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TO NEWS SHOCKS

Paul Beaudry  
Patrick Fève  
Alain Guay  
Franck Portier

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### **ABSTRACT**

When the VAR representation of a times series has a non-fundamental representation, standard SVAR techniques cannot be used to exactly identify the effects of structural shocks. This problem is known to potentially arise when one of the structural shocks represents news about the future. However, as we shall show, in many cases the non-fundamental representation of a time series may be very close to its fundamental representation implying that standard SVAR techniques may provide a very good approximation of the effects of structural shocks even when the non-fundamentalness is formally present.

This leads to the question: When is non-fundamentalness a real problem? In this paper we derive and illustrate a diagnostic based on a  $\chi^2$  test which provides a simple means of detecting whether non-fundamentalness is likely to be a quantitatively important problem in an applied setting. We use the identification of technological news shocks in US data as our running example.

Paul Beaudry  
Vancouver School of Economics  
University of British Columbia  
997-1873 East Mall  
Vancouver, B.C.  
Canada, V6T 1Z1  
and University of British Columbia  
and also NBER  
paulbe@interchange.ubc.ca

Patrick Fève  
Toulouse School of Economics  
21, Allée de Brienne  
31000 Toulouse, France  
patrick.feve@tse-fr.eu

Alain Guay  
Université du Québec à Montréal  
Dép. Sc. Économique, UQAM  
CP 8888 Centre ville, Montréal, Canada  
H3C 3P8  
guay.alain@uqam.ca

Franck Portier  
Toulouse School of Economics  
University of Toulouse  
Manufacture des Tabacs  
21 Allée de Brienne  
31000 Toulouse, FRANCE  
and CEPR  
franck.portier@tse-fr.eu

# When is Nonfundamentalness in VARs A Real Problem? An Application to News Shocks.

Paul Beaudry\*, Patrick Fève†, Alain Guay‡  
and Franck Portier§¶

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## Abstract

When a structural model has a nonfundamental VAR representation, standard SVAR techniques cannot be used to properly identify the effects of structural shocks. This problem is known to potentially arise when one of the structural shocks represents news about the future. However, as we shall show, in many cases the nonfundamental representation of a time series may be very close to its fundamental representation implying that standard SVAR techniques may provide a very good approximation of the effects of structural shocks even when the nonfundamentalness is formally present. This leads to the question: When is nonfundamentalness a real problem? In this paper we derive and illustrate a diagnostic based on a  $R^2$  which provides a simple means of detecting whether nonfundamentalness is likely to be a quantitatively important problem in an applied settings. We use the identification of technological news shocks in US data as our running example.

**Key Words :** News, Business Cycles, Nonfundamentalness, SVARs

**JEL Class. :** E3

## 1 Introduction

Nonfundamentalness in times series arises when the economic variables used by an econometrician do not contain enough information to recover the economy's structural shocks. In such a case, standard Structural VAR (SVAR) techniques cannot be used to properly recover

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\*Vancouver School of Economics, University of British Columbia and NBER. (paulbe@mail.ubc.ca)

†Toulouse School of Economics (patrick.feve@TSE-fr.eu)

‡Université du Québec À Montréal, CIRPÉ and CIREQ (guay.alain@uqam.ca)

§Toulouse School of Economics and CEPR (franck.portier@TSE-fr.eu)

¶We benefited from discussion with Mario Forni, Luca Gambetti, Morten Ravn and Luca Sala. Forni, Gambetti and Sala kindly shared their codes and data with us.

structural disturbances since the economic variables do not allow for a VAR representation where the residuals are linear combinations of the underlying structural shocks.<sup>1,2</sup> Lütkepohl [2012], among others, has pointed out that such a nonfundamentalness problem is quite likely to arise in the presence of new shocks, or in other words, in situation where agents actions reflect news regarding future events. More recently, Forni and Gambetti [2014] have proposed a test for nonfundamentalness. This is a test of the sufficiency of the econometrician’s information set to identify some structural shocks. In follow up work, Forni, Gambetti, and Sala [2014] used this sufficient information test to explore whether the nonfundamentalness is present in simple SVARs used of identify news shocks.<sup>3</sup>

Nonfundamentalness is clearly an important concept in time series econometrics, but is it quantitatively relevant in applied problems? That is, even if a time series has only a nonfundamental VAR representation, does it mean that SVAR techniques applied to the system are un-informative about structural shocks of interest? In this paper we will *(i)* illustrate that even in the presence of nonfundamentalness, SVAR techniques can be very informative about structural shocks as the fundamental representation may be close to the nonfundamental representation implying that the nonfundamentalness problem may cause only minor bias in the estimation of structural impulses <sup>4</sup> and *(ii)* propose a simple diagnostic based on an  $R^2$  that helps determine whether or not nonfundamentalness, even when it is found to be present, is likely to alter substantially the results obtained from SVAR

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<sup>1</sup>The first critiques of structural VAR models on the basis of potential nonfundamentalness are Hansen and Sargent [1991] and Lippi and Reichlin [1993].

<sup>2</sup> As shown in Beaudry and Portier [2014], on a qualitative ground, a model with news shocks may give rise to a nonfundamental representation but does not necessarily do so. It depends on the precise properties of the news process as well as the variables available to the econometrician.

<sup>3</sup> In order to explore the importance of nonfundamentalness for the identification of news shocks, Forni, Gambetti, and Sala [2014] use a dataset composed of 107 US quarterly macroeconomic series and estimate its principal components. They begin by identifying news shocks following the identification strategy proposed in Barsky and Sims [2011] and show that the resulting estimated shocks are not orthogonal to the estimated principal components, which is evidence for nonfundamentalness. They then explain why estimating an augmented VAR that includes principal components from a large data set offers a means of overcoming a nonfundamentalness problem.

<sup>4</sup> We are not the first to recognize that standard SVAR methods may work well in identifying shocks even when the underlying system suffers from nonfundamentalness. This point has been previously illustrated in Sims [2012] which looks at the issue in a quantitatively reasonable DSGE model. Beaudry, Portier, and Seymen [2013] have also shown that the two prominent structural VAR approaches in the news literature (Beaudry and Portier [2006] and Barsky and Sims [2011]) are in general capable of recuperating news shocks dynamics from artificially generated data.

techniques. In particular, we will show that a sufficient information test will generally detect significant nonfundamentalness even when its economic importance is very minor. We will also show when the  $R^2$  associated with the aforementioned sufficient information test is a better indication of the quantitative relevance of the nonfundamentalness problem than the significance level of the test itself.

The paper will begin by illustrating, using a simple example, why nonfundamentalness should be viewed as a quantitative issue not an either/or issue with respect to its implications for SVAR exercises. To do so, we exploit a simple Lucas' tree model with news. We will show that for reasonable discount factors nonfundamentalness, while present, is not a quantitative problem in this setup. Nonetheless, we show that a sufficient information test for nonfundamentalness will detect significant nonfundamentalness. This example will help motivate our interest in the  $R^2$  associated with their test as a measure of the relevance of nonfundamentalness. We then turn to formally showing when the  $R^2$  from this test is indeed a measure of the bias in the identification of structural shocks, which will explain the results we have obtained with the simulated Lucas' tree model. We will show that the relative bias in recovering the true structural shocks is of the order of half the  $R^2$  of the projection of the misspecified structural shocks on the true ones. Finally, we will complete the exercise by examining whether the identification of technological news shocks is likely subject to a quantitatively important nonfundamentalness problem. To do so, we replicate the procedure suggested by Forni, Gambetti, and Sala [2014] of including the principal components extracted from a large data set into an SVAR as a means of overcoming the potential nonfundamentalness problem. To identify technological news shocks, we use a procedure that extends the logic of Beaudry and Portier [2006], with a new one identified as the only shock that can have a permanent effect on TFP but that does not have an impact effect on TFP. It has been argued elsewhere that this procedure, while different to that used by Forni, Gambetti, and Sala [2014], provides a robust way of identifying technological news shocks (see Beaudry and Portier [2014] and Portier [2014]).<sup>5</sup> Our finding is that nonfundamentalness is not a

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<sup>5</sup>The object of this work is not to debate whether our preferred strategy for the identification of news shocks is better or not to that proposed in Barsky and Sims [2011] and used by Forni, Gambetti, and Sala [2014]. The issue is whether previous results which have suggested the importance of news shocks in business cycles could have been misleading due to the problem of nonfundamentalness. For this argument to

serious problem in the identification of technological news shocks, even though there may remain debate on how best to use SVAR techniques to identify the effects of news.<sup>6</sup>

## 2 On the severity of the nonfundamentalness problem in a stylized model

In this section, we simulate the simple Lucas’ tree model with news presented in Forni, Gambetti, and Sala [2014]. In this model, nonfundamentalness is always present. We use this model to show that for modest to high values of the discount factor, the nonfundamentalness is not quantitatively important.

