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ABSTRACT

This paper provides a new and simple model of endogenous horizontal product differentiation based on a standard demand structure derived from quadratic utility. One objective of the paper is to explain the "empirical Bertrand paradox" – the failure to observe homogeneous product Bertrand oligopoly, while homogeneous product Cournot oligopoly has significant empirical relevance. In our model firms invest in product differentiation if differentiation investments are sufficiently effective (i.e. if differentiation is not too costly). The threshold level of differentiation effectiveness needed to induce such investments is an order of magnitude less for Bertrand firms than for Cournot firms. Thus there is a wide range over which Bertrand firms differentiate their products but Cournot firms do not. If Cournot firms do choose to differentiate their products, corresponding Bertrand firms always differentiate more. We also establish the important insight that if product differentiation is endogenous Bertrand firms may charge higher prices and earn higher profits than corresponding Cournot firms, in contrast to the general presumption that Bertrand behavior is more competitive than Cournot behavior. Interestingly, consumer surplus increases with differentiation in the Cournot model but, due to sharply increasing prices, decreases with differentiation in the Bertrand model.

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1. Introduction

One of the classic topics in oligopoly theory is the "Bertrand Paradox", which dates from Bertrand's (1883) review of Cournot (1838). Bertrand suggested a model in which symmetric price-setting duopoly firms produce a homogenous product at constant marginal cost. The resulting (Nash) equilibrium, in which price equals marginal cost, seems unreasonable. As stated by Tirole (1988, pp. 210-211): "We call this the Bertrand *paradox* because it is hard to believe that firms in industries with few firms never succeed in manipulating the market price to make profits." It also seems implausible that the price should be completely unaffected by the number of firms as we go from two firms in a market to an arbitrarily large number.

A related puzzle, referred to as the "empirical Bertrand paradox", is that we rarely if ever observe an apparent Bertrand equilibrium in an oligopoly producing a homogeneous product, whereas homogeneous product Cournot oligopolies appear to be empirically relevant. Homogeneous product oligopolies are much more likely to be well approximated by the Cournot than the Bertrand model. Slade (1995, 9. 381) summarizes empirical work using Cournot and Bertrand models and reports no cases in which the Bertrand model is applied to the homogeneous product case. In contrast, the Cournot model is commonly applied in such cases. Recent examples of such Cournot applications include Carvajal, Deb, Fenske, & Quah (2013) for petroleum and Jansen, van Lier, van Witteloostuijn, & von Ochssée (2012) for natural gas.

Our first objective in this paper is to develop a model that can explain the empirical Bertrand paradox. There is a substantial prior literature suggesting possible resolutions of the Bertrand paradox, but our approach, based on endogenous horizontal product differentiation with a standard demand structure arising from quadratic utility, is new. A main contribution is to provide a natural and very simple explanation of why homogeneous product oligopoly is rarely, if ever, observed, and at the same time, to explain why homogeneous product Cournot models *are* likely to be empirically relevant.

We emphasize that our model does not seek to encompass in a single model all important resolutions of the Bertrand paradox. Nor is it an attempt to fully explain the empirical record of any particular industry. In particular, we acknowledge that vertical differentiation is empirically important in many industries. Our objective here is to abstract from various important phenomena that have been studied before, including vertical product differentiation, so as to focus on the role of costly horizontal product differentiation. We believe that focusing attention in this way allows for the development of valuable insights that would be obscured by a more complete model.

A second objective of this paper is to determine whether Bertrand industries are necessarily more competitive than corresponding Cournot industries once we allow for endogenous horizontal product differentiation. The conventional wisdom of oligopoly theory is that Bertrand industries are more competitive than Cournot industries in the sense of having lower prices and lower profits. However, this conventional wisdom is based on models in which firms have a common level of product differentiation. We show that Bertrand firms can charge higher prices and earn higher profits than corresponding Cournot firms due to greater horizontal product differentiation under Bertrand competition. We also examine implications for consumer surplus.

A classic example of investment in horizontal product differentiation is provided by the much-studied rivalry between Coca-Cola and Pepsi, which has been estimated using a Bertrand specification in Gasmi et al. (1992). As has been well-established in the marketing literature, many people cannot tell Coke and Pepsi apart in blind taste tests, yet high levels of advertising and other marketing activities create strong perceived product differences from the point of view of consumers. A classic study of this type is Woolfolk et al. (1983) which demonstrates that stated preferences by Pepsi and Coke "loyalists" depend strongly on whether they are given a Coke bottle or a Pepsi bottle, regardless of whether Coke or Pepsi is actually in the bottle.[1]

These advertising investments are economically important and both Coca-Cola and Pepsi are among the world's largest advertisers.[2] It is the investment in advertising, packaging, and other marketing activities that "creates" product differentiation in this case. In an interesting paper using MRI brain imaging, McClure et al. (2004) shows that the brain responds very differently to Coke and Pepsi when brand cues are available. As stated in the article: "In the brand-cued experiment, brand knowledge for one of the drinks had a dramatic effect on expressed behavioral preferences and on the measured brain responses." Thus the enormous advertising investments undertaken by Coke and Pepsi apparently do have a significant measurable effect on the brain's responses to perceptual stimuli. Horizontal product differentiation is also consistent with major empirical examples such as breakfast cereals.[3]

Our model is based on a standard quadratic utility function with two substitutable goods and a numeraire good. The model nests the two extremes of homogeneous goods and unrelated goods and allows for easy comparisons between Bertrand and Cournot duopoly outcomes. As in much of the related literature, the model has two stages. Firms simultaneously decide how much to

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[1] See Tremblay and Polasky (2002) for an economic analysis of using advertising to create both vertical and horizontal product differentiation.

[2] See, for example, Adbrands.net at www.adbrands.net/top_global_advertisers.htm which has Coke and Pepsi at 7th and 11th respectively for total global advertising expenditure in 2012.

[3] The packaged breakfast cereal market is dominated by two large firms – General Mills and Kellogg's, with a couple of other significant firms (Post and Quaker) and a number of small firms.

invest in horizontal product differentiation in stage 1 taking into account the equilibrium outcome of the subsequent price (Bertrand) or quantity (Cournot) game in stage 2.

A feature of our model is the introduction of an exogenous parameter, denoted β, that shifts down the effectiveness of differentiation investment or equivalently, shifts down the cost of achieving a given level of differentiation. Use of this parameter allows us to determine the threshold at which the effectiveness of such investment is just high enough to induce investment under each mode of competition. This approach provides additional insights not found in the existing literature. For example, in the vertical differentiation literature, Motta (1993) assumes a plausible cost of investment and finds that Cournot and Bertrand firms always invest in quality so as to make their products distinct. [4] Our more general approach allows for the important possibility that there is some sufficiently low level of differentiation effectiveness at which firms choose not to differentiation their products.

We show that there is a very wide range of values of β (and hence a wide range of the cost of achieving a given level of differentiation) under which Bertrand firms invest in horizontal differentiation, but Cournot firms do not. Thus in accordance with the "empirical Bertrand paradox", our model can explain why we might commonly observe homogeneous product industries that are consistent with the Cournot model, but rarely if ever do so in the Bertrand case. In addition, for any given value of β, we show that Bertrand firms differentiate their products more than corresponding Cournot firms, making it possible for them to charge higher prices and earn higher profits in equilibrium.

[4] Motta (1993) assumes a cost of investment, $u^2/2$, where u denotes the quality resulting from the investment.

Our paper also provides what we view as valuable insights concerning the welfare comparison of Bertrand and Cournot models. Specifically we show that, in the Bertrand case, increasing differentiation is always associated with reduced consumer surplus, despite the fact that consumers like variety. Consumer surplus falls because horizontal product differentiation significantly increases prices and these higher prices more than offset the benefits of increased variety. Product differentiation in the Cournot case has the opposite effect: an increase in product differentiation is always associated with increased consumer surplus so that the benefits of increased variety are larger than the loss due to higher prices.

Section 2 is devoted to a review and discussion of the related literature and Section 3 sets out the basic model structure. Sections 4 and 5 respectively develop the implications of horizontal product differentiation for Bertrand and Cournot duopolies, while Section 6 provides comparative results. Section 7 contains concluding remarks.

