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ABSTRACT

Value stocks are more exposed to disaster risk than growth stocks. Embedding disasters into an investment-based asset pricing model induces strong nonlinearity in the pricing kernel. Our single-factor model reproduces the failure of the CAPM in explaining the value premium in finite samples in which disasters are not materialized, and its relative success in samples in which disasters are materialized. The relation between pre-ranking market betas and average returns is flat in simulations, despite a strong positive relation between true market betas and expected returns. Evidence in the long U.S. sample from 1926 to 2014 lends support to the model's key predictions.

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1 Introduction

Fama and French (FF, 1992, 1993) show that despite their similar market betas, firms with high book-to-market equity (value firms) earn higher average stock returns than firms with low book-to-market equity (growth firms). This stylized fact is commonly referred to as the value premium puzzle. However, consistent with Ang and Chen (2007), we show that the FF evidence is specific to the post-1963 sample. The Capital Asset Pricing Model (CAPM) captures the value premium in the long U.S. sample from July 1926 to June 2014. The value premium is on average 0.52% per month ($t = 2.60$), its CAPM alpha is 0.23% ($t = 1.18$), and its market beta is 0.45 ($t = 3.52$).

We study whether accounting for rare disasters helps explain the value premium puzzle. We embed disasters into a standard investment-based asset pricing model. Our framework features three key ingredients: (i) severe but rare declines in aggregate consumption and productivity growth; (ii) asymmetric adjustment costs; and (iii) recursive preferences. The single-factor model explains the failure of the CAPM in capturing the value premium in finite samples in which disasters are not materialized, as well as its relative success in samples in which disasters are materialized.

Intuitively, with asymmetric adjustment costs, when a disaster hits, value firms are burdened with more unproductive capital, and find it more difficult to reduce capital than growth firms. As such, value firms are more exposed to the disaster risk than growth firms. Combined with higher marginal utilities of the agent in disasters, the model implies a sizeable value premium.

More important, the disaster risk induces strong nonlinearity in the pricing kernel, making the linear CAPM a poor empirical proxy to fully capture its pricing power. In particular, when disasters are not materialized in a finite sample, estimate market betas only measure the weak covariation of the value-minus-growth returns with market excess returns during normal times. However, the value premium is mainly driven by the higher exposures of value stocks to the disaster risk than growth stocks. Consequently, the CAPM fails to explain the value premium in normal times. In contrast, when disasters are materialized in a finite sample, the estimated market betas provide an adequate account for the large covariation between the value-minus-growth returns and the pricing kernel. As such, the CAPM captures the value premium in the sample with disasters. In all, we provide a disaster-based explanation for the value premium puzzle.

FF (2006) refute the Ang and Chen (2007) evidence that the CAPM captures the value premium in the long sample, and argue that the CAPM's general problem is that cross-sectional variation in the market beta goes unrewarded. We confirm the flat relation between the market beta and average returns, known as the beta anomaly, both in the post-1963 sample and in the long sample. More important, we show that our single-factor model is largely consistent with the beta "anomaly." In simulated samples both with and without disasters, sorting on pre-ranking market betas yields average return spreads that are small and insignificant, post-ranking beta spreads that are significantly positive, and CAPM alpha spreads that are negative and sometimes significant.

The crux is that rolling market betas are a poor proxy for the true betas. Intuitively, based on prior 60-month rolling windows, pre-ranking betas are average betas over the prior five years. In contrast, true betas accurately and immediately reflect changes in aggregate and firm-specific conditions. In simulations, true betas often mean-revert within a given rolling window, giving rise to negative correlations with rolling betas, especially in samples without disasters.

Our work has important implications. Most important, FF's (1992, 1993) claim of the death of the CAPM might have been exaggerated, at least in the context of the value premium. Instead, we lend credence to the largely overlooked work by Ang and Chen (2007). FF's (2006) refutation of their evidence based on pre-ranking beta sorts is suspect. Firm-level beta estimates come with a great deal of measurement errors. We offer a fully specified model, in which rolling beta sorts yield no average return spreads, yet the CAPM captures the value premium in time series regressions.

Building on Berk, Green, and Naik (1999), early investment-based asset pricing models reproduce the level of the value premium with only one aggregate shock (e.g., Carlson, Fisher, and Giammarino (2004), Zhang (2005), and Cooper (2006)). However, the CAPM roughly holds in these models. More recent models try to break the tight link between the pricing kernel and market excess returns by introducing multiple aggregate shocks.[1] Although successful in explaining the failure of the CAPM in the post-1963 sample, these models are at odds with the long-sample evidence. In contrast, while retaining the single-shock structure to fit the long-sample evidence, our model manages to fail the CAPM via disaster-induced nonlinearity.

Our work also expands the disaster literature, which uses disasters to explain asset pricing puzzles, so far mostly at the aggregate level. Barro (2006, 2009) revives the disaster explanation of the equity premium in Rietz (1988), by calibrating the disaster model to international macroeconomic data. Gourio (2012) embeds disasters into a production economy to jointly explain the equity premium and business cycles. Gabaix (2012) uses time-varying exposures of cash flow to disasters, and Wachter (2013) uses time-varying disaster probability to explain time-varying risk premiums. We integrate the disaster literature with investment-based asset pricing, and study the impact of the disaster risk on the cross section of returns. In particular, we show how embedding disasters into a standard investment model enables it to explain the failure of the CAPM in the post-1963 sample.

The rest of the paper is organized as follows. Section 2 presents the stylized facts. Section 3 constructs the model. Section 4 contains the quantitative results. Finally, Section 5 concludes.

2 Stylized Facts

Table 1 reports the monthly CAPM regressions for the ten book-to-market deciles and the value-minus-growth decile in the U.S. data. The monthly returns data for the deciles, the value-weighted market portfolio, and the one-month Treasury bill rate are from Kenneth French's data library.

Confirming the well-known evidence reported in FF (1992, 1993), Panel A shows that the CAPM fails to explain the value premium in the sample after July 1963. Moving from the growth decile to the value decile, the average excess return rises from 0.42% per month to 0.93%, and the average

[1] Prominent examples include short-run and long-run risks in Ai and Kiku (2013), investment-specific technological shocks in Kogan and Papanikolaou (2013), stochastic adjustment costs in Belo, Lin and Bazdresch (2014), shocks to financing costs in Belo, Lin, and Yang (2014), and uncertainty shocks in Koh (2014).

return spread (the value premium) is 0.51% ($t = 2.75$). Despite the nearly monotonic relation between book-to-market and average excess returns, the market beta is largely flat across the deciles. The value-minus-growth decile has only a tiny market beta of 0.01 ($t = 0.07$). Accordingly, its CAPM alpha is 0.51% ($t = 2.26$), which is identical in magnitude to the average value premium.

However, consistent with the evidence first reported (to our knowledge) in Ang and Chen (2007), Panel B shows that the CAPM captures the value premium in the long sample from July 1926 to June 2014. The average excess returns vary from 0.57% per month for the growth decile to 1.09% for the value decile. The value premium is on average 0.52% ($t = 2.60$), which is close to that in the post-1963 sample. More important, the CAPM captures the average value premium, with an insignificant alpha of 0.23% ($t = 1.18$) and a significant market beta of 0.45 ($t = 3.52$). From Panel C, the CAPM also performs well from July 1926 to June 1963. The average value premium is still 0.52% per month, albeit insignificant ($t = 1.31$). The market beta is large, 0.74 ($t = 5.33$). As such, the CAPM alpha becomes negative, -0.11% ($t = -0.34$).

To shed light on the differences across the post-1963 sample and the long sample, Table 2 reports large market swings with market excess returns below 1.5 and above 98.5 percentiles of the empirical distribution, as well as the corresponding months and the value-minus-growth returns. There are in total 32 such observations, 24 of which are from the Great Depression. When market excess returns are low, the value-minus-growth returns tend to be low, and when market excess returns are high, the value-minus-growth returns tend to be high. Their correlation is 0.73 across these observations. In particular, the lowest value premium is -22.67% in March 1938, which comes with an abysmally low market excess return of -23.80%. The highest value premium is 69.99% in August 1932, which comes with an exuberantly high market excess return of 36.41%. More recently, following the bankruptcy of Lehman Brothers, the market excess return is -17.23% in October 2008, in which the value-minus-growth return is -11.93%.

Figure 1 presents the scatter plots and fitted market regression lines for the value-minus-growth returns for the long sample (Panel A) and the post-1963 sample (Panel B). In Panel A, we highlight in red the observations with monthly market excess returns below 1.5 and above 98.5 percentiles of the empirical distribution. These observations clearly contribute to the market beta of 0.45 ($t = 3.52$) for the value-minus-growth decile in the long sample. In contrast, Panel B shows that large swings in the stock market are scarce in the post-1963 sample, giving rise to a flat regression line. In all, the CAPM does a good job in explaining the value premium in the long sample that includes the Great Depression, but that the CAPM fails in the short post-1963 sample.

While large swings in the stock market have a large impact on the performance of the CAPM, its relative success in the long sample is robust to extreme observations. Table 3 evaluates the robustness of the long-term performance of the CAPM to extreme observations. From Panel A, even after we remove the observations with extreme 3% of market excess returns, the CAPM continues to do well with an alpha of 0.17% per month ($t = 0.81$) and a market beta of 0.20 ($t = 2.54$) for the value-minus-growth decile. However, its average return, 0.30%, is only marginally significant. Removing extreme 10% observations of market excess returns, we obtain an average value premium of 0.43% ($t = 2.72$), an insignificant alpha of 0.28%, and a significant beta of 0.20. Perhaps surprisingly, only after we remove 50% of market excess returns, do we observe a significant value-minus-growth alpha of 0.56% ($t = 2.18$). The average return is 0.63% ($t = 3.33$), and the market beta is 0.07 ($t = 0.53$). The evidence suggests that the CAPM performs quite well in the long sample, and that the Great Depression is important, with its risk priced in the value-minus-growth decile, even after the extreme observations are removed (perhaps due to investor expectations of the disaster risk).

FF (2006) refute the evidence that the CAPM captures the value premium in the long sample (Table 1, Panel A). FF argue that the CAPM's problem is that cross-sectional variation in the market beta goes unrewarded. This flat relation between the market beta and average returns, known as the beta anomaly, has a long tradition in empirical finance (e.g., Fama and MacBeth (1973), FF (1992, 1993), and Frazzini and Pedersen (2014)). Table 4 presents evidence on this flat relation.

At the end of June of each year t, we sort all stocks into deciles based on the NYSE breakpoints of pre-ranking betas, which are estimated from rolling-window regressions of firm-level excess returns on the market excess returns in the prior 60 months (24 months minimum). We calculate value-weighted decile returns, and the deciles are rebalanced in June. Table 4 reports the average returns and the CAPM regressions for the market beta deciles. The sample starts in July 1928 because we use the data from the first two years to estimate the pre-ranking betas in June of 1928.

Panel A shows that, contradicting the CAPM, the relation between the market beta and the average return in the post-1963 sample is largely flat. Moving from the low to high beta decile, the average excess return rises from 0.51% per month to 0.63%, and the spread of only 0.12% is within 0.5 standard errors from zero. More seriously, the pre-ranking beta sorts yield a large spread in the post-ranking beta, 1.04 ($t = 11.41$), across the extreme deciles. As a result, the CAPM alpha for the high-minus-low decile is large and negative, -0.40%, albeit insignificant ($t = -1.48$).

From Panel B, the evidence from the long sample seems even stronger. The average excess return rises from 0.57% per month for the low beta decile to only 0.81% for the high beta decile. The average return spread of 0.24% is within one standard error from zero. More important, the post-ranking beta moves from 0.57 to 1.70, and the spread of 1.12 is more than 18 standard errors from zero. As a result, the CAPM alpha for the high-minus-low beta decile is significantly negative, -0.47% ($t = -2.40$). Panel C reports largely similar evidence for the pre-1963 sample. The average return for the high-minus-low beta decile is 0.40% ($t = 0.88$), and its post-ranking beta is 1.18 ($t = 16.04$). The CAPM alpha is large, -0.56%, albeit only marginally significant ($t = -1.94$).

3 An Investment Model with Disasters

Our model is constructed by combining elements from the investment-based asset pricing model of Zhang (2005) and the Rietz-Barro disasters model (e.g., Rietz (1988) and Barro (2006, 2009)). The economy consists of a representative agent with the Epstein-Zin (1989) utility and heterogenous firms. The firms take the representative agent's intertemporal rate of substitution as given when determining their optimal policies. The production technology is subject to both aggregate and firm-specific shocks. The aggregate shock contains a disaster state as well as a recovery state.

