NBER WORKING PAPER SERIES

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Working Paper 21888
http://www.nber.org/papers/w21888

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
January 2016

We appreciate helpful comments and assistance with data from Josh Coval, Ben Friedman, Xavier Gabaix, Tina Liu, Matteo Maggiori, Robert Merton, Richard Roll, Steve Ross, Emil Siriwardane, and Glen Weyl, and participants in the macroeconomics seminar at Harvard University. This research project involves no outside funding or conflicts of interest. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 21888
January 2016
JEL No. E44,G12,G13

ABSTRACT

A new options-pricing formula applies to far-out-of-the money put options on the overall stock market when disaster risk is the dominant force, the size distribution of disasters follows a power law, and the economy has a representative agent with Epstein-Zin utility. In the applicable region, the elasticity of the put-options price with respect to maturity is close to one. The elasticity with respect to exercise price is greater than one, roughly constant, and depends on the difference between the power-law tail parameter and the coefficient of relative risk aversion, γ. The options-pricing formula conforms with data from 1983 to 2015 on far-out-of-the-money put options on the U.S. S&P 500 and analogous indices for other countries. The analysis uses two types of data—indicative prices on OTC contracts offered by a large financial firm and market data provided by OptionMetrics, Bloomberg, and Berkeley Options Data Base. The options-pricing formula involves a multiplicative term that is proportional to the disaster probability, p. If γ and the size distribution of disasters are fixed, time variations in p can be inferred from time fixed effects. The estimated disaster probability peaks particularly during the recent financial crisis of 2008-09 and the stock-market crash of October 1987.

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We derive a new options-pricing formula that applies when disaster risk is the dominant force, when the size distribution of disasters is characterized by a power law, and when the economy has a representative agent with Epstein-Zin utility with a constant coefficient of relative risk aversion. Specifically, we consider far-out-of-the-money put options on the overall stock market, corresponding empirically to the S&P 500 in the United States and analogous indices for other countries. The pricing formula applies when the option is sufficiently far out of the money (operationally, a relative exercise price or moneyness of 0.9 or less) and when the maturity length is not too long (operationally, up to 6 months).

In the prescribed region, the elasticity of the put-options price with respect to maturity is close to one. The elasticity with respect to the exercise price is greater than one, roughly constant, and depends on the difference between the power-law tail parameter, denoted α, and the coefficient of relative risk aversion, γ. (This difference has to be positive for various rates of return not to blow up.)

The options-pricing formula involves a multiplicative term that is proportional to the disaster probability, p, measured per year. This term depends also on three other parameters: γ, α, and the threshold disaster size, z_0. If these three parameters are fixed, the variations over time in the multiplicative term reveal the time variations in p. The estimation of the time series for p can then be carried out through standard fixed effects for calendar time. However, a full analysis requires amendments to the options-pricing formula to allow for time-varying p.

We show that the theoretical formula conforms with data from 1983 to 2015 on far-out-of-the-money put options on the U.S. S&P 500 and analogous indices over shorter periods for other countries. Our analysis relies on two types of data—indicative prices on over-the-counter (OTC) contracts offered to clients by a large financial firm and market data provided by

OptionMetrics, Bloomberg, and Berkeley Options Data Base. A key advantage of the OTC source is its provision of a rich array of contracts by exercise price and maturity. In particular, the relative exercise price goes down to 0.5, and the maturity can be 12 months or more. A downside of these data is that the reported prices do not necessarily correspond to actual trades. An advantage of the market data is the correspondence with actual trades, but there are problems with stale prices and sizes of bid-ask spreads. The most serious disadvantage of these data is the limited information on far-out-of-the-money options, which rarely trade. The market data (and trades) are also concentrated on short maturities; for example, about half of the OptionMetrics contracts have maturity of two months or less. In any event, we find that the main results are similar from the two types of data sources.

Extensions of the empirical analysis would allow for second-order terms. These terms involve the possibility of multiple disasters, the presence of a diffusion term, allowances for discounting and expected growth, and the potential for default on options contracts. We think that the most important extension, stressed in Seo and Wachter (2015), involves the effects on stock-options prices from potentially changing disaster probabilities.

I. Baseline Disaster Model and Previous Results

We use a familiar setup based on rare-macroeconomic disasters, as developed in Rietz (1988) and Barro (2006, 2009). The model is set up for convenience in discrete time. Real GDP, Y, is generated from

$$(1) \qquad \log(Y_{t+1}) = \log(Y_t) + g + u_{t+1} + v_{t+1},$$

where, $g \geq 0$ is the deterministic part of growth, u_{t+1} (the diffusion term) is an i.i.d. normal shock with mean 0 and variance σ^2, and v_{t+1} (the jump term) is a disaster shock. Disasters arise from a

Poisson process with probability of occurrence p per period. When a disaster occurs, GDP falls by the fraction b, where $0<b\leq 1$. The distribution of disaster sizes is time invariant. (The baseline model includes disasters but not bonanzas.) This jump-diffusion process for GDP is analogous to the one posited for stock prices in Merton (1976, equations [1]-[3]).[1]

In the underlying Lucas (1978)-tree model, which assumes a closed economy, no investment, and no government purchases, consumption, C_t, equals GDP, Y_t. The implied expected growth rate of C and Y is given, if the period length is very short, by

(2) $$g^* = g + (1/2)\cdot \sigma^2 - p\cdot Eb,$$

where Eb is the mean of b. In this and subsequent formulas, we use an equal sign, rather than approximately equal, when the equality holds as the period length shrinks to zero.

The representative agent has Epstein-Zin/Weil utility,[2] as in Barro (2009):

(3) $$[(1-\gamma)U_t]^{\left(\frac{1-\theta}{1-\gamma}\right)} = C_t^{1-\theta} + \left(\frac{1}{1+\rho}\right)\cdot [(1-\gamma)E_t U_{t+1}]^{\left(\frac{1-\theta}{1-\gamma}\right)},$$

where $\gamma>0$ is the coefficient of relative risk aversion, $\theta>0$ is the reciprocal of the intertemporal-elasticity-of-substitution (IES) for consumption, and $\rho>0$ is the rate of time preference. As shown in Barro (2009) (based on Giovannini and Weil [1989] and Obstfeld [1994]), with i.i.d. shocks and a representative agent, the attained utility ends up satisfying the form:

(4) $$U_t = \Phi \cdot C_t^{1-\gamma}/(1-\gamma),$$

[1]Related jump-diffusion models appear in Cox and Ross (1976).
[2]Epstein and Zin (1989) and Weil (1990).

where the constant Φ>0 depends on the parameters of the model. Using equations (3) and (4), the first-order condition for optimal consumption over time follows from a perturbation argument as

$$(5)\qquad \left[E_t\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]^{\left(\frac{\gamma-\theta}{\gamma-1}\right)} = \left(\frac{1}{1+\rho}\right)\cdot E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\cdot R_{t+1}\right],$$

where R_{t+1} is the gross rate of return on any available asset from time t to time t+1. When $\gamma=\theta$—the familiar setting with time-separable power utility—the term on the left-hand side of equation (5) equals one.

The process for C and Y in equation (1) implies, if the period length is very short:

$$(6)\qquad E_t\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma} = 1+(1-\gamma)g - p + p\cdot E(1-b)^{1-\gamma} + \left(\tfrac{1}{2}\right)(1-\gamma)^2\sigma^2.$$

This condition can be used along with equation (5) to price various assets, including a risk-free bond and an equity claim on a perpetual flow of consumption (that is, the Lucas tree). The constant risk-free interest rate is, if the period length is short:

$$(7)\quad r^f = \rho + \theta g^* - p\cdot\left[E(1-b)^{-\gamma} - \left(\frac{\gamma-\theta}{\gamma-1}\right)E(1-b)^{1-\gamma} - \theta\cdot Eb + \left(\frac{1-\theta}{\gamma-1}\right)\right] - \left(\tfrac{1}{2}\right)\gamma(1+\theta)\sigma^2$$

Let P_t be the price at the start of period t of an unlevered equity claim on the Lucas tree. Let V_t be the dividend-price ratio; that is, the ratio of P_t to consumption, C_t. In the present model with i.i.d. shocks, V_t equals a constant, V. (This condition implies that the growth rate of P_t equals the growth rate of C_t.) The reciprocal of V equals the constant dividend-price ratio and can be determined from equations (5) and (6), if the period length is short, to be:

$$(8)\qquad \frac{1}{V} = \rho - (1-\theta)g^* + p\cdot\left[\left(\frac{1-\theta}{\gamma-1}\right)E(1-b)^{1-\gamma} - (1-\theta)\cdot Eb - \left(\frac{1-\theta}{\gamma-1}\right)\right] + \left(\tfrac{1}{2}\right)\gamma(1-\theta)\sigma^2.$$

The constant expected rate of return on equity, r^e, is the sum of the dividend yield, 1/V, and the expected rate of capital gain on equity, which equals g*, the expected growth rate of the dividend (consumption). Therefore, r^e is the same as equation (8) except for the elimination of the term -g*.[3] The constant equity premium is given from equations (8) and (9) by:

$$(9) \qquad r^e - r^f = \gamma\sigma^2 + p \cdot [E(1-b)^{-\gamma} - E(1-b)^{1-\gamma} - Eb].$$

In equation (9), we can think of the rates of return; the variance, σ^2, of the diffusion process; and the disaster (jump) probability, p, as all measured per year.

The diffusion term, $\gamma\sigma^2$, in equation (9) is analogous to the expression for the equity premium in Mehra and Prescott (1985) and is negligible compared to the observed average equity premium if γ and σ^2 take on empirically reasonable values. For many purposes—including the pricing of far-out-of-the-money stock options—this term can be ignored.

The disaster or jump term in equation (9) is proportional to the disaster probability, p. The expression in brackets that multiplies p depends on the size distribution of disasters, b, and the coefficient of relative risk aversion, γ. The overall disaster term was calibrated in Barro (2006) and Barro and Ursua (2012) by using the long-term history of macroeconomic disasters for up to 40 countries to pin down p and the distribution of b. The resulting term accords roughly with an observed average (unlevered) equity premium of 0.04-0.05 per year if γ is around 3-4.

[3]The transversality condition, which ensures that the value of tree equity is positive and finite, is $r^e > g^*$.

II. Pricing Stock Options

A. Setup for pricing options

We now discuss the pricing of stock options within our model, which fits into the class of jump-diffusion models. Options pricing within this general class goes back to Merton (1976) and Cox and Ross (1976). The use of prices of far-out-of-the-money put options to infer disaster probabilities was pioneered by Bates (1991). This idea has been applied recently by, among others, Bollerslev and Todorov (2011); Backus, Chernov, and Martin (2011); Seo and Wachter (2015); and Siriwardane (2015).

We derive a pricing solution for far-out-of-the-money put options under the assumption that disaster events (i.e. jumps) are the dominant force to consider. Key underlying conditions for the validity of the solution are that the option be sufficiently far out of the money and that the maturity not be too long. Under these conditions, we derive a simple pricing formula that reflects the underlying Poisson nature of disaster events, combined with the assumed power-law distribution for the sizes of disasters. This formula generates testable hypotheses—which we subsequently test—on the relation of put-options prices to maturity and exercise price. The formula also allows, if some key parameters can be treated as constants, for a straightforward time-fixed-effects procedure to back out a time series for disaster probability.

Consider a put option on equity in the Lucas tree. To begin, suppose that the option has a maturity of one period and can be exercised only at the end of the period (a European option). The exercise price or strike on the put option (that is, the price at which one can sell) is

$$\text{exercise price} = \varepsilon \cdot P_t, \tag{10}$$

where we assume $0<\varepsilon\leq 1$. We refer to ε, the ratio of the exercise price to the stock price, as the relative exercise price (also described as "moneyness").

