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ABSTRACT

Governments are present-biased toward spending. Fiscal rules are deficit limits that trade off commitment to not overspend and flexibility to react to shocks. We compare coordinated rules – chosen jointly by a group of countries – to uncoordinated rules. If governments' present bias is small, coordinated rules are tighter than uncoordinated rules: individual countries do not internalize the redistributive effect of interest rates. However, if the bias is large, coordinated rules are slacker: countries do not internalize the disciplining effect of interest rates. Surplus limits enhance welfare, and increased savings by some countries or outside economies can hurt the rest.

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The integration of capital markets in Europe following the adoption of the Euro in 1999 led to a convergence of interest rates and a rise of imbalances across the region. Tight fiscal policy in Northern European countries such as Germany contributed to a region-wide decline in interest rates, which induced increased borrowing by other European countries such as Greece.[1] Between 2003 and 2008, for example, Germany's government deficit declined from 4.2% to 0.2% of GDP, whereas Greece's government deficit increased from 7.8% to 10.2% of GDP.[2]

How should countries coordinate their fiscal policies in an integrated capital market? Is there a benefit to the joint design of fiscal rules? Over the past 30 years, more than 90 countries have adopted fiscal rules, including rules applying to individual countries and rules applying to groups of countries. In 2013, of the 97 countries that had fiscal rules in place, 49 were subject to national rules, 48 to supranational rules, and 14 to both.[3] For example, Germany was constrained not only by the guidelines of the European Union's Stability and Growth Pact (SGP), but also by its own constitutionally mandated "debt brake" which imposed a tighter limit on the government's structural deficit than the SGP.[4]

In this paper, we study the optimal design of *coordinated* fiscal rules, which are chosen jointly by a group of countries, and compare them to *uncoordinated* fiscal rules, which are chosen independently by each country. Are coordinated rules tighter or more lax than uncoordinated rules? What happens if some countries — like Germany in the case of the European Union — can supplement coordinated rules with additional fiscal constraints?

Our theory of fiscal rules is motivated by a fundamental tradeoff between

[1]The impact of Germany's fiscal reforms on its declining government deficit is documented in Breuer (2015), and the impact of the government deficit on the current account is discussed in Kollmann et al. (2015) for Germany and Abbas et al. (2011) more broadly. Draghi (2016) addresses the depressing effect of Germany's current account on interest rates.

[2]See `https://data.oecd.org/gga/general-government-deficit.htm`. Fernández-Villaverde, Garicano and Santos (2013) argue that the drop in interest rates that followed European integration led to the abandonment of reforms and institutional deterioration in the peripheral European countries.

[3]See IMF Fiscal Rules Data Set, 2013 and Budina et al. (2012). The treaties that encompass the supranational rules correspond to the European Union's Stability and Growth Pact, the West African Economic and Monetary Union, the Central African Economic and Monetary Community, and the Eastern Caribbean Currency Union.

[4]See Truger and Will (2012). Other countries with both national and supranational rules in 2013 were Austria, Bulgaria, Croatia, Denmark, Estonia, Finland, Lithuania, Luxembourg, Poland, Slovak Republic, Spain, Sweden, and the United Kingdom.

commitment and flexibility: on the one hand, rules provide valuable commitment as they can limit distorted incentives in policymaking that result in a spending bias and excessive deficits; on the other hand, there is a cost of reduced flexibility as fiscal constitutions cannot spell out policy prescriptions for every single shock or contingency, and some discretion may be optimal. Under uncoordinated fiscal rules, each country resolves this commitment-versus-flexibility tradeoff independently. Under a coordinated fiscal rule, countries resolve this tradeoff jointly.

We consider a two-period model in which a continuum of identical governments choose deficit-financed public spending. At the beginning of the first period, each government receives an idiosyncratic shock to the social value of spending in this period. Governments are benevolent ex ante, prior to the realization of the shock, but present-biased ex post, when it is time to choose spending. This preference structure results naturally from the aggregation of heterogeneous, time-consistent citizens' preferences (Jackson and Yariv, 2015, 2014), or as a consequence of turnover in political economy models (e.g., Aguiar and Amador, 2011).[5] We assume that the shock to the value of spending is a government's private information, or *type*, capturing the fact that not all contingencies are contractible or observable. The combination of a present bias and private information implies that governments face a tradeoff between commitment and flexibility. We define a fiscal rule in this context as a fully enforceable deficit limit, imposed prior to the realization of the shock.

Our environment is the same as that considered in Amador, Werning and Angeletos (2006) and Halac and Yared (2014). These papers characterize optimal uncoordinated fiscal rules, which are chosen independently by each government taking global interest rates as given. We depart by studying coordinated fiscal rules, which are chosen by a central authority representing all governments, taking into account the impact that fiscal rules have on global interest rates. Coordinated rules internalize the fact that lowering flexibility affects countries not only directly by limiting their borrowing and spending, but also indirectly by reducing interest rates.[6]

[5]See also Persson and Svensson (1989), Alesina and Tabellini (1990), Alesina and Perotti (1994), Lizzeri (1999), Tornell and Lane (1999), Battaglini and Coate (2008), and Caballero and Yared (2010). Our formulation of governments' preferences corresponds to the quasi-hyperbolic consumption model; see Laibson (1997).

[6]In our model, a government's debt exerts an externality on other governments solely through

An optimal uncoordinated fiscal rule is a deficit limit such that, on average, the distortion above the limit is zero. Specifically, consider a government that, ex post, would like to borrow more than allowed by the imposed limit. If the government experienced a relatively low shock to the value of spending, it will be overborrowing compared to its ex-ante optimum, as the government is present-biased ex post. On the other hand, if the government experienced a relatively high shock, it will be underborrowing, as the government is constrained by the deficit limit. For a fixed interest rate, an optimal deficit limit equalizes the marginal benefit of providing more flexibility to underborrowing types to the marginal cost of providing more discretion to overborrowing types.

Our results contrast these uncoordinated rules with coordinated rules. Consider first a situation in which governments are not present-biased, so that an optimal uncoordinated fiscal rule grants full flexibility. The optimal coordinated fiscal rule in this case is tighter than the uncoordinated one, and hence interest rates are lower under coordination. The reason is intuitive: governments choosing rules independently do not internalize the fact that by allowing themselves more flexibility, they increase interest rates, thus redistributing resources away from governments that borrow more toward governments that borrow less. Committing ex ante to tighter constraints is socially beneficial: the cost of reducing flexibility is mitigated by the drop in the interest rate, which benefits more indebted countries whose marginal value of borrowing is higher. This redistributive effect of the interest rate is also present in other models with incomplete markets, such as Yared (2013) and Azzimonti, de Francisco and Quadrini (2014).

Our main result, however, shows that when governments' present bias is sufficiently large, the optimal coordinated fiscal rule is slacker than the uncoordinated one, and hence interest rates are higher under coordination. The reason is that interest rates also have a natural *disciplining effect*. Governments choosing rules independently do not internalize the fact that by reducing their own discretion, they lower interest rates, thus increasing governments' desire to borrow

the interest rate. Coordinated rules may differ from uncoordinated rules for reasons different from those studied here if higher debt by some governments entails other externalities, such as a higher risk of crisis and contagion, inflation, or future fiscal transfers. Beetsma and Uhlig (1999) and Chari and Kehoe (2007) study settings in which the existence of a common monetary policy generates an externality.

and worsening fiscal discipline for all. Committing ex ante to more flexibility is socially beneficial: the cost of increasing discretion for overborrowing countries is mitigated by the rising interest rate, which induces everyone to borrow less. Paradoxically, in some cases, the externality is large enough that all countries can be made ex ante better off by abandoning their uncoordinated fiscal rules and allowing themselves full flexibility. More generally, we show that whether the optimal coordinated fiscal rule grants more or less flexibility than the uncoordinated one depends on the relative strength of the redistributive and disciplining effects of the interest rate, which in turn depends on governments' present bias.

We discuss a number of extensions of our baseline model. A natural question, in light of our main result, is whether additional instruments can enhance welfare when governments' present bias is large and thus the disciplining effect of the interest rate dominates the redistributive effect. While linear taxes have no effect in our setting, we show that using a coordinated fiscal rule where governments are not allowed to exceed a maximum surplus limit (in addition to a maximum deficit limit) can improve upon using only deficit limits. Surplus limits are never used in an optimal uncoordinated fiscal rule, as these limits force low government types which are overborrowing to borrow even more. However, surplus limits also serve to increase interest rates, and through this channel they can improve overall fiscal discipline.

This logic is in fact general, and has further implications when governments' present bias is large. For instance, suppose a small subset of countries are able to supplement coordinated rules with additional fiscal constraints. By increasing their savings, this subset of countries will reduce the global interest rate and worsen fiscal discipline in the remaining countries — a result that resonates with the experiences of Germany and Greece mentioned above. Moreover, we show that the optimal response of the central authority is to tighten the fiscal rule in these remaining countries whose welfare declines. Similarly, consider an extension of our model in which countries can borrow from an outside economy. We find that an external supply of funds can also lower welfare by depressing interest rates and increasing countries' overborrowing, and lead to the tightening of fiscal constraints. These effects arise when the optimal coordinated fiscal rule is slacker than the uncoordinated one under a large present bias, a result that we show to

be robust to ex-ante heterogeneity across countries and an infinite time horizon.

This paper is related to several literatures. First, the paper fits into the mechanism design literature that studies the tradeoff between commitment and flexibility in self-control settings, including Athey, Atkeson and Kehoe (2005), Amador, Werning and Angeletos (2006), and Halac and Yared (2014).[7,8] Unlike this literature, we endogenize the effective price of the temptation good — which in our environment corresponds to the interest rate — and we show how this price can serve as a natural disciplining device, affecting the optimal mechanism for a group of agents. Our analysis and results can be applied to different self-control problems; see Section 5 for a discussion. Second, the paper is related to an extensive literature on the political economy of fiscal policy.[9] Most closely related is Azzimonti, Battaglini and Coate (2016), which considers the quantitative welfare implications of a balanced budget rule when the government is present-biased. In contrast to this work, we study the design of fiscal rules in a global economy in which individual rules affect global interest rates. In this regard, our paper is related to the literature on policy coordination across countries, including Chari and Kehoe (1990) and Persson and Tabellini (1995).[10] Whereas these papers emphasize the benefits of coordinating policies, our interest is in the coordination of rules, namely how countries can benefit from choosing the set of allowable policies jointly. Finally, more broadly, our paper contributes to the literature on hyperbolic discounting and the benefits of commitment devices.[11]

[7] These papers solve for the optimal mechanism, whereas for most of our analysis we restrict attention to rules that take the form of deficit limits (exploring variations in Section 4.1). Deficit limits can be shown to correspond to the optimal uncoordinated mechanism under weak conditions. Characterizing the optimal coordinated mechanism, however, is difficult because the problem is not convex.

[8] See also Sleet (2004), Ambrus and Egorov (2013), and Bond and Sigurdsson (2015), as well as Bernheim, Ray and Yeltekin (2015) which considers the self-enforcement of commitment contracts. More generally, the paper relates to the literature on delegation in principal-agent settings, including Holmström (1977, 1984), Alonso and Matouschek (2008), Amador and Bagwell (2013), and Ambrus and Egorov (2015).

[9] In addition to the work previously cited, see Krusell and Rios-Rull (1999), Acemoglu, Golosov and Tsyvinski (2008), Yared (2010), Azzimonti (2011), and Song, Storesletten and Zilibotti (2012).

[10] See also the discussion in fn. 6 and, among others, Rogoff (1985), Alesina and Barro (2002), Cooley and Quadrini (2003), Cooper and Kempf (2004), Aguiar et al. (2015), and Chari, Dovis and Kehoe (2016).

[11] See, for example, Phelps and Pollak (1968), Laibson (1997), Barro (1999), Krusell and Smith, Jr. (2003), Krusell, Kruscu and Smith, Jr. (2010), Lizzeri and Yariv (2016), and Bisin,

1 Model

1.1 Setup

We study a simple model of fiscal policy in which a continuum of governments each make a spending and borrowing decision. Our setup is the same as that analyzed in Amador, Werning and Angeletos (2006), with the exception that we allow for multiple governments and an endogenous interest rate.

There are two periods and a unit mass of ex-ante identical governments.[12] At the beginning of the first period, each government observes a shock to its economy, $\theta > 0$, which is the government's private information or *type*. θ is drawn from a bounded set $\Theta \equiv \left[\underline{\theta}, \overline{\theta}\right]$ with a continuously differentiable distribution function $F(\theta)$, normalized so that $\mathbb{E}[\theta] = 1$.

Following the realization of θ, each government chooses first-period public spending g and second-period assets x subject to a budget constraint:

$$g + \frac{x}{R} = \tau, \tag{1}$$

where τ is the revenue of the government in the initial period and R is the endogenously determined gross interest rate.

The government's welfare *prior* to the realization of its type θ is

$$\mathbb{E}\left[\theta U(g) + W(x)\right], \tag{2}$$

where $U'(\cdot) > 0$, $U''(\cdot) < 0$, $W'(\cdot) > 0$, and $W''(\cdot) < 0$. $U(g)$ represents the government's utility from first-period spending g and $W(x)$ is the government's continuation value associated with carrying forward assets x.[13] Note that a higher value of θ corresponds to a higher marginal benefit of first-period spending. As in Amador, Werning and Angeletos (2006), we take θ to be a taste shock multiplying first-period utility. This is a tractable way to introduce a value for flexibility, as

Lizzeri and Yariv (2015).

[12] We purposely abstract away from heterogeneity in order to study differences between coordinated and uncoordinated fiscal rules that are not due to countries having different characteristics. We show the robustness of our results to ex-ante heterogeneity in Section 4.4.

[13] Here $W(\cdot)$ is simply taken to be the second-period utility of assets, including any discount factor. In Section 4.5, we provide a microfoundation for $W(\cdot)$ in an infinite horizon economy.

we explain subsequently. Flexibility is also valuable if instead the shock is to the government's revenue τ. Indeed, such a revenue shock yields the same welfare representation as in (2) if $U(\cdot)$ is exponential (see Section 5.4 of Amador, Werning and Angeletos, 2006).

The government's welfare *after* the realization of its type θ, when choosing spending g and assets x, is

$$\theta U(g) + \beta W(x), \tag{3}$$

where $\beta \in (0, 1]$.

Because the world consists of a continuum of governments which can only borrow and lend from one another, total spending in the aggregate must equal the value of total resources available. Let $g(\theta, R)$ be the level of first-period spending chosen by a government of type θ when the interest rate is R. Note that since governments are ex-ante identical, the distribution of realized types across governments is the same as the distribution of types for each government. Thus, given that the density function is $f(\theta)$ and each government has resources τ, the global resource constraint in the first period is

$$\int_{\underline{\theta}}^{\overline{\theta}} g(\theta, R) f(\theta) d\theta = \tau. \tag{4}$$

The interest rate R must adjust so that governments' spending decisions satisfy (4). Equations (1) and (4) imply that the global resource constraint is satisfied in the second period, in the sense that assets held globally equal zero. That is, letting $x(\theta, R)$ be the level of assets chosen by a government of type θ when the interest rate is R, the second-period resource constraint holds: $\int_{\underline{\theta}}^{\overline{\theta}} x(\theta, R) f(\theta) d\theta = 0$.

