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PARETO AND PIKETTY:
THE MACROECONOMICS OF TOP INCOME AND WEALTH INEQUALITY

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ABSTRACT

Since the early 2000s, research by Thomas Piketty, Emmanuel Saez, and their coauthors has revolutionized our understanding of income and wealth inequality. In this paper, I highlight some of the key empirical facts from this research and comment on how they relate to macroeconomics and to economic theory more generally. One of the key links between data and theory is the Pareto distribution. The paper describes simple mechanisms that give rise to Pareto distributions for income and wealth and considers the economic forces that influence top inequality over time and across countries. For example, it is in this context that the role of the famous $r-g$ expression is best understood.

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Since the early 2000s, research by Thomas Piketty and Emmanuel Saez (and their coauthors, including Anthony Atkinson and Gabriel Zucman) has revolutionized our understanding of income and wealth inequality. The crucial point of departure for this revolution is the extensive data they have used, based largely on administrative tax records. Piketty's (2014) *Capital in the Twenty-First Century* is the latest contribution in this line of work, especially with the new data it provides on capital and wealth. Piketty also proposes a framework for describing the underlying forces that affect inequality and wealth, and unlikely as it seems, a bit of algebra that plays an important role in Piketty's book has even been seen on T-shirts: $r > g$.

In this paper, I highlight some of the key empirical facts from this research and describe how they relate to macroeconomics and to economic theory more generally. One of the key links between data and theory is the Pareto distribution. The paper explains simple mechanisms that give rise to Pareto distributions for income and wealth and considers the economic forces that influence top inequality over time and across countries.

To organize what follows, recall that GDP can be written as the sum of "labor income" and "capital income." This split highlights several kinds of inequality that we can explore. In particular, there is *within* inequality for each of these components: How much inequality is there within labor income? How much inequality within capital income — or, more appropriately here, among the wealth itself for which capital income is just the annual flow? And there is also *between* inequality related to the split of GDP between capital and labor. This between inequality takes on particular relevance given the "within" inequality fact that most wealth is held by a small fraction of the population; anything that increases between inequality therefore is very likely to increase overall inequality.¹ In the three main sections of this paper, I consider each of these concepts in turn. I first highlight some of the key facts related to each type of inequality. Then I use economic theory to shed light on these facts.

The central takeaway of the analysis is summarized by the first part of the title of the paper, "Pareto and Piketty." In particular, there is a tight link between the share of income going to the top 1 percent or top 0.1 percent and the key parameter of a Pareto distribution. Understanding why top inequality takes the form of a Pareto distribution

¹One could also productively explore the correlation of the two within components: are people at the top of the labor income distribution also at the top of the capital income and wealth distributions?

and what economic forces can cause the key parameter to change is therefore central to understanding the facts. As just one example, the central role that Piketty assigns to $r - g$ has given rise to some confusion, in part because of its familiar presence in the neoclassical growth model, where it is not obviously related to inequality. The relationship between $r - g$ and inequality is much more easily appreciated in models that explicitly generate Pareto wealth inequality.

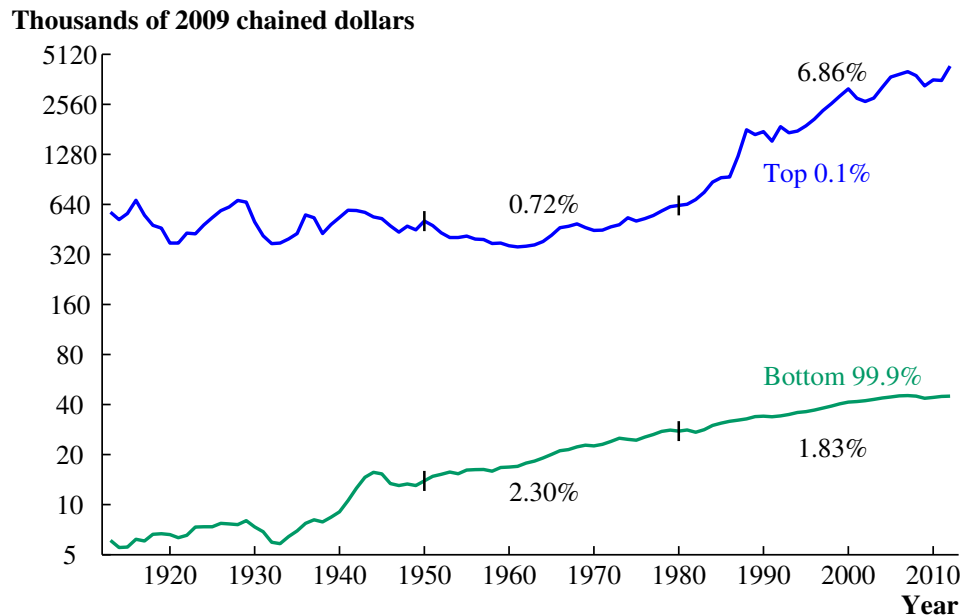
Capital in the Twenty-First Century, together with the broader research agenda of Piketty and his coauthors, opens many doors by assembling new data on top income and wealth inequality. The theory that Piketty develops to interpret these data and make predictions about the future is best viewed as a first attempt to make sense of the evidence. Much like Marx, Piketty plays the role of provocateur, forcing us to think about new ideas and new possibilities. As I explain below, the extent to which $r - g$ is the fundamental force driving top wealth inequality, both in the past and in the future, is unclear. But by encouraging us to entertain these questions and by providing a rich trove of data in which to study them, Piketty and his coauthors have made a tremendous contribution.

Before we begin, it is also worth stepping back to appreciate the macroeconomic consequences of the inequality that Piketty and his coauthors write about. For example, consider Figure 1. This figure is constructed by merging two famous data series: one is the Piketty-Saez top inequality data (about which we'll have more to say shortly) and the other is the long-run data on GDP per person for the United States that comes from Angus Maddison (pre-1929) and from the Bureau of Economic Analysis.

To set the stage, note that GDP per person since 1870 looks remarkably similar to a straight line when plotted on a log scale, exhibiting a relatively constant average growth rate of around 2 percent per year. Figure 1 applies the Piketty-Saez inequality shares to average GDP per person to produce an estimate of GDP per person for the top 0.1% and the bottom 99.9%.² Two key results stand out. First, until recently, there is remarkably little growth in the average GDP per person at the top: the value in 1913 is actually *lower* than the value in 1977. Instead, all the growth until around 1960 occurs in the bottom 99.9%. The second point is that this pattern changed in recent decades.

²It is important to note that this estimate is surely imperfect. GDP likely does not follow precisely the same distribution as Adjusted Gross Income: health benefits are more equally distributed, for example. However, even with these caveats, the estimate still seems useful.

Figure 1: GDP per person, Top 0.1% and Bottom 99.9%



Note: This figure displays an estimate of average GDP per person for the top 0.1% and the bottom 99.9%. Average annual growth rates for the periods 1950–1980 and 1980–2007 are also reported. Source: Aggregate GDP per person data are taken from the Bureau of Economic Analysis (since 1929) and Angus Maddison (pre-1929). The top income share used to divide the GDP is from the October 2013 version of the world top incomes database, from <http://g-mond.parisschoolofeconomics.eu/topincomes/>.