The payoff on the put option at the start of period t+1 is zero if $P_{t+1} \geq \varepsilon \cdot P_t$. If $P_{t+1} < \varepsilon \cdot P_t$, the payoff is $\varepsilon P_t - P_{t+1}$. If $\varepsilon < 1$, the put option is initially out of the money. We focus empirically on options that are sufficiently far out of the money (ε sufficiently below one) so that the diffusion term, u, in equation (1) has a negligible effect on the chance of getting into the money over one period. The value of the put option then hinges on the disaster term, v, in equation (1). More specifically, the value of the put option will depend on the probability, p, of experiencing disasters and the distribution of disaster sizes, b. Further, what will mostly matter is the likelihood of experiencing one disaster. As long as the period (the maturity of the option) is not too long, the chance of two or more disasters has a second-order pricing impact that can be ignored as a good approximation.

Let the price of the put option at the start of period t be $\Omega \cdot P_t$. (For convenience, we omit a time subscript on Ω.) We refer to Ω, the ratio of the options price to the stock price, as the relative options price. The gross rate of return, R^o_{t+1}, on the put option is given by

(11) $$R^o_{t+1} = 0 \text{ if } \frac{P_{t+1}}{P_t} \geq \varepsilon$$

$$R^o_{t+1} = \frac{1}{\Omega} \cdot \left(\varepsilon - \frac{P_{t+1}}{P_t}\right) if \ \frac{P_{t+1}}{P_t} < \varepsilon \, .$$

Suppose that we neglect the diffusion term, u, in equation (1) and also neglect the possibility of two or more disasters during a period. In that case, if there is one disaster of size b, the put option is in the money at the start of period t+1 if[4]

$$\frac{P_{t+1}}{P_t} = (1+g) \cdot (1-b) < \varepsilon \, .$$

[4]This condition would be modified if the underlying stock pays out dividends and if the stock-options contract does not adjust for this payout.

In most cases, the length of the period (maturity of the put option) will be short enough so that, for reasonable growth rates, we can ignore the term g.

Let π be the probability of the put option getting into the money, conditional on experiencing one disaster over the period. Given g and ε, π depends on the distribution of disaster sizes, b. In Barro and Jin (2011), this size distribution was found empirically to conform well to a power-law density for the transformed variable $z \equiv 1/(1-b)$, which can be viewed as the ratio of normal to disaster consumption. The condition $0<b\leq 1$ translates into $z>1$, with z tending to infinity as b tends to 1. The probability of getting into the money on the put option, conditional on having one disaster, is:

(12) $$\pi = 1 - Prob.\left[z < \frac{1+g}{\varepsilon}\right].$$

Since p is the probability of having a disaster in a period, the overall probability of getting into the money over one period is $p\pi$, where π is given in equation (12). (Again, we are neglecting the chance of two or more disasters.)

When expressed in terms of z, the gross rate of return on the put option is modified from equation (11) to:

(13) $$R^o_{t+1} = \frac{1}{\Omega} \cdot \left(\varepsilon - \frac{1+g}{z}\right) if\ 1\ disaster\ occurs\ and\ z > (1+g)/\varepsilon\ ,$$

$$R^o_{t+1} = 0 \text{ otherwise.}$$

To determine Ω, we use the first-order condition from equation (5), with R_{t+1} given by R^o_{t+1} from equation (13). The results depend on the form of the distribution for z, to which we now turn.

B. Power-law distribution of disaster sizes

Based on the findings for the distribution of observed macroeconomic disaster sizes in Barro and Jin (2011), we assume that the density function for z conforms to a power law:[5]

(14) $$f(z) = Az^{-(1+\alpha)}, where\ A > 0, \alpha > 0, and\ z \geq z_0 > 1\ .$$

The general notion of this type of power law was applied by Pareto (1897) to the distribution of high incomes. The power-law distribution has since been applied widely in physics, economics, computer science, and other fields. For surveys, see Mitzenmacher (2003) and Gabaix (2009), who discusses underlying growth forces that can generate power laws. Examples of applications include sizes of cities (Gabaix and Ioannides [2004]), stock-market activity (Gabaix, et al. [2003, 2006]), CEO compensation (Gabaix and Landier [2008]), and firm size (Luttmer [2007]). The power-law distribution has been given many names, including heavy-tail distribution, Pareto distribution, Zipfian distribution, and fractal distribution.

The parameter $z_0>1$ in equation (14) is the threshold beyond which the power-law density applies. For example, in Barro and Ursua (2012), the floor disaster size of $b_0=0.095$ corresponds to $z_0=1.105$. We treat z_0 as a constant. The condition that f(z) integrate to one from z_0 to infinity implies $A = \alpha z_0^{\alpha}$. Therefore, the power-law density function in equation (14) becomes

(15) $$f(z) = \alpha z_0^{\alpha} \cdot z^{-(1+\alpha)},\ z \geq z_0 > 1\ .$$

The key parameter in the power-law distribution is α, which governs the thickness of the right tail. A smaller α implies a thicker tail.

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[5]In Kou (2002, p. 1090), a power-law distribution is ruled out because the expectation of next period's asset price is infinite. This property applies because Kou allows for favorable jumps (bonanzas) and, more importantly, he assumes that the power-law shock enters directly into the log of the stock price. This problem does not arise in our context because we consider disasters and not bonanzas, and, more basically, because our power-law shock multiplies the level of GDP (and consumption and the stock price), rather than adding to the log of GDP.

The probability of drawing a transformed disaster size above z is given by

$$(16) \qquad 1 - F(z) = (\frac{z}{z_0})^{-\alpha}.$$

Thus, the probability of seeing an extremely large transformed disaster size, z (expressed as a ratio to the threshold, z_0), declines with z in accordance with the tail exponent $\alpha>0$.

We can use equation (15) to compute π, the probability of getting into the money on the put option, conditional on experiencing one disaster:

$$(17) \qquad \pi = z_0^{\alpha}(1+g)^{-\alpha}\varepsilon^{\alpha}\ .$$

We assumed here $\varepsilon<(1+g)/z_0$, meaning that the put option is sufficiently far out of the money so that one disaster of threshold size is not enough to get the option into the money. (Otherwise, we would have $\pi=1$.) Equation (17) implies that π rises with ε in accordance with the exponent α. The overall effect of α on π is negative (given the condition $\varepsilon<[1+g]/z_0$); that is, a thinner tail makes getting into the money less likely.

One issue about the power-law density is that some moments related to the transformed disaster size, z, might be unbounded. For example, in equation (7), the risk-free rate of return, r^f, depends negatively on the term $E(1-b)^{-\gamma}$. Heuristically (or exactly with time-separable power utility), we can think of this term as representing the expected marginal utility of consumption in a disaster state relative to that in a normal state. When $z\equiv 1/(1-b)$ is distributed according to f(z) from equation (15), we can compute

$$(18) \qquad E(1-b)^{-\gamma} = E(z^{\gamma}) = \left(\frac{\alpha}{\alpha-\gamma}\right)\cdot z_0^{\gamma} \quad \text{if } \alpha>\gamma.$$

The term in equation (18) is larger when γ is larger (more risk aversion) or α is smaller (fatter tail for disasters). But, if $\alpha\leq\gamma$, the tail is fat enough, relative to the degree of risk aversion, so that the term blows up. In this case, r^f heads toward minus infinity in equation (7), and the equity premium heads toward plus infinity in equation (9). Of course, in the data, proxies for the risk-free rate are not minus infinity, and measures of the equity premium are not plus infinity. Therefore, the empirical application of the power-law density in Barro and Jin (2011) confined γ to a range that avoided unbounded outcomes, given the value of α that was estimated from the observed distribution of disaster sizes. That is, the estimate of the unknown γ had to satisfy $\gamma<\alpha$ in order for the model to have any chance to accord with observed average rates of return.[6] In this range, the values of r^f, 1/V, and r^e-r^f given in equations (7)-(9) are well defined. The same condition turns out to enter into our analysis of far-out-of-the-money put-options prices.

Barro and Jin (2011, Table 1) estimated the power-law tail parameter, α, in single power-law specifications (and also considered double power laws). The estimation was based on macroeconomic disaster events of size 10% or more computed from the long history for many countries of per capita personal consumer expenditure (the available proxy for consumption, C) and per capita GDP, Y. The estimated values of α in the single power laws were 6.3, with a 95% confidence interval of (5.0, 8.1), for C and 6.9, with a 95% confidence interval of (5.6, 8.5), for Y.[7] Thus, the observed macroeconomic disaster sizes suggest a range for α of roughly 5-8.

Some results depend on another term, $E(1-b)^{1-\gamma}$. Heuristically, this term corresponds to the expectation of the product of the proportionate decline in GDP (and consumption and

[6] With constant absolute risk aversion and a power-law distribution of disaster sizes, the relevant term has to blow up. The natural complement to constant absolute risk aversion is an exponential distribution of disaster sizes. In this case, the relevant term is bounded if the parameter in the exponential distribution is larger than the coefficient of absolute risk aversion. With an exponential size distribution and constant relative risk aversion, the relevant term is always finite.

[7] Barro and Jin (2011, Table 1) found that the data could be fit better with a double power law. In these specifications, with a threshold of z_0=1.105, the tail parameter, α, was smaller in the part of the distribution with the largest disasters than in the part with the smaller disasters. The cutoff value for the two parts was at a value of z around 1.4.

stock price) during a disaster and the ratio of disaster to normal marginal utility. With the power-law density for z in equation (15), we can derive

$$\text{(19)} \qquad \mathrm{E}(1-b)^{1-\gamma} = E(z^{\gamma-1}) = \left(\frac{\alpha}{1+\alpha-\gamma}\right) \cdot z_0^{\gamma-1} \quad \text{if } 1+\alpha>\gamma \,.$$

The condition $\gamma<\alpha$, mentioned before, guarantees that the term in equation (19) is well defined.[8]

C. Options-pricing formula

To get the formula for Ω, the relative options price, we use the first-order condition from equations (5) and (6), with the gross rate of return, R_{t+1}, corresponding to the return R_{t+1}^{o} from put options in equation (13). We can rewrite the first-order condition in this context as

$$\text{(20)} \qquad 1+\hat{\rho} = (1+g)^{-\gamma} \cdot E_t(z^{\gamma} R_{t+1}^{o}),$$

where $z \equiv 1/(1-b)$ is the transformed disaster size and $1+\hat{\rho}$ is an overall discount term, given from equations (5) and (6) (when the period length is short and the diffusion term is negligible) by

$$\text{(21)} \qquad 1+\hat{\rho} = 1+\rho-(\gamma-\theta)g + p\cdot\left(\frac{\gamma-\theta}{\gamma-1}\right)\cdot\left[E(1-b)^{1-\gamma}-1\right] .$$

We could substitute out for the term $E(1-b)^{1-\gamma}$ on the right-hand side of equation (21) from equation (19).

