OPTIMAL MONETARY GROWTH WITH
ACCOMODATING FISCAL POLICY
IN A SMALL OPEN ECONOMY

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Policy in a Small Open Economy

ABSTRACT

This paper emphasizes how the choice of the optimal monetary growth rate in a small open economy under perfect capital mobility depends upon the accommodating policy chosen to maintain the overall budget constraint in the economy. When this occurs through lump sum taxation, the optimal monetary growth rate is shown to be the "distorted" Friedman monetary rule. If the adjustment occurs through the income tax rate, the optimal monetary growth rate involves a Phelps-type tradeoff between the income tax rate and the inflation tax rate. The framework is suited for analyzing optimal macroeconomic policy in general and the latter part of the paper considers an optimal monetary-fiscal package.

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The question of the optimal rate of monetary growth has been extensively discussed for many years now; see e.g. Bailey (1956), Friedman (1969, 1971), Tobin (1968), Brock (1974, 1975), Turnovsky and Brock (1980), and others. This literature, which has approached the issue from various viewpoints, has been restricted entirely to a closed economy. The present paper has two objectives. The first is to investigate the issue for a small open economy. The second, which we shall see is a natural extension of the first, is to determine the optimal rate of monetary growth as part of a broader optimal macroeconomic policy package for such an economy. The framework we employ is one in which the behavioral relationships are derived from optimizing behavior on the part of private agents in the economy.[1] Optimal policy is then determined to maximize the welfare of the representative utility-maximizing individual in the domestic economy.

Following much of the current literature, the analysis is based on the assumption of perfect capital mobility, in the sense that uncovered interest rate parity (UIP) holds.[2] As other authors have shown, with a constant rate of time discount (as we also assume) these utility maximizing models imply that the economy must always be in steady state equilibrium.[3] This in turn imposes constraints on the policy options available to the economy. Appropriate accommodation, which we take to be undertaken by some fiscal instrument, is required to ensure that the equilibrium is sustained within the aggregate budget constraint facing the economy. One of our main results is to show that the optimal rate of monetary growth depends critically upon what the accommodating fiscal policy variable is chosen to be. In this respect, the extension to the small open economy is important, since such accommodation is not required to sustain equilibrium in a closed economy.

The fact that equilibrium requires some fiscal accommodation leads naturally into the second issue, namely the determination of optimal fiscal policy in conjunction with the optimal rate of monetary growth. Specifically, the paper determines the joint determination of (i) the optimal monetary growth rate; (ii) the optimal tax rate; (iii) the optimal level of government expenditure.[4]

A second simplifying assumption underlying the analysis is that of a single traded good so that purchasing power parity (PPP) holds. This assumption is made mainly for analytical convenience and is not critical. But it does mean that determining the optimal rate of optimal monetary growth is equivalent to determining the optimal rate of exchange depreciation or exchange rate crawl. The crawling peg system has often been advocated as being an attractive regime in that it combines stability with flexibility. On the one hand, steady adjustments in the exchange rate minimize fluctuations, yet at the same time the crawling peg may allow individual countries to pursue independent monetary policies. Different rates of exchange depreciation have different welfare implications for an economy. Some years ago, Mathieson (1976) addressed the question of the optimal exchange rate crawl from the viewpoint of consumption maximization under the assumptions of PPP and UIP, using a model based on arbitrarily specified behavioral relationships.[5] Our analysis can be viewed as addressing this issue using a natural framework for this type of welfare analysis.

I. THE MACROECONOMIC FRAMEWORK

The model we shall employ is adapted from Turnovsky (1985). It consists of three sectors: (i) consumers, (ii) firms, and (iii) the domestic government. All consumers and firms are assumed to be identical, enabling us to focus on the representative individual in each group.

A. Structure of Economy

We assume that the domestic economy is small. It produces a single traded good, the foreign price of which is given on the world market. In percentage change terms, the PPP relationship implies

$$p = q + e$$

where

p = rate of inflation of the good in domestic currency,

q = rate of inflation of the good in foreign currency,

e = actual rate of exchange depreciation, which under perfect foresight, also equals the anticipated rate of exchange depreciation.

Domestic residents may hold two assets. The first is domestic money, which is not held by foreigners. Secondly, they may hold a traded bond, denominated in foreign currency, and which may be issued either by the domestic government or by foreigners. Under perfect foresight, the UIP relationship is

$$r = r^* + e$$

where

r = domestic nominal interest rate,

r^* = foreign nominal interest rate.