2. Literature Review

Following Hotelling (1929), a well-known approach to horizontal product differentiation is to interpret the characteristics of a product as determined by the location of the product on a line or a circle.[5] Each consumer buys at most one unit of the product. As shown by d'Aspremont, Gabszewicz, & Thisse (1979), if firms first choose their locations on a Hotelling line and then choose prices, they will choose to locate as far apart as possible, at the ends of the line, violating the principle of minimum differentiation proposed by Hotelling (1929).

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[5] Location choice may have either a horizontal or vertical interpretation. If all locations are equivalent to each other, then location choice corresponds to horizontal differentiation. However, if some locations are better than others then location choice can have a vertical interpretation.

One important difference between our analysis and the location choice literature is that we require firms to make costly investments in order to differentiate their products rather than simply choosing their location. The role of investment in product differentiation seems fundamental in many cases – making the analogy with location choice potentially misleading. Our approach also allows for a straightforward comparison of Bertrand with Cournot competition, whereas issues of existence and multiple equilibria arise in the location choice literature.[6] More generally, our consideration of a quadratic utility function in which consumers can purchase different quantities of each product extends our understanding of horizontal product differentiation to a different form of demand.

Another major approach to the Bertrand paradox is based on vertical product differentiation in which firms choose quality as pioneered by Shaked & Sutton (1982, 1983) and Motta (1993). Consumers purchase just one unit of a good that varies in quality. Firms commit to distinct or different quality levels in a first stage so as to avoid cutthroat Bertrand competition in the second stage. In Motta (1993), costly investment results in a wider gap in qualities (and higher profit in the fixed cost of quality model) under Bertrand than Cournot competition. Interestingly, however, Boccard & Wauthy (2010) show that introducing a capacity choice prior to the choice of quality can eliminate quality differentiation.

We argue that horizontal product differentiation provides an alternative means by which Bertrand firms can avoid extreme price competition and, at the same time, explains the choice of Cournot firms to produce homogeneous products. Horizontal product differentiation has the

[6] Quadratic costs with respect to distance ensure the existence of Bertrand equilibrium prices (see d'Aspremont, Gabszewicz, & Thisse, 1979), but the existence of a unique Cournot equilibrium requires that demand be elastic (perhaps by adding an outside good) and a specification as to which locations get served (see Anderson, De Palma, and Thisse, 1992, p. 332).

advantage of simplicity in that, unlike models of vertical integration, it does not require asymmetry at the solution.

A further approach to the Bertrand paradox focuses on the possibility that the mode of conduct itself (Bertrand or Cournot) might be an endogenous choice variable as in the classic treatment of Singh & Vives (1984). If so, depending on the details of model specification, the combination of homogeneous products and Bertrand conduct might be ruled out. If firms know that they will produce homogeneous products, they would opt for quantity-setting behavior rather than price-setting behavior. Following Singh & Vives (1984), this literature relies primarily on an assumed ability of firms to sign binding price contracts or binding quantity contracts that make the mode of conduct endogenous. Instead of taking the mode of conduct as exogenous and the extent of product differentiation as endogenous, the mode of conduct choice literature does the reverse, taking the extent of differentiation as exogenous and the mode of conduct as endogenous.

Although not specifically modeled, our analysis is not incompatible with the endogenous choice of mode of conduct. Firms knowing that it is costly to them to differentiate their products may try to commit to quantity-setting behavior. Once commitment to a mode of conduct has been made, our analysis then shows that the incentive for a firm to differentiate its product depends on the mode of competition: for the same effectiveness of investment in product differentiation, Cournot competition will result in less product differentiation than Bertrand competition.

Based on the work of Kreps & Scheinkman (1983), Friedman (1983, p. 47), Shapiro (1989, pp. 350-351), and Loertscher (2008), it is also possible that the mode of conduct is largely determined by the nature of technology in an industry and therefore properly viewed as exogenous with respect to the product differentiation decision. Specifically, if quantity is hard to change – as when it is determined by capacity constraints – and price adjusts to clear the market, then the Cournot model is appropriate[7]. If on the other hand, quantity can be readily changed to clear the market, then it becomes plausible that firms in an oligopoly would differentiate their products and engage in Bertrand price-setting behavior. Thus the mode of conduct can be exogenous – or at least hard to change relative to product differentiation decisions. For simplicity we do not model the barrier to entry that maintains the oligopoly. Some products, such as butter or sugar are hard to differentiate and do not have natural capacity constraints. Pure competition would result without government market support programs.

We do not take a position here on the relative empirical significance of endogenous mode of conduct models. They do seem to require the existence of contracts that, while not uncommon, are far from the norm and that can be rendered undesirable by realistic transaction costs or by uncertainties of various types. We simply argue that in many markets changes in technology are slow-moving relative to product differentiation induced through advertising (as with soft drinks) or minor changes in product specification (as with automobiles). If so, treating the mode of conduct as exogenous would often be appropriate. As mentioned above, it is conceptually possible for our model to be extended to allow for an endogenous mode of conduct decision combined with a decision to invest in horizontal product differentiation.

Yet another potential resolution of the Bertrand paradox is based on the possibility of implicit collusion in repeated price-setting games as noted in Tirole (1988) and extensively studied in the subsequent literature. Also, experimental work on the Bertrand model, including

[7] The automobile industry provides an example where capacity constraints are important, but there is also scope for significant product differentiation. Our model suggests that if automobile producers were not capacity constrained, they would be involved in even more product differentiation.

Bruttel (2009), suggests the possibility that the Bertrand paradox may be avoided for behavioral reasons or for reasons related to bounded rationality. The Bertrand paradox can also be avoided if cost is uncertain and firms are risk averse, as in Wamback (1999), resulting in prices above marginal cost.

The comparative competitiveness properties of Bertrand and Cournot models have been addressed by several authors. The basic finding, provided by Cheng (1985), Singh & Vives (1984), Vives (1985), Hsu and Wang (2005), and others is that if Bertrand and Cournot duopolies face the same demand and cost conditions, then the Bertrand industry would generate lower profits, lower prices, and more consumer surplus. Also Tremblay and Tremblay (2011) compares the stability characteristics of Cournot and Bertrand models under product differentiation.

Qiu (1997) provides an important extension in which cost is made endogenous through the introduction of endogenous process R&D. The paper finds that the conventional ranking of the two models may be reversed. Cournot firms will often have a stronger incentive to invest in R&D, causing costs to fall and possibly providing more consumer surplus (and more total surplus after including profits) than in the Bertrand case. Symeondis (2003) examines product R&D that can improve product quality and, like Qui (1997), finds that the Cournot model can generate more total surplus if there are strong R&D spillovers and products are not too strongly differentiated in equilibrium. Our model differs from Qui (1997) because investment causes product differentiation not cost reduction and differs from both Qiu (1997) and Symeondis (2003) in that it is not based on R&D spillovers. In our setting, investment in product differentiation reduces, but never eliminates, the higher consumer surplus generated by Bertrand competition. Also, Bertrand competition can be more profitable than Cournot competition.

3. Model of Horizontal Product Differentiation

We assume a duopoly model in which firm 1 produces quantity x_1 and firm 2 produces quantity x_2. Goods x_1 and x_2 can range between being perfect substitutes (homogeneous) to being totally unrelated. Using M to represent consumption of a numeraire good, the aggregate or representative utility function is taken to be

$$U = a(x_1 + x_2) - (b/2)(x_1^2 + x_2^2) - sx_1x_2 + M\,. \qquad (1)$$

Since M is additively separable, there are no income effects of demand. The parameter s represents the degree of substitutability between the products x_1 and x_2.[8]

Without loss of generality we undertake an algebraically convenient normalization and rescaling of variables such that b = 1. If b =1 then the feasible range for s is between 0 and 1. If s = 0, then demand for each good is independent. Since products are unrelated, each firm has a monopoly with respect to its good. If $s = 1$ (or, more generally, if $s = b$), goods are perfect substitutes and are, in effect, identical or homogenous. To measure the degree of differentiation, we define a parameter $v = 1 - s$ (v for "variety") where $0 \le v \le 1$. However, it is convenient to use s in the specification of the demand structure, yielding the following inverse demand functions:

$$p_1 = \partial U/\partial x_1 = a - x_1 - sx_2.$$

$$p_2 = \partial U/\partial x_2 = a - x_2 - x_1. \qquad (2)$$

[8] We do not examine the possibility considered in Singh and Vives (1984) that the products could be complements (s < 0).Such an extension addresses what, in our view, is an essentially different issue – coordination of complementary products rather than competition between substitutes.

