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# MONETARY GROWTH, INFLATION, AND ECONOMIC ACTIVITY IN A DYNAMIC MACRO MODEL

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## ABSTRACT

This paper analyzes the effects of an increase in the monetary growth rate within a dynamic optimizing macroeconomic model. Both the short-run and long-run effects, and therefore the adjustments along the transitional path, depend critically upon the tax structure and the firm's corresponding optimal financial decisions. With all bond financing, the effects depend upon the extent to which interest payments are tax deductible for corporations. If this is sufficiently high, the effects of an increase in the monetary growth rate are generally expansionary. With low interest deductibility, or if the tax structure induces equity financing, the effects are generally contractionary.

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## 1. INTRODUCTION

Some years ago, Tobin (1965) established the proposition that an increase in the inflation rate will increase the degree of capital intensity. The economy he considered was one in which all taxes were lump sum, with the impact of inflation operating through the portfolio adjustment effect. However, recent work, most notably by Feldstein, emphasizes the interaction between the tax structure and inflation in determining its effects on capital accumulation. In a series of papers he argues that for plausible parameter values, an increase in the rate of inflation is likely to lower the real rate of return to savers, thereby lowering the rate of savings, ultimately leading to a reduction in the long-run capital stock.[1] He also demonstrates how under existing tax rules a permanent increase in the expected rate of inflation can have adverse effects on the price of shares which also leads to contractionary effects on the capital stock. He contrasts this latter view with those obtained by other economists, who reach the opposite conclusion; see, e.g., Fama (1981).

As Feldstein notes, his analyses are based on restrictive models with specific assumptions. First, the analysis is restricted to steady-state behavior and therefore abstracts from the intertemporal aspects of the accumulation process. Yet clearly, the interaction between the tax structure and the inflation rate plays a critical role in determining the entire time path of capital accumulation and thereby determining the adjustment of the economy over time. Secondly, the savings functions employed in these studies are typically arbitrarily specified. To incorporate rational behavior it is important that these, along with other behavioral relationships, be derived from an intertemporal optimization.

In this paper, we examine the impact of permanent increases in inflation within the context of a dynamic general equilibrium model, embodying intertemporal optimization on the part of private agents. The model we consider is an adaptation of the perfect foresight equilibrium model developed by Brock and Turnovsky (1981). A key feature of this model is the incorporation of various tax rates into the cost of capital and how the interaction of these tax rates with the rate of inflation impacts on the financial decisions of the firm. Indeed, the crucial driving force of the model is how an increase in the monetary growth rate--the proxy we employ for the permanent expected rate of inflation--influences the long-run cost of capital. This determines the adjustment in the steady-state capital-labor ratio, which under the assumption of perfect foresight, in turn conditions the short-run adjustment in the economy and its transitional dynamics. These effects in turn depend at least in part upon whether the existing tax structure induces firms to finance its investments by issuing bonds or by issuing equities. In the case where equity financing is optimal, an increase in the monetary growth rate has the generally contractionary effects suggested by Feldstein. On the other hand, when the tax structure favors bond financing, the effect depends critically upon the extent to which interest payments on bonds are tax deductible to corporations.

The question of the relationship between the long-run rate of inflation (or equivalently the monetary growth rate) and the capital stock has received a lot of attention in the monetary growth literature, beginning with the work by Tobin, noted earlier. In the optimal monetary growth model developed by Sidrauski (1967), money was shown to be super-neutral, meaning that the long-run stock of capital is independent of the monetary growth rate. Using descriptive models, other authors have shown that an increase in the monetary growth rate is associated with a higher capital-labor ratio (or capital stock); see, e.g., Burmeister and Dobell (1970), Fischer (1979a). On the other hand using a cash-in-advance model of the economy, Stockman (1981) shows that the long-run capital stock is inversely related to the rate of inflation. The present analysis fits into this literature by showing that the relationship between the long-run capital stock and the long-run rate of inflation depends critically upon the existing tax structure and the implied optimal financial decisions for firms. While this aspect was briefly considered in the Brock-Turnovsky analysis, the present study goes far beyond this by studying the intertemporal adjustment, as well as the broader macroeconomic implications.

## 2. PERFECT FORESIGHT EQUILIBRIUM

The model developed by Brock and Turnovsky involves a good deal of structure and is discussed in detail elsewhere; see Brock and Turnovsky (1981), Turnovsky (1982). For present purposes it suffices to review its main features in sufficient detail to make the analysis as self contained as possible.

The framework consists of three basic sectors--households, firms, and the government--all of which are interrelated through their respective budget constraints. The representative consumer chooses his consumption, labor supply, the rates at which he wishes to add to his real holdings of money balances, government bonds, corporate bonds, and equities so as to maximize the following intertemporal utility function

(1) $$\int_0^\infty [U(c,l) + V(m)]\, e^{-\beta t}\, dt \qquad \begin{matrix} U_c > 0, & U_l < 0, & V' > 0 \\ U_{cc} < 0, & U_{ll} < 0, & V'' < 0 \end{matrix}$$

subject to his budget constraint and initial endowments of assets.[2] The instantaneous utility function is assumed to be concave in its arguments and for expositional simplicity is assumed to be additively separable in consumption $c$, and labor $l$, on the one hand, and in real money balances, $m$, on the other.[3] The parameter $\beta$ represents the consumer's rate of time preference.

Firms are assumed to produce output subject to a production function, which is assumed to have the usual neoclassical properties of positive, but diminishing, marginal physical product and constant returns to scale. Thus we may write it in the form

(2) $$y = F(k,l) = lf(k/l) \qquad f' > 0, f'' < 0$$

where $y$ denotes supply, and $k$ denotes capital. The firm's production decisions, together with its financial decisions--choosing the mix between debt and equity financing, and dividend policy--are made to maximize the initial market value of the firm's outstanding securities. This can be expressed in the form

(3) $$Z(0) = \int_0^\infty \left[ \exp - \int_0^t x(\tau) d\tau \right] \gamma(t) dt$$

where $Z(0)$ is the market value of the firm's securities at time 0, $\gamma(t)$ = real net cash flow, and $x(t)$ is the instantaneous cost of capital at time $t$. The derivation of the actual expression appearing in (3) is complicated and is given in detail in Brock and Turnovsky (1981; pp. 189-192).

Finally, the government makes expenditure decisions, sets the tax rates on ordinary personal income, on capital gains income, and on corporate income, as well as making financial decisions, subject to its budget constraint. We assume that the monetary authority has a policy of pegging the nominal monetary growth rate, which we take to be a proxy for the expected permanent inflation rate. The government is also assumed to issue an interest bearing asset to finance its deficit. The analysis assumes the existence of perfect certainty, so that in equilibrium all income earning assets must pay the same after-tax real rate of return.[4]

The perfect foresight equilibrium derived by Brock and Turnovsky and which we shall consider, is defined as a situation in which the demand for output, labor, and the various securities in the economy all equal the corresponding real supplies, and all expectations are realized. This is summarized by the following set of equations:

(4a) $$U_c(c,l) = \alpha$$

(4b) $$\frac{U_l(c,l)}{U_c(c,l)} = -F_l(k,l)(1-\tau_y)$$

(4c) $$\frac{V'(m)}{U_c(c,l)} = \theta + p$$

(4d) $$(1-\tau_p)f'(k/l) = x^*$$

(4e) $$x^* = \theta + \min\left[\frac{(\tau_y - z\tau_p)(\theta+p)}{1-\tau_y}, \frac{(\theta+p)\tau_c + \bar{i}(\tau_y-\tau_c)}{1-\tau_c}\right]$$

(5a) $$\dot{\alpha} = \alpha(\beta - \theta)$$

(5b) $$\dot{k} = lf(k/l) - c - g$$

(5c) $$\dot{m} + \dot{b}_g = g + \theta b_g - \tau_y lf(k/l) - mp + [\theta - (1-\tau_y)f']k$$

(5d) $$\dot{m} = (\mu - p)m$$

where

$c$ = real consumption in equilibrium,

$l$ = real employment in equilibrium,

$\alpha$ = marginal utility of consumption,

$k$ = capital,

$\theta$ = after-tax, real rate of return to consumers,

$x^*$ = minimized cost of capital,

$m$ = real stock of money,

$b_g$ = real stock of government bonds,

$p$ = rate of inflation,

$\mu$ = rate of nominal monetary growth, taken to be fixed,

$g$ = real government expenditure,

$\bar{l}$ = dividend payout rate (taken to be exogenous),

$\tau_y$ = 'ordinary' personal income tax; i.e., tax rate levied on wage, dividend, and interest income,

$\tau_c$ = rate of tax levied on nominal capital gains on equities,

$\tau_p$ = rate of corporate income tax,

$z$ = fraction of interest payments deductible from corporate income tax.

