NBER WORKING PAPER SERIES

# FISCAL POLICIES AND REAL EXCHANGE RATES IN THE WORLD ECONOMY

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Working Paper No. 2065

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 1986

This paper was prepared for the Macroeconomics Division, DRD, World Bank. We thank Thomas Krueger and Kei-Mu Yi for efficient research assistance. The research reported here is part of the NBER's research programs in International Studies and Economic Fluctuations. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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## ABSTRACT

This paper examines the effects of fiscal policies on the evolution of real rates of interest and real exchange rates in the interdependent world economy. We construct an analytical framework suitable for a detailed examination of the various channels through which these variables are influenced by government spending and by tax policies. The analytical framework employs a general equilibrium approach highlighting the roles played by wealth effects and by temporal and intertemporal substitution effects. The general principle illustrated by the analysis of the dynamic effects of budget deficits is that the consequences of temporary tax policies stretch beyond the period during which the temporary policies are in effect. The counterpart to these dynamic implications is the rise in the economy's external debt induced by the budget deficit the service of which stretches into the indefinite future. By series of examples, allowing for both distortionary and non-distortionary taxes and for various patterns of government spending, it is shown that the quantitative and qualitative effects of fiscal policies on real exchange rates, real interest rates, debt accumulation and the like depend critically on the commodity composition of government spending and its intertemporal allocations on the one hand, and on the details of government debt issue and tax structure, including the timing of taxes and borrowing and the types of taxes used to finance the budget, on the other hand.



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This paper deals with the effects of fiscal policies on the evolution of real rates of interest and real exchange rates in the interdependent world economy. The main purpose of the analysis is to construct an analytical framework suitable for a detailed examination of the various channels through which government spending and tax policies influence interest rates and real exchange rates. Such an analysis clarifies the mechanisms through which the effects of fiscal policies are transmitted internationally.

Our analysis is motivated by developments occuring in the world economy during the first half of the 1980's. During this period changes in national fiscal policies were unsynchronized, real rates of interest were high and volatile and real exchange rates exhibited diverging trends and were subject to large fluctuations. The course of fiscal policies undertaken by the major industrial countries affected the rest of the world through the integrated goods and capital markets and resulted in increased concern in each country over policy measures taken in the rest of the world. In addition to the interactions among the major economies, the debt-ridden countries faced with high world interest rates adopted fiscal policy measures that influenced their competitive position and external-debt accumulation. Our analysis provides a framework useful for interpreting such developments.

The analytical framework employs a general equilibrium intertemporal approach. One of its key implications is that a proper analysis of the effects of fiscal policies on real exchange rates, real interest rates, debt accumulation and other important macroeconomic variables must indicate the timing and composition of fiscal policies. Accordingly, the analysis must specify the details of the commodity composition of government spending and its intertemporal allocations on the one hand, and the details of government debt issue and of the tax structure including the timing of taxes and borrowing and the types of taxes used to finance the budget, on the other hand.

The emphasis on the intertemporal considerations underlying fiscal policies permits the adoption of a long-term perspective in an evaluation of the consequences of current policies. In this perspective the short, medium, and long-term implications of policies are intimately linked to each other through the forward-looking behavior of economic agents who are subject to intertemporal budget constraints.

One of the implications of the analysis is that even though the real rates of interest and real exchange rates which are at the center of our analysis are influenced by fiscal policies, they are not policy instruments directly manipulated by governments. Rather, these variables may be viewed as useful indicators for macroeconomic policies.

The paper is divided into three main parts. Part I contains an analysis of the effects of government spending. In this context we develop the analytical framework for the two-country model of the world economy. To highlight the key mechanisms and channels through which fiscal policies operate we abstract from monetary considerations. The subsequent two parts deal with the effects of budget deficits induced by tax policies. Accordingly, we analyse in Part II the effects of distortionary taxes and in Part III the effects of non-distortionary taxes. The former operate primarily through temporal and intertemporal substitution effects and the latter through wealth effects. In this context we first examine the small country case with special emphasis on the dynamics of the real exchange rate, and then turn to the two-country case with special emphasis on the cross-country comovements of real exchange rates. The paper is concluded in Part IV which contains a detailed summary of the key results.

## I. Government Spending Policies

In this part of the paper we analyse the effects of government spending on the real exchange rate. To focus on the unique role played by the temporal and the intertemporal allocations of government expenditures, we specify the model so as to ensure that the details of government finance, particularly the timing of taxes and borrowing, are immaterial. Accordingly, we assume in this part of the paper that all taxes are lump-sum and that the horizon governing private and public-sectors' behavior coincide. Issues concerning the details of government finance are dealt with in subsequent parts of this paper. We start with an outline of the analytical framework for a two-country model of the world economy. This framework is then applied to an analysis of the effects of government spending on the world rate of interest and on the real exchange rates in the two countries.

### I.1: The Analytical Framework

Throughout we assume that there are two composite goods: an internationally tradable good, denoted by $x$, and a non-tradable good, denoted by $n$. To allow for intertemporal considerations we assume for simplicity a two-period model, period 0 and period 1.[1] The relative price of the non-tradable good (the inverse of the real exchange rate) in period t is denoted by $p_{nt}$, the exogeneously given output of that good is $\bar{Y}_{nt}$, government purchases of the non-tradable good are $G_{nt}$, and private-sector demand is $c_{nt}$ $(t=0,1)$.

The private- sector life-time budget constraint is

$$(I.1)\qquad (c_{x0}+p_{n0}c_{n0}) + \alpha_{x1}(c_{x1}+p_{n1}c_{n1})$$

$$= (\bar{Y}_{x0}+p_{n0}\bar{Y}_{n0}) + \alpha_{x1}(\bar{Y}_{x1}+p_{n1}\bar{Y}_{n1}) - (1+r_{x,-1})B^{p}_{-1} = W_0$$

where $\alpha_{x1} = 1/(1+r_{x0})$ denotes the discount factor and where $c_{xt}$ and $\bar{Y}_{xt}$ denote, respectively, the levels of consumption and the exogeneously given level of production of tradable goods, in period $t$ $(t=0,1)$, $W_0$ denotes wealth, $r_{xt}$ $(t=-1,0)$ denotes the world rate of interest and $B_t^p$ denotes private-sector debt in period $t$ $(t= -1,0)$. The values of wealth, debt, and the rates of interest are measured in terms of tradable goods.

In addition to the private-sector life-time budget constraint the economy's overall constraint incorporates the government budget constraint. Accordingly, the government present-value intertemporal budget constraint is

$$(G_{x0} + p_{n0}G_{n0}) + \alpha_{x1}(G_{x1} + p_{n1}G_{n1}) = T_0 + \alpha_{x1}T_1 - (1+r_{x,-1})B_{-1}^g \tag{I.2}$$

where $G_{xt}$ and $G_{nt}$ denote respectively government purchases of tradable and non-tradable goods, and where $T_t$ and $B_t^g$ denote, respectively, lump-sum taxes and government debt in period t. Consolidating the private sector life-time constraint (I.1) with that of the government (I.2) and imposing equality between consumption and production of non-tradable goods in each period, yields the economy's consolidated constraint:

$$c_{x0} + \alpha_{x1}c_{x1} = (\bar{Y}_{x0}-G_{x0}) + \alpha_{x1}(\bar{Y}_{x1}-G_{x1}) - (1+r_{x,-1})B_{-1} \tag{I.3}$$

where $B_t = B_t^p+B_t^g$ denotes the economy's external debt in period t.

The individual maximizes life-time utility subject to the life-time budget constraint (I.1). We assume that the life-time utility function can be expressed as a function of two linearly homogeneous sub-utility functions $C_0(c_{x0},c_{n0})$ and $C_1(c_{x1},c_{n1})$. Hence, life-time utility is $U(C_0,C_1)$. The maximization of this utility function subject to the life-time constraint (I.1) is carried out in two stages where the first stage optimizes the composition of spending within each period and the second stage optimizes the intertemporal allocation of spending between periods.

The optimization of the intertemporal allocation of (the consumption-based) real spending yields the demand functions for each period real spending $C_t = C_t((\alpha_{c1}, W_{c0})$ where $\alpha_{c1}$ is the (consumption-based) real discount factor, and where $W_{c0}$ is (the consumption-based) real wealth. Expressed in terms of tradable goods, the level of spending in each period is $P_t C_t$ where $P_t$ is the consumption-based price index (the "true" price deflator). Thus, $\alpha_{c1} = \alpha_{x1} P_1/P_0$ and $W_{c0} = W_0/P_0$. Obviously, the price index in each period depends on the temporal relative price $p_{nt}$ with an elasticity that equals the relative share of expenditure on non-tradable goods, $\beta_{nt}$. Within each period the (sub) utility-maximizing allocation of spending between goods depends on the relative price $p_{nt}$.[2]

The market for non-tradable goods must clear in each country during each period. Accordingly the market clearing conditions for the domestic non-tradable goods are

$$c_{n0}(p_{n0}, P_0 C_0(\alpha_{c1}, W_{c0})) = \bar{Y}_{n0} - G_{n0} \tag{I.4}$$

$$c_{n1}(p_{n1}, P_1 C_1(\alpha_{c1}, W_{c0})) = \bar{Y}_{n1} - G_{n1} \tag{I.5}$$

where the left-hand-sides of these equilibrium conditions show the demand functions and the right-hand sides show the supply net of government purchases. As seen, the demand function depends on the relative price, $p_{nt}$, and on spending, $P_t C_t$ , where $P_t$ is the consumption-based price index, and $C_t$ is (the consumption-based) real spending. As indicated the level of real spending depends on the (consumption-based) real discount factor, $\alpha_{c1}$. We assume that the utility function is homothetic so that the elasticity of consumption demand with respect to spending as well as the elasticity of spending with respect to wealth are unity.

Market clearing requires that in each period changes in the demand for non-tradable goods (induced by various shocks) are equal to changes in the supply net of government purchases. Accordingly, differentiating equations (I.4)-(I.5) for given levels of output and evaluating around $G = 0$ (reflecting the assumption that the initial level of government spending is zero) yields

$$\text{(I.6)} \qquad (\eta_{n_0 p_{n0}} + \eta_{P_0 p_{n0}}) \hat{p}_{n0} + \eta_{C_0 \alpha} \hat{\alpha}_{c1} + \hat{W}_{c0} = - \varphi_{n0} \beta^g_n (1-\gamma^g_s) dG$$

$$\text{(I.7)} \qquad (\eta_{n_1 p_{n1}} + \eta_{P_1 p_{n1}}) \hat{p}_{n1} + \eta_{C_1 \alpha} \hat{\alpha}_{c1} + \hat{W}_{c0} = - \varphi_{n1} \beta^g_{n1} \gamma^g_s dG$$

where $\eta$ denotes the elasticity of the variable indicated by the first subscript with respect to the variable indicated by the second subscript, $\varphi_{nt}$ denotes the inverse of the value of private consumption of non-tradable goods in period t, that is, $\varphi_{nt} = 1/p_{nt}c_{nt}$ $(t=0,1)$, and where a circumflex denotes a logarithmic derivative. The intertemporal and the temporal allocations of government spending are governed by the government saving propensity, $\gamma^g_s$, defined as the ratio of future government spending (in present value terms) to the discounted sum of government spending, and by the relative share of government spending on non-tradables in total government spending in period t, $\beta^g_{nt}$. The left-hand-sides of equations (I.6)-(I.7) show that private-sector demand is altered through changes in temporal prices ($p_{nt}$), intertemporal prices ($\alpha_{c1}$), and real wealth ($W_{c0}$). The right-hand-sides of these equations show that the net supply of non-tradable goods is altered through changes in output (supply shocks) and through changes in government purchases.

In what follows we use the market clearing conditions in order to analyse the international transmission of fiscal policies. In the present case, since the only tradable good is a single composite commodity, the international price which effects the transmission mechanism is the world rate of interest. The analysis proceeds in two stages. In the first, we determine the effects of fiscal policies on the time paths of the real exchange rate and of private consumption of tradable goods under the assumption that the world rate of interest is given. Similarly, we also determine the effects of changes in the world rate of interest on the paths of the real exchange rate and of private consumption of tradable goods under the assumption that fiscal policies are given. In the second stage we use a two-country framework and combine these partial results in order to determine the equilibrium relations between fiscal policies, the world rate of interest, and the time paths of the domestic and foreign real exchange rates.

## I.2: Government Spending, the Rate of Interest and The Real Exchange Rate

In this section we analyse the effects of government spending on the world rate of interest and on the paths of the domestic and foreign real exchange rates. This analysis identifies the precise mechanism of the international transmission of the effects of government spending on both tradable and non-tradable goods.

