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THE REAL EFFECTS OF FOREIGN INFLATION IN THE PRESENCE OF CURRENCY SUBSTITUTION

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ABSTRACT

The paper explores optimizing models of small open economies that hold foreign money balances. Particular attention is paid to the impact of foreign inflation on the real exchange rate and other real variables. At first, an environment in which foreign money is the only traded asset is explored. This is compared to a more general setting in which many assets can be traded. The effect of foreign inflation on domestic real variables depends on: 1) the degree to which it causes a substitution out of traded assets as a whole and into non-traded assets, and 2) the change in real returns on the portfolio of traded assets held by domestic residents.

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1. Introduction

Residents of many countries that actively engage in international trade must hold foreign currency balances. These holdings provide an important link for transmission of inflation from abroad for economies that otherwise might be insulated through flexible exchange rates.

James Tobin, in his classic work on money and growth (1965), proposed that changes in the inflation rate might have a real economic effect even if nominal prices are completely flexible. Higher inflation would induce a shift in people's desired portfolio away from money toward capital, which in turn would have the effect of increasing investment. Consider, now, a country that is not very self-sufficient, and that carries on many transactions using foreign currencies. In such a country, a shift in the real demand for domestic versus foreign currency has a real economic impact. A change in the inflation rate abroad can change economic conditions at home through what is essentially an international version of the Tobin effect -- where the relevant tradeoff is between domestic and foreign money balances, rather than money balances and capital.

This paper considers a perfect-foresight intertemporal optimizing model of a small open economy in which just such an international Tobin effect is present. An increase in foreign inflation causes consumers to wish to shift out of holding foreign money and into domestic money. When foreign money is the only internationally traded asset, this implies a direct effect on the current account. Foreign balances are run down over time through a current account deficit, which implies that domestic consumption of traded goods rises relative to production. This is accomplished through a decrease in the price of traded goods relative to non-traded goods.

Although the model presented here does verify that there is a tendency for a real appreciation domestically in the face of higher foreign inflation through the international Tobin effect, there is a mitigating factor. Residents want to run down their hodings of foreign balances, but they do not necessarily have to run a current account deficit to do so. The increase in foreign inflation will increase the rate at which foreign money's value is eroded in real terms. The capital losses from this source may more than satisfy people's desire to dispose of real foreign money holdings, so a trade deficit and its attendant real appreciation may not be necessary.

A further mitigating factor is added when bonds as well as foreign money may be traded internationally. Then the impact of the portfolio rearrangement on the current account may be lessened or eliminated. As foreign inflation rises, holdings of foreign money need not be reduced gradually. Another channel is to trade foreign money for foreign bonds.

The channels through which foreign inflation have real effects on the domestic economy in models of currency substitution can be described in more general terms. What is essential is that home country residents hold traded assets whose real return varies with changes in the foreign inflation rate. The impact on the real exchange rate depends on: 1) the degree to which foreign inflation causes a substitution out of traded assets as a whole and into non-traded assets through a Tobin effect, and 2) the change in real returns on the portfolio of traded assets held by domestic residents.

Section 2 of this paper presents a basic optimizing model and discusses some of the key assumptions. An economy which trades only goods and foreign money is examined in section 3. The fourth section adds traded bonds. The fifth section discusses the real effects of foreign inflation in general terms. The models of sections 2, 3 and 4 are special cases that help to illustrate the conclusions drawn in the fifth section.

A recent paper by Daniel (1985) considers the international transmission of inflation under currency substitution. Her model is in many ways similar to the ones considered here. She discusses the effect of real capital losses on foreign money balances as a factor that allows dimunition of foreign money holdings without a change in the trade balance (p. 134). However, since foreign money is the only traded asset in her model, she does not consider the possibility of asset trade as an avenue for changing foreign currency balances. Her model also has the disadvantage that consumption decisions are not explicitly optimizing. This is important here because higher foreign inflation has real effects from reducing real income and thereby affecting consumption choices.

The models of sections 2-4 are closely related to that of Calvo and Rodriguez (1977), and are essentially optimizing versions of their set-up. Money plays a role because it is an argument of the utility function. In fact, real domestic balances and real foreign balances appear separately. The models, therefore, are extremely similar to those of Liviatan (1981) and Calvo (1985). The models of Rogers (1986), Sen (1986), and Obstfeld (1981) also share some of the characteristics of the ones considered here. None of these papers considers international trade in both foreign money and bonds, but that environment has been explored in the cash - in - advance models of, for example, Helpman (1981), Helpman and Razin (1982), and Svennson (1985). The work of Feenstra (1986) suggests that many properties of models in which money enters in some fundamental way (such as the cash - in - advance approach) are captured with money in the utility function. However, this paper makes several simplifying assumptions for concreteness, so that it cannot claim the generality of some of the aforementioned papers.