### 2.1 A simple Lucas’ tree model with news

Consider an endowment economy with one representative tree and one representative household. The tree produces each period a dividend  $\Theta_t$ , that we call TFP. The process of  $\Theta$  has a news component  $\nu$  and a surprise one  $u$ :

$$\Theta_t = \Theta_{t-1} + \nu_{t-2} + u_t,$$

where  $\nu_t$  and  $u_t$  are gaussian with unit variance.<sup>7</sup> Preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t c_t,$$

such that the equilibrium price of a tree, that we call SP for stock prices, is given, after solving the model, by

$$SP_t = \frac{\beta}{1-\beta} \Theta_t + \frac{\beta}{1-\beta} (\beta \nu_t + \nu_{t-1}).$$

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be relevant it should be applied to identification strategies that claim to provide evidence in support of news shocks (as the one we use here), not to an identification strategy such as used by Barsky and Sims [2011] that have suggested that such shocks may be rather unimportant even when disregarding the nonfundamentalness problem. See Beaudry, Nam, and Wang [2011] for an in depth discussion of the robustness of Barsky and Sims’s [2011] results.

<sup>6</sup> Using our chosen identification strategy, and using the factors of Forni, Gambetti, and Sala [2014], we first show that the identified news shocks do not pass the fundamentalness test they proposed. However, we find that the  $R^2$  of this regression is small. Accordingly, when we re-estimate our VARs with the principal components included, we find that the impulse responses associated with news shocks are not significantly modified. In particular the news shock obtained after including the principal components to the VAR continue to generate business cycle type fluctuations, as consumption, hours and investment increase in the short run, while TFP increases only in the long run.

<sup>7</sup>Our results are robust to a change of the TFP process (longer or shorter news) and to the relative size of the variances.

The structural moving average representation of the solution is given by

$$\begin{pmatrix} \Delta\Theta_t \\ \Delta SP_t \end{pmatrix} = \begin{pmatrix} L^2 & 1 \\ \frac{\beta^2}{1-\beta} + \beta L & \frac{\beta}{1-\beta} \end{pmatrix} \begin{pmatrix} \nu_t \\ u_t \end{pmatrix}.$$

The determinant of the moving average coefficients matrix is

$$-\frac{\beta^2}{1-\beta} - \beta L + \frac{\beta}{1-\beta} L^2.$$

The roots of that determinant are 1 and  $-\beta$ . We assume that only current and past values of TFP and SP are observed by the econometrician. As  $|\beta| < 1$ , the shocks  $\nu_t$  and  $u_t$  are nonfundamental for the variables  $\Delta\Theta_t$  and  $\Delta SP_t$ , and cannot be recovered by an econometrician. Note that when  $\beta$  goes to one, in economic terms when the stock price weights future  $\Theta$ s a lot, we get closer to the fundamental case. What do we mean by “closer”? The metric we will consider is the distance between the estimated impulse response functions to the identified news shock  $\tilde{\nu}_t$  and their theoretical counterpart. <sup>8</sup>

## 2.2 Simulation results

We simulate the above model, and recover the observables TFP and SP. We then estimate a bivariate VAR and use the simple Beaudry and Portier [2006] short run identification scheme. The news is identified as the shock that has no effect on impact on TFP. We then compare the IRF of TFP and SP to a news shock, and perform the Forni and Gambetti’s [2014] sufficient information test for nonfundamentalness. In our context, the test consists in projecting  $\tilde{\nu}_t$  (the identified news shock) on a constant and the past of the true  $\nu_t$  and  $u_t$  that have been used to generate the data. As the shocks are *i.i.d.* and orthogonal one to each other, the  $R^2$  of that regression should be 0 if  $\tilde{\nu}_t$  was indeed equal to  $\nu_t$ .

We consider three values for  $\beta$  : .01, .7 and .99. The nonfundamentalness problem is expected to be “less severe” when  $\beta$  is large, as agents react more strongly to news shocks.

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<sup>8</sup>This illustrate a result found in more general linear rational expectations models by Ljungqvist and Sargent [2004] and Mertens and Ravn [2010]. In such models, the effect of news shocks on the MA representation of the model solution involves a parameter referred to as the anticipation rate. The anticipation rate measures the rate at which rational forward looking agents discount future innovations. In this Lucas’ tree model, this rate depends only on discount rate  $\beta$ . In more general models (see for example Mertens and Ravn [2010]), it involves other parameters, such as intertemporal elasticity of substitution (which is one here), Frisch elasticity of labour supply, capital share, etc...

We consider estimations over  $T = 250$  periods (result are similar with  $T = 1000$ ). The VAR has five lags. For the test for nonfundamentalness, we project  $\tilde{\nu}_t$  on a constant and one or four lags of  $\nu_t$  and  $u_t$ . For each of those experiments, we run 10,000 simulations.

In Figures 1 to 3, we display the 95% bands and the median for the estimated responses of TFP and SP to an identified news shock, together with the theoretical response. The visual inspection of these IRF is a first indicator of nonfundamentalness severity. We also present more formal measures. First, we plot the empirical density of the p-value of the nonfundamentalness test. This test is the F-test that the coefficients of  $\nu_t$  and  $u_t$  lags are all null. Second, we plot the empirical density of the  $R^2$  over the 10,000 simulations for each model. The  $R^2$  is the one of the regression of  $\tilde{\nu}_t$  on a constant the past of  $\nu_t$  and  $u_t$ . Note that the p-value will answer the question “Is there nonfundamentalness?” while the size of the  $R^2$  will be more indicative of another question, namely “Does it matter?”. Results are presented in Figures 1 to 3 and are very telling.

When  $\beta = .1$  (Figure 1), nonfundamentalness is a real problem. Estimated IRF are far from the theoretical ones (what matters are the two first periods, as the model responses are flat after and therefore quite easy to catch with the VAR). The p-value of the nonfundamentalness test are always equal to zero: fundamentalness is always rejected. Finally,  $R^2$ s are close to 1: estimated shocks can be very well predicted by current theoretical innovations: nonfundamentalness does matter.