Since other things equal (i.e. holding quantities x_1, x_2, and M constant), we obtain $\partial U/\partial s = -x_1x_2 < 0$ and since $v = 1 - s$, it follows that consumers gain from variety or equivalently from greater product differentiation. This property could reflect a taste for variety at the individual level or some distribution of tastes captured in aggregate utility function U.

Firms 1 and 2 can each choose to increase the degree of product differentiation (variety) by making a differentiation investment, denoted k_1 and k_2. The combined effect, $K = k_1 + k_2$, of the investments of both firms determines the value of v. We make the simplifying assumption that the differentiation investment affects only the degree of differentiation (or the degree of substitutability) with no effects on other aspects of demand. One possibility is to interpret the differentiation investment as an advertising cost aimed at making the product more distinct from the other product in the eyes of consumers. Another possibility is to interpret the investment as the cost of changing some physical characteristic of the product that differentiates it from the other product, as when breakfast cereal companies come up with additional variations in taste, texture, and packaging or when car manufacturers adopt new colors and or new body shapes for cars or undertake other differentiation activities of a costly but essentially horizontal nature.

We model the effect of differentiation investments on the degree of differentiation (variety) experienced by consumers using the following convenient functional form:

$$v = 1 - s \text{ where } s = 1/e^{\beta K} = e^{-\beta K} \tag{3}$$

for $\beta > 0$ and $K = k_1 + k_2$. If neither firm invests in differentiation, then $K = 0$ and variety $v = 0$, so the products are effectively identical (substitutability, s, is 1). If either firm invests in differentiation (such as by advertising or by superficial product adjustment) then $v > 0$ ($s < 1$) and the products are differentiated. An increase in differentiation investment by either firm increases v at a decreasing rate.

An important feature of Equation (3) is that differentiation expenditures by one firm have a symmetric effect on the other firm in the sense that s increases equally for both firms. This is an intrinsic feature of horizontal product differentiation. If product A is differentiated relative to product B it follows that product B is equally differentiated relative to product A. This point is particularly obvious in the case of locational differentiation where if B moves further from A then A must also become further from B and by exactly the same amount.

In practice, when firms make differentiation investments they might also seek to create a better product (i.e. vertically differentiate) and take business from the rival. This could be captured in our model by allowing the intercept of demand, parameter a in Equation 2, to be firm specific with a_1 increasing in k_1 and decreasing in k_2 and vice versa for a_2. However, as we wish to focus on the effects of pure horizontal product differentiation, we abstract from such considerations here.

The functional form (3) captures the empirically reasonable property that there is no finite amount of advertising or other investment aimed at differentiating products that can make substitutable products into completely unrelated products. The monopoly outcome in which product are unrelated ($v = 1$ and $s = 0$) is reached only in the limit as the combined differentiation investment, K, approaches infinity. Strictly speaking v can never equal 1 as that would require an infinite investment, so the admissible range for v is the half-closed interval [0,1). In the context of the coke and pepsi example, this property means that no amount of advertising can make consumers think that coke and pepsi are completely unrelated.

The parameter β represents the effectiveness of investment in creating product differentiation. We imagine that β could take on different values depending on the nature of the product. If β is small, then investment has little effect in changing variety as perceived by consumers whereas, if β is large, even a small amount of investment can have a substantial effect. As previously mentioned, by exogenously varying β we can determine the threshold at which investment becomes profitable under each mode of competition. As a result, we can characterize the range of values of β under which Cournot firms will produce homogeneous products while Bertrand firms differentiate their products. This region provides our resolution of the "empirical Bertrand Paradox".

The two-stage game played by firms gives rise to a sub-game perfect Nash equilibrium. Each firm simultaneously commits to its investment in stage 1 taking the investment of the other firm as given. In making its investment, each firm correctly anticipates the effects of its investment on the outcome of the Bertrand-Nash or Cournot-Nash equilibrium in stage 2.

4. Bertrand Competition

Suppose first that the firms act as Bertrand competitors. We solve for the second stage equilibrium conditional on s, and then show how the equilibrium changes with product differentiation, $v = 1 - s$. We then consider the first stage in which product differentiation is determined by the choices of k_1 and k_2.

4.1 *Second Stage – Pricing Decisions*

In the second stage, each firm maximizes variable profit with respect to its own price treating the other firm's price as exogenous and treating k_1, k_2, and therefore s as predetermined. Variable profit for firm i for i = 1,2, denoted V_i, excludes the differentiation investments, k_i, sunk at stage 1:

$$V_i \equiv (p_i - c)x_i \tag{4}$$

To express outputs as functions of prices and s, we rewrite the inverse demand functions (2) as $x_1 + sx_2 = a - p_1$ and $sx_1 + x_2 = a - p_2$ and solve to obtain:

$$x_1 = [(a - p_1) - (a - p_2)s]/(1 - s^2);$$

$$x_2 = [(a - p_2) - (a - p_1)s]/(1 - s^2) \qquad (5)$$

The demand equations (5) require $s < 1$, which applies if the products are not homogenous. If products are homogenous ($s = 1$), consumers will buy from only one firm if that firm has a strictly lower price. If the firms charge the same price, we adopt the standard convention that they share the quantity demanded equally.

We next maximize variable profit (4) using (5) to solve for the Bertrand equilibrium prices and quantities. Conditional on an exogenous (or pre-determined) level of product differentiation, the properties of this model are known, but we report and prove the specific results for our setting. At the stage 2 Bertrand equilibrium, each firm has the same price, output, and variable profit. As shown in Appendix 1, these common values, denoted p, x, and V respectively, depend on s and hence on $v = 1$-s as follows:

$$p = p^B(s) = (a\text{-}c)(1\text{-}s)/(2\text{-}s) + c$$

$$x = x^B(s) = (a\text{-}c)/[(2\text{-}s)(1+s)]$$

$$V = V^B(s) = (1\text{-}s^2)(x^B(s))^2 \qquad (6)$$

A superscript B identifies functional relationships as depending on Bertrand competition.

As (6) shows, in order for output to be positive the maximum willingness to pay, a, must exceed marginal cost, c. We impose $a > c$ as a regularity condition for all subsequent analysis. If $s = 1$ (homogeneous products), then (6) implies $p = c$ and $x = (a\text{-}c)/2$. This is the standard Bertrand solution with homogeneous products. If $s = 0$ (separate monopolies for goods 1 and 2), then the equilibrium prices are higher: $p = (a + c)/2$, which exceeds c due to the requirement that $a > c$. Interestingly, however, the quantities are the same in this dual monopoly case as in the homogeneous product case: $x = (a-c)/2$. Each firm produces the same amount but consumers are willing to pay more because the products are differentiated.

As shown in Proposition 1, increases in product differentiation cause the Bertrand price to rise for the admissible range of v (from 0 to 1). Correspondingly, reductions in v – less variety – cause price to fall. Also, starting with homogeneous products ($v = 0$), quantities initially fall as differentiation increases, reach a minimum at $v = ½$, then increase as differentiation increases further. Greater product differentiation always increases (variable) profits.

Proposition 1: Under Bertrand competition, an increase in product differentiation, v, causes

i) prices to rise,

ii) outputs to fall if $0 \leq v < ½$, reach a minimum at $v = 1/2$, and then rise for $½ < v < 1$.

iii) variable profits to rise.

