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EXCHANGE RATES, INTEREST RATES, AND THE RISK PREMIUM

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ABSTRACT

The well-known uncovered interest parity puzzle arises from the empirical regularity that, among developed country pairs, the high interest rate country tends to have high expected returns on its short term assets. At the same time, another strand of the literature has documented that high real interest rate countries tend to have currencies that are strong in real terms - indeed, stronger than can be accounted for by the path of expected real interest differentials under uncovered interest parity. These two strands - one concerning short-run expected changes and the other concerning the level of the real exchange rate - have apparently contradictory implications for the relationship of the foreign exchange risk premium and interest-rate differentials. This paper documents the puzzle, and shows that existing models appear unable to account for both empirical findings. The features of a model that might reconcile the findings are discussed.

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There are two well-known empirical relationships between interest rates and foreign exchange rates, one concerning the rate of change of the exchange rate and the other concerning the level of the exchange rate. Each of these empirical relationships presents challenges to traditional economic models in international finance, and each has spurred advances in the modeling of investor behavior and macroeconomic relationships. Both are important for understanding the role of openness in financial markets and aggregate economic relationships. What has been heretofore unnoticed is that the two relationships taken together constitute a paradox – the explanations advanced for one empirical finding are completely inadequate for explaining the other.

The interest parity (or forward premium) puzzle in foreign exchange markets finds that over short time horizons (from a week to a quarter) when the interest rate (one country relative to another) is higher than average, the short-term deposits of the high-interest rate currency tend to earn an excess return. That is, the high interest rate country tends to have the higher expected return in the short run. The empirical literature on the forward premium anomaly is vast. Classic early references include Bilson (1981) and Fama (1984). Engel (1996, 2014) surveys the empirical work that establishes this puzzle, and discusses the problems faced by the literature that tries to account for the regularity. A risk-based explanation of this anomaly requires that the short-term deposits in the high-interest rate country are relatively riskier (the risk arising from exchange rate movements, since the deposit rates in their own currency are taken to be riskless), and therefore incorporate an expected excess return as a reward for risk-bearing. The ex ante risk premium must therefore be time-varying and covary with the interest differential.

Standard exchange rate models, such as the textbook Mundell-Fleming model or the well-known Dornbusch (1976) model, assume that interest parity holds – that there are no ex ante excess returns from holding deposits in one country relative to another. Those models have a prediction about the level of the exchange rate. The level of the exchange rate is important in international macroeconomics because it will help to determine demand for traded goods, especially when some nominal prices are sticky. These models predict that when a country has a higher than average relative interest rate, the price of foreign currency should be lower than average. This relationship is borne out in the data, but the strength of the home currency tends to be greater than is warranted by rational expectations of future short-term interest differentials as the models posit under interest parity – there is excess comovement or volatility. One way to rationalize this finding is to appeal to the influence of expected future risk premiums on the level of the exchange rate. That is, the country with the relatively high interest rate has the lower risk premium and hence the stronger currency. When a country's interest rate is high, its currency is appreciated not only because its deposits pay a higher interest rate but also because they are less risky.[1]

These two predictions about risk go in opposite directions: the high interest rate country has higher expected returns in the short run, but a stronger currency in levels. The former implies the high interest rate currency is riskier, the latter that it is less risky. That is the central puzzle of this paper. This study confirms these empirical regularities in a unified framework for the exchange rates of the G7 countries (Canada, France, Germany, Italy, Japan and the U.K.) relative to the U.S.

It is helpful to express this puzzle mathematically. Let ρ_{t+j+1} be the difference between the return between period $t+j$ and $t+j+1$ on a foreign short-term deposit and the home short-term deposit, inclusive of the return from currency appreciation. This study always takes the U.S. to be the home country. Let $r_t^* - r_t$ be the difference in the ex ante real (inflation adjusted) interest rate in the foreign country and the U.S. We use the * notation throughout to denote the foreign country.

The literature on interest parity has struggled to account for the robust empirical finding that $\text{cov}(E_t\rho_{t+1}, r_t^* - r_t) > 0$. Here, "cov" refers to the unconditional covariance, and $E_t\rho_{t+1}$ to the conditional expectation of ρ_{t+1}. The ex ante excess return on the foreign deposit is positively correlated with the foreign less U.S. interest differential. This is a correlation between two variables known at time *t*: the risk premium and the interest rate differential. It is not a correlation between two unexpected returns, which may be the source of a risk premium. Instead it is an unconditional correlation between two ex ante returns, suggesting that the factor(s) that drive time variation in the foreign exchange risk premium and the factor(s) that drive time variation in the interest rate differential have a common component. An analogy would be a finding that the risk premium on stocks is positively correlated with the short-term interest rate. Models with standard preferences in a setting of undistorted financial markets are unable to account for this empirical finding by appealing to a risk premium arising from foreign exchange fluctuations. The consumption variances and covariances that drives $E_t\rho_{t+1}$ in such models do not also lead to an interest differential that covaries positively with $E_t\rho_{t+1}$.[2]

Recent advances have found that the interest parity puzzle can be explained with the same formulations of non-standard preferences that have been used to account for other asset-pricing anomalies. These studies model the ex ante excess return as a risk premium related to the variances of consumption in the home and foreign country. Verdelhan (2010) builds on the model of external habits of Campbell and Cochrane (1999), and Colacito (2009), Colacito and Croce (2011, 2013) and Bansal and

[1] Hodrick (1989) and Obstfeld and Rogoff (2002) incorporate risk into macroeconomic models of the level of the exchange rate. The latter includes a role for risk in a micro-founded model similar to a Dornbusch sticky-price model.
[2] On this point, see for example Bekaert et al. (1997) and Backus et al. (2001). Also see the surveys of Engel (1996, 2014).

Shaliastovich (2007, 2013) develop the model of preferences in Epstein and Zin (1989) and Weil (1990) to account for this anomaly. Those studies show how the foreign exchange risk premium can be related to the difference in the conditional variance of consumption in the foreign country relative to the home country, in a setting of undistorted, complete financial markets. Lustig et. al. (2011) uses Epstein-Zin-Weil preferences to show how differential responses to a common component in the variances of home and foreign consumption can generate the empirical relationship. These papers are important not only to our understanding of the interest parity puzzle, but also to our understanding of asset pricing more generally because they show the power of a single model of preferences to account for a number of asset pricing regularities.

A different approach to explaining the interest parity puzzle advances an explanation akin to the model of rational inattention of Mankiw and Reis (2002) and Sims (2003). This explanation builds on a standard model of exchange rates such as Dornbusch (1976). A monetary contraction increases the interest rate and leads to an appreciation of the currency. However, some investors are slow to adjust their portfolios, perhaps because it is costly to monitor and gather information constantly. As more investors learn of the monetary contraction, they purchase home assets, leading to a further home appreciation. So when the home interest rate increases, the return on the home asset increases both from the higher interest rate and the currency appreciation. This model of portfolio dynamics was proposed informally by Froot and Thaler (1990) and called "delayed overshooting." Eichenbaum and Evans (1995) provide empirical evidence that is consistent with this hypothesis, and Bacchetta and van Wincoop (2010) develop a rigorous model.

In the data for currencies of major economies relative to the U.S., when $r_t^* - r_t$ is high (relative to its mean), the level of the foreign currency tends to be stronger (appreciated). Dornbusch (1976) and Frankel (1979) are the original papers to draw the link between real interest rates and the level of the exchange rate in the modern, asset-market approach to exchange rates. The connection has not gone unchallenged, principally because the persistence of exchange rates and interest differentials makes it difficult to establish their comovement with a high degree of uncertainty. For example, Meese and Rogoff (1988) and Edison and Pauls (1993) treat both series as non-stationary and conclude that evidence in favor of cointegration is weak. However, more recent work that examines the link between real interest rates and the exchange rate, such as Engel and West (2006), Alquist and Chinn (2008), and Mark (2009), has tended to reestablish evidence of the empirical link. Another approach connects surprise changes in interest rates to unexpected changes in the exchange rate. There appears to be a strong link of the exchange rate to news that alters the interest differential – see, for example, Faust et al. (2006), Andersen et. al. (2007) and Clarida and Waldman (2008).

It is widely recognized that exchange rates are excessively volatile relative to the predictions of monetary models that assume interest parity or no foreign exchange risk premium. Frankel and Meese (1987) and Rogoff (1996) are prominent papers that make this point. Evans (2011) refers to the “exchange-rate volatility puzzle” as one of six major empirical challenges in the study of exchange rates. Recent contributions that examine aspects of this excess volatility include Engel and West (2004), Bacchetta and van Wincoop (2006), and Evans (2012).

This excessive volatility in the level of the exchange rate arises (by definition) from the effect of deviations from uncovered interest parity on the level of the exchange rate. This effect is forward looking, and can be summarized in the variable $E_t\sum_{j=0}^{\infty}(\rho_{t+j+1}-\overline{\rho})$. We use the overbar notation, as in $\overline{x}$, to denote the unconditional mean of a variable x_t. When this sum of the ex ante risk premiums on foreign deposits increases, the home currency appreciates. The second empirical finding we focus on can be summarized as $\operatorname{cov}\left(E_t\sum_0^{\infty}\rho_{t+j+1}, r_t^*-r_t\right)<0$. That means that when $r_t^*-r_t$ is high (relative to its mean), the home currency is strong for two reasons: the influence of interest rates under uncovered interest parity (as in Dornbusch and Frankel) and the influence of deviations from uncovered interest parity.

It is clear from examining the two covariances that are at the heart of the empirical puzzle of this paper, it must be the case that while the interest parity puzzle has $\operatorname{cov}(E_t\rho_{t+1}, r_t^*-r_t)>0$, for some period in the future (that is, for some $j>0$), $\operatorname{cov}(E_t\rho_{t+j+1}, r_t^*-r_t)<0$, the reverse sign.

Neither modern models of the foreign exchange risk premium nor of delayed overshooting can account for the finding concerning the level of the exchange rate, that $\operatorname{cov}\left(E_t\sum_0^{\infty}\rho_{t+j+1}, r_t^*-r_t\right)<0$. We explain why these models are not capable of accounting for both puzzles. The very features that make them able to account for the interest parity puzzle work against explaining the level puzzle. As we show, both the models of the risk premium and of delayed overshooting imply a sort of muted adjustment in financial markets, which can account for the interest parity puzzle, but the excess comovement puzzle requires a sort of magnified adjustment.

We describe the features of a model that can reconcile the empirical findings. We suggest that there may be multiple factors that drive the relationship between interest rates and exchange rates. We embed a simple model of liquidity risk based on Nagel (2014) within a standard open-economy macroeconomic model. In that framework, an asset may earn a liquidity premium that increases as nominal interest rates rise, or as there are shocks to the financial system. Both the macroeconomic shocks (for example, to monetary policy) that drive interest rates as well as financial shocks to liquidity play a role in the exchange rate – interest rate nexus, and could potentially account for both empirical findings.

Section 1 develops the approach of this paper. Section 2 presents empirical results. Section 3 explains why the empirical findings constitute a puzzle. We discuss the difficulties encountered by asset pricing approaches such as representative agent models of the risk premium, and models of "delayed overshooting".[3] Then this section proposes the model of the liquidity premium that can potentially encompass both empirical findings.

The study of risk premiums in foreign exchange markets sheds light on important questions in asset pricing that go beyond the narrow interest of specialists in international asset markets. The foreign exchange rate is one of the few, if not the only, aggregate asset for an economy whose price is readily measurable, so its pricing offers an opportunity to investigate some key predictions of asset pricing theories. For example, in the absence of arbitrage, the rate of real depreciation of the home country's currency equals the log of the stochastic discount factor (*s.d.f.*) for foreign returns relative to the log of the corresponding *s.d.f.* for home returns, while the risk premium (as conventionally measured) is proportional to the conditional variance of the log of the *s.d.f.* for home relative to the variance of the *s.d.f.* for foreign returns.[4] Thus, the behavior of the foreign exchange rate may give direct evidence on the fundamental building blocks of equilibrium asset pricing models.

1. Excess Returns and Real Exchange Rates

We develop here a framework for examining behavior of ex ante excess returns and the level of the exchange rate. Our set-up will consider a home and a foreign country. In the empirical work of section 2, we always take the U.S. as the home country (as does the majority of the literature), and consider other major economies as the foreign country. Let i_{t+j} be the home one-period nominal interest for deposits in period $t+j$ that pay off in period $t+j+1$ and i^*_{t+j} is the corresponding foreign interest rate. s_t denotes the log of the foreign exchange rate, expressed as the U.S. dollar price of foreign currency. The excess return on the foreign deposit held from period $t+j$ to period $t+j+1$, inclusive of currency return is given by:

(1) $$\rho_{t+j+1} \equiv i^*_{t+j} + s_{t+j+1} - s_{t+j} - i_{t+j}\,.$$

This definition of excess returns corresponds with the definition in the literature. We can interpret this as a first-order log approximation of the excess return in home currency terms for a foreign security. As Engel (1996) notes, the first-order log approximation may not really be adequate for appreciating the implications of economic theories of the *expected* excess return. For example, if the

[3] "Representative agent models" may be an inadequate label for models of the risk premium that are developed off of the Euler equation of a representative agent under complete markets, generally taking the consumption stream as exogenous.
[4] The *s.d.f.*s for home and foreign returns are unique when asset markets are complete. See Backus et al. (2001), Brandt et al. (2006), and section 3.1 below.

exchange rate is conditionally log normally distributed, then $\ln\left(E_t(S_{t+1}/S_t)\right) = E_t s_{t+1} - s_t + \frac{1}{2}\text{var}_t(s_{t+1})$, where $\text{var}_t(s_{t+1})$ refers to the conditional variance of the log of the exchange rate and S_t is the level (not log) of the exchange rate. Engel (1996) points out that this second-order term is approximately the same order of magnitude as the risk premiums implied by some economic models. However, we proceed with analysis of excess returns defined according to equation (1) both because it is the object of almost all of the empirical analysis of excess returns in foreign exchange markets, and because the theoretical literature that we consider in section 3 seeks to explain expected excess returns defined precisely as $E_t\rho_{t+j+1}$.

The well-known uncovered interest parity puzzle comes from the empirical finding that the change in the log of the exchange rate is negatively correlated with the home less foreign interest differential, $i_t - i_t^*$. That is, estimates of $\text{cov}(s_{t+1} - s_t, i_t - i_t^*) = \text{cov}(E_t s_{t+1} - s_t, i_t - i_t^*)$ tend to be negative. As Engel (1996, 2014) surveys, this finding is consistent over time among pairs of high-income, low-inflation countries. From equation (1), we note that the relationship $\text{cov}(E_t s_{t+1} - s_t, i_t - i_t^*) < 0$ is equivalent to $0 < \text{var}(i_t - i_t^*) < \text{cov}(E_t\rho_{t+1}, i_t^* - i_t)$. That is, when the foreign interest rate is relatively high, so $i_t^* - i_t$ is above average, the excess return on foreign assets also tends to be above average. This is considered a puzzle because it has been very difficult to find plausible economic models that can account for this relationship.

While almost all of the empirical literature on the interest parity puzzle has documented evidence concerning $\text{cov}(E_t\rho_{t+1}, i_t^* - i_t)$, we recast the puzzle in terms of $\text{cov}(E_t\rho_{t+1}, r_t^* - r_t)$. r_t is the home (i.e., U.S.) ex ante real interest rate, defined as $r_t = i_t - E_t\pi_{t+1}$, where $\pi_{t+1} \equiv p_{t+1} - p_t$ and p_t denotes the log of the consumer price index in the home country. r_t^* is defined analogously. This is an approximation of the real interest rate. Analogous to the discussion above of the exchange rate, a different approximation would include a term for the variance of inflation. In essence, that variance is treated as a constant here.

We conduct empirical work using real interest rates for three reasons. First, the theoretical discussion of the interest parity puzzle usually builds models to explain $\text{cov}(E_t\rho_{t+1}, r_t^* - r_t)$, essentially assuming there is no inflation risk. Second, in high inflation countries, the evidence that $\text{cov}(E_t\rho_{t+1}, i_t^* - i_t) > 0$ is less robust – see Bansal and Dahlquist (2000) and Frankel and Poonawala (2010). The puzzle arises mostly among country pairs where inflation is low and stable. Third, the theoretical models of the level of the exchange rate, such as Dornbusch (1976) and Frankel (1979), relate the stationary component of the exchange rate to real interest differentials.

To measure the relation between the interest differential and the level of the exchange rate, begin by rearranging (1), subtracting off unconditional means, and iterating forward to get:

(2) $$s_t^T = s_t^{IP} - E_t \sum_{j=0}^{\infty} (\rho_{t+j+1} - \overline{\rho}) .$$

where $s_t^T \equiv s_t - \lim_{k\to\infty}(E_t s_{t+k} - k(\overline{s_{+1} - s}))$ and $s_t^{IP} \equiv E_t \sum_{j=0}^{\infty}\left(i_{t+j}^* - i_{t+j} - (\overline{i^* - i})\right)$. In deriving this expression, we have assumed that the interest differential and the ex ante excess return are stationary random variables.[5]

The $\lim_{k\to\infty}(E_t s_{t+k} - k(\overline{s_{+1} - s}))$ term is the permanent component of the exchange rate according to the decomposition of Beveridge and Nelson (1981). That is, assuming that the nominal exchange rate is stationary in first differences, the Beveridge-Nelson decomposition allows us to define a permanent component that follows a pure mean zero random walk, and a stationary or transitory component. Therefore, s_t^T is the transitory component. When we talk about the effect of risk on the level of the exchange rate, we refer to this component – the actual log of the exchange rate, normalized by its permanent component. If interest parity held, so that $E_t \rho_{t+j+1} = 0$ for all $j \geq 0$, the transitory component of the exchange rate is equal to the infinite sum of the expected nominal interest differentials, which we have denoted by s_t^{IP} (the *IP* superscript referring to *interest parity*.) The effect of ex ante excess returns on the level of the exchange rate is given in the term $E_t \sum_{j=0}^{\infty} (\rho_{t+j+1} - \overline{\rho})$.

The Dornbusch and Frankel models that assume interest parity imply $\text{cov}\left(s_t^{IP}, r_t^* - r_t\right) > 0$. We show empirically that $\text{cov}\left(E_t \sum_0^{\infty} \rho_{t+j+1}, r_t^* - r_t\right) < 0$. From (2), it follows that there is excess comovement in the level of the stationary component of the exchange rate: $\text{cov}\left(s_t^T, r_t^* - r_t\right) > \text{cov}\left(s_t^{IP}, r_t^* - r_t\right)$. That is, $\text{cov}\left(s_t^T, r_t^* - r_t\right) - \text{cov}\left(s_t^{IP}, r_t^* - r_t\right) = -\text{cov}\left(E_t \sum_0^{\infty} \rho_{t+j+1}, r_t^* - r_t\right) > 0$.

Some more insight into equation (2) can be gleaned by looking at the relationship of exchange rates to consumer price levels. The real exchange rate is given by $q_t \equiv s_t + p_t^* - p_t$. Assume for simplicity there is no drift in the real exchange rate. We can rewrite (2) by adding and subtracting prices appropriately:

(3) $$q_t - \lim_{k\to\infty}(E_t q_{t+k}) = E_t \sum_{j=0}^{\infty} r_{t+j}^* - r_{t+j} - (\overline{r^* - r}) - E_t \sum_{j=0}^{\infty} (\rho_{t+j+1} - \overline{\rho}),$$

When purchasing power parity holds in the long run, so the real exchange rate is stationary as we will find for our data, $\lim_{k\to\infty}(E_t q_{t+k})$ is simply the unconditional mean of the real exchange rate. Then equation (3) says the real exchange rate is above its mean either when the sum of current and future expected real

[5] Specifically, these variables are square summable, so that the sums on the right side of the equation exist.

interest differentials (foreign less home) are above average, or when the sum of expected current and future excess returns are above average. In the Dornbusch model, an increase in the current real interest rate influences the level of the real exchange rate through the term involving current and expected future real interest rate differentials: $E_t \sum_{j=0}^{\infty} r_{t+j}^* - r_{t+j} - (\overline{r^* - r})$. Our empirical finding that $\text{cov}\left(E_t \sum_0^\infty \rho_{t+j+1}, r_t^* - r_t\right) < 0$ implies that there is excessive volatility in the real exchange rate level.