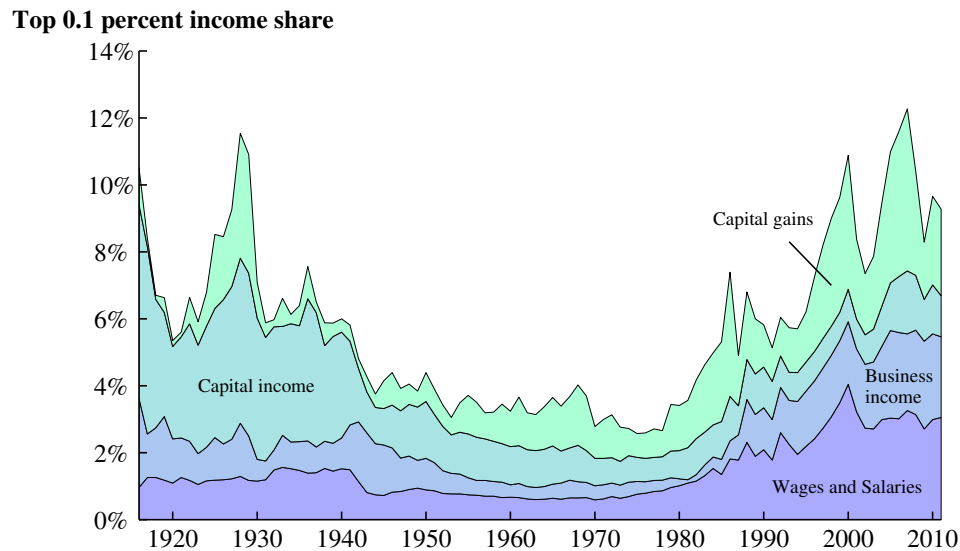
For example, average growth in GDP per person for the bottom 99.9% declined by around half a percentage point, from 2.3% between 1950 and 1980 to only 1.8% between 1980 and 2007. In contrast, after being virtually absent for 50 years, growth at the top accelerated sharply: GDP per person for the top 0.1% exhibited growth more akin to China's economy, averaging 6.86% since 1980. Changes like this clearly have the potential to matter for economic welfare and merit the attention they've received.

1. Labor Income Inequality

1.1. Basic Facts

One of the key papers documenting the rise in top income inequality is Piketty and Saez (2003), and it is appropriate to start with an updated graph from their paper. Figure 2 shows the share of income going to the top 0.1 percent of families in the United States,

Figure 2: The Composition of U.S. Income Inequality



Note: The figure shows the composition of the top 0.1 percent income share. Source: These data are taken from the “data-Fig4B” tab of the September 2013 update of the spreadsheet appendix to Piketty and Saez (2003).

along with the composition of this income. Piketty and Saez emphasize three key facts seen in this figure. First, top income inequality follows a U-shaped pattern in the long term: high prior to the Great Depression, low and relatively steady between World War II and the mid-1970s, and then rising since then, ultimately reaching similar levels today to the high levels of top income inequality experienced in the 1910s and 1920s. Second, much of the decline in top inequality in the first half of the 20th century was associated with capital income. Third, much of the rise in top inequality during the last several decades is associated with labor income, particularly if one includes “business income” in this category.

1.2. Theory

The next section of the paper will discuss wealth and capital income inequality. Here, motivated by the facts just discussed for the period since 1970, I’d like to focus on labor income inequality. In particular, what are the economic determinants of top labor income inequality, and why might they change over time and differ across countries?

At least since Pareto (1896) first discussed income heterogeneity in the context of his eponymous distribution, it has been appreciated that incomes at the top are well characterized by a power law. That is, apart from a proportionality factor to normalize units, $\Pr[\text{Income} > y] = y^{-1/\eta}$ — the fraction of people with incomes greater than some cutoff is proportional to the cutoff raised to some power. This is the defining characteristic of a Pareto distribution.

We can easily connect this distribution to the Piketty and Saez “top share” numbers. In particular, for the Pareto distribution just given, the fraction of income going to the top p percentiles equals $(\frac{100}{p})^{\eta-1}$. In other words, the top share varies directly with the key exponent of the Pareto distribution, η . With $\eta = 1/2$, the share of income going to the top 1 percent is $100^{-1/2} = .10$, or 10 percent, while if $\eta = 2/3$, this share is $100^{-1/3} \approx 0.22$, or 22 percent. An increase in η leads to a rise in top income shares. Hence this parameter is naturally called a measure of Pareto inequality. In the U.S. economy today, η is approximately 0.6.

A theory of top income inequality, then, needs to explain two things: (i) why do top incomes obey a Pareto distribution, and (ii) what economic forces determine η ? The economics literature in recent years includes a number of papers that ask related questions. For example, Gabaix (1999) studies the so-called Zipf’s Law for city populations: why does the population of cities follow a Pareto distribution, and why is the inequality parameter very close to 1? Luttmer (2007) asks the analogous question for firms: why is the distribution of employment in U.S. firms a Pareto distribution with an inequality parameter very close to 1? Here, the questions are slightly different: Why might the distribution of income be well-represented by a Pareto distribution, and why does the inequality parameter change over time and differ across countries? Interestingly, it turns out that there is a lot more inequality among city populations or firm employment than there is among incomes (their η ’s are close to 1.0 instead of 0.6). Also, the size distribution of cities and firms is surprisingly stable when compared to the sharp rise in U.S. top income inequality.

From this recent economics literature as well as from an earlier literature on which it builds, we learn that the basic mechanism for generating a Pareto distribution is surprisingly simple: *exponential growth that occurs for an exponentially-distributed*

*amount of time leads to a Pareto distribution.*³

To see how this works, we first require some heterogeneity. Suppose people are exponentially distributed across some variable x , which could denote age or experience or talent. For example, $\Pr[\text{Age} > x] = e^{-\delta x}$, where δ denotes the death rate in the population. Next, we need to explain how income varies with age in the population. A natural assumption is exponential growth: suppose income rises exponentially with age (or experience or talent) at rate μ : $\text{Income} = e^{\mu x}$. In this case, the log of income is just proportional to age, so the log of income obeys an exponential distribution with parameter δ/μ .

Next, we use an interesting property: if the log of income is exponential, then the level of income obeys a Pareto distribution:⁴

$$\Pr[\text{Income} > y] = y^{-\delta/\mu}.$$

Recall from our earlier discussion that the Pareto inequality measure is just the inverse of the exponent in this equation, which gives

$$\eta_{\text{income}} = \frac{\mu}{\delta}. \tag{1}$$

The Pareto exponent is increasing with μ , the rate at which incomes grow with age and decreasing in the death rate δ . Intuitively, the lower is the death rate, the longer some lucky people in the economy can benefit from exponential growth, which widens Pareto inequality. Similarly, faster exponential growth across ages (which might be interpreted as a higher return to experience) also widens inequality.

This simple framework can be embedded in a richer model to produce a theory of top income inequality. For example, Jones and Kim (2014) build a model along these lines in which both μ and δ are endogenous variables that respond to changes in economic policy or technology. In their setup, x corresponds to the human capital of entrepreneurs. Entrepreneurs who put forth more effort cause their incomes to grow more rapidly, corresponding to a higher μ . The death rate δ is an endogenous rate of

³Excellent introductions to Pareto models can be found in Mitzenmacher (2004), Gabaix (2009), Benhabib (2014), and Moll (2012b). Benhabib traces the history of Pareto-generating mechanisms and attributes the earliest instance of a simple model like that outlined here to Cantelli (1921).

⁴This derivation is explained in more detail in the appendix at the end of the paper, also available at <http://www.stanford.edu/~chadj/SimpleParetoJEP.pdf>.

creative destruction by which one entrepreneur is displaced by another. Technological changes that make a given amount of entrepreneurial effort more effective, such as information technology or the world wide web, will increase top income inequality. Conversely, exposing formerly closed domestic markets to international competition may increase creative destruction and reduce top income inequality. Finally, the model also incorporates an important additional role for luck: the richest people are those who not only avoid the destruction shock for long periods, but also those who benefit from the best idiosyncratic shocks to their incomes. Both effort and luck play central roles at the top, and models along these lines combined with data on the stochastic income process of top earners can allow us to quantify their comparative importance.