3.1 The Representative Agent

The representative agent has the recursive utility, U_t, defined over aggregate consumption, C_t:

$$U_t = \left[(1-\iota)C_t^{1-\frac{1}{\psi}} + \iota\left(E_t\left[U_{t+1}^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \tag{1}$$

in which ι is the time discount factor, ψ is the intertemporal elasticity of substitution, and γ is the relative risk aversion. The pricing kernel is given by:

$$M_{t+1} = \iota\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{U_{t+1}^{1-\gamma}}{E_t\left[U_{t+1}^{1-\gamma}\right]}\right)^{\frac{1/\psi-\gamma}{1-\gamma}}. \tag{2}$$

Consumption Dynamics

The log consumption growth, denoted $g_{ct} \equiv \log(C_t/C_{t-1})$ is specified as:

$$g_{ct} = \overline{g} + g_t, \tag{3}$$

in which g_t follows a first-order autoregressive process per Mehra and Prescott (1985). Specifically,

$$g_{t+1} = \rho_g g_t + \sigma_g \epsilon_{t+1}, \tag{4}$$

in which ϵ_{t+1} is a standard normal shock, and the unconditional mean of g_t is zero.

After calibrating the parameter values of ρ_g and σ_g to the consumption growth data in normal times (Section 4.1), we use the Rouwenhorst (1995) procedure to discretize g_t into a grid with five values $\{g_1, g_2, g_3, g_4, g_5\}$. The discretization also produces a transition matrix, $P_\star$, given by:

$$P_\star = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{15} \\ p_{21} & p_{22} & \cdots & p_{25} \\ \vdots & \vdots & \ddots & \vdots \\ p_{51} & p_{52} & \cdots & p_{55} \end{bmatrix}, \tag{5}$$

in which p_{ij}, for $i, j = 1, \ldots, 5$, is the probability of $g_{t+1} = g_j$ conditional on $g_t = g_i$.

To incorporate disaster risk into the model, we modify directly the discretized g_t grid and its transition matrix, following Danthine and Donaldson (1999). Specifically, we insert into the g_t grid a disaster state, $g_0 = \lambda_D$, in which $\lambda_D < 0$ is the disaster size, as well as a recovery state, $g_6 = \lambda_R$, in which $\lambda_R > 0$ is the recovery size. Accordingly, we form the transition matrix, P, by modifying $P_\star$ to incorporate the disaster and recovery states as follows:

$$P = \begin{bmatrix} \theta & 0 & 0 & \ldots & 0 & 1-\theta \\ \eta & p_{11}-\eta & p_{12} & \ldots & p_{15} & 0 \\ \eta & p_{21} & p_{22}-\eta & \ldots & p_{25} & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \eta & p_{51} & p_{52} & \ldots & p_{55}-\eta & 0 \\ 0 & (1-\nu)/5 & (1-\nu)/5 & \ldots & (1-\nu)/5 & \nu \end{bmatrix}. \tag{6}$$

In the modified transition matrix, η is the probability of entering the disaster state from any of the normal states, and θ is the probability of remaining in the disaster state next period conditional on the economy being in the disaster state in the current period. As such, θ is the persistence of the disaster state. Similarly, ν is the persistence of the recovery state. In addition, in constructing the transition matrix, we have implicitly assumed that the economy can only enter the recovery state following a disaster. Once in the recovery state, the economy can enter any of the normal states with an equal probability, $(1-\nu)/5$, but cannot fall immediately back into the disaster state. Finally, it should be emphasized that our model retains only one aggregate state, g_t, in the economy.

3.2 Firms

The economy is also populated by heterogeneous firms, indexed by i.

Production

Firms produce output with capital, and are subject to both aggregate and firm-specific shocks. Operating profits, $\Pi_{it} \equiv \Pi(K_{it}, Z_{it}, X_t)$, for firm i are given by:

$$\Pi_{it} = (X_t Z_{it})^{1-\xi} K_{it}^{\xi} - f K_{it}, \tag{7}$$

in which $\xi > 0$ is the curvature parameter, X_t is the aggregate productivity, Z_{it} is the firm-specific productivity, and K_{it} is capital. The fixed costs of production are fK_{it}, with $f > 0$, and are scaled by capital to ensure that these costs do not become trivially small along the balanced growth path.

The aggregate productivity growth, $g_{xt} \equiv \log(X_t/X_{t-1})$, is linked to the consumption growth:

$$g_{xt} = \overline{g} + \phi g_t, \tag{8}$$

in which $\phi > 0$. The firm-specific productivity for firm i, Z_{it}, has a transition function given by:

$$z_{it+1} = (1 - \rho_z)\overline{z} + \rho_z z_{it} + \sigma_z e_{it+1}, \tag{9}$$

in which $z_{it} \equiv \log Z_{it}$, $\overline{z}$ is the unconditional mean of z_{it} common to all firms, and e_{it+1} is an independently and identically distributed standard normal shock. We assume that e_{it+1} and e_{jt+1} are uncorrelated for any $i \neq j$, and ϵ_{t+1} and e_{it+1} are uncorrelated for all i.

Adjustment Costs

Let I_{it} denote firm i's investment at time t. Capital accumulates as follows:

$$K_{it+1} = I_{it} + (1 - \delta)K_{it}, \tag{10}$$

in which δ is the rate of depreciation. Capital investment entails asymmetric adjustment costs:

$$\Phi_{it} \equiv \Phi(I_{it}, K_{it}) = \begin{cases} a^+ K_{it} + \frac{c^+}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it} & \text{for} \quad I_{it} > 0 \\ 0 & \text{for} \quad I_{it} = 0 \\ a^- K_{it} + \frac{c^-}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it} & \text{for} \quad I_{it} < 0 \end{cases}, \tag{11}$$

in which $a^- > a^+ > 0$ and $c^- > c^+ > 0$ capture the nonconvex adjustment costs.

Value Maximization

Upon observing current period productivity levels X_t and Z_{it}, firm i makes optimal investment decision, I_{it}, and optimal exit decision, χ_{it}, to maximize its market value of equity. Let $D_{it} \equiv \Pi_{it} - I_{it} - \Phi(I_{it}, K_{it})$ denote dividends. The cum-dividend market value of equity, V_{it}, is given by:

$$V_{it} \equiv V(K_{it}, Z_{it}, X_t) = \max_{\{\chi_{it}\}} \left(\max_{\{I_{it}\}} D_{it} + E_t \left[M_{t+1} V(K_{it+1}, X_{t+1}, Z_{it+1}) \right], \ sK_{it} \right), \qquad (12)$$

in which $s > 0$ is the liquidation value parameter, subject to the capital accumulation equation (10).

When $V_{it} \geq sK_{it}$, which is the exit threshold, firm i does not exit from the economy, i.e., $\chi_{it} = 0$. For incumbent firms that remain in the economy, evaluating the value function at the optimum yields $V_{it} = D_{it} + E_t[M_{t+1}V_{it+1}]$. Equivalently, $E_t[M_{t+1}R_{it+1}] = 1$, in which $R_{it+1} \equiv V_{it+1}/(V_{it} - D_{it})$ is the stock return, and $E_t[R_{it+1}] = r_{ft} + \beta_{it}^M \lambda_{Mt}$, in which $r_{ft} \equiv 1/E_t[M_{t+1}]$ is the real interest rate, $\beta_i^M \equiv -\text{Cov}_t[r_{it+1}^S, M_{t+1}]/\text{Var}_t[M_{t+1}]$ is the true beta, and $\lambda_{Mt} \equiv \text{Var}_t[M_{t+1}]/E_t[M_{t+1}]$ is the price of risk.

When $V_{it} < sK_{it}$, firm i exits from the economy at the beginning of time t, i.e., $\chi_{it} = 1$. We set its stock return from the beginning of period $t-1$ to the beginning of t, R_{it}, to be a predetermined delisting return. Shumway (1997) shows that the average delisting return is -30% for stocks on New York Stock Exchange and American Stock Exchange. Shumway and Warther (1999) show that the average delisting return is -55% for stocks on National Association of Securities Dealers Automated Quotations. We set the delisting return to be -42.5%, which is the average estimate.

When firm i exits from the economy at the beginning of time t, we assume that it enters an immediate reorganization process. The current shareholders of the firm receive sK_{it} as the liquidation value and leaves, and the old firm ceases to exit. New shareholders take over the remainder of the firm's capital, $(1 - s - \kappa)K_{it}$, in which $\kappa \in (0, 1 - s)$ is the reorganization cost parameter. For simplicity, we assume that the reorganization process occurs instantaneously. At the beginning of

t, the old firm is replaced by a new firm with an initial capital of $(1-s-\kappa)K_{it}$ owned by new shareholders. The new firm draws a firm-specific log productivity, z_{it}, from its unconditional distribution, which is normal with a mean of zero and a standard deviation of $\sigma_z/\sqrt{1-\rho_z^2}$. For computational tractability, our modeling of entry and exit keeps the number of firms constant in the economy.

4 Quantitative Results

We calibrate the model in Section 4.1, present the results on the CAPM in Section 4.2, discuss the key intuition in Section 4.3, conduct comparative statics on the CAPM performance in Section 4.4, and examine the model's implications on the beta anomaly in Section 4.5.

4.1 Calibration

We calibrate the model in monthly frequency. Table 5 reports the parameter values for the benchmark calibration. For the preference parameters, we set the intertemporal elasticity of substitution, ψ, to be 1.5 as in Bansal and Yaron (2004). The relative risk aversion, γ, is five. The time discount factor, ι, is set to be 0.99035 to generate an average risk free rate about 1.8% per annum.

For the parameters that govern the consumption dynamics in normal times, we set the average growth rate, $\overline{g}$, to be 0.019/12, the persistence $\rho_g = 0.6$, and the conditional volatility $\sigma_g = 0.0025$ to be consistent with the data of per capita real consumption (nondurables and services) growth from 1947 to 2013.[2] The implied consumption growth volatility is 1.43% per annum in simulated quarterly data, and is not far from 1.02% in the real data. In simulated annual data, the consumption growth volatility is 1.7%, which is not far from 1.25% in the real data.[3] The first-order autocorrelation is 0.49 in simulated quarterly data and 0.26 in simulated annual data. While the former is higher than 0.30 in the real quarterly data, the latter is lower than 0.49 in the real annual data.

For the parameters that govern the disaster dynamics, we set the disaster persistence, $\theta = 0.97$,

[2] The quarterly and annual consumption data are from National Income and Product Accounts (NIPA) Table 7.1.

[3] We have also experimented with an alternative estimation, in which the consumption growth volatility is 1.8% per annum from 1947 to 2013 in the real data. In this calculation, we scale nondurables and services from NIPA Table 1.1.5 by the consumer price index from Bureau of Labor Statistics and then by population from NIPA Table 7.1.

which is the probability that the economy remains in the disaster state in the next month conditional on it being in the disaster state in the current month. This monthly persistence accords with the quarterly persistence of $0.97^3 = 0.914$ in Gourio (2012). As such, the average duration of disasters is $1/(1 - 0.97) = 33$ months (roughly three years), consistent with the evidence in Barro and Ursua (2008). We set the disaster probability η to be 0.028/12, which implies an annual disaster probability of 2.8% estimated in Nakamura, Steinsson, Barro and Ursua (2013). Gourio uses a quarterly disaster probability of 0.72%, also consistent with an annual probability of 2.8%.

We still have three disaster parameters remaining, including the disaster size, λ_D, the recovery size, λ_R, and the recovery persistence, ν. We experiment with different values so that the impulse response of log consumption to a disaster shock in the model mimics that in the real data (e.g., Nakamura, Steinsson, Barro and Ursua (2013)). This procedure yields $\lambda_D = -0.0275$, $\lambda_R = 0.0325$, and $\nu = 0.95$. Figure 4 reports that the model's impulse response is roughly comparable with that in the data. In particular, the maximum short-term effect of disasters in the model is a fall of about 23% in log consumption, which is somewhat smaller than 29% in the data reported by Nakamura et al. (Table 2). The log-term negative effect is 15% fall in log consumption, which is close to 14% in the data.

The remaining parameters govern the various technologies in the economy. We set the curvature parameter in the production function, $\xi = 0.65$, which is close to the estimate of Hennessy and Whited (2007). The monthly depreciation rate, δ, is 0.01, which implies an annual rate of 12% as estimated by Cooper and Haltiwanger (2006). The persistence, ρ_z, and conditional volatility, σ_z, of the firm-specific productivity are set to be 0.985 and 0.5, respectively, which are somewhat larger than the corresponding values in Zhang (2005) after adjusting for the curvature parameter ξ. We do so to enlarge the cross-sectional dispersion of firms. The long-run mean of log firm-specific productivity, $\bar{z}$, is -9.75, so that the long-run average detrended capital is around unity in simulations.

We set the liquidation parameter, $s = 0$, meaning that shareholders receive nothing in the event of bankruptcy. The reorganizational cost parameter, κ, is 0.25. The leverage for the aggregate productivity, ϕ, is set to be unity to generate an annual output growth volatility of 2.43% in normal times. This volatility is close to 2.30% in the data, in which output is the per capita real gross domestic product from 1947 to 2013 (NIPA Table 7.1). Finally, we set the adjustment cost parameters $a^+ = 0.035$, $a^- = 0.05$, $c^+ = 75$, $c^- = 150$, and the fixed costs parameter, $f = 0.005$. Because of the lack of direct evidence on their values, we calibrate these parameters to match the properties of the book-to-market deciles. Section 4.4 contains extensive comparative statics on the quantitative impact of these parameter values on our key results.