The equilibrium value of wealth $W_0$, is obtained by substituting the government present-value budget constraint (I.2) into the corresponding private-sector budget constraint. Accordingly,

$$W_0 = \left[p_{n0}(\bar{Y}_{n0}-G_{n0}) + (\bar{Y}_{x0}-G_{x0})\right] + \alpha_{x1}\left[\bar{Y}_{n1}-G_{n1}) + (\bar{Y}_{x1}-G_{x1})\right] - (1+r_{x,-1})B_{-1} \quad . \tag{I.8}$$

Starting from a zero level of initial government spending, consider a rise in the discounted sum of government spending by dG. This change in aggregate government spending falls in part on non-tradable goods as indicated by the relevant terms on the right-hand-side of equations (I.6)-(I.7). For a given value of the world rate of interest (measured in terms of tradable goods) the effect of the rise in government spending on the time path of the real exchange rate, $p_{n0}/p_{n1}$, is found by subtracting equation (I.7) from (I.6) and using the Slutsky decomposition. If the expenditure shares of the private sector do not vary over time, this yields[3]

$$\frac{d \log (p_{n0}/p_{n1})}{dG} = \frac{\varphi_{n0}\beta_n(1-\gamma_s)}{\beta_n\sigma + (1-\beta_n)\sigma_{nx}} \left[\frac{\beta^g_{n0}(1-\gamma^g_s)}{\beta_n (1-\gamma_s)} - \frac{\beta^g_{n1}\gamma^g_s}{\beta_n\gamma_s}\right] \tag{I.9}$$

where $\sigma$ and $\sigma_{nx}$ denote the intertemporal and the temporal elasticities of substitution, and where $\gamma_s$, defined as the ratio of private sector future consumption (in present-value terms) to the discounted sum of private-sector spending, denotes the private-sector saving propensity.

Equation (I.9) reveals that the direction of the change in the path of the real exchange rate depends on the temporal and the intertemporal allocations of government demand for non-tradable goods relative to the corresponding allocations of private-sector demand. If the ratio of the relative share of government spending on non-tradable goods in the current period, $\beta^g_{no}(1-\gamma^g_s)$, to the private-sector share, $\beta_n(1-\gamma_s)$, exceeds the corresponding ratio in the future period, $\beta^g_{n1}\gamma^g_s/\beta_n\gamma_s$, then a rise in government spending raises the percentage rate of change of the real exchange rate and vice versa.

This result can be interpreted in terms of a "transfer-problem" criterion relating the temporal and the intertemporal spending patterns of the government and the domestic private sector. Accordingly, the rise in government spending raises the current price of non-tradable goods relative to its future price, if the pattern of government spending is biased towards current non-tradable goods in comparison with the pattern of private-sector spending.

We turn next to determine the effects of government spending on the path of private-sector consumption of tradable goods. Analogously to the previous specification, the demand functions for tradable goods are

$$c_{x0} = c_{x0}(p_{n0}\ ,\ \ P_0C_0(\alpha_{c1}\ ,\ \ W_{c0})) \tag{I.10}$$

$$c_{x1} = c_{x1}(p_{n1}\ ,\ \ P_1C_1(\alpha_{c1}\ ,\ \ W_{c0})) \tag{I.11}$$

Obviously, in contrast to the markets for non-tradable goods, the consumption of tradable goods in any given period is not limited by the available domestic supply. In determining the percentage change in the ratio $c_{x0}/c_{x1}$ we differentiate equations (I.10)-(I.11) and use the Slutsky decomposotion. Accordingly,

$$\frac{d\ \log\ (c_{x0}/c_{x1})}{d\ \log\ (p_{n0}/p_{n1})} = \beta_n(\sigma_{nx} - \sigma). \tag{I.12}$$

Equation (I.12) shows that the qualitative effects of a rise in the price ratio $p_{n0}/p_{n1}$ on the tradable-good consumption ratio, $c_{x0}/c_{x1}$, depends only on whether the temporal elasticity of substitutions, $\sigma_{nx}$, exceeds or falls short of the intertemporal elasticity of substitution $\sigma$. A rise in the relative price of non-tradable goods, $p_{nt}$, induces substitution of consumption of tradable goods for non-tradable goods <u>within</u> the period. The magnitude of this temporal substitution is indicated by $\sigma_{nx}$. Further, if $p_{n0}$ rises by more than $p_{n1}$ (so that the ratio $p_{n0}/p_{n1}$, rises), then the extent of the temporal substitution within the current period exceeds the corresponding substitution within the future period. As a result the ratio of current to future consumption of tradable goods rises. This is reflected by the positive term $\beta_n\sigma_{nx}$ in equation (I.12). The same rise in the intertemporal price ratio $p_{n0}/p_{n1}$ raises the (consumption-based) real rate of interest (and lowers the corresponding real discount factor, $\alpha_{c1}$). This rise in the real rate of interest induces substitution of spending <u>between</u> periods: from the present to the future period. The magnitude of this intertemporal substitution is indicated by the negative term $-\beta_n\sigma$ in equation (I.12). Finally, we note that the change in the intertemporal consumption ratio does not depend on private wealth. This reflects the homotheticity assumption which implies that the tax-induced fall in wealth lowers current and future demand for tradable goods by the same proportion.

Combining equations (I.9) with (I.12) yields

$$\text{(I.13)} \qquad \frac{d \log (c_{x0}/c_{x1})}{dG} = \frac{\beta_n^2\varphi_{n0}(1-\gamma_s)(\sigma_{nx}-\sigma)}{\beta_n\sigma+(1-\beta_n)\sigma_{nx}} \left[\frac{\beta^g_{n0}(1-\gamma^g_s)}{\beta_n(1-\gamma_s)} - \frac{\beta^g_{n1}\gamma^g_s}{\beta_n\gamma_s}\right]$$

Equation (I.13) shows that the direction of the effect of a rise in government spending on the path of tradable-goods consumption, $c_{x0}/c_{x1}$, depends on the product of two factors. First, the government-induced temporal-intertemporal bias in demand relative to the private sector (indicated by the term in the squared brackets in (I.13)), and second, the temporal-intertemporal substitution bias in private-sector demand (indicated by $\sigma_{nx}-\sigma$). The first determines the effect of the rise in G on the price ratio and the second translates the change in the price ratio into changes in the consumption ratio.

We turn next to determine the effects of changes in the world rate of interest (or equivalently the world discount factor) under the assumption that government spending remains intact. Using equations (I.6)-(I.7) it can be shown that the percentage changes in the periodic price of non-tradable goods arising from a given percentage change in the world discount factor are

$$\text{(I.14)} \qquad \hat{p}_{n0} = \left[\frac{\gamma_s \sigma}{\beta_n \sigma + (1-\beta_n)\sigma_{nx}} + \frac{\gamma_s(\mu_1 - 1)}{(1-\beta_n)\sigma_{nx}}\right] \hat{\alpha}_{x1}$$

$$\text{(I.15)} \qquad \hat{p}_{n1} = \left[\frac{-(1-\gamma_s)\sigma}{\beta_n \sigma + (1-\beta_n)\sigma_{nx}} + \frac{\gamma_s(\mu_1 - 1)}{(1-\beta_n)\sigma_{nx}}\right] \hat{\alpha}_{x1}$$

where $\mu_1 = \left[p_{n1}(\bar{Y}_{n1}-G_n) + (\bar{Y}_{x1}-G_{x1})\right]/(p_{n1}c_{n1} + c_{x1})$ denotes the ratio of future net output to private consumption, and where $\gamma_s$ denotes the private-sector relative share of saving out of wealth.

It is seen that the link between $p_{nt}$ and $\alpha_{x1}$ operates through two channels. The first channel (indicated by the first term) is the intertemporal substitution effect which is positive in its effect on the current price and negative in its effect on the future price. Accordingly, a rise in the world discount factor (that is, a fall in the rate of interest) induces substitution of spending from the future towards the present. Part of the rise in current aggregate demand falls on the non-tradable goods and drives up their current price. Likewise, the decline in future demand reduces the future price of non-tradable goods.

The second channel (indicated by the second term in equations (I.14)-(I.15)) is the wealth effect. As is evident, the wealth effect induced by changes in the world rate of interest may be positive or negative depending on whether the country is a borrower or lender. Accordingly, if $\mu_1 > 1$, that is if in the future period GDP net of government spending exceeds private spending (so that in the future period the private-sector runs a trade account surplus), then the rise in the world discount factor (the fall in the rate of interest) induces a positive wealth effect. On the other hand, if $\mu_1 < 1$, the same rise in the discount factor induces a negative wealth effect. In this context it is relevant to note that the influence of the wealth effect on $p_{n0}$ is identical to its influence on $p_{n1}$; this reflects the assumption that the wealth elasticity of spending is unitary and that the expenditure share, $\beta_n$ , and the temporal elasticity of substitution, $\sigma_{nx}$, do not vary over time.

The dependence of the wealth effect on whether the <u>future</u> value of GDP net of government spending exceeds or falls short of future private spending (that is on whether $\mu_1$ exceeds or falls short of unity) is interpreted by reference to the economy's consolidated budget constraint (I.3). This constraint implies that the discounted sum of the trade account surpluses, $(TA)_0 + \alpha_{x1}(TA)_1$, equals the historically given external debt commitment, $(1+r_{x,-1})B_{-1}$, where $(TA)_t$ denotes the trade-account surplus in period t ($t=0,1$). If $\mu_1 > 1$, then $(TA)_1 > 0$ and, therefore, the sum of the present-period current account deficit $\left[-(TA)_0 + r_{x,-1}B_{-1}\right]$ and the amortization of the historical debt $B_{-1}$ must be positive. This sum is the private-sector's present-period <u>borrowing needs</u>, to which the current rate of interest applies. Thus, the direction of the wealth effect induced by the fall in the current rate of interest depends on whether the borrowing needs (current account deficit plus amortization) are positive or negative.

Equations (I.14)-(I.15) show that if the current borrowing needs are positive ($\mu_1>1$), then the fall in the world rate of interest must raise $p_{n0}$ since in that case both the substitution and the wealth effects operate in the same direction. The two effects exert, however, conflicting influences on the future price, $p_{n1}$. If the current borrowing needs are negative ($\mu_1<1$), then the fall in the world rate of interest lowers the future price $p_{n1}$ (since both the wealth and substitution effects operate in the same direction), but its impact on the current price depends on the relative magnitudes of the (negative) wealth and (positive) substitution effects. If the current borrowing needs are zero (so that $\mu_1=1$) then the fall in the rate of interest does not exert any wealth effect and, therefore, it must induce a rise in $p_{n0}$ and a fall in $p_{n1}$ through a pure intertemporal substitution.

The multitude of possibilities arising from alternative assumptions about the sign and magnitude of the wealth effect does not impact on the analysis of the time profile of the real exchange rate, nor does it influence the analysis of changes in the (consumption-based) real rate of interest. The homotheticity assumption together with the assumption that the expenditure share, $\beta_n$, and the temporal elasticity of substitution, $\sigma_{nx}$, do not vary over time, imply that the intertemporal ratio, $p_{n0}/p_{n1}$, is independent of wealth. Therefore, we proceed by analysing the effects of changes in the world rate of interest on the time-path of the real exchange rate.

Subtracting equations (I.15) from (I.14) yields

$$\text{(I.16)} \qquad \frac{d \log (p_{n0}/p_{n1})}{d \log \alpha_{x1}} = \frac{\sigma}{\beta_n \sigma + (1-\beta_n)\sigma_{nx}}$$

Equation (I.16) indicates that changes in the world rate of interest influence the path of the real exchange rate only through the intertemporal substitution effect. Accordingly a rise in the world rate of interest (a fall in $\alpha_{x1}$) induces intertemporal substitution of spending towards the future and, thereby, lowers the current price of non-tradable goods relative to the future price (that is, it decelarates the rate of increase of the real exchange rate from period zero to period one). As before, the wealth-induced proportional change in the current demand and, thereby, in the current price of non-tradable goods, $p_{n0}$, equals the corresponding changes in the future demand and price. Therefore, these wealth effects do not influence the price ratio $p_{n0}/p_{n1}$.