It is important to note that $E_t \sum_{j=0}^{\infty} i_t^* - i_t - (\overline{i^* - i})$ is not the interest differential on long-term bonds, even hypothetical infinite-horizon bonds. It is the difference between the expected return from holding an infinite sequence of short-term foreign bonds and the expected return from the infinite sequence of short-term home bonds. An investment that involves rolling over short term assets has different risk characteristics than holding a long-term asset, which might include a holding-period risk premium.[6]

In the next section, we present evidence that $\text{cov}(E_t \rho_{t+1}, r_t^* - r_t) > 0$ and $\text{cov}\left(E_t \sum_0^\infty \rho_{t+j+1}, r_t^* - r_t\right) < 0$. The short-run ex ante excess return on the foreign security, $E_t \rho_{t+1}$, is negatively correlated with the real interest differential, consistent with the many empirical papers on the uncovered interest parity puzzle. But the sum of current and expected future returns is positively correlated.

The empirical approach of this paper can be described simply. We estimate vector error-correction models (VECMs) in the variables s_t, $i_t - i_t^*$, and $p_t - p_t^*$. From the VECM estimates, we construct measures of $E_t\left(i_t^* - i_t - (\pi_{t+1}^* - \pi_{t+1})\right) = r_t^* - r_t$. Using standard projection formulas, we can also construct estimates of s_t^T and s_t^{IP}. To measure $E_t \sum_{j=0}^{\infty} (\rho_{t+j+1} - \bar{\rho})$, we take the difference of s_t^T and s_t^{IP}. From these ECM estimates, we calculate our estimates of the covariances just discussed.[7] Our approach of estimating undiscounted expected present values of interest rates is presaged in Campbell and Clarida (1987), Clarida and Gali (1994) and more recently in Froot and Ramadorai (2005), Mark (2009) and Brunnermeier et. al. (2009).[8] [9]

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[6] Chinn and Meredith (2004) find that uncovered interest parity holds relatively well for long-term bonds. Under some further assumptions (e.g., that the expectations hypothesis of the term structure holds and the interest rate differential is a first-order Markov process), this would imply the reversal in the sign of the covariance we highlight.
[7] We also consider VECMs that are augmented with data on stock market returns, oil prices, gold prices and long-term interest rates, which are included solely for the purpose of improving the forecasts of future interest rates and inflation rates.
[8] This method does not require estimation of expected long-term real interest rates, about which there is some controversy. See Bansal, et al. (2012).

2. Empirical Results

We investigate the behavior of exchange rates and interest rates for the U.S. relative to the other six countries of the G7: Canada, France, Germany, Italy, Japan, and the U.K. We also consider the behavior of U.S. variables relative to an aggregate weighted average of the variables from these six countries, with weights measured as the value of each country's exports and imports as a fraction of the average value of trade over the six countries. This set of seven countries are particularly interesting for examining these exchange rate puzzles because these countries have had floating exchange rates among themselves since the early 1970s, little foreign exchange intervention in the market for each currency relative to the dollar, very low capital controls, very little default risk, low inflation (especially for each country relative to the U.S.), and very deep markets in foreign exchange. These facts narrow the possible explanations for the puzzles.

Our study uses monthly data. Foreign exchange rates are noon buying rates in New York, on the last trading day of each month, culled from the daily data reported in the Federal Reserve historical database. The price levels are consumer price indexes from the Main Economic Indicators on the OECD database. Nominal interest rates are taken from the last trading day of the month, and are the midpoint of bid and offer rates for one-month Eurorates, as reported on Intercapital from Datastream. The interest rate data begin in June 1979, and the empirical work uses the time period June 1979 to October 2009. The choice of an end date of October 2009 represents a compromise. On the one hand, it is important for our purposes to include these data well into 2009 because it has been noted in some recent papers that there was a crash in the "carry trade" in 2008, so it would perhaps bias our findings if our sample ended prior to this crash.[10] On the other hand, it seems plausible that there were some changes in the driving processes for interest rates and exchange rates during the turbulent period from late 2008 until early 2013 because of the global financial crisis and the European debt crisis.

2.1 Fama regressions

The "Fama regression" (see Fama, 1984) is the basis for the forward premium puzzle. It is usually reported as a regression of the change in the log of the exchange rate between time t+1 and t on the time t interest differential:

$$s_{t+1} - s_t = \zeta_s + \tilde{\beta}_s (i_t - i_t^*) + u_{s,t+1}$$

Under uncovered interest parity, $\zeta_s = 0$ and $\tilde{\beta}_s = 1$. We can rewrite this regression as:

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[9] See the appendix for a detailed discussion of the relation of the empirical work in this paper to Froot and Ramadorai (2005).

[10] See, for example, Brunnermeier, et al. (2009) and Jordà and Taylor (2012).

(4) $\rho_{t+1} = \zeta_s + \beta_s(i_t^* - i_t) + u_{s,t+1}$,

where $\beta_s \equiv 1 - \tilde{\beta}_s$. The left-hand side of the regression is the ex post excess return on the foreign security. If $\beta_s > 0$, there is a positive correlation between the excess return on the foreign currency and the foreign-home interest differential.

We estimate the Fama regression for our currencies as a preliminary exercise. Table 1 reports the 90% confidence interval for the regression coefficients from (4), based on Newey-West standard errors. For all of the currencies, the point estimates of β_s are positive. The 90% confidence interval for β_s lies above zero for four (Italy and France being the exceptions, where the confidence interval for the latter barely includes zero.) For four of the six, zero is inside the 90% confidence interval for the intercept term, ζ_s. (In the case of the U.K., the confidence interval barely excludes zero, while for Japan we find strong evidence that ζ_s is greater than zero.)

The G6 exchange rate (the weighted average exchange rate, defined in the data section) appears to be less noisy than the individual exchange rates. In all of our tests, the standard errors of the coefficient estimates are smaller for the G6 exchange rate than for the individual country exchange rates, suggesting that some idiosyncratic movements in country exchange rates get smoothed out when we look at averages. The weights in the G6 exchange rate are constant. We can think of this exchange rate as the dollar price of a fixed basket of currencies, and can interpret our tests as examining the properties of expected returns on this asset. Our discussion focuses on the returns on this asset because its returns appear to be more predictable than for the individual currencies. Examining the behavior of the returns on the weighted portfolio is a more appealing way of aggregating the data and reducing the effects of the idiosyncratic noise in the country data than estimating the Fama regression as a panel using all six exchange rates. There is not a good theoretical reason to believe that the coefficients in the Fama regression are the same across currencies, and the gains from panel methods are likely to be small from a panel that would impose no restrictions across the equations on the coefficients. Table 1 reports that the 90% confidence interval for this exchange rate lies well above zero, with a point estimate of 2.467. The intercept coefficient, on the other hand is very near zero, and the 90% confidence interval easily contains zero.

2.2 Vector error correction model

A building block in our estimates of the central statistics of this study, $\text{cov}(E_t\rho_{t+1}, r_t^* - r_t)$ and $\text{cov}\left(E_t\sum_0^\infty \rho_{t+j+1}, r_t^* - r_t\right)$, is a vector error correction model from which we extract measures of expected inflation (in order to construct the short-term real interest rate differential, $r_t^* - r_t$), and the ex ante excess returns for future periods ($E_t \rho_{t+j}$).

Let

(5) $$x_t = \begin{bmatrix} s_t \\ p_t - p_t^* \\ i - i_t^* \end{bmatrix},$$

where s_t is the log of the dollar price of foreign currency, $p_t - p_t^*$ is the log of the US consumer price level relative to the foreign price level, and $i_t - i_t^*$ is the US interest rate less the foreign interest rate expressed as monthly returns.

We estimate:

(6) $$x_t - x_{t-1} = C_0 + Gx_{t-1} + C_1(x_{t-1} - x_{t-2}) + C_2(x_{t-2} - x_{t-3}) + C_3(x_{t-3} - x_{t-4}) + u_t$$

While the *C* matrices are unrestricted, the *G* matrix takes the form:

$$G = \begin{bmatrix} g_{11} & -g_{11} & g_{13} \\ g_{21} & -g_{21} & g_{23} \\ g_{31} & -g_{31} & g_{33} \end{bmatrix}.$$

This form implies that if there is cointegration, the nominal exchange rate and relative inflation rate adjust to misalignments in the real exchange rate. The estimated coefficients of equation (6) are reported in Appendix table A.1. This system can be estimated efficiently using equation-by-equation ordinary least squares. We take as uncontroversial the presumption that the nominal interest rate differential is stationary. For all the currency pairs, the point estimate of g_{33} is large (in absolute value) and negative, consistent with stationarity.

A large literature has been devoted to the question of whether the nominal exchange rate and the nominal price differential are cointegrated, or whether the real exchange rate is stationary. Three recent studies of uncovered interest parity, Froot and Ramadorai (2005), Mark (2009) and Brunnermeier, et al. (2009) estimate statistical models that assume the real exchange rate is stationary, but do not test for a unit root. Jordà and Taylor (2012) demonstrate that there is a profitable carry-trade strategy that exploits the uncovered interest parity puzzle when the trading rule is enhanced by including a forecast that the real exchange rate will return to its long-run level when its deviations from the mean are large.

We find fairly strong evidence of mean reversion in the real exchange rate in this sample. The strength of this evidence may arise from using the interest rate differential in the VECM as a covariate, which previous studies have not included. We construct bootstrap distributions of the estimates of g_{11}, g_{12}, and $g_{11} - g_{21}$. Table 2 reports these distributions and the estimated coefficients. Subtracting the second equation from the first, the estimate of $g_{11} - g_{21}$ measures the monthly mean reversion in the real exchange rate. We find the estimated difference is significant at the 5 percent level for three of the currencies, and at the 10 percent level for two more currency pairs. A sixth case, Italy, is nearly significant at the ten percent level. Only the Canadian dollar shows no clear signs of cointegration. The estimates of $g_{11} - g_{21}$ range from approximately 0.02 (for the Canadian dollar) to 0.04 (for the U.K. pound), implying a strong tendency for the real exchange rate to mean revert. The estimate of $g_{11} - g_{21}$ for the G6 average currency is approximately 0.03, and significant at the 5 percent level.

The general formulation of the VECM in (6) allows us to construct a measure of the permanent component of the nominal exchange rate (as required in the calculation given in equation (2)) whether or not the nominal exchange rate and relative nominal prices are cointegrated. Given the evidence presented in Table 2, we proceed under the assumption of cointegration so that there is a permanent common component to the nominal exchange rate and the price differential.

Below, we primarily report results from a VECM with three lags. We have calculated the Bayes Information Criterion for the optimal lag length in the VECM, and found that for all currency pairs, the optimal lag length is 1. However, we stick with the longer VECM because it seems possible that there are important dynamic relationships that are not captured in a model with a single lag. In fact, below we also report results of a VECM with 12 lags. We also report results from VECMs that include the dollar price of oil and gold, relative (foreign to U.S.) long-term interest rates and relative stock returns. These variables are included because they are forward looking and may improve forecasts of ρ_{t+j}. However, in practice, there is a penalty for using larger VECMs (i.e., more lags and more variables) because many more parameters are estimated. We find that while the pattern of our estimates of $\text{cov}(E_t \rho_{t+j}, r_t^* - r_t)$ are the same across all of the VECMs, the standard errors of some of the estimates of that covariance increase as the number of coefficients estimated increase.

2.3 Fama regressions in real terms

A counterpart to equation (4) in real terms can be written as:

(7) $$q_{t+1} - q_t + \hat{r}_t^* - \hat{r}_t = \zeta_q + \beta_q \left(\hat{r}_t^* - \hat{r}_t \right) + u_{q,t+1}.$$

We use a ^ over the real interest rate variables to emphasize that these variables are estimated from our VECM. The dependent variable in this regression is equal to $\rho_{t+1} + u_{p,t+1}$, where $u_{p,t+1}$ is the relative inflation forecast error, $u_{p,t+1} \equiv p_{t+1}^* - p_t^* - \left(p_{t+1} - p_t \right) - \hat{E}_t \left(p_{t+1}^* - p_t^* - \left(p_{t+1} - p_t \right) \right)$, where we use the notation $\hat{E}_t$ to designate our estimate of the expected inflation differential. Much of the theoretical literature on the foreign exchange risk premium builds models that interpret the Fama regression as one in which the dependent and independent variables are defined in real terms as in (7), and assume no inflation risk.

There are two senses in which our measures of $\hat{r}_t^* - \hat{r}_t$ are estimates. The first is that the parameters of the VECM are estimated. But even if the parameters were known with certainty, we would still only have estimates of $\hat{r}_t^* - \hat{r}_t$ because we are basing our measures of $\hat{r}_t^* - \hat{r}_t$ on linear projections. Agents certainly have more sophisticated methods of calculating expectations, and use more information than is contained in our VECM.

The findings for regression (7) in real terms are similar to those when the regression is estimated on nominal variables. For all currencies, the estimates of β_q, reported in Table 3, are positive, which implies that the high real interest rate currency tends to have high real expected excess returns. The estimated coefficient for the G6 aggregate is close to 2.

Table 3 and all of the subsequent tables report the Newey-West standard errors from regression (7), and also report two sets of confidence intervals for each parameter estimate based on bootstraps.. The Newey-West standard errors ignore the fact that $\hat{r}_t^d$ is a generated regressor. The first bootstrap uses percentile intervals and the second percentile-t intervals.[11]

The 95 percent confidence interval for β_q lies above zero for Germany, Japan, and the U.K. The 90 percent confidence interval contains zero for Canada and Italy. The 95 percent confidence interval contains zero for France, but the 90 percent confidence interval lies above zero for the first bootstrap. These findings are similar to those from the Fama regressions in nominal returns (reported in Table 1), with the exception of the Canadian dollar which was significantly positive in the nominal regression.

The fact that all six of the estimates of β_q for the separate countries are positive conveys more information than the individual tests of significance. A test of the joint null that all β_q are less than or equal to zero can be rejected at a confidence level greater than 99.9%, using a bootstrapped statistic based on the joint distribution of the residuals from the regressions in Table 3.

The findings are clear using the G6 average exchange rate: the coefficient estimate is 1.98 and both confidence intervals lie above zero. The estimate of ζ_q is very close to zero, and the confidence intervals contain zero.

In summary, the evidence on the interest parity puzzle is similar in real terms as in nominal terms. The point estimates of the coefficient β_q are positive and tend to be statistically significantly greater than

[11] See Hansen (2010). The Appendix describes the bootstraps in more detail.

zero, $\mathrm{cov}(E_t \rho_{t+1}, r_t^* - r_t) > 0$. Even in real terms, the country with the higher interest rate tends to have short-run excess returns (i.e., excess returns and the interest rate differential are positively correlated.)

2.4 The real exchange rate, real interest rates, and the level risk premium

Table 4 reports estimates from

(8) $$q_t = \zeta_Q + \beta_Q \left(\hat{r}_t^* - \hat{r}_t\right) + u_{Q,t}.$$

In all cases, the coefficient estimate is positive. In almost all cases, although the confidence intervals are wide, the coefficient is significantly positive. This regression confirms the well-known relationship that the U.S. dollar tends to be stronger when the U.S. real interest rate is relatively high. Our chief interest is not this relationship, but whether the real exchange rate responds more or less to the real interest differential than predicted by uncovered interest parity. That is, we are interested in the sign of $\mathrm{cov}(E_t \sum_{j=0}^{\infty} \rho_{t+j+1}, r_t^* - r_t)$.

Our central empirical finding is reported in Table 5. This table reports the regression:

(9) $$\hat{E}_t \sum_{j=0}^{\infty} (\rho_{t+j+1} - \bar{\rho}) = \zeta_\rho + \beta_\rho \left(\hat{r}_t^* - \hat{r}_t\right) + u_{\rho t}.$$

$\hat{E}_t \sum_{j=0}^{\infty} (\rho_{t+j+1} - \bar{\rho})$ is calculated as the difference between our measure of the transitory component of the exchange rate, s_t^T , and the estimated sum of current and expected future interest differentials, s_t^{IP} , following equation (2).

For all of the currency pairs, the estimated slope coefficient is negative, implying $\mathrm{cov}\left(E_t \sum_0^{\infty} \rho_{t+j+1}, r_t^* - r_t\right) < 0$. The 90 percent confidence intervals are wide, but with only a few exceptions, lie below zero. We can reject the joint null that this covariance is greater than or equal to zero for all six currencies at a confidence level greater than 99.9 percent.

The confidence interval for the G6 average strongly excludes zero. To get an idea of magnitudes, a one percentage point difference in annual rates between the foreign and home real interest rates equals a 1/12th percentage point difference in monthly rates. The coefficient of around -31 reported for the regression when we take the U.S. relative to the average of the other G7 countries then translates into around a 2.6% effect on $\hat{E}_t \sum_{j=0}^{\infty} (\rho_{t+j+1} - \bar{\rho})$ of a one percentage point change in the interest rate differential. From Table 4, we can see that if the U.S. real rate increases one annualized percentage point above the real rate in the other countries, the dollar is predicted to be 3.6% (43.7/12) stronger. Of that 3.6%, 1.0% is the amount attributable to the higher real interest differential under uncovered interest parity (as in the Dornbusch (1976) model), and the remaining 2.6% represents the effect of the cumulated expected excess return on dollar deposits on the exchange rate.

This finding that $\text{cov}\left(E_t \sum_0^\infty \rho_{t+j+1}, r_t^* - r_t\right) < 0$ is surprising in light of the well-known uncovered interest parity puzzle. We have documented that when $r_t^* - r_t$ is above average, foreign deposits tend to have expected excess returns relative to U.S. deposits. That seems to imply that the high interest rate currency is the riskier currency. But the estimates from equation (9) deliver the opposite message – the high interest rate currency has the lower level risk premium. Since we have found $\text{cov}(E_t \rho_{t+1}, r_t^* - r_t) > 0$ and $\text{cov}\left(E_t \sum_0^\infty \rho_{t+j+1}, r_t^* - r_t\right) < 0$, we must have $\text{cov}\left(E_t \rho_{t+j}, r_t^* - r_t\right) < 0$ for at least some $j > 0$. That is, we must have a reversal in the correlation of the expected one-period excess returns with $r_t^* - r_t$ as the horizon extends.

This is illustrated in Figure 1, which plots estimates of the slope coefficient in a regression of $\hat{E}_t(\rho_{t+j})$ on $\hat{r}_t^* - \hat{r}_t$ for $j = 1, \ldots, 120$:

$$\hat{E}_t(\rho_{t+j}) = \zeta_j + \beta_j \left(\hat{r}_t^* - \hat{r}_t\right) + u_t^j$$

For the first few *j*, this coefficient is positive, but it eventually turns negative at longer horizons.

To summarize, when the U.S. real interest rate relative to the foreign real interest rate is higher than average, the transitory component of the dollar is stronger than average. Crucially, it is even stronger than would be predicted by a model of uncovered interest parity. Ex ante excess returns or the foreign exchange risk premium contribute to this strength. This implies that the expected sum of future excess returns on the foreign asset must increase when the U.S. real interest rate rises, which is a reversal of the correlation between the interest differential and expected returns in the short run.

Figure 2 shows that a similar pattern occurs when we estimate

$$\hat{E}_t(\rho_{t+j}) = \zeta_j + \beta_j \left(i_t^* - i_t\right) + u_t^j .$$

The usual empirical work on the interest parity puzzle, as in Table 1, reports on the correlation of the excess return with the nominal interest differential. These findings imply $\text{cov}\left(E_t \rho_{t+1}, i_t^* - i_t\right) > 0$. In Figure 2, we see nonetheless that $\text{cov}\left(E_t \rho_{t+j}, i_t^* - i_t\right) < 0$ as the horizon extends out to greater than 20 months. The figure plots results for the G6 average exchange rate, but the pattern is similar for the individual bilateral exchange rates. The evidence is less strong that $\text{cov}\left(E_t \sum_0^\infty \rho_{t+j+1}, i_t^* - i_t\right) < 0$ than it is for the analogous relationship involving real interest rate differentials.