We can evaluate the right-hand side of equation (20) using the density f(z) from equation (15) along with the expression for R_{t+1}^{o} from equation (13). The result involves integration over the interval $z \geq (1+g)/\varepsilon$ where, conditional on having a disaster, the disaster size is

[8] The mean disaster size equals $1-\left(\frac{\alpha}{1+\alpha}\right)\cdot\frac{1}{z_0}$.

large enough to get the put option into the money. The formula depends also on the probability, p, of having a disaster. Specifically, we have:

(22) $$(1+\hat{\rho})(1+g)^{\gamma}=\frac{p}{\Omega}\cdot\int_{(\frac{1+g}{\varepsilon})}^{\infty}\left\{z^{\gamma}\cdot\left[\varepsilon-\frac{1+g}{z}\right]\cdot\alpha z_0^{\alpha}z^{-(1+\alpha)}\right\}dz\ .$$

Evaluating the integral (assuming $\gamma<\alpha$ and $\varepsilon<[1+g]/z_0$) leads to a closed-form formula for the relative options price:

(23) $$\Omega=\frac{\alpha z_0^{\alpha}}{(1+\hat{\rho}+\alpha g)}\cdot\frac{p\varepsilon^{1+\alpha-\gamma}}{(\alpha-\gamma)(1+\alpha-\gamma)}\ .$$

D. Maturity of the option

The result in equation (23) applies when the maturity of the put option is one "period." We now take account of the maturity of the option. In continuous time, the parameter p, measured per year, is the Poisson hazard rate for the occurrence of a disaster. Let T, in years, be the maturity of the (European) put option. The density, h, for the number of hits (disasters) over T is given by[9]

(24) $$h(0)=e^{-pT},$$

$$h(1)=pTe^{-pT},$$

$$\ldots$$

$$h(x)=\frac{(pT)^{x}e^{-pT}}{x!},x=0,1,\ldots$$

If pT is much less than 1, the contribution to the options price from two or more disasters will be second-order, relative to that from one disaster. For given p, this condition requires a consideration of maturities, T, that are not "too long." In this range, we can proceed as in our

[9]See Hogg and Craig (1965, p. 88).

previous analysis to consider just the probability and size of one disaster. Then, in equation (23), p will be replaced as a good approximation by pT.

The discount rate, $\hat{\rho}$, and growth rate, g, in equation (23) will be replaced (approximately) by $\hat{\rho}$T and gT, where $\hat{\rho}$ and g are measured per year. For given $\hat{\rho}$ and g, if T is not "too long," we can neglect these discounting and growth terms in equation (23). (Under similar conditions, effects from dividend payouts can also be ignored.) Basically, the impacts of these terms are of the same order as the effect from two or more disasters, which we have already neglected.

When T is short enough to neglect multiple disasters and the discounting and growth terms, the formula for the relative options price changes from equation (23) to:[10]

$$\Omega = \frac{\alpha z_0^{\alpha} \cdot pT \cdot \varepsilon^{1+\alpha-\gamma}}{(\alpha-\gamma)(1+\alpha-\gamma)} . \qquad (25)$$

Here are some properties of the options-pricing formula in equation (25):

- The formula for Ω, the ratio of the options price to the stock price, is well-defined if α>γ, the condition noted before that ensures the finiteness of various rates of return.

[10]The possibility of two disasters turns out to introduce into equation (25) the multiplicative term:

$$1+pT\cdot\left\{-1+0.5\cdot\left[\frac{\alpha z_0^{\alpha}[1+2(\alpha-\gamma)+(\alpha-\gamma)(1+\alpha-\gamma)[\log\left(\frac{1}{\varepsilon}\right)-2\log(z_0)]}{(\alpha-\gamma)(1+\alpha-\gamma)}\right]\right\},$$

assuming $\frac{1}{\varepsilon} > z_0^2$, so that two disasters just at the threshold size are not sufficient to get the option into the money. The full term inside the large brackets has to be positive, so that this multiplicative term is increasing in T. The effects from the discount rate and growth rate add multiplicative terms that look like (1-positive constant·$\hat{\rho}$T) and (1-positive constant·gT). Hence, these multiplicative terms are decreasing in T. The overall effect of T implied by the combination of the three multiplicative terms is unclear. That is, it is unclear how the full result for Ω would deviate from unit elasticity with respect to T.

- The exponent on maturity, T, equals 1.
- The exponent on the relative exercise price, ε, equals 1+α-γ, which is constant and greater than 1 because α>γ. We noted before that α ranged empirically between 5 and 8. The corresponding range for γ (needed to replicate an average unlevered equity premium of 0.04-0.05 per year) is between 2.5 and 5.5, with lower γ associating with lower α. The implied range for α-γ (taking account of the association between γ and α) is between 2.5 and 4.5, implying a range for the exponent on ε between 3.5 and 5.5.
- For given T and ε, Ω depends on the disaster probability, p; the shape of the power-law density, as defined by the tail coefficient, α, and the threshold, z_0; and the coefficient of relative risk aversion, γ. If we maintain the assumptions that α, z_0, and γ are fixed, we can think of the whole expression for Ω as having a multiplier proportional to p. That is, a multiplicative fixed effect for calendar time would reveal the proportionate variations over time in p. (An important caveat is that our derivation of the options-pricing formula in equation [25] ignored the possibility of changing p.)

We can look at the results in terms of the "risk-neutral probability," p^n, defined as the value of p that would generate a specified relative options price, Ω, when γ=0. The formula for the ratio of the risk-neutral to the objective probability, p^n/p, implied by equation (25) is:

(26) $$\frac{p^n}{p} = \frac{\alpha(1+\alpha)}{(\alpha-\gamma)(1+\alpha-\gamma)} \cdot \varepsilon^{-\gamma} \ .$$

Note that p^n/p depends on the relative exercise price, ε, but not on the maturity, T. If we assume parameter values consistent with the previous discussion—for example, α=7 and γ=3.5—the implied p^n/p is 5.1 when ε=0.9, 7.8 when ε=0.8, 12.4 when ε=0.7, 21.3 when ε=0.6, and 40.3 when ε=0.5. Hence, the relative risk-neutral probability associated with far-out-of-the-money put options is sharply above one.

To view it another way, the relative options price, Ω, may seem far too high at low ε, when assessed in terms of the (risk-neutral) probability needed to justify this price. Thus, people who are paying these prices to insure against the risk of an enormous disaster may appear to be irrational. In contrast, the people writing these far-out-of-the-money puts may seem to be getting free money by insuring against something that is virtually impossible. Yet the pricing is reasonable if people have roughly constant relative risk aversion with γ around 3.5 (assuming a tail parameter, α, for disaster size around 7). The people writing these options will have a comfortable income almost all the time, but will suffer tremendously during the largest rare disasters (when the marginal utility of consumption is extremely high).

E. Diffusion term

Recall that the derivation of the formula for Ω, the relative options price, in equation (25) neglected the diffusion term, u, in the process for GDP (and consumption and the stock price) in equation (1). This omission is satisfactory if the put option is sufficiently far out of the money so that, given a reasonable variance σ^2 in the diffusion term, the chance of getting into the money over the maturity T is negligible. In other words, the tail for the normal process is not fat enough to account by itself for, say, 10% or greater declines in stock prices over periods up to, say, a few months. Operationally, our main empirical analysis applies the options-pricing formula in equation (25) to options that are at least 10% out of the money ($\varepsilon \leq 0.9$) and to maturities, T, that range up to 6 months.

If we consider put options at or close to the money, the diffusion term would have a first-order impact on the value of the option. If we neglect the disaster (jump) term—which will be satisfactory here—we would be in the standard Black-Scholes world. In this setting (with i.i.d. shocks), a key property of the normal distribution is that the variance of the stock price over interval T is proportional to T, so that the standard deviation is proportional to the square root of T. This property led to the result in Brenner and Subrahmanyam (1988) that the value of an at-the-money put option would be roughly proportional to the square root of the maturity.

We, therefore, have two results concerning the impact of maturity, T, on the relative options price, Ω. For put options far out of the money (operationally for $\varepsilon \leq 0.9$), the exponent on T is close to 1. For put options close to the money (operationally for $\varepsilon = 1$), the exponent on T is close to one-half. These predictions turn out to hold empirically for put options on the S&P 500 and on analogous market indices for eight other countries.

We could carry out a more general analysis that includes simultaneously diffusion and jump (disaster) risks, though the simplicity of equation (25) would be lost. We can anticipate that, in a range for the relative exercise price, ε, between 0.9 and 1.0, the exponent on T in the formula for Ω would range between 1.0 and 0.5, with values closer to 0.5 applying when the sample was weighted toward options that were close to the money.

For some purposes, we are mainly interested in isolating time-varying disaster probabilities, p, that apply over the short term to large disasters, such as 10% or more declines in per capita GDP. In this context, the main information would likely come from far-out-of-the-money put options, such as where $\varepsilon \leq 0.9$. In this range, the diffusion term would likely have a minor impact, and the formula in equation (25) would provide a satisfactory approximation for the relative options price, Ω. Therefore, the fixed-effects estimates corresponding to this formula should be informative about the time-varying p associated with large disasters.

F. Variations in the disaster probability

The asset-pricing formulas were derived under the assumption that the disaster probability, p, and other parameters were fixed.[11] However, we can get some idea of how persistent changes in p affect stock prices and, thereby, relative stock-options prices by considering the effect from a once-and-for-all (permanent) change in p on the price-dividend ratio, V, given in equation (8). This equation implies (if we hold fixed g, rather than g*, in equation [2]):

$$\frac{1}{V}\frac{\partial V}{\partial p} = -\left(\frac{1-\theta}{\gamma-1}\right)\cdot V\cdot\left[E(1-b)^{1-\gamma}-1\right]. \tag{27}$$

We assume γ>1, which is needed to have any chance of explaining the average equity premium. In this case, as stressed by Bansal and Yaron (2004) and Barro (2009), the sign of the effect of p on V in equation (27) depends on whether θ (the reciprocal of the IES) is less than or greater than one.[12] The "normal result," whereby more uncertainty reduces stock prices, requires θ<1. Bansal and Yaron (2004) assume IES=1/θ=1.5, whereas Barro (2009) focuses on IES=1/θ=2.

If we assume θ=0.5 and use ranges for the tail parameter, α, and coefficient of relative risk aversion, γ, discussed before, we can get an idea of the magnitude of the right-hand side of

[11]Kelly and Jiang (2014, p. 2842) assume a power-law density for returns on individual securities. Their power law depends on a cross-sectional parameter and also on aggregate parameters that shift over time. In the latter part of their analysis, they assume time dependence in the economy-wide values of the tail parameter, analogous to our α, and the threshold, analogous to our z_0. (Their threshold corresponds to the fifth percentile of observed monthly returns.)

[12]This condition on θ also determines the effect on the stock price from a change in σ or in the power-law tail parameter, α, which determines the size distribution of disasters, b.

equation (27).[13] It turns out that a reasonable range is between 5 and 7; that is, an increase in p by 0.01 would generate a proportionate decline in the stock price by between 5% and 7%. However, the conclusions are sensitive to the parameter θ. If θ were close to 1, the effect of a change in p on the stock price would be minor.

A change in p by 0.01 should be viewed in relation to the average p of around 0.04 per year, gauged by the macroeconomic disaster data (Barro and Ursua [2008]). That is, a change in p by 0.01 is large in the sense of constituting 25% of the average p. Moreover, the effects inferred from equation (27) correspond to permanent shifts in p. Nevertheless, it is clear that effects from changing p might contribute substantially to pricing of far-out-of-the-money put options. A further consideration is that the volatility of p (that is, the chance of p rising or falling) is likely to move around. Hence, the effects from potentially shifting p on prices of far-out-of-the-money put-options prices may be substantially higher at some points in time (when volatility is greater) than others.