The issue of the effect of an increase in domestic money growth on the real exchange rate, a matter which is of primary concern to Calvo and Rodriguez (1977), Liviatan (1981) and Calvo (1985) (and in slightly different contexts, Obstfeld (1981) and Daniel (1985)), is also briefly considered.

In some respects, the work of Cuddington (1983) is also relevant here. Although his approach differs in that he investigates a partial equilibrium, descriptive Keynesian two-country model, he is also concerned with behavior in an economy in which domestic residents' portfolio includes both foreign money and foreign bonds. He asks the question of whether economic behavior is much different in such an economy as compared to one in which only bonds are traded internationally, and concludes that it is not. Although this paper is really concerned primarily with a different set of questions, in the economy explored here currency substitution is essential.

2. The Model

In this section, the basic model is laid out, and the dynamics of the system are briefly described. The economy is small and open. It produces and consumes a traded good and a non-traded good. People hold domestic money and foreign money. Foreign bonds may or may not be traded -- both cases are examined. Factors of production are not mobile across borders. Producers maximize profit. Consumers, who have infinite horizons, maximize utility. Utility is a function of the goods they consume, and of the two monies. The foreign exchange rate is flexible.

This model is meant to describe a small country whose imports and exports are a large fraction of domestic consumption and output. When this country engages in international trade, most of the transactions are denominated in foreign currency. For example, the small trading partners of the U.S. deal primarily in dollars. Residents of such a country must hold an inventory of foreign currency balances for international transactions, as well as domestic currency. A more thorough analysis would identify the economic motives for choosing an invoice currency (see, for example, Bilson (1983) or Krugman (1984)), and would construct an economic motive for holding balances of each currency based on transactions costs. Here, the ad hoc assumption that real money balances are in the utility function is made. This assumption seems adequate for deriving the key insights in this paper. The results here rest on two main premises -- first, that consumption choices are made optimally and with knowledge of the correct budget constraint (which includes capital losses on nominal assets from inflation), and, second, that higher inflation of prices in one currency on the margin makes that money less attractive to hold. A more rigorous but more complicated analysis would still have these features.

Because factors of production are not internationally mobile, there is a fixed production possibility frontier for this economy. Output of non-traded and traded goods is completely determined by the price of home goods relative to non-traded goods. To introduce some notation, let

$P_T \equiv$ the nominal price in home currency of traded goods,

$P_H \equiv$ the nominal price in home currency of home goods,

and,

$$p \equiv P_H / P_T .$$

Real output of home goods, y_H, and real output of traded goods, y_T, are functions only of p:

$$y_H = y_H(p),\ y'_H > 0; \text{ and } y_T = y_T(p),\ y'_T < 0.$$

Consumers get satisfaction from both goods, so instantaneous utility depends on c_H and c_T,

$$u = u(c_H, c_T).$$

Both monies are also in the utility function. The utility function is assumed separable in consumption goods and the money balances. (This is a simplifying assumption, but see Calvo (1985) and Feenstra (1986).) Nominal domestic money balances are deflated by the domestic home price of non-traded goods, while foreign nominal balances are deflated by the foreign currency price of traded goods. (This assumption differs from Liviatan (1981) and Calvo (1985), who deflate both monies by the traded goods price.) This choice of deflators seems approximately correct if all transactions in the home good are denominated in the domestic currency and all international dealings are in the foreign currency. In practice, some buying or selling of the traded good will be in terms of the home currency, but it is assumed this fraction is negligible. So, the instantaneous utility of money services, v, depends on m and f:

$$v = v(m, f),$$

where

$$m \equiv M / P_H ,$$

$$f \equiv F / P_T^* ,$$

and

$M \equiv$ nominal domestic money balances,

$F \equiv$ domestic nominal holdings of foreign money, and

$P_T^* \equiv$ nominal price of traded goods in foreign currency terms.

The value of the foreign exchange rate is given by the ratio of traded goods prices in home currency to traded goods prices in foreign currrency -- P_T/P_T^*. Foreigners do not hold the domestic money. It will also be assumed that Inada conditions hold on u and v so that positive amounts of both goods are consumed, and positive amounts of both monies are held.

Consumers in general can hold wealth in three forms - domestic money, foreign money or an internationally traded bond, b, that is a claim on traded goods and pays a given real interest rate r. The individual's total assets, then, are given by

(1) $$a = mp + f + b,$$

in terms of the traded good. Section 3 considers the case when bonds are not traded, so $b = 0$.

Consumers earn income from interest paid on bonds and (negative income) from capital losses on nominal money holdings from inflation. They also earn income from the output of the goods, and they receive transfers of money, τ, from the government. So, their assets accumulate according to

(2) $$\dot{a} = ra - (\pi+r)mp - (\pi^{*}+r)f + \tau + y_T - c_T + p(y_H - c_H),$$

where

$$\pi = \dot{P}_T\ ,\ \pi^* = \dot{P}_T^{*}\ ,$$

and $\dot{x}$ represents the time derivative of x.