In Figure 3, for the G6 currency, we plot the slope coefficient estimates from the regression

$$\rho_{t+j} = \zeta_j + \beta_j \left(\hat{r}_t^* - \hat{r}_t \right) + u_t^j .$$

That is, the dependent variable in the regression is actual ex post excess returns on the foreign deposit, rather than the measure of ex ante excess returns calculated on forecasts from the VECM. The same pattern emerges as in the previous plots. For small values of *j*, we find $\text{cov}\left(\rho_{t+j}, \hat{r}_t^* - \hat{r}_t\right) > 0$, but as the horizon increases, the sign of the covariance reverses. The pattern of coefficient estimates is not as smooth because the regressions use ex post rather than ex ante returns, but the reversal of sign is unmistakable. It is not possible, of course, to calculate $\text{cov}\left(\sum_0^\infty \rho_{t+j+1}, r_t^* - r_t\right)$, because there are only a finite number of realizations of ex post returns in our sample. Although we could calculate a truncated sum, the disadvantages of this procedure are well known from the literature. This problem partly motivated the development of the Campbell and Shiller (1987, 1988) technique, which is closely related to our approach.

Figure 4 is analogous to Figure 3, except that the regressor is the nominal interest rate differential. Figure 4 plots the slope coefficients from the regression

$$\rho_{t+j} = \zeta_j + \beta_j \left(i_t^* - i_t \right) + u_t^j .$$

Again we can see the pattern of initially positive slope estimates, and then a reversal of sign. These regressions are notable because they do not rely on our VECM analysis at all. The initial slope coefficient estimated ($j = 1$) is exactly the estimate reported for the G6 average exchange rate in Table 1. Valchev (2014) finds very similar results for a panel regression of excess returns for 18 countries against the U.S. dollar, using data spanning 1976-2013, imposing the same slope coefficient across currencies.

We consider two extensions of the empirical analysis to see if augmenting the simple VECM estimated here can sharpen the forecasts of future short-term real interest rates. The results reported so far are from a VECM with three lags, using monthly data. We estimated the model using 12 lags, which might capture longer run dynamics in the monthly data.

The second extension added four variables to the VECM for each country. We include a stock price index and a measure of long-term nominal government bond yields. The long-term bond yields are from the IMF's International Financial Statistics, "interest rates, government securities and government bonds." The stock price indexes are monthly, from Datastream.[12] The yields and stock prices are taken relative to the corresponding U.S. variable. We also include data on the dollar price of oil and the dollar

[12] The Datastream codes are TOTMKCN(PI), TOTMKFR(PI), TOTMKIT(PI), TOTMKUK(PI), TOTMKBD(PI), TOTMKJP(PI), and TOTMKUS(PI)

price of gold.[13] As we have noted above, our VECM estimates of expected inflation and expected future interest rates are estimates both because the coefficients of the VECM must be estimated, but also because the simple VECM does not include all of the information and news the market uses to forecast future inflation and interest rates. The point of including these four variables is that they are asset prices that respond quickly to news about the future state of the economy. For example, the gold price is believed to be sensitive to news about U.S. monetary policy. Oil prices are thought to react to expectations of global economic growth, which in turn may influence expectations of inflation and interest rates. The stock prices and long-term bond rates from each economy may contain information about local monetary policy and economic growth prospects.

We estimated the same parameters as reported in Tables 2, 3, and 4 for each of these models. The point estimates for the augmented models were very similar to those reported for the baseline model. The confidence intervals for the slope coefficients for the regressions reported in the tables based on expectations generated from the VECM with 12 lags were wider than for the VECM with 3 lags. These wider confidence intervals might reflect the greater imprecision in coefficient estimates in the VECM with 12 lags. The more parsimonious specification has far fewer coefficients to estimate. On the other hand, the findings when the additional informational variables are included in the VECM are not very different than those reported in the tables. That is, not only the signs but the statistical significance of the coefficient estimates (based on Newey-West statistics and on both bootstraps) are similar.

Simlarly to Figure 1, Figure 5 plots the slope coefficient estimates for the regression

$$\hat{E}_t(\rho_{t+j}) = \zeta_j + \beta_j\left(\hat{r}_t^* - \hat{r}_t\right) + u_t^j$$

for the 12-lag VECM. Figure 6 plots these slope coefficients for the VECM augmented with information variables. The overall message is the same as in the previous specifications.

We turn now to the implications of these empirical findings for models of the foreign exchange risk premium.

3. The Puzzle

We have found that there is excess comovement of the level of the exchange rate and the interest differential, in the sense that the covariance of the stationary component of the exchange rate with the foreign less U.S. interest rate is more negative than would hold under interest parity: $\text{cov}\left(E_t\sum_0^\infty \rho_{t+j+1}, r_t^* - r_t\right) < 0$. This finding of excess comovement is not surprising in itself, and corresponds to previous findings of excess volatility in the literature. The difficulty resides in reconciling

[13] These data are from the Federal Reserve Bank of St. Louis database. The oil price is the spot price of West Texas Intermediate crude oil, and the gold price is the Gold Fixing Price in the London Bullion Market.

this finding with the well-known interest parity "puzzle" that finds $\text{cov}(E_t\rho_{t+1}, r_t^* - r_t) > 0$. A complete theory of exchange rate and interest rate behavior needs to explain not only the interest parity puzzle, but also why $\text{cov}(E_t\rho_{t+j}, r_t^* - r_t) < 0$ for some $j > 1$.

Recent work has made progress in developing economic models to account for the uncovered interest parity puzzle. The models we review below – one set based on risk averse behavior of investors, the other on rational inattention – rely on a curtailed adjustment by markets to a change in interest rates to explain the interest parity puzzle. Suppose some shock raises $r_t^* - r_t$. In one set of models, the shock also increases the riskiness of foreign assets for home investors relative to the riskiness of home assets for foreign investors. Investors' desire to increase investment in the foreign deposits because of the increase in $r_t^* - r_t$ is muted by the increase in foreign exchange risk, which implies an increase in the risk premium on the foreign deposits. In the other set of models, there is initially partial adjustment by investors, not based on risk aversion, but by slow reaction to news of the increase in $r_t^* - r_t$. Some investors do not adjust their portfolios immediately, which then generates higher expected returns on the foreign deposits in the short run before all investors rebalance their portfolios.

These models do not allow for a channel of amplified adjustment, by which the high interest rate currency is considered more desirable by investors, leading to a lower expected return on that currency. That is, there is no channel by which $\text{cov}(E_t\rho_{t+j}, r_t^* - r_t) < 0$ for any *j*.

3.1 Models of the foreign exchange risk premium under complete markets

Almost since the initial discovery of the interest-parity puzzle, there have been attempts to account for the behavior of expected returns in foreign exchange markets without relying on any market imperfections, such as market incompleteness or deviations from rational expectations.[14] The literature has built models of risk premiums based on risk aversion of a representative agent. Those models formulate preferences in order to generate volatile risk premiums which are important not only for understanding the uncovered interest parity puzzle, but also a number of other puzzles in asset pricing regarding returns on equities and the term structure. See, for example, Bansal and Yaron (2004).

Here we briefly review the basic theory of foreign exchange risk premiums in complete-markets models and relate the factors driving the risk premium to the state variables driving stochastic discount factors. See, for example, Backus et al. (2001) or Brandt et al. (2006).

When markets are complete, there is a unique stochastic discount factor, M_{t+1} that prices returns denominated in units of the home consumption basket. The returns on any asset *j* denominated in units of

[14] See Engel (1996, 2014) for surveys of the theoretical literature.

home consumption satisfy $1 = E({}_t M_{t+1} e^{r_{j,t+1}})$ for all *j*. Likewise, there is a unique *s.d.f.*, M^*_{t+1} that prices returns expressed in units of the foreign consumption basket. As Backus et. al. (2001) show, when the *s.d.f.* and returns are log-normally distributed (or, as an approximation), we can write:

(10) $$E_t \rho_{t+1} = \tfrac{1}{2}(\text{var}_t\, m_{t+1} - \text{var}_t\, m^*_{t+1})$$

(11) $$r^*_t - r_t = E_t(m_{t+1} - m^*_{t+1}) + \tfrac{1}{2}(\text{var}_t(m_{t+1}) - \text{var}_t(m^*_{t+1}))$$

The lower case variables, m_{t+1} and m^*_{t+1} are the logs of M_{t+1} and M^*_{t+1}, respectively.

We focus attention on the "long-run risks" model of Bansal and Yaron (2004), based on Epstein-Zin (1989) preferences. Colacito and Croce (2011) have recently applied the model to understand several properties of equity returns, real exchange rates and consumption. Bansal and Shaliastovich (2007, 2012), Lustig and Verdelhan (2007), Colacito (2009), and Backus et al. (2010) demonstrate how the "long-run risks" model can account for the interest-parity anomaly. Colacito and Croce (2013) build a general equilibrium two-good, two-good endowment economy in which agents in both countries have Epstein-Zin preferences, under both complete markets and portfolio autarky, and are able to account for the interest-parity puzzle as well as other asset-pricing anomalies.

These papers directly extend equilibrium closed-economy models to a two-country open-economy setting. The closed economy models assume an exogenous stream of endowments, with consumption equal to the endowment. The open-economy versions assume an exogenous stream of consumption in each country. These could be interpreted either as partial equilibrium models, with consumption given but the relation between consumption and world output unmodeled. Or they could be interpreted as general equilibrium models in which each country consumes an exogenous stream of its own endowment and there is no trade between countries.[15] Under the latter interpretation, the real exchange rate is a shadow price, since in the absence of any trade in goods, there can be no trade in assets that have any real payoff.

In each country, households are assumed to have Epstein-Zin preferences. The home agent's utility is defined by the recursive relationship:

(12) $$U_t = \left\{(1-\beta)C_t^\rho + \beta\left[E_t\left(U_{t+1}^\alpha\right)^{\rho/\alpha}\right]\right\}^{1/\rho}.$$

In this relationship, β measures the patience of the consumer, $1-\alpha > 0$ is the degree of relative risk aversion, and $1/(1-\rho) > 0$ is the intertemporal elasticity of substitution. Assume, as in Bansal and Yaron (2004) that $\alpha < \rho$, which corresponds to the case in which agents prefer an early resolution of risk, and $0 < \rho < 1$, so the intertemporal elasticity of substitution is greater than one.

[15] Colacito and Croce's (2013) two-good model does allow for trade in goods.

We will consider a somewhat more general version of the long-run risks model than is present in the literature, but one that nests several models. We present only a version in which real interest rates are determined, but discuss extensions to the nominal interest rate.

Assume an exogenous path for consumption in each country. In the home country (with $c_t \equiv \ln(C_t)$):

(13) $$c_{t+1} - c_t = \mu + l_t + \sqrt{u_t^h + u_t^c}\,\varepsilon_{t+1}^x .$$

The conditional expectation of consumption growth is given by $\mu + l_t$. The component l_t represents a persistent consumption growth component modeled as a first-order autoregression:

(14) $$l_{t+1} = \varphi_l l_t + \sqrt{w_t^h + w_t^c}\,\varepsilon_{t+1}^l .$$

The innovations, ε_{t+1}^x and ε_{t+1}^l are assumed to be uncorrelated within each country, distributed $i.i.d.\,N(0,1)$, but each shock may be correlated with its foreign counterpart (ε_{t+1}^{*x} and ε_{t+1}^{*l}, which are mutually uncorrelated.)

In the foreign country, we have:

(15) $$c_{t+1}^* - c_t^* = \mu^* + l_t^* + \sqrt{u_t^f + u_t^c}\,\varepsilon_{t+1}^{*x}$$

(16) $$l_{t+1}^* = \varphi_l^* l_{t+1}^* + \sqrt{w_t^f + w_t^c}\,\varepsilon_{t+1}^{*l}$$

The conditional variances are written as the sum of two independent components. The component with the *h* superscript is idiosyncratic to the home country. An *f* superscript refers to the foreign idiosyncratic component. The one with the *c* superscript is common to the home and foreign country. Conditional variances are stochastic and follow first-order autoregressive processes:

(17) $$u_{t+1}^i = (1-\varphi_u)\theta_u^i + \varphi_u u_t^i + \sigma_u \varepsilon_{t+1}^{iu}, \quad i = h, f, c$$

(18) $$w_{t+1}^i = (1-\varphi_w)\theta_w^i + \varphi_w w_t^i + \sigma_w \varepsilon_{t+1}^{iw}. \qquad i = h, f, c$$

The innovations, ε_{t+1}^{iu} and ε_{t+1}^{iw}, $i = h, f, c$ are assumed to be uncorrelated, distributed *i.i.d.* with mean zero and unit variance.

We can log linearize the first-order conditions as in Hansen et al. (2008). We will ignore terms that are not time-varying or that do not affect both the conditional means and variances of the stochastic discount factors, lumping those variables into the catchall terms Ξ_t and Ξ_t^*.

The home discount factor is given by:

(19) $$-m_{t+1} = \gamma_u^r(u_t^h + u_t^c) + \gamma_w^r(w_t^h + w_t^c) + \lambda_u^r\sqrt{u_t^h + u_t^c}\,\varepsilon_{t+1}^x + \lambda_w^r\sqrt{w_t^h + w_t^c}\,\varepsilon_{t+1}^l + \Xi_t .$$

The foreign discount factor is given by:

(20) $$-m_{t+1}^* = \gamma_u^{*r}(u_t^f + u_t^c) + \gamma_w^{*r}(w_t^f + w_t^c) + \lambda_u^{*r}\sqrt{u_t^f + u_t^c}\,\varepsilon_{t+1}^{*x} + \lambda_w^{*r}\sqrt{w_t^f + w_t^c}\,\varepsilon_{t+1}^{*l} + \Xi_t^* .$$

The parameters in these log-linearization are:

$$\gamma_u^r = \alpha(\alpha-\rho)/2 \qquad \gamma_u^{*r} = \alpha^*(\alpha^*-\rho^*)/2 \qquad \gamma_w^r = \alpha(\alpha-\rho)\omega_w^2/2 \qquad \gamma_w^{*r} = \alpha^*(\alpha^*-\rho^*)\omega_w^{*2}/2$$

$$\lambda_u^r = 1-\alpha \qquad \lambda_u^{*r} = 1-\alpha^* \qquad \lambda_w^r = -(\alpha-\rho)\omega_w \qquad \lambda_w^{*r} = -(\alpha^*-\rho^*)\omega_w^*$$

$$\omega_w = \beta/(1-\beta\varphi_w) \qquad \omega_w^* = \beta^*/(1-\beta^*\varphi_w)$$

Bansal and Shaliastovich (2007, 2012), Colacito (2009), and Backus et al. (2010) assume identical parameters in the two countries. Applying (10) and (11), we find:

(21) $$E_t\rho_{t+1} = \tfrac{1}{2}\left[\left(\lambda_u^r\right)^2\left(u_t^h-u_t^f\right)+\left(\lambda_w^r\right)^2\left(w_t^h-w_t^f\right)\right]$$

(22) $$r_t^*-r_t = \left[-\gamma_u^r+\tfrac{1}{2}\left(\lambda_u^r\right)^2\right]\left(u_t^h-u_t^f\right)+\left[-\gamma_w^r+\tfrac{1}{2}\left(\lambda_w^r\right)^2\right]\left(w_t^h-w_t^f\right) .$$

Under the parameter restrictions listed above ($1-\alpha>0$, $0<\rho<1$, $\alpha<\rho$), it is straightforward to verify $\text{cov}\left(E_t\rho_{t+1}, r_t^*-r_t\right)>0$ in this model, providing a resolution of the interest parity puzzle. Intuitively, under these parameter restrictions, when the relative variance of the home consumption stream ($u_t^h-u_t^f$ or $w_t^h-w_t^f$) is high, there are two effects. First, as Bansal and Shaliastovich (2012) put it, there is a "flight to quality" – home investors shift their portfolios to less risky assets. The increase in volatility "increases the uncertainty about future growth, so the demand for risk-free assets increases, and in equilibrium, real yields fall", that is $r_t^*-r_t$ rises. Second, foreign exchange risk for home investors rises more than for foreign investors, leading to an increase in $E_t\rho_{t+1}$.

Lustig et al. (2011) consider a case where attitudes toward risk are different in the two countries. That study emphasizes the importance of different responses to common shocks, rather than focusing on idiosyncratic shocks to consumption volatility. Suppose these country-specific shocks are set to zero ($u_t^h=u_t^f=w_t^h=w_t^f=0$), and there are differences in risk aversion ($\alpha\neq\alpha^*$) but other parameters of preferences (β and ρ) are identical. Then:

(23) $$E_t\rho_{t+1} = \tfrac{1}{2}\left[\left(\left(\lambda_u^r\right)^2-\left(\lambda_u^{*r}\right)^2\right)u_t^c+\left(\left(\lambda_w^r\right)^2-\left(\lambda_w^{*r}\right)^2\right)w_t^c\right]$$

(24) $$r_t^*-r_t = \left[\gamma_u^{*r}-\gamma_u^r+\tfrac{1}{2}\left(\left(\lambda_u^r\right)^2-\left(\lambda_u^{*r}\right)^2\right)\right]u_t^c+\left[\gamma_w^{*r}-\gamma_w^r+\tfrac{1}{2}\left(\left(\lambda_w^r\right)^2-\left(\lambda_w^{*r}\right)^2\right)\right]w_t^c .$$

Under the same set of parameter assumptions as above ($1-\alpha>0$, $1-\alpha^*>0$, $0<\rho<1$, $\alpha<\rho$), one finds again $\text{cov}\left(E_t\rho_{t+1}, r_t^*-r_t\right)>0$. As Lustig et al. (2011) explain, "When precautionary saving demand is strong enough, an increase in the volatility of consumption growth (and, consequently, of marginal utility growth) lowers interest rates." When $\alpha^*>\alpha$ (for example), the home country is more risk averse, and the precautionary effect is larger in the home country, so $r_t^* - r_t$ commoves positively with u_t^c and w_t^c. Also, the foreign exchange risk for home residents exceeds that for foreign investors, so $E_t\rho_{t+1}$ commoves positively with the common shocks u_t^c and w_t^c.

Under either specification – idiosyncratic shocks and identical preferences, or different preferences and common shocks – the interest rate differential and the foreign exchange risk premium are responding to changes in the variance of consumption growth. A precautionary motive that drives down the home interest rate also increases the foreign exchange risk premium. There is an under-adjustment of investors to the lower home interest rate. They do not flock to foreign deposits to the same extent that risk neutral investors would because of aversion to foreign exchange risk. But this muted adjustment implies that there is no force to account for the negative correlation of $E_t\sum_0^\infty \rho_{t+j+1}$ with $r_t^* - r_t$. Under the first specification,

$$E_t\sum_0^\infty \rho_{t+j+1} = \tfrac{1}{2}\left[\left(\lambda_u^r\right)^2 \frac{u_t^h - u_t^f}{1-\varphi_u} + \left(\lambda_w^r\right)^2 \frac{w_t^h - w_t^f}{1-\varphi_w}\right],$$

and under the second specification,

$$E_t\sum_0^\infty \rho_{t+j+1} = \tfrac{1}{2}\left[\left(\left(\lambda_u^r\right)^2 - \left(\lambda_u^{*r}\right)^2\right)\frac{u_t^c}{1-\varphi_u} + \left(\left(\lambda_w^r\right)^2 - \left(\lambda_w^{*r}\right)^2\right)\frac{w_t^c}{1-\varphi_w}\right].$$

It is clear that under either specification, $\operatorname{cov}\left(E_t\sum_0^\infty \rho_{t+j+1}, r_t^* - r_t\right) > 0$, contravening our empirical findings.