The induced change in the price ratio of non-tradable goods together with the change in the world rate of interest influence the intertemporal-consumption ratio of tradable goods, $c_{x0}/c_{x1}$ according to

$$\text{(I.17)} \qquad \frac{d \log (c_{x0}/c_{x1})}{d \log \alpha_{x1}} = \frac{\sigma_{nx}\,\sigma}{\beta_n\sigma + (1-\beta_n)\sigma_{nx}} .$$

Equation (I.17) shows that the only factors governing the change in this intertemporal-consumption ratio are pure temporal and intertemporal substitution effects. As before the wealth effects do not influence this ratio. In this case, however, (and in contrast with the effects of the rise in government spending analysed in equation (I.13)), both the temporal and the intertemporal elasticities of substitution operate in the same direction. This is evident by noting from the definition of the (consumption-based) real discount factor and from equation (I.16) that

$$\text{(I.18)} \qquad \frac{d \log \alpha_{c1}}{d \log \alpha_{x1}} = 1-\beta_n\frac{d \log (p_{n0}/p_{n1})}{d \log \alpha_{x1}} = \frac{(1-\beta_n)\sigma_{nx}}{\beta_n\sigma + (1-\beta_n)\sigma_{nx}}$$

Thus, a rise in the world discount factor raises the (consumption-based) real discount factor (but by a smaller proportion in view of the rise in $p_{n0}/p_{n1}$). This rise induces intertemporal substitution of spending towards the present and raises the consumption ratio $c_{x0}/c_{x1}$. Further, a rise in the price of non-tradable goods induces within each period substitution in consumption towards tradable goods. Since the rise in the discount factor raises $p_{n0}/p_{n1}$, the temporal substitution in consumption is stronger in the current period and, therefore, also operates to raise the ratio $c_{x0}/c_{x1}$.

Equations (I.13) and (I.17) summarize the results of the first stage of the analysis. They provide the ingredients for the second stage in which we determine the equilibrium relation between the world rate of interest and the level of government spending. In order to determine the equilibrium in the world economy we need to consider the factors governing world demand and world supply of tradable goods. The foreign economy is assumed to be characterized by a structure of demand and supply similar to that of the domestic economy. Thus, the relative world supply of tradable goods net of government purchases, z, is

$$z = \frac{(\bar{Y}_{x0}-G_{x0}) + (\bar{Y}^{*}_{x0}-G^{*}_{x0})}{(\bar{Y}_{x1}-G_{x1}) + (\bar{Y}^{*}_{x1}-G^{*}_{x1})} \tag{I.19}$$

The analysis of the equilibrium in the world economy is carried out with the aid of Figure 1. Panel I of Figure 1 shows the <u>relative</u> intertemporal world supply, S, and the <u>relative</u> intertemproal domestic, D, foreign, $D^{*}$, and world, $D^{w}$, demands for tradable goods. The world relative demand is a weighted average of the domestic and foreign relative demands. That is,

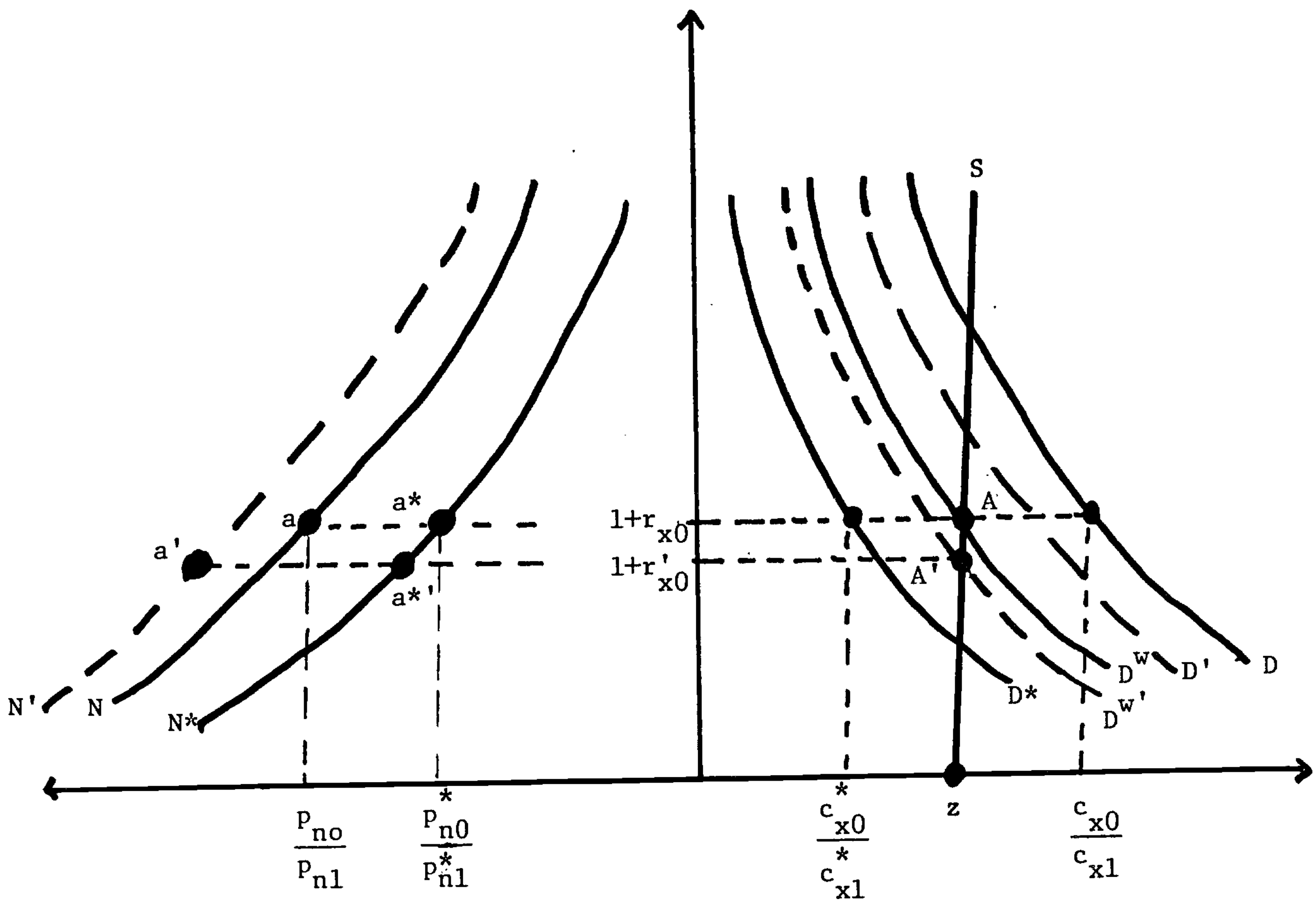


Figure 1: The Effects of Government Spending on the World Rate of Interest and on the Paths of the Real Exchange Rates.

Data: $\sigma > \sigma_{nx}$ , $\dfrac{1-\gamma^g_s}{1-\gamma_s} > \dfrac{\gamma^g_s}{\gamma_s}$ , $\beta^g_n = 1$

$D^{w} = \dfrac{c_{x0}+c^{*}_{x0}}{c_{x1}+c^{*}_{x1}} = \mu \dfrac{c_{x0}}{c_{x1}} + (1-\mu) \dfrac{c^{*}_{x0}}{c^{*}_{x1}}$ where $\mu = c_{x1}/(c_{x1}+c^{*}_{x1})$ . The relative demand schedules relate the desired consumption ratio of tradable goods to the rate of interest. Their slope reflects the negative relation embodied in equation (I.17). These demand schedules are drawn for a given level of government spending. The relative supply schedule is drawn with a zero interest elasticity since we abstract from investment. This schedule is also drawn for a given level of government spending.

The schedules $N$ and $N^{*}$ in Panel II of Figure 1 show the relation between the world rate of interest and the internal relative price structure (the path of the real exchange rate) in each country. The negative slope of these schedules reflects the relation embodied in equation (I.16).

The initial equilibrium is described in Panel I by point A in which the world rate of interest is $r_{x0}$. The periodic percentage changes of the domestic and the foreign real exchange rates associated with the initial equilibrium are shown in Panel II by points a and $a^{*}$.

Consider the effects of a rise in the level of the domestic government spending. This change alters the domestic relative demand (and, thereby, the world relative demand), as well as the world relative supply. As shown in equation (I.13) the direction of the change in the relative demand schedules depend on the government-induced bias in the intertemporal net supply of non-tradable goods and on the bias in the temporal-intertemporal substitution in private-sector demand. In order to determine the direction of the change in the relative supply schedule we differentiate equation (I.19). Accordingly,

$$\text{(I.20)} \qquad \frac{d \log z}{dG} = \lambda^{g}_{x1}(1-\beta^{g}_{n1})\,\gamma^{g}_{s} - \lambda^{g}_{x0}(1-\beta^{g}_{n0})(1-\gamma^{g}_{s})$$

where $\lambda^g_{xt}$ denotes the reciprocal of the world output of tradable goods net of government purchases of these goods in period t (t=0,1). Thus, $\lambda^g_{xt} = 1/(\bar{Y}_{xt} - G_{xt} + \bar{Y}^*_{xt} - G^*_{xt})$. Equation (I.20) indicates that the direction of the change in the relative supply reflects the bias in the intertemporal allocation of government spending on tradable goods.

Since the direction of the shift of the world relative demand and supply depends on the assumed magnitude of the various parameters, we cannot determine on a-priori grounds whether the rise in government spending raises or lowers the equilibrium world rate of interest. Similar considerations apply to the effects of government spending on the paths of the domestic and foreign real exchange rates. As indicated by equation (I.9), depending on the temporal and intertemporal pattern of government spending, the rise in government spending may induce a rightwards or leftwards shift of the N schedule in Panel II of Figure 1.

In order to illustrate the working of the model we consider in Figure 1 the effects of government spending for a benchmark case in which the intertemporal elasticity of substitution, $\sigma$, exceeds the temporal elasticity, $\sigma_{nx}$, the ratio of the shares of government spending to private spending in the current period, $(1-\gamma^g_s)/(1-\gamma_s)$ exceeds the corresponding ratio of future spending, $\gamma^g_s/\gamma_s$, and government spending falls entirely on non-tradable goods (so that $\beta^g_n = 1$). As indicated by equation (I.13), in this benchmark case, the domestic and, thereby, the world relative demand schedules shift leftwards from the position indicated by D and $D^w$ to the position indicated by D′ and $D^{w\prime}$, respectively. Further, as indicated by equation (I.20) with $\beta^g_n = 1$ the relative supply of world tradable goods does not change. It follows that in this case the equilibrium point shifts from point A to point A′ in Panel I of Figure 1, and the world rate of interest falls from $r_{x0}$ to $r'_{x0}$.

In Panel II of Figure 1 we show the effects of the rise in government spending on the paths of the domestic and foreign real exchange rates. As indicated by equation (I.9), in this benchmark case, the N schedule shifts leftwards and, given the new lower world rate of interest, the domestic and foreign equilibrium points shift from a and $a^*$ to $a'$ and $a^{*\prime}$, respectively. Accordingly the percentage change (per unit of time) of the real exchange rates increases in both countries. In concluding the presentation of this benchmark case we note that since the world rate of interest (measured in terms of tradable goods) falls and since in both countries the time-paths of the real exchange rates steepen, it follows that in both countries the consumption-based real rates of interest fall (even though, in general, the magnitude of this decline need not be the same for both countries).

It is important to note that we chose this specific benchmark case in which the rise in government spending lowers the world rate of interest in order to highlight the implications of government spending on non-tradable goods. In fact, if government spending falls entirely on tradable goods, so that $\beta_n^g = 0$, then the rise in spending does not alter the relative demand schedules in Figure 1 (as seen from equation (I.13) with $\beta_n^g = 0$) but it induces a leftwards shift of the relative supply schedule (as seen from equation (I.20) for the case $\beta_n^g = 0$ and $\left[(1-\gamma_s^g)/(1-\gamma_s)\right] > (\gamma_s^g/\gamma_s)$. Thus, under such circumstances, the rise in government spending raises the equilibrium rate of interest.

The more general configurations of the effects of government spending on the world rate of interest, as implied by equations (I.13) and (I.19), are summarized in Table 1. The Table demonstrates that if the commodity composition of government spending is strongly biased towards goods that are internationally tradable (so that $\beta_n^g$ is small), then the key factor determining the direction of the change in the world rate of interest is the intertemporal allocation of

TABLE 1

THE EFFECTS OF A RISE IN GOVERNMENT SPENDING ON THE WORLD RATE OF INTEREST

| Relation Between Temporal and Intertemporal Elasticities if Substitution | Intertemporal and Temporal Allocations of Government Spending | | | |
|---|---|---|---|---|
| | $(1-\gamma_s^g)/(1-\gamma_s) > \gamma_s^g/\gamma_s$ | | $(1-\gamma_s^g)/(1-\gamma_s) < \gamma_s^g/\gamma_s$ | |
| | $\beta_n^g=0$ | $\beta_n^g=1$ | $\beta_n^g=0$ | $\beta_n^g=1$ |
| $\sigma_{nx} > \sigma$ | + | + | - | - |
| $\sigma_{nx} = \sigma$ | + | 0 | - | 0 |
| $\sigma_{nx} < \sigma$ | + | - | - | + |

Note: The world rate of interest is measured in terms of internationally tradable goods.

government and private-sector spending. If government spending is biased towards the current period relative to private-sector spending, so that $(1-\gamma_s^g)/(1-\gamma_s)$ exceeds $\gamma_s^g/\gamma_s$, then the world rate of interest rises and vice versa. On the other hand if the commodity composition of government spending is strongly biased towards non-tradable goods (so that $\beta_n^g$ is close to unity), then the direction of the change in the interest rate depends on the interaction between the intertemporal allocation of government spending relative to the private sector and the difference between the temporal and the intertemporal elasticities of substitution of the domestic private-sector. In fact, since in this case the effects of government spending operate only through changes in the relative demand schedules, the rate of interest rises if $(\sigma_{nx}-\sigma)\left[(1-\gamma_s^g)/(1-\gamma_s) - \gamma_s^g/\gamma_s\right]$ is positive, and vice versa.