Proof: i) Differentiating (6) with respect to s yields $dp^B/ds = -(a-c)/(2-s)^2 < 0$. Since $s = v - 1$, it follows that $dp^B/dv = (dp^B/ds)(ds/dv) = -dp^B/ds > 0$.

ii) Differentiating output as in (6) with respect to s yields

$$dx^B/ds = x(2s-1)/[(2-s)(1+s)] \qquad (7)$$

It can be seen from (7) that dx^B/ds is positive if $s > ½$, zero if $s = ½$ and negative if $s < ½$. Since $v = 1 - s$, it follows that dx^B/dv is negative if $v < ½$, zero at $v = ½$ and positive if $v > ½$.

iii) From (6) and (7), we obtain

$$dV^B/ds = -2x^2(1 - s + s^2)/(2-s) < 0 \qquad (8)$$

As $dV^B/dv = - dV^B/ds$ it follows from (8) that $dV^B/dv > 0$. ***

4.2 *First Stage – Investments in Product Differentiation*

In stage 1, firm 1 chooses k_1 and firm 2 chooses k_2 to maximize profit taking the investment of the other firm as given. Due to sequential rationality as implied by the subgame perfect Nash equilibrium, Bertrand firms understand that a failure to differentiate their products will yield a second stage outcome in which profits are zero. This understanding provides a strong incentive to undertake positive differentiation investments in the first stage.

The differentiation investments k_1 and k_2 jointly determine $K = k_1 + k_2$, which in turn determines the degree of differentiation, $v = 1\text{-}s$ where $s = e^{-\beta K}$ (see (3)). The partial effect of each firm's investment on s, taking the investment of the other firm as given reduces substitutability (and increases the degree of differentiation)

$$\partial s/\partial k_1 = \partial s/\partial k_2 = ds/dK = - \beta e^{-\beta K} = - \beta s < 0 \tag{9}$$

Recognizing that variables will take on equilibrium values (6) in the second stage, the first stage profit for firm i can be written as:

$$\pi_i = V^B(s) - k_i = (1\text{-}s^2)(x^B(s))^2 - k_i \tag{10}$$

where s and $v = 1\text{-}s$ depend on k_1 and k_2 as in (3). In setting k_i, each firm i for i = 1, 2 correctly anticipates the effect of k_i on its variable profit at the second stage equilibrium, but takes the investment of the other firm as fixed. Using (8) and (9), the first order condition for an interior solution ($k_i > 0$) to firm i's profit maximization problem is therefore

$$\partial \pi_i/\partial k_i = (dV^B/ds)(\partial s/\partial k_i) – 1 = 2\beta s(x^B(s))^2(1 – s + s^2)/(2\text{-}s) – 1 = 0 \tag{11}$$

A corner solution in which there is no investment in differentiation arises if $\partial\pi_i/\partial k_i \leq 0$ at $k_i = 0$. In order for Equation (11) to characterize a maximum rather than a minimum it is necessary that the second order conditions are satisfied, which is shown in Appendix 2.

Since equilibrium output, x, varies with s, equation (11) is a complicated function of s, making a closed form solution for s and for the common level of investment, $k = k_1 = k_2$, difficult to obtain. However, we are able to determine important characteristics of the solution. In particular, Proposition 2 indicates the threshold level of β below which product differentiation is prohibitively costly. Our assumption of an exponential functional form (see (3)) to specify the relationship between variety, v, and investment is not critical. If, for example, instead of (3), we assume a power function: $s = (1 + k_1 + k_2)^{-\beta}$, exactly the same threshold level applies.[9]

Proposition 2: Under Bertrand competition, both firms choose to differentiate their products at stage 1 if and only if $\beta > 2/(a\text{-}c)^2$. If $\beta \leq 2/(a\text{-}c)^2$ then no differentiation investment takes place and products are homogeneous at stage 2.

Proof: No differentiation ($s = 1$) takes place if and only $k_1 = k_2 = 0$, which occurs if and only if $\partial\pi_i/\partial k_i \leq 0$ at $k_i = 0$ and $s = 1$ (i.e. at $v = 0$). Substituting $s = 1$ into (11) and using $x = (a\text{-}c)/2$ from (6) shows that $\partial\pi_i/\partial k_i = \beta(a\text{-}c)^2/2 - 1 \leq 0$ if and only if $\beta \leq 2/(a\text{-}c)^2$.***

In the case of prohibitively expensive differentiation costs, it is still feasible for both firms to enter, so we cannot rule out homogeneous product Bertrand oligopoly. However, in that case each firm is indifferent about whether to produce or whether to withdraw from the industry.

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[9] If $s = (1 + k_1 + k_2)^{-\beta}$, then $\partial s/\partial k_i = -\beta s/(1 + k_1 + k_2)$. If $k_1 = k_2 = 0$, then $s = 1$ and $\partial s/\partial k_i = -\beta$, which setting $s = 1$ in (10) is the same as if we had assumed $s = e^{-\beta K}$. The proof of Proposition 2 then goes through as before. We could also use a Taylor's series expansion to approximate $s = e^{-\beta K}$ by $s = 1 - \beta K + \beta^2 K^2/2$, which implies that $\partial s/\partial k_i = -\beta(1-\beta K) < 0$ for $\beta K < 1$. Since $\partial s/\partial k_i = -\beta$ at $k_1 = k_2 = 0$, Proposition 2 again applies.

As a result, any positive entry cost or fixed cost would prevent the Bertrand outcome. Provided β exceeds the threshold level, Bertrand firms will necessarily differentiate their products.

5. The Cournot Model

Suppose now that the firms are Cournot competitors. We first examine the second stage choice of output before considering the choice of differentiation investment in stage 1.

5.1 *Second Stage – Quantity Decisions*

In the second stage, we take k_i as given from stage 1. Setting output, x_1, to maximize variable profit, V_1, as in (4), holding x_2 fixed and setting x_2 to maximize V_2 holding x_1 fixed and using the demand functions given by (2), we obtain the first order conditions:

$$\partial V_1/\partial x_1 = a - c - 2x_1 - sx_2 = 0$$

$$\partial V_2/\partial x_2 = a - c - sx_1 - 2x_2 = 0 \tag{12}$$

Equations (13) define the Cournot equilibrium values of output, which are unique and symmetric. Equilibrium prices and variable profits then follow from (2) and (7). Using $v = 1\text{-}s$ and a superscript C to identify functional relationships associated with Cournot competition, we express the common output, price and profit at the stage 2 Cournot equilibrium as follows:

$$x = x^C(s) = (a\text{-}c)/(2 + s)$$

$$p = p^C(s) = (a\text{-}c)/(2+s) + \text{c}$$

$$V = V^C(s) = (x^C(s))^2 \tag{13}$$

The effects of variation in product differentiation are set out in Proposition 3.

Proposition 3: Under Cournot competition, an increase in product differentiation v, causes outputs to rise, prices to rise, and variable profits to rise.

Proof: From (13), we obtain

$$dx^C/ds = -(a-c)/(2+s)^2 = -x/(2+s),$$

$$dp^C/ds = -(a-c)/(2+s)^2 \text{ and}$$

$$dV^C/ds = -2(x^C(s))^2/(2+s) = -2(a-c)^2/(2+s)^3 \quad (14)$$

Since $dv/ds = -1$, the result follows. ***

In the limit as v approaches 1, each firm is effectively a monopolist over an independent good, produces output level, $x = (a-c)/2$, and charges the common price, $p = (a + c)/2$, as in the Bertrand case. If $v = 0$, then products are homogenous and output and price are at the standard homogenous product Cournot levels: $x = (a-c)/3$ and $p = (a + 2c)/3$. It is notable that each firm sets a higher output (and charges a higher price) at the monopoly outcome with independent products than if products are homogeneous.

5.2 *First Stage – Investments in Product Differentiation*

Recognizing that variables will take on equilibrium values (14) in the second stage, the first stage profit for firm i can be written as: $\pi_i = V^C(s) - k_i = (x^C(s))^2 - k_i$. As with the Bertrand model, in the first stage each firm i sets its differentiation investment, k_i, to maximize profit, understanding its effect on variable profit at the second stage equilibrium, but taking as exogenous the differentiation investment of the other firm. Using (14) and (9), the associated first order condition for an interior solution for firm i is given by

$$\partial\pi_i/\partial k_i = (dV^C/ds)(\partial s/\partial k_i) - 1 = 2\beta s(x^C(s))^2/(2+s) - 1 = 0 \quad (15)$$

where $x^C(s) = (a\text{-}c)/(2 + s)$ from (13). (See Appendix 2 for a proof that second order conditions are satisfied.) The corner solution in which $k_i = 0$ occurs if $d\pi_i/dk_i \leq 0$ at $k_i = 0$. Proposition 5 identifies the threshold level of β below which product differentiation does not occur.