The interest parity puzzle is sometimes portrayed as a relation between currency depreciation and the interest differential. Using real exchange rates and real interest rates, the puzzle is expressed as $\operatorname{cov}\left(q_{t+1} - q_t, r_t^* - r_t\right) > 0$. This is a stronger relationship than $\operatorname{cov}\left(E_t\rho_{t+1}, r_t^* - r_t\right) > 0$. It can be expressed as $\operatorname{cov}\left(E_t\rho_{t+1}, r_t^* - r_t\right) > \operatorname{var}\left(r_t^* - r_t\right)$. Because the condition is stronger, the empirical relationship is not as robust, but it is found to hold for many currencies and time periods nonetheless (see the surveys of Engel, 1996, 2014). The model with identical preferences and idiosyncratic shocks is able to account for this relationship if risk aversion is strong enough. It is straightforward to see that if the coefficient of relative risk aversion is greater than one ($\alpha < 0$), the model implies $\operatorname{cov}\left(q_{t+1} - q_t, r_t^* - r_t\right) > 0$. Similarly in the model with common shocks, but in which $\alpha \neq \alpha^*$, if relative risk aversion is greater than one in both countries, we find $\operatorname{cov}\left(q_{t+1} - q_t, r_t^* - r_t\right) > 0$.

It is notable that both formulations of the model that derive $\operatorname{cov}\left(q_{t+1}-q_t, r_t^*-r_t\right)>0$ imply (using equation (3)) $\operatorname{cov}\left(q_t-\lim_{k\to\infty}(E_t q_{t+k}), r_t^*-r_t\right)<0$, in contradiction to the ample evidence reported in Table 4. That is, when the models are able to account for the stronger form of the interest parity puzzle, they imply that the country with the higher interest rate tends to have the weaker currency: the transitory component of the exchange rate is negatively correlated with the interest differential. This prediction of the models is not noted in the literature, and is in striking contrast to the widely accepted empirical prediction of the Dornbusch-style models that a higher real interest rate is associated with a stronger currency.

In the Dornbusch model, investors are risk neutral. A monetary contraction in the foreign country raises the relative foreign real interest rate because nominal prices are sticky. Uncovered interest parity holds, so the increase in $r_t^*-r_t$ is accompanied by an expected real depreciation of the foreign currency – a fall in $E_t q_{t+1}-q_t$. In order to generate the expected fall in the real exchange rate, q_t rises initially (and then falls over time) so there is a real appreciation of the foreign currency. The intuition for the predictions of the risk-premium models is similar, except that the exchange rate behavior is reversed. Because of risk aversion, the shock that drives up $r_t^*-r_t$ is accompanied by an expected real appreciation if investors are sufficiently risk averse – an increase in $E_t q_{t+1}-q_t$. To generate this expected appreciation of the foreign currency, the real exchange rate initially falls relative to its permanent component. There is a real depreciation of the foreign currency when $r_t^*-r_t$ is high.

In economic terms, in the Dornbusch model, when $r_t^*-r_t$ is positive, the foreign deposit is attractive to investors, who buy the foreign currency which leads to a positive correlation between the real exchange rate and $r_t^*-r_t$. In these models of risk-averse behavior, $r_t^*-r_t$ is driven by shocks to the variance of consumption growth. When investors are sufficiently risk averse, a shock that causes $r_t^*-r_t$ to rise also makes the foreign deposit so risky that investors are attracted to home deposits, leading to a stronger home currency and a negative correlation between the real exchange rate and $r_t^*-r_t$. The models explain the strong version of the interest parity puzzle with strong risk aversion – but the prediction of the model for the level of the real exchange rate is the opposite of the data.

Some papers extend the above model to allow for changes in inflation, and are able to generate predictions about the correlation of the nominal interest rate differential with $E_t\rho_{t+1}$.[16] In Bansal and Shaliastovich (2012), for example, inflation processes are exogenous, but higher inflation is assumed to lead to lower consumption growth. Hence, inflation volatility influences the risk premium and interest

[16] Bansal and Shaliastovich (2012), Backus et al. (2010), Lustig et al. (2011).

rates through its influence on real consumption. The analysis is similar to that for changes in the variance of real consumption shocks – an increase in home inflation variance lowers the real and nominal interest rate through a precautionary effect and increases the risk premium on foreign deposits.

Verdelhan (2010) builds a two-country endowment model, with a representative agent in each country whose preferences are of the form first proposed by Campbell and Cochrane (1999).[17] In Verdelhan's approach, the real interest rate differential and the foreign exchange risk premium are driven by a factor that is related to the consumption "habit" in each country. Each agent's utility depends on the "surplus" - his consumption relative to an aggregate habit level that is determined as a function of the aggregate consumption level. Similar to the model with Epstein-Zin preferences, the real interest differential and the foreign exchange risk premium are determined by the same driving factor, in this case the surplus. When the surplus is small in the home country, a precautionary effect leads to a lower home interest rate. But also, home investors find foreign deposits riskier relative to the riskiness of home deposits for foreign investors, so the foreign exchange risk premium is high. They underreact to a relatively high foreign interest rate because of foreign exchange risk. The Appendix shows in more detail that this model cannot account for the main empirical findings of this paper because the single factor that drives the risk premium and the interest differential does not allow for any source of excess adjustment by investors.

3.2 Delayed overshooting/ delayed reaction

The behavior of exchange rates and interest rates described here seems related to the notion of "delayed overshooting". The term was coined by Eichenbaum and Evans (1995), but is used to describe a hypothesis first put forward by Froot and Thaler (1990). Froot and Thaler's explanation of the forward premium anomaly was that when, for example, the home interest rate rises, the currency appreciates as it would in a model of interest parity such as Dornbusch's (1976) classic paper. They hypothesize that the full reaction of the market is delayed, perhaps because some investors are slow to react to changes in interest rates, so that the currency keeps on appreciating in the months immediately following the interest rate increase. Bacchetta and van Wincoop (2010) build a model based on this intuition. Much of the empirical literature that has documented the phenomenon of delayed overshooting has focused on the impulse response of exchange rates to identified monetary policy shocks, though in the original context, the story was meant to apply to any shock that leads to an increase in relative interest rates.[18]

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[17] Moore and Roche (2010) also use Campbell-Cochrane preferences to provide a solution to the interest parity puzzle. Stathopolous (2012) examines other international asset pricing puzzles in a two-good equilibrium model that assumes these preferences.

[18] See, for example, Eichenbaum and Evans (1995), Kim and Roubini (2000), Faust and Rogers (2003), Scholl and Uhlig (2008), and Bjornland (2009).

Froot and Thaler (1990) present a descriptive model of delayed overshooting that, they say, can explain the interest parity puzzle:

> Consider as an example, the hypothesis that at least some investors are slow in responding to changes in the interest differential. It may be that these investors need some time to think about trades before executing them, or that they simply cannot respond quickly to recent information. These investors might also be called "central banks," who seem to "lean against the wind" by trading in such a way as to attenuate the appreciation of a currency as interest rates increase. Other investors in the model are fully rational, albeit risk averse, and even may try to exploit the first group's slower movements. A simple story along these lines has the potential for reconciling the above facts. First, it yields negative coefficient estimates of -3 as long as some changes in nominal interest differentials also reflect changes in real interest differentials. While changes in nominal interest rates have different instantaneous effects on the exchange rate across different exchange-rate models, most of these models predict that an increase in the dollar real interest rate (all else equal) should lead to instantaneous dollar appreciation. If only part of this appreciation occurs immediately, and the rest takes some time, then we might expect the exchange rate to appreciate in the period subsequent to an increase in the interest differential. (p. 188)

Some intuition can be gained in the case in which $i_t^* - i_t$ follows a first-order autoregression as does the interest differential in Bacchetta and van Wincoop (2010),

(25) $i_t^* - i_t = \theta\left(i_{t-1}^* - i_{t-1}\right) + \varepsilon_t,\ 0 \le \theta < 1.$

Then using the definition of s_t^{IP} from equation (2), we have:

(26) $s_t^{IP} = \theta s_{t-1}^{IP} + \varepsilon_t / \left(1-\theta\right).$

For simplicity, assume inflation is constant in both countries, so the distinction between real and nominal rates is not important.

While the model of Bacchetta and van Wincoop is complex and requires numerical solution, the essence of it can be described as a model in which the exchange rate only gradually approaches the level that would hold under interest parity, as in the Froot and Thaler story. The gradual adjustment can be modeled as

(27) $s_t - s_t^{IP} = \delta(s_{t-1} - s_{t-1}^{IP}) + \alpha\varepsilon_t, \qquad 0 \le \delta < 1.$

Assume that there is initial underreaction so $\alpha < 0$. We find

$$\operatorname{cov}(s_{t+1} - s_t, i_t^* - i_t) = -\operatorname{var}\left(i_t - i_t^*\right)\left(1 + \frac{\alpha(1-\delta)(1-\theta^2)}{1-\theta\delta}\right).$$

The model will deliver the well-known result from the Fama regression, $\operatorname{cov}(s_{t+1} - s_t, i_t^* - i_t) > 0$ if $\alpha\left(1-\delta\right)\left(1-\theta^2\right) < \theta\delta - 1$. This condition can be satisfied if α is large enough in absolute value, so the initial underreaction of the exchange rate is sufficiently large. In order for the home currency to appreciate initially when the home interest rate rises, we must have $-\alpha\left(1-\theta\right) < 1$. It is necessary that $\theta > \delta$ for both conditions to be satisfied. Under these conditions, we also find $\operatorname{cov}(s_t, i_t^* - i_t) > 0$, as we find strongly in the data (in real terms.) This contrasts with the risk premium models in the previous section, which could account for $\text{cov}\left(q_{t+1}-q_t, r_t^*-r_t\right)>0$ but imply $\text{cov}\left(q_t-\lim_{k\to\infty}(E_t q_{t+k}), r_t^*-r_t\right)<0$

It is easy to see that the impulse response function of the exchange rate to shocks to the interest rate differential will take on the hump shape that is found in the empirical estimates of Eichenbaum and Evans (1995) and replicated in the model of Bacchetta and van Wincoop (2010). Nonetheless, in this model, $\text{cov}(E_t\rho_{t+j}, i_t^*-i_t)$ is positive for all *j*.

The model is built to explain the uncovered interest parity puzzle, and delivers $\text{cov}(E_t\rho_{t+1}, i_t^*-i_t)=-\alpha\left(1-\delta\right)/\left(1-\theta\delta\right)>0$. But $\text{cov}(E_t\rho_{t+j}, i_t^*-i_t)=\delta^{j-1}\text{cov}(E_t\rho_{t+1}, i_t^*-i_t)>0$, so the covariance does not switch signs. This model has a single shock, the monetary shock, that drives both the interest rate differential and the expected excess return. There is underreaction to the monetary shock, and that underreaction dissipates over time, but there is no source of magnified adjustment.

3.3 Liquidity return

A model that can successfully account for the empirical findings of this paper may need to incorporate both a source of under-adjustment and a source of over-adjustment to changes in interest rates. One model with such properties includes short-term assets valued not only for their return but also for some role they play as liquid assets. Here we sketch the implications of considering the role of liquidity return. Our model is based on Nagel (2014).

Consider the home country investor. Following Nagel, we assume the investor has three assets to choose among – demand deposits which are considered to be money, a near-money asset such as interest bearing Eurocurrency deposits denominated in the home currency, and an asset that is not as liquid for the home investor – in this case, the foreign Eurocurrency deposit. The near-money asset is not as liquid as money, but the demand deposit pays a lower rate of interest, assumed to be zero. When the interest rate rises (for example, because of a monetary policy shock), the opportunity cost of holding money rises, so investors shift their portfolios toward more near-money. The near-money is now more valued for its liquidity. The near-money is in greater demand for two reasons – it pays a higher interest rate, and it is more prized for its non-pecuniary liquidity services.

This is the source of a sort of amplified adjustment by investors to a change in the interest rate. The demand for the asset changes more than it would if investors were concerned only with the pecuniary return.

Nagel also presents evidence that there are volatile shocks to the liquidity value of near-money. We append the model of liquidity return to a simple standard New Keynesian open economy macroeconomic model. These liquidity shocks are the source of the partial adjustment by investors. A negative shock that reduces the liquidity of near-money in the home country weakens demand for home assets, leading to a depreciation of the home currency. In our model, the monetary policymaker reacts to the depreciation by raising the home interest rate – either because the policymaker wants to stabilize exchange rates, or else because it is concerned about the inflationary impact of the depreciation. So when there is a negative liquidity shock to home near-money, the home interest rate rises which increases demand for that asset but at the same time the home asset is less liquid, so the increase in demand is muted.

The foreign country investors are symmetric to the home investors. They consider the foreign currency Eurocurrency deposit to be a liquid near-money. For simplicity, we assume parameters are the same in the home and foreign countries, so we can express the key relationships in relative home to foreign terms.

In a standard symmetric two-country New Keynesian model, the dynamics of exchange rates, interest rates and prices are summarized by three equations: a price adjustment equation (or open-economy Phillips curve); a monetary policy rule; and an equation that summarizes financial market equilibrium, typically uncovered interest parity. It is the last equation that we will modify here. We replace the interest parity equation with a relationship for excess returns:

(28) $$E_t \rho_{t+1} = -\alpha\left(i_t^* - i_t\right) + \eta_t, \qquad \alpha > 0.$$

The first component relates the pecuniary excess return to the foreign less home interest differential. As described above, if the home interest rate rises, the liquidity services of home Eurocurrency deposits rise. As a result, the ex-ante excess return on the foreign deposit must increase. In other words, the home and foreign deposits pay the same effective expected return, but the relative liquidity return on the home asset rises, requiring an increase in the relative expected pecuniary return on the foreign asset. η_t represents an independent component to the liquidity return of home relative to foreign assets that might arise from financial market shocks (as in Nagel.) A positive realization of η_t raises the expected return on the foreign short-term asset relative to the expected return on the home asset. This represents an increase in the liquidity value of the home asset (relative to the foreign.)[19]

Consumer price inflation can be summarized by the equation:

(29) $$\pi_t^* - \pi_t = -\delta\left(q_t - \bar{q}_t\right) + \beta E_t(\pi_{t+1}^* - \pi_{t+1}), \qquad 0 < \beta < 1.$$

[19] Valchev (2014) presents a model to account for the puzzle of this paper, also based on liquidity returns, but the economic mechanisms in that paper are very different than those here. Gabaix and Maggiori (2014) is a recent study in which a liquidity premium is introduced into a model of exchange rates. However, the notion of liquidity in that paper refers to the cost of making international financial transactions, as opposed to the liquidity of assets as in this model and Valchev's. Also, the model of Gabaix and Maggiori is unable to explain the puzzle of this paper.

The household's utility discount factor is β. The parameter governing the speed of adjustment of prices, δ, depends on two underlying parameters in the model: β, and the probability that a firm will not change its price in a given period (in a Calvo pricing framework), θ. Specifically, $\delta \equiv (1-\theta)(1-\theta\beta)/\theta$, so that δ is decreasing in θ. $\overline{q}_t$ is the equilibrium real exchange rate – the value the real exchange rate would take on if nominal prices were flexible. It is driven by exogenous productivity shocks in the two countries, so that an increase in relative home productivity leads to a real home appreciation. Engel (2012) shows how such an equation can be derived in a model with producer-currency price stickiness and home bias in preferences. When $q_t - \overline{q}_t$ is positive, prices of goods favored by foreign consumers are relatively high, so foreign inflation will fall relative to home inflation over time. Because productivity shocks are persistent, we assume $\overline{q}_t$ follows a first-order autoregressive process with a high degree of serial correlation.

Monetary policy is specified as a simple Taylor rule:

(30) $$i_t^* - i_t = \sigma(\pi_t^* - \pi_t) + \phi\left(i_{t-1}^* - i_{t-1}\right). \qquad \sigma > 0,\ 0 < \phi < 1.$$

Equation (30) assumes in each country the policymaker targets its own consumer price inflation, and that the instrument rules are identical. The policy rule includes interest-rate smoothing, embodied in the lagged interest rate term. Almost all estimated Taylor rules find significant evidence of interest rate smoothing.

While a closed-form solution for this simple system is not possible, it is helpful to develop intuition by solving the model as if it were static – solving in terms of current shocks, expected future values, and the lagged interest rate. In particular, we can use equations (1) and (28)-(30) to find:

$$E_t\rho_{t+1} = -\frac{\alpha\sigma\delta}{D}\overline{q}_t + \frac{1+\sigma\delta}{\Delta}\eta_t - \frac{\alpha\phi}{\Delta}\left(i_{t=1}^* - i_{t-1}\right) + \frac{\alpha\sigma\delta}{\Delta}E_t q_{t+1} - \frac{\alpha\sigma\left(\delta+\beta\right)}{\Delta}E_t\left(\pi_{t+1}^* - \pi_{t+1}\right)$$

$$i_t^* - i_t = \frac{\sigma\delta}{\Delta}(\overline{q}_t + \eta_t) + \frac{\phi}{\Delta}\left(i_{t-1}^* - i_{t-1}\right) - \frac{\sigma\delta}{\Delta}E_t q_{t+1} + \frac{\sigma\left(\delta+\beta\right)}{\Delta}E_t\left(\pi_{t+1}^* - \pi_{t+1}\right)$$

where $\Delta \equiv 1 + \sigma\delta\left(1+\alpha\right)$.

First consider a positive realization of the shock to the equilibrium real exchange rate, $\overline{q}_t > 0$. This tends to push up foreign inflation, inducing a foreign monetary tightening, which directly increases $i_t^* - i_t$. From the model of liquidity returns, equation (28) implies $E_t\rho_{t+1}$ must fall. The liquidity return of the foreign bond has increased, so its relative pecuniary return declines. Holding the expected future exchange rate constant, both the increase in $i_t^* - i_t$ and the decline in $E_t\rho_{t+1}$ work to depreciate the home currency in real terms, so q_t rises. This increase in q_t lowers foreign relative inflation, $\pi_t^* - \pi_t$. In response, monetary policymakers lower $i_t^* - i_t$, which offsets some of the initial increase in the interest differential. Hence the effect of a $\bar{q}_t$ shock on $i_t^* - i_t$ is less than one-for-one in equilibrium.

If there is a positive liquidity shock in the home country, so η_t is positive, the ex-ante excess return on the foreign deposits, $E_t \rho_{t+1}$, rises. Holding the expectation of the future real exchange rate constant, and holding interest rates constant, this induces an appreciation of the home currency (a drop in q_t.) This in turn causes foreign inflation to rise relative to home inflation, inducing policymakers to increase i_t^* relative to i_t. The increase in $i_t^* - i_t$, *ceteris paribus*, contributes to a drop in $E_t \rho_{t+1}$, partially offsetting the direct effect on an increase in η_t, so $E_t \rho_{t+1}$ rises less than one-for-one in equilibrium.

The shock to the equilibrium real exchange rate contributes toward a negative correlation between the short-run ex ante excess return on the foreign deposit and the foreign less home interest differential, while the liquidity shock contributes to a positive correlation. Can this model reproduce the empirical findings that $\text{cov}(E_t \rho_{t+1}, r_t^* - r_t) > 0$ and $\text{cov}(E_t \sum_{j=0}^{\infty} \rho_{t+j+1}, r_t^* - r_t) < 0$? It can because there is protracted adjustment of the nominal interest rate differential, arising from interest-rate smoothing and from persistence in the underlying shocks to inflation sourced from the equilibrium real exchange rate.

If the liquidity shock, η_t, has a high enough variance relative to the monetary shock, it will be the dominant factor in determining the covariance between the interest differential and the short-run ex ante excess return, and will deliver $\text{cov}(E_t \rho_{t+1}, r_t^* - r_t) > 0$. At longer horizons, the effect of the interest rate differential on the liquidity premium will dominate, and we will find $\text{cov}(E_t \rho_{t+j}, r_t^* - r_t) < 0$.