III. Empirical Analysis

The model in the previous section delivers some testable predictions. First, the elasticity of the price of far out-of-the-money put options with respect to maturity, T—denoted β_T—is close to one. This result applies in the region where diffusion risk is negligible compared to disaster risk and when the effects from multiple disasters can be ignored. Second, in the same region, the elasticity of the price of far-out-of-the-money put options with respect to the relative exercise price, ε—denoted β_ε—is greater than one and constant if the coefficient of relative risk aversion, γ, and the disaster-size-distribution parameters, α and z_0, are fixed. Moreover, the estimated elasticity can be compared with that implied by estimated parameter values from the

[13]This analysis requires an estimate of the price-dividend ratio, V, determined by equation (8). We assume here, based on the analysis in Barro and Ursua (2008), that the rate of time preference, ρ, is 0.04 per year.

rare-disasters literature. Finally, if γ, α, and z_0 are fixed, time variations in the full menu of prices of far-out-of-the-money put prices reflect fluctuations in disaster probability, p_t, which we recover with our model. We test these theoretical predictions empirically by analyzing prices of far-out-of-the-money put options on the U.S. S&P 500 and analogous broad indices for other countries.

A. Data and methodology

Our primary data source is a broker-dealer with a sizable market-making operation in global equities. We utilize over-the-counter (OTC) options prices for nine equity-market indices for developed and emerging markets—S&P 500 (U.S.), FTSE (U.K.), DAX (Germany), Euro Stoxx 50 (Euro zone), Nikkei (Japan), OMX (Sweden), SMI (Switzerland), Nifty (India), and Bovespa (Brazil). We check the results with OTC data against those with market-based information from Option Metrics for the United States and from Bloomberg for the United States and other countries. This check is useful because, as mentioned, the OTC data do not necessarily correspond to actual trades.

Our primary data derive from implied-volatility surfaces generated by the broker-dealer for the purpose of analysis, pricing, and marking-to-market.[14] These surfaces are constructed from transactions prices of options and OTC derivative contracts.[15] The dealer interpolates these observed values to obtain implied volatilities for strikes ranging from 50% to 150% of spot and for a range of maturities from 15 days to 2 years and more. Even at very low strikes, for which the associated options seldom trade, the estimated implied volatilities need to be accurate for the

[14]A common practice in OTC trading is for executable quotes to be given in terms of implied volatility instead of the actual price of an option. Once the implied volatility is agreed on, the options price is determined from the Black-Scholes formula based on the readily observable price of the underlying security. Since the Black-Scholes formula provides a one-to-one link between price and volatility, quotes can be given equivalently in terms of implied volatility or price.

[15]Dealers observe prices through own trades and from indications by inter-dealer brokers. It is also a common practice for dealers to ask clients how their prices compare to other market makers in OTC transactions.

correct pricing of OTC derivatives such as variance swaps and structured retail products. Therefore, sell-side dealers have strong incentives to maintain the accuracy of their implied-volatility surfaces.

As mentioned, the OTC data source is superior to market-based alternatives in the breadth of coverage for exercise prices and maturities. Notably, the market data tend to be unreliable or entirely unavailable for options that are far out of the money and for long maturities. For example, OptionMetrics has very limited information on far out-of-the-money put options prices due to the lack of market transactions and methodological challenges. Specifically, their volatility surface is mainly limited to 20-delta options volatilities at the extreme, which correspond to options that are close to the money,[16] whereas the OTC data contain implied volatilities for 5-delta and even 1-delta options.

The broad range of strikes in the broker-dealer data is important for our analysis because it is the prices of far-out-of-the-money put options that will mainly reflect disaster risk. In practice, we focus on put options with exercise prices of 50%, 60%, 70%, 80%, and 90% of spot; that is, we exclude options within 10% of spot.

For maturities, we focus on a range between 30 days and 6 months; specifically, for 30 days, 60 days, 90 days, and 6 months.[17] Our main analysis excludes options with maturities greater than six months because the prices in this range may be influenced significantly by the possibility of multiple disaster realizations and also by dividend payouts and discounting.

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[16]A 20-delta option has a price that changes by 0.20% for a 1% change in the underlying security price. OptionMetrics Volatility Surface uses interpolation to generate the implied volatility for each security on each day, based on a kernel-smoothing algorithm. The lower bound of this volatility surface is 20-delta. In our use of the OptionMetrics data, we expand on the range of option strikes by applying linear interpolation whenever there are two or more observations for a single trading date. This procedure enlarges the volatility surface.

[17]We omit 15-day options because we think measurement error is particularly serious in this region in pinning down the precise maturity. Even the VIX index, which measures short-dated implied volatility, does not track options with maturity less than 23 days.

However, in practice, the results for 1-year maturity accord reasonably well with those for shorter maturities.

Using the data on implied volatilities, we re-construct options prices from the standard Black-Scholes formula, assuming a zero discount rate and no dividend payouts. We should emphasize that the use of the Black-Scholes *formula* to translate implied volatilities into options prices does not bind us to the Black-Scholes *model* of options prices. The formula is used only to convert the available data expressed as implied volatilities into options prices. Our calculated options prices are comparable to directly quoted prices (subject to approximations related to discounting and dividend payouts).

B. Basic model fit

We estimate the model based on equation (25) with non-linear least-squares regression. In this form, we think of the error term as additive with a constant variance. Log-linearization with a constant-variance error term (that is, a shock proportional to price) is problematic because it understates the typical error in extremely far-out-of-the-money put prices, which are close to zero. That is, this specification gives undue weight to puts with extremely low exercise prices.

In the non-linear regression, we allow for multiplicative time fixed effects to capture time-varying probabilities of disasters. The deterministic part of the regression setup is:

$$\Omega_{it} = \Phi_t \cdot T_{it}^{\beta_T} \cdot \varepsilon_{it}^{\beta_\varepsilon} \,, \tag{28}$$

where i denotes a security (a put option on a broad market index) and t is time, Ω_{it} is the ratio of options price to stock price, Φ_t is the time fixed effect, T_{it} is the maturity, and ε_{it} is the ratio of exercise price to stock price. In the model, the time fixed effect, Φ_t, corresponds to the term:

$$\Phi_t = p_t \alpha z_0^\alpha / [(\alpha - \gamma)(1 + \alpha\text{-}\gamma)]. \tag{29}$$

We think of γ (coefficient of relative risk aversion), α (thickness of the disaster tail), and z_0 (threshold size for disasters) as fixed over time. However, we allow p_t to vary over time; that is, we view the time-varying fixed effect as reflecting solely changes in disaster probability. In our implementation, we also allow the time series of p_t and, hence, Φ_t, to differ across countries.

The missing element in the analysis is that variations in p_t, if persisting, affect stock prices and, therefore, Ω_{it}. We return later to this consideration. With respect to the exponents in equation (29), the model implies β_T=1 and $\beta_\epsilon=1+\alpha-\gamma$, which is constant (over time and across securities) and greater than one.

We sample the data at monthly frequency, selecting only month-end dates, to allow for ease of computation with a non-linear solver. The selection of mid-month dates yields similar results. The sample period for the United States in our main analysis is August 1994-June 2015. Because of lesser data availability, the samples for the other countries are shorter. We expand the sample period for the U.S. back to 1983 as a robustness check, although we do not emphasize this longer-term sample because the data quality before 1994 are considerably poorer. Table 1 shows the model estimation using non-linear least-squares regression. The specification allows for different time fixed effects for each country.

The estimated elasticities with respect to maturity, β_T, are close to one. For example, the estimated coefficient for the United States is 0.978 (s.e.=0.036) and that for all nine countries jointly is 0.938 (s.e.=0.038).[18] These results indicate that far-out-of-the-money prices of put options on broad market indices are roughly proportional to maturity, in accordance with our rare-disasters model. This nearly proportional relationship between options price and maturity for far-out-of-the-money put options is a newly documented fact that cannot be explained under

[18]However, the joint estimates correspond to an unbalanced panel that gives greater weight to later periods (which have more data available).

the Black-Scholes model. To our knowledge, other theoretical models of options prices also do not predict this behavior.

The results with respect to maturity can be visualized in Figure 1, Panel A, which plots ratios of put prices to spot prices against maturity, assuming an exercise price of 80% of spot. The blue curve corresponds to the historical data that underlie Table 1. The red curve shows values generated by the Black-Scholes model, assuming a log-normal distribution of shocks and a constant volatility of 30% (chosen to accord with the average observed level of put prices). Most importantly, the Black-Scholes model predicts that these far-out-of-the-money put prices will have a convex relationship with maturity. This pattern deviates from the nearly linear relationship shown by the historical data.

In contrast, as discussed in Brenner and Subrahmanyam (1988), prices of at-the-money put options in the Black-Scholes model are roughly proportional to the square root of the maturity. This result arises because, with a diffusion process driven by i.i.d. normal shocks, the variance of the log of the stock price is proportional to time and, therefore, the standard deviation is proportional to the square root of time. This pattern implies the concave relation between put price and maturity as shown by the red curve in Figure 1, Panel B. In this case, the Black-Scholes prediction accords with the historical data, shown by the blue curve in Panel B.

Table 2 provides detailed regression estimates for the nine countries for at-the-money put prices. The estimated coefficient on maturity is 0.518 (s.e.=0.007) for the United States and 0.495 (0.007) for the nine countries jointly. Hence, as predicted by Black-Scholes, these coefficients are very close to 0.5.

If one considers exercise prices between 80% and 100% of spot, the Black-Scholes prediction for the relation between put price and maturity shifts from convex to concave at around 90% of spot (with the exact shift point depending on the underlying volatility). The predicted relation turns out to be nearly linear for an exercise price around 90% of spot. In contrast, as implied by Table 1 and Figure 1, Panel A, the relation in the data is roughly linear in maturity for a broad range of exercise prices below 90%—down to at least 50%. These results accord with the rare-disasters model but not with Black-Scholes.

To summarize, the fit of the Black-Scholes model is good for at-the-money put options but poor for put options with exercise prices below 90% of spot. These patterns arise because the diffusion component of shocks dominates pricing of at-the-money put options, whereas disaster risk, not captured in Black-Scholes, dominates the pricing of far-out-of-the-money put options. As discussed earlier in the modeling section, the roughly proportional relationship between far-out-of-the-money put prices and maturity arises because, in a Poisson context, the probability of a disaster is proportional to maturity. The resulting formula is only approximate because it neglects the potential for multiple disasters within the time frame of an option's maturity, omits a diffusion term entirely, and also ignores discounting and dividend payouts. However, for options that are not "too long," these approximations will be reasonably accurate.

Table 1 also shows estimates of the elasticity with respect to the relative exercise price, β_ϵ. This coefficient corresponds in the model to $1 + \alpha - \gamma$, where α is the tail coefficient and γ is the coefficient of relative risk aversion. The estimates are all positive and greater than one, as predicted by the model. The estimated coefficients are similar across countries, falling in a range from 5.16 (s.e.=0.39) for Brazil to 6.06 (0.23) for Switzerland.[19] The joint estimate across the nine countries is 5.83 (0.24).

Rare-disasters research with macroeconomic data, such as Barro and Ursua (2008) and Barro and Jin (2011), suggested that a γ of 3-4 would accord with observed average (unlevered)

[19]However, statistical tests reject the hypothesis of equal coefficients at less than the 1% critical level.

equity premia. With this range for γ, the estimated values of β_ε=1-α-γ from Table 1 imply tail coefficients, α, between 7 and 9. This finding compares with a direct estimate for α based on macroeconomic data on consumption in Barro and Jin (2011, Table 1) of 6.3 (s.e.=0.8). That is, the estimates from Table 1 suggest a thinner tail (higher α) than those found from observation of the size distribution of macroeconomic disasters (based on GDP or consumption). As discussed later, the thinner tail goes along with an implied average probability of disaster that is higher than that inferred from the macroeconomic data.