The representative consumer's plans are obtained by solving the following intertemporal optimization problem

(1a) $Max \int_0^\infty U(c, l, m, g)e^{-(r^*-q)t}dt \quad U_c > 0, U_l < 0, U_g > 0$

subject to

(1b) $\dot{m} + \dot{b} + c = (1-\tau)(wl + \pi) + (r^* - q)b - (q + e)m - T$

and initial conditions

(1c) $m(0) = \dfrac{M_0}{P(0)} = \dfrac{M_0}{Q_0 E(0)} \; ; b(0) = \dfrac{E(0)B_0}{Q_0 E(0)} = \dfrac{B_0}{Q_0}$

where

c = real consumption,

m = real money balances; M = nominal money balances,

b = real stock of (traded) bonds; B = nominal stock of bonds,

l = labor,

w = real wage rate,

π = real profit,

τ = income tax rate,

T = real lump sum tax,

g = real government expenditure,

P = domestic price level,

Q = foreign price level,

E = exchange rate, measured in units of domestic currency per unit of foreign currency.

Several features of this specification merit comment. First, the consumer's rate of time preference is taken to be the (after-tax) real rate of interest, $r^* - q$. As is well known, as long as the rate of time preference is assumed to be constant, then this is the only value which under perfect capital mobility is consistent with the ultimate attainment of a steady state equilibrium.[6]

Secondly, the utility function is assumed to be concave in its four arguments, c, g, l, and m. Consumers are assumed to derive positive marginal utility from the consumption of both private and public goods, but positive marginal disutility from providing labor services. We shall assume that for given values of c, g, and l, the marginal utility of money balances satisfies

$$sgn \; U_m = sgn\,(m^* - m)$$

where m^* denotes the corresponding satiation level of real money balances, such as in Friedman (1969). For $m < m^*$, the marginal utility of holding money is positive; for $m > m^*$ the holding costs outweigh the benefits and the net marginal utility of holding money balances becomes negative.

In determining his optimal plans for c, l, m, and b, the representative consumer is assumed to take $g, e, q, \pi, w, r^*, E, Q, P$, as given. These decisions are made subject to the budget constraint (1b), which is expressed in real flow terms. This relationship is

straightforward, the only point worth noting is that for simplicity, we assume that interest income is untaxed. The initial conditions (1c) relate to the initial stocks of real bonds and money held by the consumers. By definition, these are the corresponding initial nominal stocks, divided by the initial price level. In the case of the real money stock, this is endogenously determined through an appropriate initial jump in the exchange rate. By contrast, the initial stock of real bonds is predetermined by past accumulation.

Firms are assumed to produce output by hiring labor to maximize real profit

$$(2) \qquad \pi = f(l) - wl$$

where $f(l)$ is the firm's production function, assumed to possess the usual property of positive but diminishing marginal product of labor.

The domestic government's budget constraint, expressed in real terms is

$$(3) \qquad \dot{m} + \dot{a} = g + (r^* - q)a - \tau[wl + \pi] - (q + e)m - T$$

where

        a = real stock of traded bonds, issued by the domestic government (= A/P, where

        A = nominal stock of bonds).

Subtracting (3) from (1b) and noting (2), the sectoral budget constraints imply

$$(4) \qquad \dot{n} = f(l) - c - g + (r^* - q)n$$

where

        $n \equiv b - a$ = stock of net credit of the domestic economy.

That is, the rate of change of net credit of the domestic economy (the rate of accumulation of traded bonds) equals the balance of payments on current account, which in turn equals the balance of trade plus the net interest income earned on the traded bonds. With a single traded commodity, the balance of trade is simply the excess of domestic production over domestic absorption by the domestic private and government sectors.

B. Macroeconomic Equilibrium

We shall analyze the economy in perfect foresight equilibrium. This is defined to occur where the planned demand and supply functions derived from the optimizations, consistent with the accumulation equations, clear all markets at all points of time. We now proceed to develop these conditions for the economy.

We begin with consumers and write the optimization problem (1a), (1b) in Lagrangian form

(5) $\quad L \equiv e^{-(r^*-\eta)t} U(c,l,m,g) + \mu e^{-(r^*-\eta)t}\{(1-\tau)(wl+\pi) + (r^*-q)b - (q+e)m - T - \dot{m} - \dot{b} - c\}$

where $\mu e^{-(r^*-\eta)t}$ is the associated discounted Lagrange multiplier. The optimality conditions for consumers are

(6a) $\qquad\qquad\qquad\qquad U_c(c,l,m,g) = \mu$

(6b) $\qquad\qquad\qquad\qquad U_l(c,l,m,g) = -(1-\tau)w\mu$

(6c) $\qquad\qquad\qquad\qquad U_m(c,l,m,g) = (r^*+e)\mu - \dot{\mu}$

(6d) $\qquad\qquad\qquad\qquad \dot{\mu} = 0 \; ; \; i.e., \mu = constant$

together with the budget constraint (1b) and initial conditions (1c). In addition, there are the transversality conditions

(6e) $\qquad\qquad\qquad \lim_{t\to\infty} \mu m e^{-(r^*-\eta)t} = \lim_{t\to\infty} \mu b e^{-(r^*-\eta)t} = 0$

The equilibrium conditions for the representative firm are simply the usual marginal product condition