4.2 Explaining the Performance of the CAPM

We simulate 2,000 artificial samples from the model, each with 5,000 firms and $1,500$ months. We start each simulation by setting the initial capital stocks of all firms at unity and by drawing their firm-specific productivity from the unconditional distribution of Z_{it}. We drop the first 444 months to neutralize the impact of the initial condition. The remaining $1,056$ months of simulated data are treated as those from the economy's stationary distribution. The sample size is comparable to the period from July 1926 to June 2014 in the real data (Table 1). We calculate the model moments on each artificial sample and report cross-simulation averaged results.

Table 6 reports the model's key results under the benchmark calibration. Panel A shows the CAPM regressions for ten book-to-market deciles in the samples with at least one disaster episode (1,847 out of 2,000 samples). The average returns of the deciles increase with book-to-market from 0.74% per month for the growth decile to 1.19% for the value decile. The average value-minus-growth return is 0.45% per month, which is not far from 0.52% in the data (Table 1, Panel A). However, despite the lower value premium in the model, its t-statistic is 5.83, which is more than twice as large as 2.60 in the data. As such, the volatility of the value-minus-growth returns in the model is less than one half of that in the data. Similarly, the t-statistics for the individual deciles in the model are more than three times larger than those in the data. As such, stock return volatilities in our model are lower than those in the data, in line with the aggregate-level results in Barro (2006, 2009).

The CAPM holds in the model's artificial samples with disasters. The value-minus-growth alpha is -0.21% ($t = -1.72$), and its 95% confidence interval across simulations spans from -0.46% to 0.06%. Accordingly, the market beta is estimated to be 0.82 ($t = 6.82$). The beta also aligns monotonically with book-to-market, rising from 0.82 for the growth decile to 1.64 for the value decile.

Panel B reports the CAPM regressions for the book-to-market deciles in the samples without disasters (153 out of 2,000 samples). The average returns continue to align monotonically with book-to-market, going from 0.77% per month to 1.21%. The average return of the value-minus-growth decile, 0.45% per month, is identical to that from the disaster samples. More important, Panel B shows that the CAPM fails badly in the no-disaster samples. The CAPM regression of the value-minus-growth decile yields an alpha of 0.47% ($t = 3.71$). The 95% confidence interval for the alpha spans from 0.22% to 0.75%. In addition, the market beta for the value-minus-growth decile is insignificantly negative, -0.03 ($t = -0.24$). Finally, the R^2 is in effect zero. In all, the quantitative results in Table 6 on the CAPM regressions are largely aligned with the evidence in Table 1.

4.3 Inspecting the Mechanism

What is the driving force behind the quantitative results in Table 6? To shed light on this important issue, we first show that value stocks are more exposed to disaster risk than growth stocks. We then illustrate how the disaster risk is reflected in the CAPM regressions of the value-minus-growth returns. Finally, we examine how the disaster risk induces strong nonlinearity in the pricing kernel.

The Impact of Disaster Risk on Firm-level Risk and Risk Premiums

We characterize the impact of disaster risk using the model's solution and impulse responses. First, we use the model's solution on the $\widehat{K}_{it}$-g_t-z_{it} grid to plot the true beta, β_{it}^M, and the expected risk premium, $E_t[R_{it+1}] - r_{ft}$, against the detrended capital, $\widehat{K}_{it}$, and the log firm-specific productivity, z_{it}. To examine the impact of disaster risk, we make the plots for two values of the detrended consumption growth, g_t, including the disaster state, λ_D, and the mean of normal states (zero).

Panel A in Figure 3 shows that in disasters, low-z firms, which tend to be value firms, are substantially riskier than high-z firms, which tend to be growth firms. The mechanism is similar in nature to, but more powerful quantitatively than that in Zhang (2005) and Cooper (2006). Briefly, because of asymmetric adjustment costs, value firms are burdened with more unproductive capital, finding it more difficult to downsize than growth firms. As such, value firms are riskier than growth firms in bad times. While prior studies describe the working of the mechanism in recessions, Panel A extends the mechanism to rare disasters. Accordingly, Panel B shows that low-z firms charge substantially higher risk premiums than high-z firms in the disaster state.

In contrast, Panels C and D show that risk and risk premiums are largely flat across firms in the normal states. When the overall economy is doing fine, even low-z value firms do not have strong incentives to disinvest. As such, the mechanism of asymmetric adjustment costs fails to take strong effect, giving rise to weak spreads in risk and risk premiums across firms. Also, in each panel, only a portion of the $\widehat{K}_{it}$-z_{it} grid is plotted. This missing region is where firms exit the economy. Naturally, low-z firms are more likely to exit than high-z firms. In addition, because the fixed costs of production are proportional to capital, high-$\widehat{K}$ firms are more likely to exit than low-$\widehat{K}$ firms. In simulations, value firms tend to have lower z but higher $\widehat{K}$ than growth firms.

We also quantify the impact of disaster via impulse responses. We simulate the economy for 150,000 months, a period that contains slightly more than 300 disaster episodes. An episode starts with the economy switching from a normal state to the disaster state, and ends with it switching from the recovery state to a normal state. We trace the responses in risk and risk premiums for 25 years after an initial disaster shock, and calculate the average responses across the disaster episodes.

Figure 4 reports the mean responses for the value decile, the growth decile, and the averages across all ten book-to-market deciles. From Panel A, the true value beta jumps up by 0.37, and the true growth beta up by almost 0.20, in response to a disaster shock. To get a sense of the magnitude, the long-term average is around 0.04 for the value beta and 0.02 for the growth beta. As such, disasters cause an enormous jump in risk, more so for value than for growth firms. Accordingly, Panel B shows that in response to a disaster shock, the expected risk premium of the value decile jumps up by more than 400% over the subsequent two years, and that of the growth decile up by almost 100% over the same period. Again, to get a sense of the magnitude, the long-term average is about 15% per annum for the value decile and 9% for the growth decile. In all, consistent with the evidence in Section 2, our model implies that value stocks are more exposed to disaster risk than growth stocks.

Nonlinearity in the CAPM Regressions

The disaster risk has important implications on the CAPM regressions. Figure 5 reports the scatter plots for the CAPM regressions of the value-minus-growth decile. Panel A is the scatter plot from stacking the simulated data for all the 1,847 out of 2,000 artificial samples (the same samples used in Table 6) with at least one disaster episode. Panel B is the scatter plot from stacking the simulated data for the remaining 153 samples without disasters.

The basic patterns mimic those in Figure 1 based on the long U.S. sample. From Panel A of Figure 5, the value premium covaries strongly with market excess returns in disaster samples. Both returns are large and negative in disasters, and large and positive in the subsequent recoveries. As a result, the market beta for the value-minus-growth decile is 0.86, which is a population moment because of the large number of simulations. However, the CAPM alpha is -0.24% per month, meaning that the CAPM does not hold exactly in our dynamic single-factor model, even though the CAPM alpha is insignificant in finite samples (Table 6, Panel A).

In contrast, Panel B of Figure 5 shows that the value premium does not covary at all with market excess returns in artificial samples with no disasters. Without the large swings in the same direction in the value premium and market excess returns in disasters as well as in subsequent recoveries, the CAPM regression line of the value premium is flat. As population moments, the market beta is in effect zero, and the CAPM alpha is 0.45% per month. This simulation result reproduces the U.S. evidence from 1963 to 2014 (Figure 1, Panel B).

Nonlinearity in the Pricing Kernel

The disaster risk induces strong nonlinearity in the pricing kernel, nonlinearity which in turn implies that the CAPM is a poor proxy of the pricing kernel. If the CAPM holds exactly, the pricing kernel can be expressed as a linear function of the market excess return, R_{Mt+1}, i.e., $M_{t+1} = l_0 + l_1 R_{Mt+1}$, in which l_0 and l_1 are constants (e.g., Cochrane (2005)). Panels C and D of Figure 5 show that the pricing kernel in the model is far from a linear function of the market excess return.

In particular, Panel C reports the scatter plot for regressing the pricing kernel on the market excess return based on the disaster samples. (The results from all the 2,000 artificial samples are quantitatively similar.) The regression yields a slope of -0.35 but an R^2 of only 25%, despite the model's single-factor structure. The linear CAPM fits poorly the observations from the disaster state, with very high realizations of the pricing kernel (marginal utilities), and the observations from the recovery state, with very low realizations of the pricing kernel. Panel D shows that the CAPM is an even worse proxy for the pricing kernel in the samples without disasters. The slope from regressing the pricing kernel on the market excess return is only -0.03, falling in magnitude from -0.35 from the disaster samples. The R^2 also drops from 25% to 11%.

4.4 Comparative Statics

To gain further insights into the working of the key mechanism, we conduct comparative static experiments on a wide array of parameters. We group the parameters into three categories: (i) disaster dynamics: the disaster size, λ_D, the disaster persistence, θ, the disaster probability, η, the recovery persistence, ν, and the recovery size, ν; (ii) technology: the adjustment costs parameters, a^+, a^-, c^+, and c^-, the curvature in production, ξ, the fixed costs parameter, f, the productivity growth leverage, ϕ, the liquidation value parameter, s, the reorganizational costs parameter, κ, and the delisting return, $\widetilde{R}$; as well as (iii) preferences: the risk aversion, γ, and the intertemporal elasticity of substitution, ψ. In each experiment, we only vary one parameter, while keeping all the others unchanged from the benchmark calibration. The only exception is that, when necessary, we adjust the time discount factor, ι, to pin the average interest rate at around 1.8% per annum. Finally, for each parameter, we consider two values, with one above and the other below the benchmark value.

Disaster Dynamics

From the first four columns in the upper panel of Table 7, increasing the disaster size and persistence raises the value premium, and worsens the failure of the CAPM in samples without disasters. Intuitively, a higher magnitude for λ_D and θ strengthens the nonlinear, disaster dynamics, making the linear CAPM an even poorer proxy for the pricing kernel, especially in normal times. The effect of raising the disaster probability, η, goes in the same direction, but its quantitative impact is minimal. Intuitively, η mainly determines the percentage of samples with at least one disaster episode out of 2,000 simulations. This percentage is 92% in the benchmark calibration with $\eta = 0.23\%$, and 74% with $\eta = 0.13\%$ and 97% with $\eta = 0.33\%$. However, conditional on at least one disaster appearing in a given sample, the nonlinear dynamics are mostly governed by the disaster size and persistence.

The recovery size and persistence have little impact on the magnitude of the value premium and the performance of the CAPM. Intuitively, risk and risk premiums are mostly determined by the model's dynamics in bad times, particularly rare disasters, in which the representative agent's marginal utilities are the highest. In contrast, in the recovery state, in which marginal utilities are at their lowest level, the risk and risk premium spreads between value and growth firms are tiny.

Technology

The last eight columns in the upper panel of Table 7 report the comparative statics with the four adjustment costs parameters. The upward nonconvex costs parameter, a^+, and its downward counterpart, a^-, work in the opposite direction. While increasing a^+ reduces the value premium and its CAPM alpha in normal times, increasing a^- does the opposite. The a^- effect works through the familiar Zhang-Cooper asymmetry mechanism. A high value of a^- means that value firms face a higher hurdle in reducing their unproductive capital, giving rise to higher risk and risk premiums.

Why does the upward nonconvex costs parameter, a^+, work differently? Intuitively, firms disinvest very infrequently. In a typical simulation, only 0.5% of the firm-month observations have negative investment. Such a low disinvestment frequency means that a^+ is the main parameter that determines the magnitude of nonconvex costs of adjustment, a^+K_{it}. A lower value of a^+ means that firms would in general have higher capital, including value firms. When a disaster hits, value firms would be burdened with more unproductive capital, reinforcing the asymmetry mechanism. As such, a lower a^+ value increases the value premium and its CAPM alpha in no-disaster samples.

The upward convex costs parameter, c^+, works in the same direction as its downward counterpart, c^-. A higher value of c^- restricts the flexibility of value firms in reducing their unproductive capital. Although applying to positive investment, c^+ also limits value firms in adjusting their capital. However, the c^+ effect works in the opposite direction as the a^+ effect. The crux is that c^+ involves quadratic adjustment costs, which do not reduce capital as directly as a^+ in nonconvex costs.

From the first two columns in the lower panel of Table 7, decreasing the curvature in the production function by increasing ξ from 0.6 to 0.7 raises the value premium and the market beta of the value-minus-growth decile, especially in disaster samples. Intuitively, the sensitivity of profits to the aggregate productivity is $\partial\Pi_{it}/\partial X_t = (1-\xi)Z_{it}^{1-\xi}K_{it}^{\xi}X_t^{-\xi}$. In general, the relation between this sensitivity and ξ is ambiguous. However, the relation tends to be positive for value firms with high K_{it} and low Z_{it}, but negative for growth firms with low K_{it} and high Z_{it}. Also, the relation is more likely to be positive when X_t is particularly low, as in disasters.