Consumers are assumed to have a constant discount rate equal to the world interest rate. They maximize

$$\int_0^{\infty}[u(c_H,c_T) + v(m,f)]e^{-rt}dt$$

subject to (1) and (2). When bonds are traded there must be an additional constraint

$$\lim_{t\to\infty} a_t e^{-rt} \geq 0.$$

In the absence of this condition, it would be optimal to borrow an arbitrarily large amount from abroad at any time, and borrow more in the future to meet interest obligations.

Consumers set their marginal rate of substitution equal to the price ratio, or

(3) $$u_H(c_H,c_T) = pu_T(c_H,c_T)$$

where the subscript refers to the derivative with respect to the corresponding argument of the utility function. Equilibrium in the home goods market requires that

$$c_H = y_H(p),$$

so that c_H, and from eq. (3), c_T are functions only of p. It also follows from eq. (3) that u_H and u_T are functions only of p. It will be useful to assume that

$c'_T > 0$; $u'_T < 0$; and $u'_H > 0$.

If both goods are normal, the first of these two assumptions are met. Slightly stronger conditions are required for the third. (The Appendix contains a short discussion of these conditions.)

The other first order conditions can be written

$$v_f/u_T = r + \pi^* - (u'_T\dot{p}/u_T), \tag{4}$$

and,

$$v_m/u_H = r + \pi - (u'_T\dot{p}/u_T). \tag{5}$$

These exhaust the optimality conditions when only foreign money and goods are traded, but when bonds are traded there is the additional condition

$$\dot{p} = 0. \tag{6}$$

It is worthwile considering eqs. (4), (5) and (6) together. (These equations can be thought of as either the steady-state of the economy without bonds or as the state at any point in time of the economy with traded bonds.) They give us home and foreign money demands as functions of domestic and foreign nominal interest rates and consumption expenditures on non-traded and traded goods. Linearized money demand functions can be gotten by totally differentiating (4) and (5) using (6) to get

$$\begin{aligned} dm = \; & [(v_{ff}(r+\pi)u_{HH} - v_{fm}(r+\pi^*)u_{HT})/D]dc_H \\ & + [(v_{ff}(r+\pi)u_{HT} - v_{fm}(r+\pi^*)u_{TT})/D]dc_T \\ & + (v_{ff}u_H/D)d(r+\pi) \quad - \quad (v_{fm}u_T/D)d(r+\pi^*), \end{aligned}$$

and,

$$
\begin{aligned}
df = \; & [(v_{mm}(r+\pi^*)u_{HT} - v_{fm}(r+\pi)u_{HH})/D]dc_H \\
& + [(v_{mm}(r+\pi^*)u_{TT} - v_{fm}(r+\pi)u_{HT})/D]dc_T \\
& + (v_{mm}u_T/D)d(r+\pi^*) \quad - \quad (v_{fm}u_H/D)d(r+\pi),
\end{aligned}
$$

where

$$D \equiv v_{mm}v_{ff} - v_{fm}^2 \; > 0.$$

Assuming concavity of the instantaneous utility functions, we have $v_{mm} < 0$, $v_{ff} < 0$, $u_{HH} < 0$, and $u_{TT} < 0$. Assuming homogeneity of gives us that $u_{HT} > 0$. This leaves us with the question of the sign of v_{fm}. Inspection of the money demand equations suggests that $v_{fm} < 0$ might be a reasonable assumption. It would mean that home money demand is positively related to expenditure on home goods and demand for foreign money is positively related to expenditure on traded goods. It would further mean that as foreign inflation rises, demand for domestic money rises and vice-versa. It is this shift from one money to the other that is the essence of the economic behavior that we wish to describe, so we will assume $v_{fm} < 0$.

It should be mentioned that the sign of v_{fm} is critical to Liviatan's (1981) criticism of Calvo and Rodriguez (1977). He assumes that this derivative has positive sign, and then shows that the Calvo-Rodriguez (CR) results are reversed in an optimizing model. Had he assumed a negative sign, as assumed here, the CR results would hold up. The negative sign is consistent with the notion that demand for foreign real balances should rise when home inflation rises, which CR assume. Liviatan mistakenly argues along the following lines that the positive derivative is consistent with CR. He notes that the ratio of foreign to home money demand does not change in CR if there is no change in nominal interest rates. He then totally differentiates (4) using (6) while inexplicably ignoring eq. (5). He sets $d(r+\pi) = dc_H = dc_T = 0$ to get

$$v_{mm}dm + v_{fm}df = 0.$$

Using the assumption that m/f is constant, this implies

$$(v_{mm} + (f/m)v_{fm})dm = 0.$$

He then concludes from this equation that v_{fm} must be positive. The problem with this argument is that if (4), (5) and (6) hold, and the nominal interest rates and consumption of the goods cannot change, then there is no way for demand for either money to change. It is not the coefficient in front of dm in Liviatan's equation that must be zero, but instead dm itself must vanish. Liviatan's argument then says nothing about the correct sign for v_{fm}. In choosing a sign for this derivative, Liviatan's stated principle is that it should be taken in such a way as to emulate the behavior in CR's money demand function. It is argued above that this principle is consistent with a negative sign on the derivative, but perhaps the most compelling argument that Liviatan chose the wrong sign according to this guideline is that his conclusions are exactly the opposite of CR.