(7a) $\qquad\qquad\qquad\qquad f'(l) = w$

together with the definition of profit

(7b) $\qquad\qquad\qquad\qquad \pi = f(l) - wl$

To complete the description of macroeconomic equilibrium, government policy needs to be specified. In general, the government has five policy instruments available to it: $M, A, \tau, T,$ and g, any four of which can be chosen independently. While in general these can vary quite arbitrarily over time, following Turnovsky and Brock (1980), one can show that for the perfect foresight equilibrium of the present model, the optimal policies turn out to be constant

over time. Thus one can, without any loss of generality, restrict ourselves to constant policies at the outset.[7] Specifically, we shall assume that the government specifies monetary policy in terms of a constant rate of nominal monetary growth, θ, say. The rate of real monetary grwoth is therefore

(8a) $$\dot{m} = (\theta - q - e)m$$

with the corresponding change in real bonds being

(8b) $$\dot{a} = g + (r^* - q)a - \tau[wl + \pi] - \theta m - T$$

The issue we shall analyze in detail below concerns the optimal choice of θ, and how this depends upon the other accommodating policies.

Combining the optimality conditions (6a)-(6d), (7a), (7b), together with the accumulation equations (1b), (8a), (4), the perfect foresight equilibrium can be specified by the following set of equations

(9a) $$U_c(c, l, m, g) = \mu$$
(9b) $$U_l(c, l, m, g) = -f'(l)(l-\tau)\mu$$
(9c) $$U_m(c, l, m, g) = (r^* + e)\mu$$
(9d) $$\dot{m} = (\theta - q - e)m$$
(9e) $$\dot{b} = (1-\tau)f(l) - c - T + (r^* - q)b - \theta m$$
(9f) $$\dot{n} = f(l) - c - g + (r^* - q)n$$

where μ is constant.

Equations (9a)-(9c) can be solved to determine

(10a) $$c = c(m, \mu; g, \tau)$$
(10b) $$l = l(m, \mu; g, \tau)$$
(10c) $$e = e(m, \mu; g, \tau)$$

Substituting (10c) into (9d) yields

$$\dot{m} = (\theta - q - e(m, \mu; g, \tau))m$$

From the concavity of U, it can be shown that $e_m < 0$, which in turn implies a locally unstable time path for real money balances. We shall restrict our attention to an equilibrium in which

the real money stock remains bounded. This occurs if and only if

$$\theta = q + e$$

so that $\dot{m} \equiv 0$. Thus, in order for the real money stock to be ultimately bounded, it must in fact remain constant at all times. Given that the money stock is predetermined, the required (constant) real stock is attained by an appropriate initial jump in the nominal exchange rate E.

With m constant, equations (10a)-(10c) imply that c, l, and e are also constant. While the consumer's transversality condition for money will be satisfied, we must also ensure that the corresponding condition for bonds will be met.[8] With c, l, constant, the solution for consumer's holding of real bonds, obtained by integrating (9e) is

$$b(t) = e^{(r^*-q)t} \left[b_0 + \frac{(1-\tau)f(l) - c - T - \theta m}{r^* - q} (1 - e^{-(r^*-q)t}) \right]$$

For the transversality condition in (6e) to hold, we require

(11) $$(r^* - q)b_0 + (1-\tau)f(l) - c - T - \theta m = 0$$

where b_0 is the initially given real stock of bond holdings.

One further requirement we impose is the intertemporal budget constraint on the economy, ruling out the possibility that the country can run up indefinite credit or debt with the rest of the world. This is expressed by

$$\lim_{t \to \infty} n(t)e^{-(r^*-q)t} = 0$$

and solving n(t) from (9f) this reduces to the condition

(12) $$(r^* - q)(b_0 - a_0) + f(l) - c - g = 0$$

Thus, the perfect foresight equilibrium reduces to the following five equations:

(13a) $$U_c(c, l, m, g) = \mu$$
(13b) $$U_l(c, l, m, g) = -f'(l)(l-\tau)\mu$$
(13c) $$U_m(c, l, m, g) = (r^* + \theta - q)\mu$$
(13d) $$(r^* - q)b_0 + (1-\tau)f(l) - T - \theta m - c = 0$$
(13e) $$(r^* - q)(b_0 - a_0) + f(l) - c - g = 0$$

Equations (13a)-(13c) describe the usual marginal rate of substitution conditions necessary for consumer optimality. The remaining two equations are budget constraints. With the economy always being in steady state equilibrium, no accumulation of assets occurs. Equation (13d) requires that consumers' real interest income from their inherited bond holdings plus income from current production, less income tax, lump sum tax, and inflation tax, must equal current consumption. Equation (13e) is the analogous budget constraint for the overall economy. With b_0 being predetermined by previous accumulation, and a_0 being the initial stock of government bonds outstanding, these five equations determine c, l, m, μ, together with *one* of the policy instruments, namely θ, T, τ, or g.