C. Estimated disaster probabilities

We can use the estimated time fixed effects for each country from the regressions in Table 1, along with equation (29), to construct time series of (objective) disaster probabilities, p_{jt}, where j now denotes the country. The critical assumption here is that, aside from p_{jt}, the other parameters on the right-hand side of equation (29) are constant over time for country j. In that case, the estimated p_{jt} will be proportional to the time fixed effect, Φ_{jt}, for country j.

To get a ballpark idea of the level of p_{jt}, we assume that, in each country, the threshold for disaster sizes is fixed at z_0 =1.1 (as in Barro and Jin [2011]) and that the coefficient of relative risk aversion is γ=3. We allow the tail coefficient, α_j, to differ across countries; that is, we allow countries to differ with respect to the size distribution of potential disasters. We use the estimated coefficients from Table 1 for β_ε (which equals 1+α-γ in the model) to back out the implied tail coefficient, α_j, for country j. (These values range from 7.2 to 8.1.) Figure 2 presents the resulting time series of disaster probabilities for each country, and Table 3 provides summary statistics for these probabilities. Note that the levels of the series, but not the time patterns, depend on our assumed parameter values.

The disaster probabilities shown in Figure 2 have high correlations across the countries**,** with an average pair-wide correlation of 0.88. This property indicates that a large part of the inferred disaster probability can be attributed to the chance of a common (global) disaster. The median disaster probability is high, around 13% per year for the S&P 500. Similar medians apply to the other countries. These high probabilities are much greater than that—3-4% per year—estimated from macroeconomic data on rare disasters (see, for example, Barro and Ursua [2008]). We think that this overstatement of average disaster probability goes along with the understatement of tail risk in disaster sizes, as noted before. That is, the estimated probabilities are too high, and the estimated sizes are too low.

The disaster probabilities in Figure 2 are volatile and right-skewed with spikes during crisis periods. The U.S. disaster probability hit a peak of 70% per year in November 2008. Other countries had their highest disaster probabilities in the range of 60% to 70% in October and November 2008. Japan had the highest peak disaster probability—93% in October 2008. The overall patterns mirror the options-derived U.S. equity premia in Martin (2015) and the U.S. disaster probabilities found by Siriwardane (2015).

Figure 2 suggests a lower bound on disaster probability around 3% per year. This value is close to the (constant) disaster probability of 3.6-3.7% per year found in macroeconomic data by Barro and Ursua (2008).

A first-order AR(1) coefficient for the U.S. disaster probability in Figure 2 is 0.88 (applying at a monthly frequency). This coefficient implies that rare-disaster shocks have an average half-life of 5.4 months. The persistence of disaster probabilities for the other countries is similar to that for the United States, with the AR(1) coefficients ranging from 0.81 for Japan to 0.89 for Sweden.

Although we attributed the time pattern shown in Figure 2 to variable disaster probability, p_{jt}, the variations in the multiplicative time fixed effects may also reflect changes in the other parameters contained in the model's multiplicative disaster term, $p\alpha Z_0^\alpha/[(\alpha-\gamma)(1+\alpha-\gamma)]$.[20] For example, outward shifts in the size distribution of disasters, generated by reductions in the tail parameter, α, or increases in the threshold disaster size, z_0, work like increases in p. Similarly, increases in the coefficient of relative risk aversion, γ, would raise the overall multiplicative term. This kind of change in risk preference, possibly due to habit formation, has been stressed by Campbell and Cochrane (1999). Separation of changes in the parameters of the disaster distribution from those in risk aversion require simultaneous consideration of asset-pricing effects (reflected in Figure 2) with information on the actual incidence and size of disasters (based, for example, on movements of macroeconomic variables).

Another issue is that the underlying asset-pricing theory assumed that the multiplicative term $p\alpha z_0^\alpha/[(\alpha-\gamma)(1+\alpha-\gamma)]$ in equation (25) was constant because each parameter in this expression, including p, was constant. Therefore, there is a disconnect from the theory in using the empirically estimated model to gauge the time variations in the multiplicative term and then attribute the changes to shifting p_{jt}. Surprisingly, although the empirical estimates strongly reject the hypothesis that the multiplicative term is constant over time for each country (as is clear from Figure 2), the estimated coefficients β_T and β_ε in Table 1 conformed well in major respects with the underlying theory (which assumed a fixed p).

The missing element in the asset-pricing theory is that potential changes in p_{jt} have effects on options prices that combine with those from potential realizations of disasters. Specifically, an increase in p_{jt} —to the extent that it has persistence—typically lowers the spot

[20]Note that, in this model with i.i.d. shocks, the multiplicative term does not depend on the intertemporal elasticity of substitution for consumption, 1/θ, or the rate of time preference, ρ,

stock price[21] (and, thereby, raises the ratio of exercise to spot price for an existing option). From a pricing perspective, put prices on options on the overall stock market tend to be higher if future changes in p_{jt} are more likely, if the distribution of the sizes of these changes has a bigger right tail, and if a change in p_{jt} has a larger magnitude of effect on stock prices (possibly because the changes are more persistent).[22] This effect is likely to be of first-order importance and, therefore, should be brought into the analysis.

We think the reason that the empirical model in Table 1 works well in some respects despite the neglect of price effects from potentially changing disaster probabilities is that the time pattern of these probabilities (Figure 2) looks like a rare-disaster process. That is, on rare occasions, p_{jt} moves sharply (and temporarily) higher, and the form of the distribution of the sizes of these changes may resemble a power law. However, even if this interpretation is correct, the problem is that the inferred p_{jt} are actually combinations of levels of disaster probability with effects from potentially changing p_{jt}. This perspective may explain why the inferred p_{jt} are too high on average (compared to the incidence of macroeconomic disasters), whereas the estimated disaster tails are too thin (that is, the estimated coefficients α_j are too high). The underestimation of disaster sizes is effectively a compensation for the overestimation of disaster probabilities.

D. Model robustness

Tables 4-9 explore the robustness of the baseline model from Table 1 under various scenarios. Table 4 presents estimates of different elasticities β_ε over the four ranges of exercise price ratios, ε, used in the baseline model: 0.5 to 0.6, 0.6 to 0.7, 0.7 to 0.8, and 0.8 to 0.9. The

[21]In the theory, this sign applies if the intertemporal elasticity of substitution, $1/\theta$, exceeds one. See Bansal and Yaron (2004) and Barro (2009).

[22]Seo and Wachter (2015) argue that time-varying disaster probability is central for the pricing of options. They also argue that a failure to incorporate this changing disaster probability accounts for some puzzling findings in Backus, Chernov, and Martin (2011). Specifically, for a given level of disaster probability, the model underestimates the prices of options. An analogous force operates in our model.

estimates fall into a fairly narrow range over the first three intervals; for example, for the United States, the estimated coefficients are 5.5 (s.e.=0.1), 5.3 (0.2), and 5.7 (0.2). However, in the highest range, the estimated coefficients tend to be larger, 6.3 (0.2) for the United States. For all countries jointly, the estimated values are 4.7 (0.2), 5.0 (0.2), 5.5 (0.2), and 6.2 (0.2). Possibly the higher β_ε coefficients in the range for ε between 0.8 and 0.9 arises because the diffusion term is non-negligible in this range.

Table 5 applies the analogous methodology to examine elasticities β_T over the three ranges of maturity, T, used in the baseline model: 30 to 60 days, 60 to 90 days, and 90 days to 6 months. There is a tendency for β_T to fall with T, particularly in the highest range. For example, for the United States, the estimates in the three ranges are 1.19 (s.e.=0.05), 1.03 (0.04), and 0.92 (0.03). Similarly, for all countries jointly, the results are 1.14 (0.05), 0.99 (0.04), and 0.88 (0.04). These effects might relate to possibilities for multiple disasters and to discounting.

Table 6 carries out a related analysis in which the structure of included maturities goes out to one year. In this case, the single estimated elasticity, β_T, for each country is lower than the corresponding value in Table 1. This result reflects the tendency found in Table 5 of β_T to fall with T. For example, for the United States, the estimated β_T in Table 6 is 0.93 (s.e.=0.03), compared with 0.98 (0.04) in Table 1. Similarly, for all countries jointly, the estimate in Table 6 is 0.89 (0.03), compared with 0.94 (0.04) in Table 1. Note that, even with the inclusion of maturities as long as one year, the estimated β_T coefficients are close to the unit elasticity predicted by the baseline model.

Table 7 takes a different view of maturity elasticities, β_T, by allowing for variation with respect to exercise price, ε, rather than T. As noted before, prices of at-the-money put options (dominated by diffusion risk) are proportional to the square root of maturity, whereas prices of far-out-of-the-money put options (dominated by disaster risk) are roughly linear in maturity. Thus, in a broad sense, the maturity elasticity, β_T, is declining in relative exercise price, ε. Table 7 shows that this inverse pattern applies throughout the range of ε from 0.5 to 0.9. For example, for the United States, the estimated β_T is 1.97 (s.e.=0.11) when ε=0.5, 1.65 (0.09) when ε=0.6, 1.43 (0.07) when ε=0.7, 1.20 (0.05) when ε=0.8, and 0.91 (0.03) when ε=0.9. Similarly, for all countries jointly, the respective estimates are 1.81 (0.16), 1.62 (0.13), 1.40 (0.09), 1.15 (0.06), and 0.86 (0.03). Therefore, the results suggest that the values close to 1.0 for the estimated β_T in Table 1 are averages of values that are actually declining with ε. We think it important to extend the baseline model to account for this richer pattern of β_T.

Table 8 explores the stability of the baseline results from Table 1 with regard to sample period. Results apply to the pre-financial-crisis period before 2008 (1994-2007 for the United States), the crisis period of 2008-2010, and the post-crisis period of 2011-2015. The estimated coefficients β_T and βε are similar in the three periods, although the values are somewhat lower in the crisis interval of 2008-2010. For example, for the United States, the estimates of β_T are 1.03 (s.e.=0.04), 0.88 (0.05), and 1.12 (0.08), respectively, for the three periods. For all countries jointly, the corresponding estimates are 0.98 (0.03), 0.85 (0.06), and 1.10 (0.05). Possibly the low estimated β_T during the crisis period can be explained by a disaster probability, p_{jt}, that was unusually high in the very short term but projected to fall in the near future.

The estimates of βε for the three samples for the United States are 6.78 (s.e.=0.26), 5.25 (0.28), and 6.34 (0.36), whereas those for all countries jointly are 6.39 (0.20), 5.06 (0.33), and 6.70 (0.29). Possibly the low values of β_ε during the crisis interval can be explained by the unusually high volatility of stock prices—if we think of this high volatility as accompanied by a perceived fatter tail for bad outcomes (represented by a low tail exponent α and a correspondingly low value of β_ε, which equals $1+\alpha-\gamma$).

Table 9 shows how the results from Table 1 change with the use of alternative data sources. Panel A uses U.S. data for October 2010-July 2014, over which market data are available from OptionMetrics and Bloomberg. The main finding is that the estimated coefficients β_T and β_ε from these two alternative data sources are similar to those for the OTC data. Panel B considers longer samples, although the sample available from Bloomberg is shorter than those for the other two sources. Again, the conclusion is that the main results are similar for the different data sources. Panel C shows results with OTC and Bloomberg data for eight countries over the common sample period October 2010-June 2015. (Data from Bloomberg are unavailable for Brazil.) Most of the results are similar for the OTC and Bloomberg sources, except for some puzzling results for the estimated β_ε coefficients for Japan and India.

