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ABSTRACT

We ask two questions related to how access to credit affects the nature of business cycles. First, does the standard theory of unsecured credit account for the high volatility and procyclicality of credit and the high volatility and countercyclicality of bankruptcy filings found in U.S. data? Yes, it does, but only if we explicitly model recessions as displaying countercyclical earnings risk (i.e., rather than having all households fare slightly worse than normal during recessions, we ensure that more households than normal fare very poorly). Second, does access to credit smooth aggregate consumption or aggregate hours worked, and if so, does it matter with respect to the nature of business cycles? No, it does not; in fact, consumption is 20 percent more volatile when credit is available. The interest rate premia increase in recessions because of higher bankruptcy risk discouraging households from using credit. This finding contradicts the intuition that access to credit helps households to smooth their consumption.

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1. Introduction

Significant developments have taken place in the unsecured consumer credit market since the 1980s. The balance of gross total unsecured credit increased from 5 percent of GDP in 1980 to 8 percent of GDP in the 2000s. At the same time, the number of bankruptcy filings increased, from 0.36 percent of all households in 1980 to more than 1.4 percent in the early 2000s.[1] Yet surprisingly, the business cycle properties of households' access to credit have not been studied in the literature.

We ask two questions in this paper:

1. Does the standard theory of unsecured credit, as posed, for example, by Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) and Livshits, MacGee, and Tertilt (2007), suitably extended to address business cycles, offer correct predictions about the cyclical behavior of unsecured household credit and bankruptcies? Both variables are highly volatile in the data (3.5 and 6.5 times the volatility of output), with unsecured credit being somewhat procyclical (having a 0.3 correlation with output) and bankruptcies being somewhat countercyclical (-0.2 correlation with output).

2. Does the large increase in credit in the consumer credit market affect the nature of business cycles? In particular, we are interested in whether the main macroeconomic aggregates behave differently — and if so, to what extent — as a result of the existence of household access to credit under the U.S. legal system. The variables that we examine are aggregate consumption and investment and, to a lesser extent, aggregate hours worked.

From an ex-ante point of view, the contribution of households' increased access to borrowing and default to aggregate fluctuations is not clear. One might be tempted to think that increased access

[1] These numbers represent total consumer bankruptcy filings. For Chapter 7 bankruptcy filings, the proportion is 0.26 percent in 1980 and about 1 percent in the early 2000s.

to credit allows households to smooth their consumption and more easily endure fluctuations in the business cycle. Herkenhoff (2014), however, argues that it allows households to prolong job searches, thereby inducing deeper and longer recessions. Alternatively, one might conclude, in light of the Great Recession, that the financial sector destabilizes the economy by encouraging oscillations in private consumption.

We investigate these issues by posing model economies in which heterogeneous agents are subject to idiosyncratic shocks to their earning possibilities. We compare the properties of economies in which households have access to credit and bankruptcy (a feature intrinsically associated with unsecured credit) with the properties of economies in which households either do not have access to credit or do have access, but without the possibility of filing for bankruptcy. The types of economies we pose are essentially a combination of the economies with bankruptcy à la Chatterjee et al. (2007) or Livshits et al. (2007) and those with aggregate shocks à la Krusell and Smith Jr. (1998).

Regarding our first question, we find that the model economies share the main cyclical properties of borrowing and default with the U.S. economy: Borrowing is procyclical, and bankruptcy filings are countercyclical. Bankruptcy filings are quite volatile, but credit is less volatile than in the data, leading us to believe that the volatility of consumer credit may itself be subject to shocks, rather than being solely the result of households' choices in response to aggregate shocks to earnings.

With respect to the second question, we find that the availability of credit makes consumption more volatile, about 20 percent more volatile. The behavior of hours worked is more subdued. The availability of credit increases only slightly the volatility of hours, about 2 percent more. Consequently, output does not vary differently across credit regimes, thereby implying that the larger volatility of consumption when there is access to credit is accompanied by a smaller volatility of investment. Note that the behavior of consumption is the opposite to what one could think that credit is, an additional tool to smooth consumption. Then, why does its availability lead to a higher consumption volatility?

