NBER WORKING PAPER SERIES

THE POLITICAL ECONOMY OF TRADE AND INTERNATIONAL LABOR MOBILITY

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Working Paper 21274
http://www.nber.org/papers/w21274

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 2015, Revised August 2018

Previously circulated as "The Political Economy of Trade and Labor Mobility in a Ricardian World." We appreciate the suggestions made by participants in presentations given at Georgetown University, Trento Summer School 2015, UECE Lisbon Meetings 2015, SAET Meetings 2016, Banco de Mexico, and Berlin Workshop on New Insights into Political Economy 2016. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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JEL No. F13,F22

ABSTRACT

We explore the political economy of trade and migration policies in several models of international trade. We show that in a Ricardian world, free trade and no international labor mobility is a Nash equilibrium outcome, but free trade and free international labor mobility is not. The result holds under different assumptions about the set of goods, preferences and the number of countries. An analogous result also holds in multifactor economies such as: a version of the standard two-sector Heckscher-Ohlin model, the Ricardo-Vinner specific factors model, and a three-sector model with a non-tradeable sector. We also study three extensions of our model in which free trade and at least partial labor mobility is a Nash equilibrium outcome. One extension introduces increasing returns to scale. Another an extractive elite. Finally, we allow the recipient country to charge an immigration fee in the form of an income tax and distribute the proceeds among domestic workers, which induces a Pareto improvement for the global economy.

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in advanced and low-income countries differ by a factor of 10 or more (see, for example, Rodrik 2002, and Freeman 2006); which suggests that restrictions on labor mobility could be the largest distortion in the world economy (see, for example, Clements 2011). Moreover, there are at least two good reasons to study the political economy of trade and migration policies simultaneously rather than as two independent policies.

First, theoretical trade models explicitly or implicitly imply that international trade and factor mobility could be complements and/or substitutes. If they are complements, why do we observe harsher restrictions to international migrations than to international trade? If they are substitutes, why do we observe illegal migration between countries that engage in free trade? Second, countries can potentially engage in free international trade and labor mobility, impose trade and migration restrictions, choose a combination of free trade and migration restrictions or vice versa. Indeed, as Brexit reminds us, the political economy of trade and migration cannot be simply mapped into a one-dimension policy decision (say, more, or less globalization). On the contrary, it seems that the pro-Brexit coalition favors free trade, but opposes free labor mobility.[1]

We develop an international political economy model for the determination of trade and labor mobility policies. First, all countries simultaneously decide on their trade and migration policies with the aim of maximizing the wellbeing of domestic workers. In order to keep things as simple as possible, we assume that only a fraction of the labor endowment of each country is mobile and that trade and migration policies can adopt only two values: free trade and no trade, and free labor mobility and no labor mobility.[2] In the case of two countries, no trade is equivalent to complete autarky and no labor mobility implies that mobile workers must stay in their countries of origin. In the multi-country case, each country must select its trade and labor mobility partners. In other words, the trade policy of any given country can be described as the set of countries with which that country agrees to engage in free trade. The situation with respect to migration policy is analogous. Second, mobile workers decide where they will reside and work. Finally, countries engage in trade, production and consumption.

We consider two families of economies (and several variations within each family): Ricardian economies and factor proportions models. For all these models, we obtain a rather negative result. We show that, although it is always possible to find conditions under which free trade and no labor mobility is a Nash equilibrium outcome of the political economy game, free trade and free labor mobility is never a Nash equilibrium outcome. From a positive point of view, this may help to explain why international labor markets are much less integrated than international markets of commodities are. However, this suggests a significant misallocation of resources in the global economy as well as a substantial failure of international institutions. From a normative point of view, this result could also be judged undesirable

[1] See, for example, https://voxeu.org/article/brexit-four-freedoms-and-indivisibility-dogma

[2] In Section 5 we relax this assumption.

based on fairness considerations. For this reason, we also explore several ways of overcoming this negative result. In particular, we show that if quantitative migration restrictions are replaced by a migration fee implemented as an income tax on migrants whose revenue is distributed among domestic workers in the recipient country, workers all over the world will be better off. Thus, free trade and free labor mobility will become a Nash equilibrium outcome.

1.1 The challenge of achieving free trade and international labor mobility

1.1.1 Ricardian economies

In a Ricardian economy, trade is not a source of conflict because everybody gains when countries specialize in their comparative advantages and engage in international trade. Thus, free trade and no labor mobility is a Nash equilibrium outcome of the political economy game that we studied. In a Ricardian world, free trade does not produce real wage equalization because countries use different production technologies. As a consequence, if countries that engage in trade also allow free labor mobility, workers will move from poor to rich countries until real wages are equalized. Then, under free trade, labor mobility reduces wages in rich countries and, hence, workers from rich countries prefer to block migration flows. Free trade and free labor mobility is not a Nash equilibrium outcome of the political economy game.

We show that this logic applies not only to a simple Ricardian model, but also to more complex Ricardian economies, including those with a continuum of goods (as in Dornbusch, Fischer and Samuelson, 1977), non-homothetic preferences (as in Matsuyama, 2000) and multiple countries (as in Eaton and Kortum, 2002). In a nutshell, we confirm the basic results obtained using the simple Ricardian model under several variations in the model's assumptions (set of goods, preferences, and set of countries). For each Ricardian model, we also deduce the conditions under which free trade induces partial convergence, full divergence and a reversal of fortune in terms of the well-being of workers in each country. In all cases, free trade and free labor mobility lead to full convergence, but this is never a Nash equilibrium of the political game. Thus, in a Ricardian world, the lack of convergence in levels of well-being across countries can be attributed to an international political equilibrium that blocks free labor mobility.

1.1.2 Factor proportions models

A Ricardian world is by no means the unique economic environment in which free trade does not preclude international migrations, but political interactions lead to no labor mobility. Several multifactor models produce analogous results. We study three of them: a version of the standard two-sector Heckscher-Ohlin model, the Ricardo-Vinner specific factors model, and a three-sector model (two tradeable goods and one non-tradeable good).[3] In a two country Heckscher-Ohlin model with country-specific total factor

[3] See, Galiani, Heymann and Magud (2017), Galiani, Schofield and Torrens (2014), and Galiani and Somaini (2018).

productivity, free trade does not induce full wage equalization, which opens the door to migration flows. However, free trade and free labor mobility is never a Nash equilibrium outcome. The reason is that domestic workers in the country with high real wages under free trade prefer to block free labor mobility because the flow of immigrants would reduce their real wages. We also show that in a Heckscher-Ohlin economy is more complicated than in a Ricardian economy to support free trade and no labor mobility as a Nash equilibrium outcome. Only when the country with high real wages under free trade is fully specialized in the non-labor intensive good (which requires that the countries are endowed with very dissimilar relative factors endowments), workers in both countries are willing to support free trade.

In a Ricardo-Vinner specific factors economy, free trade and free labor mobility is not a Nash equilibrium outcome either. Once again, under free trade, real wages decrease with the size of the labor force, inducing workers in the country with high real wages to favor blocking labor mobility. In order to keep things as simple as possible, we assume that both industries are equally labor-intensive, which makes free trade and no labor mobility a Nash equilibrium outcome. On the contrary, if industries differ in their labor intensities, workers in the country with a comparative advantage in the non-labor-intensive industry might prefer to block free trade.

Finally, we explore trade and migration policies in a model with three sectors (two tradeable and one non-tradeable) and three factors of production (e.g., capital, natural resources and labor). This model allows us to capture a different channel through which workers are in favor of free trade, namely, the role of the non-tradeable sector. In the poor country, which specializes in the natural resource-intensive good, workers are employed in the non-tradeable sector. They support free trade because it increases the demand of the non-tradeable good and, hence, wages. In the rich country, which is diversified, workers are employed in the capital-intensive manufacturing sector and in the non-tradeable sector. They support free trade because it expands the demands for the manufactured good and the non-tradeable good, inducing higher wages. Thus, free trade and no labor mobility is a Nash equilibrium outcome. Regarding international labor mobility, workers in the poor country clearly support free labor mobility because they want to be able to migrate to the rich country. Even if some workers are immobile, they will also support free labor mobility since a reduction in the labor force of the poor country will increase wages and improve the terms of trade. However, workers in the rich country prefer to block immigration because any increase in the domestic labor force reduces their wages and deteriorates the country's terms of trade. Thus, free trade and free labor mobility is not a Nash equilibrium outcome.

1.1.3 Inducing free trade and free labor mobility

We also study three alternative ways of overcoming this negative result, that is, we explore changes in the economic and political environment as well as in the set of policy instruments available, that make trade and labor mobility a Nash equilibrium outcome.

First, we introduce differentiated products, monopolistic competition and increasing returns to scale, à la Krugman (1979) and Helpman (1981), in one of the tradeable sectors of a simple Ricardian economy. In this environment, workers from a scarcely populated rich country may prefer to allow migration flows, at least for a certain range of the population, because a bigger labor force induces an expansion of the number of varieties produced and a reduction in the price of each variety. Indeed, we deduce conditions under which free trade and some, but not full, free international labor mobility is a Nash equilibrium outcome of the political economy game. This may help to explain why immigration policies in the U.S. were very liberal until the end of the 19th century, but then became much more restrictive in the early 20th century.

A second way of generating free labor mobility is to introduce an elite that extracts income from workers as in Acemoglu and Robinson (2012). In particular, we show that, if the extractive elite in the rich country is powerful enough, then free labor mobility is possible. The reason for this is that, although the extractive elite in the rich country is hurt by the reduction in wages induced by an inflow of workers, immigrants are also a new source of income for the elite (more workers to extract from). Hence, if the second effect outweighs the first one and the elite is relatively powerful, then the rich country will permit an inflow of workers. Trade, however, is not a source of conflict between workers and the elite because, in a Ricardian world, everybody gains from trade. In other words, workers in a rich country only care about wages, while the extractive elite cares about total labor income. This produces a conflict of interest between workers and the elite in regard to labor mobility but not with regard to trade policy.

Finally, we introduce an immigration fee. In particular, we assume that the recipient country can charge an income tax to immigrant workers and pay transfers to domestic workers. Under free trade, the flow of immigrants has an ambiguous effect on the wellbeing of workers from the rich country because, on the one hand, it reduces real wages in the rich country, but, on the other hand, it increases the revenue from immigration fees. Moreover, we find conditions under which the second effect dominates the first one and, therefore, workers from the rich country are better off charging an immigration fee that induces positive migration flows rather than completely blocking migration. Mobile workers in the poor country are always better off if they can migrate to the rich country, while immobile workers from the poor country benefit from the rise in real wages caused by the outflow of workers. Thus, free trade and international labor mobility subject to a positive, but not prohibitive immigration fee could be supported as a Nash equilibrium outcome.

1.2 Related literature

As we have already mentioned, the economic and political economy literature has devoted much less attention to migration than to international trade in both theoretical and empirical terms (Facchini, 2004, and Facchini, Mayda and Mishra, 2007). Nevertheless, there is a very interesting body of literature on labor mobility, migration policies and the political economy of migration. First of all, there is a well-established body of literature on international factor mobility and, in particular, on the determinants and effects of migration (see, among others, Mundell 1957, Markusen 1983, Grossman 1983, and Wong 1995).

Second, several works have shed light on the crucial role played by international labor mobility in the convergence of living standards between Europe and America during the 19th and early 20th centuries (e.g., Taylor and Williamson, 1997, O'Rourke and Sinnott, 2003, O'Rourke, 2004, and Hatton and Williamson, 2005).[4] These studies suggest that international labor mobility has a strong influence on cross-country convergence of levels of well-being. Numerous authors have also documented the fact that migration policies became much more restrictive in the 20th century and that, nowadays, international labor markets are much less integrated than international markets of commodities are (e.g., O'Rourke, 2004 and Hatton and Williamson, 2007, Rodrik 2002, Freeman 2006 and Clements 2011).[5]

Third, there is an empirical literature that investigates the effects of several shocks that affected migration flows on wages and employment of domestic workers. See, among others, Nickell and Saleheen (2009) for a recent evaluation of the literature and new estimates for Great Britain. Unfortunately, these works focus on relatively small shocks in migration flows and, therefore, they are not necessarily informative on the effects of unrestricted mass migration. Indeed, in a Ricardian economy with a finite number of goods, there is a simple theoretical argument that suggests that under free trade small migration flows may have no effect on wages, while free labor mobility has a significant impact on wages (for details,

[4] Taylor and Williamson (1997) find that international real wage dispersion declined by 28% from 1870 to 1910, but that without the mass migrations that occurred during this period, wage dispersion would have increased by 7%. In the same vein, O'Rourke (2004) reports that wages rose in emigration countries during the late 19th and early 20th centuries, converging with countries of immigration (see also Hatton and Williamson, 2005). O'Rourke and Sinnott (2003) argue that "One hundred years ago mass emigration raised living standards significantly in countries such as Ireland, Italy and Sweden, enabling them to converge on the core countries of the day, Britain and the U.S. Indeed, mass migration can account for as much as 70% of the convergence in living standards worldwide which occurred during the late 19th century."

[5] O'Rourke (2004) and Hatton and Williamson (2007) show that liberal policies on immigration during the 19th century came to an end in the early 20th century, when many popular destination countries began imposing severe restrictions on immigration. Moreover, they document that these restrictions have remained in place even after many restrictions on the movement of goods have been eliminated. Freeman (2006) examines the degree of international economic integration in labor compared with other factors and concludes that the labor market is the least developed facet of globalization. In line with this view, see also Rodrik (2002) and Clements (2011).

see section 3.2). There is also evidence that large trade shocks have significant effects on wages and employment (see, among others, Autor, Dorn and Hanson 2016), suggesting that large migration flows also produce large effects on wages. Indeed, in a Heckscher-Ohlin model with country-specific total factor productivity, the effect of trade and labor mobility on real wages is greater than the effect of trade alone (see, section 4.1).

Fourth, although the political economy of labor mobility has not been fully explored, several works have discussed different economic and political determinants of migration policies. Foreman-Peck (1992) develops a simple political economy model of factor mobility focused on the receiving country. He shows that if the government of the receiving country gives a great deal of weight to the interests of landowners (workers) and if land and labor are complements, then immigration policies will tend to be open (restrictive). Along the same lines, Benhabib (1996) shows that, under majority voting, immigrants with skills that are complementary with those of the median voter will be selected. Razin and Sadka (1999) study the political economy of immigration when the receiving country has a pay-as-you-go pension system, and Razin, Sadka, and Swagell (2000) investigate how unskilled immigration affects redistribution policies in the host country. Hatton and Williamson (2007) emphasize that trade is based on comparative advantage, while migration is based on absolute advantage. They also mention the spread of democracy and the decline of empires as an explanation for the change in migration policies in the 20th century. Mayda (2007), in line with our multifactor model with a non-tradeable sector, attributes the observed differences in attitudes toward trade and immigration (today, people are more pro-trade than pro-immigration) to the influence exerted by individuals working in non-tradeable sectors. However, to the best of our knowledge, there is no formal international political economy model that explains the existence of very few restrictions on international trade in conjunction with severe restrictions on international labor mobility. In this paper we develop a simple but formal model of trade and labor mobility policies.

Finally, our extension with an immigration fee used to finance transfers to domestic workers is related to and Eichenberger and Stadelmann (2017), who explore labor mobility with residency fees, and Posner and Weyl (2018), who propose a system for democratizing immigration visas that could generate benefits for unskilled workers in developed countries.

The rest of this paper is organized as follows. Section 2 presents an international political economy model for the determination of trade and labor mobility policies. Sections 3 and 4 characterize the equilibrium of the model for different Ricardian economies and for factor proportions models, respectively. Section 5 develops three extensions in which free trade and at least some labor mobility is a Nash equilibrium. Section 5.1 introduces monopolistic competition, differentiated products and economies of scale in one of the sectors of the simple Ricardian model. Section 5.2 introduces an extractive elite in the political economy model. Section 5.3 introduces migration fees. Section 6 concludes.

2 A simple model of trade and labor mobility policies

In this section we present a simple model of international trade and labor mobility. All the models in the paper are particular cases of this framework.

2.1 Countries, endowments, technologies and preferences

Consider a world integrated by J countries indexed by $j = 1, ..., J$. Each country is characterized by its labor endowment $\bar{L}^j$, its endowment of other factors $\bar{F}^j = \left(\bar{F}_1^j, ..., \bar{F}_F^j\right)$ and technologies for the production of a set of goods Z. Let Q_z^j indicate the aggregate production of good $z \in Z$ in country j and L_z^j ($F_{f,z}^j$) the amount of labor (factor f) employed in industry z in country j. All agents have the same utility function $u\left(c^j\right)$, where c^j is the consumption plan of an agent in country j.

2.2 Factor mobility

Only a fraction $m \in [0, 1]$ of the labor endowment of each country is mobile at zero cost. The rest of the labor endowment and all other factors are completely immobile. Let $\mathbf{J}_M$ be a partition of J. A set $J_M \in \mathbf{J}_M$ is a subset of countries that allow labor mobility among them, but do not allow it with the rest of the world. Then, for each $j \in J_M$, the labor force of country j, denoted by L^j, is given by $L^j = (1 - m)\,\bar{L}^j + \theta^j m \sum_{i \in J_M} \bar{L}^i$, where θ^j is the proportion of the mobile labor force in J_M that selects to locate in country j. Naturally, $\theta^j \in [0, 1]$ and $\sum_{i \in J_M} \theta^i = 1$. Mobile workers locate in the country in which they get a higher utility. Thus, choice-of-location decisions come from selecting $\left(\theta^i\right)_{i \in J_M}$ in order to maximize $\sum_{i \in J_M} \theta^i v^i$, where v^i is the indirect utility of a worker located in country i. Although all workers located in country j will earn the same utility v^j, not all workers originally from country j necessarily get the same utility. For this reason, we will denote by $v^{j,m}$ ($v^{j,im}$) the indirect utility of a mobile (immobile) worker originally from country j.

2.3 Trade

Let $Z_T \subset Z$ be the set of tradeable goods and $Z_N \subset Z$ the set of non-tradeable goods. Logically, $Z_T \cup Z_N = Z$. Let $\mathbf{J}_T$ be a partition of J. A set $J_T \in \mathbf{J}_T$ is a subset of countries that allow trade among themselves, but do not allow it with the rest of the world. p_z^j denotes the price of good z in country j and C_z^j indicates the aggregate consumption of good z in country j. Then, for $z \in Z_T$, $p_z^j = p_z$ for $j \in J_T$ and $\sum_{j \in J_T} p_z \left(Q_z^j - C_z^j\right) = 0$, and for $z \in Z_N$, $Q_z^j = C_z^j$.

2.4 Politics

In each country there is a government which selects trade and migration policies in order to maximize the welfare of its native worker (mobile and immobile workers of the country). When we refer to the "trade and labor mobility policies" of a country, we are talking about whether the country allows international trade and labor mobility with each of the other countries in the world or not. When we refer to the "welfare of native workers", we are saying that the government of each country selects its trade and labor mobility policies with the aim of maximizing the utilitarian social welfare function

$$W_G^j = mv^{j,m} + (1-m)\, v^{j,im}$$

that gives exactly the same weight to every native-born worker (mobile and immobile) and no weight at all to foreign immigrants. The implicit assumption is that potential foreign immigrants do not have political power to influence domestic decisions on trade and labor mobility.

The timing of events is as follows: 1) Governments simultaneously determine trade and labor mobility policies. 2) Mobile workers choose their location. 3) Given the labor force of each country, countries produce, trade and consume.

2.5 Equilibrium

We define the equilibrium as a combination of market equilibrium[6] for the economy and Nash equilibrium for policy decisions. Formally, an equilibrium is: (i) a pair of partitions of the set of countries $(\mathbf{J}_T, \mathbf{J}_M)$; (ii) for each $(\mathbf{J}_T, \mathbf{J}_M)$, a distribution of the labor force into countries $\left(L^j\right)_{j\in J}$; and (iii) for each $(\mathbf{J}_T, \mathbf{J}_M)$ and $\left(L^j\right)_{j\in J}$, prices and an allocation of the labor force and other factors to sectors $\left(p_z^j, L_z^j, F_{f,z}^j\right)_{z\in Z, f\in F, j\in J}$ such that:

1. Collective decisions: For all $j \in J$, $W_G^j(\mathbf{J}_T, \mathbf{J}_M) \geq W_G^j\left(\mathbf{J}_T^j, \mathbf{J}_M^j\right)$ for all $\left(\mathbf{J}_T^j, \mathbf{J}_M^j\right)$ obtained by a unilateral deviation of $j's$ policy decisions from $(\mathbf{J}_T, \mathbf{J}_M)$.

2. Labor mobility: For each pair of partitions $(\mathbf{J}_T, \mathbf{J}_M)$, $\left(L^j\right)_{j\in J}$ such that each mobile worker cannot be better off by changing his or her location.

3. Production, trade and consumption: For each $\left(L^j\right)_{j\in J}$, $\left(p_z^j, L_z^j, F_{f,z}^j\right)_{z\in Z, f\in F, j\in J}$ is a market equilibrium.

[6]For homogenous goods, market equilibrium equates with perfect competition, while in the case of differentiated goods, it equates with monopolistic competition.

3 Trade and labor mobility policies in Ricardian economies

This section studies international trade and labor mobility in a Ricardian world.

3.1 A simple Ricardian economy

Consider an economy with two countries ($J = 2$), two tradeable goods ($Z_T = \{1, 2\}$) and one non-tradeable good ($Z_N = \{3\}$). Production functions are $Q_z^j = L_z^j / a_{L,z}^j$, where $a_{L,z}^j > 0$ is the unit labor requirement in industry $z \in Z = Z_T \cup Z_N$ in country $j \in J$. Let $A_z = a_{L,z}^2 / a_{L,z}^1$ and assume $A_1 > A_2$, i.e., country 1 has a comparative advantage in good 1. Let $\bar{L}^j$ and L^j be the labor endowment and the labor force of country j, respectively. Only a fraction $m \in [0, 1]$ of $\bar{L}^j$ is mobile. All agents have the same preferences, given by $u\left(c^j\right) = \sum_{z \in Z} \alpha_z \ln\left(c_z^j\right)$, with $\alpha_z > 0$ and $\sum_{z \in Z} \alpha_z = 1$. Let w^j and p_z^j denote the wage rate and the price of good z in country j, respectively. Thus, the indirect utility function is $v^j = C + \sum_{z \in Z} \alpha_z \ln\left(w^j / p_z^j\right)$, where $C = \sum_{z \in Z} \alpha_z \ln(\alpha_z)$.

Lemma 1 characterizes the effects of trade and labor mobility on the relative well-being of workers in both countries. Let $v^j(\lambda_T, \lambda_M)$ be the utility of a worker located in country j, where $\lambda_T = 1$ if $\mathbf{J}_T = \{\{1, 2\}\}$ and $\lambda_T = 0$, otherwise, and $\lambda_M = 1$ if $\mathbf{J}_M = \{\{1, 2\}\}$ and $\lambda_M = 0$, otherwise. Thus, $\lambda_T = 1$ indicates that both countries accept free trade, while $\lambda_T = 0$ indicates that at least one country refuses to trade and, hence, both countries operate under autarky. $\lambda_M = 1$ indicates that both countries accept free labor mobility, while $\lambda_M = 0$ indicates that at least one country does not accept free labor mobility and, hence, mobile workers are forced to remain in their own country.

Lemma 1 ***(simple Ricardian economy)****. Let $T^j = -\sum_{z \in Z} \alpha_z \ln\left(a_{L,z}^j\right)$ be the average productivity of country j and assume $T^1 > T^2$. Define $\Delta = (\alpha_1 + \alpha_2) \ln\left(\alpha_1 \bar{L}^2 / \alpha_2 \bar{L}^1\right) + \left(T_N^1 - T_N^2\right)$, where $T_N^j = -\alpha_3 \ln\left(a_{L,3}^j\right)$ is the productivity of country j in the production of non-tradeable goods. Suppose that $A_1 > \frac{\alpha_1\left(\bar{L}^2 + m\bar{L}^1\right)}{\alpha_2(1-m)\bar{L}^1} > (A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}} > \frac{\alpha_1(1-m)\bar{L}^2}{\alpha_2\left(\bar{L}^1 + m\bar{L}^2\right)} > A_2$.*

1. *If $\Delta > T^1 - T^2$, then trade produces divergence in real wages, but trade and labor mobility undo the divergence, inducing complete convergence. Formally: $v^1(1,0) - v^2(1,0) > v^1(0,0) - v^2(0,0) > v^1(1,1) - v^2(1,1) = 0$.*

2. *If $0 < \Delta < T^1 - T^2$, then trade produces convergence in real wages, and trade and labor mobility produces complete convergence. Formally: $v^1(0,0) - v^2(0,0) > v^1(1,0) - v^2(1,0) > v^1(1,1) - v^2(1,1) = 0$.*

3. *If $\Delta < 0$, then trade produces a reversal of fortune but trade and labor mobility undo this reversal inducing complete convergence in real wages. Formally: $v^1(0,0) - v^2(0,0) > v^1(1,1) - v^2(1,1) = 0 > v^1(1,0) - v^2(1,0)$.* ***Proof****: see Appendix A.1.* ■

Under free trade, labor mobility induces complete convergence in real wages. The reason for this is that mobile workers migrate from the poor country to the rich country until real wages are the same in both places. There are two implicit assumptions behind this result. First, there must be a sufficient number of mobile workers. Otherwise, even if all mobile workers in the poor country migrate to the rich country, real wages will not be fully equalized. Second, real wages must be equalized before the rich country starts producing both tradeable goods. $A_1 > \alpha_1 \left(\bar{L}^2 + m\bar{L}^1\right)/\alpha_2 (1-m)\bar{L}^1 > (A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}} > \alpha_1 (1-m)\bar{L}^2/\alpha_2 \left(\bar{L}^1 + m\bar{L}^2\right) > A_2$ ensures that both conditions are satisfied. Trade alone has a positive effect on the well-being of everybody, but an ambiguous effect in terms of convergence. While, under autarky, differences in real wages depend only on relative aggregate productivity (real wages are higher in country 1 when $T^1 > T^2$), under free trade, they depend on relative labor abundance, expenditure shares and relative levels of productivity in the non-tradeable sector, but not on relative levels of productivity in the tradeable sector (real wages are higher in country 1 when $\Delta = (\alpha_1 + \alpha_2)\ln\left(\alpha_1\bar{L}^2/\alpha_2\bar{L}^1\right) + \left(T_N^1 - T_N^2\right) > 0$). That is, under free trade, country 1 is relatively richer than country 2 if it is relatively labor-scarce, it specializes in a good with a relatively high expenditure share and it is relatively more productive in the non-tradeable sector. Indeed, it is even possible for trade to produce a reversal of fortunes if the country with higher aggregate productivity is relatively labor-abundant, it specializes in a good with a low expenditure share and productivity differences in the non-tradeable sector are small.

Let $W_G^j(\lambda_T, \lambda_M)$ denotes the social welfare function of country j when trade and migration policies are (λ_T, λ_M). Proposition 1 characterizes the political equilibrium.

Proposition 1 ***(simple Ricardian economy)***. *Suppose that* $A_1 > \frac{\alpha_1\left(\bar{L}^2+m\bar{L}^1\right)}{\alpha_2(1-m)\bar{L}^1} > (A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}} > \frac{\alpha_1(1-m)\bar{L}^2}{\alpha_2\left(\bar{L}^1+m\bar{L}^2\right)} > A_2$, $T^1 > T^2$ *and* $\Delta \neq 0$. *Then, the trade and labor mobility game has three Nash equilibrium outcomes: (i) neither trade nor labor mobility; (ii) no trade and free labor mobility; and (iii) free trade and no labor mobility.*[7] *Moreover:*

1. $W_G^1(1,0) > W_G^1(0,1) = W_G^1(0,0)$, *i.e., for country* 1, *free trade and no labor mobility is better than no trade and free labor mobility or no trade and no labor mobility;*

2. $\left\{W_G^2(1,0), W_G^2(0,1)\right\} > W_G^2(0,0)$, *while* $W_G^2(1,0) > W_G^2(0,1)$ *if and only if:*

$$\alpha_1 \ln\left(\frac{A_1\alpha_2\bar{L}^1}{\alpha_1\bar{L}^2}\right) > m\left(T^1 - T^2\right), \tag{1}$$

[7] Note that some of the Nash equilibrium outcomes can be reached using different Nash equilibrium strategy profiles. For example, the outcome free trade and no labor mobility can be reached by both countries accepting trade and blocking free labor mobility or by both countries accepting trade and only the rich country under free trade blocking free labor mobility. The same applies to all the propositions in the paper.

In other words, for country 2, free trade and no labor mobility and no trade and free labor mobility are better than no trade and no labor mobility, while free trade and no labor mobility are better than no trade and free labor mobility when productivity differences are not too large. ***Proof****: see Appendix A.1.* ■

No trade and no labor mobility is a Nash equilibrium outcome because if one country decides to isolate itself, there is nothing that the other country can do to change the political equilibrium. This is a very unlikely equilibrium, however. In fact, free trade and no labor mobility is also a Nash equilibrium outcome, and it is strictly preferred to complete isolation by both countries.[8] No trade and free labor mobility is also a Nash equilibrium outcome. If one country decides to restrict trade, free trade is impossible no matter what policy is chosen by the other country. In such circumstances, domestic workers in the rich country and immobile workers in the poor country do not worry about immigration because real wages are fully determined by domestic productivity and do not depend on labor endowments. Mobile workers in the poor country are eager to migrate to the rich country because productivity and, hence, real wages are higher there. For the rich country, this equilibrium outcome is always dominated by free trade and no labor mobility. For the poor country, the comparison between the two equilibria is ambiguous (condition (1)). On the one hand, free trade generates gains from trade for all citizens. On the other hand, free labor mobility generates an increase in real wages for mobile workers equal to the aggregate productivity differences between the countries. Note, however, that no trade and free labor mobility is not a robust equilibrium outcome. For example, if workers in the rich country have a slight aversion to immigration, then no trade and free labor mobility ceases to be a Nash equilibrium outcome. Finally, free trade and free labor mobility is not a Nash equilibrium outcome. Under free trade, domestic workers in a rich country (a labor-scarce country that specializes in the production of tradeable goods with a high expenditure share and with high productivity in the non-tradeable sector) are opposed to labor mobility because the flow of immigrants would reduce their real wages. Proposition 1, in conjunction with Lemma 1, suggests that part of the lack of convergence in real wages among countries can be attributed to a political equilibrium that allows trade but bans migration.

[8]Bagwell and Staiger (1999) study a two-country game in which governments simultaneously select their tariffs. They show that Nash equilibrium tariffs are inefficient because neither government internalizes the negative terms-of-trade effect that an increase in its import duties has on the other country. They also show how trade agreements could partially resolve this problem, inducing a Pareto improvement from the Nash equilibrium. We can reinterpret autarky in our model as the equilibrium without a trade agreement (high tariffs) and free trade as the equilibrium with a trade agreement (low tariffs).

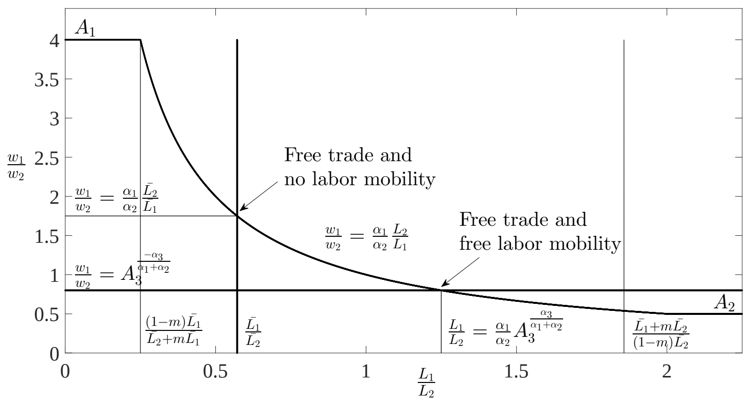

Fig. 1. Trade and labor mobility in the simple Ricardian economy. Note: $a^1_{L,z} = 1$ for $z = 1, 2, 3$, $a^1_{L,1} = 4$, $a^1_{L,2} = 0.50$, $a^1_{L,3} = 1.25$, $\bar{L}^1 = 1$, $\bar{L}^2 = 1.75$, $m = 0.75$, $\alpha_1 = \alpha_2 = 0.25$, and $\alpha_3 = 0.50$.

Next we show that essentially the same results hold for more complex Ricardian Economies. In particular, we explore three changes in the economy. First, we briefly discuss various changes in the set of goods. Introducing more tradeable and/or non-tradeable goods does not affect the results regardless of whether we are dealing with a continuum of goods with exogenous non-tradeable goods or endogenous non-tradeable goods induced by iceberg transportation costs (Dornbusch, Fischer and Samuelson, 1977). Second, we consider a Ricardian model with non-homothetic preferences, following Matsuyama (2000). Finally, we study the determination of trade and labor mobility policies in a multi-country Ricardian model with a continuum of goods, along the lines of Eaton and Kortum (2002).

3.2 Multiple goods (Dornbusch, Fischer and Samuelson, 1977)

The results obtained in section 3.1 can easily be extended to a finite set of tradeable and non-tradeable goods (see Appendix A.2 for details). The key complication is that, under free trade, the marginal tradeable good in the chain of comparative advantage may or may not be produced by both countries. If, under no labor mobility in the trading equilibrium, each country produces a different set of tradeable goods, then labor mobility increases real wages in the poor country and decreases them in the rich one. As a consequence, workers in the rich country are opposed to free labor mobility. Conversely, if, under no labor mobility in the trading equilibrium, both countries are producing the marginal good, then small changes in the country-allocation of mobile workers do not affect either the set of goods produced by each country or real wages in either country. As a consequence, the rich country will be willing to allow some immigrants to enter. However, once the poor country stops producing the marginal good and that good's production is fully relocated to the rich country, real wages in the rich country start decreasing and, hence, workers in the rich country no longer accept labor mobility. This could explain why several empirical works find that small migration flows have no effect on domestic wages, while historical studies that focus on mass migration suggest that there are significant effects on real wages and convergence.

A more elegant way of introducing multiple goods is to consider a continuum of goods as in Dornbusch, Fischer and Samuelson (1977) (see Appendix A.2 for details). Indeed, in this setting, it is always the case that, under free trade, any reallocation of mobile workers to other countries changes the marginal industry and, as a consequence, increases real wages in one country and decreases them in the other. This holds in the case of exogenous non-tradeable goods and also in the case of endogenous non-tradeable goods induced by iceberg transportation costs. Hence, with a continuum of goods, under free trade, labor mobility always reduces real wages in the rich country, which implies that workers in the rich country are opposed to labor mobility.

3.3 Non-homothetic preferences (Matsuyama, 2000)

Until this point, we have assumed Cobb-Douglas preferences. It is well-known that many trade and development patterns can be explained much more accurately when using non-homothetic preferences. In order to verify that our results are consistent with those preferences, in this section we consider a Ricardian economy with non-homothetic preferences as in Matsuyama (2000) (see Appendix A.3 for details).

Consider an economy with two countries ($J = \{1, 2\}$) and a continuum of tradeable goods $Z = [0, \infty)$ indexed by z (in order to stress the role of non-homothetic preferences we assume that all goods are tradeable.) Country 1 has a comparative advantage in higher indexed goods. Specifically, assume that $A_z = a^2_{L,z}/a^1_{L,z}$ is a continuously differentiable strictly increasing function, $A_0 < 1$ and $\lim_{z\to\infty} A_z > 1$. Each agent owns 1 unit of labor and is either mobile or immobile. Goods come in discrete units and each agent can consume a unit or no unit of each good. Specifically, the utility function is given by $u\left(c^j\right) = \int_0^\infty b_z c^j_z dz$, where $b_z > 0$ is the utility weight of good z and $c^j_z = 1$ if good z is consumed and $c^j_z = 0$ if it is not. Moreover, $b_z/a^j_{L,z}$ is a decreasing function of z for each j. Proposition 2 characterizes the political equilibrium.

Proposition 2 ***(non-homothetic preferences).*** *Let $\left(\bar{z}, \bar{v}^1, \bar{v}^2\right)$ be the unique solution to $\int_{\bar{z}}^{\bar{v}^1} a^1_{L,z} dz = 1 - \left[\bar{L}^2/A_{\bar{z}}\left(\bar{L}^1 + \bar{L}^2\right)\right]$, $\int_{\bar{z}}^{\bar{v}^2} a^1_{L,z} dz = \bar{L}^1/A_{\bar{z}}\left(\bar{L}^1 + \bar{L}^2\right)$ and $\int_0^{\bar{z}} a^2_{L,z} dz = \bar{L}^2/\left(\bar{L}^1 + \bar{L}^2\right)$. Suppose that: (i) $\frac{\bar{L}^2 + m\bar{L}^1}{\bar{L}^1+\bar{L}^2} > \int_0^{\hat{z}} a^2_{L,z} dz > \frac{(1-m)\bar{L}^2}{\bar{L}^1+\bar{L}^2}$, where $A_{\hat{z}} = 1$; (ii) $A_z > \int_0^z a^2_{L,z} dz$ for $z \in [z_L, z_H]$, where z_L and z_H are given by $\int_0^{z_L} a^2_{L,z} dz = (1-m)\bar{L}^2/\left(\bar{L}^1 + \bar{L}^2\right)$ and $\int_0^{z_H} a^2_{L,z} dz = \left(\bar{L}^2 + m\bar{L}^1\right)/\left(\bar{L}^1 + \bar{L}^2\right)$, respectively; and (iii) $\tilde{v}^1 > \tilde{v}^2$, where $\int_0^{\tilde{v}^j} a^j_{L,z} dz = 1$. Then, the trade and labor mobility game has three Nash equilibrium outcomes: (i) neither trade nor labor mobility; (ii) no trade and free labor mobility; and (iii) free trade and no labor mobility. Moreover:*

1. $W_G^1(1,0) > W_G^1(0,1) = W_G^1(0,0)$, *i.e., for country* 1, *free trade and no labor mobility are better than no trade and free labor mobility or no trade and no labor mobility;*

2. $\left\{W_G^2(1,0), W_G^2(0,1)\right\} > W_G^2(0,0)$, *while* $W_G^2(1,0) > W_G^2(0,1)$ *if and only if*

$$\int_{\tilde{v}^2}^{\bar{v}^2} b_z dz > m \int_{\tilde{v}^2}^{\tilde{v}^1} b_z dz \tag{2}$$

i.e., for country 2, *free trade and no labor mobility and no trade and free labor mobility are better than no trade and no labor mobility, while free trade and no labor mobility are better than no trade and free labor mobility when productivity differences are not too large.* ***Proof:*** *see Appendix A.3.* ■

Proposition 2 suggests that our results continue to hold even when preferences are non-homothetic. Free trade and free labor mobility is not a Nash equilibrium outcome because workers in the rich country under free trade prefer to block labor mobility. All other outcomes are Nash equilibria. For the rich country under autarky (country 1), free trade and no labor mobility prevail over all other possible equilibrium outcomes. For the poor country under autarky (country 2), free trade and no labor mobility prevail over no trade and free labor mobility when condition (2) holds, i.e., when the gains from trade ($\int_{\tilde{v}^2}^{\bar{v}^2} b_z dz$) are greater than the productivity gains under autarky for mobile workers ($m \int_{\tilde{v}^2}^{\tilde{v}^1} b_z dz$).

3.4 Multiple countries (Eaton and Kortum, 2002)

In this section, we introduce multiple countries. We base our analysis on the Ricardian model developed by Eaton and Kortum (2002) (see Appendix A.4 for details). Consider an economy with a finite set of countries, J and a continuum of tradeable goods, $Z_T = [0,1]$. Assume there are no geographic barriers that limit the mobility of tradeable goods, i.e., we consider the zero-gravity case. However, as in previous sections, not all workers are mobile. Let $\bar{L}^j$ and L^j be the labor endowment and the labor force of country j, respectively. Only a fraction $m \in [0,1]$ of $\bar{L}^j$ is mobile. Preferences are identical for all agents in every country. Specifically, $u(c^j) = \left[\int_0^1 \left(c_z^j\right)^\rho dz\right]^{\frac{1}{\rho}}$, where $\sigma = (1-\rho)^{-1} > 1$. Let $a_{L,z}^j$ be the unit labor requirement of good z in country j. Labor productivity is a random draw from a Frechet distribution, i.e., the cumulative distribution function of $a_{L,z}^j$ is given by $\Pr\left(a_{L,z}^j \leq a\right) = 1 - e^{-T^j a^\theta}$, where $T^j > 0$ and $\theta > \sigma - 1$. These distributions are independent across goods and countries.

Proposition 3 characterizes the political equilibrium when each country can decide to trade or not and can decide to allow labor mobility or not with each other country. The only restriction that we impose is that if country i accepts free trade (labor mobility) with country j and country j accepts free trade (labor mobility) with country k, then country i must accept free trade (labor mobility) with country k.

Proposition 3 ***(multiple countries)***. *Suppose that* $\min_{i\in J}\left\{T^i/\left(1-m\right)\bar{L}^i\right\} \geq \sum_{i\in J}T^i/\sum_{i\in J}\bar{L}^i$ *and assume that* $T^j \neq T^k$ *and* $T^j/\bar{L}^j \neq T^k/\bar{L}^k$ *for all* $j,k \in J$ *and* $j \neq k$. *Then:*

1. *No trade and any pattern of labor mobility is a Nash equilibrium outcome. Moreover, among those equilibria, no trade and complete free labor mobility prevail over the other equilibrium outcomes for all countries.*

2. *No labor mobility and any pattern of trade is a Nash equilibrium outcome. Moreover, among those equilibria, no labor mobility and complete free trade dominates the other equilibrium outcomes for all countries.*

3. $W_G^j\left(1,0\right) > W_G^j\left(0,1\right)$ *if and only if:*

$$\ln\left[\frac{\sum_{i\in J}\left(T^i\right)^{\frac{1}{1+\theta}}\left(\bar{L}^i\right)^{\frac{\theta}{1+\theta}}}{\left(T^j\right)^{\frac{1}{1+\theta}}\left(\bar{L}^j\right)^{\frac{\theta}{1+\theta}}}\right] > m\ln\left(\frac{\max_{i\in J}\left\{T^i\right\}}{T^j}\right) \tag{3}$$

   In other words, if the above condition holds, then for country j, *complete free trade and no labor mobility are better than complete free factor mobility and no trade.*

4. *Any pattern of trade policy other than complete autarky and any pattern of labor mobility policy within the countries that trade with each other other than no labor mobility are not a Nash equilibrium outcome. In particular, complete free trade and any pattern of labor mobility policy other than no mobility are not a Nash equilibrium outcome.* ***Proof***: *see Appendix A.4.* ■

Proposition 3 confirms our main results in a multi-country setting. Although there are many equilibrium outcomes, note that complete free trade is incompatible with any form of labor mobility. The reason for this is that, under free trade, workers in a relatively rich country always prefer to block labor mobility from relatively poor countries. This produces a cascade effect. The richest country does not want to accept labor mobility. Then, the second-richest country also prefers to block labor mobility and so on until labor mobility is completely blocked. Moreover, it is not possible to divide the world into zones that allow free trade and free labor mobility within the zone but do not allow trade or labor mobility with the rest of world. The problem is that each free trade zone is a miniature version of the world under complete free trade. Hence, the richer countries within the zone will prefer to block intra-zone labor mobility.

Proposition 3 also extends our results concerning how countries rank different equilibrium outcomes. If there is no trade at all, then all countries will be at least as well off with full labor mobility as with any other equilibrium outcome that restricts labor mobility. Mobile workers will relocate to the most productive country in the world, while immobile workers all over the world and all workers in the most productive country in the world will not be affected by migration flows, since their wages are only determined by the level of productivity. If there is no labor mobility at all, then all countries are better off with complete free trade rather than with any other equilibrium outcome that imposes restrictions on trade. This is a standard gains-from-trade argument in a Ricardian model. The larger the set of countries that engage in free trade, the larger the gains from comparative advantage and trade. Finally, we can compare free trade and no labor mobility with no trade and free labor mobility. Equation (3) is the key condition. The left-hand side of it represents the gains for country j associated with moving from complete autarky to complete free trade. The right-hand side represents the gains for country j associated with moving from no labor mobility to full labor mobility (all the while under autarky). Note that, for the richest country in the world under autarky (the country with the highest T^j), condition (3) always holds because the gains from trade are positive, while labor mobility under autarky does not have any effect on the country. In general, in this model, gains from trade are relatively high for a sparsely populated country with low productivity ($\left(T^j\right)^{\frac{1}{1+\theta}}\left(\bar{L}^j\right)^{\frac{\theta}{1+\theta}}$ low), while gains from labor mobility under autarky are relatively high for a country with low productivity (T^j low). Since some very poor countries are also very populous, it is not clear which countries will prefer free trade and no labor mobility to no trade and free labor mobility.

4 Trade and labor mobility policies in factor proportions models

In a Ricardian economy, trade policy is not a source of internal conflict because everybody gains when countries engage in international trade. On the contrary, in a multiple factors economy, potentially, there are winners and losers from international trade. This section studies trade and migration policies in several models with multiple factors of production. First, we consider a Heckscher-Ohlin model with country-specific total factor productivity. Second, we explore a Ricardo-Vinner specific factors model. Finally, in the context of a single small open economy, Galiani, Schofield and Torrens (2014), Galiani, Heymann and Magud (2017), and Galiani and Somaini (2018) develop several versions of multi-factors models, in which workers employed in the non-tradeable sector do not support protectionist policies. We build on these models, extending them to the case of two economies that could potentially engage in trade of goods as well as into labor mobility between them.

In all cases, we assume that each government selects trade and migration policies to maximize the utility of domestic workers. For example, this could be the case of two democracies, where workers are the majority of the voters. Formally, the objective function of country j's government is given by $W_G^j = m v_L^{j,m} + (1-m)\, v_L^{j,im}$, where $v_L^{j,m}$ ($v_L^{j,im}$) denotes the utility of a mobile (immobile) worker in country j.[9]

[9] Recall that $\lambda_T = 1$ indicates that both countries accept free trade, while $\lambda_T = 0$ indicates that at least one country refuses to trade. $\lambda_M = 1$ indicates that both countries accept free labor mobility, while $\lambda_M = 0$ indicates that at least one

4.1 Heckscher-Ohlin economy

Consider an economy with two countries ($J = 2$), two tradeable goods ($Z_T = \{1, 2\}$) and two factors of production (F and labor L). Production functions are given by $Q_z^j = T^j \left(F_z^j\right)^{b_z} \left(L_z^j\right)^{1-b_z}$ for $z = 1, 2$, with $1 > b_1 > b_2 > 0$ and $T^1 > T^2$, where T^j is total factor productivity in country j. Factor endowments in country j are $\left(\bar{F}^j, \bar{L}^j\right) > (0, 0)$. All agents have the same preferences, given by $u\left(c^j\right) = \prod_{z\in Z} \left(c_z^j\right)^{\alpha_z}$, with $\alpha_z > 0$ and $\sum_{z\in Z} \alpha_z = 1$. In the appendix we fully characterize the equilibrium under autarky and free trade. The following proposition characterizes the political equilibrium.

Proposition 4 ***(Heckscher-Ohlin economy).*** *Let* $\bar{f}^1 = T^1\bar{F}^1 / \left(T^1\bar{F}^1 + T^2\bar{F}^2\right)$ *and* $\bar{l}^1 = T^1\bar{L}^1 / \left(T^1\bar{L}^1 + T^2\bar{L}^1\right)$. *Assume that* $T^1 > T^2$, $T^1/T^2 \neq \left[\left(1 - \bar{f}^1\right) \bar{l}^1 / \bar{f}^1 \left(1 - \bar{l}^1\right)\right]^{\tilde{\alpha}}$ *and* $\bar{f}^1 \neq \bar{l}^1$. *Then:*

1. *Suppose that under free trade and no labor mobility the country relatively well endowed with factor F is diversified. Then, the trade and labor mobility game has a unique Nash equilibrium outcome: no trade and no labor mobility.*

2. *Suppose that under free trade and no labor mobility the country relatively well endowed with factor F specializes in the F-intensive good (i.e., good 1). Then, there is a set κ such that:*

   (a) If $\left(\bar{l}^1, \bar{f}^1\right) \in \kappa$, then the trade and labor mobility game has two Nash equilibrium outcomes: (i) no trade and no labor mobility; and (ii) free trade and no labor mobility. Moreover, $W_G^j(1, 0) > W_G^j(0, 0)$ for $j = 1, 2$.

   (b) If $\left(\bar{l}^1, \bar{f}^1\right) \notin \kappa$, then the trade and labor mobility game has a unique Nash equilibrium outcome: no trade and no labor mobility. ***Proof****: see Appendix B.1.* ■

Proposition 4 extends the main result in Section 3 to a Heckscher-Ohlin economy. In a Heckscher-Ohlin economy free trade and free labor mobility is never a Nash equilibrium outcome (see Fig 2.a). The reason is that domestic workers in the country with high real wages under free trade prefer to block free labor mobility because the flow of immigrants would reduce their real wages. For example, within the diversification cone, the utility of a worker in country j under free trade is $v_L^j(1, \lambda_M) = T^j C \left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\tilde{\alpha}} \left(\frac{T^1\bar{F}^1+T^2\bar{F}^2}{T^1L^1+T^2L^2}\right)^{\tilde{\alpha}}$. Thus, if mobile workers are allowed to choose their location, they will locate in country 1 (since $T^1 > T^2$), reducing the utility of workers in country 1 (note that $v_L^1(1, \lambda_M)$ is decreasing in L^1/L^2).

There are also some differences between the political equilibria of the trade and labor mobility game under a Heckscher-Ohlin and a Ricardian economy. First, in a Heckscher-Ohlin economy is more complicated than in a Ricardian economy to make free trade and no labor mobility a Nash equilibrium outcome.

country does not accept free labor mobility.

For example, within the diversification cone (regions a_1 and a_2 in Fig 2.b), the workers of the country importing the labor-intensive good always prefer to block international trade. Indeed, the only way in which workers in both countries are willing to support free trade is when the relative factors endowments of the countries are very different (regions b_2, c_2, e_2 and f_2 in Fig. 2.b), inducing the country with high real wages under free trade to fully specialized in the non-labor intensive good.

Second, unlike in a Ricardian economy, in a Heckscher-Ohlin economy, no trade and free labor mobility is not a Nash equilibrium outcome. In a Ricardian economy, under autarky, real wages are not fully determined by the productivity of the country and, hence, they are not affected by the size of the labor force. On the contrary, in a Heckscher-Ohlin economy, under autarky, real wages depend on the country's total factor productivity, but also on factor endowments (ceteris paribus, real wages decrease with $\bar{F}^j/L^j$). Therefore, workers in the country with high real wages under autarky oppose labor mobility.

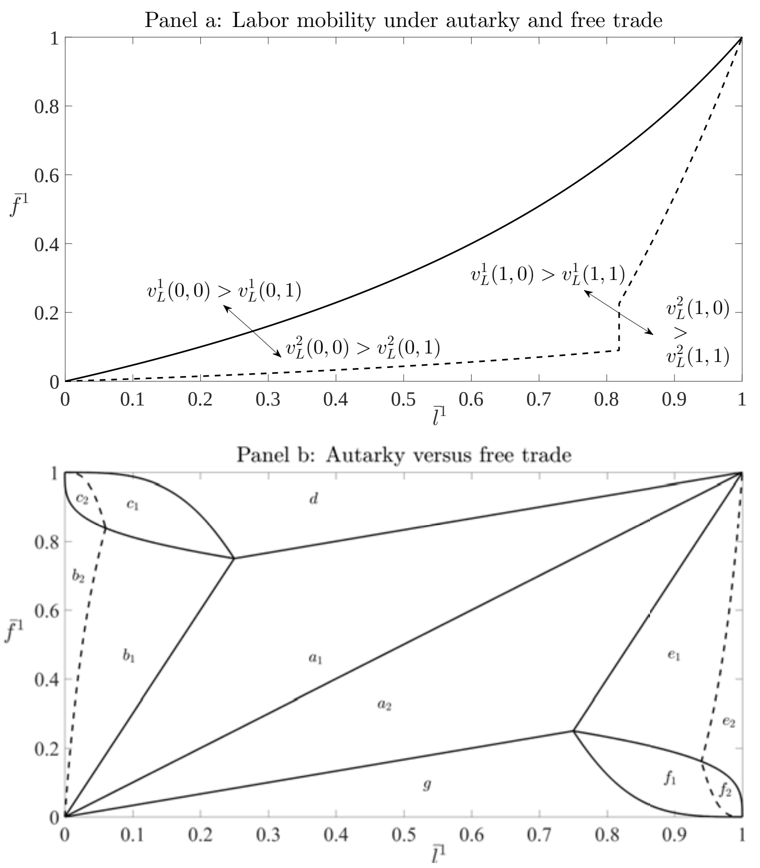

Fig. 2. Trade and labor mobility in the Heckscher-Ohlin model. **a**. Labor mobility under autarky and free trade. **b**. Autarky versus free trade. In regions a_1, b_1, c_1, and d, $v_L^1(0,0) > v_L^1(1,0)$; in regions a_2, e_1, f_1, and g, $v_L^2(0,0) > v_L^2(1,0)$; and, in regions b_2, c_2, e_2 and f_2, $v_L^1(1,0) > v_L^1(0,0)$ and $v_L^2(1,0) > v_L^2(0,0)$. In region $a = a_1 \cup a_2$ both countries are diversified; in region $b = b_1 \cup b_2$ country 1

is specialized in good 1 and country 2 is diversified; in region $c = c_1 \cup c_2$ country 1 is specialized in good 1 and country 2 in good 2; in region d country 1 is diversified and country 2 is specialized in good 2; in region $e = e_1 \cup e_2$ country 1 is diversified and country 2 is specialized in good 1; in region $f = f_1 \cup f_2$ country 1 is specialized in good 2 and country 2 is good 1; and, finally, in region g country 1 is specialized in good 2 and country 2 is diversified.

4.2 Ricardo-Vinner specific factors economy

Consider an economy with two countries ($J = 2$), two tradeable goods ($Z_T = \{1, 2\}$) and three factors of production (F_1, F_2 and labor L). Production functions are given by $Q_z^j = \left(F_z^j\right)^b \left(L_z^j\right)^{1-b}$ for $z = 1, 2$, with $b \in (0, 1)$. Factor endowments in country j are $\left(\bar{F}_1^j, \bar{F}_2^j, \bar{L}^j\right) > (0, 0, 0)$. All agents have the same preferences, given by $u\left(c^j\right) = \prod_{z \in Z} \left(c_z^j\right)^{\alpha_z}$, with $\alpha_z > 0$ and $\sum_{z \in Z} \alpha_z = 1$. In the appendix we fully characterize the equilibrium under autarky and free trade. The following proposition characterizes the political equilibrium.

Proposition 5 ***(Ricardo-Vinner specific factors economy).*** *Assume that* $\bar{F}_1^1/\bar{F}_2^1 > \bar{F}_1^2/\bar{F}_2^2$, $\left(\bar{F}_1^1/\bar{F}_1^2\right)^{\alpha_1} \left(\bar{F}_2^1/\bar{F}_2^2\right)^{(1-\alpha_1)} \neq \bar{L}^1/\bar{L}^2$ *and* $\left[\bar{F}_1^1 \bar{F}_2^1 + (1 - \alpha_1) \bar{F}_1^2 \bar{F}_2^1 + \alpha_1 \bar{F}_2^2 \bar{F}_1^1\right] / \left(\bar{F}_1^1 + \bar{F}_1^2\right) \left(\bar{F}_2^1 + \bar{F}_2^2\right) \neq \bar{L}^1 / \left(\bar{L}^2 + \bar{L}^2\right)$. *Then, the trade and labor mobility game has two Nash equilibrium outcomes: (i) no trade and no labor mobility; and (ii) free trade and no labor mobility. Moreover,* $W_G^j(1, 0) > W_G^j(0, 0)$ *for* $j = 1, 2$. ***Proof:*** *see Appendix B.2.* ■

As in a Heckscher-Ohlin economy, in a Ricardo-Vinner specific factors economy, free trade and free labor mobility is not a Nash equilibrium outcome. The intuition is analogous. Real wages under free trade decline with the size of the labor force and, hence, workers in the country with high real wages under free trade prefer to restrict labor mobility. Under autarky, real wages also fall as workers relocate to the country with high real wages under autarky. Therefore, no trade and free labor mobility is not a Nash equilibrium outcome, either. Finally, free trade and no labor mobility is always a Nash equilibrium outcome. The reason is that we have assumed that both industries are equally labor-intensive. Otherwise, the workers in the country with a comparative advantage in the non-labor-intensive industry might prefer to block free trade.

4.3 Multiple factors and non-tradeable goods

Consider an economy with two countries ($J = 2$), two tradeable goods (a rural good F and manufactures M), one non-tradeable good (services N) and three factors of production (capital K, natural resources F and labor L). Production functions are given by $Q_F^j = T_F^j \left(F^j\right)^b \left(K_F^j\right)^{1-b}$, $Q_M^j = T_M^j \left(L_M^j\right)^b \left(K_M^j\right)^{1-b}$, and $Q_N^j = T_N^j L_N^j$, where T_z^j is total factor productivity in sector $z = F, M, N$ in country j, F^j is the quantity of natural resources employed in sector F in country j, K_z^j is the quantity of capital employed in sector $z = F, M$ in country j, L_z^j is the quantity of labor employed in sector $z = M, N$ in country j, and $b \in (0,1)$.[10] Factor endowments in country j are $\left(\bar{F}^j, \bar{K}^j, \bar{L}^j\right) > (0,0,0)$. All agents have the same preferences, given by $u\left(c^j\right) = \prod_{z \in Z}\left(c_z^j\right)^{\alpha_z}$, with $\alpha_z > 0$ and $\sum_{z \in Z} \alpha_z = 1$. In the appendix we fully characterize the equilibrium under autarky and free trade. Moreover, we find conditions under which country 1 has a comparative advantage in manufactures and country 2 specializes in rural products. The following proposition characterizes the political equilibrium.

Proposition 6 ***(multiple factors and non-tradeable goods)****.* *There is a vector of thresholds $(\bar{\kappa}_1, \bar{\kappa}_2, \bar{\kappa}_3)$ such that, if $\bar{K}^1/\bar{K}^2 > \bar{\kappa}_1$, $\bar{L}^1\bar{F}^2/\bar{L}^2\bar{F}^1 > \bar{\kappa}_2$ and $T_N^1/T_N^2 > \bar{\kappa}_3$, then, the trade and labor mobility game has only two Nash equilibrium outcomes: no trade and no labor mobility and free trade and no labor mobility. Moreover, $W_G^j(1,0) > W_G^j(0,0)$ for $j = 1,2$.* ***Proof****: see Appendix B.3.* ■

As in previous sections no trade and no labor mobility is a Nash equilibrium outcome. Nothing can be done if one country decides to fully isolate itself. Free trade and no labor mobility is a Nash equilibrium outcome because workers in both countries gain from free trade. Workers in country 1 are better off accepting international trade than under autarky for two reasons. First, in country 1 free trade leads to an expansion of the manufacturing sector and a contraction of the rural sector. This induces an increase in the demand of labor. Second, free trade expands the aggregate income in the tradeable sectors, which increases the demand of the non-tradeable good and, hence, the demand of labor. Both effects operate in the same direction, pushing wages up. The situation for workers in country 2 is more complicated. On the one hand, free trade produces a contraction in the manufacturing sector and an expansion in the rural sector. This leads to a decrease in labor demand. On the other hand, free trade expands the demand of the non-tradeable good and, therefore, labor demand. This effect is bigger the higher the equilibrium terms of trade. The first condition in Proposition 6 assures that the equilibrium terms of trade are high enough for the second effect to be dominant. The second condition in Proposition 6 implies that workers in country 1 are richer than workers in country 2. Thus, if international labor mobility is allowed, there will be migrations to country 1. This will lead to an increase in the labor supply of country 1, depressing domestic wages. In addition, the increase in the labor supply will expand the production of manufactures, inducing export-biased growth in country 1 and a corresponding decline in the terms of trade of country 1. In order to avoid these effects, workers in country 1 will oppose international labor mobility. As a consequence, free trade and free labor mobility is not a Nash equilibrium outcome. No trade and free

[10] We could have easily assumed different coefficients in the production function of each tradeable sector. Qualitative results do not depend on this simplification.

labor mobility is not a Nash equilibrium outcome either. The workers in the high-wage country under autarky will block migration flows because any increase in the domestic labor force depresses wages.

The logic behind Proposition 6 applies beyond the particular model we studied in this section. Workers employed in the non-tradeable sector gain from free trade because the demand of non-tradeable goods is maximized when the country engages in international trade. If workers in the non-tradeable sector are an important proportion of the labor force, as it is the case in many postindustrial economies, in each country there is a solid majority that supports free trade. International labor mobility, however, increases labor supply in high-wage countries, pushing wages down. As a consequence, workers in high-wage countries oppose massive immigration.

5 Inducing free trade and free labor mobility

In this section we study three ways to support free trade and free labor mobility as a Nash equilibrium outcome of the trade and labor mobility game. First, we introduce product differentiation, monopolistic competition and increasing returns to scale as in Krugman (1979) in one of the tradeable industries of an, otherwise, simple Ricardian economy. This makes free trade more complicated, but it relaxes the incentives to block international labor mobility. Second, keeping the economic environment of Section 3 fixed, we explore a change in the political game. Following Acemoglu and Robinson (2012), we introduce an extractive elite in each country. Under the proper distribution of political power in each country, this could induce a political equilibrium in which both countries agree to trade and allow free labor mobility. Finally, we expand the policy instruments available to the governments, allowing the recipient country to charge a migration fee to foreign workers that decide to locate in the country. If the revenue from migration fees is used to pay transfers to domestic workers, free trade and free labor mobility could be supported as a Nash equilibrium outcome.

5.1 Increasing returns to scale (Krugman, 1979)

Consider an economy with two countries ($J = 2$), two tradeable goods ($Z_T = \{1, 2\}$) and one non-tradeable good ($Z_N = \{3\}$). Goods 2 and 3 are homogenous products, but good 1 is a differentiated product. All agents have the same preferences given by $u\left(c^j\right) = \sum_{z \in Z} \alpha_z \ln\left(c_z^j\right)$ and $c_1^j = \left[\int_0^n c_1^j\left(i\right)^\rho di\right]^{\frac{1}{\rho}}$, where $\alpha_z \in (0,1)$, $\sum_{z \in Z} \alpha_z = 1$, $\rho \in (0,1)$, $\sigma = (1-\rho)^{-1}$, and $c_1^j\left(i\right)$ indicates the quantity of variety $i \in [0, n]$ that is consumed. The production function in sector 1 is $L_1^j\left(i\right) = a_{L,1}^j Q_1^j\left(i\right) + f$, where $L_1^j\left(i\right)$ is the labor employed in the production of variety i of good 1, $Q_1^j\left(i\right)$ is the production of variety i of good 1, $f > 0$ is the fixed cost of producing a variety of good 1. Production functions in sectors 2 and 3 are as in the simple Ricardian model, i.e., $L_2^j = a_{L,2}^j Q_2^j$, $L_3^j = a_{L,3}^j Q_3^j$. Let $A_z = a_{L,z}^2 / a_{L,z}^1$ and assume

$A_1 > A_2$, i.e., country 1 has a comparative advantage in good 1. As in previous sections, $\bar{L}^j$ indicates the labor endowment of country j and only a fraction $m \in [0,1]$ of $\bar{L}^j$ is mobile. Contrary to previous sections, suppose that countries can place partial restrictions on labor mobility. In particular, assume that each country can choose among three migration policies. Let $\lambda_M = 0$ indicate that at least one country does not accept labor mobility and, hence, mobile workers are forced to stay in their country, while $\lambda_M = 1$ indicates that both countries accept complete free labor mobility. Let $\lambda_M = \bar{\lambda}_M < 1$ indicate that the recipient country accepts, at most, no more than a fraction $\bar{\lambda}_M$ of the mobile workers from the other country, while the country of origin allows at least some portion $\bar{\lambda}_M$ of the mobile workers of that country to emigrate. As in previous sections, trade policy can be no trade ($\lambda_T = 0$) or complete free trade ($\lambda_T = 1$).

The following proposition characterizes the political equilibrium.