2. Wealth Inequality

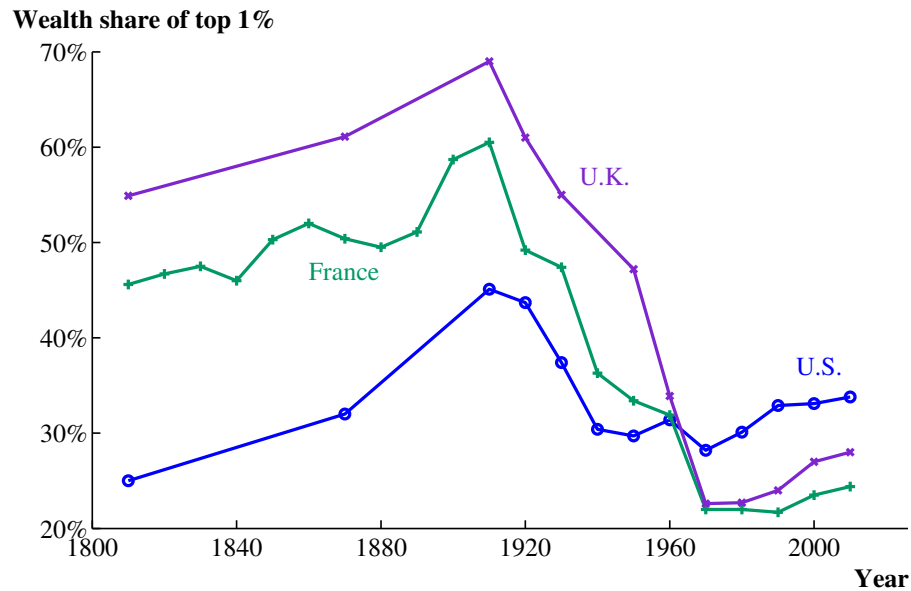
2.1. Basic Facts

Up until this point, we've focused on inequality in labor income. Piketty's (2014) book, in contrast, is primarily about wealth, which turns out to be a more difficult subject. Models of wealth are conceptually more complicated because wealth accumulates gradually over time. In addition, data on wealth are more difficult to obtain. Income data are "readily" (in comparison only!) available from tax authorities, while wealth data are gathered less reliably. For example, common sources include estate taxation, which affects an individual infrequently, or surveys, in which wealthy people may be reluctant to share the details of their holdings. With extensive effort, Piketty assembles the wealth inequality data shown in Figure 3, and several findings stand out immediately.

First, wealth inequality is much greater than income inequality. The top 1 percent of families possess around 35 or 40 percent of wealth in the United States in 2010, versus around 17 percent of income. Put another way, the income cutoff for the top 1 percent is about \$330,000 — in the ballpark of the top salaries for academics. In contrast, according to the latest data from Saez and Zucman (2014), the wealth cutoff for the top 1 percent is an astonishing \$4 million! Note that both groups include about 1.5 million families.

Second, wealth inequality in France and the United Kingdom is dramatically lower today than it was at any time between 1810 and 1960. The share of wealth going to the

Figure 3: Wealth Inequality



Note: The figure shows the share of aggregate wealth held by the richest 1 percent of the population. Source: Supplementary Table S10.1 for Chapter 10 of Piketty (2014), <http://piketty.pse.ens.fr/en/capital21c2>.

top 1 percent is around 25 or 30 percent today, versus peaks in 1910 of 60 percent or more. Two world wars, the Great Depression, the rise of progressive taxation — some combination of these and other events led to an astonishing drop in wealth inequality both there and in the United States between 1910 and 1965.

Third, wealth inequality has increased during the last 50 years, although the increase seems small in comparison to the declines just discussed. An important caveat to this statement applies to the United States: the data shown are those used by Piketty in his book, but Saez and Zucman (2014) have recently assembled what they believe to be superior data in the United States, and these data show a rise to a 40 percent wealth share for the top 1 percent by 2010, much closer to the earlier U.S. peak in the first part of the 20th century.

2.2. Theory

A substantial and growing body of economic theory seeks to understand the determinants of wealth inequality.⁵ Pareto inequality in wealth readily emerges through the same mechanism we discussed in the context of income inequality: exponential growth that occurs over an exponentially-distributed amount of time. In the case of wealth inequality, this exponential growth is fundamentally tied to the interest rate, r : in a standard asset accumulation equation, the return on wealth is a key determinant of the growth rate of an individual's wealth. On the other hand, this growth in an individual's wealth occurs against a backdrop of economic growth in the overall economy. To obtain a variable that will exhibit a stationary distribution, one must normalize an individual's wealth level by average wealth per person or income per person in the economy. If average wealth grows at rate g — which in standard models will equal the growth rate of income per person and capital per person — the normalized wealth of an individual then grows at rate $r - g$. This logic underlies the key $r - g$ term for wealth inequality that makes a frequent appearance in Piketty's book. Of course, r and g are potentially endogenous variables in general equilibrium so — as we will see — one must be careful in thinking about how they might vary independently.

To be more specific, imagine an economy of heterogeneous people. The details of the model we describe next are given in the appendix at the end of the paper.⁶ But the logic is straightforward to follow. To keep it simple, assume there is no labor income and that individuals consume a constant fraction α of their wealth. As discussed above, wealth earns a basic return r . However, wealth is also subject to a wealth tax: a fraction τ is paid to the government every period. With this setup, the individual's wealth grows exponentially at a constant rate $r - \tau - \alpha$. Next, assume that average wealth per person (or capital per person) grows exogenously at rate g , for example in the context of some macro growth model. The individual's normalized wealth then grows exponentially at rate $r - g - \tau - \alpha > 0$. This is the basic “exponential growth” part of the requirement for a Pareto distribution.

Next, we obtain heterogeneity in the simplest possible fashion: assume that each

⁵References include Wold and Whittle (1957), Stiglitz (1969), Huggett (1996), Quadrini (2000), Castaneda, Diaz-Gimenez and Rios-Rull (2003), Benhabib and Bisin (2006), Cagetti and Nardi (2006), Nirei (2009), Benhabib, Bisin and Zhu (2011), Moll (2012a), Piketty and Saez (2012), Aoki and Nirei (2013), Moll (2014), and Piketty and Zucman (2014).

⁶See also <http://www.stanford.edu/~chadj/SimpleParetoJEP.pdf>.

person faces a constant probability of death, \bar{d} , in each period. Because Piketty (2014) emphasizes the role played by changing rates of population growth, we'll also include population growth, assumed to occur at rate \bar{n} . Each new person born in this economy inherits the same amount of wealth, and the aggregate inheritance is simply equal to the aggregate wealth of the people who die each period. It is straightforward to show that the steady-state distribution of this birth-death process is an exponential distribution, where the age distribution is $\Pr[\text{Age} > x] = e^{-(\bar{n}+\bar{d})x}$. That is, the age distribution is governed by the (gross) birth rate, $\bar{n} + \bar{d}$. The intuition behind this formulation is that a fraction $\bar{n} + \bar{d}$ of new people are added to the economy each instant.

We now have exponential growth occurring over an exponentially-distributed amount of time. The model we presented in the context of the income distribution suggested that the Pareto inequality measure equals the ratio of the “growth rate” to the “exponential distribution parameter” and that logic also holds for this model of the wealth distribution. In particular, wealth has a steady-state distribution that is Pareto with

$$\eta_{\text{wealth}} = \frac{r - g - \tau - \alpha}{\bar{n} + \bar{d}}. \quad (2)$$

An equation like this is at the heart of many of Piketty's statements about wealth inequality, for example as measured by the share of wealth going to the top 1 percent. Other things equal, an increase in $r - g$ will increase wealth inequality: people who are lucky enough to live a long time — or are part of a long-lived dynasty — will accumulate greater stocks of wealth. Also, a higher wealth tax will lower wealth inequality. In richer frameworks that include stochastic returns to wealth, the super-rich are also those who benefit from a lucky run of good returns, and a higher variance of returns will increase wealth inequality.