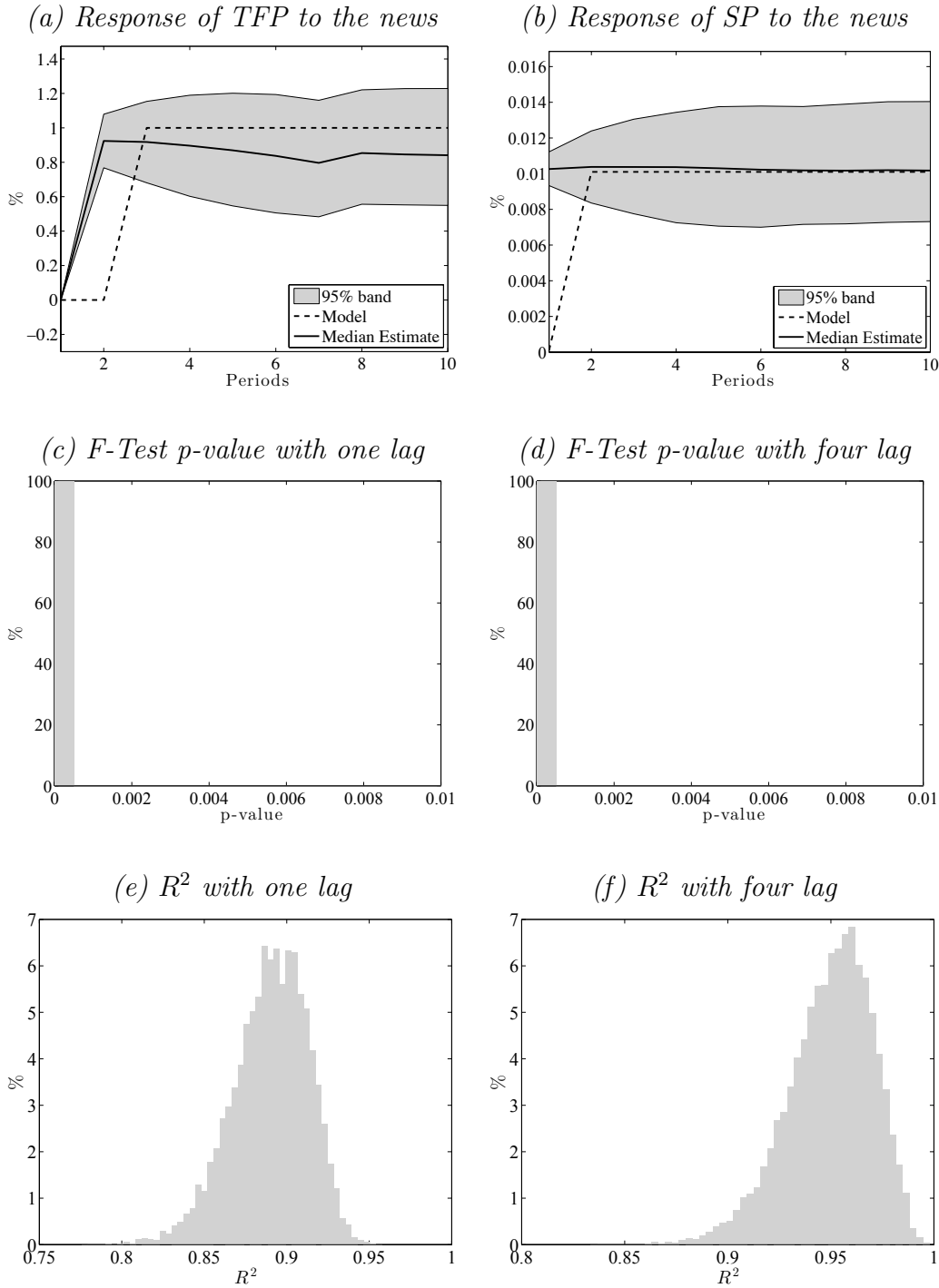
With a more realistic value of  $\beta = .99$  typically used in quarterly models, SP data put much more weight on expectations of  $\Theta$  in the data generating model. As shown in Figure 2, the theoretical nonfundamentalness now has little quantitative bite: estimated IRF are very close to that implied by the model. Nevertheless, the theoretical news shock is found nonfundamental when performing the F-test with four lags (although not with one lag). But even if fundamentalness is rejected,  $R^2$ s are less than 0.1, which suggests that nonfundamentalness may not matter quantitatively, and which is confirmed by the inspection of the IRF.

Finally, Figure 3 shows a case where  $R^2$ s are around .3. This is obtained for  $\beta = .7$ . The IRF are very well captured, except for a blip in TFP in period 2. Nevertheless, testing for



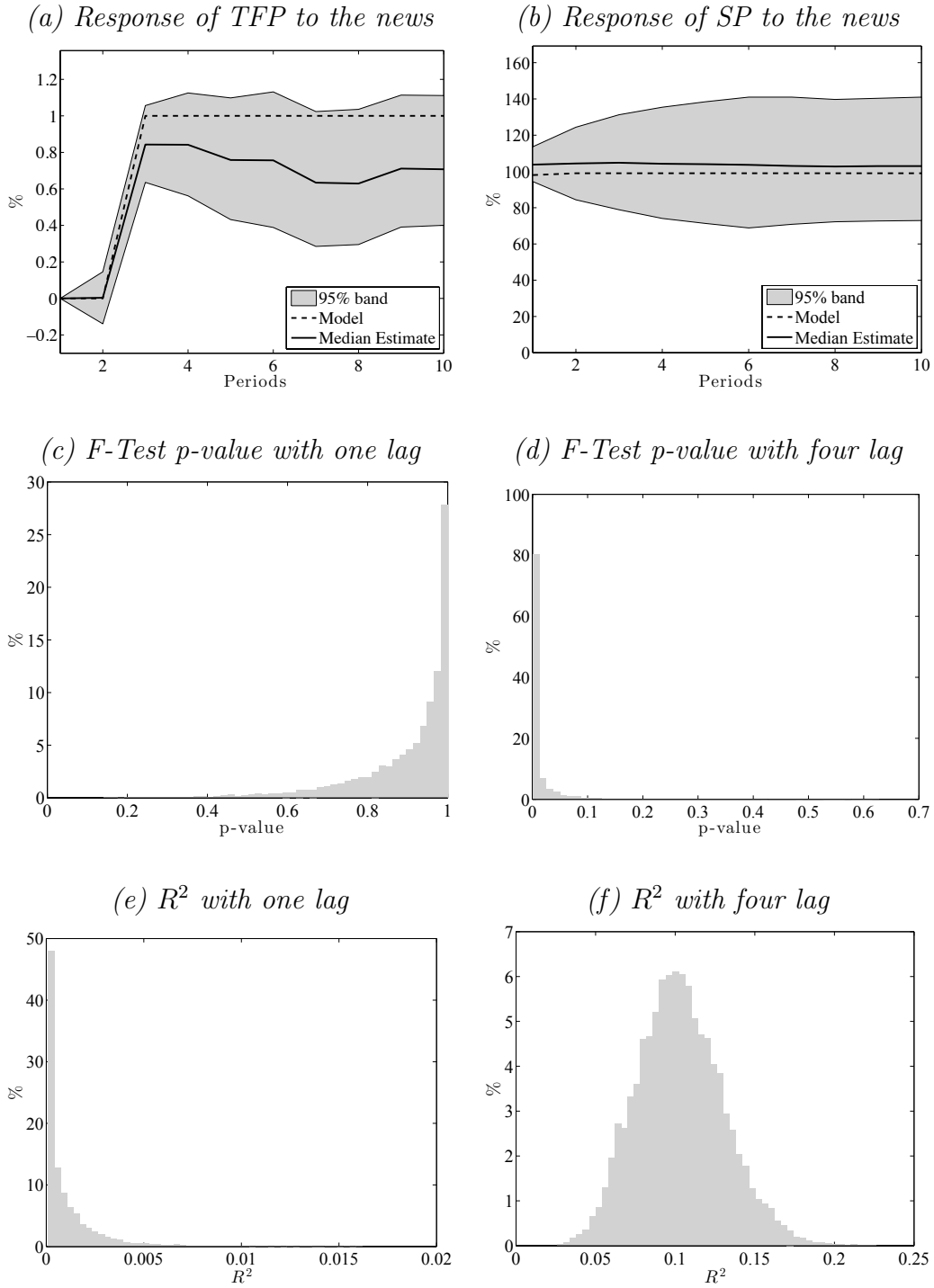
fundamentalness will systematically reject it, as the p-values of the F-test are always less than 1%, and are actually zero more than 95% of the simulations. How can we explain that IRF are so well estimated and nevertheless fundamentalness rejected? This arises because the  $R^2$ s of the regression of  $\tilde{\nu}_t$  on past values of the  $\nu_{ts}$  are small (around .1 with one lag, around .3 with four lags).

