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Robert Shimer
Iván Werning

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ABSTRACT

We study pairwise trading mechanisms in the presence of private information and limited commitment, whereby either trader can walk away from a proposed trade when he learns the trading price. We show that when one trader's information is relevant for the other trader's value of the asset, optimal trading arrangements may necessarily conceal the traders' information. While limited commitment itself may not be costly, it shapes how prices transmit information.

Robert Shimer
Department of Economics
University of Chicago
1126 East 59th Street
Chicago, IL 60637
and NBER
shimer@uchicago.edu

Iván Werning
Department of Economics, E17-234
MIT
77 Massachusetts Avenue
Cambridge, MA 02139
and NBER
iwerning@mit.edu

1 Introduction

How does dispersed information get transmitted and aggregated by financial markets? What are the impediments to information revelation? Is information passed on from informed to uninformed traders? Is information transmission always desirable?

Economists have important paradigms to address these classical questions. Grossman (1976) first showed that rational expectation models aggregate dispersed information through prices, formalizing the ideas in Hayek (1945). Indeed, under some conditions, prices may reveal information perfectly. This leads to a paradox: if private information is instantly revealed then there are no incentives to gather it. In response, Grossman and Stiglitz (1980) and Kyle (1985) introduced noise traders, alongside informed and uninformed traders. Informed traders do their part to reveal information, but their effect on the price is distorted by the presence of noise traders. An alternative recent approach considers decentralized settings where information takes time to percolate through the entire market (see e.g. Duffie and Manso, 2007; Duffie, Malamud and Manso, 2014). Traders learn from each other in each meeting, but it takes time for this information to get around. In all these theories information transmission enhances the efficiency of the market, but it may be hindered by noise traders or delayed by illiquid markets.

The purpose of this paper is to propose and explore a different paradigm for thinking about these questions. We also focus on decentralized over-the-counter markets; however, we take a step back from the slow diffusion of information across the market in order to zoom in on how information is transmitted within a bilateral meeting. To avoid ad hoc assumptions on trading arrangements, we use a mechanism design approach (Kennan and Wilson, 1993). Any specific trading arrangement, such as a particular bargaining protocol, is a special case of the mechanisms we consider. Our main results question the assumption that information is revealed in bilateral trades. Indeed, we argue that efficient trading arrangements, including many forms of bargaining, prevent the revelation of information in order to enhance the bilateral gains from trade. Information revelation is imperfect by design, not because of external impediments or constraints.

Our approach is firmly rooted in classical microeconomic information theory and recent advances in mechanism design. Indeed, we will leverage several useful concepts and results from this literature. However, our focus and perspective is quite different, since the existing literature has focused on the impediments to efficient trading due to private information, rather than addressing the questions about information transmission in trade.

We focus on a bilateral trade situation under incomplete information. A single buyer and a single seller meet and there are gains from trade. Each trader may have some private information that is possibly relevant to both traders' valuations. The two traders negotiate over a trading price, but each retains the option to walk away from the negotiation at any time. We ask whether the negotiation process, including the trading price, reveals each trader's information to the other. We find that the answer depends on how negotiations are structured, by which we mean the mechanism that links the structure of the model, including the traders' private information, to the terms of trade. There are mechanisms that fully reveal each trader's information to the other. However, sometimes Pareto efficient mechanisms must hide some information. As a result, Pareto efficient mechanisms may sometimes allow one trader to profit at the expense of the other. This is true even though there exist feasible (but Pareto inefficient) mechanisms where all information is revealed and no individual ever regrets a trade. If existing arrangements strive for trading efficiency, then our results imply that information transmission may be purposefully prevented in the marketplace.

We believe this insight may be useful for understanding trading patterns and information revelation in over-the-counter securities markets. A key attribute of these markets is that many buyers and sellers are financially sophisticated traders who, under certain circumstances, may have private information about an asset's cash flow. The information may not be directly about the quality of the asset they are trading, but instead might be about the underlying economic environment. For example, traders may disagree about how to model the stochastic process for a new security's cash flow. Over time, ample historical data may limit the scope for disagreement, but then a highly unusual stream of cash flows—a rare event—may lead traders to conclude that the old cash flow model is misspecified. Sophisticated traders will respond to this uncertainty by undertaking research to improve their model. They will also recognize that other sophisticated traders will perform their own research, obtaining their own model. These competing models are the traders' private information.

This discussion also suggests that both sides of the market may be privately informed with useful information. While it may be reasonable that a car's owner has unique insight into the car's quality to the extent a car is an experience good, this assumption seems less plausible for many financial assets.[1] Any trader can try to model an asset's cash flow, whether he owns that asset or not. Although we start with the simpler one-sided private information scenario, we will also consider a situation with two-sided private information.

The critical issue is the winner's curse: a rational buyer will fear that if a seller is willing to sell him an asset at a low price, the seller's model suggests that the assets's value is low, reducing the buyer's willingness to pay. The more information is conveyed through prices, the more acute this fear will be and the greater will be the reduction in the willingness to pay. Similarly a seller will rationally worry that if a buyer is willing to purchase an asset at a high price, the buyer's model suggests that that the asset's value is high. These concerns can lead to a breakdown in trade, even if it is common knowledge that there are gains from trade at some price. If less information is conveyed through prices, the response of willingness-to-pay to price is weaker, potentially allowing for more trade.

Trading mechanisms that obscure private information mitigate this problem. If a buyer knows only that his own signal is low, his willingness-to-pay is higher than if he knows both signals are low; and similarly a seller who knows only

[1] A large literature applies the Akerlof 'lemons' model to macroeconomic settings. A non-exhaustive list includes Eisfeldt (2004); Daley and Green (2012); Philippon and Skreta (2012); Tirole (2012); Kurlat (2013b); Chari, Shourideh and Zetlin-Jones (2014); Chiu and Koeppl (2014); Guerrieri and Shimer (2014); Camargo and Lester (2014); Kurlat (2013a); Chang (2014); Guerrieri and Shimer (2015). To our knowledge all of these papers assume that buyers are either uninformed or have knowledge that is only a subset of the sellers'.

that her own signal is high has a lower reservation price than a seller who knows both signals are high. In fact, under certain conditions, a constant sales price will ensure that trade always occurs yet will reveal no information between the trading partners, while a fully revealing trading mechanism will necessitate a complete breakdown in trade. Randomized prices are another instrument for concealing information and may be optimal in some cases. This illustrates the tension between trading efficiency and information revelation.

This tension might seem counterintuitive. A reduction in asymmetric information alleviates the lemons problem in Akerlof (1970), so one might expect that Pareto optimal trading mechanisms would induce traders to reveal their information to each other. This conclusion would be warranted if there were policies that could costlessly force traders to reveal their information. In our environment, however, information revelation is endogenous. It is possible to construct trading mechanisms that induce traders to reveal their information to each other, but we find that Pareto optimal mechanisms generally do not have this property.

In the standard mechanism design approach to bargaining (Myerson, 1979), each trader observes his own signal and makes a report to the mechanism, which then instructs the traders on whether and at what price to trade as a function of those reports. Mechanisms must satisfy participation constraints, so both traders are willing to enter the mechanism *ex ante*, before observing their signal, or at the *interim* stage, after observing their signal.

A distinguishing feature of our approach is that we also impose *ex post* participation constraints: after the mechanism recommends that trade takes place at a particular price, both traders have the option to walk away, given all the information available at that time. For example, if a buyer is only willing to buy an asset at some price in the event that the seller would refuse to sell at that same price, trade will not occur at that price. We assume this lack of commitment because we believe it is natural in over-the-counter markets. In these settings, it find it difficult to imagine market participants being committed to trade before they know the price at which trade takes place. It is also an immediate implications of any standard bargaining protocol.

The lack of commitment assumption is critical to our results and puts information transmission at center stage. The key issue is that ex post constraints depend on the information revealed by the price. If traders are committed to trade before the mechanism gives its instructions, it is costless for the mechanism to reveal each trader's information to the other. Ex post participation constraints capture how learning from prices constrains the set of feasible trades. Formally, we look at the set of veto-incentive-compatible mechanisms (Forges, 1999). Veto incentive compatibility constraints allow for the possibility that an trader misreports his type and then decides not to trade upon learning the recommendation of the mechanism.

Interestingly, the lack of commitment not to walk away may or may not affect the efficiency of trade. In some cases the maximum feasible gains from trade is unaffected by whether traders have a commitment technology. That lack of commitment is costless may explain why real-world trading arrangements allow traders to back away from a trade and do not introduce institutional arrangements to create more commitment.

Even if the lack of commitment does not affect attainable allocations or the gains from trade, it is crucial for our main conclusion, that information revelation is costly. Thus, it potentially affects the way we implement allocations, to hide information. One way to hide information is through randomization: we find that the optimal price should at times be a noisy function of the reported signals, so as to reduce the ability to deduce others' information. Interestingly, rather than a nuisance due to noisy traders as in Grossman and Stiglitz (1980), efficient trading between rational trader may require noisy prices.

We do not view our paper as offering actual proposals for a mechanism that will improve the efficiency of trade, although it may be possible to use our approach to construct such a mechanism. Instead, we are interested in understanding what a pair of traders can accomplish on their own and what features real-world trading outcomes might have. Our main conclusions are (i) we should not expect that pairwise optimal trading mechanisms will induce information revelation; (ii) we should not be surprised by the ability of either party to walk away from a trade based on the information they learn through bargaining; and (iii) we should not expect that equilibrium prices will depend only on fundamentals, including the economic environment and the trader's signals, but instead they may be random.

Also important for our results is that traders hold information that the other party cares about. We prove in Section 4 that revealing information is costless in pure private values environment, such as Myerson and Satterthwaite (1983). This is because traders never learn anything from the mechanism which changes their willingness to trade at a particular price. The trade-off between information transmission and trading efficiency is only important when at least one of the traders has information that is valuable to the other. Nevertheless, we believe that this common values case is the most natural one in asset markets since everyone would like to have a better model of an asset's cash flow.

This paper is related to a growing literature that examines information diffusion in over-the-counter markets. To focus on information diffusion, these papers often assume that traders share all of their information in every meeting. Some treat this assumption as a primitive (Duffie, Malamud and Manso, 2009), but others generate this from the trading mechanism. For example, Duffie and Manso (2007) and Duffie, Giroux and Manso (2010) assume that traders observe each others' bids in a second price auction. They can invert those to infer each others' beliefs, which they use to update their own beliefs in future meetings. Duffie, Malamud and Manso (2014) assume pairwise meetings with prices determined by a double auction. Under the assumption that bids are monotone in beliefs, each trader can again invert the other's bid to update their own beliefs.

In other papers, not all information is transmitted in every meeting. The classic example of such a situation is Wolinsky (1990). A large number of individuals meet in pairs to bargain over an indivisible asset. If they reach an agreement, they trade and each exits the market. If they fail to reach an agreement, they wait one period and are matched with another randomly selected trader in the following period. Wolinsky (1990) proves that, even in the limit as the delay between trading rounds disappears, trading prices do not reflect all of the available information. In contrast, Green (1991) shows that it is feasible for a patient uninformed trader to elicit the private information held by informed competitive traders. In a more recent paper, Golosov, Lorenzoni and Tsyvinski (2014) relax the assumption that assets are indivisible and that traders exit after trade. They conclude that information gradually diffuses through the economy, with the value of information converging to zero.

Our environment is much simpler than the ones in these papers, since there is only one buyer and one seller. But in contrast to the existing literature, we study optimal trading arrangements to explore the tension between information diffusion and trading efficiency. Any particular trading protocol may not reveal information fully, but this leaves open the possibility that other better mechanisms reveal more information. Our results show that the very best mechanisms purposefully do not reveal information fully. We leave the issue of merging these two approaches to future research.

An important recent theoretical contribution exploring ex post participation constraints and veto incentive compatible mechanisms is Gerardi, Hörner and Maestri (2014), who consider an environment with one-sided private information. They compare the set of allocations attainable in this environment to those attainable in the standard mechanism design problem with commitment. Although we use the same theoretical concepts, such as veto incentive compatibility, their focus on allocations is quite different from our focus on information revelation, indeed, in some sense the opposite. For example, even in cases where the no-commitment and commitment allocation coincide, the implications for information revelation are different. Their main result provides a characterization of veto incentive compatibility constraints showing these can be reduced to a set of linear constraints expressed directly in terms of the allocation, casting information transmission entirely to one side.

The remainder of this paper proceeds as follows. Section 2 considers the one-sided private information case. Section 3 sets up the two-sided information case. Section 4 then discusses the special case of private values while Section 5 treats the case of common values. We conclude briefly in Section 6.

2 One-Sided Private Information

There is a single asset and two traders. One of the traders, the seller, initially owns the asset. The other trader, the buyer, has some cash that he could use to purchase the asset. Each trader is risk-neutral and their valuations for the asset depend on signals that each receives.