We note that our setting does not allow for cross-subsidization across types. Specifically, the net present value of spending and assets cannot be different for a lower type relative to a higher type,[14] and hence fiscal transfers across countries are ruled out. Also, to simplify the exposition and without loss of generality, we have abstracted away from borrowing and lending of the household sector.[15]

[14] This is in contrast to other models such as Thomas and Worrall (1990) and Atkeson and Lucas (1992).

[15] Our model is identical to one in which households in an economy do not have access to external financial markets, and the government can borrow and lend on their behalf. The model can be extended to introduce a subset of households that can access external financial

1.2 Fiscal Rules

There are two frictions in our setting. First, if $\beta < 1$, a government's objective (3) following the realization of its type does not coincide with its objective (2) prior to this realization. In particular, the government is present-biased: its welfare after θ is realized overweighs the importance of current spending compared to its welfare before θ is realized. As mentioned in the Introduction, this structure arises naturally when the government's preferences aggregate heterogeneous citizens' preferences, even if the latter are time consistent (see Jackson and Yariv, 2015, 2014). This formulation can also be motivated by political turnover; for instance, preferences such as these emerge in settings with political uncertainty where policymakers place a higher value on public spending when they hold power and can make spending decisions (see Aguiar and Amador, 2011).

The second friction in our setting is that the realization of θ — which affects the marginal social utility of first-period spending — is privately observed by the government. One possible interpretation is that θ is not verifiable ex post by a rule-making body; therefore, even if it is observable, fiscal rules cannot explicitly depend on the value of θ. An alternative interpretation is that the exact cost of public goods is only observable to the policymaker, who may be inclined to overspend on these goods.[16]

The combination of these two frictions leads to a tradeoff between commitment and flexibility. Specifically, note that ex ante, as a function of its type θ and the interest rate R, each government would like to choose first-period spending $g^{ea}(\theta, R)$ and second-period assets $x^{ea}(\theta, R)$ satisfying

$$\theta U'\left(g^{ea}(\theta, R)\right) = RW'\left(x^{ea}(\theta, R)\right) \tag{5}$$

under the budget constraint (1). However, this ex ante optimum cannot be implemented with full flexibility: if the government were given full flexibility to choose spending and borrowing, ex post it would choose $g^f(\theta, R)$ and $x^f(\theta, R)$ satisfying

$$\theta U'\left(g^f(\theta, R)\right) = \beta RW'\left(x^f(\theta, R)\right), \tag{6}$$

markets without affecting our main results. Details are available from the authors upon request.

[16] A third possibility is that citizens have heterogeneous preferences or information on the optimal level of public spending, and only the government sees the aggregate. See Sleet (2004).

and hence a present-biased government would overborrow relative to (5). In addition, the ex ante optimum cannot be achieved with full commitment: a spending plan cannot be made explicitly contingent on the realization of the government's type θ, and hence (5) cannot be implemented by fully committing the government to a contingent plan. Therefore, a tradeoff between commitment and flexibility arises, and the optimal mechanism is then not trivial.

We define a fiscal rule as a cutoff $\theta^* \in [0, \overline{\theta}]$ such that if the government's type is $\theta > \theta^*$, its first-period spending and second-period assets are $g^f(\theta^*, R)$ and $x^f(\theta^*, R)$ respectively, whereas if the government's type is $\theta \leq \theta^*$, spending and assets are $g^f(\theta, R)$ and $x^f(\theta, R)$ (where $g^f(\cdot)$ and $x^f(\cdot)$ are given by (1) and (6)). This fiscal rule can be implemented using a maximum deficit limit, spending limit, or debt limit. Under such an implementation, all types $\theta \leq \theta^*$ can make their full-flexibility ex-post optimal choices within the limit, whereas types $\theta > \theta^*$ are constrained and thus choose spending at the limit. Deficit limits capture aspects of many of the fiscal rules observed in practice. Moreover, under weak conditions on the distribution function $F(\theta)$, deficit limits correspond to the optimal mechanism when the interest rate is exogenous (see Amador, Werning and Angeletos, 2006).

Our interest is in comparing the case in which the fiscal rule θ^* is uncoordinated — chosen independently by each government — and the case in which this rule is coordinated — chosen by a central authority representing all governments. Whereas each government takes the interest rate R as given when choosing its optimal uncoordinated rule, the central authority takes into account the impact of θ^* on the interest rate R when choosing the optimal coordinated rule.

Throughout our analysis, we assume non-increasing absolute risk aversion:

Assumption 1. *$-U''(g)/U'(g)$ and $-W''(x)/W'(x)$ are non-increasing in g and x respectively.*

Let $R(\theta^*)$ denote the level of the interest rate when fiscal rule θ^* applies to all governments. The next lemma follows from Assumption 1.

Lemma 1. *$R(\theta^*)$ is strictly increasing in θ^* for all $\theta^* < \overline{\theta}$.*

Lemma 1 describes how the tightness of fiscal rules impacts the level of global interest rates. The higher is the value of the cutoff θ^*, the more flexible is the

fiscal rule, so the higher is the level of borrowing and, as a result, the higher is the interest rate. This relationship between the fiscal rule and the interest rate plays a central role in our analysis of coordinated versus uncoordinated rules.

Regarding implementation, it is worth noting that when the interest rate is endogenously determined, the mapping from θ^* to a spending or borrowing limit need not be monotonic. To see why, consider a fiscal rule θ^*, associated with a maximum allowable level of public spending $g^f\left(\theta^*, R\left(\theta^*\right)\right)$. Holding the interest rate fixed, the direct effect of an increase in θ^* is to increase $g^f\left(\theta^*, R\left(\theta^*\right)\right)$. But there is also an indirect effect: when θ^* increases, $R\left(\theta^*\right)$ increases, and depending on the relative strength of income and substitution effects, $g^f\left(\theta^*, R\left(\theta^*\right)\right)$ can decrease. For some cases — like the case of log preferences that we study in some of our extensions — one can ensure that the direct effect outweighs the indirect effect, so that $g^f\left(\theta^*, R\left(\theta^*\right)\right)$ is monotonically increasing in θ^*.

2 Uncoordinated Fiscal Rules

We begin by analyzing uncoordinated fiscal rules. Each government independently chooses a fiscal rule to maximize its expected welfare, subject to the budget constraint and taking the interest rate as given:

$$\max_{\theta^* \in [0,\overline{\theta}]} \left\{ \begin{array}{l} \int_{\underline{\theta}}^{\theta^*} \left(\theta U(g^f(\theta, R)) + W(x^f(\theta, R))\right) f(\theta) d\theta \\ + \int_{\theta^*}^{\overline{\theta}} \left(\theta U(g^f(\theta^*, R)) + W(x^f(\theta^*, R))\right) f(\theta) d\theta \end{array} \right\} \tag{7}$$

$$\text{subject to (1) and (6).}$$

This program takes into account that, given a fiscal rule θ^*, all types $\theta \leq \theta^*$ exert full discretion and thus choose spending $g^f(\theta, R)$ and assets $x^f(\theta, R)$ (defined by (1) and (6)), whereas all types $\theta > \theta^*$ have no discretion and thus choose $g^f(\theta^*, R)$ and $x^f(\theta^*, R)$.

Note that program (7) allows for any positive cutoff $\theta^* \leq \overline{\theta}$, and given this, one can show that the solution is a cutoff $\theta_u^* > 0$ that satisfies the first-order

condition with equality. This condition yields

$$\int_{\theta_u^*}^{\overline{\theta}} \left(\theta U'\left(g^f\left(\theta_u^*, R\right)\right) - RW'\left(x^f\left(\theta_u^*, R\right)\right)\right) f\left(\theta\right) d\theta = 0, \tag{8}$$

where $f(\theta) = 0$ for $\theta < \underline{\theta}$. Equation (8) shows that the optimal uncoordinated fiscal rule sets a cutoff θ_u^* such that the average distortion above this cutoff is zero. Specifically, given the cutoff, there exists $\widehat{\theta} > \theta_u^*$ such that if the government's type is $\theta \in [\theta_u^*, \widehat{\theta})$, then

$$\theta U'\left(g^f\left(\theta_u^*, R\right)\right) < RW'\left(x^f\left(\theta_u^*, R\right)\right),$$

and hence the government overborrows relative to its ex-ante optimum (defined in (5)). If instead the government's type is $\theta \in (\widehat{\theta}, \overline{\theta}]$, then

$$\theta U'\left(g^f\left(\theta_u^*, R\right)\right) > RW'\left(x^f\left(\theta_u^*, R\right)\right),$$

and hence the government underborrows relative to its ex-ante optimum. The optimal uncoordinated rule specifies θ_u^* so that the marginal benefit of providing more flexibility to types $\theta > \widehat{\theta}$ which are underborrowing is equal to the marginal cost of providing more discretion to types $\theta < \widehat{\theta}$ which are overborrowing.

By substituting (6) into (8), we obtain the following result.

Proposition 1. *For any given interest rate R, the optimal uncoordinated fiscal rule specifies a cutoff θ_u^* satisfying*

$$\frac{\mathbb{E}\left[\theta | \theta \geq \theta_u^*\right]}{\theta_u^*} = \frac{1}{\beta}. \tag{9}$$

Equation (9) shows that the optimal uncoordinated fiscal rule is independent of the form of the utility functions and the level of the interest rate. If $\beta = 1$, (9) implies $\theta_u^* = \overline{\theta}$, so the optimal uncoordinated rule entails full flexibility. Intuitively, in the absence of a present bias, there is no benefit to the government from constraining its borrowing and spending. At the other extreme, if $\beta \leq \underline{\theta}$, (9) implies $\theta_u^* \leq \underline{\theta}$, so the government grants itself minimal discretion. That is, all types are bunched at the same spending level, which corresponds to what

would be the government's flexible spending level if its type were θ_u^*. Finally, if $\beta \in (\underline{\theta}, 1)$, (9) implies that the optimal uncoordinated rule is bounded discretion with a cutoff $\theta_u^* \in (\underline{\theta}, \overline{\theta})$.

Under mild restrictions on the distribution function $F(\theta)$, Proposition 1 yields that the level of discretion in the optimal uncoordinated fiscal rule is monotonically decreasing in the government's present bias:

Corollary 1. *If $F(\theta)$ satisfies*

$$\frac{d \log \mathbb{E}\left[\theta | \theta \geq \theta^*\right]}{d \log \theta^*} < 1 \text{ for all } \theta^* \leq \overline{\theta}, \tag{10}$$

then θ_u^* is strictly increasing in β.

Condition (10) is satisfied by all log-concave densities, which includes several familiar distributions (Bagnoli and Bergstrom, 2005).

Proposition 1 characterizes the fiscal rule θ_u^* that each government chooses when taking the interest rate as given. At the same time, note that this rule effectively determines the level of the interest rate: as described in Section 1, R must adjust so that the global resource constraint (4) is satisfied. If $\beta = 1$ and thus $\theta_u^* = \overline{\theta}$, the interest rate is such that $\int_{\underline{\theta}}^{\overline{\theta}} g^f(\theta, R) f(\theta) d\theta = \tau$. At the other extreme, if $\beta \leq \underline{\theta}$ and thus $\theta_u^* \leq \underline{\theta}$, each government runs a balanced budget, so the first-order condition (8) implies an interest rate $R = U'(\tau)/W'(0)$.[17] Finally, if $\beta \in (\underline{\theta}, 1)$ and thus $\theta_u^* \in (\underline{\theta}, \overline{\theta})$, the interest rate is pinned down by

$$\int_{\underline{\theta}}^{\theta_u^*} g^f\left(\theta, R\left(\theta_u^*\right)\right) f(\theta) d\theta + \int_{\theta_u^*}^{\overline{\theta}} g^f\left(\theta_u^*, R\left(\theta_u^*\right)\right) f(\theta) d\theta = \tau,$$

where recall θ_u^* is independent of R.[18] Lemma 1 implies that if governments are present-biased (i.e. $\beta < 1$), then the interest rate that is induced by the uncoordinated fiscal rules θ_u^* is lower than the one that would prevail were all governments granted full flexibility.

[17] If $\theta_u^* \leq \underline{\theta}$, all types' first-period spending is $g^f(\theta_u^*, R)$, and so by (4) we have $g^f(\theta_u^*, R) = \tau$.

[18] As for implementation, on the other hand, note that θ_u^* is associated with a maximum allowable spending limit $g^f\left(\theta_u^*, R\left(\theta_u^*\right)\right)$ which does depend on R.

3 Coordinated Fiscal Rules

We now proceed to the main part of our analysis, which considers the optimal coordinated fiscal rule. This rule is chosen by a central authority that represents all governments and takes into account the impact that rules have on the interest rate, as characterized in Lemma 1.

3.1 Solving for the Optimal Coordinated Fiscal Rule

An optimal coordinated fiscal rule maximizes total expected welfare subject to each government's budget constraint and the global resource constraint:

$$\max_{\theta^* \in [0,\overline{\theta}]} \left\{ \begin{array}{c} \int_{\underline{\theta}}^{\theta^*} \left(\theta U(g^f(\theta, R(\theta^*))) + W(x^f(\theta, R(\theta^*))) \right) f(\theta) d\theta \\ + \int_{\theta^*}^{\overline{\theta}} \left(\theta U(g^f(\theta^*, R(\theta^*))) + W(x^f(\theta^*, R(\theta^*))) \right) f(\theta) d\theta \end{array} \right\} \tag{11}$$

$$\text{subject to (1), (4), and (6).}$$

This program is identical to program (7) which solves for the optimal uncoordinated fiscal rule, with the exception that (11) takes into account that the interest rate is a function of the cutoff θ^*. That is, given $g^f(\theta, R(\theta^*))$ and $x^f(\theta, R(\theta^*))$ defined by (1) and (6), the interest rate $R(\theta^*)$ is defined by the global resource constraint (4), and is characterized in Lemma 1.