Figure 1: A case with serious nonfundamentalness ( $\beta = .1$ )



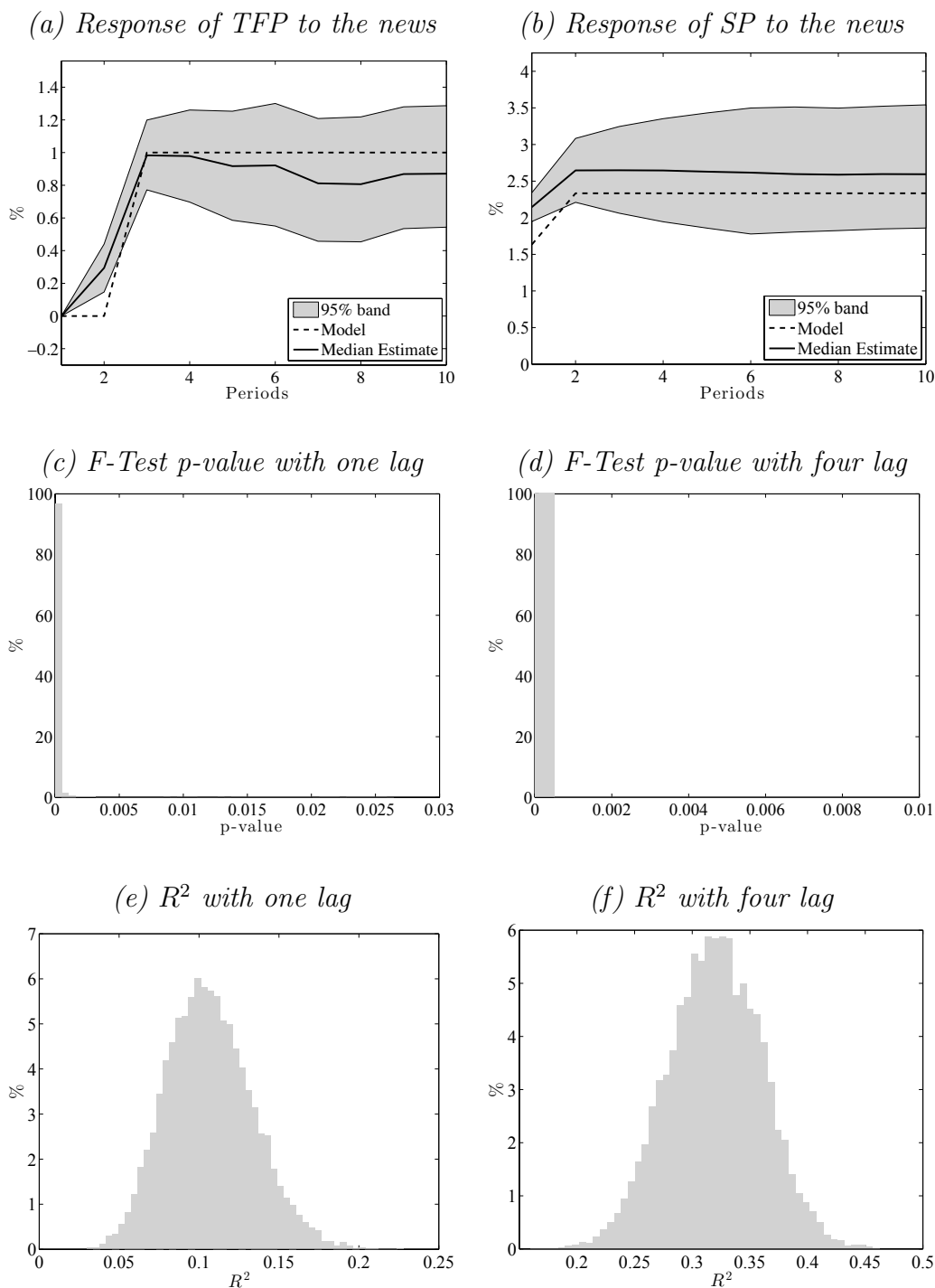
This figures are derived from the simulation of a simple Lucas' tree model with news. In panel (a) and (b) are represented the theoretical responses to a news shock (dashed lines) together with the median and 95% confidence interval obtained from the repeated estimation (10,000 replications) of a VAR 2 impulse response to the shock that does not affect TFP on impact. The estimated news shock is then projected on one or four lags of the two theoretical shocks, and we test for orthogonality. Panels (c) and (d) show the distribution of the p-value for the orthogonality test. Panels (e) and (f) show the distribution of the  $R^2$ s of those regressions.

Figure 2: A case with no serious nonfundamentalness ( $\beta = .99$ )



This figures are derived from the simulation of a simple Lucas' tree model with news. In panel (a) and (b) are represented the theoretical responses to a news shock (dashed lines) together with the median and 95% confidence interval obtained from the repeated estimation (10,000 replications) of a VAR 2 impulse response to the shock that does not affect TFP on impact. The estimated news shock is then projected on one or four lags of the two theoretical shocks, and we test for orthogonality. Panels (c) and (d) show the distribution of the p-value for the orthogonality test. Panels (e) and (f) show the distribution of the R<sup>2</sup>s of those regressions.

Figure 3: A case with  $R^2$  around .2 : nonfundamentalness is not much of a problem ( $\beta = .7$ )



This figures are derived from the simulation of a simple Lucas' tree model with news. In panel (a) and (b) are represented the theoretical responses to a news shock (dashed lines) together with the median and 95% confidence interval obtained from the repeated estimation (10,000 replications) of a VAR 2 impulse response to the shock that does not affect TFP on impact. The estimated news shock is then projected on one or four lags of the two theoretical shocks, and we test for orthogonality. Panels (c) and (d) show the distribution of the p-value for the orthogonality test. Panels (e) and (f) show the distribution of the  $R^2$ s of those regressions.

### 3 A $R^2$ diagnosis for the severity of nonfundamentalness

In this section, we propose a theoretical explanation for our simulation results. In a Factor Augmented VAR (FAVAR) model, we consider a misspecified VAR representation that would omit the factors. Using that misspecified model to identify structural shocks, we show that the bias in recovering these shocks is of the size of the  $R^2$  of the projection of (misspecified) structural shocks on the past of the factors, which corresponds to the regression proposed by Forni and Gambetti [2014]. We also show that a small  $R^2$  is compatible with a clear rejection of fundamentalness. In such a case, nonfundamentalness is of little quantitative importance.

#### 3.1 The Econometric Setup

Assume that data are generated according to the following FAVAR(1) model,<sup>9</sup> labeled as  $\mathcal{M}_0$ :

$$\begin{aligned} Y_t &= B_y Y_{t-1} + B_f f_{t-1} + \epsilon_{y,t}, \\ f_t &= C_f f_{t-1} + \epsilon_{f,t}, \end{aligned} \tag{\mathcal{M}_0}$$

where the vector  $Y_t$  contains  $n$  variables of interest and  $f_t$  is a vector of (observed) relevant  $q$  factors. As usual, we assume that the variance of each factor is normalized to unity.<sup>10</sup> Our goal is to assess the quantitative effects of omitting the relevant set of factors  $f_t$  at the estimation stage and thus when identifying the true structural shocks. The analysis is then similar to a standard omitting variable problem in linear regression.

Assume that there is a unique linear transformation that maps innovations  $\epsilon_{y,t}$  into structural shocks  $\eta_t$  according to

$$\epsilon_{y,t} = A_0 \eta_t,$$

where  $A_0$  is a non-singular matrix. As usual, we impose the normalization assumption that  $E(\eta_t \eta_t') = I_n$ . This orthogonality/normalization assumption is not sufficient (except if  $n = 1$ ) to identify the structural shock, since  $E(\epsilon_{y,t} \epsilon_{y,t}') = A_0 A_0'$  is symmetric. At least  $n(n - 1)/2$

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<sup>9</sup>For the clarity of the presentation, we consider a FAVAR model with one lag only. Results can be easily extended to a more general lags structure.

<sup>10</sup>This representation adds factors in a VAR representation of the data and thus differs from a more general representation of dynamic factor models (see Stock and Watson [2005]).

restrictions have to be imposed to identify  $A_0$ . To fix ideas, we assume that identification is achieved by imposing point restrictions in the form

$$R \text{vec}(A_0) = r, \tag{1}$$

where  $R$  is a  $(m \times n^2)$  selection matrix and  $r$  a  $(m \times 1)$  vector of  $m$  restrictions.<sup>11</sup> Combining with the covariance matrix, these additional restrictions allow to identify each elements of  $A_0$ .  $r = 0$  corresponds to a case of zero impact restriction, which is often assumed in the SVAR literature. When  $n = 2$ , a single zero restriction in  $A_0$  is sufficient to uncover shocks. This is for example the case of Beaudry and Portier [2006], in which technological news are identified by imposing that they have no contemporaneous effect on the level of TFP. In what follows, we do not need to be explicit about the identifying restrictions  $R$ , and will keep the matrix  $A_0$  unspecified.

### 3.2 The misspecified model

We assume that the factors are not used and/or observed by the econometrician, who therefore estimates the following VAR(1) model  $\mathcal{M}_1$ :

$$Y_t = \tilde{B}_y Y_{t-1} + \tilde{\epsilon}_{yt}, \tag{\mathcal{M}_1}$$

whereas  $\mathcal{M}_0$  constitutes the Data Generating Process of  $Y_t$ . We further assume that the econometrician uses the restriction (1) to identify the structural shocks.