Throughout this section, we assume that the seller privately observes a signal and so has more information than the buyer. This is the classical "lemons" assumption. For example, the asset may be an experience good, and so the seller has learned its quality by owning it, as might be the case for automobiles (Akerlof, 1970). The one-sided private information assumption is perhaps harder to justify in the context of financial markets, since buyers and sellers can each observe some public information about the asset and can conduct research that may grant them access to other signals. One case where the one-sided private information assumption might be reasonable is for a mortgage originator, who might have private information about the quality of a mortgage pool.[2]

We assume without loss of generality that the seller's signal is her expected value of the asset s. Let F denote the cumulative distribution of s with convex support $[\underline{s}, \bar{s}]$. The buyer's expected value conditional on the seller's signal is $b(s)$, which we assume is nondecreasing with $b(s) \geq s$ for all s. The seller privately observes s, while the buyer only knows the functions F and b.

Rather than study any particular trading or bargaining protocol, we adopt a mechanism design approach and focus on constrained-efficient mechanisms. Following Kennan and Wilson (1993), we view the mechanism design approach as informative about what traders may accomplish through any mechanism, including bargaining, given the restrictions implied by private information and technology. One-sided private information is a useful theoretical starting point because Samuelson (1984) provides an exhaustive study of Pareto efficient interim

[2]For some empirical evidence on this hypothesis, see Keys, Mukherjee, Seru and Vig (2010), Demiroglu and James (2012), Jiang, Nelson and Vytlacil (2014a), Jiang, Nelson and Vytlacil (2014b), and Piskorski, Seru and Witkin (2015).

mechanisms in this context. Although we are interested in mechanisms which satisfy an additional constraint, that the uninformed buyer is willing to participate in the mechanism after learning whatever information is revealed by the mechanism, our results still build on his characterization.

2.1 Interim Optimal Mechanisms

We start by analyzing Pareto optimal trading mechanisms. Using the revelation principle (Myerson, 1979), we know that any trading mechanism is payoff-equivalent to a direct revelation mechanism. The seller observes s and makes a report $\hat{s}$ to the mechanism. The mechanism then instructs the seller to give the asset to the buyer with probability $q(\hat{s})$ and instructs the buyer to give the seller a transfer of $t(\hat{s})$. Moreover, without loss of generality, the mechanism ensures that the seller truthfully reports her type, $\hat{s} = s$.

Any interim optimal mechanism solves the following problem:

$$\begin{aligned}
&\max_{\{t(s),q(s)\}} \int_{\underline{s}}^{\bar{s}} (t(s) - q(s)s)dF(s), \qquad (1)\\
&\text{subject to } \int_{\underline{s}}^{\bar{s}} (q(s)b(s) - t(s))dF(s) \geq u,\\
&\qquad t(s) - q(s)s \geq 0 \text{ for all } s,\\
&\qquad t(s) - q(s)s \geq t(\hat{s}) - q(\hat{s})s \text{ for all } s, \hat{s}.
\end{aligned}$$

The objective is to maximize the seller's expected profit. The first constraint imposes that the buyer's expected profit is at least u; by varying u we trace out the Pareto frontier between the expected profit for the seller and buyer. The second constraint is that the seller's expected profit must be nonnegative conditional on her signal s. The third constraint is that a seller with signal s weakly prefers reporting it rather than any other signal $\hat{s}$, the incentive compatibility constraint. We treat u as a free parameter, but it makes sense to restrict attention to $u \geq 0$ to ensure the buyer's voluntary participation. The case with $u = 0$ corresponds to the seller-optimal allocation. We also restrict attention to low values of u for which the constraint set is nonempty.[3]

The following result follows immediately from Samuelson (1984) and so we omit its proof:

Proposition 1. *If* $u > \max_s \int_{\underline{s}}^{s} (b(v) - s) dF(v)$, *the constraint set in problem* (1) *is empty. Otherwise, there exists numbers* s_1, s_2, *and* q *with* $\underline{s} \leq s_1 \leq s_2 \leq \bar{s}$ *and* $q \in (0, 1)$ *such that the following policy solves problem* (1)*:*

$$q(s) = \begin{cases} 1 \\ \bar{q} \\ 0 \end{cases} \quad \text{and } t(s) = \begin{cases} (1-\bar{q})s_1 + \bar{q}s_2 & \underline{s} \leq s \leq s_1 \\ \bar{q}s_2 & \text{if} \quad s_1 < s \leq s_2 \\ 0 & s_2 < s \leq \bar{s}. \end{cases}$$

Intuitively, keeping q piecewise constant is useful because it reduces the number of incentive constraints. The incentive constraints $t(s) - q(s)s \geq t(s') - q(s')s$ and $t(s') - q(s')s' \geq t(s) - q(s)s'$ imply $t(s) = t(s')$ whenever $q(s) = q(s')$. This implies that if a seller with some other signal s'' has an incentive to truthfully report s'' rather than s, he also has an incentive to truthfully report s'' rather than s'. In contrast, if the three signals s, s', and s'' all result in different trading probabilities, we must separately verify each pair of incentive constraints. We emphasize that this logic has nothing to do with the transmission of information from seller to the buyer, but instead with reducing the dimensionality of the constraints on the seller.

2.2 Information Revelation and Ex Post Participation

The mechanism design problem (1) is a technical device for characterizing possible trading arrangement given the constraints implied by private information. In our view, problem (1) ignores an important constraint on real world trading arrangements. The buyer can refuse to purchase the asset if, *after* he learns the terms of trade, he anticipates losing money by buying it. In making this decision, the buyer

[3]The constraint set is nonempty when $u = 0$ since $q(s) = t(s) = 0$ is always feasible.

can use any information that he learns from the mechanism, in particular the terms of trade. To capture this, we modify problem (1) by adding an ex post participation constraint for the buyer. In this section we describe the constraint intuitively and defer a precise statement until we get to the two-sided private information problem in Section 3.

Ex post participation constraints are tightly connected to information transmission. Problem (1) is silent about the buyer's beliefs after trade occurs. For example, the mechanism could tell the buyer the seller's report, or it could tell the buyer the transfer $t(s)$ and trading probability $q(s)$, or it could try to hide some of that information from the buyer, for example by using a lottery. In contrast, modeling ex post participation constraints requires us to be explicit about what a buyer learns from the mechanism. This puts information transmission front and center.

Problem (1) already imposes that the buyer's ex ante profit is at least equal to $u \geq 0$. Thus on average the buyer is willing to trade. If a trading mechanism can hide all of the seller's information, there is no additional constraint coming from the requirement that buyers are willing to trade ex post. We require that, at a bare minimum, a mechanism reveals one piece of information to the buyer, the trading price when there is supposed to be trade. In fact, optimal mechanisms only give the buyer this piece of information.

More precisely, a mechanism with ex post participation constraints takes the seller's report and recommends either a (possibly random) trading price or no trade. If the mechanism recommends trading at a price p, the buyer uses Bayes rule and his understanding of the economic environment and the mechanism to update his beliefs about the seller's signal. He then decides whether he is willing to accept the trade given his posterior belief about the seller's signal. The ex post participation constraint imposes that the buyer earns nonnegative profits at any trading price p, an additional constraint on feasible trading mechanisms.

The ex post participation constraint makes information revelation costly. The more prices that a mechanism uses, the more a buyer learns about the seller's report and so the tighter are the ex post participation constraints. This is the tension between information transmission and trading efficiency.

Adding ex post participation constraints to problem (1) leads to three logical possibilities. First, the constraints may have no impact on either the trading mechanism or the allocation. Second, the constraints may affect the way the trading mechanism implements an allocation, requiring a careful partial disclosure of information to the buyer, but not affect the allocation itself. Third, it may render a given allocation infeasible, requiring a change in the allocation.

The next result provides necessary and sufficient condition to be in the first two cases.

Proposition 2. *Assume* $u \leq \max_s \int_{\underline{s}}^{s}(b(v) - s)dF(v)$ *so the constraint set in problem* (1) *is nonempty. It is feasible to implement the allocation solving problem* (1) *while satisfying the buyer's ex post participation constraints if and only if*

$$\int_{s_1}^{s_2}(b(v) - s_2)dF(v) \geq 0, \tag{2}$$

where $s_1 \leq s_2$ *are the thresholds described in Proposition 1. When condition* (2) *holds, there exists a feasible implementation that uses at most two trading prices.*

We view condition (2) as being quite weak. It holds trivially whenever $s_1 = s_2$, so the interim mechanism never involves probabilistic trading. This is the case, for example, for the buyer-optimal interim mechanism. It is also the case when $F(s)$ is uniform on $[0, 1]$ and $b(s)$ is linear. The condition may also hold when $s_1 < s_2$, but it is possible to construct examples where condition (2) fails.

The finding that buyer's ex post participation constraint often does not affect the value of trade might be surprising. It suggests an explanation for why real-world trading mechanisms allow traders to walk away after learning the transaction price: this lack of commitment may be costless. Nevertheless, the lack of information is still important for shaping how information is transmitted through trade, as illustrated in our proof of the result, which we now proceed to sketch to emphasize this point (the full proof can be found in the appendix).

Our proof of sufficiency is constructive. When condition (2) holds, we construct mechanisms that recommend the buyer and seller either trade or don't trade. When the mechanism recommends trade, it proposes either a single trading price or one of two prices. The buyer learns the recommended trading price and updates his beliefs about the seller's signal using that price. We prove that the buyer is still willing to purchase the asset at the proposed price conditional on his updated beliefs, and that the expected transfers and trading probabilities coincide with those in Proposition 1. Crucially, the buyer is left with a coarser information set than the seller, for example sometimes learning only that the seller's expected valuation is below the trading price. Our proof of the necessity of condition (2) leverages results in Gerardi, Hörner and Maestri (2014). In the interim optimal mechanism, the buyer pays an average price of s_2 whenever the seller's signal lies between s_1 and s_2. When condition (2) is violated, buyers are unwilling to pay this price in order to buy an asset in this quality range. This leads to a violation of the buyer's ex post participation constraint.

When condition (2) fails, optimal mechanisms satisfying ex post participation constraints must reveal a bit more information. For example, they may involve trade a low price $\underline{p}$ if the seller's signal is below some low threshold s_1, probabilistic trade at the buyer's valuation $b(s)$ if the seller's signal lies in between the thresholds s_1 and a slightly higher threshold s_1', and probabilistic trade at a still higher price $\bar{p}$ if the seller's signal is in between s_1' and a higher threshold s_2, with no trade above s_2. With such a mechanism, the buyer is left with a coarser information set than the seller, sometimes knowing only that the seller's report was smaller than s_1, other times knowing only that it lay in between s_1' and s_2. Optimal trading mechanisms with ex post participation constraints hide information.

2.3 Cost of Full Information Revelation

Proposition 2 establishes that, under certain circumstances, it is possible to implement the solution to problem (1) while satisfying the buyer's ex post participation constraint. To do so, the optimal mechanism recommends either no trade or trade at one of at most two prices. Here we address a related issue: suppose we confine attention to mechanisms that fully reveal the seller's information to the buyer and that satisfy the buyer's ex post participation constraint. For example, the mechanism recommends trade at a higher price whenever the seller reports a higher signal. The buyer must also be willing to trade at that price, correctly interpreting how that price was influenced by the seller's signal. We prove that full information revelation reduces the value of the seller.

To show this, we modify problem (1) by adding one more constraint,

$$q(s)b(s) - t(s) \geq 0 \text{ for all } s. \tag{3}$$

The buyer must earn nonnegative profit at each value of the seller's signal.[4] We first prove that whenever $b(\underline{s}) = \underline{s}$, the solution to the full revelation problem, given by problem (1) with the additional constraint (3), has no trade, except possible at $s = \underline{s}$:

Proposition 3. *Assume $b(\underline{s}) = \underline{s}$ and consider the full revelation problem. Then $u > 0$ is not feasible and the unique solution with $u = 0$ has $q(s) = t(s) = 0$ for all $s > \underline{s}$.*

In contrast, it is easy to construct examples with trade in the interim problem, even when $b(\underline{s}) = \underline{s}$.[5] In such cases, full information revelation completely destroys all the gains from trade. By implication, there is a strict benefit from preventing information revelation. Efficient trading mechanisms must hide some information.

The logic of Proposition 3 is simple. At the lowest valuation, there are no gains from trade. The buyer's and seller's participation constraint then pins down the trading price, $t(\underline{s})/q(\underline{s}) = b(\underline{s}) = \underline{s}$. For trade to take place at higher signals, $s > \underline{s}$, the price must be higher than the seller's signal. But then the seller with the lowest signal prefers to misreport her signal. The only feasible allocation involves zero

[4]We do not directly impose that different reports induce different prices, although this is an important practical way for prices to reveal information. Instead, we allow for the possibility that the mechanism directly transmits the seller's report to the buyer. This approach is more convenient but has little practical impact on our results.