The first-order condition of the coordinated program yields:

Lemma 2. *The optimal coordinated fiscal rule specifies a cutoff θ_c^*, with associated interest rate $R = R(\theta_c^*)$, which whenever $\theta_c^* < \overline{\theta}$ satisfies*

$$\frac{\mathbb{E}[\theta | \theta \geq \theta_c^*]}{\theta_c^*} = \frac{1}{\beta} + \frac{R'(\theta_c^*)}{(1 - F(\theta_c^*)) \theta_c^* U'(g^f(\theta_c^*, R)) \frac{\partial g^f(\theta_c^*, R)}{\partial \theta_c^*}} (\rho + \lambda), \tag{12}$$

where

$$\rho = -\frac{1}{R} \left[\begin{array}{c} \int_{\underline{\theta}}^{\theta_c^*} W'\left(x^f(\theta, R)\right) x^f(\theta, R) f(\theta) d\theta \\ + \int_{\theta_c^*}^{\overline{\theta}} W'\left(x^f(\theta_c^*, R)\right) x^f(\theta_c^*, R) f(\theta) d\theta \end{array} \right] \geq 0 \tag{13}$$

and

$$\lambda = \left[\begin{array}{c} \int_{\underline{\theta}}^{\theta_c^*} \left(RW' \left(x^f \left(\theta, R \right) \right) - \theta U' \left(g^f \left(\theta, R \right) \right) \right) \frac{dg^f(\theta,R)}{dR} f\left(\theta \right) d\theta \\ + \int_{\theta_c^*}^{\bar{\theta}} \left(RW' \left(x^f \left(\theta_c^*, R \right) \right) - \theta U' \left(g^f \left(\theta_c^*, R \right) \right) \right) \frac{dg^f(\theta_c^*,R)}{dR} f\left(\theta \right) d\theta \end{array} \right] \gtreqless 0. \quad (14)$$

Comparing Lemma 2 and Proposition 1 shows how the optimal coordinated fiscal rule θ_c^* differs from the optimal uncoordinated fiscal rule θ_u^*. The difference is that the second term in (12) does not appear in expression (9). This term is associated with two factors, ρ and λ, which capture the effects that the interest rate has on the allocation. As we explain subsequently, ρ captures the *redistributive effect* of the interest rate, while λ is the *disciplining effect*. These effects are internalized by a coordinated rule but not by an uncoordinated rule.

The redistributive effect of the interest rate, ρ, is positive. This effect captures the fact that higher interest rates hurt first-period borrowers by increasing their debt in the second period. Countries of higher type θ borrow more in the first period and therefore benefit more from a reduction in the interest rate than countries of lower type. Moreover, because of their higher spending in the first period, higher type countries also have a higher marginal cost of debt in the second period than lower type countries. Hence, the central authority (which cares about average welfare) weighs higher type countries by more, and as a result finds it optimal to commit to a lower interest rate to redistribute resources from lower type to higher type countries.

To understand the consequences of the redistributive effect, suppose that condition (10) holds, so that the left-hand side of (12) is decreasing in θ_c^*. Then holding all else fixed, (12) shows that a higher value of ρ implies a lower value of θ_c^*.[19] That is, the redistributive effect puts downward pressure on the optimal level of discretion: by lowering flexibility, the coordinated rule induces a lower interest rate, thus redistributing resources from countries that borrow less to those that borrow more. This redistribution is ex-ante beneficial for all countries.

The redistributive effect of the interest rate arises even in the absence of a present bias, i.e. even if $\beta = 1$. As mentioned in the Introduction, this effect is present in other models that abstract from self-control issues and consider in-

[19] Note that $\frac{R'(\theta_c^*)}{\theta_c^*(1-F(\theta_c^*))U'(g^f(\theta_c^*,R))\frac{\partial g^f(\theta_c^*,R)}{\partial \theta_c^*}} > 0$ by Lemma 1.

stead incomplete market economies with heterogenous agents. The redistributive channel reflects the fact that, absent perfect insurance markets, distortions such as deficit limits can improve social welfare.

Consider next the disciplining effect of the interest rate, λ. This effect captures the fact that the level of the interest rate affects the level of borrowing and spending that governments choose when given discretion. As shown in (14), λ may be positive or negative; its sign depends on how borrowing and spending change with R and how this in turn affects low versus high θ types. For intuition, suppose $dg^f(\theta, R)/dR < 0$, so that higher interest rates induce governments to borrow less. A higher interest rate in this case is beneficial for countries whose type is relatively low, as these countries overborrow relative to their ex-ante optimum. On the other hand, a higher interest rate harms countries whose type is high because these countries underborrow relative to their ex-ante optimum.

To understand the consequences of the disciplining effect, suppose again that condition (10) holds, so the left-hand side of (12) is decreasing in θ_c^*, and maintain the assumption that $dg^f(\theta, R)/dR < 0$. It can be verified that if θ_c^* in expression (14) were to take the value of θ_u^* given in (9), then λ would be negative. Intuitively, if the cutoff is chosen at the uncoordinated optimum θ_u^*, then as discussed in Section 2, the average distortion above the cutoff is zero: on average, the constrained types $\theta > \theta_u^*$ are neither overborrowing nor underborrowing relative to the ex-ante optimum. This means that the disciplining effect is determined by the unconstrained types $\theta \leq \theta_u^*$, and since these types are overborrowing, a higher interest rate can improve welfare by increasing discipline. It follows that λ is negative, and by (12) this effect increases the cutoff θ_c^*. That is, a negative disciplining effect puts upward pressure on the optimal level of discretion: by increasing flexibility, the coordinated rule induces a higher interest rate, thus improving fiscal discipline for overborrowing governments. This higher level of discipline is ex-ante beneficial for all countries.

Example. As an illustration, suppose $W(x) = x$. This example does not satisfy our assumption that $W(x)$ is strictly concave and hence it is not covered by our model; yet, we find it instructive to show the extent of the disciplining effect of the interest rate. The optimal uncoordinated fiscal rule in this case is still a cutoff

θ_u^* that satisfies (9), so that $\theta_u^* < \overline{\theta}$ for $\beta < 1$. However, note that since assets held globally equal zero, the ex-ante optimal allocation maximizes

$$\int_{\underline{\theta}}^{\overline{\theta}} \left(\theta U(g(\theta, R)) + W(x(\theta, R))\right) f(\theta) d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \theta U(g(\theta, R)) f(\theta) d\theta,$$

and thus it equalizes the marginal utility of first-period spending across all government types. Moreover, by (6), this allocation can be implemented by granting full flexibility to all governments, so that each type θ chooses $g^f(\theta, R)$ satisfying $\theta U'(g^f(\theta, R)) = \beta R$. It follows that the optimal coordinated fiscal rule is a cutoff $\theta_c^* = \overline{\theta}$ for all $\beta \leq 1$. Intuitively, since the marginal utility of assets is constant when $W(x) = x$, the redistributive effect of the interest rate ρ is zero, whereas the disciplining effect λ is negative and large enough that full flexibility is always optimal. As a result, in this stark example, the optimal coordinated fiscal rule is slacker than the uncoordinated one whenever $\beta < 1$.

More generally, we find that the level of discretion in the optimal coordinated fiscal rule, and how it compares to that in the optimal uncoordinated fiscal rule, depends on the relative strength of the redistributive and disciplining effects of the interest rate. The next section shows that which of the two effects is dominant depends on governments' present bias.

3.2 Coordination and Present Bias

The following proposition states our main result.

Proposition 2. *There exist $\overline{\beta}, \underline{\beta} \in [\underline{\theta}, 1]$, $\overline{\beta} > \underline{\beta}$, such that if $\beta \geq \overline{\beta}$, then $\theta_c^* < \theta_u^*$, whereas if $\beta \leq \underline{\beta}$, then $\theta_c^* > \theta_u^*$ and $\theta_c^* > \underline{\theta}$. That is, the optimal coordinated fiscal rule provides less flexibility than the optimal uncoordinated fiscal rule if governments' present bias is small enough, but it provides more flexibility than the optimal uncoordinated fiscal rule if governments' present bias is large enough.*

When governments' present bias is small, the optimal coordinated fiscal rule is more stringent than the uncoordinated one, and hence the interest rate is lower under coordination. To see the logic, take $\beta = 1$, so that governments are not present-biased. The optimal uncoordinated rule in this case entails full

flexibility, with a cutoff $\theta_u^* = \overline{\theta}$. In fact, there is no disciplining effect of the interest rate, as no government overborrows relative to the ex-ante optimum. Since the redistributive effect of the interest rate is positive, it follows that social welfare can be improved by imposing a tighter fiscal rule, $\theta_c^* < \theta_u^*$, which reduces the interest rate. This tighter rule lowers flexibility, but it benefits all countries from an ex-ante perspective by redistributing resources from lower types to higher types which borrow more and are harmed by high interest rates.

In contrast, when governments' present bias is large, the optimal coordinated fiscal rule is more lax than the uncoordinated one, and hence the interest rate is higher under coordination.[20] To see why this is the case, take some $\beta \leq \underline{\theta}$. The optimal uncoordinated rule then entails minimal discretion, with a cutoff $\theta_u^* \leq \underline{\theta}$. Given the endogenous interest rate, it follows that any rule $\theta_c^* = \theta_u^* \leq \underline{\theta}$ yields the same allocation as $\theta_c^* = \underline{\theta}$,[21] and thus, to prove the claim, it suffices to show that setting $\theta_c^* = \underline{\theta}$ is not optimal. The proof of Proposition 2 rests on showing that at $\theta_c^* = \underline{\theta}$, increasing flexibility is socially beneficial.

To illustrate, combine (13) and (14) to write the sum of the redistributive and disciplining effects of the interest rate as

$$\rho + \lambda = \int_{\underline{\theta}}^{\theta_c^*} \left[\begin{array}{c} -\frac{1}{R} W' \left(x^f \left(\theta, R \right) \right) x^f \left(\theta, R \right) \\ + \left(RW' \left(x^f \left(\theta, R \right) \right) - \theta U' \left(g^f \left(\theta, R \right) \right) \right) \frac{dg^f(\theta,R)}{dR} \end{array} \right] f \left(\theta \right) d\theta$$
$$+ \int_{\theta_c^*}^{\overline{\theta}} \left[\begin{array}{c} -\frac{1}{R} W' \left(x^f \left(\theta_c^*, R \right) \right) x^f \left(\theta_c^*, R \right) \\ + \left(RW' \left(x^f \left(\theta_c^*, R \right) \right) - \theta U' \left(g^f \left(\theta_c^*, R \right) \right) \right) \frac{dg^f(\theta_c^*,R)}{dR} \end{array} \right] f \left(\theta \right) d\theta.$$

The first integral corresponds to the redistributive and disciplining effects on government types whose spending is unconstrained; the second integral corresponds to these effects on types that are constrained by the fiscal rule. Suppose the cutoff is chosen at the uncoordinated optimum, $\theta_c^* = \theta_u^*$, and, for intuition, take β approaching $\underline{\theta}$ from above, so that θ_u^* and θ_c^* also approach $\underline{\theta}$ from above. As explained in Section 3.1, the disciplining effect on constrained types is zero at $\theta_c^* = \theta_u^*$; moreover, as β goes to $\underline{\theta}$, the redistributive effect goes to zero because all

[20] For some parametric examples, we find that there exists a unique $\beta^* \in (\underline{\theta}, 1)$ such that $\theta_c^* < \theta_u^*$ if $\beta > \beta^*$ whereas $\theta_c^* > \theta_u^*$ and $\theta_c^* > \underline{\theta}$ if $\beta < \beta^*$.

[21] Analogous to the discussion in Section 2, if $\theta_c^* \leq \underline{\theta}$, then all types' first-period spending is $g^f(\theta_c^*, R) = \tau$.

types' assets go to zero. As for the unconstrained types, their mass goes to zero as β approaches $\underline{\theta}$; however, the redistributive and disciplining effects on these types differ in this limit: the redistributive effect vanishes, but the disciplining effect is strictly negative.[22] Thus, it is possible to induce governments of type θ close to $\underline{\theta}$ to save more at little interest cost to higher government types.

Proposition 2 shows that when the present bias is large, governments independently prefer tight rules. Governments do not internalize the fact that by allowing themselves less flexibility, they reduce interest rates. By increasing discretion and therefore raising R, the central authority can provide flexibility while at the same time guaranteeing more discipline as a consequence of the higher interest rate that induces governments to borrow less. We find that in some cases, in fact, the externality is large enough that all countries can be made ex ante better off by jointly abandoning their uncoordinated rules and allowing themselves full flexibility. This is true, for example, for many parameter values under log preferences and a uniform distribution of types.[23]

As for the magnitude of the welfare gains, note that the benefits of coordination when governments' present bias is large are bounded from above by the benefits of stabilization. That is, the welfare gains from allowing greater flexibility stem from the ability to smooth out macroeconomic fluctuations; as observed by Lucas (1987), these gains are thus quantitatively small. On the other hand, it is worth noting that coordination can have quantitatively large effects on the interest rate. For instance, suppose governments' present bias is sufficiently large so that $\beta \leq \underline{\theta}$. Then under the optimal uncoordinated fiscal rule the interest rate is $R = U'(\tau)/W'(0)$, whereas under the optimal coordinated fiscal rule it is $R > (U'(\tau)/W'(0))\,\underline{\theta}/\beta$, which grows without bound as β goes down.

[22] Note that in this limit, the unconstrained types' first-period spending is decreasing in R as their assets are zero and hence there is no income effect of the interest rate.

[23] E.g., when $U(g) = \log(g)$, $W(x) = \log(\tau + x)$, $\Theta = [0.5, 1.5]$, $f(\theta) = 1$, and $\beta = 0.5$. The claim of course is also true in the quasi-linear case described in Section 3.1.

4 Discussion

4.1 Other Instruments

We have shown in Section 3 that if governments' present bias is large enough, the optimal coordinated fiscal rule provides more flexibility than the optimal uncoordinated fiscal rule. By increasing flexibility, the coordinated rule induces some government types to spend and borrow more, which increases the interest rate and therefore leads other types to spend and borrow less. A natural question in light of this result is whether other instruments can achieve a similar effect.

In particular, can it be optimal to *force* some government types to spend and borrow more? Within our framework with no transfers, consider using a fiscal rule that imposes a maximum surplus limit in addition to a maximum deficit limit. The rule specifies two cutoffs, $\theta^* \in [0, \overline{\theta}]$ and $\theta^{**} \in [0, \theta^*]$, such that: for types $\theta < \theta^{**}$, the levels of first-period spending and second-period assets are $g^f(\theta^{**}, R)$ and $x^f(\theta^{**}, R)$; for types $\theta \in [\theta^{**}, \theta^*]$, these levels are $g^f(\theta, R)$ and $x^f(\theta, R)$; and for types $\theta > \theta^*$, these levels are $g^f(\theta^*, R)$ and $x^f(\theta^*, R)$. Hence, only types $\theta \in [\theta^{**}, \theta^*]$ have full discretion; all other types are constrained by the rule and therefore choose spending either at the maximum deficit limit — thus spending less than in their flexible optimum — or at the maximum surplus limit — thus spending more than in their flexible optimum.

It is immediate that an optimal uncoordinated fiscal rule always sets $\theta_u^{**} \leq \underline{\theta}$, so the government is not constrained by a maximum surplus limit. For an individual government that takes the interest rate as fixed, the only effect of setting a surplus limit is to force low types to borrow more. Since these types are already overborrowing relative to the ex-ante optimum in the absence of a surplus limit, a binding limit can only reduce the country's expected welfare.

In contrast, an optimal coordinated fiscal rule may set $\theta_c^{**} > \underline{\theta}$, so the government is constrained by a maximum surplus limit. A coordinated rule takes into account not only the direct effect of surplus limits on borrowing by low types, but also the indirect effect that operates through the interest rate. By increasing R, a surplus limit has a disciplining effect, and this effect can more than compensate for the distortions caused by the increased overborrowing by low types.