The various possibilities concerning the relative magnitudes of the key parameters also imply that the effects of government spending on the time-path of the domestic and foreign real exchange rates are not clear cut. The possible outcomes are summarized in Table 2. The results in the Table show that if the commodity composition of government spending is strongly biased towards internationally tradable goods (so that $\beta_n^g$ is about zero) then, as implied by equation (I.9), the change in government spending does not displace the N schedule in Panel II of Figure 1. Therefore, the induced change in the path of the domestic real exchange rate mirrors only the change in the rate of interest since it involves a movement along the given N schedule. It follows that, with $\beta_n^g$ small, the change in the domestic time-path of the real exchange rate is inversely related to the change in the world rate of interest. This inverse relation is verified from a comparison between the entries appearing in Tables 1 and 2 in the columns corresponding to the case of $\beta_n^g=0$.

TABLE 2

THE EFFECTS OF A RISE IN GOVERNMENT SPENDING ON THE PATHS OF DOMESTIC AND FOREIGN REAL EXCHANGE RATES

| Relation Between Temporal and Intertemporal Elasticities of Substitution | The Real Exchange Rate in the | Intertemporal and Temporal Allocations of Government Spending: $\frac{1-\gamma_s^g}{1-\gamma_s} > \frac{\gamma_s^g}{\gamma_s}$, $\beta_n^g=0$ | $\frac{1-\gamma_s^g}{1-\gamma_s} > \frac{\gamma_s^g}{\gamma_s}$, $\beta_n^g=1$ | $\frac{1-\gamma_s^g}{1-\gamma_s} < \frac{\gamma_s^g}{\gamma_s}$, $\beta_n^g=0$ | $\frac{1-\gamma_s^g}{1-\gamma_s} < \frac{\gamma_s^g}{\gamma_s}$, $\beta_n^g=1$ |
|---|---|---|---|---|---|
| $\sigma_{nx} > \sigma$ | Domestic Economy | - | ? | + | ? |
| | Foreign Economy | - | - | + | + |
| $\sigma_{nx} = \sigma$ | Domestic Economy | - | + | + | - |
| | Foreign Economy | - | 0 | + | 0 |
| $\sigma_{nx} < \sigma$ | Domestic Economy | - | + | + | - |
| | Foreign Economy | - | + | + | - |

Note: The paths of the real exchange rates are measured by $p_{n0}/p_{n1}$ and $p^*_{n0}/p^*_{n1}$.

In the other extreme case, in which government spending falls mainly on non-tradable goods (so that $\beta_n^g$ is close to unity) then, as long as the temporal elasticity of substitution, $\sigma_{nx}$, does not exceed the intertemporal elasticity of substitution, $\sigma$, the key factor determining whether the path of the real exchange rate steepens or flattens is the intertemporal allocation of government spending. If government spending is biased towards the current period relative to private-sector spending, so that $(1-\gamma_s^g)/(1-\gamma_s)$ exceeds $\gamma_s^g/\gamma_s$, then the rise in spending accelerates the time-path of the real exchange rate and vice versa. On the other hand if $\sigma_{nx}$ exceeds $\sigma$, then the time-path of the real exchange rate is influenced by two conflicting forces, the one operating through a movement along the N schedule (induced by the change in the rate of interest) and the other operating through a shift of the N schedule (induced by the direct effect of government spending on the relative supply of non-tradable goods).

Finally, we note that since the foreign schedule, $N^*$, is not affected by domestic government spending, the time-path of the foreign real exchange rate, $p_{n0}^*/p_{n1}^*$, is always related negatively to the world rate of interest. On the other hand, since the correlation between the time-path of the domestic real exchange rate and the world rate of interest may be positive, zero, or negative (as may be varified by comparing the results reported in Table 1 and 2) it follows that the cross-country correlations between the paths of the real exchange rates and between the (consumption-based) real rates of interest may also be negative, zero, or positive. The analysis underlying Tables 1 and 2 identifies the main factors governing the signs of the various cross-country correlations.

## II: Budget Deficits With Distortionary Tax Policies

In the previous part of this paper we analysed the effects of government spending under the assumption that taxes are non-distortionary. We turn next to examine the effects of changes in taxes under the assumption that government spending remains intact. Thus, we analyse the effects of a budget deficit arising from a change in the time-profile of taxes. Such a tax policy influences the effective (tax adjusted) rate of interest and impacts on private-sector behavior. The altered behavior influences the equilibrium of the world and, thereby, transmits the effects of the domestic tax policy to the rest of the world. In this part of the paper we analyse these mechanisms using an extension of the formulation outlined in section I.1.

Since the key mechanism through which the tax policy influences private-sector behavior operates through alterations of the effective (tax adjusted) discount factor, we start by incorporating taxes into the definition of the discount factor. For this purpose we illustrate the mechanism by focusing on a specific tax: the value added tax system (VAT) under which export is exempt. Evidently, this tax is equivalent to a consumption tax. In the presence of such a tax the effective (tax adjusted) discount factor is denoted by $\alpha_{x\tau 1}$ which is related to the undistorted world discount factor, $\alpha_{x1}$. according to

$$\alpha_{x\tau 1} = \frac{1+\tau_1}{1+\tau_0}\alpha_{x1}$$

where $\tau_t$ is the ad-valorem consumption tax rate in period t (t=0,1). Correspondingly the effective (consumption-based) real discount factor is denoted by $\alpha_{c\tau 1}$ where

$$\alpha_{c\tau 1} = \alpha_{x\tau 1} \frac{P_1}{P_0} \quad .$$

With such taxes, private-sector demands depend on $\alpha_{c\tau 1}$ rather than on $\alpha_{c1}$ and, therefore, changes in the time profile of taxes alter private-sector behavior. To simplify we assume that foreign-government spending and taxes are zero.

A budget deficit arising from a current-period tax cut (a reduction in $\tau_0$) must be accompanied by a corresponding rise in future taxes (a rise in $\tau_1$) so as to maintain government solvency as long as government spending policies remain intact. The effects of such a change in the time profile of taxes are analysed with the aid of Figure 2. The initial equilibrium is described in Panel I by point A in which the world rate of interest is $r_{x0}$. For convenience of exposition we assume that in the initial situation the time-profile of taxes is "flat" (that is $\tau_0 = \tau_1$), so that initially, the domestic and foreign rates of interest (in terms of tradable goods) are equal to each other. The time-paths of the domestic and foreign real exchange rates associated with the initial equilibrium is indicated in Panel II by points a and $a^*$ along the N and the $N^*$ schedules. Thus, the initial equilibrium is identical to the one portrayed in Figure 1.

Consider the effects of a budget deficit arising from a tax cut. Given the initial value of the world rate of interest, $r_{x0}$, the reduction in $\tau_0$ and the increase in $\tau_1$ (implied by the government budget constraint) raise the domestic effective discount factor, $\alpha_{x\tau 1}$, and induce an upwards displacement of the domestic relative demand schedule from D to D′. The proportional vertical displacement of the schedule equals the proportional change in the effective discount factor. This displacement is necessary in order to offset the effect of the tax-induced reduction in the effective rate of interest on the desired

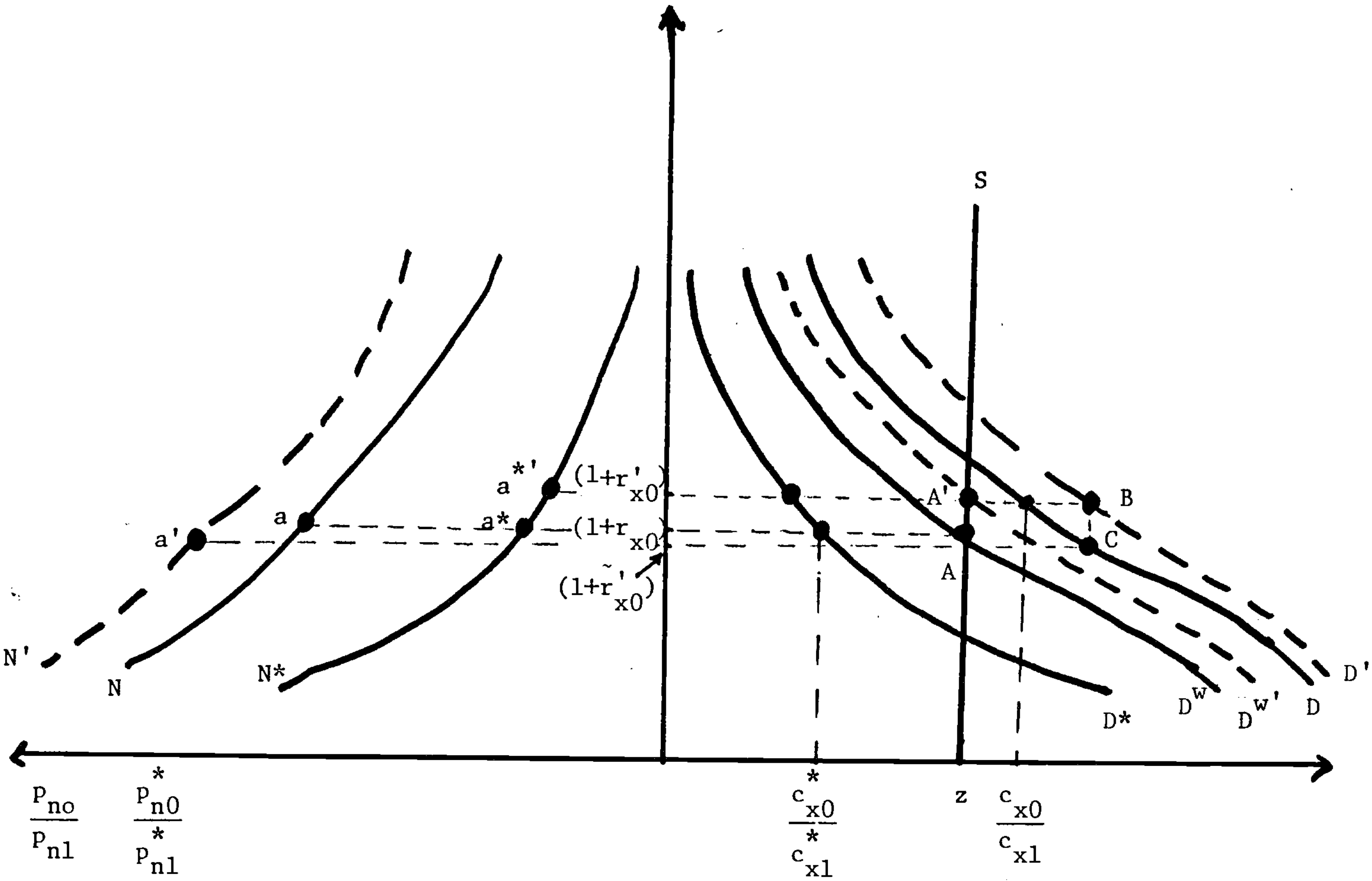


Figure 2: The Effects of a Budget Deficit Arising From a Cut in a Value Added Tax on the World Rate of Interest and on the Paths of the Real Exchange Rates.

domestic consumption ratio. Corresponding to the new domestic schedule D', the world relative-demand schedule shifts from $D^{w}$ to $D^{w'}$. The new equilibrium is described by point A' in Panel I of Figure 2. Hence the world rate of interest rises from $r_{x0}$ to $r'_{x0}$. The proportional vertical displacement of the world relative-demand schedule, $D^{w}$, (indicated by the distance AA'), is smaller than the corresponding displacement of the domestic schedule, D (indicated by the distance BC) since the world schedule is a weighted average of the domestic and the given foreign schedules. It follows that the domestic effective rate of interest must fall from $r_{x0}$ to a lower level such as $\tilde{r}'_{x0}$.