Equation (4a) is simply a shorthand notation for the marginal utility of consumption. Equation (4b) equates the instantaneous marginal rate of substitution between consumption and leisure to the after-tax real wage, which in perfect foresight equilibrium is simply the after-tax marginal physical product of labor. The third equation requires that the marginal utility derived from holding a dollar in cash balances equals the marginal utility from spending the dollar on consumption. Given that $\theta$ measures the after-tax real rate of return to consumers, this equation can also be interpreted as saying that the marginal rate of substitution between money balances and consumption equals the after-tax nominal rate of return, $\theta + p$.

Equation (4d) describes the marginal product condition for capital. This requires that the marginal physical product of capital, net of the corporate income tax, equal the minimized cost of capital, $x^*$.

The latter variable is the critical one in the model and is specified in (4c). This relationship embodies the optimizing behavior of firms in choosing their capital structure. With perfect certainty, and therefore with bonds and equities necessarily being perfect substitutes, firms adopt either all bond financing, or all equity financing, depending upon which is cheaper. As shown by Brock and Turnovsky, the cost of financing through issuing bonds (debt capital), $x_b^*$, is

(6a) $$x_b^* = \theta + \frac{(\tau_y - z\tau_p)(\theta + p)}{1 - \tau_y}$$

while the cost of equity capital, $x_e^*$, is

(6b) $$x_e^* = \theta + \frac{(\theta + p)\tau_c + \bar{i}(\tau_y - \tau_c)}{1 - \tau_c}$$

In the absence of all taxation $\tau_y = \tau_c = \tau_p = 0$ in which case both (6a) and (6b) reduce to

(6c) $$x_b^* = x_e^* = \theta$$

In this case, the cost of debt capital equals the cost of equity capital and firms will be indifferent between them. In the presence of taxes, firms will adopt all bond financing or all equity financing according as $x_b^* \lessgtr x_e^*$, i.e.,

(7) $$\frac{(\tau_y - z\tau_p)(\theta + p)}{1 - \tau_y} \lessgtr \frac{\tau_c(\theta + p) + \bar{i}(\tau_y - \tau_c)}{1 - \tau_c}$$

This is the significance of (4c). Thus the firm's financial decision depends critically upon the tax structure.

These results are just statements of Modigliani and Miller (1958) and Miller (1977)-type optimality conditions and are indeed perfectly consistent with them. Assuming that the tax on capital gains is less than on other kinds of personal income ($\tau_c < \tau_y$), a property of most economies, then as is familiar, firms find it optimal to minimize their dividend payout rate $i$, by setting $i = \bar{i}$, where $\bar{i}$ is taken to be some exogenously set legal minimum payout rate.[5]

We should note that in contrast to Brock and Turnovsky, who assume $z = 1$, the specification of the cost of debt capital (6a) allows only partial deductibility of interest payments from corporate profit taxation. The reason for this is that with $z = 1$ and a positive equilibrium rate of inflation ($\mu > 0$), the long-run taxable corporate profits become negative; firms are in effect subsidized by the government indefinitely to issue bonds. Not surprisingly, this encourages bond financing, but it is also unrealistic since interest deductibility applies only against positive corporate profits. To avoid this difficulty we restrict $z$ to lie in the range

$$(8) \qquad 0 < z < z^* \equiv \frac{\theta + p\tau_y}{\theta + p} < 1$$

where $z^*$ defines the fraction of interest deductibility at which corporate profits are zero.[6] Note that if $p = 0$, then $z^* = 1$ and full deductibility is consistent with nonnegative taxable corporate income. In most economies, firms typically have full deductibility of interest payments up to the zero profit point defined by $z^*$, after which no further deductions are allowed. Our specification of partial deductibility $z$ can be viewed as approximating an average where some firms earning less than this level of profit have full deductibility, while others earning profit in excess of this level, at the margin receive no further deductions.

In fact, the upper bound on $z$ in (8) is not as restrictive as may at first appear. Full deductibility of interest payments is consistent with nonnegative profits if, for example, each representative firm is supplied inelastically with a specific factor $h$ say, such as a patent, which adds to production, but the costs of which are not deductible.[7] Specifying the production function by say

$$y = lf(k/l) + h$$

we can show that full deductibility, $z = 1$, is consistent with nonnegative profits if the returns to the fixed factor are sufficiently large to satisfy

$$h \geq pk/(1 - \tau_p)$$

But since the precise magnitude of the upper bound $z^*$ in (8) is itself unimportant, for convenience and without essential loss of generality, we can consider $h = 0$. With this in mind, the reader is free to allow $z = 1$, if he so wishes.

The five equations (4a)-(4e) may be used to solve for the short-run solutions to the five variables $l$, $c$, $p$, $\theta$, and $x^*$ in terms of the variables $\alpha$, $k$, $m$, which together with $b_g$ evolve in accordance with the dynamic equations (5). The first of these is simply the consumer's optimality condition

$$\theta = \beta - \dot{U}_c / U_c$$

familiar from optimal consumption models.[8] Equation (5b) describes the rate of capital accumulation required to maintain product market equilibrium, while (5c) describes the government budget constraint. The derivation of this form has involved the use of the optimality conditions for both firms and households, as well as the linear homogeneity of the production function; see Brock and Turnovsky (1981). The fiscal variables $g$, $\tau_y$, $\tau_c$, $\tau_p$, are all taken to be fixed exogenously over time. Finally, the dynamics of the model is completed by the specification of government financial policy, which we assume to be to fix the monetary growth rate at the constant rate $\mu$.

The system of equations (4) and (5) provide the basis for the short-run and long-run analysis of monetary growth undertaken in subsequent sections. At this point two points should be noted. First, the equilibrium real stocks of corporate bonds ($b_p$) and equities ($qE$) are determined by the optimality condition (7). Apart from knife-edge cases, these imply either $b_p = k$ or $qE = k$, depending upon whether the optimal structure calls for all bond or all equity financing.

Secondly, the nominal rate of return on the financial securities satisfy the following equation

$$r_g(1 - \tau_y) - p = \theta = \max \left[ r_b(1 - \tau_y) - p, \;\; \bar{i}(1 - \tau_y) + \dot{q}(1 - \tau_c)/q - \tau_c p \right] \tag{9}$$

where

$r_g$ = nominal rate of interest on government bonds,

$r_b$ = nominal rate of interest on corporate bonds,

$q$ = real price of equities.

This equation asserts that in equilibrium, the nominal rate of interest paid on government bonds must be such as to equate their after-tax real rate of return to investors to the after-tax real rate of return on the existing private security. If bond-financing is optimal for firms, then $r_g = r_b$. Otherwise, in the case of equity financing $r_g$ is determined by the rate of return on equities, which equals the after-tax return on dividends plus the after-tax real rate of capital gain.[9]

## 3. STEADY STATE EFFECTS OF MONETARY EXPANSION

Since the analysis is based on the assumption of perfect foresight, the transitional dynamic adjustment of the system is determined in part by the expectations of the long-run steady state. It is therefore convenient to begin with a consideration of the steady state and the long-run effects of a monetary expansion.