Can this class of models explain why wealth inequality was so high historically in France and the United Kingdom relative to today? Or why wealth inequality was historically much higher in Europe than in the United States? Qualitatively, two of the key channels that Piketty emphasizes are at work in this framework: either a low growth rate income per person, g , or a low rate of population growth, \bar{n} — both of which applied in the 19th century — will lead to higher wealth inequality.

Piketty (2014, p. 232) summarizes the logic underlying models like this with char-

acteristic eloquence: “[I]n stagnant societies, wealth accumulated in the past takes on considerable importance.” On the role of population growth, for example, Piketty notes that an increase means that inherited wealth gets divided up by more offspring, reducing inequality. Conversely, a decline in population growth will concentrate wealth. A related effect occurs when the economy’s per capita growth rate rises. In this case, inherited wealth fades in value relative to new wealth generated by economic growth. Silicon Valley in recent decades is perhaps an example worth considering. Reflections of these stories can be seen in the factors that determine η for the distribution of wealth in the equation above.

2.3. General Equilibrium

Whether changes in the parameters of models in this genre can explain the large changes in wealth inequality that we see in the data is an open question. However, one cautionary note deserves mention: the comparative statics just provided ignore the important point that arguably all the parameters considered so far are endogenous. For example, changes in the economy’s growth rate g or the rate of the wealth tax τ can be mirrored by changes in the interest rate itself, potentially leaving wealth inequality unchanged.⁷ To take another example, the fraction of wealth that is consumed, α , will naturally depend on the rate of time preference and the death rate in the economy.

Because the parameters that determine Pareto wealth inequality are interrelated, it is unwise to assume that the direction of changing any single parameter will have an unambiguous effect on the distribution of wealth. General equilibrium forces matter and can significantly alter the fundamental determinants of Pareto inequality.

As one example, if tax revenues are used to pay for government services that enter utility in an additively separable fashion, the formula for wealth inequality in this model reduces to $\eta_{\text{wealth}} = \frac{\bar{n}}{\bar{n}+d}$; see the appendix for the details.⁸ Remarkably, in this formulation the distribution of wealth is invariant to wealth taxes. In addition,

⁷This relationship can be derived from a standard Euler equation for consumption with log utility, which delivers the result that $r - g - \tau = \rho$, where ρ is the rate of time preference. With log utility, the substitution and income effects from a change in growth or taxes offset and change the interest rate one for one.

⁸There are two key reasons that deliver this result. The first is the Euler equation point made earlier, that $r - g - \alpha$ will be pinned down by exogenous parameters. The second is that the substitution and income effect from taxes cancel each other out with log utility, so the tax rate does not matter. For these two reasons, the numerator of the Pareto inequality measure for wealth, $r - g - \tau - \alpha$, simplifies to just \bar{n} .

the effect of population growth on wealth can actually go in the opposite direction from what we've seen so far. The intuition for this result is interesting: while in partial equilibrium, the growth rate of normalized wealth is $r - g - \tau - \alpha$, in general equilibrium, the only source of heterogeneity in the model is population growth. Newborns in this economy inherit the wealth of the people who die. Because of population growth, there are more newborns than people who die, so newborns inherit less than the average amount of wealth per capita. This dilution of the inheritance via population growth is the key source of heterogeneity in the model, and this force ties the distribution of wealth across ages at a point in time to population growth. Perhaps a simpler way of making the point is this: if there were no population growth in the model, newborns would each inherit the per capita amount of wealth in the economy. The accumulation of wealth by individuals over time would correspond precisely to the growth in the per capita wealth that newborns inherit, and there would be no inequality in the model despite the fact that $r > g$!

More generally, other possible effects on the distribution of wealth need to be considered in a richer framework. Examples include bequests, social mobility, progressive taxation, transition dynamics, and the role of both macroeconomic and microeconomic shocks. The references cited earlier make progress on these fronts.

To conclude this section, I think two points are worth appreciating. First, in a way that is easy to overlook because of our general lack of familiarity with Pareto inequality, Piketty is right to highlight the link between $r - g$ and top wealth inequality. That connection has a firm basis in economic theory. On the other hand, as I've tried to show, the role of $r - g$, population growth, and taxes is more fragile than this partial equilibrium reasoning suggests. For example, it is not necessarily true that a slowdown in either per capita growth or population growth in the future will increase inequality. There are economic forces working in that direction in partial equilibrium. But from a general equilibrium standpoint, these effects can easily be washed out depending on the precise details of the model. Moreover, these research ideas are relatively new, and the empirical evidence needed to sort out such details is not yet available.

3. “Between” Inequality: Capital vs Labor

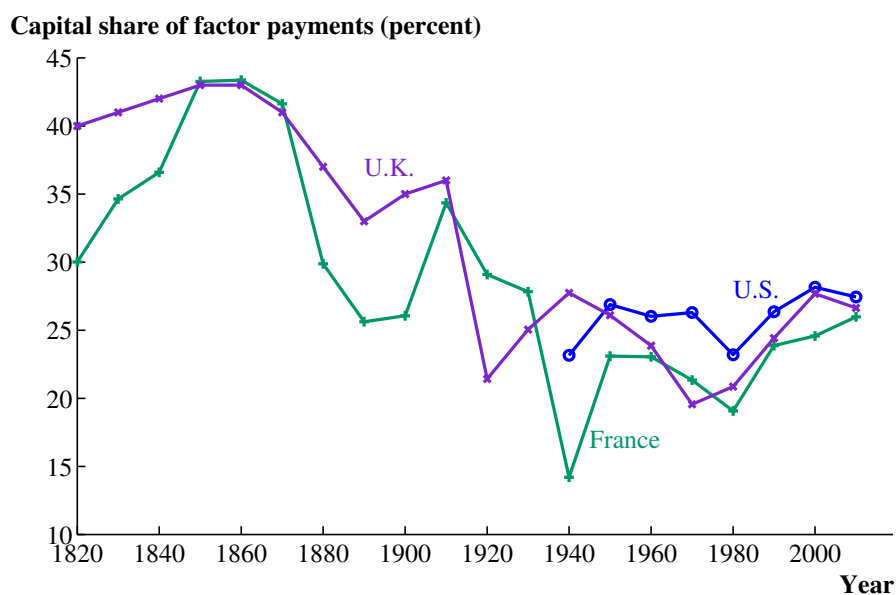
We next turn to “between” inequality: how is income to capital versus income to labor changing, and how is the wealth-income ratio changing? This type of inequality takes on particular importance given our previous fact about within inequality: most of wealth is held by a small fraction of the population, which means that changes in the share of national income going to capital (e.g. rK/Y) or in the aggregate capital-output ratio also contribute significantly to inequality. Whereas Pareto inequality describes how inequality at the top of the distribution is changing, this between inequality is more about inequality between the top 10 percent of the population (who hold around 3/4 of the wealth in the United States according to Saez and Zucman (2014)) and the bottom 90 percent.

3.1. Basic Facts

At least since Kaldor (1961), a key stylized fact of macroeconomics has been the relative stability of factor payments to capital as a share of GDP. Figure 4 shows the long historical time series for France, the United Kingdom, and the United States that Piketty (2014) has assembled. A surprising point emerges immediately: prior to World War II, the capital share exhibits a substantial negative trend, falling from around 40 percent in the mid-1800s to below 30 percent. By comparison, the data since 1940 show some stability, though with a notable rise between 1980 and 2010. In Piketty’s data, the labor share is simply one minus the capital share, so the corresponding changes in labor’s share of factor payments can be read from this same graph.