A lot of analysis of options pricing, going back to Bates (1991), suggests that the nature of this pricing changed in character following the 1987 stock-market crash. In particular, a "smile" in graphs of implied volatility against exercise price is thought to apply only post-1987. To assess this idea, we expanded our analysis to the period June 1983 to December 1995, using quotes on S&P 500 index options from the Berkeley Options Data Base.[23] These data derive from CBOE's Market Data Retrieval tapes. Because of the limited number of quotes on far-out-of-the-money options in this data base, we form our monthly panel by aggregating quotes from the last five trading days of each month.

[23] Direct access to this database has been discontinued. We are thankful to Josh Coval for sharing his version of the data.

Table 10, Panel A presents the regression estimates for 1983-1995 in the context of our baseline model. The estimate for β_T is 1.01 (s.e. =0.086) and that for β_ϵ is 6.79 (0.354). These results are close to those obtained in Table 1 with U.S. OTC data on the S&P 500 for 1994-2015. Therefore, the basic structure for pricing of far-out-of-the-money put prices on the S&P 500 seems similar for 1983-1995 and 1994-2015.

As before, we back out a time series for disaster probability, p_t, based on time fixed effects, assuming that parameters other than p_t in the multiplicative term for options prices are fixed. We also use levels for these other parameters as specified before. Figure 3 graphs the time series of estimated disaster probability. Readily apparent is the dramatic jump in p_t at the time of the October 1987 crash (in which the S&P 500 declined by 20.5% in a single day). The estimated p_t reached 259% but fell rapidly thereafter. The Persian Gulf War of 1990-1991 caused another rise in disaster probability to a high of 42%.

Table 10, Panel B shows statistics associated with the time series in Figure 3. A comparison pre-crash (June 1983-Sept 1987) and post-crash (Oct 1988-Dec 1995) shows a long-run increase in the typical size of the estimated disaster probability, p_t. For example, the pre-crash mean and median are 0.064 and 0.062, respectively, whereas the post-crash values are 0.108 and 0.085. Moreover, the minimum value pre-crash, 0.021, is substantially lower than that, 0.037, post-crash. Thus, the overall pattern is that the October 1987 crash raised the average disaster probability and also increased the minimum level to which the disaster probability tended to revert. These changes likely account for the introduction of a smile into the graph of implied volatility versus exercise price following the October 1987 stock-market crash.

IV. Conclusions

Options prices contain rich information on market perceptions of rare disaster risks. We develop a new options-pricing formula that applies when disaster risk is the dominant force, the size distribution of disasters follows a power law, and the economy has a representative agent with Epstein-Zin utility. The formula is simple but its main implications about maturity and exercise price accord with U.S. and other data from 1983 to 2015 on far-out-of-the-money put options on overall stock markets. If the coefficient of relative risk aversion and the size distribution of disasters are fixed, the regression estimates of time fixed effects provide information on the evolution of disaster probability. The estimated disaster probability peaks during the recent financial crisis of 2008-09 and in the stock-market crash of October 1987. This market-based assessment of disaster risk is a valuable indicator of aggregate economic shocks that can be used by practitioners, macroeconomists, and policymakers.

One important future extension would allow for effects of time-varying disaster probability in the options-pricing formula. Other extensions involve variation in risk aversion and the size distribution of disasters. We plan also to allow for the usual diffusion risk.

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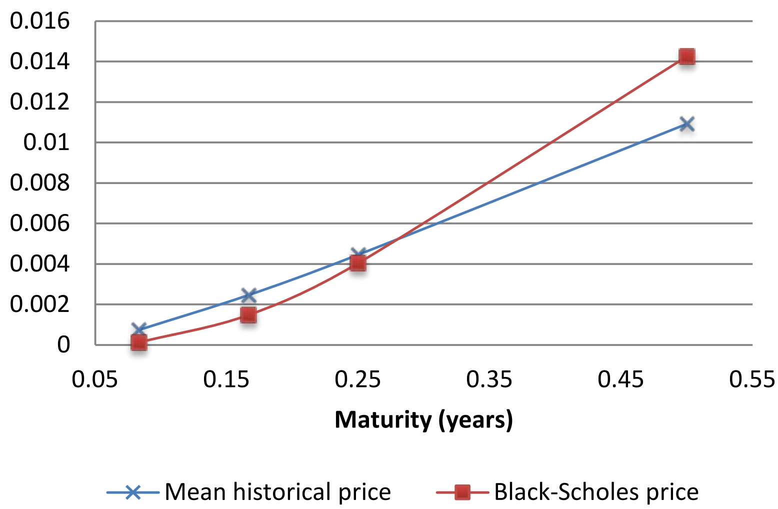
Figure 1: Comparison of prices with Black-Scholes predicted prices

This figure compares the mean of observed put prices across maturities with predicted prices from the Black-Scholes model. Black-Scholes prices are generated assuming flat volatility across maturities (30% for out-of-the-money options and 19% for at-the-money options). For ease of comparison, the volatilities are chosen so that the prices scale appropriately to historical prices. Panel A graphs relative put prices on the S&P 500 with strike of 80% of spot. Panel B graphs relative put prices with strike of 100% of spot.

Panel A: Out-of-the-Money Puts



Panel B: At-the-Money Puts

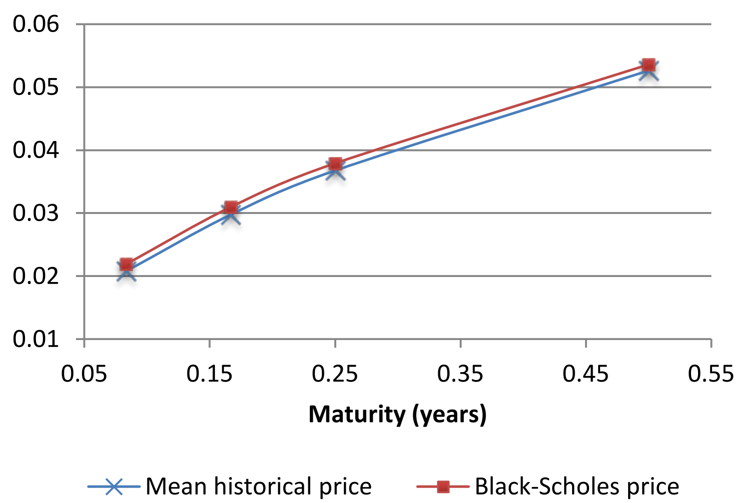


Figure 2: Disaster probabilities

This figure graphs the estimated disaster probabilities associated with the regressions in Table 1. The annualized disaster probability, p_{jt} for country j, is calculated from the multiplicative time fixed-effect coefficients in equation (29), assuming $z_0 = 1.1$ and γ=3. With γ pinned down, the estimates of $\beta_\varepsilon=1+\alpha_j-\gamma$ imply that the tail coefficients, α_j, range from 7.16 to 8.06.

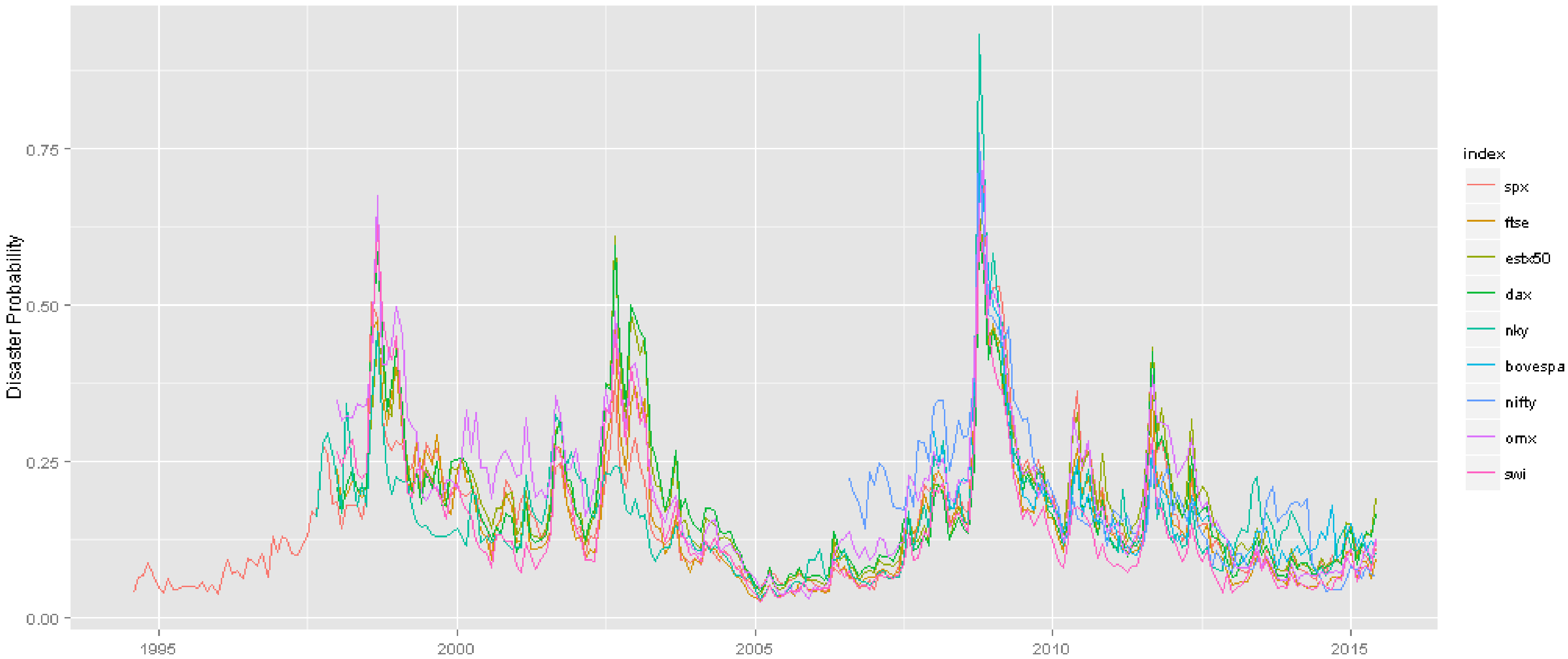


Figure 3: S&P 500 disaster probabilities: 1983-1995

This figure presents the disaster probabilities associated with the regressions in Table 10, based on the Berkeley Option Database. Annualized disaster probability is calculated from the multiplicative time fixed-effect coefficients in equation (29), assuming $z_0 = 1.1$ and γ=3.