The change in the time-profile of taxes which (for any given level of the world rate of interest) raises the effective discount factor, also alters the position of the domestic schedule N in Panel II of Figure 2. In analogy to the previous analysis of the displacement of the relative demand schedule, the proportional vertical displacement of the N schedule equals the percentage change in the effective discount factor. As indicated by equation (I.16), this displacement is necessary in order to offset the effects of the tax-induced reduction in the effective rate of interest on the time-path of the domestic real exchange rate. Hence, given the new domestic effective rate of interest $\tilde{r}'_{x0}$, the rate of increase of the domestic real exchange rate from period zero to period one accelerates (as $p_{n0}/p_{n1}$ rises). Likewise, given the new world rate of interest, $r'_{x0}$, the rate of increase of the foreign real exchange rate decelarates (as $p^{*}_{n0}/p^{*}_{n1}$ falls). These changes are indicated in Panel II of Figure 2 by the displacement of the equilibrium points $a^{*}$ and a to $a^{*\prime}$ and a' , respectively.

The foregoing analysis implies that the budget deficit arising from the cut in taxes raises the world rate of interest and lowers the domestic effective rate of interest (both measured in terms of tradable goods). Further, the deficit raises the periodic percentage change of the domestic real exchange rate and lowers the corresponding foreign percentage change. These changes in the time-paths of relative prices imply from equation (I.18) that the foreign (consumption-based) real rate of interest rises and that the domestic (consumption-based) effective real rate of interest falls. Further, the magnitudes of the changes in the rates of interest are smaller if the rates of interest are measured in terms of the consumption baskets than if they are measured in terms of internationally tradable goods (the absolute difference between the two magnitudes rises with the ratio of the temporal to the intertemporal elasticities of substitution, $\sigma_{nx}/\sigma$). We conclude that the budget deficit results in a negative cross-country correlation between changes in the domestic and the foreign (consumption-based) real rates of interest.

## III: Budget Deficits with Non-Distortionary Tax Policies

In this part of the paper we analyse the effects of budget deficits for situations in which taxes are non-distortionary. In contrast with the analysis contained in the previous part in which budget deficits influence the economy primarily through temporal and intertemporal substitution effects induced by the distortions, here the main mechanism operates through wealth effects.

To generate the wealth effects of budget deficits we allow for differences between the time horizons relevant for individual decision making and for the society at large. These differences result in discrepancies between the private and public-sectors' cost of borrowing and, thereby, render importance to the timing of taxes. Thus, we introduce a "myopic" element as in Blanchard (1985).[4] Accordingly, there are overlapping generations of rational individuals but due to mortality each individual has a finite horizon. The coefficient of "myopia" reflects the finiteness of the horizon. Suppose that $\gamma$

is the probability that an individual survives from one period to the next and let $\gamma < 1$. The magnitude of $\gamma$ influences savings in two ways. First, it introduces a risk premium which raises the rate of interest applicable to individuals above the world rate. Hence, if the world rate of interest measured in terms of tradable goods is $r_x = R-1$, then the effective (risk adjusted) rate of interest is $(R-\gamma)/\gamma$. This impacts on wealth through the heavier discounting of future disposable incomes. Second, it lowers the effective saving propensity. For example, assuming a logarithmic utility function and denoting the (constant) subjective discount factor by $\delta$, it can be shown that the saving propensity in the absence of mortality risk is $\delta$ which is reduced to $\gamma\delta$ in the presence of such a risk.

The subsequent analysis is divided into two sections. In the first we examine the small-country case and focus on the dynamics of the real exchange rate. In the second we examine the two-country case with special emphasis on the consequences of budget deficits on the cross-country comovements of the real exchange rates.

## III.1: The Small Country Case

Consider a small country facing a constant world rate of interest, $r_x$. The assumption that the utility function is logarithmic implies that aggregate private-sector spending in period t, $Z_t$, is related to private-sector wealth, $W_t$, according to

$$Z_t = (1-\gamma\delta)W_t \tag{III.1}$$

where $\gamma\delta$ denotes the constant effective saving propensity, and where spending and wealth are measured in terms of tradable goods. The value of wealth can be expressed as

$$\text{(III.2)} \qquad W_t = \frac{R}{R-\gamma}\,\overline{Y}_x + \sum_{v=0}^{\infty}\left(\frac{\gamma}{R}\right)^v p_{n,t+v}\,\overline{Y}_n - \sum_{v=0}^{\infty}\left(\frac{\gamma}{R}\right)^v T_{t+v} - RB^P_{t-1} \quad .$$

Equation (III.2) defines wealth as the discounted sum of disposable income net of debt commitment. As seen, GDP in period t (measured in terms of tradable goods) is $\overline{Y}_x + p_{nt}\overline{Y}_n$ where $\overline{Y}_x$ and $\overline{Y}_n$ are respectively, the fixed levels of production of tradable and non-tradable goods, and $p_{nt}$ is the relative price of non-tradable goods--the inverse of the real exchange rate. Disposable income is GDP minus taxes and the discounted sum of disposable income is obtained by employing the effective discount factor $\gamma/R$ (where $R = 1+r_x$).

Total private-sector spending is composed of purchases of tradable and non-tradable goods. Thus, $Z_t = C_{xt} + p_{nt}C_{nt}$ where $C_{xt}$ and $C_{nt}$ denote, respectively, consumption of tradable and non-tradable goods. The assumed form of the utility function implies that these quantities are proportional to total spending according to

$$\text{(III.3)} \qquad C_{xt} = (1-\beta_n)Z_t$$

$$\text{(III.4)} \qquad p_{nt}C_{nt} = \beta_n Z_t$$

where as before $\beta_n$ denotes the expenditure share of non-tradable goods.

Equilibrium in the market for non-tradable goods requires that in each period, t, private-sector demand for non-tradable goods equals the value of output net of government purchases of these goods, $G_{nt}$. Thus, using equations (III.1) and (III.4) the equilibrium condition is

(III.5) $$(1-\gamma\delta)\beta_n W_t = p_{nt}(\bar{Y}_{nt} - G_{nt}) \quad .$$

Private-sector debt in each period reflects the debt commitment in the recent past plus the flow of new debt arising from the excess of spending over disposable income. Accordingly, using equations (III.1) and (III.5), the dynamic evolution of private-sector debt implies that

(III.6) $$B_t^p = RB_{t-1}^p + \frac{1}{\beta_n} p_{nt} (\bar{Y}_n - G_{nt}) - (\bar{Y}_x + p_{nt} \bar{Y}_n - T_t) \quad .$$

For given paths of government spending and taxes, the system of equations (III.2), (III.5) and (III.6) supplemented by the solvency condition according to which at the limit all debt is repaid, can be solved to yield the equilibrium paths of the real exchange rate and of private-sector debt. Once these paths are known the rest of the model can be solved to yield the equilibrium paths of spending on tradable and non-tradable goods as well as for the evolution of the current account of the balance of payments.

It can be shown that the equilibrium paths of the price of one-tradable goods (the inverse of the real exchange rate) and of private-sector debt are

(III.7) $$p_{nt} = \bar{p}_n + \tilde{\nu}\,(B_{-1}^p - B^p)\lambda_1^t$$

and

(III.8) $$B_t^p = B^p + (B_{-1}^p - B^p)\lambda_1^{t+1}$$

where $\lambda_1$ is a positive fraction and $\tilde{\nu}$ is a negative expression. The magnitudes of $\lambda_1$, $\tilde{\nu}$, $\bar{p}_n$ and $B^p$ depend on the parameters of the model.[5] To facilitate the subsequent analysis of budget deficits we assume that the conditions necessary for the existence of steady-state equilibrium are satisfied. In that case $\bar{p}_n$ is positive and the sign of $B^p$ depends on whether $\delta R$ exceeds or falls short of unity. Suppose that $\delta R$ falls short of unity. In that case the economy is a net debtor in the steady state. If the initial debt $B^p_{-1}$ is smaller than the steady-state debt then, as indicated by equation (III.8), the economy keeps on borrowing along the path to the steady state until its net debtor position reaches $B^p$. Along this path, as indicated by equation (III.7), the relative price of non-tradable goods falls monotonically until it reaches the steady-state level, $\bar{p}_n$. The monotonic rise in the economy's net-debtor position lowers wealth and reduces spending. As a result along the path, the current account of the balance of payments improves. The reduced demand for non-tradable goods consequent on the reduced wealth reflects itself in a downwards trend of the relative price of these goods, that is, a rise in the real exchange rate.

The foregoing analysis concludes the specification of the dynamics of the key economic variables in the small open economy. We turn next to analyse the dynamic response of these variables to a budget deficit resulting from an unanticipated change in the time-profile of taxes. Consider an initial steady state disturbed by a tax cut, occuring in period zero, that is accompanied by a corresponding tax hike in period one, and suppose that in all other future periods taxes remain at their initial level of zero. Following the usual mechanism, the current-period budget deficit raises the existing population's wealth and stimulates spending. This raises the relative price of non-tradable goods. On the other hand, once the rise in period-one taxes occurs, the wealth of the population existing in period one falls. The level of wealth in period one is lower since the segment of the population that enjoyed the initial tax cut raised its spending in period zero and, thereby, consumed part of its wealth while the newly-born segment of the population inherited a larger tax liability.

The reduction in period-one wealth induces a fall in spending and lowers the relative price of non-tradable goods. Finally, throughout the periods subsequent to period one, the relative weight of these two segments in the total population declines due to mortality while at the same time the relative weight of the new born not subject to the higher taxes of period one rises. The changes in the composition of the population occuring with the passage of time are associated with an upwards trend in aggregate wealth, spending and the relative price of non-tradable goods. Ultimately, since the relative weight of those subject to the change in the tax profile approaches zero, the initial steady-state equilibrium is restored.

The pattern of response discussed above is illustrated in Figure 3 which shows the dynamic effects that an unanticipated fall in taxes in period zero accompanied by a fully anticipated corresponding rise in taxes in period one exert on the relative price of non-tradable goods and on consumption. We also note that since total spending is related positively to the relative price of non-tradable goods, the general pattern exhibitied by the path of $p_n$ also applies to the path of spending in Figure 3.

In Figure 4 we show the path of the economy's external debt. Since by assumption government debt in the initial equilibrium was zero, and since the tax profile ensures that from period two onward government debt is also zero, it follows that during these periods private-sector debt, $B_t^p$ , coincides with the economy's external debt, $B_t$. In this context we note that since the steady-state value of the economy's external debt remains intact, the discounted sum of current and prospective trade surpluses (adjusted for the initial debt commitment)--all evaluated at the long run--also remains unchanged. Further, since the levels of output are fixed it follows that the discounted sum of current and future consumption--evaluated at the same long run--also remains intact. It

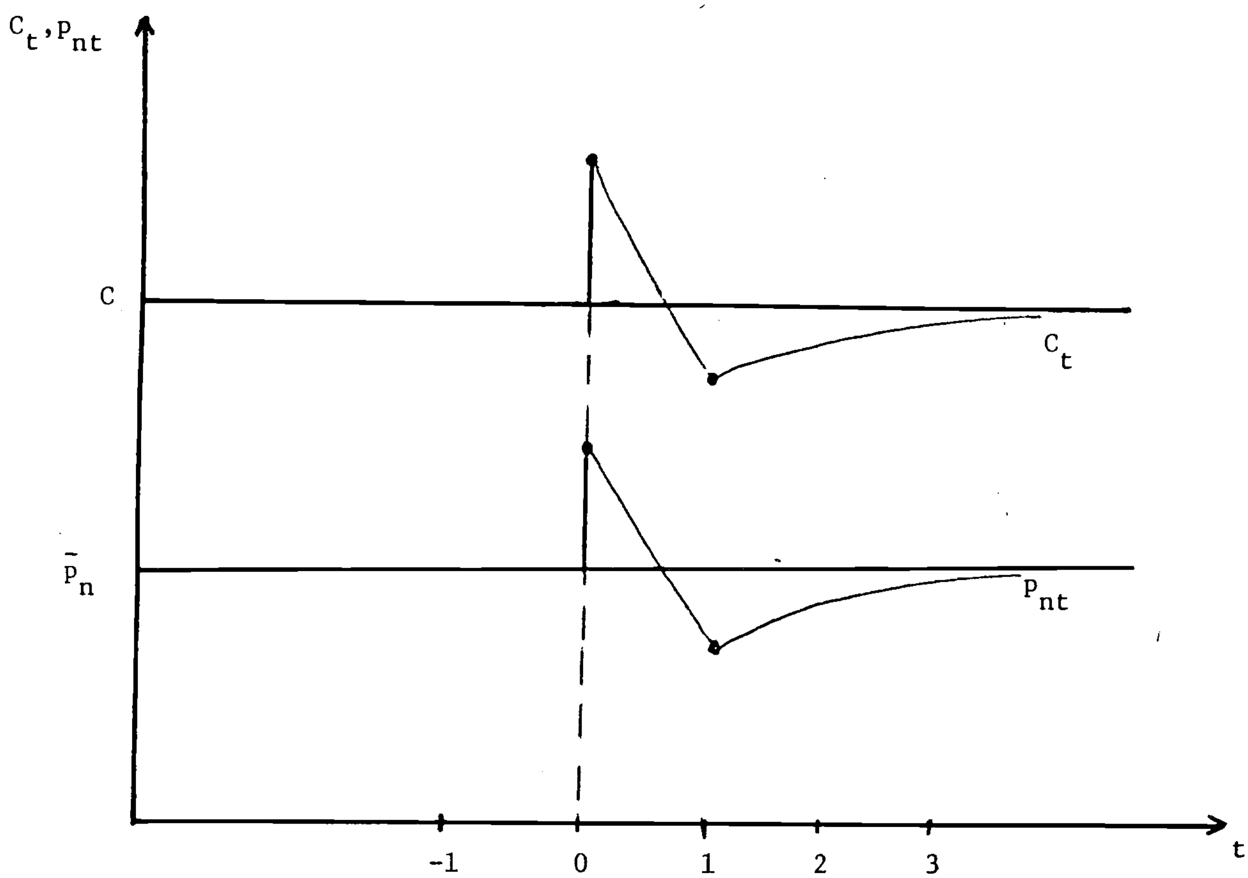


Figure 3: The Effects of an Unanticipated Government-Budget Deficit on Consumption and on the Relative Price of Non-tradable Goods.

follows that if we truncate the path of consumption in Figure 3 at any arbitrary period short of the steady state, the discounted sum of the post-deficit consumption exceeds the corresponding quantity obtained along the benchmark path with no taxes.