[5]Assume $b(s) > s$ for almost all s and there exists an $s^* > \underline{s}$ with $\int_{\underline{s}}^{s^*} (b(v) - s^*) dF(s) \geq 0$. This is the case when $F(s) = s$ on $[0, 1]$ and $b(s) = ks$ for some $k \geq 2$. Then setting $q(s) = 1$ and $t(s) = s^*$ if $s \leq s^*$ and $q(s) = t(s) = 0$ otherwise is feasible when $u \leq \int_{\underline{s}}^{s^*} (b(v) - s^*) dF(s)$ and gives the seller positive expected value.

trade. In contrast, mechanisms that hide information from the buyer allow for the possibility that the buyer loses money when the seller has the lowest signal. And prices above the seller's signal are useful for providing the seller with the incentive to truthfully reveal her signal.

Even if there are gains from trade, $b(s) > s$ for all s, full revelation reduces the highest value that the informed seller can attain:

Proposition 4. *Assume $u = 0$, $b(\underline{s}) < \bar{s}$, and there exists an $\varepsilon > 0$ such that $b(s) \geq s + \varepsilon$ for all s. The addition of constraint* (3) *strictly reduces the value of problem* (1)*.*

This result is driven by the following observations. When $u = 0$ the buyer must not gain from trade ex ante. This implies that in a fully revealing mechanism, the buyer must not get any gain from trade ex post, after learning the seller's valuation. This effectively restricts the price to equal the buyer's valuation, while incentive compatibility for the seller requires that the trading probability is always strictly positive. This implies that the seller has a positive gain from trade, even when he has the highest signal. However, in the solution to the interim problem (1), the seller has zero gain from trade when she has the highest signal.

Once again, this Proposition implies that there are strict benefits from hiding information. Efficient trading mechanisms avoid full revelation. Recall from the discussion surrounding Proposition 2 that this is accomplished by a combination of not having the price not react fully to the seller's information (e.g. pooling at one or two prices) and sometimes by making the price noisy (random prices).

Although these two propositions establish that information revelation may be costly, there are a few cases when it is costless to transmit the seller's information to the buyer. For example, if the buyer's value of the asset for any possible signal is higher than the seller's value for any possible signal, $b(\underline{s}) \geq \bar{s}$, then the first best allocation can be obtained. The buyer is willing to pay some price regardless of the seller's signal and the seller is willing to accept the same price, regardless of her signal. But in most interesting cases, the model with one-sided private information highlights the tension between trading efficiency and information revelation.

A standard concept of informational efficiency in asset markets is that "security prices fully reflect all available information" (Fama, 1970, 1991). Of course Fama focuses on exchanges and other markets with many participants, while we look at a market with two traders, and so the analogy we develop here is inexact. Nevertheless, one might conjecture from the existing literature that Pareto optimal mechanisms would be fully-revealing. We reject that hypothesis. Restricting attention to fully-revealing mechanisms generally leads to inferior allocations.

3 Two-Sided Private Information

We turn next to the realistic case where both the buyer and seller have private information. The basic structure of the model is unchanged, except for the fact that the buyer now also has a signal. After briefly describing the economic environment, we formally discuss the constraints on mechanisms that arise from private information and the lack of commitment from both parties. We then provide a characterization that allows us to limit attention to mechanisms that randomize over a finite number of prices. We end the section by discussing the additional constraints imposed by full information revelation.

3.1 Preferences and Information

Both the buyer and seller receive a signal and we assume for analytical convenience that the signal is binary, $b \in \{0, 1\}$ denotes the buyer's signal and $s \in \{0, 1\}$ denotes the seller's. Let π_{bs} denote the ex ante joint probability that the buyer receives signal b and the seller receives signal s. To avoid trivial cases, we assume $\pi_{bs} > 0$ for all b and s.

The buyer's expected value for the asset is v^B_{bs} when the buyer's signal is b and the seller's signal is s. The seller's expected value is v^S_{bs}. If the seller gives the asset to the buyer with probability q in return for a certain cash transfer of t, the buyer's expected profit is $qv^B_{bs} - t$ and the seller's is $t - qv^S_{bs}$, where we normalize the profit from no trade to zero.

The buyer privately observes his signal b and the seller privately observes her signal s. In the private values case, v_{bs}^B depends only on b and v_{bs}^S depends only on s, so the willingness of a trader to accept a transfer t in exchange for trade with probability q depends only on her own signal. But more generally, each trader's willingness to trade depends on their belief about the other traders' signals. Different trading mechanisms allow a trader to refuse to trade based on different information sets.

3.2 Feasible Mechanisms

We are interested in understanding the set of feasible trades given the constraints imposed both by private information and by the ability of either trader to walk away from the deal after learning the terms of trade. This again motivates a mechanism design approach. Using the Revelation Principle (Myerson, 1979), we focus without loss of generality on a direct revelation mechanism, where each trader is induced to truthfully report his signal to a mechanism.

More precisely, the traders observe their signals and make a report to the mechanism. The mechanism then either recommends that they trade at some (possibly random) price p or that they don't trade. If the mechanism recommends trade at price p, both traders must be willing to trade in order for trade to occur. In making this decision, each knows her own signal and report, the outcome recommended by the mechanism, and the structure of the mechanism itself. She may use this to update her beliefs about the other trader's signal and hence about her own desire to trade.

The equilibrium expected profit of the traders' are

$$V^B = \mathbb{E}^B(v_{bs}^B - p) \text{ and } V^S = \mathbb{E}^S(p - v_{bs}^S), \tag{4}$$

where the buyer's expectations $\mathbb{E}^B$ and the seller's expectations $\mathbb{E}^S$ are taken over the buyer's signal b, the seller's signal s, their truthful reports $\hat{b} = b$ and $\hat{s} = s$, and the price p. The price depends on the mechanism and is potentially a random variable conditional on the buyer's and seller's reports.

We focus attention on mechanisms that satisfy ex post participation constraints. Each trader must be willing to trade after he learns the trading price, under the assumption that the other trader truthfully reported her signal. This imposes

$$\mathbb{E}^B(v^B_{bs} - p|b, p) \geq 0 \text{ and } \mathbb{E}^S(v^B_{bs} - p|s, p) \geq 0, \tag{5}$$

where now the buyer's expectations $\mathbb{E}^B$ are now taken with respect to the seller's signal, conditional on the buyer's signal b, truthful report $\hat{b} = b$, and the price p. The seller's expectations $\mathbb{E}^S$ are similarly constructed. We contrast this with the standard interim constraints,

$$\mathbb{E}^B(v^B_{bs} - p|b) \geq 0 \text{ and } \mathbb{E}^S(v^B_{bs} - p|s) \geq 0,$$

which require that the trader's expected profits are nonnegative only at the interim stage, before they learn the price recommended by the mechanism. The ex post participation constraints imply the interim participation constraints, since the former has to hold for every signal and price combination, while the latter holds only signal by signal.

Finally, the veto incentive compatibility constraint states that the buyer and seller earn higher profits by truthfully reporting their signal rather than misrepresenting it, conditional on the other trader truthfully reporting her signal:

$$\mathbb{E}^B(v^B_{bs} - p|b) \geq \mathbb{E}^B\left[\max\left\{\mathbb{E}^B(v^B_{bs} - p|b, \hat{b}, p), 0\right\}|b, \hat{b}\right] \tag{6a}$$

$$\mathbb{E}^S(p - v^S_{bs}|s) \geq \mathbb{E}^S\left[\max\left\{\mathbb{E}^S(p - v^S_{bs}|s, \hat{s}, p), 0\right\}|s, \hat{s}\right] \tag{6b}$$

The left hand side is the expected payoff conditional on the trader's truthfully reported signal, taken with respect to the other trader's signal and the recommended price. The inner expectation on the right hand side is taken with respect to the other trader's signal, conditional on the trader's own signal and report as well as the recommended price. The max operator reflects the possibility that a trader misreports her signal and then refuses to trade upon observing a particular price recommendation. The outer expectation is taken over prices, conditioning only on the trader's own signal and report.

In the standard interim incentive problem, we drop the possibility of refusing to trade after observing the recommended price. This reduces the value of the right hand side of inequalities (6), relaxing the constraint. Using the law of iterated expectations, we can express the interim incentive constraint as

$$\mathbb{E}^B(v^B_{bs} - p|b) \geq \mathbb{E}^B(v^B_{bs} - p|b, \hat{b}) \text{ and } \mathbb{E}^S(p - v^S_{bs}|s) \geq \mathbb{E}^S(p - v^S_{bs}|s, \hat{s}),$$

where the left hand side is the trader's expected profit if he truthfully reports his signal and the right hand side is his expected profit if he makes any other report.

A Pareto optimal veto incentive compatible mechanism maximizes a Pareto weighted average of V^B and V^S, defined in equation (4) subject to the ex post participation constraint (5) and the veto incentive compatibility constriant (6).[6] As explained above, this is a constrained version of the standard mechanism design problem, which imposes only an interim participation constraint and an interim incentive compatibility constraint. We exploit the fact that the interim problem is a relaxed version of the veto incentive compatible problem in our characterization of optimal trading mechanisms.

3.3 Formal Characterization of Mechanisms

This section provides a formal characterization of veto incentive compatible mechanisms. We start by noting that mechanisms only need use a finite number of prices:

Proposition 5. *Take any feasible veto incentive compatible mechanism. There exists a feasible veto incentive compatible mechanism with the same trading probabilities and expected payoffs conditional on any signals* (b, s)*, in which the recommendation is always of the form* $p \in \{p_1, p_2, \ldots, p_{13}, \varnothing\}$.

[6] Formally, the buyer's ex post participation constraint follows from the buyer's veto incentive compatibility constraints with $b = \hat{b}$, and similarly for the seller. We find it conceptually convenient to keep this constraints separate.

The finding that we use at most 13 prices depends on the dimension of the signals that the buyer and seller receive. In general, if the buyer has N^B possible signals and the seller has N^S possible signals, we need at most $N \equiv (N^B)^2 + (N^S)^2 + N^B N^S + 1$ prices. In practice, we find that far fewer prices are sufficient in our examples.

Our proof shows that any veto incentive compatible mechanism is completely characterized by the payoff of a trader who receives some signal and makes a (possibly different report) and by the trading probabilities conditional on the signal pair (b, s). This outcome is of dimension $N - 1$, linear in the probability measure over prices. We can therefore express the outcome as the weighted average of at most N extreme points of this set, i.e. by putting weight onto at most N such prices, in addition to the option not to trade.

Building on this proposition, we denote a veto incentive compatible mechanism by a set of prices p_n, $n \in \{1, \ldots, N\}$, and probabilities $\omega_{n|bs}$ such that the mechanism recommends trading at price p_n with probability $\omega_{n|bs}$ when the buyer reports signal b and the seller reports signal s. Naturally $\omega_{n|bs} \geq 0$ for all (b, s). We allow that $\sum_n \omega_{n|bs} < 1$, in which case $1 - \sum_n \omega_{n|bs}$ is the probability that the mechanism recommends no trade following the reports (b, s).

We use this notation to write the veto incentive compatibility problem as

$$\max_{\{p_n,\omega_{n|bs}\}} \sum_{n,b,s} \left(\phi(p_n - v^S_{bs}) + (1-\phi)(v^B_{bs} - p_n)\right)\omega_{n|bs}\pi_{bs} \tag{7}$$

$$\begin{aligned}
\text{subject to } & \sum_s (v^B_{bs} - p_n)\omega_{n|bs}\pi_{bs} \geq 0 \text{ for all } b \text{ and } n, \\
& \sum_b (p_n - v^S_{bs})\omega_{n|bs}\pi_{bs} \geq 0 \text{ for all } s \text{ and } n, \\
& \sum_{n,s} (v^B_{bs} - p_n)\omega_{n|bs}\pi_{bs} \\
& \qquad \geq \sum_n \max\left\{\sum_s (v^B_{bs} - p_n)\omega_{n|\hat{b}s}\pi_{bs}, 0\right\} \text{ for all } b \text{ and } \hat{b}, \text{ and} \\
& \sum_{n,b} (p_n - v^S_{bs})\omega_{n|bs}\pi_{bs} \\
& \qquad \geq \sum_n \max\left\{\sum_b (p_n - v^S_{bs})\omega_{n|b\hat{s}}\pi_{bs}, 0\right\} \text{ for all } s \text{ and } \hat{s}.
\end{aligned}$$

The objective function is equal to $\phi V^S + (1-\phi)V^B$, where the seller's and buyer's expected profits are defined in equation (4) and ϕ is the seller's Pareto weight. The exact expression uses the observation that $\omega_{n|bs}\pi_{bs}$ is the probability that the price is p_n, the buyer's signal is b, and the seller's signal is s.