We assume that with θ being chosen to optimize welfare, the necessary accommodation is undertaken by one of the tax parameters τ or T, and we refer to this as *fiscal* accommodation. But it is also possible that the accommodating variable is government expenditure. This possibility is not discussed but instead, we focus on the optimal determination of g (along with θ) in Section 5 below. A further means of sustaining equilibrium is for the monetary authorities to set the appropriate initial values of a_0 and b_0 through an initial open market operation with domestic residents. This, however, is equivalent to accommodation by means of lump sum taxation.[9]

Other combinations of these adjustments exist, but the key point to observe is that the viability of the perfect foresight equilibrium with perfect capital mobility requires an accommodation of this sort on the part of the domestic policy maker. This is necessary to rule out the possibility that would otherwise exist for the small economy to borrow indefinitely from abroad. This possibility does not exist either with imperfect capital mobility or in a closed economy and in such cases, this accommodation is not required; see e.g. Turnovsky and Brock (1980), Turnovsky (1985).

II. OPTIMAL GOVERNMENT POLICIES: A GENERAL CHARACTERIZATION

We now state the general conditions characterizing the optimal choice of government policy. We assume that the domestic government seeks to determine policies which maximize the intertemporal welfare function of the representative individual, (1a), subject to the equilibrium constraints, (13). Since everything is stationary, this can be accomplished by maximizing the instantaneous utility function. This problem can be formulated expressing consumer utility as an indirect utility function, in terms of government policy variables, and then optimizing. Alternatively, it can be expressed in terms of maximizing the following Lagrangian expression

$$L \equiv U(c, l, m, g) + \nu_1[\mu - U_c(c, l, m, g)]$$
$$- \nu_2[U_l(c, l, m, g) + f'(l)(1-\tau)\mu] - \nu_3[U_m(c, l, m, g) - (r^* + \theta - q)\mu]$$
$$+ \nu_4[(r^* - q)b_0 + (1-\tau)f(l) - T - \theta m - c]$$
$$+ \nu_5[(r^* - q)(b_0 - a_0) + f(l) - c - g]$$

We present the optimality conditions in two parts: the first pertains to the private sector variables, c, l, m, μ, while the second relates to the policy variables, although only a subset of the latter will in general be optimized.

The optimality conditions are:

Private Sector Variables

(14a)
$$\frac{\partial L}{\partial c} \equiv U_c - \nu_1 U_{cc} - \nu_2 U_{lc} - \nu_3 U_{mc} - \nu_4 - \nu_5 = 0$$

(14b)
$$\frac{\partial L}{\partial l} \equiv U_l - \nu_1 U_{cl} - \nu_2[f''(1-\tau)\mu + U_{ll}] - \nu_3 U_{ml} + \nu_4(1-\tau)f' + \nu_5 f' = 0$$

(14c)
$$\frac{\partial L}{\partial m} \equiv U_m - \nu_1 U_{cm} - \nu_2 U_{lm} - \nu_3 U_{mm} - \theta \nu_4 = 0$$

(14d)
$$\frac{\partial L}{\partial \mu} \equiv \nu_1 - \nu_2 f'(1-\tau) + \nu_3(r^* + \theta - q) = 0$$

Policy Variables

Some subset of:

(15a)
$$\frac{\partial L}{\partial \theta} \equiv \nu_3 \mu - m \nu_4 = 0$$

(15b)
$$\frac{\partial L}{\partial T} \equiv \nu_4 = 0$$

(15c)
$$\frac{\partial L}{\partial \tau} \equiv \nu_2 f'\mu - \nu_4 f = 0$$

(15d)
$$\frac{\partial L}{\partial g} \equiv U_g - \nu_1 U_{cg} - \nu_2 U_{lg} - \nu_3 U_{mg} - \nu_5 = 0$$

In addition, there are the underlying equilibrium constraints (13a)-(13e).

III. OPTIMAL MONETARY GROWTH

We now use the general characteristics contained in (13)-(15) to determine the optimal monetary growth rate, showing how it varies under the two choices with respect to the accommodating fiscal instrument.

A. Accommodation Through Lump Sum Taxation

We begin with the case where the domestic government chooses to maintain the equilibrium by means of levying an appropriate lump sum tax, T. In this case the relevant conditions are (15a), (15b), which together yield

$$\nu_4 = 0$$
$$\nu_3\mu = 0$$

The second equation implies two possibilities. First if, $\mu = 0$, then (13a)-(13c) imply $U_c = U_l = U_m = 0$, yielding constant optimal values of $\hat{c}, \hat{l}, \hat{m}$. But with a_0 and b_0 assumed to be given, these optimal values $\hat{c}, \hat{l}$ are in general inconsistent with the national budget constraint (13e).

