calvo	7	4.6	3.0	2.0	1.4	1.1	0.4	0.4	0.4	0.3	0.4	0.4
4. + Taylor rule depends on output growth	8	4.7	3.5	2.7	2.2	2.0	1.5	1.6	1.6	1.8	1.6	1.6
5. + Taylor rule is smoothed	9	4.7	3.4	2.8	2.4	2.1	1.4	1.5	1.6	-	1.5	1.6
6. + investment shock	10	4.8	3.5	2.9	2.5	2.2	1.5	1.6	1.7	-	1.6	1.7
7. + monetary shock	11	4.8	3.6	2.9	2.5	2.3	1.5	1.7	1.8	-	1.7	1.8
8. + intertemporal preference shock	12	4.7	3.4	2.8	2.4	2.1	1.3	1.5	1.6	-	1.5	-

Table 7: Disaster Models - Return on Equity (% annualized) - simulation average

Model	state vars	Perturbation					Taylor Projection			Smolyak collocation		
		1st	2nd	3rd	4th	5th	1st	2nd	3rd	1st	2nd	3rd
1. Benchmark with EZ and disasters	4	5.3	5.3	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4
2. + capital adjustment costs	5	5.5	5.6	5.7	5.8	5.8	5.8	5.8	5.8	-	6.2	5.8
3. + Calvo	7	5.5	5.6	5.8	5.9	6.0	5.7	5.7	5.7	5.1	5.7	5.7
4. + Taylor rule depends on output growth	8	5.4	5.6	5.8	5.9	5.9	5.7	5.7	5.7	4.7	5.7	5.7
5. + Taylor rule is smoothed	9	5.4	5.6	5.9	6.1	6.1	5.6	5.7	5.8	-	5.7	5.7
6. + investment shock	10	5.5	5.7	6.1	6.2	6.2	5.8	5.8	5.9	-	5.8	5.9
7. + monetary shock	11	5.5	5.7	6.1	6.2	6.2	5.7	5.8	5.9	-	5.8	5.9
8. + intertemporal preference shock	12	5.4	5.5	5.9	6.1	6.1	5.6	5.7	5.8	-	5.7	-

Table 8: Run time (seconds)

Model	state vars	Perturbation					Taylor Projection			Smolyak collocation		
		1st	2nd	3rd	4th	5th	1st	2nd	3rd	1st	2nd	3rd
1. Benchmark with EZ and disasters	4	0.0	0.0	0.0	0.0	0.3	0.5	0.7	1.6	0.3	0.5	1.0
2. + capital adjustment costs	5	0.0	0.0	0.0	0.3	1.6	0.6	1.0	2.0	-	1.1	4.6
3. + Calvo	7	0.0	0.0	0.0	0.9	4.5	0.5	1.3	7.7	0.4	3.3	83.2
4. + Taylor rule depends on output growth	8	0.0	0.0	0.0	1.2	6.6	0.6	1.5	13.7	0.4	5.7	308.4
5. + Taylor rule is smoothed	9	0.0	0.0	0.0	1.5	10.3	0.5	1.8	29.2	-	12.7	622.3
6. + investment shock	10	0.0	0.0	0.1	3.4	18.4	0.5	2.2	54.3	-	23.9	1444.0
7. + monetary shock	11	0.0	0.0	0.1	4.4	31.7	0.5	2.7	100.4	-	31.0	4067.2
8. + intertemporal preference shock	12	0.0	0.0	0.1	5.4	60.3	0.6	3.5	161.5	-	62.2	-

We work on a Dell computer with an Intel(R) Core(TM) i7-5600U Processor and 16GB RAM.