Before delving too deeply into these numbers, it is worth appreciating another pattern documented by Piketty (2014). Figure 5 shows the capital-output ratio — the ratio of the economy’s stock of machines, buildings, roads, land, and other forms of physical capital to the economy’s gross domestic product — for this same group of countries, back to 1870. The movements are once again striking. France and the United Kingdom exhibit a very high capital-output ratio around 7 in the late 1800s. This ratio falls sharply and suddenly with World War I, to around 3, before rising steadily after World War II to around 6 today. The destruction associated with the two World Wars and the subsequent transition dynamics as Europe recovers are an obvious interpretation of

Figure 4: Capital Shares



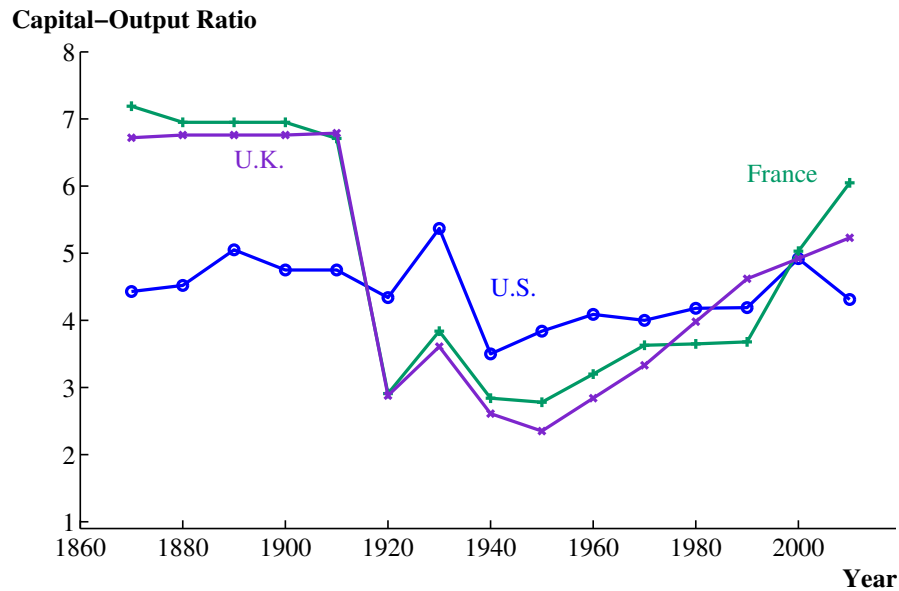
Note: Capital shares (including land rents) for each decade are averages over the preceding ten years. Source: Supplementary tables for Chapter 6 of Piketty (2014), <http://piketty.pse.ens.fr/en/capital21c2> for France and the U.K. The U.S. shares are taken from Piketty and Zucman (2014).

these facts. The capital-output ratio in the United States appears relatively stable in comparison, though still showing a decline during the Great Depression and a rise from 3.5 to 4.5 in the post-World War II period. These are wonderful new facts that were not broadly known prior to Piketty's efforts.

Delving into the detailed data underlying these graphs — which Piketty (2014) generously and thoroughly provides — highlights an important feature of the data. By focusing on only two factors of production, capital and labor, Piketty includes land as a form of capital. Of course, the key difference between land and the rest of capital is that the quantity of land is fixed, while the quantity of other forms of capital is not. For the purpose of understanding inequality between the top and the rest of the distribution, including land as a part of capital is eminently sensible. On the other hand, for connecting the data to macroeconomic theory, one must be careful.

For example, in the 18th and early 19th centuries, Piketty notes that rents paid to landlords averaged around 20 percent of national income. His capital income share for the United Kingdom before 1910 is taken from Allen (2007), with some adjustments,

Figure 5: The Capital-Output Ratio



Source: Supplementary Table S4.5 for Chapter 4 of Piketty (2014), <http://piketty.pse.ens.fr/en/capital21c2>.

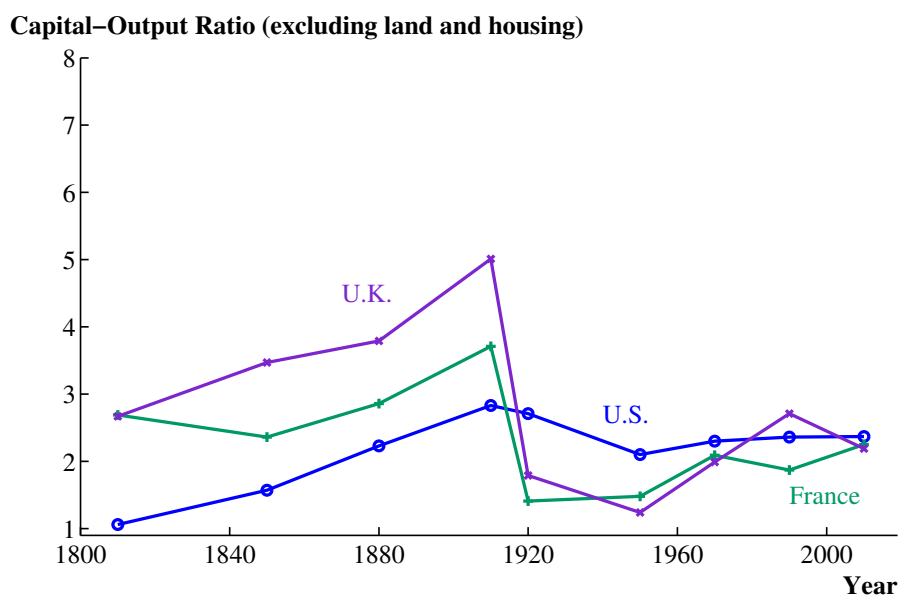
and shows a sharp decline in income from land rents (down to only 2 percent by 1910), which masks a rise in income from reproducible capital.

Similarly, much of the large swing in the European capital-output ratios shown in Figure 5 are due to land as well. (In Piketty's book, Figures 3.1 and 3.2 make this clear.) For example, in 1700 in France, the value of land equals almost 500 percent of national income versus only 12 percent by 2010. Moreover, the rise in the capital-output ratio since 1950 is to a great extent due to housing, which rises from 85 percent of national income in 1950 to 371 percent in 2010. Bonnet, Bono, Chapelle and Wasmer (2014) document this point in great detail, going further to show that the rise in recent decades is primarily due to a rise in housing prices rather than to a rise in the quantity of housing.

As an alternative, consider what is called reproducible, non-residential capital, that is the value of the capital stock excluding land and housing. This concept corresponds much more closely to what we think of when we model physical capital in macro models. Data for this alternative are shown in Figure 6.

In general, the movements in this measure of the capital-output ratio are more muted — especially during the second half of the 20th century. There is a recovery

Figure 6: The Capital-Output Ratio, Excluding Land and Housing



Source: Supplementary Tables S3.1, S3.2, and S4.2 for Chapters 3 and 4 of Piketty (2014), <http://piketty.pse.ens.fr/en/capital21c2>.

following the destruction of capital during World War II, but otherwise the ratio seems relatively stable in the latter period. In contrast, it is striking that the value in 2010 is actually lower than the value in several decades in the 1800s for both France and the United Kingdom. Similarly, the value in the United States is generally lower in 2010 than it was in the first three decades of the 20th century. I believe this is something of a new fact to macroeconomics — it strikes me as surprising and worthy of more careful consideration. I would have expected the capital-output ratio to be higher in the 20th century than in the 19th.

Stepping back from these discussions of the facts, an important point related to the “fundamental tendencies of capitalist economies,” to use Piketty’s language, needs to be appreciated. From the standpoint of overall wealth inequality, the declining role of land and the rising role of housing is not necessarily relevant. The inequality of wealth exists independent of the form in which the wealth is held. In the Pareto models of wealth inequality discussed in the preceding section, it turns out not to matter whether the asset that is accumulated is a claim on physical capital or a claim on a fixed aggregate quantity of land: the role of $r - g$ in determining the Pareto inequality measure

η , for example, is the same in both setups.⁹ However, if one wishes to fit Piketty's long-run data to macroeconomic growth models — to say something about the shape of production functions — then it becomes crucial to distinguish between land and physical capital.