The first two constraints are the ex post participation constraints (5). Using Bayes rule,

$$\Pr(s|b, p_n) = \frac{\Pr(p_n|b,s)\Pr(s|b)}{\sum_{s'} \Pr(p_n|b,s')\Pr(s'|b)},$$

where $\Pr(s|b, p_n)$ is the probability that the seller's signal is s conditional on the buyer's signal and report b and the price p_n; $\Pr(p_n|b,s) = \omega_{n|bs}$ is the probability that the price is p_n given the the buyer's signal and report b and the seller's signal s; and $\Pr(s|b) = \pi_{bs} / \sum_{s'} \pi_{bs'}$ is the probability that the seller's signal is s given the buyer's signal b. It follows that

$$\mathbb{E}^B(v^B_{bs} - p_n | b, p_n) = \sum_s (v^B_{bs} - p_n)\Pr(s|b,p_n) = \frac{\sum_s (v^B_{bs} - p_n)\omega_{n|bs}\pi_{bs}}{\sum_s \omega_{n|bs}\pi_{bs}}.$$

The first constraint drops the irrelevant denominator. The second constraint is an analogous condition for the seller.

The final two constraints are the veto incentive compatibility constraints (6). The proof is similar, again using Bayes rule and dropping irrelevant terms from the denominator. There are two important differences. First, the participation constraint holds after the traders learn the trading price, while the incentive compatibility constraint holds before they learn the trading price and so sums across the possible prices p_n. Second, the right hand side distinguishes between the trader's signal, b or s, which affects the probability distribution over the other trader's signal through π, and the trader's report $\hat{b}$ or $\hat{s}$, which affect the probability distribution over the price through ω.

We can also write the analogous interim problem as

$$\max_{\{p_n,\omega_{n|bs}\}} \sum_{n,b,s} \big(\phi(p_n - v^S_{bs}) + (1-\phi)(v^B_{bs} - p_n)\big)\omega_{n|bs}\pi_{bs} \tag{8}$$

$$\begin{aligned}
\text{subject to } & \sum_{n,s}(v^B_{bs} - p_n)\omega_{n|bs}\pi_{bs} \geq 0 \text{ for all } b, \\
& \sum_{n.b}(p_n - v^S_{bs})\omega_{n|bs}\pi_{bs} \geq 0 \text{ for all } s, \\
& \sum_{n,s}(v^B_{bs} - p_n)\omega_{n|bs}\pi_{bs} \geq \sum_{n,s}(v^B_{bs} - p_n)\omega_{n|\hat{b}s}\pi_{bs} \text{ for all } b \text{ and } \hat{b}, \text{ and} \\
& \sum_{n,b}(p_n - v^S_{bs})\omega_{n|bs}\pi_{bs} \geq \sum_{n,b}(p_n - v^S_{bs})\omega_{n|b\hat{s}}\pi_{bs} \text{ for all } s \text{ and } \hat{s}.
\end{aligned}$$

As we have already argued, the interim problem is a relaxed version of the veto incentive compatible problem.

3.4 Fully-Revealing Mechanisms

A fully-revealing veto incentive compatible mechanism is a feasible veto incentive compatible mechanism in which the mechanism is constrained to reveal each trader's report to the other trader before a trade is consummated. If the mechanism recommends trading at a price p_{bs} when the buyer's report is b and the seller's report is s, then both the buyer and seller must be willing to trade at that price knowing the other trader's signal. This imposes $v^B_{bs} \geq p_{bs} \geq v^S_{bs}$. It is straightforward to verify that a fully-revealing mechanism needs to use at most one price per report pair (b, s), i.e. at most four prices in our scenario.

We write the full revelation problem as

$$\max_{\{p_{bs}, \omega_{bs}\}} \sum_{n,b,s} \left(\phi(p_{bs} - v^S_{bs}) + (1-\phi)(v^B_{bs} - p_{bs})\right)\omega_{bs}\pi_{bs} \tag{9}$$

subject to $(v^B_{bs} - p_{bs})\omega_{bs} \geq 0$ for all b and s,

$(p_{bs} - v^S_{bs})\omega_{bs} \geq 0$ for all b and s,

$\sum_s (v^B_{bs} - p_{bs})\omega_{bs}\pi_{bs} \geq \sum_s \max\{v^B_{bs} - p_{\hat{b}s}, 0\}\omega_{\hat{b}s}\pi_{bs}$ for all b and $\hat{b}$, and

$\sum_b (p_{bs} - v^S_{bs})\omega_{bs}\pi_{bs} \geq \sum_s \max\left\{p_{\hat{b}s} - v^S_{bs}, 0\right\}\omega_{b\hat{s}}\pi_{bs}$ for all s and $\hat{s}$.

Here ω_{bs} is the probability that the mechanism recommends trade when the buyer's report is b and the seller's report is s. The objective function is essentially unchanged from the interim and veto-incentive compatible problems. The first two constraints impose that the buyer and seller earn nonnegative profits conditional on both reports. The final two constraints ensure that neither trader wishes to misrepresent his signal, allowing for the possibility that the trader then refuses to trade following certain reports by the other trade.

4 Private Values

We briefly comment on the private values case, a common simplifying assumption in the mechanism design literature (Myerson and Satterthwaite, 1983; McAfee and Reny, 1992). We show that information revelation is costless in this case:

Proposition 6. *Assume private values, $v^B_{bs} = v^B_b$ and $v^S_{bs} = v^S_s$ for all b and s. The value of problems* (7) *and* (9) *are the same.*

Note that the buyer's and seller's value may still be correlated because their signals are correlated; however, neither trader cares directly about the other trader's signal.

The proof shows that the constraint set is the same in the two problems. Intuitively, the benefit of hiding information is that doing so prevents traders from learning something that makes them not want to trade. A buyer with signal b may in general only be willing to pay a price p if he believes the seller's signal takes on a particular value of s. But with private values, this is not the case. The buyer's willingness to pay depends only on his own signal and so telling him the seller's signal does not alter the ex post participation constraint. The same is true for the seller.

This highlights the importance of common values for the trade-off between information revelation and trading efficiency. Suppose both traders' valuations are increasing in the other trader's signal. If the buyer knows the seller has a low signal, he will be unwilling to pay a high price for the asset. Similarly if the seller knows the buyer has a high signal, she will be unwilling to accept a low price for the asset. This may imply that prices are sensitive to traders' reports, which makes it difficult to induce truthful information revelation. The solution is to penalize a seller who reports a low signal or a buyer who reports a high signal by not trading, even if trading is efficient. In contrast, it may be possible to sustain more efficient trading arrangements by keeping some information private. The remainder of the paper shows how this works.

5 Common Values

We now turn to the case with common values. We start by introducing a useful example which parametrizes the information structure in a way that lends itself easily to interpretation and assumes a constant gain from trade. We then solve for mechanisms that maximize the sum of utilities, or equivalently, the gains from trade. In our example, this is equivalent to maximizing the probability of trade. We then turn to mechanisms that impose full information revelation and show that this restriction is costly. Finally, we study other efficient mechanisms that trace out the Pareto frontier between the buyer and seller.

5.1 An Example

We focus on a particular example which illustrates some general properties of the model. Assume that the traders' binary signals are correlated:

$$\pi_{bs} = \begin{cases} \frac{1}{2}(\alpha^2 + (1-\alpha)^2) & \text{if} \quad b = s \\ \alpha(1-\alpha) & \phantom{\text{if}} \quad b \neq s, \end{cases}$$

where $\frac{1}{2} < \alpha < 1$. The seller's payoff is

$$v_{bs}^S = \begin{cases} \frac{(1-\alpha)^2}{\alpha^2+(1-\alpha)^2} & b = s = 0 \\ \frac{1}{2} & \text{if} \quad b \neq s \\ \frac{\alpha^2}{\alpha^2+(1-\alpha)^2} & b = s = 1, \end{cases}$$

while the buyer's payoff is $v_{bs}^B = \gamma + v_{bs}^S$ for all (b, s) where $\gamma > 0$.

As motivation for the payoff structure, suppose that the state of the world is $\delta \in \{0, 1\}$, taking on each value with equal probability. The seller values the asset at δ, while the buyer values it at $\delta + \gamma$. Here γ denotes the constant gains from trade. Neither trader knows the true value of δ, but instead each receives a binary signal. The signals are independent conditional on the asset's payoff, but they are only imperfectly correlated with the payoff. In particular, the buyer (seller) receives the "accurate" signal $b = \delta$ ($s = \delta$) with probability α and otherwise receives the inaccurate signal $b = 1 - \delta$ ($s = 1 - \delta$). One can easily verify that these signals give rise to the correlation structure and signal-conditional expected values defined above.

To put a more economic interpretation on the example, we view the buyer's and seller's signals as models of the asset's cash flow. Each trader is more likely to believe the asset is valuable if it is valuable, but their models are distinct and so each can potentially learn from the other. Although it is common knowledge that there are gains from trade, $\gamma > 0$, private information may prevent them from recognizing the gains from trade at any particular price p. Nevertheless, we show below that obscuring that private information is better for trade than the alternative possibility, setting up a fully revealing trading mechanism.

5.2 Trade Maximization

We start by describing the solution to the veto incentive compatible problem (7) with equal Pareto weights, $\phi = 1/2$. Since the difference between the buyer's and seller's valuation is constant in this example, the equal Pareto-weighted problem is equivalent to maximizing the probability of trade, a natural benchmark. The following proposition characterizes the optimal mechanism:

Proposition 7. *Consider the example in Section 5.1. There exists a solution to problem* (7) *with $\phi = 1/2$ that uses at most three prices, v_{00}^B, $\frac{1}{2}(1+\gamma)$, and v_{11}^S:*

1. *If $\gamma \geq 2\alpha - 1$, trade occurs at price $\frac{1}{2}(1+\gamma)$ for sure.*

2. *If $2\alpha - 1 > \gamma > \frac{\alpha(\alpha(3-2\alpha)-1)}{(1-\alpha(1-\alpha))(1-2\alpha(1-\alpha))}$, an optimal mechanism is as follows:*

   $(\hat{b}, \hat{s}) = (0,0) \Rightarrow p = v_{00}^B$ *with prob.* μ *and* $p = \frac{1}{2}(1+\gamma)$ *with prob.* $1-\mu$;

   $(\hat{b}, \hat{s}) = (1,1) \Rightarrow p = v_{11}^S$ *with prob.* μ *and* $p = \frac{1}{2}(1+\gamma)$ *with prob.* $1-\mu$;

   $(\hat{b}, \hat{s}) = (1,0) \Rightarrow p = \frac{1}{2}(1+\gamma)$;

   $(\hat{b}, \hat{s}) = (0,1) \Rightarrow p = \frac{1}{2}(1+\gamma)$ *with prob.* λ *and no trade with prob.* $1-\lambda$.

3. *If $\frac{\alpha(\alpha(3-2\alpha)-1)}{(1-\alpha(1-\alpha))(1-2\alpha(1-\alpha))} \geq \gamma > 0$, an optimal mechanism is as follows:*

   If $(\hat{b}, \hat{s}) = (0,0) \Rightarrow p = v_{00}^B$ *with prob.* λ *and no trade with prob.* $1-\lambda$;

   If $(\hat{b}, \hat{s}) = (1,1) \Rightarrow p = v_{11}^S$ *with prob.* λ *and no trade with prob.* $1-\lambda$;

   If $(\hat{b}, \hat{s}) = (1,0) \Rightarrow p = \frac{1}{2}(1+\gamma)$;

   If $(\hat{b}, \hat{s}) = (0,1) \Rightarrow$ *no trade.*

Figure 1 shows the three regions of the parameter space. When the gains from trade are large, $\gamma \geq 2\alpha - 1$, the first best can be attained by trading at a constant price. In this case, a buyer who knows only that he received the signal b is willing to accept the price $\frac{1}{2}(1+\gamma)$ if he doesn't know the seller's signal, and similarly for the seller with signal s who does not know the buyer's signal. A constant price successfully keeps this information hidden from each trader.

Note, however, that if a buyer had the low signal and knew the seller had the low signal, she would refuse to trade at this price whenever $v^B_{00} < \frac{1}{2}(1+\gamma)$ or equivalently $\gamma < \frac{\alpha^2-(1-\alpha)^2}{\alpha^2+(1-\alpha)^2}$. Similarly a seller with the high signal would refuse to sell to a buyer with the high signal if he knew this information whenever $v^S_{11} > \frac{1}{2}(1+\gamma)$. This reduces to the same parameter restriction. Since $\frac{\alpha^2-(1-\alpha)^2}{\alpha^2+(1-\alpha)^2} > 2\alpha - 1$, information revelation is costly in some of this region. The next section explores this insight more carefully.

For intermediate gains from trade, the previous trading arrangement breaks down. A seller with the high signal would refuse to sell at the intermediate price if she doesn't know the buyer's signal, and similarly a buyer with the low signal would refuse to buy at the intermediate price if he doesn't know the seller's signal. Any veto incentive compatible trading arrangement must reveal some information. The optimal mechanism reveals as little as possible.