This model incorporates incentives for both under-adjustment and over-adjustment by investors. It is too simple to take directly to the data, but for plausible parameter values, the model can roughly reproduce the regression results reported in Tables 3 and 5. As a baseline set of parameters, we take $\beta = 0.998$, $\delta = 0.014$, $\sigma = 0.1275$, $\phi = 0.915$, $\alpha = 0.15$. The serial correlation of the equilibrium real exchange rate is set to 0.99 in monthly data. We set the shocks to liquidity have a variance equal to four percent of the variance of the equilibrium real exchange rate. The Appendix elaborates on this calibration and examines the model under different parameter values. The parameter for time preference, β, is calibrated at a standard level for a monthly frequency. The price adjustment parameter δ implies an expected life of a nominal price in the Calvo model of nine months, which is a standard calibration. The parameters for the policymaker's interest rate rule, σ and ϕ imply a long-run increase in the interest rate of 1.5 basis points for each basis point increase in inflation, which is consistent with U.S. data. The smoothing parameter is based on an estimated Taylor rule for the U.S. relative to the G6 over our data sample. The serial correlation of the equilibrium real exchange rate is set to produce a half-life of five years, in line with the very persistent real exchange rates among high-income countries. The value of α, the response of the liquidity premium to changes in the interest rate, is based on estimates reported by Nagel for spreads of various less liquid assets over the T-bill rate. Nagel reports a value of around 0.10, but it is adjusted upward slightly here to capture the idea that foreign deposits are somewhat more illiquid for domestic investors. The variance of the liquidity shocks is more difficult to calibrate, and is treated as a free parameters here, though Nagel (2014) notes these are relatively volatile. While it may seem that a variance of four percent of the equilibrium real exchange rate is small, this refers to the unconditional variance of the latter. Given the persistence of the equilibrium real exchange rate, and that the liquidity shocks have no persistence, the variance of innovations to the liquidity shock equal twice the variance of innovations to the equilibrium real exchange rate. This parameter mainly affects the comovement of the short-run excess return with the real interest differential.

For these parameter values, we find the model produces a coefficient of 1.81 for the regression of $E_t\rho_{t+1}$ on $r_t^* - r_t$, corresponding to the regression results reported in Table 3. That table reports a coefficient of 1.98 for the U.S. relative to the G6. The regression of $E_t\sum_{j=0}^{\infty}\rho_{t+j+1}$ on $r_t^* - r_t$ produces a coefficient of -20.66 in the model compared to -30.89 reported in Table 5 for the U.S. relative to the G6. The model generates reasonable looking behavior for interest rates and inflation. One of the important elements of a Keynesian model is its ability to replicate the high correlation of real and nominal interest rates, which is 0.79 in our data (for the U.S. relative to the G6) and 0.77 in the model.

As the appendix shows, a greater persistence in nominal interest rates, particularly arising from more persistent shocks to inflation, generates a larger (in absolute value) regression coefficient of $E_t\sum_{j=0}^{\infty}\rho_{t+j+1}$ on $r_t^* - r_t$. More volatile liquidity shocks lead to a larger regression coefficient of $E_t\rho_{t+1}$ on $r_t^* - r_t$.

The model illustrates the forces that a model requires to resolve the puzzle. The shocks that lead to muted adjustment by investors must be volatile enough that they determine the short run positive correlation of $E_t\rho_{t+1}$ and $r_t^* - r_t$. But their effect relative to the forces of amplified adjustment must weaken over time, so that $E_t\rho_{t+j}$ is negatively correlated with $r_t^* - r_t$ for a large enough *j*.

4. Conclusions

To summarize: A large literature has been devoted to explaining the uncovered interest parity puzzle which implies $\text{cov}(E_t \rho_{t+1}, r_t^* - r_t) > 0$. Another stylized fact that is generally accepted is that when a country's real interest rate is relatively high, its currency is relatively strong. However, exchange rates appear to be more volatile than can be accounted for if uncovered interest parity holds, suggesting $\text{cov}\left(E_t \sum_0^\infty \rho_{t+j+1}, r_t^* - r_t\right) < 0$. Our empirical findings confirm these relationships.

These findings pose a puzzle. Models that have been built to account for the uncovered interest parity puzzle cannot also account for $\text{cov}\left(E_t \sum_0^\infty \rho_{t+j+1}, r_t^* - r_t\right) < 0$. Neither models of delayed reaction to monetary shocks nor models of foreign exchange risk premiums work because they imply a dampened response, not an excessively volatile response to interest rates.

We suggest a possible avenue to explain our findings by introducing a non-pecuniary liquidity return on assets. When a country's assets become more valued for their liquidity, the country's currency appreciates. This eases inflationary pressure, allowing policymakers to lower interest rates. This provides the under-adjustment part of the story – investors trade off the lower pecuniary return from the lower interest rate with the higher liquidity return. On the other hand, when interest rates rise, then liquid interest-bearing assets are more valued for their liquidity. This is the over-adjustment part of the story – higher interest rates are accompanied by higher liquidity return, giving investors extra incentive to buy the high-interest rate asset.

There may be other possible resolutions to the empirical puzzles presented here. Several recent papers have explored the implications for rare, large currency depreciations for the uncovered interest parity puzzle. Farhi and Gabaix (2014) present a full general equilibrium model of rare disasters and real exchange rates. Their model implies that when the home real interest rate is high, the home currency is weak in real terms, and so cannot account for the levels puzzle presented here.[20] This correlation occurs during "normal times" in their model – the anticipation of a future disaster increases interest rates and weakens the currency of the country that is expected to experience the future problems. Nonetheless, there are two caveats that must be considered in light of Farhi and Gabaix and the related literature. The first is that if rare disasters are important, than the linear VAR technology used in this paper may not correctly capture the stochastic process for real exchange rates and real interest rates. Farhi et. al. (2013) and Burnside et. al. (2011a, 2011b) extract information from options to infer expectations about rare large movements in exchange rates. Moreover, if these large rare events are important, then the lognormal approximations that lie behind our analysis of the risk premium in section 3.1 are not correct. Higher

[20] See also Gourio et al. (2013).

order cumulants matter for the risk premium in that case.[21] In fact, since our technique only takes a first-order approximation to the solution in terms of means and variances, we may be missing some higher-order effects coming from a more general solution, as in the general equilibrium model of Colacito and Croce (2013).

It may be that it is necessary to abandon the assumption that all agents have fully rational expectations. Some version of the model proposed by Hong and Stein (1999) may account for the empirical results uncovered here, which perhaps could be described as a combination of overreaction and momentum trading. That is, the short-term behavior of the real exchange rate under high interest rates incorporates overreaction in that the currency appreciates more than it would under interest parity. But perhaps momentum trading leads to expectations of further appreciation in the short run when the interest rate is high. Burnside et al. (2011c), and Gourinchas and Tornell (2004), are recent approaches that have relaxed the assumption of full rationality in some way. Ilut (2012) adopts an optimizing approach in which ambiguity averse agents may underreact to good news and overreact to bad news.

This study brings two strands of the literature together. The uncovered interest parity puzzle has, in recent years, been primarily addressed as a "finance" puzzle – it has been shown that models with exotic preferences account for the empirical regularity as reaction to foreign exchange risk. The second puzzle – the excess comovement of the exchange rate with interest rates – has been addressed more as an "economics" puzzle. The literature has noted, though not successfully explained, the high volatility of the level of the exchange rate. In bringing the two strands of the literature together, we uncover a striking conflict in the implications of the two puzzles that poses challenges to both lines of research.

[21] See Martin (2013).

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Table 1

Fama Regressions: $s_{t+1} - s_t + i_t^* - i_t = \zeta_s + \beta_s(i_t^* - i_t) + u_{s,t+1}$

1979:6-2009:10

Country	$\hat{\zeta}_s$	90% c.i. $(\hat{\zeta}_s)$	$\hat{\beta}_s$	90% c.i. $(\hat{\beta}_s)$
Canada	-0.045	(-0.250,0.160)	2.271	(1.186,3.355)
France	-0.028	(-0.346,0.290)	1.216	(-0.171,2.603)
Germany	0.192	(-0.136,0.520)	2.091	(0.599,3.583)
Italy	0.032	(-0.325,0.389)	0.339	(-0.680,1.359)
Japan	0.924	(0.504,1.343)	3.713	(2.390,5.036)
United Kingdom	-0.410	(-0.768,-0.051)	3.198	(1.170,5.225)
G6	0.054	(-0.184,0.292)	2.467	(0.769,4.164)

Notes: 90 percent confidence intervals in parentheses based on Newey-West standard errors.

Table 2

Bootstrapped distribution of g_{11} , g_{12}, and $g_{11} - g_{12}$ from VECM

Country	g_{11} Estimates	left tail 1%	5%	10%
Canada	-0.0187 (0.0117)	-0.0545	-0.0381	-0.0311
France	-0.0283 (0.0113)	-0.0510	-0.0340	-0.0276
Germany	-0.0335 (0.0105)	-0.0479	-0.0324	-0.0250
Italy	-0.0221 (0.0117)	-0.0503	-0.0336	-0.0268
Japan	-0.0242 (0.0105)	-0.0423	-0.0281	-0.0203
United Kingdom	-0.0400 (0.0159)	-0.0486	-0.0327	-0.0265
G6	-0.0297 (0.0105)	-0.0485	-0.0295	-0.0234

Country	g_{21} Estimates	right tail 1%	5%	10%
Canada	0.0022 (0.0013)	0.0051	0.0032	0.0024
France	0.0022 (0.0009)	0.0022	0.0013	0.0009
Germany	0.0029 (0.0012)	0.0029	0.0324	0.0013
Italy	0.0036 (0.0012)	0.0027	0.0018	0.0011
Japan	0.0010 (0.0007)	0.0031	0.0014	0.0014
United Kingdom	0.0007 (0.0025)	0.0040	0.0027	0.0019
G6	0.0031 (0.0010)	0.0022	0.0014	0.0010

Country	$g_{11} - g_{21}$ Estimates	1%	left tail 5%	**10%**
Canada	-0.0209	-0.0563	-0.0382	-0.0318
	(0.0116)			
France	-0.0305	-0.0510	-0.0352	-0.0279
	(0.0112)			
Germany	-0.0364	-0.0480	-0.0328	-0.0257
	(0.0105)			
Italy	-0.0258	-0.0497	-0.0339	-0.0266
	(0.0117)			
Japan	-0.0250	-0.0416	-0.0289	-0.0207
	(0.0104)			
United Kingdom	-0.0408	-0.0482	-0.0333	-0.0272
	(0.0162)			
G6	-0.0328	-0.0492	-0.0299	-0.0235
	(0.0105)			

Table 3

Fama Regression in Real Terms: $q_{t+1} - q_t + \hat{r}_t^* - \hat{r}_t = \zeta_q + \beta_q\left(\hat{r}_t^* - \hat{r}_t\right) + u_{q,t+1}$

1979:6-2009:10

Country	$\hat{\zeta}_q$	*95% c.i.(* $\hat{\zeta}_q$ *)*	*90% c.i.(* $\hat{\zeta}_q$ *)*	$\hat{\beta}_q$	*95% c.i.(* $\hat{\beta}_q$ *)*	*90% c.i.(* $\hat{\beta}_q$ *)*
Canada	0.030	(-0.182,0.208)	(-0.141,0.171)	0.722	(-1.103,3.065)	(-0.670,2.665)
	(0.111)	(-0.151,0.200)	(-0.118,0.162)	(0.768)	(-1.004,2.749)	(-0.673,2.492)
France	-0.071	(-0.321,0.124)	(-0.274,0.072)	1.482	(-0.237,3.283)	(0.076,3.004)
	(0.186)	(-0.312,0.107)	(-0.266,0.061)	(1.089)	(-0.834,3.881)	(-0.353,3.514)
Germany	-0.040	(-0.274,0.099)	(-0.232,0.065)	1.733	(0.321,4.896)	(0.643,4.531)
	(0.183)	(-0.257,0.087)	(-0.229,0.058)	(1.112)	(0.246,4.740)	(0.546,4.405)
Italy	0.069	(-0.222,0.278)	(-0.153,0.255)	0.431	(-1.154,2.542)	(-0.881,2.227)
	(0.186)	(-0.182,0.262)	(-0.122,0.244)	(0.971)	(-1.478,2.633)	(-1.125,2.196)
Japan	0.110	(-0.018,0.367)	(0.024,0.332)	2.360	(0.593,4.595)	(0.985,4.320)
	(0.195)	(-0.007,0.363)	(0.023,0.331)	(0.946)	(0.297,4.958)	(0.815,4.558)
United	-0.165	(-0.521,0.029)	(-0.447,-0.028)	1.850	(0.288,4.055)	(0.654,3.771)
Kingdom	(0.211)	(-0.527,0.016)	(-0.492,-0.024)	(0.886)	(0.176,4.144)	(0.465,3.913)
G6	-0.050	(-0.238,0.127)	(-0.194,0.091)	1.983	(0.394,4.335)	(0.644,3.969)
	(0.143)	(-0.218,0.110)	(-0.190,0.078)	(0.976)	(0.091,4.241)	(0.570,3.934)

Notes: The Newey-West standard error is reported below the coefficient estimate in parentheses. The confidence intervals are bootstrapped. The first reports a percentile interval bootstrap and the second a percentile-t interval bootstrap. See Appendix for details.

Table 4

Regression of q_t on $\hat{r}_t^* - \hat{r}_t$: $q_t = \zeta_Q + \beta_Q\left(\hat{r}_t^* - \hat{r}_t\right) + u_{q,t+1}$

1979:6-2009:10

Country	$\hat{\beta}_Q$	*95% c.i.(* $\hat{\beta}_Q$ *)*	*90% c.i.(* $\hat{\beta}_Q$ *)*
Canada	46.996 (8.688)	(25.157,95.390) (21.633,145.736)	(31.793,90.162) (29.714,128.420)
France	20.372 (10.854)	(-1.998,46.182) (-11.709,65.220)	(3.549,42.051) (-5.951,57.766)
Germany	52.410 (12.415)	(.616,91.078) (-0.073,132.108)	(28.470,87.010) (13.828,118.588)
Italy	38.359 (8.042)	(10.971,73.668) (8.031,93.569)	(15.560,68.766) (13.416,84.040)
Japan	19.650 (6.582)	(-2.817,47.262) (0.515,50.100)	(3.032,42.822) (5.013,45.018)
United Kingdom	15.744 (7.875)	(-0.793,36.283) (-8.155,48.868)	(4.006,32.573) (-3.824,43.002)
G6	43.702 (10.124)	(17.664,80.124) (10.506,98.543)	(23.549,75.480) (19.449,89.191)

Notes: The Newey-West standard error is reported below the coefficient estimate in parentheses. The confidence intervals are bootstrapped. The first reports a percentile interval bootstrap and the second a percentile-t interval bootstrap. See Appendix for details.

Table 5

Regression of $\hat{E}_t \sum_{j=0}^{\infty}(\rho_{t+j+1} - \overline{\rho})$ on $\hat{r}_t^* - \hat{r}_t$: $\hat{E}_t \sum_{j=0}^{\infty}(\rho_{t+j+1} - \overline{\rho}) = \zeta_\rho + \beta_\rho\left(\hat{r}_t^* - \hat{r}_t\right) + u_{\rho t}$

1979:6-2009:10

Country	$\hat{\beta}_\rho$	*95% c.i.(* $\hat{\beta}_\rho$ *)*	*90% c.i.(* $\hat{\beta}_\rho$ *)*
Canada	-24.762 (5.523)	(-60.281,-10.757) (-98.321,-10.812)	(-52.700,-15.414) (-68.054,-14.849)
France	-13.983 (8.268)	(-39.998,3.105) (-45.244,8.814)	(-34.960,0.200) (-40.468,4.248)
Germany	-33.895 (10.365)	(-62.299,-5.924) (-87.170,3.844)	(-58.804,-10.621) (-73.809,-4.432)
Italy	-26.556 (6.206)	(-54.355,-4.446) (-64.174,-4.848)	(-49.863,-10.649) (-57.032,-9.335)
Japan	-15.225 (6.487)	(-41.927,2.218) (-42.394,-0.325)	(-37.617,-2.177) (-38.379,-3.176)
United Kingdom	-10.717 (8.565)	(-31.865,3.436) (-42.105,13.602)	(-27.130,1.060) (-37.710,9.599)
G6	-30.890 (8.352)	(-59.899,-9.893) (-68.593,-9.665)	(-56.359,-14.642) (-60.065,-13.478)

Notes: The Newey-West standard error is reported below the coefficient estimate in parentheses. The confidence intervals are bootstrapped. The first reports a percentile interval bootstrap and the second a percentile-t interval bootstrap. See Appendix for details.