3.2. Theory

The macroeconomics of the capital-output ratio is arguably the best-known theory within all of macroeconomics, with its essential roots in the analysis of Solow (1956) and Swan (1956). The familiar formula for the steady-state capital-output ratio is $s/(n+g+\delta)$, where s is the (gross) investment share of GDP, n denotes population growth, g is the steady-state growth rate of income per person, and δ is the rate at which capital depreciates. Notice that this expression pertains to the ratio of *reproducible* capital — machines, buildings, and highways — and therefore is not strictly comparable to the graphs that Piketty reports, which include land.

In this framework, a higher rate of investment s will raise the steady-state capital-output ratio, while increases in population growth n , a rise in the growth rate of income per person g , or a rise in the capital depreciation rate δ would tend to reduce that steady-state ratio. Partly for expositional purposes, Piketty simplifies this formula to another that is mathematically equivalent: \tilde{s}/\tilde{g} , where $\tilde{g} = n + g$ and \tilde{s} now denotes the investment rate net of depreciation, $\tilde{s} = s - \delta K/Y$. This more elegant equation is helpful for a general audience and gets the qualitative comparative statics right: in particular, Piketty emphasizes that a slowdown in growth — whether in per capita terms or in population growth — will raise the capital-output ratio in the long-run. Piketty occasionally uses the simple formula to make *quantitative* statements, e.g. if the growth rate falls in half, then the capital-output ratio will double (for example, see the discussion beginning on page 170). This statement is not correct and takes the simplification too far.¹⁰

It is plausible that some of the decline in the capital-output ratio in France and the United Kingdom since the late 1800s is due to a rise in the rate of population growth and the growth of income per person — that is, to a rise in $n + g$ — and it is possible

⁹The background models in the appendix provide the details supporting this claim.

¹⁰In particular, it ignores the fact that \tilde{s} will change when the growth rate changes, via the $\delta K/Y$ term.

that a slowing growth rate of aggregate GDP in recent decades and in the future could contribute to a rise in the capital-output ratio. However, the quantitative magnitude of these effects is significantly mitigated by taking depreciation into account. These points are discussed in detail in Krusell and Smith (2014).

To see an example, consider a depreciation rate of 7 percent, a population growth rate of 1 percent, and a growth rate of income per person of 2 percent. In this case, in the extreme event that all growth disappears, the $n + g + \delta$ denominator of the Solow expression falls from 10 percent to 7 percent, so that the capital-output ratio increases by a factor of $10/7$, or around 40 percent. That would be a large change, but it is nothing like the changes we see for France or the United Kingdom in Figure 5.

One may also worry that these comparative statics hold the saving rate s constant. Fortunately, the case with optimizing saving is also easy to analyze and gives similar results. For example, with Cobb-Douglas production, $(r + \delta)K/Y = \alpha$, where α is the exponent on physical capital. With log utility, the Euler equation for consumption gives $r = \rho + g$. Therefore the steady state for the capital-output ratio is $\alpha/(\rho + g + \delta)$, which features similarly small movements in response to changes in per capita growth g . The bottom line from these examples is that qualitatively it is plausible that slowdowns in growth can increase the capital-output ratio in the economy, but the magnitudes of these effects should not be exaggerated.

The effect on between inequality — i.e. on the share of GDP paid as a return to capital — is even less clear. In the Cobb-Douglas example, of course, this share is constant. How then do we account for the empirical rise in capital's share since the 1980s? The research on this question is just beginning and there are not yet clear answers.¹¹

Piketty himself offers one possibility, suggesting that the elasticity of substitution between capital and labor may be greater than one (as opposed to equaling one in the Cobb-Douglas case outlined above).¹² To understand this claim, look back at Figures 4 and 5. The fact that the capital share and the capital-output ratio move together, at least broadly over the long swing of history, is taken as suggestive evidence that the elasticity of substitution between capital and labor is greater than one. Given the importance of

¹¹Recent papers studying the rise in the capital share in the last two decades include Karabarounis and Neiman (2013), Elsby, Hobijn and Şahin (2013), and Bridgman (2014).

¹²For example, see the discussion starting on page 220.

land in both of these time series, however, I would be hesitant to make too much of this correlation. The state-of-the-art in the literature on this elasticity is inconclusive, with some papers arguing for an elasticity greater than one but others arguing for less than one; for example, see Karabarbounis and Neiman (2013) and Oberfield and Raval (2014).

4. Conclusion

Through extensive data work, particularly with administrative tax records, Piketty and Saez and their coauthors have shifted our understanding of inequality in an important way. To a much greater extent than we've appreciated before, the dynamics of top income and wealth inequality are crucial. Future research combining this empirical evidence with models of top inequality is primed to shed light on this phenomenon.¹³

In *Capital in the Twenty-First Century*, Piketty suggests that the fundamental dynamics of capitalism will create a strong tendency toward greater inequality of wealth and even dynasties of wealth in the future, unless this tendency is mitigated by the enactment of policies like a wealth tax. This claim is inherently more speculative. Although the concentration of wealth has risen in recent decades, the causes are not entirely clear and include a decline in saving rates outside the top of the income distribution (as discussed by Saez and Zucman, 2014), the rise in top labor income inequality, and a general rise in real estate prices. The theoretical analysis behind Piketty's prediction of rising wealth inequality often includes a key simplification in the relationships between variables: for example, assuming that changes in the growth rate g will not be mirrored by changes in the rate of return r , or that the saving rate net of depreciation won't change over time. If these theoretical simplifications do not hold — and there are reasons to be dubious — then the predictions of a rising concentration of wealth are mitigated. The future evolution of income and wealth, and whether they are more or less unequal, may turn on a broader array of factors.

I'm unsure about the extent to which $r - g$ will be viewed a decade or two from now as the key force driving top wealth inequality. However, I am certain that our

¹³In this vein, it is worth noting that the Statistics of Income division of the Internal Revenue Service makes available random samples of detailed tax records in their public use microdata files, dating back to the 1960s (for more information on these data, see <http://users.nber.org/~taxsim/gdb/>).

understanding of inequality will have been enhanced enormously by the impetus — both in terms of data and in terms of theory — that Piketty and his coauthors have provided.

Appendix: Simple Models of Pareto Inequality

This appendix seeks to illustrate the simplest models of Pareto inequality. The model for income is about as simple as it can get and is quite useful for intuition and for understanding where Pareto distributions come from. The model for wealth builds on the key insight of the income model. However, it is more complicated, partly by nature and partly so that it can speak to the roles of “ $r - g$ ” and population growth that Piketty (2014) highlights in his book.

A Income Inequality

The simplest models of Pareto inequality are surprisingly easy to understand. Pareto inequality emerges from *exponential growth that occurs for an exponentially-distributed amount of time*. Excellent introductions to Pareto models can be found in Mitzenmacher (2004), Gabaix (2009), Benhabib (2014), and Moll (2012b). Benhabib traces the history of Pareto-generating mechanisms and attributes the earliest instance of a simple model like that outlined here to Cantelli (1921).

To see how this works, we first require some heterogeneity. Suppose people are exponentially distributed across some variable x , which could denote age or experience or talent. For example, $\Pr[\text{Age} > x] = e^{-\delta x}$, where δ denotes the death rate in the population.

Next, we need to explain how income varies with age in the population. A natural assumption is exponential growth: suppose income y rises exponentially with age (or experience or talent) at rate μ : $y = e^{\mu x}$. Inverting this assumption gives us the age at which an individual earns income y : $x(y) = 1/\mu \cdot \log y$.