Proposition 3. *Consider fiscal rules consisting of a maximum deficit limit and a maximum surplus limit, given by cutoffs* $\theta^* \in [0, \overline{\theta}]$ *and* $\theta^{**} \in [0, \theta^*]$ *respectively. In an optimal uncoordinated fiscal rule,* $\theta_u^{**} \leq \underline{\theta}$*. There exist* $\left(U(\cdot), W(\cdot), F(\theta), \tau, \beta\right)$ *such that an optimal coordinated fiscal rule sets* $\theta_c^* > \theta_u^*$*,* $\theta_c^* > \underline{\theta}$*, and* $\theta_c^{**} > \underline{\theta}$*.*

If governments' present bias is large enough, then for some specifications of our model, the optimal coordinated fiscal rule will set a strictly higher maximum deficit limit and a strictly lower maximum surplus limit than the optimal uncoordinated fiscal rule. To see the idea, combine the first-order condition for θ_c^* given in (12) with the analog of that condition for θ_c^{**}. We obtain that if θ_c^* and θ_c^{**} are interior, then

$$\theta_c^{**} U'\left(g^f\left(\theta_c^{**}, R\right)\right)\left(\frac{\mathbb{E}\left[\theta | \theta \leq \theta_c^{**}\right]}{\theta_c^{**}} - \frac{1}{\beta}\right) = \theta_c^* U'\left(g^f\left(\theta_c^*, R\right)\right)\left(\frac{\mathbb{E}\left[\theta | \theta \geq \theta_c^*\right]}{\theta_c^*} - \frac{1}{\beta}\right).$$

The left-hand side is the average distortion due to overborrowing by low types; the right-hand side is the average distortion due to overborrowing by high types. The optimal coordinated rule specifies $(\theta_c^*, \theta_c^{**})$ to equalize these costs. Thus, under a large present bias, committing to overborrowing by low types can boost welfare by increasing the interest rate and reducing overborrowing by high types.

Maximum deficit and surplus limits are simple policy instruments which do not require the use of transfers.[24] More broadly, one could depart from our setting to allow for other instruments that imply transfers, like (interior) taxes. We make two observations. First, one may conjecture that a Pigouvian tax on borrowing or the associated interest income could be used by the central authority to increase fiscal discipline. However, in a closed economy with only one asset like ours, a linear tax would have no effect on the equilibrium allocation (see, e.g., Diamond, 1967; Hart, 1975; Stiglitz, 1982; Geanakoplos and Polemarchakis, 1986). Intuitively, since the endogenous interest rate adjusts, a linear tax would only result in a proportional change in R so that the effective cost of borrowing and the allocation are kept unchanged.[25] Second, one can show that the use of

[24] As noted previously, solving for the optimal coordinated mechanism without transfers in full generality is difficult, as the problem is not convex when the interest rate is endogenous. This is in contrast to the uncoordinated problem, which, as shown in Amador, Werning and Angeletos (2006), can be ensured to be convex under weak conditions.

[25] That is, consider a linear tax ϕ on borrowing, which raises no revenue since aggregate net

non-linear taxes, on the other hand, can enhance welfare. This is not surprising: deficit and surplus limits are extreme forms of non-linear taxes which induce a particularly large distortion on the highest and lowest government types. This distortion can be reduced, for example, by allowing governments to exceed a limit by paying an interior tax fee. Since a general study of mechanisms with transfers would give rise to a whole new set of issues that are beyond the scope of our paper, we leave these questions for future work.

4.2 Interaction of Coordinated and Uncoordinated Rules

Our analysis so far has considered two extreme cases: either all countries choose fiscal rules independently, or a central authority chooses a fiscal rule that applies to all countries. However, as discussed in the Introduction, reality may be in between these two extremes. Examples like that of the European Union and Germany suggest that even when a group of countries agree on a common rule, some of these countries may be able (and want) to enforce additional fiscal constraints. We investigate this possibility in this section.

Consider a coordinated fiscal rule θ_c^* and, to fix ideas, assume that this rule is implemented with a spending limit $g_c^* \equiv g^f(\theta_c^*, R(\theta_c^*))$. Suppose that a fraction ψ of governments can individually impose a different rule on themselves. Because the coordinated rule is fully enforceable, governments cannot implement a cutoff $\theta^* > \theta_c^*$; that is, all countries must respect the spending limit g_c^*. However, some governments may be able to commit to a cutoff $\theta^* < \theta_c^*$, thus restricting themselves to lower spending in the first period than allowed by the central authority. Enforcing these additional fiscal constraints requires strong institutions; we are interested in the case in which only a fraction ψ of countries have the necessary institutional environment to set $\theta^* < \theta_c^*$.

If governments' present bias is small, the possibility of supplementing the coordinated rule with additional fiscal constraints is irrelevant: by Proposition 2, individual governments prefer slacker constraints than those optimally imposed

borrowing is zero. The budget constraint (1) becomes $g + \frac{x}{R}(1-\phi) = \tau$ and flexible spending $g^f(\theta, R)$ is now given by $(1-\phi)\theta U'\left(g^f(\theta,R)\right) = \beta R W'\left(x^f(\theta,R)\right)$. Because R adjusts so that the global resource constraint (4) holds, the equilibrium cost of borrowing $R/(1-\phi)$, and thus the equilibrium allocation, are unchanged by ϕ. Note also that if governments are left to choose their fiscal rules without coordination, the rules they would choose are invariant to R and ϕ.

by the central authority. If governments' present bias is large, on the other hand, Proposition 2 implies that governments would want to impose stricter rules on themselves than imposed centrally. In this case, the fraction ψ of governments which have the ability to implement additional constraints would choose to adopt their optimal uncoordinated fiscal rule $\theta_u^* < \theta_c^*$. What is the impact on the world economy? How would the central authority respond?

Arguments analogous to those in Lemma 1 imply that when a fraction ψ of governments adopt tighter fiscal rules, the interest rate declines. If the coordinated fiscal rule is kept unchanged, with a spending limit g_c^*, the lower interest rate then induces higher borrowing and spending by the remaining governments whose rules have not changed. That is, by imposing more discipline on themselves, the fraction ψ of governments worsen fiscal discipline everywhere else.

In response to this, however, the central authority would optimally change the coordinated spending limit g_c^*. Under certain conditions, we are able to solve the central authority's problem when a fraction ψ of governments choose their optimal uncoordinated rule $\theta_u^* < \theta_c^*$, and we find that the optimal level of discretion for the remaining fraction of countries is decreasing in ψ.

Proposition 4. *Consider fiscal rules for a set of countries when a fraction ψ can choose θ_u^* if the central authority chooses $\theta_c^* > \theta_u^*$. There exist $\underline{\beta} \in [\underline{\theta}, 1]$ and $\overline{\psi} \in (0,1)$ such that if $\beta \leq \underline{\beta}$ and $\psi \leq \overline{\psi}$, then $\theta_c^* > \theta_u^*$ and $\theta_c^* > \underline{\theta}$. Moreover, if $U(g) = \log(g)$, $W(x) = \log(\tau + x)$, and θ_c^* is a unique and interior global optimum with $\theta_c^* > \theta_u^*$, then a marginal increase in ψ causes θ_c^* to decline.*

When the optimal coordinated fiscal rule is slacker than the uncoordinated one, an inefficiency arises if some governments can adopt tighter fiscal rules than those imposed centrally. As described above, the tighter rules depress global interest rates and reduce fiscal discipline for the rest of the governments. Moreover, note that under log preferences, equations (13) and (14) yield

$$\rho + \lambda = \frac{1 - R\left(\theta_c^*\right)}{R\left(\theta_c^*\right)\left(1 + R\left(\theta_c^*\right)\right)}, \tag{15}$$

and thus the sum of the redistributive and disciplining effects of the interest rate is decreasing in $R\left(\theta_c^*\right)$. Intuitively, the redistributive effect is stronger on the margin when interest rates are low: when $R\left(\theta_c^*\right)$ declines, all types shift spending towards

the first period, implying that their marginal utility-weighted debt in the second period increases and, as implied by (13), ρ increases. It follows that if a fraction of governments adopt more stringent rules and thus the interest rate declines, the redistributive effect becomes more powerful relative to the disciplining effect. As a result, the optimal response of the central authority is to tighten restrictions for the remaining governments, whose welfare declines.[26]

In sum, if governments' present bias is large, so that the optimal coordinated fiscal rule provides more flexibility than the uncoordinated one, then the ability of some countries to impose greater fiscal restrictions on themselves has clear externalities on others. We find that these countries will adopt such restrictions, and all countries will face lower interest rates and less flexibility as a consequence.

4.3 Open World Economy

We have studied optimal coordinated fiscal rules for an entire world economy. In practice, though, coordinated rules are chosen by groups of countries within a larger world system. In this section, we explore what this consideration implies for the optimal design of rules, and how changes in outside economies affect a group of countries which choose their rules jointly.

Denote by E the group of countries coordinating on fiscal rules and by A the rest of the world, where for simplicity we let countries in A have mass one like those in E. Suppose A-countries lend an exogenous (positive or negative) amount L to E-countries in the first period and are repaid RL in the second period, where R is the common world interest rate. Since each individual government in E faces the same budget constraint and the same welfare function as in our baseline model, we define type θ's flexible level of spending $g^f(\theta, R)$ as in (6), along with a fiscal rule θ^*. What is different in this open world economy is that the interest rate must reflect borrowing and lending between countries in E and A. The first-period global resource constraint (4) under a common rule θ^* therefore

[26]Note that θ_u^* is independent of θ_c^*, and hence setting $\theta_u^* < \theta_c^*$ is a best response for the fraction ψ of countries.

becomes[27]

$$\int_{\underline{\theta}}^{\theta^*} g^f\left(\theta, R\left(\theta^*\right)\right) f\left(\theta\right) d\theta + \int_{\theta^*}^{\overline{\theta}} g^f\left(\theta^*, R\left(\theta^*\right)\right) f\left(\theta\right) d\theta = \tau + L. \tag{16}$$

Proposition 1 applies to this setting by analogous arguments as those in our baseline model: taking the interest rate as given, each individual government in E chooses an optimal uncoordinated fiscal rule θ_u^* satisfying (9). The implied allocation, of course, depends on the value of external lending L. For example, if $\beta \leq \underline{\theta}$, all governments in E allow themselves no flexibility and thus choose second-period assets $x = -RL$, i.e. they borrow L in the initial period. By the same logic as in Section 2, the interest rate then satisfies $R = U'(\tau+L)/W'(-RL)$.

Regarding the optimal coordinated fiscal rule, results analogous to those in Proposition 2 also apply to this open world economy. In particular, provided that initial lending from A-countries is not too high, we can show that the optimal coordinated rule is slacker than the uncoordinated one when governments' present bias is large, since then the disciplining effect of the interest rate outweighs the redistributive effect. The result relies on lending L being below a level $\overline{L} > 0$ as there is now an additional redistributive effect of the interest rate: a decline in R redistributes resources from creditor A-countries to debtor E-countries.[28]

Now given a large present bias, how do changes in external funds from A affect countries in E? An increase in lending L has a depressing effect on global interest rates, which not only worsens fiscal discipline but can also lead to reduced flexibility in E.

[27] The second-period global resource constraint is

$$\int_{\underline{\theta}}^{\theta^*} x^f\left(\theta, R\left(\theta^*\right)\right) f\left(\theta\right) d\theta + \int_{\theta^*}^{\overline{\theta}} x^f\left(\theta^*, R\left(\theta^*\right)\right) f\left(\theta\right) d\theta = -RL.$$

[28] We focus here on the large present bias case. Our baseline results under a small present bias also apply to this open world economy provided that L is above a level $\underline{L} < 0$.

Proposition 5. *Consider fiscal rules for a set of countries that borrow an initial amount L from external sources. There exist $\underline{\beta} \in [\underline{\theta}, 1]$ and $\overline{L} > 0$ such that if $\beta \leq \underline{\beta}$ and $L \leq \overline{L}$, then $\theta_c^* > \theta_u^*$ and $\theta_c^* > \underline{\theta}$. Moreover, if $U(g) = \log(g)$, $W(x) = \log(\tau + x)$, and θ_c^* is a unique and interior global optimum with $\theta_c^* > \theta_u^*$, then a marginal increase in L causes θ_c^* to decline.*

As discussed in the previous section, under log preferences, the redistributive effect of the interest rate becomes more powerful relative to the disciplining effect as the interest rate declines. As a result, if lending from A increases and thus R goes down, the optimal response of a central authority in E is to tighten restrictions for E-governments. Countries end up with lower interest rates, less flexibility, and lower welfare as a consequence of the increase in external funds.

4.4 Heterogeneity

In order to identify differences between coordinated and uncoordinated fiscal rules which would not be due to countries having different characteristics, we considered homogenous countries in our analysis. In this section, we show that our results are robust to ex-ante heterogeneity across countries.

There are two main factors that complicate the analysis of heterogeneity. First, the program in (11) that solves for the optimal coordinated fiscal rule need not always admit a unique global optimum, and thus global comparative statics are difficult to establish using implicit differentiation.[29] Second, even in the presence of a unique global optimum, the competing redistributive and disciplining effects of the interest rate in (12) become analytically intractable when countries are heterogeneous. Thus, while in principle heterogenous countries may optimally be subject to heterogeneous rules under coordination, a general analysis of such rules is difficult.

To make progress, we consider an environment in which countries are ex-ante heterogeneous but must adopt a common spending limit g^* when coordinating their rules. Our motivation stems from real-world fiscal rules: in practice, supranational rules typically apply uniformly across countries, even if countries have

[29] More generally, the welfare function need not be globally concave; for example, if $\beta \leq \underline{\theta}$, then this function is convex at $\theta^* = \underline{\theta}$. Without a unique global optimum for all parameters, the Implicit Function Theorem cannot be applied to large perturbations in parameters.

different ex-ante characteristics. To simplify the exposition, we model heterogeneity by allowing for two groups of countries, labeled N and S, which differ along some dimension, as we describe next. A fraction $\psi \in (0,1)$ of countries are in group N and the remaining fraction $1-\psi$ of countries are in group S.

Suppose first that the two groups of countries N and S have different distributions of shocks θ, with density functions $f_N(\theta)$ and $f_S(\theta)$ respectively. Maintain the assumption that $\mathbb{E}_N[\theta] = \mathbb{E}_S[\theta] = 1$, and define $g^f(\theta, R)$ as in (6). The arguments in Proposition 1 apply without change, and yield that a government's optimal uncoordinated fiscal rule satisfies (9) with the expectation taken under $f_N(\theta)$ or $f_S(\theta)$ depending on the government's group. Moreover, to solve for the optimal coordinated fiscal rule, note that the problem of a utilitarian central authority is analogous to (11), with the only difference that the density function is now $\psi f_N(\theta) + (1-\psi) f_S(\theta)$ instead of $f(\theta)$. The same proof strategy as in our baseline model therefore applies, implying that the results in Proposition 2 extend to countries which are heterogeneous in their shock distributions.[30]

Suppose next that the two groups of countries N and S have different levels of present bias, denoted by β_N and β_S respectively, in addition to potentially different shock distributions. The optimal uncoordinated fiscal rule for a government in group $i = N, S$ solves equation (9) with present-bias parameter β_i and the expectation taken under $f_i(\theta)$. To study the optimal coordinated fiscal rule, define the variable γ to take the value θ/β_i if the government is of type θ and belongs to group i. The flexible level of spending for a type γ, $g^f(\gamma, R)$, is then given by

$$\gamma U'\left(g^f(\gamma, R)\right) = RW'\left(x^f(\gamma, R)\right) \tag{17}$$

under the budget constraint (1). For $i = N, S$, define $\theta_i(\gamma) \equiv \beta_i \gamma$ and $h_i(\gamma) \equiv \frac{1}{\beta_i} f_i(\beta_i \gamma)$, where the support of γ in group i has limits $\underline{\gamma}_i \equiv \underline{\theta}_i/\beta_i$ and $\overline{\gamma}_i \equiv \overline{\theta}_i/\beta_i$. Note that the optimal uncoordinated rule for a government in group i can be written as γ^*_{ui} solving $\mathbb{E}_i[\theta_i(\gamma) | \gamma \geq \gamma^*_{ui}] = \gamma^*_{ui}$, with the expectation under $h_i(\gamma)$.