The steady-state equilibrium of the system is reached when $\dot{\alpha} = \dot{k} = \dot{m} = \dot{b}_g = 0$ implying that $\theta = \beta$, $p = \mu$, and $y = c + g$. That is, in steady-state equilibrium, the after-tax real rate of return to consumers equals their rate of time preference; the inflation rate equals the monetary growth rate; and output equals consumption plus government expenditures. Accordingly, the long-run equilibrium of the system can be reduced to the following four equations

(10a) $$\frac{U_l[F(k,l)-g,l]}{U_c[F(k,l)-g,l]} = -F_l(k,l)(1-\tau_y)$$

(10b) $$\frac{V'(m)}{U_c[F(k,l)-g,l]} = \beta + \mu$$

(10c) $$(1-\tau_y)f'(k/l) = x^* = \beta + \min\left[\frac{(\tau_y - z\tau_p)(\beta+\mu)}{1-\tau_y},\ \frac{(\beta+\mu)\tau_c + \bar{i}(\tau_y - \tau_c)}{1-\tau_c}\right]$$

(10d) $$g + \beta b_g - \tau_y l f(k/l) + [\beta - (1-\tau_y)f']k - m\mu = 0$$

which determines the four endogenous variables $k$, $l$, $m$, and $b_g$ in terms of the various policy parameters, including the monetary growth rate $\mu$. In addition, the nominal rates of return on the financial assets satisfy

(11) $$r_g(1-\tau_y) - \mu = \beta = \max\,[r_b(1-\tau_y) - \mu,\ \bar{i}(1-\tau_y) + \dot{q}(1-\tau_c)/q - \tau_c\mu]$$

The long-run equilibrium is obtained in the following recursive manner. Given the parameters $\beta$,

$\mu$, $\tau_y$, $\tau_p$, $\tau_i$, and $\bar{r}$, (10c) yields the long-run cost of capital, and hence the marginal physical product of capital. The precise relationship depends upon whether the firm is employing all bond or all equity financing.[10] Given the linear homogeneity of the production function, this establishes the capital-labor ratio, which in turn determines the real wage rate. Having determined $k/l$, the two marginal rate of substitution conditions (10a) and (10b) together determine the employment of labor and the real stock of money balances. With $k/l$ and $l$ now fixed, the real stock of capital is known, while the level of output follows from the production function. The government budget constraint then determines the real stock of government bonds required to balance the budget. Finally, the nominal returns on the financial assets are then obtained from (11).

Our concern is to analyze the long-run effects of an increase in the monetary growth rate (rate of inflation) on the economy. These effects are summarized in Table 1. Those which stem directly, via the capital-labor ratio, from the marginal productivity condition (10c) (the effects on the cost of capital $x^*$, the real wage rate $w$, and the capital-labor ratio) hold quite generally. The effect on $r_g$ is also general. However, the responses of $k$, $l$, and $y$, are based on the plausible assumption that the marginal utility of consumption decreases with work (increases with leisure); i.e., $U_{cl} < 0$. The key to the entire set of results is the response of the cost of capital, which in turn depends upon the financial decision of the firm. Having determined this, all subsequent effects can be derived from it, in terms of the same functional relationship in the two cases.

A. *Bond-Financing*

For bond financing to be optimal, $x_b^* < x_e^*$, in which case the firm's cost of capital and marginal productivity condition (10c) becomes[11]

(10c') $$(1 - \tau_p)f'(k/l) = \frac{\beta(1 - z\tau_p) + (\tau_y - z\tau_p)\mu}{1 - \tau_y} \equiv x_b^*$$

where $z$ is restricted to lie in the range

(8')
$$0 < z < \frac{\beta + \mu\tau_y}{\beta + \mu}$$

An increase in the monetary growth rate $\mu$ changes the cost of debt capital by an amount

$$\frac{dx_b^*}{d\mu} = \frac{\tau_y - z\tau_p}{1 - \tau_y}$$

the qualitative effects of which depend critically upon $(\tau_y - z\tau_p)$. If $z$ is taken to be constant, satisfying the inequalities

(12)
$$\frac{\tau_y}{\tau_p} < z; \quad z < \frac{\beta + \mu\tau_y}{\beta + \mu}$$

then an increase in the monetary growth rate will lead to a long-run reduction in the cost of debt capital, leading to an increase in the long-run capital-labor (and output-labor) ratio. As a consequence, the real wage rate rises. Taking $\tau_y = .2$, $\tau_p = .5$, $\beta = .05$, $\mu = .05$ as being representative of real world parameters, this will apply for values of $z$ between .4 and .6.

On the other hand, if

(12')
$$z < \tau_y/\tau_p$$

(but sufficiently large to render bond financing optimal), then an increase in the monetary growth rate raises the cost of debt capital and these effects are reversed. Finally, we may note that if $z$ is taken to be proportional to $z^*$, the maximum deductibility, we again find that an increase in the monetary growth rate raises the cost of debt capital. However, this representation of $z$ is probably less typical of real world tax structures.

These examples serve to highlight how an expansion in the monetary growth rate may quite plausibly be either expansionary or contractionary, depending upon the extent to which interest payments are tax deductible. For the sake of being concrete, we shall focus on the case where $z$ satisfies (12) and the monetary expansion is expansionary. The contractionary case can be argued analogously and is qualitatively identical to the case of equity financing discussed below.

To consider the effects of the monetary expansion on $y$ and $l$ it is convenient to take differentials of (10a) together with the production function. This yields the pair of equations

(13a) $$[U_{ll} + F_l(1 - \tau_y)U_{cl}]dl + [U_{lc} + F_l(1 - \tau_y)U_{cc}]dy = -U_c(1 - \tau_y)dw$$

(13b) $$dy - fdl = ldf$$

where $dw > 0$ and $df > 0$ are the increases in the real wage rate and in the output-labor ratio, both resulting from the rise in the capital-labor ratio, induced by the monetary expansion. The fact that output must increase can be established by the following observation. If, instead, $dy < 0$, it follows from the marginal utility condition (13a) that employment $l$ must necessarily rise. However, it then follows from the production function (13b) that a rise in both $f$ and $l$ is inconsistent with the assumed fall in $y$. Output must therefore rise.

The effect on employment, however, is indeterminate. Because of the higher capital-labor ratio, the increased output is consistent with either higher or lower employment But, irrespective of the response of $l$, the increase in the capital-labor ratio can be shown to be sufficient to ensure that the total stock of capital must rise. Finally, since the long-run after-tax real rate of return on all securities equals the rate of time preference $\beta$ and is therefore fixed, the after-tax nominal interest rate on government bonds $r_g(1 - \tau_y)$ must increase by the amount of the monetary expansion. The before-tax nominal rate $r_g$ must therefore increase more than proportionately.

B. *Equity Financing*

We turn now to the case of all equity financing. For this to be optimal, $x_e^* < x_b^*$ so that the firm's cost of capital and marginal productivity condition becomes

(10c") $$(1 - \tau_p)f'(k/l) = \beta + \frac{(\beta + \mu)\tau_c + \bar{l}(\tau_y - \tau_c)}{1 - \tau_c} = x_e^*$$

It is immediately seen that an increase in the monetary growth rate now increases the long-run cost of capital unambiguously, leading to a fall in the long-run capital-labor ratio and in the real wage. The effects on employment, output, capital, are related to changes in the capital-labor ratio precisely as before. But since the response of the latter is now reversed, these other effects are reversed as well. The only effect that remains unchanged from before is the after-tax nominal interest rate on government bonds, which again increases the amount of the monetary expansion.