The next two columns in the lower panel show that reducing the fixed costs parameter, f, increases the value premium and its CAPM alpha in no-disaster samples. At first glance, this result seems to contradict Zhang (2005), who reports that higher fixed costs imply a higher value premium. The crux is that the fixed costs are proportional to capital in our economy with balanced growth, whereas the fixed costs are constant, independent of capital, in Zhang's stationary economy. While the constant fixed costs work as operating leverage in his model, the fixed costs work in a similar way as the upward nonconvex costs of adjustment, a^+, in our model.

Increasing the productivity growth leverage, ϕ, raises the value premium and its CAPM alpha in normal times. Intuitively, a higher ϕ implies a proportionally larger aggregate shock, including that in the disaster state. More severe disasters in aggregate productivity increases the value premium, and strengthens the disaster dynamics, giving rise to a higher CAPM alpha in no-disaster samples.

The next three technological parameters involve entry and exit, including the liquidation value parameter, s, the reorganizational costs parameter, κ, and the delisting return, $\widetilde{R}$. Increasing s reduces the value premium and its CAPM alpha in no-disaster samples. Intuitively, in the event of exit, existing shareholders extract a higher liquidation value of sK_{it}, which is in effect a free abandonment option, acting as an insurance hedge against the disaster risk. The abandonment option is especially attractive for shareholders of value firms, which tend to have more unproductive capital than growth firms. Consequently, instead of facing asymmetric adjustment costs in disasters, the shareholders opt to exit, thereby reducing the risk for value firms relative to growth firms. In contrast to the strong s effect, the impact of the reorganizational costs parameter and the delisting return is quantitatively negligible. With $s = 0$, meaning existing shareholders receive nothing when exiting the economy, the percentage of exit firms is very low, only about 0.09% in simulations.

Preferences

The last four columns in the lower panel report the results for the risk aversion, γ, and the intertemporal elasticity of substitution, ψ. Not surprisingly, increasing either parameter strengthens the nonlinear disaster dynamics, raising the value premium and its CAPM alpha in no-disaster samples.

4.5 Further Implications on the Beta Anomaly

Our model is also largely consistent with the flat beta-return relation. Mimicking the empirical procedure in Table 4, we sort stocks in simulated panels at the end of each June based on pre-ranking market betas from prior 60-month rolling windows. We calculate value-weighted decile returns, and rebalance the deciles in June. Panel A of Table 8 shows that in artificial samples with disasters, the average excess return rises from 0.76% per month for the low rolling beta decile only to 0.79% for the high decile, and the spread of 0.04% is within 0.5 standard errors from zero. The pre-ranking market beta sorts also yield a spread in the post-ranking betas, although its magnitude, 0.30 ($t = 2.49$), is smaller than that in the data. The CAPM alpha of the high-minus-low decile is -0.21%, albeit insignificant ($t = -1.73$). From Panel B, results from no-disaster samples are quantitatively similar. The high-minus-low beta decile has an average return of -0.06% ($t = -0.93$), a CAPM alpha of -0.44% ($t = -3.49$), and a post-ranking market beta of 0.47 ($t = 3.49$).

It is perhaps surprising that our risk-based model can reproduce the flat beta-return relation in simulations. The crux is that the rolling market beta contains a great deal of measurement errors, and is, consequently, a poor proxy for the true market beta. Because of our single-factor structure, all aggregate variables are conditionally perfectly correlated, including the pricing kernel and the expected market risk premium, $E_t[R_{Mt+1}] - r_{ft}$. As such, the conditional CAPM holds exactly in theory (but not the unconditional CAPM), meaning that the true market beta can be backed out as $(E_t[R_{it+1}] - r_{ft})/(E_t[R_{Mt+1}] - r_{ft})$. The true market beta differs from the true beta with respect to the pricing kernel, β_{it}^M, which is calculated as $(E_t[R_{it+1}] - r_{ft})/\lambda_{Mt}$.

Panels C and D in Table 8 show that, not surprisingly, sorting on true market betas yields large average return spreads, over 1.2% per month, across extreme deciles in the model. The (unconditional) CAPM fails to price these deciles both in disaster samples and no-disaster samples. In disaster samples, the post-ranking rolling betas overshoot, giving rise to a negative CAPM alpha of -0.73% ($t = -1.98$). In no-disaster samples, the rolling betas move in the opposite direction as the true market betas, with a spread of -1.23 ($t = -11.21$). Accordingly, the CAPM alpha is 2.23% ($t = 21.52$), which is substantially higher than the average return spread of 1.22%.

To illustrate measurement errors in rolling market betas, Figure 6 presents the scatter plots of true market betas against their corresponding rolling betas, using deciles formed on rolling market betas as the testing portfolios. From Panel A, these deciles show substantial variations in rolling betas, ranging from -40 to 40, in disaster samples. However, such substantial variations only transform to small variations in true market betas from 0.5 to 2.5. In no-disaster samples, Panel B shows that rolling betas range from -20 to 20, and true market betas from 0.5 to 1.8. The correlation between the true and rolling market betas is weakly negative, -0.03, in disaster samples, but strongly negative, -0.35, in no-disaster samples.

Panels C and D present the scatter plots using deciles formed on true market betas as the testing portfolios. The variations in true market betas range from zero to 12 in disaster samples, and from zero to four in no-disaster samples. Rolling market betas remain a poor proxy for true market betas. Their correlations is weakly positive, 0.08, in disaster samples, but reliably negative, -0.20, in no-disaster samples. It is surprising how poor the rolling betas are as a proxy for true market betas, especially in no-disaster samples. Intuitively, based on 60-month rolling windows, the rolling betas are basically prior five-year averaged betas. In contrast, true market betas accurately and immediately reflect changes in aggregate and firm-specific conditions. Within a given rolling window, true market betas can even mean-revert, giving rise to opposite rankings in rolling betas.

5 Conclusion

Rare disasters help explain the value premium puzzle that value stocks earn higher average returns than growth stocks, despite their similar market betas. In an investment model augmented with disasters, we show that value stocks are more exposed to disaster risk than growth stocks. More important, the disaster risk induces strong nonlinearity in the pricing kernel. In finite samples, in which disasters are materialized, the CAPM does an adequate job in accounting for the value premium. More important, in finite samples without disasters, estimated market betas fail to measure the higher exposures of value stocks to the disaster risk than growth stocks. This nonlinearity allows the model to replicate the failure of the CAPM in the post-1963 sample. In addition, due to severe measurement errors in rolling betas, the relation between pre-ranking betas and average returns is flat in the model's simulations, despite a strong positive relation between true betas and expected returns. As such, the model is also consistent with the beta anomaly.

References

Ai, Hengjie, and Dana Kiku, 2013, Growth to value: Option exercise and the cross section of equity returns, *Journal of Financial Economics* 107, 325–349.

Ang, Andrew, and Joseph Chen, 2007, CAPM over the long run: 1926-2001, *Journal of Empirical Finance* 14, 1–40.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.

Barro, Robert J., 2006, Rare disasters and asset markets in the twentieth century, *Quarterly Journal of Economics* 121, 823–866.

Barro, Robert J., 2009, Rare disasters, asset prices, and welfare costs, *American Economic Review* 99, 243–264.

Barro, Robert J., and José F. Ursúa, 2008, Macroeconomic crises since 1870, *Brookings Papers on Economic Activity* 1, 255–335.

Belo, Frederico, Xiaoji Lin, and Santiago Bazdresch, 2014, Labor hiring, investment, and stock return predictability in the cross section, *Journal of Political Economy* 122, 129–177.

Belo, Frederico, Xiaoji Lin, and Fan Yang, 2014, External equity financing shocks, financial flows, and asset prices, working paper, The Ohio State University.

Berk, Jonathan B., Richard C. Green, and Vasant Naik, 1999, Optimal investment, growth options and security returns, *Journal of Finance* 54, 1153-1607.

Carlson, Murray, Adlai Fisher, and Ron Giammarino, 2004, Corporate investment and asset price dynamics: Implications for the cross-section of returns, *Journal of Finance* 59, 2577-2603.

Cochrane, John H., 2005, *Asset Pricing*, Princeton University Press, Princeton: New Jersey.

Cooper, Ilan, 2006, Asset pricing implications of nonconvex adjustment costs and rrreversibility of investment, *Journal of Finance* 61, 139-170.

Cooper, Russell W., and John C. Haltiwanger, 2006, On the nature of capital adjustment costs, *Review of Economic Studies* 73, 611–633.

Danthine, Jean-Pierre, and John B. Donaldson, 1999, Non-falsified expectations and general equilibrium asset pricing: The power of the peso, *Economic Journal* 109, 607–635.

Epstein, Larry G., and Stanley E. Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937-969.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3-56.

Fama, Eugene F., and Kenneth R. French, 2006, The value premium and the CAPM, *Journal of Finance* 61, 2163–2185.

Fama, Eugene F., and James D. MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, *Journal of Political Economy* 71, 607-636.

Frazzini, Andrea, and Lasse H. Pedersen, 2014, Betting against beta, *Journal of Financial Economics* 111, 1-25.

Gabaix, Xavier, 2012, An exactly solved framework for ten puzzles in macro-finance, *Quarterly Journal of Economics* 127, 645–700.

Gourio, Francois, 2012, Disaster risk and business cycles, *American Economic Review* 102, 2734–2766.

Hennessy, Christopher A., and Toni M. Whited, 2007, How costly is external financing? Evidence from a structural estimation, *Journal of Finance* 62, 1705-1745.

Kogan, Leonid, and Dimitris Papanikolaou, 2013, Firm characteristics and stock returns: The role of investment-specific shocks, *Review of Financial Studies* 26, 2718–2759.

Koh, Woo Hwa, 2014, The impact of uncertainty shocks on the cross-section of returns, working paper, The Ohio State University.

Mehra, Rajnish, and Edward C. Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145–161.

Nakamura, Emi, Jon Steinsson, Robert J. Barro, and Jose Ursua, 2013, Crises and recoveries in an empirical model of consumption disasters, *American Economic Journal: Macroeconomics* 5, 35–74.

Rietz, Thomas A., 1988, The equity risk premium: A solution, *Journal of Monetary Economics* 22, 117–131.

Rouwenhorst, K. Geert, 1995, Asset pricing implications of equilibrium business cycle models, in Thomas Cooley, ed. *Frontiers of Business Cycle Research*, Princeton University Press, Princeton: New Jersey.

Shumway, Tyler G., 1997, The delisting bias in CRSP data, *Journal of Finance* 52, 327–340.

Shumway, Tyler G., and Vincent A. Warther, 1999, The delisting bias in CRSP's Nasdaq data and its implications for the size effect, *Journal of Finance* 54, 2361–2379.

Wachter, Jessica A., 2013, Can time-varying risk of rare disasters explain aggregate stock market volatility? *Journal of Finance* 68, 987–1035.

Zhang, Lu, 2005, The value premium, *Journal of Finance* 60, 67–103.

A Detrending

Because the model features a balanced growth path, we first detrend the model before solving for its stationary equilibrium. We define the following stationary variables: $\widehat{U}_t \equiv U_t/C_t$, $\widehat{\Pi}_{it} \equiv \Pi_{it}/X_{t-1}$, $\widehat{V}_{it} \equiv V_{it}/X_{t-1}$, $\widehat{K}_{it} \equiv K_{it}/X_{t-1}$, $\widehat{I}_{it} \equiv I_{it}/X_{t-1}$, $\widehat{\Phi}_{it} \equiv \Phi_{it}/X_{t-1}$, and $\widehat{D}_{it} \equiv D_{it}/X_{t-1}$. We then rewrite the model's key equations in terms of the detrended, stationary variables as follows:

The utility-to-consumption ratio:

$$\widehat{U}_t = \left[(1-\iota) + \iota \left(E_t\left[\widehat{U}_{t+1}^{1-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}. \tag{A.1}$$

The pricing kernel:

$$M_{t+1} = \iota\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{\widehat{U}_{t+1}^{1-\gamma}(C_{t+1}/C_t)^{1-\gamma}}{E_t\left[\widehat{U}_{t+1}^{1-\gamma}(C_{t+1}/C_t)^{1-\gamma}\right]}\right)^{\frac{1/\psi-\gamma}{1-\gamma}}. \tag{A.2}$$

Profits:

$$\widehat{\Pi}_{it} = \left(\frac{X_t}{X_{t-1}}\right)^{1-\xi} Z_{it}^{1-\xi}\widehat{K}_{it}^{\xi} - f\widehat{K}_{it}. \tag{A.3}$$

Capital accumulation:

$$\widehat{K}_{it+1}\left(\frac{X_t}{X_{t-1}}\right) = (1-\delta)\widehat{K}_{it} + \widehat{I}_{it}. \tag{A.4}$$