Figure 1

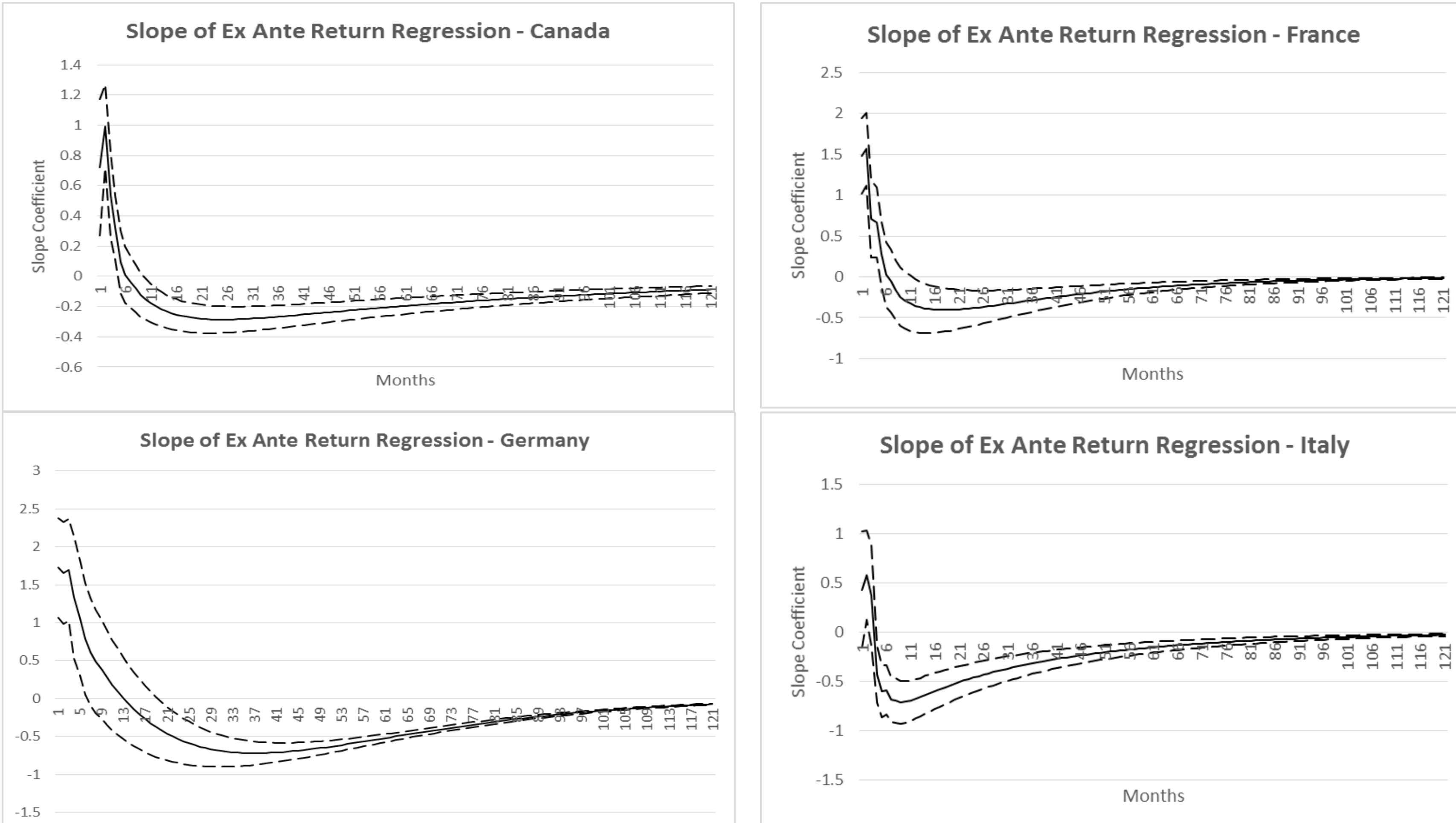
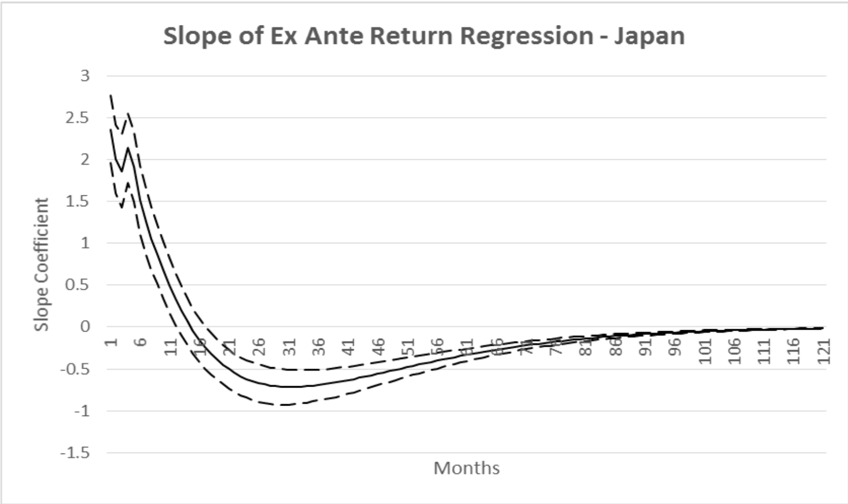
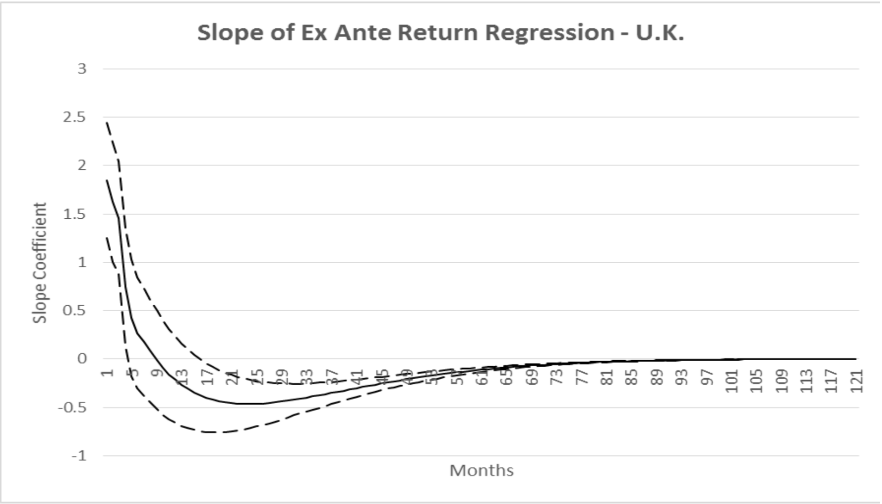
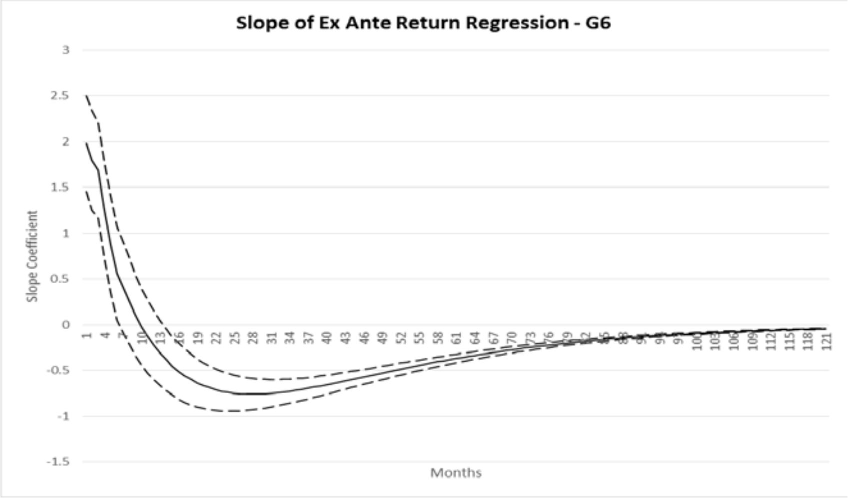

Slope of Ex Ante Return Regression - Canada
Slope Coefficient
Months
Slope of Ex Ante Return Regression - France
Slope Coefficient
Months
Slope of Ex Ante Return Regression - Germany
Slope of Ex Ante Return Regression - Italy
Slope Coefficient
Months

Figure 1







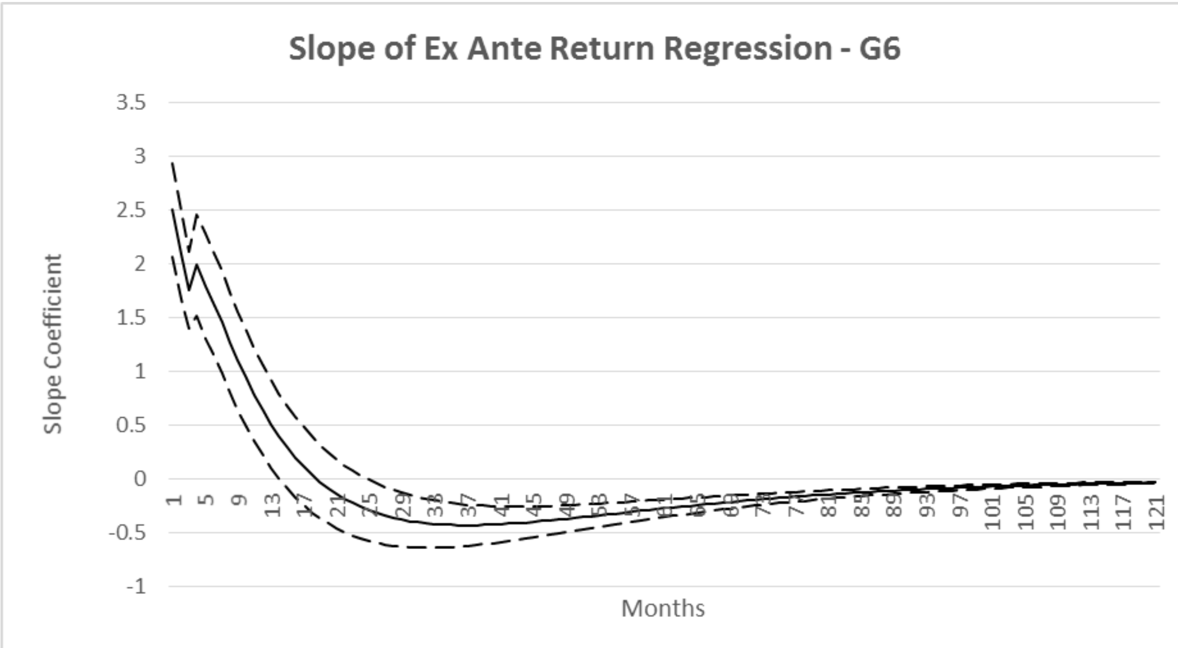
Figures plot the slope coefficients and 90% confidence interval of these regressions: $\hat{E}_t(\rho_{t+j}) = \zeta_j + \beta_j\left(r_t^* - r_t\right) + u_t^j$

Monthly data, 1979:6-2009:10

Confidence intervals calculated from Newey-West standard errors.

Figure 2

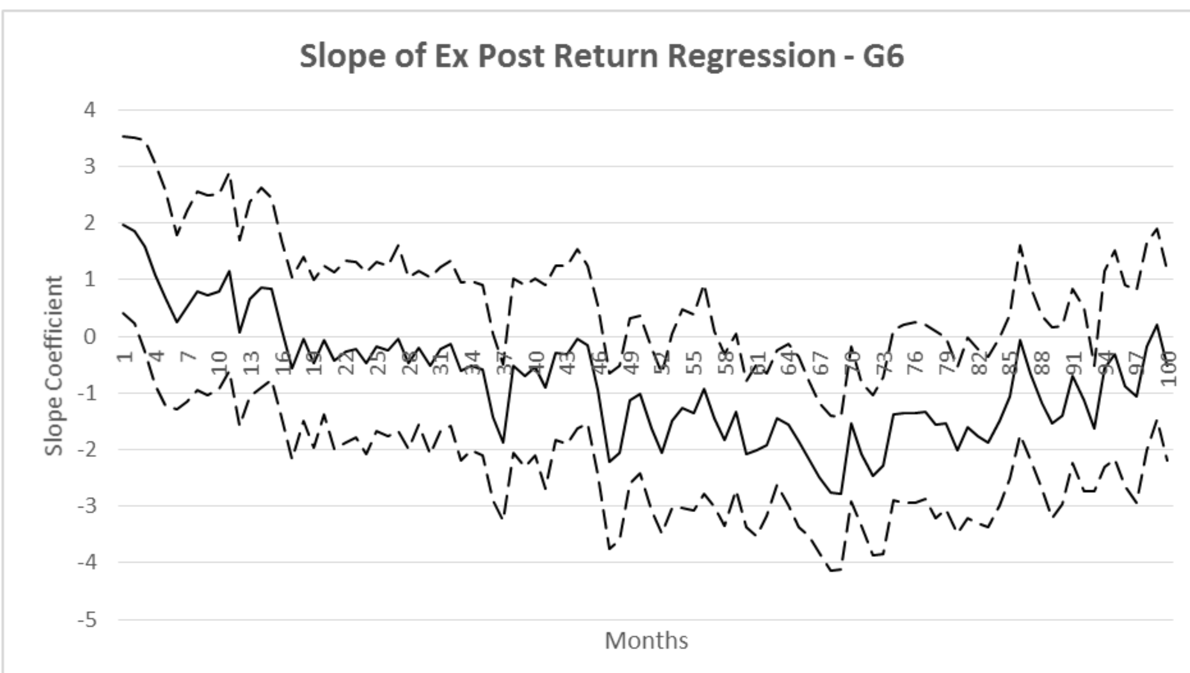
Slope coefficients and 90% confidence interval of the regression: $\hat{E}_t(\rho_{t+j}) = \zeta_j + \beta_j\left(i_t^* - i_t\right) + u_t^j$



Notes: Monthly data, 1979:6-2009:10. Confidence intervals calculated from Newey-West standard errors.

Figure 3

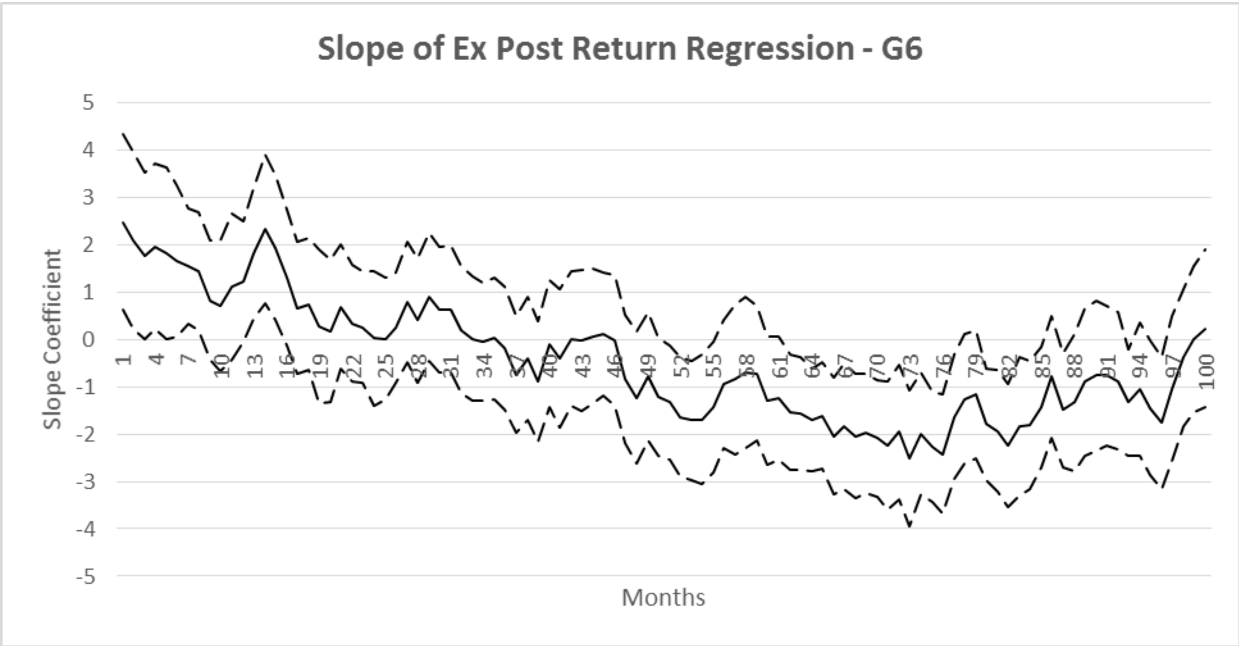
Slope coefficients and 90% confidence interval of the regression: $\rho_{t+j} = \zeta_j + \beta_j\left(\hat{r}_t^* - \hat{r}_t\right) + u_t^j$



Notes: Monthly data, 1979:6-2009:10. Confidence intervals calculated from Newey-West standard errors.

Figure 4

Slope coefficients and 90% confidence interval of the regression: $\rho_{t+j} = \zeta_j + \beta_j \left(i_t^* - i_t \right) + u_t^j$

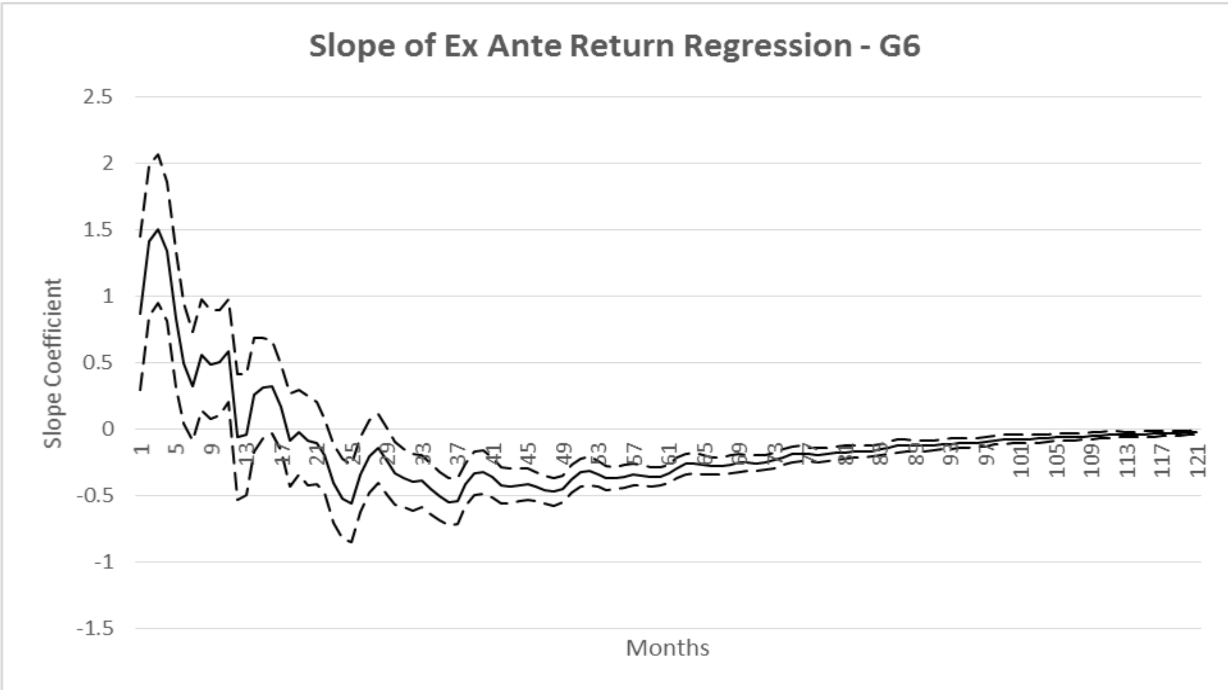


Notes: Monthly data, 1979:6-2009:10. Confidence intervals calculated from Newey-West standard errors.

Figure 5

Slope coefficients and 90% confidence interval of the regression: $\hat{E}_t(\rho_{t+j}) = \zeta_j + \beta_j \left(\hat{r}_t^* - \hat{r}_t \right) + u_t^j$

12-Lag VECM

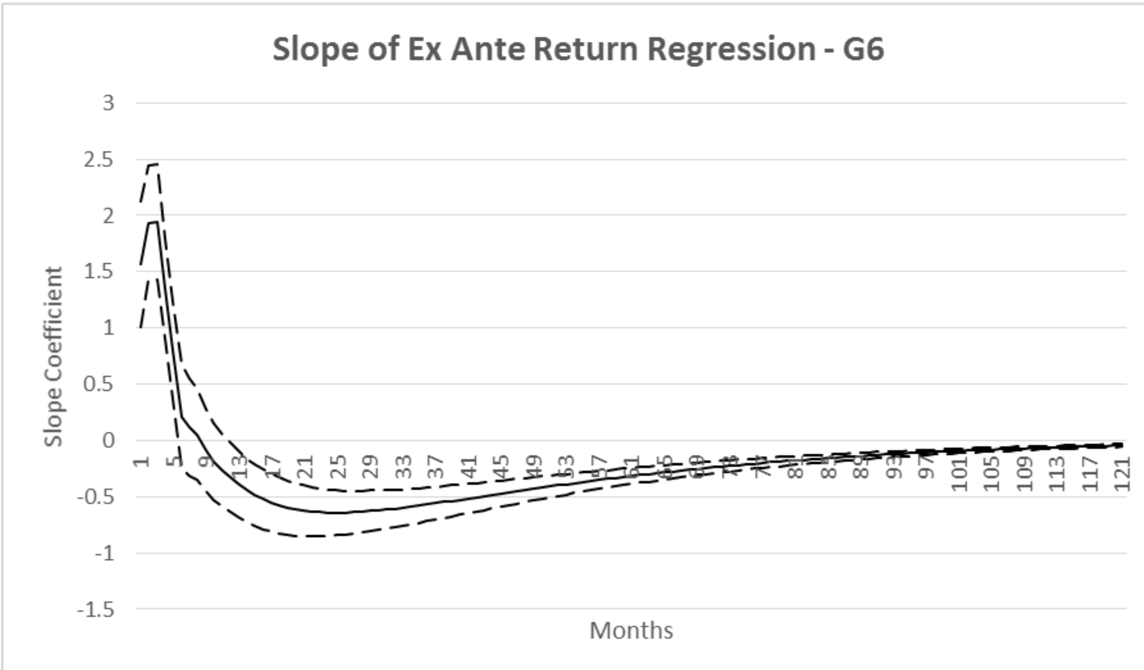


Notes: Monthly data, 1979:6-2009:10. Confidence intervals calculated from Newey-West standard errors.

Figure 6

Slope coefficients and 90% confidence interval of the regression: $\hat{E}_t(\rho_{t+j}) = \zeta_j + \beta_j\left(\hat{r}_t^* - \hat{r}_t\right) + u_t^j$

Stock prices, Long-term bond yields, Gold price, Oil price included in VECM



Notes: Monthly data, 1979:6-2009:10. Confidence intervals calculated from Newey-West standard errors.