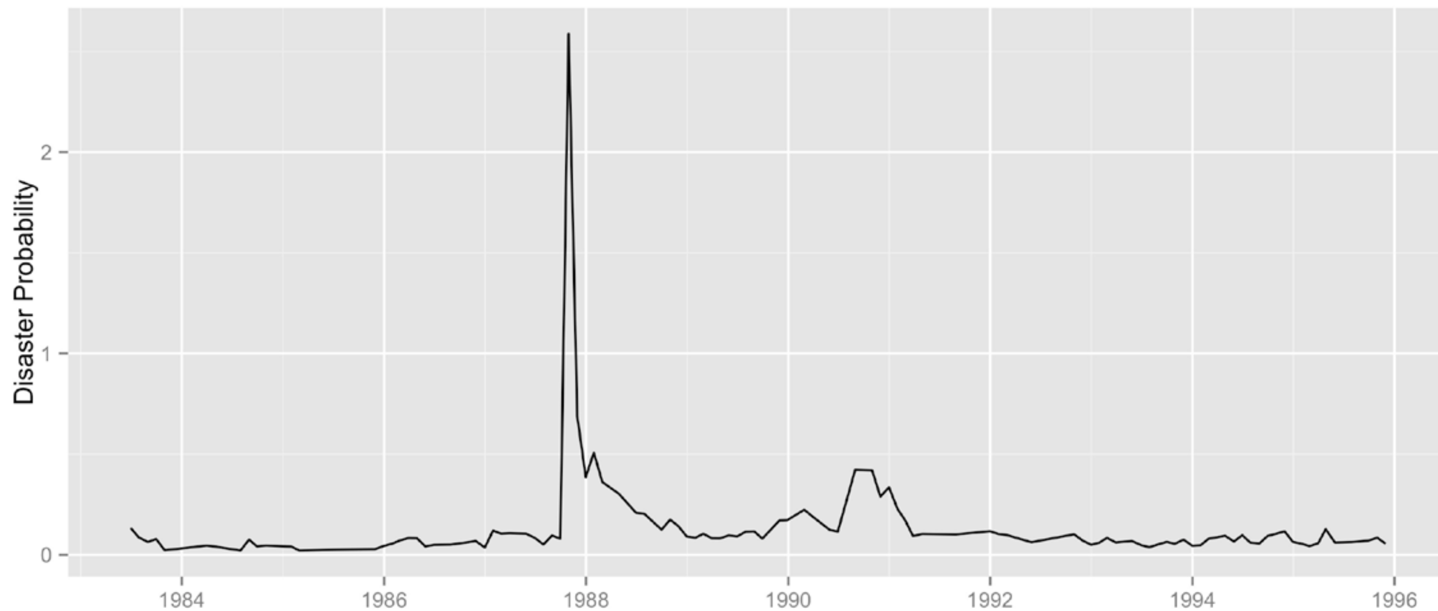


Table 1: Baseline regression estimates

This table presents non-linear least square regression estimates of our main model. We use put option prices with maturity ranging from one month to six months and strike ranging from 50% to 90% of spot price. We apply time-fixed effects to capture variations in disaster probability, which are shown in Figure 2. Time-clustered standard errors are provided in parentheses. The p-value associated with the hypothesis that β_ϵ and β_T are the same for all countries is less than 0.01.

Index	spx	ftse	estx50	dax	nky	omx	swi	bovespa	nifty	all
	US	UK	EURO	GE	JP	SWE	CH	BR	IND	
	Aug94 - Jun15	Jan98 - Jun15	Jun98 - Jun15	Jan98 - Jun15	Sep97 - Jun15	Jan98 - Jun15	Jan98 - Jun15	Jan08 - Jun15	Aug06 - Jun15	Aug94 - Jun15
β_T	0.978	0.987	0.934	0.936	0.871	0.913	0.997	0.829	0.908	0.938
	(0.036)	(0.031)	(0.029)	(0.030)	(0.069)	(0.029)	(0.033)	(0.067)	(0.057)	(0.038)
β_ϵ	6.049	5.792	5.789	5.586	5.865	5.974	6.061	5.161	5.628	5.828
	(0.224)	(0.167)	(0.175)	(0.174)	(0.478)	(0.234)	(0.228)	(0.392)	(0.297)	(0.241)

Table 2: Estimation with at-the-money put options only

This table presents non-linear least square regression estimates of our main model for at-the-money options only. We use put option prices with maturity ranging from two weeks to six month and strike equal to spot price. We apply time-fixed effects to capture variations in disaster probability. Time-clustered standard errors are in parentheses.

Index	spx	ftse	estx50	dax	nky	omx	swi	bovespa	nifty	all
	US	UK	EURO	GE	JP	SWE	CH	BR	IND	
	Aug94 - Jun15	Jan98 - Jun15	Jun98 - Jun15	Jan98 - Jun15	Sep97 - Jun15	Jan98 - Jun15	Jan98 - Jun15	Jan08 - Jun15	Aug06 - Jun15	Aug94- Jun15
β_T	0.518	0.511	0.486	0.496	0.469	0.477	0.510	0.470	0.501	0.495
	(0.007)	(0.007)	(0.007)	(0.008)	(0.010)	(0.007)	(0.008)	(0.019)	(0.014)	(0.007)

Table 3: Mean, standard deviation, quantiles of disaster probabilities

This table presents summary statistics on disaster probabilities associated with the regression results in Table 1. The disaster probabilities are calculated as indicated in the notes to Figure 2.

index	# obs	median	mean	sd	min	1%	10%	25%	50%	75%	99%	max
spx (US)	251	0.130	0.156	0.106	0.038	0.042	0.059	0.079	0.130	0.202	0.529	0.696
Ftse (UK)	210	0.133	0.158	0.107	0.027	0.035	0.053	0.076	0.133	0.211	0.474	0.628
estx50 (EURO)	205	0.164	0.187	0.113	0.039	0.045	0.075	0.109	0.164	0.236	0.574	0.649
dax (GE)	210	0.158	0.182	0.113	0.043	0.053	0.071	0.104	0.158	0.220	0.582	0.617
nky (JP)	214	0.137	0.163	0.108	0.031	0.035	0.067	0.108	0.137	0.198	0.572	0.933
omx (SWE)	210	0.187	0.203	0.126	0.029	0.038	0.068	0.102	0.187	0.266	0.639	0.731
swi (CH)	210	0.114	0.149	0.113	0.025	0.035	0.048	0.072	0.114	0.189	0.566	0.628
bovespa (BR)	90	0.152	0.185	0.117	0.079	0.085	0.103	0.116	0.152	0.199	0.675	0.724
nifty (IND)	107	0.175	0.197	0.121	0.042	0.044	0.078	0.122	0.175	0.238	0.594	0.774

Table 4: ϵ-elasticities varying by relative exercise price, ε

These regressions allow for different elasticities β_ε over ranges of relative exercise prices, ε. The specification is:

$$\Omega_{it} = c_t \cdot \varepsilon_1^{\beta_{\varepsilon 1}} T_{it}^{\beta_T} \cdot \left[\begin{array}{c} D_{1it} + D_{2it}\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{\beta_{\varepsilon 2}} + D_{3it}\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{\beta_{\varepsilon 2}}\left(\frac{\varepsilon_3}{\varepsilon_2}\right)^{\beta_{\varepsilon 3}} + D_{4it}\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{\beta_{\varepsilon 2}}\left(\frac{\varepsilon_3}{\varepsilon_2}\right)^{\beta_{\varepsilon 3}}\left(\frac{\varepsilon_4}{\varepsilon_3}\right)^{\beta_{\varepsilon 4}} \\ +D_{5it}(\frac{\varepsilon_2}{\varepsilon_1})^{\beta_{\varepsilon 2}}(\frac{\varepsilon_3}{\varepsilon_2})^{\beta_{\varepsilon 3}}(\frac{\varepsilon_4}{\varepsilon_3})^{\beta_{\varepsilon 4}}(\frac{\varepsilon_5}{\varepsilon_4})^{\beta_{\varepsilon 5}} \end{array} \right],$$

where D_{1it} is a dummy variable corresponding to $\varepsilon_1 = 0.5$, D_{2it} is a dummy variable corresponding to $\varepsilon_2 = 0.6$, etc. Standard errors, shown in parentheses, are calculated by applying multivariate delta-method and clustered by time. The p-values associated with the hypothesis that the β_ϵ's for different ranges of ε are the same are all less than 0.01.

	spx	ftse	estx50	dax	nky	omx	swx	bovespa	nifty	all
	US	UK	EURO	GE	JP	SWE	CH	BR	IND	
	Aug94 - Jun15	Jan98 - Jun15	Jun98 - Jun15	Jan98 - Jun15	Sep97 - Jun15	Jan98 - Jun15	Jan98 - Jun15	Jan08 - Jun15	Aug06 - Jun15	Aug94 - Jun15
β_T	0.978	0.986	0.934	0.936	0.871	0.913	0.997	0.829	0.908	0.938
	(0.036)	(0.031)	(0.029)	(0.030)	(0.069)	(0.029)	(0.033)	(0.067)	(0.057)	(0.038)
β_ϵ (ϵ =0.5 to 0.6)	5.543	5.071	4.710	3.744	4.189	4.947	5.081	4.523	4.691	4.663
	(0.135)	(0.092)	(0.147)	(0.299)	(0.424)	(0.206)	(0.257)	(0.302)	(0.292)	(0.162)
β_ϵ (ϵ =0.6 to 0.7)	5.293	5.085	5.038	4.768	4.735	5.212	5.214	4.743	5.005	5.021
	(0.163)	(0.117)	(0.164)	(0.239)	(0.436)	(0.241)	(0.204)	(0.362)	(0.281)	(0.209)
β_ϵ (ϵ =0.7 to 0.8)	5.653	5.462	5.496	5.362	5.377	5.732	5.657	5.010	5.390	5.502
	(0.203)	(0.151)	(0.169)	(0.155)	(0.450)	(0.241)	(0.208)	(0.387)	(0.283)	(0.229)
β_ϵ (ϵ =0.8 to 0.9)	6.348	6.086	6.084	5.919	6.331	6.222	6.388	5.366	5.887	6.147
	(0.235)	(0.181)	(0.175)	(0.176)	(0.459)	(0.220)	(0.238)	(0.391)	(0.295)	(0.240)

Table 5: T-elasticities varying by maturity

These regressions allow for different elasticities β_T over ranges of maturities, T. The specification is:

$$\Omega_{it} = c_t \cdot \varepsilon_{it}{}^{\beta_\varepsilon} T_1^{\beta_{T1}} \cdot \left[D_{1it} + D_{2it}\left(\frac{T_2}{T_1}\right)^{\beta_{T2}} + D_{3it}\left(\frac{T_2}{T_1}\right)^{\beta_{T2}}\left(\frac{T_3}{T_2}\right)^{\beta_{T3}} + D_{4it}\left(\frac{T_2}{T_1}\right)^{\beta_{T2}}\left(\frac{T_3}{T_2}\right)^{\beta_{T3}}\left(\frac{T_4}{T_3}\right)^{\beta_{T4}}\right]$$

where D_{1it} is a dummy variable corresponding to maturity $T_1 = 30\ days$, D_{2it} is a dummy variable corresponding to $T_2 = 60\ days$, etc. Standard errors, shown in parentheses, are calculated by applying multivariate delta-method and clustered by time. The p-values associated with the hypothesis that the β_T's for different maturity ranges are the same are all less than 0.01.

	spx	ftse	estx50	dax	nky	omx	swx	bovespa	nifty	all
	US	UK	EURO	GE	JP	SWE	CH	BR	IND	
	Aug94 - Jun15	Jan98 - Jun15	Jun98 - Jun15	Jan98 - Jun15	Sep97 - Jun15	Jan98 - Jun15	Jan98 - Jun15	Jan08 - Jun15	Aug06 - Jun15	Aug94 - Jun15
β_ϵ	6.050	5.793	5.790	5.587	5.866	5.976	6.062	5.162	5.630	5.829
	(0.225)	(0.168)	(0.176)	(0.175)	(0.478)	(0.235)	(0.229)	(0.393)	(0.298)	(0.241)
β_T (30D-60D)	1.192	1.223	1.148	1.153	1.043	1.129	1.264	0.957	1.058	1.139
	(0.051)	(0.041)	(0.035)	(0.040)	(0.113)	(0.035)	(0.050)	(0.087)	(0.073)	(0.054)
β_T (60D-90D)	1.034	1.053	0.990	1.002	0.913	0.948	1.065	0.852	0.960	0.991
	(0.041)	(0.036)	(0.033)	(0.036)	(0.089)	(0.036)	(0.040)	(0.077)	(0.063)	(0.044)
β_T (90D-6M)	0.924	0.927	0.876	0.875	0.820	0.860	0.933	0.792	0.861	0.884
	(0.034)	(0.030)	(0.028)	(0.028)	(0.057)	(0.028)	(0.031)	(0.061)	(0.055)	(0.035)

Table 6: Inclusion of puts with longer maturities

This table corresponds to Table 1 except that the maturity range is from 30 days to 1 year. The p-value associated with the hypothesis that β_ϵ and β_T are the same for all countries is less than 0.01.