We follow similar steps to those above to solve the central authority's problem under coordination. Let $\underline{\gamma} \equiv \min\{\underline{\gamma}_N, \underline{\gamma}_S\}$, $\overline{\gamma} \equiv \max\{\overline{\gamma}_N, \overline{\gamma}_S\}$, $h(\gamma) \equiv \psi h_N(\gamma) + (1-\psi) h_S(\gamma)$, and $\theta(\gamma) \equiv \left(\psi h_N(\gamma)\theta_N(\gamma) + (1-\psi) h_S(\gamma)\theta_S(\gamma)\right)/h(\gamma)$. The

[30] If the expectation of θ also varies across countries, then our main result under a large present bias continues to hold provided that this variation is not too large.

optimal coordinated fiscal rule γ_c^* solves:

$$\max_{\gamma^* \in [0,\overline{\gamma}]} \left\{ \begin{array}{l} \int_{\underline{\gamma}}^{\gamma^*} \left(\theta(\gamma) U(g^f(\gamma, R(\gamma^*))) + W(x^f(\gamma, R(\gamma^*)))\right) h(\gamma) d\gamma \\ + \int_{\gamma^*}^{\overline{\gamma}} \left(\theta(\gamma) U(g^f(\gamma^*, R(\gamma^*))) + W(x^f(\gamma^*, R(\gamma^*)))\right) h(\gamma) d\gamma \end{array} \right\} \tag{18}$$

$$\text{subject to (1), (17), and}$$

$$\int_{\underline{\gamma}}^{\gamma^*} g^f(\gamma, R(\gamma^*)) h(\gamma) d\gamma + \int_{\gamma^*}^{\overline{\gamma}} g^f(\gamma^*, R(\gamma^*)) h(\gamma) d\gamma = \tau.$$

The solution yields analogous results to those in Proposition 2, showing that our findings are robust to heterogeneity in shock processes and present biases.

Proposition 6. *Consider fiscal rules for a set of heterogeneous countries: a fraction $\psi \in (0,1)$ of countries have parameters $\{f_N, \beta_N\}$ and the remaining fraction $1-\psi$ have parameters $\{f_S, \beta_S\}$, where $\mathbb{E}_N[\theta] = \mathbb{E}_S[\theta] = 1$. There exist $\overline{\beta}, \underline{\beta} \in [\underline{\theta}, 1]$, $\overline{\beta} > \underline{\beta}$, such that if $\min\{\beta_N, \beta_S\} \geq \overline{\beta}$, then $\gamma_c^* < \min\{\gamma_{uN}^*, \gamma_{uS}^*\}$, whereas if $\max\{\beta_N, \beta_S\} \leq \underline{\beta}$, then $\gamma_c^* > \max\{\gamma_{uN}^*, \gamma_{uS}^*\}$ and $\gamma_c^* > \underline{\gamma}$.*

4.5 Infinite Horizon

We have studied a two-period model in which a government's continuation welfare as a function of assets, $W(x)$, is exogenously specified. In practice, this continuation welfare depends on future fiscal rules, policies, and interest rates, where the interest rates in turn depend on the policies adopted across countries. Consider an infinite horizon setting with independent and identically distributed (i.i.d.) shocks, where the government's utility of spending in period $t \in \{0, 1, \ldots\}$ is $\theta_t U(g_t)$. The analysis of uncoordinated fiscal rules in such a setting is still simple, as one can subsume within the function $W(\cdot)$ the future sequence of fiscal rules and interest rates. Analogous to the results in Amador, Werning and Angeletos (2006), we show in the Online Appendix that the optimal uncoordinated fiscal rule in fact coincides with that in our two-period setting. Specifically, in each period t, each government implements a time invariant cutoff θ_u^* satisfying

(9), such that all types $\theta_t \leq \theta_u^*$ have full flexibility at t and all types $\theta_t > \theta_u^*$ spend at the flexible level corresponding to type θ_u^* at t.

Under coordination, on the other hand, countries must take into account how fiscal rules affect the interest rate, which is non-trivial with an infinite horizon. To see why, suppose countries use a time-invariant rule θ^*. For a given deterministic sequence of interest rates, the rule can be implemented as a sequence of history-dependent borrowing limits for each country, inducing a wealth distribution across countries at every date. The difficulty is that in equilibrium, the sequence of interest rates must be such that the net world wealth is zero in each period, and this implies a fixed point problem that in general cannot be solved analytically.

To make the problem tractable, we follow Halac and Yared (2014) by assuming that the government's utility of spending in period t is $\theta_t U(g_t) = \theta_t \log(g_t)$, and considering the limit of a T-period economy as $T \to \infty$. These assumptions imply that if countries adopt a time-invariant rule θ^*, then the interest rate is constant over time and increasing in θ^*. Of course, in principle, countries may choose a time-varying rule that depends in a complicated manner on the sequence of assets in each country and the sequence of world wealth distribution. However, we show in the Online Appendix that if countries restrict themselves to rules $\theta^*(t)$ that apply to all countries symmetrically (independently of their assets), then a time-invariant cutoff θ_c^* is optimal under coordination. This allows us to extend our analysis of coordinated rules to an infinite horizon, and to show that our results are robust: as described in the Online Appendix, we find that in an infinite horizon setting with i.i.d. shocks and log preferences, $\theta_c^* < \theta_u^*$ if β is high enough whereas $\theta_c^* > \theta_u^*$ and $\theta_c^* > \underline{\theta}$ if β is low enough.[31]

5 Conclusion

This paper presented a theoretical framework to compare coordinated and uncoordinated fiscal rules. We established that whether the optimal coordinated

[31] Given an infinite horizon setting, one could also study the self-enforcement of fiscal rules. That is, while we have assumed throughout our paper that governments are able to commit to abiding a given rule, in a dynamic setting, rules may be respected even in the absence of such commitment power. As discussed in Halac and Yared (2014, pp. 1582–83), if governments place a sufficiently high value on the future, a deficit limit can be self-enforced in equilibrium by using the threat of reversion to full flexibility upon a violation of the limit.

fiscal rule is more or less constraining than the optimal uncoordinated fiscal rule depends on governments' present bias. In particular, if the present bias is large, a central authority optimally imposes a slacker deficit limit than that chosen by individual governments: by increasing flexibility, the coordinated rule leads to a higher interest rate, which naturally increases fiscal discipline in all countries. We showed that imposing a maximum surplus limit in addition to a maximum deficit limit can boost welfare by increasing interest rates further and harnessing the power of their disciplining effect. Finally, we studied the effects of some countries being able to supplement coordinated rules with additional fiscal constraints, as well as the effects of an external supply of funds, showing how they influence governments' deficits and the optimal level of discretion.

Although our focus has been on fiscal policy, our analysis applies more generally to any group of households, firms, or countries that face a tradeoff between commitment and flexibility. For instance, households choose forced savings plans as a means to commit to not overspend; firms impose investment rules on themselves to prevent over-expansion; and countries set environmental quotas to limit pollution. These parties face a commitment-versus-flexibility tradeoff, as they also value having discretion to respond to possible contingencies. Furthermore, in all these circumstances, the price of the temptation good — the interest rate for households, the price of investment goods for firms, and the price of polluting materials for countries — is endogenous to the rules that parties choose. Specifically, the more flexible are the rules, the higher is the price of the temptation good. As such, an ex-ante commitment to flexibility, while not necessarily privately beneficial for the parties involved, can allow to increase overall discipline and lead to higher social welfare.

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A Appendix

A.1 Proof of Lemma 1

Consider a fiscal rule θ^* applying to all governments. Type θ's first-period spending is $g^f(\theta^*, R(\theta^*))$ if $\theta > \theta^*$ and $g^f(\theta, R(\theta^*))$ if $\theta \leq \theta^*$. Substituting into the first-period global resource constraint given in (4) yields

$$\int_{\underline{\theta}}^{\theta^*} g^f(\theta, R(\theta^*)) f(\theta) d\theta + \int_{\theta^*}^{\overline{\theta}} g^f(\theta^*, R(\theta^*)) f(\theta) d\theta = \tau. \tag{19}$$

Differentiating this equation with respect to θ^*, we obtain

$$R'(\theta^*) = \frac{-(1 - F(\theta^*)) \frac{\partial g^f(\theta^*, R)}{\partial \theta^*}}{\int_{\underline{\theta}}^{\theta^*} \frac{dg^f(\theta, R)}{dR} f(\theta) d\theta + \int_{\theta^*}^{\overline{\theta}} \frac{dg^f(\theta^*, R)}{dR} f(\theta) d\theta}. \tag{20}$$

To determine the sign of $R'(\theta^*)$, note that differentiating (6) with respect to θ gives

$$\theta U''\left(g^f(\theta, R(\theta^*))\right) \frac{dg^f(\theta, R(\theta^*))}{d\theta} + U'\left(g^f(\theta, R(\theta^*))\right) = \beta R(\theta^*) W''\left(x^f(\theta, R(\theta^*))\right) \frac{dx^f(\theta, R(\theta^*))}{d\theta}.$$

Using (1) to substitute for $\frac{dx^f(\theta,R(\theta^*))}{d\theta}$ and (6) to substitute for β and rearranging terms, this equation yields

$$\frac{dg^f\left(\theta,R\left(\theta^*\right)\right)}{d\theta}=\frac{1}{\theta}\frac{1}{-\frac{U''\left(g^f(\theta,R(\theta^*))\right)}{U'\left(g^f(\theta,R(\theta^*))\right)}-R\left(\theta^*\right)\frac{W''\left(x^f(\theta,R(\theta^*))\right)}{W'\left(x^f(\theta,R(\theta^*))\right)}}>0, \tag{21}$$

where the strict inequality follows from the fact that $U(\cdot)$ and $W(\cdot)$ are strictly increasing and concave. Equation (21) implies that the numerator on the right-hand side of (20) is strictly negative for $\theta^*<\overline{\theta}$.

To sign the denominator in (20), differentiate (6) with respect to R and follow similar steps as above to obtain

$$\begin{aligned}\frac{dg^f\left(\theta,R\left(\theta^*\right)\right)}{dR} &= \frac{1}{R\left(\theta^*\right)}\frac{-1}{-\frac{U''\left(g^f(\theta,R(\theta^*))\right)}{U'\left(g^f(\theta,R(\theta^*))\right)}-R\left(\theta^*\right)\frac{W''\left(x^f(\theta,R(\theta^*))\right)}{W'\left(x^f(\theta,R(\theta^*))\right)}}\\ &\quad+\frac{1}{R\left(\theta^*\right)}\mu\left(\theta,R\left(\theta^*\right)\right)\left(\tau-g^f\left(\theta,R\left(\theta^*\right)\right)\right),\end{aligned} \tag{22}$$

where

$$\mu\left(\theta,R\left(\theta^*\right)\right)\equiv\frac{-R\left(\theta^*\right)\frac{W''\left(x^f(\theta,R(\theta^*))\right)}{W'\left(x^f(\theta,R(\theta^*))\right)}}{-\frac{U''\left(g^f(\theta,R(\theta^*))\right)}{U'\left(g^f(\theta,R(\theta^*))\right)}-R\left(\theta^*\right)\frac{W''\left(x^f(\theta,R(\theta^*))\right)}{W'\left(x^f(\theta,R(\theta^*))\right)}}.$$

The first term in (22) is strictly negative whereas the sign of the second term is ambiguous and depends on the sign of $\tau-g^f\left(\theta,R\left(\theta^*\right)\right)$. Since the denominator in (20) is equal to the integral of (22) over θ, this denominator therefore consists of a strictly negative term plus the following term:

$$\begin{aligned}&\frac{1}{R\left(\theta^*\right)}\int_{\underline{\theta}}^{\theta^*}\mu\left(\theta,R\left(\theta^*\right)\right)\left(\tau-g^f\left(\theta,R\left(\theta^*\right)\right)\right)f\left(\theta\right)d\theta\\ +&\frac{1}{R\left(\theta^*\right)}\int_{\theta^*}^{\overline{\theta}}\mu\left(\theta^*,R\left(\theta^*\right)\right)\left(\tau-g^f\left(\theta^*,R\left(\theta^*\right)\right)\right)f\left(\theta\right)d\theta.\end{aligned} \tag{23}$$

To determine the sign of (23), note that if $\theta''>\theta'$, then (1) and (21) imply $g^f\left(\theta'',R\left(\theta^*\right)\right)>g^f\left(\theta',R\left(\theta^*\right)\right)$ and $x^f\left(\theta'',R\left(\theta^*\right)\right)<x^f\left(\theta',R\left(\theta^*\right)\right)$. Moreover, by

Assumption 1, it follows that

$$-\frac{U''\left(g^f(\theta'',R)\right)}{U'\left(g^f(\theta'',R)\right)} \leq -\frac{U''\left(g^f(\theta',R)\right)}{U'\left(g^f(\theta',R)\right)}, \quad \text{and} \quad -\frac{W''\left(x^f(\theta'',R)\right)}{W'\left(x^f(\theta'',R)\right)} \geq -\frac{W''\left(x^f(\theta',R)\right)}{W'\left(x^f(\theta',R)\right)}.$$

Hence, we obtain that if $\theta'' > \theta'$, then $\tau - g^f\left(\theta'', R\left(\theta^*\right)\right) < \tau - g^f\left(\theta', R\left(\theta^*\right)\right)$ and $\mu\left(\theta'', R\left(\theta^*\right)\right) \geq \mu\left(\theta', R\left(\theta^*\right)\right)$. It follows that $\tau - g$ and μ are weakly negatively correlated, and given (19) the expected value of $\tau - g$ is equal to zero. Therefore, the sign of (23) is weakly negative, implying that the denominator in (20) is strictly negative. Since we had established that the numerator in (20) is also strictly negative for $\theta^* < \overline{\theta}$, we obtain $R'(\theta^*) > 0$ for $\theta^* < \overline{\theta}$.