Calvo (1985) essentially takes the neutral position that the derivatives of the instantaneous utility functions cannot be signed. In general the axioms underlying utility theory would not give any insight into mundane questions about the sign of a cross-derivative. But these same axioms could also not justify putting real money balances in the utility function to begin with. The only purpose of such an assumption is to derive plausible ad hoc money demand functions in an otherwise utility maximizing framework. To assume money is an argument of the utility function but then to refuse to make the assumptions on the shape of the utility function that render reasonable money demand functions is like pouring a glass of water but then refusing to drink it. The assumption here that v_{fm} is negative is based solely on the premise that it yields well-behaved money demand equations.

To complete the description of the model, the evolution of domestic money and of foreign assets must be described. Monetary authorities keep a constant growth rate of money μ, so

(7) $$\dot{m} = m(\mu - \pi - \dot{p}/p).$$

The rate of accumulation of foreign assets is given by the current account less the capital losses on foreign money from inflation of foreign traded goods prices. The current account, in turn is the trade balance plus interest receipts on holdings of foreign bonds. So,

(8) $$\dot{z} = y_T(p) - c_T(p) + rz - (\pi^* + r)f,$$

where

$z \equiv b + f.$

In section 3, there are no foreign bonds so $z \equiv f$.

The equations in this section have been written in general enough form that they apply whether or not foreign bonds are traded. The next section examines short- and long-run behavior of the economy when only money is traded, while section 4 considers the full complement of assets.

3. Foreign Money is the Only Traded Asset

Foreign money plays the dual role of a transactions medium and an internationally traded store of value if there are no traded bonds. When might such a description of the economy be plausible? One case might be when all the foreign assets held within the country are very liquid for the purposes of making trades. For example, for a country that holds its international assets almost exclusively in the form of very short term Eurocurrency deposits, it would not be necessary to distinguish between foreign money and bonds.

Let money be defined as an asset that can be used as a medium of exchange, but that pays a lower (although not necessarily zero) rate of return than less liquid assets. To examine the question of how the effects of foreign inflation are transmitted in a model of currency substitution, should the model include both foreign money and bonds? The answer hinges on how the relative returns to money and bonds change with a change in inflation. For foreign inflation to have any real effect, it must change the real return on one or both types of assets. If it changes the real return on both types of assets equally (i.e., the nominal interest rates on foreign money and foreign bonds change equally, but not as much as the foreign inflation rate), then there is no change in the desired mix of foreign money and bonds. These could then be considered a composite asset. The model of this section allows only foreign money, which for simplicity is assumed to pay a zero nominal return. The real return falls one for one with an increase in foreign inflation. (The models of Daniel (1985), Liviatan (1981) and Calvo (1985) are also this type.)

On the other hand, if higher foreign inflation does change the relative returns on foreign money and bonds (such as when foreign money pays a zero rate of return, and the real rate of return on bonds is unaffected by the change in inflation) then money and bonds must both be in the model, as in section 4.

The primary concern of this section is how a change in foreign inflation affects the real economic variables: the real exchange rate, the current account, real domestic money demand, real hoarding and the demands and supplies of the goods. (We are concerned strictly with the effects of an unanticipated permanent increase in foreign inflation. The nominal foreign price level is held constant, so the foreign monetary authorities might be contracting the level of the money supply at the same time they increase its rate of change.) First, steady-state relationships are derived and then the dynamic response of these variables to foreign inflation rate and domestic money growth changes are studied.

Setting $\dot{m}$, $\dot{p}$ and $\dot{z}$ equal to zero in eqs. (4), (5) ,(7) and (8) yield the following long-run conditions:

(9) $$\bar{y}_T - \bar{c}_T - \pi^*\bar{f} = 0,$$

(10) $$\bar{v}_f - (r + \pi^*)\bar{u}_T = 0, \text{ and}$$

(11) $$\bar{v}_m - (r + \mu)\bar{u}_H = 0.$$

The "-" over a variable denotes its long-run level. There are a couple of things to note about these equations. First, from eq. (9), the long-run current account (which is the same as the long-run trade balance because there are no interest earning assets that are traded) must be in surplus if the foreign inflation rate is positive. The steady-state value of the real exchange rate, $\bar{p}$, depends only on the steady-state capital losses on nominal foreign balances, $\pi^*\bar{f}$. Second, the long-run conditions for equilibrium in the money markets are the same as the instantaneous equilibrium conditions when bonds are traded. When bonds are introduced, there is no adjustment time - the economy immediately jumps to its long-run equilibrium.