Proposition 4: With Cournot competition, firms choose to differentiate their products at stage 1 if and only if $\beta > 13.5/(a\text{-}c)^2$. If $\beta \leq 13.5/(a\text{-}c)^2$ then no investment takes place and products are homogeneous at stage 2.

Proof: No differentiation ($s = 1$) takes place if and only if and only if $\partial\pi_i/\partial k_i \leq 0$ at $k_i = 0$ and $s = 1$ (i.e. at $v = 0$) for i = 1,2. Substituting $s = 1$ into (15) and using $x = (a\text{-}c)/3$ from (13) shows that $\partial\pi_i/\partial k_i \leq 0$ if and only if $\beta \leq 13.5/(a\text{-}c)^2$. ***

As with the Bertrand case, firms undertake less product differentiation than would be needed to maximize joint profits.

6. Comparing the Cournot and Bertrand Models

6.1 The Empirical Bertrand paradox

Proposition 6 provides a characterization of the range of differentiation effectiveness, β, for which product differentiation occurs or does not occur in the Bertrand and Cournot models. The proposition also establishes that if Bertrand firms have an incentive to differentiate, they will always invest more than their Cournot counterparts leading to greater product differentiation under Bertrand than Cournot competition. To facilitate the exposition, we use superscripts *B* and *C* to represent Bertrand and Cournot outcomes respectively. Thus p^B and x^B represent the (common) price and (common) level of output and $v^B = 1 - s^B$ denotes the level of product differentiation under Bertrand competition.

Proposition 5:

(i) If $\beta \leq 2/(a\text{-}c)^2$, then products are homogeneous under both Bertrand and Cournot competition.

(ii) If $2/(a\text{-}c)^2 < \beta \leq 13.5/(a - c)^2$, then products are differentiated under Bertrand competition and are homogenous under Cournot competition: $v^B > v^C = 0$.

(iii) If $\beta > 13.5/(a\text{-}c)^2$, then products are differentiated under both Bertrand and Cournot competition, but are more differentiated under Bertrand than Cournot competition: $v^B > v^C > 0$.

Proof: Parts (i) and (ii) follow directly from Propositions 2 and 4. (iii) If $\beta > 13.5/(a\text{-}c)^2$, then $k^C_i > 0$ from Proposition 5. From $k^C_i > 0$, (15) and (14), we have $d\pi_i^C/dk_i = 0$ which implies $2\beta s^C(x^C)^2 = 2+s^C$ where $s^C = 1/e^{\beta K} > 0$ for $K = k^C_1 + k^C_2$. Setting $K = k^C_1 + k^C_2$ and $s = s^C$ in (11), we obtain $\partial\pi_i^B/\partial k_i = 2\beta s^C(x^B)^2(1 - s^C + (s^C)^2)/(2\text{-}s^C) - 1$. For the same value of s > 0, it follows from (6) and (13) that $x^B = (2 + s)x^C/(2\text{-} s)(1+ s) > x^C$ and hence $2\beta s^C(x^B)^2 > 2\beta s^C(x^C)^2 = 2+s^C$ and $d\pi_i^B/dk_i > (2+s^C)(1 - s^C + (s^C)^2)/(2\text{-}s^C) - 1$ at $K = k^C_1 + k^C_2$. Since $(2+s^C)(1 - s^C + (s^C)^2) - (2\text{-}s^C) = (s^C)^2(1 + s^C) > 0$, we obtain $\partial\pi_i^B/\partial k_i > 0$ for $K = k^C_1 + k^C_2$, which implies $k^B_i > k^C_i$ and $s^B > s^C$ for $\beta > 13.5/(a\text{-}c)^2$ ***

Proposition 5 indicates that for both Bertrand and Cournot competition, whether the firms undertake differentiation expenditures depends on the effectiveness of differentiation investments, measured by β, relative to the difference, $(a - c)$ between the demand intercept and marginal cost – a reflection of the strength of demand. The stronger is demand, the less effective differentiation investments needs to be to justify differentiation. Letting $\beta^B \equiv 2/(a\text{-}c)^2$ represent the critical level of differentiation effectiveness needed for Bertrand firms to undertake differentiation and $\beta^C \equiv 13.5/(a\text{-}c)^2$ the corresponding critical value for the Cournot case, Table 1 shows the values of β^B and β^C as the strength of demand increases.

Table 1: Critical Values of Differentiation Effectiveness, β

Demand ($a - c$)	Bertrand (β^B)	Cournot (β^C)
2	0.50	3.38
4	0.13	0.84
8	0.031	0.21
12	0.014	0.094
16	0.0078	0.053
20	0.0050	0.034
30	0.0022	0.015
50	0.00080	0.0054

As can be seen from Table 1, the critical values, β^B and β^C rapidly get smaller as demand increases, but since $\beta^C/\beta^B = 6.75$ independent of demand, the critical value of differentiation effectiveness required to induce differentiation under Cournot competition is always a factor of 6.75 higher than for the Bertrand model. Consequently, our results explain the empirical Bertrand paradox in a way that is robust to the level of demand: there is a very wide range of parameter values under which products are homogeneous under Cournot competition, but differentiated under Bertrand competition.

6.2 Endogenous Product Differentiation and Comparative Competitiveness

It is well-known that if products are homogeneous, Bertrand competition is more intense than Cournot competition. As shown by Singh and Vives (1984), this insight generalizes to any common level of product differentiation short of being completely unrelated. Using a demand structure similar to ours, Singh and Vives (1984) demonstrate that, for any (exogenous) common level of differentiation less than unrelated products ($v < 1$), the Bertrand model generates higher output, lower prices and lower profits than the Cournot model. If products are unrelated ($v = 1$), then prices and outputs are at the monopoly level in both models.

However, when we allow for the highly relevant possibility that product differentiation is endogenous, the comparison of Cournot and Bertrand is more difficult and very different in its implications. If any differentiation at all occurs, Bertrand firms choose a higher level of product differentiation than Cournot firms. This consideration tends to raise prices and lower output in the Bertrand case to an extent that can offset the inherently greater competitiveness of Bertrand behaviour conditional on a given common level of differentiation.

Proposition 6 establishes that, even with endogenous product differentiation, the Bertrand model generates higher output than the Cournot model. However, prices and profits are *not* necessarily lower in the Bertrand case. Therefore, allowing for endogenous product differentiation is an important limitation on the general presumption that Bertrand industries are "more competitive" than corresponding Cournot industries.

Proposition 6:

i) Independent of whether product differentiation is the same across modes of competition or is chosen endogenously, output is higher under Bertrand than Cournot competition: $x^B > x^C$.

ii) For the same level of product differentiation, both price and profit are strictly lower under Bertrand than Cournot competition: $p^B < p^C$ and $\pi^B < \pi^C$.

iii) With endogenous product differentiation it is possible for Bertrand firms to charge higher prices and earn more profit than corresponding Cournot firms. For example, for an effectiveness of investment parameter, $\beta = 13.5/(a\text{-}c)^2$, Bertrand firm undertake more differentiation ($v^B = 0.60823 > v^C = 0$), charge higher prices and earn more profit than corresponding Cournot firms.