In summary, these results highlight the fact that the long-run effects of an increase in the monetary growth rate on the economy depend upon the existing tax rates and the firm's associated optimal financial policy. If it favors equity financing, a higher monetary growth rate will lead to a reduction in the capital-labor ratio, the effects of which are generally contractionary. If it favors bond financing, it may lead either to an increase in the capital-labor ratio, the effects are which are generally expansionary, or a decrease, in which case they are contractionary, depending upon the extent to which interest payments are deductible for corporations. In the latter case, bond financing may be more or less contractionary than equity financing. The lower tax rate on capital gains favors equity financing, but this must be balanced against the deductibility of interest payments which favors bond financing.[12]

## 4. TRANSITIONAL DYNAMICS: BOND FINANCING

We turn now to a consideration of the transitional dynamic adjustment path. With firms employing all bond financing, the short-run cost of capital and corresponding marginal product condition is

$$(1 - \tau_p) f'(k/l) = \theta + \frac{(\tau_y - z\tau_p)(\theta + p)}{1 - \tau_y} = x_b^* \tag{4c'}$$

Equations (4a)-(4d), (4c') determine the short-run solutions for $c$, $l$, $\theta$, $p$, and $x_b^*$ in terms of $\alpha$, $m$, and $k$, the latter being determined by (5a), (5b) and (5d). In addition, there is the dynamic adjustment of real government bonds, $b_g$. But since this is determined residually and does not interact with the remaining variables, it can be ignored insofar as these variables are concerned.[13]

### A. *Instantaneous Solutions*

From these short-run equations, we see that the solutions for $c$, $l$, $\theta$, and $p$, are of the form[14]

(14a) $$l = l(\alpha, k); \qquad l_\alpha > 0, \quad l_k > 0$$

(14b) $$c = c(\alpha, k); \qquad c_\alpha < 0, \quad c_k < 0$$

(14c) $$\theta = \theta(\alpha, k, m); \qquad \theta_\alpha \gtrless 0, \theta_k < 0; \quad \theta_m \lesseqgtr 0$$

(14d) $$p = p(\alpha, k, m); \qquad p_\alpha < 0, \quad p_k > 0, \quad p_m < 0$$

The signs of the partial derivatives are as indicated and are derived in an appendix to this paper, available on request.

Intuitively, a *ceteris paribus* increase in the marginal utility of consumption, $\alpha$, means that consumption is reduced and the supply of labor rises. With the real stock of money fixed, the marginal rate of substitution condition (4c) implies that the nominal after-tax rate of return to consumers, $\theta + p$, must fall. In addition, with capital fixed instantaneously, the increase in employment means that the capital-labor ratio falls, leading to an increase in the marginal product of capital and hence in the short-run equilibrium real cost of capital. This in turn requires either a rise in $\theta$ or a fall in $p$. In fact the latter must occur. If instead, $p$ were to rise, then for $x_i^*$ to rise, $\theta$ would also have to rise, leading to an unambiguous rise in $\theta + p$, inconsistent with the required fall. Hence the short-run inflation rate must fall, while the effect on $\theta$ remains unambiguous.

An increase in the real stock of capital $k$ tends to raise the real wage thereby increasing labor supply. Given $\alpha$, this causes consumption to fall. The nominal interest rate $\theta + p$ remains unchanged, so that $d\theta + dp = 0$, implying that the after-tax real rate of return changes by an amount $d\theta = -dp$. Although employment rises, it does so by a lesser amount than capital, so that the capital-labor ratio rises. The marginal product of capital therefore falls, so that the equilibrium cost of capital also falls, implying that the rate of inflation $p$ rises, with the consumer rate of return falling by an exactly offsetting amount.

Given the separability of the utility function, an increase in the real money stock leaves consumption and employment unchanged. The fall in the marginal utility of holding money leads to a fall in the nominal interest rate. With $k$ and $l$ fixed, the capital-labor ratio remains unchanged, so that the short-run equilibrium cost of capital $x^*$ remains fixed. This in turn requires the consumer rate of return $\theta$ and the rate of inflation to change in the same direction if $\tau_y - z\tau_p < 0$, or in opposite directions otherwise. Indeed the only response consistent with the required fall in the nominal interest rate is for $p$ to fall, with the response of $\theta$ depending upon $(\tau_y - z\tau_p)$.

B. *Dynamic Structure*

Linearizing the differential equations (5a), (5b), (5d) about the steady state equilibrium, the dynamics may be written in the form

(15)
$$\begin{bmatrix} \dot{\alpha} \\ \dot{m} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} -\alpha\theta_\alpha & -\alpha\theta_m & -\alpha\theta_k \\ -mp_\alpha & -mp_m & -mp_k \\ F_l l_\alpha - c_\alpha & 0 & F_l l_k - c_k + F_k \end{bmatrix} \begin{bmatrix} \alpha - \tilde{\alpha} \\ m - \tilde{m} \\ k - \tilde{k} \end{bmatrix}$$

where $\tilde{\alpha}$, $\tilde{m}$, $\tilde{k}$, denote the steady-state equilibria determined in Section 3 and the partial derivatives appearing in (15) are evaluated at steady state. It is clear that the corporate finance decisions influence the dynamics through the effects on the inflation rate $p$, and the rate of return to consumers, $\theta$.

It can be shown that under a mild restriction the dynamic system (16) includes one negative (stable) root and two positive (unstable) roots.[15] Eliminating the latter by invoking the transversality conditions, the stable solutions for $k$, $m$, and $\alpha$ are of the form

(16a)
$$k(t) = (k_0 - \tilde{k})e^{\lambda_1 t} + \tilde{k}$$

(16b)
$$m(t) = (m(0) - \tilde{m})e^{\lambda_1 t} + \tilde{m}$$

(16c)
$$\alpha(t) = (\alpha(0) - \tilde{\alpha})e^{\lambda_1 t} + \tilde{\alpha}$$

where $\lambda_1 < 0$ is the stable root. We assume that capital evolves continuously from its initial stock $k_0$, while the initial values of $m(0)$, $\alpha(0)$ are determined endogenously. With the nominal stock of money determined by the monetary growth rule, $\mu$, the initial nominal stock $M_0$ is predetermined, so that the initial real stock $m(0)$ is determined by some appropriate initial jump in the price level. The initial jump in $\alpha(0)$ takes place through initial jumps in consumption and the employment of labor. Omitting details, the initial jumps in $\alpha(0)$, $m(0)$ required to ensure that $\alpha$, $m$ follow stable paths, are given by the expressions

(17a) $$\alpha(0) = \tilde{\alpha} + \phi_1(k_0 - \tilde{k})$$

(17b) $$m(0) = \tilde{m} + \phi_2(k_0 - \tilde{k})$$

where

$$\phi_1 \equiv \frac{(mV''(m) + \alpha\lambda_1)p_k}{J} < 0$$

$$\phi_2 \equiv \frac{(V'(m) - \alpha\lambda_1)mp_k}{\alpha J} > 0$$

$$J \equiv (\alpha\theta_\alpha + \lambda_1)(mp_m + \lambda_1) - m_\alpha p_\alpha \theta_m > 0$$

And the expressions for the partial derivatives $\theta_i$, $p_i$, $i = \alpha, m, k$, are available in the Appendix.

## C. *Short-Run Comparative Statics*

Our objective is to analyze the effects of an increase in the monetary growth rate on the dynamic adjustment path of the economy. The most critical determinant of this is how the initial jumps in $\alpha(0)$, $m(0)$ respond to the monetary expansion. This is because these initial jumps transmit the impact of the monetary expansion to the other short-run variables, via the short-run equilibrium relationships (14a)-(14d).

Differentiating equations (17a), (17b) and using the expressions for the steady-state effects given in Table 1, yields

(18a) $$\frac{d\alpha(0)}{d\mu} = (\eta - \phi_1\psi)\frac{d\widetilde{(k/l)}}{d\mu}$$

(18b)
$$\frac{dm(0)}{d\mu} = \left[\frac{V'}{\alpha V''}\eta - \phi_2\psi\right]\frac{d(\tilde{k/l})}{d\mu} + \frac{\alpha\eta}{V''}$$

where $\eta < 0$, $\psi > 0$, are defined in Table 1. Evaluating $(\eta - \phi_1\psi)$ we can show that [16]

(19)
$$\frac{d\alpha(0)}{d\mu} = \frac{-\lambda_1 l\Omega}{D}\frac{d(\tilde{k/l})}{d\mu}$$

where $\Omega \equiv U_{cc}[U_{ll} + \alpha F_{ll}(1-\tau_y)] - U_{cl}^2 > 0$, and $D > 0$ is defined in Table 1. Thus, an increase in the monetary growth rate increases the marginal utility of consumption in the short run if and only if it raises the long-run capital-labor ratio. The corresponding expression for $dm(0)/d\mu$ is not so simple and hence is not reported.