Appendix

A.1

Table A1
Coefficient estimates from VECMs
1979:6-2009:10

Canada

	$s_t - s_{t-1}$	$p_t^R - p_{t-1}^R$	$i_t^R - i_{t-1}^R$
(Intercept)	-0.4584	0.0733	-0.0144
$s_{t-1} - p_{t-1}^R$	-0.0187	0.0022	-0.0004
i_{t-1}^R	-1.3804	0.3659	-0.1102
$s_{t-1} - s_{t-2}$	-0.0101	0.0324	0.0040
$p_{t-1}^R - p_{t-2}^R$	-0.0281	0.0566	-0.0170
$i_{t-1}^R - i_{t-2}^R$	-4.6764	0.3031	-0.0848
$s_{t-2} - s_{t-3}$	0.0636	0.0053	0.0043
$p_{t-2}^R - p_{t-3}^R$	-0.1542	0.0368	0.0195
$i_{t-2}^R - i_{t-3}^R$	0.3412	-0.2015	0.0166
$s_{t-3} - s_{t-4}$	0.0390	-0.0063	-0.0010
$p_{t-3}^R - p_{t-4}^R$	0.3320	0.0543	-0.0212
$i_{t-3}^R - i_{t-4}^R$	-0.5013	0.3728	0.0190
R^2	0.05	0.08	0.13

France

	$s_t - s_{t-1}$	$p_t^R - p_{t-1}^R$	$i_t^R - i_{t-1}^R$
(Intercept)	-4.8745	0.3984	0.1067
$s_{t-1} - p_{t-1}^R$	-0.0283	0.0022	0.0007
i_{t-1}^R	-0.5996	0.2497	-0.1243
$s_{t-1} - s_{t-2}$	0.0521	0.0180	0.0016
$p_{t-1}^R - p_{t-2}^R$	0.1457	0.2221	0.0148
$i_{t-1}^R - i_{t-2}^R$	-0.1733	0.0804	-0.3283
$s_{t-2} - s_{t-3}$	0.0483	0.0093	0.0046
$p_{t-2}^R - p_{t-3}^R$	0.7696	-0.0421	0.0206
$i_{t-2}^R - i_{t-3}^R$	-0.5356	0.1822	-0.0430
$s_{t-3} - s_{t-4}$	0.0820	-0.0016	0.0004
$p_{t-3}^R - p_{t-4}^R$	0.6618	0.1393	-0.0739
$i_{t-3}^R - i_{t-4}^R$	1.5395	-0.0271	0.0032
R^2	0.05	0.22	0.22

Germany

	$s_t - s_{t-1}$	$p_t^R - p_{t-1}^R$	$i_t^R - i_{t-1}^R$
(Intercept)	-1.4739	0.2004	0.0067
$s_{t-1} - p_{t-1}^R$	-0.0335	0.0029	0.0001
i_{t-1}^R	-2.2007	0.4995	-0.0381
$s_{t-1} - s_{t-2}$	0.0486	0.0199	0.0016
$p_{t-1}^R - p_{t-2}^R$	0.6154	0.0814	0.0055
$i_{t-1}^R - i_{t-2}^R$	-2.0641	0.2419	0.1724
$s_{t-2} - s_{t-3}$	0.0403	0.0060	-0.0004
$p_{t-2}^R - p_{t-3}^R$	-0.2929	0.0291	0.0056
$i_{t-2}^R - i_{t-3}^R$	0.7971	0.1484	-0.1350
$s_{t-3} - s_{t-4}$	0.0639	-0.0031	0.0001
$p_{t-3}^R - p_{t-4}^R$	0.5742	-0.1026	-0.0048
$i_{t-3}^R - i_{t-4}^R$	536820	-0.1789	-0.0276
R^2	0.06	0.12	0.07

Italy

	$s_t - s_{t-1}$	$p_t^R - p_{t-1}^R$	$i_t^R - i_{t-1}^R$
(Intercept)	-16.2931	2.6931	-0.0715
$s_{t-1} - p_{t-1}^R$	-0.0221	0.0036	-0.0001
i_{t-1}^R	0.1056	0.4070	-0.1151
$s_{t-1} - s_{t-2}$	0.0773	0.0113	0.0003
$p_{t-1}^R - p_{t-2}^R$	-0.3443	0.3778	-0.0068
$i_{t-1}^R - i_{t-2}^R$	-1.7433	0.1120	-0.1153
$s_{t-2} - s_{t-3}$	0.0714	0.0028	0.0024
$p_{t-2}^R - p_{t-3}^R$	0.7718	-0.0683	0.0043
$i_{t-2}^R - i_{t-3}^R$	0.1222	-0.0132	0.1360
$s_{t-3} - s_{t-4}$	0.0774	-0.0008	0.0039
$p_{t-3}^R - p_{t-4}^R$	0.6177	0.0512	0.0186
$i_{t-3}^R - i_{t-4}^R$	1.7392	0.2441	-0.1464
R^2	0.06	0.40	0.15

Japan

	$s_t - s_{t-1}$	$p_t^R - p_{t-1}^R$	$i_t^R - i_{t-1}^R$
(Intercept)	-10.0751	0.5716	0.1129
$s_{t-1} - p_{t-1}^R$	-0.0242	0.0008	0.0002
i_{t-1}^R	-3.1859	0.3014	-0.0525
$s_{t-1} - s_{t-2}$	0.0143	-0.0140	0.0004
$p_{t-1}^R - p_{t-2}^R$	-0.5921	0.0647	-0.0018
$i_{t-1}^R - i_{t-2}^R$	0.3740	-0.0520	0.1390
$s_{t-2} - s_{t-3}$	0.0207	0.0111	0.0001
$p_{t-2}^R - p_{t-3}^R$	-0.0698	-0.1941	0.0122
$i_{t-2}^R - i_{t-3}^R$	3.0030	0.6690	0.0940
$s_{t-3} - s_{t-4}$	0.0136	0.0020	0.0005
$p_{t-3}^R - p_{t-4}^R$	-0.0416	-0.1583	-0.0237
$i_{t-3}^R - i_{t-4}^R$	3.9180	0.0092	0.0418
R^2	0.05	0.11	0.10

United Kingdom

	$s_t - s_{t-1}$	$p_t^R - p_{t-1}^R$	$i_t^R - i_{t-1}^R$
(Intercept)	1.8366	-0.0188	-0.0323
$s_{t-1} - p_{t-1}^R$	-0.0400	0.0007	0.0004
i_{t-1}^R	-1.9722	0.3071	-0.0674
$s_{t-1} - s_{t-2}$	0.0941	0.0255	0.0008
$p_{t-1}^R - p_{t-2}^R$	-0.1056	0.0290	0.0032
$i_{t-1}^R - i_{t-2}^R$	-0.3261	-0.3684	0.0684
$s_{t-2} - s_{t-3}$	0.0309	0.0106	0.0004
$p_{t-2}^R - p_{t-3}^R$	0.3732	-0.0583	-0.0023
$i_{t-2}^R - i_{t-3}^R$	-0.8468	0.5442	0.0627
$s_{t-3} - s_{t-4}$	0.0238	0.0094	0.0002
$p_{t-3}^R - p_{t-4}^R$	0.0332	-0.1061	0.0037
$i_{t-3}^R - i_{t-4}^R$	-1.6905	1.0814	-0.0531
R^2	0.06	0.08	0.06

G6

	$s_t - s_{t-1}$	$p_t^R - p_{t-1}^R$	$i_t^R - i_{t-1}^R$
(Intercept)	-6.3496	0.7123	0.0534
$s_{t-1} - p_{t-1}^R$	-0.0297	0.0031	0.0002
i_{t-1}^R	-2.2496	0.3228	-0.0641
$s_{t-1} - s_{t-2}$	0.0589	0.0196	0.0011
$p_{t-1}^R - p_{t-2}^R$	0.1636	0.1552	-0.0094
$i_{t-1}^R - i_{t-2}^R$	-0.2604	0.2365	-0.0349
$s_{t-2} - s_{t-3}$	0.0368	0.0098	0.0022
$p_{t-2}^R - p_{t-3}^R$	0.3971	-0.0207	0.0058
$i_{t-2}^R - i_{t-3}^R$	0.1497	0.2370	0.0393
$s_{t-3} - s_{t-4}$	0.0489	-0.0019	0.0003
$p_{t-3}^R - p_{t-4}^R$	0.8704	-0.0538	-0.0203
$i_{t-3}^R - i_{t-4}^R$	3.3658	0.1735	-0.0417
R^2	0.05	0.12	0.07

A.2 Verdelhan (2010) Model

In Verdelhan (2010) there are two symmetric countries. The objective of Home household *i* is to maximize

(31) $$E_t \sum_{j=0}^{\infty} \beta^j (C_{i,t+j} - H_{t+j})^{1-\gamma} / (1-\gamma),$$

where γ is the coefficient of relative risk aversion, and H_t represents an external habit. H_t is defined implicitly by defining the "surplus", $s_t \equiv \ln\left((C_t - H_t)/C_t\right)$, where C_t is aggregate consumption, and s_t is assumed to follow the stochastic process:

(32) $$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \mu(s_t)(c_{t+1} - c_t - g), \quad 0 < \phi < 1.$$

Here, ϕ and $\bar{s}$ are parameters, and $c_t \equiv \ln(C_t)$ is assumed to follow a simple random walk:

(33) $$c_{t+1} = g + c_t + u_{t+1}, \text{ where } u_{t+1} \sim i.i.d.\ N(0,\sigma^2).$$

$\mu(s_t)$ represents the sensitivity of the surplus to consumption growth, and is given by:

(34) $$\mu(s_t) \equiv \frac{1}{\bar{S}}\sqrt{1-2(s_t-\bar{s})} - 1, \text{ when } s_t \le s_{\max}, \text{ 0 elsewhere.}$$

The log of the stochastic discount factor is given by:

(35) $$m_{t+1} = \ln(\beta) - \gamma\left[g + (\phi-1)(s_t - \bar{s}) + (1+\mu(s_t))(c_{t+1} - c_t - g)\right]$$

When the parameters $\bar{S}$ and $s_{\max}$ are suitably normalized, Verdelhan shows we can write the expected rate of depreciation as:

(36) $$r_t^* - r_t = \left[\gamma(1-\phi) - \left(\gamma^2\sigma^2 / \bar{S}^2\right)\right](s_t - s_t^*),$$

where s_t^* is the Foreign surplus. The excess return is given by:

(37) $$E_t\rho_{t+1} = -(\gamma^2\sigma^2 / \bar{S}^2)(s_t - s_t^*).$$

Under the assumption of Verdelhan (2010) that $\gamma(1-\phi) < \gamma^2\sigma^2 / \bar{S}^2$, this model can account for the empirical finding of $\text{cov}(E_t\rho_{t+1}, r_t^* - r_t) > 0$. However, because $s_t - s_t^*$ follows a first-order autoregressive process, the model also implies $\text{cov}\left(E_t \sum_0^{\infty} \rho_{t+j+1}, r_t^* - r_t\right) > 0$, contrary to our empirical findings.

A.3 Bootstraps

This appendix describes five bootstrap methods. The first, to construct the standard errors reported in Table 2, are constructed under the null hypothesis of no cointegration between relative prices and the nominal exchange rate. Tables 3, 4 and 5 construct bootstraps from pseudo-data generated from the estimated VECMs, using the percentile and the percentile-t methods. We use a fourth method of bootstrapping to test the joint null hypotheses in Tables 2, 3, 4, and 5. These bootstraps draw from the joint empirical distribution for the G6 currencies. The fifth method, which is not reported in the text, but reported in this appendix, uses Kilian's (1998) "bootstrap after bootstrap" method to correct for possible small sample biases in the coefficient estimates of the parameters of the VECM that are used to construct the percentile and percentile-t confidence intervals for Tables 3, 4 and 5.

For Table 2, we first estimate a VAR with four lags in the variables $s_t - s_{t-1}$, $p_t - p_t^* - \left(p_{t-1} - p_{t-1}^*\right)$, and $i_t - i_t^*$. In other words, this VAR is estimated under the assumption that s_t and $p_t - p_t^*$ each have a unit root but $i_t - i_t^*$ is stationary. We use the estimates from this VAR to construct pseudo-data. Initial values are set at the sample means. We draw the error terms with equal probability from the vector of error terms estimated by the VAR at each date. We generate samples of 500+*T* observations, then use the last *T* observations, where *T* corresponds to the length of the time series we use in estimation. For each pseudo-sample, we then estimate the VECM of equation (6). Table 2 reports the 1%, 5% and 10% left tails of the empirical distribution of g_{11}, g_{12} and $g_{11} - g_{12}$.

For both bootstraps in the results reported in Tables 3, 4, and 5, we construct pseudo-samples using the VECM estimates. Initial values are set as described in the previous paragraph. We draw the error terms with equal probability from the vector of error terms estimated by the VECM at each date. For each pseudo-sample, we estimate the VECM. We estimate all of the regression coefficients reported in Tables 3, 4 and 5, and calculate the Newey-West standard errors for each of those regressions. We repeat this exercise 1000 times.

The first confidence interval based on the bootstraps uses the coefficient estimates reported in the tables. Let $\hat{\beta}$ refer to any of the coefficient estimates reported in Tables 3, 4 and 5. From the regressions on the pseudo-samples, we order the coefficient estimates from these 1000 replications from smallest to largest - $\hat{\beta}_1$ is the smallest and $\hat{\beta}_{1000}$ be the largest. The confidence interval reported in the tables is based on ($[\hat{\beta} - (\hat{\beta}_{950} - \hat{\beta}), \hat{\beta} + (\hat{\beta} - \hat{\beta}_{50})]$. That is, the reported confidence interval corrects for the asymmetry in the distribution of $\hat{\beta}_i$ from the regressions on the pseudo-samples.

Hansen (2010) argues that the first bootstrap method performs poorly when the $\hat{\beta}_i$ do not have a symmetric distribution. Instead, he recommends the following procedure. As above, let $\hat{\beta}$ refer to the estimated coefficient in the data, and $\hat{\sigma}$ to be the Newey-West standard error in the data. For each pseudo-sample *i*, we will record analogous estimates: $\hat{\beta}_i$ and $\hat{\sigma}_i$. θ_i is defined by: $\theta_i = \frac{\hat{\beta}_i - \hat{\beta}}{\hat{\sigma}_i}$. We arrange these θ_i from smallest to largest, so that θ_1 is the smallest and θ_{1000} is the largest. The third confidence interval reported for each coefficient estimate is given by $[\hat{\beta} - \hat{\sigma}\theta_{950}, \hat{\beta} - \hat{\sigma}\theta_{50}]$. It turns out that our two bootstraps generally produce very similar confidence intervals.

In each of Tables 3, 4, and 5, the slope coefficient estimates for each of the G6 currencies are all of the same sign. This allows us to construct a very simple test of the joint null hypotheses reported in the text. We take the errors generated from the estimated VECMs, as we did in constructing the confidence intervals in tables 3, 4 and 5, described above. However, now we take the vector of errors for each date from all six of the individual currency VECMs jointly as a point in the empirical distribution. That is, in constructing pseudo-data, we draw from the 18x1 vector of errors for each date – 3x1 for each of six countries. With this data, we then estimate VECMs as in equation (6) for each country (country by country). We estimate the coefficients of Tables 3, 4 and 5 using each pseudo-sample, repeating this process 2000 times. We record for example for Table 3 (but analogously for Tables 4 and 5), the proportion of times all six coefficient estimates are negative, which gives us the probability reported in the text.

Kilian (1998) suggests that bootstrap distributions based on VAR (or VECM) estimates may be biased in small samples. We follow Kilian's procedure for producing unbiased distributions. Note that this is only a problem for bootstraps such as in Tables 3 and 5 that are not constructed under the null hypothesis, as opposed to those reported in Table 2 in which the pseudo-data is constructed under the null. Here is a concise account of how Kilian's method works: Estimate the VECM. Construct pseudo samples based on data generated using the estimated coefficients from the VECM. For each pseudo sample, re-estimate the VECM. Construct the mean parameter estimate from the pseudo samples for each parameter. Then adjust the original parameter estimates. For example, if the original estimate of parameter a_{ij} is given by $\hat{a}_{ij}^0$, and the mean of the parameter estimate from the pseudo sample is $\bar{a}_{ij}^0$, then construct the adjusted parameter as: $\hat{a}_{ij}^1 = 2\hat{a}_{ij}^0 - \bar{a}_{ij}^0$. Then construct new pseudo-samples using the adjusted parameters, and proceed as before.

We do not report the results using this method in the text because it turns out that the bootstrap distributions for the coefficient estimates reported in Tables 3 and 5 are not much different under the original bootstrap and the Kilian bootstrap. Here we report the median and mean parameter bias in the estimates from Tables 3 and 5 for the "bias corrected" bootstraps and compared the bootstraps used in Tables 3 and 5. (The bias is reported as the mean or median coefficient estimate from the bias corrected bootstrap less the corresponding estimate from the original bootstrap.) In all cases, the bias correction in those coefficient estimates is small and does not affect our conclusions:

	Canada	France	Germany	Italy	Japan	U.K.	G6
Table 3							
Coefficient estimate	0.722	1.482	1.733	0.431	2.360	1.850	1.983
Mean bias	0.137	0.076	0.198	0.190	0.148	0.043	-0.037
Median bias	0.153	0.079	0.210	0.190	0.138	0.070	-0.056
Table 5							
Coefficient estimate	-24.762	-13.983	-33.895	-26.556	-15.225	-10.717	-30.890
Mean bias	2.34	-0.933	-1.291	-2.979	-3.949	0.890	-1.8822
Median bias	2.856	-2.039	-3.491	-2.368	-3.519	0.761	-2.961

Kilian, Lutz. 1998. "Small-Sample Confidence Intervals for Impulse Response Functions." Review of Economics and Statistics 80, 218-230.

A.4 Model Simulations

As stated in the text, the baseline parameters are given by $\beta = 0.998$, $\delta = 0.014$, $\sigma = 0.1275$, $\phi = 0.915$, $\alpha = 0.15$, and $\xi = 0.99$, where the latter is the serial correlation of $\bar{q}_t$ assuming that variable follows a first-order autoregressive process. In addition, the variance of η_t is set equal to 0.04 times the variance of $\bar{q}_t$: $\text{var}(\eta_t) / \text{var}(\bar{q}_t) = 0.04$.

The price stickiness parameter is set as a compromise. According to Monacelli (2004), a standard parameterization assumes that the expected price duration is one year. However, Bils and Klenow (2004) find that the half-life of prices in the data for the U.S. consumer price index is around 5.5 months, excluding sale items, implying an expected duration of around 8 months. The parameter δ is calculated from the formula $\delta \equiv (1-\theta)(1-\theta\beta)/\theta$, where θ is the probability of not adjusting the price in any period in the Calvo model.

We do not observe the equilibrium real exchange rate. This calibration assumes a half-life of five years for the equilibrium real exchange rate. This is based, first, on Rogoff's (1996) well-cited claim that the consensus is that among high-income countries, the half-life of real exchange rates is 3 to 5 years. The upper end of that range is appropriate because Rogoff's consensus applies to the actual real exchange rate, rather than the equilibrium rate, so some of the adjustment involves convergence of the disequilibrium component that might be due to price stickiness. Rogoff's "purchasing power parity puzzle", however, refers in part to the fact that the actual real exchange rate is too persistent to be accounted for by adjustment of the disequilibrium component. Second, Engel (2000) decomposes the real exchange rate for the U.S. relative to the U.K. into a component identified as an equilibrium component – based on the relative price of nontraded to traded goods – and a disequilibrium component. The equilibrium component has a serial correlation of 0.97 in quarterly data, implying a serial correlation of around 0.99 in monthly data.

The value of α is based on Nagel's (2014) estimates in a regression of his measure of the liquidity premium on the U.S. fed funds rate. That parameter ranges from a low of around 0.05 to a high of around 0.11, depending on the regression specification and the measure of the liquidity premium. The liquidity premium refers to the spread between the repo rate on repurchase agreements with Treasury collateral and the T-bill rate, which Nagel argues captures mostly liquidity return because there is very little risk in the repos. We choose a value of α slightly larger than Nagel's estimates in order to capture the idea that the foreign deposits may be even less liquid than these repos.

In the table below, we alter each parameter individually, leaving the others unchanged. (In the case of the Taylor rule, in the first three rows, we consider changes in the smoothing parameter, ϕ, leaving the long-run response of policy interest rate to inflation unchanged ($\phi / (1-\sigma) = 1.5$.) We report the slope coefficient for the regression of short-run excess returns on the real interest differential as in Table 3, the regression of the cumulated expected excess returns on the real interest differential, as in Table 5, and the correlation of real and nominal interest rates (which is 0.79 in our data for the real and nominal interest rates of the U.S. relative to the G6 average.)