The adjustment costs function:

$$\widehat{\Phi}_{it} = \begin{cases} a^+\widehat{K}_{it} + \frac{c^+}{2}\left(\frac{\widehat{I}_{it}}{\widehat{K}_{it}}\right)^2\widehat{K}_{it} & \text{for} \quad \widehat{I}_{it} > 0 \\ 0 & \text{for} \quad \widehat{I}_{it} = 0 \\ a^-\widehat{K}_{it} + \frac{c^-}{2}\left(\frac{\widehat{I}_{it}}{\widehat{K}_{it}}\right)^2\widehat{K}_{it} & \text{for} \quad \widehat{I}_{it} < 0 \end{cases}. \tag{A.5}$$

Dividends:

$$\widehat{D}_{it} = \widehat{\Pi}_{it} - \widehat{I}_{it} - \widehat{\Phi}_{it}. \tag{A.6}$$

The value function, $\widehat{V}_{it} = \widehat{V}\left(\widehat{K}_{it}, Z_{it}, X_t/X_{t-1}\right)$:

$$\max_{\{\chi_{it}\}}\left[\max_{\{\widehat{I}_{it}\}} \widehat{D}_{it} + E_t\left[M_{t+1}\widehat{V}\left(\widehat{K}_{it+1}, \frac{X_{t+1}}{X_t}, Z_{it+1}\right)\right]\left(\frac{X_t}{X_{t-1}}\right), s\widehat{K_{it}}\right]. \tag{A.7}$$

Finally, the stock return for an incumbent firm:

$$R_{it+1} = \frac{\widehat{V}_{it+1}\left(X_t/X_{t-1}\right)}{\widehat{V}_{it} - \widehat{D}_{it}}. \tag{A.8}$$

B Computation

To solve for the pricing kernel, we first iterate on the log utility-to-consumption ratio, $\widehat{u}_t \equiv \log(\widehat{U}_t)$:

$$\exp(\widehat{u}_t) = \left[(1-\iota) + \iota E_t\left[\exp\left[(1-\gamma)(\widehat{u}_{t+1} + \bar{g} + g_{t+1})\right]\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \tag{B.1}$$

as a function of g_t. The pricing kernel can then be rewritten as:

$$M_{t+1} \equiv M(g_t, g_{t+1}) = \iota \exp\left(-\frac{1}{\psi}(\bar{g} + g_{t+1})\right)\left(\frac{\exp\left[(1-\gamma)(\widehat{u}_{t+1} + \bar{g} + g_{t+1})\right]}{E_t\left[\exp\left[(1-\gamma)(\widehat{u}_{t+1} + \bar{g} + g_{t+1})\right]\right]}\right)^{\frac{1/\psi-\gamma}{1-\gamma}}. \tag{B.2}$$

We iterate on the following recursion to compute the value and policy functions:

$$\widehat{V}(\widehat{K}_{it}, Z_{it}, g_t) = \max_{\{\chi_{it}\}}\left[\max_{\{\widehat{I}_{it}\}} \ \widehat{D}_{it} + E_t\left[M_{t+1}\widehat{V}(\widehat{K}_{it+1}, Z_{it+1}, g_{t+1})\right]\exp(\overline{g} + \phi g_t), \ s\widehat{K}_{it}\right]. \tag{B.3}$$

We discretize the state space using 100 grid points for K_{it}, 13 points for z_{it}, and seven grid points for g_t. We specify the lower bound of the capital grid to be 0.01 and the upper bound to be 25. The interval is so large enough that the two bounds are never binding in simulations. We construct the capital grid recursively, following McGrattan (1999), i.e., $K_j = K_{j1} + c_{k1}\exp(c_{k2}(j-2))$, in which $j = 1, ..., 100$ is the index of grid points, and c_{k1} and c_{k2} are two constant parameters chosen to provide the desired number of grid points and the upper bound of the capital grid, given a predetermined lower bound of 0.01. We use the piecewise linear interpolation to obtain firm value, investment, and expected returns that do not lie directly on the grid points. Finally, we use a simple global search routine in maximizing the right-hand side of equation (B.3). The objective function is computed on a grid of K_{it+1} between 0.01 and 25 with 12,000 points, and a simple maximum is taken. We solve for the value function sequentially. After the value function is obtained on the coarse K_{it+1} grid, we use it as the initial condition in the next stage of maximization with a finer grid of K_{it+1}. The final results are from a K_{it+1} grid with 120,000 points.

Table 1 : The Market Regressions for the Book-to-market Deciles, July 1926–June 2014

For each report the average excess return (m), the CAPM alpha, the market beta, their t-statistics adjusted for heteroscedasticity and autocorrelations, and the R^2. "V−G" is the value-minus-growth decile.

	Growth	2	3	4	5	6	7	8	9	Value	V−G
					Panel A: July 1963 to June 2014						
m	0.42	0.52	0.55	0.55	0.54	0.59	0.68	0.71	0.79	0.93	0.51
t_m	2.00	2.72	2.95	2.89	2.97	3.25	3.81	3.88	4.07	3.91	2.75
α	−0.12	0.01	0.06	0.06	0.08	0.13	0.24	0.27	0.32	0.39	0.51
t_α	−1.28	0.15	0.92	0.57	0.83	1.45	2.27	2.28	2.92	2.50	2.26
β	1.06	1.01	0.98	0.99	0.91	0.93	0.88	0.88	0.94	1.07	0.01
t_β	40.79	46.28	34.55	29.35	27.50	28.34	22.89	17.53	20.85	15.44	0.07
R^2	0.86	0.92	0.90	0.87	0.83	0.85	0.78	0.76	0.76	0.66	0.00
					Panel B: July 1926 to June 2014						
m	0.57	0.68	0.68	0.68	0.73	0.76	0.77	0.92	1.03	1.09	0.52
t_m	3.24	4.09	4.05	3.67	4.12	4.04	3.86	4.43	4.36	3.82	2.60
α	−0.09	0.06	0.05	−0.01	0.08	0.07	0.05	0.18	0.21	0.14	0.23
t_α	−1.27	1.20	0.89	−0.13	1.02	0.93	0.58	1.93	1.83	0.94	1.18
β	1.00	0.95	0.97	1.06	1.00	1.05	1.09	1.14	1.27	1.45	0.45
t_β	47.76	28.52	59.63	20.06	27.74	16.04	16.14	15.40	13.46	13.38	3.52
R^2	0.90	0.92	0.92	0.90	0.89	0.87	0.84	0.82	0.79	0.72	0.14
					Panel C: July 1926 to June 1963						
m	0.78	0.90	0.86	0.85	0.99	0.99	0.88	1.21	1.36	1.30	0.52
t_m	2.56	3.06	2.81	2.42	2.93	2.68	2.18	2.84	2.74	2.20	1.31
α	−0.04	0.11	0.03	−0.09	0.08	0.02	−0.18	0.09	0.09	−0.15	−0.11
t_α	−0.48	1.56	0.36	−1.07	0.80	0.15	−1.28	0.74	0.43	−0.55	−0.34
β	0.96	0.92	0.97	1.10	1.06	1.13	1.24	1.30	1.48	1.70	0.74
t_β	42.94	20.05	50.80	14.68	26.14	12.58	16.54	16.82	14.50	14.24	5.33
R^2	0.94	0.92	0.94	0.92	0.93	0.89	0.89	0.88	0.84	0.77	0.32

Table 2 : Large Swings in the Stock Market and the Corresponding Value-minus-growth Returns, July 1926–June 2014

This table reports market excess returns, MKT, that are below 1.5 and above 98.5 percentiles in the long U.S. sample. V−G is the value premium. Both MKT and V−G are in monthly percent.

Month	MKT	V−G	Month	MKT	V−G
November 1928	11.79	−0.41	August 1933	12.03	4.92
October 1929	−20.07	7.57	January 1934	12.63	34.10
June 1930	−16.25	−3.54	September 1937	−13.57	−10.90
May 1931	−13.16	−3.09	March 1938	−23.80	−22.67
June 1931	13.75	14.80	April 1938	14.49	8.76
September 1931	−29.07	−5.03	June 1938	23.77	15.22
December 1931	−13.42	−16.73	September 1939	16.94	56.61
April 1932	−17.98	−2.85	May 1940	−21.93	−15.49
May 1932	−20.44	3.61	October 1974	16.10	−13.58
July 1932	33.47	45.73	January 1975	13.66	19.70
August 1932	36.41	69.99	January 1976	12.16	15.04
October 1932	−13.09	−12.97	March 1980	−12.90	−9.02
February 1933	−15.06	−7.45	January 1987	12.47	−2.98
April 1933	37.93	22.41	October 1987	−23.24	−1.21
May 1933	21.36	45.01	August 1998	−16.08	6.33
June 1933	13.05	10.29	October 2008	−17.23	−11.93

Table 3 : The Performance of the CAPM, July 1926–June 2014, Sensitivity Analysis

In Panel A, we delete monthly observations with market excess returns below 1.5 and 98.5 percentiles of the empirical distribution. As such, we remove in total 3% of extreme observations for market excess returns. The procedures in other panels are defined analogously.

	Growth	2	3	4	5	6	7	8	9	Value	V−G	[t]
					Panel A: 3%							
m	0.60	0.71	0.69	0.65	0.70	0.73	0.67	0.82	0.88	0.90	0.30	1.85
α	−0.08	0.06	0.06	0.00	0.09	0.10	0.03	0.15	0.15	0.09	0.17	0.81
β	1.04	1.00	0.97	0.98	0.93	0.96	0.99	1.01	1.11	1.23	0.20	2.54
R^2	0.86	0.90	0.89	0.86	0.83	0.82	0.78	0.76	0.73	0.63	0.03	
					Panel B: 5%							
m	0.62	0.74	0.72	0.67	0.72	0.74	0.70	0.83	0.91	0.93	0.31	1.99
α	−0.09	0.06	0.06	0.00	0.08	0.09	0.03	0.15	0.16	0.10	0.19	0.97
β	1.03	1.00	0.97	0.98	0.94	0.95	0.99	1.01	1.10	1.22	0.18	2.48
R^2	0.85	0.89	0.88	0.85	0.81	0.81	0.76	0.75	0.72	0.60	0.02	
					Panel C: 10%							
m	0.64	0.78	0.76	0.76	0.80	0.83	0.80	0.90	1.00	1.07	0.43	2.72
α	−0.13	0.04	0.04	0.03	0.11	0.11	0.08	0.17	0.20	0.15	0.28	1.42
β	1.03	1.00	0.97	0.98	0.93	0.97	0.98	0.99	1.09	1.24	0.20	2.61
R^2	0.82	0.87	0.86	0.83	0.79	0.79	0.73	0.72	0.68	0.57	0.02	
					Panel D: 20%							
m	0.74	0.85	0.83	0.82	0.85	0.88	0.83	0.97	1.10	1.19	0.45	2.81
α	−0.12	0.02	0.00	0.01	0.09	0.12	0.02	0.13	0.22	0.17	0.29	1.47
β	1.04	1.01	1.01	0.99	0.93	0.92	0.98	1.01	1.07	1.23	0.19	2.29
R^2	0.77	0.82	0.82	0.77	0.73	0.72	0.68	0.67	0.61	0.48	0.01	
					Panel E: 30%							
m	0.81	0.90	0.90	0.86	0.85	0.92	0.87	1.00	1.15	1.22	0.41	2.46
α	−0.11	0.03	0.01	−0.01	0.03	0.10	0.01	0.09	0.23	0.16	0.27	1.30
β	1.05	0.99	1.02	0.99	0.93	0.92	0.98	1.03	1.06	1.22	0.17	1.72
R^2	0.71	0.76	0.77	0.71	0.66	0.66	0.59	0.60	0.52	0.40	0.01	
					Panel F: 40%							
m	0.82	0.89	0.93	0.88	0.87	0.94	0.97	1.09	1.24	1.34	0.52	2.98
α	−0.16	−0.03	0.03	0.01	0.03	0.12	0.10	0.17	0.28	0.20	0.36	1.65
β	1.08	1.01	0.99	0.96	0.92	0.90	0.96	1.01	1.05	1.25	0.18	1.63
R^2	0.65	0.70	0.69	0.63	0.60	0.57	0.52	0.51	0.45	0.34	0.01	
					Panel G: 50%							
m	0.83	0.90	0.94	0.91	0.89	0.98	1.01	1.14	1.34	1.46	0.63	3.33
α	−0.23	−0.09	−0.00	0.00	0.07	0.14	0.15	0.26	0.41	0.34	0.56	2.18
β	1.12	1.06	1.00	0.97	0.87	0.90	0.92	0.93	0.99	1.19	0.07	0.53
R^2	0.59	0.64	0.59	0.54	0.49	0.48	0.40	0.36	0.32	0.24	0.00	

Table 4 : The CAPM Regressions for the Market Beta Deciles, July 1928–June 2014

For each market beta decile, we report the average return (m), the CAPM alpha, the post-ranking market beta, their t-statistics adjusted for heteroscedasticity and autocorrelations, and the R^2. "H" is the highest pre-ranking market beta decile, "L" the lowest decile, and "H−L" the high-minus-low decile.