Having analysed the path of debt during period two onward, we return to elaborate on the evolution of the economy's external debt during periods zero and one--the periods during which the tax policy is in effect. In period zero, the rise in total spending consequent on the budget deficit must be financed through external borrowing as reflected in the increased size of the economy's external debt. This change in the economy's net debtor position reflects primarily the rise in government debt necessary to finance the budget deficit. In fact private-sector debt in period zero falls (since only a fraction of the rise in wealth is spent). The rise in government debt exceeds the reduction in private-sector debt and, therefore, the economy's external debt in period zero rises.

In period one the economy's external debt can be written as

$$\text{(III.9)} \qquad \acute{B}_1 = R^2 B_{-1} + (1-\beta_n)(RZ_0 + Z_1) - (R\bar{Y}_x + \bar{Y}_x) \quad .$$

Equation (III.9) expresses the value of external debt in period one as the present value of the initial debt commitment plus the excess of consumption of tradable goods over the production these goods, all evaluated in present value as of period one. Our previous arguments imply that the post-deficit external debt in period one (a period that falls short of the steady state) exceeds the corresponding value along the benchmark path with no taxes since the value of $RZ_0 + Z_1$ along the post-deficit path exceeds the corresponding quantity along

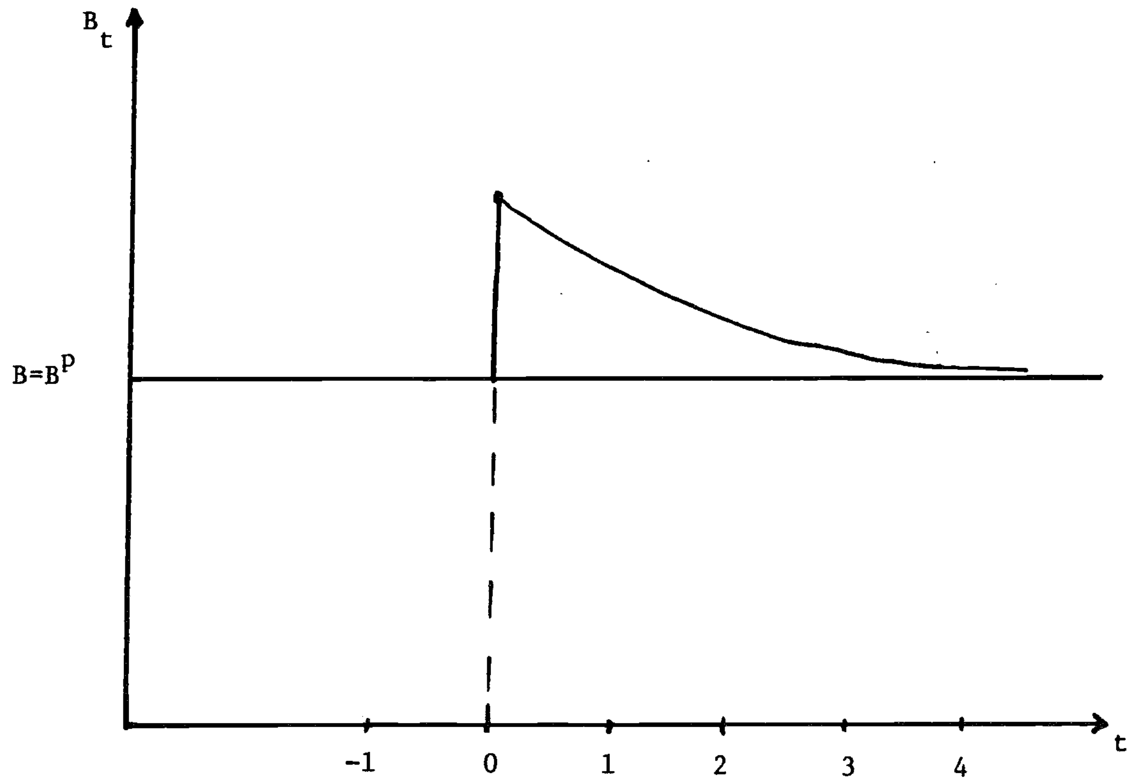


Figure 4: The Dynamic Effects of an Unanticipated Government-Budget Deficit on the Economy's External Debt.

the benchmark path with no taxes. This completes our analysis of the dynamic effects of budget deficits as they operate in the small open economy.

## III.2 <u>The Two-Country World</u>

In this section we extend the analysis to the two-country case. Such an extension illuminates the nature of the international transmission mechanism of tax policies. In our analysis we continue to assume that all taxes are non distortionary. As a result the mechanism responsible for the real consequences of budget deficits operates through the wealth effects.

To simplify the exposition we divide the horizon into two: the current period and the furure period. All quantities pertaining to the current period are indicated by a zero subscript and the paths of the exogenous variables are assumed stationary across future periods.[6]

Equilibrium necessitates that in the current period world output of tradable goods is demanded and the discounted sum of future outputs of tradable goods equals the discounted sums of furure domestic and foreign demands. Likewise, in each country current and future period outputs of non-tradable goods must be demanded. In what follows we outline the complete two-country model. The aggregate consumption functions at home and abroad are $Z_t = (1-\gamma\delta)W_t$ and $Z_t^* = (1-\gamma\delta^*)W_t^*$ where as before the saving propersities are $\gamma\delta$ and $\gamma\delta^*$ (where the survival probability, $\gamma$, is assumed to be equal across countries). Domestic and foreign wealth are defined as

$$(\text{III.10}) \qquad W_0 = (\bar{Y}_{x0} + p_{n0}\bar{Y}_{n0} - T_0) + \frac{\gamma}{R-\gamma}(\bar{Y}_x + p_n\bar{Y}_n - T) + (1+r_{x,-1})(B^g_{-1} - B_{-1})$$

and

$$(\text{III.11}) \qquad W_0^* = (\bar{Y}_{x0}^* + p_{n0}^*\bar{Y}_{n0}^* - T_0^*) + \frac{\gamma}{R-\gamma}(\bar{Y}_x^* + p_n^*\bar{Y}_n^* - T^*) + (1+r_{x,-1})(B^{*g}_{-1} + B_{-1}).$$

As seen, equations (III.10)-(III.11) express wealth as the sum of the present values of current and future disposable incomes plus net asset positions. Also in these equations the term $\gamma/(R-\gamma)$ denotes the present value of an annuity (commencing at period t=1) evaluated by using the discount factor relevant for private decision making, $\gamma/R$.

The market clearing conditions for the domestic non-tradable goods require that

$$\beta_n(1-\gamma\delta)W_0 = P_{n0}\left[\bar{Y}_{n0} - \beta_n^g(1-\gamma_s^g)G\right] \tag{III.12}$$

and

$$\beta_n\left[\gamma\delta W_0 + \frac{1-\gamma}{R-1}\,\frac{R}{R-\gamma}\,(\bar{Y}_x + P_n\bar{Y}_n - T)\right] = \frac{1}{R-1}\left[P_n\bar{Y}_n - \beta_n^g\gamma_s^g G\right] \tag{III.13}$$

where, as before, G denotes the discounted sum of government spending and where $\beta^g$ and $\gamma_s^g$ indicates the government's temporal and intertemporal spending patterns. Equation (III.12) specifies the equilibrium condition in the current-period market while equation (III.13) states that the discounted sum of domestic demand for future non-tradable goods equals the discounted sum of future supply net of government absorption.

Analogously, equations (III.14)-(III.15) describe the corresponding equilibrium conditions in the foreign markets for non-tradable goods.

$$\beta_n^*(1-\gamma\delta^*)W_0^* = P_{no}^*\ \bar{Y}_{n0}^* - \beta_n^{*g}(1-\gamma_s^{*g})G^* \tag{III.14}$$

and

$$\beta_n^*\left[\gamma\delta^* W_0^* + \frac{1-\gamma}{R-1}\,\frac{R}{R-\gamma}\,(\bar{Y}_x^* + P_n^*\bar{Y}_n^* - T^*)\right] = \frac{1}{R-1}\left[P_n^*\bar{Y}_n^* - \beta_n^{*g}\gamma_s^{*g}G\right] \tag{III.15}$$

Finally, the equilibrium conditions in the <u>world</u> market for tradable goods are specified in equations (III.16)-(III.17) where the first of the two pertains to the current period and the second pertains to the discounted sums of demand and supply in all future periods.

$$\text{(III.16)} \qquad (1-\beta_n)(1-\gamma\delta)W_0 + (1-\beta_n^*(1-\gamma\delta^*)W_0^* =$$

$$\bar{Y}_x - (1-\beta_n)(1-\gamma_s^g)G + \bar{Y}_x^* - (1-\beta_n^{*g})(1-\gamma_s^{*g})G^*$$

and

$$\text{(III.17)} \qquad (1-\beta_n)\left[\gamma\delta W_0 + \frac{1-\gamma}{R-1}\,\frac{R}{R-\gamma}\,(\bar{Y}_x + p_n\bar{Y}_n - T)\right]$$

$$+ (1-\beta_n^*)\left[\gamma\delta^* W_0^* + \frac{1-\gamma}{R-1}\,\frac{R}{R-\gamma}\,(\bar{Y}_x^* + p_n^*\bar{Y}_n^* - T^*)\right] =$$

$$\frac{1}{R-1}\left[\bar{Y}_x - (1-\beta_n^g)\gamma_s^g + \bar{Y}_x^* - (1-\beta_n^{*g})\gamma_s^{*g}G^*\right] \quad .$$

The system of equations (III.10)-(III.17) can be solved for the equilibrium values of the domestic and foreign current-period wealth, $W_0$ and $W_0^*$, current and future prices of non-tradable goods (the inverse of the corresponding real exchange rates), $p_{n0}$, $p_{n0}^*$, $p_n$, $p_n^*$, and for the world rate of interest, R-1. As usual, the eight-equations system (III.10)-(III.17) is linearly dependent and thus, by Walras's Law one of these equations can be left out. In what follows we leave out equation (III.10) specifying the equilibrium value of domestic wealth.

The system can be reduced to two basic equilibrium conditions. These conditions state that the world markets for tradable goods clear in both the current period as well as in the (consolidated) future period. These equations, derived explicitely in the Appendix, are reduced-form equations--they incorporate the requirement that in each country and in all periods the markets for non-tradable goods clear. Accordingly,

$$\text{(III.18)} \qquad (1-\beta_n)(1-\gamma\delta)W_0 + (1-\beta_n^*)(1-\gamma\delta^*)W_0^* = \bar{Y}_x + \bar{Y}_x^*$$

$$\text{(III.19)} \qquad (1-\beta_n)\left[\gamma\delta W_0 + \frac{(1-\gamma)R}{(R-1)(R-\gamma)} I\ (R,W_0,T) + \right.$$

$$\left.(1-\beta_n^*)\left[\gamma\delta^* W_0^*(R) + \frac{(1-\gamma)R}{(R-1)(R-\gamma)} I^*(R)\right] = \frac{1}{R-1}\ (\bar{Y}_x + \bar{Y}_x^*)\right.$$

Equation (III.18) states that the sum of world private demand for current tradable goods equals world supply. In this equation $(1-\beta_n)(1-\gamma\delta)W_0$ is the home country's private demand and $(1-\beta_n^*)(1-\gamma\delta^*)W_0^*$ is the corresponding foreign demand. The foreign wealth is expressed as a negative function of the rate of interest reflecting the role of the latter in discounting future incomes and in influencing the real exchange rate used to evaluate the income streams. It is noteworthy that this reduced-form functional dependence of wealth on the rate of interest is not shown explicitely for the domestic wealth since we have ommited the explicit domestic-wealth equation (III.10) by Walras's Law. This choice makes the equilibrium determination of domestic wealth (along with the world rate of interest) the focus of the subsequent analysis.