This model improperly ignores the role played by the factors  $f_t$ . We are in a typical case of missing relevant variables in VARs.<sup>12</sup> The omitted variables problem will affect the misspecified VAR model  $\mathcal{M}_1$  in various ways. First, by omitting the factor  $f_{t-1}$ , the VAR(1) model will not properly uncover the size of the shock, because part of the identified structural shocks will be polluted by the missing factors  $f_{t-1}$ . Second, the omitting factor  $f_{t-1}$  will affect the dynamics of  $y_t$  and the matrix  $\tilde{B}_y$  does not properly summarize the true dynamic structure of the economy. Third, the covariance structure of the variables  $y_t$  and  $f_t$  can affect the proper measurement of the auto-regressive matrix  $\tilde{B}_y$  at the estimation stage. In what follows, we explicitly measure the bias introduced by these elements.

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<sup>11</sup>These restrictions can be generalized to other identification schemes.

<sup>12</sup>See Stock and Watson [2001], [2005], Canova [2006] and Lütkepohl [2005] for a discussion of this issue.

The restricted structural shocks (the ones obtained from  $\mathcal{M}_1$ ) are denoted  $\tilde{\epsilon}_{yt} = \tilde{A}_0 \tilde{\eta}_t$ . We impose the same normalization assumption  $E(\tilde{\eta}_t \tilde{\eta}_t') = I_n$  and the same additional restrictions

$$R \text{vec} \left( \tilde{A}_0 \right) = r,$$

where  $R$  and  $r$  are the same as in model  $\mathcal{M}_0$ . Denoting  $\tilde{\Sigma} = E(\tilde{\epsilon}_{yt} \tilde{\epsilon}_{yt}')$  and  $\Sigma = E(\epsilon_{y,t} \epsilon_{y,t}')$ , we deduce  $\tilde{A}_0 \tilde{A}_0' = \tilde{\Sigma} \geq \Sigma = A_0 A_0'$  in the matrix sense and  $\|\tilde{A}_0\| \geq \|A_0\|$ , because the canonical residual omits the factor  $f_{t-1}$ . We now examine in more details the effects of omitting  $f_{t-1}$  in the estimation of model  $\mathcal{M}_1$  and for the identification of structural shocks.

### 3.3 Consequences for identification

The vector of the residuals from the estimation of model  $\mathcal{M}_1$  is given by

$$\widehat{\tilde{\epsilon}}_y = M_Y \tilde{\epsilon}_y = M_Y Y,$$

where  $\tilde{\epsilon}_y$  is the  $T \times n$  of error terms for each of the  $n$  equations,  $Y$  is the  $T \times n$  matrix of the corresponding  $Y_t$  and  $M_Y = I - Y_{-1} (Y_{-1}' Y_{-1})^{-1} Y_{-1}'$  is the orthogonal projection matrix to  $Y_{-1}$ , *i.e.* the matrix containing the lagged values of  $Y$ . Using model  $\mathcal{M}_0$  and the same notations, we deduce

$$M_Y Y = M_Y F_{-1} B_f' + M_Y \epsilon_y.$$

This implies for the estimated misspecified structural shocks

$$\widehat{\tilde{\eta}} \widehat{\tilde{A}}_0' = M_Y F_{-1} B_f' + M_Y \epsilon_y.$$

Since  $\widehat{\tilde{\epsilon}}_{yt} = \tilde{\epsilon}_{yt} + o_p(1)$ ,  $\widehat{\tilde{\eta}} = \tilde{\eta} + o_p(1)$  and  $\widehat{\tilde{A}}_0 = \tilde{A}_0 + o_p(1)$ , we obtain <sup>13</sup>

$$\tilde{\eta} \tilde{A}_0' = M_Y F_{-1} B_f' + M_Y \epsilon_y + o_p(1).$$

Since  $M_Y \epsilon_y = \epsilon_y + o_p(1)$ , we can write

$$\tilde{\eta} = M_Y F_{-1} B_f' \left( \tilde{A}_0' \right)^{-1} + \epsilon_y \left( \tilde{A}_0' \right)^{-1} + o_p(1),$$

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<sup>13</sup>This holds for each element of the matrix  $\tilde{A}_0$ . The expression  $o_p(1)$  means that this term converges in probability to zero

where  $F_{-1}$  is a  $T \times q$  matrix containing the lagged values of the factors. Using the linear relation between the canonical residuals and the structural shocks, this finally yields

$$\tilde{\eta} = M_Y F_{-1} B'_f \left( \tilde{A}'_0 \right)^{-1} + \eta A_0 \left( \tilde{A}'_0 \right)^{-1} + o_p(1).$$

Now suppose for the sake of exposition that we are interested in one specific structural shock  $\eta_{it}$ . Using above expression, one gets

$$\tilde{\eta}_{it} = e'_i \tilde{\eta}_t \equiv e'_i \tilde{A}_0^{-1} B_y \hat{f}_{t-1} + e'_i \tilde{A}_0^{-1} A_0 \eta_t + o_p(1), \quad (2)$$

with  $e_i$  a selecting vector that is composed of zeros and one at the  $i$ th element. The variable  $\hat{f}_{t-1}$  is obtained from

$$\hat{f}'_{t-1} = f'_{t-1} - Y'_{t-1} \left( \sum_{t=2}^T Y_{t-1} Y'_{t-1} \right)^{-1} \sum_{t=2}^T Y_{t-1} f'_{t-1}.$$

We can rewrite equation (2) under the form

$$\tilde{\eta}_{it} = e'_i \tilde{\eta}_t = \delta'_i \hat{f}_{t-1} + e'_i \tilde{A}_0^{-1} A_0 \eta_t + o_p(1). \quad (3)$$

We now define  $v_{it} = e'_i \tilde{A}_0^{-1} A_0 \eta_t$ . In a matrix form, equation (3) rewrites:

$$\tilde{\eta}_i = M_Y F_{-1} \delta_i + v_i + o_p(1).$$

### 3.4 Testing for nonfundamentality

Testing for nonfundamentality can be achieved by regressing the structural shock of interest  $\tilde{\eta}_i$  on the lags of the factors and then perform a Granger causality test on equation (3), as proposed by Forni and Gambetti [2014]. As  $Y_{-1}$  is correlated with the factors  $F_{-1}$  (a natural result, because factors are extracted from macroeconomic variables in  $y_t$ ), this does not yield a consistent estimator of  $\delta_i$ . Therefore, we regress the structural shocks  $\tilde{\eta}_i$  on the lags of the factors orthogonal to  $Y_{t-1}$ , namely  $M_Y F_{-1}$ .

The corresponding Wald statistic  $W_T$  for the Granger Causality test is:

$$W_T = \frac{\hat{\delta}'_i (F'_{-1} M_Y F_{-1}) \hat{\delta}_i}{\hat{\sigma}_{vi}^2},$$



where  $\hat{\delta}_i$  is the consistent estimator of  $\delta_i$  and  $\hat{\sigma}_{v_i}^2$  is the estimator of the variance of  $v_{it}$ , i.e. the error term of the regression of  $\hat{\eta}_{it}$  on  $\hat{f}_{t-1}$ . The coefficient of determination  $R_i^2$  associated to the linear regression of  $\hat{\eta}_{it}$  on  $\hat{f}_{t-1}$  is given by:

$$R_i^2 = \frac{\hat{\delta}_i' (F_{-1}' M_Y F_{-1}) \hat{\delta}_i}{\hat{\delta}_i' (F_{-1}' M_Y F_{-1}) \hat{\delta}_i + \hat{v}_i' \hat{v}_i}.$$

Using  $\hat{v}_i' \hat{v}_i = T \times \hat{\sigma}_{v_i}^2$ , we obtain the following proposition:

**Proposition 1** *For a given sample size  $T$ , the following relation holds between the Wald statistics  $W_T$  and the  $R^2$  of the projection of the (misspecified) structural shocks  $\tilde{\eta}_i$  on the lags of the factors orthogonal to  $Y_{t-1}$ :*

$$W_T = T \frac{R_i^2}{(1 - R_i^2)}.$$

The Wald statistic is composed of two terms. The first term  $T$  (the size of the sample) refers to the precision of the estimation, since the covariance matrix of the factors has been normalized to identity. As the sample size increases, the precision of the estimate, i.e. the inverse of its variance, becomes larger. The second term  $R^2$  accounts for the explanatory power of the factors.