Index	spx	ftse	estx50	dax	nky	omx	swi	bovespa	nifty	all
	US	UK	EURO	GE	JP	SWE	CH	BR	IND	
	Aug94 - Jun15	Jan98 - Jun15	Jun98 - Jun15	Jan98 - Jun15	Sep97 - Jun15	Jan98 - Jun15	Jan98 - Jun15	Jan08 - Jun15	Aug06 - Jun15	Aug94 - Jun15
β_T	0.933	0.930	0.877	0.879	0.840	0.880	0.933	0.832	0.877	0.892
	(0.027)	(0.024)	(0.022)	(0.023)	(0.045)	(0.022)	(0.026)	(0.049)	(0.041)	(0.027)
β_ϵ	4.857	4.549	4.585	4.489	4.985	4.666	4.870	4.100	4.460	4.664
	(0.127)	(0.097)	(0.097)	(0.101)	(0.246)	(0.115)	(0.148)	(0.223)	(0.139)	(0.127)

Table 7: *T*-elasticities varying by relative exercise price, ε

These regressions allow for different elasticities β_T over ranges of relative exercise prices, ε. The specification is:

$$\Omega_{it} = c_t \cdot \varepsilon_{it}{}^{\beta_\varepsilon} \cdot \left[D_{1it}T_{it}^{\beta_{T1}} + D_{2it}T_{it}^{\beta_{T2}} + D_{3it}T_{it}^{\beta_{T3}} + D_{4it}T_{it}^{\beta_{T4}} + D_{5it}T_{it}^{\beta_{T5}}\right]$$

where D_{1it} is a dummy variable corresponding to $\varepsilon_1 = 0.5$, D_{2it} is a dummy variable corresponding to $\varepsilon_2 = 0.6$, etc. Standard errors, shown in parentheses, are calculated by applying multivariate delta-method and clustered by time. The p-values associated with the hypothesis that the β_T's for different ranges of ε are the same are all less than 0.01.

Index	spx	ftse	estx50	dax	nky	omx	swx	bovespa	nifty	all
	US	UK	EURO	GE	JP	SWE	CH	BR	IND	
	Aug94 - Jun15	Jan98 - Jun15	Jun98 - Jun15	Jan98 - Jun15	Sep97 - Jun15	Jan98 - Jun15	Jan98 - Jun15	Jan08 - Jun15	Aug06 - Jun15	Aug94 - Jun15
β_ϵ	4.070	3.617	3.588	3.501	4.242	3.700	3.855	3.288	3.641	3.799
	(0.130)	(0.095)	(0.094)	(0.118)	(0.185)	(0.098)	(0.157)	(0.208)	(0.109)	(0.107)
$\beta_T(\epsilon = 0.5)$	1.972	2.083	1.947	1.708	1.310	1.988	2.033	1.738	1.824	1.807
	(0.106)	(0.089)	(0.119)	(0.097)	(0.306)	(0.172)	(0.160)	(0.259)	(0.263)	(0.159)
$\beta_T(\epsilon = 0.6)$	1.651	1.760	1.701	1.650	1.310	1.712	1.760	1.480	1.599	1.620
	(0.093)	(0.080)	(0.092)	(0.082)	(0.234)	(0.126)	(0.103)	(0.198)	(0.190)	(0.126)
$\beta_T(\epsilon = 0.7)$	1.431	1.494	1.443	1.424	1.221	1.443	1.514	1.234	1.359	1.404
	(0.074)	(0.065)	(0.066)	(0.060)	(0.168)	(0.084)	(0.072)	(0.144)	(0.134)	(0.091)
$\beta_T(\epsilon = 0.8)$	1.195	1.217	1.161	1.151	1.059	1.146	1.241	0.994	1.106	1.153
	(0.054)	(0.048)	(0.045)	(0.044)	(0.110)	(0.050)	(0.051)	(0.098)	(0.088)	(0.060)
$\beta_T(\epsilon = 0.9)$	0.909	0.906	0.853	0.853	0.810	0.835	0.920	0.749	0.832	0.863
	(0.034)	(0.029)	(0.026)	(0.028)	(0.060)	(0.026)	(0.031)	(0.060)	(0.051)	(0.034)

Table 8: Alternative Sample Periods

This table corresponds to Table 1 except for the sample periods. The p-values associated with the hypothesis that the β_T and β_ϵ are the same across periods are all less than 0.01.

		spx	ftse	estx50	dax	nky	omx	swx	bovespa	nifty	all
		US	UK	EURO	GE	JP	SWE	CH	BR	IND	
		Aug94 - Jun15	Jan98 - Jun15	Jun98 - Jun15	Jan98 - Jun15	Sep97 - Jun15	Jan98 - Jun15	Jan98 - Jun15	Jan08 - Jun15	Aug06 - Jun15	Aug94 - Jun15
Pre-Crisis	β_T	1.027	0.994	0.928	0.922	1.004	0.917	1.016	--	1.032	0.978
		(0.037)	(0.034)	(0.040)	(0.037)	(0.029)	(0.032)	(0.035)		(0.043)	(0.030)
	β_ϵ	6.785	5.919	6.121	5.623	7.616	6.359	6.143	--	7.236	6.391
		(0.260)	(0.162)	(0.205)	(0.215)	(0.294)	(0.289)	(0.225)		(0.174)	(0.201)
2008-2010	β_T	0.885	0.935	0.904	0.925	0.718	0.857	0.931	0.762	0.812	0.849
		(0.054)	(0.060)	(0.050)	(0.061)	(0.096)	(0.059)	(0.068)	(0.070)	(0.061)	(0.063)
	β_ϵ	5.248	5.423	5.088	5.189	4.431	5.209	5.526	4.714	5.061	5.063
		(0.276)	(0.319)	(0.295)	(0.327)	(0.436)	(0.392)	(0.446)	(0.370)	(0.289)	(0.333)
2011???-2015	β_T	1.125	1.137	1.022	1.035	1.056	1.052	1.123	1.069	1.207	1.101
		(0.083)	(0.073)	(0.049)	(0.058)	(0.038)	(0.050)	(0.073)	(0.039)	(0.037)	(0.048)
	β_ϵ	6.336	6.478	6.220	6.339	7.023	6.257	7.959	6.966	6.890	6.700
		(0.359)	(0.355)	(0.311)	(0.382)	(0.255)	(0.233)	(0.486)	(0.213)	(0.138)	(0.286)

Table 9: Comparison of alternative data sources

This table corresponds to Table 1 but with different data sources. Panels A and B are for U.S. data. Panel A uses a common sample (October 2010-July 2014) for three data sources: the broker-dealer (OTC) data, OptionMetrics, and Bloomberg. Column 1 uses the broker-dealer source. Column 2 uses OptionMetrics data, applying a bivariate linear interpolation on the implied volatility surface to obtain put prices with the same maturities and strikes as in column 1. Column 3 uses OptionMetrics data with more granular strikes at every 5% moneyness interval. Column 4 uses Bloomberg data constructed from Bloomberg's implied volatility surface with the same strikes and maturities as the broker-dealer**.** Panel C compares the broker-dealer results with those from Bloomberg for all countries except Brazil (which lacks Bloomberg data). The sample is October 2010-June 2015. Time-clustered standard errors are in parentheses.

Panel A, U.S. data, October 2010-July 2014				
	(1)	**(2)**	**(3)**	**(4)**
	Broker-dealer	**OptionMetrics**	**OptionMetrics (granular strikes)**	**Bloomberg**
	Oct 2010-July 2014	**Oct 2010-July 2014**	**Oct 2010-July 2014**	**Oct 2010-July 2014**
	(1)	(2)	(3)	(4)
β_T	1.121	1.086	1.129	1.201
	(0.084)	(0.083)	(0.088)	(0.084)
β_ϵ	6.238	6.096	6.002	6.590
	(0.342)	(0.338)	(0.331)	(0.311)
N	920	760	1395	920

Panel B, U.S. data, varying samples				
	(1)	**(2)**	**(3)**	**(4)**
	Broker-dealer	**OptionMetrics**	**OptionMetrics (granular strikes)**	**Bloomberg**
	Jan 1996-July 2014	**Jan 1996-July 2014**	**Jan 1996-July 2014**	**Jan 2005-July 2014**
	(1)	(2)	(3)	(4)
β_T	0.972	0.935	0.971	0.963
	(0.036)	(0.032)	(0.035)	(0.056)
β_ϵ	6.016	5.913	5.826	6.292
	(0.224)	(0.218)	(0.214)	(0.275)
N	4460	3139	5753	1472

Panel C, multiple countries, OTC and Bloomberg data, October 2010-June 2015																
	SPX (US)		FTSE (UK)		ESTX (EURO)		DAX (GE)		NKY (JP)		OMX (SWE)		SWX (CH)		NIFTY (IND)	
	Broker	BBG	Broker	BBG	Broker	BBG	Broker	BBG	Broker	BBG	Broker	BBG	Broker	BBG	Broker	BBG
β_T	1.137	1.207	1.145	1.112	1.028	1.012	1.045	1.020	1.072	1.086	1.067	1.037	1.129	1.081	1.218	1.163
	(0.079)	(0.077)	(0.068)	(0.067)	(0.046)	(0.048)	(0.057)	(0.061)	(0.038)	(0.047)	(0.051)	(0.068)	(0.070)	(0.074)	(0.036)	(0.056)
β_ϵ	6.368	6.682	6.517	6.506	6.213	6.117	6.366	6.865	7.021	5.403	6.329	7.205	8.030	8.536	6.910	11.193
	(0.338)	(0.297)	(0.333)	(0.340)	(0.285)	(0.282)	(0.366)	(0.414)	(0.239)	(0.557)	(0.239)	(0.306)	(0.476)	(0.608)	(0.133)	(0.464)
N	1140	1140	1140	1140	1140	1140	1140	1140	1140	1140	1140	1140	1140	1140	1140	1140

Table 10: Regression estimates for early sample period

This table presents baseline regression estimates of our main model and summary statics on the associated disaster probabilities for the period June 1983 to December 1995. The data are from the Berkeley Option Database. We form monthly panels of put-options prices by aggregating quotes from the last five trading days of each month. Consistent with the methodology used to analyze OptionMetrics data, we apply a bivariate linear interpolation on the implied volatility surface to obtain put prices with granular strikes at every 5% moneyness interval and maturities ranging from one to six months. The time-fixed effects capture the variations in disaster probability, which are shown in Figure 3 and summarized in Panel B. Time-clustered standard errors are provided in parentheses.

Panel A: Coefficient estimates

	SPX (US)
	June 1983-Dec 1995
β_T	1.014
	(0.086)
β_ϵ	6.790
	(0.354)
N	502

Panel B: Disaster probabilities, summary statistics

	June 1983- Dec 1995	Pre-crash June 1983-Sep 1987	Oct 1987- Sep 1988	Post crash Oct 1988-Dec 1995
min	0.021	0.021	0.124	0.037
25%	0.056	0.041	0.208	0.062
median	0.082	0.062	0.361	0.085
mean	0.135	0.064	0.597	0.108
75%	0.116	0.083	0.507	0.116
max	2.588	0.133	2.588	0.422