Proposition 7 ***(increasing returns to scale)***. *Suppose that:*

$$T^1 - T^2 > \frac{\alpha_1}{\sigma - 1} \ln \left[\frac{\bar{L}^2 + m\bar{L}^1}{(1-m)\,\bar{L}^1} \right] \tag{4}$$

$$A_1 > \frac{\alpha_1 \left(\bar{L}^2 + m\bar{L}^1\right)}{\alpha_2 (1-m)\,\bar{L}^1} > \frac{\alpha_1 \left(1 - \bar{\lambda}_M m\right) \bar{L}^2}{\alpha_2 \left(\bar{L}^1 + \bar{\lambda}_M m \bar{L}^2\right)} > (A_3)^{\frac{-\alpha_3}{\alpha_1 + \alpha_2}} > \frac{\alpha_1 (1-m)\,\bar{L}^2}{\alpha_2 \left(\bar{L}^1 + m\bar{L}^2\right)} > A_2 \tag{5}$$

$$\alpha_1 \ln \left[\frac{A_1 \alpha_2 \left(\bar{L}^1 + \bar{\lambda}_M m \bar{L}^2\right)}{\alpha_1 \left(1 - \bar{\lambda}_M m\right) \bar{L}^2} \right] > \frac{\alpha_1}{\sigma - 1} \ln \left[\frac{\alpha_1 \bar{L}^2}{(1-\alpha_3)\left(\bar{L}^1 + \bar{\lambda}_M m \bar{L}^2\right)} \right] \tag{6}$$

Then[11]*:*

[11] (4) ensures that, under autarky, country 1 is richer than country 2 no matter where mobile workers decide to go. This allows us to avoid dealing with multiple economic equilibria. If complete or partial labor mobility is allowed, mobile workers will always decide to go to or stay in country 1. It is possible to relax this condition to $T^1 - T^2 > [\alpha_1/(\sigma-1)] \ln \left[(1-m)\,\bar{L}^2/\left(\bar{L}^1 + m\bar{L}^2\right)\right]$, provided that we assume that, when there are two equilibria, all mobile workers decide to go to or stay in country 1. In other words, the cost of relaxing this condition is the imposition of an equilibrium selection assumption.

(5) deals with the effects of labor mobility under free trade. First, it ensures that there are enough mobile workers to equalize real wages under free trade and that real wages are equalized before the rich country starts producing both tradeable goods. Second, it rules out uninteresting cases in which partial labor mobility is not binding because even when $\lambda_M = \bar{\lambda}_M$, enough workers can move to equalize real wages in the countries. Finally, to simplify the analysis, (5) also rules out the possibility of a reversal of fortune. Thus, under free trade, it is always the case that $L^2/L^1 \geq (\alpha_2/\alpha_1)\,(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$, which implies that country 1 is never poorer than country 2.

(6) ensures that immobile workers in country 2 are better off under free trade and partial labor mobility than under autarky and no labor mobility. Moving from autarky and no labor mobility to free trade and partial labor mobility has two effects on immobile workers in country 2. The first term of (6) captures the standard gains from trade coming from specialization in the sector with a comparative advantage. Moreover, migration to country 1 only reinforces this effect. The second term of (6) captures the effects on the number of varieties produced in equilibrium. Under autarky and no labor

1. *No trade and no labor mobility is a Nash equilibrium outcome.*

2. *No trade and partial labor mobility is a Nash equilibrium outcome if and only if* $\bar{\lambda}_M \left\{ T^1 - T^2 + \frac{\alpha_1}{\sigma-1} \ln \left[\frac{\bar{L}^1 + \bar{\lambda}_M m \bar{L}^2}{(1-\bar{\lambda}_M m)\bar{L}^2} \right] \right\} \geq -\frac{\alpha_1}{\sigma-1} \ln \left(1 - \bar{\lambda}_M m\right)^{\frac{1}{m}}$. *Moreover, if no trade and partial labor mobility is an equilibrium outcome, then* $W_G^j\left(0, \bar{\lambda}_M\right) \geq W_G^j(0,0)$ *for* $j = 1, 2$.

3. *No trade and free labor mobility is a Nash equilibrium outcome if and only if* $\bar{\lambda}_M \left\{ T^1 - T^2 + \frac{\alpha_1}{\sigma-1} \ln \left[\frac{\bar{L}^1 + \bar{\lambda}_M m \bar{L}^2}{(1-\bar{\lambda}_M m)\bar{L}^2} \right] \right\} \leq \left(T^1 - T^2\right) + \frac{\alpha_1}{\sigma-1} \ln \left[\frac{\left(\bar{L}^1 + m\bar{L}^2\right)(1-m)^{\frac{1}{m}}}{(1-m)\bar{L}^2\left(1-\bar{\lambda}_M m\right)^{\frac{1}{m}}} \right]$. *Moreover, if no trade and free labor mobility is an equilibrium outcome, then* $W_G^j(0,1) \geq \max\left\{ W_G^j(0,0), W_G^j\left(0, \bar{\lambda}_M\right) \right\}$ *for* $j = 1, 2$.

4. *Free trade and no labor mobility is a Nash equilibrium if and only if* $\left(\frac{1-\alpha_3}{\alpha_2}\right)^{\frac{1}{\sigma}} (A_1)^{\frac{\sigma-1}{\sigma}} > \frac{\alpha_1 \bar{L}^2}{\alpha_2 \bar{L}^1}$.

5. *Suppose that* $\left(\bar{L}^1 + \bar{\lambda}_M m \bar{L}^2\right)^{\phi} \left[\left(1 - \bar{\lambda}_M m\right) \bar{L}^2\right]^{\alpha_2} > \max\left\{ \left(\bar{L}^1\right)^{\phi} \left(\bar{L}^2\right)^{\alpha_2}, \left(L^1\right)^{\phi} \left(L^2\right)^{\alpha_2} \right\}$,*where* $\phi = \frac{\alpha_1 - \alpha_2(\sigma-1)}{\sigma-1}$, $L^1 = \frac{\alpha_1\left(\bar{L}^1+\bar{L}^2\right)}{\alpha_1+\alpha_2(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}}$ *and* $L^2 = \frac{\alpha_2(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}\left(\bar{L}^1+\bar{L}^2\right)}{\alpha_1+\alpha_2(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}}$. *Then, free trade and partial labor mobility is a Nash equilibrium outcome, but free trade and free labor mobility is not a Nash equilibrium outcome.* ***Proof****: see Appendix C.1.* ■

As in previous sections, no trade and no labor mobility is always a Nash equilibrium outcome because there is no way that a country can change the world equilibrium with a unilateral move in policy. No trade and partial (free) labor mobility is also a Nash equilibrium outcome. The intuition is as follows. Under autarky, country 1 favors labor mobility because this brings more workers to the country and, hence, more varieties of good 1 can be produced in equilibrium. Under autarky, immobile workers in country 2 are opposed to labor mobility because mobile workers will leave the country, thereby reducing the varieties of good that can be produced in equilibrium. Conversely, mobile workers in country 2 support labor mobility because they are better off if they relocate to country 1. The condition in part 2 (3) assures that the gains from partial (free) labor mobility enjoyed by mobile workers in country 2 more than compensate for the losses sustained by immobile workers. Note also that, when these equilibria exist, the less restrictive equilibrium outcome always dominates more restrictive one. Free trade and no labor mobility is always a Nash equilibrium outcome. Since country 1 specializes in the sector that operates under increasing returns to scale, in addition to the standard gains from trade coming from the specialization in the sector with a comparative advantage, free trade allows country 1 to further exploits

mobility, country 2 produces $n^2 = \alpha_1 \bar{L}^2/\sigma f$ varieties of good 1, while under free trade and partial labor mobility in the trading equilibrium, $n = (1-\alpha_3)\left(\bar{L}^1 + \bar{\lambda}_M m \bar{L}^2\right)/\sigma f$ varieties are produced. Note that, if $n \geq n^2$, then (6) holds trivially; while if $n < n^2$, (6) simply means that the comparative advantage effect always prevails over the variety effect.

economies of scale in sector 1. For country 2, free trade has two effects. First, country 2 also enjoys the standard gains from trade. Second, under autarky, country 2 produces $n^2 = \alpha_1 \bar{L}^2/\sigma f$ varieties of good 1, while in the trading equilibrium $n = (1-\alpha_3)\,\bar{L}^1/\sigma f$ varieties are produced. The condition in part 4 ensures that the comparative advantage effect prevails over the variety effect.

The most interesting result in Proposition 7 is part 5. Free trade and partial labor mobility is a Nash equilibrium outcome, but free trade and complete free labor mobility is not. The intuition is as follows. Under free trade, labor mobility has two effects on country 1. First, there is a terms-of-trade effect. As workers move to country 1, L^1 increases and L^2 decreases, which leads to a deterioration in the terms of trade of country 1. Second, there is a scale effect. As L^1 increases, more varieties of good 1 are produced in the trading equilibrium. Thus, under free trade, labor mobility has an ambiguous effect on the well-being of country 1. The condition in part 5 of the proposition simply states that the scale effect prevails over the terms-of-trade effect when country 1 allows partial labor mobility, but the opposite is true when there is completely free labor mobility.

5.2 Extractive elites (Acemoglu and Robinson, 2012)

Consider an economy with two countries ($J = 2$), two tradeable goods ($Z_T = \{1, 2\}$) and one non-tradeable good ($Z_N = \{3\}$). Production functions, endowments and preferences are as in section 3. The novelty is that each of the countries is populated by two types of agents: L^j workers, each of whom owns one unit of labor, and E^j elite members. The elite is a purely extractive one with no productive role in society (Acemoglu and Robinson, 2012). The elite simply appropriates a fraction $\beta^j \in (0, 1)$ of L^j. Moreover, expropriation is socially costly. A fraction $\delta \in (0, 1)$ of each unit expropriated by the elite is lost in the process. Formally, after expropriation, each worker keeps $\left(1-\beta^j\right)$ units of labor and each member of the elite gets $\delta\beta^j L^j/E^j$ units of labor. Thus, the effective labor force of country j is $\tilde{L}^j = B^j L^j = \left[1-\beta^j(1-\delta)\right] L^j$. As in previous sections, only a fraction $m \in [0, 1]$ of the labor force of each country is mobile at zero cost. The rest of the labor force and elite members are immobile.

Each government selects trade and migration policies in order to maximize an utilitarian welfare function W_G^j, whose weight depends on the size of each group as well as on how influential they are in the political process. In particular, we assume that each government maximizes $W_G^j = \frac{\bar{L}^j\left[(1-m)v^{j,im}+mv^{j,m}\right]+E^j\left(1+\varphi^j\right)v^{j,e}}{\bar{L}^j+E^j(1+\varphi^j)}$, where $v^{j,im}$, $v^{j,m}$, $v^{j,e}$ are the indirect utility functions of an immobile worker, a mobile worker and a member of the elite, respectively.[12] $\varphi^j \in [-1, \infty]$ is a measure of the political power of the elite. If $\varphi^j = -1$ ($\varphi^j = \infty$), then the government is a perfect agent for domestic workers (the elite). φ^j can also be considered a measure of how unequal political institutions are (Ace-

[12] Grossman and Helpman (2001) provide several sets of micro-foundations for a welfare function of this type, including a combination of a probabilistic voting model with a lobby model.

moglu and Robinson, 2012). Note also that W_G^j takes into account the welfare of emigrants, but not the welfare of immigrants. The reason for this is that immigrants tend to have much less political influence than domestic workers do, particularly with regard to immigration policies. The timing of events is as in previous sections, and we assume that the elite groups collect their shares after migration has taken place but before production, trade and consumption decisions have been made.

The following proposition characterizes the political equilibrium.

Proposition 8 ***(extractive elite)***. *Suppose that:*[13]

$$\left(\frac{B^1}{B^2}\right) A_1 > \frac{\alpha_1\left(\bar{L}^2 + m\bar{L}^1\right)}{\alpha_2\left(1-m\right)\bar{L}^1} > \left(\frac{1-\beta^2}{1-\beta^1}\right)^{\frac{1}{\alpha_1+\alpha_2}} \left(A_3\right)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}} > \frac{\alpha_1\left(1-m\right)\bar{L}^2}{\alpha_2\left(\bar{L}^1 + m\bar{L}^2\right)} > A_2\left(\frac{B^1}{B^2}\right) \tag{7}$$

$$\beta^2 > \beta^1 \tag{8}$$

$$\alpha_2 \ln\left[\frac{B^2\Gamma^2}{A_2 B^1 \Gamma^1 \left(A_3\right)^{\frac{\alpha_3}{\alpha_1+\alpha_2}}}\right] > \ln\left[\frac{\left(\bar{L}^1 + m\bar{L}^2\right)\left(\alpha_1\Gamma^1 + \alpha_2\Gamma^2\left(A_3\right)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}\right)}{\left(\bar{L}^1 + \bar{L}^2\right)\alpha_1\Gamma^1}\right] \tag{9}$$

$$\alpha_1 \ln\left[\frac{A_1 B^1 L^1 \Gamma^1}{B^2\Gamma^2\left(A_3\right)^{\frac{\alpha_3}{\alpha_1+\alpha_2}}}\right] > m\ln\left(\frac{1-\beta^1}{1-\beta^2}\right) \tag{10}$$

Assume that $\bar{L}^2/\bar{L}^1 \neq \left(\alpha_2\Gamma^2/\alpha_1\Gamma^1\right)\left(A_3\right)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$. *Let* $L^1 = \frac{\alpha_1\Gamma^1\left(\bar{L}^1+\bar{L}^2\right)}{\alpha_1\Gamma^1+\alpha_2\Gamma^2\left(A_3\right)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}}$ *and* $L^2 = \frac{\alpha_2\Gamma^2\left(A_3\right)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}\left(\bar{L}^1+\bar{L}^2\right)}{\alpha_1\Gamma^1+\alpha_2\Gamma^2\left(A_3\right)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}}$. *Then:*

1. *No trade and no labor mobility is always a Nash equilibrium outcome.*

2. *Free trade and no labor mobility is always a Nash equilibrium outcome. Moreover,* $W_G^j(1,0) \geq W_G^j(0,0)$ *for* $j = 1, 2$.

3. *No trade and free labor mobility is a Nash equilibrium outcome if and only if* $1+\varphi^2 \leq \frac{m\bar{L}^2 \ln\left(\frac{1-\beta^1}{1-\beta^2}\right)}{-E^2\ln(1-m)}$.

[13] (7) implies that, regardless of migration flows, if countries trade, country j specializes in good $z = j$, and that, under free trade, labor mobility induces full wage convergence. (8) simply means that the elite of country 1 is less extractive than the elite of country 2, in the sense that it appropiates a lower fraction of its labor force. (9) ensures that the elite of the rich country under autarky (country 1) prefers free trade and free labor mobility to autarky and free labor mobility. Intuitively, for the elite of country 1, when there is free labor mobility, gains from trade prevail over the negative effect that trade liberalization has on the size of the labor force of country 1. Finally, (10) states that workers in the poor country under autarky (country 2) prefer free trade and free labor mobility to autarky and free labor mobility. Intuitively, the gains from trade for all workers in country 2 are higher than the productivity gains under autarky for mobile workers.

4. *If* $\bar{L}^2/\bar{L}^1 > \left(\alpha_2\Gamma^2/\alpha_1\Gamma^1\right)(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$, *free trade and free labor mobility is a Nash equilibrium outcome if and only if:*

   (a) $\left(L^1\right)^{(1-\alpha_2)}\left(L^2\right)^{\alpha_2} > \left(\bar{L}^1\right)^{(1-\alpha_2)}\left(\bar{L}^2\right)^{\alpha_2}$ *and* $1+\varphi^1 > \bar{\varphi}^1 = \dfrac{\bar{L}^1\alpha_2\ln\left(\frac{L^1\bar{L}^2}{\bar{L}^1L^2}\right)}{E^1\ln\left[\frac{(L^1)^{(1-\alpha_2)}(L^2)^{\alpha_2}}{(\bar{L}^1)^{(1-\alpha_2)}(\bar{L}^2)^{\alpha_2}}\right]}$.

   (b) $\left(L^1\right)^{\alpha_1}\left(L^2\right)^{1-\alpha_1} > \left(\bar{L}^1\right)^{\alpha_1}\left(\bar{L}^2\right)^{1-\alpha_1}$ *or* $\left(L^1\right)^{\alpha_1}\left(L^2\right)^{1-\alpha_1} < \left(\bar{L}^1\right)^{\alpha_1}\left(\bar{L}^2\right)^{1-\alpha_1}$ *and* $1+\varphi^2 < \bar{\varphi}^2 = \dfrac{-\bar{L}^2\alpha_1\ln\left(\frac{L^1\bar{L}^2}{\bar{L}^1L^2}\right)}{E^2\ln\left[\frac{(L^1)^{\alpha_1}(L^2)^{1-\alpha_1}}{(\bar{L}^1)^{\alpha_1}(\bar{L}^2)^{1-\alpha_1}}\right]}$.

5. *If* $\bar{L}^2/\bar{L}^1 < \left(\alpha_2\Gamma^2/\alpha_1\Gamma^1\right)(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$, *free trade and free labor mobility is a Nash equilibrium outcome if and only if:*

   (a) $\left(L^1\right)^{\alpha_1}\left(L^2\right)^{1-\alpha_1} > \left(\bar{L}^1\right)^{\alpha_1}\left(\bar{L}^2\right)^{1-\alpha_1}$ *and* $1+\varphi^2 > \bar{\varphi}^2$;

   (b) $\left(L^1\right)^{(1-\alpha_2)}\left(L^2\right)^{\alpha_2} > \left(\bar{L}^1\right)^{(1-\alpha_2)}\left(\bar{L}^2\right)^{\alpha_2}$ *or* $\left(L^1\right)^{(1-\alpha_2)}\left(L^2\right)^{\alpha_2} < \left(\bar{L}^1\right)^{(1-\alpha_2)}\left(\bar{L}^2\right)^{\alpha_2}$ *and* $1+\varphi^1 < \bar{\varphi}^1$. ***Proof***: *see Appendix C.2.* ■

No trade and no labor mobility is always a Nash equilibrium outcome because, if one country decides to isolate itself, there is nothing that the other country can do to change the political equilibrium. This is also a very unlikely equilibrium, however. In fact, free trade and no labor mobility is also always a Nash equilibrium outcome which dominates complete isolation. No trade and free labor mobility is a Nash equilibrium outcome when workers in the poor country under autarky (country 2) are politically powerful (φ^2 low). The intuition is straightforward. If one country decides to restrict trade, free trade is impossible no matter what policy is chosen by the other country. Under autarky, workers in country 1 and immobile workers in country 2 do not worry about immigration because real wages do not depend on labor endowments. Mobile workers in country 2 are eager to migrate to country 1 because real wages are higher there. The elite in country 1 favors labor mobility because it brings more workers to the country. The only group that will oppose free labor mobility is the elite in country 2, which would not be able to extract resources from mobile workers anymore. As a consequence, autarky and free labor mobility is an equilibrium if the elite in country 2 do not have sufficient political power.

Free trade and free labor mobility is a Nash equilibrium outcome when the elite of the high-wage country under free trade and the workers in the low-wage country under free trade are politically powerful.[14] The intuition behind this result is straightforward. Domestic workers in a high-wage country

[14] Real wages in country 1 are higher than in country 2 under free trade if and only if $\left(\alpha_1\Gamma^1\bar{L}^2/\alpha_2\Gamma^2\bar{L}^1\right)(A_3)^{\frac{\alpha_3}{\alpha_1+\alpha_2}} > 1$, i.e., when country 1 has a relative shortage of labor ($\bar{L}^2/\bar{L}^1$ high), it has a relatively less extractive elite (Γ^1/Γ^2 high), it specializes in goods with a high expenditure share (α_1/α_2 high) and it has relatively high level of productivity in non-tradeable goods (A_3 high).

under free trade are opposed to labor mobility because the flow of immigrants would reduce their wages. The extractive elite is also hurt by the reduction in the wage rate, but immigrants are also a new source of income (more workers to extract resources from). If the second effect outweighs the first, then the elite is in favor of labor mobility. As a consequence, labor mobility will be allowed only when the elite is powerful enough to impose its views. Domestic workers in a low-wage country under free trade are in favor of labor mobility because the flow of emigrants would increase their wages. Higher wages also benefit the extractive elite, but as mobile workers migrate to the other country, the elite has a smaller domestic labor force to extract resources from. If the second effect outweighs the first one, the elite is opposed to free labor mobility. Therefore, labor mobility will be allowed only when workers are powerful enough to impose their views. Thus, labor mobility can be a major source of conflict between workers and the elite. In the high-wage country, workers prefer to block labor mobility, while the elite prefer to allow it. Conversely, in the low-wage country, workers prefer to allow labor mobility, while the elite prefer to block it. Trade, on the other hand, is not a source of conflict between workers and the elite. The main reason for this is that, in the Ricardian model, there is only one factor of production and, hence, free trade makes it possible to exploit the gains from specialization and trade without generating any redistributive effect. Recall, however, that labor mobility produces different effects under autarky and free trade and, hence, the elite of country 1 and workers in country 2 might prefer autarky and free labor mobility to free trade and free labor mobility, which is ruled out under Assumption 7.

An historical application of the model in this section is Argentina from 1875 to 1930, when the country was ruled by a landowning-elite. With the pattern of land ownership determined by the political history of the country, and the prices of exports, imports and capital set by international markets, total land rents depended on the labor supply. Therefore, immigration policy became a critical variable under the control of the government. Not surprisingly, the Argentine elite chose to promote immigration. This decreased wages and increased the return on land. Indeed, Taylor (1997) calibrates a general equilibrium model to estimate the impact on wages of the massive flow of immigration to Argentina up to the First World War. His calibration suggests that the flow of immigration reduced real wages by approximately 20% from what wage levels would have been if immigration had not taken place. During the same period, Australian immigration policies were substantially different from those of Argentina. Despite having similar factor endowments, land was more concentrated in Argentina than in Australia, where family-operated, medium-sized farms were relatively more common. Therefore, landowners in Australia never controlled the government as they did in Argentina (Hirst, 1979). Additionally, the Australian mining sector significantly contributed to the emergence of a unionized labor force across the economy. Trade unions and entrepreneurs involved with mining formed a powerful coalition that opposed the creation of a ruling land-owning elite as well as free immigration (Diaz Alejandro, 1984).

5.3 Citizenship for sale

Proposition 8 shows that, if we introduce a powerful elite that cares about aggregate income (wL), then free trade and free labor mobility could be a Nash equilibrium outcome. More generally, all we need is a mechanism through which the payoff for each citizen in the rich country depends on aggregate income. Next, we eliminate the extractive elite and assume that rich countries charge an entry fee to foreign mobile workers, which is then distributed among domestic workers.

Consider an economy with two countries ($J = 2$), two tradeable goods ($Z_T = \{1, 2\}$) and one non-tradeable good ($Z_N = \{3\}$). Production functions, endowments and preferences are as in the simple Ricardian economy. Let $\tau^j \in [0, 1]$ be the income tax rate that country j charges to foreign mobile workers that decide to locate in j. Since there is no tax imposed on domestic workers, τ^j can be interpreted as an entry or immigration fee. The revenue from immigration fees is used to pay transfers to domestic workers. Let $Tf^j \geq 0$ indicates the transfer per capita received by country j's workers. The government of country j selects its trade and migration policies in order to maximize $W_G^j = mv_L^{j,m} + (1-m)\, v_L^{j,im}$ subject to $Tf^j \bar{L}^j = \tau^j \max\left\{L^j - \bar{L}^j, 0\right\}$, government j's budget constraint. As in previous sections, trade policy is $\lambda_T \in \{0, 1\}$, where $\lambda_T = 1$ indicates that both countries accept free trade and $\lambda_T = 0$ that at least one country refuses to trade. Migration policy is slightly more complicated. Let $\lambda_M \in \{0, (1, \tau)\}$ with $\tau \in [0, 1]$, where $\lambda_M = 0$ indicates that at least one country does not accept free labor mobility and $\lambda_M = (1, \tau)$ indicates that both countries accept labor mobility with the recipient country charging a proportional income tax $\tau \in [0, 1]$ to foreign mobile workers.

The following proposition characterizes the political equilibrium.

Proposition 9 ***(citizenship for sale).*** *Suppose that* $A_1 > \frac{\alpha_1\left(\bar{L}^2+m\bar{L}^1\right)}{\alpha_2(1-m)\bar{L}^1} > \frac{\alpha_1\bar{L}^2}{\alpha_2\bar{L}^1} > (A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}} > \frac{\alpha_1(1-m)\bar{L}^2}{\alpha_2\left(\bar{L}^1+m\bar{L}^2\right)} > A_2$ *and* $T^1 > T^2$. *Then:*

1. *(i) Neither trade nor labor mobility; (ii) no trade and immigration fee* $\hat{\tau} = 1 - e^{-(T_1 - T_2)} > 0$; *and (iii) free trade and no labor mobility are always Nash equilibrium outcomes. Moreover:*

   (a) $\left\{W_G^1(1, 0), W_G^1(0, (1, \hat{\tau}))\right\} > W_G^1(0, 0)$ *while* $W_G^1(1, 0) > W_G^1(0, (1, \hat{\tau}))$ *if and only if* $\left(\frac{\alpha_1\bar{L}^2}{A_2\alpha_2\bar{L}^1}\right)^{\alpha_2} > \frac{\bar{L}^1+\hat{\tau}m\bar{L}^2}{\bar{L}^1}$.

   (b) $W_G^2(1, 0) > W_G^2(0, (1, \hat{\tau})) = W_G^2(0, 0)$.

2. *(iv) Free trade and immigration fee* $\tau^* = \arg\max_{0 \leq \tau \leq \bar{\tau}} \left\{\left[\frac{1-\psi(\tau)}{\psi(\tau)}\right]^{\alpha_2}\left[1 + \tau\frac{\psi(\tau)\left(\bar{L}^1+\bar{L}^2\right)-\bar{L}^1}{\bar{L}^1}\right]\right\} > 0$ *is*

a Nash equilibrium outcome if and only if

$$\alpha_2 \ln \left[\frac{\alpha_1 (1-\psi(\tau^*))}{A_2 \alpha_2 \psi(\tau^*)}\right] \geq \ln \left[\frac{\bar{L}^1 + \hat{\tau} m \bar{L}^2}{\bar{L}^1 + \tau^* \left(\psi(\tau^*)\left(\bar{L}^1 + \bar{L}^2\right) - \bar{L}^1\right)}\right] \tag{11}$$

$$\alpha_1 \ln \left[\frac{A_1 \alpha_2 \psi(\tau^*)}{\alpha_1 (1-\psi(\tau^*))}\right] \geq m \max\left\{\ln(1-\tau^*) + T^1 - T^2, 0\right\} \tag{12}$$

where $\psi(\tau) = \frac{\alpha_1 (1-\tau)^{1/1-\alpha_3}}{\alpha_1 (1-\tau)^{1/1-\alpha_3} + \alpha_2 (A_3)^{-\alpha_3/1-\alpha_3}}$ *and* $\bar{\tau} = 1 - \left(\alpha_2 \bar{L}^1 / \alpha_1 \bar{L}^2\right)^{1-\alpha_3} (A_3)^{-\alpha_3}$.[15] *Moreover:*

(a) $W_G^1(1,(1,\tau^*)) \geq W_G^1(1,0) > W_G^1(0,0)$ *and* $W_G^1(1,(1,\tau^*)) \geq W_G^1(0,(1,\hat{\tau}))$.

(b) $W_G^2(1,(1,\tau^*)) \geq W_G^2(1,0) > W_G^2(0,(1,\hat{\tau})) = W_G^2(0,0)$.

(c) If $\left(\frac{\alpha_1 \bar{L}^2}{\alpha_2 \bar{L}^1}\right)\left[\frac{\bar{L}^2 - (\bar{L}^1 + \bar{L}^2)\alpha_2}{\bar{L}^2}\right]^{\frac{1}{1-\alpha_3}} > (A_3)^{\frac{-\alpha_3}{1-\alpha_3}}$, *then* $\tau^* < \bar{\tau}$, $L^1 = \psi(\tau^*)\left(\bar{L}^1 + \bar{L}^2\right) > \bar{L}^1$, *and* $W_G^j(1,(1,\tau^*)) > W_G^j(1,0)$ *for* $j = 1, 2$. ***Proof****: see Appendix C.3.* ■

As in Proposition 1 and, for exactly the same reasons, no trade and no labor mobility is always a Nash equilibrium outcome. It is also an outcome dominated by other Nash equilibrium outcomes. Indeed, as in Proposition 1, free trade and no labor mobility is always a Nash equilibrium outcome and it is strictly preferred to no trade and no labor mobility by both countries. There are also important differences between Propositions 1 and 9.

First, when the recipient country cannot charge an immigration fee, no trade and free labor mobility is a Nash equilibrium outcome (Proposition 1). Under autarky, domestic workers in the rich country (country 1) and immobile workers in the poor country (country 2) are indifferent with any migration policy because real wages are fully determined by domestic productivity, while mobile workers in the poor country are better off migrating to the rich country because real wages are higher there. When the recipient country can charge an immigration fee, no trade and free labor mobility with immigration fee $\hat{\tau} = 1 - e^{-(T_1 - T_2)} > 0$ is a Nash equilibrium outcome (Proposition 9 - Part 1). Under autarky, domestic workers in the rich country are better off allowing immigration because their real wages are fully determined by domestic productivity and they receive the transfers from immigration fees, while workers from the poor country are indifferent with any migration policy. For immobile workers in the poor country, real wages are fully determined by domestic productivity. Mobile workers from the poor country locate in the rich country and, hence, earn a high real wage. However, they must also pay an immigration fee equal to the difference between real wages in the rich and the poor country.

Second, when the recipient country cannot charge an immigration fee, free trade and free labor mobility is not a Nash equilibrium outcome (Proposition 1). Under free trade, domestic workers in a

[15] $\bar{\tau} = 1 - \left(\alpha_2 \bar{L}^1 / \alpha_1 \bar{L}^2\right)^{1-\alpha_3} (A_3)^{-\alpha_3}$ is the prohibitive immigration fee. That is, $\tau \geq \bar{\tau}$ implies $L^1 = \bar{L}^1$ and $L^2 = \bar{L}^2$.

rich country are opposed to labor mobility because the flow of immigrants would reduce their real wages. On the contrary, when the recipient country can charge an immigration fee, free trade and free labor mobility with immigration fee $\tau^* > 0$ is a Nash equilibrium outcome provided that conditions (11) and (12) hold (Proposition 9 - Part 2). Moreover, whenever $(\lambda_T, \lambda_M) = (1, (1, \tau^*))$ is a Nash equilibrium outcome, it is strictly preferred to all other Nash equilibrium outcomes by both countries. Under free trade, the flow of immigrants reduces real wages in the rich country (country 1), but it increases the revenue from immigration fees and, hence, the transfer received by each worker in the rich country. When $\left(\frac{\alpha_1 \bar{L}^2}{\alpha_2 \bar{L}^1}\right)\left[\frac{\bar{L}^2-(\bar{L}^1+\bar{L}^2)\alpha_2}{\bar{L}^2}\right]^{\frac{1}{1-\alpha_3}} > (A_3)^{\frac{-\alpha_3}{1-\alpha_3}}$, the second effect dominates the first one for high values of the immigration fee and, hence, workers from the rich country are better off charging an immigration fee that induces positive migration flows rather than imposing the prohibitive immigration fee. Mobile workers in the poor country (country 2) are also better off if they are allowed to migrate to country 1, while immobile workers from the poor country benefit from the rise in real wages caused by the outflow of workers.

Note, however, that $(\lambda_T, \lambda_M) = (1, (1, \tau^*))$ is not always a Nash equilibrium outcome. There are two potential complications. First, it is possible that the rich country prefers to deviate to no trade and free labor mobility with immigration fee $\hat{\tau}$. The advantage of doing so is that the revenue from immigration fees could be higher under $(\lambda_T, \lambda_M) = (0, (1, \hat{\tau}))$ than under $(\lambda_T, \lambda_M) = (1, (1, \tau^*))$. The disadvantage is that gains from trade are lost. (11) assures that gains from trade are high enough, so this deviation is not profitable. Second, it is possible that the poor country prefers to deviate to $(\lambda_T, \lambda_M) = (0, (1, \tau^*))$. The advantage of doing so is that for mobile workers from the poor country, when $\tau^* < \hat{\tau}$ and countries do not trade, migration leave a positive rent. The disadvantage is that gains from trade are lost. (12) assures that gains from trade for workers in country 2 are high enough, so they do not have an incentive to deviate from $(\lambda_T, \lambda_M) = (1, (1, \tau^*))$.

6 Conclusions

We have shown that, by combining a Ricardian economy with a simple international political economy model, we can explain the salient stylized facts about international trade and labor mobility. In a Ricardian world, countries use different technologies and, as a consequence, there is no wage equalization under free trade. This wedge in real wages opens the door to migration flows, which, combined with free trade, induce full wage convergence. However, workers in rich countries block immigration in order to protect their high wages. In contrast, nobody is willing to block free trade because, in a Ricardian world, everybody gains from international trade. Thus, our model naturally explains the present world equilibrium: few restrictions on international trade in goods and very restrictive barriers to international labor mobility.

We have also shown that several multifactor models lead to an analogous result. In particular, we have studied a version of the standard two-sector Heckscher-Ohlin model, the Ricardo-Vinner specific factors model, and a three-sector model (two tradeable goods and one non-tradeable good). In all cases, we have shown that, although it is always possible to find conditions under which free trade and no labor mobility is a Nash equilibrium outcome of the political economy game, free trade and free labor mobility is never a Nash equilibrium outcome.

Given this rather negative result, we have explored three extensions of our model in which free trade and, at least, some migration flows, can be sustained as a Nash equilibrium outcome. One possibility is to introduce increasing returns to scale. Then, workers in a sparsely populated rich country might prefer, at least during one phase in the development process, to allow immigration because a bigger labor force increases the number of varieties of the differentiated good that can be produced in equilibrium, which can offset the negative effect of immigration flows on wages. Another possibility is to introduce an extractive elite that prefers to have a bigger labor force to extract resources from. A third alternative is to introduce an immigration fee in rich countries, implemented as an income tax on immigrants, and use the revenue generated to pay transfers to domestic workers, which formally backs some recent proposals to reform immigration policies in developed countries (see, for example, Eichenberger and Stadelmann 2017 and Posner and Weyl 2018).

Apart from explaining broad patterns of trade and migration policies, this study points to profound implications for the political economy of development. First, according to our model, workers in rich countries constitute a very conservative force worldwide. Does this imply that less inclusive political institutions in rich countries could be beneficial for the world as a whole? A naive interpretation of our extension involving an extractive elite would lead us to conclude that this is the case. Indeed, as the extractive elite in the rich country becomes more powerful (φ increases), it is more likely that, in equilibrium, there will be free trade and free labor mobility, which would induce full convergence in the levels of well-being of workers across the world. An important limitation of this interpretation, however, is that it fails to take into account the endogenous link between political and economic institutions. As the extractive elite in the rich country becomes more powerful, we should expect to see economic institutions deteriorate (β increases). In other words, our model highlights the fact that a socioeconomic group can play a very progressive internal role by favoring inclusive domestic economic institutions and, at the same time, a very conservative role in terms of worldwide equilibrium by supporting policies that restrict cross-country convergence in levels of well-being.

Second, the model illustrates a more general principle. Institutional differences combined with no factor mobility induce large differences in economic development across locations, while institutional differences and free factor mobility are associated with smaller or no differences in economic development across locations and a greater concentration of resources in locations that have properly functioning institutions. Thus, institutional differences point to a theory of differential development under no factor mobility, while they point to a theory of concentration of economic activity under free factor mobility. However, it is worth noting that, even when there is free factor mobility, better institutions in any location induce better economic outcomes. In other words, under free factor mobility, the quality of institutions does not account for differences in economic development across locations, but institutional change could still be a key determinant of global economic development over time.

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Online Appendix to "The Political Economy of Trade and International Labor Mobility"

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August 2018

In this appendix we prove all the lemmas, propositions and results discussed in the paper.

A. Trade and labor mobility policies in Ricardian economies

Appendix A presents the proofs of all the results in Section 3. It also develops the details of the results discussed in Section 3.2 (multiple goods).

A.1 A simple Ricardian economy

Consider an economy with two countries ($J = 2$), two tradeable goods ($Z_T = \{1, 2\}$) and one non-tradeable good ($Z_N = \{3\}$). Production functions are $Q^j_z = L^j_z / a^j_{L,z}$, where $a^j_{L,z} > 0$ is the unit labor requirement in industry z in country j. Let $A_z = a^2_{L,z}/a^1_{L,z}$ and assume $A_1 > A_2$. Let $\bar{L}^j$ and L^j be the labor endowment and the labor force of country j, respectively. Only a fraction $m \in [0, 1]$ of $\bar{L}^j$ is mobile. All agents have the same preferences, given by $u\left(c^j\right) = \sum_{z\in Z} \alpha_z \ln\left(c^j_z\right)$, with $\alpha_z > 0$ and $\sum_{z\in Z} \alpha_z = 1$.

Under autarky all goods must be produced domestically and, hence, $p^j_z = w^j a^j_{L,z}$ for all $z \in Z$ and $j = 1, 2$. The indirect utility of a worker who owns one unit of labor in country j is $v^j = C + T^j$, where $T^j = -\sum_{z\in Z} \alpha_z \ln\left(a^j_{L,z}\right)$ is a measure of the aggregate productivity of country j. If labor mobility is allowed, all mobile workers go to or stay in the country with the highest aggregate productivity T^j.

Under free trade, if $A_1 > \alpha_1 L^2/\alpha_2 L^1 > A_2$, then country j specializes in good $z = j \in \{1, 2\}$. Thus, $p_1 = w^1 a^1_{L,1}$, $p_2 = w^2 a^2_{L,2}$, $p^j_3 = w^j a^j_{L,3}$ and the balanced trade condition is $\alpha_2 w^1 L^1 = \alpha_1 w^2 L^2$. Therefore, the indirect utility of a worker who owns one unit of labor is $v^1 = C + T^1 + \alpha_2 \ln\left(\alpha_1 L^2/A_2\alpha_2 L^1\right)$ in country 1 and $v^2 = C + T^2 + \alpha_1 \ln\left(A_1\alpha_2 L^1/\alpha_1 L^2\right)$ in country 2. If labor mobility is not allowed, then $L^j = \bar{L}^j$. If labor mobility is allowed, mobile workers will go to or stay in the country with the highest v^j. Since v^1 is decreasing in L^1/L^2 while v^2 is increasing in L^1/L^2, if there are enough mobile workers, they will relocate until $v^1 = v^2$. This implies $L^2/L^1 = (\alpha_2/\alpha_1)(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$ and, hence, $v^1 = v^2 = C + T^2 + \alpha_1 \ln(A_1) + [\alpha_1\alpha_3/(\alpha_1 + \alpha_2)] \ln(A_3)$. Moreover, there will be migrations to country 1 whenever $\bar{L}^2/\bar{L}^1 >$

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$(\alpha_2/\alpha_1)(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$ and migrations to country 2 whenever $\bar{L}^2/\bar{L}^1 < (\alpha_2/\alpha_1)(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$. Finally, the distribution of mobile workers between the countries does not violate either $L^j \in \left[(1-m)\bar{L}^j, \bar{L}^j + m\bar{L}^{-j}\right]$ or $A_1 > \alpha_1 L^2/\alpha_2 L^1 > A_2$, provided that the following assumption holds.

Assumption 1 (simple Ricardian economy). *Regardless of migration flows, if countries trade, country j specializes in good $z = j$ for $j = 1, 2$. Moreover, under free trade labor mobility induces full wage convergence. Formally* $A_1 > \frac{\alpha_1\left(\bar{L}^2+m\bar{L}^1\right)}{\alpha_2(1-m)\bar{L}^1} > (A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}} > \frac{\alpha_1(1-m)\bar{L}^2}{\alpha_2\left(\bar{L}^1+m\bar{L}^2\right)} > A_2$.

Lemma 1 characterizes the effects of trade and labor mobility on the relative well-being of workers in both countries.

Lemma 1 (simple Ricardian economy). *Let* $T^j = -\sum_{z\in Z}\alpha_z \ln\left(a^j_{L,z}\right)$ *and assume* $T^1 > T^2$. *Define* $\Delta = (\alpha_1+\alpha_2)\ln\left(\alpha_1\bar{L}^2/\alpha_2\bar{L}^1\right) + \left(T^1_N - T^2_N\right)$, *where* $T^j_N = -\alpha_3 \ln\left(a^j_{L,3}\right)$. *Suppose that assumption 1 holds.*

1. *If* $\Delta > T^1 - T^2$, *then* $v^1(1,0) - v^2(1,0) > v^1(0,0) - v^2(0,0) > v^1(1,1) - v^2(1,1) = 0$.
2. *If* $0 < \Delta < T^1 - T^2$, *then* $v^1(0,0) - v^2(0,0) > v^1(1,0) - v^2(1,0) > v^1(1,1) - v^2(1,1) = 0$.
3. *If* $\Delta < 0$, *then* $v^1(0,0) - v^2(0,0) > v^1(1,1) - v^2(1,1) = 0 > v^1(1,0) - v^2(1,0)$.

Proof: Under autarky, regardless of the mobile workers' location decision, $v^j(0,\lambda_M) = C + T^j$. Thus, $v^1(0,0) - v^2(0,0) = T^1 - T^2$, which is positive by assumption. Under free trade, assumption 1 implies $v^1(1,\lambda_M) = C + T^1 + \alpha_2\ln\left(\alpha_1 L^2/A_2\alpha_2 L^1\right)$ and $v^2(1,\lambda_M) = C + T^2 + \alpha_1 \ln\left(A_1\alpha_2 L^1/\alpha_1 L^2\right)$. If $\lambda_M = 0$, then $L^2/L^1 = \bar{L}^2/\bar{L}^1$ and, hence, $v^1(1,0) - v^2(1,0) = T^1_N - T^2_N + (\alpha_1+\alpha_2)\ln\left(\alpha_1\bar{L}^2/\alpha_2\bar{L}^1\right)$. If $\lambda_M = 1$, then $L^2/L^1 = \alpha_2/\alpha_1 (A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$ (by assumption 1 $\left(\bar{L}^2 + m\bar{L}^1\right)/(1-m)\bar{L}^1 > L^2/L^1 > (1-m)\bar{L}^2/\left(\bar{L}^1 + m\bar{L}^2\right)$), which implies $v^1(1,1) - v^2(1,1) = 0$. Simple comparisons complete the proof of the lemma. ■

Proposition 1 (simple Ricardian economy). *Suppose that assumption 1 holds,* $T^1 > T^2$ *and* $\Delta \neq 0$. *Then, the trade and labor mobility game has three Nash equilibrium outcomes: (i)* $(\lambda_T, \lambda_M) = (0,0)$; *(ii)* $(\lambda_T,\lambda_M) = (0,1)$; *and (iii)* $(\lambda_T,\lambda_M) = (1,0)$. *Moreover:*

1. $W^1_G(1,0) > W^1_G(0,1) = W^1_G(0,0)$.
2. $\left\{W^2_G(1,0), W^2_G(0,1)\right\} > W^2_G(0,0)$, *while* $W^2_G(1,0) > W^2_G(0,1)$ *if and only if* $\alpha_1\ln\left[\left(A_1\alpha_2\bar{L}^1\right)/\left(\alpha_1\bar{L}^2\right)\right] > m\left(T^1 - T^2\right)$.

Proof: Government 1 payoffs are given by $W^1_G(0,0) = W^1_G(0,1) = C + T^1$, $W^1_G(1,0) = C + T^1 + \alpha_2\ln\left(\alpha_1\bar{L}^2/A_2\alpha_2\bar{L}^1\right)$, and $W^1_G(1,1) = C + T^1 - \alpha_2\ln A_2 - [\alpha_2/(\alpha_1+\alpha_2)]\left(T^1_N - T^2_N\right)$. Government 2 payoffs are given by $W^2_G(0,0) = C + T^2$, $W^2_G(0,1) = C + mT^1 + (1-m)T^2$, $W^2_G(1,0) = C + T^2 + \alpha_1\ln\left[\left(A_1\alpha_2\bar{L}^1\right)/\left(\alpha_1\bar{L}^2\right)\right]$ and $W^2_G(1,1) = C + T^2 + \alpha_1\ln A_1 + [\alpha_1/(\alpha_1+\alpha_2)]\left(T^1_N - T^2_N\right)$. (These calculations are implicitly based on Assumption 1.)

$(\lambda_T,\lambda_M) = (0,0)$ is always a Nash equilibrium outcome because, if both countries are selecting no trade and no labor mobility, then, conditionally on the decision of the other country, there is no policy that can move the economy toward free trade or toward free labor mobility. Thus, any unilateral deviation will not change the payoff for any player.

$(\lambda_T, \lambda_M) = (1,0)$ is a Nash equilibrium outcome when $W_G^1(1,0) \geq W_G^1(0,0)$ and $W_G^2(1,0) \geq W_G^2(0,0)$. From assumption 1 $\left(\alpha_1 \bar{L}^2 / \alpha_2 \bar{L}^1\right) > A_2$, which implies $W_G^1(1,0) - W_G^1(0,0) = \alpha_2 \ln\left(\alpha_1 \bar{L}^2 / A_2 \alpha_2 \bar{L}^1\right) > 0$. From assumption 1 $A_1 > \left(\alpha_1 \bar{L}^2 / \alpha_2 \bar{L}^1\right)$, which implies $W_G^2(1,0) - W_G^2(0,0) = \alpha_1 \ln\left(A_1 \alpha_2 \bar{L}^1 / a_{L,1}^1 \alpha_1 \bar{L}^2\right) > 0$.

$(\lambda_T, \lambda_M) = (0,1)$ is a Nash equilibrium outcome when $W_G^1(0,1) \geq W_G^1(0,0)$ and $W_G^2(0,1) \geq W_G^2(0,0)$. $W_G^1(0,0) = W_G^1(0,1)$, while $W_G^2(0,1) - W_G^2(0,0) = m\left(T^1 - T^2\right)$, which is positive by assumption.

$(\lambda_T, \lambda_M) = (1,1)$ is not a Nash equilibrium outcome, provided that $\Delta = (\alpha_1 + \alpha_2)^{-1}\left(T_N^1 - T_N^2\right) - \ln\left(\alpha_2 \bar{L}^1 / \alpha_1 \bar{L}^2\right) \neq 0$. Note that $W_G^1(1,1) - W_G^1(1,0) = -\alpha_2 \Delta$ and $W_G^2(1,1) - W_G^2(1,0) = \alpha_1 \Delta$, which implies that either $W_G^1(1,1) - W_G^1(1,0) < 0$ (when $\Delta > 0$) or $W_G^1(1,1) - W_G^1(1,0) < 0$ (when $\Delta < 0$).

We have already proved that $W_G^1(1,0) > W_G^1(0,1) = W_G^1(0,0)$ and $\left\{W_G^2(1,0), W_G^2(0,1)\right\} > W_G^2(0,0)$. Finally, note that $W_G^2(1,0) - W_G^2(0,1) = \alpha_1 \ln\left(A_1 \alpha_2 \bar{L}^1 / \alpha_1 \bar{L}^2\right) - m\left(T^1 - T^2\right) > 0$ if and only if $\alpha_1 \ln\left(A_1 \alpha_2 \bar{L}^1 / \alpha_1 \bar{L}^2\right) > m\left(T^1 - T^2\right)$, which completes the proof of the proposition. ■

A.2 Multiple goods

A.2.1 A finite set of goods

Assume that $Z_T = \{1, 2, ... z_T\}$, $Z_N = \{z_{T+1}, ..., z_{T+N}\}$ and $Q_z^j = L_z^j / a_{L,z}^j$. Let $A_z = a_{L,z}^2 / a_{L,z}^1$ be strictly decreasing for $z \in Z_T$ with $A_1 > 1$ and $A_{z_T} < 1$. Under autarky, $p_z^j = w^j a_{L,z}^j$ for all $z \in Z$ and $j = 1, 2$. The indirect utility of a worker who owns one unit of labor in country j is $v^j = C + T^j$, where $T^j = -\sum_{z \in Z} \alpha_z \ln\left(a_{L,z}^j\right)$. If labor mobility is allowed, all mobile workers will go to or stay in the country with the highest T^j.

Under free trade, there exists $\bar{z} \in Z_T$, such that $p_z = w^1 a_{L,z}^1$ for $z = 1, ..., \bar{z} - 1$, $p_z = w^2 a_{L,z}^2$ for $z = \bar{z} + 1, ..., z_T$, and $p_z^j = w^j a_{L,z}^j$ for $z \in Z_N$. In order to determine $\bar{z}$ and w^1/w^2, it is useful to define the following function. $F(x) = A_z$ if $\left[\left(\sum_{k=1}^{k=z-1} \alpha_k\right) / \left(\sum_{k=z}^{k=z_T} \alpha_k\right) A_z\right] \leq x \leq \left[\left(\sum_{k=1}^{k=z} \alpha_z\right) / \left(\sum_{k=z+1}^{k=z_T} \alpha_z\right) A_z\right]$, while $F(x) = \left(\sum_{k=1}^{k=z} \alpha_k\right) / \left(\sum_{k=z+1}^{k=z_T} \alpha_k\right) x$ if $\left[\left(\sum_{k=1}^{k=z} \alpha_k\right) / \left(\sum_{k=z+1}^{k=z_T} \alpha_k\right) A_z\right] < x < \left[\left(\sum_{k=1}^{k=z} \alpha_k\right) / \left(\sum_{k=z+1}^{k=z_T} \alpha_k\right) A_{z+1}\right]$. $F(x)$ is a decreasing and continuous function of x. In the trading equilibrium $w^1/w^2 = F(L_1/L_2)$ and $\bar{z}$ is such that $A_{\bar{z}} \geq w^1/w^2 > A_{\bar{z}+1}$. Moreover, $1 < \bar{z} < z_T$ if and only if $A_1 > F(L_1/L_2) > A_{z_T}$, which we assume holds. Also note that if $\left[\left(\sum_{k=1}^{k=\bar{z}-1} \alpha_k\right) / \left(\sum_{k=\bar{z}}^{k=z_T} \alpha_k\right) A_{\bar{z}}\right] \leq L^1/L^2 \leq \left[\left(\sum_{k=1}^{k=\bar{z}} \alpha_k\right) / \left(\sum_{k=\bar{z}+1}^{k=z_T} \alpha_k\right) A_{\bar{z}}\right]$, then both countries produce good $\bar{z}$, $w^1/w^2 = F\left(L^1/L^2\right) = A_{\bar{z}}$, and $p_{\bar{z}} = w^1 a_{L,\bar{z}}^1 = w^2 a_{L,\bar{z}}^2$; while if $\left[\left(\sum_{k=1}^{k=\bar{z}} \alpha_k\right) / \left(\sum_{k=\bar{z}+1}^{k=z_T} \alpha_k\right) A_{\bar{z}}\right] < L^1/L^2 < \left[\left(\sum_{k=1}^{k=\bar{z}} \alpha_k\right) / \left(\sum_{k=\bar{z}+1}^{k=z_T} \alpha_k\right) A_{\bar{z}+1}\right]$, then only country 1 produces good $\bar{z}$, $w^1/w^2 = F\left(L^1/L^2\right) = \left[\left(\sum_{k=1}^{k=\bar{z}} \alpha_k\right) / \left(\sum_{k=\bar{z}+1}^{k=z_T} \alpha_k\right)\right]\left(L^2/L^1\right)$ and $p_{\bar{z}} = w^1 a_{L,\bar{z}}^1$. The indirect utility of a worker who owns one unit of labor is given by $v^1 = C + T^1 + \sum_{z=\bar{z}+1}^{z=z_T} \alpha_z \ln\left(F\left(L^1/L^2\right)/A_z\right)$ in country 1 and $v^2 = C + T^2 + \sum_{z=1}^{z=\bar{z}} \alpha_z \ln\left(A_z / F\left(L^1/L^2\right)\right)$ in country 2, where $\bar{z}$ is such that $A_{\bar{z}} \geq w^1/w^2 = F\left(L^1/L^2\right) > A_{\bar{z}+1}$.

If labor mobility is not allowed, then $L^j = \bar{L}^j$, which implies $\bar{z}$ is such that $A_{\bar{z}} \geq w^1/w^2 = F\left(\bar{L}^1/\bar{L}^2\right) > A_{\bar{z}+1}$. If labor mobility is allowed, mobile workers will go to or stay in the country with the highest v^j.

If there are enough mobile workers, they will relocate until $v^1 = v^2$. As a consequence, $\ln\left(w^1/w^2\right) = -\left(1-\alpha_N\right)^{-1}\left(T_N^1 - T_N^2\right)$, where $T_N^j = -\sum_{z\in Z_N}\alpha_z \ln\left(a_{L,z}^j\right)$ is the productivity of country j in non-tradeable goods and $\alpha_N = \sum_{z\in Z_N}\alpha_z$ is the expenditure share in non-tradeable goods. Thus:

$$\begin{aligned} v^1 = v^2 &= C + T^1 - \left(1-\alpha_N\right)^{-1}\left(T_N^1 - T_N^2\right)\left(\textstyle\sum_{z=\hat{z}+1}^{z=z_T}\alpha_z\right) - \textstyle\sum_{z=\hat{z}+1}^{z=z_T}\alpha_z \ln\left(A_z\right) \\ &= C + T^2 + \textstyle\sum_{z=1}^{z=\hat{z}}\alpha_z \ln\left(A_z\right) + \left(1-\alpha_N\right)^{-1}\left(T_N^1 - T_N^2\right)\left(\textstyle\sum_{z=1}^{z=\hat{z}}\alpha_z\right) \end{aligned}$$

In order to determine the marginal industry $\hat{z}$ and the labor allocation L_1/L_2 under free labor mobility, we need to consider two possible cases.

a. Suppose there exists $z \in (1, z_T)$, such that $\ln\left(A_z\right) = -\left(1-\alpha_N\right)^{-1}\left(T_N^1 - T_N^2\right)$. Then, the marginal industry is determined by $\ln\left(A_{\hat{z}}\right) = -\left(1-\alpha_N\right)^{-1}\left(T_N^1 - T_N^2\right)$, both countries produce $\hat{z}$, and L^1/L^2 is such that $\left[\left(\sum_{z=1}^{z=\hat{z}-1}\alpha_z\right)/\left(\sum_{z=\hat{z}}^{z=z_T}\alpha_z\right)A_{\hat{z}}\right] \le L^1/L^2 \le \left[\left(\sum_{z=1}^{z=\hat{z}}\alpha_z\right)/\left(\sum_{z=\hat{z}+1}^{z=z_T}\alpha_z\right)A_{\hat{z}}\right]$ and $\left(\bar{L}^2 + m\bar{L}^1\right)/\left(1-m\right)\bar{L}^1 \le L^1/L^2 \le \left(1-m\right)\bar{L}^2/\left(\bar{L}^1 + m\bar{L}^2\right)$.

b. Suppose there is no $z \in (1, z_T)$, such that $\ln\left(A_z\right) = -\left(1-\alpha_N\right)^{-1}\left(T_N^1 - T_N^2\right)$. Then, the marginal industry is $\hat{z}$ such that $\ln\left(A_{\hat{z}}\right) > -\left(1-\alpha_N\right)^{-1}\left(T_N^1 - T_N^2\right) > \ln\left(A_{\hat{z}+1}\right)$ and the labor allocation is given by $\left(L^1/L^2\right) = \left[\left(\sum_{z=1}^{z=\hat{z}}\alpha_z\right)/\left(\sum_{z=\hat{z}+1}^{z=z_T}\alpha_z\right)\right]\left(w^2/w^1\right)$ where $\ln\left(w^1/w^2\right) = -\left(1-\alpha_N\right)^{-1}\left(T_N^1 - T_N^2\right)$.

Finally, the distribution of mobile workers between the countries does not violate either $L^j \in \left[\left(1-m\right)\bar{L}^j, \bar{L}^j + m\bar{L}^{-j}\right]$ or $A_1 > F\left(L_1/L_2\right) > A_{z_T}$, provided that the following assumption holds.

Assumption 1 (finite set of goods). *Labor mobility induces full wage convergence. Formally:* $A_1 > F\left(\frac{(1-m)\bar{L}^1}{\bar{L}^2 + m\bar{L}^1}\right) > \prod_{z\in Z_N}\left(A_z\right)^{\frac{-\alpha_z}{1-\alpha_N}} > F\left(\frac{\bar{L}^1 + m\bar{L}^2}{(1-m)\bar{L}^2}\right) > A_{z_T}$.

Lemma 1 characterizes the effects of trade and labor mobility on the countries' relative wages.

Lemma 1 (finite set of goods). *Let* $T^j = -\sum_{z\in Z}\alpha_z \ln\left(a_{L,z}^j\right)$ *and assume* $T^1 > T^2$. *Define* $\Delta = \left(1-\alpha_N\right)\ln\left(F\left(\bar{L}^1/\bar{L}^2\right)\right) + \left(T_N^1 - T_N^2\right)$, *where* $T_N^j = -\sum_{z\in Z_N}\alpha_z \ln\left(a_{L,z}^j\right)$. *Suppose that Assumption 1 holds.*

1. *If* $\Delta > T^1 - T^2$, *then* $v^1(1,0) - v^2(1,0) > v^1(0,0) - v^2(0,0) > v^1(1,1) - v^2(1,1) = 0$.
2. *If* $0 < \Delta < T^1 - T^2$, *then* $v^1(0,0) - v^2(0,0) > v^1(1,0) - v^2(1,0) > v^1(1,1) - v^2(1,1) = 0$.
3. *If* $\Delta < 0$, *then* $v^1(0,0) - v^2(0,0) > v^1(1,1) - v^2(1,1) = 0 > v^1(1,0) - v^2(1,0)$.

Proof: Under autarky, regardless of mobile workers' location decisions, $v^j(0,\lambda_M) = C + T^j$. Thus, $v^1(0,0) - v^2(0,0) = T^1 - T^2$, which is positive by assumption. Under free trade and no labor mobility, $v^1(1,0) = C + T^1 + \ln\left(F\left(\bar{L}^1/\bar{L}^2\right)\right)\left(\sum_{z=\bar{z}+1}^{z=z_T}\alpha_z\right) - \sum_{z=\bar{z}+1}^{z=z_T}\alpha_z \ln\left(A_z\right)$, and $v^2(1,0) = C + T^2 + \sum_{z=1}^{z=\bar{z}}\alpha_z \ln\left(A_z\right) - \ln\left(F\left(\bar{L}^1/\bar{L}^2\right)\right)\left(\sum_{z=1}^{z=\bar{z}}\alpha_z\right)$, where $\bar{z}$ is such that $A_{\bar{z}} \ge F\left(\bar{L}^1/\bar{L}^2\right) > A_{\bar{z}+1}$. Thus, $v^1(1,0) - v^2(1,0) = \left(1-\alpha_N\right)\ln\left(F\left(\bar{L}^1/\bar{L}^2\right)\right) + \left(T_N^1 - T_N^2\right)$. Under free trade and free labor mobility, assumption 1 implies $v^1(1,1) = v^2(1,1) = C + T^1 - \left(1-\alpha_N\right)^{-1}\left(T_N^1 - T_N^2\right)\left(\sum_{z=\hat{z}+1}^{z=z_T}\alpha_z\right) - \sum_{z=\hat{z}+1}^{z=z_T}\alpha_z \ln\left(A_z\right)$. Simple comparisons complete the proof of the lemma. ■

Proposition 1 characterizes the political equilibrium.

Proposition 1 (finite set of goods). *Suppose that Assumption 1 holds, $T^1 > T^2$ and $\Delta \neq 0$. Then, the trade and labor mobility game has three Nash equilibrium outcomes: (i) $(\lambda_T, \lambda_M) = (0,0)$; (ii) $(\lambda_T, \lambda_M) = (0,1)$; and (iii) $(\lambda_T, \lambda_M) = (1,0)$. Moreover:*

1. $W_G^1(1,0) > W_G^1(0,1) = W_G^1(0,0)$.
2. $\left\{W_G^2(1,0), W_G^2(0,1)\right\} > W_G^2(0,0)$, *while* $W_G^2(1,0) > W_G^2(0,1)$ *if and only if* $\sum_{z=1}^{z=\bar{z}} \alpha_z \ln\left(A_z / F\left(\bar{L}^1/\bar{L}^2\right)\right) > m\left(T^1 - T^2\right)$.

Proof: Government 1 payoffs are given by $W_G^1(0,0) = W_G^1(0,1) = C + T^1$, $W_G^1(1,0) = C + T^1 + \ln\left(F\left(\bar{L}^1/\bar{L}^2\right)\right)\left(\sum_{z=\bar{z}+1}^{z=z_T} \alpha_z\right) - \sum_{z=\bar{z}+1}^{z=z_T} \alpha_z \ln(A_z)$, and $W_G^1(1,1) = C + T^1 - (1-\alpha_N)^{-1}\left(T_N^1 - T_N^2\right)\left(\sum_{z=\hat{z}+1}^{z=z_T} \alpha_z\right) - \sum_{z=\hat{z}+1}^{z=z_T} \alpha_z \ln(A_z)$. Government 2 payoffs are given by $W_G^2(0,0) = C + T^2$, $W_G^2(0,1) = C + mT^1 + (1-m)T^2$, $W_G^2(1,0) = C + T^2 + \sum_{z=1}^{z=\bar{z}} \alpha_z \ln(A_z) - \ln\left(F\left(\bar{L}^1/\bar{L}^2\right)\right)\left(\sum_{z=1}^{z=\bar{z}} \alpha_z\right)$ and $W_G^2(1,1) = C + T^2 + \sum_{z=1}^{z=\hat{z}} \alpha_z \ln(A_z) + (1-\alpha_N)^{-1}\left(T_N^1 - T_N^2\right)\left(\sum_{z=1}^{z=\hat{z}} \alpha_z\right)$. (These calculations are implicitly based on Assumption 1).

$(\lambda_T, \lambda_M) = (0,0)$ is always a Nash equilibrium outcome.

$(\lambda_T, \lambda_M) = (1,0)$ is a Nash equilibrium outcome when $W_G^1(1,0) \geq W_G^1(0,0)$ and $W_G^2(1,0) \geq W_G^2(0,0)$. $W_G^1(1,0) - W_G^1(0,0) = \sum_{z=\bar{z}+1}^{z=z_T} \alpha_z \ln\left(F\left(\bar{L}^1/\bar{L}^2\right)/A_z\right)$, where $A_{\bar{z}} \geq F\left(\bar{L}^1/\bar{L}^2\right) > A_{\bar{z}+1}$. Since $F\left(\bar{L}^1/\bar{L}^2\right) > A_z$ for all $z = \bar{z}+1, ..., z_T$, and since Assumption 1 implies $1 < \bar{z} < z_T$, it must be the case that $W_G^1(1,0) - W_G^1(0,0) > 0$. $W_G^2(1,0) - W_G^2(0,0) = \sum_{z=1}^{z=\bar{z}} \alpha_z \ln\left(A_z / F\left(\bar{L}^1/\bar{L}^2\right)\right)$, where $A_{\bar{z}} \geq F\left(\bar{L}^1/\bar{L}^2\right) > A_{\bar{z}+1}$. Since $A_z > F\left(\bar{L}^1/\bar{L}^2\right)$ for all $z = 1, ..., \bar{z}$, and since Assumption 1 implies $1 < \bar{z} < z_T$, it must be the case that $W_G^2(1,0) - W_G^2(0,0) > 0$.

$(\lambda_T, \lambda_M) = (0,1)$ is a Nash equilibrium outcome when $W_G^1(0,1) \geq W_G^1(0,0)$ and $W_G^2(0,1) \geq W_G^2(0,0)$. $W_G^1(0,0) = W_G^1(0,1)$, while $W_G^2(0,1) - W_G^2(0,0) = m\left(T^1 - T^2\right)$, which is positive by assumption.