Equation (19), together with the short-run solutions described by (14a), (14b), immediately imply the following short-run effects of an increase in the monetary growth rate on the real part of the system:

(20a)
$$\frac{dl(0)}{d\mu} = l_\alpha \frac{d\alpha(0)}{d\mu}$$

(20b)
$$\frac{dc(0)}{d\mu} = c_\alpha \frac{d\alpha(0)}{d\mu}$$

(20c)
$$\frac{dy(0)}{d\mu} = [f - f'(k/l)]\frac{dl(0)}{d\mu}$$

(20d)
$$\frac{dw(0)}{d\mu} = (f''k/l^2)\frac{dl(0)}{d\mu}$$

(20e)
$$\frac{dx^*(0)}{d\mu} = -(1-\tau_p)f''\frac{k}{l^2}\frac{dl(0)}{d\mu}$$

(20f)
$$\frac{d\dot{k}(0)}{d\mu} = \frac{dy(0)}{d\mu} - \frac{dc(0)}{d\mu}$$

The signs of all these expressions depend upon the long-run response of the capital-labor ratio; see (19). In the case that this rises, the immediate effect of an increase in the monetary growth rate is to increase employment and output, reduce consumption and wages, and increase the cost of capital and investment. These impact effects are all reversed if the capital-labor ratio falls.

To obtain the short-run effects on the financial variables $p$, $\theta$, and $\theta + p = r_g(1 - \tau_y)$, we consider (4e'). Substituting (4c) into this relationship yields

$$p = \left[\frac{1 - z\tau_p}{1 - \tau_y}\right]\frac{V'(m)}{\alpha} - (1 - \tau_p)f'(k/l) \tag{21}$$

Next, differentiating (21) at time 0 with respect to the monetary growth rate leads to

$$\frac{dp(0)}{d\mu} = \frac{1}{\alpha^2}\left[\frac{1 - z\tau_p}{1 - \tau_y}\right]\left[\alpha V''(m)\frac{dm(0)}{d\mu} - V'(m)\frac{d\alpha(0)}{d\mu}\right] + (1 - \tau_p)f''\frac{k}{l^2}\frac{dl(0)}{d\mu}$$

The change in the monetary growth rate impinges on the inflation rate in two ways. The first is through jumps in the nominal interest rate, $\theta + p$, stemming from jumps in $\alpha(0)$ and $m(0)$; secondly through initial jumps in the employment of labor resulting from the jump in $\alpha(0)$. Using (i): equations (18a), (18b) and (20a); (ii) the definitions of $\phi_1$, $\phi_2$; and (iii) the steady-state effects summarized in Table 1, we can establish:

$$\frac{dp(0)}{d\mu} - 1 = \left[\frac{\tau_y - z\tau_p}{1 - \tau_y}\right]\left[1 + \frac{\lambda_1 p_k(V' + mV'')\psi(1 - z\tau_p)}{\alpha f''J(1 - \tau_p)(1 - \tau_y)} + l_\alpha\frac{k}{l^2}(\eta - \phi_1\psi)\right] \tag{22}$$

Consider now the expression given in (22). It can also be shown by direct evaluation that $1 + l_\alpha(k/l^2)\eta > 0$. Therefore a sufficient, but not necessary, condition for the term in parentheses to be positive is

$$\epsilon = \frac{mV''(m)}{V'(m)} > -1 \tag{23}$$

That is, the elasticity of the marginal utility of money with respect to real money balances must be less than 1 in magnitude. This condition is a mild one and is certainly met if the concave function $V$ is homogeneous in $m$, as often assumed. Invoking (23), we find that

$$\frac{dp(0)}{d\mu} \lesseqgtr 1 \quad \text{according as} \quad \tau_y - z\tau_p \lesseqgtr 0 \tag{24}$$

If $\tau_y - z\tau_p < 0$, so that the increase in the monetary growth rate has an expansionary effect on the long-run capital-labor ratio, then the short-run rate of inflation rises by less than the monetary growth rate. There is therefore partial instantaneous adjustment of the inflation rate toward its new steady state. By contrast, if $\tau_y - z\tau_p > 0$, there is initial overadjustment of the inflation rate.

The effect on the after-tax nominal rate of return, $\theta + p$, follows by differentiating (4d) and yields (dropping the index $t = 0$)

(25)
$$\frac{d(\theta + p)}{d\mu} = 1 + \frac{1}{\alpha^2}(V'\phi_1 - \alpha V''\phi_2)\frac{d\tilde{k}}{d\mu}$$

Recalling the definition of $\phi_1$, $\phi_2$, and imposing (23) we derive

(26)
$$\frac{dr_g(1 - \tau_y)}{d\mu} \lesseqgtr 1 \quad \text{according as} \quad \tau_y - z\tau_p \lesseqgtr 0$$

so that the after-tax nominal interest rate initially underadjusts or overadjusts its ultimate response, depending upon whether $\tau_y - z\tau_p \lesseqgtr 0$.

It is evident from the above analysis that the driving force behind the initial short-run responses are the long-run expectations and the jumps that these impose on the initial marginal utility of consumption $\alpha(0)$ and the real stock of money balances $m(0)$. A more intuitive explanation for the behavior runs as follows. We discuss this for the generally expansionary case where $z > \tau_y/\tau_p$. Under these conditions an increase in the monetary growth rate increases the steady state capital stock, $\tilde{k}$, while reducing the steady state marginal utility of consumption $\tilde{\alpha}$. The reduction in $\tilde{\alpha}$ causes $\alpha(0)$ to fall, while the increase in $\tilde{k}$, causes it to rise; see (17) above. On balance, the capital stock effect dominates and instantaneously the marginal utility of consumption rises.[17]

From our earlier discussion, this induced rise in the marginal utility is associated with a decline in consumption and a rise in employment. With the stock of capital fixed in the short run, the increase in employment leads to a rise in output. It also leads to an instantaneous fall in the capital-labor ratio, which leads to a fall in the real wage, a rise in the marginal physical product of capital, and in the cost of capital. Finally the monetary expansion raises both the inflation rate and also the after-tax nominal interest rate, although by amounts less than their respective long-run magnitudes.

These instantaneous effects generate dynamic adjustments. First, the increase in output, coupled with the reduction in consumption, means that investment must rise, leading to additional capital stock. Secondly, the fact that the short-run inflation is less than the monetary expansion means that the real money stock begins to increase. Thirdly, the increase in the real rate of return implies that the marginal utility $\alpha$ starts to fall. All these changes generate further changes in the short-run variables. The fall in $\alpha$ causes employment to begin falling, although this is offset by the stimulus to employment resulting from the increase in capital stock. Similar effects (although reversed in direction) apply to consumption. Furthermore, the increases in the real money stock and capital, together with the fall in $\alpha$, cause the inflation rate and the after-tax nominal rate of return to rise further. These adjustments continue until the new steady-state equilibrium is attained. At that point some of the initial responses will be more than reversed by offsetting effects during the transitional adjustment path, so that the short-run effect is ultimately reversed. Take for example the capital-labor ratio. In the short run, with the physical capital stock fixed, the increase in employment leads to a fall in this ratio. The subsequent accumulation of capital along the adjustment path more than compensates for the adjustment of labor and the capital-labor ratio eventually increases. The cost of capital and the wage rate, which depend directly upon the capital-labor ratio, are therefore both also reversed between the short and long run. While employment is stimulated in the short run, its long-run response is ambiguous. It is possible that it will increase further during the course of the transition, in which case the long-run increase in output exceeds the short-run expansion. However, it is possible that employment declines to a level which may either exceed or be less than the original level. In this case the long-run response of output may either exceed or be less than the short-run response, depending upon the degree of capital accumulation during the transition.