When the mechanism recommends trading at the extreme prices v^B_{00} or v^S_{11}, the two reports are common knowledge. In contrast, some uncertainty remains when the mechanism recommends trading at the intermediate price, with each trader updating his beliefs about the other trader's signal. The mechanism randomizes across recommendations so as to leave a seller with the the high signal and a buyer with the low signal indifferent about trading when instructed to trade at the intermediate price. A buyer with the low signal is sufficiently confident that the seller has the high signal that he is willing to pay a high price, and conversely for a seller with the high signal.

This mechanism partially aligns the expected trading price with the traders reports. This gives traders and incentive to lie to get a better price. To keep the traders honest, the mechanism punishes them when their reports differ in the

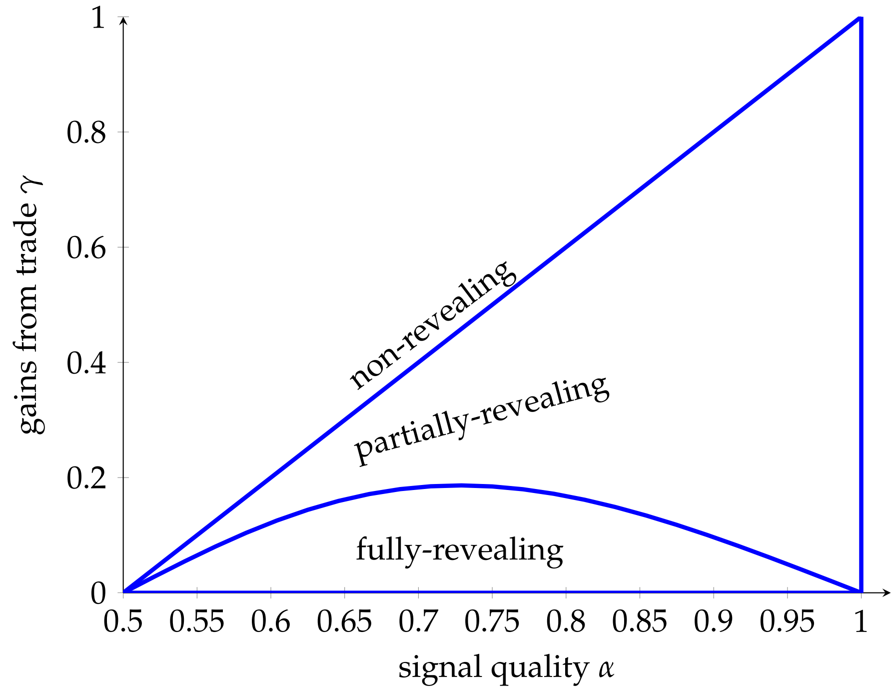

Figure 1: Regions of the parameter space in which different types of veto incentive compatible mechanisms are trade-maximizing. The details of the mechanisms are given in Proposition 7.

direction of self interest. When the seller reports a high signal and the buyer reports a low signal, there is a possibility that trade breaks down.

Finally, with low gains from trade, the mechanism is deterministic and fully-revealing. In this case, the probability of trade is low. Hiding information is feasible but turns out to be counterproductive.

Our proof of Proposition 7 is constructive. We define the probabilities λ and μ and show that the proposed mechanism satisfies the constraints of the veto incentive compatible problem (7). We then solve the relaxed interim problem (8) and prove that the maximized value of this problem is the same as the value we obtained for an interim mechanism. This recalls Proposition 2, where we proved that the veto incentive compatibility constraints need not affect the value of the one-sided private information problem. In that environment, the constraints were important for understanding information transmission, and this remains true with two-sided private information.

5.3 Fully Revealing Mechanisms

We highlight the cost of information revelation by solving the full revelation problem (9). With very large or very small gains from trade, the solution in Proposition 7 is consistent with full information revelation, but otherwise there is a cost from such a policy.

To prove this, first assume that $\gamma \geq \frac{2\alpha-1}{\alpha^2+(1-\alpha)^2}$ so $v^S_{bs} \leq v^B_{b's'}$ for all b, b', s, and s'. This is a tighter restriction than the first case in Proposition 7. A constant price $\frac{1}{2}(v^S_{22} + v^B_{11})$ achieves the first best, always trading, even if both traders know the other trader's signal. Conversely, in the third case in Proposition 7, we proposed implementing the optimum using a fully revealing mechanism. It follows that there is no cost to information revelation.

But at intermediate values of the gains from trade,

$$\frac{2\alpha - 1}{\alpha^2 + (1-\alpha)^2} > \gamma > \frac{\alpha(\alpha(3-2\alpha)-1)}{(1-\alpha(1-\alpha))(1-2\alpha(1-\alpha))},$$

trade maximization necessarily entails hiding information. The solution to problem (9) is as follows:

$(\hat{b}, \hat{s}) = (0, 0) \Rightarrow p = v^B_{00}$;

$(\hat{b}, \hat{s}) = (1, 1) \Rightarrow p = v^S_{11}$;

$(\hat{b}, \hat{s}) = (1, 0) \Rightarrow p = \frac{1}{2}(1+\gamma)$;

$(\hat{b}, \hat{s}) = (0, 1) \Rightarrow p = \frac{1}{2}(1+\gamma)$ with prob. λ and no trade with prob. $1-\lambda$.

In contrast, the solution to problem (7) puts positive weight onto the intermediate price $\frac{1}{2}(1+\gamma)$ in the first two cases. This is impossible with a fully revealing mechanism. If the buyer knew when both traders received the low signal, he

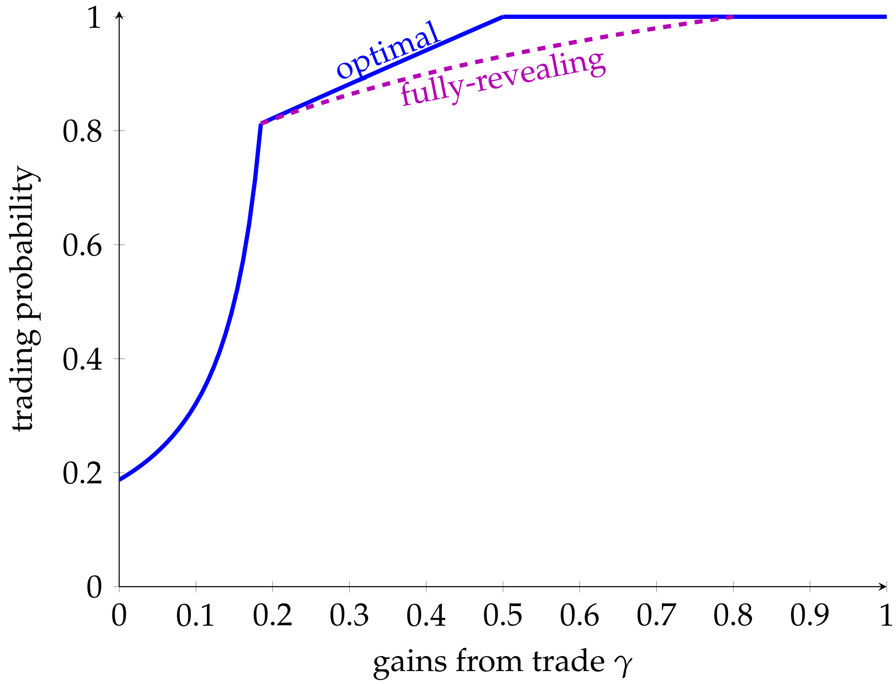

Figure 2: Trading Probability with $\alpha = 3/4$. The solid line shows the trading probability under the trade-maximizing veto incentive compatible mechanism, the solution to problem (7). The dashed line imposes the additional constraint that the mechanism is fully-revealing, the solution to problem (9).

would refuse to pay more than v^B_{00}; and if the seller knew when both traders received the high signal, he would refuse to accept less than v^S_{11}. The probability of trading when the buyer reports a low signal and the seller reports a high signal, λ, is set so as to ensure that the seller has an incentive to report a low signal when he receives one and similarly for a buyer who receives a low signal. We illustrate this through the incentive constraint of a seller who receives the low signal:

$$\left(v^B_{00} - v^S_{00}\right)\pi_{00} + \left(\tfrac{1}{2}(1+\gamma) - v^S_{10}\right)\pi_{10} = \left(\tfrac{1}{2}(1+\gamma) - v^S_{00}\right)\lambda\pi_{00} + \left(v^S_{11} - v^S_{10}\right)\pi_{10}.$$

The left hand side is the expected profit of a seller who truthfully reports the low signal. The probability that the buyer gets the low signal conditional on the seller's low signal is $\pi_{00}/(\pi_{00}+\pi_{10})$. In this event, the seller's gain from trade under the proposed mechanism is $v^B_{00} - v^S_{00}$. Alternatively, with complementary probability, the buyer gets the high signal and the seller's gain from trade is the intermediate price minus the intermediate valuation. If the seller misrepresents his signal, he changes the price and trading probability but doesn't change the buyer's signal distribution or his own valuation. This is illustrated on the right hand side of the equality.

This incentive constraint is tighter in the fully revealing mechanism than the optimal mechanism because prices are more sensitive to reports. As a result, the trading probability λ is lower when the buyer has the low signal and the seller has the high signal. For example, when $\alpha = 3/4$ and $\gamma = 1/2$, the probability of trade is 1 under the optimal mechanism and falls to 0.88 under the best fully-revealing mechanism. Figure 2 shows the increased efficiency of a partially-revealing mechanism for this value of the accuracy parameter α.

5.4 Pareto Frontier

We close by exploring the set of feasible payoffs more generally. We maintain the assumption that the traders' signals are binary but we drop the particular payoff structure in Section 5.1. We find that the solutions to the veto incentive compatible problem (7) and the standard interim problem (8) do not generally coincide. The additional constraints on transfers and trading probabilities in the former problem sometimes have a real bite.

Proposition 8. *Assume v^B_{bs} and v^S_{bs} are nondecreasing in b and s. Consider any mechanism $\{p, \omega\}$ in the constraint set of problem* (7)*. Let $q_{bs} \equiv \sum_n \omega_{n|bs}$ and $t_{bs} \equiv \sum_n p_n \omega_{n|bs}$ denote the trading probability and expected transfer conditional on the reports in this mechanism. Then*

$$t_{bs} \in [v^S_{0s} q_{bs}, v^B_{b1} q_{bs}] \textit{ for all } (b, s) \in \{0, 1\}^2. \tag{10}$$

In addition, if $v_{11}^S q_{11} > t_{11}$*,*

$$\frac{v_{01}^B q_{01} - t_{01}}{v_{11}^S q_{11} - t_{11}} \geq \frac{\pi_{11}\left(v_{01}^B q_{11} - t_{11}\right)}{\pi_{01}\left(t_{11} - v_{01}^S q_{11}\right)}; \tag{11}$$

and if $t_{00} > v_{00}^B q_{00}$*,*

$$\frac{t_{01} - v_{01}^S q_{01}}{t_{00} - v_{00}^B q_{00}} \geq \frac{\pi_{00}\left(t_{00} - v_{01}^S q_{00}\right)}{\pi_{01}\left(v_{01}^B q_{00} - t_{00}\right)}. \tag{12}$$

The restrictions (10)–(12) are necessary for implementing the solution to the interim problem in a manner consistent with limited commitment. In numerical simulations, we have not found any other binding constraints.

Condition (10) is the analog of condition (2) from the model with one-sided private information. Consider a seller who receives signal s. She believes that the value of the asset is at least v_{0s}^S, regardless of the buyer's signal, and so would never accept a lower price in any veto incentive compatible mechanism. That is, for any $p_n < v_{0s}^S$, the seller's ex post participation constraint implies $\omega_{n|bs} = 0$ for all buyer reports b. Since $t_{bs} = \sum_n p_n \omega_{n|bs}$ and $q_{bs} = \sum_n \omega_{n|bs}$, it follows that $t_{bs}/q_{bs} \geq v_{01}^S$ whenever $q_{bs} > 0$. This proves $t_{bs} \geq v_{0s}^S q_{bs}$. The proof that $t_{bs} \leq v_{b1}^B q_{bs}$ is symmetric, using the fact that a buyer who receives a signal b would never pay more than v_{b1}^B for the asset, regardless of his beliefs.

The constraints (11) and (12) have no analog in the one-sided problem but may also bind. The basic issue is that an optimal mechanism in the interim problem may set a higher expected price when the buyer reports the low signal than when he reports the high signal: $t_{0s}/q_{0s} > t_{1s}/q_{1s}$ for some s. But this means that the seller with signal s can infer from a low trading price that the buyer most likely got a high signal, which raises the minimum price that the seller is willing to accept above v_{0s}^S. This places an additional restriction on the relationship between transfers and trading probabilities which those constraints capture.