	L	2	3	4	5	6	7	8	9	H	H−L
					Panel A: July 1963 to June 2014						
m	0.51	0.52	0.52	0.56	0.66	0.54	0.68	0.53	0.62	0.63	0.12
t_m	3.64	3.46	3.11	3.15	3.46	2.67	3.06	2.25	2.33	1.92	0.43
α	0.22	0.17	0.11	0.12	0.17	0.02	0.10	−0.08	−0.06	−0.18	−0.40
t_α	2.03	1.75	1.32	1.39	1.89	0.22	1.17	−0.83	−0.47	−0.90	−1.48
β	0.57	0.68	0.81	0.87	0.98	1.03	1.14	1.22	1.35	1.61	1.04
t_β	12.29	16.79	19.13	20.74	27.23	30.22	46.72	41.42	34.60	30.04	11.41
R^2	0.54	0.68	0.77	0.79	0.86	0.86	0.88	0.86	0.84	0.78	0.43
					Panel B: July 1928 to June 2014						
m	0.57	0.63	0.64	0.73	0.82	0.71	0.80	0.72	0.82	0.81	0.24
t_m	4.80	4.51	4.23	4.33	4.36	3.55	3.69	2.99	3.04	2.59	0.94
α	0.21	0.17	0.12	0.14	0.16	0.01	0.04	−0.13	−0.11	−0.26	−0.47
t_α	2.68	2.18	2.04	2.37	2.29	0.11	0.54	−1.53	−1.09	−1.80	−2.40
β	0.57	0.74	0.82	0.93	1.05	1.12	1.22	1.36	1.49	1.70	1.12
t_β	22.94	29.62	35.56	40.62	41.07	40.12	46.51	36.08	26.55	40.59	18.46
R^2	0.67	0.81	0.85	0.88	0.90	0.90	0.91	0.90	0.88	0.85	0.58
					Panel C: July 1928 to June 1963						
m	0.66	0.79	0.81	0.98	1.05	0.96	0.99	1.00	1.11	1.06	0.40
t_m	3.15	3.00	2.91	3.02	2.85	2.44	2.32	2.07	2.06	1.76	0.88
α	0.19	0.16	0.14	0.19	0.16	0.01	−0.04	−0.17	−0.17	−0.36	−0.56
t_α	1.73	1.53	1.72	2.41	1.70	0.09	−0.42	−1.14	−0.91	−1.68	−1.94
β	0.58	0.77	0.83	0.98	1.10	1.17	1.28	1.45	1.58	1.76	1.18
t_β	20.61	28.37	30.76	50.20	30.53	29.28	31.99	34.52	21.46	34.34	16.04
R^2	0.79	0.90	0.92	0.94	0.93	0.93	0.93	0.93	0.90	0.89	0.72

Table 5 : Parameter Values in the Monthly Benchmark Calibration

Parameters	Value	Description
		Panel A: Preferences
ι	0.99035	The subjective discount factor
γ	5	The relative risk aversion
ψ	1.5	The elasticity of intertemporal substitution
		Panel B: Consumption dynamics
$\bar{g}$	0.019/12	The average consumption growth
ρ_g	0.6	The persistence of consumption growth
σ_g	0.0025	The conditional volatility of consumption growth
η	0.028/12	The disaster probability
λ_D	-0.0275	The disaster size
θ	$0.914^{1/3}$	The disaster persistence
λ_R	0.0325	The recovery size
ν	0.95	The recovery persistence
		Panel C: Technology
ξ	0.65	The curvature parameter in the production function
δ	0.01	The capital depreciation rate
f	0.005	Fixed costs of production
$\overline{z}$	-9.75	The long-run mean of log firm-specific productivity
ρ_z	0.985	The persistence of log firm-specific productivity
σ_z	0.5	The conditional volatility of log firm-specific productivity
ϕ	1	The leverage of productivity growth
a^+	0.035	Upward nonconvex adjustment costs
a^-	0.05	Downward nonconvex adjustment costs
c^+	75	Upward convex adjustment costs
c^-	150	Downward convex adjustment costs
s	0	The liquidation value parameter
κ	0.25	The reorganizational cost parameter
$\widetilde{R}$	-0.425	The delisting return

Table 6 : The CAPM Regressions for the Book-to-market Deciles in the Model

Results are averaged across 2,000 simulated economies. The mean excess returns, denoted m, and the CAPM alphas are in monthly percent. We also report the 2.5 and 97.5 percentiles for the alphas and betas. The t-statistics are adjusted for heteroscedasticity and autocorrelations.

	Growth	2	3	4	5	6	7	8	9	Value	V−G
	Panel A: Samples with disasters										
m	0.74	0.74	0.73	0.74	0.75	0.77	0.81	0.86	0.96	1.19	0.45
t_m	13.83	13.61	13.43	13.24	13.15	13.07	13.10	13.07	13.17	13.61	5.83
α	0.08	0.06	0.04	0.01	−0.00	−0.03	−0.05	−0.09	−0.13	−0.13	−0.21
t_α	1.38	1.09	0.76	0.32	−0.08	−0.53	−0.82	−1.15	−1.40	−1.38	−1.72
$\alpha, 2.5\%$	−0.04	−0.05	−0.06	−0.08	−0.11	−0.15	−0.20	−0.25	−0.32	−0.34	−0.46
$\alpha, 97.5\%$	0.20	0.17	0.14	0.12	0.10	0.08	0.07	0.07	0.07	0.11	0.06
β	0.82	0.85	0.87	0.90	0.94	1.00	1.08	1.19	1.37	1.64	0.82
t_β	18.82	23.74	28.64	33.03	33.72	30.60	26.47	20.63	16.00	18.05	6.82
$\beta, 2.5\%$	0.64	0.73	0.79	0.83	0.86	0.89	0.93	0.99	1.07	1.38	0.47
$\beta, 97.5\%$	0.98	0.97	0.97	1.00	1.07	1.19	1.32	1.48	1.69	1.87	1.10
R^2	0.45	0.47	0.49	0.50	0.52	0.55	0.57	0.60	0.64	0.65	0.24
	Panel B: Samples without disasters										
m	0.77	0.76	0.75	0.75	0.75	0.77	0.80	0.85	0.95	1.21	0.45
t_m	18.58	18.43	18.09	17.98	18.11	18.57	19.32	20.52	22.55	24.99	7.10
α	−0.01	−0.02	−0.06	−0.11	−0.10	−0.08	−0.02	0.08	0.23	0.46	0.47
t_α	−0.07	−0.24	−0.73	−1.26	−1.24	−0.95	−0.18	0.99	2.83	5.02	3.71
$\alpha, 2.5\%$	−0.23	−0.16	−0.23	−0.25	−0.26	−0.24	−0.19	−0.11	0.04	0.27	0.22
$\alpha, 97.5\%$	0.15	0.14	0.10	0.03	0.04	0.07	0.15	0.23	0.39	0.67	0.75
β	0.95	0.96	1.00	1.05	1.05	1.05	1.00	0.95	0.89	0.92	−0.03
t_β	11.07	10.87	11.32	11.98	11.89	11.88	11.47	10.95	9.85	9.00	−0.24
$\beta, 2.5\%$	0.79	0.79	0.83	0.89	0.87	0.88	0.84	0.77	0.68	0.71	−0.29
$\beta, 97.5\%$	1.15	1.13	1.19	1.21	1.23	1.25	1.15	1.12	1.07	1.11	0.24
R^2	0.11	0.12	0.12	0.14	0.14	0.13	0.13	0.11	0.10	0.08	0.00

Table 7 : Comparative Statics

Results are averaged across 2,000 simulations. In each experiment, we vary one parameter, while keeping all the others unchanged from the benchmark calibration. For each parameter, for example, the disaster size, λ_D, we consider two values, with one above, −0.03, and the other below, −0.025, the benchmark value, −0.0275. See Table 5 for the descriptions of the symbols.

	λ_D		θ		η		ν		λ_R		a^+		a^-		c^+		c^-	
	−0.025	−0.03	0.955	0.985	0.13%	0.33%	0.935	0.965	2.75%	3.75%	0.025	0.045	0.035	0.065	50	100	100	200
Disaster samples																		
m	0.34	0.55	0.29	0.47	0.42	0.46	0.46	0.43	0.45	0.44	0.48	0.29	0.25	0.47	0.37	0.49	0.39	0.46
t_m	4.78	6.75	4.49	5.62	5.72	5.78	5.94	5.61	5.83	5.70	6.57	3.75	3.73	5.97	4.64	6.59	5.25	5.90
α	−0.22	−0.20	−0.21	−0.16	−0.21	−0.21	−0.20	−0.22	−0.21	−0.21	−0.25	−0.23	−0.21	−0.23	−0.24	−0.20	−0.21	−0.22
t_α	−1.98	−1.51	−2.08	−1.33	−1.60	−1.89	−1.66	−1.80	−1.75	−1.78	−1.91	−2.05	−1.66	−1.82	−2.04	−1.61	−1.82	−1.80
β	0.77	0.86	0.74	0.77	0.79	0.86	0.85	0.78	0.85	0.81	0.96	0.64	0.61	0.89	0.73	0.90	0.76	0.85
t_β	6.65	7.11	6.56	7.39	6.01	7.80	6.74	6.75	6.74	7.04	6.42	6.19	4.32	6.77	7.23	6.57	6.57	6.85
R^2	0.24	0.24	0.25	0.21	.21	0.27	0.23	0.25	0.24	0.25	0.33	0.16	0.15	0.27	0.20	0.27	0.22	0.25
Samples without disasters																		
m	0.33	0.54	0.28	0.55	0.42	0.46	0.45	0.43	0.44	0.45	0.45	0.28	0.22	0.46	0.38	0.49	0.39	0.46
t_m	5.63	8.08	5.24	7.89	6.74	7.29	7.09	6.90	6.99	7.06	8.48	4.24	3.84	7.32	5.54	8.42	6.35	7.29
α	0.24	0.71	0.07	0.86	0.43	0.50	0.49	0.45	0.47	0.47	0.63	0.14	0.26	0.49	0.27	0.62	0.41	0.49
t_α	2.14	4.96	0.67	5.66	3.38	3.97	3.88	3.52	3.72	3.77	5.39	1.10	2.16	3.83	1.98	5.12	3.27	3.89
β	0.12	−0.19	0.32	−0.33	−0.02	−0.05	−0.05	−0.02	−0.04	−0.04	−0.23	0.16	−0.04	−0.04	0.13	−0.17	−0.02	−0.04
t_β	0.89	−1.35	2.56	−2.32	−0.13	−0.35	−0.41	−0.19	−0.31	−0.34	−1.71	1.20	−0.32	−0.31	0.96	−1.25	−0.15	−0.36
R^2	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

	ξ		f		ϕ		s		κ		$\widetilde{R}$		γ		ψ	
	0.6	0.7	0	0.015	0.8	1.2	0.15	0.3	0	0.5	−0.3	−0.55	3.5	6.5	1	2
Disaster samples																
m	0.44	0.51	0.47	0.40	0.20	0.62	0.20	−0.03	0.45	0.45	0.47	0.44	0.18	0.57	−0.06	0.51
t_m	5.15	7.24	6.30	4.86	3.50	6.83	2.95	−0.28	5.79	5.87	6.08	5.64	2.61	7.19	−2.67	5.74
α	−0.18	−0.27	−0.21	−0.22	−0.21	−0.21	−0.27	−0.35	−0.21	−0.21	−0.19	−0.23	−0.23	−0.11	−0.29	−0.18
t_α	−1.49	−2.20	−1.71	−1.89	−2.31	−1.62	−2.72	−3.80	−1.72	−1.73	−1.58	−1.89	−2.48	−0.81	−10.00	−1.60
β	0.72	1.02	0.87	0.75	0.72	0.82	0.63	0.48	0.82	0.83	0.83	0.83	0.75	0.67	1.74	0.67
t_β	6.69	7.53	6.61	7.25	5.84	8.46	7.06	6.57	6.84	6.80	6.75	6.85	6.24	5.29	10.34	8.43
R^2	0.19	0.39	0.27	0.19	0.15	0.31	0.16	0.09	0.24	0.24	0.24	0.24	0.37	0.13	0.24	0.28
Samples without disasters																
m	0.44	0.48	0.45	0.40	0.18	0.63	0.27	0.10	0.44	0.45	0.46	0.44	0.15	0.60	−0.07	0.50
t_m	6.14	9.90	7.78	5.81	3.65	9.01	4.37	1.68	7.07	7.11	7.27	7.03	3.18	8.47	−3.15	7.10
α	0.52	0.52	0.54	0.31	−0.10	0.89	0.34	0.20	0.47	0.47	0.48	0.47	−0.12	0.96	−0.31	0.82
t_α	3.77	4.59	4.42	2.31	−1.09	5.79	2.78	1.74	3.68	3.69	3.76	3.65	−1.64	5.87	−11.70	5.23
β	−0.09	−0.04	−0.10	0.11	0.47	−0.25	−0.09	−0.13	−0.03	−0.03	−0.03	−0.03	0.51	−0.34	1.98	−0.30
t_β	−0.70	−0.35	−0.77	0.76	3.58	−1.88	−0.66	−1.01	−0.22	−0.23	−0.22	−0.25	4.35	−2.45	17.67	−2.27
R^2	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.01	0.26	0.01

Table 8 : The CAPM Regressions for Deciles Formed on Rolling Market Betas and True Market Betas in the Model

Results are based on 2,000 simulations.