Consider instead $\nu_3 = 0$. The equilibrium now reduces to:

(16a) $(U_c f' + U_l) - \nu_1 [U_{cc} f' + U_{cl}] - \nu_2 [U_{lc} f' + U_{ll} + f'(1-\tau)U_c] = 0$

(16b) $\nu_1 - \nu_2 f'(1-\tau) = 0$

(16c) $U_m - \nu_1 U_{cm} - \nu_2 U_{lm} = 0$

(16d) $f'(1-\tau)U_c + U_l = 0$

(16e) $U_m - (r^* + \theta - q)U_c = 0$

(16f) $$(r^* - q)b_0 + (1-\tau)f(l) - \theta m - c - T = 0$$

(16g) $$(r^* - q)(b_0 - a_0) + f(l) - c - g = 0$$

Equations (16a)-(16d), (16g) determine the optimal values for c, l, m, ν_1, ν_2. Given these values, (16e) determines the optimal rate of monetary growth, while finally (16f) determines the lump sum tax T necessary to sustain the equilibrium.

The optimal rate of monetary growth contained in the solution is virtually identical to that derived by Turnovsky and Brock for a closed economy. In particular, we can solve (16a), (16b), (16d) for ν_1, ν_2,

(17a) $$\nu_1 = \frac{\tau(1-\tau)(f')^2 U_c}{\Delta}$$

(17b) $$\nu_2 = \frac{\tau f' U_c}{\Delta}$$

where

$$\Delta \equiv f'(1-\tau)[U_{cc}f' + U_{cl}] + U_{cc}f' + U_{ll} + f''(1-\tau)U_c$$

Given the concavity of U in c and l, and that of f in l, a sufficient condition for $\Delta < 0$ is that $U_{cl} < 0$; i.e., that the marginal utility of consumption increases with leisure. This assumption is plausible and shall be maintained. Substituting these solutions for ν_1, ν_2, into (16c) and using (16e), the optimal rate of monetary growth is given by

(18) $$\hat{\theta} = \frac{\tau f' U_c}{\Delta} \frac{\partial}{\partial m}\left[\frac{U_l}{U_c}\right] - (r^* - q)$$

From this relationship, we observe that the optimal rate of monetary growth is identical to the celebrated Friedman full liquidity rule,

$$\theta = -(r^* - q), \text{ or equivalently } r = r^* + e = 0$$

if and only if: either (i) $\tau = 0$, or (ii) the marginal rate of substitution of consumption for labor is independent of the real stock of money balances. In the former case there are no taxes to distort the consumption-leisure choice; in the latter case, the utility function is weakly separable in m, which therefore leaves the choice between consumption and leisure

unaffected.

If, for example, $\partial(U_l/U_c)/\partial m < 0$, by reducing m below m^*, the marginal rate of substitution of labor for consumption will be increased; workers will be willing to supply more labor, output will increase and the utility from consumption will rise correspondingly. The opposite is true if $\partial(U_l/U_c)/\partial m > 0$. In effect, the optimal policy calls for a balancing of the direct utility of money on the one hand, with its indirect effects resulting from its interaction with consumption and leisure. Turnovsky and Brock have referred to (18) as describing a 'distorted' Friedman liquidity rule. Using the steady-state relationship, $\theta = q + e$, (18) translates immediately into an equivalent relationship describing the optimal crawling peg, $\hat{e}$,

$$(18') \qquad \hat{e} = \tau f' \cdot \frac{U_c}{\Delta} \frac{\partial}{\partial m} \left[\frac{U_l}{U_c} \right] - r^*$$

B. Accommodation Through Income Tax Rate

We now determine the optimal monetary growth rate under the assumption that the stabilization authorities maintain equilibrium by adusting the income tax rate, τ, rather than through lump sum taxation. The main point we shall show is that this leads to the kind of tradeoff between inflation tax on the one hand, and income tax, on the other, first discussed by Phelps (1973). Using (15c), the equilibrium conditions are summarized by

$$(19a) \qquad (U_c f' + U_l) - \nu_1[U_{cc} f' + U_{cl}] - \nu_2[U_{lc} f' + U_{ll} + f''(1-\tau)U_c]$$
$$- \nu_3[U_{mc} f' + U_{ml}] - \nu_4 \tau f' = 0$$

$$(19b) \qquad U_m - \nu_1 U_{cm} - \nu_2 U_{lm} - \nu_3 U_{mm} - \theta \nu_4 = 0$$

$$(19c) \qquad \nu_1 - \nu_2 f'(1-\tau) + \nu_3(r^* + \theta - q) = 0$$

$$(19d) \qquad \nu_3 \mu - m\nu_4 = 0$$

$$(19e) \qquad \nu_2 f' \mu - \nu_4 f = 0$$

together with the constraints (13a)-(13e).