The second reduced-form equation (III.19) states that the discounted sum of domestic and foreign demands for future tradable goods equals the discounted sum of future world supply. The first term is the product of the consumption share of tradable goods $(1-\beta_n)$ and total domestic future consumption. The latter equals the sum of the savings of those alive in period zero, $\gamma\delta W_0$ , and the discounted sum of the demand for future goods of those who will be born in the future and whose disposable income in each period is I. This reduced-form future disposable income (in terms of tradable goods) is expressed as a negative function of future taxes, T, and a positive function of the future relative price of non-tradable goods. The latter, in turn, depends negatively on R (through its effect on future wealth of those yet unborn) and positively on $W_0$ (through its effect on the demand of those alive). An analogous interpretation applies to the foreign disposable income, $I^*$. The dependence of $I^*$ on R only reflects the assumption that foreign taxes are zero and incorporates the negative dependence of $W_0^*$ on R.

Equations (III.18)-(III.19) yield the equilibrium values of the home country's initial wealth, $W_0$, and the world rate of interest, $r_x = R-1$, for any given values of the parameters. In equilibrium the demand for non-tradable goods $\beta_n(1-\gamma\delta)W_0$ equals the value of the supply, $p_{n0}\bar{Y}_n$. Hence, the equilibrium price (the inverse of the real exchange rate) is

$$\text{(III.20)} \qquad p_{n0} = \frac{\beta_n(1-\gamma\delta)W_0}{\bar{Y}_n} \quad .$$

The equilibrium of the system is analysed by means of Figure 5. The PP schedule drawn in panel I of Figure 5 shows combinations of $r_x$ and $p_{n0}$ that clears the market for present tradable goods. It is positively sloped since a rise in the rate of interest lowers foreign demand (by lowering $W_0^*$) and a rise in $p_{n0}$ raises domestic demand (br raising $W_0$). Future tradable-goods market clears along the FF schedule. This schedule is negatively sloped since a rise in the rate of interest creates an excess demand for future tradable goods which must be offset by a fall in $W_0$ (and therefore $p_{n0}$).[7] Panel II of the figure shows the negative relation between the equilibrium rate of interest and the foreign relative price of non-tradable goods. This relation is based on equation (III.21) which is the foreign-country analogue to equation (III.20). Thus,

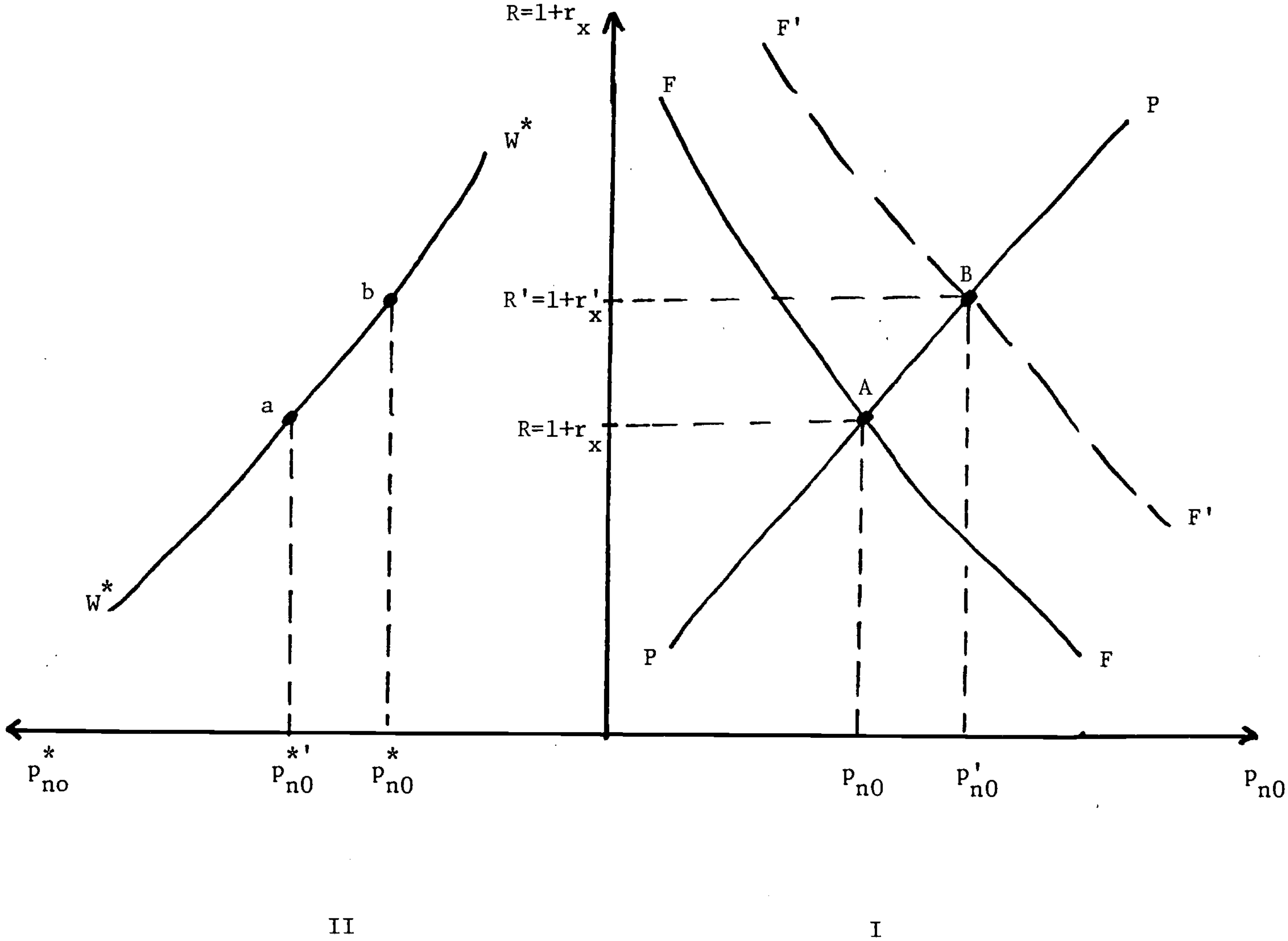


Figure 5: Budget Deficits, Real Exchange Rates and the Real Rate of Interest.

$$p_{n0}^{*} = \frac{\beta_{n}^{*}(1-\gamma\delta^{*})}{\bar{Y}_{n}} W_{0}^{*}(R) \qquad . \tag{III.21}$$

The equilibrium of the system is shown by point A in panel I and point a in panel II of Figure 5. Accordingly, the current equilibrium relative price of domestic tradable goods is $p_{n0}$, the foreign equilibrium relative price is $p_{n0}^{*}$, and the corresponding equilibrium rate of interest is $r_x$. In what follows we analyse the effects of a domestic budget deficit on the world rate of interest and on the equilibrium real exchange rates.

A domestic budget deficit arising from a current tax cut necessitates a corresponding rise in future taxes, T. As seen from equation (III.19) the rise in future taxes lowers future domestic disposable income, I, and thereby lowers the demand for future goods. For a given world rate of interest the fall in demand can be eliminated by a rise in $W_0$. As implied by equation (III.20) the rise in $W_0$ is associated with a rise in $p_{n0}$. Thus the FF schedule shifts to the right to F'F'. As is evident the horizontal shift of the FF schedule is proportional to $(1-\gamma)$; if $\gamma=1$ the position of the schedule as well as the characteristics of the initial equilibrium remain intact (the Ricardian equivalence case). In general, as indicated in panel I the new equilibrium obtains at point B with a <u>higher rate of interest, a higher domestic relative price of non-tradable goods, $p_{n0}$, and a higher level of domestic wealth and consumption</u>. <u>The new equilibrium is indicated in panel II by point b where it is seen that the higher rate of interest lowers foreign wealth and consumption and reduces the foreign relative price of non-tradable goods</u>. Thus, on the basis of the correlations between domestic and foreign private-sector spending and between domestic and foreign real exchange rates, <u>the international transmission of the budget deficit is negative</u>. As an interpretation we note that the wealth effects induced by the domestic budget deficit arises from the redistribution of income across generations. Since the deficit transfers income from future generations (whose propensity to consume present goods is zero) to the current generation (whose propensity to spend on present goods is positive) it creates an excess demand for present tradable goods resulting in a rise in their intertemporal relative price--the rate of interest. Likewise, it creates an excess demand for domestic non-tradable goods and an excess supply of foreign non-tradable goods. These excess demand and supply alter the temporal relative prices of these goods--the real exchange rates.

## IV: Summary

In this part of the paper we provide a detailed summary of the key results emerging from the analysis of the effects of government spending and tax policies on the equilibrium in the interdependent world economy. This summary focuses on the induced changes in the real exchange rates and in the rates of interest.

In analyzing the effects of government spending it was shown that these effects depend critically on two biases: the bias in the intertemporal allocation of government spending relative to the domestic private sector and the bias in the commodity composition of government purchases relative to the domestic private sector. If government spending is strongly biased towards purchases of tradable goods, then the key factor determining whether the world rate of interest rises or falls is the intertemporal pattern of government spending relative to the private sector: if the latter is biased towards current spending then the rate of interest rises, and vice versa. These adjustments in the rate of interest reflect the changes in the country's borrowing needs that arise from the intertemporal pattern of government spending.

The analysis also provides information about the time-paths of the domestic and foreign real exchange rates. If the relative share of government spending on tradable goods is high, then a rise in government spending decelerates the rate of change of the domestic and foreign real exchange rates if the intertemporal allocation of government spending (relative to the private sector) is biased towards the present. On the other hand if the intertemporal allocation of government spending (relative to the private sector) is biased towards the future, then the rates of change of the real exchange rates accelarate. It follows that in this case government spending induces positive cross-country correlations between the time-paths of the real exchange rates as well as between the (consumption-based) real rates of interest.

In contrast, if the commodity composition of government spending is strongly biased towards purchases of non-tradable goods, then the interest-rate effects depend on the interaction between the bias in the intertemporal allocation of government spending relative to the private sector and the temporal-intertemporal substitution bias of the domestic private-sector. Accordingly, in the absence of knowledge of the magnitudes of the saving propensities of the government and of the private sector, as well as on the private-sector temporal and intertemporal elasticities of substitution, there is no presumption as to whether a rise in government spending raises or lowers world rates of interest.

The lack of an a-priori presumption is even more pronounced when we consider the effects of government spending (that are strongly biased towards non-tradable goods) on the time-paths of the real exchange rates. We showed that the correlation between the domestic and foreign paths of the real exchange rates is positive if the intertemporal elasticity of substitution exceeds the temporal elasticity, and is zero if the temporal and intertemporal elasticities of substitution are equal to each other. This correlation may be positive, zero, or negative if the temporal elasticity of substitution exceeds the intertemporal elasticity. Since there is no presumption about the cross-country correlations between the paths of the real exchange rates, it follows that there is also no presumption about the cross-country correlation between the (consumption-based) real rates of interest.

The sharpest contrast between the implications of alternative biases in the commodity-composition of government spending arises in situations in which the intertemporal elasticity of substitution of the domestic private sector exceeds the temporal elasticity of substitution. In that case the direction of the effects of government spending on the world rate of interest and on the time-paths of the domestic and foreign real exchange rates is reversed as the commodity composition of government spending changes from one extreme to the other. For example, if the intertemporal allocation of government spending relative to the domestic private sector is biased towards the present, then a rise in government spending on tradable goods raises the world rate of interest, and decelerates the rates of change of the domestic and foreign real exchange rates. On the other hand, if the rise in government spending falls on non-tradable goods, then it lowers the world rate of interest and accelerates the rates of change of the domestic and foreign real exchange rates. These results demonstrate the important insights obtained from a disaggregation of the spectrum of commodities into those that are internationally tradable and those that are non-tradable. Indeed, such a disaggregation modifies some of the clear-cut results obtained in Frenkel and Razin (1985) in which all goods were internationally tradable.

It is important to emphasize that once tradable and non-tradable goods are present, then even though there is no a-priori presumption concerning the precise effects of government spending on the world rate of interest and on the time-paths of the real exchange rates, our analysis identified the key parameters whose relative magnitudes determine these effects.