$(\lambda_T, \lambda_M) = (1,1)$ is not a Nash equilibrium outcome, provided that $\Delta \neq 0$. Note that:

$$W_G^1(1,1) - W_G^1(1,0) = \sum_{z=\hat{z}+1}^{z=z_T} \alpha_z \ln\left(\frac{F\left(L^1/L^2\right)}{A_z}\right) - \sum_{z=\bar{z}+1}^{z=z_T} \alpha_z \ln\left(\frac{F\left(\bar{L}^1/\bar{L}^2\right)}{A_z}\right)$$
$$W_G^2(1,1) - W_G^2(1,0) = \sum_{z=1}^{z=\hat{z}} \alpha_z \ln\left(\frac{A_z}{F\left(L^1/L^2\right)}\right) - \sum_{z=1}^{z=\bar{z}} \alpha_z \ln\left(\frac{A_z}{F\left(\bar{L}^1/\bar{L}^2\right)}\right)$$

where L^1/L^2 is the labor allocation under free trade and free labor mobility. Suppose that $\Delta > 0$. Then $v^1(1,0) > v^2(1,0)$ and $L_1/L_2 > \bar{L}^1/\bar{L}^2$. Since F is decreasing and $\Delta > 0$, $F\left(\bar{L}^1/\bar{L}^2\right) > F\left(L^1/L^2\right)$ and, hence, $\hat{z} \geq \bar{z}$. Finally, $F\left(\bar{L}^1/\bar{L}^2\right) > A_z$ for $z = \bar{z}+1, ..., z_T$. Thus, $W_G^1(1,1) - W_G^1(1,0) < 0$. Suppose that $\Delta < 0$. Then $v^1(1,0) < v^2(1,0)$ and $L_1/L_2 < \bar{L}^1/\bar{L}^2$. Since F is decreasing and $\Delta < 0$, $F\left(\bar{L}^1/\bar{L}^2\right) < F\left(L^1/L^2\right)$. Hence, $\hat{z} \leq \bar{z}$. Finally, $F\left(\bar{L}^1/\bar{L}^2\right) < A_z$ for $z = 1, ...\bar{z}$. Thus, $W_G^2(1,1) - W_G^2(1,0) < 0$. Note, however, that if $F\left(\bar{L}^1/\bar{L}^2\right) = A_{\bar{z}}$, a small reallocation of mobile workers does not change the marginal industry and, as consequence, there is no effect on w^1/w^2. In other words, $\Delta \neq 0$ implies that $(\lambda_T, \lambda_M) = (1,1)$ is not a Nash equilibrium outcome, but if $F\left(\bar{L}^1/\bar{L}^2\right) = A_{\bar{z}}$ free trade and partial labor mobility can be a Nash equilibrium outcome.

We have already proved that $W_G^1(1,0) > W_G^1(0,1) = W_G^1(0,0)$ and $\{W_G^2(1,0), W_G^2(0,1)\} > W_G^2(0,0)$. Finally, note that $W_G^2(1,0) - W_G^2(0,1) = \sum_{z=1}^{z=\bar{z}} \alpha_z \ln\left(A_z/F\left(\bar{L}^1/\bar{L}^2\right)\right) > m\left(T^1 - T^2\right)$ if and only if $\sum_{z=1}^{z=\bar{z}} \alpha_z \ln\left(A_z/F\left(\bar{L}^1/\bar{L}^2\right)\right) > m\left(T^1 - T^2\right)$, which completes the proof of the proposition. ■

A.2.2 A continuum of goods

Assume that $Z_T = [0,k)$, $Z_N = [k,1]$ and $Q_z^j = L_z^j/a_{L,z}^j$. Let $A_z = a_{L,z}^2/a_{L,z}^1$ be a continuously differentiable strictly decreasing function for $z \in [0,k)$ that satisfies $A_0 > 1$ and $A_k < 1$. All agents have the same preferences, given by $u\left(c^j\right) = \int_0^1 \alpha_z \ln\left(c_z^j\right) dz$, with $\int_0^1 \alpha_z dz = 1$. Thus, the indirect utility function is given by $v^j = C + \int_0^1 \alpha_z \ln\left(w^j/p_z^j\right) dz$, where $C = \int_0^1 \alpha_z \ln\left(\alpha_z\right) dz$.

Under autarky, all goods must be produced domestically and, hence, $p_z^j = w^j a_{L,z}^j$ for all $z \in Z$ and $j \in J$. The indirect utility of a worker who owns one unit of labor in country j is $v^j = C + T^j$, where $T^j = -\int_0^1 \alpha_z \ln\left(a_{L,z}^j\right) dz$ is a measure of the productivity of country j. If labor mobility is allowed, all mobile workers go to or stay in the country with the higher T^j.

Under free trade, in the trading equilibrium country 1 produces lower-indexed tradeable goods $z \in [0,\bar{z}] \subset Z_T$ and non-tradeable goods $z \in Z_N = [k,1]$ and country 2 produces higher-indexed tradeable goods $z \in [\bar{z},k)$ and non-tradeable goods $z \in Z_N = [k,1]$. The marginal tradeable industry $\bar{z}$ is given by $A_{\bar{z}} = w^1/w^2$. The balanced trade condition is $w^1/w^2 = \left[\alpha(\bar{z})/\left(1 - \alpha_N - \alpha(\bar{z})\right)\right]\left(L^2/L^1\right)$, where $\alpha_N = \int_k^1 \alpha_z dz$ is the portion of income spent on non-tradeable goods and $\alpha(z) = \int_0^z \alpha_z dz$ is the portion of world income spent on tradeable goods in the range $[0,z]$. There exists a unique $\left(\bar{z}, w^1/w^2\right)$, with $\bar{z} \in (0,k)$ and $w^1/w^2 > 0$, that simultaneously satisfies the marginal tradeable industry condition and the balanced trade condition.[1] Since $p_z = w^1 a_{L,z}^1$ for $z \in [0,\bar{z}]$, $p_z = w^2 a_{L,z}^2$ for $[\bar{z},k)$, $p_z^j = w^j a_{L,z}^j$ for $z \in [k,1]$ and $j = 1,2$, the indirect utility of a worker who owns one unit of labor is $v^1 = C + T^1 + \int_{\bar{z}}^k \alpha_z \ln\left(\frac{A_{\bar{z}}}{A_z}\right) dz$ in country 1 and $v^2 = C + T^2 + \int_0^{\bar{z}} \alpha_z \ln\left(\frac{A_z}{A_{\bar{z}}}\right) dz$ in country 2, where $A_{\bar{z}} = \left[\alpha(\bar{z})/\left(1 - \alpha_N - \alpha(\bar{z})\right)\right]\left(L^2/L^1\right)$. If labor mobility is not allowed, then $L^j = \bar{L}^j$. If labor mobility is allowed, mobile workers will go to or stay in the country with higher v^j. If $v^1 > v^2$, then mobile workers will move from country 2 to 1, v^1 will decrease and v^2 will increase. Analogously, if $v^2 > v^1$, then mobile workers will move from country 1 to 2, v^2 will decrease and v^1 will increase. Therefore, and, provided that there are enough mobile workers, they will relocate until $v^1 = v^2$, which implies $\ln\left(w^1/w^2\right) = -\left(1 - \alpha_N\right)^{-1}\left(T_N^1 - T_N^2\right)$, where $T_N^j = -\int_k^1 \alpha_z \ln\left(a_{L,z}^j\right) dz$ is the average productivity of country j in the production of non-tradeable goods. This expression determines the ratio w^1/w^2 which makes mobile workers indifferent to the possibility of settling in one country or the other. It depends only on the productivity differences in the non-tradeable industries. Once we know w^1/w^2, we can use $A_{\hat{z}} = w^1/w^2$ to determine the marginal tradeable industry $\hat{z}$. Then, the balanced trade condition implies $L^2/L^1 = A_{\hat{z}}\left(1 - \alpha_N - \alpha(\hat{z})\right)/\alpha(\hat{z})$. The utility of a worker who owns one unit of labor is given by:

$$v^1 = v^2 = C + T^1 + \int_{\hat{z}}^k \alpha_z \ln\left(\frac{A_{\hat{z}}}{A_z}\right) dz = C + T^2 + \int_0^{\bar{z}} \alpha_z \ln\left(\frac{A_z}{A_{\hat{z}}}\right) dz,$$

[1] It is simple to verify that there exists a unique $\bar{z} \in (0,k)$ that satisfies $A_{\bar{z}} = \left[\alpha(\bar{z})/\left(1 - \alpha_N - \alpha(\bar{z})\right)\right]\left(L^2/L^1\right)$. A_z is a continuous and strictly decreasing function and $A_0 > 0$. $B(z) = \left[\alpha(z)/\left(1 - \alpha_N - \alpha(z)\right)\right]\left(L^2/L^1\right)$ is a continuous and strictly increasing function, $B(0) = 0$ and $\lim_{z \to k} B(z) = \infty$.

where $(1-\alpha_N)\ln(A_{\hat{z}}) = -\left(T_N^1 - T_N^2\right)$. Moreover, there will be migration to country 1 whenever $\bar{L}^2/\bar{L}^1 > A_{\hat{z}}(1-\alpha_N-\alpha(\hat{z}))/\alpha(\hat{z})$ and migration to country 2 whenever $\bar{L}^2/\bar{L}^1 < A_{\hat{z}}(1-\alpha_N-\alpha(\hat{z}))/\alpha(\hat{z})$. Finally, we must verify that the distribution of mobile workers between the countries does not violate $L^j \in \left[(1-m)\bar{L}^j, \bar{L}^j + m\bar{L}^{-j}\right]$. In order to avoid such a situation, we impose the following assumption.

Assumption 1 (continuum of goods). *Labor mobility induces full wage convergence. Formally:* $\frac{\bar{L}^2+m\bar{L}^1}{(1-m)\bar{L}^1} > \frac{A_{\hat{z}}(1-\alpha_N-\alpha(\hat{z}))}{\alpha(\hat{z})} > \frac{(1-m)\bar{L}^2}{\bar{L}^1+m\bar{L}^2}$, *where* $\hat{z}$ *is implicitly given by* $(1-\alpha_N)\ln(A_{\hat{z}}) = -\left(T_N^1 - T_N^2\right)$.

Lemma 1 characterizes the effects of trade and labor mobility on relative wages.

Lemma 1 (continuum of goods). *Let* $T^j = -\int_0^1 \alpha_z \ln\left(a_{L,z}^j\right) dz$ *be the average productivity of country* j *and assume* $T^1 > T^2$. *Define* $\Delta_{\bar{z}} = (1-\alpha_N)\ln(A_{\bar{z}}) + \left(T_N^1 - T_N^2\right)$, *where* $A_{\bar{z}} = [\alpha(\bar{z})/(1-\alpha_N-\alpha(\bar{z}))]\left(\bar{L}^2/\bar{L}^1\right)$ *and* $T_N^j = -\int_k^1 \alpha_z \ln\left(a_{L,z}^j\right) dz$ *is the average productivity of country* j *in the production of non-tradeable goods. Suppose that assumption 2 holds. Then:*

1. *If* $\Delta_{\bar{z}} > \left(T^1 - T^2\right)$, *then* $v^1(1,0) - v^2(1,0) > v^1(0,0) - v^2(0,0) > v^1(1,1) - v^2(1,1) = 0$.

2. *If* $0 < \Delta_{\bar{z}} < \left(T^1 - T^2\right)$, *then* $v^1(0,0) - v^2(0,0) > v^1(1,0) - v^2(1,0) > v^1(1,1) - v^2(1,1) = 0$.

3. *If* $\Delta_{\bar{z}} < 0$, *then* $v^1(0,0) - v^2(0,0) > v^1(1,1) - v^2(1,1) = 0 > v^1(1,0) - v^2(1,0)$.

Proof: Under autarky, regardless of mobile workers' location decisions, $v^j(0,\lambda_M) = C + T^j$. Thus, $v^1(0,0) - v^2(0,0) = T^1 - T^2$, which is positive by assumption. Under free trade, if there is no labor mobility $v^1(1,0) = C + T^1 + \int_{\bar{z}}^k \alpha_z \ln(A_{\bar{z}}/A_z)\, dz$ and $v^2(1,0) = C + T^2 + \int_0^{\bar{z}} \alpha_z \ln(A_z/A_{\bar{z}})\, dz$. Hence, $v^1(1,0) - v^2(1,0) = T_N^1 - T_N^2 + (1-\alpha_N)\ln(A_{\bar{z}})$. Under free trade, if there is free labor mobility $v^1(1,1) = C + T^1 + \int_{\hat{z}}^k \alpha_z \ln(A_{\hat{z}}/A_z)\, dz$ and $v^2(1,1) = C + T^2 + \int_0^{\hat{z}} \alpha_z \ln(A_z/A_{\hat{z}})\, dz$. Hence $v^1(1,1) - v^2(1,1) = T_N^1 - T_N^2 + (1-\alpha_N)\ln(A_{\hat{z}})$. Provided that assumption 1 holds, we have $\ln(A_{\hat{z}}) = \ln\left(w^1/w^2\right) = -(1-\alpha_N)^{-1}\left(T_N^1 - T_N^2\right)$ and, hence, $v^1(1,1) - v^2(1,1) = 0$. Simple comparisons complete the proof of the lemma. ■

As in the simple Ricardian model, under free trade, labor mobility leads to a complete convergence in real wages. The reason for this is that mobile workers move from the poor country to the rich country until they equalize real wages (with Assumption 1 ensuring that there are enough mobile workers to make this happen). Trade alone has an ambiguous effect on convergence. While, under autarky, the wage difference depends on the average productivity differential (real wages are higher in country 1 when $T^1 > T^2$), under free trade, it depends on the productivity differential in the marginal tradeable industry as well as on the average productivity differential in non-tradeable industries (real wages are higher in country 1 when $\Delta_{\bar{z}} = (1-\alpha_N)\ln(A_{\bar{z}}) + \left(T_N^1 - T_N^2\right) > 0$). In turn, the productivity differential in the marginal industry $(A_{\bar{z}})$ is high when country 1 is relatively labor-scarce and expenditure shares in low-indexed tradeable goods are high. Note that we can interpret $\Delta_{\bar{z}}$ as a measure of the productivity differential under free trade. When it is higher than the average productivity differential, free trade induces divergence; when it is positive but lower than the average productivity differential, free trade induces partial convergence; and when it is negative, free trade leads to a reversal of fortune.

Proposition 1 characterizes the political equilibrium.

Proposition 1 (continuum of goods). *Suppose that Assumption 1 holds, $T^1 > T^2$, and $\Delta_{\bar{z}} \neq 0$. Then, the trade and labor mobility game has three Nash equilibrium outcomes: (i) $(\lambda_T, \lambda_M) = (0,0)$; (ii) $(\lambda_T, \lambda_M) = (0,1)$; and (iii) $(\lambda_T, \lambda_M) = (1,0)$. Moreover:*

1. $W_G^1(1,0) > W_G^1(0,1) = W_G^1(0,0)$.
2. $\{W_G^2(1,0), W_G^2(0,1)\} > W_G^2(0,0)$, *while* $W_G^2(1,0) > W_G^2(0,1)$ *if and only if* $\int_0^{\bar{z}} \alpha_z \ln(A_z/A_{\bar{z}})\, dz > m(T^1 - T^2)$.

Proof: Government 1 payoffs are given by $W_G^1(0,0) = W_G^1(0,1) = C + T^1$, $W_G^1(1,0) = C + T^1 + \int_{\bar{z}}^k \alpha_z \ln(A_{\bar{z}}/A_z)\, dz$, and $W_G^1(1,1) = C + T^1 + \int_{\hat{z}}^k \alpha_z \ln(A_{\hat{z}}/A_z)\, dz$. Government 2 payoffs are given $W_G^2(0,0) = C + T^2$, $W_G^2(0,1) = C + mT^1 + (1-m)T^2$, $W_G^2(1,0) = C + T^2 + \int_0^{\bar{z}} \alpha_z \ln(A_z/A_{\bar{z}})\, dz$ and $W_G^2(1,1) = C + T^2 + \int_0^{\hat{z}} \alpha_z \ln(A_z/A_{\hat{z}})\, dz$. (These calculations are implicitly based on Assumption 1.)

$(\lambda_T, \lambda_M) = (0,0)$ is always a Nash equilibrium outcome.

$(\lambda_T, \lambda_M) = (0,1)$ is a Nash equilibrium outcome when $W_G^1(0,1) \geq W_G^1(0,0)$ and $W_G^2(0,1) \geq W_G^2(0,0)$. $W_G^1(0,1) = W_G^1(0,0) = C + T^1$, while $W_G^2(0,1) - W_G^2(0,0) = m(T^1 - T^2) > 0$, which is positive by assumption.

$(\lambda_T, \lambda_M) = (1,0)$ is a Nash equilibrium outcome when $W_G^1(1,0) \geq W_G^1(0,0)$ and $W_G^2(1,0) \geq W_G^2(0,0)$. $A_z > A_{\bar{z}}$ for $z \in [0, \bar{z}]$ while $A_z < A_{\bar{z}}$ for $z \in [\bar{z}, k]$. Then $W_G^1(1,0) - W_G^1(0,0) = \int_{\bar{z}}^k \alpha_z \ln(A_{\bar{z}}/A_z)\, dz > 0$ and $W_G^2(1,0) - W_G^2(0,0) = \int_0^{\bar{z}} \alpha_z \ln(A_z/A_{\bar{z}})\, dz > 0$.

$(\lambda_T, \lambda_M) = (1,1)$ is not a Nash equilibrium outcome (provided that $\Delta_{\bar{z}} = (1-\alpha_N)^{-1}(T_N^1 - T_N^2) - \ln\left[(1-\alpha_N - \alpha(\bar{z}))\bar{L}^1/\alpha(\bar{z})\bar{L}^2\right] \neq 0$). In order to prove this, note that $W_G^1(1,1) - W_G^1(1,0) = \int_{\hat{z}}^k \alpha_z \ln(A_{\hat{z}}/A_z)\, dz - \int_{\bar{z}}^k \alpha_z \ln(A_{\bar{z}}/A_z)\, dz$ and $W_G^2(1,1) - W_G^2(1,0) = \int_0^{\hat{z}} \alpha_z \ln(A_z/A_{\hat{z}})\, dz - \int_0^{\bar{z}} \alpha_z \ln(A_z/A_{\bar{z}})\, dz$. If $\hat{z} > \bar{z}$, then $A_{\hat{z}} < A_{\bar{z}}$ and, hence, $\ln(A_{\hat{z}}/A_z) < \ln(A_{\bar{z}}/A_z)$. Therefore, $W_G^1(1,1) - W_G^1(1,0) < 0$. If $\hat{z} < \bar{z}$, then $A_{\hat{z}} > A_{\bar{z}}$ and, hence, $\ln(A_z/A_{\hat{z}}) < \ln(A_z/A_{\bar{z}})$. Therefore, $W_G^2(1,1) - W_G^2(1,0) < 0$. If $\hat{z} = \bar{z}$, then $A_{\hat{z}} = A_{\bar{z}}$ and, hence, $W_G^1(1,1) = W_G^1(1,0)$ and $W_G^2(1,1) = W_G^2(1,0)$. However, $\hat{z} = \bar{z}$ if and only if $(1-\alpha_N)^{-1} \int_k^1 \alpha_z \ln(A_z)\, dz + \ln\left[(1-\alpha_N - \alpha(\bar{z}))\bar{L}^1/\alpha(\bar{z})\bar{L}^2\right] = 0$, i.e., whenever $\Delta_{\bar{z}} = 0$, which we rule out by assumption.

We have already proved that $W_G^1(1,0) > W_G^1(0,1) = W_G^1(0,0)$ and $\{W_G^2(1,0), W_G^2(0,1)\} > W_G^2(0,0)$. Finally, note that $W_G^2(1,0) - W_G^2(0,1) = \int_0^{\bar{z}} \alpha_z \ln(A_z/A_{\bar{z}})\, dz - m(T^1 - T^2) > 0$ if and only if $\int_0^{\bar{z}} \alpha_z \ln(A_z/A_{\bar{z}})\, dz > m(T^1 - T^2)$, which completes the proof of the proposition. ■

Proposition 1 (continuum of goods) is the analogous of Proposition 1 (finite set of goods) when there is a continuum of goods. Other than free trade and free labor mobility, any other outcome is a Nash equilibrium outcome. As in the previous section, free trade and free labor mobility are not a Nash equilibrium outcome because workers in the rich country under free trade prefer to block labor mobility (since the inflow of labor reduces real wages). Free trade and no labor mobility always dominates other Nash equilibria for the rich country under autarky (country 1). The same is true for the poor country under autarky (country 2) when $\int_0^{\bar{z}} \alpha_z \ln(A_z/A_{\bar{z}})\, dz > m(T^1 - T^2)$ holds. The logic behind this result is as follows. Under autarky, labor mobility does not affect workers in country 1 because their wages are fully determined by the aggregate productivity of the country and they will not relocate even if they have the chance to do so. Since all workers in country 1 gain from trade, free trade and no labor mobility dominates other equilibria for country 1. Under autarky, mobile workers in country 2 will relocate to country 1. Compared with a situation in which there is no trade and no labor mobility, this will produce a gain of $\left(T^1 - T^2\right)$ per mobile worker. Compared with a situation in which there is no trade and no labor mobility, free trade will produce a gain of $\int_0^{\bar{z}} \alpha_z \ln\left(A_z/A_{\bar{z}}\right) dz$ for each worker in country 2. Thus, country 2 prefers free trade and no labor mobility to autarky and free labor mobility when gains from trade for all (mobile and immobile) workers are higher than the productivity gains for mobile workers.

A.2.3 Endogenous non-tradeable goods (iceberg trade costs)

As in the case of exogenous non-tradeable goods, consider an economy with two countries ($J = 2$) and a continuum of goods ($Z = [0, 1]$). The production functions are $Q_z^j = L_z^j / a_{L,z}^j$, where $a_{L,z}^j > 0$ is the unit labor requirement in industry z in country j. Let $A_z = a_{L,z}^2 / a_{L,z}^1$ and assume that country 1 has a comparative advantage in lower-indexed goods. Specifically, for $z \in [0, 1]$ A_z is a continuously differentiable strictly decreasing function. In contrast with the previous section, all goods are assumed to be tradeable, but there are transportation costs. A fraction $g < 1$ of each good shipped from one country to the other is lost.

Under autarky, the analysis is the same as in the previous section. Under free trade, in a trading equilibrium, country 1 produces low-indexed goods $z \in [0, \bar{z}_L]$, country 2 produces high-indexed goods $z \in [\bar{z}_H, 1]$ and both countries produce goods $z \in [\bar{z}_L, \bar{z}_H]$. Goods in the ranges $[0, \bar{z}_L]$ and $[\bar{z}_H, 1]$ are tradeable, while goods in the range $[\bar{z}_L, \bar{z}_H]$ are non-tradeable. The marginal industries $0 < \bar{z}_L < \bar{z}_H < 1$ are given by $gA_{\bar{z}_L} = w^1/w^2 = A_{\bar{z}_H}/g$. The balanced trade condition implies $w^1/w^2 = \left[\alpha\left(\bar{z}_L\right) / \left(1 - \alpha\left(\bar{z}_H\right)\right)\right] \left(L^2/L^1\right)$, where $\alpha(z) = \int_0^z \alpha_z dz$. There is a unique tuple $\left(\bar{z}_L, \bar{z}_H, w^1/w^2\right)$ with $0 < \bar{z}_L < \bar{z}_H < 1$ and $w^1/w^2 > 0$ that simultaneously satisfies the two marginal industry conditions and the balanced trade condition. Then, the indirect utility of a worker who owns one unit of labor is $v^1 = C + T^1 + \int_{\bar{z}_H}^1 \alpha_z \ln\left(A_{\bar{z}_H}/A_z\right) dz$ in country 1 and $v^2 = C + T^2 + \int_0^{\bar{z}_L} \alpha_z \ln\left(A_z/A_{\bar{z}_L}\right) dz$ in country 2, where $gA_{\bar{z}_L} = \left[\alpha\left(\bar{z}_L\right)/1 - \alpha\left(\bar{z}_H\right)\right] \left(L^2/L^1\right) = A_{\bar{z}_H}/g$ and $T^j = -\int_0^1 \alpha_z \ln\left(a_{L,z}^j\right) dz$. If labor mobility is not allowed, then $L^j = \bar{L}^j$. If labor mobility is allowed, mobile workers will go to or stay in the country with higher v^j. If $v^1 > v^2$, then mobile workers will move from country 2 to 1, v^1 will decrease and v^2 will increase. Analogously, if $v^2 > v^1$, then mobile workers will move from country 1 to 2, v^2 will decrease and v^1 will increase. Therefore, and, provided that there are enough mobile workers, they will relocate until $v^1 = v^2$, which implies: $\ln\left(\frac{w^1}{w^2}\right) = \frac{1}{\alpha(\hat{z}_L)} \left[-\int_{\hat{z}_L}^{\hat{z}_H} \alpha_z \ln\left(A_z\right) dz - \left[1 - \alpha\left(\hat{z}_H\right)\right] \ln\left(A_{\hat{z}_H}\right)\right] + \ln g$, where $gA_{\hat{z}_L} = w^1/w^2 = A_{\hat{z}_H}/g$. Once we have determined $\left(\hat{z}_L, \hat{z}_H, w^1/w^2\right)$, the balanced trade condition determines the country allocation of mobile workers, which is given by $L^2/L^1 = \left(w^1/w^2\right) \left[1 - \alpha\left(\hat{z}_H\right)/\alpha\left(\hat{z}_L\right)\right]$. Then, indirect utilities are given by:

$$v^1 = v^2 = C + T^1 + \int_{\hat{z}_H}^1 \alpha_z \ln\left(\frac{A_{\hat{z}_H}}{A_z}\right) dz$$

Finally, we must verify that the distribution of mobile workers between countries does not violate $L^j \in \left[(1 - m)\bar{L}^j, \bar{L}^j + m\bar{L}^{-j}\right]$. In order to avoid such a situation, we impose the following assumption.

Assumption 1 (continuum of goods and iceberg trade costs) *Labor mobility induces full convergence. Formally:* $\frac{\bar{L}^2 + m\bar{L}^1}{(1-m)\bar{L}^1} > gA_{\hat{z}_L}\left[\frac{1-\alpha(\hat{z}_H)}{\alpha(\hat{z}_L)}\right] > \frac{(1-m)\bar{L}^2}{\bar{L}^1 + m\bar{L}^2}$.

Lemma 1 characterizes the effects of trade and labor mobility on relative wages.

Lemma 1 (continuum of goods and iceberg trade costs). *Let* $T^j = -\int_0^1 \alpha_z \ln\left(a_{L,z}^j\right) dz$ *be the average productivity of country* j *and assume* $T^1 > T^2$. *Define* $\Delta_{\bar{z}_L, \bar{z}_H} = \alpha\left(\bar{z}_L\right) \ln\left(A_{\bar{z}_L}\right) +$

$[1-\alpha(\bar{z}_H)]\ln(A_{\bar{z}_H}) + T_N^1 - T_N^2$, *where* $gA_{\bar{z}_L} = [\alpha(\bar{z}_L)/1-\alpha(\bar{z}_H)]\left(\bar{L}^2/\bar{L}^1\right) = A_{\bar{z}_H}/g$ *and* $T_N^j = \int_{\bar{z}_L}^{\bar{z}_H}\alpha_z \ln\left(a_{L,z}^j\right)dz$ *is the average productivity of country* j *in the production of non-tradeable goods. Suppose that Assumption 1 holds. Then:*

1. *If* $\Delta_{\bar{z}_L,\bar{z}_H} > \left(T^1 - T^2\right)$, *then* $v^1(1,0) - v^2(1,0) > v^1(0,0) - v^2(0,0) > v^1(1,1) - v^2(1,1) = 0$.
2. *If* $0 < \Delta_{\bar{z}_L,\bar{z}_H} < \left(T^1 - T^2\right)$, *then* $v^1(0,0) - v^2(0,0) > v^1(1,0) - v^2(1,0) > v^1(1,1) - v^2(1,1) = 0$.
3. *If* $\Delta_{\bar{z}_L,\bar{z}_H} < 0$, *then* $v^1(0,0) - v^2(0,0) > v^1(1,1) - v^2(1,1) = 0 > v^1(1,0) - v^2(1,0)$.

Proof: Under autarky, regardless of mobile workers' location decisions, we have $v^{j,im}(0,\mu) = C + A^j$. Thus, $v^{1,im}(0,0) - v^{2,im}(0,0) = A^1 - A^2$, which is positive by assumption. Under free trade, if there is no factor mobility, $v^{1,im}(1,0) = C + A^1 + \int_{\bar{z}_H}^1 \alpha_z \ln\left(\frac{A_{\bar{z}_H}}{A_z}\right)dz$ and $v^{2,im}(1,0) = C + A^2 + \int_0^{\bar{z}_L}\alpha_z \ln\left(\frac{A_z}{A_{\bar{z}_L}}\right)dz$. Hence, $v^{1,im}(1,0) - v^{2,im}(1,0) = \int_{\bar{z}_L}^{\bar{z}_H}\alpha_z \ln(A_z) + \ln\left[(A_{\bar{z}_L})^{\alpha(\bar{z}_L)}(A_{\bar{z}_H})^{1-\alpha(\bar{z}_H)}\right]$. If there is free factor mobility, $v^1 = C + A^1 + \int_{\hat{z}_H}^1 \alpha_z \ln\left(\frac{A_{\hat{z}_H}}{A_z}\right)dz$ and $v^2 = C + A^2 + \int_0^{\hat{z}_L}\alpha_z \ln\left(\frac{A_z}{A_{\hat{z}_L}}\right)dz$. Hence, $v^{1,im}(1,1) - v^{2,im}(1,1) = \int_{\hat{z}_L}^{\hat{z}_H}\alpha_z \ln(A_z) + \ln\left[(A_{\hat{z}_L})^{\alpha(\hat{z}_L)}(A_{\hat{z}_H})^{1-\alpha(\hat{z}_H)}\right]$. Provided that assumption 1 holds, in equilibrium it must be that $\ln\left[(A_{\hat{z}_L})^{\alpha(\hat{z}_L)}(A_{\hat{z}_H})^{1-\alpha(\hat{z}_H)}\right] = -\int_{\hat{z}_L}^{\hat{z}_H}\alpha_z \ln(A_z)$. Therefore, $v^{1,im}(1,1) - v^{2,im}(1,1) = 0$. Simple comparisons complete the proof of the lemma. ■

Free trade and free labor mobility lead to full convergence in real wages, and trade has an ambiguous effect on relative wages. Trade can produce, divergence, partial convergence, or even a reversal of fortune, depending on the values of $\Delta_{\bar{z}_L,\bar{z}_H}$ and $\left(T^1 - T^2\right)$. Once again, we can interpret $\Delta_{\bar{z}_L,\bar{z}_H}$ as a measure of the productivity differential between the countries under free trade. There are, however, two novelties. First, since there are two marginal industries, we must average these productivity differentials. Second, the average productivity of country j in the production of non-tradeable goods is now endogenous.

Proposition 1 characterizes the political equilibrium.

Proposition 1 (continuum of goods and iceberg trade costs). *Suppose that Assumption 3 holds,* $T^1 > T^2$ *and* $\Delta_{\bar{z}_L,\bar{z}_H} \neq 0$. *Then, the trade and labor mobility game has three Nash equilibria: (i)* $(\lambda_T, \lambda_M) = (0,0)$; *(ii)* $(\lambda_T, \lambda_M) = (0,1)$; *and (iii)* $(\lambda_T, \lambda_M) = (1,0)$. *Moreover:*

1. $W_G^1(1,0) > W_G^1(0,1) = W_G^1(0,0)$.
2. $\left\{W_G^2(1,0), W_G^2(0,1)\right\} > W_G^2(0,0)$, *while* $W_G^2(1,0) > W_G^2(0,1)$ *if and only if* $\int_0^{\bar{z}_L}\alpha_z \ln(A_z/A_{\bar{z}_L})dz > m\left(T^1 - T^2\right)$.

Proof*:* Government 1 payoffs are given by $W_G^1(0,0) = W_G^1(0,1) = C + A^1$, $W_G^1(1,0) = C + A^1 + \int_{\bar{z}_H}^1 \alpha_z \ln(A_{\bar{z}_H}/A_z)dz$, and $W_G^1(1,1) = C + A^1 + \int_{\hat{z}_H}^1 \alpha_z \ln(A_{\hat{z}_H}/A_z)dz$. Government 2 payoffs are given $W_G^2(0,0) = C + A^2$, $W_G^2(0,1) = C + mA^1 + (1-m)A^2$, $W_G^2(1,0) = C + A^2 + \int_0^{\bar{z}_L}\alpha_z \ln\left(\frac{A_z}{A_{\bar{z}_L}}\right)dz$ and $W_G^2(1,1) = C + A^2 + \int_0^{\hat{z}_L}\alpha_z \ln\left(\frac{A_z}{A_{\hat{z}_L}}\right)dz$. (These calculations are implicitly based on Assumption 2).

$(\lambda_T, \lambda_M) = (0,0)$ is always a Nash equilibrium.

$(\lambda_T, \lambda_M) = (0,1)$ is a Nash equilibrium when $W_G^1(0,1) \geq W_G^1(0,0)$ and $W_G^2(0,1) \geq W_G^2(0,0)$. $W_G^1(0,1) = W_G^1(0,0) = C + T^1$, while $W_G^2(0,1) - W_G^2(0,0) = m\left(T^1 - T^2\right) > 0$, which is positive by assumption.

$(\lambda_T, \lambda_M) = (1,0)$ is a Nash equilibrium when $W_G^1(1,0) \geq W_G^1(0,0)$ and $W_G^2(1,0) \geq W_G^2(0,0)$. $A_z > A_{\bar{z}_L}$ for $z \in [0, \bar{z}_L)$ while $A_z < A_{\bar{z}_H}$ for $z \in (\bar{z}_H, 1]$. Then $W_G^1(1,0) - W_G^1(0,0) = \int_{\bar{z}_H}^1 \alpha_z \ln(A_{\bar{z}_H}/A_z)\, dz > 0$ and $W_G^2(1,0) - W_G^2(0,0) = \int_0^{\bar{z}_L} \alpha_z \ln(A_z/A_{\bar{z}_L})\, dz > 0$.

$(\lambda_T, \lambda_M) = (1,1)$ is not a Nash equilibrium. In order to prove this, note that $W_G^1(1,1) - W_G^1(1,0) = \int_{\hat{z}_H}^1 \alpha_z \ln(A_{\hat{z}_H}/A_z)\, dz - \int_{\bar{z}_H}^1 \alpha_z \ln(A_{\bar{z}_H}/A_z)\, dz$ and $W_G^2(1,1) - W_G^2(1,0) = \int_0^{\hat{z}_L} \alpha_z \ln(A_z/A_{\hat{z}_L})\, dz - \int_0^{\bar{z}_L} \alpha_z \ln(A_z/A_{\bar{z}_L})\, dz$. Since, in equilibrium, $A_{\bar{z}_H} = g^2 A_{\bar{z}_L}$, either $\hat{z}_L > \bar{z}_L$ and $\hat{z}_H > \bar{z}_H$ or $\hat{z}_L < \bar{z}_L$ and $\hat{z}_H < \bar{z}_H$, or $\hat{z}_L = \bar{z}_L$ and $\hat{z}_H = \bar{z}_H$. If $\hat{z}_L > \bar{z}_L$ and $\hat{z}_H > \bar{z}_H$, then $A_{\hat{z}_H} < A_{\bar{z}_H}$. Thus, $W_G^1(1,1) - W_G^1(1,0) < 0$. If $\hat{z}_L < \bar{z}_L$ and $\hat{z}_H < \bar{z}_H$, then $A_{\hat{z}_L} > A_{\bar{z}_L}$. Thus, $W_G^2(1,1) - W_G^2(1,0) < 0$. Finally, note that $\hat{z}_L = \bar{z}_L$ and $\hat{z}_H = \bar{z}_H$ only if $\Delta_{\bar{z}_L, \bar{z}_H} = \ln\left[(A_{\bar{z}_L})^{\alpha(\bar{z}_L)} (A_{\bar{z}_H})^{1-\alpha(\bar{z}_H)}\right] + \int_{\bar{z}_L}^{\bar{z}_H} \alpha_z \ln(A_z) = 0$, which we rule out by assumption.

We have already proved that $W_G^1(1,0) > W_G^1(0,1) = W_G^1(0,0)$ and $\left\{W_G^2(1,0), W_G^2(0,1)\right\} > W_G^2(0,0)$. Finally, note that $W_G^2(1,0) - W_G^2(0,1) = \int_0^{\bar{z}_L} \alpha_z \ln(A_z/A_{\bar{z}_L})\, dz - m\left(T^1 - T^2\right) > 0$ if and only if $\int_0^{\bar{z}_L} \alpha_z \ln(A_z/A_{\bar{z}_L})\, dz > m\left(T^1 - T^2\right)$, which completes the proof of the proposition. ■

Proposition 1 shows that iceberg transportation costs do not affect the political economy of trade and labor mobility. Free trade and free labor mobility are not a Nash equilibrium. All the other outcomes are Nash equilibria, but free trade and no labor mobility always dominate other equilibria for the rich country under autarky (country 1), while the same is true for the poor country under autarky (country 2) when $\int_0^{\bar{z}_L} \alpha_z \ln(A_z/A_{\bar{z}_L})\, dz > m\left(T^1 - T^2\right)$ holds, i.e., when aggregate productivity differentials between the countries are lower than the gains from trade for country 2.

A.3 Non-homothetic preferences

Consider an economy with two countries ($J = \{1, 2\}$) and a continuum of tradeable goods $Z = [0, \infty)$ indexed by z. Assume that $A_z = a_{L,z}^2 / a_{L,z}^1$ is a continuously differentiable strictly increasing function, $A_0 < 1$ and $\lim_{z\to\infty} A_z > 1$. Each agent owns 1 unit of labor and is either mobile or immobile. Goods come in discrete units and each agent can consume a unit or no unit of each good. The utility function is given by $u\left(c^j\right) = \int_0^\infty b_z c_z^j dz$, where $b_z > 0$ is the utility weight of good z and $c_z^j = 1$ if good z is consumed and $c_z^j = 0$ if it is not. Moreover, $b_z / a_{L,z}^j$ is a decreasing function of z for each j.

Under autarky, all goods must be produced domestically. Hence, if good z is produced, $p_z^j = a_{L,z}^j w^j$, where w^j is the wage rate in country j. A worker in country j tries to maximizes $u\left(c^j\right) = \int_0^\infty b_z c_z^j dz$ subject to $\int_0^\infty p_z^j c_z^j dz = w^j$. Since $b_z / a_{L,z}^j$ is decreasing, the worker selects $c_z^j = 1$ for $z \in \left[0, \tilde{v}^j\right]$ and $c_z^j = 0$ for $z \in \left(v^j, \infty\right)$, where $\tilde{v}^j$ is the unique solution of $\int_0^{\tilde{v}^j} a_{L,z}^j dz = 1$. Therefore, the indirect utility function of a worker in country j is $v^j(0,0) = \int_0^{\tilde{v}^j} b_z dz$. If labor mobility is allowed mobile workers will go to or stay in the country with the highest v^j.

Under free trade $p_z = \min\left\{a_{L,z}^1 w^1, a_{L,z}^2 w^2\right\}$. Since A_z is continuous and strictly increasing, in the trading equilibrium country 1 produces high-indexed goods $z \in [\bar{z}, \infty)$ and country 2 produces low-indexed goods $z \in [0, \bar{z}]$. The marginal industry is given by $A_{\bar{z}} = w^1 / w^2$. A worker in country j tries to maximizes $u\left(c^j\right) = \int_0^{\infty} b_z c_z^j dz$ subject to $\int_0^{\infty} p_z c_z^j dz = w^j$. Since $b_z/a_{L,z}^j$ is decreasing for each j, it must be the case that for any w^1 and w^2, b_z/p_z is decreasing in z. Hence, a worker in country j selects $c_z^j = 1$ for $z \in \left[0, \bar{v}^j\right]$ and $c_z^j = 0$ for $z \in \left(\bar{v}^j, \infty\right)$, where $\bar{v}^1$ and $\bar{v}^2$ are given by (assuming that $\bar{v}^j > \bar{z}$) $(A_{\bar{z}})^{-1} \int_0^{\bar{z}} a_{L,z}^2 dz + \int_{\bar{z}}^{\bar{v}^1} a_{L,z}^1 dz = 1 = \int_0^{\bar{z}} a_{L,z}^2 dz + A_{\bar{z}} \int_{\bar{z}}^{\bar{v}^2} a_{L,z}^1 dz$. The supply of good $z \in [0, \bar{z}]$ is $Q_z^2 = L_z^2/a_{L,z}^2$. Since $\bar{v}^j > \bar{z}$, each agent demands 1 unit of $z \in [0, \bar{z}]$, which implies that the aggregate demand of z is $C_z^1 + C_z^2 = L^1 + L^2$. Since $\int_0^{\bar{z}} L_z^2 dz = L^2$, we obtain $\int_0^{\bar{z}} a_{L,z}^2 dz = L^2/\left(L^1 + L^2\right)$. Once we have determined $\bar{z}$, relative wages are given by $w^1/w^2 = A_{\bar{z}}$, while $\bar{v}^1$ and $\bar{v}^2$ are the solutions of $\int_{\bar{z}}^{\bar{v}^1} a_{L,z}^1 dz = 1 - \left[L^2/A_{\bar{z}}\left(L^1 + L^2\right)\right]$ and $\int_{\bar{z}}^{\bar{v}^2} a_{L,z}^1 dz = L^1/A_{\bar{z}}\left(L^1 + L^2\right)$, respectively. Note that $\bar{v}^2 > \bar{z}$ because $L^1/A_{\bar{z}}\left(L^1 + L^2\right) > 0$, $\bar{v}^1 > \bar{z}$ if and only if $A_{\bar{z}} > L^2/\left(L^1 + L^2\right)$. The indirect utility of a worker is $v^j = \int_0^{\bar{v}^j} b_z dz$. If labor mobility is not allowed, then $L^j = \bar{L}^j$. If labor mobility is allowed, then mobile workers will go to or stay in the country with the highest v^j (equivalently the highest $\bar{v}^1$). If $A_{\bar{z}} > 1$, then $\bar{v}^1 > \bar{v}^2$ and, hence, mobile workers will move from country 2 to 1. As a consequence, $\bar{v}^1$ will decrease and v^2 will increase. Conversely, if $A_{\bar{z}} < 1$, then $\bar{v}^1 > \bar{v}^2$ and, hence, mobile workers will move from country 1 to 2. As a consequence, v^2 will decrease and v^1 will increase. Therefore, provided that there are enough mobile workers, they will relocate until $v^1 = v^2$, which implies $\bar{v}^1 = \bar{v}^2 = \hat{v}$ and $A_{\hat{z}} = w^1/w^2 = 1$. Once we have determined $\hat{z}$, the country-allocation of mobile workers is given by $L^2 = \left(\bar{L}^1 + \bar{L}^2\right) \int_0^{\hat{z}} a_{L,z}^2 dz$. The indirect utility of a worker is $v^1 = v^2 = \int_0^{\hat{v}} b_z dz$, where $\hat{v}$ and $\hat{z}$ are implicitly given by $\int_{\hat{z}}^{\hat{v}} a_{L,z}^1 dz = 1 - \int_0^{\hat{z}} a_{L,z}^2 dz$ and $A_{\hat{z}} = 1$. Finally, we must verify that the distribution of mobile workers between the countries does not violate $L^j \in \left[(1-m)\bar{L}^j, \bar{L}^j + m\bar{L}^{-j}\right]$. This holds when $\frac{\bar{L}^2 + m\bar{L}^1}{\bar{L}^1 + \bar{L}^2} > \int_0^{\hat{z}} a_{L,z}^2 dz > \frac{(1-m)\bar{L}^2}{\bar{L}^1 + \bar{L}^2}$, where $A_{\hat{z}} = 1$. We must also verify that $\bar{v}^1 > \bar{z}$, which holds when $A_z > \int_0^z a_{L,z}^2 dz$ for $z \in [z_L, z_H]$, where z_L and z_H are given by $\int_0^{z_L} a_{L,z}^2 dz = (1-m)\bar{L}^2/\left(\bar{L}^1 + \bar{L}^2\right)$ and $\int_0^{z_H} a_{L,z}^2 dz = \left(\bar{L}^2 + m\bar{L}^1\right)/\left(\bar{L}^1 + \bar{L}^2\right)$, respectively. Thus, we impose the following assumption.

Assumption 2 (non-homothetic preferences). *Labor mobility induces full wage convergence. Formally:* $\frac{\bar{L}^2 + m\bar{L}^1}{\bar{L}^1 + \bar{L}^2} > \int_0^{\hat{z}} a_{L,z}^2 dz > \frac{(1-m)\bar{L}^2}{\bar{L}^1 + \bar{L}^2}$, *where* $A_{\hat{z}} = 1$. *Moreover,* $A_z > \int_0^z a_{L,z}^2 dz$ *for* $z \in [z_L, z_H]$, *where* z_L *and* z_H *are give by* $\int_0^{z_L} a_{L,z}^2 dz = (1-m)\bar{L}^2/\left(\bar{L}^1 + \bar{L}^2\right)$ *and* $\int_0^{z_H} a_{L,z}^2 dz = \left(\bar{L}^2 + m\bar{L}^1\right)/\left(\bar{L}^1 + \bar{L}^2\right)$, *respectively.*

Lemma 2 characterizes the effects of trade and labor mobility on relative wages in the countries.

Lemma 2 (non-homothetic preferences). *Suppose that Assumption 2 holds and* $\tilde{v}^1 > \tilde{v}^2$, *where* $\int_0^{\tilde{v}^j} a_{L,z}^j dz = 1$. *Then:*

1. *If* $A_{\bar{z}} > 1$ *and* $\int_{\tilde{v}^1}^{\bar{v}^1} b_z dz > \int_{\tilde{v}^2}^{\bar{v}^2} b_z dz$, *then* $v^1(1,0) - v^2(1,0) > v^1(0,0) - v^2(0,0) > v^1(1,1) - v^2(1,1) = 0$.

2. *If* $A_{\bar{z}} > 1$ *and* $\int_{\tilde{v}^1}^{\bar{v}^1} b_z dz < \int_{\tilde{v}^2}^{\bar{v}^2} b_z dz$ *then,* $v^1(0,0) - v^2(0,0) > v^1(1,0) - v^2(1,0) > v^1(1,1) - v^2(1,1) = 0$.

3. *If* $A_{\bar{z}} < 1$, *then* $v^1(0,0) - v^2(0,0) > v^1(1,1) - v^2(1,1) = 0 > v^1(1,0) - v^2(1,0)$.

Proof: Under autarky, the indirect utility of a worker in country j is given by $v^j(0,0) = \int_0^{\tilde{v}^j} b_z dz$, where $\int_0^{\tilde{v}^j} a_{L,z}^j dz = 1$. Since $\tilde{v}^1 > \tilde{v}^2$ and $b_z > 0$ for all $z \in Z$, it must be the case that $v^1(0,0) - v^2(0,0) =$

$\int_{\tilde{v}^2}^{\tilde{v}^1} b_z dz > 0$. Under free trade and no labor mobility, the indirect utility of a worker in country j is given by $v^j(1,0) = \int_0^{\bar{v}^j} b_z dz$, where $\int_{\bar{z}}^{\bar{v}^1} a_{L,z}^1 dz = 1 - \left[\bar{L}^2/A_{\bar{z}}\left(\bar{L}^1 + \bar{L}^2\right)\right]$, $\int_{\bar{z}}^{\bar{v}^2} a_{L,z}^1 dz = \bar{L}^1/A_{\bar{z}}\left(\bar{L}^1 + \bar{L}^2\right)$ and $\int_0^{\bar{z}} a_{L,z}^2 dz = \bar{L}^2/\left(\bar{L}^1 + \bar{L}^2\right)$. Since $b_z > 0$ for all $z \in Z$, $v^1(1,0) - v^2(1,0) = \int_{\bar{v}^2}^{\bar{v}^1} b_z dz > 0$ if and only if $\bar{v}^1 > \bar{v}^2$. Since $\int_{\bar{v}^2}^{\bar{v}^1} a_{L,z}^1 dz = 1 - 1/A_{\bar{z}}$ and $a_{L,z}^1 > 0$ for all $z \in Z$, $\bar{v}^1 > \bar{v}^2$ if and only if $A_{\bar{z}} > 1$. We have already seen that, under free trade and free labor mobility, the indirect utility of a worker is the same in both countries, i.e., $v^1(1,1) - v^2(1,1) = 0$. Finally, $v^1(1,0) - v^2(1,0) > v^1(0,0) - v^2(0,0)$ if and only if $\int_{\bar{v}^2}^{\bar{v}^1} b_z dz > \int_{\tilde{v}^2}^{\tilde{v}^1} b_z dz$ or, which amounts to the same thing, $\int_{\tilde{v}^1}^{\bar{v}^1} b_z dz > \int_{\tilde{v}^2}^{\bar{v}^2} b_z dz$. ■

Trade and labor mobility induce full convergence because workers move from the poor country under free trade to the rich country under free trade until the level of well-being in the two countries is equalized. When $A_{\bar{z}} > 1$, under free trade, country 1 is richer than country 2 and, hence, some workers will migrate to country 2. Since we are assuming that country 1 is the rich country under autarky, $A_{\bar{z}} > 1$ implies that free trade will no alter the countries' relative positions. However, it is possible that free trade will reduce or amplify the difference between the countries' levels of well-being. Without specifying b_z, it is difficult to determine when trade will induce convergence beyond the general condition that the gains from trade in the rich country must be lower than in the poor country ($\int_{\tilde{v}^1}^{\bar{v}^1} b_z dz < \int_{\tilde{v}^2}^{\bar{v}^2} b_z dz$).[2]

Proposition 2 characterizes the political equilibrium.

Proposition 2 (non-homothetic preferences). *Suppose that Assumption 2 holds and $\tilde{v}^1 > \tilde{v}^2$, where $\int_0^{\tilde{v}^j} a_{L,z}^j dz = 1$. Then, the trade and labor mobility game has three Nash equilibrium outcomes: (i) $(\lambda_T, \lambda_M) = (0,0)$; (ii) $(\lambda_T, \lambda_M) = (0,1)$; and (iii) $(\lambda_T, \lambda_M) = (1,0)$. Moreover:*

1. $W_G^1(1,0) > W_G^1(0,1) = W_G^1(0,0)$.

2. $\left\{W_G^2(1,0), W_G^2(0,1)\right\} > W_G^2(0,0)$, *while* $W_G^2(1,0) > W_G^2(0,1)$ *if and only if* $\int_{\tilde{v}^2}^{\bar{v}^2} b_z dz > m\int_{\tilde{v}^2}^{\tilde{v}^1} b_z dz$.

Proof: Since $\tilde{v}^1 > \tilde{v}^2$ implies $v^1 = \int_0^{\tilde{v}^1} b_z dz > \int_0^{\tilde{v}^2} b_z dz = v^2$, under autarky and free labor mobility, all mobile workers go to or stay in country 1. Due to assumption 2, under free trade and free labor mobility, mobile workers relocate until $v^1 = v^2 = \int_0^{\hat{v}} b_z dz$. The payoffs of government 1 are given by: $W_G^1(0,0) = W_G^1(0,1) = \int_0^{\tilde{v}^1} b_z dz$, where $\tilde{v}^1$ is given by $\int_0^{\tilde{v}^1} a_{L,z}^1 dz = 1$; $W_G^1(1,0) = \int_0^{\bar{v}^1} b_z dz$, where $\bar{v}^1$ is given by $\frac{1}{A_{\bar{z}}}\int_0^{\bar{z}} a_{L,z}^2 dz + \int_{\bar{z}}^{\bar{v}^1} a_{L,z}^1 dz = 1$; and $W_G^1(1,1) = \int_0^{\hat{v}} b_z dz$, where $\hat{v}$ is given by $\int_{\hat{z}}^{\hat{v}} a_{L,z}^1 dz = 1 - \int_0^{\hat{z}} a_{L,z}^2 dz$. The payoffs of government 2 are given by: $W_G^2(0,0) = \int_0^{\tilde{v}^2} b_z dz$, where $\tilde{v}^2$ is given by $\int_0^{\tilde{v}^2} a_{L,z}^2 dz = 1$; $W_G^2(0,1) = m\int_0^{\tilde{v}^1} b_z dz + (1-m)\int_0^{\tilde{v}^2} b_z dz$; $W_G^2(1,0) = \int_0^{\bar{v}^2} b_z dz$, where $\bar{v}^2$ is given by $\int_0^{\bar{z}} a_{L,z}^2 dz + A_{\bar{z}}\int_{\bar{z}}^{\bar{v}^2} a_{L,z}^1 dz = 1$; and $W_G^1(1,1) = \int_0^{\hat{v}} b_z dz$.

$(\lambda_T, \lambda_M) = (0,0)$ is a always a Nash equilibrium outcome.

[2] For example, if $b_z = 1/z$, then there will be convergence whenever $\bar{v}^1/\tilde{v}^1 < \bar{v}^2/\tilde{v}^2$, i.e., if free trade leads to a higher percentage increase in the range of goods consumed by the poor country than it does in the rich country. If we also specify $a_{L,z}^1$ and $a_{L,z}^2$, we can characterize the conditions for convergence in greater detail. Let $a_{L,z}^2 = 1$ for all z, $a_{L,z}^1 = \gamma/e^z$ with $1 < \gamma < e/(e-1)$. Then $A_z = e^z/\gamma$, $\tilde{v}^1 = \ln[\gamma/(\gamma-1)]$, $\tilde{v}^2 = 1$, $\bar{z} = \bar{L}^2/\left(\bar{L}^1 + \bar{L}^2\right)$, $\bar{v}^1 = \ln\left[\gamma e^{\bar{z}}/\left(\gamma(1+\bar{z}) - e^{\bar{z}}\right)\right]$, $\bar{v}^2 = \ln\left[e^{\bar{z}}/\bar{z}\right]$, $\hat{z} = \ln(\gamma)$, $\hat{v} = \ln(\gamma/\hat{z})$. Thus, free trade will induce convergence in real wages if and only if $\gamma < e^{\bar{z}}$ and $\ln\left[\gamma e^{\bar{z}}/\left(\gamma(1+\bar{z}) - e^{\bar{z}}\right)\right] < \ln[\gamma/(\gamma-1)]\ln\left[e^{\bar{z}}/\bar{z}\right](\gamma) < z_H$ and $z < e^z/\gamma < 1 + z$ for $z \in [z_L, z_H]$, where $z_L = (1-m)\bar{L}/\left(\bar{L}^1 + \bar{L}^2\right)$ and $z_H = \left(\bar{L}^2 + m\bar{L}^1\right)/\left(\bar{L}^1 + \bar{L}^2\right)$.

$(\lambda_T, \lambda_M) = (0,1)$ is a Nash equilibrium outcome when $W_G^j(0,1) \geq W_G^j(0,0)$ for $j = 1,2$. $W_G^1(0,1) = W_G^1(0,0)$, while $W_G^2(0,1) - W_G^2(0,0) = m\int_{\tilde{v}^2}^{\tilde{v}^1} b_z dz > 0$ because $b_z > 0$ for all z and $\tilde{v}^1 > \tilde{v}^2$.

$(\lambda_T, \lambda_M) = (1,0)$ is a Nash equilibrium outcome when $W_G^j(1,0) \geq W_G^j(0,0)$ for $j = 1,2$. The indirect utility of a worker in country 1 under autarky is $v^1(0,0) = \int_0^{\tilde{v}^1} b_z dz$, where $\tilde{v}^1$ is given by $\int_0^{\tilde{v}^1} a_{L,z}^1 dz = 1$. The indirect utility of a worker in country 1 under free trade and no labor mobility is $v^1(1,0) = \int_0^{\bar{v}^1} b_z dz$, where $\bar{v}^1$ is given by $(A_{\bar{z}})^{-1} \int_0^{\bar{z}} a_{L,z}^2 dz + \int_{\bar{z}}^{\bar{v}^1} a_{L,z}^1 dz = 1$. Therefore, $(A_{\bar{z}})^{-1} \int_0^{\bar{z}} a_{L,z}^2 dz + \int_{\bar{z}}^{\bar{v}^1} a_{L,z}^1 dz = \int_0^{\tilde{v}^1} a_{L,z}^1 dz$. This implies $\int_0^{\bar{z}} a_{L,z}^1 (A_z/A_{\bar{z}})\, dz + \int_{\bar{z}}^{\bar{v}^1} a_{L,z}^1 dz = \int_0^{\tilde{v}^1} a_{L,z}^1 dz$, which, after some simple algebra, implies $\int_{\tilde{v}^1}^{\bar{v}^1} a_{L,z}^1 dz = \int_0^{\bar{z}} a_{L,z}^1 [1 - (A_z/A_{\bar{z}})]\, dz$. Since $A_z < A_{\bar{z}}$ for $z \in [0, \bar{z})$ and $a_{L,z}^1 > 0$ for all z, it must be the case that $\int_0^{\bar{z}} a_{L,z}^1 [1 - (A_z/A_{\bar{z}})]\, dz > 0$. Since $a_{L,z}^1 > 0$ for all z, $\int_{\tilde{v}^1}^{\bar{v}^1} a_{L,z}^1 dz > 0$ implies $\bar{v}^1 > \tilde{v}^1$. Finally, since $b_z > 0$ for all z, $\bar{v}^1 > \tilde{v}^1$ implies $W_G^1(1,0) - W_G^1(0,0) = \int_{\tilde{v}^1}^{\bar{v}^1} b_z dz > 0$. The indirect utility of a worker in country 2 under autarky is $v^2(0,0) = \int_0^{\tilde{v}^2} b_z dz$, where $\tilde{v}^2$ is given by $\int_0^{\tilde{v}^2} a_{L,z}^2 dz = 1$. The indirect utility of a worker in country 2 under free trade and no labor mobility is $v^2(1,0) = \int_0^{\bar{v}^2} b_z dz$, where $\bar{v}^2$ is given by $\int_0^{\bar{z}} a_{L,z}^2 dz + A_{\bar{z}} \int_{\bar{z}}^{\bar{v}^2} a_{L,z}^1 dz = 1$. Therefore, $\int_0^{\bar{z}} a_{L,z}^2 dz + A_{\bar{z}} \int_{\bar{z}}^{\bar{v}^2} a_{L,z}^1 dz = \int_0^{\tilde{v}^2} a_{L,z}^2 dz$. This implies $\int_0^{\bar{z}} a_{L,z}^2 dz + \int_{\bar{z}}^{\bar{v}^2} (A_{\bar{z}}/A_z)\, a_{L,z}^2 dz = \int_0^{\tilde{v}^2} a_{L,z}^2 dz$, which, after some simple algebra, implies $\int_{\bar{z}}^{\bar{v}^2} (A_{\bar{z}}/A_z)\, a_{L,z}^2 dz = \int_{\bar{z}}^{\tilde{v}^2} a_{L,z}^2 dz$. Since $A_z > A_{\bar{z}} > 0$ for $z > \bar{z}$ and $a_{L,z}^2 > 0$ for all z, it must be the case that $\bar{v}^2 > \tilde{v}^2$. Finally, since $b_z > 0$ for all z, $\bar{v}^2 > \tilde{v}^2$ implies $W_G^2(1,0) - W_G^2(0,0) = \int_{\tilde{v}^2}^{\bar{v}^2} b_z dz > 0$.

$(\lambda_T, \lambda_M) = (1,1)$ is not a Nash equilibrium outcome. Given L^1 and L^2, the equilibrium $(\bar{v}^1, \bar{v}^2, \bar{z})$ is determined by $\int_{\bar{z}}^{\bar{v}^1} a_{L,z}^1 dz = 1 - L^2/A_{\bar{z}} (L^1 + L^2)$, $\int_{\bar{z}}^{\bar{v}^2} a_{L,z}^1 dz = L^1/A_{\bar{z}} (L^1 + L^2)$, and $\int_0^{\bar{z}} a_{L,z}^2 dz = L^2/(L^1 + L^2)$. Differentiating these expressions with respect to $L^1/(L^1 + L^2)$, we obtain

$$\frac{\partial \bar{v}^1}{\partial [L^1/(L^1 + L^2)]} = \frac{-A'_{\bar{z}} L^2}{(A_{\bar{z}})^2 (L^1 + L^2)\, a_{L,\bar{v}^1}^1 a_{L,\bar{z}}^2} < 0$$
$$\frac{\partial \bar{v}^2}{\partial [L^1/(L^1 + L^2)]} = \frac{A'_{\bar{z}} L^1}{(A_{\bar{z}})^2 (L^1 + L^2)\, a_{L,\bar{v}^2}^1 a_{L,\bar{z}}^2} > 0$$

Thus, as the proportion of the labor force that decides to relocate to country 1 increases (decreases), $\bar{v}^1$ decreases (increases) and $\bar{v}^2$ increases (decreases). We have already shown that $\bar{v}^1 > \bar{v}^2$ if and only if $A_{\bar{z}} > 1$. Therefore, when $A_{\bar{z}} > 1$, if there is free labor mobility, $L^1 > \bar{L}^1$ and $L^2 < \bar{L}^2$. Then, $\bar{v}^1 < \hat{v} < \bar{v}^2$, which implies $W_G^1(1,0) - W_G^1(1,1) = \int_{\hat{v}}^{\bar{v}^1} b_z dz > 0$ and $W_G^2(1,1) - W_G^2(1,0) = \int_{\bar{v}^2}^{\hat{v}} b_z dz > 0$. Conversely, when $A_{\bar{z}} < 1$, if there is free labor mobility, $L^1 < \bar{L}^1$ and $L^2 > \bar{L}^2$. Then, $\bar{v}^2 < \hat{v} < \bar{v}^1$, which implies $W_G^1(1,1) - W_G^1(1,0) = \int_{\bar{v}^1}^{\hat{v}} b_z dz > 0$ and $W_G^2(1,0) - W_G^2(1,1) = \int_{\hat{v}}^{\bar{v}^2} b_z dz > 0$. Thus, provided that $A_{\bar{z}} \neq 1$, free trade and free labor mobility will never be a Nash equilibrium outcome.

We have already shown that $W_G^1(1,0) > W_G^1(0,0) = W_G^1(0,1)$ and $W_G^2(1,0) > W_G^1(0,0)$ and $W_G^2(0,1) > W_G^2(0,0)$. The last step is to compare $W_G^2(1,0) = \int_0^{\bar{v}^2} b_z dz$ and $W_G^2(0,1) = m\int_0^{\tilde{v}^1} b_z dz + (1-m)\int_0^{\tilde{v}^2} b_z dz$. $W_G^2(1,0) > W_G^2(0,0)$ if and only if $\int_{\tilde{v}^2}^{\bar{v}^2} b_z dz > m\int_{\tilde{v}^2}^{\tilde{v}^1} b_z dz$. This completes the proof of the proposition. ■

A.4 Multiple countries

Consider an economy with a finite set of countries J countries, indexed by $j = 1, ..., J$, and a continuum of tradeable goods, $Z_T = [0,1]$ indexed by z. Assume there are no geographic barriers that limit the mobility of tradeable goods. Let $\bar{L}^j$ and L^j be the labor endowment and the labor force of country j, respectively. Only a fraction $m \in [0,1]$ of $\bar{L}^j$ is mobile. Preferences are given by $u\left(c^j\right) = \left[\int_0^1 \left(c_z^j\right)^\rho dz\right]^{\frac{1}{\rho}}$, with $\sigma = (1-\rho)^{-1} > 1$, for all agents in every countries. Let $a_{L,z}^j$ be the unit labor requirement of good z in country j. The cumulative distribution function of $a_{L,z}^j$ is given by $\Pr\left(a_{L,z}^j \leq a\right) = 1 - e^{-T^j a^\theta}$, where $T^j > 0$ and $\theta > \sigma - 1$. These distributions are independent across goods and countries.

Given $a_{L,z}^j$, if good z is produced in country j, its price will be $p_z^j = a_{L,z}^j w^j$, where w^j is the wage rate in country j. Thus, if good z is produced in country j, its price distribution will be $G_z^j(p) = \Pr\left(p_z^j \leq p\right) = 1 - e^{-T^j\left(\frac{p}{w^j}\right)^\theta}$. Under autarky, all goods can only be produced domestically and, hence, the price distribution of good z in country j is $G_z^j(p) = 1 - e^{-T^j\left(\frac{p}{w^j}\right)^\theta}$. For the CES utility function, the exact price index is given by $P^j = \left[\int_0^1 \left(p_z^j\right)^\rho dz\right]^{\frac{1}{\rho}} = \left[\int_0^\infty (p)^\rho dG_z^j(p)\right]^{\frac{1}{\rho}} = \gamma\left(T^j\right)^{-\frac{1}{\theta}} w^j$, where $\gamma = \left[\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)\right]^{\frac{1}{1-\sigma}}$, Γ is the Gamma function and $1 + \theta > \sigma$. Hence, the real wage rate in country j is $w^j/P^j = \left(T^j\right)^{\frac{1}{\theta}}/\gamma$ or, equivalently, the indirect utility function of a worker in country j is $v^j = -\ln(\gamma) + \frac{1}{\theta}\ln\left(T^j\right)$. As a consequence, if migration is allowed, all mobile workers will go to or stay in the country with the highest T^j.