If we assume that variables that are on the order of the square of the interest rate - i.e., π^{*2}, μ^2, r^2, $\pi^*\mu$, π^*r, and μr - are approximately zero, then the Appendix shows that $\bar{f}$ falls and $\bar{m}$ rises with an increase in foreign inflation, as expected. The response of the long-run real exchange rate is key to understanding the steady-state changes in other economic quantities. This in turn depends on how π^*f changes:

(12) $$\frac{d\bar{p}}{d\pi^*} = \frac{\bar{f} + \pi^*(d\bar{f}/d\pi^*)}{(\bar{y}'_T - \bar{c}'_T)} .$$

Since $y'_T - c'_T$ is negative, $\bar{p}$ would fall with an increase in foreign inflation if π^* were initally very small. An increase in foreign inflation would mean a capital loss on nominal foreign money holdings. To maintain real foreign money holdings, the steady-state current account surplus must rise, which calls for a real depreciation (a fall in $\bar{p}$). It will be assumed that this is the normal case -- that $\pi^*\bar{f}$ rises when π^* rises. However, since $\bar{f}$ also falls, the change in $\pi^*\bar{f}$ is in general ambiguous. So, normally there is a long-run real depreciation if foreign inflation rises, but if initial foreign inflation is high enough there may be a long-run real appreciation.

If there is a long-run real depreciation, then domestic output of home goods falls while output of traded goods rises. The consumption of traded goods falls and of non-traded goods rises.

The system of equations, (4),(5),(7) and (8), which describe the dynamics is very complex and non-linear. It is helpful to linearize the system near the steady-state described in equations (9), (10) and (11):

$$\dot{p} \simeq (r+\pi)(p-\bar{p}) - (v_{fm}/u'_T)(m-\bar{m}) - (v_{ff}/u'_T)(f-\bar{f}), \tag{13}$$

$$\dot{m} \simeq (\mu-\pi^*)m(u'_H/u_H)(p-\bar{p}) + (m/u_H)[v_{fm}(u'_H/u'_T)-v_{mm}](m-\bar{m}) \tag{14}$$

$$+ (m/u_H)[v_{ff}(u'_H/u'_T)-v_{fm}](f-\bar{f}), \text{ and}$$

$$\dot{f} \simeq (y'_T-c'_T)(p-\bar{p}) - \pi^*(f-\bar{f}). \tag{15}$$

By again assuming that constants that are of the same size as the square of the interest rate are approximately zero, the system can be shown to be saddle stable. We have

$$\dot{\mathbf{x}} \simeq \theta(\mathbf{x}-\bar{\mathbf{x}}), \tag{16}$$

where $\mathbf{x}$ is the vector $\mathbf{x}' = (p,m,f)$, and $\theta < 0$.

Foreign money balances cannot adjust instantaneously in this model. Because of higher inflation of foreign prices, domestic residents wish to shift out of foreign money balances.. They would, for example, prefer to denominate more transactions in domestic currency since the cost of holding foreign balances has risen. It follows immediately that df/dt must be negative:

$$d\dot{f}/d\pi^* \simeq -\theta(d\bar{f}/d\pi^*).$$

Even though the country does begin to run down its foreign balances immediately, it is not umambiguously clear that this implies the current account must fall into deficit. There are two ways for the country to lose real money balances -- by running a trade deficit, and by letting their balances erode in real terms from the higher foreign inflation. Thus, there need not be a real appreciation (an increase in p, the price of home goods relative to traded goods) initially. Mathematically this is shown by the two terms within the brackets of

$$dp/d\pi^* = [f - \theta(d\bar{f}/d\pi^*)]/(y'_T - c'_T).$$

Since the real exchange rate change is ambiguous, it is of course not possible to say how real output or consumption of either good changes.

It is possible to show, as in the Appendix (and assuming that initially the inflation rates are the same at home and abroad), that if there is an initial depreciation it is not as large as the eventual depreciation. That is, in the case of depreciation, there is undershooting. In any event, after the initial response the exchange rate depreciates over time.

Residents also acquire domestic money balances over time, so the initial increase in home real balances is smaller than the long run rise. Actually, the perverse result of an initial decline in domestic real money balances cannot be ruled out.