In this context it is relevant to note that the analytical framework which allows for tradable and non-tradable goods, can be reinterpreted and applied to the analysis of the effects of fiscal policies on real wages in a model with variable labor supply. The reinterpretation of the model treats leisure as the non-tradable good, the real wage as the real exchange rate, and the temporal elasticity of substitution as the (compensated) elasticity of labor supply. With this interpretation, government hiring of labor is viewed as government purchases of non-tradable goods, and the relative share of government expenditure on non-tradable goods, $\beta_n^g$, corresponds to the relative share of wages in the government budget. Likewise, the private-sector expenditure share, $\beta_n$, is viewed as the relative share of leisure in private-sector total spending (inclusive of the imputed value of leisure). This interpretation suggests that the effects of government spending on the rate of interest and on the time-path of real wages depend critically on the relative importance of wages in the government budget. Our analysis implies that this dependence is especially pronounced in cases in which labor is inelastically supplied.

The second part of the paper was devoted to the analysis of distortionary tax policies. In this context we examined the effects of a budget deficit arising from a current cut in value-added taxes followed by a corresponding rise in future taxes. It was shown that the budget deficit raises the world rate of interest and lowers the domestic effective interest rate. In addition, the deficit accelerates the rate of change of the domestic real exchange rate and de- celerates the corresponding foreign rate of change. Thereby, the deficit lowers the domestic (consumption-based) effective real rate of interest, and raises the corresponding foreign real rate of interest. These changes result in a negative cross-country correlation between the (consumption-based) real effective rates of interest. It was also shown that with high intertemporal elasticities of substitution the budget deficit raises domestic spending and lowers the contemporaneous domestic real exchange rate; at the same time the deficit lowers foreign spending and raises the contemporaneous foreign real exchange rate. These effects result in negative cross-country correlations between spending as well as between real exchange rates.

In the third part of the paper we analyzed the consequences of budget deficits on the real exchange rate operating through the mechanism of the pure wealth effect. To focus on this mechanism rather than the mechanisms of substitution effects we assumed that all taxes are non-distortionary and that the temporal and intertemporal elasticities of substitution are unity. The wealth effects of budget deficits stem from the assumption that, due to mortality, the individual's horizon is finite.

Our analysis of the small-country case focused on the dynamic consequences of budget deficits. In general, alterations in the time profile of taxes induce modifications of the intertemporal patterns of private-sector spending which reflect themselves in the time-profile of the real exchange rate. Specifically, a budget deficit arising from a current tax cut that is accompanied by a subsequent tax hike raises current wealth, stimulates current consumption and lowers the relative price of tradable goods--the real exchange rate. The corresponding tax hike that follows exerts the opposite influence: it lowers private-sector spending and raises the real exchange rate.

The changes occurring during the period of the tax hike more than offset the changes occurring during the period of the tax cut. As a result, the stage is set for the dynamics of private-sector spending as well as of the real exchange rate during all subsequent periods. Accordingly during the periods following the tax hike the path of private-sector spending lies below the path obtained in the absence of the tax policy but, over time it converges back to the initial path. Correspondingly, throughout the period following the tax hike the path of the real exchange rate lies above the path obtained in the absence of the tax policy but it converges to it over time.

These dynamic effects of the budget deficit illustrate a general principle: the consequences of temporary tax policies stretch beyond the period during which the temporary policy is in effect. The counterpart to these dynamic implications is the rise in the economy's external debt induced by the budget deficit the service of which stretches into the indefinite future.

Our analysis of the two-country focused on the consequences of budget deficits on the domestic and the foreign real exchange rates. In this case the international transmission mechanism operates through the effects of the budget deficit on the world rate of interest. With non-distortionary taxes, the wealth effect induced by the budget deficit raises the domestic demand for present-period goods and thereby lowers the domestic real exchange rate and raises the world rate of interest. The rise in the world rate of interest transmits the effects of the domestic deficit to the rest of the world: it lowers foreign spending and raises the foreign real exchange rate. Accordingly the budget deficit induces negative cross-country correlations between private-sector spending and between the real exchange rates.

The analysis in Part II of the paper illustrated the mechanism of temporal-intertemporal substitution induced by distortionary finance of government spending by considering the implications of a specific tax system--the value-added tax system. With this illustration the qualitative effects of a budget deficit on the world rate of interest were similar to those obtained in Part III in which the key mechanism was that of the wealth effect: in both cases a current budget deficit raises the world rate of interest. It is important to emphasize, however, that this similarity is not general. Indeed, as shown in Frenkel-Razin (1986c) there is a class of distortionary taxes (such as taxes on labor income and on capital income) that yield opposite interest-rate effects.

We conclude by stating that one of the more general implications demonstrated by this paper is that a proper analysis of the effects of fiscal policies on the world economy must specify on the one hand the details of the commodity composition of government spending and its intertemporal allocations and, on the other hand, the details of the tax structure, including the timing of taxes and the types of taxes used to finance the budget.

# APPENDIX

## REAL EXCHANGE RATES IN A TWO-COUNTRY WORLD

In this Appendix we derive the reduced form equations (III.18)-(III.19) of the text. Throughout we omit equation (III.10) of the text by Walras's Law. Using equations (III.3) and (III.5) of the text and solving for the future values of production of non-tradable goods yields

$$(\text{A-1}) \qquad p_n \bar{Y}_n = A\left[(R-1)\gamma\delta W_0 + \frac{(1-\gamma)R}{R-\gamma}(\bar{Y}_x - T)\right]$$

$$(\text{A-2}) \qquad p_n^* \bar{Y}_n^* = A^*\left[(R-1)\gamma\delta^* W_0^* + \frac{(1-\gamma)R}{R-\gamma}(\bar{Y}_x^* - T^*)\right]$$

where $$\theta = \frac{\beta_n^g \gamma_s^g G}{p_n \bar{Y}_n} \quad , \quad \theta^* = \frac{\beta_n^{*g} \gamma_s^{*g} G^*}{p_n^* \bar{Y}_n^*}$$

$$A = \frac{\beta_n}{(1-\theta)-\beta_n(1-\gamma)R/(R-\gamma)} \quad , \quad A^* = \frac{\beta_n^*}{(1-\theta^*)-\beta_n^*(1-\gamma)R/(R-\gamma)}$$

The requirement that in equilibrium there is positive consumption of non-tradable goods which command a positive price impose the feasibility conditions according to which

$$(\text{A-3}) \qquad A \geq 0 \quad , \quad A^* \geq 0$$

Substituting (A-2) along with equation (III.14) of the text into equation (III.11) of the text yields

$$\text{(A-4)}\qquad W_0^* = D^*\left[\bar{Y}_x^* - T_0^* + \frac{\gamma}{R-\gamma}\left(1+A^*\frac{(1-\gamma)R}{R-\gamma}\right)(\bar{Y}_x^* - T^*) + (1+r_{x,-1})(B_{-1}^{*g} + B_{-1})\right]$$

where

$$D^* = (1-\theta^*)\left\{(1-\theta)-\beta_n^*\left[\frac{(R-1)\gamma^2\delta^*}{(1-\theta^*)(R-\gamma)-\beta_n^*(1-\gamma)R} + \frac{1-\gamma\delta^*}{1-\theta^*}\right]\right\}^{-1}$$

The requirement that in equilibrium there is positive wealth imposes the additional feasibility constraint according to which

$$\text{(A-5)}\qquad D^* \geq 0$$

Substituting equation (A-4) into equation (III.16) of the text yields

$$\text{(A-6)}\qquad (1-\beta_n)(1-\gamma\delta)W_0 + (1-\beta_n^*)(1-\gamma\delta^*)D^*\left[Y_x^* - T_0^* + \frac{\gamma}{R-\gamma}\left(1+A^*\frac{(1-\gamma)R}{R-\gamma}\right)(\bar{Y}_x^* - T^*) + (1+r_{x,-1})(B_{-1}^{*g} + B_{-1})\right]$$

$$= \bar{Y}_x - (1-\beta_n^g)(1-\gamma_s^g)G + \bar{Y}_x^* - (1-\beta_n^{*g})(1-\gamma_s^{*g})G^*$$

Substituting equations (A-1)-(A-2) for $p_n\bar{Y}_n$ and $p_n^*\bar{Y}_n^*$ into equation (III.17) of the text yields

$$(A\text{-}7) \qquad (1-\beta_n)\left[\gamma\delta W_0 + \frac{(1-\gamma)R}{(R-1)(R-\gamma)}\left\{A(R-1)\gamma\delta W_0 + (1 + A\,\frac{(1-\gamma)R}{R-\gamma})(\bar{Y}_x - T)\right\}\right]$$

$$+ (1-\beta_n^*)\left[\gamma\delta^* W_0^* + \frac{(1-\gamma)R}{(R-1)(R-\gamma)}\left\{A^*(R-1)\gamma\delta^* W_0^* + (1 + A^*\,\frac{(1-\gamma)R}{R-\gamma})(\bar{Y}_x^* - T^*)\right\}\right]$$

$$= \frac{1}{R-1}\left[\bar{Y}_x - (1-\beta_n^g)\gamma_s^g G + \bar{Y}_x^* - (1-\beta_n^{*g})\gamma_s^{*g} G^*\right] \quad .$$

The system of equations (A-4), (A-6) and (A-7) can be used to solve for the equilibrium values of $W_0$, $W_0^*$ and R.

To derive the more compact formulation of the reduced-form equilibrium conditions of the text we focus on the role of domestic tax policy by assuming that $G=G^*=T^*=0$. We first note that for a given value of the parameters the equilibrium value of foreign wealth shown in equation (A-4) can be expressed implicitely as

$$(A\text{-}8) \qquad W_0^* = W_0^*(R) \quad , \quad \partial W_0^*/\partial R < 0$$

Equation (A-8) expresses foreign current wealth as a negative function of the rate of interest. This reduced-form relationship incorporates the equilibrium conditions in the markets for current and future non-tradable goods. The negative dependence on the rate of interest reflects the role of the rate of interest in discounting future incomes and in influencing the real exchange rates used to evaluate the income streams. Next, we define the domestic and foreign reduced-form future disposable incomes

$$(A\text{-}9) \qquad I(R,\ W_0,\ T) = (R-1)A\gamma\delta W_0 + (1 + \frac{(1-\gamma)RA}{R-\gamma})(\bar{Y}_x - T)$$

and

$$(A\text{-}10) \qquad I^*(R) = (R-1)A\gamma\delta W_0^*(R) + (1 + \frac{(1-\gamma)RA^*}{R-\gamma})\ \bar{Y}_x^*$$

Equation (A-9) expresses disposable income (in terms of tradable goods) as a negative function of future taxes, T, and a positive function of the relative price of non-tradable goods, $p_n$. The latter in turn depends negatively on R (through its effect on future wealth of those yet unborn) and positively on $W_0$ (through its effect on the demand of those alive). An analogous interpretation applies to the foreign disposable income, $I^*$, where in (A-10) we incorporate the functional dependence of $W_0^*$ on R and the assumption that foreign taxes are zero.

Substituting equations (A-9)-(A-10) into (A-6)-(A-7) together with the assumption that $G=G^*=T^*=0$ yields

$$\text{(A-11)} \qquad (1-\beta_n)(1-\gamma\delta)W_0 + (1-\beta_n^*)(1-\gamma\delta^*)W_0^*(R) = \bar{Y}_x + \bar{Y}_x^*$$

$$\text{(A-12)} \qquad (1-\beta_n)\left[\gamma\delta W_0 + \frac{(1-\gamma)R}{(R-1)(R-\gamma)} I(R, W_0, T)\right] +$$

$$(1-\beta_n^*)\left[\gamma\delta^* W_0^*(R) + \frac{(1-\gamma)R}{(R-1)(R-\gamma)} I^*(R)\right] = \frac{1}{R-1}(\bar{Y}_x + \bar{Y}_x^*)$$

Equations (A-11)-(A-12) are the reduced-form equilibrium conditions (III.18)-(III.19) of the text. These equations underlie the diagrammatical analysis of the text.

## FOOTNOTES

[1]The analysis in this part of the paper supplements the two-country model in Frenkel and Razin (1985) in which all goods were internationally tradable. For a related analysis of fiscal policies and the real exchange rate within a small-country model see Buiter (1986).

[2]A similar procedure is developed in Svensson and Razin (1983).

[3]An Appendix containing the derivation of this and of some other formulae used in this paper can be obtained from the authors upon request.

[4]Blanchard's approach extends and applies Yaari's (1965) uncertain lifetime model to an overlapping generations macroeconomic model.

[5]This result is based on Helpman and Razin (1985).

[6]The procedure of time-aggregation is developed in Frenkel and Razin (1986a). The extension of the model incorporating non-tradable goods draws on Frenkel and Razin (1986b).

[7]A similar figure linking real interest rates to real exchange rates is used also by Branson (1986) and Dornbusch (1983).

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