To establish the tradeoff, it is convenient to assume that utility is additively separable in m, when under lump sum taxation, the Friedman full liquidity rule, $\theta = -(r^* - q)$, is obtained. The critical relationship is (19b), which with $U_{cm} = U_{lm} = 0$ simplifies to

(19b')
$$U_m - \nu_3 U_{mm} - \theta \nu_4 = 0$$

Solving (19) and (13) for ν_3 and ν_4, we find

$$\nu_4 = \tau(f')^2 U_c^2 / \Omega$$
$$\nu_3 = \tau(f')^2 U_{mm}/\Omega = m\nu_4/U_c$$

where

$$\Omega \equiv f'[f(1-\tau) - m\theta][U_{cc}f' + U_{cl}] + ff'U_{cl} + fU_{ll}$$
$$+ [ff''(1-\tau) + \tau(f')^2]U_c$$

Under mild restrictions $\Omega < 0$, in which case $\nu_3 < 0$, $\nu_4 < 0$.[10] Thus (19) and (13) together imply

(20)
$$(r^* + \theta - q)(U_c - \nu_4) = \nu_3 U_{mm} - (r^* - q)\nu_4$$

which given the signs of ν_3, ν_4, U_c and U_{mm} implies $\theta > -(r^* - q)$.

In other words, the optimal rate of monetary growth (contraction) falls short of the Friedman full liquidity rule, $\theta = -(r^* - q)$. The reason for this can be seen by subtracting (13e) from (13d) and writing the steady state domestic government budget constraint in the form

$$\theta m + T + \tau f(l) = g + (r^* - q)a_o$$

The Friedman full liquidity rule implies a constant rate of monetary contraction, yielding an inflation subsidy rather than inflation tax. This means that additional revenue must be raised through the income tax τ. But this will reduce the employment of labor, and therefore the level of output and consumption, thereby increasing the distortions caused by this form of taxation. In effect, the optimal monetary growth requires a balancing of the distortions caused by the required adjustment in τ on the utility level from real activity on the one hand, against the cost θ imposes on liquidity on the other. In the process of achieving this balance, only partial adjustment to the Friedman rule is achieved. This conclusion is essentially an extension of the Phelps result to a small open economy.

IV. OPTIMAL MONETARY-FISCAL PACKAGE

Up until this point, we have assumed that government expenditure remains fixed. We now

consider the situation where the government chooses its optimal expenditure g in conjunction with the monetary growth rate θ.

We assume that the accommodation required to sustain stady state equilibrium is carried out by lump sum taxation, in which case the optimality conditions consist simply of appending the marginal condition on g, (15d), to the set (16a)-(16g). Having previously established that $\nu_3 = \nu_4 = 0$ (by virtue of the accommodating lump sum tax T), the marginal condition (15d) reduces to

$$(15d') \qquad U_g - \nu_1 U_{cg} = \nu_2 U_{lg} - \nu_5 = 0$$

Combining this equation with (14a), ν_5 may be eliminated from this equation to yield

$$(15d'') \qquad U_g - U_c - \nu_1[U_{cg} - U_{cc}] - \nu_2[U_{lg} - U_{lc}] = 0$$

Now we have also established above that with accommodating lump sum taxation, ν_1, ν_2, are given by (17a), (17b). Substituting these expressions into (15d''), the optimality condition for government expenditure is given by

$$(21) \qquad U_g = U_c \left[1 + \frac{\tau(1-\tau)(f')^2}{\Delta} (U_{cg} - U_{cc}) + \frac{\tau f'}{\Delta} (U_{lg} - U_{lc}) \right]$$

Using (16d) this can be written in the equivalent form

$$(22) \qquad U_g = U_c \left\{ 1 + \frac{\tau f' U_c}{\Delta} \left[\frac{d}{dg} (\frac{U_l}{U_c}) - \frac{d}{dc} (\frac{U_l}{U_c}) \right] \right\}$$

In essence, this condition equates the marginal utility of government expenditure to an adjusted marginal utility of private consumption. The adjustment reflects the distortions on the work-leisure decision due to the presence of the non-zero income tax rate, τ.

The expression for the optimal monetary growth rate is still given by (18). The only difference is that in general, the optimal choice of g will impact on the marginal rate of substitution between consumption and labor. If the utility function is additively separable in

g, then this expression is independent of g and the optimal monetary growth rate $\hat{\theta}$ is invariant with respect to the chosen level of government activity.[11]

The fact that income taxes are distortionary is familiar. Suppose, finally, that the government chooses the optimal tax rate, τ, in addition to the rate of monetary growth and its level of expenditure. In this case, (15c) must be further added to the above optimality conditions (16a)-(16g), (15d'). With $\nu_4 = 0$, (15c) now implies $\nu_2 = 0$, which from (16b) yields in turn $\nu_1 = 0$. From (17a), (17b) we obtain further $\tau = 0$, and the optimality conditions for this overall policy optimization reduce to