It is easy to construct examples of economies in which these constraints bind. Figure 3 illustrates one such case, using the payoff structure in Section 5.1 and setting $\alpha = 0.6$ and $\gamma = 0.2$. The lightest shaded region illustrates the entire set

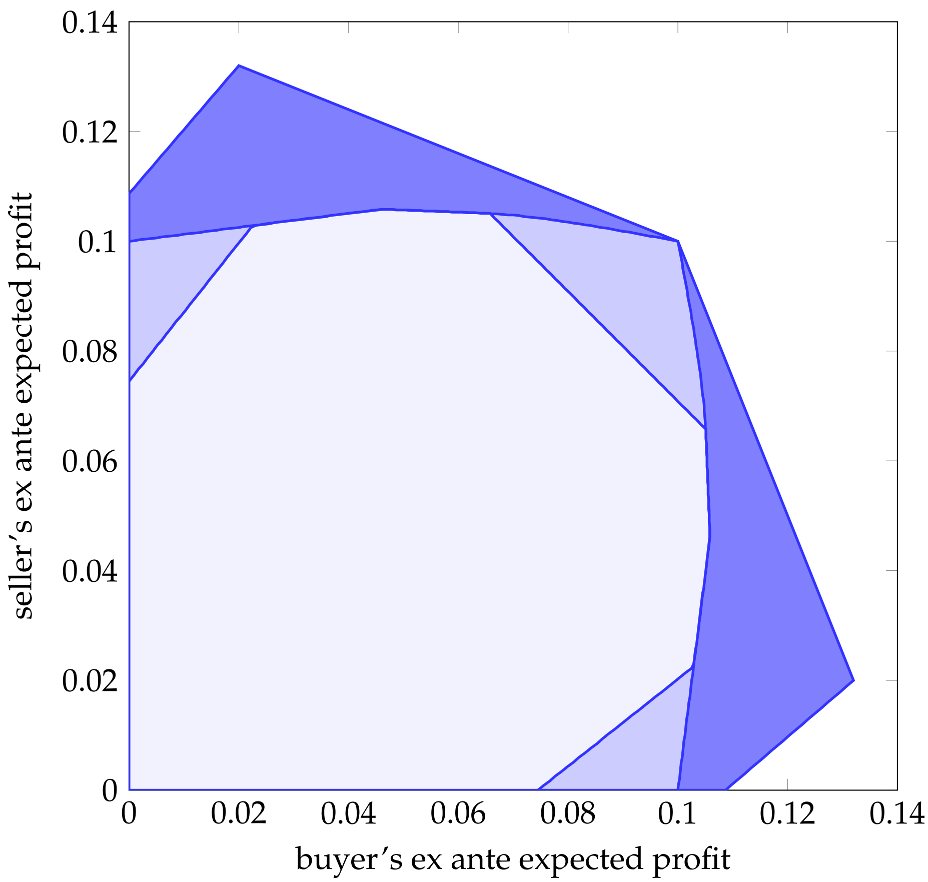

Figure 3: The set of feasible payoffs with 2-sided private information. The figure assumes the payoff structure in Section 5.1 with $\alpha = 0.6$ and $\gamma = 0.2$. The lightest shaded area is the set of payoffs obtainable using a fully revealing veto incentive compatible mechanism. The intermediate shaded area is the set of payoffs that can additional be obtained using an arbitrary veto incentive compatible mechanism. The darkest area shows the set of payoffs that can additionally be obtained using an interim mechanism.

of payoffs that can be obtained using a feasible fully-revealing mechanism, i.e. a feasible policy in problem (9). The intermediate shaded region is the set of payoffs that can additionally be obtained using a veto incentive compatible mechanism, i.e. a feasible policy in problem (7). And the darkest shaded region is the set of payoffs that can be obtained using an interim mechanism, a feasible policy in problem (8).

The constraints (10), (11), and (12) sometimes bind in this example. The possibility that the first constraint binds could be anticipated from Gerardi, Hörner and Maestri (2014), and so we focus on the more novel constraints. Consider the problem of maximizing the seller's profit subject to giving the buyer at least 0.09. In the interim problem, we find $q_{00} = q_{11} = q_{10} = 1$ and $q_{01} = 0.875$, with $t_{00} = 0.512$, $t_{11} = 0.543$, $t_{10} = 0.708$, and $t_{01} = 0.622$. This gives the seller expected profit 0.214. But this policy violates the constraint (12) for the buyer (and none of the other constraints).

Instead, the veto-incentive compatible problem yields a lower trading probability, $q_{00} = q_{11} = q_{10} = 1$ and $q_{01} = 0.830$, with $t_{00} = 0.564$ and $t_{11} = 0.616$, both higher, and $t_{10} = 0.625$ and $t_{01} = 0.520$, both lower. The reduction in the trading probability implies that the seller's profit is lower, 0.212. It is still the case that the expected trading price t_{bs}/q_{bs} is higher when the buyer and seller have different signals than when they both have the high signal, but the gap is smaller than in the interim mechanism.

Figure 3 also highlights the important role played by concealing information in veto incentive compatible mechanisms. The smallest shaded region indicates the set of payoffs obtainable by a fully-revealing veto incentive compatible mechanism. With fully-revealing mechanisms, a trader always knows his trading partner's signal, and so the veto incentive compatibility constraints require that prices are sensitive to information. To offset this, the incentive constraints reduce the probability of trade when the buyer reports the low signal or the seller reports the high signal, reducing overall efficiency.

6 Conclusion

This paper highlights the transmission of information within a single trade. We find a tension between pairwise efficient trading arrangements and information revelation. In some cases, we can obtain the unconstrained optimum by trading at a constant price, independent of the traders' signals. This blocks all information transmission. In other cases, the unconstrained optimum is unattainable, but efficient trading arrangements still conceal information by making the trading price insensitive to information and possibly random conditional on all available information. In many cases, mechanisms that fully reveal all information reduce trading efficiency.

We close with some thoughts on how this single trade can fit into a dynamic environment with learning and retrading. First, suppose that the same buyer and seller trade in two or more periods without obtaining any new information. Optimality requires that they use a single trading mechanism, with reports in the first period dictating trades in both period. Thus our basic approach trivially carries over to this environment.

A more interesting case occurs when the buyer in one period has an option to resell the asset in a later period. For example, a mortgage originator with private information about the quality of a mortgage pool may sell a mortgage-backed security to an insurance company. The insurance company might later decide to sell the security to a pension fund. In this case, the insurance company has an incentive to credibly design a first period mechanism that prevents it from learning the mortgage originator's information. Doing so eliminates private information in the secondary market, which simplifies the resale process (Gorton and Pennacchi, 1990). The primary market is then an example of one in which there is no tension between trading efficiency and information transmission. Minimizing the information that the insurance company learns from the mortgage originator enhances both trading and retrading efficiency.

It is also easy to construct examples in which information transmission is privately and socially useful. For example, the buyer's belief that house prices are over-valued may reduce the price of mortgage-backed securities as well as the amount of new mortgage origination. On the one hand, an optimal economy-wide trading mechanism in this environment trades off the benefits of placing the already-created security against the Hayekian benefit of information transmission through prices. On the other hand, contracts that are privately optimal bilaterally may not internalize the proper value of information transmission and may look closer to the contracts studied in the present paper. Although a complete dynamic analysis is outside our present reach, we conjecture, extrapolating from our current results, that in some cases information transmission may come to a halt, with diffusion stopping short of full revelation even in the long run, in contrast to the results in Duffie and Manso (2007) and Golosov, Lorenzoni and Tsyvinski (2014). Studying such extensions is an interesting avenue for future research.

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Appendix

A Proofs

Proof of Proposition 2. First assume $s_1 = s_2$. In this case, Samuelson (1984) notes that the optimal mechanism can be implemented through a single price $p = s_1$, with the seller allowed to decide whether to sell the asset at that price. Given the choice between selling for s_1 or not selling, the seller chooses to sell the asset whenever his signal is smaller than the price, $s \leq p$. The buyer's expected profit conditional on knowing that the seller is willing to sell the asset is therefore

$$\frac{\int_{\underline{s}}^{s_1}(b(v) - s_1)dF(v)}{F(s_1)}.$$

Using the first constraint in problem (1) and the fact that there is no trade when $s > s_1$, this is at least equal to $u/F(s_1)$. This proves the buyer is willing to trade even conditional on the information that the seller is willing to trade.

From now we assume $s_1 < s_2$. In the next step, assume that condition (2) holds and

$$\int_{\underline{s}}^{s_1}(b(v) - (1-q)s_1 - qs_2)dF(v) \geq 0$$

The outcome of the optimal mechanism can be implemented using two prices, $p_1 = (1-q)s_1 + qs_2$ and $p_2 = s_2$. If the seller reports $s \leq s_1$, trade occurs at the low price p_1. If the seller reports $s_1 < s \leq s_2$, trade occurs the high price with probability q and otherwise there is no trade. Now the trading price conveys information to the buyer, but the buyer is still willing to buy at either price, even though he understands that the price conveys information about the seller's signal.

In the third step, we assume that condition (2) holds and

$$\int_{\underline{s}}^{s_1}(b(v) - (1-q)s_1 - qs_2)dF(v) < 0.$$

We still implement the optimal mechanism using two prices. If the seller reports $s \leq s_1$, trade occurs at a low price p_1 with probability $1-\rho$ and at the high price p_2 with probability ρ. If the seller reports $s_1 < s \leq s_2$, trade occurs the high price with probability q and otherwise there is no trade. The high price is still $p_2 = s_2$, so the expected transfer when the seller reports $s_1 < s \leq s_2$ is qs_2. The low price ensures that the buyer breaks even when offered the low price,

$$p_1 = \frac{\int_{\underline{s}}^{s_1} b(v)dF(v)}{F(s_1)}.$$

Note that this implies $p_1 < (1-q)s_1 + qs_2$, since we have assumed that the buyer would lose money if forced to pay $(1-q)s_1 + qs_2$ to a seller with signal $s \leq s_1$. Finally, the trading probability ρ ensures that the expected transfer when the seller reports a signal $s \leq s_1$ is $(1-q)s_1 + qs_2$:

$$\rho = \frac{(1-q)s_1 + qs_2 - p_1}{p_2 - p_1}.$$

This defines $\rho \in (0,1)$. We can verify that the buyer's profit is nonnegative when offered the high price,

$$\rho \int_{\underline{s}}^{s_1} (b(v) - p_2)dF(v) + q \int_{s_1}^{s_2} (b(v) - p_2)dF(v) \geq u,$$

since the buyer earns zero profits in the other circumstances.

Finally, suppose condition (2) is violated. Gerardi, Hörner and Maestri (2014) prove that an ex post mechanism can implement the interim optimal solution if and only if

$$\int_{s}^{\bar{s}} (q(v)b(v) - t(v))dF(v) \geq 0 \text{ for all } s.$$

We break this condition into three regions. First, if $s \geq s_2$, this holds trivially.

Second, if $s_2 > s \geq s_1$, the condition reduces to

$$q \int_s^{s_2} (b(v) - s_2) dF(v) \geq 0.$$

The integrand is equal to zero when $s = s_2$ and it is concave in s since b is nondecreasing; Therefore if the condition holds at $s = s_1$, it holds at all $s \in [s_1, s_2]$. In the third region, $s_1 > s \geq \underline{s}$ and the condition reduces to

$$\int_s^{s_1} (b(v) - (1-q)s_1 - qs_2) dF(v) + q \int_{s_1}^{s_2} (b(v) - s_2) dF(v) \geq 0.$$

Again, the first integrand is equal to zero when $s = s_1$ and it is concave in s; therefore if this condition holds at $s = \underline{s}$, it holds at all $s \in [\underline{s}, s_1]$. But the condition at $s = \underline{s}$ is just the requirement that the buyer earns nonnegative profits, a condition that we know holds. □

Proof of Proposition 3*.* Observe

$$t(\underline{s}) - q(\underline{s})\underline{s} \geq t(s) - q(s)\underline{s} \geq q(s)(s - \underline{s}),$$

where the first inequality uses the incentive compatibility constraint of a seller with signal $\underline{s}$ and the second uses the participation constraint of a seller with signal s. Moreover, $t(\underline{s}) = q(\underline{s})\underline{s}$ whenever $b(\underline{s}) = \underline{s}$: the buyer's and seller's participation constraints at signal $\underline{s}$ jointly imply $q(\underline{s})b(\underline{s}) - t(\underline{s}) \geq 0 \geq q(\underline{s})\underline{s} - t(\underline{s})$, and both inequalities bind when $b(\underline{s}) = \underline{s}$. This proves $q(s)(s - \underline{s}) \leq 0$, so $q(s) = 0$ for all $s \geq \underline{s}$. The buyer's and seller's participation constraints at s then imply $t(s) = 0$. □

Proof of Proposition 4*.* We first prove that the buyer's profit is zero in problem (1) when $u = 0$. First, Myerson (1979) proves that the seller's incentive constraint is equivalent to imposing that $q(s)$ is nonincreasing and

$$t(s) = q(s)s + \int_{s}^{\bar{s}} q(v)dv + t(\bar{s}) - q(\bar{s})\bar{s} \text{ for all } s. \tag{13}$$

Substitute this into problem (1) and use integration-by-parts to rewrite it as

$$\begin{aligned}
&\max_{\{t(\bar{s}),q(s)\}} \int_{\underline{s}}^{\bar{s}} q(v)F(v)dv + t(\bar{s}) - q(\bar{s})\bar{s},\\
&\text{subject to } \int_{\underline{s}}^{\bar{s}} q(v)(b(v)-v)dF(v) - \int_{\underline{s}}^{\bar{s}} q(v)F(v)dv \geq t(\bar{s}) - q(\bar{s})\bar{s},\\
&\qquad t(\bar{s}) - q(\bar{s})\bar{s} \geq 0,\\
&\qquad q(s) \text{ nonincreasing,}
\end{aligned}$$

where we simplify the first constraint using $u = 0$. If the first constraint were slack, we could increase $t(\bar{s})$ and raise the value of the objective function. It follows that the buyer's gain from trade is zero in any solution to problem (1).