All the variables in the economy, being tied to the state variables, evolve toward steady state in accordance with a first-order differential equation. In Figure 1 we illustrate the dynamics for three of the critical variables; the capital-labor ratio $k/l$, the rate of inflation $p$, and the level of output $y$. If initially, the economy has a steady-state capital-labor ratio $\overline{k/l}$, then at time 0, when the monetary expansion takes place, the capital-labor ratio drops to point $A$ in Figure 1a. Thereafter, it rises monotonically toward the new steady-state level $\tilde{k}/l$. Likewise, if the initial rate of inflation is $p = \bar{\mu}$, at time 0 it jumps to $p(0)$, denoted by $B$ in Figure 1b. Thereafter, it rises exponentially toward the new higher steady state level $\tilde{\mu}$. Finally, output follows one of the two adjustment paths illustrated in Figure 1c, depending upon whether the short-run response exceeds or is less than the long-run response.

The case where $z < \tau_y/\tau_p$ so that bond financing is contractionary can be analyzed analogously. Indeed it is similar to the case of equity financing to which we now turn.[18]

## 5. TRANSITIONAL DYNAMICS: EQUITY FINANCING

The case where the optimality conditions induce firms to employ all equity financing is formally similar to the previous case of all bond financing, so that our treatment can be brief. The key difference is that the short-run cost of capital, determining the marginal product of capital, is

(4c")
$$(1-\tau_p)f'(k/l) = x_e^* = \theta + \frac{\tau_c(\theta + p) + \bar{i}(\tau_y - \tau_c)}{1-\tau_c}$$

The short-run solutions for $c$, $l$, $\theta$, $p$, and $x_e^*$ are now determined in (4a)-(4d), (4c") in terms of $\alpha$, $m$, and $k$, which evolve according to (5a), (5b), and (5d).

Since the short-run solutions for the real part of the system (consumption and employment) are determined independently of the short-run cost of capital, the solutions for $c$ and $l$ are again given by (14a) and (14b), respectively, with the partial derivatives being as before. The solutions for the financial variables, on the other hand, are still of similar form to (14c), (14d), namely,

(14c')
$$\theta = \theta(\alpha, k, m) \quad \theta_\alpha < 0,\ \ \theta_k < 0,\ \ \theta_m > 0$$

(14d')
$$p = p(\alpha, k, m) \quad p_\alpha < 0,\ \ p_k > 0,\ \ p_m < 0$$

The difference is that the magnitudes, and in some cases the signs of the partial derivatives change. These expressions are available in the Appendix.

Formally, the linearized dynamics of the system about its steady state are again described in (15), although as noted, the partial derivatives of the functions $p(\cdot)$ and $\theta(\cdot)$ are now different. With $\theta$ now being negative, an additional mild restriction is now required to ensure that the dynamic system contains one stable and two unstable roots, as before.[19]

With this condition, the stable solution is precisely of the same form as in the bond-financed case, with $k$, $m$, and $\alpha$, being determined by (16). As before, an increase in the monetary growth rate impacts on the dynamics of the economy through the steady state, and how this in turn influences the initial jumps in $\alpha(0)$ and $m(0)$, as determined by (17a), (17b). In this case, we find

$$\frac{d\alpha(0)}{d\mu} = -\lambda_1 \frac{l\Omega}{D} \frac{d(\tilde{k/l})}{d\mu} < 0 \tag{27}$$

The jump in the initial value $\alpha(0)$ resulting from the increase in the monetary growth rate is reversed from the previous case. This is because all the short-run impact effects are proportional to the effects on the steady-state capital-labor ratio. And as we have shown previously, the long-run effect of the monetary expansion on the latter is reversed between the two cases. Thus, equation (27) together with the short-run solutions (14a) and (14b) implies that the short-run effects of an increase in the monetary growth rate is to reduce employment and output, increase consumption and wages, and reduce the cost of capital and investment. The short-run effects on the nominal and financial variables $p$, $\theta + p$, and $\theta$, are obtained as before; in particular, we can show

$$\frac{dp(0)}{d\mu} > 1 \tag{28}$$

$$\frac{dr_g(1-\tau_y)}{d\mu} = \frac{d(\theta + p)}{d\mu} > 1 \tag{29}$$

Thus equations (28) and (29) yield the important implication that in the short run, both the rate of inflation and the after-tax nominal rate of return overshoot the long-run effects of the monetary expansion. In addition, the real consumer rate of return, $\theta$, can be shown to fall.

The following dynamic adjustments are generated by these initial responses. First, the fall in output, together with the rise in consumption, means that investment falls so that the capital stock starts to decline. Secondly, the overshooting of the inflation rate causes the real money stock to decline, while the fall in the real rate of return means that the marginal utility of consumption starts to increase. As in the case of bond financing, further changes in the short-run variables result. The increase in $\alpha$ stimulates employment, although this is offset by the decline in employment resulting from the decline in capital stock. Consumption is affected in a parallel way. The fall in the real money stock and capital, together with the rise in $\alpha$, causes the inflation rate and the after-tax nominal rate of return to fall. These variables therefore begin to correct for the initial over-adjustment.

Figure 2 illustrates the dynamics for the capital-labor ratio, the rate of inflation, and the level of output.[20] Starting from an initial capital-labor ratio $k/l$, at time 0, when the expansion in the monetary growth rate takes place, the capital-labor ratio increases to the point $D$ in Figure 2a. Thereafter, it falls monotonically towards the new steady-state level $k/l$, which lies below the original level. The increase in the capital-labor ratio is therefore more than reversed during the transitional path. The path for the inflation rate is illustrated in Figure 2b. Starting from an initial inflation rate $p = \bar{\mu}$, at the time of the monetary expansion it jumps to $E$, thereby overshooting the new long-run equilibrium. Thereafter, the inflation rate begins to fall, converging monotonically to it new steady-state level. Finally, output follows either of the two adjustment paths indicated in Figure 2c. As indicated previously, output falls both in the short run and in the long run. If the former exceeds the latter, the output must rise during the transition and vice versa.

## 6. CONCLUSIONS

In this paper we have analyzed the effects of a monetary expansion within a dynamic optimizing macroeconomic model. We have shown that both the short-run and long-run effects of an increase in the monetary growth rate, and therefore the effects along the transitional path, depend critically upon the tax structure and the firm's optimal financial decisions which correspond to them.

Because the analysis deals with a world of perfect certainty, the optimal financial decisions call for all bond financing or all equity financing, depending upon the tax rates on personal income, corporate income, and on capital gains. If the tax structure induces firms to employ all bond financing, the effects of an increase in the monetary growth rate depend critically upon the extent to which interest payments are tax deductible for corporations. In the case where the degree of deductibility is constant and exceeds the ratio of the personal to the corporate profit tax rate, the effects of an increase in the monetary growth rate are generally expansionary. In the short run, employment and output both rise. With capital fixed instantaneously, the capital-labor ratio falls instantaneously, although thereafter it rises monotonically over time toward a higher level. In the long run, output and capital rise, while employment, although indeterminate, will probably rise as well. The short-run inflation rate and after-tax nominal rate of return increases partially and during the subsequent transition complete their adjustment to the new equilibrium.

By contrast, with low interest deductibility of interest payments and bond financing, or if the tax structure induces equity financing, the effects of an increase in the monetary growth rate are generally contractionary. In the short run employment and output fall; the capital-labor ratio rises instantaneously, although it falls over time toward a level below the original. Long-run output and capital fall, while employment will probably, but not necessarily, fall as well. The short-run inflation rate and after-tax rate of return overshoot initially, thereafter falling steadily towards their new equilibrium levels.