Parameter	value	Slope Coefficient for Table 3	Slope Coefficient for Table 5	Correlation of real and nominal interest rates
baseline		1.81	-20.66	0.77
ϕ	0.95	1.09	-10.86	0.61
ϕ	0.90	2.09	-23.52	0.81
ϕ	0.85	2.93	-29.21	0.89
ϕ	0.95	1.93	-16.98	0.91
ϕ	0.90	1.63	-18.23	0.62
ϕ	0.88	1.03	-1.25	0.23
σ	0.10	1.04	-8.06	0.41
σ	0.20	2.53	-19.70	0.95
σ	0.30	3.18	-16.11	0.98
δ	0.01	1.21	-22.10	0.78
δ	0.015	1.94	-20.37	0.77
δ	0.02	2.66	-18.83	0.76
ξ	0.995	7.62	-49.81	0.85
ξ	0.98	0.39	-6.74	0.66
ξ	0.90	0.05	-0.11	0.45
α	0.10	1.55	-13.51	0.77
α	0.20	2.10	-27.70	0.77
α	0.30	2.78	-41.44	0.77
$\mathrm{var}(\eta_t)/\mathrm{var}(\bar{q}_t)$	0.02	0.78	-21.89	0.77
$\mathrm{var}(\eta_t)/\mathrm{var}(\bar{q}_t)$	0.05	2.32	-20.06	0.77
$\mathrm{var}(\eta_t)/\mathrm{var}(\bar{q}_t)$	0.10	4.79	-17.11	0.77

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A.5 Relation to Froot and Ramadorai (2005)

On the surface, there appears to be a strong relationship between some of the empirical work in Froot and Ramadorai (2005, hereinafter FR), and the empirical findings of this paper. FR write out an expression for the real exchange rate similar to (3), and estimate a VAR that is similar to the VECM estimated in this paper.

The main focus of FR is on the contribution of institutional-investor currency flows to the determination of exchange rates. They ask whether those flows are contributing to movements of the exchange rate because they contribute to the component determined by the sum of expected real interest differentials or to the sum of expected excess returns. They also ask whether it is news about future trades that affects the exchange rate, or the actual current trades.

There is a small section of the paper (Tables 5 and 6, pp. 1558-1560) that focuses on comovements of real interest rates and excess returns. However, their focus is on the comovement of innovations of real interest rates and excess returns at different horizons, while this paper focuses on the unconditional covariances of ex ante returns and real interest rates, $\text{cov}(E_t \rho_{t+1}, r_t^* - r_t)$ and $\text{cov}\left(E_t \sum_0^\infty \rho_{t+j+1}, r_t^* - r_t\right)$. That is, this study is concerned with unconditional covariances of returns that are known at time *t*, while FR are concerned with covariances of innovations of returns with innovations in real interest rates. While these covariances are not completely unrelated, as we will show here, they look at different aspects of the data. FR do not cite any of the literature on the interest parity puzzle, and do not relate their findings to that work. They do have a notion of overreaction of the exchange rate to interest rate changes, but that is different than our notion of excess comovement – again, it reflects the different between covariances of shocks versus unconditional covariances.

The comparison of the moments estimated in the two papers is made more difficult by a problem with FR's estimation. Their system is overidentified, meaning that there are an infinite number of estimates of the moments they are concerned with that could be generated from their VAR. This presents a difficulty in comparing the moments they estimate because their Tables V and VI label moments using the notation for the sample moments they calculate, but the overidentification causes some murkiness in translating those to population moments.

This section of the appendix proceeds in three parts. In part A.5.a, some general observations are made about the different information that is embodied in unconditional correlations and correlations of shocks. Part A.5.b discusses some of the problems with FR's estimation, and some general issues with estimating this type of model. Part A.5.c then compares the moments estimated in this paper to the moments estimated in FR.

A.5.a

A useful place to point to the distinction between unconditional covariances and covariances of innovations is in the context of the model of delayed overshooting of section 3.2 of this paper. In that section, nominal prices are held constant, and the interest differential follows an exogenously given process. The uncovered interest parity puzzle concerns $\mathrm{cov}(E_t\rho_{t+1}, i_t^* - i_t)$, or $\mathrm{cov}(E_t s_{t+1} - s_t, i_t^* - i_t)$. The delayed overshooting literature has focused on impulse response functions from identified monetary shocks. If the monetary shock were the only shock, we could express these innovations as $\mathrm{cov}\left(s_{t+j} - E_{t-1}s_{t+j}, i_t^* - i_t - E_{t-1}\left(i_t^* - i_t\right)\right)$.

It is common in the literature to invoke a "hump shape" in the impulse response function as an explanation for the Fama regression finding of $\mathrm{cov}(E_t s_{t+1} - s_t, i_t^* - i_t) > 0$. (Note that the literature usually expresses both the Fama regression and the impulse responses in terms of $i_t - i_t^*$ rather than $i_t^* - i_t$ as in this paper.) The "delayed overshooting" literature finds that initially the domestic currency appreciates as the home interest rate increases, so $\mathrm{cov}\left(s_t - E_{t-1}s_t, i_t^* - i_t - E_{t-1}\left(i_t^* - i_t\right)\right) > 0$. The impulse response of future changes in the exchange rate is in the direction of further appreciation: $\mathrm{cov}\left(s_{t+1} - s_t - E_{t-1}\left(s_{t+1} - s_t\right), i_t^* - i_t - E_{t-1}\left(i_t^* - i_t\right)\right) > 0$, for example. The impulse responses for subsequent changes in the exchange rate have the same sign for a number of periods before reversing sign, so that the maximum impulse response does not occur in the first period. This is in contrast to the implications of the Dornbusch model, and has been called delayed overshooting.

Does the finding of $\mathrm{cov}\left(s_{t+1} - s_t - E_{t-1}\left(s_{t+1} - s_t\right), i_t^* - i_t - E_{t-1}\left(i_t^* - i_t\right)\right) > 0$ offer an explanation for $\mathrm{cov}(E_t s_{t+1} - s_t, i_t^* - i_t) > 0$? Not necessarily. The model of section 3.2 offers a nice illustration of the point. It was noted in that section that $\mathrm{cov}(s_{t+1} - s_t, i_t - i_t^*) < 0$ holds if and only if $\alpha\left(1-\delta\right)\left(1-\theta^2\right) < \theta\delta - 1$. Recall that α determines the initial response of the exchange rate to the interest-rate shock, θ measures the persistence of the interest differential (modeled as a first-order autoregression), and δ determines the speed of adjustment of the exchange rate to the equilibrium exchange rate. We assumed $0 \le \delta < 1$, $0 \le \theta < 1$, and $\alpha < 0$. On the other hand, $\mathrm{cov}\left(s_{t+1} - s_t - E_{t-1}\left(s_{t+1} - s_t\right), i_t^* - i_t - E_{t-1}\left(i_t^* - i_t\right)\right) > 0$ holds if and only if $\alpha\left(1-\delta\right) < -1$. The two conditions happen to be equivalent if $\theta = \delta$, but there is no economic reason that condition should hold.

Notice that the condition for positive covariance of the impulse response does not depend at all on the persistence of the interest rate differential. Even if the impulse response function is hump-shaped, if the interest differential is persistent enough, it will not account for the uncovered interest parity puzzle in the form $\mathrm{cov}(E_t s_{t+1} - s_t, i_t^* - i_t) > 0$. That is not a trivial consideration given the persistence of interest rates among the

G7 countries. So a hump-shaped impulse response function is not sufficient to account for the interest-parity puzzle.

A.5.b

FR consider a VAR with the vector of variables $z_t' = [\rho_t \quad f_t \quad d_t \quad q_t]$. Here, ρ_t is defined as in this paper, and corresponds to FR's variable r_t . f_t is FR's measure of institutional investor flows, and does not play a role in this discussion. We adopt FR's notation for d_t, which is defined as:

$$d_t \equiv i_{t-1}^* - \pi_t^* - (i_{t-1} - \pi_t) \ .$$

q_t is the real exchange rate as defined in this paper, and corresponds to the variable δ_t in FR.

FR specify a VAR for z_t of the form:

$$z_t = \Gamma z_{t-1} + u_t \ .$$

FR do not impose any restrictions on the matrix Γ.

Recall $\rho_t \equiv i_{t-1}^* + s_t - s_{t-1} - i_{t-1} = d_t + q_t - q_{t-1}$.

Using FR's notation where ej is a column vector with a one in position *j* and 0 elsewhere, there should be a restriction of the form $e1'z_t = (e3' + e4')z_t - e4'z_{t-1}$. The excess return ρ_t is defined in terms of d_t, q_t and q_{t-1}, and so cannot have unrestricted dynamics that are defined independently. A key variable in FR's analysis is the innovation in the excess return, $\rho_t - E_{t-1}\rho_t$. Given the definition of ρ_t, we have $\rho_t - E_{t-1}\rho_t = d_t - E_{t-1}d_t + q_t - E_{t-1}q_t$, or $e1'u_t = (e3' + e4')u_t$. FR do not impose any restriction that insures this equality holds. So, one could use as measures of $\rho_t - E_{t-1}\rho_t$ either $e1'u_t$ or $(e3' + e4')u_t$ or any weighted average of the two. FR report only one of these measures, and do not test whether the overidentifying restriction is valid.

Actually, had FR estimated the system by OLS, equation-by-equation, the restriction would be imposed by the estimation. The covariance matrix for the system would be singular, but that is how it should be. To see this, write out the system in detail:

$$\begin{bmatrix} \rho_t \\ f_t \\ d_t \\ q_t \end{bmatrix} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \gamma_{14} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \gamma_{24} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} \\ \gamma_{41} & \gamma_{42} & \gamma_{43} & \gamma_{44} \end{bmatrix} \begin{bmatrix} \rho_{t-1} \\ f_{t-1} \\ d_{t-1} \\ q_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix} .$$

Let a ^ represent the OLS estimate of the parameter. Since $q_t - q_{t-1} = \rho_t - d_t$, one estimated relationship from this equation has:

$$q_t - q_{t-1} = (\hat{\gamma}_{11} - \hat{\gamma}_{31})\rho_{t-1} + (\hat{\gamma}_{12} - \hat{\gamma}_{32}) f_{t-1} + (\hat{\gamma}_{13} - \hat{\gamma}_{33}) d_{t-1} + (\hat{\gamma}_{14} - \hat{\gamma}_{34}) q_{t-1} + \hat{u}_{1t} - \hat{u}_{3t} .$$

Another estimated relationship is:

$$q_t - q_{t-1} = \hat{\gamma}_{41}\rho_{t-1} + \hat{\gamma}_{42} f_{t-1} + \hat{\gamma}_{43} d_{t-1} + \left(\hat{\gamma}_{44} - 1\right) q_{t-1} + \hat{u}_{4t} .$$

From the properties of OLS estimators, it is the case that $\hat{\gamma}_{41} = \hat{\gamma}_{11} - \hat{\gamma}_{31}$, $\hat{\gamma}_{42} = \hat{\gamma}_{12} - \hat{\gamma}_{32}$, $\hat{\gamma}_{43} = \hat{\gamma}_{13} - \hat{\gamma}_{33}$, and $\hat{\gamma}_{44} - 1 = \hat{\gamma}_{14} - \hat{\gamma}_{34}$, so $\hat{u}_{4t} = \hat{u}_{1t} - \hat{u}_{3t}$. However, it is apparent from the discussion of estimation in FR on pages 1544-1545 that the overidentifying restrictions are not imposed, either deliberately or by dint of the estimation technique.

In fact, FR report moments involving the residuals of the first three elements of u_t but not the fourth. It is tempting to believe that an easy fix to the problem is for FR to drop q_t from their VAR. However, that points to another issue. Suppose their VAR only contains ρ_t and d_t but not q_t (we ignore f_t for simplicity.) So let $x_t' = \left[\rho_t \quad d_t\right]$, and consider a VAR of the form $x_t = \mathrm{B}x_{t-1} + v_t$. In this case, there is only one measure of $\rho_t - E_{t-1}\rho_t$ given by $e1'u_t$. However, FR base their analysis on this version of equation (3) above:

$$q_t = E_t \sum_{j=0}^{\infty} \left(r^*_{t+j} - r_{t+j}\right) - E_t \sum_{j=0}^{\infty} \rho_{t+j+1} ,$$

where, to make the correspondence between notation clear, $E_t\left(r^*_{t+j} - r_{t+j}\right) = E_t d_{t+j+1}$ for $j \geq 0$. One can construct measures of $E_t \sum_{j=0}^{\infty} \left(r^*_{t+j} - r_{t+j}\right)$ and $E_t \sum_{j=0}^{\infty} \rho_{t+j+1}$ from the VAR, as FR do from their VAR. However, the value of q_t constructed this way does not satisfy the identity $q_t = q_{t+1} + d_{t+1} - \rho_{t+1}$. Crucially for FR it therefore does not satisfy the relationship $\rho_t - E_{t-1}\rho_t = d_t - E_{t-1}d_t + q_t - E_{t-1}q_t$. Why not? The easiest way to see this is that the VAR involving the variables ρ_t and d_t is equivalent to a VAR in the two variables d_t and $q_t - q_{t-1}$, since ρ_t is constructed as the sum of d_t and $q_t - q_{t-1}$. However, such a VAR allows $q_t - q_{t-1}$ to be nonstationary, contrary to FR's assumption that it is stationary. FR's forward looking equation for the real exchange rate is not correct if the real exchange rate is nonstationary. It requires the additional term $\lim_{k \to \infty}(E_t q_{t+k})$, the permanent component of the real exchange rate, that appears in equation (3) above. That is,

$$q_t = \lim_{k \to \infty}(E_t q_{t+k}) + E_t \sum_{j=0}^{\infty} \left(r^*_{t+j} - r_{t+j}\right) - E_t \sum_{j=0}^{\infty} \rho_{t+j+1} .$$

If the VAR were instead defined in terms of d_t and q_t, which is a simplified version of the specification in this paper, no problem arises. In this specification ρ_t is a residual, calculated to satisfy $\rho_t = d_t + q_t - q_{t-1}$, and it follows that the relationship $\rho_t - E_{t-1}\rho_t = d_t - E_{t-1}d_t + q_t - E_{t-1}q_t$ holds, and there are no overidentifying restrictions that are ignored.

A.5.c

In order to compare the moments calculated in this paper with those that FR intended to calculate, following the logic of the previous paragraph, consider an example of a stochastic system involving d_t and q_t. It is useful to consider an MA representation. Assume:

$$q_t = \sum_{i=0}^{\infty} \alpha_i \varepsilon_{t-i} + \sum_{i=0}^{\infty} \beta_i u_{t-i}$$

$$d_t = \sum_{i=0}^{\infty} \gamma_i \varepsilon_{t-i} + \sum_{i=0}^{\infty} \delta_i u_{t-i} \; .$$

Here ε_t and u_t are i.i.d., mean zero, mutually uncorrelated random variables. We specify d_t and q_t as the sum of two moving average processes to facilitate intuition for the empirical findings in this paper.

The two moments we focus on in this paper are $\text{cov}(E_t \rho_{t+1}, r_t^* - r_t)$ and $\text{cov}\left(E_t \sum_0^{\infty} \rho_{t+j+1}, r_t^* - r_t\right)$, which equal $\text{cov}(E_t \rho_{t+1}, E_t d_{t+1})$ and $\text{cov}\left(E_t \sum_0^{\infty} \rho_{t+j+1}, E_t d_{t+1}\right)$, respectively. With a bit of work, one finds:

$$\text{cov}(E_t \rho_{t+1}, E_t d_{t+1}) = \text{var}(\varepsilon) \cdot \sum_{i=1}^{\infty} \gamma_i (\alpha_i - \alpha_{i-1} - \gamma_i) + \text{var}(u) \cdot \sum_{i=1}^{\infty} \delta_i (\beta_i - \beta_{i-1} - \delta_i)$$

$$\text{cov}\left(E_t \sum_0^{\infty} \rho_{t+j+1}, E_t d_{t+1}\right) = \text{var}(\varepsilon) \left[\sum_{i=1}^{\infty} \gamma_i \sum_{j=i}^{\infty} \gamma_i - \sum_{i=1}^{\infty} \gamma_i \alpha_{i-1} \right] + \text{var}(u) \left[\sum_{i=1}^{\infty} \delta_i \sum_{j=i}^{\infty} \delta_i - \sum_{i=1}^{\infty} \delta_i \beta_{i-1} \right].$$

The finding that $\text{cov}(E_t \rho_{t+1}, r_t^* - r_t)$ is positive and $\text{cov}\left(E_t \sum_0^{\infty} \rho_{t+j+1}, r_t^* - r_t\right)$ is negative is easy to reconcile in this general unstructured setting. With two economic forces driving real exchange rates and interest rates, one might be more important in the short run and one more important in the longer run. For example we might have $\text{var}(\varepsilon) > \text{var}(u)$, but the coefficients on ε in the moving average processes for d_t and q_t might die out more quickly than those on *u*.

FR's Tables V and VI calculate covariances of four innovations related to excess returns with four innovations related to interest rate differentials. In terms of our MA example, the innovations in excess returns (reading across the top row of FR's tables) are:

$$\rho_t - E_{t-1}\rho_t = (\alpha_0 + \gamma_0)\varepsilon_t + (\beta_0 + \delta_0) u_t$$

$$(E_t - E_{t-1})(\rho_{t+1} + \rho_{t+2} + \ldots + \rho_{t+k}) = \left(\alpha_k - \alpha_0 + \sum_{i=1}^{k} \gamma_i \right) \varepsilon_t + \left(\beta_k - \beta_0 + \sum_{i=1}^{k} \delta_i \right) u_t$$

$$(E_t - E_{t-1})(\rho_{t+k+1} + \rho_{t+k+2} + \ldots) = \left(-\alpha_k + \sum_{i=k+1}^{\infty} \gamma_i \right) \varepsilon_t + \left(-\beta_k + \sum_{i=k+1}^{\infty} \delta_i \right) u_t$$

$$(E_t - E_{t-1})(d_t + d_{t+1} + \ldots) = \sum_{i=0}^{\infty} \gamma_i \varepsilon_t + \sum_{i=0}^{\infty} \delta_i u_t$$

The innovations in interest rates (reading down the column in FR's tables) are:

$$d_t - E_{t-1}d_t = \gamma_0\varepsilon_t + \delta_0 u_t$$

$$(E_t - E_{t-1})(d_{t+1} + d_{t+2} + \ldots + d_{t+k}) = \sum_{i=1}^{k}\gamma_i\varepsilon_t + \sum_{i=1}^{k}\delta_i u_t$$

$$(E_t - E_{t-1})(\rho_{t+k+1} + \rho_{t+k+2} + \ldots) = \sum_{i=k+1}^{\infty}\gamma_i\varepsilon_t + \sum_{i=k+1}^{\infty}\delta_i u_t$$

$$(E_t - E_{t-1})(d_t + d_{t+1} + \ldots) = \sum_{i=0}^{\infty}\gamma_i\varepsilon_t + \sum_{i=0}^{\infty}\delta_i u_t$$

(Note that the last innovation in each group is the same, but in FR is different because of their overidentification. Also, in deriving the expressions from FR, it is helpful to note a typo in that paper. In the lower part of page 1543, FR state $v_{er,t} = e1'\Phi u_t$ and $v_{iv,t} = e1'\Psi u_t$. But given how $v_{er,t}$ and $v_{iv,t}$ are defined on page 1541, these are expressions for $v_{er,t-1}$ and $v_{iv,t-1}$.)

Inspecting the covariances of the first set of innovations with the second set, we find no moments that correspond to $\text{cov}(E_t\rho_{t+1}, E_t d_{t+1})$ and $\text{cov}\left(E_t\sum_0^\infty \rho_{t+j+1}, E_t d_{t+1}\right)$ as calculated above. This is not surprising given the example in A.5.a, which showed how covariances of innovations may give quite different information than unconditional covariances.