$$\tag{23a} \tau = 0$$

$$\tag{23b} \theta = -(r^* - q)$$

$$\tag{23c} U_c f' + U_l = 0$$

$$\tag{23d} U_m = 0$$

$$\tag{23e} U_g = U_c$$

$$\tag{23f} (r^* - q)(b_0 - a_0) + f(l) - c - g = 0$$

$$\tag{23g} (r^* - q)b_0 + f(l) - \theta m - c = T$$

In this optimally integrated macroeconomic policy package, the income tax rate is set to zero, thereby eliminating this source of distortion. As a consequence, the optimality condition for government expenditure simply reduces to equating its marginal utility to that of private goods. Moreover, with $\tau = 0$, the optimal monetary growth rate should now be set at the Friedman full liquidity rule, $\theta = -(r^* - q)$. The corresponding optimal levels of $\hat{l}$, $\hat{c}$, $\hat{m}$, $\hat{g}$, are jointly determined by the marginal conditions (23c)-(23e), together with the national budget constraint (23f). Finally, the required accommodation in lump sum taxation, necessary to sustain this equilibrium, is determined residually by the government budget constraint (23g).[12]

V. CONCLUSIONS

This paper has emphasized how the choice of the optimal monetary growth rate (exchange rate crawl) in a small open economy under perfect capital mobility depends upon the

accommodating policy chosen to maintain the overall budget constraint for the economy. Two alternatives have been considered.

First, if the government accommodates through lump sum taxation, the optimal monetary growth rate is the `distorted´ Friedman monetary growth rule. This reduces to the Friedman full liquidity rule when the consumer's utility function is additively separable in real money balances. In this case, the optimal monetary growth rule drives the domestic nominal interest rate to zero. That is the domestic monetary growth rate is equal and opposite to the real rate of interest, or equivalently the domestic exchange rate should appreciate at a steady rate equal to the foreign nominal interest rate. Secondly, if the domestic government accommodates by adjusting the income tax rate, the optimal monetary growth rate falls short of the distorted Friedman rule. We derive an analogous tradeoff between the income tax rate and the inflation tax rate to that obtained by Phelps some years ago.

The framework we have employed is a natural one for analyzing optimal macroeconomic policy in general. In the latter part of the paper, we have considered an optimal monetary-fiscal package. The optimal integrated government policy--by which we mean the optimal choice of θ, g and τ--is determined simply as follows. The optimal income tax rate $\tau = 0$; the optimal rate of monetary growth $\theta = -(r^* - q)$; the optimal level of government expenditure is determined by equating its marginal utility is that of consumption. These results are analogous to those obtained by Turnovsky and Brock for a closed economy, the one difference the accommodation in lump sum taxes required to sustain the steady state equilibrium.

We conclude by commenting on the roles of the two special assumptions underlying our analysis. The PPP assumption, while convenient is not essential. If this is relaxed, the perfect foresight equilibrium will still require the economy to be continuously in steady state equilibrium, with some accommodation by the fiscal authorities. The only difference is that the appropriate relative price for the two commodities needs to be established. The extension

to imperfect capital mobility is more difficult, in that one cannot incorporate two alternative bonds satisfactorily, without introducing uncertainty. The polar case of zero capital mobility can be shown to reduce essentially to the Turnovsky-Brock model for a closed economy. In this case the optimal monetary growth rate drives the economy to the distorted Friedman rule, without any policy accommodation being necessary.

APPENDIX

STATIONARITY OF OPTIMAL POLICIES IN PERFECT FORESIGHT EQUILIBRIUM

In this Appendix we demonstrate the stationarity of the optimal policy in Perfect Foresight Equilibrium, thereby justifying the procedure in the text.

Suppose initially that the monetary growth rate $\theta(t)$ is allowed to be a function of time. The Perfect Foresight Equilibrium is

(A.1a)
$$U_c(c, l, m, g) = \mu$$

(A.1b)
$$U_l(c, l, m, g) = -f'(l)(1-\tau)\mu$$

(A.1c)
$$U_m(c, l, m, g) = (r^* + e)\mu$$

(A.1d)
$$\dot{m} = (\theta(t) - q - e)m$$

(A.1e)
$$\dot{b} = (1-\tau)f(l) - c - T + (r^* - q)b - \theta(t)m$$

(A.1f)
$$\dot{b} - \dot{a} = f(l) - c - g + (r^* - q)n$$

The policy problem is to choose the monetary growth rate so as to

(A.2)
$$Max \int_0^\infty U(c, l, m, g)e^{-(r^* - q)t}dt$$

subject to the constraints (A.1a)-(A.1f) and the transversality conditions.