The qualitative properties of the model appear to be generally consistent with the observed behavior of the U.S. economy during the past decade or so. This period has seen a general shift by firms towards more equity financing. The falling inflation rate, together with the decline in investment, during the period 1979-1982, following the increase in the monetary growth rate during the earlier period 1974-78, is consistent with the overshooting of the inflation rate, as well as the longer-run decline in activity, shown to be associated with equity financing. Moreover, the increase in real interest rates during 1980-83, following the earlier monetary expansion, is also consistent with the predicted responses under equity financing. Finally, the recent improvement in the U.S. economy, accompanied by the move towards more bond financing might also be viewed as being broadly consistent with the theoretical analysis. However, these suggestions are only tentative, and it is hoped that the present framework may provide a useful approach for the detailed empirical investigation of the dynamic behavior of the economy.

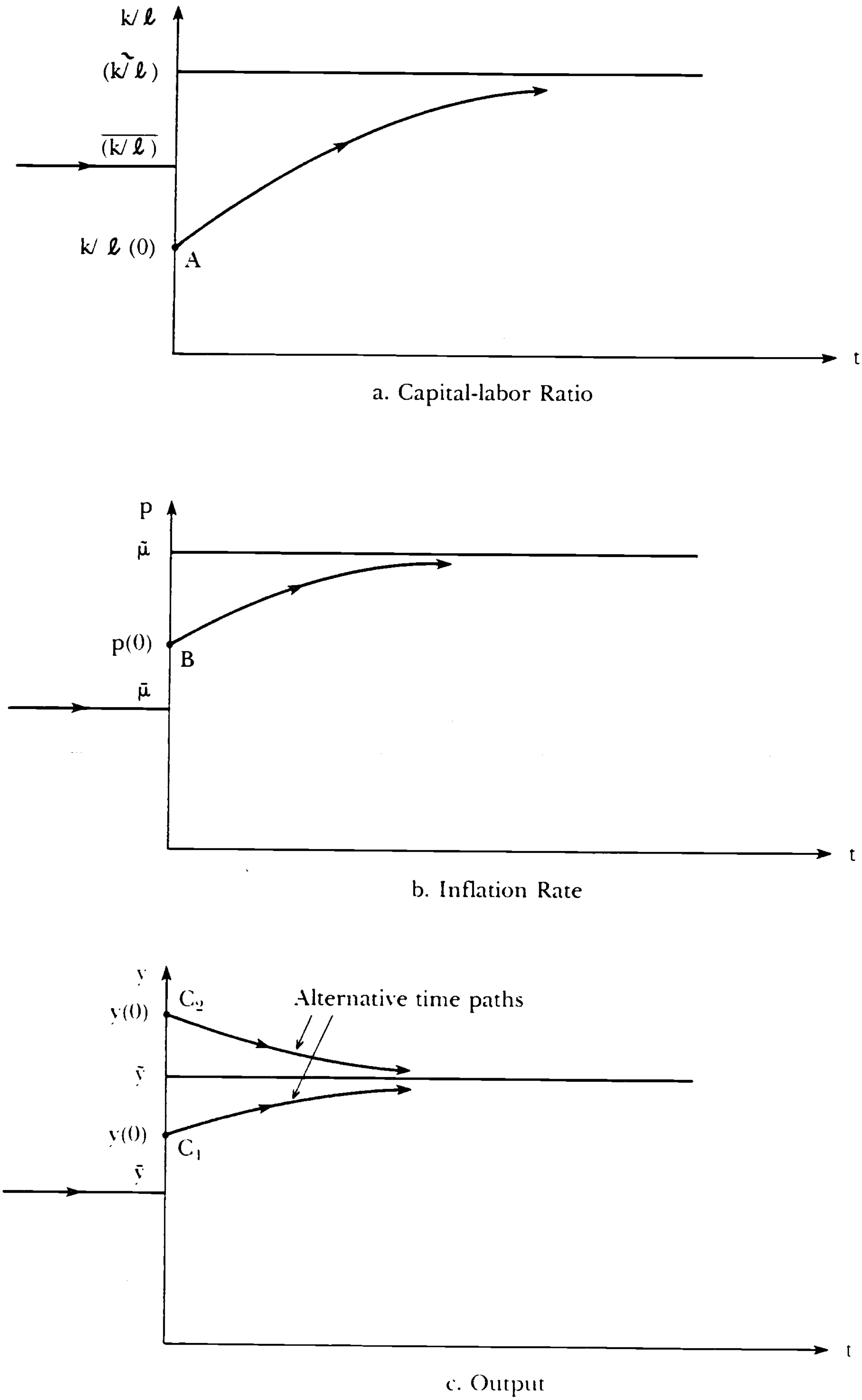


Figure 1: Time Path for Economy with Bond Financing: Expansionary Case

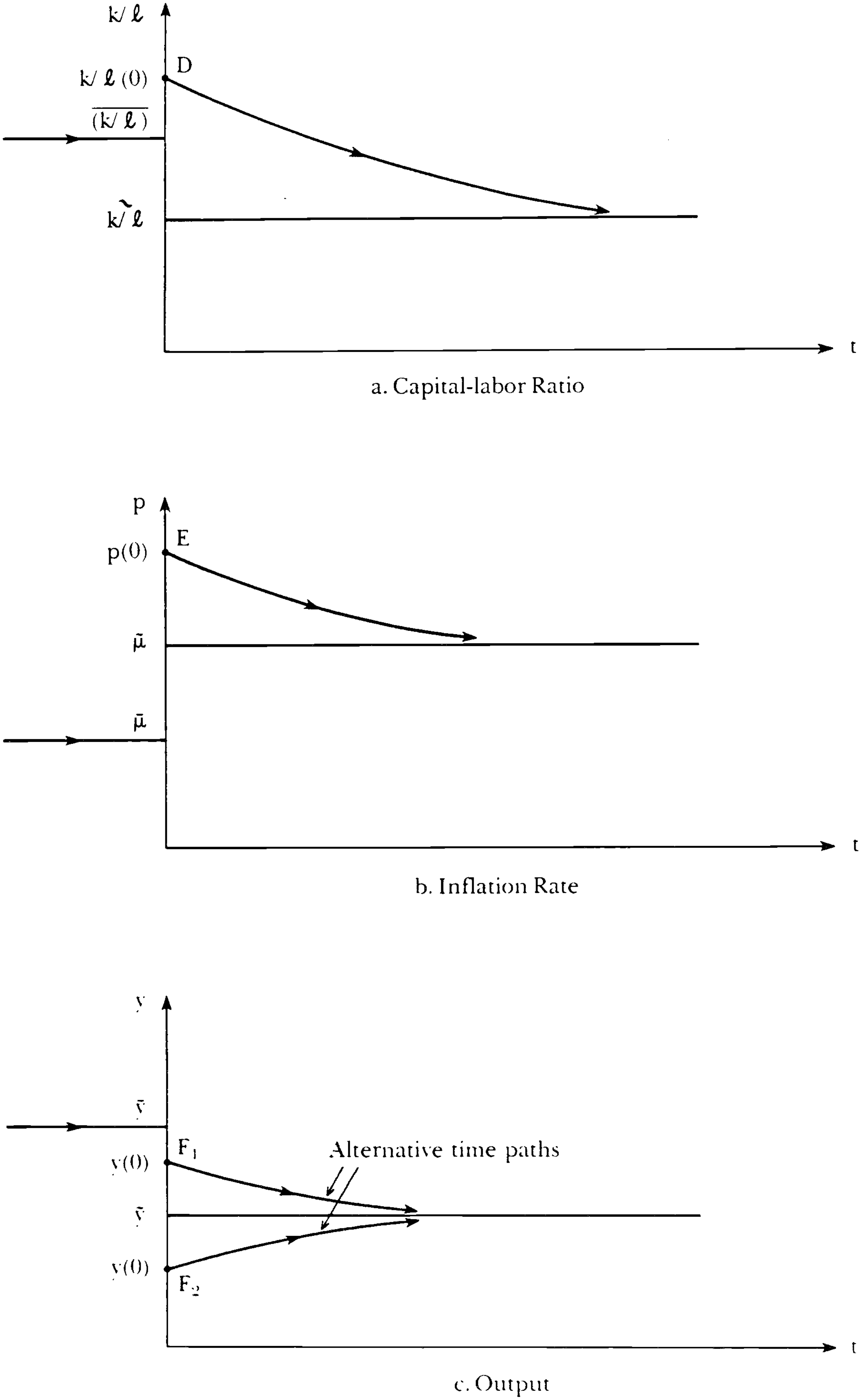


Figure 2: Time Path for Economy with Equity Financing

Table 1

STEADY STATE EFFECTS OF INCREASE IN MONETARY GROWTH RATE

Effect on

1. *Cost of Capital*

$$\frac{dx^*}{d\mu} = \frac{\tau_y - z\tau_p}{1-\tau_y} \qquad \text{bond-financing}$$