Figure 1: Model residuals across the ergodic set

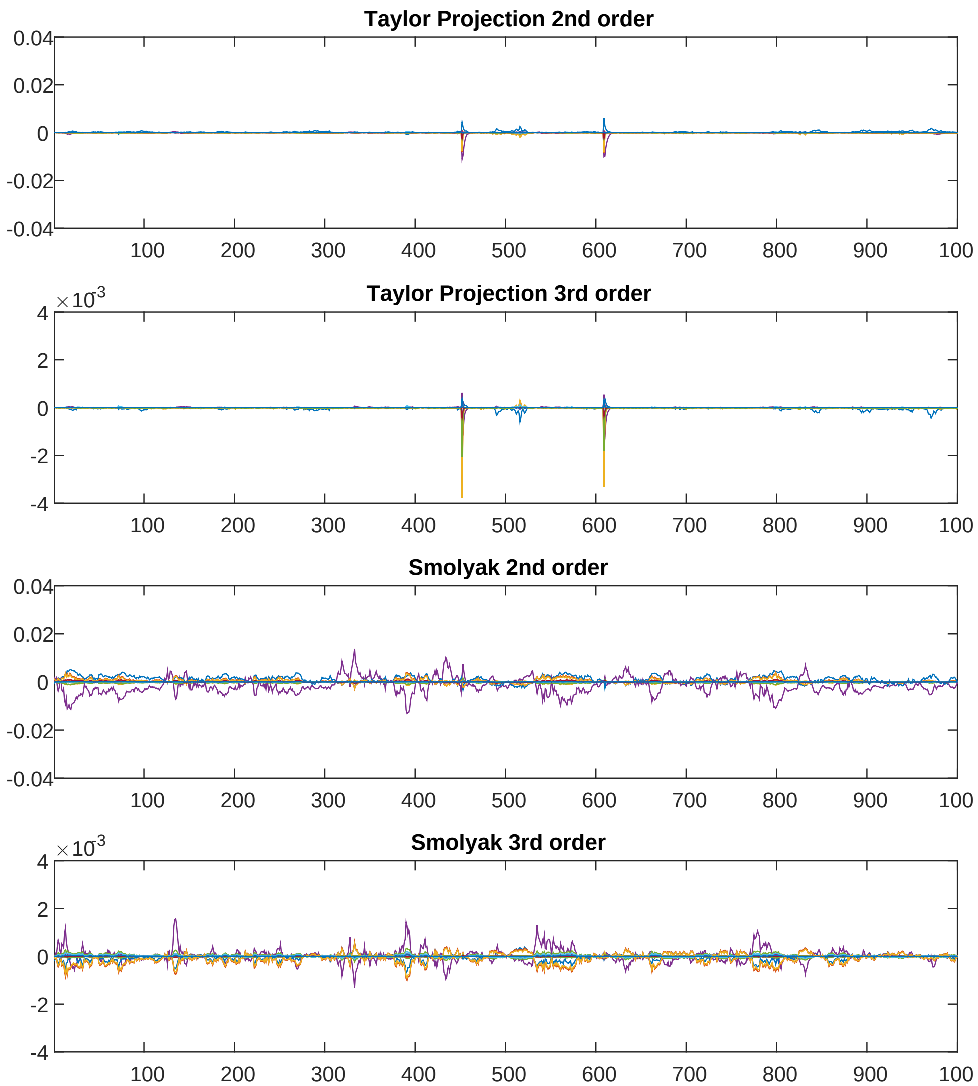

Figure 2: Impulse response functions to a disaster shock

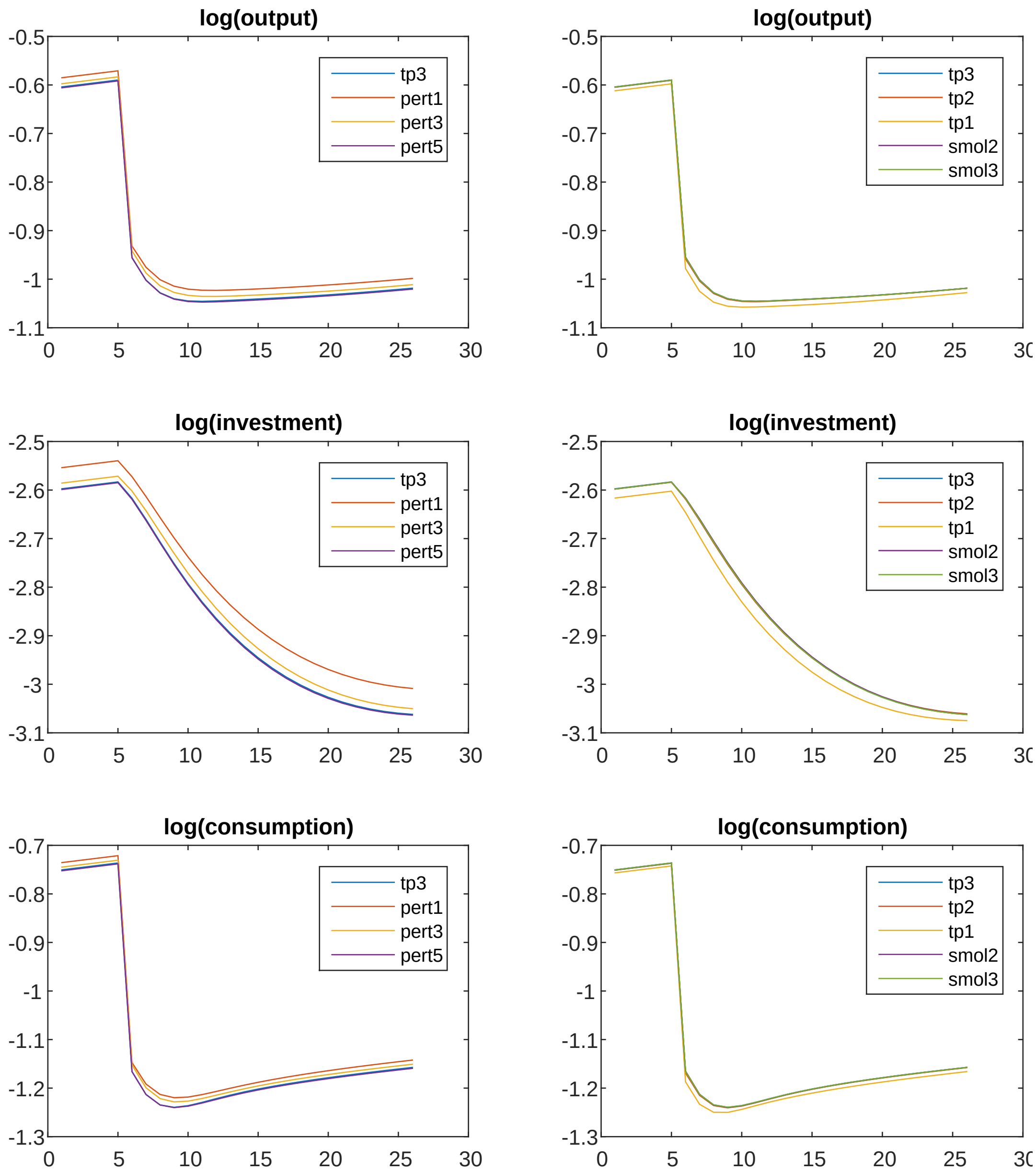