That's it, and the Pareto distribution then emerges easily:

$$\begin{aligned}\Pr[\text{Income} > y] &= \Pr[\text{Age} > x(y)] \\ &= e^{-\delta x(y)} \\ &= y^{-\frac{\delta}{\mu}}\end{aligned}\tag{3}$$

Recall that the Pareto inequality index is just the inverse of the exponent in this equation, which gives

$$\eta_{\text{income}} = \frac{\mu}{\delta}.\tag{4}$$

The Pareto exponent is increasing with μ , the rate at which incomes grow with age (or experience or talent) and decreasing in the death rate δ . Intuitively, the lower is the death rate, the longer some lucky people in the economy can benefit from exponential growth, which widens Pareto inequality. Similarly, faster exponential income growth across age (a higher return to experience?) also widens inequality. Jones and Kim (2014) build a richer model of labor income inequality along these lines that endogenizes μ and δ .

B Wealth Inequality

A Pareto distribution of wealth can be obtained using a similar logic. Richer models of wealth inequality that motivate the simple model below can be found in Wold and Whittle (1957), Stiglitz (1969), Huggett (1996), Quadrini (2000), Castaneda, Diaz-Gimenez and Rios-Rull (2003), Benhabib and Bisin (2006), Cagetti and Nardi (2006), Nirei (2009), Benhabib, Bisin and Zhu (2011), Moll (2012a), Piketty and Saez (2012), Aoki and Nirei (2013), Moll (2014), and Piketty and Zucman (2014).

B1. Individual wealth

Let a denote an individual's wealth, which accumulates over time according to

$$\dot{a} = ra - \tau a - c\tag{5}$$

where r is the interest rate, τ is a wealth tax, and c is the individual's consumption. Assume consumption is a constant fraction α of wealth (e.g. as it will be with log utility), which yields

$$\dot{a} = (r - \tau - \alpha)a. \quad (6)$$

With this law of motion, the wealth of an individual of age x at date t is

$$a_t(x) = a_{t-x}(0) e^{(r-\tau-\alpha)x} \quad (7)$$

where $a_{t-x}(0)$ is the initial wealth of a newborn at date $t - x$, described further below.

B2. Heterogeneity through a birth-death process

The simple birth-death process here is a canonical model of the demography literature; for example, see Tuljapurkar (2008) or do a google search for “stable population theory”. The number of people born at date t is

$$B_t = B_0 e^{\bar{n}t}. \quad (8)$$

Death is a Poisson process with arrival rate \bar{d} . As shown at the end of this note, the stationary distribution for this birth-death process is exponential:

$$\Pr[\text{Age} > x] = e^{-(\bar{n}+\bar{d})x}. \quad (9)$$

To see the intuition behind this equation, notice that the (long-run) birth rate for this process is $\bar{b} \equiv \bar{n} + \bar{d}$.¹⁴ That is, a fraction \bar{b} of the population is newly born at each instant, some to compensate for deaths and some representing net population growth. The age distribution then declines exponentially at rate \bar{b} .

¹⁴The law of motion for the population is

$$\frac{\dot{N}_t}{N_t} = \frac{B_t}{N_t} - \bar{d}.$$

So population growth is constant if and only if N grows at the same rate as B , i.e. at rate \bar{n} . In this case, $B/N = \bar{n} + \bar{d}$.

B3. The wealth distribution in partial equilibrium

Newborns equally inherit the wealth of the people who die in this economy:

$$\begin{aligned} a_t(0) &= \frac{\bar{d}K_t}{(\bar{n} + \bar{d})N_t} \\ &= \bar{a}k_t \end{aligned} \tag{10}$$

where $\bar{a} \equiv \bar{d}/(\bar{n} + \bar{d})$ and $k_t \equiv K_t/N_t$ is capital (wealth) per person in the economy. To understand this equation, consider the first line. The numerator in the first part of this equation, $\bar{d}K_t$, equals aggregate wealth of the people who die, and the denominator is the number of newborns. In the second line, notice that because of population growth, newborns inherit less than the average amount of capital per person in the economy, and this fraction is given by \bar{a} .

Assume that the macroeconomy is in steady state, so that capital per person grows at a constant and exogenous rate, g , over time: $k_t = k_0 e^{gt}$. Equation (10) can be used to help characterize the cross-section distribution of wealth at date t . In particular, the amount of wealth that a person of age x at date t inherited when they were born (at date $t - x$) is

$$a_{t-x}(0) = \bar{a}k_{t-x} = \bar{a}k_t e^{-gx}. \tag{11}$$

And substituting this expression into (7), we obtain the cross-section of wealth at date t by age:

$$a_t(x) = \bar{a}k_t e^{(r-g-\tau-\alpha)x} \tag{12}$$

This is the exponential growth process that is one of the two key ingredients that delivers a Pareto distribution for normalized wealth, and one can already see that $r - g$ plays a role. The other key ingredient is the exponential age distribution in equation (9), providing the heterogeneity. Together, these two building blocks give us our requirement: exponential growth occurs over an exponentially-distributed amount of time.

Inverting equation (12) gives the age at which a person in the cross-section achieves wealth a :

$$x(a) = \frac{1}{r - g - \tau - \alpha} \log \left(\frac{a}{\bar{a}k_t} \right). \tag{13}$$

Then the wealth distribution is

$$\begin{aligned}
 \Pr[\text{Wealth} > a] &= \Pr[\text{Age} > x(a)] \\
 &= e^{-(\bar{n}+\bar{d})x(a)} \\
 &= \left(\frac{a}{\bar{a}k_t}\right)^{-\frac{\bar{n}+\bar{d}}{r-g-\tau-\alpha}}.
 \end{aligned} \tag{14}$$

Recall that Pareto inequality is measured by the inverse of the exponent in the expression above, which gives our first main result for wealth inequality:

$$\eta_{\text{wealth}} = \frac{r-g-\tau-\alpha}{\bar{n}+\bar{d}}. \tag{15}$$

B4. The consumption share of wealth, α

If expected lifetime utility is

$$\int_0^\infty e^{-(\rho+\bar{d})t} \log c_t dt \tag{16}$$

then it is straightforward to show that $c_t = (\rho + \bar{d})a_t$. That is, consumption is a constant fraction of wealth, and we have $\alpha = (\rho + \bar{d})$. The linearity of consumption in wealth applies more generally, delivering a richer formula for α ; see Moll (2014), for example.

It is worth pausing here to address a natural question: why is there no N_t or B_t in the utility function? The answer is that leaving N_t out is the simplest approach. This case corresponds to the assumption that individuals do not care about their offspring, and this is consistent with the structure of the rest of the model — namely, that newborns equally inherit the wealth of the people who die. It would be useful to consider altruism, where newborns inherit wealth from parents who care about their well-being, and such structures have been considered in the literature cited earlier.

B5. The wealth distribution in general equilibrium

We close the model in two different ways, which turn out to yield the same result for Pareto inequality in general equilibrium. Consider an “AK” production function:

$$Y_t = A_t K_t. \tag{17}$$

Our two cases are

1. **Capital model:** Here, $A_t = \bar{A}$ is constant over time, and capital accumulates endogenously: $\dot{K}_t = Y_t - C_t - T_t - \delta K_t$, where C denotes aggregate consumption and $T_t = \tau K_t$ denotes aggregate tax revenue. The fact that tax revenue enters the budget constraint (rather than being rebated lump sum) leads to substitution and income effects canceling. This case corresponds to the tax revenue being thrown away or alternatively being spent on a public good that enters utility in an additively separable fashion.
2. **Land model:** Alternatively, suppose $A_t = A_0 e^{\bar{g}t}$ and let $K_t = \bar{K}$ denote a fixed supply of land.

Both interpretations generate economic growth. The fact that they lead to identical Pareto wealth inequality highlights the fact that whether wealth is capital that accumulates or just land that does not does not matter from the standpoint of wealth inequality.