Our findings require some qualifications, two of which are common to all the economies within the class that we study: that lending institutions themselves do not contribute to economic instability (we model lenders as if they were banks with 100 percent reserves) and that the origin of the fluctuations in the business cycle is some form of total factor productivity (TFP) shock. The rest of the qualifications are more subtle. The first one is that we model recessions as a period during which all households are faring a little worse than normal, but some households are faring very poorly, as implied by the findings about the countercyclicality of earnings risk of Storesletten, Telmer, and Yaron (2004) and Guvenen, Ozkan, and Song (2012). The second qualification is that the legal system matches that in the United States, where households have easy access to filing for bankruptcy. This feature is what makes credit procyclical: It implies lower expected bankruptcy rates in good times and hence better credit terms for borrowing households. Clearly, this feature is also necessary in accounting for bankruptcies themselves. Economies with credit but without default have countercyclical credit and less volatile aggregate consumption. Another issue we explore is that there are occasionally very large recessions, which do not seem to matter, and hence the size of the recessions does not qualify our findings.

To study the interaction between unsecured credit and business cycles, we bring together the following four strands of literature for the first time. First, our model features uninsurable idiosyncratic labor income shocks (Bewley (1986), İmrohoroğlu (1989), Huggett (1993), Aiyagari (1994)). Second, we introduce unsecured credit and equilibrium bankruptcies (Chatterjee et al. (2007), Livshits et al. (2007)). Third, we introduce aggregate fluctuations to the economy in which consumers are subject to uninsured idiosyncratic shocks (Krusell and Smith Jr. (1998), Castañeda, Díaz-Giménez, and Ríos-Rull (1998)). Finally, the business cycles in our model pose countercyclical earnings risk as argued by Storesletten et al. (2004) and Guvenen et al. (2012). Two more important papers related to ours are Kiyotaki and Moore (1997) and Neumeyer and Perri (2005). In the former, credit also turns out to be procyclical, albeit for reasons very different from ours. The latter shows that consumption volatility in emerging economies is magnified by the countercyclical movement of borrowing interest. Arellano (2008) and

Chatterjee and Eyigungor (2012) provide a theory for these movements that is related to what goes on in our environment.[2] Nakajima and Ríos-Rull (2005) explored aggregate fluctuations in endowment economies where aggregate fluctuations are announced in advance. The findings there are that for those environments both bankruptcy and credit are extremely procyclical. Clearly, the choice of environment misses the relevant properties of the data. Recently, there has been some new work that explores business cycle properties of economies where households can file for bankruptcy. In Chapter 4 of his thesis, Fieldhouse (2014) (jointly written with Livshits and MacGee) explores the aggregate properties of endowment economies with aggregate endowment shocks and unsecured credit. The authors find that aggregate fluctuations in earnings generate countercyclical bankruptcy filings, but credit is also countercyclical. In addition to failing to replicate the cyclicality of credit, the volatilities in filing, interest rates, and debt are well below the data. Gordon (2013) explores how the presence of aggregate risk affects the welfare assessment of different policies when there are state contingent contracts that allow intermediaries to always have zero profits. Because the paper barely explores the business cycle properties of the model, we see this paper as complementary to ours. Corbae and D'Erasmo (2014) explore the cyclical behavior of loans in a model of the banking industry where financial intermediaries can and do default, and they also find procyclical lending.

The rest of the paper is organized as follows. Section 2 describes the business cycle properties of the main macroeconomics aggregates and those of credit and bankruptcy. Section 3 describes the model. The specification of the parameterization to map the model economies to U.S. data is in Section 4, and Section 5 briefly discusses computation arising from the complexity of the environment. The main analysis is in Section 6. Section 7 explores how our findings change when we depart from the two assumptions that we make in our baseline model economy: that there is countercyclical earnings risk and that there is a possibility of rare but large recessions. Finally, some concluding thoughts are in Section 8. Appendix A provides a detailed description of the computation of the model. Appendix B displays additional tables.

[2] While countercyclical interest rate hikes occur as in our economy, the sovereign default literature points to individual histories with persistent shocks. Instead, these hikes happen here because of countercyclical earnings risk.

2. Credit and Bankruptcy Facts Related to the Business Cycle

Table 1 describes the business cycle properties of the U.S. economy using yearly data. The data include the volume of credit and the number of personal bankruptcies under all legal forms as well as only those under Chapter 7, which is what our model replicates. The data are from the period after 1980, when data on the number of bankruptcies are available. We filter the data using the Hodrick-Prescott (H-P) filter with the smoothing parameter of 6.25, following Ravn and Uhlig (2002). In 2005, a major bankruptcy reform made filing more difficult and thereby induced many people to file for bankruptcy preemptively. In 2008, the Great Recession began. For both of these reasons — bankruptcy reform and the onset of the Great Recession — we report the data in two different panels. Panel A reports the data up to 2004, and Panel B reports up to the most recent available year.