Next, substitute the binding constraint into the objective function to rewrite the previous problem as

$$\begin{aligned}
&\max_{\{t(\bar{s}),q(s)\}} \int_{\underline{s}}^{\bar{s}} q(v)(b(v)-v)dF(v),\\
&\text{subject to } \int_{\underline{s}}^{\bar{s}} q(v)(b(v)-v)dF(v) - \int_{\underline{s}}^{\bar{s}} q(v)F(v)dv \geq t(\bar{s}) - q(\bar{s})\bar{s},\\
&\qquad t(\bar{s}) - q(\bar{s})\bar{s} \geq 0,\\
&\qquad q(s) \text{ nonincreasing.}
\end{aligned}$$

If $t(\bar{s}) > q(\bar{s})\bar{s}$, reducing $t(\bar{s})$ relaxes the first constraint. Thus we can impose

$t(\bar{s}) = q(\bar{s})\bar{s}$ without loss of generality. The problem reduces to

$$\begin{aligned}
&\max_{\{q(s)\}} \int_{\underline{s}}^{\bar{s}} q(v)(b(v) - v)dF(v), \\
\text{subject to } &\int_{\underline{s}}^{\bar{s}} q(v)(b(v) - v)dF(v) - \int_{\underline{s}}^{\bar{s}} q(v)F(v)dv \geq 0, \\
&q(s) \text{ nonincreasing.}
\end{aligned}$$

Note, however, that the solution only necessarily has $t(\bar{s}) = q(\bar{s})\bar{s}$ when the first constraint in this problem binds.

If the first constraint is slack, the solution to the problem is $q(s) = 1$ for all s. Using integration by parts on the constraint that this is the first constriant holds when $q(s) = 1$ for all s if and only if $\int_{\underline{s}}^{\bar{s}} b(v)dF(v) \geq \bar{s}$. When $\int_{\underline{s}}^{\bar{s}} b(v)dF(v) < \bar{s}$, $t(\bar{s}) = q(\bar{s})\bar{s}$.

Now consider problem (1) and suppose, in order to find a contradiction, that the addition of constraint (3) does not affect the solution. First assume $\int_{\underline{s}}^{\bar{s}} b(v)dF(v) \geq \bar{s}$, so the solution to problem (1) has $q(s) = 1$. Condition (3) evaluated at $s = \underline{s}$ requires $b(\underline{s}) \geq t(\underline{s})$. The third constraint of problem (1) evaluated at $s = \underline{s}$ and $\hat{s} = \bar{s}$ requires $t(\underline{s}) \geq t(\bar{s})$. And the second constraint evaluated at $s = \bar{s}$ requires $t(\bar{s}) \geq \bar{s}$. This contradicts the assumption that $b(\underline{s}) < \bar{s}$. The addition of constraint (3) strictly reduces the value of problem (1) when $\int_{\underline{s}}^{\bar{s}} b(v)dF(v) \geq \bar{s} > b(\underline{s})$.

Alternatively, assume $\int_{\underline{s}}^{\bar{s}} b(v)dF(v) < \bar{s}$. We have proven that the buyer's gain from trade, $\int_{\underline{s}}^{\bar{s}}(q(v)b(v) - t(v))dF(v)$, is zero and $t(\bar{s}) = q(\bar{s})\bar{s}$ in the interim problem. Suppose we can implement the same value in problem (1) with the additional constraint (3). Since this problem imposes $q(s)b(s) \geq t(s)$ for all s, the constraint must bind for all s. In particular, $t(\bar{s}) = q(\bar{s})b(\bar{s})$. This coincides with the solution to the interim problem if and only if $q(\bar{s}) = 0$.

To rule this out, again note that the seller's incentive constraint implies equation (13). Eliminate $t(s)$ between this equation and $t(s) = q(s)b(s)$ and differentiate to get $dq(s)(b(s)-s)+q(s)db(s)=0$ or

$$q(s)=k\exp\left(-\int_{\underline{s}}^{s}\frac{1}{b(v)-v}db(v)\right),$$

where k is a constant of integration. We require $k\in[0,1]$ in order for $q(\underline{s})$ to be a valid probability. It is easy to verify that setting $k=1$ maximizes (and $k=0$ minimizes) the objective function in problem (1) with constraint (3) within this class of policies. Since $b(v)>v$, $q(\bar{s})>0$, completing the contradiction. □

Proof of Proposition 5*.* A deterministic veto incentive compatible mechanism is a report-conditional measure μ_{bs} over sets of prices. In particular, for any $P\subset\mathbb{R}_+$, let $\mu_{bs}(P)$ denote the probability that the mechanism recommends trading at a price $p\in P$ conditional on the reports (b,s), with $1-\mu_{bs}(\mathbb{R}_+)$ denoting the conditional probability of no trade.

Take any measures $\{\mu_{bs}\}$ that satisfy the ex-post participation and nonnegative constraints:

$$\int_P\sum_s(v^B_{bs}-p)\pi_{bs}d\mu_{bs}(p)\geq 0\text{ for all }b\text{ and }P\subset\mathbb{R}_+,$$
$$\int_P\sum_b(p-v^S_{bs})\pi_{bs}d\mu_{bs}(p)\geq 0\text{ for all }s\text{ and }P\subset\mathbb{R}_+,$$

and $\int_P d\mu_{bs}(p)\geq 0$ for all $P\subset\mathbb{R}_+$. Define

$$\bar{V}^B_{bb'}\equiv\int_{\mathbb{R}_+}\max\left\{\sum_s(v^B_{bs}-p)\pi_{bs}d\mu_{b's}(p),0\right\},$$
$$\bar{V}^S_{ss'}\equiv\int_{\mathbb{R}_+}\max\left\{\sum_b(p-v^S_{bs})\pi_{bs}d\mu_{bs'}(p),0\right\},$$
$$\text{and }Q_{bs}\equiv\int_{\mathbb{R}_+}d\mu_{bs}(p).$$

These are the profit of a buyer who receives signal b and reports b', the profit of a seller who receives signal s and reports s', and the probability of trade when the buyer reports signal b and the seller reports signal s. The outcome

$$x \equiv (\{\bar{V}^B_{bb'}\}, \{\bar{V}^S_{ss'}\}, \{Q_{bs}\}),$$

a vector of dimension $(N^B)^2 + (N^S)^2 + N^B N^S$, contains all the information necessary to evaluate a mechanism's feasibility and compute the welfare for the buyer and seller. In particular, the measures are a feasible veto incentive compatible mechanisms if and only if

$$\bar{V}^B_{bb} \geq \bar{V}^B_{bb'}, \quad \bar{V}^S_{ss} \geq \bar{V}^S_{ss'}, \text{ and } \quad 1 \geq Q_{bs}$$

for all b, b', s, and s'; and the expected payoffs from the mechanism are $V^B = \sum_b \bar{V}^B_{bb}$ and $V^S = \sum_s \bar{V}^S_{ss}$.

Now define the set of prices $\bar{P}$ such that $p \in \bar{P} \Leftrightarrow \{d\mu_{bs}(p)\} \neq 0$. For each $p \in \bar{P}$, define

$$\tilde{x}(p) \equiv (\{\bar{V}^B_{bb'}(p)\}, \{\bar{V}^S_{ss'}(p)\}, \{\gamma_{bs}\}),$$

where

$$\tilde{V}^B_{bb'}(p) \equiv \max\left\{\sum_s (v^B_{bs} - p)\pi_{bs}\gamma_{b's}(p), 0\right\},$$

$$\tilde{V}^S_{ss'}(p) \equiv \max\left\{\sum_b (p - v^S_{bs})\pi_{bs}\gamma_{bs'}(p), 0\right\},$$

$$\text{and } \gamma_{bs}(p) = \frac{d\mu_{bs}(p)}{\sum_{b's'} d\mu_{b's'}(p)}.$$

Note that for each $p \in \bar{P}$, either $\tilde{V}^B_{bb'}(p) = \frac{\sum_s (v^B_{bs} - p)\pi_{bs} d\mu_{b's}(p)}{\sum_{b''}\sum_s d\mu_{b''s}(p)}$ or $\tilde{V}^B_{bb'}(p) = 0$, and likewise for $\tilde{V}^S_{ss'}(p)$. This implies that

$$x = \int_{\bar{P}} \tilde{x}(p) d\mu(p),$$

where $\mu(P) \equiv \sum_{bs} \mu_{bs}(P)$ for all $P \subset \bar{P}$. Thus $\tilde{x}$ is a Radon-Nikodym derivative and the outcome x is obtained by choosing a measure μ over the vectors $\tilde{x}$ indexed by $p \in \bar{P}$. In other words, x is in the convex cone generated by the vectors $\{\tilde{x}(p)\}_{p\in\bar{P}}$. It follows by Carathéodory's Theorem that there are is a finite subset $(p_1, p_2, \ldots, p_N) \subset \bar{P}$, with $N \equiv (N^B)^2 + (N^S)^2 + N^B N^S + 1$, such that

$$x = \sum_{n=1}^{N} \tilde{x}(p_n)\omega_n$$

for $\omega_n \geq 0$ and $\sum_{n=1}^{N} \omega_n = 1$. □

Proof of Proposition 6*.* Take any mechanism $\{p_n, \omega_{n|bs}\}$ that satisfies the constraints of problem (7). We construct a fully revealing mechanism $\{p_{bs}, \omega_{bs}\}$ that satisfies the constraints of problem (9) and achieves the same value of the objective function. We propose setting $\omega_{bs} = \sum_n \omega_{n|bs}$ and

$$p_{bs} = \frac{\sum_n p_n \omega_{n|bs}}{\omega_{bs}}$$

whenever $\omega_{bs} > 0$, with p_{bs} arbitrary when $\omega_{bs} = 0$. With these assumptions and the private values restrictions, the two participation constraints in problem (7) reduce to the two participation constraints in problem (9). Similarly, the two incentive constraints in problem (7) reduce to the two incentive constraints in problem (9). This proves that any payoff that can be obtained in the veto incentive compatible program can also be obtained using a fully revealing mechanism. Conversely, the mechanism $\{p_{bs}, \omega_{bs}\}$ is clearly feasible in problem (7), proving the two constraint sets are equivalent. □

Proof of Proposition 7*.* We divide the parameter space into three regions. In each region, we first verify that the proposed mechanism is feasible and then prove that it achieves the same trading probability as the trade-maximizing interim mechanism.

Non-revealing Region: Assume $\gamma \geq 2\alpha - 1$. Under the proposed mechanism, the participation constraint of a buyer with the low signal $b = 0$ and a seller with the high signal $s = 1$ hold if and only if $\gamma \geq 2\alpha - 1$. The remaining participation constraints in problem (7) always hold, and the incentive constraints hold trivially. This mechanism is therefore feasible. It obtains the first best amount of trade and hence solves problem (7).

Partially-Revealing Region: Next assume

$$2\alpha - 1 > \gamma > \frac{\alpha(\alpha(3 - 2\alpha) - 1)}{(1 - \alpha(1 - \alpha))(1 - 2\alpha(1 - \alpha))}. \tag{14}$$

We first determine the probabilities μ and λ. These are pinned down by two equations. The first is the the binding participation constraint of the buyer who receives and reports the low signal $b = 0$ and is instructed to trade at $\frac{1}{2}(1 + \gamma)$:

$$\frac{\left(v^B_{00} - \frac{1}{2}(1 + \gamma)\right)(1 - \mu)\pi_{00} + \left(v^B_{01} - \frac{1}{2}(1 + \gamma)\right)\lambda\pi_{01}}{\pi_{00} + \pi_{01}} = 0.$$

The buyer believes that with probability $\pi_{00}/(\pi_{00} + \pi_{01})$, seller also received the low signal, in which case they were instructed to trade at the intermediate price with probability $1 - \mu$; and he believes the seller received the high signal with probability $\pi_{11}/(\pi_{00} + \pi_{01})$, in which case they were instructed to trade at the intermediate price with probability λ. Thus the left hand side is the buyer's expected profit conditional on this event.