**Corollary 1** *Suppose that the Wald statistics is above its critical value, so that the test rejects the fundamentalness of the residuals (or of identified structural shocks from the wrong model). Such a rejection is compatible with arbitrarily low level of the  $R^2$ , and therefore with little quantitative importance on the nonfundamentalness problem, as long as the sample size  $T$  is large enough.*

Let us illustrate Corollary 1. Consider a single factor ( $q = 1$ ) in the regression and a sample of size  $T = 200$ , as very usual in applied time series macroeconomics. In this case, the limiting distribution of the Wald statistic under the hypothesis  $B_f = 0$  is a chi-square statistic with one degree of freedom. Its critical value at 5% is 3.84. This implies an associated critical  $R^2$  equals to 0.0192. In words, it is possible to reject fundamentalness even though the factor  $F_{-1}$ <sup>14</sup> explain only less than 2% of the variance of the identified structural shock.

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<sup>14</sup>More precisely  $M_Y F_{-1}$  the orthogonal projection of the factor on the past of  $Y$ .

Proposition 2 formalizes the relationship between the  $R^2$  of the projection of (misspecified) structural residuals and the distance to the true model. <sup>15</sup>

**Proposition 2** *The  $R^2$  statistics we have constructed is :*

(i) *a consistent estimator of the distance between the misspecified impact matrix of structural shocks  $\tilde{A}_0$  and the true one  $A_0$ . Indeed, when  $R^2$  is small,  $\tilde{A}_0$  is close to  $A_0$  and at the limit when  $R^2 \rightarrow 0$ ,  $\tilde{A}_0$  tends to the true one  $A_0$ .*

(ii) *a consistent estimator of the distance between the misspecified variance decomposition on impact and the true one.*

The meaning of Proposition 2 is that if  $R^2$  is small, the distance between the two models is small even if the Wald test rejects fundamentalness.

### 3.5 Characterization of biases in the canonical bivariate model of Beaudry and Portier [2006]

Consider the identification of technological news shocks in the bivariate model with Total Factor Productivity (*TFP*) and a measure of Stock Prices (*SP*). Following Beaudry and Portier [2006], the technological news  $\eta_{2,t}$  is the shock that is orthogonal to current *TFP*. The true model is  $\mathcal{M}_0$  with  $Y = (TFP, SP)'$ , while the econometrician is estimating  $\mathcal{M}_1$ . According to the structural assumption,  $A_0$  is lower triangular and given by

$$A_0 = \begin{bmatrix} a_{0,11} & 0 \\ a_{0,21} & a_{0,22} \end{bmatrix}.$$

Under the misspecified model  $\mathcal{M}_1$ , we maintain the same identifying restriction, such that the misspecified impact matrix  $\tilde{A}_0$  is given by

$$\tilde{A}_0 = \begin{bmatrix} \tilde{a}_{0,11} & 0 \\ \tilde{a}_{0,21} & \tilde{a}_{0,22} \end{bmatrix}.$$

Using the same logic than before, we can derive proposition 3.

**Proposition 3** *The two identified structural shocks in the misspecified model  $\mathcal{M}_1$  are given by*

$$\begin{aligned} \tilde{\eta}_{1t} &= \delta'_1 \hat{f}_{t-1} + \frac{a_{0,11}}{\tilde{a}_{0,11}} \eta_{1t} + o_p(1), \\ \tilde{\eta}_{2t} &= \delta'_2 \hat{f}_{t-1} + \Theta \eta_{1t} + \frac{a_{0,22}}{\tilde{a}_{0,22}} \eta_{2t} + o_p(1), \end{aligned}$$

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<sup>15</sup>Proofs are gathered in an appendix

with  $\Theta = \begin{bmatrix} \frac{a_{0,21}}{\tilde{a}_{0,22}} - \frac{a_{0,11}}{\tilde{a}_{0,11}} \frac{\tilde{a}_{0,21}}{\tilde{a}_{0,22}} \end{bmatrix}$  The identification of the true structural shocks therefore depends on the relative biases of the three elements of the matrix  $\tilde{A}_0$ . Those relative biases satisfy, when  $R_1^2$  and  $R_2^2$  are small:

$$\begin{aligned} \frac{\hat{\tilde{a}}_{0,11} - a_{0,11}}{a_{0,11}} &\simeq \frac{1}{2}R_1^2, \\ \frac{\hat{\tilde{a}}_{0,22} - a_{0,22}}{a_{0,22}} &\leq \frac{1}{2}R_2^2, \\ \hat{\Theta} &\leq (1 - R_2^2)^{1/2}. \end{aligned}$$

Therefore, the relative bias impact response to a news shock is smaller than half of the  $R^2$ .

This proposition makes explicit that from a quantitative point of view, it is not the value of the Wald statistics but the size of the  $R^2$  that matters for the bias caused by nonfundamentalness.

## 4 Application to the identification of TFP news shocks in U.S. data

In this section, we show that the results of Beaudry and Portier [2006] and [2014] are robust to the nonfundamentalness critique.

### 4.1 Baseline results

In the following, we use the same sample as used by Forni, Gambetti, and Sala [2014] and use the data described in Beaudry and Portier [2014].<sup>16</sup> TFP is corrected for utilisation, consumption is total consumption (including durable) and investment is total investment (see the data appendix).

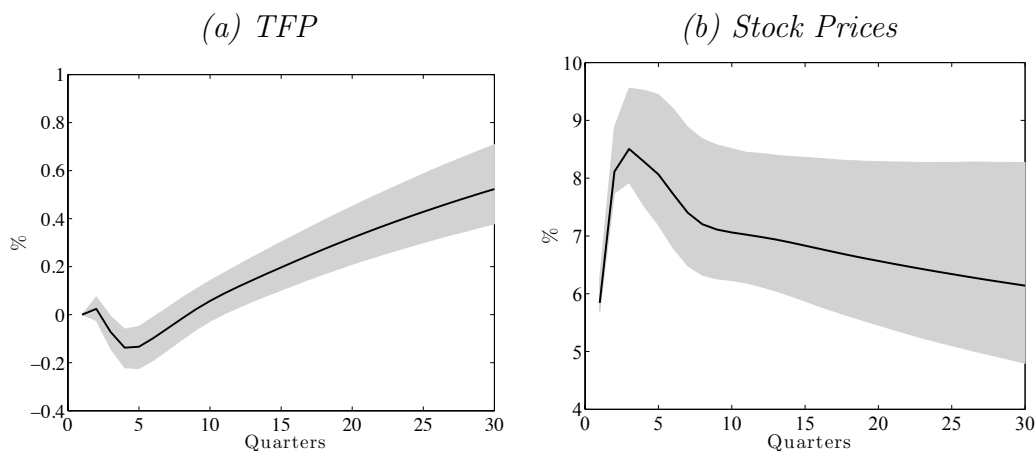
We first consider the basic Beaudry and Portier's [2006] VAR 2. Whereas the small dimension of the VAR might be a weakness, this VAR has the advantage of being simple and, as discussed in Beaudry and Portier [2014], gives results that are robust to various extensions. The two variables in the system are TFP and Stock Prices. The single identifying restriction is that the identified news has no impact effect on TFP, which correspond to a

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<sup>16</sup>Note that the results of our VARs are robust to a longer sample (1946-2013), but the factors are only available on the shorter sample.

Choleski decomposition in which TFP is the first variable and the news shock the second shock. Figure 4 shows that we indeed identify a diffusion news. TFP does not increase for about 10 quarters <sup>17</sup>, but does in the long run.