It is helpful to compare briefly the effects in this model of an increase in domestic money growth on the real exchange rate with those derived by Liviatan. The key difference arises from the fact that with the assumption of $v_{fm} < 0$, foreign money holdings increase in the long run with an increase in long-run domestic inflation. As in Liviatan, we have

$$dp/d\mu = -[\theta(d\bar{f}/d\mu)]/(y'_T - c'_T),$$

but here this derivative is negative, while in Liviatan it is greater than zero. So, under the assumption than $v_{fm} < 0$, an increase in μ leads unambiguously to a real depreciation in the short run. This is the same conclusion reached by Calvo and Rodriguez (1977). A minor difference with Liviatan arises from the fact that an increase in domestic money growth causes a long-run depreciation through its effect on $\pi^*\bar{f}$. Since $\bar{f}$ rises, $\bar{p}$ falls according to:

$$d\bar{p}/d\mu = [\pi^*(d\bar{f}/d\mu)]/(y'_T - c'_T).$$

4. The Model with Traded Bonds

The model of section 3 shows that when foreign inflation increases domestic residents wish to decrease their real holdings of foreign money. There are only two avenues to accomplish this -- by running a current account deficit, or by allowing the real balances to erode through the higher inflation of foreign prices. This section shows that there is a third channel if there is another traded asset -- in this case, bonds. Residents can divest their holdings of foreign money by exchanging them for bonds.

In the model presented here, because the rate of time preference is constant and equal to the world rate of interest, people are always satisfied with their current level of total wealth and claims on foreigners. That is, they do not save or dissave, and they do not borrow or lend internationally. This assumption, though quite unrealistic, helps to highlight the role of the availability of foreign bonds. Because the increase in foreign inflation does not affect overall saving, residents merely swap foreign money for bonds. The other two channels for disposing of foreign real money balances that were of prime importance in the model of section 3 are superfluous here.

There may, however, be an effect on the trade balance and the real exchange rate. From eq. (8), since $\dot{z} = 0$

$$(17) \qquad y_T(p) - c_T(p) = (\pi^* + r)f - rz.$$

Since rz will not change with higher inflation, the direction of change of the trade balance and the real exchange rate depends on the change in $(\pi^*+r)f$:

$$(18) \qquad dp/d\pi^* = [f + (\pi^* + r)(df/d\pi^*)]/(y'_T - c'_T).$$

(Compare with eq. (12).) Thus, if $(\pi^* + r)f$ rises there will be a trade surplus and a real depreciation.

The equations describing this economy at any point in time closely resemble the equations for steady state in the model without bonds. Equations (4), (5), (6) and (8) imply eq. (17) and

$$(19) \qquad v_m/u_H = \pi + r$$

$$(20) \qquad v_f/u_T = \pi^* + r.$$

Notice the strong similarity of eqs. (9), (10) and (11) with eqs. (17), (19) and (20).

Because the transversality conditions of the optimization problem are sufficient to guarantee an optimum (along with the first order conditions), a path for the economy that goes to the steady state will be optimal. For this economy, there is instability unless real money holdings adjust instantaneously to their long-run level. Using eqs. (6), (7) and (19),

$$\dot{m} = (\mu + r - (v_m/u_H))m.$$

Linearizing near steady-state and using eq. (20),

$$\dot{m} = \alpha(m - \bar{m}),$$

where $\alpha \equiv -(m/v_{ff}u_H)(v_{ff}v_{mm} - v_{fm}^2) > 0$. Thus, nominal prices will adjust so that m always equals its long-run value.

Equation (20) shows that foreign money holdings, f, are a function of m and p, both of which always equal their long-run levels. Equation (8) can then be linearized as

$$\dot{z} = r(z-\bar{z}),$$

which shows that claims on foreigners, z, will not change over time (i.e., z must equal $\bar{z}$). So, when there is a shock to the system -- for example, a change in foreign inflation -- residents may trade foreign money for bonds, but their total foreign assets will not change ever.

The result of this analysis is that the response of variables in the short run to a change in foreign inflation is virtually identical to the steady-state response of those variables in the model of section 3. Foreign money holdings immediately decline, and the real value of domestic money holdings, m, rises.

The change in the real exchange rate is given by eq. (18), and the explanation of this equation is very similar to that of eq. (12). The quantity $(\pi^* + r)f$ represents the loss in real terms of holding foreign money instead of bonds. As foreign inflation rises there are two effects on this term. On the one hand, there will be a greater inflation erosion of existing money balances, but on the other hand the higher inflation induces smaller holdings of foreign money which implies a smaller loss. The net effect is ambiguous. Since residents wish to maintain the value of their real claims on the rest of the world, if $(\pi^* + r)f$ does change it must induce a change in the trade balance. If, for example, $(\pi^* + r)f$ rises, then the country must run a trade surplus and there must be a real depreciation.

Turning to the question posed by Cuddington (1983), if there were no foreign money held by domestic residents, would there be any difference in the model? For the particular issues studied here the answer is a most definite yes. This section has examined a change in the foreign rate of inflation holding the real interest rate constant. If there were no foreign money, such a change would have no effect in this model. As it stands, the only effect of higher foreign inflation is through its effect on lost real returns from holding foreign money instead of foreign bonds -- i.e., through the term $\pi^* f$. If f were zero, of course $\pi^* f$ is zero.