The table displays the standard business cycle facts: Consumption is less volatile than investment, and all aggregates are highly correlated. With respect to credit and bankruptcy, we see that consumer credit is quite volatile — about three times more volatile than output — and that it is clearly procyclical, although less strongly than consumption or investment. Bankruptcies are extremely volatile — about twice as volatile as investment — and are slightly countercyclical. Both properties are exacerbated if we include the data up to 2013.

At this stage, we should point out that the procyclicality of credit does not square well with its possible role as an instrument to smooth consumption in bad times. Still, before passing judgment, we will wait to see what the model tells us.

3. The Model

The model extends Chatterjee et al. (2007) by introducing aggregate shocks. The environment is inhabited by a mass of households that can save or borrow and that can and often do default on their

Table 1: Cyclical Properties of the U.S. Economy

Variable	SD%	Relative SD%[a]	Auto-corr	Cross-Correlation of Output with x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}
A. U.S. Economy: 1980-2004, H-P Parameter 6.25								
Output	1.18	1.00	0.25	–0.48	0.25	1.00	0.25	–0.48
Consumption	0.91	0.77	0.36	–0.44	0.42	0.87	0.21	–0.28
Investment	4.83	4.09	0.11	–0.30	0.24	0.92	0.07	–0.73
Capital share	1.12	0.95	0.43	0.27	0.04	–0.08	–0.27	–0.39
Average hours	0.31	0.27	–0.04	–0.14	0.31	0.74	–0.23	–0.67
Aggregate hours	1.42	1.20	0.36	–0.44	0.15	0.92	0.44	–0.41
Consumer credit	**3.93**	3.33	0.45	–0.19	0.14	**0.32**	0.45	0.23
All bankruptcy	**7.80**	6.61	0.44	0.20	–0.21	**–0.25**	0.23	0.52
Chapter 7 bankruptcy	**8.69**	7.37	0.44	0.19	–0.14	**–0.18**	0.21	0.48
B. U.S. Economy: 1980-2013, H-P Parameter 6.25								
Output	1.22	1.00	0.31	–0.39	0.31	1.00	0.31	–0.39
Consumption	0.97	0.79	0.39	–0.39	0.43	0.91	0.30	–0.28
Investment	5.77	4.72	0.24	–0.17	0.35	0.93	0.14	–0.59
Capital share	1.31	1.07	0.36	0.38	0.38	0.05	–0.26	–0.36
Average hours	0.38	0.31	0.15	–0.12	0.37	0.80	–0.03	–0.55
Aggregate hours	1.79	1.46	0.44	–0.44	0.16	0.90	0.55	–0.21
Consumer credit	**3.81**	3.11	0.45	–0.28	–0.12	**0.15**	0.45	0.41
All bankruptcy	**17.06**	13.94	0.06	0.28	–0.19	**–0.37**	–0.19	0.08
Chapter 7 bankruptcy	**21.02**	17.17	0.04	0.28	–0.16	**–0.34**	–0.20	0.05

Note: Logs of the data are filtered using the H-P filter with a smoothing parameter of 6.25. Output: real GDP. Consumption: real private consumption expenditures; Investment: real gross domestic investment; Consumer credit: gross revolving consumer credit (Flow of Funds), deflated by GDP deflator; Bankruptcy filings: Consumer bankruptcy filings per household.
[a] Relative to the standard deviation of output.

debt. The punishment for defaulting in the model captures the punishment for Chapter 7 consumer bankruptcy in the United States. There is a large number of financial intermediaries that extend loans to households, adjusting the terms and conditions of loans for different aggregate states and different types of households, depending on the expected default probability and the expected rate of return for risk-free assets. Generally, high-risk borrowers are charged a high default premium, and, because of their persistence, recessions increase the default premium for all borrowers. In equilibrium, predicted default probabilities that are used to compute the default premium for different types of loans are consistent with the optimal default decision of borrowers, given the default premium offered by financial intermediaries.

3.1. Bankruptcy Filings in the Model

The procedure and the consequences of bankruptcy filings are modeled to capture those of the Chapter 7 consumer bankruptcy filings in the United States. Let $h \in \{0, 1\}$ denote the credit history; $h = 1$ indicates a record of a bankruptcy filing in the household's recent credit history; and $h = 0$ indicates the absence of any such record. We will refer to h as simply the household's credit history, with the rating either good ($h = 0$) or bad ($h = 1$). A household with a good credit history can borrow and can default on its debt. Upon bankruptcy, the filing household experiences the following events:

1. Creditors can garnish a fraction ξ of the labor income. This garnishment represents both partial repayment by the borrower before defaulting and the creditors' attempt to garnish income before the borrower files for bankruptcy.
2. The unsecured debt balance of the borrower that is not covered by the garnished labor income is discharged. The creditors lose any future claims to the discharged debts.
3. When defaulting, the household is not permitted to save a positive amount in the current period. The assumption is a simple way to recognize that a household's attempt to accumulate assets during the period of bankruptcy filing results in those assets being seized by creditors.
4. The household begins the next period with a bad credit history ($h = 1$).
5. A household with a bad credit history ($h = 1$) cannot get any new loans. This assumption is broadly consistent with the experience of bankrupt individuals reported in Musto (1999). There are no restrictions on saving.
6. There is a positive probability λ that a credit history of a household is cleared, and the household

will start the next period with a good credit history ($h = 0$). This is a simple way of modeling the fact that a history of a bankruptcy filing remains on an individual's credit history for only a finite number of years. After the history of defaulting is cleared, the past default has no consequences.

Since loans might not be repaid because of the option of a bankruptcy, profit-maximizing financial intermediaries have to take into account the probability of repayment in determining the terms of loans. Moreover, since borrowers with different opportunities for future earnings and with different amounts of debts have potentially different probabilities of repayment, the terms of the unsecured loans are a function of the borrower's characteristics, the state of the economy, and the size of the loans. Further details are discussed in the section following the household's problem.

3.2. Demographics

There is a continuum of households of measure one at each point in time. Each household faces an exogenous survival probability π. In each period, measure $(1 - \pi)$ households are born and replace the deceased households, keeping the size of the population constant. We assume a complete annuity market for survival risk. This concise life-cycle structure ensures that households are born with zero assets and therefore do not have time to accumulate as many precautionary assets as they would want, thus creating the conditions for an active loans market.

3.3. Preferences

The preferences of a household are given by the expected value of a discounted sum of period utilities:

$$E_0 \left\{ \sum_{t=0}^{\infty} (\beta\pi)^t \gamma_t \, u(c_t, 1 - \ell_t) \right\}, \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, $\pi \in (0,1)$ is the survival probability, $\gamma_t \in [0,1]$ is an idiosyncratic preference shock that follows a finite-state Markov process with Markov transition matrix $\Gamma^{\gamma}_{\gamma,\gamma'}$, c_t is consumption in period t, and ℓ_t is hours worked. E_0 is the expectation operator taken with respect to information in period 0. The utility function u is continuous, strictly increasing, and strictly concave in both arguments.

The use of preference shocks as a trigger for indebtedness deserves some discussion. Expenditure shocks have often been used (e.g., Chatterjee et al. (2007), Livshits et al. (2007)) to capture events such as large uninsured medical expenditures, divorces, and unwanted births that may trigger defaults without borrowing. In that case, the volume of indebtedness of households does not closely depend on the risk premium or on the aggregate state of the economy. A household that is hit by a preference shock, however, chooses to borrow a certain amount and may then subsequently default if such a choice turns out to be optimal. Preference shocks that increase the marginal utility of consumption interact with the aggregate state of the economy to determine how much is borrowed and how much is defaulted — a feature that turns out to be crucial in generating the procyclicality of credit in our model economies.

3.4. Technology

There is one good produced via $z\,F(K,L)$, where z is an aggregate productivity shock that follows a finite-state Markov process with Markov transition matrix $\Gamma^{z}_{z,z'}$, K and L are aggregate capital and labor measured in efficiency units, and F is a constant returns to scale aggregate production function, strictly increasing, strictly concave, and satisfying Inada conditions. The good can be either consumed or invested in physical capital. The aggregate resource constraint is

$$C_t + K_{t+1} = (1-\delta)K_t + z_t F(K_t, L_t), \tag{2}$$

where C_t is aggregate consumption in period t and δ is the depreciation rate of physical capital.

3.5. Endowments

A household is born with zero assets and is endowed with one unit of time each period. Following Storesletten et al. (2004), individual labor productivity consists of three components. The distribution of the permanent shock e is denoted by Γ^e_e, and the persistent shock p follows a finite-state Markov process with Markov transition matrix $\Gamma^p_{p,p'|z'}$, explicitly depending on z'. We use this feature when we calibrate the model so that earnings risk is countercyclical. There is also a transitory shock t with distribution Γ^t_t.

3.6. Market Arrangements

Households cannot trade state-contingent securities but can smooth consumption by changing their hours worked, by saving or borrowing, and by defaulting on their debt. The asset position of households is denoted by $a \in \mathbb{A}$, with $a < 0$ indicating that the household is borrowing.