A.2 Proof of Proposition 1 and Corollary 1

The proof of Proposition 1 follows from the claims in the text and the fact that $\theta_u^* < \overline{\theta}$ for $\beta < 1$. To see the latter, note that the second derivative of the objective evaluated at $\theta^* = \overline{\theta}$ is

$$-\frac{\partial g^f\left(\theta^*, R\right)}{\partial \theta^*}|_{\theta^*=\overline{\theta}}\left(\overline{\theta}U'\left(g^f\left(\overline{\theta}, R\right)\right) - RW'\left(x^f\left(\overline{\theta}, R\right)\right)\right) f\left(\overline{\theta}\right),$$

which is strictly positive for $\beta < 1$.

To prove Corollary 1, note that to establish that θ_u^* is strictly increasing in β, it is sufficient to show that the left-hand side of (9) is strictly decreasing in θ_u^*, since the right-hand side is strictly decreasing in β. The derivative of the left-hand side of (9) for a cutoff θ^* is

$$\frac{d\left(\mathbb{E}\left[\theta|\theta \geq \theta^*\right]/\theta^*\right)}{d\theta^*} = \frac{1}{\theta^*}\left(\frac{d\mathbb{E}\left[\theta|\theta \geq \theta^*\right]}{d\theta^*} - \frac{\mathbb{E}\left[\theta|\theta \geq \theta^*\right]}{\theta^*}\right). \tag{24}$$

Condition (10) implies that the right-hand side of (24) is strictly negative.

A.3 Proof of Lemma 2

The first-order condition of program (11) yields

$$\frac{\partial g^f\left(\theta_c^*,R\right)}{\partial\theta_c^*}\int_{\theta_c^*}^{\overline{\theta}}\left(\theta U'\left(g^f\left(\theta_c^*,R\right)\right)-RW'\left(x^f\left(\theta_c^*,R\right)\right)\right)f\left(\theta\right)d\theta$$
$$+R'\left(\theta_c^*\right)\left(\begin{array}{c}\int_{\underline{\theta}}^{\theta_c^*}W'\left(x^f\left(\theta,R\right)\right)\left(\tau-g^f\left(\theta,R\right)\right)f\left(\theta\right)d\theta\\+\int_{\theta_c^*}^{\overline{\theta}}W'\left(x^f\left(\theta_c^*,R\right)\right)\left(\tau-g^f\left(\theta_c^*,R\right)\right)f\left(\theta\right)d\theta\end{array}\right)$$
$$-R'\left(\theta_c^*\right)\left(\begin{array}{c}\int_{\underline{\theta}}^{\theta_c^*}\left(RW'\left(x^f\left(\theta,R\right)\right)-\theta U'\left(g^f\left(\theta,R\right)\right)\right)\frac{dg^f(\theta,R)}{dR}f\left(\theta\right)d\theta\\+\int_{\theta_c^*}^{\overline{\theta}}\left(RW'\left(x^f\left(\theta_c^*,R\right)\right)-\theta U'\left(g^f\left(\theta_c^*,R\right)\right)\right)\frac{dg^f(\theta_c^*,R)}{dR}f\left(\theta\right)d\theta\end{array}\right)=0.$$

Substitution of (1) and (6) and simple algebraic manipulations yield (12)−(14).

A.4 Proof of Proposition 2

To prove the first part of the proposition ($\beta\geq\overline{\beta}$), take $\beta=1$. By Proposition 1, $\theta_u^*=\overline{\theta}$. Now consider θ_c^*. By Lemma 1, $R'\left(\theta^*\right)>0$ for $\theta^*<\overline{\theta}$, implying that there is no loss of generality in maximizing (11) with respect to the interest rate.[32] Given $\beta=1$ and using the Implicit Function Theorem, first-order conditions yield

$$\frac{\partial g^f\left(\theta_c^*,R\right)}{\partial\theta_c^*}\frac{1}{R'\left(\theta_c^*\right)}\int_{\theta_c^*}^{\overline{\theta}}\left(\theta U'\left(g^f\left(\theta_c^*,R\right)\right)-RW'\left(x^f\left(\theta_c^*,R\right)\right)\right)f\left(\theta\right)d\theta$$
$$+\left(\begin{array}{c}\int_{\underline{\theta}}^{\theta_c^*}W'\left(x^f\left(\theta,R\right)\right)\left(\tau-g^f\left(\theta,R\right)\right)f\left(\theta\right)d\theta\\+\int_{\theta_c^*}^{\overline{\theta}}W'\left(x^f\left(\theta_c^*,R\right)\right)\left(\tau-g^f\left(\theta_c^*,R\right)\right)f\left(\theta\right)d\theta\end{array}\right)\geq 0, \tag{25}$$

which holds with equality if θ_c^* is interior. Suppose now that $\theta_c^*=\overline{\theta}$. Note that using (20), we can rewrite the first term on the left-hand side of (25) as

$$-\left(\int_{\underline{\theta}}^{\theta_c^*}\frac{dg^f\left(\theta,R\left(\theta_c^*\right)\right)}{dR}f\left(\theta\right)d\theta+\int_{\theta_c^*}^{\overline{\theta}}\frac{dg^f\left(\theta^*,R\left(\theta_c^*\right)\right)}{dR}f\left(\theta\right)d\theta\right)$$
$$\times\mathbb{E}\left[\theta U'\left(g^f\left(\theta_c^*,R\right)\right)-RW'\left(x^f\left(\theta_c^*,R\right)\right)|\theta\geq\theta_c^*\right],$$

[32] The same analysis holds by taking the derivative with respect to θ_c^*; we pursue this route to simplify the steps.

which is equal to zero at $\theta_c^* = \overline{\theta}$ (as the expectation is equal to zero given $\beta = 1$). To sign the second term on the left-hand side of (25), note that by (1) and (21), $g^f(\theta, R)$ is strictly increasing in θ whereas $x^f(\theta, R)$ is strictly decreasing in θ. This implies that $W'\left(x^f(\theta, R)\right)$ and $\left(\tau - g^f(\theta, R)\right)$ are negatively correlated. Given (19), the expected value of $\tau - g$ is equal to zero; thus, it follows that the second term on the left-hand side of (25) is strictly negative. This implies that if $\theta_c^* = \overline{\theta}$, the left-hand side of (25) is strictly negative, a contradiction. Therefore, we must have $\theta_c^* < \overline{\theta} = \theta_u^*$.

To prove the second part of the proposition ($\beta \leq \underline{\beta}$), take $\beta \leq \underline{\theta}$. By Proposition 1, $\theta_u^* \leq \underline{\theta}$. Note that any rule $\theta_c^* \leq \underline{\theta}$ would yield the same allocation and hence the same welfare as a rule $\theta_c^* = \underline{\theta}$. Therefore, to prove the proposition, it suffices to show that $\theta_c^* = \underline{\theta}$ is not optimal. This is what we prove next.

Consider a fiscal rule $\theta^* = \underline{\theta}$ with associated interest rate $R = R(\theta^*)$. Welfare under this rule is given by (11). The first derivative with respect to θ^* is

$$\begin{aligned}
&\frac{\partial g^f(\theta^*, R)}{\partial \theta^*} \int_{\theta^*}^{\overline{\theta}} \left(\theta U'\left(g^f(\theta^*, R)\right) - RW'\left(x^f(\theta^*, R)\right)\right) f(\theta)\, d\theta \\
&+ R'(\theta^*) \begin{pmatrix} \int_{\underline{\theta}}^{\theta^*} \frac{dg^f(\theta,R)}{dR} \left(\theta U'\left(g^f(\theta, R)\right) - RW'\left(x^f(\theta, R)\right)\right) f(\theta)\, d\theta \\ + \int_{\theta^*}^{\overline{\theta}} \frac{dg^f(\theta^*,R)}{dR} \left(\theta U'\left(g^f(\theta^*, R)\right) - RW'\left(x^f(\theta^*, R)\right)\right) f(\theta)\, d\theta \end{pmatrix} \\
&+ R'(\theta^*) \begin{pmatrix} \int_{\underline{\theta}}^{\theta^*} W'\left(x^f(\theta, R)\right)\left(\tau - g^f(\theta, R)\right) f(\theta)\, d\theta \\ + \int_{\theta^*}^{\overline{\theta}} W'\left(x^f(\theta^*, R)\right)\left(\tau - g^f(\theta^*, R)\right) f(\theta)\, d\theta \end{pmatrix}.
\end{aligned}$$

Using (20) to substitute for $R'(\theta^*)$ in the second line and rearranging terms yields

$$\begin{aligned}
&\frac{\partial g^f(\theta^*, R)}{\partial \theta^*} \left\{ \begin{array}{c} \int_{\theta^*}^{\overline{\theta}} \left(\theta U'\left(g^f(\theta^*, R)\right) - RW'\left(x^f(\theta^*, R)\right)\right) f(\theta)\, d\theta \\ - \frac{(1 - F(\theta^*))}{\int_{\underline{\theta}}^{\theta^*} \frac{\frac{dg^f(\theta,R)}{dR}}{\frac{dg^f(\theta^*,R)}{dR}} f(\theta)\, d\theta + \int_{\theta^*}^{\overline{\theta}} f(\theta)\, d\theta} \begin{pmatrix} \int_{\underline{\theta}}^{\theta^*} \frac{\frac{dg^f(\theta,R)}{dR}}{\frac{dg^f(\theta^*,R)}{dR}} \left(\theta U'\left(g^f(\theta, R)\right) - RW'\left(x^f(\theta, R)\right)\right) f(\theta)\, d\theta \\ + \int_{\theta^*}^{\overline{\theta}} \left(\theta U'\left(g^f(\theta^*, R)\right) - RW'\left(x^f(\theta^*, R)\right)\right) f(\theta)\, d\theta \end{pmatrix} \end{array} \right\} \\
&+ R'(\theta^*) \begin{pmatrix} \int_{\underline{\theta}}^{\theta^*} W'\left(x^f(\theta, R)\right)\left(\tau - g^f(\theta, R)\right) f(\theta)\, d\theta \\ + \int_{\theta^*}^{\overline{\theta}} W'\left(x^f(\theta^*, R)\right)\left(\tau - g^f(\theta^*, R)\right) f(\theta)\, d\theta \end{pmatrix}. \qquad (26)
\end{aligned}$$

If $\theta^* = \underline{\theta}$, then $g^f(\theta^*, R) = \tau$ and $x^f(\theta^*, R) = 0$, and it can therefore be verified that each of the two lines in (26) equals zero at $\theta^* = \underline{\theta}$. This means that welfare is at a local maximum or a local minimum. We will now establish that the second derivative of the objective at $\theta^* = \underline{\theta}$ is strictly positive, implying that the objective is at a local minimum at this point.

Before taking second-order conditions, note that substituting with $\theta^* = \underline{\theta}$ in (20) yields

$$R'(\theta^*) = -\frac{\frac{\partial g^f(\theta^*, R)}{\partial \theta^*}}{\frac{dg^f(\theta^*, R)}{dR}}. \tag{27}$$

Hence, for $\theta^* = \underline{\theta}$,

$$\frac{\partial g^f(\theta^*, R(\theta^*))}{\partial \theta^*} + \frac{dg^f(\theta^*, R(\theta^*))}{dR} R'(\theta^*) = 0.$$

Moreover, note that $\frac{\partial^2 g^f(\theta^*, R)}{\partial {\theta^*}^2}$, $\frac{\partial^2 g^f(\theta^*, R)}{\partial \theta^* \partial R}$, $\frac{d^2 g^f(\theta^*, R)}{dR^2}$, and $R''(\theta^*)$ are bounded. Using these observations and the fact that each line in (26) equals zero, the second derivative of the objective evaluated at $\theta^* = \underline{\theta}$ is equal to

$$\begin{aligned} &-\frac{\partial g^f(\theta^*, R)}{\partial \theta^*}\left(\theta^* U'\left(g^f(\theta^*, R)\right) - RW'\left(x^f(\theta^*, R)\right)\right) f(\theta^*) \\ &+\frac{\partial g^f(\theta^*, R)}{\partial \theta^*} f(\theta^*) \int_{\underline{\theta}}^{\overline{\theta}} \left(\theta U'\left(g^f(\theta^*, R)\right) - RW'\left(x^f(\theta^*, R)\right)\right) f(\theta)\, d\theta. \end{aligned} \tag{28}$$

Note that $\frac{\partial g^f(\theta^*, R)}{\partial \theta^*} > 0$ and

$$\theta^* U'\left(g^f(\theta^*, R)\right) - RW'\left(x^f(\theta^*, R)\right) < \int_{\underline{\theta}}^{\overline{\theta}} \left(\theta U'\left(g^f(\theta^*, R)\right) - RW'\left(x^f(\theta^*, R)\right)\right) f(\theta)\, d\theta \leq 0,$$

since the marginal type $\theta^* = \underline{\theta}$ overspends by more than average. Hence, (28) is strictly positive, implying that $\theta^* = \underline{\theta}$ is a local minimum.

B Online Appendix

In this Online Appendix, we provide proofs for the results in Section 4 of the paper. Since several of our claims are for the case of log preferences, we begin by restating our problem under this preference structure.

B.1 Log Preferences

Take $U(g) = \log(g)$ and $W(x) = \delta \log(\tau + x)$ for $\delta > 0$. Equations (1) and (6) imply

$$g^f(\theta, R) = \frac{\theta}{\theta + \beta\delta}\left(\tau + \frac{\tau}{R}\right), \tag{29}$$

$$x^f(\theta, R) = \frac{\beta\delta}{\theta + \beta\delta} R\left(\tau + \frac{\tau}{R}\right). \tag{30}$$

Equivalently, letting $s^f(\theta) = \frac{\beta\delta}{\theta+\beta\delta}$ be type θ's savings rate under full flexibility, we have

$$g^f(\theta, R) = \left(1 - s^f(\theta)\right)\left(\tau + \frac{\tau}{R}\right).$$

For a given rule $\theta^* \in [0, \overline{\theta}]$, the aggregate savings rate in the economy is

$$S(\theta^*) = \int_{\underline{\theta}}^{\theta^*} \frac{\beta\delta}{\theta + \beta\delta} f(\theta) d\theta + \int_{\theta^*}^{\overline{\theta}} \frac{\beta\delta}{\theta^* + \beta\delta} f(\theta) d\theta.$$

The coordinated program in (11) can be written as

$$\max_{\theta^* \in [0,\overline{\theta}]} \left\{ \begin{array}{c} \int_{\underline{\theta}}^{\theta^*} \left(\theta \log\left(\frac{\theta}{\theta+\beta\delta}\right) + \delta \log\left(\frac{\beta\delta}{\theta+\beta\delta}\right)\right) f(\theta) d\theta \\ + \int_{\theta^*}^{\overline{\theta}} \left(\theta \log\left(\frac{\theta^*}{\theta^*+\beta\delta}\right) + \delta \log\left(\frac{\beta\delta}{\theta^*+\beta\delta}\right)\right) f(\theta) d\theta \\ - \log\left(1 - S(\theta^*)\right) - \delta \log\left(S(\theta^*)\right) \end{array} \right\}. \tag{31}$$

The uncoordinated program in (7), on the other hand, reduces to the first two lines of (31).