Proof: i) From $x^B = (a\text{-}c)/(2\text{-}s^B)(1+s^B)$ and $x^C = (a\text{-}c)/(2 + s^C)$ (see (6) and (13)), we obtain

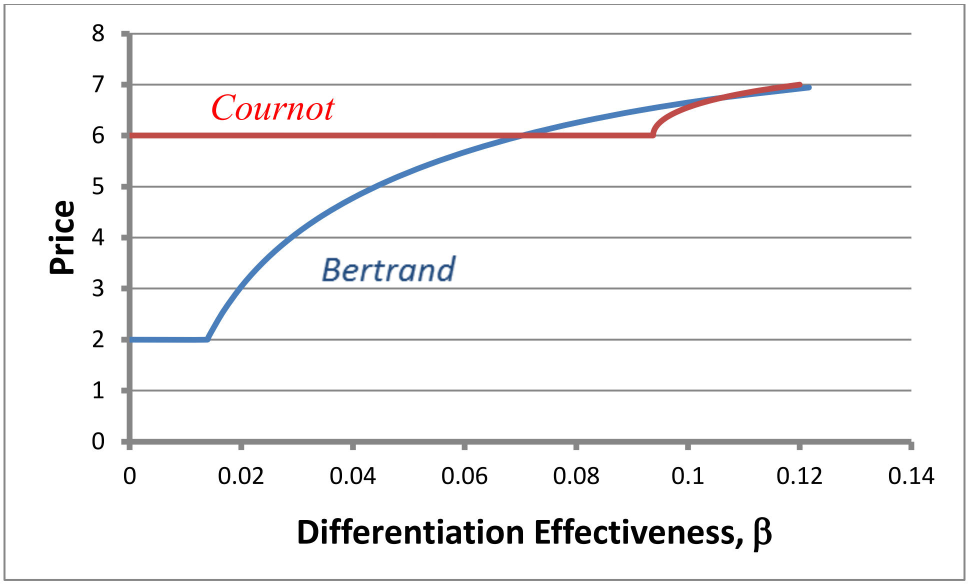
$x^B = (2+s^C)x^C/(2-s^B)(1+s^B)$, which implies $x^B > x^C$ if and only if $2+s^C - (2-s^B)(1+s^B) = s^C - s^B + (s^B)^2 > 0$. Since $s^C \geq s^B > 0$ from Proposition 6, the condition holds and $x^B > x^C$.

ii) It follows immediately from downward sloping demand and $x^B > x^C$ (see part i) that the Bertrand price must be less than the Cournot price for the same v for $v = 1 - s < 1$. Now examining profits, if $k = 0$ (homogeneous products), then $s = 1$ and $v = 1 - s = 0$ and from (6) and (13), we obtain $V^C(1) > V^B(1) = 0$ and hence $\pi^C > \pi^B = 0$. If $k^B = k^C > 0$, then $s^B = s^C = s < 1$ and $V^B(s) = V^C(s)(1-s)(2+s)^2/(1+s)(2-s)^2$ (from (6) and (13)). Letting $\psi(s) \equiv (1+s)(2-s)^2 - (1-s)(2+s)^2$, we obtain $\pi^C - \pi^B = V^C(s)\psi(s)/(1+s)(2-s)^2 > 0$ since $\psi(s) > 0$ for all $s \in (0,1]$.

iii) If $\beta = 13.5/(a-c)^2$, then from Proposition 5(ii), products are differentiated under Bertrand Competition ($v^B > 0$ and $k^B > 0$) and homogeneous under Cournot competition($v^C = 0$ and $k^C = 0$). Since $k^B > 0$, it follows from (11) that $d\pi^B_i/dk_i = 2\beta s(x^B)^2(1 - s + s^2)/(2-s) - 1 = 0$ where $x^B = (a-c)/(2-s)(1+s)$ from (6). Setting $\beta = 13.5/(a-c)^2$ and solving for s we obtain $s^B = 0.39177$ and $v^B = 1 - s^B = 0.60823$. We have $p^B > p^C$ since substituting $v^B = 0.60823$ into (6) and $v^C = 0$ into (13), yields $p^B = 0.378(a-c) + c$ and $p^C = (a-c)/3 + c$. Now examining profits, for $\beta = 13.5/(a-c)^2$, we obtain $x^B = 0.44677(a-c)$, $V^B = 0.168968(a-c)^2$, $V^C(1) = (a-c)^2/9$ from (6) and (13). Since $k^B = -\ln(s^B)/2\beta$ (from $s = e^{-\beta K}$), we further obtain $k^B = 0.0347(a-c)^2$. Thus $\pi^B = V^B(s^B) - k^B = 0.13427(a-c)^2 > \pi^C = (a-c)^2/9 = 0.1111(a-c)^2$. ***

Figure 1 illustrates the variation in the prices charged by Bertrand and Cournot firms as β is increased, raising the effectiveness of investment as a means to create product differentiation (or equivalently lowering the cost of a given level of product differentiation). We assume that the intercept of demand and marginal cost are given by $a = 14$ and $c = 2$, so $a - c = 12$. For a wide range of parameter values, $\beta \leq \beta^C = 0.094$, Cournot firms will not invest in product differentiation, corresponding to the flat part of the Cournot price response curve in Figure 1. Bertrand firms begin to differentiate at the much lower level of $\beta = \beta^B = 0.014$.

Figure 1: The effect of differentiation effectiveness, β, on price



As Figure 1 shows, when products are homogeneous ($v = 0$), Cournot firms charge \$6, whereas Bertrand firms set the much lower price of \$2, which equals marginal cost. At $\beta = \beta^B = 0.014$, Bertrand firms start investing, causing the Bertrand price to rise. At $\beta = 0.0703$ (approx.), the Bertrand price surpasses the Cournot price of \$6 and remains higher for a significant range of values of β, illustrating the result in Proposition 7, part iii, that price can be higher under Bertrand than Cournot competition. Eventually, Cournot firms start investing (at $\beta = \beta^C = 0.094$) and the Cournot price again exceeds the Bertrand price for $\beta \geq 0.107$ (approx.), but the difference is sufficiently slight that the two prices are virtually identical. Ultimately, as β becomes very large, products become unrelated and both prices approach the monopoly price of \$8.

The Bertrand price is much more sensitive to product differentiation than the Cournot price. We do not show price as a function of differentiation, *v*, in the diagram. However, changing *v* from 0 (homogeneous products) to 0.4 raises the Cournot price from \$6.00 to \$6.62 – a modest increase of about 10%. For Bertrand firms, an increase in product differentiation from 0 to 0.4 causes price to rise dramatically – from $2 to $5.43 – an increase of over 150%! The ability of Bertrand firms to aggressively raise prices makes variable profits under price competition much more sensitive to product differentiation than are variable profits under Cournot competition. As a result, Bertrand firms undertake differentiation investments at much lower values of β than do Cournot firms. Bertrand firms have a powerful incentive to invest in product differentiation since it enables them to rapidly move away from the cutthroat price competition that prevails when products are homogeneous.

A comparison of the effects of product differentiation on output is also interesting. Again for $a - c = 12$, Figure 2 shows output per firm for both Bertrand and Cournot competition as the degree of product differentiation, v, varies. The qualitative relationship between output and product differentiation does not depend on the magnitude of $a – c$. If products are homogeneous products ($v = 0$), each Bertrand firm produces $x^B = (a-c)/2 = 6$, whereas Cournot firms produce a lower quantity, $x^C = (a-c)/3 = 4$. As products become more differentiated, Cournot output increases steadily until it reaches the monopoly output of 6 at $v = 1$. Thus Cournot firms exploit the increased demand arising from greater product variety by raising both quantity and price (see Figure 1). In contrast, Bertrand firms initially reduce output as products become differentiated, with output reaching a minimum at $v = 0.5$. Since output falls, it follows that Bertrand firms initially realize the benefits of product differentiation solely through aggressive price increases. Despite this pattern of output response, output is always higher under Bertrand than Cournot competition with or without endogenous product differentiation (see Proposition 6, part i).