Figure 3: Impulse response functions to a disaster risk shock

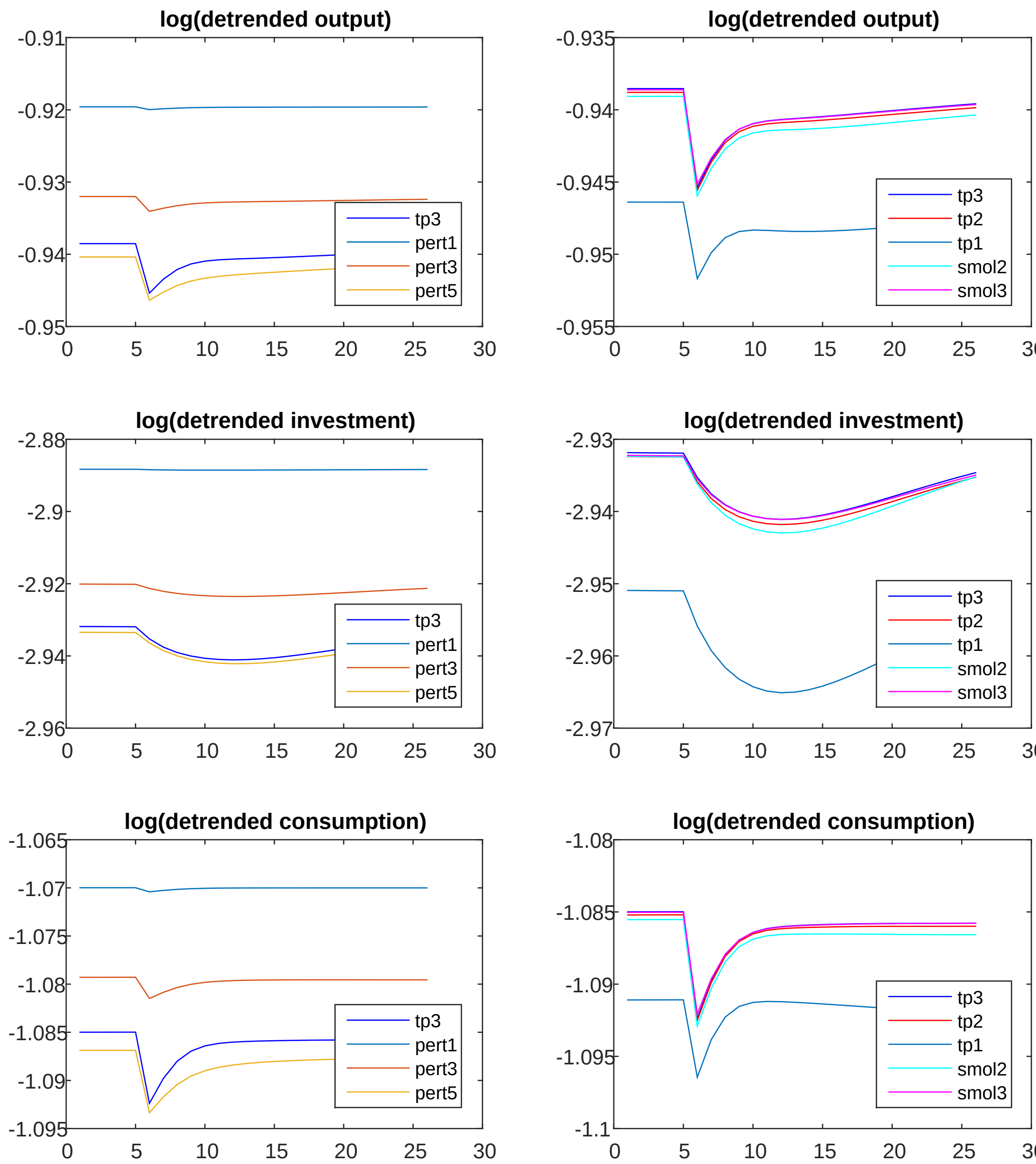

Figure 4: Impulse response functions to small (left) and big (right) disaster risk shocks