As explained in the paper, the difference between the optimal coordinated

fiscal rule and the optimal uncoordinated fiscal rule can be expressed as a function of the redistributive and disciplining effects of the interest rate. Under log preferences, the sum of these two effects is

$$\rho + \lambda = \frac{1 - \delta R\left(\theta_c^*\right)}{R\left(\theta_c^*\right)\left(1 + R\left(\theta_c^*\right)\right)}. \tag{32}$$

Equation (32) is the same as equation (15) but allowing for any $\delta > 0$. This equation shows that the redistributive effect of the interest rate dominates the disciplining effect if and only if $R\left(\theta_c^*\right) < 1/\delta$. As discussed in the paper, the redistributive effect is stronger on the margin when interest rates are low.[33]

B.2 Proof of Proposition 3

We prove the results by example. Take log preferences. Analogous to the expressions in Section B.1 above, given cutoffs $\theta^* \in [0, \overline{\theta}]$ and $\theta^{**} \in [0, \theta^*]$, the aggregate savings rate in the economy is

$$S\left(\theta^*, \theta^{**}\right) = \int_{\underline{\theta}}^{\theta^{**}} \frac{\beta\delta}{\theta^{**} + \beta\delta} f(\theta) d\theta + \int_{\theta^{**}}^{\theta^*} \frac{\beta\delta}{\theta + \beta\delta} f(\theta) d\theta + \int_{\theta^*}^{\overline{\theta}} \frac{\beta\delta}{\theta^* + \beta\delta} f(\theta) d\theta,$$

and the coordinated program can be written as

$$\max_{\theta^* \in [0,\overline{\theta}], \theta^{**} \in [0,\theta^*]} \left\{ \begin{array}{l} \int_{\underline{\theta}}^{\theta^{**}} \left(\theta \log\left(\frac{\theta^{**}}{\theta^{**}+\beta\delta}\right) + \delta \log\left(\frac{\beta\delta}{\theta^{**}+\beta\delta}\right)\right) f\left(\theta\right) d\theta \\ + \int_{\theta^{**}}^{\theta^*} \left(\theta \log\left(\frac{\theta}{\theta+\beta\delta}\right) + \delta \log\left(\frac{\beta\delta}{\theta+\beta\delta}\right)\right) f\left(\theta\right) d\theta \\ + \int_{\theta^*}^{\overline{\theta}} \left(\theta \log\left(\frac{\theta^*}{\theta^*+\beta\delta}\right) + \delta \log\left(\frac{\beta\delta}{\theta^*+\beta\delta}\right)\right) f\left(\theta\right) d\theta \\ - \log\left(1 - S\left(\theta^*, \theta^{**}\right)\right) - \delta \log\left(S\left(\theta^*, \theta^{**}\right)\right) \end{array} \right\}. \tag{33}$$

Take $\delta = 1$ and $F(\theta)$ to be exponential with parameter 0.0785 and set $\left[\underline{\theta}, \overline{\theta}\right]$

[33] The relevant threshold for $R(\theta_c^*)$ depends on δ because a reduction in δ has a similar effect as a reduction in $R(\theta_c^*)$: all types shift spending to the present when δ declines.

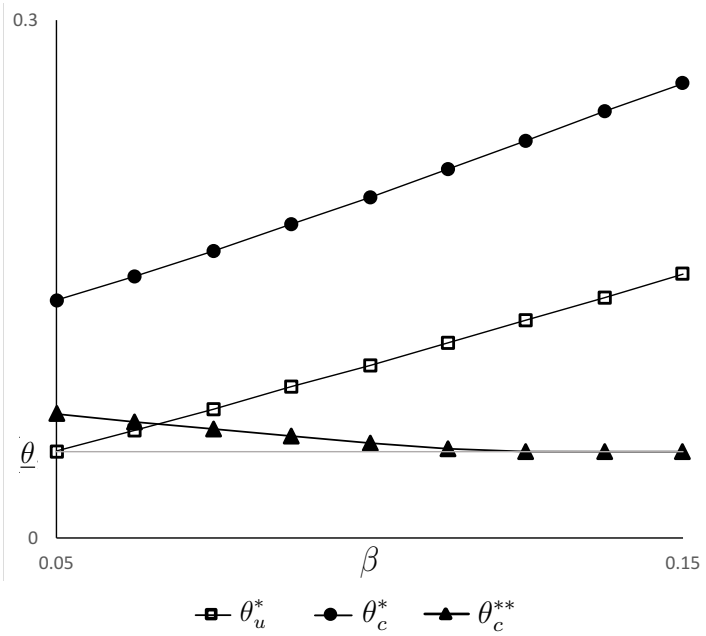

Figure 1: Maximum surplus limits.

given by $[0.05, 2]$. The parameter and truncation we choose ensure that $\mathbb{E}[\theta] = 1$. For a range of β, Figure 1 depicts the cutoff θ_u^* in the optimal uncoordinated rule and the cutoffs θ_c^* and θ_c^{**} in the optimal coordinated rule, as a function of β. By the arguments in the text, $\theta_u^{**} \leq \underline{\theta}$ always holds. Hence, as shown in the figure, we find that there exist $\big(U(\cdot), W(\cdot), F(\theta), \tau, \beta\big)$ such that $\theta_c^* > \theta_u^*$, $\theta_c^* > \underline{\theta}$, and $\theta_c^{**} > \underline{\theta} \geq \theta_u^{**}$.

B.3 Proof of Proposition 4

To prove the first part of the proposition, we follow analogous steps as in the proof of Proposition 2 for the case of $\beta \leq \underline{\theta}$. We show that setting a coordinated cutoff $\theta^* = \underline{\theta}$ is not optimal. Note that by Proposition 2, if $\psi = 0$, then $\theta^* = \underline{\theta}$ is indeed not optimal. Moreover, since the objective function of the coordinated problem is continuous in θ^* and ψ, it follows that for $\psi = \varepsilon$, $\varepsilon > 0$ arbitrarily small, $\theta^* = \underline{\theta}$ is not optimal either. Therefore, given $\beta \leq \underline{\theta}$, there exists $\overline{\psi} \in (0, 1)$ such that if $\psi \leq \overline{\psi}$, then $\theta_c^* > \theta_u^*$ and $\theta_c^* > \underline{\theta}$.

To prove the second part of the proposition, take log preferences and assume θ_c^* is a unique and interior global optimum with $\theta_c^* > \theta_u^*$. We consider the program

that solves for the optimal coordinated fiscal rule taking into account that a fraction ψ of governments choose θ_u^*. Analogous to the analysis in Section B.1 above, given a rule $\theta^* > \theta_u^*$, the aggregate savings rate in the economy is (we allow here for any $\delta > 0$; the statement of Proposition 4 takes $\delta = 1$):

$$\begin{aligned} S(\theta^*, \psi) &= (1-\psi)\left(\int_{\underline{\theta}}^{\theta^*} \frac{\beta\delta}{\theta+\beta\delta} f(\theta)d\theta + \int_{\theta^*}^{\overline{\theta}} \frac{\beta\delta}{\theta^*+\beta\delta} f(\theta)d\theta\right) \\ &\quad +\psi\left(\int_{\underline{\theta}}^{\theta_u^*} \frac{\beta\delta}{\theta+\beta\delta} f(\theta)d\theta + \int_{\theta_u^*}^{\overline{\theta}} \frac{\beta\delta}{\theta_u^*+\beta\delta} f(\theta)d\theta\right). \end{aligned} \tag{34}$$

Note that $dS(\theta^*, \psi)/d\psi > 0$ for $\theta_u^* < \theta^*$. The coordinated program, taking the heterogeneity into account, can be written as

$$\max_{\theta^* \in [0,\overline{\theta}]} \left\{ \begin{array}{c} (1-\psi)\left[\begin{array}{c} \int_{\underline{\theta}}^{\theta^*} \left(\theta \log\left(\frac{\theta}{\theta+\beta\delta}\right) + \delta \log\left(\frac{\beta\delta}{\theta+\beta\delta}\right)\right) f(\theta)\, d\theta \\ + \int_{\theta^*}^{\overline{\theta}} \left(\theta \log\left(\frac{\theta^*}{\theta^*+\beta\delta}\right) + \delta \log\left(\frac{\beta\delta}{\theta^*+\beta\delta}\right)\right) f(\theta)\, d\theta \end{array} \right] \\ +\psi\left[\begin{array}{c} \int_{\underline{\theta}}^{\theta_u^*} \left(\theta \log\left(\frac{\theta}{\theta+\beta\delta}\right) + \delta \log\left(\frac{\beta\delta}{\theta+\beta\delta}\right)\right) f(\theta)\, d\theta \\ + \int_{\theta_u^*}^{\overline{\theta}} \left(\theta \log\left(\frac{\theta_u^*}{\theta_u^*+\beta\delta}\right) + \delta \log\left(\frac{\beta\delta}{\theta_u^*+\beta\delta}\right)\right) f(\theta)\, d\theta \end{array} \right] \\ -\log\left(1 - S(\theta^*, \psi)\right) - \delta \log\left(S(\theta^*, \psi)\right) \end{array} \right\}$$

subject to (9).

The first-order condition, assuming an interior optimum, is

$$\int_{\theta_c^*}^{\overline{\theta}} \left(\frac{\theta}{\theta_c^*} - \frac{\theta+\delta}{\theta_c^*+\beta\delta}\right) f(\theta)\, d\theta - \left(\int_{\theta_c^*}^{\overline{\theta}} \frac{\beta\delta}{(\theta_c^*+\beta\delta)^2} f(\theta)d\theta\right)\left(\frac{1}{1-S(\theta_c^*, \psi)} - \delta\frac{1}{S(\theta_c^*, \psi)}\right) = 0. \tag{35}$$

Since θ_c^* is the unique global optimum, we can determine its comparative statics with respect to ψ by implicit differentiation of (35). Since the program is locally

concave, the derivative of the left-hand side of (35) with respect to θ_c^* is negative. If we can establish that the derivative of the left-hand side of (35) with respect to ψ is negative, then this implies that θ_c^* is locally decreasing in ψ. We find that this is indeed the case: the derivative of the left-hand side of (35) with respect to ψ is

$$-\left(\int_{\theta_c^*}^{\overline{\theta}} \frac{\beta\delta}{\left(\theta_c^* + \beta\delta\right)^2} f(\theta) d\theta\right) \frac{dS\left(\theta_c^*, \psi\right)}{d\psi} \left(\frac{1}{\left(1 - S\left(\theta_c^*, \psi\right)\right)^2} + \delta \frac{1}{\left(S\left(\theta_c^*, \psi\right)\right)^2}\right) < 0 \ ,$$

where we have taken into account that $dS\left(\theta_c^*, \psi\right)/d\psi > 0$ since $\theta_u^* < \theta_c^*$.

B.4 Proof of Proposition 5

To prove the first part of the proposition, we follow the same steps as in the proof of Proposition 2 for the case of $\beta \leq \underline{\theta}$, taking into account that (4) is now replaced by (16). We show that setting a coordinated cutoff $\theta^* = \underline{\theta}$ is not optimal. Note that $R'\left(\theta^*\right)$ continues to satisfy (20), and it satisfies (27) when $\theta^* = \underline{\theta}$. The first-order condition of the coordinated problem must therefore satisfy equation (26). If $\theta^* = \underline{\theta}$, then $g^f\left(\theta^*, R\right) = \tau + L$ and $x^f\left(\theta^*, R\right) = -RL$, so that (26) becomes

$$-R'\left(\underline{\theta}\right) \left(\int_{\underline{\theta}}^{\overline{\theta}} W'\left(x^f\left(\theta^*, R\right)\right) f\left(\theta\right) d\theta\right) L.$$

Recall that $R'\left(\underline{\theta}\right) > 0$. Thus, if $L < 0$, the expression above is strictly positive, implying that $\theta^* = \underline{\theta}$ is not optimal as an increase in θ^* would increase welfare. If instead $L = 0$, then by the proof of Proposition 2, $\theta^* = \underline{\theta}$ is not optimal either. Hence, given $\beta \leq \underline{\theta}$, we obtain $\theta_c^* > \theta_u^*$ and $\theta_c^* > \underline{\theta}$ for $L \leq 0$. Finally, since the objective function of the coordinated problem is continuous in θ^* and L, it follows that for $L = \varepsilon$, $\varepsilon > 0$ arbitrarily small, $\theta^* = \underline{\theta}$ is not optimal either. Therefore, given $\beta \leq \underline{\theta}$, there exists $\overline{L} > 0$ such that if $L \leq \overline{L}$, then $\theta_c^* > \theta_u^*$ and $\theta_c^* > \underline{\theta}$.

To prove the second part of the proposition, we consider the problem under log preferences as in Section B.1, but with (4) now replaced by (16). The program in (31) becomes (we allow here for any $\delta > 0$; the statement of Proposition 5 takes

$\delta = 1$):

$$\max_{\theta^* \in [0,\overline{\theta}]} \left\{ \begin{array}{c} \int_{\underline{\theta}}^{\theta^*} \left(\theta \log\left(\frac{\theta}{\theta+\beta\delta}\right) + \log\left(\frac{\beta\delta}{\theta+\beta\delta}\right) \right) f(\theta)\, d\theta \\ + \int_{\theta^*}^{\overline{\theta}} \left(\theta \log\left(\frac{\theta^*}{\theta^*+\beta\delta}\right) + \delta \log\left(\frac{\beta\delta}{\theta^*+\beta\delta}\right) \right) f(\theta)\, d\theta \\ - \log\left(1 - S(\theta^*)\right) - \delta \log\left(S(\theta^*) + L/\tau\right) \end{array} \right\}.$$

The first-order condition, assuming an interior optimum, is

$$\int_{\theta_c^*}^{\overline{\theta}} \left(\frac{\theta}{\theta_c^*} - \frac{\theta+\delta}{\theta_c^*+\beta\delta} \right) f(\theta)\, d\theta - \left(\int_{\theta_c^*}^{\overline{\theta}} \frac{\beta\delta}{(\theta_c^*+\beta\delta)^2} f(\theta) d\theta \right) \left(\frac{1}{1 - S(\theta^*)} - \delta \frac{1}{S(\theta^*) + L/\tau} \right) = 0. \tag{36}$$

Since by assumption θ_c^* is the unique global optimum given L, we can determine its comparative statics with respect to L by implicit differentiation of (36). Since the program is locally concave, the derivative of the left-hand side of (36) with respect to θ_c^* is negative. If we can establish that the derivative of the left-hand side of (36) with respect to L is negative, then this implies that θ_c^* is locally decreasing in L. We find that this is indeed the case: the derivative of the left-hand side of (36) with respect to L is

$$- \left(\int_{\theta_c^*}^{\overline{\theta}} \frac{\beta\delta}{(\theta_c^*+\beta\delta)^2} f(\theta) d\theta \right) \left(\frac{\delta/\tau}{(S(\theta^*) + L/\tau)^2} \right) < 0.$$

B.5 Proof of Proposition 6

Define γ_{ui}^* as the optimal uncoordinated rule for country group i with parameters $\{f_i, \beta_i\}$, and let γ_c^* be the optimal coordinated rule for both country groups, given $\{f_N, \beta_N, f_S, \beta_S, \psi\}$. We begin by proving the first part of the proposition ($\beta \geq \overline{\beta}$). By analogous reasoning as in the proof of the first part of Proposition 2, if $\beta_S = \beta_N = 1$, then

$$\gamma_c^* < \min\left\{\gamma_{uN}^*, \gamma_{uS}^*\right\}. \tag{37}$$

To prove the second part of the proposition ($\beta \leq \underline{\beta}$), take $\beta_i \leq \underline{\theta}_i$ for $i = N, S$.