Figure 2: Product differentiation and output per firm

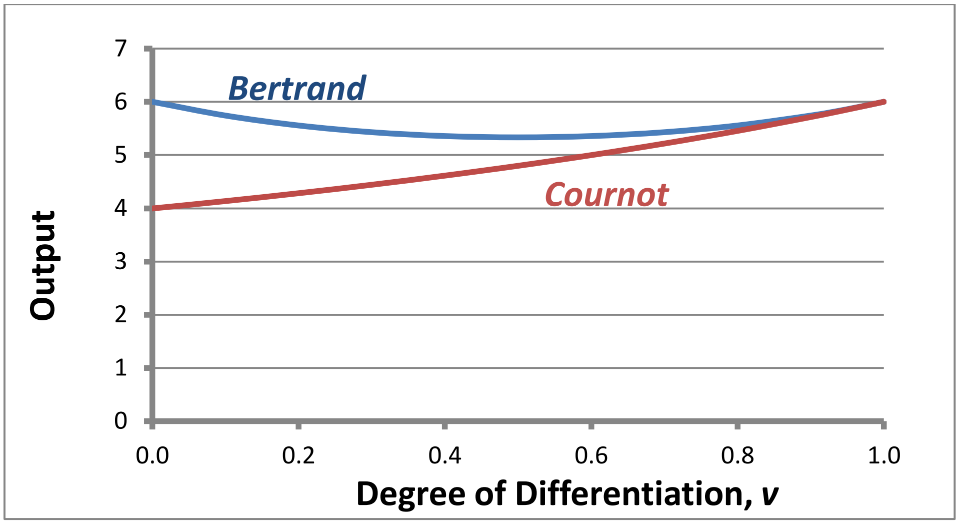
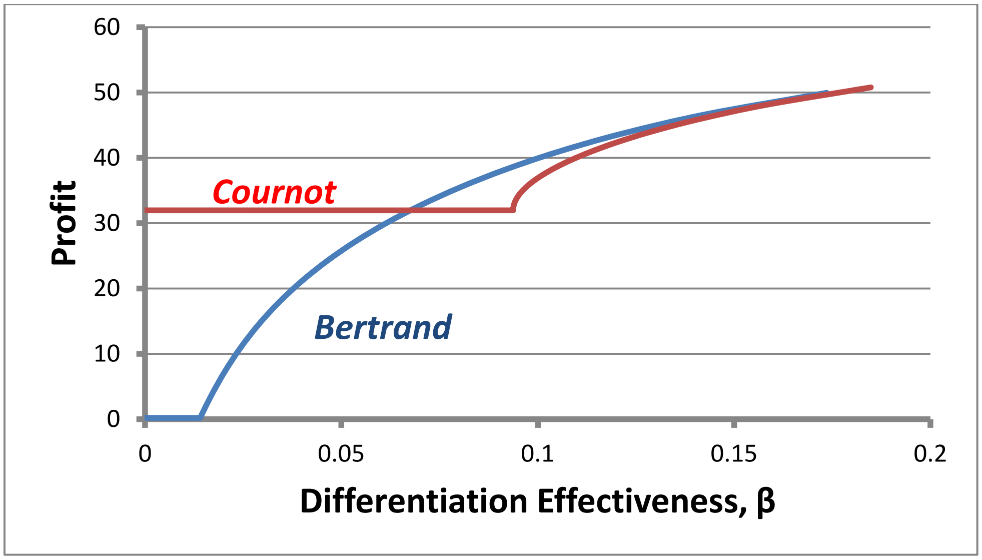


Figure 3 compares the variation in industry profit under Bertrand and Cournot competition as β is increased making investment more effective as a means to create product differentiation. For Figure 3, we again assume $a - c = 12$, but profit has the same qualitative relationship with β for any values of a - c. As shown by the initial flat portion of the Bertrand line in Figure 3, products are homogeneous ($v = 0$) and profit is zero under Bertrand competition for $\beta \leq \beta^B = 0.014$. Under Cournot competition, products are homogeneous for a much wider range of β, namely $\beta \leq \beta^C = 0.09375$, but each Cournot firm earns \$16, with a combined profit of \$32 for the industry. For $\beta > \beta^B = 0.014$, Bertrand profits rise due to product differentiation. Cournot firms invest for $\beta > \beta^C = 0.09375$, but, already at $\beta = \beta^C$, Bertrand firms earn a combined profit of \$38.6, which exceeds the \$32 in industry profit earned under Cournot competition. At higher levels of β, profit under Bertrand competition continues to exceed profit under Cournot competition, but profit levels become almost indistinguishable.

Figure 3: The effect of differentiation effectiveness, β, on profits



6.3 Endogenous Product Differentiation and Consumer Surplus.

There is a significant literature comparing the effects of Bertrand and Cournot competition on consumers. The effects of different levels of investment in process R&D under the two modes of competition have been considered, but the literature does not address the effects of different levels of investment in horizontal product differentiation.

Letting $G \equiv U - (p_1x_1 - p_2x_2 - M)$ denote consumer surplus (or "gains"), then from (1) for b = 1, (2) and the equality of outputs and prices across firms, we obtain

$$G = 2(a\text{-}p)x - (1+s)x^2 = (1+s)x^2 \qquad (16)$$

Since the relationship between output and product differentiation has a different functional form depending on the mode of competition (see (7) and (14)), we define $G^B = G^B(\mathrm{s}) = (1+s)(x^B(s))^2$ and $G^C = G^C(s) = (1+s)(x^C(s))^2$ to examine consumer surplus as a function of product differentiation, v, where $v = 1 - s$. Proposition 7 follows.

Proposition 7:

i) For any given level of product differentiation, v, an increase in v:

(a) reduces consumer surplus under Bertrand competition.

(b) increases consumer surplus under Cournot competition.

ii) Whatever the levels of product differentiation, v^B and v^C, other than unrelated products ($v = v^B = v^C = 1$), consumer surplus is always higher under Bertrand than Cournot competition.

Proof: i) From (16), $\mathrm{d}x^B/\mathrm{d}s = - x^B(1\text{-}2s)/(2\text{-}s)(1+s)$ (see (7)), $\mathrm{d}x^C/\mathrm{d}s = - x^C/(2+s)$ (see (14)) and $s > 0$ (from $v = 1\text{-} s < 1$), we obtain

$$\mathrm{d}G^B/\mathrm{d}s = 3\mathrm{s}(x^B(\mathrm{s}))^2/(2 - s) > 0$$

$$\mathrm{d}G^C/\mathrm{d}s = -s(x^C(\mathrm{s}))^2/(2+s) < 0 \qquad (17)$$

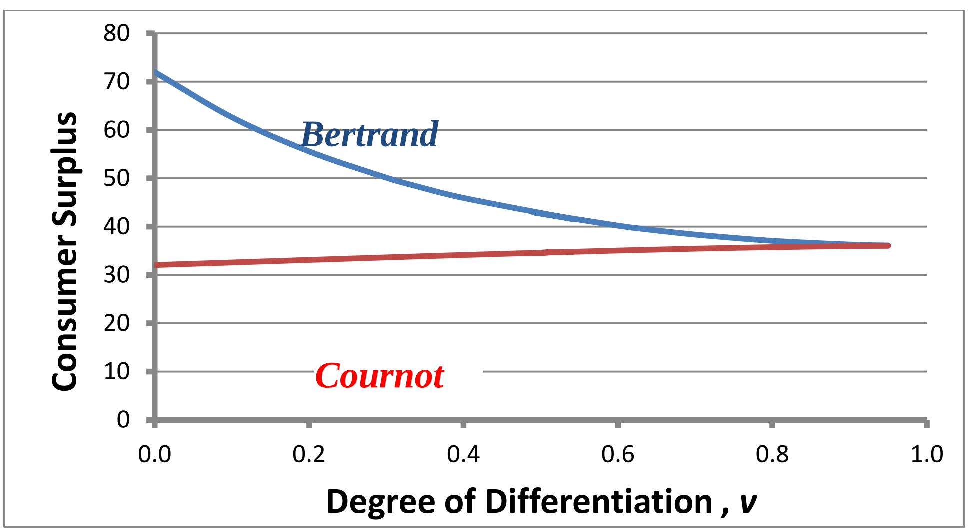
Since $v = 1\text{-} s$ and $\mathrm{d}G/\mathrm{d}v = - \mathrm{d}G/\mathrm{d}s$, it follows from (17) that $\mathrm{d}G^B/\mathrm{d}v < 0$ proving (a) and $\mathrm{d}G^C/\mathrm{d}v > 0$ proving (b). ii) If $v < 1$, then $s = 1 - v > 0$. Since $\mathrm{d}G^B/\mathrm{d}s > 0$ and $\mathrm{d}G^C/\mathrm{d}s < 0$ for $s > 0$ (see (17)), we have $G^B > G^C$ for any v^B, $v^C \in [0,1]$ provided $v^B < 1$ or $v^C < 1$. If $v = 1$(products are unrelated), then $x^B = x^C = (a\text{-}c)/2$ (see (7) and (13)) and we obtain $G^B = G^C$ from (16) ***

Proposition 7 makes the striking point that when Bertrand firms produce differentiated products rather than homogeneous products, consumers are made worse off. Even though consumers get more utility from differentiated products at given prices, Bertrand firms take advantage of product differentiation to raise prices sufficiently that the price increase more than offsets the direct gain in utility experienced by consumers. By contrast, the price increases arising from increases in product differentiation under Cournot competition are relatively modest and are not sufficient to offset the consumer gains from greater variety.