Under complete free trade, consumers will buy from the less expensive producer. The lowest price is lower than p unless each country is selling at a higher price. Therefore, the price-distribution that consumers actually face for good z is $G_z(p) = 1 - \prod_{j=1}^J \left[1 - G_z^j(p)\right] = 1 - e^{-\sum_{j=1}^J T^j\left(\frac{p}{w^j}\right)^\theta}$. The probability that country j is the lowest cost supplier of good z is $q_z^j = \int_0^\infty \prod_{i\neq j}\left[1 - G_z^i(p)\right] dG_z(p) = T^j\left(\frac{\gamma w^j}{P}\right)^{-\theta}$, where $P = \left[\int_0^1 (p_z)^\rho dz\right]^{\frac{1}{\rho}} = \left[\int_0^\infty (p)^\rho dG_z(p)\right]^{\frac{1}{\rho}} = \gamma\left(\sum_{j=1}^J T^j\left(w^j\right)^{-\theta}\right)^{-\frac{1}{\theta}}$ is the exact price index. Since there is a continuum of goods, q_z^j is also the fraction of goods that each country buys from j. Thus, the balanced trade conditions are given by $w^j L^j = q_z^j \sum_{i=1}^J w^i L^i$ for $j = 1, ..., J$. Solving these equations, we obtain $w^k/w^j = \left(\frac{T^k}{L^k}/\frac{T^j}{L^j}\right)^{\frac{1}{1+\theta}}$. Hence, the real wage rate in country j is $w^j/P = (1/\gamma)\left(T^j/L^j\right)^{\frac{1}{1+\theta}}\left[\sum_{i=1}^J \left(T^i\right)^{\frac{1}{1+\theta}}\left(L^i\right)^{\frac{\theta}{1+\theta}}\right]^{\frac{1}{\theta}}$ or, equivalently, the indirect utility of a worker in country j is $v^j = -\ln(\gamma) + \frac{1}{1+\theta}\ln\left(\frac{T^j}{L^j}\right) + \frac{1}{\theta}\ln\left[\sum_{i\in J}\left(T^i\right)^{\frac{1}{1+\theta}}\left(L^i\right)^{\frac{\theta}{1+\theta}}\right]$. If migration is not allowed, then $L^j = \bar{L}^j$ for all $j \in J$. If migration is allowed and enough workers are mobile, the equilibrium allocation of workers will be $L^j = \left(T^j/\sum_{i\in J}T^i\right)\left(\sum_{i\in J}\bar{L}^i\right)$. Then, regardless of the location of a given worker, the indirect utility of that worker under free trade and free labor mobility will be given by $v^j = -\ln(\gamma) + \frac{1}{\theta}\ln\left(\sum_{i\in J}T^i\right)$. Finally, we must verify that the distribution of mobile workers across countries satisfies $L^j \geq (1-m)\bar{L}^j$ for all $j \in J$, i.e., we need to impose the following assumption.

Assumption 3 (multiple countries). *Complete labor mobility induces full wage convergence. Formally:* $\min_{i\in J}\left\{T^i/(1-m)\bar{L}^i\right\} \geq \sum_{i\in J}T^i/\sum_{i\in J}\bar{L}^i$.

Lemma 3 characterizes the effects of trade and labor mobility on the relative wages.

Lemma 3 (multiple countries). *Suppose that Assumption 3 holds and* $T^j > T^k$. *Then:*

1. *If* $\ln\left(\bar{L}^j/\bar{L}^k\right) < (\theta)^{-1}\ln\left(T^k/T^j\right)$, *then trade induces divergence in real wages between countries j and k, but trade and labor mobility undo the divergence, inducing complete convergence. Formally:* $v^j(1,0) - v^k(1,0) > v^j(0,0) - v^k(0,0) > v^j(1,1) - v^k(1,1) = 0$.

2. *If* $(\theta)^{-1}\ln\left(T^k/T^j\right) < \ln\left(\bar{L}^j/\bar{L}^k\right) < \ln\left(T^j/T^k\right)$, *then trade induces convergence in real wages between countries j and k and trade and labor mobility induce complete convergence. Formally:* $v^j(0,0) - v^k(0,0) > v^j(1,0) - v^k(1,0) > v^j(1,1) - v^k(1,1) = 0$.

3. *If* $\ln\left(\bar{L}^j/\bar{L}^k\right) > \ln\left(T^j/T^k\right)$, *then trade induces a reversal of fortune between countries j and k but trade and labor mobility undo this reversal, inducing complete convergence in real wages. Formally:* $v^j(0,0) - v^k(0,0) > v^j(1,1) - v^k(1,1) = 0 > v^j(1,0) - v^k(1,0)$.

Proof: Under autarky, regardless of mobile workers' location decisions, we have $v^j(0,\mu) = -\ln(\gamma) + (1/\theta)\ln T^j$. Thus, $v^j(0,0) - v^k(0,0) = \theta\ln\left(T^j/T^k\right)$, which is positive by assumption. Under free trade, if there is no labor mobility, $v^j(1,0) = -\ln(\gamma) + (1+\theta)^{-1}\ln\left(T^j/\bar{L}^j\right) + (1/\theta)\ln\left[\sum_{i\in J}\left(T^i\right)^{\frac{1}{1+\theta}}\left(\bar{L}^i\right)^{\frac{\theta}{1+\theta}}\right]$. Hence, $v^j(1,0) - v^k(1,0) = (1+\theta)^{-1}\left[\ln\left(T^j/L^j\right) - \ln\left(T^k/L^k\right)\right]$. If there is free labor mobility and Assumption 3 holds, $v^j(1,1) = -\ln(\gamma) + (1/\theta)\ln\left(\sum_{i\in J}T^i\right)$. Hence, $v^j(1,1) - v^k(1,1) = 0$. Simple comparisons complete the proof of the lemma. ■

Under free trade, labor mobility induces full convergence in real wages in all of the countries in the world. The reason is that mobile workers migrate from the relatively poor countries under free trade (those with low $T^i/\bar{L}^i$) to the relatively rich countries under free trade (those with high $T^i/\bar{L}^i$) until real wages are the same in all locations. There is only one implicit condition behind this result: There must be enough mobile workers in poor countries so that migration from poor to rich countries is sufficient to fully equalize T^i/L^i in all countries. Assumption 3 ensures that this is the case even for the poorest country in the world (lowest $T^i/\bar{L}^i$). Formally, Assumption 3 implies that, if all the mobile workers in the poorest country in the world decide to migrate to other countries, the real wage of a worker in that country will be higher than in the rest of world. Trade alone has an ambiguous effect on convergence. While, under autarky, differences in real wages depend only on relative levels of productivity (real wages are higher in country j than in country k when $T^j > T^k$), under free trade, they also depend on relative labor abundance (real wages are higher in country j than in country k when $T^j/\bar{L}^j > T^k/\bar{L}^k$). Since it is perfectly possible that $T^j > T^k$, but $T^j/\bar{L}^j < T^k/\bar{L}^k$, trade can lead to a reversal of fortune if the country with a higher aggregate level of productivity is relatively labor-abundant.

So far, we have considered polar cases, i.e., "free trade" means that all countries are allowing free trade and "autarky" means that there is no trade at all. The same is true of our consideration of labor mobility. However, it is possible that countries are trading only with some countries. Similarly, countries may accept labor mobility only with a restricted group of countries. Proposition 3 characterizes the political equilibrium when each country can decide to trade or not and can decide to allow labor mobility or not with each other country. The only restriction that we impose is that if country i accepts free trade (labor mobility) with country j and country j accepts free trade (labor mobility) with country k, then country i must accept free trade (labor mobility) with country k.

Proposition 3 (multiple countries). *Suppose that assumption 3 holds and assume that $T^j \neq T^k$ and $T^j/\bar{L}^j \neq T^k/\bar{L}^k$ for all $j,k \in J$ and $j \neq k$. Then:*

1. *No trade and any pattern of labor mobility policy is a Nash equilibrium outcome. Moreover, among those equilibria, no trade and complete free labor mobility prevail over the other equilibrium outcomes for all countries.*

2. *No labor mobility and any pattern of trade policy is a Nash equilibrium outcome. Moreover, among those equilibria, no labor mobility and complete free trade prevail over the other equilibrium outcomes for all countries.*

3. $W_G^j(1,0) > W_G^j(0,1)$ *if and only if:*

$$\ln\left[\frac{\sum_{i\in J}\left(T^i\right)^{\frac{1}{1+\theta}}\left(\bar{L}^i\right)^{\frac{\theta}{1+\theta}}}{\left(T^j\right)^{\frac{1}{1+\theta}}\left(\bar{L}^j\right)^{\frac{\theta}{1+\theta}}}\right] > m\ln\left(\frac{\max_{i\in J}\left\{T^i\right\}}{T^j}\right)$$

   In other words, if the above condition holds, for country complete free trade and no labor mobility are better than complete free factor mobility and no trade.

4. *Any pattern of trade policy other than complete autarky and any pattern of labor mobility policy within the countries that trade with each other other than no labor mobility are not a Nash equilibrium outcome. In particular, complete free trade and any pattern of labor mobility policy other than no mobility are not a Nash equilibrium outcome.*

Proof: No trade and any pattern of labor mobility policy is a Nash equilibrium outcome. Suppose that no country allows free trade and consider any partition $\mathbf{J}_M$ of the set of countries J. Each element of $\mathbf{J}_M$ is a set of countries that allow factor mobility among them, but do not allow it with the rest of world. Arbitrarily select $j \in J_M \in \mathbf{J}_M$. Since all countries are blocking trade, there is no unilateral move by j that can induce trade. Since countries in $J - J_M$ do not accept labor mobility with countries in J_M, there is no unilateral move by j that can induce labor mobility outside J_M. Therefore, the only relevant decision for j is about labor mobility with countries in J_M. In fact, given that mobile workers can move from one country to another at no cost, j must decide between accepting free labor mobility with all countries in J_M or with none of them. Regardless of j's decision, immobile workers in country j get $v^j = -\ln\gamma + (1/\theta)\ln T^j$. If j does not allow labor mobility, then mobile workers in j also get $v^j = -\ln\gamma + (1/\theta)\ln T^j$. Conversely, if j allows labor mobility, mobile workers in j will relocate in the richest country in J_M. Thus, they will get $v^j = -\ln\gamma + (1/\theta)\max_{i\in J_M}\left\{\ln T^i\right\}$. Since $W_G^j = mv^{j,m} + (1-m)\,v^{j,im}$, it follows that W_G^j when j allows labor mobility will be higher than or equal to W_G^j when j blocks labor mobility. Finally, since $j \in J_M \in \mathbf{J}_M$ were also selected arbitrarily, the same analysis applies to any $j \in J_M \in \mathbf{J}_M$, which completes the proof that no trade and any pattern of labor mobility are a Nash equilibrium outcome. Finally, note that $W_G^j = -\ln\gamma + (m/\theta)\max_{i\in J_M}\left\{\ln T^i\right\} + \left[(1-m)/\theta\right]\ln T^j$ is nondecreasing in J_M. Indeed, W_G^j is strictly increasing in J_M for all countries except the one with the highest T^j, for which $\max_{i\in J_M}\left\{\ln T^i\right\}$ does not change with J_M. Thus, under complete autarky, the pattern of labor mobility that maximizes W_G^j for all $j \in J$ is $J_M = J$. This completes the proof of part 1.

No labor mobility and any pattern of trade policies is a Nash equilibrium outcome. Suppose that no country allows migration and consider any partition $\mathbf{J}_T$ of the set of countries J. Each element of $\mathbf{J}_T$ is a set of countries that allows free trade within that group, but does not allow it with the rest of the world. Arbitrarily select $j \in J_T \in \mathbf{J}_T$. Since no country is offering free labor mobility, there is nothing that country j can do to bring about labor mobility. Suppose that country j decides to trade only with a subset $\tilde{J}_T \subset J_T$. Then, $W_G^j = v^j = -\ln(\gamma) + (1+\theta)^{-1}\ln\left(T^j/\bar{L}^j\right) + (1/\theta)\ln\left[\sum_{i\in\tilde{J}_T}\left(T^i\right)^{\frac{1}{1+\theta}}\left(\bar{L}^i\right)^{\frac{\theta}{1+\theta}}\right]$. Since W_G^j increases with the number of countries in J_T, the best response for country j is to set $\tilde{J}_T = J_T$. Since we have picked an arbitrary country, the same analysis applies to all $j \in J_T \in \mathbf{J}_T$. Therefore, complete free trade and any pattern of labor mobility policy are a Nash equilibrium outcome. Finally, note that $W_G^j = -\ln\gamma + (1+\theta)^{-1}\ln\left(T^j/\bar{L}^j\right) + (1/\theta)\ln\left[\sum_{i\in J_T}\left(T^i\right)^{\frac{1}{1+\theta}}\left(\bar{L}^i\right)^{\frac{\theta}{1+\theta}}\right]$ is increasing in J_T. Thus, when there is no labor mobility, the pattern of trade that maximizes W_G^j for all $j \in J$ is $J_T = J$. This completes the proof of part 2.

The payoff of the government of country j under autarky and free factor mobility is given by $W_G^j(0,1) = -\ln\gamma + (m/\theta)\max_{i\in J}\left\{\ln T^i\right\} + \left[(1-m)/\theta\right]\ln T^j$, while, under complete free trade and no labor mobility, it is $W_G^j(1,0) = -\ln\gamma + (1+\theta)^{-1}\ln\left(T^j/\bar{L}^j\right) + \frac{1}{\theta}\ln\left[\sum_{i\in J_T}\left(T^i\right)^{\frac{1}{1+\theta}}\left(\bar{L}^i\right)^{\frac{\theta}{1+\theta}}\right]$. Therefore, $W_G^j(1,0) > W_G^j(0,1)$ if and only if $\ln\left[\frac{\sum_{i\in J}\left(A^i\right)^{\frac{1}{1+\theta}}\left(\bar{L}^i\right)^{\frac{\theta}{1+\theta}}}{\left(A^j\right)^{\frac{1}{1+\theta}}\left(\bar{L}^j\right)^{\frac{\theta}{1+\theta}}}\right] > m\ln\left(\frac{\max_{i\in J}\left\{A^i\right\}}{A^j}\right)$. This completes the proof of part 3.

Free trade and any pattern of labor mobility other than no mobility is not a Nash equilibrium outcome. Assume that all countries are allowing free trade and consider any arbitrary labor mobility partition $\mathbf{J}_M$ for which at least one set J_M has more than one country. Among the countries in J_M, let j be the one with the highest $T^j/\bar{L}^j$ ratio. Next, we show that, if country j blocks labor mobility with countries in J_M, then W_G^j increases. The utility of a worker in country $j \in J_M \in \mathbf{J}_M$ is given by:

$$v^j = -\ln(\gamma) + \frac{1}{1+\theta}\ln\left(\frac{T^j}{L^j}\right) + \frac{1}{\theta}\ln\left[\sum_{i\in J_M}\left(T^i\right)^{\frac{1}{1+\theta}}\left(L^i\right)^{\frac{\theta}{1+\theta}} + \sum_{i\in J-J_M}\left(T^i\right)^{\frac{1}{1+\theta}}\left(L^i\right)^{\frac{\theta}{1+\theta}}\right],$$

where L^i is the labor force of country i. Let us reorder the countries in J_M in such a way that $T^1/\bar{L}^1 > T^2/\bar{L}^2 > ... > T^{n_M}/\bar{L}^{n_M}$, where $n_M = \#J_M \geq 2$ is the number of countries in J_M. Mobile worker will go to or stay in the country with the highest T^j/L^j. Thus, mobile workers in J_M will first relocate to country 1 until $T^1/L^1 = T^2/(1-m)\bar{L}^2$. Then, they will relocate to countries 1 and 2 until $T^1/L^1 = T^2/L^2 = T^3/(1-m)\bar{L}^3$. The relocation of mobile workers will end when (a) $T^1/L^1 = ... = T^l/L^l > T^{l+1}/(1-m)\bar{L}^2 > ... > T^{n_M}/(1-m)\bar{L}^{n_M}$ and $\sum_{i=1}^{l}L^i = \sum_{i=1}^{l}\bar{L}^i + m\sum_{i=l+1}^{n_M}\bar{L}^i$; (b) $T^1/L^1 = T^2/L^2 = ... = T^{n_M}/L^{n_M}$ and $\sum_{i=1}^{n_M}L^i = \sum_{i=1}^{n_M}\bar{L}^i$. In case (a), we obtain $L^j = \left(T^j/\sum_{i=1}^{l}T^i\right)\left(\sum_{i=1}^{l}\bar{L}^i + m\sum_{i=l+1}^{n_M}\bar{L}^i\right)$ for $j = 1, ..., l$ and $L^j = (1-m)\bar{L}^j$ for $j = l+1, ..., n_M$. In order for this to be the equilibrium allocation of mobile workers we must verify that $L^j \geq (1-m)\bar{L}^j$ for $j = 1, ..., n_M$ and $T^1/L^1 = ... = T^l/L^l > T^{l+1}/(1-m)\bar{L}^{l+1}$, which holds if and only if $T^l/(1-m)\bar{L}^l \geq \sum_{i=1}^{l}T^i/\left(\sum_{i=1}^{l}\bar{L}^i + m\sum_{i=l+1}^{n_M}\bar{L}^i\right) > T^{l+1}/(1-m)\bar{L}^{l+1}$. We must also check that there is not another $l' < l$ for which the above condition holds. In case (b), we obtain $L^j = \left(T^j/\sum_{i=1}^{n_M}T^i\right)\left(\sum_{i=1}^{n_M}\bar{L}^i\right)$ for $j = 1, ..., n_M$. In order for this to be the equilibrium allocation of mobile workers we must verify that

$L^j \geq (1-m)\,\bar{L}^j$ for $j=1,...,n_M$, which holds if and only if $T^{n_M}/\,(1-m)\,\bar{L}^{n_M} \geq \sum_{i=1}^{n_M} T^i / \sum_{i=1}^{n_M} \bar{L}^i$. Summing up, the equilibrium allocation of mobile workers is $L^j = \left(T^j/\sum_{i=1}^{l} T^i\right)\left(\sum_{i=1}^{l} \bar{L}^i + m\sum_{i=l+1}^{n_M} \bar{L}^i\right)$ for $j=1,...,l\,(J_M)$ and $L^j = (1-m)\,\bar{L}^j$ for $j = l\,(J_M)+1,...,n_M$, where $l\,(J_M)$ is given by the first time that $T^l/\,(1-m)\,\bar{L}^l \geq \sum_{i=1}^{l} T^i/\left(\sum_{i=1}^{l} \bar{L}^i + m\sum_{i=l+1}^{n_M} \bar{L}^i\right) > T^{l+1}/\,(1-m)\,\bar{L}^{l+1}$ holds .

Result 1: $l\,(J_M - \{1\}) \geq l\,(J_M)$. Proof: It suffices to show that $\sum_{i=1}^{l} T^i/\left(\sum_{i=1}^{l} \bar{L}^i + m\sum_{i=l+1}^{n_M} \bar{L}^i\right) > \sum_{i=2}^{l} T^i/\left(\sum_{i=2}^{l} \bar{L}^i + m\sum_{i=l+2}^{n_M} \bar{L}^i\right)$ for each l.

$$\begin{aligned}
\frac{T^1}{\bar{L}^1} > \frac{\sum_{i=2}^{l} T^i}{\sum_{i=2}^{l} \bar{L}^i} \Rightarrow \frac{T^1}{\bar{L}^1} &> \frac{\sum_{i=2}^{l} T^i}{\sum_{i=2}^{l} \bar{L}^i + m\sum_{i=l+2}^{n_M} \bar{L}^i} \\
\Leftrightarrow \frac{T^1 + \sum_{i=2}^{l} T^i}{\bar{L}^1 + \sum_{i=2}^{l} \bar{L}^i + m\sum_{i=l+2}^{n_M} \bar{L}^i} &> \frac{\sum_{i=2}^{l} T^i}{\sum_{i=2}^{l} \bar{L}^i + m\sum_{i=l+2}^{n_M} \bar{L}^i} \\
\Leftrightarrow \frac{\sum_{i=1}^{l} T^i}{\sum_{i=1}^{l} \bar{L}^i + m\sum_{i=l+1}^{n_M} \bar{L}^i} &> \frac{\sum_{i=2}^{l} T^i}{\sum_{i=2}^{l} \bar{L}^i + m\sum_{i=l+1}^{n_M} \bar{L}^i}
\end{aligned}$$

The first line relies on the fact that we have reordered countries in such a way that $T^1/\bar{L}^1 > T^2/\bar{L}^2 > ... > T^{n_M}/\bar{L}^{n_M}$. The other lines are simple algebra.

Result 2: $\bar{L}^1 < L^1\,(J_M)$ and $L^j\,(J_M - \{1\}) \geq L^j\,(J_M)$ for $j = 2,...,n_M$. Proof:

$$\begin{aligned}
\frac{T^1}{\bar{L}^1} > \frac{\sum_{i=1}^{l} T^i}{\sum_{i=1}^{l} \bar{L}^i} \Rightarrow \frac{T^1}{\bar{L}^1} &> \frac{\sum_{i=1}^{l} T^i}{\sum_{i=1}^{l} \bar{L}^i + m\sum_{i=l+1}^{n_M} \bar{L}^i} \\
&\Rightarrow \left(\frac{T^1}{\sum_{i=1}^{l} T^i}\right)\left(\sum_{i=1}^{l} \bar{L}^i + m\sum_{i=l+1}^{n_M} \bar{L}^i\right) > \bar{L}^1 \\
&\Rightarrow L^1\,(J_M) > \bar{L}^1
\end{aligned}$$

The first line relies on the fact that we have reordered countries in such a way that $T^1/\bar{L}^1 > \frac{A^2}{\bar{L}^2} > ... > \frac{A^{n_M}}{\bar{L}^{n_M}}$. The last line introduces the equilibrium value of $L^1\,(J_M)$.

$$\begin{aligned}
\frac{T^1}{\bar{L}^1} > \frac{\sum_{i=2}^{l} T^i}{\sum_{i=2}^{l} \bar{L}^i} \Rightarrow \frac{T^1}{\bar{L}^1} &> \frac{\sum_{i=2}^{l} T^i}{\sum_{i=2}^{l} \bar{L}^i + m\sum_{i=l+1}^{n_M} \bar{L}^i} \\
&\Rightarrow \left(\frac{T^j}{\sum_{i=2}^{l} T^i}\right)\left(\sum_{i=2}^{l} \bar{L}^i + m\sum_{i=l+1}^{n_M} \bar{L}^i\right) > \left(\frac{T^j}{\sum_{i=1}^{l} T^i}\right)\left(\sum_{i=1}^{l} \bar{L}^i + m\sum_{i=l+1}^{n_M} \bar{L}^i\right) \\
&\Rightarrow L^j\,(J_M - \{1\}) \geq L^j\,(J_M) \text{ for } j \leq l\,(J_M - \{j\})
\end{aligned}$$

The first line relies on the fact that we have reordered countries in such a way that $T^1/\bar{L}^1 > T^2/\bar{L}^2 > ... > T^{n_M}/\bar{L}^{n_M}$. The last line introduces the equilibrium values of $L^j\,(J_M - \{1\})$ and $L^j\,(J_M)$ for $j \leq l\,(J_M - \{j\})$. Finally, for $l > l\,(J_M - \{j\})$, we have $L^j\,(J_M) = (1-m)\,\bar{L}^j$. Thus, it must be the case that $L^j\,(J_M - \{1\}) \geq (1-m)\,\bar{L}^j$ for $l > l\,(J_M - \{j\})$.

Result 3: v^j is decreasing in L^j and increasing in L^i. Proof: v^j is given by:

$$v^j = -\ln(\gamma) + \frac{1}{1+\theta}\ln\left(\frac{T^j}{L^j}\right) + \frac{1}{\theta}\ln\left[\sum_{i\in J_M}\left(T^i\right)^{\frac{1}{1+\theta}}\left(L^i\right)^{\frac{\theta}{1+\theta}} + \sum_{i\in J-J_M}\left(T^i\right)^{\frac{1}{1+\theta}}\left(L^i\right)^{\frac{\theta}{1+\theta}}\right]$$

Taking the derivative of v^j with respect to L^j and L^i, we obtain:

$$\frac{\partial v^j}{\partial L^j} = \frac{1}{(1+\theta)L^j}\left[-1 + \frac{\left(T^j\right)^{\frac{1}{1+\theta}}\left(L^j\right)^{\frac{\theta}{1+\theta}}}{\sum_{i\in J_M}\left(T^i\right)^{\frac{1}{1+\theta}}\left(L^i\right)^{\frac{\theta}{1+\theta}}}\right] < 0$$

$$\frac{\partial v^j}{\partial L^i} = \frac{1}{(1+\theta)L^i}\frac{\left(T^i\right)^{\frac{1}{1+\theta}}\left(L^i\right)^{\frac{\theta}{1+\theta}}}{\sum_{i\in J_M}\left(T^i\right)^{\frac{1}{1+\theta}}\left(L^i\right)^{\frac{\theta}{1+\theta}}} > 0 \; i \neq j, i \in J_M$$

Thus, v^j is decreasing in L^j and increasing in L^i.

Results 1-3 imply that v^1 prefers to block labor mobility. Thus, under complete free trade, the richest country in a set of countries that are allowing free labor mobility within that set prefers to deviate and block labor mobility. This completes the proof that complete free trade and any pattern of labor mobility policy other than no mobility is not a Nash equilibrium outcome. This result can also be used to prove that any pattern of trade policy except complete autarky and any pattern of labor mobility policy within the trading-partner countries other than no labor mobility is not a Nash equilibrium outcome. Take any partition $\mathbf{J}_T$ and consider any arbitrary set $J_T \in \mathbf{J}_T$. Then, any pattern of labor mobility within J_T other than no mobility is not part of a Nash equilibrium outcome. The proof arises immediately from the previous results because each J_T can be treated as a world economy with J_T countries among which there is complete free trade. This completes the proof of part 4. ■

B Trade and labor mobility policies in factor proportions models

Appendix B presents the proofs of all the results in Section 4. Throughout Appendix B we employ the following notation: $v^j(\lambda_T, \lambda_M)$, $v_L^{j,m}(\lambda_T, \lambda_M)$ and $v_L^{j,im}(\lambda_T, \lambda_M)$ denote the utility of a worker located in country j, the utility of a mobile worker from country j and the utility of an immobile worker from country j, respectively, under trade and labor mobility regime (λ_T, λ_M). $W_G^j(\lambda_T, \lambda_M) = m v_L^{j,m}(\lambda_T, \lambda_M) + (1-m) v_L^{j,im}(\lambda_T, \lambda_M)$ indicates the welfare of domestic workers in country j under trade and labor mobility regime (λ_T, λ_M).

B.1 Heckscher-Ohlin economy

Consider an economy with two countries ($J = 2$), two tradeable goods ($Z_T = \{1, 2\}$) and two factors of production (F and labor L). Production functions are given by $Q_z^j = T^j \left(F_z^j\right)^{b_z} \left(L_z^j\right)^{1-b_z}$ for $z = 1, 2$, with $1 > b_1 > b_2 > 0$ and $T^1 > T^2$, where T^j is total factor productivity in country j. Factor endowments in country j are $\left(\bar{F}^j, \bar{L}^j\right) > (0, 0)$. All agents have the same preferences, given by $u\left(c^j\right) = \prod_{z \in Z} \left(c_z^j\right)^{\alpha_z}$, with $\alpha_z > 0$ and $\sum_{z \in Z} \alpha_z = 1$.

Lemma 4 characterizes the equilibrium of a Heckscher-Ohlin economy under autarky.

Lemma 4 (Heckscher-Ohlin economy under autarky). *Assume there is no trade in goods, i.e.,* $\lambda_T = 0$.

1. *Suppose there is no labor mobility, i.e.,* $\lambda_M = 0$. *Then,* $v_L^j(0,0) = CT^j \left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\tilde{\alpha}} \left(\frac{\bar{F}^j}{\bar{L}^j}\right)^{\tilde{\alpha}}$ *where* $C = (\alpha_1)^{\alpha_1} (\alpha_2)^{\alpha_2} (b_1)^{\alpha_1 b_1} (1-b_1)^{\alpha_1(1-b_1)} (b_2)^{\alpha_2 b_2} (1-b_2)^{\alpha_2(1-b_2)}$ *and* $\tilde{\alpha} = \alpha_1 b_1 + \alpha_2 b_2$.

2. *Suppose there is free labor mobility, i.e.,* $\lambda_M = 1$.

   (a) *If* $\frac{T^1}{T^2} > \left[\frac{\bar{F}^2\left(\bar{L}^1 + m\bar{L}^2\right)}{\bar{F}^1(1-m)\bar{L}^2}\right]^{\tilde{\alpha}}$, *then* $L^1 = \left(\bar{L}^1 + m\bar{L}^2\right)$, $L^2 = (1-m)\bar{L}^2$, $v_L^1(0,1) = CT^1 \left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\tilde{\alpha}} \left(\frac{\bar{F}^1}{\bar{L}^1 + m\bar{L}^2}\right)^{\tilde{\alpha}}$ *and* $v_L^2(0,1) = CT^2 \left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\tilde{\alpha}} \left[\frac{\bar{F}^2}{(1-m)\bar{L}^2}\right]^{\tilde{\alpha}}$.

   (b) *If* $\left[\frac{\bar{F}^2(1-m)\bar{L}^1}{\bar{F}^1\bar{L}^2 + \bar{F}^1(1-m)\bar{L}^1}\right]^{\tilde{\alpha}} \leq \frac{T^1}{T^2} \leq \left[\frac{\bar{F}^2\left(\bar{L}^1 + m\bar{L}^2\right)}{\bar{F}^1(1-m)\bar{L}^2}\right]^{\tilde{\alpha}}$, *then* $L^1 = \frac{\left(T^1\right)^{1/\tilde{\alpha}}\bar{F}^1\left(\bar{L}^1 + \bar{L}^2\right)}{\left(T^1\right)^{1/\tilde{\alpha}}\bar{F}^1 + \left(T^2\right)^{1/\tilde{\alpha}}\bar{F}^2}$, $L^2 = \frac{\left(T^2\right)^{1/\tilde{\alpha}}\bar{F}^2\left(\bar{L}^1 + \bar{L}^2\right)}{\left(T^1\right)^{1/\tilde{\alpha}}\bar{F}^1 + \left(T^2\right)^{1/\tilde{\alpha}}\bar{F}^2}$ *and* $v_L^1(0,1) = v_L^2(0,1) = C\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\tilde{\alpha}} \left[\frac{\left(T^1\right)^{1/\tilde{\alpha}}\bar{F}^1 + \left(T^2\right)^{1/\tilde{\alpha}}\bar{F}^2}{\bar{L}^1 + \bar{L}^2}\right]^{\tilde{\alpha}}$.

   (c) *If* $\frac{T^1}{T^2} < \left[\frac{\bar{F}^2(1-m)\bar{L}^1}{\bar{F}^1\bar{L}^2 + \bar{F}^1(1-m)\bar{L}^1}\right]^{\tilde{\alpha}}$, *then* $L^1 = (1-m)\bar{L}^1$, $L^2 = \bar{L}^2 + m\bar{L}^1$, $v_L^1(0,1) = CT^1 \left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\tilde{\alpha}} \left[\frac{\bar{F}^1}{(1-m)\bar{L}^1}\right]^{\tilde{\alpha}}$ *and* $v_L^2(0,1) = CT^2 \left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\tilde{\alpha}} \left[\frac{\bar{F}^2}{\bar{L}^2 + m\bar{L}^1}\right]^{\tilde{\alpha}}$.

3. *If* $\frac{T^1}{T^2} > \left(\frac{\bar{F}^2\bar{L}^1}{\bar{F}^1\bar{L}^2}\right)^{\tilde{\alpha}}$, *then* $v_L^1(0,0) > v_L^1(0,1)$, *while if* $\frac{T^1}{T^2} < \left(\frac{\bar{F}^2\bar{L}^1}{\bar{F}^1\bar{L}^2}\right)^{\tilde{\alpha}}$ *then* $v_L^2(0,0) > v_L^2(0,1)$.

Proof. Let p_z^j denotes the price of good $z=1,2$ in country j, r^j the return of factor F in country j and w^j the wage rate in country j. Under autarky the equilibrium conditions in country j are:

$$L_1^j = \frac{(1-b_1)\left[(1-b_2)\bar{F}^j - \left(w^j/r^j\right) b_2 L^j\right]}{(b_1-b_2)\left(w^j/r^j\right)},\ L_2^j = \frac{(1-b_2)\left[b_1\left(w^j/r^j\right)L^j - (1-b_1)\bar{F}^j\right]}{(b_1-b_2)\left(w^j/r^j\right)}$$

$$F_1^j = \frac{b_1\left[(1-b_2)\bar{F}^j - \left(w^j/r^j\right) b_2 L^j\right]}{(b_1-b_2)},\ F_2^j = \frac{b_2\left[b_1\left(w^j/r^j\right)L^j - (1-b_1)\bar{F}^j\right]}{(b_1-b_2)}$$

$$Q_1^j = T^j (b_1)^{b_1}(1-b_1)^{1-b_1}\frac{(1-b_2)\bar{F}^j - \left(w^j/r^j\right)b_2 L^j}{\left(w^j/r^j\right)^{1-b_1}(b_1-b_2)}$$

$$Q_2^j = T^j (b_2)^{b_2}(1-b_2)^{1-b_2}\frac{b_1\left(w^j/r^j\right)L^j - (1-b_1)\bar{F}^j}{\left(w^j/r^j\right)^{1-b_2}(b_1-b_2)}$$

$$\frac{w^j}{p_z^j} = (1-b_z)T^j\left(\frac{F_z^j}{L_z^j}\right)^{b_z},\ \frac{r^j}{p_z^j} = b_z T^j\left(\frac{L_z^j}{F_z^j}\right)^{1-b_z}\quad \text{for } z=1,2$$

$$\frac{p_2^j}{p_1^j} = \frac{(b_1)^{b_1}(1-b_1)^{1-b_1}}{(b_2)^{b_2}(1-b_2)^{1-b_2}}\left(\frac{w^j}{r^j}\right)^{b_1-b_2}$$

$$\alpha_z\left(p_1^j Q_1^j + p_2^j Q_2^j\right) = p_z^j Q_z^j \text{ for } z=1,2$$

Therefore, in equilibrium, $w^j/r^j = (1-\tilde{\alpha})\bar{F}^j/\tilde{\alpha}L^j$ and, hence, the utility of a worker in country j is $v_L^j = (\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2} w^j/\left(p_1^j\right)^{\alpha_1}\left(p_2^j\right)^{\alpha_2} = T^j C\left[(1-\tilde{\alpha})/\tilde{\alpha}\right]^{\tilde{\alpha}}\left(\bar{F}^j/L^j\right)^{\tilde{\alpha}}$, where $\tilde{\alpha} = \alpha_1 b_1 + \alpha_2 b_2$ and $C = (\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2}(b_1)^{\alpha_1 b_1}(1-b_1)^{\alpha_1(1-b_1)}(b_2)^{\alpha_2 b_2}(1-b_2)^{\alpha_2(1-b_2)}$.

When there is no labor mobility, $L^j = \bar{L}^j$. When there is free labor mobility, mobile workers locate in the country with the highest v_L^j. We must distinguish three possible cases.

a. Suppose that $T^1/T^2 > \left[\bar{F}^2\left(\bar{L}^1 + m\bar{L}^2\right)/(1-m)\bar{F}^1\bar{L}^2\right]^{\tilde{\alpha}}$. Then, $v_L^1 > v_L^2$ for all $L^1 \in \left[(1-m)\bar{L}^1, \bar{L}^1 + m\bar{L}^2\right]$ and $L^2 \in \left[(1-m)\bar{L}^2, \bar{L}^2 + m\bar{L}^1\right]$. Therefore, in equilibrium, $L^1 = \left(\bar{L}^1 + m\bar{L}^2\right)$ and $L^2 = (1-m)\bar{L}^2$.

b. Suppose that $\left\{\bar{F}^2(1-m)\bar{L}^1/\left[\bar{F}^1\bar{L}^2 + \bar{F}^1(1-m)\bar{L}^1\right]\right\}^{\tilde{\alpha}} \leq \frac{T^1}{T^2} \leq \left[\bar{F}^2\left(\bar{L}^1 + m\bar{L}^2\right)/\bar{F}^1(1-m)\bar{L}^2\right]^{\tilde{\alpha}}$. Then, $v_L^1 \geq v_L^2$ for $L^1 = (1-m)\bar{L}^1$ and $L^2 = \bar{L}^2 + m\bar{L}^1$, while $v_L^1 \leq v_L^2$ for $L^1 = \bar{L}^1 + m\bar{L}^2$ and $L^2 = (1-m)\bar{L}^2$. Moreover, v_L^j is decreasing in L^j. Therefore, in equilibrium, it must be the case that $v_L^1 = v_L^2$, which implies $L^1 = \left(T^1\right)^{1/\tilde{\alpha}}\bar{F}^1\left(\bar{L}^1 + \bar{L}^2\right)/\left[\left(T^1\right)^{1/\tilde{\alpha}}\bar{F}^1 + \left(T^2\right)^{1/\tilde{\alpha}}\bar{F}^2\right]$ and $L^2 = \left(T^2\right)^{1/\tilde{\alpha}}\bar{F}^2\left(\bar{L}^1 + \bar{L}^2\right)/\left[\left(T^1\right)^{1/\tilde{\alpha}}\bar{F}^1 + \left(T^2\right)^{1/\tilde{\alpha}}\bar{F}^2\right]$.

c. Suppose that $T^1/T^2 < \left\{\bar{F}^2(1-m)\bar{L}^1/\left[\bar{F}^1\bar{L}^2 + \bar{F}^1(1-m)\bar{L}^1\right]\right\}^{\tilde{\alpha}}$. Then, $v_L^2 > v_L^1$ for all $L^1 \in \left[(1-m)\bar{L}^1, \bar{L}^1 + m\bar{L}^2\right]$ and $L^2 \in \left[(1-m)\bar{L}^2, \bar{L}^2 + m\bar{L}^1\right]$. Therefore, in equilibrium $L^1 = (1-m)\bar{L}^1$ and $L^2 = \bar{L}^2 + m\bar{L}^1$. Summing up, the utility of a worker in country j under autarky and free labor mobility is given by:

$$v_L^1(0,1)=C\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\tilde{\alpha}}\begin{cases}\left[\frac{\left(T^1\right)^{1/\tilde{\alpha}}\bar{F}^1}{\bar{L}^1+m\bar{L}^2}\right]^{\tilde{\alpha}} & \text{if } \frac{T^1}{T^2}>\left[\frac{\bar{F}^2\left(\bar{L}^1+m\bar{L}^2\right)}{\bar{F}^1(1-m)\bar{L}^2}\right]^{\tilde{\alpha}}\\ \left[\frac{\left(T^1\right)^{1/\tilde{\alpha}}\bar{F}^1+\left(T^2\right)^{1/\tilde{\alpha}}\bar{F}^2}{\bar{L}^1+\bar{L}^2}\right]^{\tilde{\alpha}} & \text{if } \left[\frac{\bar{F}^2(1-m)\bar{L}^1}{\bar{F}^1\bar{L}^2+\bar{F}^1(1-m)\bar{L}^1}\right]^{\tilde{\alpha}}\le\frac{T^1}{T^2}\le\left[\frac{\bar{F}^2\left(\bar{L}^1+m\bar{L}^2\right)}{\bar{F}^1(1-m)\bar{L}^2}\right]^{\tilde{\alpha}}\\ \left[\frac{\left(T^1\right)^{1/\tilde{\alpha}}\bar{F}^1}{(1-m)\bar{L}^1}\right]^{\tilde{\alpha}} & \text{if } \frac{T^1}{T^2}<\left[\frac{\bar{F}^2(1-m)\bar{L}^1}{\bar{F}^1\bar{L}^2+\bar{F}^1(1-m)\bar{L}^1}\right]^{\tilde{\alpha}}\end{cases}$$

$$v_L^2(0,1)=C\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\tilde{\alpha}}\begin{cases}\left[\frac{\left(T^2\right)^{1/\tilde{\alpha}}\bar{F}^2}{(1-m)\bar{L}^2}\right]^{\tilde{\alpha}} & \text{if } \frac{T^1}{T^2}>\left[\frac{\bar{F}^2\left(\bar{L}^1+m\bar{L}^2\right)}{(1-m)\bar{F}^1\bar{L}^2}\right]^{\tilde{\alpha}}\\ \left[\frac{\left(T^1\right)^{1/\tilde{\alpha}}\bar{F}^1+\left(T^2\right)^{1/\tilde{\alpha}}\bar{F}^2}{\bar{L}^1+\bar{L}^2}\right]^{\tilde{\alpha}} & \text{if } \left[\frac{\bar{F}^2(1-m)\bar{L}^1}{\bar{F}^1\bar{L}^2+\bar{F}^1(1-m)\bar{L}^1}\right]^{\tilde{\alpha}}<\frac{T^1}{T^2}\le\left[\frac{\bar{F}^2\left(\bar{L}^1+m\bar{L}^2\right)}{(1-m)\bar{F}^1\bar{L}^2}\right]^{\tilde{\alpha}}\\ \left[\frac{\left(T^2\right)^{1/\tilde{\alpha}}\bar{F}^2}{\bar{L}^2+m\bar{L}^1}\right]^{\tilde{\alpha}} & \text{if } \frac{T^1}{T^2}<\left[\frac{\bar{F}^2(1-m)\bar{L}^1}{\bar{F}^1\bar{L}^2+\bar{F}^1(1-m)\bar{L}^1}\right]^{\tilde{\alpha}}\end{cases}$$

Finally, note that $T^1/T^2 > \left(\bar{F}^2\bar{L}^1/\bar{F}^1\bar{L}^2\right)^{\tilde{\alpha}}$ implies $v_L^1(0,0) > v_L^1(0,1)$, while $T^1/T^2 < \left(\bar{F}^2\bar{L}^1/\bar{F}^1\bar{L}^2\right)^{\tilde{\alpha}}$ implies $v_L^2(0,0) > v_L^2(0,1)$. This completes the proof of the lemma. ■

Lemma 5 characterizes the equilibrium of a Heckscher-Ohlin economy under free trade.

Lemma 5 (Heckscher-Ohlin economy under free trade). *Assume there is free trade in goods, i.e.,* $\lambda_T = 1$.

1. *Let* $\bar{f}^1 = T^1\bar{F}^1/\left(T^1\bar{F}^1+T^2\bar{F}^2\right)$, $l^1 = T^1L^1/\left(T^1L^1+T^2L^2\right)$ *and*

$$\eta_1\left(l^1\right)=\frac{(1-\tilde{\alpha})\,b_2l^1}{\tilde{\alpha}\,(1-b_2)},\ \eta_2\left(l^1\right)=\frac{-\alpha_2\,(b_1-b_2)+b_1\,(1-\tilde{\alpha})\,l^1}{\tilde{\alpha}\,(1-b_1)}$$

$$\eta_3\left(l^1\right)=\frac{(1-\tilde{\alpha})\,b_1l^1}{\tilde{\alpha}\,(1-b_1)},\ \eta_4\left(l^1\right)=\frac{\alpha_1\,(b_1-b_2)+b_2\,(1-\tilde{\alpha})\,l^1}{\tilde{\alpha}\,(1-b_2)}$$

$$\eta_5\left(l^1\right)=\frac{\left(\frac{\alpha_1B_{11}}{\alpha_2B_{21}}\right)^{\frac{1}{b_1}}\left(1-l^1\right)^{\frac{1-b_1}{b_1}}}{\left(l^1\right)^{\frac{1-b_1}{b_1}}+\left[\frac{\alpha_1B_{11}}{\alpha_2B_{21}}\right]^{\frac{1}{b_1}}\left(1-l^1\right)^{\frac{1-b_1}{b_1}}},\ \eta_6\left(l^1\right)=\frac{\left(\frac{\alpha_1B_{12}}{\alpha_2B_{22}}\right)^{\frac{1}{b_2}}\left(1-l^1\right)^{\frac{1-b_2}{b_2}}}{\left(l^1\right)^{\frac{1-b_2}{b_2}}+\left(\frac{\alpha_1B_{12}}{\alpha_2B_{22}}\right)^{\frac{1}{b_2}}\left(1-l^1\right)^{\frac{1-b_2}{b_2}}}$$

$$\eta_7\left(l^1\right)=\frac{\left(\frac{\alpha_2B_{22}}{\alpha_1B_{12}}\right)^{\frac{1}{b_2}}\left(1-l^1\right)^{\frac{1-b_2}{b_2}}}{\left(l^1\right)^{\frac{1-b_2}{b_2}}+\left(\frac{\alpha_2B_{22}}{\alpha_1B_{12}}\right)^{\frac{1}{b_2}}\left(1-l^1\right)^{\frac{1-b_2}{b_2}}},\ \eta_8\left(l^1\right)=\frac{\left(\frac{\alpha_2B_{21}}{\alpha_1B_{11}}\right)^{\frac{1}{b_1}}\left(1-l^1\right)^{\frac{1-b_1}{b_1}}}{\left(l^1\right)^{\frac{1-b_1}{b_1}}+\left(\frac{\alpha_2B_{21}}{\alpha_1B_{11}}\right)^{\frac{1}{b_1}}\left(1-l^1\right)^{\frac{1-b_1}{b_1}}}$$

where $B_{11} = (b_1)^{b_1}(1-b_1)^{1-b_1}$, $B_{12} = (b_1)^{b_2}(1-b_1)^{1-b_2}$, $B_{21} = (b_2)^{b_1}(1-b_2)^{1-b_1}$, *and* $B_{22} = (b_2)^{b_2}(1-b_2)^{1-b_2}$. *Then:*

   (a) *If* $\max\left\{\eta_1\left(l^1\right),\eta_2\left(l^1\right)\right\} < \bar{f}^1 < \min\left\{\eta_3\left(l^1\right),\eta_4\left(l^1\right)\right\}$, *then,* $v_L^j(1,\lambda_M) = T^jC\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\tilde{\alpha}}\left(\frac{T^1\bar{F}^1+T^2\bar{F}^2}{T^1L^1+T^2L^2}\right)^{\tilde{\alpha}}$.

(b) *If* $\eta_3\left(l^1\right) \leq \bar{f}^1 < \eta_5\left(l^1\right)$, *then,* $v_L^1\left(1,\lambda_M\right) = T^1 C\left[\frac{(1-b_1)\bar{F}^1}{b_1 L^1}\right]^{b_1}\left(\frac{r^2}{w^2}\right)^{\alpha_2(b_1-b_2)}$ *and* $v_L^2\left(1,\lambda_M\right) = T^2 C\left(\frac{w^2}{r^2}\right)^{\tilde{\alpha}}$, *where* $\frac{w^2}{r^2}$ *is the unique solution to* $\frac{\alpha_2 T^1\left(\bar{F}^1\right)^{b_1}\left(L^1\right)^{1-b_1}}{B_{11}} = \frac{\tilde{\alpha}T^2\left(w^2/r^2\right)L^2-(1-\tilde{\alpha})T^2\bar{F}^2}{\left(w^2/r^2\right)^{1-b_1}(b_1-b_2)}$.

(c) *If* $\eta_5\left(l^1\right) \leq \bar{f}^1 \leq \eta_6\left(l^1\right)$, *then,* $v_L^1\left(1,\lambda_M\right) = \frac{C\left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_2}(1-b_1)\left[T^1\left(\bar{F}^1\right)^{b_1}\left(L^1\right)^{(1-b_1)}\right]^{\alpha_1}\left[T^2\left(\bar{F}^2\right)^{b_2}\left(L^2\right)^{1-b_2}\right]^{\alpha_2}}{\left(B_{11}\right)^{\alpha_1}\left(B_{22}\right)^{\alpha_2}L^1}$ *and* $v_L^2\left(1,\lambda_M\right) = \frac{C\left(\frac{\alpha_2}{\alpha_1}\right)^{\alpha_1}(1-b_2)\left[T^1\left(\bar{F}^1\right)^{b_1}\left(L^1\right)^{(1-b_1)}\right]^{\alpha_1}\left[T^2\left(\bar{F}^2\right)^{b_2}\left(L^2\right)^{1-b_2}\right]^{\alpha_2}}{\left(B_{11}\right)^{\alpha_1}\left(B_{22}\right)^{\alpha_2}L^2}$.

(d) *If* $\bar{f}^1 \geq \eta_4\left(l^1\right)$ *and* $\bar{f}^1 > \eta_6\left(l^1\right)$, *then,* $v_L^1\left(1,\lambda_M\right) = T^1 C\left(\frac{w^1}{r^1}\right)^{\tilde{\alpha}}$ *and* $v_L^2\left(1,\lambda_M\right) = T^2 C\left(\frac{w^1}{r^1}\right)^{\alpha_1(b_1-b_2)}\left[\frac{(1-b_2)\bar{F}^2}{b_2 L^2}\right]^{b_2}$, *where* $\frac{w^1}{r^1}$ *is the unique solution to* $\frac{T^1\left[(1-\tilde{\alpha})\bar{F}^1-\tilde{\alpha}\left(w^1/r^1\right)L^1\right]}{\left(w^1/r^1\right)^{1-b_2}(b_1-b_2)} = \frac{\alpha_1 T^2\left(F^2\right)^{b_2}\left(L^2\right)^{1-b_2}}{B_{22}}$.

(e) *If* $\eta_8\left(l^1\right) < \bar{f}^1 \leq \eta_2\left(l^1\right)$, *then,* $v_L^1\left(1,\lambda_M\right) = T^1 C\left(\frac{w^1}{r^1}\right)^{\tilde{\alpha}}$ *and* $v_L^2\left(1,\lambda_M\right) = T^2 C\left[\frac{(1-b_1)\bar{F}^2}{b_1 L^2}\right]^{b_1}\left(\frac{r^1}{w^1}\right)^{\alpha_2(b_1-b_2)}$, *where* $\frac{w^1}{r^1}$ *is the unique solution to* $\frac{\alpha_2 T^2\left(\bar{F}^2\right)^{b_1}\left(L^2\right)^{1-b_1}}{B_{11}} = \frac{T^1\left[\tilde{\alpha}\left(w^1/r^1\right)L^1-(1-\tilde{\alpha})\bar{F}^1\right]}{(b_1-b_2)\left(w^1/r^1\right)^{1-b_1}}$.

(f) *If* $\eta_7\left(l^1\right) \leq \bar{f}^1 \leq \eta_8\left(l^1\right)$, *then,* $v_L^1\left(1,\lambda_M\right) = \frac{C\left(\frac{\alpha_2}{\alpha_1}\right)^{\alpha_1}(1-b_2)\left[T^2\left(\bar{F}^2\right)^{b_1}\left(L^2\right)^{1-b_1}\right]^{\alpha_1}\left[T^1\left(\bar{F}^1\right)^{b_2}\left(L^1\right)^{1-b_2}\right]^{\alpha_2}}{\left(B_{11}\right)^{\alpha_1}\left(B_{22}\right)^{\alpha_2}L^1}$ *and* $v_L^2\left(1,\lambda_M\right) = \frac{C(1-b_1)\left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_2}\left[T^2\left(\bar{F}^2\right)^{b_1}\left(L^2\right)^{1-b_1}\right]^{\alpha_1}\left[T^1\left(\bar{F}^1\right)^{b_2}\left(L^1\right)^{1-b_2}\right]^{\alpha_2}}{\left(B_{11}\right)^{\alpha_1}\left(B_{22}\right)^{\alpha_2}L^2}$.

(g) *If* $\bar{f}^1 \leq \eta_1\left(l^1\right)$ *and* $\bar{f}^1 < \eta_7\left(l^1\right)$, *then,* $v_L^1\left(1,\lambda_M\right) = T^1 C\left[\frac{(1-b_2)\bar{F}^1}{b_2 L^1}\right]^{b_2}\left(\frac{w^2}{r^2}\right)^{\alpha_1(b_1-b_2)}$ *and* $v_L^2\left(1,\lambda_M\right) = T^2 C\left(\frac{w^2}{r^2}\right)^{\tilde{\alpha}}$, *where* $\frac{w^2}{r^2}$ *is the unique solution to* $\frac{\alpha_1 T^1\left(\bar{F}^1\right)^{b_2}\left(L^1\right)^{1-b_2}}{B_{22}} = \frac{(1-\tilde{\alpha})T^2\bar{F}^2-\tilde{\alpha}\left(w^2/r^2\right)T^2 L^2}{\left(w^2/r^2\right)^{1-b_2}(b_1-b_2)}$.

2. *Let* $\bar{l}^1 = T^1\bar{L}^1/\left(T^1\bar{L}^1+T^2\bar{L}^2\right)$ *and*

$$\eta_9\left(l^1\right) = \frac{(1-\tilde{\alpha})\,b_2 l^1}{\alpha_1\left(\frac{T^1}{T^2}\right)^{\frac{1-b_2}{b_2}}(b_1-b_2)\,l^1+(1-\tilde{\alpha})\,b_2 l^1+\tilde{\alpha}\left(\frac{T^1}{T^2}\right)^{\frac{1}{b_2}}(1-b_2)\left(1-l^1\right)}$$

$$\eta_{10}\left(l^1\right) = \frac{\tilde{\alpha}\,(1-b_1)\,l^1-(b_1-b_2)\,\alpha_2\left(1-l^1\right)\left(\frac{T1}{T^2}\right)}{\tilde{\alpha}\,(1-b_1)\,l^1-(b_1-b_2)\,\alpha_2\left(\frac{T^1}{T^2}\right)\left(1-l^1\right)+b_1\,(1-\tilde{\alpha})\left(1-l^1\right)\left(\frac{T^1}{T^2}\right)^{\frac{1}{b_1}}}$$

$$\bar{l} = \frac{\left(\frac{T^1}{T^2}\right)\alpha_2\,(1-b_2)}{\alpha_1\,(1-b_1)+\left(\frac{T^1}{T^2}\right)\alpha_2\,(1-b_2)}$$

Assume that and $T^1 > T^2$. *Then:*

(a) *Suppose that* $\bar{f}^1 > \eta_9\left(\bar{l}^1\right)$ *for* $\bar{l}^1 < \bar{l}$; *or* $\bar{f}^1 > \eta_{10}\left(\bar{l}^1\right)$ *for* $\bar{l}^1 \geq \bar{l}$. *Then* $v_L^1(1,0) > v_L^1(1,1)$.

(b) *Suppose that* $\bar{f}^1 < \eta_9\left(\bar{l}^1\right)$ *for* $\bar{l}^1 < \bar{l}$*; or* $\bar{f}^1 < \eta_{10}\left(\bar{l}^1\right)$ *for* $l^1 \geq \bar{l}$*. Then,* $v_L^2(1,0) > v_L^2(1,1)$*.*

(c) *Suppose that* $\bar{f}^1 = \eta_9\left(\bar{l}^1\right)$ *for* $l^1 < \bar{l}$*; or* $\bar{f}^1 = \eta_{10}\left(\bar{l}^1\right)$ *for* $l^1 > \bar{l}$*; or* $\bar{f}^1 \in \left[\eta_9\left(\bar{l}^1\right), \eta_{10}\left(\bar{l}^1\right)\right]$ *for* $l^1 = \bar{l}$*. Then,* $v_L^1(1,0) = v_L^1(1,1) = v_L^2(1,0) = v_L^2(1,1)$*.*

Proof of Part 1: Under free trade the equilibrium conditions in country j are:

$$L_1^j = \max\left\{\frac{(1-b_1)\left[(1-b_2)\bar{F}^j - \left(w^j/r^j\right)b_2L^j\right]}{(b_1-b_2)\left(w^j/r^j\right)}, 0\right\}$$

$$L_2^j = \max\left\{\frac{(1-b_2)\left[b_1\left(w^j/r^j\right)L^j - (1-b_1)\bar{F}^j\right]}{(b_1-b_2)\left(w^j/r^j\right)}, 0\right\}$$

$$F_1^j = \max\left\{\frac{b_1\left[(1-b_2)\bar{F}^j - \left(w^j/r^j\right)b_2L^j\right]}{(b_1-b_2)}, 0\right\},\ F_2^j = \max\left\{\frac{b_2\left[b_1\left(w^j/r^j\right)L^j - (1-b_1)\bar{F}^j\right]}{(b_1-b_2)}, 0\right\}$$

$$Q_1^j = \max\left\{T^jB_{11}\frac{(1-b_2)\bar{F}^j - \left(w^j/r^j\right)b_2L^j}{\left(w^j/r^j\right)^{1-b_1}(b_1-b_2)}, 0\right\}$$

$$Q_2^j = \max\left\{T^jB_{22}\frac{b_1\left(w^j/r^j\right)L^j - (1-b_1)\bar{F}^j}{\left(w^j/r^j\right)^{1-b_2}(b_1-b_2)}, 0\right\}$$

$$\frac{w^j}{p_z} = (1-b_z)T^j\left(\frac{F_z^j}{L_z^j}\right)^{b_z},\ \frac{r^j}{p_z} = b_zT^j\left(\frac{L_z^j}{F_z^j}\right)^{1-b_z} \quad \text{for } z = 1,2 \text{ and } F_z^j > 0 \text{ and } L_z^j > 0$$

$$p_z \geq \frac{\left(r^j\right)^{b_z}\left(w^j\right)^{1-b_z}}{T^j\left(b_z\right)^{b_z}\left(1-b_z\right)^{1-b_z}} \text{ for } z = 1,2 \text{ and } j = 1,2$$

$$\alpha_2p_1\left(Q_1^1 + Q_1^2\right) = \alpha_1p_2\left(Q_2^1 + Q_2^2\right)$$

We must consider seven possible cases.

a. Suppose that both countries are diversified. Then, in equilibrium, $L_1^j = (1-b_1)\left[(1-b_2)\bar{F}^j - (w/r)b_2L^j\right]/(b_1-b_2)(w/r)$ and $L_2^j = (1-b_2)\left[b_1(w/r)L^j - (1-b_1)\bar{F}^j\right]/(b_1-b_2)(w/r)$ for $j = 1,2$, $F_1^j = b_1\left[(1-b_2)\bar{F}^j - (w/r)b_2L^j\right]/(b_1-b_2)$ and $F_2^j = b_2\left[b_1(w/r)L^j - (1-b_1)\bar{F}^j\right]/(b_1-b_2)$ for $j = 1,2$, $Q_1^j = T^jB_{11}\left[(1-b_2)\bar{F}^j - (w/r)b_2L^j\right]/(w/r)^{1-b_1}(b_1-b_2)$ and $Q_2^j = T^jB_{22}\left[b_1(w/r)L^j - (1-b_1)\bar{F}^j\right]/(w/r)^{1-b_2}(b_1-b_2)$ for $j = 1,2$, $w^j/p_z = (1-b_z)T^j\left(F_z^j/L_z^j\right)^{b_z}$ and $r^j/p_z = b_zT^j\left(L_z^j/F_z^j\right)^{1-b_z}$ for $j = 1,2$ and $z = 1,2$, $p_2/p_1 = (B_{11}/B_{22})\left(\frac{w}{r}\right)^{b_1-b_2}$, and $w/r = (1-\tilde{\alpha})\left(T^1\bar{F}^1 + T^2\bar{F}^2\right)/\tilde{\alpha}\left(T^1L^1 + T^2L^2\right)$. For this to be an equilibrium we need $(1-b_1)\bar{F}^j/b_1L^j < w/r < (1-b_2)\bar{F}^j/b_2L^j$ for $j = 1,2$. Since $b_1 > b_2$, we require $[\bar{F}^1/L^1 \geq \bar{F}^2/L^2$ and $(1-b_1)\bar{F}^1/b_1L^1 < w/r < (1-b_2)\bar{F}^2/b_2L^2]$ or $[\bar{F}^1/L^1 \leq \bar{F}^2/L^2$ and $(1-b_1)\bar{F}^2/b_1L^2 < w/r < (1-b_2)\bar{F}^1/b_2L^1]$. $[\bar{F}^1/L^1 \geq \bar{F}^2/L^2$ and $(1-b_1)\bar{F}^1/b_1L^1 < w/r < (1-b_2)\bar{F}^2/b_2L^2]$ is equivalent to $l^1 \leq \bar{f}^1 < \min\left\{(1-\tilde{\alpha})b_1l^1/\tilde{\alpha}(1-b_1), \left[\alpha_1(b_1-b_2) + b_2(1-\tilde{\alpha})l^1\right]/\tilde{\alpha}(1-b_2)\right\}$, while $[\bar{F}^1/L^1 \leq \bar{F}^2/L^2$ and $(1-b_1)\bar{F}^2/b_1L^2 < w/r < (1-b_2)\bar{F}^1/b_2L^1]$ is equivalent to $\max\left\{(1-\tilde{\alpha})b_2l^1/\tilde{\alpha}(1-b_2), \left[-\alpha_2(b_1-b_2) + b_1(1-\tilde{\alpha})l^1\right]/\tilde{\alpha}(1-b_1)\right\} < \bar{f}^1 \leq l^1$, where

$\bar{f}^j = T^j \bar{F}^j / \left(T^1 \bar{F}^1 + T^2 \bar{F}^2\right)$ and $l^j = T^j L^j / \left(T^1 L^1 + T^2 L^2\right)$. Thus, we require

$$\max\left\{\frac{(1-\tilde{\alpha})\, b_2 l^1}{\tilde{\alpha}\,(1-b_2)}, \frac{-\alpha_2\,(b_1-b_2)+b_1\,(1-\tilde{\alpha})\, l^1}{\tilde{\alpha}\,(1-b_1)}\right\} < \bar{f}^1 < \min\left\{\frac{(1-\tilde{\alpha})\, b_1 l^1}{\tilde{\alpha}\,(1-b_1)}, \frac{\alpha_1\,(b_1-b_2)+b_2\,(1-\tilde{\alpha})\, l^1}{\tilde{\alpha}\,(1-b_2)}\right\}$$

Then, the utility of a worker in country j is $v_L^j\,(1,\lambda_M) = (\alpha_1)^{\alpha_1}\,(\alpha_2)^{\alpha_2}\, w^j/\,(p_1)^{\alpha_1}\,(p_2)^{\alpha_2} = T^j C\left[(1-\tilde{\alpha})\,/\tilde{\alpha}\right]^{\tilde{\alpha}}\left[\left(T^1\bar{F}^1+T^2\bar{F}^2\right)/\left(T^1L^1+T^2L^2\right)\right]^{\tilde{\alpha}}$. Finally, note that $v_L^1\,(1,\lambda_M) > v_L^2\,(1,\lambda_M)$ if and only if $T^1 > T^2$.