Because the details are somewhat involved, we'll just report the main result first. In both cases, we assume that taxes are taken out of the economy and thrown away. In each, the interest rate in general equilibrium satisfies:

$$r - g - \tau - \alpha = \bar{n}, \quad (18)$$

so wealth inequality in general equilibrium is

$$\eta_{\text{wealth}} = \frac{\bar{n}}{\bar{n} + \bar{d}}. \quad (19)$$

What is going on here? The first intuition comes from the standard Euler equation for the standard neoclassical growth model with log utility, e.g. $r - g = \rho$. In particular, the interest rate moves one-for-one with the growth rate, and $r - g$ is just a constant. Another feature of log utility is that substitution and income effects offset. This, together with the fact that we are throwing away the tax revenue in this setup, delivers the result that the tax rate does not matter for long-run inequality. If taxes are rebated lump sum, the tax parameter will matter once again for inequality in general; I suspect that the progressivity of the tax on wealth could also matter more generally.¹⁵

¹⁵The lump-sum rebate case makes the model more complicated, in that a lump sum rebate adds a form of income that is not directly proportional to wealth, so we lose the simple exponential growth that makes this model so easy, though results should still go through asymptotically. Heathcote, Storesletten and Violante (2014) highlight a related point and note that similar issues arise with progressive taxation.

A second intuition is even more appropriate here. Recall that $r - g - \tau - \alpha$ is the growth rate of an individual's normalized wealth. It is this growth rate that turns out to equal the rate of population growth, \bar{n} . To see why, look back at equation (10) and recall that each newborn inherits less than the average amount of capital per person in the economy; in fact, they get the fraction $\frac{\bar{d}}{\bar{n} + \bar{d}}$. Apart from this cohort effect, each person in this economy is essentially the same. In particular, in this setup, the size of each cohort grows at rate \bar{n} , so that the per capita wealth of each generation falls at rate \bar{n} as we look at younger and younger cohorts. But this is just another way of saying that normalized wealth — i.e. taking out macroeconomic growth at rate g — grows over time at rate \bar{n} . This is why the general equilibrium requires $r - g - \tau - \alpha = \bar{n}$.

An important implication of this reasoning can now be seen: if there were no population growth in the model, newborns would each inherit the per capita amount of wealth in the economy. The accumulation of wealth by individuals over time would correspond precisely to the growth in the per capita wealth that newborns inherit, and there would be no inequality in the model!

This section illustrates very nicely an important point about models of Pareto inequality: the general equilibrium of the model must be considered, and it can change the comparative statics. For example, we already noted that in partial equilibrium, an increase in the population growth rate \bar{n} lowers Pareto inequality, as the concentration of wealth gets diluted by more offspring. In general equilibrium, the effect works in the opposite direction for the reasons discussed above. Similarly, $r - g$ and τ no longer matter for inequality in general equilibrium.

B6. Details of the Capital Model

Since individual consumption is proportional to wealth, aggregate consumption is as well: $C = \alpha K$. For the baseline case, we assume that tax revenue is used to pay for government services that enter utility in an additively separable way, so the aggregate resource constraint for this economy is $Y = C + I + T$, where I is gross investment. The capital accumulation equation then implies that aggregate growth is $g_Y = A - \delta - \tau - \alpha$, and therefore per capita growth is $g = A - \delta - \tau - \alpha - \bar{n}$.

The equilibrium interest rate in this model is just the net marginal product of capital: $r = A - \delta$. Combining these last two equations gives the key result needed above:

$$r - g - \tau - \alpha = \bar{n}.$$

Lump-sum rebate of tax revenue: The case in which the wealth tax is rebated lump sum is different, however. In this case, the exponential growth of normalized wealth across ages breaks down, except at the very top for the wealthiest people: the lump sum rebate is a vanishing fraction of the wealth for the richest households. So the partial equilibrium equation for η continues to apply, but only at the very top of the wealth distribution. Now, however, the aggregate resource constraint is $Y = C + I$, so that all of the tax revenue comes back into the economy as consumption or investment. In this case, aggregate growth is $g_Y = A - \delta - \alpha$, which is invariant to the tax rate in the log utility case. Now $r - g - \tau - \alpha = \bar{n} - \tau$, and top wealth inequality is given by

$$\eta_{\text{wealth}}^{\text{lumpsum}} = \frac{\bar{n} - \tau}{\bar{n} + \bar{d}}. \quad (20)$$

So what happens to the tax revenue matters crucially for the effect of wealth taxes on top wealth inequality.¹⁶

B7. Details of the Land Model

For the land model, let P_t denote the price (measured in units of output) of one unit of land. Aggregate wealth is then $W_t = P_t \bar{K}$. The price of land satisfies a standard arbitrage equation:

$$r = \frac{A_t}{P_t} + \frac{\dot{P}_t}{P_t}. \quad (21)$$

That is, one can invest P units of output in the bank and earn interest on it, or one can buy a unit of land, earn the dividend A_t , and then sell it, pocketing the capital gain. Along a balanced growth path (no bubbles), this equation implies the capital gain term equals the growth rate of A , \bar{g} , so the price of land is pinned down by

$$P_t = \frac{A_t}{r - \bar{g}}. \quad (22)$$

Aggregate consumption in this economy can be computed in two ways, and this

¹⁶This analysis requires $\bar{n} \geq \tau$.

allows us to solve for the interest rate. First,

$$C = \alpha W = \alpha P_t \bar{K} = \frac{\alpha}{r - \bar{g}} \cdot A_t \bar{K} = \frac{\alpha}{r - \bar{g}} \cdot Y_t. \quad (23)$$

Alternatively,

$$\begin{aligned} C_t &= Y_t - T_t = Y_t - \tau W_t = Y_t - \tau P_t \bar{K} \\ &= Y_t - \frac{\tau}{r - \bar{g}} \cdot A_t \bar{K} = \left(1 - \frac{\tau}{r - \bar{g}}\right) Y_t \end{aligned} \quad (24)$$

Equating these two expressions for consumption and noting that $g_Y = \bar{g}$ so that $g = \bar{g} - \bar{n}$ gives the required solution for the interest rate: $r - g - \tau - \alpha = \bar{n}$. Wealth inequality is therefore given by equation (15).

Lump-sum rebate: If tax revenues are rebated lump sum, then $C = Y$. Then from (23), we must have $r - \bar{g} = \alpha$, so that $r - g - \alpha = \bar{n}$ and therefore $r - g - \tau - \alpha = \bar{n} - \tau$, and inequality with lump sum rebates is also given by equation (20) in the land version of the model.

B8. The stationary distribution of the simple birth-death process

Let $G(x, t) = \Pr[\text{Age} > x]$ denote the complementary form of the age distribution at time t . With population growth rate \bar{n} and death rate \bar{d} , the distribution evolves over a small time interval Δt as

$$G(x, t + \Delta t) = \frac{1 - \bar{d}\Delta t}{1 + \bar{n}\Delta t} \cdot G(x, t) + G(x - \Delta x, t) - G(x, t). \quad (25)$$

The first term captures the change from deaths and population growth (to keep the distribution proper), while the last two terms capture the inflow of younger people into the higher ages.

Using a Taylor expansion for $\frac{1}{1 + \bar{n}\Delta t} \approx 1 - \bar{n}\Delta t$ and ignoring the higher order terms leads to

$$\frac{G(x, t + \Delta t) - G(x, t)}{\Delta t} = -(\bar{n} + \bar{d})G(x, t) - \frac{G(x, t) - G(x - \Delta x, t)}{\Delta x}, \quad (26)$$

where we've also used the fact that $\Delta x = \Delta t$.

Taking the limits as $\Delta t \rightarrow 0$ gives

$$\frac{\partial G(x, t)}{\partial t} = -(\bar{n} + \bar{d})G(x, t) - \frac{\partial G(x, t)}{\partial x} \quad (27)$$

Setting the time derivative equal to zero and solving for the stationary distribution yields the desired result:

$$G(x) = e^{-(\bar{n} + \bar{d})x}. \quad (28)$$

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