The essence of the difference with Cuddington can be seen by examining his claim that nothing is lost by aggregating foreign money and bonds into a single term for foreign assets. Equation (8) tells us that this cannot be done in this model. The rate of accumulation of real foreign assets depends on the mix between foreign money and bonds. Hence, the trade balance in general depends upon both the rate of accumulation of real foreign assets and the amount of each type of asset in investors' portfolio. These issues do not arise with Cuddington because he completely separates the asset choice problem from the rest of the economic system.

Taking up the question posed by Calvo and Rodriguez (1977) and Liviatan (1981), how does a change in domestic money growth affect the real exchange rate? The answer here is that there must be a real depreciation (assuming $v_{fm} < 0$), but the reasons are entirely different than those explored by either Calvo-Rodriguez or Liviatan. Here, because an increase in μ causes a shift into foreign money, there will be additional losses from foregone interest. That is $(\pi^*+r)f$ will rise, which implies a greater trade surplus and a real depreciation.

5. Conclusion

The models of sections 3 and 4 illustrate some points which can be made more generally. The trade balance, which is determined by the real exchange rate, equals the rate of accumulation of real traded assets less the real return on traded assets. (Here the real return is meant to be the nominal interest rate on the asset less the rate of inflation. In particular, it does not include such implicit returns that arise from the value of the asset as a transactions medium.) The effect of the foreign inflation on real variables can be traced through its impact on these two components of the trade balance.

In a wide variety of models that exhibit saddle-stability, the total value of traded assets in terms of traded goods will evolve according to an equation

$$\dot{z} = \beta(z - \bar{z}), \quad \beta < 0. \tag{21}$$

Since z itself cannot jump at the instant of a change in foreign inflation -- it can only adjust gradually over time -- the change in the rate of real traded asset accumulation, $\dot{z}$, depends only on the change in long-run real traded asset holdings, $\bar{z}$. If higher foreign inflation causes domestic residents to wish to shift out of traded assets generally, that can only be accomplished gradually as traded goods are purchased with these assets. The term $\bar{z}$ can be thought of as their target holdings of traded assets, to which they adjust slowly over time.

Many dynamic models, although not all, yield an equation such as (21). It certainly holds in the models of sections 3 and 4. When foreign money is the only traded asset, as in section 3 (and the model of Daniel (1985)), then higher inflation of foreign prices causes domestic residents to lower their target traded asset holdings -- that is, $\bar{z} = \bar{f}$ declines. In the model of section 4, higher inflation does not induce any shift out of traded assets, so $\bar{z}$ is unchanged.

A model similar to that presented in sections 2-4, but in which the discount rate is endogenized as in Obstfeld (1981) yields ambiguous results for the change in $\bar{z}$ (see Rogers (1986)). An equation such as (21) could appear in many types of descriptive models of economies with currency substitution, with or without sticky prices. These models would need to specify a behavioral equation to determine $\bar{z}$.

Having determined $\dot{z}$ is not enough to tell how the trade balance and real exchange rate react. We can write generally

$$TB(p) = \dot{z} - (i_j^* - \pi^*)\cdot\mathbf{z}, \tag{22}$$

where $\mathbf{z}$ is the vector of traded assets and i_j^* is the nominal rate of return on the j^{th} asset. This equation shows that the trade balance, TB, and hence the real exchange rate, p, depends not only on $\dot{z}$, but also on the total real returns from traded assets, $(i_j^* - \pi^*)\cdot\mathbf{z}$. These real returns may change when π^* rises for two reasons: 1) the real rate of return on asset j may change, and 2) within the portfolio of traded assets there may be a shift toward those whose real return has risen. In the models of both sections 3 and 4 the direction of this change was uncertain.

The models of this paper suggest some previously unexplored avenues for the transmission of foreign inflation. Most previous studies of currency substitution -- particularly the empirical ones (see, for example, Cuddington (1983) and the references cited therein) -- ignore the effects of foreign inflation on flow economic variables, such as saving and the trade balance. Yet the theoretical propositions developed here suggest that these are the keys to understanding the role of currency substitution in subjecting an economy to real consequences of foreign inflation shocks.

APPENDIX

The appendix demonstrates the propositions in the text page-by-page.

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From the first-order condition, eq. (3), and the condition for equilibrium in the home goods market, one can derive expressions for c_T, c_H, u, u_T and u_H as functions of p. Taking these derivatives we find

$$c'_T = [u_T + (pu_{HT} - u_{HH})y'_H]/(u_{HT} - pu_{TT}).$$

If both goods are normal, the two expressions in parentheses are positive which implies $c'_T > 0$.

We also have

$$u'_T = [(u_{HT}{}^2 - u_{HH}u_{TT})y'_H + u_{TT}u_T]/(u_{HT} - pu_{TT}).$$

This is negative, again assuming both goods are normal.

The next derivative is given by

$$u'_H = [p(u_{HT}{}^2 - u_{HH}u_{TT})y'_H + u_{HT}u_T]/(u_{HT} - pu_{TT}).$$

In general, the sign of this derivative is ambiguous. However, if production effects are sufficiently small (so y_H is near zero) and u is homogenous (so u_{HT} is positive), then under the normality assumption the derivative is positive.