b. Suppose that country 1 is specialized in good 1 and country 2 is diversified. Then, in equilibrium, $L_1^1 = L^1$, $L_2^1 = 0$, $L_1^2 = (1-b_1)\left[(1-b_2)\,\bar{F}^2 - \left(w^2/r^2\right) b_2 L^2\right]/\,(b_1-b_2)\left(w^2/r^2\right)$, $L_2^2 = (1-b_2)\left[b_1\left(w^2/r^2\right) L^2 - (1-b_1)\,\bar{F}^2\right]/\,(b_1-b_2)\left(w^2/r^2\right)$, $F_1^1 = \bar{F}^1$, $F_2^1 = 0$, $F_1^2 = b_1\left[(1-b_2)\,\bar{F}^2 - \left(w^2/r^2\right) b_2 L^2\right]/\,(b_1-b_2)$, $F_2^2 = b_2\left[b_1\left(w^2/r^2\right) L^2 - (1-b_1)\,\bar{F}^2\right]/\,(b_1-b_2)$, $Q_1^1 = T^1\left(\bar{F}^1\right)^{b_1}\left(L^1\right)^{1-b_1}$, $Q_1^2 = T^2 B_{11}\left[(1-b_2)\,\bar{F}^2 - \left(w^2/r^2\right) b_2 L^2\right]/\left(w^2/r^2\right)^{1-b_1}(b_1-b_2)$, $Q_2^1 = 0$, $Q_2^2 = A^2 B_{22}\left[b_1\left(w^2/r^2\right) L^2 - (1-b_1)\,\bar{F}^2\right]/\left(w^2/r^2\right)^{1-b_2}(b_1-b_2)$, $w^1/p_1 = (1-b_1)\, T^1\left(\bar{F}^1/L^1\right)^{b_1}$, $r^1/p_1 = b_1 T^1\left(L^1/\bar{F}^1\right)^{1-b_1}$, $w^2/p_z = (1-b_z)\, T^2\left(F_z^2/L_z^2\right)^{b_z}$, $r^2/p_z = b_z T^2\left(L_z^2/F_z^2\right)^{1-b_z}$ for $z = 1, 2$, $p_2/p_1 = (B_{11}/B_{22})\left(w^2/r^2\right)^{b_1-b_2}$, and w^2/r^2 is the unique solution to $\alpha_2 T^1\left(\bar{F}^1\right)^{b_1}\left(L^1\right)^{1-b_1}/B_{11} = \left[\tilde{\alpha}T^2\left(w^2/r^2\right)L^2 - (1-\tilde{\alpha})\, T^2\bar{F}^2\right]/\left(w^2/r^2\right)^{1-b_1}(b_1-b_2)$. For this to be an equilibrium we need $w^1/r^1 \geq w^2/r^2$ and $(1-b_1)\,\bar{F}^2/b_1L^2 < w^2/r^2 < (1-b_2)\,\bar{F}^2/b_2L^2$. $w^1/r^1 \geq w^2/r^2$ if and only if $\bar{f}^1 \geq b_1\,(1-\tilde{\alpha})\, l^1/\tilde{\alpha}\,(1-b_1)$. $w^2/r^2 > (1-b_1)\,\bar{F}^2/b_1L^2$ if and only if $\left[T^1\left(\bar{F}^1\right)^{b_1}\left(L^1\right)^{1-b_1}/T^2\left(\bar{F}^2\right)^{b_1}\left(L^2\right)^{1-b_1}\right](b_1-b_2) > (b_2-b_1)$, which always holds. $w^2/r^2 < (1-b_2)\,\bar{F}^2/b_2L^2$ if and only if $\alpha_2 T^1\left(\bar{F}^1\right)^{b_1}\left(L^1\right)^{1-b_1}/B_{11} < \alpha_1 T^2\left(\bar{F}^2\right)^{b_1}\left(L^2\right)/^{1-b_1}B_{21}$. Thus, we require

$$\frac{b_1\,(1-\tilde{\alpha})\, l^1}{\tilde{\alpha}\,(1-b_1)} \leq \bar{f}^1 < \frac{\left(\frac{\alpha_1 B_{11}}{\alpha_2 B_{21}}\right)^{\frac{1}{b_1}}\left(1-l^1\right)^{\frac{1-b_1}{b_1}}}{\left(l^1\right)^{\frac{1-b_1}{b_1}} + \left(\frac{\alpha_1 B_{11}}{\alpha_2 B_{21}}\right)^{\frac{1}{b_1}}\left(1-l^1\right)^{\frac{1-b_1}{b_1}}}$$

Then, the utility of a worker in country 1 is $v_L^1\,(1,\lambda_M) = (\alpha_1)^{\alpha_1}\,(\alpha_2)^{\alpha_2}\, w^1/\,(p_1)^{\alpha_1}\,(p_2)^{\alpha_2} = CT^1\left[(1-b_1)\,\bar{F}^1/b_1L^1\right]^{b_1}\left(r^2/w^2\right)^{\alpha_2(b_1-b_2)}$ and the utility of a worker in country 2 is $v_L^2\,(1,\lambda_M) = (\alpha_1)^{\alpha_1}\,(\alpha_2)^{\alpha_2}\, w^2/\,(p_1)^{\alpha_1}\,(p_2)^{\alpha_2} = CT^2\left(w^2/r^2\right)^{\tilde{\alpha}}$. Finally, note that $v_L^1\,(1,\lambda_M) > v_L^2\,(1,\lambda_M)$ if and only if $T^1\left[(1-b_1)\,\bar{F}^1/b_1L^1\right]^{b_1} > T^2\left(w^2/r^2\right)^{b_1}$. Since in this region $w^1/r^1 = (1-b_1)\,\bar{F}^1/b_1L^1 \geq w^2/r^2$, $T^1 > T^2$ implies $v_L^1\,(1,\lambda_M) > v_L^2\,(1,\lambda_M)$.

c. Suppose that country 1 is specialized in good 1 and country 2 is specialized in good 2. Then, in equilibrium, $L_1^1 = L^1$, $L_2^1 = 0$, $L_1^2 = 0$, $L_2^2 = L^2$, $F_1^1 = \bar{F}^1$, $F_2^1 = 0$, $F_1^2 = 0$, $F_2^2 = \bar{F}^2$, $Q_1^1 = T^1\left(\bar{F}^1\right)^{b_1}\left(L^1\right)^{1-b_1}$, $Q_2^1 = 0$, $Q_1^2 = 0$, $Q_2^2 = T^2\left(\bar{F}^2\right)^{b_2}\left(L^2\right)^{1-b_2}$, $w^1/p_1 = (1-b_1)\, T^1\left(\bar{F}^1/L^1\right)^{b_1}$, $r^1/p_1 = b_1T^1\left(L^1/\bar{F}^1\right)^{1-b_1}$, $w^2/p_2 = (1-b_2)\, T^2\left(\bar{F}^2/L^2\right)^{b_2}$, $r^2/p_2 = b_2T^2\left(L^2/\bar{F}^2\right)^{1-b_2}$ and $p_2/p_1 = \alpha_2 T^1\left(\bar{F}^1\right)^{b_1}\left(L^1\right)^{1-b_1}/\alpha_1 T^2\left(\bar{F}^2\right)^{b_2}\left(L^2\right)^{1-b_2}$. For this to be an equilibrium we need $\left(r^1\right)^{b_2}\left(w^1\right)^{1-b_2}/T^1 \geq \left(r^2\right)^{b_2}\left(w^2\right)^{1-b_2}/T^2$ and $\left(r^1\right)^{b_1}\left(w^1\right)^{1-b_1}/T^1 \leq \left(r^2\right)^{b_1}\left(w^2\right)^{1-b_1}/T^2$. $\left(r^1\right)^{b_2}\left(w^1\right)^{1-b_2}/T^1 \geq \left(r^2\right)^{b_2}\left(w^2\right)^{1-b_2}/T^2$ if and only if $T^2\left(\bar{F}^2\right)^{b_2}\left(L^2\right)^{1-b_2}/B_{22} \geq \alpha_2 T^1\left(\bar{F}^1\right)^{b_2}\left(L^1\right)^{1-b_2}/\alpha_1 B_{12}$, while $\left(r^1\right)^{b_1}\left(w^1\right)^{1-b_1}/T^1 \leq \left(r^2\right)^{b_1}\left(w^2\right)^{1-b_1}/T^2$ if and only if

$\alpha_1 T^2 \left(\bar{F}^2\right)^{b_1} \left(L^2\right)^{1-b_1} / B_{21} \leq \alpha_2 T^1 \left(\bar{F}^1\right)^{b_1} \left(L^1\right)^{1-b_1} / B_{11}$. Thus, we require

$$\frac{\left(\frac{\alpha_1 B_{11}}{\alpha_2 B_{21}}\right)^{\frac{1}{b_1}} \left(1-l^1\right)^{\frac{1-b_1}{b_1}}}{\left(l^1\right)^{\frac{1-b_1}{b_1}} + \left(\frac{\alpha_1 B_{11}}{\alpha_2 B_{21}}\right)^{\frac{1}{b_1}} \left(1-l^1\right)^{\frac{1-b_1}{b_1}}} \leq \bar{f}^1 \leq \frac{\left(\frac{\alpha_1 B_{12}}{\alpha_2 B_{22}}\right)^{\frac{1}{b_2}} \left(1-l^1\right)^{\frac{1-b_2}{b_2}}}{\left(l^1\right)^{\frac{1-b_2}{b_2}} + \left(\frac{\alpha_1 B_{12}}{\alpha_2 B_{22}}\right)^{\frac{1}{b_2}} \left(1-l^1\right)^{\frac{1-b_2}{b_2}}}$$

Then, the utility of a worker in country 1 is $v_L^1(1, \lambda_M) = (\alpha_1)^{\alpha_1} (\alpha_2)^{\alpha_2} w^1 / (p_1)^{\alpha_1} (p_2)^{\alpha_2} = C(\alpha_1/\alpha_2)^{\alpha_2} (1-b_1) \left[T^1 \left(\bar{F}^1\right)^{b_1} \left(L^1\right)^{(1-b_1)}\right]^{\alpha_1} \left[T^2 \left(\bar{F}^2\right)^{b_2} \left(L^2\right)^{1-b_2}\right]^{\alpha_2} / (B_{11})^{\alpha_1} (B_{22})^{\alpha_2} L^1$ and the utility of a worker in country 2 is $v_L^2(1, \lambda_M) = (\alpha_1)^{\alpha_1} (\alpha_2)^{\alpha_2} w^2 / (p_1)^{\alpha_1} (p_2)^{\alpha_2} = C(\alpha_2/\alpha_1)^{\alpha_1} (1-b_2) \left[T^1 \left(\bar{F}^1\right)^{b_1} \left(L^1\right)^{(1-b_1)}\right]^{\alpha_1} \left[T^2 \left(\bar{F}^2\right)^{b_2} \left(L^2\right)^{1-b_2}\right]^{\alpha_2} / (B_{11})^{\alpha_1} (B_{22})^{\alpha_2} L^2$. Finally, note that $v_L^1(1, \lambda_M) > v_L^2(1, \lambda_M)$ if and only if $T^1 \left(1-l^1\right) \alpha_1 (1-b_1) / \alpha_2 (1-b_2) > T^2 l^1$. Since in this region $l^1 < \alpha_1 (1-b_1) / (1-\tilde{\alpha})$, $T^1 > T^2$ implies $v_L^1(1, \lambda_M) > v_L^2(1, \lambda_M)$.

d. Suppose that country 1 is diversified and country 2 is specialized in good 2. Then, in equilibrium, $L_1^1 = (1-b_1)\left[(1-b_2)\bar{F}^1 - \left(w^1/r^1\right) b_2 L^1\right] / (b_1-b_2)\left(w^1/r^1\right)$, $L_2^1 = (1-b_2)\left[b_1 \left(w^1/r^1\right) L^1 - (1-b_1)\bar{F}^1\right] / (b_1-b_2)\left(w^1/r^1\right)$, $L_1^2 = 0$, $L_2^2 = L^2$, $F_1^1 = b_1\left[(1-b_2)\bar{F}^1 - \left(w^1/r^1\right) b_2 L^1\right] / (b_1-b_2)$, $F_2^1 = b_2\left[b_1\left(w^1/r^1\right) L^1 - (1-b_1)\bar{F}^1\right] / (b_1-b_2)$, $F_1^2 = 0, F_2^2 = \bar{F}^2$, $Q_1^1 = T^1 B_{11}\left[(1-b_2)\bar{F}^1 - \left(w^1/r^1\right) b_2 L^1\right] / \left(w^1/r^1\right)^{1-b_1} (b_1-b_2)$, $Q_1^2 = 0$, $Q_2^1 = T^1 B_{22}\left[b_1\left(w^1/r^1\right) L^1 - (1-b_1)\bar{F}^1\right] / \left(w^1/r^1\right)^{1-b_2} (b_1-b_2)$, $Q_2^2 = T^2 \left(F^2\right)^{b_2} \left(L^2\right)^{1-b_2}$, $w^1/p_z = (1-b_z) T^1 \left(F_z^1/L_z^1\right)^{b_z}$, $r^1/p_z = b_z T^1 \left(L_z^1/F_z^1\right)^{1-b_z}$ for $z = 1, 2$, $w^2/p_2 = (1-b_2) T^2 \left(F^2/L^2\right)^{b_2}$, $r^2/p_2 = b_2 T^2 \left(L^2/F^2\right)^{1-b_2}$, $p_2/p_1 = (B_{11}/B_{22})\left(w^1/r^1\right)^{b_1-b_2}$, and w^1/r^1 is the unique solution to $T^1\left[(1-\tilde{\alpha})\bar{F}^1 - \tilde{\alpha}\left(w^1/r^1\right) L^1\right] / \left(w^1/r^1\right)^{1-b_2} (b_1-b_2) = \alpha_1 T^2 \left(F^2\right)^{b_2} \left(L^2\right)^{1-b_2} / B_{22}$. For this to be an equilibrium we need $w^1/r^1 \geq w^2/r^2$ and $(1-b_1)\bar{F}^1/b_1 L^1 < w^1/r^1 < (1-b_2)\bar{F}^1/b_2 L^1$. $w^1/r^1 \geq w^2/r^2$ if and only if $\bar{f}^1 \geq \left[\alpha_1(b_1-b_2) + b_2(1-\tilde{\alpha}) l^1\right] / \tilde{\alpha}(1-b_2)$. $(1-b_1)\bar{F}^1/b_1 L^1 < w^1/r^1$ if and only if $\alpha_1 T^2 \left(F^2\right)^{b_2} \left(L^2\right)^{1-b_2} / B_{22} < \alpha_2 T^1 \left(F^1\right)^{b_2} \left(L^1\right)^{1-b_2} / B_{12}$. $w^1/r^1 < (1-b_2)\bar{F}^1/b_2 L^1$ if and only if $\left[T^2 \left(\bar{F}^2\right)^{b_2} \left(L^2\right)^{1-b_2} / T^1 \left(\bar{F}^1\right)^{b_2} \left(L^1\right)^{1-b_2}\right] (b_1-b_2) > (b_2-b_1)$, which always holds. Thus, we require

$$\bar{f}^1 \geq \frac{\alpha_1(b_1-b_2) + b_2(1-\tilde{\alpha}) l^1}{\tilde{\alpha}(1-b_2)} \text{ and } \bar{f}^1 > \frac{\left(\frac{\alpha_1 B_{12}}{\alpha_2 B_{22}}\right)^{\frac{1}{b_2}} \left(1-l^1\right)^{\frac{1-b_2}{b_2}}}{\left(l^1\right)^{\frac{1-b_2}{b_2}} + \left(\frac{\alpha_1 B_{12}}{\alpha_2 B_{22}}\right)^{\frac{1}{b_2}} \left(1-l^1\right)^{\frac{1-b_2}{b_2}}}$$

Then, the utility of a worker in country 1 is $v_L^1(1, \lambda_M) = (\alpha_1)^{\alpha_1} (\alpha_2)^{\alpha_2} w^1 / (p_1)^{\alpha_1} (p_2)^{\alpha_2} = T^1 C \left(w^1/r^1\right)^{\tilde{\alpha}}$ and the utility of a worker in country 2 is $v_L^2(1, \lambda_M) = (\alpha_1)^{\alpha_1} (\alpha_2)^{\alpha_2} w^2 / (p_1)^{\alpha_1} (p_2)^{\alpha_2} = T^2 C \left(w^1/r^1\right)^{\alpha_1(b_1-b_2)} \left[(1-b_2)\bar{F}^2/b_2 L^2\right]^{b_2}$. Finally, note that $v_L^1(1, \lambda_M) > v_L^2(1, \lambda_M)$ if and only if $T^1 \left(w^1/r^1\right)^{b_2} > T^2 \left[(1-b_2)\bar{F}^2/b_2 L^2\right]^{b_2}$. Since in this region $w^1/r^1 \geq w^2/r^2 = (1-b_2)\bar{F}^2/b_2 L^2$, $T^1 > T^2$ implies $v_L^1(1, \lambda_M) > v_L^2(1, \lambda_M)$.

e. Suppose that country 1 is diversified and country 2 is specialized in good 1. Then, in equilibrium, $L_1^1 = (1-b_1)\left[(1-b_2)\bar{F}^1 - \left(w^1/r^1\right) b_2 L^1\right] / (b_1-b_2)\left(w^1/r^1\right)$, $L_2^1 = (1-b_2)\left[b_1\left(w^1/r^1\right) L^1 - (1-b_1)\bar{F}^1\right] / (b_1-b_2)\left(w^1/r^1\right)$, $L_1^2 = L^2$, $L_2^2 = 0$, $F_1^1 = b_1\left[(1-b_2)\bar{F}^1 - \left(w^1/r^1\right) b_2 L^1\right] / (b_1-b_2)$, $F_2^1 = b_2\left[b_1\left(w^1/r^1\right) L^1 - (1-b_1)\bar{F}^1\right] / (b_1-b_2)$,

$F_1^2 = \bar{F}^2$, $F_2^2 = 0$, $Q_1^1 = T^1 B_{11}\left[\left(1-b_2\right)\bar{F}^1 - \left(w^1/r^1\right) b_2 L^1\right] / \left(w^1/r^1\right)^{1-b_1}\left(b_1 - b_2\right)$, $Q_1^2 = T^2\left(\bar{F}^2\right)^{b_1}\left(L^2\right)^{1-b_1}$, $Q_2^1 = T^1 B_{22}\left[b_1\left(w^1/r^1\right)L^1 - \left(1-b_1\right)\bar{F}^1\right] / \left(w^1/r^1\right)^{1-b_2}\left(b_1-b_2\right)$, $Q_2^2 = 0$, $w^1/p_z = \left(1-b_z\right)T^1\left(F_z^1/L_z^1\right)^{b_z}$, $r^1/p_z = b_z T^1\left(L_z^1/F_z^1\right)^{1-b_z}$ for $z = 1, 2$, $w^2/p_1 = \left(1-b_1\right)T^2\left(\bar{F}^2/L^2\right)^{b_1}$, $r^2/p_1 = b_1 T^2\left(L^2/\bar{F}^2\right)^{1-b_1}$, $p_2/p_1 = \left(B_{11}/B_{22}\right)\left(w^1/r^1\right)^{b_1-b_2}$, and w^1/r^1 is the unique solution to $\alpha_2 T^2\left(\bar{F}^2\right)^{b_1}\left(L^2\right)^{1-b_1}/B_{11} = T^1\left[\tilde{\alpha}\left(w^1/r^1\right)L^1 - \left(1-\tilde{\alpha}\right)\bar{F}^1\right] / \left(b_1-b_2\right)\left(w^1/r^1\right)^{1-b_1}$. For this to be an equilibrium we need $w^1/r^1 \leq w^2/r^2$ and $\left(1-b_1\right)\bar{F}^1/b_1L^1 < w^1/r^1 < \left(1-b_2\right)\bar{F}^1/b_2L^1$. $w^1/r^1 \leq w^2/r^2$ if and only if $\bar{f}^1 \leq \left[b_1\left(1-\tilde{\alpha}\right)l^1 - \alpha_2\left(b_1-b_2\right)\right]/\tilde{\alpha}\left(1-b_1\right)$. $\left(1-b_1\right)\bar{F}^1/b_1L^1 < w^1/r^1$ if and only if $\left[T^2\left(\bar{F}^2\right)^{b_1}\left(L^2\right)^{1-b_1}/T^1\left(\bar{F}^1\right)^{b_1}\left(L^1\right)^{1-b_1}\right]\left(b_1-b_2\right) > \left(b_2-b_1\right)$, which always holds. $w^1/r^1 < \left(1-b_2\right)\bar{F}^1/b_2L^1$ if and only if $\bar{f}^1 > \left(\alpha_2B_{21}/\alpha_1B_{11}\right)^{\frac{1}{b_1}}\left(1-l^1\right)^{\frac{1-b_1}{b_1}} / \left[\left(l^1\right)^{\frac{1-b_1}{b_1}} + \left(\alpha_2B_{21}/\alpha_1B_{11}\right)^{\frac{1}{b_1}}\left(1-l^1\right)^{\frac{1-b_1}{b_1}}\right]$. Thus, we require

$$\frac{\left(\frac{\alpha_2B_{21}}{\alpha_1B_{11}}\right)^{\frac{1}{b_1}}\left(1-l^1\right)^{\frac{1-b_1}{b_1}}}{\left(l^1\right)^{\frac{1-b_1}{b_1}} + \left(\frac{\alpha_2B_{21}}{\alpha_1B_{11}}\right)^{\frac{1}{b_1}}\left(1-l^1\right)^{\frac{1-b_1}{b_1}}} < \bar{f}^1 \leq \frac{b_1\left(1-\tilde{\alpha}\right)l^1 - \alpha_2\left(b_1-b_2\right)}{\tilde{\alpha}\left(1-b_1\right)}$$

Then, the utility of a worker in country 1 is $v_L^1\left(1,\lambda_M\right) = \left(\alpha_1\right)^{\alpha_1}\left(\alpha_2\right)^{\alpha_2} w^1/\left(p_1\right)^{\alpha_1}\left(p_2\right)^{\alpha_2} = T^1 C\left(w^1/r^1\right)^{\tilde{\alpha}}$ and the utility of a worker in country 2 is $v_L^2\left(1,\lambda_M\right) = \left(\alpha_1\right)^{\alpha_1}\left(\alpha_2\right)^{\alpha_2} w^2/\left(p_1\right)^{\alpha_1}\left(p_2\right)^{\alpha_2} = T^2 C\left[\left(1-b_1\right)\bar{F}^2/b_1L^2\right]^{b_1}\left(r^1/w^1\right)^{\alpha_2\left(b_1-b_2\right)}$. Finally, note that $v_L^1\left(1,\lambda_M\right) > v_L^2\left(1,\lambda_M\right)$ if and only if $T^1\left(w^1/r^1\right)^{b_1} > T^2\left[\left(1-b_1\right)\bar{F}^2/b_1L^2\right]^{b_1}$ or, which is equivalent

$$\bar{f}^1 > \frac{\tilde{\alpha}\left(\frac{T^2}{T^1}\right)^{\frac{1}{b_1}}\left(1-b_1\right)l^1 - \left(b_1-b_2\right)\alpha_2\left(1-l^1\right)\left(\frac{T^2}{T^1}\right)^{\frac{1-b_1}{b_1}}}{\tilde{\alpha}\left(\frac{T^2}{T^1}\right)^{\frac{1}{b_1}}\left(1-b_1\right)l^1 - \left(b_1-b_2\right)\alpha_2\left(\frac{T^2}{T^1}\right)^{\frac{1-b_1}{b_1}}\left(1-l^1\right) + b_1\left(1-\tilde{\alpha}\right)\left(1-l^1\right)}$$

f. Suppose that country 1 is specialized in good 2 and country 2 is good 1. Then, in equilibrium, $L_1^1 = 0$, $L_2^1 = L^1$, $L_1^2 = L^2$, $L_2^2 = 0$, $F_1^1 = 0$, $F_2^1 = \bar{F}^1$, $F_1^2 = \bar{F}^2$, $F_2^2 = 0$, $Q_1^1 = 0$, $Q_1^2 = T^2\left(\bar{F}^2\right)^{b_1}\left(L^2\right)^{1-b_1}$, $Q_2^1 = T^1\left(\bar{F}^1\right)^{b_2}\left(L^1\right)^{1-b_2}$, $Q_2^2 = 0$, $w^1/p_2 = \left(1-b_2\right)T^1\left(\bar{F}^1/L^1\right)^{b_2}$, $r^1/p_2 = b_2T^1\left(L^1/\bar{F}^1\right)^{1-b_2}$, $w^2/p_1 = \left(1-b_1\right)T^2\left(\bar{F}^2/L^2\right)^{b_1}$, $r^2/p_1 = b_1T^2\left(L^2/\bar{F}^2\right)^{1-b_1}$, and $p_2/p_1 = \alpha_2T^2\left(\bar{F}^2\right)^{b_1}\left(L^2\right)^{1-b_1}/\alpha_1T^1\left(\bar{F}^1\right)^{b_2}\left(L^1\right)^{1-b_2}$. For this to be an equilibrium we need $\left(r^1\right)^{b_1}\left(w^1\right)^{1-b_1}/T^1 \geq \left(r^2\right)^{b_1}\left(w^2\right)^{1-b_1}/T^2$ and $\left(r^1\right)^{b_2}\left(w^1\right)^{1-b_2}/T^1 \leq \left(r^2\right)^{b_2}\left(w^2\right)^{1-b_2}/T^2$. $\left(r^1\right)^{b_1}\left(w^1\right)^{1-b_1}/T^1 \geq \left(r^2\right)^{b_1}\left(w^2\right)^{1-b_1}/T^2$ if and only $\bar{f}^1 \leq \left(\alpha_2B_{21}/\alpha_1B_{11}\right)^{\frac{1}{b_1}}\left(1-l^1\right)^{\frac{1-b_1}{b_1}} / \left[\left(l^1\right)^{\frac{1-b_1}{b_1}} + \left(\alpha_2B_{21}/\alpha_1B_{11}\right)^{\frac{1}{b_1}}\left(1-l^1\right)^{\frac{1-b_1}{b_1}}\right]$. $\left(r^1\right)^{b_2}\left(w^1\right)^{1-b_2}/T^2 \leq \left(r^2\right)^{b_2}\left(w^2\right)^{1-b_2}/T^2$ if and only if $\bar{f}^1 \geq \left(\alpha_2B_{22}/\alpha_1B_{12}\right)^{\frac{1}{b_2}}\left(1-l^1\right)^{\frac{1-b_2}{b_2}} / \left[\left(l^1\right)^{\frac{1-b_2}{b_2}} + \left(\alpha_2B_{22}/\alpha_1B_{12}\right)^{\frac{1}{b_2}}\left(1-l^1\right)^{\frac{1-b_2}{b_2}}\right]$. Thus, we require

$$\frac{\left(\frac{\alpha_2B_{22}}{\alpha_1B_{12}}\right)^{\frac{1}{b_2}}\left(1-l^1\right)^{\frac{1-b_2}{b_2}}}{\left(l^1\right)^{\frac{1-b_2}{b_2}} + \left(\frac{\alpha_2B_{22}}{\alpha_1B_{12}}\right)^{\frac{1}{b_2}}\left(1-l^1\right)^{\frac{1-b_2}{b_2}}} \leq \bar{f}^1 \leq \frac{\left(\frac{\alpha_2B_{21}}{\alpha_1B_{11}}\right)^{\frac{1}{b_1}}\left(1-l^1\right)^{\frac{1-b_1}{b_1}}}{\left(l^1\right)^{\frac{1-b_1}{b_1}} + \left(\frac{\alpha_2B_{21}}{\alpha_1B_{11}}\right)^{\frac{1}{b_1}}\left(1-l^1\right)^{\frac{1-b_1}{b_1}}}$$

Then, the utility of a worker in country 1 is $v_L^1(1,\lambda_M) = (\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2} w^1/(p_1)^{\alpha_1}(p_2)^{\alpha_2} = C(T^2)^{\alpha_1}(T^1)^{\alpha_2}(\alpha_2/\alpha_1)^{\alpha_1}(1-b_2)(\bar{F}^2)^{\alpha_1 b_1}(L^2)^{\alpha_1(1-b_1)}(\bar{F}^1)^{\alpha_2 b_2}(L^1)^{\alpha_2(1-b_2)}/(B_{11})^{\alpha_1}(B_{22})^{\alpha_2}L^1$ and the utility of a worker in country 2 is $v_L^2(1,\lambda_M) = (\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2} w^2/(p_1)^{\alpha_1}(p_2)^{\alpha_2} = C(T^2)^{\alpha_1}(T^1)^{\alpha_2}(1-b_1)(\alpha_1/\alpha_2)^{\alpha_2}(\bar{F}^1)^{\alpha_2 b_2}(L^1)^{\alpha_2(1-b_2)}(\bar{F}^2)^{\alpha_1 b_1}(L^2)^{\alpha_1(1-b_1)}/(B_{11})^{\alpha_1}(B_{22})^{\alpha_2}L^2$. Finally, note that $v_L^1(1,\lambda_M) > v_L^2(1,\lambda_M)$ if and only if $L^2\alpha_2(1-b_2) > \alpha_1(1-b_1)L^1$ or, which is equivalent

$$l^1 < \frac{T^1\alpha_2(1-b_2)}{T^2\alpha_1(1-b_1)+T^1\alpha_2(1-b_2)}$$

g. Suppose that country 1 is specialized in good 2 and country 2 is diversified. Then, in equilibrium, $L_1^1 = 0$, $L_2^1 = L^1$, $L_1^2 = (1-b_1)\left[(1-b_2)\bar{F}^2 - \left(w^2/r^2\right)b_2L^2\right]/(b_1-b_2)\left(w^2/r^2\right)$, $L_2^2 = (1-b_2)\left[b_1\left(w^2/r^2\right)L^2-(1-b_1)\bar{F}^2\right]/(b_1-b_2)\left(w^2/r^2\right)$, $F_1^1 = 0$, $F_2^1 = \bar{F}^1$, $F_1^2 = b_1\left[(1-b_2)\bar{F}^2-\left(w^2/r^2\right)b_2L^2\right]/(b_1-b_2)$, $F_2^2 = b_2\left[b_1\left(w^2/r^2\right)L^2-(1-b_1)\bar{F}^2\right]/(b_1-b_2)$, $Q_1^1 = 0$, $Q_1^2 = T^2B_{11}\left[(1-b_2)\bar{F}^2-\left(w^2/r^2\right)b_2L^2\right]/\left(w^2/r^2\right)^{1-b_1}(b_1-b_2)$, $Q_2^1 = T^1\left(\bar{F}^1\right)^{b_2}\left(L^1\right)^{1-b_2}$, $Q_2^2 = T^2B_{22}\left[b_1\left(w^2/r^2\right)L^2-(1-b_1)\bar{F}^2\right]/\left(w^2/r^2\right)^{1-b_2}(b_1-b_2)$, $w^1/p_2 = (1-b_2)T^1\left(\bar{F}^1/L^1\right)^{b_2}$, $r^1/p_2 = b_2T^1\left(L^1/\bar{F}^1\right)^{1-b_2}$, $w^2/p_z = (1-b_z)T^2\left(F_z^2/L_z^2\right)^{b_z}$, $r^2/p_z = b_zT^2\left(L_z^2/F_z^2\right)^{1-b_z}$ for $z = 1,2$, $p_2/p_1 = (B_{11}/B_{22})\left(w^2/r^2\right)^{b_1-b_2}$, and w^2/r^2 is the unique solution to $\alpha_1T^1\left(\bar{F}^1\right)^{b_2}\left(L^1\right)^{1-b_2}/B_{22} = \left[(1-\tilde{\alpha})T^2\bar{F}^2-\tilde{\alpha}\left(w^2/r^2\right)T^2L^2\right]/\left(w^2/r^2\right)^{1-b_2}(b_1-b_2)$. For this to be an equilibrium it must be the case that $w^1/r^1 \le w^2/r^2$ and $(1-b_1)\bar{F}^2/b_1L^2 < w^2/r^2 < (1-b_2)\bar{F}^2/b_2L^2$. $w^1/r^1 \le w^2/r^2$ if and only if $\bar{f}^1 \le b_2(1-\tilde{\alpha})l^1/\tilde{\alpha}(1-b_2)$. $w^2/r^2 > (1-b_1)\bar{F}^2/b_1L^2$ if and only if $\bar{f}^1 < (\alpha_2B_{22}/\alpha_1B_{12})^{\frac{1}{b_2}}\left(1-l^1\right)^{\frac{1-b_2}{b_2}}/\left[\left(l^1\right)^{\frac{1-b_2}{b_2}}+(\alpha_2B_{22}/\alpha_1B_{12})^{\frac{1}{b_2}}\left(1-l^1\right)^{\frac{1-b_2}{b_2}}\right]$. $w^2/r^2 < (1-b_2)\bar{F}^2/b_2L^2$ if and only if $\left[T^1\left(\bar{F}^1\right)^{b_2}\left(L^1\right)^{1-b_2}/T^2\left(\bar{F}^2\right)^{b_2}\left(L^2\right)^{1-b_2}\right](b_1-b_2) > \alpha_1(b_2-b_1)$, which always holds. Thus, we need

$$\bar{f}^1 \le \frac{b_2(1-\tilde{\alpha})l^1}{\tilde{\alpha}(1-b_2)} \text{ and } \bar{f}^1 < \frac{\left(\frac{\alpha_2B_{22}}{\alpha_1B_{12}}\right)^{\frac{1}{b_2}}\left(1-l^1\right)^{\frac{1-b_2}{b_2}}}{\left(l^1\right)^{\frac{1-b_2}{b_2}}+\left(\frac{\alpha_2B_{22}}{\alpha_1B_{12}}\right)^{\frac{1}{b_2}}\left(1-l^1\right)^{\frac{1-b_2}{b_2}}}$$

Then, the utility of a worker in country 1 is $v_L^1(1,\lambda_M) = (\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2}w^1/(p_1)^{\alpha_1}(p_2)^{\alpha_2} = T^1C\left[(1-b_2)\bar{F}^1/b_2L^1\right]^{b_2}\left(w^2/r^2\right)^{\alpha_1(b_1-b_2)}$ and the utility of a worker in country 2 is $v_L^2(1,\lambda_M) = (\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2}w^2/(p_1)^{\alpha_1}(p_2)^{\alpha_2} = T^2C\left(w^2/r^2\right)^{\tilde{\alpha}}$. Finally, note that $v_L^1(1,\lambda_M) > v_L^2(1,\lambda_M)$ if and only if $T^1\left[(1-b_2)\bar{F}^1/b_2L^1\right]^{b_2} > T^2\left(w^2/r^2\right)^{b_2}$ or, which is equivalent

$$\bar{f}^1 > \frac{(1-\tilde{\alpha})b_2l^1}{\alpha_1\left(\frac{T^1}{T^2}\right)^{\frac{1-b_2}{b_2}}(b_1-b_2)l^1+(1-\tilde{\alpha})b_2l^1+\tilde{\alpha}\left(\frac{T^1}{T^2}\right)^{\frac{1}{b_2}}(1-b_2)\left(1-l^1\right)}$$

Proof of Part 2. From part 1:
a. $v_L^1(1,\lambda_M) > v_L^2(1,\lambda_M)$ if and only if:

$$\bar{f}^1 > \frac{(1-\tilde{\alpha})b_2l^1}{\alpha_1\left(T^1/T^2\right)^{\frac{1-b_2}{b_2}}(b_1-b_2)l^1+(1-\tilde{\alpha})b_2l^1+\tilde{\alpha}\left(T^1/T^2\right)^{\frac{1}{b_2}}(1-b_2)\left(1-l^1\right)}$$

for $l^1 < T^1\alpha_2(1-b_2) / \left[T^2\alpha_1(1-b_1) + T^1\alpha_2(1-b_2)\right]$; or

$$\bar{f}^1 > \frac{\tilde{\alpha}\left(T^2/T^1\right)^{\frac{1}{b_1}}(1-b_1)\,l^1 - (b_1-b_2)\,\alpha_2\left(1-l^1\right)\left(T^2/T^1\right)^{\frac{1-b_1}{b_1}}}{\tilde{\alpha}\,(T^2/T^1)^{\frac{1}{b_1}}(1-b_1)\,l^1 - (b_1-b_2)\,\alpha_2\,(T^2/T^1)^{\frac{1-b_1}{b_1}}(1-l^1) + b_1(1-\tilde{\alpha})(1-l^1)}$$

for $l^1 \geq T^1\alpha_2(1-b_2) / \left[T^2\alpha_1(1-b_1) + T^1\alpha_2(1-b_2)\right]$.

b. $v_L^1(1,\lambda_M) < v_L^2(1,\lambda_M)$ if and only if

$$\bar{f}^1 < \frac{(1-\tilde{\alpha})\,b_2l^1}{\alpha_1\,(T^1/T^2)^{\frac{1-b_2}{b_2}}(b_1-b_2)\,l^1 + (1-\tilde{\alpha})\,b_2l^1 + \tilde{\alpha}\,(T^1/T^2)^{\frac{1}{b_2}}(1-b_2)(1-l^1)}$$

for $l^1 < T^1\alpha_2(1-b_2) / \left[T^2\alpha_1(1-b_1) + T^1\alpha_2(1-b_2)\right]$; or

$$\bar{f}^1 < \frac{\tilde{\alpha}\left(T^2/T^1\right)^{\frac{1}{b_1}}(1-b_1)\,l^1 - (b_1-b_2)\,\alpha_2\left(1-l^1\right)\left(T^2/T^1\right)^{\frac{1-b_1}{b_1}}}{\tilde{\alpha}\,(T^2/T^1)^{\frac{1}{b_1}}(1-b_1)\,l^1 - (b_1-b_2)\,\alpha_2\,(T^2/T^1)^{\frac{1-b_1}{b_1}}(1-l^1) + b_1(1-\tilde{\alpha})(1-l^1)}$$

for $l^1 \geq T^1\alpha_2(1-b_2) / \left[T^2\alpha_1(1-b_1) + T^1\alpha_2(1-b_2)\right]$.

c. $v_L^1(1,\lambda_M) = v_L^2(1,\lambda_M)$ if and only if

$$\bar{f}^1 = \frac{(1-\tilde{\alpha})\,b_2l^1}{\alpha_1\,(T^1/T^2)^{\frac{1-b_2}{b_2}}(b_1-b_2)\,l^1 + (1-\tilde{\alpha})\,b_2l^1 + \tilde{\alpha}\,(T^1/T^2)^{\frac{1}{b_2}}(1-b_2)(1-l^1)}$$

for $l^1 < T^1\alpha_2(1-b_2) / \left[T^2\alpha_1(1-b_1) + T^1\alpha_2(1-b_2)\right]$; or

$$\bar{f}^1 = \frac{\tilde{\alpha}\left(T^2/T^1\right)^{\frac{1}{b_1}}(1-b_1)\,l^1 - (b_1-b_2)\,\alpha_2\left(1-l^1\right)\left(T^2/T^1\right)^{\frac{1-b_1}{b_1}}}{\tilde{\alpha}\,(T^2/T^1)^{\frac{1}{b_1}}(1-b_1)\,l^1 - (b_1-b_2)\,\alpha_2\,(T^2/T^1)^{\frac{1-b_1}{b_1}}(1-l^1) + b_1(1-\tilde{\alpha})(1-l^1)}$$

for $l^1 > T^1\alpha_2(1-b_2) / \left[T^2\alpha_1(1-b_1) + T^1\alpha_2(1-b_2)\right]$; or

$$\bar{f}^1 \in \left[\begin{array}{c}\frac{(1-\tilde{\alpha})b_2l^1}{\alpha_1(T^1/T^2)^{\frac{1-b_2}{b_2}}(b_1-b_2)l^1+(1-\tilde{\alpha})b_2l^1+\tilde{\alpha}(T^1/T^2)^{\frac{1}{b_2}}(1-b_2)(1-l^1)},\\ \frac{\tilde{\alpha}\left(T^2/T^1\right)^{\frac{1}{b_1}}(1-b_1)l^1-(b_1-b_2)\alpha_2\left(1-l^1\right)\left(T^2/T^1\right)^{\frac{1-b_1}{b_1}}}{\tilde{\alpha}(T^2/T^1)^{\frac{1}{b_1}}(1-b_1)l^1-(b_1-b_2)\alpha_2(T^2/T^1)^{\frac{1-b_1}{b_1}}(1-l^1)+b_1(1-\tilde{\alpha})(1-l^1)}\end{array}\right]$$

for $l^1 = T^1\alpha_2(1-b_2) / \left[T^2\alpha_1(1-b_1) + T^1\alpha_2(1-b_2)\right]$.

Next we prove three results that will allow us to compare $v_L^j(1,0)$ and $v_L^j(1,1)$.

Result 1: $v_L^1(1,\lambda_M)$ and $v_L^2(1,\lambda_M)$ are continuous functions of L^1 and L^2 for any T^1, T^2, $\bar{F}^1$ and $\bar{F}^2$. From a simple inspection of $v_L^1(1,\lambda_M)$ and $v_L^2(1,\lambda_M)$, it is clear that, in the interior of each of the seven regions, $v_L^1(1,\lambda_M)$ and $v_L^2(1,\lambda_M)$ are continuous. It is also easy to prove that $v_L^1(1,\lambda_M)$ and $v_L^2(1,\lambda_M)$ are continuous in the boundary of each region. For example, the intersection between the boundaries of regions a and b is $\left\{\left(l^1,\bar{f}^1\right) : l^1 \in [0,(1-b_1)\,\alpha_1/(1-\tilde{\alpha})(1-\alpha_2b_2)],\ \bar{f}^1 = b_1(1-\tilde{\alpha})\,l^1/\tilde{\alpha}(1-b_1)\right\}$ or, which is equivalent,

$L^1 = \psi\left(\bar{L}^1+\bar{L}^2\right)$, with $\psi = \tilde{\alpha}(1-b_1)T^2\bar{F}^1/\left[\alpha_2(b_1-b_2)T^1\bar{F}^1+T^2\bar{F}^2b_1(1-\tilde{\alpha})+\tilde{\alpha}(1-b_1)T^2\bar{F}^1\right]$ and $T^1\bar{F}^1 \in \left[0,(\alpha_1b_1/\alpha_2b_2)T^2\bar{F}^2\right]$. In region a, $v_L^1(1,\lambda_M) = T^1C\left[(1-\tilde{\alpha})/\tilde{\alpha}\right]^{\tilde{\alpha}}\left[\left(T^1\bar{F}^1+T^2\bar{F}^2\right)/\left(T^1L^1+T^2L^2\right)\right]^{\tilde{\alpha}}$, while in region b, $v_L^1(1,\lambda_M) = CT^1\left[(1-b_1)\bar{F}^1/b_1L^1\right]^{b_1}\left(r^2/w^2\right)^{\alpha_2(b_1-b_2)}$, where w^2/r^2 is the unique solution to $\alpha_2T^1\left(\bar{F}^1\right)^{b_1}\left(L^1\right)^{1-b_1}/B_{11} = \left[\tilde{\alpha}T^2\left(w^2/r^2\right)L^2-(1-\tilde{\alpha})T^2\bar{F}^2\right]/\left(w^2/r^2\right)^{1-b_1}(b_1-b_2)$. Therefore, in the boundary we have:

$$\lim_{L^1\to\psi\left(\bar{L}^1+\bar{L}^2\right)^+} v_L^1(1,\lambda_M) = CT^1\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\tilde{\alpha}}\left[\frac{T^1\bar{F}^1+T^2\bar{F}^2}{T^1\psi\left(\bar{L}^1+\bar{L}^2\right)+T^2(1-\psi)\left(\bar{L}^1+\bar{L}^2\right)}\right]^{\tilde{\alpha}}$$

$$\lim_{L^1\to\psi\left(\bar{L}^1+\bar{L}^2\right)^-} v_L^1(1,\lambda_M) = v_L^1(1,\lambda_M) = CT^1\left[\frac{(1-b_1)\bar{F}^1}{b_1\psi\left(\bar{L}^1+\bar{L}^2\right)}\right]^{\tilde{\alpha}}$$

Moreover, note that when $\lim_{L^1\to\psi\left(\bar{L}^1+\bar{L}^2\right)^+} v_L^1(1,\lambda_M) = \lim_{L^1\to\psi\left(\bar{L}^1+\bar{L}^2\right)^-} v_L^1(1,\lambda_M) = v_L^1(1,\lambda_M)$. Thus, $v_L^1(1,\lambda_M)$ is continuous at $L^1 = \psi\left(\bar{L}^1+\bar{L}^2\right)$.

Result 2: $v_L^1(1,\lambda_M)$ is decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$. In region a, $v_L^1(1,\lambda_M) = T^1C\left[(1-\tilde{\alpha})/\tilde{\alpha}\right]^{\tilde{\alpha}}\left[\left(T^1\bar{F}^1+T^2\bar{F}^2\right)/\left(T^1L^1+T^2L^2\right)\right]^{\tilde{\alpha}}$, which is decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$. In region b, $v_L^1(1,\lambda_M) = CT^1\left[(1-b_1)\bar{F}^1/b_1L^1\right]^{b_1}\left(r^2/w^2\right)^{\alpha_2(b_1-b_2)}$, where w^2/r^2 is the unique solution to $\alpha_2T^1\left(\bar{F}^1\right)^{b_1}\left(L^1\right)^{1-b_1}/B_{11} = \left[\tilde{\alpha}T^2\left(w^2/r^2\right)L^2-(1-\tilde{\alpha})T^2\bar{F}^2\right]/\left(w^2/r^2\right)^{1-b_1}(b_1-b_2)$. Since w^2/r^2 is increasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$, $v_L^1(1,\lambda_M)$ is decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$. In region c, $v_L^1(1,\lambda_M) = C(\alpha_1/\alpha_2)^{\alpha_2}(1-b_1)\left[T^1\left(\bar{F}^1\right)^{b_1}\left(L^1\right)^{(1-b_1)}\right]^{\alpha_1}\left[T^2\left(\bar{F}^2\right)^{b_2}\left(L^2\right)^{1-b_2}\right]^{\alpha_2}/(B_{11})^{\alpha_1}(B_{22})^{\alpha_2}L^1$, which is decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$. In region d, $v_L^1(1,\lambda_M) = CT^1\left(w^1/r^1\right)^{\tilde{\alpha}}$, where w^1/r^1 is the unique solution to $T^1\left[(1-\tilde{\alpha})\bar{F}^1-\tilde{\alpha}\left(w^1/r^1\right)L^1\right]/\left(w^1/r^1\right)^{1-b_2}(b_1-b_2) = \alpha_1T^2\left(F^2\right)^{b_2}\left(L^2\right)^{1-b_2}/B_{22}$. Since w^1/r^1 is decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$, $v_L^1(1,\lambda_M)$ is also decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$. In region e, $v_L^1(1,\lambda_M) = T^1C\left(w^1/r^1\right)^{\tilde{\alpha}}$, where w^1/r^1 is the unique solution to $\alpha_2T^2\left(\bar{F}^2\right)^{b_1}\left(L^2\right)^{1-b_1}/B_{11} = T^1\left[\tilde{\alpha}\left(w^1/r^1\right)L^1-(1-\tilde{\alpha})\bar{F}^1\right]/(b_1-b_2)\left(w^1/r^1\right)^{1-b_1}$. Since w^1/r^1 is decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$, $v_L^1(1,\lambda_M)$ is also decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$. In region f, $v_L^1(1,\lambda_M) = C(\alpha_2/\alpha_1)^{\alpha_1}(1-b_2)\left[A^2\left(\bar{F}^2\right)^{b_1}\left(L^2\right)^{1-b_1}\right]^{\alpha_1}\left[A^1\left(\bar{F}^1\right)^{b_2}\left(L^1\right)^{1-b_2}\right]^{\alpha_2}/(B_{11})^{\alpha_1}(B_{22})^{\alpha_2}L^1$, which is decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$. In region g, $v_L^1(1,\lambda_M) = T^1C\left[(1-b_2)\bar{F}^1/b_2L^1\right]^{b_2}\left(w^2/r^2\right)^{\alpha_1(b_1-b_2)}$, where w^2/r^2 is the unique solution to $\alpha_1T^1\left(\bar{F}^1\right)^{b_2}\left(L^1\right)^{1-b_2}/B_{22} = \left[(1-\tilde{\alpha})T^2\bar{F}^2-\tilde{\alpha}\left(w^2/r^2\right)T^2L^2\right]/\left(w^2/r^2\right)^{1-b_2}(b_1-b_2)$. Since w^2/r^2 is decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$, $v_L^1(1,\lambda_M)$ is also decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$. Finally, due to Result 1, $v_L^1(1,\lambda_M)$ is a continuous function and, hence, it must be decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$ not only in each region, but everywhere.

Result 3: Following the same steps we employed in Result 2, it is easy to prove that $v_L^2(1,\lambda_M)$ is decreasing in $L^2/\left(\bar{L}^1+\bar{L}^2\right)$. For example, in region e, $v_L^2(1,\lambda_M) = CT^2\left[(1-b_1)\bar{F}^2/b_1L^2\right]^{b_1}\left(r^1/w^1\right)^{\alpha_2(b_1-b_2)}$, where $\frac{w^1}{r^1}$ is the unique solution to $\alpha_2T^2\left(\bar{F}^2\right)^{b_1}\left(L^2\right)^{1-b_1}/B_{11} = T^1\left[\tilde{\alpha}\left(w^1/r^1\right)L^1-(1-\tilde{\alpha})\bar{F}^1\right]/(b_1-b_2)\left(w^1/r^1\right)^{1-b_1}$. Since w^1/r^1 is increasing in $L^2/\left(\bar{L}^1+\bar{L}^2\right)$, $v_L^2(1,\lambda_M)$ is decreasing in $L^2/\left(\bar{L}^1+\bar{L}^2\right)$.

From Results 1-3, if $v_L^1(1,0) > v_L^2(1,0)$, then $v_L^1(1,0) > v_L^1(1,1)$; if $v_L^2(1,0) > v_L^1(1,0)$, then $v_L^2(1,0) > v_L^2(1,1)$; and if $v_L^1(1,0) = v_L^2(1,0)$, then $v_L^1(1,0) = v_L^2(1,0) = v_L^1(1,0) = v_L^2(1,0)$. This completes the proof of the lemma. ■

Proposition 4 characterizes the equilibrium trade and labor mobility policies in a Heckscher-Ohlin economy.

Proposition 4 (Heckscher-Ohlin economy). *Assume that* $T^1 > T^2$, $\frac{T^1}{T^2} \neq \left[\frac{(1-\bar{f}^1)\bar{l}^1}{\bar{f}^1(1-\bar{l}^1)}\right]^{\tilde{\alpha}}$ *and* $\bar{f}^1 \neq \bar{l}^1$.

1. *Suppose that under free trade and no labor mobility the country relatively well endowed with factor F is diversified. Then, the trade and labor mobility game has a unique Nash equilibrium outcome: no trade and no labor mobility.*

2. *Suppose that under free trade and no labor mobility the country relatively well endowed with factor F specializes in the F-intensive good (i.e., good 1). Define:*

$$\kappa = \left\{ \begin{array}{cc} \multicolumn{2}{c}{\left(\bar{l}^1, \bar{f}^1\right) \in [0,1]^2 \text{ such that:}} \\ \bar{l}^1 < \bar{l}_b \text{ and } \bar{f}^1 \geq \eta_b\left(\bar{l}^1\right) & \text{when } \eta_3\left(\bar{l}^1\right) \leq \bar{f}^1 < \eta_5\left(\bar{l}^1\right) \\ \eta_c^L\left(\bar{l}^1\right) \leq \bar{f}^1 \leq \eta_c^H\left(\bar{l}^1\right) & \text{when } \eta_5\left(\bar{l}^1\right) \leq \bar{f}^1 \leq \eta_6\left(\bar{l}^1\right) \\ \bar{l}^1 > \bar{l}_e \text{ and } \bar{f}^1 < \eta_e\left(\bar{l}^1\right) & \text{when } \eta_8\left(\bar{l}^1\right) < \bar{f}^1 \leq \eta_2\left(\bar{l}^1\right) \\ \eta_f^L\left(\bar{l}^1\right) \leq \bar{f}^1 \leq \eta_f^H\left(\bar{l}^1\right) & \text{when } \eta_7\left(\bar{l}^1\right) \leq \bar{f}^1 \leq \eta_8\left(\bar{l}^1\right) \end{array} \right\}$$

   (a) *If $\left(\bar{l}^1, \bar{f}^1\right) \in \kappa$, then the trade and labor mobility game has two Nash equilibrium outcomes: (i) no trade and no labor mobility; and (ii) free trade and no labor mobility. Moreover, $W_G^j(1,0) > W_G^j(0,0)$ for $j = 1,2$.*

   (b) *If $\left(\bar{l}^1, \bar{f}^1\right) \notin \kappa$, then the trade and labor mobility game has a unique Nash equilibrium outcome: no trade and no labor mobility.*

Proof:

No trade and no labor mobility is always a Nash equilibrium outcome.

Free trade and no labor mobility could be a Nash equilibrium outcome only if, under free trade and no labor mobility, the country relatively well endowed with factor F specializes in good 1. In order to prove this we must consider seven possible cases. From Lemma 4 Part 1, $v_L^j(0,0) = CT^j\left[(1-\tilde{\alpha})/\tilde{\alpha}\right]^{\tilde{\alpha}}\left(\bar{F}^j/\bar{L}^j\right)^{\tilde{\alpha}}$ for $j = 1,2$.

a. Suppose that $\max\left\{\eta_1\left(\bar{l}^1\right), \eta_2\left(\bar{l}^1\right)\right\} < \bar{f}^1 < \min\left\{\eta_3\left(\bar{l}^1\right), \eta_4\left(\bar{l}^1\right)\right\}$. Then, from Lemma 5 Part 1-a, both countries are diversified and $v_L^j(1,0) = T^jC\left[(1-\tilde{\alpha})/\tilde{\alpha}\right]^{\tilde{\alpha}}\left[\left(T^1\bar{F}^1 + T^2\bar{F}^2\right)/\left(T^1\bar{L}^1 + T^2\bar{L}^2\right)\right]^{\tilde{\alpha}}$. Moreover, $\bar{f}^1 > \bar{l}^1$ implies $W_G^1(0,0) = v_L^1(0,0) > v_L^1(1,0) = W_G^1(1,0)$, while $\bar{f}^1 < \bar{l}^1$ implies $W_G^2(0,0) = v_L^2(0,0) > v_L^1(1,0) = W_G^2(1,0)$. Thus, free trade and no labor mobility is not a Nash equilibrium outcome.

b. Suppose that $\eta_3\left(\bar{l}^1\right) \leq \bar{f}^1 < \eta_5\left(\bar{l}^1\right)$, which implies $\bar{f}^1 > \bar{l}^1$ (i.e., country 1 is relatively well endowed with factor F). Then, from Lemma 5 Part 1-b, country 1 specializes in good 1, $v_L^1(1,0) = T^1C\left[(1-b_1)\bar{F}^1/b_1\bar{L}^1\right]^{b_1}\left(r^2/w^2\right)^{\alpha_2(b_1-b_2)}$ and $v_L^2(1,0) = T^2C\left(w^2/r^2\right)^{\tilde{\alpha}}$, where w^2/r^2 is the unique solution to $\alpha_2T^1\left(\bar{F}^1\right)^{b_1}\left(\bar{L}^1\right)^{1-b_1}/B_{11} = \left[\tilde{\alpha}T^2\left(w^2/r^2\right)\bar{L}^2 - (1-\tilde{\alpha})T^2\bar{F}^2\right]/\left(w^2/r^2\right)^{1-b_1}(b_1-b_2)$.

Moreover $W_G^1(1,0) = v_L^1(1,0) \geq v_L^1(0,0) = W_G^1(0,0)$ if and only if $w^2/r^2 \leq [(1-b_1)/b_1]^{\frac{b_1}{\alpha_2(b_1-b_2)}} [\tilde{\alpha}/(1-\tilde{\alpha})]^{\frac{\tilde{\alpha}}{\alpha_2(b_1-b_2)}} \left(\bar{F}^1/\bar{L}^1\right)$ or, which is equivalent:

$$\bar{l}^1 < \bar{l}_b = \frac{B_{11}\tilde{\alpha}\left(\frac{1-b_1}{b_1}\right)^{\frac{b_1 b_1}{\alpha_2(b_1-b_2)}}}{B_{11}\tilde{\alpha}\left(\frac{1-b_1}{b_1}\right)^{\frac{b_1 b_1}{\alpha_2(b_1-b_2)}} + \alpha_2(b_1-b_2)\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\frac{\tilde{\alpha}b_1}{\alpha_2(b_1-b_2)}}}$$

$$\bar{f}^1 \geq \eta_b\left(\bar{l}^1\right) = \frac{B_{11}(1-\tilde{\alpha})\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\frac{\tilde{\alpha}}{\alpha_2(b_1-b_2)}}\left(\frac{b_1}{1-b_1}\right)^{\frac{b_1(1-b_1)}{\alpha_2(b_1-b_2)}}\bar{l}^1}{\begin{array}{c}B_{11}(1-\tilde{\alpha})\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\frac{\tilde{\alpha}}{\alpha_2(b_1-b_2)}}\left(\frac{b_1}{1-b_1}\right)^{\frac{b_1(1-b_1)}{\alpha_2(b_1-b_2)}}\bar{l}^1 + \\ B_{11}\left(\frac{1-b_1}{b_1}\right)^{\frac{b_1 b_1}{\alpha_2(b_1-b_2)}}\left(1-\bar{l}^1\right) - \alpha_2\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\frac{\tilde{\alpha}b_1}{\alpha_2(b_1-b_2)}}(b_1-b_2)\bar{l}^1\end{array}}$$

$W_G^2(1,0) = v_L^2(1,0) > v_L^1(0,0) = W_G^2(0,0)$ if and only if $w^2/r^2 > (1-\tilde{\alpha})\bar{F}^2/\tilde{\alpha}\bar{L}^2$, which always holds. Thus, free trade and no labor mobility is a Nash equilibrium outcome if and only if $\bar{l}^1 < \bar{l}_b$ and $\bar{f}^1 \geq \eta_b\left(\bar{l}^1\right)$.

c. Suppose that $\eta_5\left(\bar{l}^1\right) \leq \bar{f}^1 \leq \eta_6\left(\bar{l}^1\right)$, which implies $\bar{f}^1 > \bar{l}^1$ (i.e., country 1 is relatively well endowed with factor F). Then, from Lemma 5 Part 1-c, country 1 specializes in good 1, $v_L^1(1,0) = C(\alpha_1/\alpha_2)^{\alpha_2}(1-b_1)\left[T^1\left(\bar{F}^1\right)^{b_1}\left(\bar{L}^1\right)^{(1-b_1)}\right]^{\alpha_1}\left[T^2\left(\bar{F}^2\right)^{b_2}\left(\bar{L}^2\right)^{1-b_2}\right]^{\alpha_2}/(B_{11})^{\alpha_1}(B_{22})^{\alpha_2}\bar{L}^1$ and $v_L^2(1,0) = C(\alpha_2/\alpha_1)^{\alpha_1}(1-b_2)\left[T^1\left(\bar{F}^1\right)^{b_1}\left(\bar{L}^1\right)^{(1-b_1)}\right]^{\alpha_1}\left[T^2\left(\bar{F}^2\right)^{b_2}\left(\bar{L}^2\right)^{1-b_2}\right]^{\alpha_2}/(B_{11})^{\alpha_1}(B_{22})^{\alpha_2}\bar{L}^2$. Moreover, $W_G^1(1,0) = v_L^1(1,0) \geq v_L^1(0,0) = W_G^1(0,0)$ if and only if

$$\bar{f}^1 \leq \eta_c^H\left(\bar{l}^1\right) = \frac{\left(\frac{\alpha_1}{\alpha_2}\right)^{\frac{1}{b_2}}\left[\frac{(1-b_1)}{(B_{11})^{\alpha_1}(B_{22})^{\alpha_2}}\left(\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\right)^{\tilde{\alpha}}\right]^{\frac{1}{\alpha_2 b_2}}\left(1-\bar{l}^1\right)^{\frac{1-b_2}{b_2}}}{\left(\bar{l}^1\right)^{\frac{1-b_2}{b_2}} + \left(\frac{\alpha_1}{\alpha_2}\right)^{\frac{1}{b_2}}\left[\frac{(1-b_1)}{(B_{11})^{\alpha_1}(B_{22})^{\alpha_2}}\left(\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\right)^{\tilde{\alpha}}\right]^{\frac{1}{\alpha_2 b_2}}\left(1-\bar{l}^1\right)^{\frac{1-b_2}{b_2}}},$$

while $W_G^2(1,0) = v_L^2(1,0) \geq v_L^2(0,0) = W_G^2(0,0)$ if and only if

$$\bar{f}^1 \geq \eta_c^L\left(\bar{l}^1\right) = \frac{\left(\frac{\alpha_1}{\alpha_2}\right)^{\frac{1}{b_1}}\left[\frac{(B_{11})^{\alpha_1}(B_{22})^{\alpha_2}}{(1-b_2)}\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\tilde{\alpha}}\right]^{\frac{1}{\alpha_1 b_1}}\left(1-\bar{l}^1\right)^{\frac{1-b_1}{b_1}}}{\left(\bar{l}^1\right)^{\frac{1-b_1}{b_1}} + \left(\frac{\alpha_1}{\alpha_2}\right)^{\frac{1}{b_1}}\left[\frac{(B_{11})^{\alpha_1}(B_{22})^{\alpha_2}}{(1-b_2)}\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\tilde{\alpha}}\right]^{\frac{1}{\alpha_1 b_1}}\left(1-\bar{l}^1\right)^{\frac{\alpha_1(1-b_1)}{\alpha_1 b_1}}}.$$

Thus, free trade and no labor mobility is a Nash equilibrium outcome if and only if $\eta_c^L\left(\bar{l}^1\right) \leq \bar{f}^1 \leq \eta_c^H\left(\bar{l}^1\right)$.

d. Suppose that $\bar{f}^1 \geq \eta_4\left(l^1\right)$ and $\bar{f}^1 > \eta_6\left(l^1\right)$, which implies $\bar{f}^1 > \bar{l}^1$ (i.e., country 1 is relatively well endowed with factor F). Then, from Lemma 5 Part 1-d, country 1 is diversified, $v_L^1(1,0) = T^1 C\left(w^1/r^1\right)^{\tilde{\alpha}}$ and $v_L^2(1,0) = T^2 C\left(w^1/r^1\right)^{\alpha_1(b_1-b_2)}\left[(1-b_2)\bar{F}^2/b_2\bar{L}^2\right]^{b_2}$, where w^1/r^1 is the unique solution to $T^1\left[(1-\tilde{\alpha})\bar{F}^1 - \tilde{\alpha}\left(w^1/r^1\right)\bar{L}^1\right]/\left(w^1/r^1\right)^{1-b_2}(b_1-b_2) = \alpha_1 T^2\left(F^2\right)^{b_2}\left(\bar{L}^2\right)^{1-b_2}/B_{22}$. Moreover, $W_G^1(0,0) = v_L^1(0,0) > v_L^1(1,0) = W_G^1(1,0)$ if and only $w^1/r^1 < (1-\tilde{\alpha})\bar{F}^1/\tilde{\alpha}\bar{L}^1$, which always holds. Thus, free trade and no labor mobility is not a Nash equilibrium outcome.

e. Suppose that $\eta_8\left(\bar{l}^1\right) < \bar{f}^1 \leq \eta_2\left(\bar{l}^1\right)$, which implies $1-\bar{f}^1 > 1-\bar{l}^1$ (i.e., country 2 is relatively well endowed with factor F). Then, from Lemma 5 Part 1-e, country 2 specializes in good 1,

$v_L^1(1,0) = T^1 C\left(w^1/r^1\right)^{\tilde{\alpha}}$ and $v_L^2(1,0) = T^2 C\left[\left(1-b_1\right)\bar{F}^2/b_1\bar{L}^2\right]^{b_1}\left(r^1/w^1\right)^{\alpha_2(b_1-b_2)}$, where w^1/r^1 is the unique solution to $\alpha_2 T^2\left(\bar{F}^2\right)^{b_1}\left(\bar{L}^2\right)^{1-b_1}/B_{11} = T^1\left[\tilde{\alpha}\left(w^1/r^1\right)\bar{L}^1 - \left(1-\tilde{\alpha}\right)\bar{F}^1\right]/\left(b_1-b_2\right)\left(w^1/r^1\right)^{1-b_1}$. Moreover, $W_G^1(1,0) = v_L^1(1,0) > v_L^1(0,0) = W_G^1(0,0)$ if and only if $w^1/r^1 > \left(1-\tilde{\alpha}\right)\bar{F}^1/\tilde{\alpha}\bar{L}^1$, which always holds. $W_G^2(1,0) = v_L^2(1,0) \geq v_L^2(0,0) = W_G^2(0,0)$ if and only if $w^1/r^1 \leq \left[\left(1-b_1\right)/b_1\right]^{\frac{b_1}{\alpha_2(b_1-b_2)}}\left[\tilde{\alpha}/\left(1-\tilde{\alpha}\right)\right]^{\frac{\tilde{\alpha}}{\alpha_2(b_1-b_2)}}\left(\bar{F}^2/\bar{L}^2\right)$ or, which is equivalent

$$\bar{l}^1 > \bar{l}_e = \frac{\left(b_1-b_2\right)\alpha_2}{B_{11}\tilde{\alpha}\left(\frac{1-b_1}{b_1}\right)^{\frac{b_1 b_1}{\alpha_2(b_1-b_2)}}\left(\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\right)^{\frac{\tilde{\alpha}b_1}{\alpha_2(b_1-b_2)}} + \left(b_1-b_2\right)\alpha_2}$$

$$\bar{f}^1 < \eta_e\left(\bar{l}^1\right) = \frac{\begin{array}{c} B_{11}\tilde{\alpha}\left(\frac{1-b_1}{b_1}\right)^{\frac{b_1}{\alpha_2(b_1-b_2)}}\left(\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\right)^{\frac{\tilde{\alpha}}{\alpha_2(b_1-b_2)}}\bar{l}^1 \\ -\left(b_1-b_2\right)\alpha_2\left(\frac{1-b_1}{b_1}\right)^{\frac{b_1(1-b_1)}{\alpha_2(b_1-b_2)}}\left(\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\right)^{\frac{\tilde{\alpha}(1-b_1)}{\alpha_2(b_1-b_2)}}\left(1-\bar{l}^1\right)\end{array}}{\begin{array}{c} B_{11}\left(1-\tilde{\alpha}\right)\left(1-\bar{l}^1\right) + \tilde{\alpha}B_{11}\left(\frac{1-b_1}{b_1}\right)^{\frac{b_1}{\alpha_2(b_1-b_2)}}\left(\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\right)^{\frac{\tilde{\alpha}}{\alpha_2(b_1-b_2)}}\bar{l}^1 \\ -\left(b_1-b_2\right)\alpha_2\left(\frac{1-b_1}{b_1}\right)^{\frac{b_1(1-b_1)}{\alpha_2(b_1-b_2)}}\left(\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\right)^{\frac{\tilde{\alpha}(1-b_1)}{\alpha_2(b_1-b_2)}}\left(1-\bar{l}^1\right)\end{array}}$$

Thus, free trade and no labor mobility is a Nash equilibrium outcome if and only if $\bar{l}^1 > \bar{l}_e$ and $\bar{f}^1 < \eta_e\left(\bar{l}^1\right)$.

f. Suppose that $\eta_7\left(\bar{l}^1\right) \leq \bar{f}^1 \leq \eta_8\left(\bar{l}^1\right)$, which implies $1-\bar{f}^1 > 1-\bar{l}^1$ (i.e., country 2 is relatively well endowed with factor F). Then, from Lemma 5 Part 1-e, country 2 specializes in good 1, $v_L^1(1,0) = C\left(\alpha_2/\alpha_1\right)^{\alpha_1}\left(1-b_2\right)\left[T^2\left(\bar{F}^2\right)^{b_1}\left(\bar{L}^2\right)^{1-b_1}\right]^{\alpha_1}\left[T^1\left(\bar{F}^1\right)^{b_2}\left(\bar{L}^1\right)^{1-b_2}\right]^{\alpha_2}/\left(B_{11}\right)^{\alpha_1}\left(B_{22}\right)^{\alpha_2}\bar{L}^1$ and $v_L^2(1,0) = C\left(1-b_1\right)\left(\alpha_1/\alpha_2\right)^{\alpha_2}\left[T^2\left(\bar{F}^2\right)^{b_1}\left(\bar{L}^2\right)^{1-b_1}\right]^{\alpha_1}\left[T^1\left(\bar{F}^1\right)^{b_2}\left(\bar{L}^1\right)^{1-b_2}\right]^{\alpha_2}/\left(B_{11}\right)^{\alpha_1}\left(B_{22}\right)^{\alpha_2}\bar{L}^2$. Moreover, $W_G^1(1,0) = v_L^1(1,0) \geq v_L^1(0,0) = W_G^1(0,0)$ if and only if

$$\bar{f}^1 \leq \eta_f^H\left(\bar{l}^1\right) = \frac{\left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{1}{b_1}}\left[\frac{(1-b_2)}{(B_{11})^{\alpha_1}(B_{22})^{\alpha_2}}\left(\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\right)^{\tilde{\alpha}}\right]^{\frac{1}{\alpha_1 b_1}}\left(1-\bar{l}^1\right)^{\frac{1-b_1}{b_1}}}{\left(\bar{l}^1\right)^{\frac{1-b_1}{b_1}} + \left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{1}{b_1}}\left[\frac{(1-b_2)}{(B_{11})^{\alpha_1}(B_{22})^{\alpha_2}}\left(\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\right)^{\tilde{\alpha}}\right]^{\frac{1}{\alpha_1 b_1}}\left(1-\bar{l}^1\right)^{\frac{1-b_1}{b_1}}},$$

while $W_G^2(1,0) = v_L^2(1,0) \geq v_L^2(0,0) = W_G^2(0,0)$ if and only if

$$\bar{f}^1 \geq \eta_f^L\left(\bar{l}^1\right) = \frac{\left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{1}{b_2}}\left[\frac{(B_{11})^{\alpha_1}(B_{22})^{\alpha_2}}{(1-b_1)}\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\tilde{\alpha}}\right]^{\frac{1}{\alpha_2 b_2}}\left(1-\bar{l}^1\right)^{\frac{1-b_2}{b_2}}}{\left(\bar{l}^1\right)^{\frac{1-b_2}{b_2}} + \left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{1}{b_2}}\left[\frac{(B_{11})^{\alpha_1}(B_{22})^{\alpha_2}}{(1-b_1)}\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\tilde{\alpha}}\right]^{\frac{1}{\alpha_2 b_2}}\left(1-\bar{l}^1\right)^{\frac{1-b_2}{b_2}}}$$

Thus, free trade and no labor mobility is a Nash equilibrium outcome if and only if $\eta_f^L\left(\bar{l}^1\right) \leq \bar{f}^1 \leq \eta_f^H\left(\bar{l}^1\right)$.

g. Suppose that $\bar{f}^1 \leq \eta_1\left(\bar{l}^1\right)$ and $\bar{f}^1 < \eta_7\left(\bar{l}^1\right)$, which implies $1-\bar{f}^1 > 1-\bar{l}^1$ (i.e., country 2 is relatively well endowed with factor F). Then, from Lemma 5 Part 1-g, country 2 is diversified, $v_L^1(1,0) = T^1 C\left[\left(1-b_2\right)\bar{F}^1/b_2\bar{L}^1\right]^{b_2}\left(w^2/r^2\right)^{\alpha_1(b_1-b_2)}$ and $v_L^2(1,0) = T^2 C\left(w^2/r^2\right)^{\tilde{\alpha}}$, where w^2/r^2 is the unique solution to $\alpha_1 T^1\left(\bar{F}^1\right)^{b_2}\left(\bar{L}^1\right)^{1-b_2}/B_{22} = \left[\left(1-\tilde{\alpha}\right)T^2\bar{F}^2 - \tilde{\alpha}\left(w^2/r^2\right)T^2\bar{L}^2\right]/\left(w^2/r^2\right)^{1-b_2}\left(b_1-b_2\right)$. Moreover,

$W_G^2(0,0) = v_L^2(0,0) > v_L^2(1,0) = W_G^2(1,0)$ if and only if $w^2/r^2 < (1-\tilde{\alpha})\bar{F}^2/\tilde{\alpha}\bar{L}^2$, which always holds. Thus, free trade and no labor mobility is not a Nash equilibrium outcome.