	L	2	3	4	5	6	7	8	9	H	H−L
	Panel A: Rolling market betas, samples with disasters										
m	0.76	0.78	0.81	0.83	0.85	0.86	0.86	0.85	0.83	0.79	0.04
t_m	13.72	14.09	14.04	13.89	13.55	13.41	13.08	12.65	11.79	11.50	0.53
α	0.04	0.06	0.06	0.04	0.01	−0.01	−0.04	−0.08	−0.16	−0.17	−0.21
t_α	0.69	1.29	1.17	0.82	0.29	−0.05	−0.49	−0.89	−1.45	−2.10	−1.73
$\alpha, 2.5\%$	−0.09	−0.05	−0.04	−0.07	−0.10	−0.13	−0.19	−0.27	−0.44	−0.41	−0.55
$\alpha, 97.5\%$	0.17	0.18	0.17	0.17	0.13	0.13	0.11	0.11	0.06	0.01	0.06
β	0.90	0.90	0.95	0.99	1.04	1.08	1.12	1.16	1.23	1.20	0.30
t_β	19.75	25.57	33.62	34.40	31.60	25.92	21.23	18.56	15.11	17.35	2.49
$\beta, 2.5\%$	0.77	0.80	0.85	0.88	0.92	0.91	0.93	0.93	0.95	0.99	−0.03
$\beta, 97.5\%$	1.08	1.03	1.04	1.08	1.14	1.21	1.28	1.38	1.56	1.47	0.67
R^2	0.53	0.53	0.54	0.55	0.56	0.57	0.58	0.59	0.60	0.61	0.06
	Panel B: Rolling market betas, samples without disasters										
m	0.80	0.82	0.84	0.85	0.85	0.86	0.84	0.82	0.79	0.74	−0.06
t_m	20.12	20.36	20.48	20.45	19.82	20.00	19.58	18.98	18.24	16.65	−0.93
α	0.01	0.12	0.17	0.19	0.15	0.16	0.11	0.03	−0.09	−0.42	−0.44
t_α	0.16	1.55	2.15	2.28	1.75	1.88	1.30	0.39	−1.06	−4.98	−3.49
$\alpha, 2.5\%$	−0.12	−0.02	0.02	0.04	−0.03	0.01	−0.06	−0.11	−0.24	−0.58	−0.68
$\alpha, 97.5\%$	0.14	0.26	0.31	0.33	0.31	0.31	0.27	0.18	0.05	−0.24	−0.19
β	0.97	0.86	0.82	0.82	0.86	0.86	0.90	0.97	1.08	1.43	0.47
t_β	11.79	10.20	9.50	9.27	9.30	9.15	9.72	10.37	11.67	15.74	3.49
$\beta, 2.5\%$	0.80	0.68	0.63	0.65	0.69	0.64	0.72	0.79	0.92	1.26	0.24
$\beta, 97.5\%$	1.13	1.04	0.98	1.00	1.05	1.04	1.09	1.15	1.23	1.61	0.76
R^2	0.13	0.10	0.09	0.09	0.09	0.09	0.10	0.11	0.14	0.23	0.01
	Panel C: True market betas, samples with disasters										
m	0.54	0.74	0.87	0.92	1.03	1.08	1.18	1.26	1.40	1.80	1.26
t_m	13.68	16.17	16.52	16.09	16.50	15.66	15.41	14.44	13.35	11.43	8.98
α	−0.04	0.07	0.07	0.05	0.04	−0.00	−0.04	−0.13	−0.28	−0.77	−0.73
t_α	−0.29	2.04	1.48	0.87	0.47	−0.23	−0.69	−1.29	−1.70	−2.30	−1.98
$\alpha, 2.5\%$	−0.16	0.00	−0.01	−0.03	−0.06	−0.13	−0.19	−0.35	−0.74	−1.87	−1.93
$\alpha, 97.5\%$	0.07	0.15	0.20	0.17	0.18	0.15	0.14	0.11	0.08	−0.07	0.07
β	0.72	0.83	0.99	1.09	1.23	1.36	1.53	1.74	2.12	3.23	2.51
t_β	11.79	38.88	26.53	25.90	21.97	18.91	16.06	13.68	10.68	8.40	5.54
$\beta, 2.5\%$	0.60	0.69	0.85	0.96	1.10	1.17	1.29	1.45	1.63	2.20	1.38
$\beta, 97.5\%$	0.87	0.90	1.08	1.17	1.33	1.51	1.76	2.09	2.83	4.99	4.31
R^2	0.68	0.69	0.73	0.74	0.78	0.77	0.79	0.78	0.77	0.76	0.58
	Panel D: True market betas, samples without disasters										
m	0.56	0.75	0.88	0.94	1.05	1.09	1.18	1.26	1.39	1.78	1.22
t_m	18.51	29.08	36.15	35.03	45.87	40.65	45.68	42.68	39.49	38.96	22.70
α	−0.89	0.25	0.56	0.58	0.80	0.80	0.93	0.99	1.06	1.34	2.23
t_α	−28.60	5.15	11.73	11.23	17.85	15.17	18.11	17.01	15.39	14.81	21.52
$\alpha, 2.5\%$	−0.96	0.14	0.41	0.45	0.64	0.62	0.78	0.80	0.82	1.05	1.75
$\alpha, 97.5\%$	−0.71	0.34	0.65	0.70	0.89	0.92	1.03	1.11	1.21	1.54	2.44
β	1.77	0.62	0.40	0.43	0.30	0.36	0.31	0.33	0.41	0.54	−1.23
t_β	54.41	12.05	7.60	7.57	6.03	6.19	5.39	5.14	5.38	5.43	−11.21
$\beta, 2.5\%$	1.56	0.51	0.28	0.33	0.20	0.23	0.20	0.20	0.24	0.34	−1.49
$\beta, 97.5\%$	1.85	0.75	0.57	0.61	0.50	0.59	0.55	0.61	0.71	0.93	−0.63
R^2	0.76	0.13	0.06	0.06	0.04	0.04	0.03	0.03	0.03	0.03	0.12

Figure 1 : The CAPM Regressions for the Value-minus-growth Decile, July 1926–June 2014

The figure presents the scatter plot and the fitted line for the market regression of the value-minus-growth decile. For the long sample in Panel A, the scatter points with the monthly market excess returns below the 1.5 and above 98.5 percentiles are dated in red.

Panel A: July 1926–June 2014 | Panel B: July 1963–June 2014

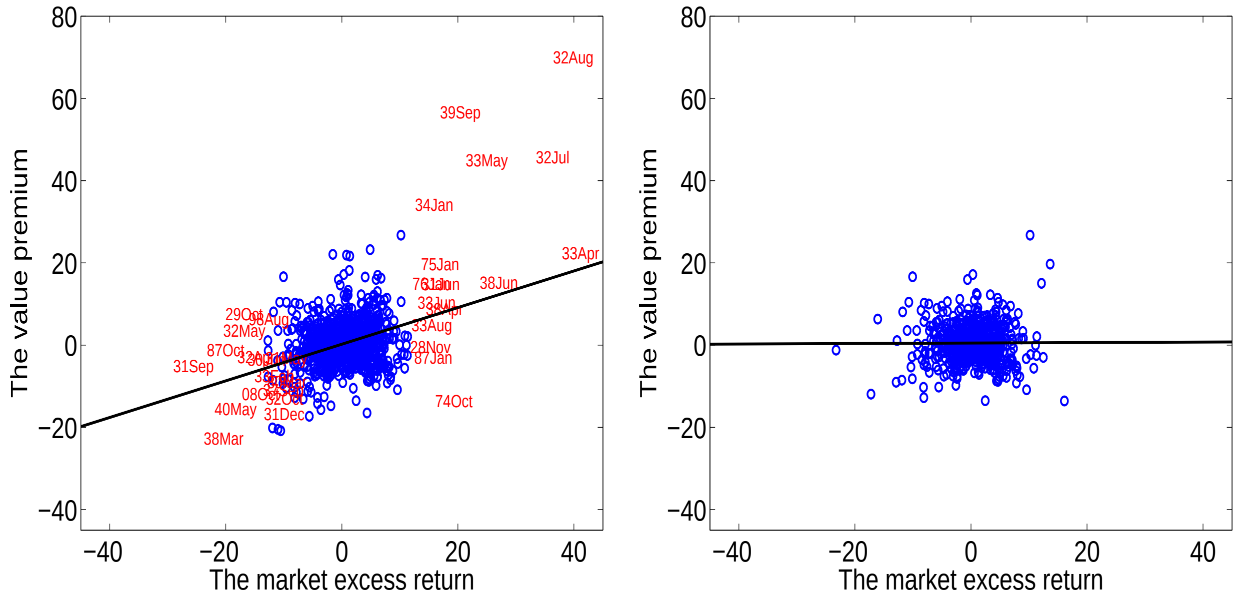


Figure 2 : The Impulse Response of Log Consumption to a Disaster Shock in the Model

In simulated data, when the economy enters the disaster state, we calculate the cumulative drop in log consumption for 25 years after the impulse. Consumption is time-aggregated from the monthly to annual frequency. The impulse response is averaged across more than 20,000 disaster episodes.

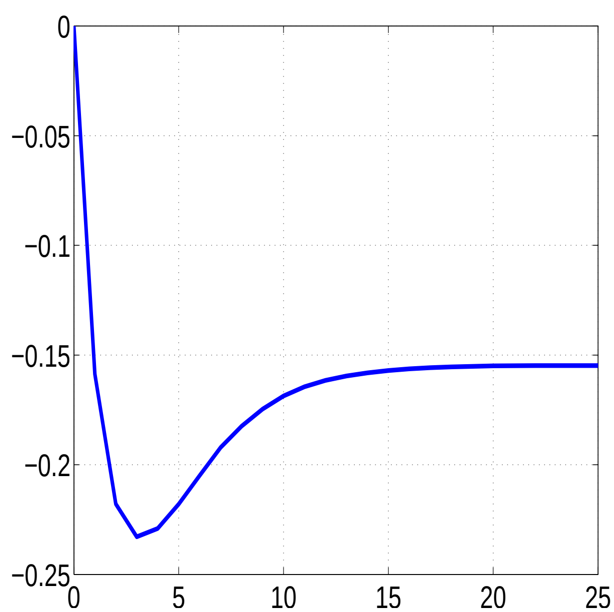


Figure 3 : The Impact of Disaster on Firm-level Risk and Risk Premiums in the Model

Results are based on the model's solution. The detrended consumption growth (g_t) is in the disaster state (λ_D) in Panels A and B, and is the mean of the normal states (zero) in Panels C and D. In each panel, capital is the detrended capital, $\widehat{K}_{it}$, and z is the log firm-specific productivity.

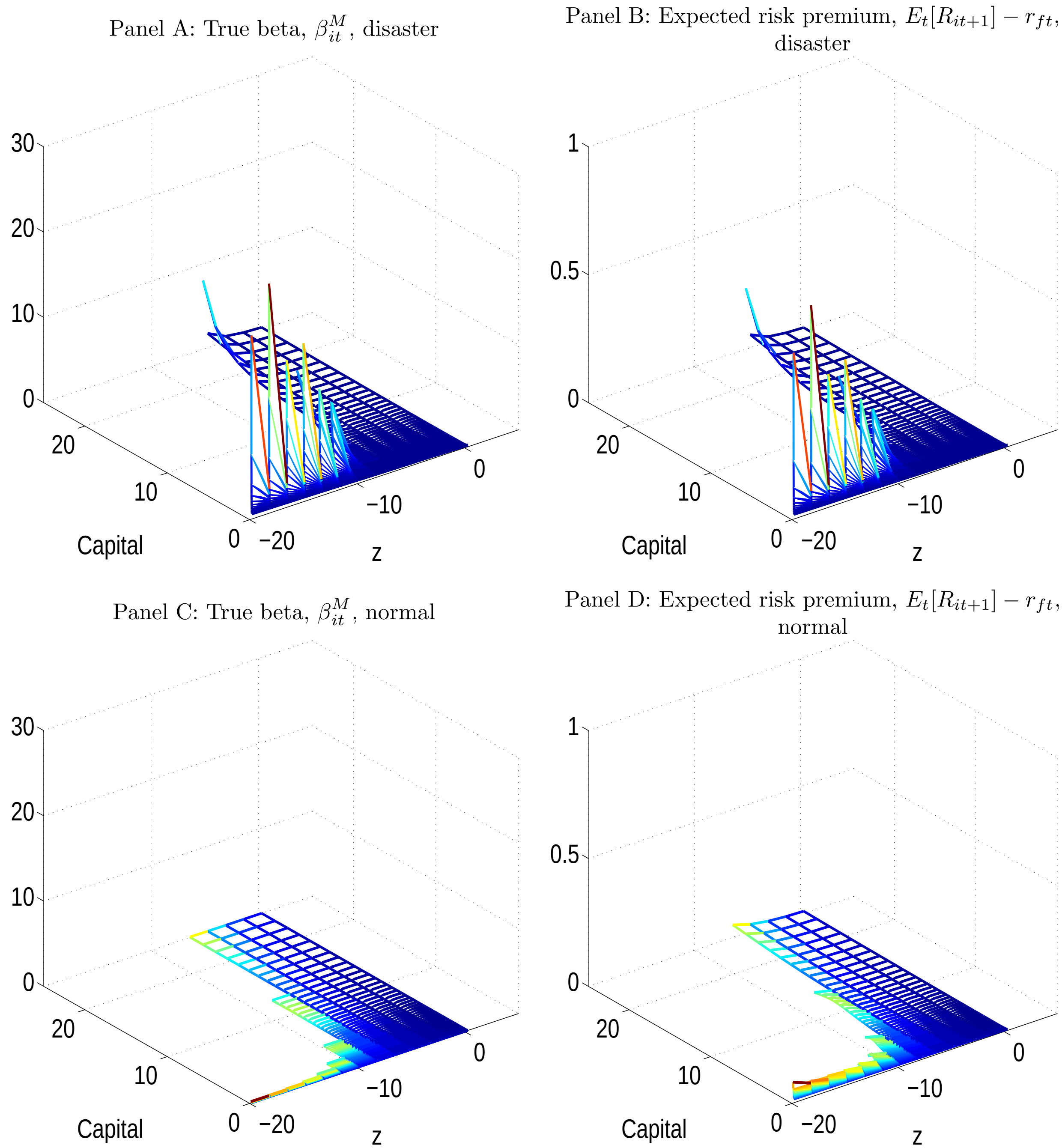


Figure 4 : The Impulse Responses for the Value and Growth Portfolios to a Disaster Shock in the Model's Simulations