Because of the symmetry of the mechanism, the participation constraint of the seller who receives the high signal and is instructed to trade at $\frac{1}{2}(1 + \gamma)$ is identical. The remaining participation constraints are slack.

The second equation is the binding incentive constraint of a seller who receives the low signal:

$$\left(v^B_{00}\mu + \tfrac{1}{2}(1+\gamma)(1-\mu) - v^S_{00}\right)\pi_{00} + \left(\tfrac{1}{2}(1+\gamma) - v^S_{10}\right)\pi_{10} = \\ \left(\tfrac{1}{2}(1+\gamma) - v^S_{00}\right)\lambda\pi_{00} + \left(v^S_{11}\mu + \tfrac{1}{2}(1+\gamma)(1-\mu) - v^S_{10}\right)\pi_{10}.$$

The left hand side is the seller's interim expected payoff conditional on receiving and reporting the signal $s = 0$. With probability $\pi_{00}/(\pi_{00} + \pi_{01})$, the buyer also received the low signal, in which case the seller earns v^B_{00} with probability μ and $\frac{1}{2}(1+\gamma)$ with probability $1-\mu$, but gives up v^S_{00}. With probability $\pi_{10}/(\pi_{00} + \pi_{01})$, the buyer received the high signal, in which case trade occurs at the intermediate price and the seller gives up v^S_{10}. The right hand side is the seller's interim expected payoff conditional on receiving the signal $s = 0$ but reporting $s = 1$. This changes the probability distribution over trade and trading prices but it doesn't affect the seller's opportunity cost.

Again, symmetry of the mechanism ensures that the incentive constraint of a buyer who receives the high signal is identical. The remaining two incentive constraints are slack.

These equations are linear in λ and μ and admit a unique solution. Under condition (14), both are valid probabilities, lying strictly between 0 and 1. Thus the proposed mechanism is feasible.

Next solve problem (8) in this region. The incentive constraints of the seller with a high signal and a buyer with a low signal are slack, as are the participation constraint of the other traders, the seller with a low signal and the buyer with a high signal. The remaining constraints bind as well as the constraints on trading probabilities $q_{00} \leq 1$, $q_{11} \leq 1$, and $q_{10} \leq 1$. This gives seven equations in eight unknowns. Generically there are a continuum of mechanisms that implement the trade-maximizing allocation, each with a different distribution of profits between the buyer and seller. Our veto incentive compatible mechanism implements a particular one, where the buyer and seller earn equal profits.

In the symmetric trade-maximizing interim mechanism, $q_{00} = q_{10} = q_{11} = 1$ and $q_{01} < 1$. The value of q_{01} is pinned down from three observations. First, symmetry implies the gains from trade are distributed equally, so $t_{00} + t_{11} = 1 + \gamma$ and $t_{01}/q_{01} = t_{10} = \frac{1}{2}(1 + \gamma)$. Second, the interim participation constraint of the buyer with the low signal binds, as does the symmetric interim participation constraint of the seller with the high signal. Third, the incentive constraint of the buyer with the high signal binds, as does the symmetric incentive constraint of the seller with the low signal. Together these results imply

$$(v^B_{00} - t_{00})\pi_{00} + (v^B_{01}q_{01} - t_{01})\pi_{01} = 0 \text{ and}$$
$$(t_{00} - v^S_{00})\pi_{00} + (t_{10} - v^S_{10})\pi_{10} = (t_{01} - v^S_{00}q_{01})\pi_{00} + (t_{11} - v^S_{10})\pi_{10}.$$

The solution to this pair of equations uniquely defines t_{00} and q_{01}, with $q_{01} \in [0, 1]$ under condition (14). Finally, these equations imply $\lambda = q_{01}$. Therefore the proposed veto incentive compatible mechanism achieves the same trade probability as the interim trade-maximizing mechanism and so is trade-maximizing among all veto incentive compatible mechanisms.

Fully-Revealing Region: Now assume

$$\frac{\alpha(\alpha(3 - 2\alpha) - 1)}{(1 - \alpha(1 - \alpha))(1 - 2\alpha(1 - \alpha))} \geq \gamma > 0. \tag{15}$$

The proposed mechanism is fully revealing and so trivially satisfies the ex post participation constraint. The binding incentive constraint of the seller who receives the low signal pins down the parameter λ:

$$(v^B_{00} - v^S_{00})\lambda\pi_{00} + \left(\tfrac{1}{2}(1 + \gamma) - v^S_{10}\right)\pi_{10} = \left(v^S_{11} - v^S_{10}\right)\lambda\pi_{10}.$$

Under condition (15), this defines $\lambda \in (0, 1]$. This is equivalent to the bidning incentive constraint of the buyer who receives the high signal.

Next turn to the interim problem. Once again, the incentive constraints of the seller with $s = 1$ and the buyer with $b = 0$ are slack, as are the participation constraint of the seller with $s = 0$ and the buyer with $b = 1$. The remaining constraints bind, as well as the constraints $q_{01} \geq 0$ and $q_{10} \leq 1$. Again, there are generically a continuum of mechanisms that implement the trade-maximizing allocation, each with a different distribution of profits between the buyer and seller. Our veto incentive compatible mechanism implements the symmetric one.

In the symmetric trade-maximizing mechanism, $q_{10} = 1$, $q_{01} = 0$, and $q_{00} = q_{11} < 1$. The value of $q_{00} = q_{11}$ is pinned down from three observations. First, symmetry implies $t_{00} + t_{11} = q_{00}(1+\gamma)$, $t_{01} = 0$, and $t_{10} = \frac{1}{2}(1+\gamma)$. Second, the participation constraint of the buyer with the low signal binds, as does the (symmetric) participation constraint of the seller with the high signal. Third, the incentive constraint of the buyer with the high signal binds, as does the (symmetric) incentive constraint of the seller with the low signal. Together these results imply

$$\left(v^B_{00} q_{00} - t_{00}\right)\pi_{00} = 0 \text{ and}$$
$$(t_{00} - v^S_{00} q_{00})\pi_{00} + (t_{10} - v^S_{10})\pi_{10} = (t_{11} - v^S_{10} q_{11})\pi_{10}.$$

The solution to this pair of equations uniquely defines $q_{11} = q_{22} = \lambda$, which again proves that the proposed veto incentive compatible mechanism is trade-maximizing among all such mechanisms. □

***Proof of Proposition 8**.* A seller who receives signal s knows that the value of the asset is at least v^S_{0s} and so the ex post participation constraint ensures that $\omega_{n|bs} = 0$ for all $p_n < v^S_{0s}$. Then either $q_{bs} = 0$, in which case $t_{bs} = 0$, or $q_{bs} > 0$ and $t_{bs}/q_{bs} \geq v^S_{0s}$. This proves $t_{bs} \geq v^S_{1s} q_{bs}$. Similarly, a buyer who receives signal b knows that the value of the asset is no more than v^B_{b1} and so the ex post participation constraint ensures that $\omega_{n|bs} = 0$ for all $p_n > v^B_{b1}$. Then either $q_{bs} = 0$, in which case $t_{bs} = 0$, or $q_{bs} > 0$ and $t_{bs}/q_{bs} \leq v^B_{b1}$. This proves $t_{bs} \leq v^B_{b2} q_{bs}$.

Now suppose $v^S_{11} q_{11} > t_{11}$. The seller's participation constraint when she has the signal $s = 1$ and is instructed to trade at the price p_n is

$$(p_n - v^S_{01})\omega_{n|01}\pi_{01} + (p_n - v^S_{11})\omega_{n|11}\pi_{11} \geq 0.$$

This implies $\omega_{n|01} = \omega_{n|11} = 0$ when $p_n < v^S_{01}$ and $\omega_{n|11} = 0$ when the inequality is weak, $p_n \leq v^S_{01}$. For $p_n \geq v^S_{01}$,

$$(v^B_{01} - p_n)\omega_{n|01} \geq \phi^S(p_n)\omega_{n|11}, \tag{16}$$

where

$$\phi^S(p) \equiv \begin{cases} \frac{\pi_{11}(v^S_{11}-p)(v^B_{01}-p)}{\pi_{01}(p-v^S_{01})} \\ 0 \end{cases} \Leftrightarrow \quad \begin{matrix} v^S_{01} < p \leq \min\{v^S_{11}, v^B_{01}\} \\ p > \min\{v^S_{11}, v^B_{01}\}. \end{matrix}$$

Then

$$\begin{aligned} t_{01} &= \textstyle\sum_{n|v^S_{01} \leq p_n \leq v^B_{01}} p_n \omega_{n|01} \\ &= v^B_{01} q_{01} - \textstyle\sum_{n|v^S_{01} \leq p_n \leq v^B_{01}} (v^B_{01} - p_n)\omega_{n|01} \\ &\leq v^B_{01} q_{01} - \textstyle\sum_{n|v^S_{01} < p_n \leq v^B_{01}} \phi^S(p_n)\omega_{n|11} \\ &= v^B_{01} q_{01} - \textstyle\sum_{n|v^S_{01} < p_n \leq v^B_{11}} \phi^S(p_n)\omega_{n|11} \\ &\leq v^B_{01} q_{01} - \phi^S\left(\frac{\sum_{n|v^S_{01} < p_n \leq v^B_{11}} p_n \omega_{n|11}}{\sum_{n|v^S_{01} < p_n \leq v^B_{11}} \omega_{n|11}}\right) \textstyle\sum_{n|v^S_{01} < p_n \leq v^B_{11}} \omega_{n|11} \\ &= v^B_{01} q_{01} - \phi^S(t_{11}/q_{11}) q_{11} \end{aligned}$$

The first equation is the definition of t_{01}, using the fact that a seller with signal $s = 1$ would never accept a price below v^S_{01} and a buyer with signal $b = 0$ would never pay a price above v^B_{01}. The second equation rewrites the sum in a more convenient form. The first inequality uses condition (16) and $\omega_{n|11} = 0$ if $p_n = v^S_{01}$. The third equality uses the definition of ϕ^S, together with the fact that $\omega_{n|01} = \omega_{n|11} = 0$ for $p_n \in (v^B_{01}, v^S_{11}]$ if $v^B_{01} < v^S_{11}$. The second inequality uses Jensen's inequality, since ϕ^S is convex. The final equality uses the definitions of t_{11} and q_{11}. Now using the definition of ϕ^S, we obtain condition (11).

Similarly, suppose $t_{00} > v^B_{00} q_{00}$. The buyer's participation constraint when he has signal $b = 0$ and is instructed to trade at the price p_n is:

$$(v^B_{00} - p_n)\omega_{n|00}\pi_{00} + (v^B_{01} - p_n)\omega_{n|01}\pi_{01} \geq 0.$$

This implies $\omega_{n|00} = 0$ for all $p_n > v^B_{01}$, $\omega_{n|01} = 0$ for all $p_n \geq v^B_{01}$, while for $p_n \leq v^B_{01}$,

$$(p_n - v^S_{01})\omega_{n|01} \geq \phi^B(p_n)\omega_{n|11}, \tag{17}$$

where

$$\phi^B(p) \equiv \begin{cases} \frac{\pi_{00}(p-v^B_{00})(p-v^S_{01})}{\pi_{01}(v^B_{01}-p)} & \Leftrightarrow \quad \max\{v^B_{00}, v^S_{01}\} \leq p < v^B_{01} \\ 0 & \quad\; p < \max\{v^B_{00}, v^S_{01}\}. \end{cases}$$

Then

$$\begin{aligned} t_{01} &= \textstyle\sum_{n|v^S_{01} \leq p_n \leq v^B_{01}} p_n \omega_{n|01} \\ &= v^S_{01} q_{01} + \textstyle\sum_{n|v^S_{01} \leq p_n \leq v^B_{01}} (p_n - v^S_{01})\omega_{n|01} \\ &\geq v^S_{01} q_{01} + \textstyle\sum_{n|v^S_{01} \leq p_n < v^B_{01}} \phi^B(p_n)\omega_{n|00} \\ &= v^S_{01} q_{01} + \textstyle\sum_{n|v^S_{00} \leq p_n < v^B_{01}} \phi^B(p_n)\omega_{n|00} \\ &\geq v^S_{01} q_{01} + \phi^B \left(\frac{\sum_{n|v^S_{00} \leq p_n < v^B_{01}} p_n \omega_{n|00}}{\sum_{n|v^S_{00} \leq p_n < v^B_{01}} \omega_{n|00}} \right) \textstyle\sum_{n|v^S_{00} \leq p_n < v^B_{01}} \omega_{n|00} \\ &= v^S_{01} q_{01} + \phi_B(t_{00}/q_{00}) q_{00} \end{aligned}$$

The steps exactly parallel the previous step, with the inequalities exploiting conditions (17) and convexity of ϕ^B. Using the definition of ϕ^B, we obtain condition (12). □