Free trade and free labor mobility is not a Nash equilibrium outcome. From Lemma 5 Part 2-a, if $\bar{f}^1 > \eta_9(\bar{l}^1)$ for $\bar{l}^1 < \bar{l}$; or $\bar{f}^1 > \eta_{10}(\bar{l}^1)$ for $\bar{l}^1 \geq \bar{l}$, then $W_G^1(1,0) = v_L^1(1,0) > v_L^1(1,1) = W_G^1(1,1)$. Therefore, workers in country 1 prefer to block labor mobility. From Lemma 5 Part 2-b, if $\bar{f}^1 < \eta_9(\bar{l}^1)$ for $\bar{l}^1 < \bar{l}$; or $\bar{f}^1 < \eta_{10}(\bar{l}^1)$ for $\bar{l}^1 \geq \bar{l}$, then $W_G^2(1,0) = v_L^2(1,0) > v_L^2(1,1) = W_G^2(1,1)$. Therefore, workers in country 2 prefer to block labor mobility. From Lemma 5 Part 2-c, if $\bar{f}^1 = \eta_9(\bar{l}^1)$ for $\bar{l}^1 < \bar{l}$; or $\bar{f}^1 = \eta_{10}(\bar{l}^1)$ for $\bar{l}^1 > \bar{l}$; or $\bar{f}^1 \in [\eta_9(\bar{l}^1), \eta_{10}(\bar{l}^1)]$ for $l^1 = \bar{l}$, then, $W_G^1(1,0) = W_G^1(1,1) = W_G^2(1,0) = W_G^2(1,1) = v_L^1(1,0) = v_L^1(1,1) = v_L^2(1,0) = v_L^2(1,1)$. Since $\bar{f}^1 = \eta_9(\bar{l}^1)$ and $\bar{l}^1 < \bar{l}$ implies $\bar{f}^1 \leq \eta_1(\bar{l}^1)$ and $\bar{f}^1 < \eta_7(\bar{l}^1)$, whenever $\bar{f}^1 = \eta_9(\bar{l}^1)$ and $\bar{l}^1 < \bar{l}$, it must be the case that $v_L^2(0,0) > v_L^2(1,0) = v_L^2(1,1)$. Thus, free trade and free labor mobility is not a Nash equilibrium. Since $\bar{f}^1 = \eta_{10}(\bar{l}^1)$ and $\bar{l}^1 > \bar{l}$ implies that $\eta_8(\bar{l}^1) < \bar{f}^1 \leq \eta_2(\bar{l}^1)$. Thus, if in addition $\bar{l}^1 > \bar{l}_e$ and $\bar{f}^1 < \eta_e(\bar{l}^1)$

$\bar{f}^1 > \frac{\bar{l}^1}{\left(\frac{T^1}{T^2}\right)^{\frac{1}{\tilde{\alpha}}}(1-\bar{l}^1)+\bar{l}^1}$ just proved that either $v_L^1(0,0) > v_L^1(1,0)$ or $v_L^2(0,0) > v_L^1(1,0)$.

No trade and free labor mobility is not a Nash equilibrium outcome. From Lemma 4 Part 3, $T^1/T^2 > \left(\bar{F}^2\bar{L}^1/\bar{F}^1\bar{L}^2\right)^{\tilde{\alpha}}$ implies that $W_G^1(0,0) = v_L^1(0,0) > v_L^1(0,1) = W_G^1(0,1)$, while $T^1/T^2 < \left(\bar{F}^2\bar{L}^1/\bar{F}^1\bar{L}^2\right)^{\tilde{\alpha}}$ implies $W_G^2(0,0) = v_L^2(0,0) > v_L^2(0,1) = W_G^2(0,1)$. Thus, no trade and free labor mobility is not a Nash equilibrium outcome.

B.2 Ricardo-Vinner specific factors economy

Consider an economy with two countries ($J = 2$), two tradeable goods ($Z_T = \{1,2\}$) and three factors of production (F_1, F_2 and labor L). Production functions are given by $Q_z^j = \left(F_z^j\right)^b \left(L_z^j\right)^{1-b}$ for $z = 1,2$, with $b \in (0,1)$. Factor endowments in country j are $\left(\bar{F}_1^j, \bar{F}_2^j, \bar{L}^j\right) > (0,0,0)$. All agents have the same preferences, given by $u(c^j) = \prod_{z\in Z}\left(c_z^j\right)^{\alpha_z}$, with $\alpha_z > 0$ and $\sum_{z\in Z}\alpha_z = 1$.

Lemma 6 characterizes the equilibrium of a Ricardo-Vinner specific factors economy under autarky.

Lemma 6 (Ricardo-Vinner specific factors economy under autarky). *Assume there is no trade in goods, i.e., $\lambda_T = 0$.*

1. *Suppose there is no labor mobility, i.e., $\lambda_M = 0$. Then, $v_L^j(0,0) = C\left(\bar{F}_1^j\right)^{b\alpha_1}\left(\bar{F}_2^j\right)^{b(1-\alpha_1)} / \left(\bar{L}^j\right)^b$, where $C = \left[(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\right]^{1-b}(1-b)$.*
2. *Suppose there is free labor mobility, i.e., $\lambda_M = 1$.*
   (a) *If $\left(\frac{\bar{F}_1^1}{\bar{F}_1^2}\right)^{\alpha_1}\left(\frac{\bar{F}_2^1}{\bar{F}_2^2}\right)^{(1-\alpha_1)} > \frac{\bar{L}^1+m\bar{L}^2}{(1-m)\bar{L}^2}$, then $L^1 = \left(\bar{L}^1+m\bar{L}^2\right)$, $L^2 = (1-m)\bar{L}^2$, $v_L^1(0,1) = \frac{C\left(\bar{F}_1^1\right)^{b\alpha_1}\left(\bar{F}_2^1\right)^{b(1-\alpha_1)}}{\left(\bar{L}^1+m\bar{L}^2\right)^b}$ and $v_L^2(0,1) = \frac{C\left(\bar{F}_1^2\right)^{b\alpha_1}\left(\bar{F}_2^2\right)^{b(1-\alpha_1)}}{\left[(1-m)\bar{L}^2\right]^b}$.*
   (b) *If $\frac{(1-m)\bar{L}^1}{\bar{L}^2+m\bar{L}^1} \leq \left(\frac{\bar{F}_1^1}{\bar{F}_1^2}\right)^{\alpha_1}\left(\frac{\bar{F}_2^1}{\bar{F}_2^2}\right)^{(1-\alpha_1)} \leq \frac{\bar{L}^1+m\bar{L}^2}{(1-m)\bar{L}^2}$, then $L^1 = \frac{\left(\bar{F}_1^1\right)^{\alpha_1}\left(\bar{F}_2^1\right)^{1-\alpha_1}\left(\bar{L}^1+\bar{L}^2\right)}{\left(\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^2\right)^{1-\alpha_1}+\left(\bar{F}_1^1\right)^{\alpha_1}\left(\bar{F}_2^1\right)^{1-\alpha_1}}$, $L^2 = \frac{\left(\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^2\right)^{1-\alpha_1}\left(\bar{L}^1+\bar{L}^2\right)}{\left(\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^2\right)^{1-\alpha_1}+\left(\bar{F}_1^1\right)^{\alpha_1}\left(\bar{F}_2^1\right)^{1-\alpha_1}}$ and $v_L^1(0,1) = v_L^2(0,1) = \left[\frac{\left(\bar{F}_1^1\right)^{\alpha_1}\left(\bar{F}_2^1\right)^{1-\alpha_1}+\left(\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^2\right)^{1-\alpha_1}}{\bar{L}^1+\bar{L}^2}\right]^b$.*

(c) *If* $\left(\frac{\bar{F}_1^1}{\bar{F}_1^2}\right)^{\alpha_1}\left(\frac{\bar{F}_2^1}{\bar{F}_2^2}\right)^{(1-\alpha_1)} < \frac{(1-m)\bar{L}^1}{\bar{L}^2+m\bar{L}^1}$, *then* $L^1 = (1-m)\bar{L}^1$, $L^2 = \bar{L}^2 + m\bar{L}^1$, $v_L^1(0,1) = \frac{C(\bar{F}_1^1)^{b\alpha_1}(\bar{F}_2^1)^{b(1-\alpha_1)}}{\left[(1-m)\bar{L}^1\right]^b}$ *and* $v_L^2(0,1) = \frac{C(\bar{F}_1^2)^{b\alpha_1}(\bar{F}_2^2)^{b(1-\alpha_1)}}{(\bar{L}^2+m\bar{L}^1)^b}$.

3. *If* $\left(\frac{\bar{F}_1^1}{\bar{F}_1^2}\right)^{\alpha_1}\left(\frac{\bar{F}_2^1}{\bar{F}_2^2}\right)^{(1-\alpha_1)} > \frac{\bar{L}^1}{\bar{L}^2}$, *then* $v_L^1(0,0) > v_L^1(0,1)$, *while if* $\left(\frac{\bar{F}_1^1}{\bar{F}_1^2}\right)^{\alpha_1}\left(\frac{\bar{F}_2^1}{\bar{F}_2^2}\right)^{(1-\alpha_1)} < \frac{\bar{L}^1}{\bar{L}^2}$, *then* $v_L^2(0,0) > v_L^2(0,1)$.

Proof: Let p_z^j denotes the price of good $z = 1,2$ in country j, r_f^j the return of factor $f = 1,2$ and w^j the wage rate. Under autarky, the equilibrium conditions in country j are:

$$p_1^j(1-b)\left(\frac{\bar{F}_1^j}{L_1^j}\right)^b = w^j = p_2^j(1-b)\left(\frac{\bar{F}_2^j}{L_2^j}\right)^b,\ L_1^j + L_2^j = L^j$$

$$p_1^j b\left(\frac{L_1^j}{\bar{F}_1^j}\right)^{1-b} = r_1^j,\ p_1^j b\left(\frac{L_1^j}{\bar{F}_1^j}\right)^{1-b} = r_2^j,\ \alpha_2 p_1^j Q_1^j = \alpha_1 p_2^j Q_2^j$$

Therefore, in equilibrium, $L_1^j = \alpha_1 L^j$, $L_2^j = \alpha_2 L^j$, $p_1^j/p_2^j = \left(\bar{F}_2^j\alpha_1/\bar{F}_1^j\alpha_2\right)^b$, $w^j/p_1^j = (1-b)\left(\bar{F}_1^j/\alpha_1 L^j\right)^b$, $w^j/p_2^j = (1-b)\left(\bar{F}_2^j/\alpha_2 L^j\right)^b$ and, hence, the utility of a worker in country j is $v_L^j(0,\lambda_M) = (\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2} w^j/\left(p_1^j\right)^{\alpha_1}\left(p_2^j\right)^{\alpha_2} = C\left(\bar{F}_1^j/L^j\right)^{b\alpha_1}\left(\bar{F}_2^j/L^j\right)^{b(1-\alpha_1)}$, where $C = \left[(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\right]^{1-b}(1-b)$.

When there is no labor mobility, $L^j = \bar{L}^j$. When there is free labor mobility, mobile workers locate in the country with the highest v_L^j. We must distinguish three possible cases.

a. Suppose that $\left(\bar{F}_1^1/\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^1/\bar{F}_2^2\right)^{(1-\alpha_1)} > \left(\bar{L}^1 + m\bar{L}^2\right)/(1-m)\bar{L}^2$. Then, $v_L^1(0,\lambda_M) > v_L^2(0,\lambda_M)$ for all $L^1 \in \left[(1-m)\bar{L}^1, \bar{L}^1 + m\bar{L}^2\right]$ and $L^2 \in \left[(1-m)\bar{L}^2, \bar{L}^2 + m\bar{L}^1\right]$. Therefore, in equilibrium, $L^1 = \left(\bar{L}^1 + m\bar{L}^2\right)$ and $L^2 = (1-m)\bar{L}^2$.

b. Suppose that $(1-m)\bar{L}^1/\left(\bar{L}^2 + m\bar{L}^1\right) \le \left(\bar{F}_1^1/\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^1/\bar{F}_2^2\right)^{(1-\alpha_1)} \le \left(\bar{L}^1 + m\bar{L}^2\right)/(1-m)\bar{L}^2$. Then, $v_L^1(0,\lambda_M) \ge v_L^2(0,\lambda_M)$ for $L^1 = (1-m)\bar{L}^1$ and $L^2 = \bar{L}^2 + m\bar{L}^1$, while $v_L^1(0,\lambda_M) \le v_L^2(0,\lambda_M)$ for $L^1 = \bar{L}^1 + m\bar{L}^2$ and $L^2 = (1-m)\bar{L}^2$. Moreover, $v_L^j(0,\lambda_M)$ is decreasing in L^j. Therefore, in equilibrium, it must be the case that $v_L^1(0,1) = v_L^2(0,1)$, which implies $L^1/\left(\bar{L}^1 + \bar{L}^2\right) = \left(\bar{F}_1^1\right)^{\alpha_1}\left(\bar{F}_2^1\right)^{1-\alpha_1}/\left[\left(\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^2\right)^{1-\alpha_1} + \left(\bar{F}_1^1\right)^{\alpha_1}\left(\bar{F}_2^1\right)^{1-\alpha_1}\right]$ and $L^2/\left(\bar{L}^1 + \bar{L}^2\right) = \left(\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^2\right)^{1-\alpha_1}/\left[\left(\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^2\right)^{1-\alpha_1} + \left(\bar{F}_1^1\right)^{\alpha_1}\left(\bar{F}_2^1\right)^{1-\alpha_1}\right]$.

c. Suppose that $\left(\bar{F}_1^1/\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^1/\bar{F}_2^2\right)^{(1-\alpha_1)} < (1-m)\bar{L}^1/\left(\bar{L}^2 + m\bar{L}^1\right)$. Then, $v_L^2(0,\lambda_M) > v_L^1(0,\lambda_M)$ for all $L^1 \in \left[(1-m)\bar{L}^1, \bar{L}^1 + m\bar{L}^2\right]$ and $L^2 \in \left[(1-m)\bar{L}^2, \bar{L}^2 + m\bar{L}^1\right]$. Therefore, in equilibrium $L^1 = (1-m)\bar{L}^1$ and $L^2 = \bar{L}^2 + m\bar{L}^1$. Summing up, the utility of a worker in country j under autarky and free labor mobility is given by:

$$v_L^1(0,1) = C \begin{cases} \left(\frac{\bar{F}_1^1}{\bar{L}^1+m\bar{L}^2}\right)^{b\alpha_1}\left(\frac{\bar{F}_2^1}{\bar{L}^1+m\bar{L}^2}\right)^{b(1-\alpha_1)} & \text{if } \left(\frac{\bar{F}_1^1}{\bar{F}_1^2}\right)^{\alpha_1}\left(\frac{\bar{F}_2^1}{\bar{F}_2^2}\right)^{(1-\alpha_1)} > \frac{\bar{L}^1+m\bar{L}^2}{(1-m)\bar{L}^2} \\ \left[\frac{\left(\bar{F}_1^1\right)^{\alpha_1}\left(\bar{F}_2^1\right)^{1-\alpha_1}+\left(\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^2\right)^{1-\alpha_1}}{\bar{L}^1+\bar{L}^2}\right]^b & \text{if } \frac{(1-m)\bar{L}^1}{\bar{L}^2+m\bar{L}^1} \leq \left(\frac{\bar{F}_1^1}{\bar{F}_1^2}\right)^{\alpha_1}\left(\frac{\bar{F}_2^1}{\bar{F}_2^2}\right)^{(1-\alpha_1)} \leq \frac{\bar{L}^1+m\bar{L}^2}{(1-m)\bar{L}^2} \\ \left[\frac{\bar{F}_1^1}{(1-m)\bar{L}^1}\right]^{b\alpha_1}\left[\frac{\bar{F}_2^1}{(1-m)\bar{L}^1}\right]^{b(1-\alpha_1)} & \text{if } \left(\frac{\bar{F}_1^1}{\bar{F}_1^2}\right)^{\alpha_1}\left(\frac{\bar{F}_2^1}{\bar{F}_2^2}\right)^{(1-\alpha_1)} < \frac{(1-m)\bar{L}^1}{\bar{L}^2+m\bar{L}^1} \end{cases}$$

$$v_L^2(0,1) = C \begin{cases} \left[\frac{\bar{F}_1^2}{(1-m)\bar{L}^2}\right]^{b\alpha_1}\left[\frac{\bar{F}_2^2}{(1-m)\bar{L}^2}\right]^{b(1-\alpha_1)} & \text{if } \left(\frac{\bar{F}_1^1}{\bar{F}_1^2}\right)^{\alpha_1}\left(\frac{\bar{F}_2^1}{\bar{F}_2^2}\right)^{(1-\alpha_1)} > \frac{\bar{L}^1+m\bar{L}^2}{(1-m)\bar{L}^2} \\ \left[\frac{\left(\bar{F}_1^1\right)^{\alpha_1}\left(\bar{F}_2^1\right)^{1-\alpha_1}+\left(\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^2\right)^{1-\alpha_1}}{\bar{L}^1+\bar{L}^2}\right]^b & \text{if } \frac{(1-m)\bar{L}^1}{\bar{L}^2+m\bar{L}^1} \leq \left(\frac{\bar{F}_1^1}{\bar{F}_1^2}\right)^{\alpha_1}\left(\frac{\bar{F}_2^1}{\bar{F}_2^2}\right)^{(1-\alpha_1)} \leq \frac{\bar{L}^1+m\bar{L}^2}{(1-m)\bar{L}^2} \\ \left(\frac{\bar{F}_1^2}{\bar{L}^2+m\bar{L}^1}\right)^{b\alpha_1}\left(\frac{\bar{F}_2^2}{\bar{L}^2+m\bar{L}^1}\right)^{b(1-\alpha_1)} & \text{if } \left(\frac{\bar{F}_1^1}{\bar{F}_1^2}\right)^{\alpha_1}\left(\frac{\bar{F}_2^1}{\bar{F}_2^2}\right)^{(1-\alpha_1)} < \frac{(1-m)\bar{L}^1}{\bar{L}^2+m\bar{L}^1} \end{cases}$$

Finally, note that $\left(\bar{F}_1^1/\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^1/\bar{F}_2^2\right)^{(1-\alpha_1)} > \bar{L}^1/\bar{L}^2$ implies $v_L^1(0,0) > v_L^1(0,1)$, while $\bar{L}^1/\bar{L}^2 < \left(\bar{F}_1^1/\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^1/\bar{F}_2^2\right)^{(1-\alpha_1)}$ implies $v_L^2(0,0) > v_L^2(0,1)$. This completes the proof of the lemma. ■

Lemma 7 characterizes the equilibrium of a Ricardo-Vinner specific factors economy under free trade.

Lemma 7 (Ricardo-Vinner specific factors economy under free trade). *Assume there is free trade of goods, i.e., $\lambda_T = 1$. Suppose that $\frac{\bar{F}_1^1}{\bar{F}_2^1} > \frac{\bar{F}_1^2}{\bar{F}_2^2}$ and let*

$$BT\left(\frac{p_1}{p_2}, \frac{L^1}{\bar{L}^2+\bar{L}^2}\right) = (1-\alpha_1)\left(\frac{p_1}{p_2}\right)\left(Q_1^1+Q_1^2\right) - \alpha_1\left(Q_2^1+Q_2^2\right)$$

where p_z is the price of good $z = 1, 2$.

1. *Suppose there is no labor mobility, i.e., $\lambda_M = 0$. Then, $v_L^j(1,0) = C\left[(\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2}\right]^b\left[\frac{\bar{F}_2^j+(\bar{p}^e)^{\frac{1}{b}}\bar{F}_1^j}{(\bar{p}^e)^{\frac{\alpha_1}{b}}\bar{L}^j}\right]^b$ where $\frac{p_1}{p_2} = \bar{p}^e$ is the unique solution to $BT\left(\frac{p_1}{p_2}, \frac{\bar{L}^1}{\bar{L}^2+\bar{L}^2}\right) = 0$.*

2. *Suppose there is free labor mobility, i.e., $\lambda_M = 1$.*

   (a) *If $\frac{\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1}{\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right)} > \frac{\bar{L}^1+m\bar{L}^2}{\bar{L}^1+\bar{L}^2}$, then, $L^1 = \bar{L}^1+m\bar{L}^2$, $L^2 = (1-m)\bar{L}^2$ and $v_L^j(1,1) = C\left[(\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2}\right]^b\left[\frac{\bar{F}_2^j+\left(p_L^e\right)^{\frac{1}{b}}\bar{F}_1^j}{\left(p_L^e\right)^{\frac{\alpha_1}{b}}L^j}\right]^b$, where $\frac{p_1}{p_2} = p_L^e$ is the unique solution to $BT\left(\frac{p_1}{p_2}, \frac{\bar{L}^1+m\bar{L}^2}{\bar{L}^1+\bar{L}^2}\right) = 0$.*

   (b) *If $\frac{(1-m)\bar{L}^1}{\bar{L}^1+\bar{L}^2} \leq \frac{\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1}{\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right)} \leq \frac{\bar{L}^1+m\bar{L}^2}{\bar{L}^1+\bar{L}^2}$, then, $L^1 = \psi\left(\bar{L}^1+\bar{L}^2\right)$, $L^2 = (1-\psi)\left(\bar{L}^1+\bar{L}^2\right)$, where $\psi = \frac{\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1}{\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right)}$ and $v_L^1(1,1) = v_L^2(1,1) = C\left[\frac{\left(\bar{F}_1^1+\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^1+\bar{F}_2^2\right)^{1-\alpha_1}}{\left(\bar{L}^1+\bar{L}^2\right)}\right]^b$.*

(c) *If* $\frac{\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1}{\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right)} < \frac{(1-m)\bar{L}^1}{\bar{L}^1+\bar{L}^2}$, *then,* $L^1 = (1-m)\,\bar{L}^1$, $L^2 = \bar{L}^2 + m\bar{L}^1$ *and,* $v_L^j(1,1) = C\left[(\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2}\right]^b \left[\frac{\bar{F}_2^j+\left(p_H^e\right)^{\frac{1}{b}}\bar{F}_1^j}{\left(p_H^e\right)^{\frac{\alpha_1}{b}}L^j}\right]^b$, *where* $\frac{p_1}{p_2} = p_H^e$ *is the unique solution to* $BT\left(\frac{p_1}{p_2}, \frac{(1-m)\bar{L}^1}{\bar{L}^2+m\bar{L}^1}\right) = 0$.

Proof: Let p_z denote the price of good $z = 1,2$, r_f^j the return of factor $f = 1,2$, and w^j the wage rate. Under autarky, the equilibrium conditions in country j are:

$$p_1(1-b)\left(\frac{\bar{F}_1^j}{L_1^j}\right)^b = w^j = p_2(1-b)\left(\frac{\bar{F}_2^j}{L_2^j}\right)^b,\ L_1^j + L_2^j = L^j \text{ for } j = 1,2$$

$$p_1 b\left(\frac{L_1^j}{\bar{F}_1^j}\right)^{1-b} = r_1,\ p_1^j b\left(\frac{L_1^j}{\bar{F}_1^j}\right)^{1-b} = r_2^j \text{ for } j = 1,2$$

$$\alpha_2 p_1\left(Q_1^1 + Q_1^2\right) = \alpha_1 p_2\left(Q_2^1 + Q_2^2\right)$$

Therefore, in equilibrium, $L_1^j = (p_1/p_2)^{\frac{1}{b}}\bar{F}_1^j L^j / \left[\bar{F}_2^j + (p_1/p_2)^{\frac{1}{b}}\bar{F}_1^j\right]$, $L_2^j = \bar{F}_2^j L^j / \left[\bar{F}_2^j + (p_1/p_2)^{\frac{1}{b}}\left(\bar{F}_1^j\right)\right]$, $w^j/p_1 = (1-b)\left\{\left[\bar{F}_2^j + (p_1/p_2)^{\frac{1}{b}}\bar{F}_1^j\right]/(p_1/p_2)^{\frac{1}{b}}L^j\right\}^b$, $w^j/p_2 = (1-b)\left\{\left[\bar{F}_2^j + (p_1/p_2)^{\frac{1}{b}}\left(\bar{F}_1^j\right)\right]/L^j\right\}^b$ and p_1/p_2 is the unique solution to $BT\left(p_1/p_2, L^1/\left(\bar{L}^1+\bar{L}^2\right)\right) = 0$, where

$$BT\left(\frac{p_1}{p_2}, \frac{L^1}{\bar{L}^1+\bar{L}^2}\right) = (1-\alpha_1)\left(\frac{p_1}{p_2}\right)\left(Q_1^1+Q_1^2\right) - \alpha_1\left(Q_2^1+Q_2^2\right)$$

$$Q_1^1\left(\frac{p_1}{p_2}, \frac{L^1}{\bar{L}^1+\bar{L}^2}\right) = \bar{F}_1^1\left[\frac{\left(\frac{p_1}{p_2}\right)^{\frac{1}{b}}L^1}{\bar{F}_2^1 + \left(\frac{p_1}{p_2}\right)^{\frac{1}{b}}\bar{F}_1^1}\right]^{1-b},\ Q_1^2\left(\frac{p_1}{p_2}, \frac{L^1}{\bar{L}^1+\bar{L}^2}\right) = \bar{F}_1^2\left[\frac{\left(\frac{p_1}{p_2}\right)^{\frac{1}{b}}\left(\bar{L}^1+\bar{L}^2-L^1\right)}{\bar{F}_2^2 + \left(\frac{p_1}{p_2}\right)^{\frac{1}{b}}\bar{F}_1^2}\right]^{1-b}$$

$$Q_2^1\left(\frac{p_1}{p_2}, \frac{L^1}{\bar{L}^1+\bar{L}^2}\right) = \bar{F}_2^1\left[\frac{L^1}{\bar{F}_2^1 + \left(\frac{p_1}{p_2}\right)^{\frac{1}{b}}\bar{F}_1^1}\right]^{1-b},\ Q_2^2\left(\frac{p_1}{p_2}, \frac{L^1}{\bar{L}^1+\bar{L}^2}\right) = \bar{F}_2^2\left[\frac{\left(\bar{L}^1+\bar{L}^2-L^1\right)}{\bar{F}_2^2 + \left(\frac{p_1}{p_2}\right)^{\frac{1}{b}}\bar{F}_1^2}\right]^{1-b}$$

In order to prove this, note that $p_1/p_2 \leq p^{A,1}$, implies $BT\left(p_1/p_2, L^1/\left(\bar{L}^1+\bar{L}^2\right)\right) < 0$, while $p_1/p_2 \geq p^{A,2}$ implies $BT\left(p_1/p_2, L^1/\left(\bar{L}^1+\bar{L}^2\right)\right) > 0$, where $p^{A,1} = \left[\bar{F}_2^1\alpha_1/\bar{F}_1^1(1-\alpha_1)\right]^b < p^{A,2} = \left[\bar{F}_2^2\alpha_1/\bar{F}_1^2(1-\alpha_1)\right]^b$. Moreover, $BT\left(p_1/p_2, L^1/\left(\bar{L}^1+\bar{L}^2\right)\right)$ is strictly increasing in p_1/p_2. Thus, there exists a unique $p_1/p_2 = p^e \in \left(p^{A,1}, p^{A,2}\right)$ such that $BT\left(p^e, L^1/\left(\bar{L}^1+\bar{L}^2\right)\right) = 0$. Therefore, in equilibrium, the utility of a worker in country j is $v_L^j(1,\lambda_M) = (\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2}w^j/(p_1)^{\alpha_1}(p_2)^{\alpha_2} = C\left[(\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2}\right]^b\left\{\left[\bar{F}_2^j + (p^e)^{\frac{1}{b}}\bar{F}_1^j\right]/(p^e)^{\frac{\alpha_1}{b}}L^j\right\}^b$.

When there is no labor mobility, $L^j = \bar{L}^j$ and, hence, $v_L^j(1,0) = C\left[(\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2}\right]^b\left\{\left[\bar{F}_2^j + (\bar{p}^e)^{\frac{1}{b}}\bar{F}_1^j\right]/(\bar{p}^e)^{\frac{\alpha_1}{b}}\bar{L}^j\right\}^b$, where $p_1/p_2 = \bar{p}^e$ is the unique solution to $BT\left(p_1/p_2, \bar{L}^1/\left(\bar{L}^1+\bar{L}^2\right)\right) = 0$. When there is free labor mobility, mobile workers locate in the country with the highest v_L^j. Next, we prove three results that will allow us to characterize the equilibrium under free labor mobility.

Result 1: p^e is decreasing in $L^1/\left(\bar{L}^2+\bar{L}^2\right)$. From the implicit function theorem, $d\left(p^e\right)/d\left[L^1/\left(\bar{L}^2+\bar{L}^2\right)\right] = -\frac{\partial BT\left(p^e,L^1/\left(\bar{L}^2+\bar{L}^2\right)\right)}{\partial\left[L^1/\left(\bar{L}^2+\bar{L}^2\right)\right]}/\frac{\partial BT\left(p^e,L^1/\left(\bar{L}^2+\bar{L}^2\right)\right)}{\partial p^e}$. We also already proved that $\partial BT\left(p^e,L^1/\left(\bar{L}^2+\bar{L}^2\right)\right)/\partial p^e > 0$. $\partial BT\left(p^e,L^1/\left(\bar{L}^2+\bar{L}^2\right)\right)/\partial\left[L^1/\left(\bar{L}^2+\bar{L}^2\right)\right] = (1-b)\left[(1-\alpha_1)p^eQ_1^1-\alpha_1Q_2^1\right]\left(\bar{L}^2+\bar{L}^2\right)^2/L^1L^2 > 0$ if and only if $(1-\alpha_1)p^eQ_1^1 > \alpha_1Q_2^1$, which always holds because $p^e > p^{A,1} = \left[\alpha_1\bar{F}_2^1/(1-\alpha_1)\bar{F}_1^1\right]^b$.

Result 2: $v_L^1(1,\lambda_M)$ is decreasing in $L^1/\left(\bar{L}^2+\bar{L}^2\right)$. $\partial v_L^1(1,\lambda_M)/\partial\left[L^1/\left(\bar{L}^2+\bar{L}^2\right)\right] < 0$ if and only if $(1/bp^e)\left\{(p^e)^{\frac{1}{b}}\bar{F}_1^1/\left[\bar{F}_2^1+(p^e)^{\frac{1}{b}}\bar{F}_1^1\right]-\alpha_1\right\}d\left(p^e\right)/d\left[L^1/\left(\bar{L}^2+\bar{L}^2\right)\right]-\left(\bar{L}^2+\bar{L}^2\right)/L^1 < 0$, which holds because $d\left(p^e\right)/d\left[L^1/\left(\bar{L}^2+\bar{L}^2\right)\right] < 0$ and $p^e > p^{A,1}$ implies $(p^e)^{\frac{1}{b}}\bar{F}_1^1 > \alpha_1\left[\bar{F}_2^1+(p^e)^{\frac{1}{b}}\bar{F}_1^1\right]$.

Result 3: $v_L^2(1,\lambda_M)$ is increasing in $L^1/\left(\bar{L}^2+\bar{L}^2\right)$. $\partial v_L^2(1,\lambda_M)/\partial\left[L^1/\left(\bar{L}^2+\bar{L}^2\right)\right] > 0$ if and only if $(1/bp^e)\left\{(p^e)^{\frac{1}{b}}\bar{F}_1^2/\left[\bar{F}_2^2+(p^e)^{\frac{1}{b}}\bar{F}_1^2\right]-\alpha_1\right\}d\left(p^e\right)/d\left[L^1/\left(\bar{L}^2+\bar{L}^2\right)\right]+\left(\bar{L}^2+\bar{L}^2\right)/L^2 > 0$, which holds because $d\left(p^e\right)/d\left[L^1/\left(\bar{L}^2+\bar{L}^2\right)\right] < 0$ and $p^e > p^{A,1}$ implies $(p^e)^{\frac{1}{b}}\bar{F}_1^2 < \alpha_1\left[\bar{F}_2^2+(p^e)^{\frac{1}{b}}\bar{F}_1^2\right]$.

We must distinguish three possible cases.

a. Suppose that $\left[\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1\right]/\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right) > \left(\bar{L}^1+m\bar{L}^2\right)/\left(\bar{L}^1+\bar{L}^2\right)$. Then, $L^1=\bar{L}^1+m\bar{L}^2$ and $L^2=(1-m)\bar{L}^2$ and, hence,

$$v_L^j(1,1) = C\left[(\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2}\right]^b\left[\frac{\bar{F}_2^j+(p_L^e)^{\frac{1}{b}}\bar{F}_1^j}{(p_L^e)^{\frac{\alpha_1}{b}}L^j}\right]^b,$$

where $p_1/p_2 = p_L^e$ is the unique solution to $BT\left(p_1/p_2,\left(\bar{L}^1+m\bar{L}^2\right)/\left(\bar{L}^1+\bar{L}^2\right)\right) = 0$. To prove this, from Results 2 and 3 $v_L^1(1,\lambda_M)$ is decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$, while $v_L^2(1,\lambda_M)$ is increasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$. Thus, if $\left[\bar{F}_2^1+(p_L^e)^{\frac{1}{b}}\bar{F}_1^1\right]/\left(\bar{L}^1+m\bar{L}^2\right) > \left[\bar{F}_2^2+(p_L^e)^{\frac{1}{b}}\bar{F}_1^2\right]/(1-m)\bar{L}^2$, then $v_L^1(1,\lambda_M) > v_L^2(1,\lambda_M)$ for all $L^1\in\left[(1-m)\bar{L}^1,\bar{L}^1+m\bar{L}^2\right]$ and $L^2=\left[(1-m)\bar{L}^2,\bar{L}^2+m\bar{L}^1\right]$. Note that $BT\left(p_L^e,\left(\bar{L}^1+m\bar{L}^2\right)/\left(\bar{L}^1+\bar{L}^2\right)\right) = 0$ if and only if $\left[\bar{F}_2^1+(p_L^e)^{\frac{1}{b}}\bar{F}_1^1\right]/\left[\bar{F}_2^2+(p_L^e)^{\frac{1}{b}}\bar{F}_1^2\right] = \left\{\left[(1-\alpha_1)(p_L^e)^{\frac{1}{b}}\bar{F}_1^1-\alpha_1\bar{F}_2^1\right]/\left[\alpha_1\bar{F}_2^2-(1-\alpha_1)(p_L^e)^{\frac{1}{b}}\bar{F}_1^2\right]\right\}^{\frac{1}{1-b}}\left[\left(\bar{L}^1+m\bar{L}^2\right)/(1-m)\bar{L}^2\right]$.
Therefore, $\left[\bar{F}_2^1+(p_L^e)^{\frac{1}{b}}\bar{F}_1^1\right]/\left(\bar{L}^1+m\bar{L}^2\right) > \left[\bar{F}_2^2+(p_L^e)^{\frac{1}{b}}\bar{F}_1^2\right]/(1-m)\bar{L}^2$ implies $p_L^e > \left[\alpha_1\left(\bar{F}_2^2+\bar{F}_2^1\right)/(1-\alpha_1)\left(\bar{F}_1^1+\bar{F}_1^2\right)\right]^b$. Due to Result 1, p^e is decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$ and, hence, $p_L^e > p_M^e = \left[\alpha_1\left(\bar{F}_2^2+\bar{F}_2^1\right)/(1-\alpha_1)\left(\bar{F}_1^1+\bar{F}_1^2\right)\right]^b$ if and only if $\bar{L}^1+m\bar{L}^2 < L^1$, where L^1 is such that $BT\left(p_M^e,L^1/\left(\bar{L}^1+\bar{L}^2\right)\right) = 0$. It is easy to prove that if $L^1/\left(\bar{L}^1+\bar{L}^2\right) = \left[\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1\right]/\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right)$, then $BT\left(p_M^e,L^1/\left(\bar{L}^1+\bar{L}^2\right)\right) = 0$. Thus, $\left[\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1\right]/\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right) > \left(\bar{L}^1+m\bar{L}^2\right)/\left(\bar{L}^1+\bar{L}^2\right)$, implies that $v_L^1(1,\lambda_M) > v_L^2(1,\lambda_M)$ for all $L^1\in\left[(1-m)\bar{L}^1,\bar{L}^1+m\bar{L}^2\right]$ and $L^2=\left[(1-m)\bar{L}^2,\bar{L}^2+m\bar{L}^1\right]$.

b. Suppose that $(1-m)\bar{L}^1/\left(\bar{L}^1+\bar{L}^2\right) \le \left[\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1\right]/\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right) \le \left(\bar{L}^1+m\bar{L}^2\right)/\left(\bar{L}^1+\bar{L}^2\right)$. Then, $L^1 = \psi\left(\bar{L}^1+\bar{L}^2\right)$, $L^2 = (1-\psi)\left(\bar{L}^1+\bar{L}^2\right)$, where $\psi =$

$\left[\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1\right]/\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right)$ and, hence,

$$v_L^1(1,1)=v_L^2(1,1)=C\left[\frac{\left(\bar{F}_1^1+\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^1+\bar{F}_2^2\right)^{1-\alpha_1}}{\left(\bar{L}^1+\bar{L}^2\right)}\right]^b.$$

Moreover, the equilibrium relative price is $p_1/p_2=p_M^e=\left[\alpha_1\left(\bar{F}_2^1+\bar{F}_2^2\right)/(1-\alpha_1)\left(\bar{F}_1^1+\bar{F}_1^2\right)\right]^b$. It is easy to show that, when $L^1=\psi\left(\bar{L}^1+\bar{L}^2\right)$ and $L^2=(1-\psi)\left(\bar{L}^1+\bar{L}^2\right)$, then, $p_1/p_2=p_M^e$ is the unique solution to $BT(p_1/p_2,\psi)=0$. Moreover, note that $(1-m)\bar{L}^1/\left(\bar{L}^1+\bar{L}^2\right)\leq\left[\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1\right]/\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right)\leq\left(\bar{L}^1+m\bar{L}^2\right)/\left(\bar{L}^1+\bar{L}^2\right)$ implies that $L^1=\psi\left(\bar{L}^1+\bar{L}^2\right)\in\left[(1-m)\bar{L}^1,\bar{L}^1+m\bar{L}^2\right]$ and $L^2=(1-\psi)\left(\bar{L}^1+\bar{L}^2\right)\in\left[(1-m)\bar{L}^2,\bar{L}^2+m\bar{L}^1\right]$. Thus, we only need to prove that, in equilibrium, the allocation of the labor force is given by $L^1=\psi\left(\bar{L}^1+\bar{L}^2\right)$ and $L^2=(1-\psi)\left(\bar{L}^1+\bar{L}^2\right)$. From case a, $\left[\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1\right]/\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right)\leq\left(\bar{L}^1+m\bar{L}^2\right)/\left(\bar{L}^1+\bar{L}^2\right)$ implies that $v_L^1(1,\lambda_M)\leq v_L^2(1,\lambda_M)$ for $L^1=\bar{L}^1+m\bar{L}^2$ and $L^2=(1-m)\bar{L}^2$. From case c (see bellow), $(1-m)\bar{L}^1/\left(\bar{L}^1+\bar{L}^2\right)\leq\left[\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1\right]/\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right)$ implies that $v_L^1(1,\lambda_M)\geq v_L^2(1,\lambda_M)$ for $L^1=(1-m)\bar{L}^1$ and $L^2=\bar{L}^2+m\bar{L}^1$. From Results 2 and 3, $v_L^1(1,\lambda_M)$ is decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$, while $v_L^2(1,\lambda_M)$ is increasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$. Therefore, in equilibrium, it must be the case that $v_L^1(1,1)=v_L^2(1,1)$, which implies $L^1/\left[\bar{F}_2^1+(p_1/p_2)^{\frac{1}{b}}\bar{F}_1^1\right]=L^2/\left[\bar{F}_2^2+(p_1/p_2)^{\frac{1}{b}}\bar{F}_1^2\right]$. Since, in equilibrium, $p_1/p_2=p_M^e$, we have $L^1=\psi\left(\bar{L}^1+\bar{L}^2\right)$ and $L^2=(1-\psi)\left(\bar{L}^1+\bar{L}^2\right)$, where $\psi=\left[\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1\right]/\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right)$.

c. Suppose that $\left[\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1\right]/\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right)<(1-m)\bar{L}^1/\left(\bar{L}^1+\bar{L}^2\right)$. Then, $L^1=(1-m)\bar{L}^1$ and $L^2=\bar{L}^2+m\bar{L}^1$ and, hence,

$$v_L^j(1,1)=C\left[(\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2}\right]^b\left[\frac{\bar{F}_2^j+(p_H^e)^{\frac{1}{b}}\bar{F}_1^j}{(p_H^e)^{\frac{\alpha_1}{b}}L^j}\right]^b,$$

$p_1/p_2=p_H^e$ is the unique solution to $BT\left(p_1/p_2,(1-m)\bar{L}^1/\left(\bar{L}^2+m\bar{L}^1\right)\right)=0$. To prove this, from Results 2 and 3 $v_L^1(1,\lambda_M)$ is decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$, while $v_L^2(1,\lambda_M)$ is increasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$. Thus, if $\left[\bar{F}_2^1+(p_H^e)^{\frac{1}{b}}\bar{F}_1^1\right]/(1-m)\bar{L}^1<\left[\bar{F}_2^2+(p_H^e)^{\frac{1}{b}}\bar{F}_1^2\right]/\left(\bar{L}^2+m\bar{L}^1\right)$, then $v_L^2(1,\lambda_M)>v_L^1(1,\lambda_M)$ for all $L^1\in\left[(1-m)\bar{L}^1,\bar{L}^1+m\bar{L}^2\right]$ and $L^2=\left[(1-m)\bar{L}^2,\bar{L}^2+m\bar{L}^1\right]$. Note that $BT\left(p_H^e,(1-m)\bar{L}^1/\left(\bar{L}^1+\bar{L}^2\right)\right)=0$ if and only if $\left[\bar{F}_2^1+(p_H^e)^{\frac{1}{b}}\bar{F}_1^1\right]/\left[\bar{F}_2^2+(p_H^e)^{\frac{1}{b}}\bar{F}_1^2\right]=\left\{\left[(1-\alpha_1)(p_H^e)^{\frac{1}{b}}\bar{F}_1^1-\alpha_1\bar{F}_2^1\right]/\left[\alpha_1\bar{F}_2^2-(1-\alpha_1)(p_H^e)^{\frac{1}{b}}\bar{F}_1^2\right]\right\}^{\frac{1}{1-b}}\left[(1-m)\bar{L}^1/\bar{L}^2+m\bar{L}^1\right]$. Therefore, $\left[\bar{F}_2^1+(p_H^e)^{\frac{1}{b}}\bar{F}_1^1\right]/(1-m)\bar{L}^1<\left[\bar{F}_2^2+(p_H^e)^{\frac{1}{b}}\bar{F}_1^2\right]/\left(\bar{L}^2+m\bar{L}^1\right)$ implies $p_H^e<\left[\alpha_1\left(\bar{F}_2^2+\bar{F}_2^1\right)/(1-\alpha_1)\left(\bar{F}_1^1+\bar{F}_1^2\right)\right]^b$. Due to Result 1, p^e is decreasing in $L^1/\left(\bar{L}^1+\bar{L}^2\right)$ and, hence, $p_H^e<p_M^e=\left[\alpha_1\left(\bar{F}_2^2+\bar{F}_2^1\right)/(1-\alpha_1)\left(\bar{F}_1^1+\bar{F}_1^2\right)\right]^b$ if and only if $\bar{L}^1(1-m)>L^1$, where L^1 is such that $BT\left(p_M^e,L^1/\left(\bar{L}^1+\bar{L}^2\right)\right)=0$. As we have already shown, if $L^1/\left(\bar{L}^1+\bar{L}^2\right)=\left[\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1\right]/\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right)$, then $BT\left(p_M^e,L^1/\left(\bar{L}^1+\bar{L}^2\right)\right)=0$. Thus, $\left[\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1\right]/\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right)<(1-m)\bar{L}^1/\left(\bar{L}^1+\bar{L}^2\right)$, implies that

$v_L^1(1,\lambda_M) > v_L^2(1,\lambda_M)$ for all $L^1 \in \left[(1-m)\bar{L}^1, \bar{L}^1 + m\bar{L}^2\right]$ and $L^2 = \left[(1-m)\bar{L}^2, \bar{L}^2 + m\bar{L}^1\right]$. This completes the proof of the lemma. ■

Proposition 5 characterizes trade and labor mobility policies in a Ricardo-Vinner specific factors economy.

Proposition 5 (Ricardo-Vinner specific factors economy). *Assume that* $\frac{\bar{F}_1^1}{\bar{F}_2^1} > \frac{\bar{F}_1^2}{\bar{F}_2^2}$, $\left(\frac{\bar{F}_1^1}{\bar{F}_1^2}\right)^{\alpha_1}\left(\frac{\bar{F}_2^1}{\bar{F}_2^2}\right)^{(1-\alpha_1)} \neq \frac{\bar{L}^1}{\bar{L}^2}$ *and* $\frac{\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1}{\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right)} \neq \frac{\bar{L}^1}{\bar{L}^2+\bar{L}^2}$. *Then, the trade and labor mobility game has two Nash equilibrium outcomes: (i) no trade and no labor mobility; and (ii) free trade and no labor mobility. Moreover,* $W_G^j(1,0) > W_G^j(0,0)$ *for* $j = 1,2$.

Proof:

No trade and no labor mobility is always a Nash equilibrium outcome.

Free trade and no labor mobility is a Nash equilibrium outcome. From Lemma 6 Part 1, $v_L^j(0,0) = C\left(\bar{F}_1^j\right)^{b\alpha_1}\left(\bar{F}_2^j\right)^{b(1-\alpha_1)}/\left(\bar{L}^j\right)^b$ or, which is equivalent, $v_L^j(0,0) = C\left[(\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2}\right]^b\left\{\left[\bar{F}_2^j + \left(p^{A,j}\right)^{\frac{1}{b}}\bar{F}_1^j\right]/\left(p^{A,j}\right)^{\frac{\alpha_1}{b}}\bar{L}^j\right\}^b$, where $p^{A,j} = \left(\bar{F}_2^j\alpha_1/\bar{F}_1^j\alpha_2\right)^b$. From Lemma 7 Part 1, $v_L^j(1,0) = C\left[(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\right]^b\left\{\left[\bar{F}_2^j + (\bar{p}^e)^{\frac{1}{b}}\bar{F}_1^j\right]/(\bar{p}^e)^{\frac{\alpha_1}{b}}\bar{L}^j\right\}^b$ where $p_1/p_2 = \bar{p}^e$ is the unique solution to $BT\left(p_1/p_2, \bar{L}^1/\left(\bar{L}^2+\bar{L}^2\right)\right) = 0$. $v_L^1(1,0) > v_L^1(0,0)$ if and only if $\left[\bar{F}_2^1 + (\bar{p}^e)^{\frac{1}{b}}\bar{F}_1^1\right]/(\bar{p}^e)^{\frac{\alpha_1}{b}} > \left[\bar{F}_2^1 + \left(p^{A,1}\right)^{\frac{1}{b}}\bar{F}_1^1\right]/\left(p^{A,1}\right)^{\frac{\alpha_1}{b}}$. Note that $\left[\bar{F}_2^1 + (p_1/p_2)^{\frac{1}{b}}\bar{F}_1^1\right]/(p_1/p_2)^{\frac{\alpha_1}{b}}$ is increasing in p_1/p_2 whenever $p_1/p_2 > p^{A,1}$. We have already proved that $\bar{p}^e > p^{A,1}$, which implies that $W_G^1(1,0) = v_L^1(1,0) > v_L^1(0,0) = W_G^1(0,0)$. $v_L^2(1,0) > v_L^2(0,0)$ if and only if $\left[\bar{F}_2^2 + (\bar{p}^e)^{\frac{1}{b}}\bar{F}_1^2\right]/(\bar{p}^e)^{\frac{\alpha_1}{b}} > \left[\bar{F}_2^2 + \left(p^{A,2}\right)^{\frac{1}{b}}\bar{F}_1^2\right]/\left(p^{A,2}\right)^{\frac{\alpha_1}{b}}$. Note that $\left[\bar{F}_2^2 + (p_1/p_2)^{\frac{1}{b}}\bar{F}_1^2\right]/(p_1/p_2)^{\frac{\alpha_1}{b}}$ is decreasing in p_1/p_2 whenever $p_1/p_2 < p^{A,2}$. We have already proved that $\bar{p}^e < p^{A,2}$, which implies that $W_G^2(1,0) = v_L^2(1,0) > v_L^2(0,0) = W_G^2(0,0)$.

Free trade and free labor mobility is not a Nash equilibrium outcome. From Lemma 7 Part 1, $v_L^j(1,0) = C\left[(\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2}\right]^b\left\{\left[\bar{F}_2^j + (\bar{p}^e)^{\frac{1}{b}}\bar{F}_1^j\right]/(\bar{p}^e)^{\frac{\alpha_1}{b}}\bar{L}^j\right\}^b$ where $p_1/p_2 = \bar{p}^e$ is the unique solution to $BT\left(p_1/p_2, \bar{L}^1/\left(\bar{L}^2+\bar{L}^2\right)\right) = 0$. From Lemma 7 Part 2-a, if $\left[\bar{F}_1^1\bar{F}_2^1 + (1-\alpha_1)\bar{F}_1^2\bar{F}_2^1 + \alpha_1\bar{F}_2^2\bar{F}_1^1\right]/\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right) > \left(\bar{L}^1+m\bar{L}^2\right)/\left(\bar{L}^1+\bar{L}^2\right)$, then $L^1 = \bar{L}^1 + m\bar{L}^2$, $L^2 = (1-m)\bar{L}^2$ and $v_L^j(1,1) = C\left[(\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2}\right]^b\left\{\left[\bar{F}_2^j + (p_L^e)^{\frac{1}{b}}\bar{F}_1^j\right]/(p_L^e)^{\frac{\alpha_1}{b}}L^j\right\}^b$, where $p_1/p_2 = p_L^e$ is the unique solution to $BT\left(p_1/p_2, \left(\bar{L}^1+m\bar{L}^2\right)/\left(\bar{L}^1+\bar{L}^2\right)\right) = 0$. $W_G^1(1,0) = v_L^1(1,0) > v_L^1(1,1) = W_G^1(1,1)$ if and only if $\left[\bar{F}_2^1 + (\bar{p}^e)^{\frac{1}{b}}\bar{F}_1^1\right]/(\bar{p}^e)^{\frac{\alpha_1}{b}}\bar{L}^1 > \left[\bar{F}_2^1 + (p_L^e)^{\frac{1}{b}}\bar{F}_1^1\right]/(p_L^e)^{\frac{\alpha_1}{b}}\left(\bar{L}^1+m\bar{L}^2\right)$, which always holds because $\bar{p}^e > p_L^e$. From Lemma 7 Part 2-b, if $(1-m)\bar{L}^1/\left(\bar{L}^1+\bar{L}^2\right) \leq \left[\bar{F}_1^1\bar{F}_2^1 + (1-\alpha_1)\bar{F}_1^2\bar{F}_2^1 + \alpha_1\bar{F}_2^2\bar{F}_1^1\right]/\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right) \leq \left(\bar{L}^1+m\bar{L}^2\right)/\left(\bar{L}^1+\bar{L}^2\right)$, then $v_L^j(1,1) = C\left[(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\right]^b\left\{\left[\bar{F}_2^j + (p_M^e)^{\frac{1}{b}}\bar{F}_1^j\right]/(p_M^e)^{\frac{\alpha_1}{b}}L^j\right\}^b$, where $L^1 = \psi\left(\bar{L}^2+\bar{L}^2\right)$, $L^2 = (1-\psi)\left(\bar{L}^2+\bar{L}^2\right)$ and $p_1/p_2 = p_M^e$ is the unique solution to $BT(p_1/p_2,\psi) = 0$. $W_G^1(1,0) = v_L^1(1,0) > v_L^1(1,1) = W_G^1(1,1)$ if and only if $\left[\bar{F}_2^1 + (\bar{p}^e)^{\frac{1}{b}}\bar{F}_1^1\right]/(\bar{p}^e)^{\frac{\alpha_1}{b}}\bar{L}^1 > \left[\bar{F}_2^1 + (p_M^e)^{\frac{1}{b}}\bar{F}_1^1\right]/(p_M^e)^{\frac{\alpha_1}{b}}\psi\left(\bar{L}^2+\bar{L}^2\right)$, which always holds whenever $\psi > \bar{L}^1/\left(\bar{L}^2+\bar{L}^2\right)$. $v_L^2(1,0) > v_L^2(1,1)$ if and only if $\left[\bar{F}_2^2+(\bar{p}^e)^{\frac{1}{b}}\bar{F}_1^2\right]/(\bar{p}^e)^{\frac{\alpha_1}{b}}\bar{L}^2 > \left[\bar{F}_2^2+(p_M^e)^{\frac{1}{b}}\bar{F}_1^2\right]/(p_M^e)^{\frac{\alpha_1}{b}}(1-\psi)\left(\bar{L}^2+\bar{L}^2\right)$, which always holds whenever $\psi < \bar{L}^1/\left(\bar{L}^2+\bar{L}^2\right)$. From Lemma 7 Part 2-c, if $\left[\bar{F}_1^1\bar{F}_2^1+(1-\alpha_1)\bar{F}_1^2\bar{F}_2^1+\alpha_1\bar{F}_2^2\bar{F}_1^1\right]/\left(\bar{F}_1^1+\bar{F}_1^2\right)\left(\bar{F}_2^1+\bar{F}_2^2\right) < (1-m)\bar{L}^1/\left(\bar{L}^1+\bar{L}^2\right)$, then $L^1=(1-m)\bar{L}^1$, $L^2=\bar{L}^2+(1-m)\bar{L}^1$ and $v_L^j(1,1) = C\left[(\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2}\right]^b\left\{\left[\bar{F}_2^j+(p_H^e)^{\frac{1}{b}}\bar{F}_1^j\right]/(p_H^e)^{\frac{\alpha_1}{b}}L^j\right\}^b$, where $p_1/p_2=p_H^e$ is the unique solution to $BT\left(p_1/p_2,(1-m)\bar{L}^1/\left(\bar{L}^1+\bar{L}^2\right)\right)=0$. $W_G^2(1,0)=v_L^2(1,0) > v_L^2(1,1)=W_G^2(1,1)$ if and only if $\left[\bar{F}_2^2+(\bar{p}^e)^{\frac{1}{b}}\bar{F}_1^2\right]/(\bar{p}^e)^{\frac{\alpha_1}{b}}\bar{L}^2 > \left[\bar{F}_2^2+(p_H^e)^{\frac{1}{b}}\bar{F}_1^2\right]/(p_H^e)^{\frac{\alpha_1}{b}}\left(\bar{L}^2+m\bar{L}^1\right)$, which always holds because $\bar{p}^e < p_H^e$.

No trade and free labor mobility is not a Nash equilibrium outcome. From Lemma 6 Part 2-c, if $\left(\bar{F}_1^1/\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^1/\bar{F}_2^2\right)^{(1-\alpha_1)} > \bar{L}^1/\bar{L}^2$, then $W_G^1(0,0)=v_L^1(0,0)>v_L^1(0,1)=W_G^1(0,1)$ and, hence, workers in country 1 prefer to block labor mobility. From Lemma 6 Part 2-c, if $\left(\bar{F}_1^1/\bar{F}_1^2\right)^{\alpha_1}\left(\bar{F}_2^1/\bar{F}_2^2\right)^{(1-\alpha_1)} < \bar{L}^1/\bar{L}^2$, then $W_G^2(0,0)=v_L^2(0,0)>v_L^2(0,1)=W_G^2(0,1)$ and, hence, workers in country 2 prefer to block labor mobility. This completes the proof of the proposition. ■

B.3 Multiple factors and non-tradeable goods

Consider an economy with two countries ($J=2$), two tradeable goods (a rural good F and manufactures M), one non-tradeable good (services N) and three factors of production (capital K, natural resources F and labor L). Production functions are given by $Q_F^j=T_F^j\left(F^j\right)^b\left(K_F^j\right)^{1-b}$, $Q_M^j=T_M^j\left(L_M^j\right)^b\left(K_M^j\right)^{1-b}$, and $Q_N^j=T_N^jL_N^j$, where T_z^j is total factor productivity in sector $z=F,M,N$ in country j, F^j is the quantity of natural resources employed in sector F in country j, K_z^j is the quantity of capital employed in sector $z=F,M$ in country j, L_z^j is the quantity of labor employed in sector $z=M,N$ in country j, and $b\in(0,1)$. Factor endowments in country j are $\left(\bar{F}^j,\bar{K}^j,\bar{L}^j\right)>(0,0,0)$. All agents have the same preferences, given by $u\left(c^j\right)=\prod_{z\in Z}\left(c_z^j\right)^{\alpha_z}$, with $\alpha_z>0$ and $\sum_{z\in Z}\alpha_z=1$.

Lemma 8 characterizes the equilibrium under autarky.

Lemma 8 (multiple factors and non-tradeable goods under autarky). *Assume there is no trade in goods, i.e., $\lambda_T=0$.*

1. *Suppose there is no labor mobility, i.e., $\lambda_M=0$. Then, in equilibrium, utilities in country j are given by:*

$$v_L^j(0,0)=B\left(\alpha_N+b\alpha_M\right)^{\alpha_F+(1-b)\alpha_M}T^j\left(\frac{\bar{F}^j}{\bar{L}^j}\right)^{b\alpha_F}\left(\frac{\bar{K}^j}{\bar{L}^j}\right)^{(1-b)(\alpha_F+\alpha_M)},$$

$$v_K^j(0,0)=\left[\frac{B(1-b)\left(\alpha_F+\alpha_M\right)}{\left(\alpha_N+b\alpha_M\right)^{b(1-\alpha_F)}}\right]T^j\left(\frac{\bar{F}^j}{\bar{K}^j}\right)^{b\alpha_F}\left(\frac{\bar{L}^j}{\bar{K}^j}\right)^{\alpha_N+b\alpha_M},$$

$$v_F^j(0,0)=\left[\frac{B\alpha_Fb}{\left(\alpha_N+b\alpha_M\right)^{\alpha_N+b\alpha_M}}\right]T^j\left(\frac{\bar{K}^j}{\bar{F}^j}\right)^{(1-b)(\alpha_F+\alpha_M)}\left(\frac{\bar{L}^j}{\bar{F}^j}\right)^{\alpha_N+b\alpha_M}.$$

where $B=\left[\frac{(b)^{b\alpha_M}}{(\alpha_F)^{b\alpha_F}(\alpha_F+\alpha_M)^{(1-b)(\alpha_F+\alpha_M)}}\right]$ *and* $T^j=\left(T_F^j\right)^{\alpha_F}\left(T_M^j\right)^{\alpha_M}\left(T_N^j\right)^{\alpha_N}$.