Results are based on 300 simulations of a disaster shock in year one, and we trace the responses for 25 years afterward. The solid blue lines are for the value decile, the broken red lines the growth deciles, and the black dotted lines are the average responses across ten book-to-market deciles.

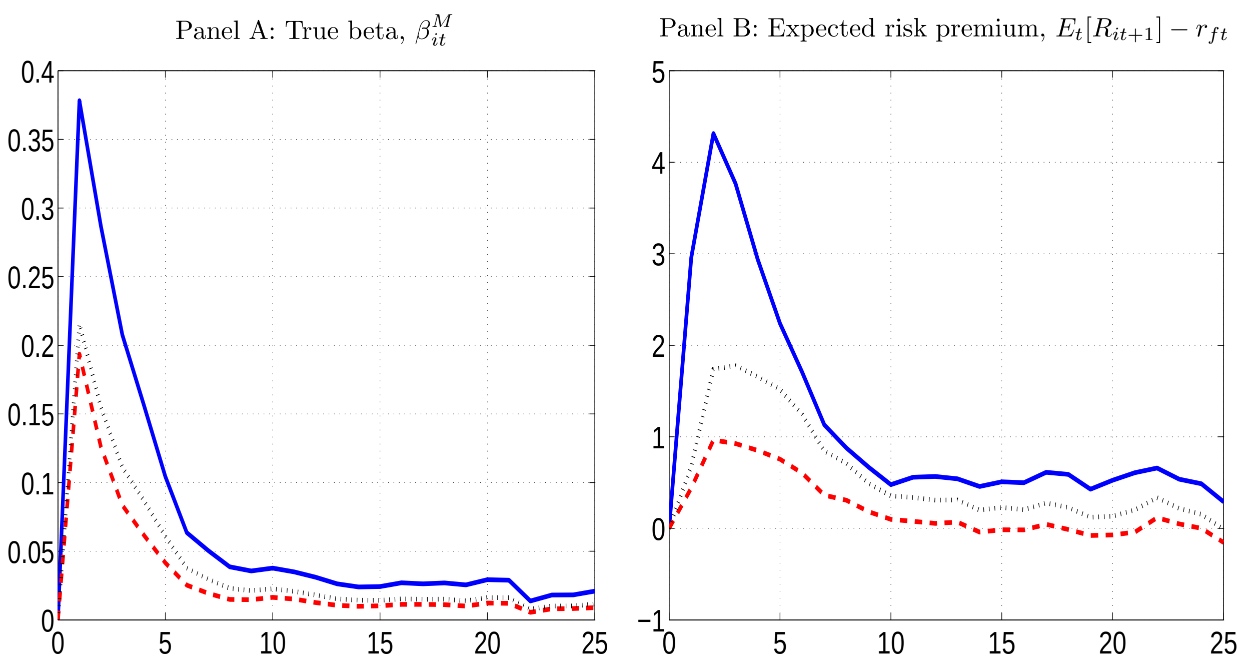


Figure 5 : The CAPM Regressions of the Value Premium and the Pricing Kernel in Artificial Samples with and without Disasters

Based on 2,000 simulations, the figure presents the scatter plots and fitted lines for the CAPM regression of the value premium (Panels A and B) and the pricing kernel (Panels C and D) in the model.

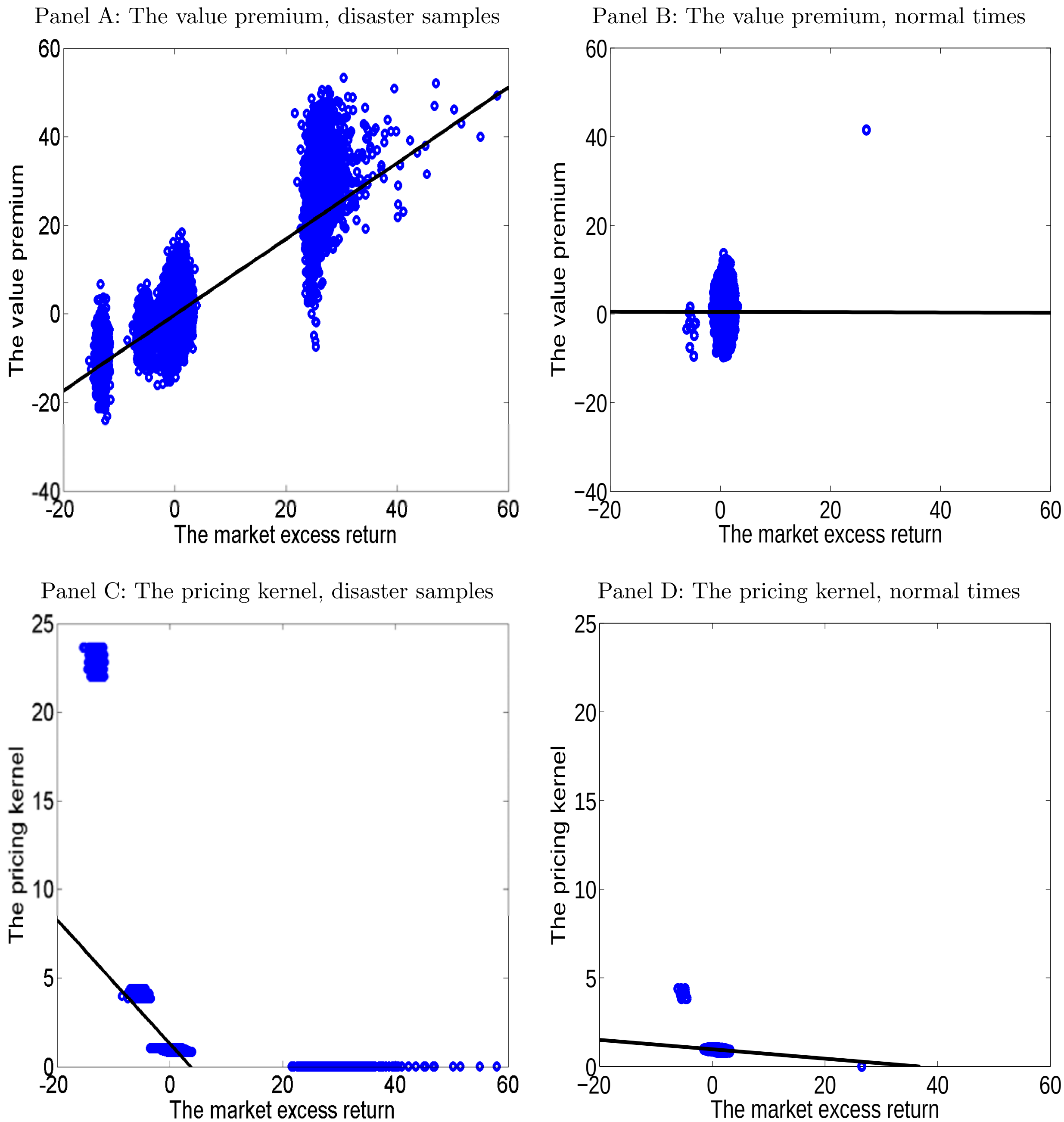
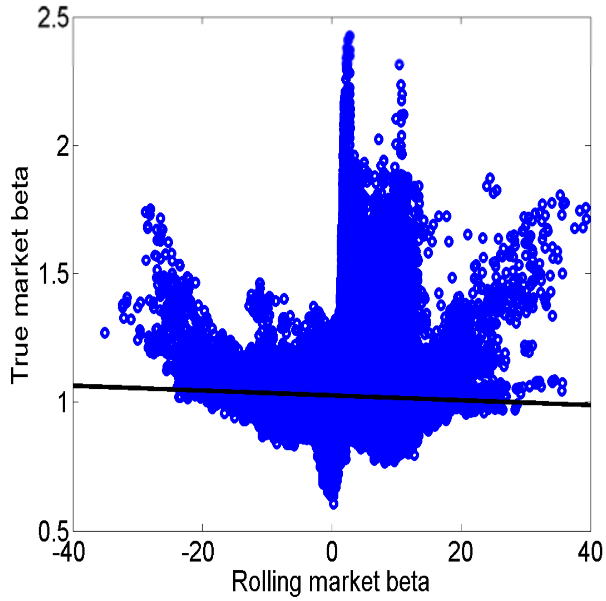


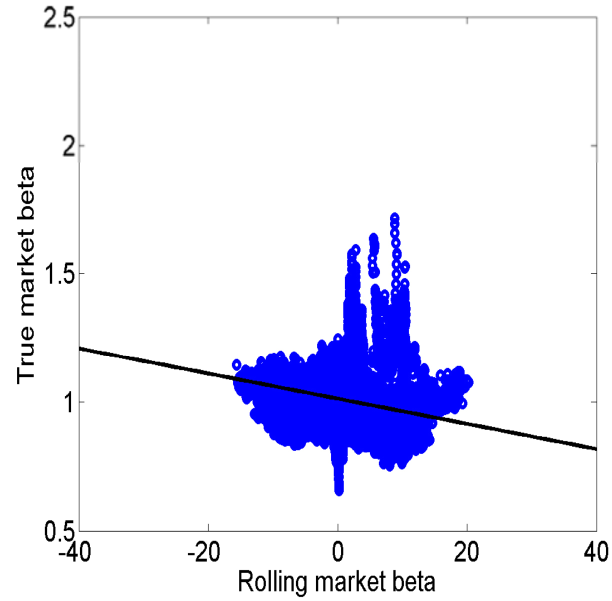
Figure 6 : True Market Betas and Rolling Market Betas in Artificial Samples with and without Disasters in the Model

The figure presents the scatter plots and fitted lines from regressing true market betas against rolling market betas in the model. The testing deciles are formed on rolling market betas in Panels A and B, and on true market betas in Panels C and D.

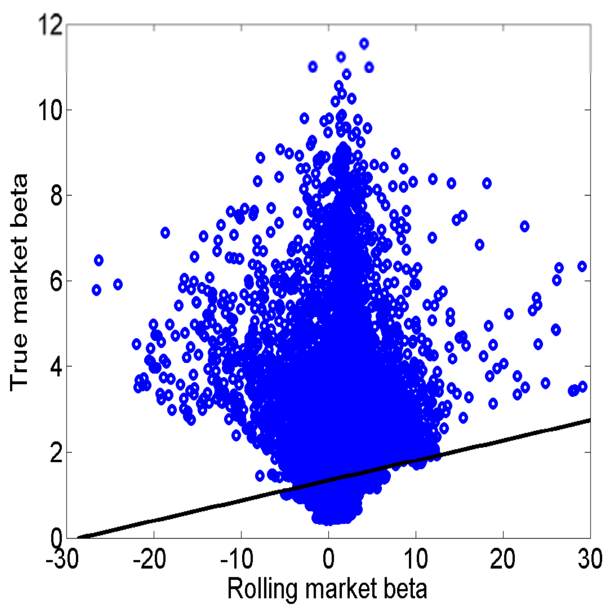
Panel A: Deciles on rolling market betas, disaster samples



Panel B: Deciles on rolling market betas, samples without disasters



Panel C: Deciles on true market betas, disaster samples



Panel D: Deciles on true market betas, samples without disasters