Figure 4: Response to a news shock in the Beaudry and Portier’s [2006] VAR 2



*Data are described in the appendix and the sample period in 1960Q1-2012Q2. The news shock is the one that does not affect TFP on impact. The VAR is estimated in levels and with 4 lags. The unit of the vertical axis is percentage deviation from the situation without shock. Grey areas correspond to the 66% confidence band. The distribution of IRF is the Bayesian simulated distribution obtained by Monte-Carlo integration with 10,000 replications, using the approach for just-identified systems discussed in Doan [1992].*

We now extend the VAR to add three extra variables: consumption, investment and hours. To identify a TFP news shock, we follow the identification strategy set out in Beaudry and Portier [2014] which is a natural extension to that introduced in Beaudry and Portier [2006]. This identification strategy only identifies a and an unrestricted technology shock, while the other shocks remain unnamed. The identifying restrictions are the following: (i) all the shocks but the unrestricted technology shock have zero impact effect on TFP, (ii) the news and the unrestricted technology shock are the only permanent shocks to TFP. In Beaudry and Portier [2014] it is shown that this identification gives robust results when one varies either the information set, the sample period and the specification.

Impulse responses are presented in Figure 5. The plain line shows the point estimates. We observe all the characteristic of a news driven economic expansion. TFP does not move in the short run, the stock market reacts instantaneously to the news, consumption, investment

<sup>17</sup>TFP actually decreases, which might be the consequence of an excessive correction for utilization.

and hours do increase on impact and subsequently, before any sizable increase in TFP. In panels (a) and (b), we also represent the responses of TFP and SP obtained from the VAR 2 (dashed-dotted gray line). Note that the response of TFP is very similar, while the response of SP is now purged from some non-news related variations.

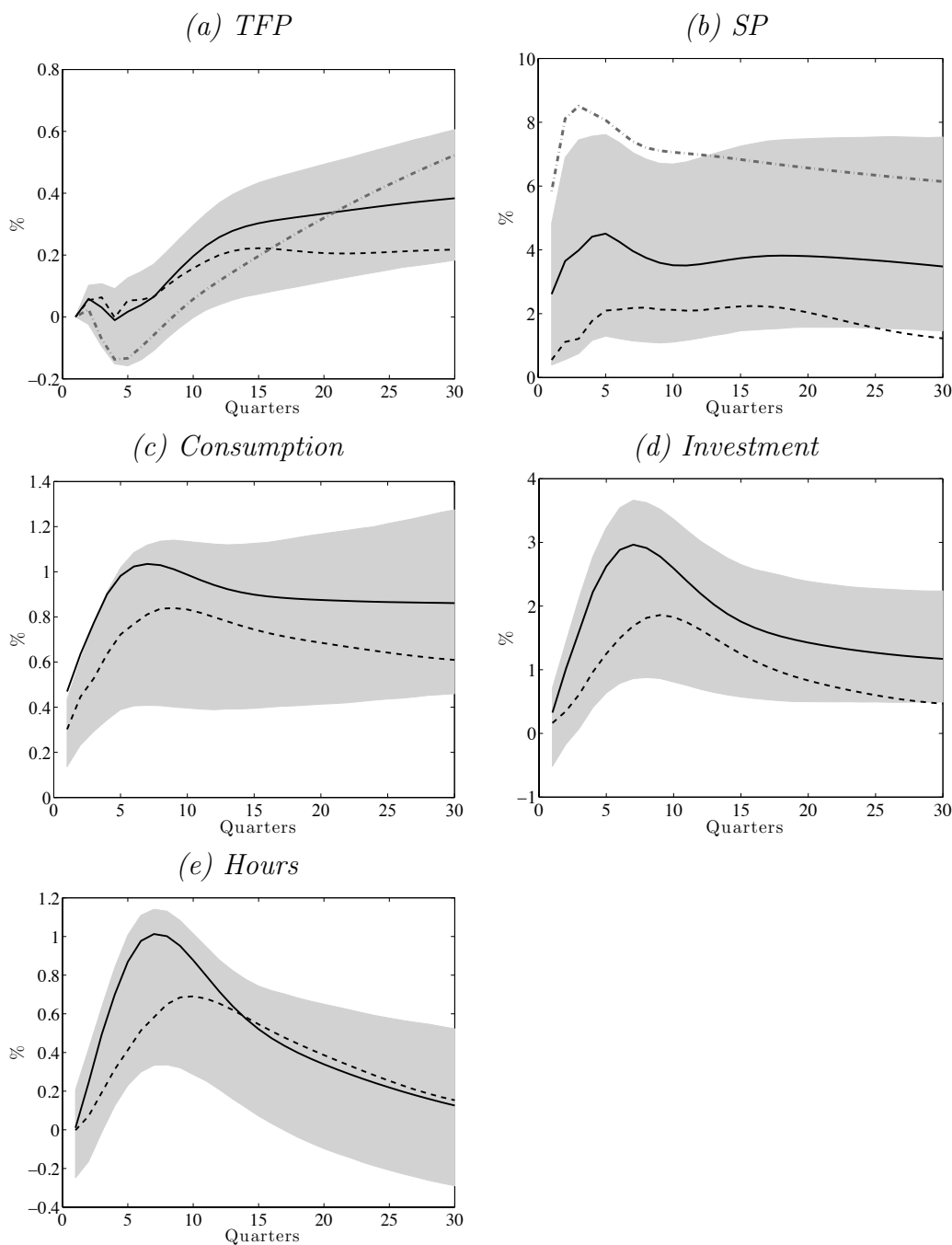
These results suggest that there are indeed news in the business cycle, but it might be the case, as pointed out by Forni, Gambetti, and Sala [2014], that the estimation suffers from nonfundamentalness. This is what we check now.

## 4.2 The quantitative unimportance of nonfundamentalness

In order to test for nonfundamentalness, we follow Forni, Gambetti, and Sala [2014]. The authors use a dataset composed of 107 US quarterly macroeconomic series, and estimate the principal components of this data set. They show that essentially all the information is contained in the first three factors. We therefore use these first three factors. We project the estimated news shock of the VAR 2 and of the VAR 5 on one lag or four lags of the first three factors, and test for the orthogonality of our news shocks to the factors. The test is a F-test, and the p-values are reported in Table 1. In all cases, the p-value is less than 5%. We therefore do agree with Forni, Gambetti, and Sala [2014] that our identified strategy is likely subject to the nonfundamentalness problem. However, does it matter for the estimation of the impulse response functions to a news shock? The answer we find is no, or at least not very much. A first element suggestive of this negative answer comes from the inspection of the  $R^2$ s associated with specification test. These are displayed in Table 1. The  $R^2$ s are never larger than .2: even though our estimated news shocks are not orthogonal to the factors, those factors explain less than 20% of the variance of the news. The simulation and theoretical results of the previous section suggest that in such a case, the nonfundamentalness should not be much of a quantitative problem.

We then re-estimate our VAR 5 by adding the three factors, so that we end up estimating a VAR 8. We use the same identification strategy, that is, : (i) all the shocks but the unrestricted technology shock have zero impact effect on TFP, (ii) the news and the unrestricted technology shock are the only permanent shocks to TFP. The estimated responses to the newly identified news shock are the black dashed lines of Figure 5. Except for the Stock

Figure 5: Comparison of the VAR 5 responses with the ones of the VAR 5 augmented with the first three factors



Data are described in the appendix and the sample period is 1960Q1-2012Q2. In the VAR 5 (the plain line), the news shock is restricted to have no impact effect on TFP but is not restricted in the long run. The dotted lines correspond to the VAR 8, that is the VAR 5 augmented with the first three factors of Forni, Gambetti, and Sala [2014]. The dashed-dotted gray lines of panels (a) and (b) are the responses to a news shock in the VAR 2 of Figure 4. The VARs are estimated in levels and with 4 lags. The unit of the vertical axis is percentage deviation from the situation without shock. Grey areas correspond to the 66% confidence band of the VAR 5. The distribution of IRF is the Bayesian simulated distribution obtained by Monte-Carlo integration with 10,000 replications, using the approach for just-identified systems discussed in Doan [1992].