These derivatives demonstrate how some functions of p change when p itself changes. These functions of p were derived assuming goods market equilibrium ($c_H = y_H$). On page 8 there is a derivative, u_{HT}, which is taken before imposing goods market equilibrium (so u_H and u_T are not functions of p). For any homogenous function this cross derivative is positive.

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Totally differentiating eqs. (9), (10) and (11) with respect to μ and π^* yields

$$\text{(A.1)} \qquad \begin{bmatrix} d\bar{p} \\ d\bar{m} \\ d\bar{f} \end{bmatrix} \simeq (A/D) \begin{bmatrix} \bar{f}d\pi^* \\ \bar{u}_T d\pi^* \\ \bar{u}_H d\mu \end{bmatrix},$$

where,

$$A \simeq \begin{bmatrix} \bar{v}_{fm}^2 - \bar{v}_{mm}\bar{v}_{ff} & -\pi^*\bar{v}_{mm} & \pi^*\bar{v}_{fm} \\ \bar{v}_{fm}(r+\pi)\bar{u}'_T - \bar{v}_{ff}(r+\mu)\bar{u}_H & \bar{v}_{fm}(\bar{y}'_T - \bar{c}'_T) & -v_{ff}(\bar{y}'_T - \bar{c}'_T) \\ \bar{v}_{fm}(r+\mu)\bar{u}'_T - \bar{v}_{mm}(r+\pi)\bar{u}'_T & -\bar{v}_{mm}(\bar{y}'_T - \bar{c}'_T) & v_{fm}(\bar{y}'_T - \bar{c}'_T) \end{bmatrix},$$

and,

$$D \simeq (\bar{y}'_T - \bar{c}'_T)(\bar{v}_{fm}^2 - \bar{v}_{ff}\bar{v}_{mm}) > 0,$$

and the $\simeq$ represents the fact that it has been assumed that constants on the order of the square of the interest rate are zero.

It follows that

$$d\bar{f}/d\pi^* < 0 \quad \text{and} \quad d\bar{m}/d\pi^* > 0.$$

The expression for $d\bar{p}/d\pi^*$ in eq. (12) comes from differentiation of eq. (9).

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The dynamic system represented by eqs. (13), (14), and (15) can be written as

$$\dot{\mathbf{x}} \simeq B(\mathbf{x} - \bar{\mathbf{x}})$$

where B is the matrix of coefficients. Again assuming the square of the interest rate, etc., are zero we have

$$\det B \simeq (\bar{y}'_T - \bar{c}'_T)(\bar{m}/\bar{u}_H\bar{u}'_T)(\bar{v}_{fm}^2 - \bar{v}_{mm}\bar{v}_{ff}) < 0,$$

$$\text{tr } B = r + (\bar{m}/\bar{u}_H)[\bar{v}_{fm}(\bar{u}'_H/\bar{u}'_T) - \bar{v}_{mm}] > 0.$$

Therefore, the system is saddle stable.

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From eqs. (15) and (16)

(A.2) $$p = \bar{p} + [(\theta + \pi^*)/(\bar{y}'_T - \bar{c}'_T)](f - \bar{f}).$$

The expression in the text for $dp/d\pi^*$ then follows immediately from eq. (12), if we evaluate the derivative at the point where f equals $\bar{f}$.

It also follows that

$$dp/d\pi^* > d\bar{p}/d\pi^*$$

if $\theta + \pi^* < 0$. If this is true, then as stated in the text, the initial real depreciation is less than the eventual real depreciation, and, from (16), $d\dot{p}/d\pi^* < 0$. To see that $\theta + \pi^* < 0$, assume initially $\mu = \pi^*$, and then note that

$$\det [B - \theta I] = -(r+\pi^*-\theta)((m/u_H)[v_{fm}(u'_H/u'_T)-v_{mm}])(\pi + \theta) + \det B = 0.$$

Next,

$$\dot{m} = af, \text{ where}$$

$$a = [(m/u_H)(v_{ff}(u'_H/u'_T)-v_{fm})]/[\theta - (m/u_H)(v_{fm}(u'_H/u'_T)-v_{mm})] < 0.$$

So $d\dot{m}/d\pi^* > 0$, from which it follows that

$$dm/d\pi^* < d\bar{m}/d\pi^*.$$

In fact $dm/d\pi^* < 0$ cannot be ruled out.

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The expression for $d\bar{p}/d\mu$ comes from differentiation of eq. (9). Using (A.2) and the expression for $d\bar{p}/d\mu$ yields $dp/d\mu$ easily.

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Total differentiation of eqs. (17), (19) and (20) using the fact that $dz = 0$, yields a system identical to (A.1) except that the A_{12} element is now $(r-\pi^{*})v_{mm}$ and the A_{13} element is $(\pi^{*}-r)v_{fm}$.

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