Performing the optimization yields the optimality conditions

(A.3a)
$$U_c - \nu_1 U_{cc} - \nu_2 U_{cl} - \nu_3 U_{mc} + s_2 = 0$$

(A.3b)
$$U_l - \nu_1 U_{cl} - \nu_2[U_{ll} + f''(1-\tau)\mu] - \nu_3 U_{ml} - s_2 f' + s_3 \tau f' = 0$$

(A.3c)
$$U_m - \nu_1 U_{cm} - \nu_2 U_{lm} - \nu_3 U_{mm} - s_1(\theta - q - e) + s_3\theta = \dot{s}_1 - (r^* - q)s_1$$

(A.3d)
$$\nu_1 - \nu_2 f'(l)(1-\tau) + \nu_3(r^* + e) = 0$$

(A.3e)
$$\nu_3\mu + s_1 m = 0$$

(A.3f)
$$\dot{s}_2 = 0$$

(A.3g)
$$\dot{s}_2 - \dot{s}_3 = 0$$

(A.3h)
$$s_3 - s_1 = 0$$

where $\nu_i e^{-(r^* - q)t}$ are the discounted Lagrange multipliers associated with the static constraints (A.1a)-(A.1c) and $s_i e^{-(r^* - q)t}$ are the discounted costate variables associated with the dynamic equations (A.1d)-(A.1f). Equations (A.3a)-(A.3f) are the optimality conditions with respect to

$c, l, m, \mu, e, b, a, \theta$. In addition, the initial jump in the exchange rate means that $m(0)$ is endogenous, implying that $s_1(0) = 0$. The ability to reoptimize at each point in time implies that $s_1(t) = 0$, so that (A.3f), (A.3g), (A.3h) imply

$$s_1(t) = s_3(t) = 0; \ s_2(t) = constant$$

Thus from the optimality conditions (A.3) and the constraints (A.1a)-(A.1c) we may write the following equations

(A.4a) $$U_c - \nu_1 U_{cc} - \nu_2 U_{cl} - \nu_3 U_{cm} + s_2 = 0$$

(A.4b) $$U_l - \nu_1 U_{cl} - \nu_2[U_{ll} + f''(1-\tau)U_c] - \nu_3 U_{ml} - s_2 f' = 0$$

(A.4c) $$U_m - \nu_1 U_{cm} - \nu_2 U_{lm} - \nu_3 U_{mm} = 0$$

(A.4d) $$\nu_1 - \nu_2 f'(1-\tau) + \nu_3(r^* + e) = 0$$

(A.4e) $$\nu_3 U_c = 0$$

(A.4f) $$U_l + f'(1-\tau)U_c = 0$$

(A.4g) $$U_m - (r^* + e)U_c = 0$$

These define seven static equations which may be solved for $c, l, m, \nu_1, \nu_2, \nu_3, e$ in terms of s_2. But from (A.3f) s_2 is constant, implying that these solutions are constant. In particular e and m are constant so that (A.1d) implies that the optimal monetary growth θ is also constant. Thus the optimal solutions is in fact stationary.

FOOTNOTES

* The constructive suggestions of the referees are gratefully acknowledged.

1. Recent work in macroeconomics in general, and in international macroeconomics in particular, has increasingly emphasized this framework. See, e.g., Obstfeld (1981,1982), Hodrick (1982), Turnovsky (1985).

2. The problem is that the case of imperfect capital mobility is difficult to treat satisfactorily in an optimizing framework without introducing uncertainty. Turnovsky (1985) introduces a domestic and foreign bond by introducing a quadratic cost on holding foreign assets. This is meant to proxy exchange risk and is obviously somewhat arbitrary.

3. See, e.g., Hodrick (1982). It is possible to generate dynamics by introducing a variable rate of time preference as is done by Obstfeld (1981).

4. These issues of optimal macroeconomic policy packages are investigated in a closed economy by Turnovsky and Brock.

5. The validity of Mathieson's optimization was called into question by Clarke and Kingston (1979), who showed that for reasonable money demand functions, including the widely used Cagan function, the rule obtained yields a consumption minimizing rather than consumption maximizing crawl. As noted in Mathieson's (1979) reply, in this case the consumption maximizing crawl is a corner solution, which turns out to be the Friedman (1969) full liquidity rule. The choice of the wealth maximizing optimal crawling peg is considered in a recent paper by Mantel and Martirena-Mantel (1982).

6. See, e.g., Hodrick (1982).

7. The constancy of the optimal policies is demonstrated in the Appendix.

8. The requirement that the real money stock be finite is sufficient for the transversality condition on money to hold. While it also seems reasonable on economic grounds, it is

not in general necessary. In order to rule out unstable time paths for real money balances, further restrictions need to be imposed on the consumer's utility function for real money balances; see Brock (1977).

9. This is essentially the Ricardian equivalence property characteristic of intertemporal utility maximizing models of this type.

10. These conditions include:

$$U_{cl} < 0$$

$$ff''(1-\tau) + \tau(f')^2 \geq 0$$

11. For the Cobb-Douglas production function $f(l) = l^\epsilon$, this last condition reduces to $\epsilon + \tau \leq 1$, which does not seem an implausible restriction.

12. One can also consider the case where the fiscal accommodation occurs through variations in the income tax rate τ. Space limitations preclude a discussion of this case.

13. In the polar case where consumers view public and private goods as perfect substitutes, (23e) holds identically and the allocation of total output between public and private goods becomes indeterminate.

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