$$= \frac{\tau_c}{1-\tau_c} > 0 \qquad \text{equity-financing}$$

2. *Capital-Labor Ratio*

$$\frac{d(k/l)}{d\mu} = \frac{1}{(1-\tau_p)f''}\frac{dx^*}{d\mu}$$

3. *Real Wage Rate*

$$\frac{dw}{d\mu} = -f''k/l\frac{d(k/l)}{d\mu}$$

4. *Consumption (Output)*

$$\frac{dc}{d\mu} = \delta\frac{d(k/l)}{d\mu}$$

$$\delta \equiv \frac{1}{D}\left[-f(1-\tau_y)U_c f''k/l - lf'[U_{ll} + F_l(1-\tau_y)U_{cl}]\right] > 0$$

5. *Employment*

$$\frac{dl}{d\mu} = \gamma\,\frac{d(k/L)}{d\mu}$$

$$\gamma \equiv \frac{1}{D}\left[lf'[U_{lc} + F_l(1-\tau_y)U_{cc}] - (1-\tau_y)U_c f''k/l\right] \lesseqgtr 0$$

6. *Capital*

$$\frac{dk}{d\mu} = \psi\frac{d(k/l)}{d\mu}$$

$$\psi \equiv -\frac{1}{D}\left[lF_l[U_{lc} + F_l(1-\tau_y)U_{cc}] + l[U_{ll} + F_l(1-\tau_y)U_{cl}] + (k/l)^2(1-\tau_y)U_c f''\right] > 0$$

7. *Marginal Utility*

$$\frac{d\alpha}{d\mu} = \eta\frac{d(k/l)}{d\mu}$$

$$\eta \equiv -\frac{1}{D}\left[lf'[U_{cc}U_{ll} - U_{cl}^2] + U_c f''(k/l)(1-\tau_y)[U_{cl} + fU_{cc}]\right] < 0$$

8. *Real Money Stock*

$$\frac{dm}{d\mu} = \frac{1}{\alpha V''}\left[V'\frac{d\alpha}{d\mu} + \alpha^2\right]$$

$$D \equiv -f[U_{lc} + F_l(1-\tau_y)U_{cc}] - [U_{ll} + F_l(1-\tau_y)U_{cl}] > 0$$

## FOOTNOTES

1. See for example, Feldstein (1976, 1980), Feldstein, Green and Sheshinski (1978). These, together with related papers, are contained in Feldstein (1983).

2. We adopt the following conventional notation. Partial derivatives are denoted by corresponding lower case letters, while total derivatives of a function of a single argument are denoted by primes.

3. As will become apparent below, the separability of the utility function in $c$ and $l$ on the one hand, and real money balances on the other, leads to a separation of determination of the real and financial variables. The introduction of $m$ into the utility function involves the familiar problem of generating a reason for holding money in a certainty world. Recent work by Feenstra (1986) has demonstrated that money in the utility function is functionally equivalent to entering money in liquidity costs in the budget constraint. Since in general, this approach does not give rise to a separable utility function, this assumption should be viewed with some caution. Finally, the Brock-Turnovsky analysis also introduced government expenditure $g$ directly into the utility function to reflect the extent to which private and public goods are viewed as substitutes by consumers. Since the present analysis treats $g$ as being fixed, its exclusion from $U$ involves no loss of generality.

4. Both private and public bonds are assumed to be infinitesimally short-lived and to be denominated in nominal terms.

5. The current proposal for tax reform in the U.S. advocates setting $\tau_c = \tau_y$. According to (7) firms will find it optimal to adopt all bond financing.

6. Real taxable corporate income is defined by

$$\pi = y - wl - zrb_p$$

where $b_p$ is the (real) stock of bonds, and $r_b$ is the nominal interest rate. Under all bond financing $b_p = k$, and under constant returns to scale $y - wl = f\,'k$, so that

$$\pi = (f\,' - zr)k$$

Using the optimality conditions (4d), (4e'), and (9), we have

$$r_b = \frac{\theta + p}{1 - \tau_y}, \quad f\,' = \frac{\theta}{1 - \tau_p} + \frac{(\tau_y - z\tau_p)(\theta + p)}{(1 - \tau_p)(1 - \tau_y)}$$

so that

$$\pi = [\theta + p\tau_y - z(\theta + p)]k/[(1 - \tau_y)(1 - \tau_p)]$$

Thus corporate taxable profit $\pi \geq 0$ if and only if

$$z \leq (\theta + p\tau_y)/(\theta + p) \equiv z^*$$

7. This suggestion was made by a referee.

8. For example, this relationship is equivalent to the optimality condition in Yaari (1964).

9. We should emphasize that following real world tax structures, $\tau_c$ is levied on the *nominal* capital gain which is $(\dot{q} + qp)E$.

10. In the absence of taxes (10c) reduces to the familiar condition $f\,'(k/l) = \beta$, which is independent of the monetary growth rate. Money is therefore super neutral. The non-superneutrality in our model therefore stems from the interaction of the monetary growth rate with the existing tax rates.

11. From (7) it can be shown that given $\tau_y > \tau_c$, a simple sufficient condition for $x_b^* < x_e^*$ for all $z$ is that $r_p < i$.

12. From (10c') and (10c") we can show

$$\frac{dx_b^*}{d\mu} - \frac{dx_e^*}{d\mu} = \frac{\tau_y - \tau_c}{(1 - \tau_c)(1 - \tau_y)} - \frac{z\tau_p}{1 - \tau_y}$$

which embodies the two effects noted in the text.

13. Note that since $b_g$ does not interact with either the short-run equilibrium or the dynamic equations (5a), (5b), and (5d), we ignore the dynamics of $b_g$, allowing it to be determined residually by the government budget constraint. As noted by Brock and Turnovsky this does raise a technical point in that in order for the solution for $b_g(t)$ to be consistent with the consumer's transversality condition for bond holdings it is necessary to assume that the monetary authority pick an appropriate initial stock of bonds by undertaking an initial open market exchange of money for bonds. The same results obtain if one assumes instead that the government continuously balances the budget by setting an appropriate lump sum tax.

14. In signing these partial derivatives we are assuming $\theta + p \geq 0$, which is equivalent to requiring that the nominal rate of return be nonnegative.

15. The mild restriction is: $mV''/V' < -(\tau_y - z\tau_p)/(1 - \tau_p)$. It is met in the case where bond financing is expansionary; it is also likely to be met in the other case as well. The two unstable roots may be either real or complex, while the single stable root is necessarily real. Thus the stable adjustment path to equilibrium is necessarily a monotonic one.

16. The derivation is available from the author.

17. This effect is demonstrated formally by (19).

18. A further question concerns the effect of an increase in the monetary growth rate on the speed of adjustment along the transitional adjustment path. This amounts to analyzing the effect of a change in $\mu$ on $\lambda_1$ and has been addressed in a simple extension of the Sidrauski (1967) model and using a specific utility function by Fischer (1979b). Because of the long-run nonsuper-neutrality of money arising from the presence of taxes, this turns out to be analytically intractible in the present model. Further analysis of this issue would require numerical simulation methods.

19. This additional mild restriction is that

$$\epsilon \equiv \frac{mV''(m)}{V'(m)} < -\tau_c$$

Given that the tax on capital gains, $\tau_c$, is typically small (in many countries zero) this condition is not an unreasonable one.

20. This figure also illustrates the behavior under bond financing in the case that $z < \tau_y/\tau_p$ so that it is contractionary.