2. *Suppose there is free labor mobility, i.e., $\lambda_M = 1$. Then, in equilibrium, the labor force in country j is*

$$L^j = \left[\left(T^j\right)\left(\bar{F}^j\right)^{b\alpha_F}\left(\bar{K}^j\right)^{(1-b)(\alpha_F+\alpha_M)}\right]^{\frac{1}{\alpha_F+(1-b)\alpha_M}}\left(\bar{T}\right)^{-1}\bar{L},$$

where $\bar{T} = \sum_{k=1,2}\left[\left(T^k\right)\left(\bar{F}^k\right)^{b\alpha_F}\left(\bar{K}^k\right)^{(1-b)(\alpha_F+\alpha_M)}\right]^{\frac{1}{\alpha_F+(1-b)\alpha_M}}$. Moreover, utilities in country $j = 1, 2$ are given by:

$$v_L^1(0,1) = v_L^2(0,1) = B\left(\alpha_N + b\alpha_M\right)^{\alpha_F+(1-b)\alpha_M}\left(\frac{\bar{T}}{\bar{L}}\right)^{b\alpha_F+(1-b)(\alpha_F+\alpha_M)},$$

$$v_K^j(0,1) = \left[\frac{B(1-b)\left(\alpha_F+\alpha_M\right)}{\left(\alpha_N+b\alpha_M\right)^{b(1-\alpha_F)}}\right]\left(T^j\right)^{\frac{1}{\alpha_F+(1-b)\alpha_M}}\left(\frac{\bar{F}^j}{\bar{K}^j}\right)^{\frac{b\alpha_F}{\alpha_F+(1-b)\alpha_M}}\left(\frac{\bar{L}}{\bar{T}}\right)^{\alpha_N+b\alpha_M},$$

$$v_F^j(0,1) = \left[\frac{B\alpha_F b}{\left(\alpha_N+b\alpha_M\right)^{\alpha_N+b\alpha_M}}\right]\left(T^j\right)^{\frac{1}{\alpha_F+(1-b)\alpha_M}}\left(\frac{\bar{K}^j}{\bar{F}^j}\right)^{\frac{(1-b)(\alpha_F+\alpha_M)}{\alpha_F+(1-b)\alpha_M}}\left(\frac{\bar{L}}{\bar{T}}\right)^{\alpha_N+b\alpha_M}.$$

Proof. Let p_z^j denotes the price of good $z = F, M, N$ in country j, r_f^j the return of factor $f = F, K$ in country j, w^j the wage rate in country j and v_L^j the utility of workers in country j. Under autarky the equilibrium conditions in country j are:

$$r_F^j = p_F^j T_F^j b\left(K_F^j/\bar{F}^j\right)^{1-b},$$
$$r_K^j = p_F^j T_F^j(1-b)\left(\bar{F}^j/K_F^j\right)^b = p_M^j T_M^j(1-b)\left(L_M^j/K_M^j\right)^b,$$
$$w^j = p_N^j T_N^j = p_M^j T_M^j b\left(K_M^j/L_M^j\right)^{1-b},$$
$$K_F^j + K_M^j = \bar{K}^j,\ L_M^j + L_N^j = L^j,$$
$$(1-\alpha_N)\,p_N^j Q_N^j = \alpha_N\left(p_F^j Q_F^j + p_M^j Q_M^j\right),\ \alpha_M p_F^j Q_F^j = \alpha_F p_M^j Q_M^j,$$
$$L^j = \bar{L}^j \text{ (when there is no labor mobility)},$$
$$v_L^1 = v_L^2,\ L^1 + L^2 = \bar{L} = \bar{L}^1 + \bar{L}^2 \text{ (when there is free labor mobility)}.$$

Equilibrium quantities: $K_M^j = \left[\alpha_M/\left(\alpha_F+\alpha_M\right)\right]\bar{K}^j$, $K_F^j = \left[\alpha_F/\left(\alpha_F+\alpha_M\right)\right]\bar{K}^j$, $L_M^j = \left[b\alpha_M/\left(\alpha_N+b\alpha_M\right)\right]L^j$, $L_N^j = \left[\alpha_N/\left(\alpha_N+b\alpha_M\right)\right]L^j$, $Q_F^j = \left[\alpha_F/\left(\alpha_F+\alpha_M\right)\right]^{1-b}T_F^j\left(\bar{F}^j\right)^b\left(\bar{K}^j\right)^{1-b}$, $Q_M^j = \left[(b)^b\alpha_M/\left(\alpha_N+b\alpha_M\right)\right]T_M^j\left(L^j\right)^b\left(\bar{K}^j\right)^{1-b}$ and $Q_N^j = \left[\alpha_N/\left(\alpha_N+b\alpha_M\right)\right]T_N^j L^j$.

Equilibrium prices: $p_F^j/p_M^j = \left[b\alpha_F/\left(\alpha_N+b\alpha_M\right)\right]^b\left(T_M^j/T_F^j\right)\left(L^j/\bar{F}^j\right)^b$ and $p_N^j/p_M^j = (b)^b\left[\left(\alpha_N+b\alpha_M\right)/\left(\alpha_F+\alpha_M\right)\right]^{1-b}\left(T_M^j/T_N^j\right)\left(\bar{K}^j/L^j\right)^{1-b}$.

Equilibrium factor prices: $w^j/p_F^j = \left[\left(\alpha_N+b\alpha_M\right)/\left(\alpha_F\right)^b\left(\alpha_F+\alpha_M\right)^{1-b}\right]T_F^j\left(\bar{F}^j/L^j\right)^b\left(\bar{K}^j/L^j\right)^{1-b}$, $w^j/p_M^j = (b)^b\left[\left(\alpha_N+b\alpha_M\right)/\left(\alpha_F+\alpha_M\right)\right]^{1-b}T_M^j\left(\bar{K}^j/L^j\right)^{1-b}$, $w^j/p_N^j = T_N^j$. $r_K^j/p_F^j = (1-b)\left[\left(\alpha_F+\alpha_M\right)/\alpha_F\right]^b T_F^j\left(\bar{F}^j/\bar{K}^j\right)^b$, $r_K^j/p_M^j = (1-b)\left[b\left(\alpha_F+\alpha_M\right)/\left(\alpha_N+b\alpha_M\right)\right]^b T_M^j\left(L^j/\bar{K}^j\right)^b$,

$r_K^j/p_N^j = (1-b)\left[(\alpha_F+\alpha_M)/(\alpha_N+b\alpha_M)\right]T_N^j\left(L^j/\bar{K}^j\right)$, $r_F^j/p_F^j = b\left[\alpha_F/(\alpha_F+\alpha_M)\right]^{1-b}T_F^j\left(\bar{K}^j/\bar{F}^j\right)^{1-b}$, $r_F^j/p_M^j = \left[\alpha_F b\,(b)^b/(\alpha_F+\alpha_M)^{1-b}(\alpha_N+b\alpha_M)^b\right]T_M^j\left(L^j/\bar{F}^j\right)^b\left(\bar{K}^j/\bar{F}^j\right)^{1-b}$ and $r_F^j/p_N^j = \left[\alpha_F b/(\alpha_N+b\alpha_M)\right]T_N^j\left(L^j/\bar{F}^j\right)$.

Utilities: $v_L^j(0,\lambda_M) = w^j/(p_F)^{\alpha_F}(p_M)^{\alpha_M}\left(p_N^j\right)^{\alpha_N}$, $v_K^j(0,\lambda_M) = r_K^j/(p_F)^{\alpha_F}(p_M)^{\alpha_M}\left(p_N^j\right)^{\alpha_N}$ and $v_F^j(0,\lambda_M) = r_F^j/(p_F)^{\alpha_F}(p_M)^{\alpha_M}\left(p_N^j\right)^{\alpha_N}$. Then:

$$\begin{aligned}
v_L^j(0,\lambda_M) &= B(\alpha_N+b\alpha_M)^{\alpha_F+(1-b)\alpha_M}T^j\left(\frac{\bar{F}^j}{L^j}\right)^{b\alpha_F}\left(\frac{\bar{K}^j}{L^j}\right)^{(1-b)(\alpha_F+\alpha_M)}, \\
v_K^j(0,\lambda_M) &= \frac{B(1-b)(\alpha_F+\alpha_M)}{(\alpha_N+b\alpha_M)^{b(1-\alpha_F)}}T^j\left(\frac{\bar{F}^j}{\bar{K}^j}\right)^{b\alpha_F}\left(\frac{L^j}{\bar{K}^j}\right)^{\alpha_N+b\alpha_M}, \\
v_F^j(0,\lambda_M) &= \frac{Bb\alpha_F}{(\alpha_N+b\alpha_M)^{\alpha_N+b\alpha_M}}T^j\left(\frac{\bar{K}^j}{\bar{F}^j}\right)^{(1-b)(\alpha_F+\alpha_M)}\left(\frac{L^j}{\bar{F}^j}\right)^{\alpha_N+b\alpha_M}.
\end{aligned}$$

where $B = \left[(b)^{b\alpha_M}/(\alpha_F)^{b\alpha_F}(\alpha_F+\alpha_M)^{(1-b)(\alpha_F+\alpha_M)}\right]$ and $T^j = \left(T_F^j\right)^{\alpha_F}\left(T_M^j\right)^{\alpha_M}\left(T_N^j\right)^{\alpha_N}$.

When there is no labor mobility $L^j = \bar{L}^j$. When there is free labor mobility, $v_L^1 = v_L^2$, which implies

$$L^j = \left[\left(T^j\right)\left(\bar{F}^j\right)^{b\alpha_F}\left(\bar{K}^j\right)^{(1-b)(\alpha_F+\alpha_M)}\right]^{\frac{1}{\alpha_F+(1-b)\alpha_M}}\left(\bar{T}\right)^{-1}\bar{L},$$

where $\bar{T} = \sum_{k=1,2}\left[\left(T^k\right)\left(\bar{F}^k\right)^{b\alpha_F}\left(\bar{K}^k\right)^{(1-b)(\alpha_F+\alpha_M)}\right]^{\frac{1}{\alpha_F+(1-b)\alpha_M}}$. Thus, under autarky, there are migrations to country 1 if and only if $\left(T^1/T^2\right) > \left(\bar{F}^2/\bar{F}^1\right)^{b\alpha_F}\left(\bar{K}^2/\bar{K}^1\right)^{(1-b)(\alpha_F+\alpha_M)}\left(\bar{L}^1/\bar{L}^2\right)^{\alpha_F+(1-b)\alpha_M}$. Finally, utilities under autarky and free labor mobility are:

$$\begin{aligned}
v_L^1(0,1) = v_L^2(0,1) &= B(\alpha_N+b\alpha_M)^{\alpha_F+(1-b)\alpha_M}\left(\frac{\bar{T}}{\bar{L}}\right)^{b\alpha_F+(1-b)(\alpha_F+\alpha_M)}, \\
v_K^j(0,1) &= \frac{B(1-b)(\alpha_F+\alpha_M)}{(\alpha_N+b\alpha_M)^{b(1-\alpha_F)}}\left(T^j\right)^{\frac{1}{\alpha_F+(1-b)\alpha_M}}\left(\frac{\bar{F}^j}{\bar{K}^j}\right)^{\frac{b\alpha_F}{\alpha_F+(1-b)\alpha_M}}\left(\frac{\bar{L}}{\bar{T}}\right)^{\alpha_N+b\alpha_M}, \\
v_F^j(0,1) &= \frac{Bb\alpha_F}{(\alpha_N+b\alpha_M)^{\alpha_N+b\alpha_M}}\left(T^j\right)^{\frac{1}{\alpha_F+(1-b)\alpha_M}}\left(\frac{\bar{K}^j}{\bar{F}^j}\right)^{\frac{(1-b)(\alpha_F+\alpha_M)}{\alpha_F+(1-b)\alpha_M}}\left(\frac{\bar{L}}{\bar{T}}\right)^{\alpha_N+b\alpha_M}.
\end{aligned}$$

This completes the proof of the lemma. ■

Lemma 9 characterizes the equilibrium under free trade.

Lemma 9 (multiple factors and non-tradeable goods under free trade). *Assume there is* *free trade of goods, i.e.,* $\lambda_T = 1$, *and the following conditions hold: .*

$$\alpha_N \alpha_F \left(\frac{\bar{L}^1}{\bar{L}^2}\right) > (1-\alpha_N)(\alpha_N + b\alpha_M)\left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}} \left(\frac{\bar{F}^1}{\bar{F}^2}\right),$$

$$\left(\frac{\bar{K}^1}{\bar{K}^2}\right)^{1-b} > \frac{\alpha_M \left[(1-\alpha_N) b + \alpha_N\right]^b \left[\alpha_N + (1-\alpha_N) b \left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}} \left(\frac{\bar{L}^2 \bar{F}^1}{\bar{L}^1 \bar{F}^2}\right)\right]^{1-b}}{\left[\alpha_N \alpha_F - (1-\alpha_N)(\alpha_M b + \alpha_N)\left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}} \left(\frac{\bar{L}^2 \bar{F}^1}{\bar{L}^1 \bar{F}^2}\right)\right]} \left(\frac{T_M^2}{T_M^1}\right)\left(\frac{\bar{L}^2}{\bar{L}^1}\right)^b.$$

1. *Suppose there is no labor mobility , i.e.,* $\lambda_M = 0$. *Then, in equilibrium, country 1 is diversified, country 2 specializes in good F and* $(p_F/p_M) = \check{p}$ *is the unique solution to* $BT\left(\check{p}, \bar{L}^1\right) = 0$.
2. *Suppose there is free labor mobility i.e.,* $\lambda_M = 1$. *Assume the following condition holds.*

$$\left(\frac{T_N^1}{T_N^2}\right)^{\frac{\alpha_N}{\alpha_M+\alpha_F}} > \frac{\alpha_N \alpha_F \left(\frac{\bar{L}^1}{\bar{L}^2}\right) - (\alpha_N + \alpha_M b)(1-\alpha_N)\left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}} \left(\frac{\bar{F}^1}{\bar{F}^2}\right)}{\alpha_M \left[(1-\alpha_N) b + \alpha_N\right]}$$

Then, in equilibrium, country 1 is diversified, country 2 specializes in good F, $p_F/p_M = \hat{p}$, *and* L^1 *are given by the solution to* $BT\left(\hat{p}, L^1\right) = 0$ *and* $FM\left(\hat{p}, L^1\right) = 0$. *Moreover,* $\hat{p} > \check{p}$ *and* $L^1 > \bar{L}_1$.

3. *Utilities are given by:*

$$v_L^1(1, \lambda_M) = \frac{\left[\frac{(b)^b [(1-\alpha_N)b+\alpha_N]^{(1-b)}}{(1-\alpha_N)^{(1-b)}}\right]^{(\alpha_M+\alpha_F)} T^1 \left(\bar{K}^1\right)^{(1-b)(\alpha_M+\alpha_F)}}{\left[L^1 + \left(\frac{p_F T_F^1}{p_M T_M^1}\right)^{\frac{1}{b}} \bar{F}^1\right]^{(1-b)(\alpha_M+\alpha_F)}} \left(\frac{T_M^1}{T_F^1}\frac{p_M}{p_F}\right)^{\alpha_F},$$

$$v_K^1(1, \lambda_M) = \frac{T^1 (1-b)}{\left(\frac{p_F T_F^1}{p_M T_M^1}\right)^{\alpha_F}} \left\{\frac{[(1-\alpha_N) b]\left[L^1 + \left(\frac{p_F T_F^1}{p_M T_M^1}\right)^{\frac{1}{b}} \bar{F}^1\right]}{[(1-\alpha_N) b + \alpha_N] \bar{K}^1}\right\}^{\alpha_N + b(\alpha_F+\alpha_M)},$$

$$v_F^1(1, \lambda_M) = \frac{T^1 \left(\frac{T_F^1}{T_M^1}\right)^{\frac{\alpha_N}{b}} \left(\frac{p_F}{p_M}\right)^{\frac{\alpha_N + b\alpha_M}{b}} \left\{[(1-\alpha_N) b + \alpha_N] \bar{K}^1 \left(\frac{p_F T_F^1}{p_M T_M^1}\right)^{\frac{1}{b}}\right\}^{(\alpha_M+\alpha_F)(1-b)}}{(b)^{-(\alpha_M+\alpha_F)} \left\{b(1-\alpha_N)\left[L^1 + \left(\frac{p_F T_F^1}{p_M T_M^1}\right)^{\frac{1}{b}} \bar{F}^1\right]\right\}^{(\alpha_M+\alpha_F)(1-b)}},$$

$$v_L^2(1, \lambda_M) = \left(\frac{\alpha_N}{1-\alpha_N}\right)^{\alpha_F+\alpha_M} \left(\frac{\bar{F}^2}{L^2}\right)^{b(\alpha_M+\alpha_F)} \left(\frac{\bar{K}^2}{L^2}\right)^{(1-b)(\alpha_M+\alpha_F)} \left(\frac{T_F^2}{T_M^2}\right)^{\alpha_M} T^2 \left(\frac{p_F}{p_M}\right)^{\alpha_M},$$

$$v_K^2(1, \lambda_M) = (1-b)(1-\alpha_N)^{\alpha_N} \left(T_N^2\right)^{\alpha_N} \left(T_F^2\right)^{\alpha_F+\alpha_M} \left(\frac{p_F}{p_M}\right)^{\alpha_M} \left(\frac{\bar{F}^2}{\bar{K}^2}\right)^{b(\alpha_F+\alpha_M)} \left(\frac{L^2}{\bar{K}^2}\right)^{\alpha_N},$$

$$v_F^2(1, \lambda_M) = b\left(\frac{1-\alpha_N}{\alpha_N}\right)^{\alpha_N} \left(T_N^2\right)^{\alpha_N} \left(T_F^2\right)^{\alpha_F+\alpha_M} \left(\frac{\bar{K}^2}{\bar{F}^2}\right)^{(1-b)(\alpha_F+\alpha_M)} \left(\frac{L^2}{\bar{F}^2}\right)^{\alpha_N} \left(\frac{p_F}{p_M}\right)^{\alpha_M},$$

where if $\lambda_M = 0$, $L^1 = \bar{L}^1$ *and* p_F/p_M *is the unique solution to* $BT\left(p, \bar{L}^1\right) = 0$, *while if* $\lambda_M = 1$, p_F/p_M, *and* L_1 *are given by the solution to* $BT\left(p, L^1\right) = 0$ *and* $FM\left(p, L^1\right) = 0$.

Proof. Under free trade equilibrium conditions are:

$$r_F^j = p_F A_F^j b \left(K_F^j/\bar{F}^j\right)^{1-b}$$

$$r_K^j = p_M T_M^j (1-b)\left(L_M^j/K_M^j\right)^b = p_F T_F^j (1-b)\left(\bar{F}^j/K_F^j\right)^b \text{ for } L_M^j > 0,$$

$$r_K^j = p_F T_F^j (1-b)\left(\bar{F}^j/K_F^j\right)^b \text{ and } \left(T_M^j\right)^{-1}\left(w^j\right)^b\left(r_K^j\right)^{1-b} > \left(T_M^{-j}\right)^{-1}\left(w^{-j}\right)^b\left(r_K^{-j}\right)^{1-b} \text{ for } L_M^j = 0,$$

$$w^j = p_N^j T_N^j = p_M T_M^j b\left(K_M^j/L_M^j\right)^{1-b} \text{ for } L_M^j > 0,$$

$$w^j = p_N^j T_N^j \text{ and } \left(T_M^j\right)^{-1}\left(w^j\right)^b\left(r_K^j\right)^{1-b} > \left(T_M^{-j}\right)^{-1}\left(w^{-j}\right)^b\left(r_K^{-j}\right)^{1-b} \text{ for } L_M^j = 0,$$

$$K_F^j + K_M^j = \bar{K}^j,\ L_M^j + L_N^j = L^j,$$

$$(1-\alpha_N)\, p_N^j Q_N^j = \alpha_N\left(p_F^j Q_F^j + p_M^j Q_M^j\right),$$

$$\alpha_M p_F\left(Q_F^1 + Q_F^2\right) = \alpha_F p_M\left(Q_M^1 + Q_M^2\right),$$

$$L^j = \bar{L}^j \text{ (when there is no labor mobility)},$$

$$v_L^1 = v_L^2,\ L^1 + L^2 = \bar{L} = \bar{L}^1 + \bar{L}^2 \text{ (when there is free labor mobility)},$$

There are two possible cases to consider: either both countries are diversified or one country is diversified and other specializes in good F. We focus on the second case and, without lose of generality, we assume that country 1 is diversified and country 2 specialized in good F.

Equilibrium quantities: Let $\tilde{p} = p_F T_F^1/p_M T_M^1$. Then, $K_F^1 = \{[(1-\alpha_N)b + \alpha_N]/(1-\alpha_N)b\}\left\{(\tilde{p})^{\frac{1}{b}}\bar{F}^1/\left[L^1 + (\tilde{p})^{\frac{1}{b}}\bar{F}^1\right]\right\}\bar{K}^1$, $K_M^1 = \left\{\left[(1-\alpha_N)bL^1 - (\tilde{p})^{\frac{1}{b}}\alpha_N\bar{F}^1\right]/\left[L^1 + (\tilde{p})^{\frac{1}{b}}\bar{F}^1\right]\right\}\bar{K}^1/(1-\alpha_N)b$, $L_M^1 = \left[(1-\alpha_N)bL^1 - \alpha_N(\tilde{p})^{\frac{1}{b}}\bar{F}^1\right]/[(1-\alpha_N)b + \alpha_N]$, $L_N^1 = \left[\alpha_N L^1 + \alpha_N(\tilde{p})^{\frac{1}{b}}\bar{F}^1\right]/[(1-\alpha_N)b + \alpha_N]$, $Q_M^1 = \frac{\left[(1-\alpha_N)bL^1 - \alpha_N(\tilde{p})^{\frac{1}{b}}\bar{F}^1\right]T_M^1\left(\bar{K}^1\right)^{1-b}}{[(1-\alpha_N)b+\alpha_N]^b(1-\alpha_N)^{1-b}(b)^{1-b}\left[L^1 + (\tilde{p})^{\frac{1}{b}}\bar{F}^1\right]^{1-b}}$, $K_F^2 = \bar{K}^2$, $K_M^2 = 0$, $L_N^2 = L^2$, $L_M^2 = 0$, $Q_F^2 = T_F^2\left(\bar{F}^2\right)^b\left(\bar{K}^2\right)^{1-b}$, $Q_M^2 = 0$, and $Q_N^2 = T_N^2\bar{L}^2$. For this to be an equilibrium we must verify that country 1 exports M or, which is equivalent, that the equilibrium relative price of F is lower under free than under autarky in country 1. Formally, $(p_F/p_M) < \left(p_F^1/p_M^1\right)^A = \{b\alpha_F/[\alpha_N + b\alpha_M]\}^b\left(T_M^1/T_F^1\right)\left(L^1/\bar{F}^1\right)^b$. We must also verify that country 2 specializes in good F. Formally, $\left(T_M^2\right)^{-1}\left(w^2\right)^b\left(r_K^2\right)^{1-b} > \left(T_M^1\right)^{-1}\left(w^1\right)^b\left(r_K^1\right)^{1-b}$ or, which is equivalent, $(p_F/p_M) > \left(p_F^2/p_M^2\right)^S = [(1-\alpha_N)b/\alpha_N]^b\left(T_M^2/T_F^2\right)\left(L^2/\bar{F}^2\right)^b$. Thus, we need the following condition

$$\left[\frac{(1-\alpha_N)b}{\alpha_N}\right]^b\left(\frac{T_M^2}{T_F^2}\right)\left(\frac{L^2}{\bar{F}^2}\right)^b = \left(\frac{p_F^2}{p_M^2}\right)^S < \frac{p_F}{p_M} < \left(\frac{p_F^1}{p_M^1}\right)^A = \left(\frac{b\alpha_F}{\alpha_N + b\alpha_M}\right)^b\left(\frac{T_M^1}{T_F^1}\right)\left(\frac{L^1}{\bar{F}^1}\right)^b$$

Note that $(p_F/p_M) < \left(p_F^1/p_M^1\right)^A$ immediately implies $L_M^1 > 0$ and $K_M^1 > 0$. Also note that $\left(p_F^2/p_M^2\right)^S <$

$\left(p_F^1/p_M^1\right)^A$ if and only if

$$\alpha_N \alpha_F \left(\frac{L^1}{L^2}\right) > (1-\alpha_N)(\alpha_N + b\alpha_M)\left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}}\left(\frac{\bar{F}^1}{\bar{F}^2}\right).$$

Equilibrium good prices: $p_N^1/p_M = (b)^b \left\{\left[(1-\alpha_N) b + \alpha_N\right]/(1-\alpha_N)\right\}^{1-b} \Big\{ \left(p_M T_M^1\right)^{\frac{1}{b}} / \Big[\left(p_M T_M^1\right)^{\frac{1}{b}} L^1 + \left(p_F T_F^1\right.$ and $p_N^2/p_F = \left[\alpha_N/(1-\alpha_N)\right]\left(T_F^2/T_N^2\right)\left(\bar{F}^2/L^2\right)^b\left(\bar{K}^2/L^2\right)^{1-b}$. $(p_F/p_M) = \check{p}$ is implicitly given by the solution to $BT\left(\check{p}, L^1\right) = 0$, where

$$BT\left(p, L^1\right) = \frac{(1-\alpha_N)^b T_M^1 \left(\bar{K}^1\right)^{1-b}\left[\alpha_F b L^1 - (\alpha_M b + \alpha_N)\left(\frac{T_F^1}{T_M^1}\right)^{\frac{1}{b}} (p)^{\frac{1}{b}} \bar{F}^1\right]}{\alpha_M (b)^{1-b}\left[(1-\alpha_N) b + \alpha_N\right]^b T_F^2 \left(\bar{F}^2\right)^b \left(\bar{K}^2\right)^{1-b}\left[L^1 + \left(\frac{T_F^1}{T_M^1}\right)^{\frac{1}{b}} (p)^{\frac{1}{b}} \bar{F}^1\right]^{1-b}} - p.$$

(We obtain $BT\left(p, L^1\right) = 0$ introducing Q_M^1, Q_F^1, Q_F^2 into the balanced trade equation $\alpha_M p_F\left(Q_F^1 + Q_F^2\right) = \alpha_F p_M\left(Q_M^1 + Q_M^2\right)$). Note that $BT\left(p, L^1\right)$ is strictly decreasing in p for $p \in \left[\left(p_F^2/p_M^2\right)^S, \left(p_F^1/p_M^1\right)^A\right]$, $BT\left(\left(p_F^1/p_M^1\right)^A, L^1\right) < 0$ and $BT\left(\left(p_F^2/p_M^2\right)^S, L^1\right) > 0$ if and only if

$$\left(\frac{\bar{K}^1}{\bar{K}^2}\right)^{1-b} > \frac{\alpha_M\left[(1-\alpha_N) b + \alpha_N\right]^b (D)^b \left[\alpha_N + D(1-\alpha_N) b \left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}}\left(\frac{L^2 \bar{F}^1}{L^1 \bar{F}^2}\right)\right]^{1-b}\left(\frac{T_M^2}{T_M^1}\right)\left(\frac{L^2}{L^1}\right)^b}{\left[\alpha_N \alpha_F - D(1-\alpha_N)(\alpha_M b + \alpha_N)\left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}}\left(\frac{L^2 \bar{F}^1}{L^1 \bar{F}^2}\right)\right]}.$$

Thus, when this condition holds $BT\left(p, L^1\right) = 0$ has a unique solution $\check{p} \in \left[\left(p_F^2/p_M^2\right)^S, \left(p_F^1/p_M^1\right)^A\right]$.

Equilibrium factor prices: Let $\tilde{p} = p_F T_F^1 / p_M T_M^1$. Then, $w^1/p_F = T_M^1\left[(b)^b/(1-\alpha_N)^{1-b}\right]\left\{\left[(1-\alpha_N) b + \alpha_N\right]/\left[L^1 + (\tilde{p})^{\frac{1}{b}} \bar{F}^1\right]\right\}^{1-b}\left(\bar{K}^1\right)^{1-b}(p_M/p_F)$, $w^1/p_M = T_M^1\left[(b)^b/(1-\alpha_N)^{1-b}\right]\left\{\left[(1-\alpha_N) b + \alpha_N\right]/\left[L^1 + (\tilde{p})^{\frac{1}{b}} \bar{F}^1\right]\right\}\left(\bar{K}^1\right)^{1-b}$, $w^1/p_N^1 = T_N^1$, $w^2/p_F = \left[\alpha_N/(1-\alpha_N)\right] T_F^2\left(\bar{F}^2/L^2\right)^b\left(\bar{K}^2/L^2\right)^{1-b}$, $w^2/p_M = \left[\alpha_N/(1-\alpha_N)\right] T_F^2\left(\bar{F}^2/L^2\right)^b\left(\bar{K}^2/L^2\right)^{1-b}(p_F/p_M)$, $w^2/p_N^2 = T_N^2$, $r_K^1/p_F = T_F^1(1-b)\left\{(1-\alpha_N) b\left[L^1 + (\tilde{p})^{\frac{1}{b}} \bar{F}^1\right]/\left[(1-\alpha_N) b + \alpha_N\right](\tilde{p})^{\frac{1}{b}} \bar{K}^1\right\}^b$, $r_K^1/p_M = T_M^1(1-b)\left\{(1-\alpha_N) b\left[L^1 + (\tilde{p})^{\frac{1}{b}} \bar{F}^1\right]/\left[(1-\alpha_N) b + \alpha_N\right]\bar{K}^1\right\}^b$, $r_K^1/p_N^1 = (1-\alpha_N) T_N^1(1-b)\left[L^1 + (\tilde{p})^{\frac{1}{b}} \bar{F}^1\right]/\left[(1-\alpha_N) b + \alpha_N\right]\bar{K}^1$, $r_K^2/p_F = (1-b) T_F^2\left(\bar{F}^2/\bar{K}^2\right)^b$, $r_K^2/p_M = (p_F/p_M)(1-b) T_F^2\left(\bar{F}^2/\bar{K}^2\right)^b$, $r_K^2/p_N^2 = (1-\alpha_N)(1-b) T_N^2 L^2/\bar{K}^2$, $r_F^1/p_F = T_F^1 b\left\{\left[(1-\alpha_N) b + \alpha_N\right]/(1-\alpha_N) b\right\}^{1-b}\left\{(\tilde{p})^{\frac{1}{b}} \bar{K}^1/\left[L^1 + (\tilde{p})^{\frac{1}{b}} \bar{F}^1\right]\right\}^{1-b}$, $r_F^1/p_M = (p_F/p_M) T_F^1 b\left\{\left[(1-\alpha_N) b + \alpha_N\right]/(1-\alpha_N) b\right\}\left\{(\tilde{p})^{\frac{1}{b}} \bar{K}^1/\left[L^1 + (\tilde{p})^{\frac{1}{b}} \bar{F}^1\right]\right\}^{1-b}$, $r_F^1/p_N^1 = T_N^1(\tilde{p})^{\frac{1}{b}}$, $r_F^2/p_F = b T_F^2\left(\bar{K}^2/\bar{F}^2\right)^{1-b}$, $r_F^2/p_M = (p_F/p_M) b T_F^2\left(\bar{K}^2/\bar{F}^2\right)^{1-b}$, and $r_F^2/p_N^2 = (1-\alpha_N) b T_N^2 L^2/\alpha_N \bar{F}^2$.

Utilities: $v_K^j(1,\lambda_M) = r_K^j/(p_F)^{\alpha_F}(p_M)^{\alpha_M}\left(p_N^j\right)^{\alpha_N}$, $v_F^j(1,\lambda_M) = r_F^j/(p_F)^{\alpha_F}(p_M)^{\alpha_M}\left(p_N^j\right)^{\alpha_N}$, and $v_L^1(1,\lambda_M) = w^j/(p_F)^{\alpha_F}(p_M)^{\alpha_M}\left(p_N^j\right)^{\alpha_N}$. Then:

$$v_L^1(1,\lambda_M) = \frac{\left[\frac{(b)^b[(1-\alpha_N)b+\alpha_N]^{(1-b)}}{(1-\alpha_N)^{(1-b)}}\right]^{(\alpha_M+\alpha_F)} T^1\left(\bar{K}^1\right)^{(1-b)(\alpha_M+\alpha_F)}}{\left[L^1+\left(\frac{p_F T_F^1}{p_M T_M^1}\right)^{\frac{1}{b}}\bar{F}^1\right]^{(1-b)(\alpha_M+\alpha_F)}}\left(\frac{T_M^1}{T_F^1}\frac{p_M}{p_F}\right)^{\alpha_F},$$

$$v_K^1(1,\lambda_M) = \frac{T^1(1-b)}{\left(\frac{p_F T_F^1}{p_M T_M^1}\right)^{\alpha_F}}\left\{\frac{[(1-\alpha_N)b]\left[L^1+\left(\frac{p_F T_F^1}{p_M T_M^1}\right)^{\frac{1}{b}}\bar{F}^1\right]}{[(1-\alpha_N)b+\alpha_N]\bar{K}^1}\right\}^{\alpha_N+b(\alpha_F+\alpha_M)},$$

$$v_F^1(1,\lambda_M) = \frac{T^1\left(\frac{T_F^1}{T_M^1}\right)^{\frac{\alpha_N}{b}}\left(\frac{p_F}{p_M}\right)^{\frac{\alpha_N+b\alpha_M}{b}}\left\{[(1-\alpha_N)b+\alpha_N]\bar{K}^1\left(\frac{p_F T_F^1}{p_M T_M^1}\right)^{\frac{1}{b}}\right\}^{(\alpha_M+\alpha_F)(1-b)}}{(b)^{-b(\alpha_M+\alpha_F)}\left\{(1-\alpha_N)\left[L^1+\left(\frac{p_F T_F^1}{p_M T_M^1}\right)^{\frac{1}{b}}\bar{F}^1\right]\right\}^{(\alpha_M+\alpha_F)(1-b)}},$$

$$v_L^2(1,\lambda_M) = \left(\frac{\alpha_N}{1-\alpha_N}\right)^{\alpha_F+\alpha_M}\left(\frac{\bar{F}^2}{L^2}\right)^{b(\alpha_M+\alpha_F)}\left(\frac{\bar{K}^2}{L^2}\right)^{(1-b)(\alpha_M+\alpha_F)}\left(\frac{T_F^2}{T_M^2}\right)^{\alpha_M}T^2\left(\frac{p_F}{p_M}\right)^{\alpha_M},$$

$$v_K^2(1,\lambda_M) = (1-b)(1-\alpha_N)^{\alpha_N}\left(T_N^2\right)^{\alpha_N}\left(T_F^2\right)^{\alpha_F+\alpha_M}\left(\frac{p_F}{p_M}\right)^{\alpha_M}\left(\frac{\bar{F}^2}{\bar{K}^2}\right)^{b(\alpha_F+\alpha_M)}\left(\frac{L^2}{\bar{K}^2}\right)^{\alpha_N},$$

$$v_F^2(1,\lambda_M) = b\left(\frac{1-\alpha_N}{\alpha_N}\right)^{\alpha_N}\left(T_N^2\right)^{\alpha_N}\left(T_F^2\right)^{\alpha_F+\alpha_M}\left(\frac{\bar{K}^2}{\bar{F}^2}\right)^{(1-b)(\alpha_F+\alpha_M)}\left(\frac{L^2}{\bar{F}^2}\right)^{\alpha_N}\left(\frac{p_F}{p_M}\right)^{\alpha_M}.$$

When there is no labor mobility $L^j = \bar{L}^j$. The required conditions for the existence of a unique equilibrium $\check{p} \in \left[\left(p_F^2/p_M^2\right)^S, \left(p_F^1/p_M^1\right)^A\right]$ become

$$\alpha_N\alpha_F\left(\frac{\bar{L}^1}{\bar{L}^2}\right) > (1-\alpha_N)(\alpha_N+b\alpha_M)\left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}}\left(\frac{\bar{F}^1}{\bar{F}^2}\right),$$

$$\left(\frac{\bar{K}^1}{\bar{K}^2}\right)^{1-b} > \frac{\alpha_M[(1-\alpha_N)b+\alpha_N]^b\left[\alpha_N+(1-\alpha_N)b\left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}}\left(\frac{\bar{L}^2\bar{F}^1}{\bar{L}^1\bar{F}^2}\right)\right]^{1-b}\left(\frac{T_M^2}{T_M^1}\right)\left(\frac{\bar{L}^2}{\bar{L}^1}\right)^b}{\left[\alpha_N\alpha_F-(1-\alpha_N)(\alpha_M b+\alpha_N)\left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}}\left(\frac{\bar{L}^2\bar{F}^1}{\bar{L}^1\bar{F}^2}\right)\right]}.$$

Furthermore, $v_L^1(1,0) > v_L^2(1,0)$ if and only if $(\alpha_M b+\alpha_N)\left(T_F^1/T_M^1\right)^{\frac{1}{b}}(p)^{\frac{1}{b}}\bar{F}^1 + (\alpha_M b/\alpha_N)[(1-\alpha_N)b+\alpha_N]\left(T_N^1/T_N^2\right)^{\frac{\alpha_N}{(\alpha_M+\alpha_F)}}\bar{L}^2 > \alpha_F b\bar{L}^1$, which holds for all $p \in \left[\left(p_F^2/p_M^2\right)^S, \left(p_F^1/p_M^1\right)^A\right]$ when

$$\left(\frac{T_N^1}{T_N^2}\right)^{\frac{\alpha_N}{(\alpha_M+\alpha_F)}} > \frac{\alpha_N\alpha_F\left(\frac{\bar{L}^1}{\bar{L}^2}\right)-(\alpha_M b+\alpha_N)(1-\alpha_N)\left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}}\left(\frac{\bar{F}^1}{\bar{F}^2}\right)}{\alpha_M[(1-\alpha_N)b+\alpha_N]}.$$

When the above condition holds, if allowed, workers will have an incentive to migrate from country 2 to country 1. This will increase L^1 and decrease L^2 and, hence, the conditions for the existence of a unique equilibrium $\check{p} \in \left[\left(p_F^2/p_M^2\right)^S, \left(p_F^1/p_M^1\right)^A\right]$ will continue to hold. Moreover, as L^1 increases, the equilibrium $\check{p}$ also increases. In order to prove this, we use the implicit function theorem to $BT\left(p, L^1\right) = 0$ to obtain $d\check{p}/dL^1 = \frac{-\partial BT\left(p,L^1\right)}{\partial L^1} / \frac{\partial BT\left(p,L^1\right)}{\partial p}$. We have already shown that $BT\left(p, L^1\right)$ is decreasing in p. Thus, $\partial BT\left(p, L^1\right)/\partial p > 0$. Differentiating $BT\left(p, L^1\right)$ with respect to L^1 we obtain

$$\frac{\partial BT\left(p, L^1\right)}{\partial L^1} = \frac{\left(1-\alpha_N\right)^b T_M^1 \left(\bar{K}^1\right)^{1-b} \left[\alpha_F b^2 L^1 + \left[\alpha_F b + \left(\alpha_N + \alpha_M b\right)\left(1-b\right)\right] \left(\frac{T_F^1}{T_M^1}\right)^{\frac{1}{b}} (p)^{\frac{1}{b}} \bar{F}^1\right]}{\alpha_M (b)^{1-b} \left[\left(1-\alpha_N\right) b + \alpha_N\right]^b T_F^2 \left(\bar{F}^2\right)^b \left(\bar{K}^2\right)^{1-b} \left[L^1 + \left(\frac{T_F^1}{T_M^1}\right)^{\frac{1}{b}} (p)^{\frac{1}{b}} \bar{F}^1\right]^{2-b}} > 0$$

Since $v_L^1\left(1, \lambda_M\right)$ is decreasing in L^1 and p_F/p_M, as workers move from country 2 to country 1, $v_L^1\left(1, \lambda_M\right)$ decreases. Since $v_L^2\left(1, \lambda_M\right)$ is decreasing in L^2 and increasing in p_F/p_M, as workers move from country 2 to country 1, $v_L^2\left(1, \lambda_M\right)$ increases. Under labor mobility, workers will move until $v_L^1(1,1) = v_L^2(1,1)$, which implies $FM\left(p, L^1\right) = 0$, where

$$FM\left(p, L^1\right) = \frac{\alpha_M \left[\left(1-\alpha_N\right) b + \alpha_N\right] \left(\frac{T_N^1}{T_N^2}\right)^{\frac{\alpha_N}{\left(\alpha_M + \alpha_F\right)}} \bar{L} + \alpha_N \left(\alpha_M b + \alpha_N\right) \left(\frac{T_F^1}{T_M^1}\right)^{\frac{1}{b}} (p)^{\frac{1}{b}} \bar{F}^1}{\alpha_N \alpha_F b + \alpha_M \left[\left(1-\alpha_N\right) b + \alpha_N\right] \left(\frac{T_N^1}{T_N^2}\right)^{\frac{\alpha_N}{\alpha_M + \alpha_F}}} - L_1.$$

Thus, the equilibrium under free labor mobility is the solution to $BT\left(p, L^1\right) = 0$ and $FM\left(p, L^1\right) = 0$. This completes the proof of the lemma. ■

Proposition 6 characterizes trade and labor mobility policies.

Proposition 6 (multiple factors and non-tradeable goods). *Suppose that governments maximize the welfare of domestic workers. Assume* $\bar{K}^1/\bar{K}^2 > \bar{\kappa}_1$, $\bar{L}^1\bar{F}^2/\bar{L}^2\bar{F}^1 > \bar{\kappa}_2$ *and* $T_N^1/T_N^2 > \bar{\kappa}_3$, *where*

$$\left(\bar{\kappa}_1\right)^{1-b} = \frac{\alpha_M \left[\left(1-\alpha_N\right) b + \alpha_N\right]^b (D)^b \left[\alpha_N + D\left(1-\alpha_N\right) b \left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}} \left(\frac{\bar{L}^2 \bar{F}^1}{\bar{L}^1 \bar{F}^2}\right)\right]^{1-b} \left(\frac{T_M^2}{T_M^1}\right) \left(\frac{\bar{L}^2}{\bar{L}^1}\right)^b}{\left[\alpha_N \alpha_F - D\left(1-\alpha_N\right)\left(\alpha_M b + \alpha_N\right) \left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}} \left(\frac{\bar{L}^2 \bar{F}^1}{\bar{L}^1 \bar{F}^2}\right)\right]}$$

$$D = \left(\frac{\alpha_F + \alpha_M}{\alpha_F}\right)^{\frac{\alpha_F}{\alpha_M}} \left(\frac{\alpha_N + b\alpha_M}{\alpha_N}\right)^{\frac{\alpha_F + (1-b)\alpha_M}{\alpha_M b}}$$

$$\bar{\kappa}_2 = \frac{\left(1-\alpha_N\right)\left(\alpha_N + b\alpha_M\right)}{\alpha_N \alpha_F} \left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}}$$

$$\left(\bar{\kappa}_3\right)^{\frac{\alpha_N}{\alpha_M + \alpha_F}} = \frac{\alpha_N \alpha_F \left(\frac{\bar{L}^1}{\bar{L}^2}\right) - \left(\alpha_M b + \alpha_N\right)\left(1-\alpha_N\right) \left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}} \left(\frac{\bar{F}^1}{\bar{F}^2}\right)}{\alpha_M \left[\left(1-\alpha_N\right) b + \alpha_N\right]}$$

Then, the trade and labor mobility game has only two Nash equilibrium outcomes: no trade and no labor mobility and free trade and no labor mobility. Moreover, $W_G^j(1,0) > W_G^j(0,0)$ for $j = 1,2$.

Proof:

No trade and no labor mobility is always a Nash equilibrium outcome.

Free trade and no labor mobility is a Nash equilibrium outcome if and only if $v_L^j(1,0) > v_L^j(0,0)$ for $j = 1,2$. $v_L^1(1,0) > v_L^1(0,0)$ if and only if

$$\frac{(\alpha_F b)^{\frac{b\alpha_F}{\alpha_M+\alpha_F}} [(1-\alpha_N) b + \alpha_N]^{(1-b)} \left(\frac{T_M^1}{T_F^1}\right)^{\frac{\alpha_F}{\alpha_M+\alpha_F}} \left(\bar{L}^1\right)^{\frac{\alpha_F+(1-b)\alpha_M}{(\alpha_M+\alpha_F)}}}{(\alpha_N + b\alpha_M)^{\frac{\alpha_F+(1-b)\alpha_M}{\alpha_M+\alpha_F}} \left(\bar{F}^1\right)^{\frac{b\alpha_F}{\alpha_M+\alpha_F}}} >$$

$$> \left(\frac{p_F}{p_M}\right)^{\frac{\alpha_F}{\alpha_M+\alpha_F}} \left[\bar{L}^1 + \left(\frac{T_F^1 p_F}{T_M^1 p_M}\right)^{\frac{1}{b}} \bar{F}^1\right]^{1-b}$$

The left hand side is increasing in (p_F/p_M). Under the conditions in the proposition, $(p_F/p_M) < \left(p_F^1/p_M^1\right)^A = [b\alpha_F/(\alpha_N + b\alpha_M)]^b \left(T_M^1/T_F^1\right)\left(L^1/\bar{F}^1\right)^b$. Finally, when we evaluate the right hand side at $\left(p_F^1/p_M^1\right)^A$, the inequality becomes an equality. $v_L^2(1,0) > v_L^2(0,0)$ if and only if

$$\frac{p_F}{p_M} > \check{p} = \frac{(b)^b (1-\alpha_N)^{b\left(\frac{\alpha_F+\alpha_M}{\alpha_M}\right)} (\alpha_N + b\alpha_M)^{\frac{\alpha_F+(1-b)\alpha_M}{\alpha_M}}}{(\alpha_F)^{\frac{b\alpha_F}{\alpha_M}} (\alpha_N)^{\frac{\alpha_F+\alpha_M}{\alpha_M}}} \left(\frac{T_M^2}{T_F^2}\right)\left(\frac{\bar{L}^2}{\bar{F}^2}\right)^b$$

$(p_F/p_M) > \check{p}$ if and only if $B\left(\check{p}, \bar{L}^1\right) > 0$ or, which is equivalent,

$$\left(\frac{\bar{K}^1}{\bar{K}^2}\right)^{1-b} > \frac{\alpha_M [(1-\alpha_N) b + \alpha_N]^b (D)^b \left[\alpha_N + D(1-\alpha_N) b \left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}} \left(\frac{\bar{L}^2 \bar{F}^1}{\bar{L}^1 \bar{F}^2}\right)\right]^{1-b} \left(\frac{T_M^2}{T_M^1}\right)\left(\frac{\bar{L}^2}{\bar{L}^1}\right)^b}{\left[\alpha_N\alpha_F - D(1-\alpha_N)(\alpha_M b + \alpha_N)\left(\frac{T_F^1 T_M^2}{T_M^1 T_F^2}\right)^{\frac{1}{b}}\left(\frac{\bar{L}^2 \bar{F}^1}{\bar{L}^1 \bar{F}^2}\right)\right]},$$

where $D = \left(\frac{\alpha_F+\alpha_M}{\alpha_F}\right)^{\frac{\alpha_F}{\alpha_M}} \left(\frac{\alpha_N+b\alpha_M}{\alpha_N}\right)^{\frac{\alpha_F+(1-b)\alpha_M}{\alpha_M b}} > 1$.

Free trade and free labor mobility is not a Nash equilibrium outcome. We have already proved that when $\left(T_N^1/T_N^2\right)^{\frac{\alpha_N}{(\alpha_M+\alpha_F)}} > \left[\alpha_N\alpha_F\left(\bar{L}^1/\bar{L}^2\right) - (\alpha_M b + \alpha_N)(1-\alpha_N)\left(T_F^1 T_M^2/T_M^1 T_F^2\right)^{\frac{1}{b}}\left(\bar{F}^1/\bar{F}^2\right)\right]/\alpha_M[(1-\alpha_N) b + \alpha_N]$, $v_L^1(1,0) > v_L^2(1,0)$ and as workers move from country 2 to country 1, $v_L^1(1,\lambda_M)$ decreases. Therefore, it must be the case that $v_L^1(1,0) > v_L^1(1,1)$.

No trade and free labor mobility is not a Nash equilibrium outcome. Suppose that $L^1 = \left[\left(T^1\right)\left(\bar{F}^1\right)^{b\alpha_F}\left(\bar{K}^1\right)^{(1-b)(\alpha_F+\alpha_M)}\right]^{\frac{1}{\alpha_F+(1-b)\alpha_M}} \left(\bar{T}\right)^{-1}\bar{L} > \bar{L}^1$. Then, under no trade and free labor mobility, workers will migrate from country 2 to country 1. Moreover $v_L^1(0,0) > v_L^1(0,1)$.if and only if $\left[\left(T^1\right)\left(\bar{F}^1\right)^{b\alpha_F}\left(\bar{K}^1\right)^{(1-b)(\alpha_F+\alpha_M)}\right]^{\frac{1}{\alpha_F+(1-b)\alpha_M}} \bar{L} > \bar{T}\bar{L}^1$. Suppose that $L^1 =$

$\left[\left(T^1\right)\left(\bar{F}^1\right)^{b\alpha_F}\left(\bar{K}^1\right)^{(1-b)(\alpha_F+\alpha_M)}\right]^{\frac{1}{\alpha_F+(1-b)\alpha_M}}\left(\bar{T}\right)^{-1}\bar{L} < \bar{L}^1$, then under no trade and free labor mobility, workers will migrate from country 1 to country 2. Moreover, $v_L^2(0,0) > v_L^2(0,1)$.if and only if $\left[\left(T^1\right)\left(\bar{F}^2\right)^{b\alpha_F}\left(\bar{K}^1\right)^{(1-b)(\alpha_F+\alpha_M)}\right]^{\frac{1}{\alpha_F+(1-b)\alpha_M}}\bar{L} > \bar{T}\bar{L}^1$. This completes the proof of the proposition. ■

C Inducing free trade and free labor mobility

Appendix C presents the proofs of all the results in Section 5.

C.1 Increasing returns to scale

Consider an economy with two countries ($J = 2$), two tradeable goods ($Z_T = \{1,2\}$) and one non-tradeable good ($Z_N = \{3\}$). All agents have the same preferences given by $u\left(c^j\right) = \sum_{z \in Z} \alpha_z \ln\left(c_z^j\right)$ and $c_1^j = \left[\int_0^n c_1^j(i)^\rho \, di\right]^{\frac{1}{\rho}}$, where $\alpha_z \in (0,1)$, $\sum_{z \in Z} \alpha_z = 1$, $\rho \in (0,1)$, $\sigma = (1-\rho)^{-1}$, and $c_1^j(i)$ indicates the quantity of variety $i \in [0,n]$ that is consumed. The production function in sector 1 is $L_1^j(i) = a_{L,1}^j Q_1^j(i) + f$, where $L_1^j(i)$ is the labor employed in the production of variety i of good 1, $Q_1^j(i)$ is the production of variety i of good 1, $f > 0$ is the fixed cost of producing a variety of good 1. Production functions in sectors 2 and 3 are $L_2^j = a_{L,2}^j Q_2^j$, $L_3^j = a_{L,3}^j Q_3^j$. Let $A_z = a_{L,z}^2 / a_{L,z}^1$ and assume $A_1 > A_2$, i.e., country 1 has a comparative advantage in good 1.

Demands functions are $C_1^j(i) = \alpha_1 \left(G^j\right)^{\sigma-1} p_1^j(i)^{-\sigma} Y^j$ for $i \in [0,n]$, $p_2^j C_2^j = \alpha_2 Y^j$, $p_3^j C_3^j = \alpha_3 Y^j$, where Y^j is aggregate income and $G^j = \left[\int_0^n p_1^j(i)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$ is the exact price index for good 1, $p_1^j(i)$ is the price of variety i, p_z^j is the price of good z and $\sigma = (1-\rho)^{-1}$. Let w^j denote the wage rate. The profits of a firm that produces variety i of good 1 are given by $\pi_1^j(i) = \left[p_1^j(i) - w^j a_{L,1}^j\right] Q_1^j(i) - w^j f$, which implies that the price that maximizes profits is $p_1^j(i) = \sigma(\sigma-1)^{-1} w^j a_{L,1}^j$. Moreover, free entry implies that in equilibrium $\pi_1^j(i) = 0$ and, hence, $L_1^j(i) = \sigma f$. Therefore, the labor allocation $\left(n^j, L_2^j, L_3^j\right)$ is the solution of maximizing $(\sigma-1) p_1^j n^j f / a_{L,1} + \sum_{z=2,3} \left(p_z^j L_z^j / a_{L,z}^j\right)$ subject to $n^j \sigma f + L_2^j + L_3^j = L^j$ and the wage rate is given by $w^j = \max\left\{(\sigma-1) p_1^j / \sigma a_{L,1}^j, p_2^j / a_{L,2}^j, p_3^j / a_{L,2}^j\right\}$.

Under autarky all goods must be produced domestically. Hence, $p_1^j = w^j a_{L,1}^j \sigma / (1-\sigma)$, $p_z^j = w^j a_{L,z}^j$ for $z = 2,3$, $n^j \sigma f = \alpha_1 L^j$, $L_2^j = \alpha_2 L^j$ and $L_3^j = \alpha_3 L^j$. Then, the indirect utility of a worker in country j is $v^j = C + T^j + [\alpha_1 / (\sigma-1)] \ln\left(\alpha_1 L^j / f\right)$, where $C = \sum_z \alpha_z \ln(\alpha_z) - \alpha_1 (\sigma-1)^{-1} \ln\left[(\sigma)^\sigma (\sigma-1)^{(\sigma-1)}\right]$ and $T^j = -\sum_{z \in Z} \alpha_z \ln\left(a_{L,z}^j\right)$. If labor mobility is not allowed, then $L^j = \bar{L}^j$. Note that v^j is increasing in L^j. As a consequence, if labor mobility is allowed, there are three possible situations. If $T^1 - T^2 > [\alpha_1 / (\sigma-1)] \ln\left[\left(\bar{L}^2 + m\bar{L}^1\right) / (1-m) \bar{L}^1\right]$, then all mobile workers will go to or stay in country 1 because, for any allocation of mobile workers to countries, we have $v^1 > v^2$. If $[\alpha_1 / (\sigma-1)] \ln\left[(1-m) \bar{L}^2 / \left(\bar{L}^1 + m\bar{L}^2\right)\right] < T^1 - T^2 < [\alpha_1 / (\sigma-1)] \ln\left[\left(\bar{L}^2 + m\bar{L}^1\right) / (1-m) \bar{L}^1\right]$, then there are two equilibria, in each of which all mobile workers go to or stay in only one country. Finally, if $T^1 - T^2 < [\alpha_1 / (\sigma-1)] \ln\left[(1-m) \bar{L}^2 / \left(\bar{L}^1 + m\bar{L}^2\right)\right]$, then all mobile workers go to or stay in country 2 because $v^2 > v^1$ for any allocation of mobile workers to countries. The intuition is straightforward. Due to the existence of increasing returns to scale in sector 1, under autarky the indirect utility of workers increases in step with the labor endowment of the country (a higher labor endowment implies that more varieties of good 1 are produced in the autarky equilibrium). As a consequence, once mobile workers start moving to one country, the indirect utility of workers in the recipient country goes up, which reinforces migration flows in the same direction.

Suppose there is free trade and $A_1 > \alpha_1 L^2/\alpha_2 L^1 > A_2$. Then, country 1 specializes in good 1 and country 2 specializes in good 2. Thus, $w^1 = (\sigma-1) p_1/\sigma a^1_{L,1} = p_3/a^1_{L,3}$, $w^2 = p_2/a^2_{L,1} = p_3/a^2_{L,3}$, $n^1 = n = (1-\alpha_3) L^1/\sigma f$, $n^2 = 0$ and the balanced trade condition implies $\alpha_2 w^1 L^1 = \alpha_1 L^2 w^2$. Then, the utility of a worker is given by $v^1 = C + T^1 + \alpha_2 \ln\left(a^1_{L,2}\alpha_1 L^2/a^2_{L,2}\alpha_2 L^1\right) + [\alpha_1/(\sigma-1)]\ln\left[(1-\alpha_3) L^1/f\right]$ in country 1 and $v^2 = C + T^2 + \alpha_1 \ln\left(a^2_{L,1}\alpha_2 L^1/a^1_{L,1}\alpha_1 L^2\right) + [\alpha_1/(\sigma-1)]\ln\left[(1-\alpha_3) L^1/f\right]$ in country 2. If labor mobility is not allowed, then $L^j = \bar{L}^j$. Under free labor mobility, mobile workers will go to or stay in the country with higher v^j. Note that, although v^1 can be increasing or decreasing in L^1, $v^1 - v^2$ is always decreasing in L^1 and increasing in L^2, as in the simple Ricardian model. Therefore, and, provided that there is enough mobile workers, they will relocate until $v^1 = v^2$, which implies $L^2/L^1 = (\alpha_2/\alpha_1)(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$. Moreover, there will be migration to country 1 whenever $\bar{L}^2/\bar{L}^1 > (\alpha_2/\alpha_1)(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$ and migration to country 2 whenever $\bar{L}^2/\bar{L}^1 < (\alpha_2/\alpha_1)(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$. The intuition is straightforward. Under free trade, country 1 specializes in the production of good 1, the sector that operates under increasing returns to scale. Therefore, the number of varieties of good 1 produced in the trading equilibrium and consumed by both countries only depends on the size of country 1's labor force. This implies that the effect that the allocation of mobile workers has on the varieties of good 1 that are produced affects both countries symmetrically. Finally, the distribution of mobile workers between the countries does not violate either $L^j \in \left[(1-m)\bar{L}^j, \bar{L}^j + m\bar{L}^{-j}\right]$ or $A_1 > \alpha_1 L^2/\alpha_2 L^1 > A_2$, provided that $A_1 > \frac{\alpha_1\left(\bar{L}^2 + m\bar{L}^1\right)}{\alpha_2(1-m)\bar{L}^1} > (A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}} > \frac{\alpha_1(1-m)\bar{L}^2}{\alpha_2\left(\bar{L}^1 + m\bar{L}^2\right)} > A_2$.

Proposition 7 characterizes trade and labor mobility policies.

Proposition 7 (increasing returns to scale). *Suppose that:*

$$T^1 - T^2 > \frac{\alpha_1}{\sigma-1}\ln\left[\frac{\bar{L}^2 + m\bar{L}^1}{(1-m)\bar{L}^1}\right]$$

$$A_1 > \frac{\alpha_1\left(\bar{L}^2 + m\bar{L}^1\right)}{\alpha_2(1-m)\bar{L}^1} > \frac{\alpha_1\left(1-\bar{\lambda}_M m\right)\bar{L}^2}{\alpha_2\left(\bar{L}^1 + \bar{\lambda}_M m\bar{L}^2\right)} > (A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}} > \frac{\alpha_1(1-m)\bar{L}^2}{\alpha_2\left(\bar{L}^1 + m\bar{L}^2\right)} > A_2$$

$$\alpha_1 \ln\left[\frac{A_1\alpha_2\left(\bar{L}^1 + \bar{\lambda}_M m\bar{L}^2\right)}{\alpha_1\left(1-\bar{\lambda}_M m\right)\bar{L}^2}\right] > \frac{\alpha_1}{\sigma-1}\ln\left[\frac{\alpha_1\bar{L}^2}{(1-\alpha_3)\left(\bar{L}^1 + \bar{\lambda}_M m\bar{L}^2\right)}\right]$$

Then:

1. *No trade and no labor mobility is a Nash equilibrium outcome.*

2. *No trade and partial labor mobility ($\lambda_M = \bar{\lambda}_M$) is a Nash equilibrium outcome if and only if $\bar{\lambda}_M\left\{T^1 - T^2 + \frac{\alpha_1}{\sigma-1}\ln\left[\frac{\bar{L}^1 + \bar{\lambda}_M m\bar{L}^2}{\left(1-\bar{\lambda}_M m\right)\bar{L}^2}\right]\right\} \geq -\frac{\alpha_1}{\sigma-1}\ln\left(1-\bar{\lambda}_M m\right)^{\frac{1}{m}}$. Moreover, if no trade and partial labor mobility is an equilibrium outcome, then $W^j_G\left(0, \bar{\lambda}_M\right) \geq W^j_G(0,0)$ for $j = 1, 2$.*

3. *No trade and free labor mobility is a Nash equilibrium outcome if and only if $\bar{\lambda}_M\left\{T^1 - T^2 + \frac{\alpha_1}{\sigma-1}\ln\left[\frac{\bar{L}^1 + \bar{\lambda}_M m\bar{L}^2}{\left(1-\bar{\lambda}_M m\right)\bar{L}^2}\right]\right\} \leq \left(T^1 - T^2\right) + \frac{\alpha_1}{\sigma-1}\ln\left[\frac{\left(\bar{L}^1 + m\bar{L}^2\right)(1-m)^{\frac{1}{m}}}{(1-m)\bar{L}^2\left(1-\bar{\lambda}_M m\right)^{\frac{1}{m}}}\right]$. More-*

over, if no trade and free labor mobility is an equilibrium outcome, then $W_G^j(0,1) \geq \max\left\{W_G^j(0,0), W_G^j\left(0,\bar{\lambda}_M\right)\right\}$ *for* $j=1,2$.

4. *Free trade and no labor mobility is a Nash equilibrium if and only if* $\left(\frac{1-\alpha_3}{\alpha_2}\right)^{\frac{1}{\sigma}}(A_1)^{\frac{\sigma-1}{\sigma}} > \frac{\alpha_1 \bar{L}^2}{\alpha_2 \bar{L}^1}$.

5. *Suppose that* $\left(\bar{L}^1+\bar{\lambda}_M m \bar{L}^2\right)^{\phi}\left[\left(1-\bar{\lambda}_M m\right)\bar{L}^2\right]^{\alpha_2} > \max\left\{\left(\bar{L}^1\right)^{\phi}\left(\bar{L}^2\right)^{\alpha_2}, \left(L^1\right)^{\phi}\left(L^2\right)^{\alpha_2}\right\}$, *where* $\phi = \frac{\alpha_1-\alpha_2(\sigma-1)}{\sigma-1}$, $L^1 = \frac{\alpha_1\left(\bar{L}^1+\bar{L}^2\right)}{\alpha_1+\alpha_2(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}}$ *and* $L^2 = \frac{\alpha_2(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}\left(\bar{L}^1+\bar{L}^2\right)}{\alpha_1+\alpha_2(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}}$. *Then, free trade and partial labor mobility is a Nash equilibrium outcome, but free trade and free labor mobility is not a Nash equilibrium outcome.*

Proof:
$(\lambda_T, \lambda_M) = (0,0)$ is a always a Nash equilibrium outcome.