By Proposition 1, $\theta_{ui}^* \leq \underline{\theta}_i$ for $i = N, S$, implying $\gamma_{ui}^* \leq \underline{\gamma}$. Note that any rule $\gamma_c^* \leq \underline{\gamma}$ would yield the same allocation and hence the same welfare as a rule $\gamma_c^* = \underline{\gamma}$. Therefore, to prove the proposition, it suffices to show that $\gamma_c^* = \underline{\gamma}$ is not optimal. To prove this, consider a fiscal rule $\gamma^* = \underline{\gamma}$ with associated interest rate $R = R(\gamma^*)$. Welfare under this rule is given by (18). The first derivative with respect to γ^* is

$$\begin{aligned}
&\frac{\partial g^f(\gamma^*, R)}{\partial \gamma^*} \int_{\gamma^*}^{\overline{\gamma}} \left(\theta(\gamma) U'\left(g^f(\gamma^*, R)\right) - RW'\left(x^f(\gamma^*, R)\right)\right) h(\gamma)\, d\gamma \\
&+ R'(\gamma^*) \left(\begin{array}{c} \int_{\underline{\gamma}}^{\gamma^*} \frac{dg^f(\gamma, R)}{dR} \left(\theta(\gamma) U'\left(g^f(\gamma, R)\right) - RW'\left(x^f(\gamma, R)\right)\right) h(\gamma)\, d\gamma \\ + \int_{\gamma^*}^{\overline{\gamma}} \frac{dg^f(\gamma^*, R)}{dR} \left(\theta(\gamma) U'\left(g^f(\gamma^*, R)\right) - RW'\left(x^f(\gamma^*, R)\right)\right) h(\gamma)\, d\gamma \end{array} \right) \\
&+ R'(\gamma^*) \left(\begin{array}{c} \int_{\underline{\gamma}}^{\gamma^*} W'\left(x^f(\gamma, R)\right)\left(\tau - g^f(\gamma, R)\right) h(\gamma)\, d\gamma \\ + \int_{\gamma^*}^{\overline{\gamma}} W'\left(x^f(\gamma^*, R)\right)\left(\tau - g^f(\gamma^*, R)\right) h(\gamma)\, d\gamma \end{array} \right).
\end{aligned}$$

The rest of the proof proceeds in the same way as the proof of our main result for homogeneous countries and is thus omitted.

B.6 Infinite Horizon

Consider an infinite horizon version of our model, with periods $t \in \{0, 1, \ldots, T\}$, $T \to \infty$, and discount factor $\delta \in (0, 1)$. The government's welfare at t before the realization of its type θ_t is

$$\mathbb{E}\left[\theta_t U(g_t) + \sum_{k=1}^{\infty} \delta^k \theta_{t+k} U(g_{t+k})\right]. \tag{38}$$

The government's welfare at t after the realization of θ_t, when choosing spending g_t, is

$$\theta_t U(g_t) + \beta \mathbb{E}\left[\sum_{k=1}^{\infty} \delta^k \theta_{t+k} U(g_{t+k})\right]. \tag{39}$$

Spending g_t satisfies the government's dynamic budget constraint:

$$g_t + \frac{x_{t+1}}{R_{t+1}} = \tau + x_t, \tag{40}$$

where x_t is the level of assets with which the government enters period t and we set $x_0 = 0$. The sum of total assets across all governments must sum to zero in each period. We assume that θ_t is i.i.d. across countries and time with an expected value $\mathbb{E}[\theta_t] = 1$. Because there are no aggregate shocks, it follows that the sequence of interest rates $\{R_t\}_{t=0}^{\infty}$ is deterministic, with $R_0 = 1$. We focus on fiscal rules at t which depend only on payoff-relevant variables: x_t and the sequence of future interest rates $\{R_{t+k}\}_{k=1}^{\infty}$.[34] We can then define

$$W_{t+1}(x_{t+1}) = \mathbb{E}\left[\sum_{k=1}^{\infty} \delta^k \theta_{t+k} U(g_{t+k})\right] \tag{41}$$

as the continuation welfare at $t+1$ associated with assets x_{t+1} and the continuation sequence of interest rates and fiscal rules. Taking this continuation welfare as given, a fiscal rule at t can be represented as a cutoff type θ^*, where the government has full flexibility if $\theta_t \leq \theta^*$ and no flexibility if $\theta_t > \theta^*$. An individual government's optimal choice of fiscal rule is analogous to that in the two-period setting:

Proposition 7. *In an infinite horizon economy with i.i.d. shocks, the optimal uncoordinated fiscal rule is a time-invariant cutoff θ_u^* satisfying (9).*

Proof. Given a deterministic sequence of interest rates, an uncoordinated fiscal rule can be represented as a cutoff sequence $\theta_u^*(t, x_t)$, which depends on time t and the assets x_t with which a government enters the period. The dependence of the rule on time captures the fact that time indexes the future path of interest rates. Moreover, with some abuse of notation, we can let $g^f(\theta_t, t, x_t)$ correspond to type θ_t's flexible level of spending given time t and assets x_t. The government's

[34] If countries do not coordinate on rules, then rules of this form are optimal under i.i.d. shocks. See Halac and Yared (2014) for a discussion.

uncoordinated problem can be written recursively as:

$$\max_{\theta_u^*(t,x_t)\in[0,\overline{\theta}]} \left\{ \begin{array}{l} \int_{\underline{\theta}}^{\theta_u^*(t,x_t)} \left(\theta_t U(g^f(\theta_t,t,x_t)) + W_{t+1}(x_{t+1}^f(\theta_t,t,x_t))\right) f(\theta_t)d\theta_t \\ + \int_{\theta_u^*(t,x_t)}^{\overline{\theta}} \left(\theta_t U(g^f(\theta_u^*(t,x_t),t,x_t)) + W_{t+1}(x_{t+1}^f(\theta_u^*(t,x_t),t,x_t))\right) f(\theta_t)d\theta_t \end{array} \right\}$$

$$\text{subject to (40) and}$$

$$g^f(\theta_t,t,x_t) = \arg\max_g \{\theta_t U(g) + W_{t+1}(R_{t+1}(\tau + x_t - g))\}.$$

Standard arguments imply that W_{t+1} is a concave and continuously differentiable function of x_{t+1}. Hence, this problem is isomorphic to that of the two-period model, and by Proposition 1 the optimal choice of $\theta_u^*(t,x_t)$ satisfies (9). □

We next study the implications of a time-invariant coordinated rule θ^* for the interest rate.

Lemma 3. *Consider an infinite horizon economy with i.i.d. shocks and* $U(g_t) = \log(g_t)$. *If all countries are subject to a time-invariant rule* θ^* *in each period, the interest rate* R_t *is constant over time and satisfies*

$$R_t = R(\theta^*) = \left[\int_{\underline{\theta}}^{\theta^*} \frac{\beta\delta/(1-\delta)}{\theta + \beta\delta/(1-\delta)} f(\theta)\,d\theta + \int_{\theta^*}^{\overline{\theta}} \frac{\beta\delta/(1-\delta)}{\theta^* + \beta\delta/(1-\delta)} f(\theta)\,d\theta\right]^{-1}. \tag{42}$$

Proof. Under log preferences, (38) can be written as

$$\mathbb{E}\left[\theta_t \log(1-s_t) + \frac{\delta}{1-\delta}\log(s_t) + \sum_{k=1}^{\infty}\delta^k\left(\theta_{t+k}\log(1-s_{t+k}) + \frac{\delta}{1-\delta}\log(s_{t+k})\right)\right] + \chi(\theta_t,t,x_t), \tag{43}$$

where s_t is a savings rate satisfying

$$g_t = (1-s_t)\left(\tau + \sum_{k=1}^{\infty}\frac{\tau}{\prod_{l=1}^{k} R_{t+l}} + x_t\right)$$

and, using the above expression, $\chi(\theta_t, t, x_t)$ satisfies

$$\chi(\theta_t, t, x_t) = \theta_t \log\left(\frac{g_t}{1-s_t}\right) + \frac{\delta}{1-\delta}\log\left(\frac{g_t}{1-s_t}\right) + \sum_{k=1}^{\infty}\frac{\delta^k}{1-\delta}\log(R_{t+k}). \quad (44)$$

Analogously, (39) can be written as

$$\theta_t \log(1-s_t) + \beta\left\{\frac{\delta}{1-\delta}\log(s_t) + \mathbb{E}\left[\sum_{k=1}^{\infty}\delta^k\left(\theta_{t+k}\log(1-s_{t+k}) + \frac{\delta}{1-\delta}\log(s_{t+k})\right)\right]\right\} + \omega_t(x_t), \quad (45)$$

where $\omega(x_t)$ satisfies

$$\omega(x_t) = \theta_t \log\left(\frac{g_t}{1-s_t}\right) + \beta\left(\frac{\delta}{1-\delta}\log\left(\frac{g_t}{1-s_t}\right) + \sum_{k=1}^{\infty}\frac{\delta^k}{1-\delta}\log(R_{t+k})\right).$$

From (45), it is clear that if given full flexibility in all periods, then a government would choose in every period a savings rate equal to

$$s^f(\theta_t) = \frac{\beta\delta}{\theta_t + \beta\delta},$$

which is a function of θ_t and does not depend on future interest rates or current assets. Now consider a time-invariant fiscal rule θ^* in a T-period economy. The analog of (45) in a finite horizon setting implies that at date $T-1$, a country chooses its flexible savings rate if $\theta_{T-1} \leq \theta^*$ and the flexible savings rate that would correspond to type θ^* if $\theta_{T-1} > \theta^*$. It then follows by backward induction that $s(\theta_t, t, x_t) = \max\left\{s^f(\theta_t), s^f(\theta^*)\right\}$ at each $t \in \{0, \ldots, T-1\}$. Taking the limit of the T-period economy as $T \to \infty$, the global resource constraint at t can therefore be written as

$$\left[\int_{\underline{\theta}}^{\theta^*}\frac{\theta_t}{\theta_t + \beta\delta/(1-\delta)}f(\theta_t)\,d\theta_t + \int_{\theta^*}^{\overline{\theta}}\frac{\theta^*}{\theta^* + \beta\delta/(1-\delta)}f(\theta_t)\,d\theta_t\right]\left(\tau + \sum_{k=1}^{\infty}\frac{\tau}{\prod_{l=1}^{k}R_{t+l}}\right) = \tau,$$

where we have taken into account that savings rates are independent of assets and the sum of assets across countries is zero in each period. The fact that this

equation holds for all periods t implies (42). □

Consider now the class of rules $\theta^*(t)$ which are possibly time-varying but apply to all countries symmetrically, independently of their assets. We show that there is an optimal coordinated fiscal rule within this class which is time-invariant. Moreover, this rule satisfies our results in Proposition 2.

Proposition 8. *Consider an infinite horizon economy with i.i.d. shocks and* $U(g_t) = \log(g_t)$*, and take fiscal rules that apply symmetrically to all countries. There exists an optimal coordinated fiscal rule* θ_c^* *that is time-invariant. Moreover, there exist* $\overline{\beta}, \underline{\beta} \in [\underline{\theta}, 1]$*,* $\overline{\beta} > \underline{\beta}$*, such that if* $\beta \geq \overline{\beta}$*, then* $\theta_c^* < \theta_u^*$*, whereas if* $\beta \leq \underline{\beta}$*, then* $\theta_c^* > \theta_u^*$ *and* $\theta_c^* > \underline{\theta}$*.*

Proof. Using the same arguments as in the proof of Lemma 3, $s(\theta_t, t, x_t) = \max\left\{s^f(\theta_t), s^f(\theta^*(t))\right\}$ under a rule $\theta^*(t)$. Define

$$S(\theta^*(t)) = \left[\int_{\underline{\theta}}^{\theta^*(t)} \frac{\beta\delta/(1-\delta)}{\theta + \beta\delta/(1-\delta)} f(\theta)\, d\theta + \int_{\theta^*(t)}^{\overline{\theta}} \frac{\beta\delta/(1-\delta)}{\theta^*(t) + \beta\delta/(1-\delta)} f(\theta)\, d\theta\right].$$

Because savings rates are independent of assets, we can write the global resource constraint at t as

$$(1 - S(\theta^*(t))) \left(\prod_{m=0}^{t-1} S(\theta^*(m))\right) \left(\prod_{m=0}^{t} R_m\right) \left(\tau + \sum_{k=1}^{\infty} \frac{\tau}{\prod_{l=1}^{k} R_l}\right) = \tau, \tag{46}$$

where $R_0 = 1$. Substituting (46) in (44) yields

$$\begin{aligned}\chi(\theta_0, 0, 0) \;=\; & -\theta_0 \log(1 - S(\theta^*(0))) - \frac{\delta}{1-\delta} \log(S(\theta^*(0))) \\ & - \sum_{t=1}^{\infty} \delta^t \log(1 - S(\theta^*(t))) - \sum_{t=1}^{\infty} \frac{\delta^t}{1-\delta} \log(S(\theta^*(t))) + \left(\theta_0 + \frac{\delta}{1-\delta}\right) \log \tau.\end{aligned}$$

Given (43), we can write welfare at date 0 as a function of the rule $\theta^*(t)$ as

$$\sum_{t=0}^{\infty}\delta^t\left[\begin{array}{c}\int_{\underline{\theta}}^{\theta^*(t)}\left(\theta\log\left(\frac{\theta}{\theta+\beta\delta/(1-\delta)}\right)+\frac{\delta}{1-\delta}\log\left(\frac{\beta\delta/(1-\delta)}{\theta+\beta\delta/(1-\delta)}\right)\right)f(\theta)\,d\theta\\+\int_{\theta^*(t)}^{\overline{\theta}}\left(\theta\log\left(\frac{\theta^*(t)}{\theta^*(t)+\beta\delta/(1-\delta)}\right)+\frac{\delta}{1-\delta}\log\left(\frac{\beta\delta/(1-\delta)}{\theta^*(t)+\beta\delta/(1-\delta)}\right)\right)f(\theta)\,d\theta\\-\log\left(1-S\left(\theta^*(t)\right)\right)-\frac{\delta}{1-\delta}\log\left(S\left(\theta^*(t)\right)\right)+\log\tau\end{array}\right].$$

Note that the term in the bracket is the same for every t, which implies that there exists a solution with a time-invariant cutoff $\theta^*(t)=\theta_c^*$. Moreover, this bracket is identical to the two-period program in (31) except that δ is replaced with $\frac{\delta}{1-\delta}$ (and there is the last term which is a constant). The results therefore follow from Proposition 2. □