Table 1: Test for nonfundamentalness and associated  $R^2$ s

Model	One lag		Four lags	
	$R^2$	F-test p-value	$R^2$	F-test p-value
VAR 2	.03	.04	.18	.05
VAR 5	.09	.01	.21	.01

*This Table presents the results of the sufficient information test proposed by Forni and Gambetti [2014]. For each VAR, the news shock is projected on one or four lags of the first three factors of Forni, Gambetti, and Sala [2014]. Table includes the p-value for the orthogonality test, as well as the  $R^2$  of those regressions. Data are described in the appendix and the sample period in 1960Q1-2012Q2. In the VAR 2, the news shock is the one that does not affect TFP on impact. In the VAR 5, the news shock is only restricted to have no impact effect on TFP but is not restricted in the long run. The VARs are estimated in levels and with 4 lags.*

Price whose response has a similar shape but is divided by a factor two, the responses of TFP, consumption, investment and hours are all very similar to that obtained in the absence of including the factors. This contrasts with Forni, Gambetti, and Sala [2014] finding that is based on the identification strategy of Barsky and Sims [2011], which itself is not very supportive of the news shocks view of business cycles. Hence, these results suggests that our chosen means of identifying news shocks generate impulse responses with properties that are robust to the nonfundamentalness critique. There may remain debate about how best to identify new shocks, but that is an issue entirely different form the issue of nonfundamentalness emphasized in Forni, Gambetti, and Sala [2014].<sup>18</sup> We therefore infer that nonfundamentalness is not likely an important factor in evaluating whether or not news shocks are relevant for business cycles.

## 5 Conclusion

In this paper we began be showing, using a simple Lucas' tree model with news, that one needs to separate the very existence of nonfundamentalness from its quantitative importance. In particular, we have shown that the relative bias in recovering the true structural shocks is of the order of half the  $R^2$  of the projection of the misspecified structural shocks on the true ones. We have then shown that when estimating the effects of technological news shocks with an identification scheme previously known to be robust, we found that nonfundamentalness is

<sup>18</sup> See Beaudry, Nam, and Wang [2011] for some answers to that question.

present, but that it does not appear to matter quantitatively. This is not of course a general result that would apply to all SVARs exercises. In fact, the test proposed by Forni and Gambetti [2014] is a useful one that macro-econometricians should systematically perform when nonfundamentalness may be present. However, this tests should not only be used to detect nonfundamentalness, but the  $R^2$  associated with the test should also be used to help assess whether nonfundamentalness is likely to be quantitative important. For technological news shocks, our findings suggest that nonfundamentalness is not likely to be a first order issue.

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## Appendix

### A Proofs

**Proof of Proposition 2 :** The first part of proposition 2 is obtained by using that  $V(\widehat{\tilde{\eta}}_{it}) = 1 = \hat{\delta}'_i (F'_{-1} M_Y F_{-1}) \hat{\delta}_i + \hat{\sigma}_{vi}^2$ , so that variance of  $\widehat{\tilde{\eta}}_{it}$  can be rewritten as <sup>19</sup>  $V(\widehat{\tilde{\eta}}_{it}) = R_i^2 + \hat{\sigma}_{vi}^2$ . The estimator  $\hat{\sigma}_{vi}^2 = (1 - R_i^2)$  is a consistent estimator of the expression  $e'_i \left( \tilde{A}_0 \right)^{-1} A_0 A'_0 \left( \tilde{A}_0 \right)^{-1'} e_i$  using the fact that  $v_{it} = e'_i \tilde{A}_0^{-1} A_0 \eta_t$  and equation (3). Thus, the  $R^2$  is a consistent estimator of the distance between the misspecified  $\tilde{A}_0$  and the true one  $A_0$ . To prove the second part of proposition 2, consider the variance of a variable  $j$  attributable to structural shock  $\eta_{it}$ . On impact, it is given by:  $e'_j \tilde{A}_0 e_i \text{Var}(\tilde{\eta}_{it}) e'_i \tilde{A}'_0 e'_j = e'_j \tilde{A}_0 e_i R_i^2 e'_i \tilde{A}'_0 e'_j + e'_j \tilde{A}_0 e_i (1 - R_i^2) e'_i \tilde{A}'_0 e'_j + o_p(1)$  with  $(1 - R^2)$  a consistent estimator of  $e'_i \left( \tilde{A}_0 \right)^{-1} A_0 A'_0 \left( \tilde{A}_0 \right)^{-1'} e_i$  as aforementioned. The  $R^2$  is then a consistent empirical measure of the discrepancy between the misspecified variance on the impact attributable to a particular shock and its true one. *QED*

**Proof of Proposition 3 :** Applying the previous computations to this simple two-variable example yields the following expression for the first structural shock :  $\tilde{\eta}_{1t} = \delta'_1 \hat{f}_{t-1} + e'_1 \tilde{A}_0^{-1} A_0 \eta_t + o_p(1) = \delta'_1 \hat{f}_{t-1} + \frac{a_{0,11}}{\tilde{a}_{0,11}} \eta_{1t} + o_p(1)$  and  $e'_1 \left( \tilde{A}_0 \right)^{-1} A_0 A'_0 \left( \tilde{A}_0 \right)^{-1'} e_1 = \left( \frac{a_{0,11}}{\tilde{a}_{0,11}} \right)^2$ . Consequently,  $(1 - R_1^2)$  is a consistent estimator of this term. Hence,  $V(\widehat{\tilde{\eta}}_{1t}) = \hat{\delta}'_1 (F'_{-1} M_Y F_{-1}) \hat{\delta}_1 + \widehat{\left( \frac{a_{0,11}}{\tilde{a}_{0,11}} \right)^2} \equiv R_1^2 + (1 - R_1^2)$ . For  $R_1^2$  small, a first order expansion this implies that  $\widehat{\tilde{a}}_{0,11} \simeq (1 + \frac{1}{2} R_1^2) a_{0,11}$ . So, the relative bias is  $\frac{\widehat{\tilde{a}}_{0,11} - a_{0,11}}{a_{0,11}} \simeq \frac{1}{2} R_1^2$ . Consider now the second structural shock  $\eta_{2t}$ . Again, using our calculations above yields  $\tilde{\eta}_{2t} = \delta'_2 \hat{f}_{t-1} + e'_2 \tilde{A}_0^{-1} A_0 \eta_t + o_p(1) = \delta'_2 \hat{f}_{t-1} + \left[ \frac{a_{0,21}}{\tilde{a}_{0,22}} - \frac{a_{0,11}}{\tilde{a}_{0,11}} \frac{\tilde{a}_{0,21}}{\tilde{a}_{0,22}} \right] \eta_{1t} + \frac{a_{0,22}}{\tilde{a}_{0,22}} \eta_{2t} + o_p(1)$ . The expression for  $\tilde{\eta}_{2t}$  is a function of the relative bias for the three terms in the matrix  $A_0$ . This implies the following variance of the second structural shock  $V(\widehat{\tilde{\eta}}_{2t}) = \hat{\delta}'_2 (F'_{-1} M_Y F_{-1}) \hat{\delta}_2 + \widehat{\Theta}^2 + \widehat{\left( \frac{a_{0,22}}{\tilde{a}_{0,22}} \right)^2} \equiv R_2^2 + (1 - R_2^2)$  where  $\Theta = \left[ \frac{a_{0,21}}{\tilde{a}_{0,22}} - \frac{a_{0,11}}{\tilde{a}_{0,11}} \frac{\tilde{a}_{0,21}}{\tilde{a}_{0,22}} \right]$ . This implies that  $\widehat{\Theta}^2 + \widehat{\left( \frac{a_{0,22}}{\tilde{a}_{0,22}} \right)^2} = (1 - R_2^2)$  Consequently,

<sup>19</sup>The unit variance of  $\widehat{\tilde{\eta}}_{it}$  is just the consequence of the normalization assumption of the structural shocks.

$$\widehat{a}_{0,22} \leq (1 - R_2^2)^{-1/2} a_{0,22} \simeq (1 + \frac{1}{2} R_2^2) a_{0,22} \text{ and } \widehat{\Theta} \leq (1 - R_2^2)^{1/2}. \text{ QED}$$

## B Data

- Hours: BLS, Series Id: PRS85006033, Nonfarm Business sector, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012
- Consumption: BEA, Table 1.1.3. Real Gross Domestic Product, Quantity Indexes, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012
- Investment: BEA, Table 1.1.3. Real Gross Domestic Product, Quantity Indexes, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012
- TFP: Utilization-adjusted quarterly-TFP series for the U.S. Business Sector, produced by John Fernald, series ID: dtfp\_util, 1947Q1-2012Q3, downloaded: 12/2012
- Stock Prices: S&P500 index deflated by CPI, obtained from the [homepage of Robert J. Shiller](#).