$(\lambda_T, \lambda_M) = \left(0, \bar{\lambda}_M\right)$ is a Nash equilibrium outcome if and only if $W_G^j\left(0,\bar{\lambda}_M\right) \geq W_G^j(0,0)$ for $j=1,2$. If $\lambda_T = 0$ and $\lambda_M = \bar{\lambda}_M$, then $\alpha_1\left(1-\bar{\lambda}_M m\right)\bar{L}^2/\alpha_2\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right) > (A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$ implies that a proportion $\bar{\lambda}_M m$ of mobile workers in country 2 will relocate to country 1. Then $W_G^1\left(0,\bar{\lambda}_M\right) = v^1\left(0,\bar{\lambda}_M\right)$ and $W_G^2\left(0,\bar{\lambda}_M\right) = \bar{\lambda}_M m v^1\left(0,\bar{\lambda}_M\right) + \left(1-\bar{\lambda}_M m\right)v^2\left(0,\bar{\lambda}_M\right)$, where $v^1\left(0,\bar{\lambda}_M\right) = C + T^1 + \left[\alpha_1/(\sigma-1)\right]\ln\left[\alpha_1\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)/f\right]$ and $v^2\left(0,\bar{\lambda}_M\right) = C + T^2 + \left[\alpha_1/(\sigma-1)\right]\ln\left[\alpha_1\left(1-\bar{\lambda}_M m\right)\bar{L}^2/f\right]$. If $\lambda_T = 0$ and $\lambda_M = 0$, then $W_G^j(0,0) = v^j(0,0) = C + T^j + \left[\alpha_1/(\sigma-1)\right]\ln\left(\alpha_1\bar{L}^j/f\right)$. Note that $W_G^1\left(0,\bar{\lambda}_M\right) - W_G^1(0,0) = \left[\alpha_1/(\sigma-1)\right]\ln\left[\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)/\bar{L}^1\right] > 0$, while $W_G^2\left(0,\bar{\lambda}_M\right) - W_G^2(0,0) \geq 0$ if and only if $\bar{\lambda}_M\left\{T^1 - T^2 + \left[\alpha_1/(\sigma-1)\right]\ln\left[\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)/\left(1-\bar{\lambda}_M m\right)\bar{L}^2\right]\right\} \geq -\left[\alpha_1/(\sigma-1)\right]\ln\left(1-\bar{\lambda}_M m\right)^{\frac{1}{m}}$. Thus, $\left(0,\bar{\lambda}_M\right)$ is a Nash equilibrium if and only if

$$\bar{\lambda}_M\left\{T^1 - T^2 + \left[\alpha_1/(\sigma-1)\right]\ln\left[\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)/\left(1-\bar{\lambda}_M m\right)\bar{L}^2\right]\right\} \geq -\left[\alpha_1/(\sigma-1)\right]\ln\left(1-\bar{\lambda}_M m\right)^{\frac{1}{m}}.$$

$(\lambda_T, \lambda_M) = (0,1)$ is a Nash equilibrium outcome if and only if $W_G^j(0,1) \geq \max\left\{W_G^j\left(0,\bar{\lambda}_M\right), W_G^j(0,0)\right\}$ for $j=1,2$. If $\lambda_T = 0$ and $\lambda_M = 1$, then $\alpha_1\left(\bar{L}^2+m\bar{L}^1\right)/\alpha_2(1-m)\bar{L}^1 > (A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$ implies that all mobile workers will relocate to country 1. Then, $W_G^1(0,1) = v^1(0,1)$ and $W_G^2(0,1) = mv^1(0,1) + (1-m)v^2(0,1)$, where $v^1(0,1) = C + T^1 + \left[\alpha_1/(\sigma-1)\right]\ln\left[\alpha_1\left(\bar{L}^1+m\bar{L}^2\right)/f\right]$ and $v^2(0,1) = C + T^2 + \left[\alpha_1/(\sigma-1)\right]\ln\left[\alpha_1(1-m)\bar{L}^2/f\right]$. Note that $W_G^1(0,1) - W_G^1(0,0) = \left[\alpha_1/(\sigma-1)\right]\ln\left[\left(\bar{L}^1+m\bar{L}^2\right)/\bar{L}^1\right] > W_G^1\left(0,\bar{\lambda}_M\right) - W_G^1(0,0) = \left[\alpha_1/(\sigma-1)\right]\ln\left[\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)/\bar{L}^1\right] > 0$. Also note that $W_G^2(0,1) - W_G^2\left(0,\bar{\lambda}_M\right) \geq 0$ if and only if $\left(T^1-T^2\right) + \left[\alpha_1/(\sigma-1)\right]\ln\left[\left(\bar{L}^1+m\bar{L}^2\right)(1-m)^{\frac{1}{m}}/(1-m)\bar{L}^2\left(1-\bar{\lambda}_M m\right)^{\frac{1}{m}}\right] \geq \bar{\lambda}_M\left\{T^1 - T^2 + \left[\alpha_1/(\sigma-1)\right]\ln\left[\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)/\left(1-\bar{\lambda}_M m\right)\bar{L}^2\right]\right\}$ and $W_G^2(0,1) - W_G^2(0,0) \geq 0$ if and only if $\left(T^1-T^2\right) + \left[\alpha_1/(\sigma-1)\right]\ln\left[\left(\bar{L}^1+m\bar{L}^2\right)(1-m)^{\frac{1}{m}}/(1-m)\bar{L}^2\left(1-\bar{\lambda}_M m\right)^{\frac{1}{m}}\right] \geq 0$. But note that $T^1 - T^2 + \left[\alpha_1/(\sigma-1)\right]\ln\left[\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)/\left(1-\bar{\lambda}_M m\right)\bar{L}^2\right] > 0$ because $T^1 - T^2 + \left[\alpha_1/(\sigma-1)\right]\ln\left[\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)/\left(1-\bar{\lambda}_M m\right)\bar{L}^2\right] > T^1 - T^2 + \left[\alpha_1/(\sigma-1)\right]\ln\left[\left(1-\bar{\lambda}_M m\right)\bar{L}^2/\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)\right] > 0$. Thus, $(0,1)$ is a Nash equilibrium outcome if and only if

$$\left(T^1-T^2\right)+[\alpha_1/(\sigma-1)]\ln\left[\left(\bar{L}^1+m\bar{L}^2\right)(1-m)^{\frac{1}{m}}/(1-m)\bar{L}^2\left(1-\bar{\lambda}_M m\right)^{\frac{1}{m}}\right]\geq$$
$$\geq\bar{\lambda}_M\left\{T^1-T^2+[\alpha_1/(\sigma-1)]\ln\left[\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)/\left(1-\bar{\lambda}_M m\right)\bar{L}^2\right]\right\}$$

$(\lambda_T,\lambda_M)=(1,0)$ is a Nash equilibrium outcome if and only if $W_G^j(1,0)\geq W_G^j(0,0)$ for $j=1,2$. If $\lambda_T=1$ and $\lambda_M=0$, then $v^1(1,0)=C+T^1+\alpha_2\ln\left(a_{L,2}^1\alpha_1\bar{L}^2/a_{L,2}^2\alpha_2\bar{L}^1\right)+[\alpha_1/(\sigma-1)]\ln\left[(1-\alpha_3)\bar{L}^1/f\right]$ and $v^2(1,0)=C+T^2+\alpha_1\ln\left(a_{L,1}^2\alpha_2\bar{L}^1/a_{L,1}^1\alpha_1\bar{L}^2\right)+[\alpha_1/(\sigma-1)]\ln\left[(1-\alpha_3)\bar{L}^1/f\right]$. Since there is no labor mobility, $W_G^j(1,0)=v^j(1,0)$. Note that $W_G^1(1,0)-W_G^1(0,0)=\alpha_2\ln\left(a_{L,2}^1\alpha_1\bar{L}^2/a_{L,2}^2\alpha_2\bar{L}^1\right)+[\alpha_1/(\sigma-1)]\ln\left[(1-\alpha_3)/\alpha_1\right]>0$, because $\alpha_1\bar{L}^2/\alpha_2\bar{L}^1>A_2$. Also note that $W_G^2(1,0)-W_G^2(0,0)=\alpha_1\ln\left(a_{L,1}^2\alpha_2\bar{L}^1/a_{L,1}^1\alpha_1\bar{L}^2\right)+[\alpha_1/(\sigma-1)]\ln\left[(1-\alpha_3)\bar{L}^1/\alpha_1\bar{L}^2\right]>0$ if and only if $\alpha_1\bar{L}^2/\alpha_2\bar{L}^1<\left[A_1(1-\alpha_3)^{\sigma-1}/(\alpha_2)^{\sigma-1}\right]^{\frac{1}{\sigma}}$, which also holds. Thus, $(1,0)$ is a Nash equilibrium outcome.

$(\lambda_T,\lambda_M)=\left(1,\bar{\lambda}_M\right)$ is a Nash equilibrium outcome if and only if $W_G^j\left(1,\bar{\lambda}_M\right)\geq\max\left\{W_G^j(1,0),W_G^j\left(0,\bar{\lambda}_M\right),W_G^j(0,0)\right\}$ for $j=1,2$. If $\lambda_T=1$ and $\lambda_M=\bar{\lambda}_M$, then $\alpha_1\left(\bar{L}^2+m\bar{L}^1\right)/\alpha_2(1-m)\bar{L}^1>(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$ implies that a proportion $\bar{\lambda}_M m$ of mobile workers in country 2 will relocate to country 1. Thus, $W_G^1\left(1,\bar{\lambda}_M\right)=v^1\left(1,\bar{\lambda}_M\right)$ and $W_G^2\left(1,\bar{\lambda}_M\right)=\bar{\lambda}_M m v^1\left(1,\bar{\lambda}_M\right)+\left(1-\bar{\lambda}_M m\right)v^2\left(1,\bar{\lambda}_M\right)$, where $v^1\left(1,\bar{\lambda}_M\right)=C+T^1+\alpha_2\ln\left[\alpha_1\bar{L}^2\left(1-\bar{\lambda}_M m\right)/A_2\alpha_2\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)\right]+[\alpha_1/(\sigma-1)]\ln\left[(1-\alpha_3)\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)/f\right]$ and $v^2\left(1,\bar{\lambda}_M\right)=C+T^2+\alpha_1\ln\left[A_1\alpha_2\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)/\alpha_1\bar{L}^2\left(1-\bar{\lambda}_M m\right)\right]+[\alpha_1/(\sigma-1)]\ln\left[(1-\alpha_3)\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)/f\right]$. Note that $W_G^1\left(1,\bar{\lambda}_M\right)-W_G^1(1,0)=\alpha_2\ln\left[\bar{L}^2\left(1-\bar{\lambda}_M m\right)/\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)\bar{L}^2\right]+[\alpha_1/(\sigma-1)]\ln\left[\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)/\bar{L}^1\right]\geq 0$ if and only if $\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)^{\frac{\alpha_1-\alpha_2(\sigma-1)}{\sigma-1}}\left[\left(1-\bar{\lambda}_M m\right)\bar{L}^2\right]^{\alpha_2}\geq\left(\bar{L}^1\right)^{\frac{\alpha_1-\alpha_2(\sigma-1)}{\sigma-1}}\left(\bar{L}^2\right)^{\alpha_2}$. Also note that $W_G^1\left(1,\bar{\lambda}_M\right)-W_G^1\left(0,\bar{\lambda}_M\right)=\alpha_2\ln\left[\alpha_1\bar{L}^2\left(1-\bar{\lambda}_M m\right)/A_2\alpha_2\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)\right]+[\alpha_1/(\sigma-1)]\ln\left[(1-\alpha_3)/\alpha_1\right]>0$ because $\alpha_1\bar{L}^2\left(1-\bar{\lambda}_M m\right)/\alpha_2\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)>A_2$. Since we have already proved that $W_G^1\left(0,\bar{\lambda}_M\right)>W_G^1(0,0)$ and $W_G^1(1,0)>W_G^1(0,0)$, then $W_G^1\left(1,\bar{\lambda}_M\right)\geq\max\left\{W_G^1(1,0),W_G^1\left(0,\bar{\lambda}_M\right),W_G^1(0,0)\right\}$ if and only if $\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)^{\frac{\alpha_1-\alpha_2(\sigma-1)}{\sigma-1}}\left[\left(1-\bar{\lambda}_M m\right)\bar{L}^2\right]^{\alpha_2}\geq\left(\bar{L}^1\right)^{\frac{\alpha_1-\alpha_2(\sigma-1)}{\sigma-1}}\left(\bar{L}^2\right)^{\alpha_2}$. $W_G^2\left(1,\bar{\lambda}_M\right)-W_G^2(1,0)\geq 0$ if and only if $\bar{\lambda}_M m\left\{T_N^1-T_N^2+\ln\left[\frac{\alpha_1\left(1-\bar{\lambda}_M m\right)\bar{L}^2}{\alpha_2\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)}\right]^{(\alpha_1+\alpha_2)}\right\}+\alpha_1\ln\left[\frac{\bar{L}^1+\bar{\lambda}_M m\bar{L}^2}{\bar{L}^1\left(1-\bar{\lambda}_M m\right)}\right]+[\alpha_1/(\sigma-1)]\ln\left[\frac{\bar{L}^1+\bar{\lambda}_M m\bar{L}^2}{\bar{L}^1}\right]>0$, which holds because $T_N^1-T_N^2+\ln\left[\alpha_1\left(1-\bar{\lambda}_M m\right)\bar{L}^2/\alpha_2\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)\right]^{\alpha_1+\alpha_2}>0$. $W_G^2\left(1,\bar{\lambda}_M\right)-W_G^2\left(0,\bar{\lambda}_M\right)=\bar{\lambda}_M m\left(v^1\left(1,\bar{\lambda}_M\right)-v^1\left(0,\bar{\lambda}_M\right)\right)+\left(1-\bar{\lambda}_M m\right)\left(v^2\left(1,\bar{\lambda}_M\right)-v^2\left(0,\bar{\lambda}_M\right)\right)$, where $v^1\left(1,\bar{\lambda}_M\right)-v^1\left(0,\bar{\lambda}_M\right)=\alpha_2\ln\left[\alpha_1\bar{L}^2\left(1-\bar{\lambda}_M m\right)/A_2\alpha_2\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)\right]+[\alpha_1/(\sigma-1)]\ln\left[(1-\alpha_3)/\alpha_1\right]>0$ because $\frac{\alpha_1\bar{L}^2\left(1-\bar{\lambda}_M m\right)}{\alpha_2\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)}>A_2$ and $v^2\left(1,\bar{\lambda}_M\right)-v^2\left(0,\bar{\lambda}_M\right)=\alpha_1\ln\left[\frac{A_1\alpha_2\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)}{\alpha_1\bar{L}^2\left(1-\bar{\lambda}_M m\right)}\right]+[\alpha_1/(\sigma-1)]\ln\left[\frac{(1-\alpha_3)\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)}{\alpha_1\left(1-\bar{\lambda}_M m\right)\bar{L}^2}\right]>0$ because $\alpha_1\ln\left[\frac{A_1\alpha_2\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)}{\alpha_1\left(1-\bar{\lambda}_M m\right)\bar{L}^2}\right]>\frac{\alpha_1}{\sigma-1}\ln\left[\frac{\alpha_1\bar{L}^2}{(1-\alpha_3)\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)}\right]$. $W_G^2\left(1,\bar{\lambda}_M\right)-W_G^2(0,0)=\bar{\lambda}_M m\left(v^1\left(1,\bar{\lambda}_M\right)-v^2\left(1,\bar{\lambda}_M\right)\right)+\left(v^2\left(1,\bar{\lambda}_M\right)-v^2(0,0)\right)$, where

$v^1\left(1,\bar{\lambda}_M\right) - v^2\left(1,\bar{\lambda}_M\right) = T_N^1 - T_N^2 + \ln\left[\frac{\alpha_1\left(1-\bar{\lambda}_M m\right)\bar{L}^2}{\alpha_2\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)}\right]^{(\alpha_1+\alpha_2)} > 0$ and $v^2\left(1,\bar{\lambda}_M\right) - v^2(0,0) = \alpha_1 \ln\left[\frac{a_{L,1}^2\alpha_2\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)}{a_{L,1}^1\alpha_1\left(1-\bar{\lambda}_M m\right)\bar{L}^2}\right] + \left[\alpha_1/(\sigma-1)\right]\ln\left[\frac{(1-\alpha_3)\left(\bar{L}^1+\bar{\mu} m\bar{L}^2\right)}{\alpha_1\bar{L}^2}\right] > 0$ because $\alpha_1\ln\left[\frac{A_1\alpha_2\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)}{\alpha_1\left(1-\bar{\lambda}_M m\right)\bar{L}^2}\right] > \frac{\alpha_1}{\sigma-1}\ln\left[\frac{\alpha_1\bar{L}^2}{(1-\alpha_3)\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)}\right]$.

Finally, $(\lambda_T,\lambda_M) = (1,1)$ is not a Nash equilibrium outcome when $\left(\bar{L}^1+\bar{\lambda}_M m\bar{L}^2\right)^{\frac{\alpha_1-\alpha_2(\sigma-1)}{\sigma-1}}\left[\left(1-\bar{\lambda}_M m\right)\bar{L}^2\right]^{\alpha_2} > \left(L^1\right)^{\frac{\alpha_1-\alpha_2(\sigma-1)}{\sigma-1}}\left(L^2\right)^{\alpha_2}$ if and only if $W_G^1\left(1,\bar{\lambda}_M\right) > W_G^1(1,1)$. This completes the proof of the proposition. ■

C.2 Extractive elites

Consider an economy with two countries ($J=2$), two tradeable goods ($Z_T=\{1,2\}$) and one non-tradeable good ($Z_N=\{3\}$). Production functions, endowments and preferences are as in the simple Ricardian economy. Each of the countries is populated by two types of agents: L^j workers, each of whom owns one unit of labor, and E^j elite members. The elite appropriates a fraction $\beta^j\in(0,1)$ of L^j. A fraction $\delta\in(0,1)$ of each unit expropriated by the elite is lost in the process. After expropriation, each worker keeps $\left(1-\beta^j\right)$ units of labor and each member of the elite gets $\delta\beta^jL^j/E^j$ units of labor. The effective labor force of country j is $\tilde{L}^j = B^jL^j = \left[1-\beta^j(1-\delta)\right]L^j$. Only a fraction $m\in[0,1]$ of the labor force of each country is mobile at zero cost. The rest of the labor force and elite members are immobile. Each government selects trade and migration policies in order to maximize $W_G^j = \frac{\bar{L}^j\left[(1-m)v^{j,im}+mv^{j,m}\right]+E^j\left(1+\varphi^j\right)v^{j,e}}{\bar{L}^j+E^j(1+\varphi^j)}$, where $v^{j,im}$, $v^{j,m}$, $v^{j,e}$ are the indirect utility functions of an immobile worker, a mobile worker and a member of the elite, respectively, and $\varphi^j\in[-1,\infty]$.

Let $T^j = -\sum_{z\in Z}\alpha_z\ln\left(a_{L,z}^j\right)$ be the productivity of country j. In order to highlight the role of the elite, assume $T^1=T^2=T$. Then, under autarky, the utility of a worker is $v^{j,h} = C+T+\ln\left(1-\beta^j\right)$ and the utility of a member of the elite is $v^{j,e} = C+T+\ln\left(\delta\beta^j\right)+\ln\left(L^j/E^j\right)$. If labor mobility is not allowed, then $L^j=\bar{L}^j$. If labor mobility is allowed, then all mobile workers go to or stay in the country with the lowest β^j.

Under free trade, if $A_1 > \alpha_1\tilde{L}^2/\alpha_2\tilde{L}^1 > A_2$, then country j specializes in good $z=j\in\{1,2\}$. Therefore, the utility of a worker is given by $v^{1,h} = C+T+\alpha_2\ln\left(\alpha_1B^2L^2/A_2\alpha_2B^1L^1\right)+\ln\left(1-\beta^1\right)$ in country 1 and $v^{2,h} = C+T+\alpha_1\ln\left(A_1\alpha_2B^1L^1/\alpha_1B^2L^2\right)+\ln\left(1-\beta^2\right)$ in country 2, while the utility of an elite member is given by $v^{1,e} = C+T+\alpha_2\ln\left(\alpha_1B^2L^2/A_2\alpha_2B^1L^1\right)+\ln\left(\delta\beta^1\right)+\ln\left(L^1/E^1\right)$ in country 1 and $v^{2,e} = C+T+\alpha_1\ln\left(A_1\alpha_2B^1L^1/\alpha_1B^2L^2\right)+\ln\left(\delta\beta^2\right)+\ln\left(L^2/E^j\right)$ in country 2. If labor mobility is not allowed, then $L^j=\bar{L}^j$. If labor mobility is allowed, mobile workers will go to or stay in the country with the highest v^j. Since v^1 is decreasing in L^1/L^2 while v^2 is increasing in L^1/L^2, if there are enough mobile workers, they will relocate until $v^1=v^2$. This implies $L^2/L^1 = \left(\alpha_2\Gamma^2/\alpha_1\Gamma^1\right)(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$, where $\Gamma^j = \left(1-\beta^j\right)^{\frac{1}{\alpha_1+\alpha_2}}/\left[1-\beta^j(1-\delta)\right]$. Moreover, there will be migration to country 1 whenever $\bar{L}^2/\bar{L}^1 > \left(\alpha_2\Gamma^2/\alpha_1\Gamma^1\right)(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$ and migration to country 2 whenever $\bar{L}^2/\bar{L}^1 < \left(\alpha_2\Gamma^2/\alpha_1\Gamma^1\right)(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$. Finally, the distribution of mobile workers between the countries does not violate either $L^j \in \left[(1-m)\bar{L}^j, \bar{L}^j + m\bar{L}^{-j}\right]$ or $A_1 > \alpha_1\tilde{L}^2/\alpha_2\tilde{L}^1 > A_2$, provided that

$$\left(\frac{B^1}{B^2}\right)A_1 > \frac{\alpha_1\left(\bar{L}^2 + m\bar{L}^1\right)}{\alpha_2(1-m)\bar{L}^1} > \left(\frac{1-\beta^2}{1-\beta^1}\right)^{\frac{1}{\alpha_1+\alpha_2}}(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}} > \frac{\alpha_1(1-m)\bar{L}^2}{\alpha_2\left(\bar{L}^1 + m\bar{L}^2\right)} > A_2\left(\frac{B^1}{B^2}\right)$$

In order to simplify the characterization of the political equilibrium, we also assume that

$$\beta^2 > \beta^1$$

$$\alpha_2 \ln\left[\frac{B^2\Gamma^2}{A_2B^1\Gamma^1(A_3)^{\frac{\alpha_3}{\alpha_1+\alpha_2}}}\right] > \ln\left[\frac{\left(\bar{L}^1 + m\bar{L}^2\right)\left(\alpha_1\Gamma^1 + \alpha_2\Gamma^2(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}\right)}{\left(\bar{L}^1 + \bar{L}^2\right)\alpha_1\Gamma^1}\right]$$

$$\alpha_1 \ln\left[\frac{A_1B^1L^1\Gamma^1}{B^2\Gamma^2(A_3)^{\frac{\alpha_3}{\alpha_1+\alpha_2}}}\right] > m\ln\left(\frac{1-\beta^1}{1-\beta^2}\right)$$

The first inequality simply means that the elite of country 1 is less extractive than the elite of country 2, in the sense that it appropiates a lower fraction of its labor force. The second inequality ensures that the elite of the rich country under autarky (country 1) prefers free trade and free labor mobility to autarky and free labor mobility. Intuitively, for the elite of country 1, when there is free labor mobility, gains from trade prevail over the negative effect that trade liberalization has on the size of the labor force of country 1. The third inequality states that workers in the poor country under autarky (country 2) prefer free trade and free labor mobility to autarky and free labor mobility. Intuitively, the gains from trade for all workers in country 2 are higher than the productivity gains under autarky for mobile workers.

Proposition 8 characterizes trade and labor mobility policies.

Proposition 8 (extractive elite). *Suppose that*

$$\left(\frac{B^1}{B^2}\right)A_1 > \frac{\alpha_1\left(\bar{L}^2 + m\bar{L}^1\right)}{\alpha_2(1-m)\bar{L}^1} > \left(\frac{1-\beta^2}{1-\beta^1}\right)^{\frac{1}{\alpha_1+\alpha_2}}(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}} > \frac{\alpha_1(1-m)\bar{L}^2}{\alpha_2\left(\bar{L}^1 + m\bar{L}^2\right)} > A_2\left(\frac{B^1}{B^2}\right)$$

$$\beta^2 > \beta^1$$

$$\alpha_2 \ln\left[\frac{B^2\Gamma^2}{A_2B^1\Gamma^1(A_3)^{\frac{\alpha_3}{\alpha_1+\alpha_2}}}\right] > \ln\left[\frac{\left(\bar{L}^1 + m\bar{L}^2\right)\left(\alpha_1\Gamma^1 + \alpha_2\Gamma^2(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}\right)}{\left(\bar{L}^1 + \bar{L}^2\right)\alpha_1\Gamma^1}\right]$$

$$\alpha_1 \ln\left[\frac{A_1B^1L^1\Gamma^1}{B^2\Gamma^2(A_3)^{\frac{\alpha_3}{\alpha_1+\alpha_2}}}\right] > m\ln\left(\frac{1-\beta^1}{1-\beta^2}\right)$$

Assume $\bar{L}^2/\bar{L}^1 \neq \left(\alpha_2\Gamma^2/\alpha_1\Gamma^1\right)(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$. *Let* $L^1 = \frac{\alpha_1\Gamma^1\left(\bar{L}^1+\bar{L}^2\right)}{\alpha_1\Gamma^1+\alpha_2\Gamma^2(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}}$ *and* $L^2 = \frac{\alpha_2\Gamma^2(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}\left(\bar{L}^1+\bar{L}^2\right)}{\alpha_1\Gamma^1+\alpha_2\Gamma^2(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}}$. *Then:*

1. *No trade and no labor mobility is always a Nash equilibrium outcome.*

2. *Free trade and no labor mobility is always a Nash equilibrium outcome. Moreover,* $W_G^j(1,0) \geq W_G^j(0,0)$ *for* $j = 1,2$.

3. *No trade and free labor mobility is a Nash equilibrium outcome if and only if* $1+\varphi^2 \leq \frac{m\bar{L}^2 \ln\left[\left(1-\beta^1\right)/\left(1-\beta^2\right)\right]}{-E^2\ln(1-m)}$.

4. *If* $\bar{L}^2/\bar{L}^1 > \left(\alpha_2\Gamma^2/\alpha_1\Gamma^1\right)(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$*, free trade and free labor mobility is a Nash equilibrium outcome if and only if:*

   (a) $\left(L^1\right)^{(1-\alpha_2)}\left(L^2\right)^{\alpha_2} > \left(\bar{L}^1\right)^{(1-\alpha_2)}\left(\bar{L}^2\right)^{\alpha_2}$ *and* $1+\varphi^1 > \bar{\varphi}^1 = \frac{\bar{L}^1\alpha_2\ln\left(\frac{L^1\bar{L}^2}{\bar{L}^1L^2}\right)}{E^1\ln\left[\frac{\left(L^1\right)^{(1-\alpha_2)}\left(L^2\right)^{\alpha_2}}{\left(\bar{L}^1\right)^{(1-\alpha_2)}\left(\bar{L}^2\right)^{\alpha_2}}\right]}$.

   (b) $\left(L^1\right)^{\alpha_1}\left(L^2\right)^{1-\alpha_1} > \left(\bar{L}^1\right)^{\alpha_1}\left(\bar{L}^2\right)^{1-\alpha_1}$ *or* $\left(L^1\right)^{\alpha_1}\left(L^2\right)^{1-\alpha_1} < \left(\bar{L}^1\right)^{\alpha_1}\left(\bar{L}^2\right)^{1-\alpha_1}$ *and* $1+\varphi^2 < \bar{\varphi}^2 = \frac{-\bar{L}^2\alpha_1\ln\left(\frac{L^1\bar{L}^2}{\bar{L}^1L^2}\right)}{E^2\ln\left[\frac{\left(L^1\right)^{\alpha_1}\left(L^2\right)^{1-\alpha_1}}{\left(\bar{L}^1\right)^{\alpha_1}\left(\bar{L}^2\right)^{1-\alpha_1}}\right]}$.

5. *If* $\bar{L}^2/\bar{L}^1 < \left(\alpha_2\Gamma^2/\alpha_1\Gamma^1\right)(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$*, free trade and free labor mobility is a Nash equilibrium outcome if and only if:*

   (a) $\left(L^1\right)^{\alpha_1}\left(L^2\right)^{1-\alpha_1} > \left(\bar{L}^1\right)^{\alpha_1}\left(\bar{L}^2\right)^{1-\alpha_1}$ *and* $1+\varphi^2 > \bar{\varphi}^2$;

   (b) $\left(L^1\right)^{(1-\alpha_2)}\left(L^2\right)^{\alpha_2} > \left(\bar{L}^1\right)^{(1-\alpha_2)}\left(\bar{L}^2\right)^{\alpha_2}$ *or* $\left(L^1\right)^{(1-\alpha_2)}\left(L^2\right)^{\alpha_2} < \left(\bar{L}^1\right)^{(1-\alpha_2)}\left(\bar{L}^2\right)^{\alpha_2}$ *and* $1+\varphi^1 < \bar{\varphi}^1$.

Proof: Note that:

$$W_G^j(0,\lambda_M) = C+T+\frac{\bar{L}^j\ln\left(1-\beta^j\right)+E^j\left(1+\varphi^j\right)\left[\ln\left(\delta\beta^j\right)+\ln\left(L^j/E^j\right)\right]}{\bar{L}^j+E^j\left(1+\varphi^j\right)}$$

$$W_G^1(1,\lambda_M) = C+T+\alpha_2\ln\left(\frac{\alpha_1\left[1-\beta^2(1-\delta)\right]L^2}{A_2\alpha_2\left[1-\beta^1(1-\delta)\right]L^1}\right)+ \\ +\frac{\bar{L}^1\ln\left(1-\beta^1\right)+E^1\left(1+\varphi^1\right)\ln\left(\delta\beta^1L^1/E^1\right)}{\bar{L}^1+E^1\left(1+\varphi^1\right)}$$

$$W_G^2(1,\lambda_M) = C+T+\alpha_1\ln\left(\frac{A_1\alpha_2\left[1-\beta^1(1-\delta)\right]L^1}{\alpha_1\left[1-\beta^2(1-\delta)\right]L^2}\right)+ \\ +\frac{\bar{L}^2\ln\left(1-\beta^2\right)+E^2\left(1+\varphi^2\right)\ln\left(\delta\beta^2L^2/E^2\right)}{\bar{L}^2+E^2\left(1+\varphi^2\right)}$$

$(\lambda_T,\lambda_M) = (0,0)$ is a always a Nash equilibrium outcome.

$(\lambda_T,\lambda_M) = (0,1)$ is a Nash equilibrium outcome if and only if $W_G^j(1,0) \geq W_G^j(0,0)$, which holds because: $W_G^1(1,0)-W_G^1(0,0) = \alpha_2\ln\left(\frac{\alpha_1B^2\bar{L}^2}{A_2\alpha_2B^1\bar{L}^1}\right) > 0$ and $W_G^1(1,0)-W_G^1(0,0) = \alpha_1\ln\left(\frac{A_1\alpha_2B^1\bar{L}^1}{\alpha_1B^2\bar{L}^2}\right) > 0$ because $\left(\frac{B^1}{B^2}\right)A_1 > \frac{\alpha_1\bar{L}^2}{\alpha_2\bar{L}^1} > A_2\left(\frac{B^1}{B^2}\right)$.

$(\lambda_T,\lambda_M)=(0,1)$ is a Nash equilibrium outcome if and only if $W_G^j(0,1)\geq W_G^j(0,0)$. $W_G^1(0,1)-W_G^1(0,0)=\frac{E^1\left(1+\varphi^1\right)\ln\left(\frac{\bar{L}^1+m\bar{L}^2}{\bar{L}^1}\right)}{\bar{L}^1+E^1(1+\varphi^1)}>0$ and $W_G^2(0,1)-W_G^2(0,0)=\frac{\bar{L}^2 m\ln\left(\frac{1-\beta^1}{1-\beta^2}\right)+E^2\left(1+\varphi^2\right)\ln(1-m)}{\bar{L}^2+E^2(1+\varphi^2)}$ if and only if $1+\varphi^2\leq\frac{m\bar{L}^2\ln\left[\left(1-\beta^1\right)/\left(1-\beta^2\right)\right]}{-E^2\ln(1-m)}$.

$(\lambda_T,\lambda_M)=(1,1)$ is a Nash equilibrium outcome if and only if $W_G^j(1,1)\geq\max\left\{W_G^j(1,0),W_G^j(0,1)\right\}$. Note that

$$\begin{aligned}W_G^1(1,1)-W_G^1(0,1)&=\alpha_2\ln\left(\frac{\alpha_1B^2L^2}{A_2\alpha_2B^1L^1}\right)+\frac{E^1\left(1+\varphi^1\right)\ln\left[L^1/\left(\bar{L}^1+m\bar{L}^2\right)\right]}{\bar{L}^1+E^1\left(1+\varphi^1\right)}\\&>\ \ln\left(\frac{\bar{L}^1+m\bar{L}^2}{L^1}\right)+\frac{E^1\left(1+\varphi^1\right)\ln\left[L^1/\left(\bar{L}^1+m\bar{L}^2\right)\right]}{\bar{L}^1+E^1\left(1+\varphi^1\right)}\\&=\frac{\bar{L}^1}{\bar{L}^1+E^1\left(1+\varphi^1\right)}\ln\left(\frac{\bar{L}^1+m\bar{L}^2}{L^1}\right)>0,\end{aligned}$$

where we have used that $\alpha_2\ln\left[\frac{B^2\Gamma^2}{A_2B^1\Gamma^1(A_3)^{\frac{\alpha_3}{\alpha_1+\alpha_2}}}\right]>\ln\left[\frac{\left(\bar{L}^1+m\bar{L}^2\right)\left(\alpha_1\Gamma^1+\alpha_2\Gamma^2(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}\right)}{\left(\bar{L}^1+\bar{L}^2\right)\alpha_1\Gamma^1}\right]$ or, which is equivalent, that $\alpha_2\ln\left(\frac{\alpha_1B^2L^2}{A_2\alpha_2B^1L^1}\right)>\ln\left(\frac{\bar{L}^1+m\bar{L}^2}{L^1}\right)$. Also note that

$$\begin{aligned}W_G^2(1,1)-W_G^2(0,1)&=\alpha_1\ln\left(\frac{A_1\alpha_2B^1L^1}{\alpha_1B^2L^2}\right)+\frac{-\bar{L}^2m\ln\left[\left(1-\beta^1\right)/\left(1-\beta^2\right)\right]+E^2\left(1+\varphi^2\right)\ln\left(L^2/\bar{L}^2\left(1-m\right)\right)}{\bar{L}^2+E^2\left(1+\varphi^2\right)}\\&>m\ln\left(\frac{1-\beta^1}{1-\beta^2}\right)+\frac{-\bar{L}^2m\ln\left[\left(1-\beta^1\right)/\left(1-\beta^2\right)\right]+E^2\left(1+\varphi^2\right)\ln\left(L^2/\bar{L}^2\left(1-m\right)\right)}{\bar{L}^2+E^2\left(1+\varphi^2\right)}\\&=\frac{E^2\left(1+\varphi^2\right)}{\bar{L}^2+E^2\left(1+\varphi^2\right)}m\ln\left(\frac{1-\beta^1}{1-\beta^2}\right)+\frac{E^2\left(1+\varphi^2\right)}{\bar{L}^2+E^2\left(1+\varphi^2\right)}\ln\left[\frac{L^2}{\bar{L}^2\left(1-m\right)}\right]>0\end{aligned}$$

where we have used that $\alpha_1\ln\left[\frac{A_1B^1L^1\Gamma^1}{B^2\Gamma^2(A_3)^{\frac{\alpha_3}{\alpha_1+\alpha_2}}}\right]>m\ln\left(\frac{1-\beta^1}{1-\beta^2}\right)$ or, which is equivalent, that $\alpha_1\ln\left(\frac{A_1\alpha_2B^1L^1}{\alpha_1B^2L^2}\right)>m\ln\left(\frac{1-\beta^1}{1-\beta^2}\right)$.

$W_G^1(1,1)-W_G^1(1,0)=\alpha_2\ln\left(\frac{\bar{L}^1L^2}{L^1\bar{L}^2}\right)+\frac{E^1\left(1+\varphi^1\right)\ln\left(L^1/\bar{L}^1\right)}{\bar{L}^1+E^1(1+\varphi^1)}$, while $W_G^2(1,1)-W_G^2(1,0)=\alpha_1\ln\left(\frac{\bar{L}^2L^1}{L^2\bar{L}^1}\right)+\frac{E^2\left(1+\varphi^2\right)\ln\left(L^2/\bar{L}^2\right)}{\bar{L}^2+E^2(1+\varphi^2)}$. Thus, we must consider several possible cases.

(i) Suppose $\bar{L}^2/\bar{L}^1>\left(\alpha_2\Gamma^2/\alpha_1\Gamma^1\right)(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$. Then $L^1>\bar{L}^1$ and $L^2<\bar{L}^2$. $W_G^1(1,1)-W_G^1(1,0)>0$ and $W_G^2(1,1)-W_G^2(1,0)>0$ if and only if $E^1\left(1+\varphi^1\right)\ln\left[\frac{\left(L^1\right)^{(1-\alpha_2)}\left(L^2\right)^{\alpha_2}}{\left(\bar{L}^1\right)^{(1-\alpha_2)}\left(\bar{L}^2\right)^{\alpha_2}}\right]>\alpha_2\bar{L}^1\ln\left(\frac{L^1\bar{L}^2}{\bar{L}^1L^2}\right)$ and $\alpha_1\bar{L}^2\ln\left(\frac{\bar{L}^2L^1}{L^2\bar{L}^1}\right)>E^2\left(1+\varphi^2\right)\ln\left[\frac{\left(\bar{L}^1\right)^{\alpha_1}\left(\bar{L}^2\right)^{1-\alpha_1}}{\left(L^1\right)^{\alpha_1}\left(L^2\right)^{1-\alpha_1}}\right]$. Conditions (a) and (b) in part 4 are equivalent to these two inequalities.

(ii) Suppose $\bar{L}^2/\bar{L}^1<\left(\alpha_2\Gamma^2/\alpha_1\Gamma^1\right)(A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}}$. Then $L^1<\bar{L}^1$ and $L^2>\bar{L}^2$. $W_G^1(1,1)-W_G^1(1,0)>0$ and $W_G^2(1,1)-W_G^2(1,0)>0$ if and only if $\alpha_2\bar{L}^1\ln\left(\frac{\bar{L}^1L^2}{L^1\bar{L}^2}\right)>E^1\left(1+\varphi^1\right)\ln\left[\frac{\left(\bar{L}^1\right)^{(1-\alpha_2)}\left(\bar{L}^2\right)^{\alpha_2}}{\left(L^1\right)^{(1-\alpha_2)}\left(L^2\right)^{\alpha_2}}\right]$ and

$E^2\left(1+\varphi^2\right)\ln\left[\frac{\left(L^1\right)^{\alpha_1}\left(L^2\right)^{1-\alpha_1}}{\left(\bar{L}^1\right)^{\alpha_1}\left(\bar{L}^2\right)^{1-\alpha_1}}\right] > \alpha_1\bar{L}^2\ln\left(\frac{L^2\bar{L}^1}{\bar{L}^2L^1}\right)$. Conditions (a) and (b) in part 5 are equivalent to these inequalities. This completes the proof of the proposition. ■

C.3 Citizenship for sale

Consider an economy with two countries ($J=2$), two tradeable goods ($Z_T=\{1,2\}$) and one non-tradeable good ($Z_N=\{3\}$). Production functions, endowments and preferences are as in the simple Ricardian economy. Let $\tau^j\in[0,1]$ be the income tax rate that country j charges to foreign mobile workers that decide to locate in j and let $Tf^j\geq 0$ be the transfer per capita received by country j's workers. The government of country j selects its trade and migration policies in order to maximize $W_G^j=mv_L^{j,m}+(1-m)\,v_L^{j,im}$ subject to $Tf^j\bar{L}^j=\tau^j\max\left\{L^j-\bar{L}^j,0\right\}$, government j's budget constraint. Trade policy is $\lambda_T\in\{0,1\}$, where $\lambda_T=1$ indicates that both countries accept free trade and $\lambda_T=0$ that at least one country refuses to trade. Migration policy is $\lambda_M\in\{0,(1,\tau)\}$ with $\tau\in[0,1]$, where $\lambda_M=0$ indicates that at least one country does not accept free labor mobility and $\lambda_M=(1,\tau)$ indicates that both countries accept labor mobility with the recipient country charging a proportional income tax $\tau\in[0,1]$ to foreign mobile workers.

Under autarky, all goods must be produced domestically. Hence, $p_z^j=w^ja_{L,z}^j$ for $z\in Z=\{1,2,3\}$. If labor mobility is not allowed, then $v^{j,im}(0,0)=v^{j,m}(0,0)=C+T^j$ and, hence, $W^j(0,0)=mv^{j,m}+(1-m)\,v^{j,im}=C+T^j$. If labor mobility is allowed, then $v^{j,im}=C+T^j+\ln\left[1+\tau^j\max\left\{L^j-\bar{L}^j,0\right\}/\bar{L}^j\right]$, where $\tau^j\in[0,1]$ is the income tax changed by country j to immigrants. Mobile workers in country 1 strictly prefer to locate in country 1 if and only if $T^1+\ln\left[1+\tau^1\max\left\{L^1-\bar{L}^1,0\right\}/\bar{L}^1\right]>T^2+\ln\left(1-\tau^2\right)$. Since $T^1>T^2$ and $\tau^j\in[0,1]$ for $j=1,2$, this inequality always holds. Thus, mobile workers in country 1 always prefer to locate in country 1. Mobile workers in country 2 prefer to locate in country 1 if and only if $T^1+\ln\left(1-\tau^1\right)\geq T^2+\ln\left[1+\tau^2\max\left\{L^2-\bar{L}^2,0\right\}/\bar{L}^2\right]$. Since $L^2\leq\bar{L}^2$, this inequality becomes $T^1+\ln\left(1-\tau^1\right)\geq T^2$. Therefore, if $T^1+\ln\left(1-\tau^1\right)\geq T^2$, then $L^1=\bar{L}^1+m\bar{L}^2$ and $L^2=(1-m)\,\bar{L}^2$, while if $T^1+\ln\left(1-\tau^1\right)<T^2$, then $L^1=\bar{L}^1$ and $L^2=\bar{L}^2$. Hence, $W_G^1\left(0,\left(1,\tau^1\right)\right)=C+T^1+\ln\left[1+\tau^1\left(L^1-\bar{L}^1\right)/\bar{L}^1\right]$ and $W_G^2\left(0,\left(1,\tau^1\right)\right)=m\left[C+\max\left\{T^1+\ln\left(1-\tau^1\right),T^2\right\}\right]+(1-m)\left(C+T^2\right)$. Finally, note that $W_G^1\left(0,\left(1,\tau^1\right)\right)$ adopts a maximum for $\tau^1=\hat{\tau}=1-e^{-(T_1-T_2)}$. Thus, $W_G^1(0,(1,\hat{\tau}))=C+T^1+\ln\left[1+\left(1-e^{-(T_1-T_2)}\right)m\bar{L}^2/\bar{L}^1\right]$ and $W_G^2(0,(1,\hat{\tau}))=C+T^2$.

Suppose there is free trade. Provided that $A_1>\alpha_1L^2/\alpha_2L^1>A_2$, country j specializes in good $z=j\in\{1,2\}$. Thus, in equilibrium, $p_1=w^1a_{L,1}^1$, $p_2=w^2a_{L,2}^2$, $p_3^j=w^ja_{L,3}^j$ and the balanced trade condition is $\alpha_2w^1L^1=\alpha_1w^2L^2$. Therefore, $v^1=C+T^1+\alpha_2\ln\left(\alpha_1L^2/A_2\alpha_2L^1\right)$ and $v^2=C+T^2+\alpha_1\ln\left(A_1\alpha_2L^1/\alpha_1L^2\right)$. If labor mobility is not allowed, then $v^{1,im}(1,0)=v^{1,m}(1,0)=W^1(1,0)=C+T^1+\alpha_2\ln\left(\alpha_1\bar{L}^2/A_2\alpha_2\bar{L}^1\right)$ and $v^{2,im}(1,0)=v^{2,m}(1,0)=W^2(1,0)=C+T^2+\alpha_1\ln\left(A_1\alpha_2\bar{L}^1/\alpha_1\bar{L}^2\right)$. If labor mobility is allowed, the indirect utility of an immobile worker who owns one unit of labor in country 1 (2) is $v^{1,im}=C+T^1+\alpha_2\ln\left(\alpha_1L^2/A_2\alpha_2L^1\right)+\ln\left[1+\tau^1\max\left\{L^1-\bar{L}^1,0\right\}/\bar{L}^1\right]$ ($v^{2,im}=C+T^2+\alpha_1\ln\left(A_1\alpha_2L^1/\alpha_1L^2\right)+\ln\left[1+\tau^2\max\left\{L^2-\bar{L}^2,0\right\}/\bar{L}^2\right]$). Mobile workers in country 1 prefer to locate in country 1 if and only if $w^1/\left(w^1a_{L,3}^1\right)^{\alpha_3}\geq w^2\left(1-\tau^2\right)/\left(w^2a_{L,3}^2\right)^{\alpha_3}$. In equilibrium, $\alpha_2w^1L^1=\alpha_1w^2L^2$, which implies that mobile workers in country 1 strictly prefer to locate in country 1 if and only if $\alpha_1L^2/\alpha_2L^1>\left(1-\tau^2\right)^{\frac{1}{1-\alpha_3}}(A_3)^{\frac{-\alpha_3}{1-\alpha_3}}$. If $\alpha_1\bar{L}^2/\alpha_2\bar{L}^1>(A_3)^{\frac{-\alpha_3}{1-\alpha_3}}$, then $\alpha_1L^2/\alpha_2L^1>\left(1-\tau^2\right)^{\frac{1}{1-\alpha_3}}(A_3)^{\frac{-\alpha_3}{1-\alpha_3}}$ holds whenever $L^2/L^1\geq\bar{L}^2/\bar{L}^1$. Thus, in equilibrium, it must be the case all mobile workers in country 1 locate in country 1, i.e., $L^1 \geq \bar{L}^1$ and $L^2 \leq \bar{L}^2$. Mobile workers in country 2 prefer to locate in country 2 if and only if $w^2/\left(w^2 a^2_{L,3}\right)^{\alpha_3} \geq w^1\left(1-\tau^1\right)/\left(w^1 a^1_{L,3}\right)^{\alpha_3}$. In equilibrium, $\alpha_2 w^1 L^1 = \alpha_1 w^2 L^2$, which implies that mobile workers in country 2 prefer to locate in country 2 if and only if $\alpha_1 L^2/\alpha_2 L^1 \leq (A_3)^{\frac{-\alpha_3}{1-\alpha_3}}/\left(1-\tau^1\right)^{\frac{1}{1-\alpha_3}}$. Therefore, we must distinguish two possible cases. If $\alpha_1 \bar{L}^2/\alpha_2 \bar{L}^1 \leq (A_3)^{\frac{-\alpha_3}{1-\alpha_3}}/\left(1-\tau^1\right)^{\frac{1}{1-\alpha_3}}$, then $\alpha_1 L^2/\alpha_2 L^1 \leq (A_3)^{\frac{-\alpha_3}{1-\alpha_3}}/\left(1-\tau^1\right)^{\frac{1}{1-\alpha_3}}$ holds for all $L^1 \geq \bar{L}^1$ and $L^2 \leq \bar{L}^2$. Then, in equilibrium, $L^1 = \bar{L}^1$ and $\bar{L}^2 = L^2$. If $\alpha_1 \bar{L}^2/\alpha_2 \bar{L}^1 > (A_3)^{\frac{-\alpha_3}{1-\alpha_3}}/\left(1-\tau^1\right)^{\frac{1}{1-\alpha_3}}$, then $\alpha_1 \bar{L}^2/\alpha_2 \bar{L}^1 > (A_3)^{\frac{-\alpha_3}{1-\alpha_3}}/\left(1-\tau^1\right)^{\frac{1}{1-\alpha_3}} > \alpha_1(1-m)\bar{L}^2/\alpha_2\left(\bar{L}^1+m\bar{L}^2\right)$ and, hence, the allocation of mobile workers is such that $\alpha_1 L^2/\alpha_2 L^1 = (A_3)^{\frac{-\alpha_3}{1-\alpha_3}}/\left(1-\tau^1\right)^{\frac{1}{1-\alpha_3}}$, which implies $L^1 = \psi\left(\tau^1\right)\left(\bar{L}^1+\bar{L}^2\right)$ and $L^2 = \left[1-\psi\left(\tau^1\right)\right]\left(\bar{L}^1+\bar{L}^2\right)$, where $\psi\left(\tau^1\right) = \alpha_1\left(1-\tau^1\right)^{\frac{1}{1-\alpha_3}}/\left[\alpha_1\left(1-\tau^1\right)^{\frac{1}{1-\alpha_3}}+\alpha_2(A_3)^{\frac{-\alpha_3}{1-\alpha_3}}\right]$. Thus, if $\tau^1 < \bar{\tau} = 1-\left(\alpha_2\bar{L}^1/\alpha_1\bar{L}^2\right)^{1-\alpha_3}(A_3)^{-\alpha_3}$, then $W^1_G\left(1,\left(1,\tau^1\right)\right) = C+T^1+\alpha_2\ln\left[\alpha_1\left(1-\psi\left(\tau^1\right)\right)/A_2\alpha_2\psi\left(\tau^1\right)\right]+\ln\left[1+\tau^1\left(\psi\left(\tau^1\right)\left(\bar{L}^1+\bar{L}^2\right)-\bar{L}^1\right)/\bar{L}^1\right]$ and $W^2_G\left(1,\left(1,\tau^1\right)\right) = C+T^2+\alpha_1\ln\left[A_1\alpha_2\psi\left(\tau^1\right)/\alpha_1\left(1-\psi\left(\tau^1\right)\right)\right]$, while if $\tau^1 \geq \bar{\tau}$, then $W^1_G\left(1,\left(1,\tau^1\right)\right) = W^1_G(1,0) = C+T^1+\alpha_2\ln\left(\alpha_1\bar{L}^2/A_2\alpha_2\bar{L}^1\right)$ and $W^2_G\left(1,\left(1,\tau^1\right)\right) = W^2_G(1,0) = C+T^2+\alpha_1\ln\left(A_1\alpha_2\bar{L}^1/\alpha_1\bar{L}^2\right)$. Note that $W^1_G\left(1,\left(1,\tau^1\right)\right)$ adopts a maximum for $\tau^1 = \tau^* = \arg\max_{0\leq\tau^1\leq\bar{\tau}}\left\{\left[\frac{1-\psi\left(\tau^1\right)}{\psi\left(\tau^1\right)}\right]^{\alpha_2}\left[1+\tau^1\frac{\psi\left(\tau^1\right)\left(\bar{L}^1+\bar{L}^2\right)-\bar{L}^1}{\bar{L}^1}\right]\right\}$. Finally, we prove that $\tau^* > 0$ and find sufficient conditions under which $\tau^* < \bar{\tau}$. Take the derivative of $W^1_G\left(1,\left(1,\tau^1\right)\right)$ with respect to τ^1.

$$\frac{\partial W^1_G\left(1,\left(1,\tau^1\right)\right)}{\partial\tau^1} = \frac{\alpha_2}{(1-\alpha_3)\left(1-\tau^1\right)}+\frac{\psi\left(\tau^1\right)\left(\bar{L}^1+\bar{L}^2\right)\left[1-\tau^1\frac{1-\psi\left(\tau^1\right)}{(1-\alpha_3)\left(1-\tau^1\right)}\right]-\bar{L}^1}{\left(1-\tau^1\right)\bar{L}^1+\tau^1\psi\left(\tau^1\right)\left(\bar{L}^1+\bar{L}^2\right)}$$

Evaluating this derivative at $\tau^1 = 0$ we obtain:

$$\frac{\partial W^1_G\left(1,\left(1,\tau^1=0\right)\right)}{\partial\tau^1} = \frac{\alpha_2}{(1-\alpha_3)}+\left[\frac{\alpha_1\bar{L}^2-\alpha_2(A_3)^{\frac{-\alpha_3}{1-\alpha_3}}\bar{L}^1}{\left[\alpha_1+\alpha_2(A_3)^{\frac{-\alpha_3}{1-\alpha_3}}\right]\bar{L}^1}\right]$$

If $\alpha_1\bar{L}^2/\alpha_2\bar{L}^1 > (A_3)^{\frac{-\alpha_3}{1-\alpha_3}}$, then $\partial W^1_G\left(1,\left(1,\tau^1=0\right)\right)/\partial\tau^1 > 0$, which implies that $\tau^* > 0$. Evaluating $\partial W^1_G\left(1,\left(1,\tau^1\right)\right)/\partial\tau^1$ at $\tau^1 = \bar{\tau}$ we obtain:

$$\frac{\partial W^1_G\left(1,\left(1,\tau^1=\bar{\tau}\right)\right)}{\partial\tau^1} = \frac{\left(\bar{L}^1+\bar{L}^2\right)\alpha_2-\bar{\tau}^1\bar{L}^2}{(1-\alpha_3)(1-\bar{\tau})\left(\bar{L}^1+\bar{L}^2\right)}$$

Note that $\partial W^1_G\left(1,\left(1,\tau^1=\bar{\tau}\right)\right)/\partial\tau^1 < 0$ if and only if $\left(\frac{\alpha_1\bar{L}^2}{\alpha_2\bar{L}^1}\right)\left[\frac{\bar{L}^2-\left(\bar{L}^1+\bar{L}^2\right)\alpha_2}{\bar{L}^2}\right]^{\frac{1}{1-\alpha_3}} > (A_3)^{\frac{-\alpha_3}{1-\alpha_3}}$. Thus, whenever $\left(\frac{\alpha_1\bar{L}^2}{\alpha_2\bar{L}^1}\right)\left[\frac{\bar{L}^2-\left(\bar{L}^1+\bar{L}^2\right)\alpha_2}{\bar{L}^2}\right]^{\frac{1}{1-\alpha_3}} > (A_3)^{\frac{-\alpha_3}{1-\alpha_3}}$, it must be the case that $\tau^* < \bar{\tau}$.

Proposition 9 characterizes trade and labor mobility policies.

Proposition 9 (citizenship for sale). *Suppose that* $A_1 > \frac{\alpha_1(\bar{L}^2+m\bar{L}^1)}{\alpha_2(1-m)\bar{L}^1} > \frac{\alpha_1\bar{L}^2}{\alpha_2\bar{L}^1} > (A_3)^{\frac{-\alpha_3}{\alpha_1+\alpha_2}} > \frac{\alpha_1(1-m)\bar{L}^2}{\alpha_2(\bar{L}^1+m\bar{L}^2)} > A_2$ *and* $T^1 > T^2$. *Then:*

1. *(i) Neither trade nor labor mobility; (ii) no trade and immigration fee* $\hat{\tau} = 1 - e^{-(T_1-T_2)} > 0$; *and (iii) free trade and no labor mobility are always Nash equilibrium outcomes. Moreover:*

   (a) $\{W_G^1(1,0), W_G^1(0,(1,\hat{\tau}))\} > W_G^1(0,0)$ *while* $W_G^1(1,0) > W_G^1(0,(1,\hat{\tau}))$ *if and only if*

$$\left(\frac{\alpha_1\bar{L}^2}{A_2\alpha_2\bar{L}^1}\right)^{\alpha_2} > \frac{\bar{L}^1 + \hat{\tau}m\bar{L}^2}{\bar{L}^1}$$

   (b) $W_G^2(1,0) > W_G^2(0,(1,\hat{\tau})) = W_G^2(0,0)$.

2. *(iv) Free trade and immigration fee* $\tau^* = \arg\max_{0\le\tau\le\bar{\tau}} \left\{\left[\frac{1-\psi(\tau)}{\psi(\tau)}\right]^{\alpha_2}\left[1+\tau\frac{\psi(\tau)(\bar{L}^1+\bar{L}^2)-\bar{L}^1}{\bar{L}^1}\right]\right\} > 0$ *is a Nash equilibrium outcome if and only if*

$$\alpha_2 \ln\left[\frac{\alpha_1(1-\psi(\tau^*))}{A_2\alpha_2\psi(\tau^*)}\right] \ge \ln\left[\frac{\bar{L}^1+\hat{\tau}m\bar{L}^2}{\bar{L}^1+\tau^*\left(\psi(\tau^*)\left(\bar{L}^1+\bar{L}^2\right)-\bar{L}^1\right)}\right]$$

$$\alpha_1 \ln\left[\frac{A_1\alpha_2\psi(\tau^*)}{\alpha_1(1-\psi(\tau^*))}\right] \ge m\max\left\{\ln(1-\tau^*) + T^1 - T^2, 0\right\}$$

   where $\psi(\tau) = \frac{\alpha_1(1-\tau)^{1/1-\alpha_3}}{\alpha_1(1-\tau)^{1/1-\alpha_3}+\alpha_2(A_3)^{-\alpha_3/1-\alpha_3}}$ *and* $\bar{\tau} = 1 - \left(\alpha_2\bar{L}^1/\alpha_1\bar{L}^2\right)^{1-\alpha_3}(A_3)^{-\alpha_3}$. *Moreover:*

   (a) $W_G^1(1,(1,\tau^*)) \ge W_G^1(1,0) > W_G^1(0,0)$ *and* $W_G^1(1,(1,\tau^*)) \ge W_G^1(0,(1,\hat{\tau}))$.

   (b) $W_G^2(1,(1,\tau^*)) \ge W_G^2(1,0) > W_G^2(0,(1,\hat{\tau})) = W_G^2(0,0)$

   (c) If $\left(\frac{\alpha_1\bar{L}^2}{\alpha_2\bar{L}^1}\right)\left[\frac{\bar{L}^2-(\bar{L}^1+\bar{L}^2)\alpha_2}{\bar{L}^2}\right]^{\frac{1}{1-\alpha_3}} > (A_3)^{\frac{-\alpha_3}{1-\alpha_3}}$, *then* $\tau^* < \bar{\tau}$, $L^1 = \psi(\tau^*)\left(\bar{L}^1+\bar{L}^2\right) > \bar{L}^1$ *and* $W_G^j(1,(1,\tau^*)) > W_G^j(1,0)$ for $j = 1,2$.

Proof:

$(\lambda_T, \lambda_M) = (0,0)$ is always a Nash equilibrium outcome.

$(\lambda_T, \lambda_M) = (0,(1,\hat{\tau}))$ with $\hat{\tau} = 1 - e^{-(T_1-T_2)}$ is a Nash equilibrium outcome. We have already proved that $W_G^1(0,(1,\hat{\tau})) > W_G^1(0,(1,\tau^1))$ for all $\tau^1 \neq \hat{\tau}$. Also note that $W_G^1(0,(1,\hat{\tau})) = C + T^1 + \ln\left[1+\left(1-e^{-(T_1-T_2)}\right)m\bar{L}^2/\bar{L}^1\right] > W_G^1(0,0) = C + T^1$. Finally, $W_G^2(0,(1,\hat{\tau})) = W_G^2(0,0)$.

$(\lambda_T, \lambda_M) = (0,(1,\tau^1))$, with $\tau^1 \neq \hat{\tau}$ is not a Nash equilibrium outcome because $W_G^1(0,(1,\hat{\tau})) > W_G^1(0,(1,\tau^1))$.

$(\lambda_T, \lambda_M) = (1,0)$ is a Nash equilibrium outcome because $W_G^1(1,0) = C + T^1 + \alpha_2\ln\left(\alpha_1\bar{L}^2/A_2\alpha_2\bar{L}^1\right) > W_G^1(0,0)$ and $W_G^2(1,0) = C + T^2 + \alpha_1\ln\left(A_1\alpha_2\bar{L}^1/\alpha_1\bar{L}^2\right) > W_G^2(0,0)$.

We have already proved that $W_G^1(0,(1,\hat{\tau})) > W_G^1(0,0)$, $W_G^1(1,0) > W_G^1(0,0)$, and $W_G^2(1,0) > W_G^2(0,(1,\hat{\tau})) = W_G^2(0,0)$. Finally, note that $W_G^1(1,0) > W_G^1(0,(1,\hat{\tau}))$ if and only if $\left(\alpha_1\bar{L}^2/A_2\alpha_2\bar{L}^1\right)^{\alpha_2} > \left(\bar{L}^1+\hat{\tau}m\bar{L}^2\right)/\bar{L}^1$, which completes the proof of part 1.

$(\lambda_T, \lambda_M) = (1, (1, \tau^*))$ could be a Nash equilibrium outcome. We have already proved that $W_G^1(1, (1, \tau^*)) \geq W_G^1(1, (1, \tau^1))$ for all $\tau^1 \in [0, 1]$, with strict inequality if $\tau^1 \neq \tau^*$. Since for $\tau^1 \geq \bar{\tau}$, $W_G^1(1, (1, \tau^1)) = W_G^1(1, 0)$, we have $W_G^1(1, (1, \tau^*)) \geq W_G^1(1, 0) > W_G^1(0, 0)$. $W_G^1(1, (1, \tau^*)) \geq W^1(0, (1, \hat{\tau}))$ if and only if

$$\alpha_2 \ln \left[\frac{\alpha_1 (1 - \psi(\tau^*))}{A_2 \alpha_2 \psi(\tau^*)} \right] \geq \ln \left[\frac{\bar{L}^1 + \hat{\tau} m \bar{L}^2}{\bar{L}^1 + \tau^* \left(\psi(\tau^*) \left(\bar{L}^1 + \bar{L}^2 \right) - \bar{L}^1 \right)} \right]$$

and we have already proved that $W_G^1(0, (1, \hat{\tau})) > W_G^1(0, (1, \tau^1))$ for all $\tau^1 \neq \hat{\tau}$. $W_G^2(1, (1, \tau^*)) = C + T^2 + \alpha_1 \ln [A_1 \alpha_2 \psi(\tau^*) / \alpha_1 (1 - \psi(\tau^*))] \geq W_G^2(1, 0) = C + T^2 + \alpha_1 \ln [A_1 \alpha_2 \bar{L}^1 / \alpha_1 \bar{L}^2]$ because $\psi(\tau^*) / (1 - \psi(\tau^*)) \geq \bar{L}^1 / \bar{L}^2$. We have already proved that $W_G^2(1, 0) > W_G^2(0, 0)$. $W_G^2(0, (1, \tau^*)) = C + m [T^1 + \ln(1 - \tau^*)] + (1 - m) T^2$ if $\tau^* < \hat{\tau}$ and $W_G^2(0, (1, \tau^*)) = W_G^2(0, 0) = C + T^2$ if $\tau^* \geq \hat{\tau}$. Thus, if $\tau^* < \hat{\tau}$, then, $W_G^2(1, (1, \tau^*)) \geq W_G^2(0, (1, \tau^*))$ if and only if

$$\alpha_1 \ln \left[\frac{A_1 \alpha_2 \psi(\tau^*)}{\alpha_1 (1 - \psi(\tau^*))} \right] \geq m \left[\ln(1 - \tau^*) + T^1 - T^2 \right]$$

while if $\tau^* \geq \hat{\tau}$, then $W_G^2(1, (1, \tau^*)) \geq W_G^2(0, (1, \tau^*))$ if and only if $\alpha_1 \ln [A_1 \alpha_2 \psi(\tau^*) / \alpha_1 (1 - \psi(\tau^*))] \geq 0$, which always holds.

Therefore, $(\lambda_T, \lambda_M) = (0, (1, \tau^*))$ is a Nash equilibrium outcome if and only if

$$\alpha_2 \ln \left[\frac{\alpha_1 (1 - \psi(\tau^*))}{A_2 \alpha_2 \psi(\tau^*)} \right] \geq \ln \left[\frac{\bar{L}^1 + \hat{\tau} m \bar{L}^2}{\bar{L}^1 + \tau^* \left(\psi(\tau^*) \left(\bar{L}^1 + \bar{L}^2 \right) - \bar{L}^1 \right)} \right]$$
$$\alpha_1 \ln \left[\frac{A_1 \alpha_2 \psi(\tau^*)}{\alpha_1 (1 - \psi(\tau^*))} \right] \geq m \max \left\{ \ln(1 - \tau^*) + T^1 - T^2, 0 \right\}$$

$(\lambda_T, \lambda_M) = (1, (1, \tau^1))$ with $\tau^1 \neq \tau^*$ is not a Nash equilibrium outcome because $W_G^1(1, (1, \tau^*)) > W_G^1(1, (1, \tau^1))$.

We have already proved that when $(\lambda_T, \lambda_M) = (0, (1, \tau^*))$ is a Nash equilibrium outcome, $W_G^1(1, (1, \tau^*)) \geq W_G^1(1, 0) > W_G^1(0, 0)$, $W_G^1(1, (1, \tau^*)) \geq W_G^1(0, (1, \hat{\tau}))$, and $W_G^2(1, (1, \tau^*)) \geq W_G^2(1, 0)$. Also note that if $\left(\frac{\alpha_1 \bar{L}^2}{\alpha_2 \bar{L}^1} \right) \left[\frac{\bar{L}^2 - (\bar{L}^1 + \bar{L}^2) \alpha_2}{\bar{L}^2} \right]^{\frac{1}{1 - \alpha_3}} > (A_3)^{\frac{-\alpha_3}{1 - \alpha_3}}$, then $\tau^* < \bar{\tau}$ and, hence, $W_G^1(1, (1, \tau^*)) > W_G^1(1, (1, \bar{\tau})) = W_G^1(1, 0)$ and $W_G^2(1, (1, \tau^*)) > W_G^2(1, (1, \bar{\tau})) = W_G^2(1, 0)$. Finally, we have also proved that $W_G^2(1, 0) > W_G^2(0, (1, \hat{\tau})) = W_G^2(0, 0)$, which completes the proof of part 2. ■