Despite the fact that consumers are made worse off by Bertrand product differentiation and better off by Cournot product differentiation, it is still true that for any given level of differentiation, consumer surplus is higher under Bertrand than Cournot competition. Indeed, as shown in Proposition 8, part ii, consumer surplus is always higher under Bertrand competition, regardless of product differentiation decisions. This last result follows from the fact that the limiting case of monopoly in which products are completely independent (variety is at $v = 1$), represents the lower bound for consumer surplus under Bertrand competition and the upper bound for consumer surplus under Cournot competition.

The results of Proposition 7 are illustrated in Figure 4 assuming, as before, that $a - c = 12$.

Figure 4: Product Differentiation and Consumer Surplus



7. Concluding Remarks

This paper provides an explanation of the empirical Bertrand paradox based on endogenous horizontal product differentiation. By the "empirical Bertrand paradox" we mean the failure to observe homogeneous product Bertrand oligopoly in practice while homogenous product Cournot oligopoly is of empirical relevance. Even though product differentiation is costly, we show that Bertrand firms have a much stronger incentive to undertake product differentiation than Cournot firms.

Our model of differentiation is sufficiently general that prohibitive differentiation costs are possible, even in the Bertrand case. The critical value of differentiation effectiveness required for differentiation under Bertrand competition is reflected by a differentiation effectiveness parameter that we label β^B. At $\beta = \beta^B$, and even at a value of β twice or three times β^B, Cournot firms will produce homogeneous products. The level of β must rise to 6.75 times β^B before Cournot firms will undertake differentiation. Consequently, there is a very wide range of differentiation effectiveness (and hence differentiation costs) over which Bertrand firms would differentiate but Cournot firms would not.

While there are other reasons why we might rarely if ever observe homogeneous product Bertrand oligopoly, we suggest that our model of endogenous horizontal product differentiation provides a natural explanation with significant empirical relevance. Examples include cases such as soft drinks – where differentiation is achieved through advertising – and automobiles, where much of the year to year changes that are made in model specification are small styling differences that have little to do with performance and much to do with creating perceived differentiation. In the case that differentiation investments are very ineffective in creating differentiation of relevance to consumers, Bertrand firms would earn zero profits and would be indifferent between producing and staying out of the market. Since even the slightest fixed costs or entry costs would generate negative profits, it is unlikely we would observe homogeneous product Bertrand oligopoly as an equilibrium outcome.

We also investigate the relative competitiveness of otherwise equivalent Bertrand and Cournot industries. Interestingly, we find that for sufficiently high values of differentiation effectiveness (i.e. low differentiation costs) Bertrand firms charge higher prices and earn larger profits than Cournot firms. For any given level of product differentiation short of completely unrelated products, variable profits are lower under Bertrand competition than Cournot competition. However, the cutthroat nature of price competition when products are homogeneous is rapidly tempered by product differentiation. Indeed, the enhanced market power enjoyed by Bertrand firms as differentiation increases more than offsets the benefits to consumers of greater variety, with the result that consumer surplus falls. Cournot competition, however, exhibits the opposite result, as greater product differentiation increases consumer surplus. Nevertheless, regardless of differences in product differentiation across the two modes of competition, consumer surplus is always higher under Bertrand competition than Cournot competition.

This paper deals only with the simultaneous move game – in which two duopolists are in a symmetric position with respect to timing. A natural extension is to consider sequential move games in which entry deterrence or, at least, entry manipulation becomes important. In such a case an incumbent might wish to make a commitment to "plant the flag" in the face of potential entry, even though it would have a post-entry incentive to differentiate its product. In such a context endogenous product differentiation is one of several types of investment that allow firm to take a particular "positon" or "niche" in the market, as in Bloch, Eaton, and Rothschild (2014). Another useful extension would be to consider multi-product firms (as in Chen and Chen (2014)) More broadly we believe that the model of horizontal product differentiation developed in this paper provides potentially useful insights for both simultaneous and sequential interactions between firms in oligopoly markets.

Appendix 1: Bertrand price, output, and profit

To prove the equilibrium conditions (6), we derive each condition as a function of s. where $s = 1 - v$. Each firm i for i = 1,2 sets its price to maximize variable profit taking the price of the other firm as given. From (4) using (5), the first order conditions are

$$dV_1/dp_1 = x_1 - (p_1 - c)/(1 - s^2) = [(a + c - 2p_1) - (a - p_2)s]/(1 - s^2) = 0$$

$$dV_2/dp_2 = x_2 - (p_2 - c)/(1 - s^2) = [(a + c - 2p_2) - (a - p_1)s]/(1 - s^2) = 0$$

These first order conditions simplify to $2p_1 - sp_2 = (a + c) - as$ and $2p_2 - sp_1 = (a + c) - as$, which imply a common price, denoted, p, where

$$p = p^B(s) = (a + c - as)/(2-s) = (a-c)(1-s)/(2-s) + c \quad \text{(A1.1)}$$

where the superscript B identifies functional relationships in the Bertrand model. Substituting (A1.1) into the demand functions (5) yields the common equilibrium quantity:

$$x = x^B(s) = (a - p)/(1 + s) = (a-c)/(2-s)(1+ s) \quad \text{(A1.2)}$$

Equations (A1.1) and (A1.2) apply for $s < 1$ (differentiated products) and for $s = 1$ (homogeneous products). Substituting (A1.1) and (A1.2) into variable profit, $V_i \equiv (p_i - c)x_i$, from (4) yields the common equilibrium variable profit at the second stage:

$$V = V^B(s) = (1-s^2)(x^B(s))^2 \quad \text{(A1.3)}$$

Appendix 2: Second order conditions for differentiation decisions.

Show $\partial^2\pi_i/(\partial k_i)^2 < 0$ for all $s \in (0,1]$ or $v = 1 - s \in [0,1)$

For both Bertrand and Cournot competition, each firm i sets k_i taking k_j for $j \neq i$ fixed and using $\partial s/\partial k_i = ds/dK = -\beta s$ from (9), we obtain $\partial\pi_i/\partial k_i = -\beta s(dV/ds) - 1 = 0$ for $k_i > 0$ from (11) and (15) and hence that

$$\partial^2\pi_i/(\partial k_i)^2 = \beta^2 s[(dV/ds) + s(d^2V/(ds)^2] \tag{A2.1}$$

For Bertrand competition, using $dV^B/ds = -2(x^B)^2(1 - s + s^2)/(2-s)$ from (8) and $dx^B/ds = -x^B(1-2s)/(2-s)(1+s)$ from (7) it can be shown that

$$d^2V^B/(ds)^2 = -6(x^B)^2[3s - 1 - s^2(1 - s)]/(2 - s)^2(1 + s) \tag{A2.2}$$

It can then be shown from (8) and (A2.2), that

$$dV^B/ds + s(d^2V^B/(ds)^2) = -2(x^B)^2\Psi^B/(2-s)^2(1+ s) \tag{A2.3}$$

where $\Psi^B \equiv (1-s+s^2)(2-s)(1+s) + 3s[3s - 1 - s^2(1-s)]$ can be expressed as

$$\Psi^B = 2(1-s)(1-s+s^2) + s^2(5+s+2s^2) > 0 \tag{A2.4}$$

From (A2.1), (A2.3) and (A2.4), we obtain $\partial^2\pi^B{}_i/(\partial k_i)^2 < 0$ for all $s \in [0,1]$.

For Cournot competition, using $dV^C/ds = -2(x^C)^2/(2+s)$ from (15) and $x^C = (a-c)/(2+s)$ from (13), we obtain

$$d^2V^C/(ds)^2 = 6(x^C)^2/(2+s)^2 \tag{A2.5}$$

It then follows from (15) and (A2.5), that

$$dV^C/ds + s(d^2V^C/(ds)^2) = -4(1-s)(x^C)^2/(2+s)^2 < 0 \tag{A2.6}$$

and hence, using (A1), that $\partial^2\pi^C{}_i/(\partial k_i)^2 < 0$ for all $s \in [0,1]$. ***

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