In order to emphasize that the interest rate of loans is specified at the time the loan is made, we keep track of the face value of the loan or promised amount to pay the following period. A borrower that promises to repay a' tomorrow (primes denote the value of variables next period), provided that it decides not to file for bankruptcy, receives $q\,a'$ in the current period, so q is the discount price of the loan. Savings are recorded in the standard way. A household that saves a' receives $(1 + r')a'$ in the following period, where r' is the rate of return on savings that is determined in the next period and is thus subject to aggregate shocks.

There is also a perfect annuity market, which insures the mortality risk: A household that saves $\pi a'$ will receive a' only if it survives. This feature has no interesting consequences; it just ensures that assets do not disappear after the death of households.

3.7. Household's Problem

The individual state variables are $\{x, h, a\}$, where we use x to summarize the exogenous individual shocks to a household (γ, e, p, t). Naturally, we use $\Gamma^{x}_{x,x'|z'}$ to denote the Markov transition probabilities of x. Recall that γ is the preference shock, e is the individual permanent productivity shock, p is the individual persistent productivity shock, and t is the individual transitory productivity shock. $h \in \{0, 1\}$ is the household's credit history, and a is the current asset position of the household. The aggregate state variables are $\{z, K, m\}$, where z is an aggregate shock to productivity, K is the aggregate capital stock, and $m(x, h, a) \in \mathbb{M}$ is a type distribution of households. Aggregate capital K is an aggregate state variable in addition to m, because m is not a sufficient statistic for K, since it depends on the type distribution in the previous period. In addition, to determine wages and rates of return, we need to know aggregate labor, L, an equilibrium object. We write $L = \phi_L(z, K, m)$ as the function that determines aggregate labor supply. The laws of motion for the state variables are $\phi_K(z, K, m)$ for aggregate capital and $\phi_m(z, z', K, m)$ for the type distribution. The latter contains z' as an argument because the distribution of the persistent productivity shock in the next period, p', and thus the type distribution in the next period, m', depends on z'. To solve their problem, households also need to know wages, $w(z, K, L, m)$, rates of return, $r(z, K, L, m)$, and prices for each type of loan, $q(z, K, m, x, a')$. We write wages and rates of return as being functions of the state variables and of labor to stress that they are (mostly; see the discussion below) marginal productivities that depend on the total amount of inputs. In a strict sense, however, factor prices depend on state variables only after substituting labor using equilibrium function ϕ_L.

Let us first consider the problem of a household with a good credit history ($h = 0$) that does not file for bankruptcy. Given pricing functions, labor function, $\phi_L(z, K, m)$, and laws of motion, $\phi_m(z, z', K, m)$ and $\phi_K(z, K, m)$, the household solves the following problem:

$$V_0(z,K,m,x,0,a)=\max_{c,\ell,a'}\left\{u(c,1-\ell)+\beta\pi\gamma\sum_{z'}\sum_{s'}\Gamma^{z}_{z,z'}\Gamma^{x}_{x,x'|z'}V(z',K',m',x',0,a')\right\} \quad (3)$$

subject to

$$\begin{aligned} c+a'\ \pi\ q(z,K,m,x,a') &= a[1+r(z,K,L,m)\mathbb{1}_{a\geq 0}]+e\ p\ t\ \ell\ w(z,K,L,m), & (4)\\ L &= \phi_L(z,K,m), & (5)\\ m' &= \phi_m(z,z',K,m), & (6)\\ K' &= \phi_K(z,K,m). & (7) \end{aligned}$$

$V_0(z,K,m,x,0,a)$ is the value for a household with a good credit history if it chooses not to file for bankruptcy. The household chooses current consumption c, hours worked ℓ, and the asset position in the next period a'. $V(z,K,m,x,h,a)$ is the value function. Equation (4) is the standard budget constraint. Asset holdings for the next period, a', are multiplied by π (perfect annuities term) and $q(.)$. For savers, $q(.)=1$, that is, there is no discount due to default risk. For borrowers who might default on their loans, $q(.)$ includes not only the inverse of the expected interest rate (the interest rate charged to risk-free borrowers), but also any premium due to the risk of default. The return on savings, $r(z,K,L,m)$, is multiplied by $\mathbb{1}_{a\geq 0}$, where $\mathbb{1}$ is the indicator function that takes the value 1(0) if the condition attached to it is true (false), because only savers earn interest. The interest charged to borrowers is implicit in $q(.)$. Equation (5) yields the equilibrium quantity of labor necessary to compute factor prices, and equations (6) and (7) are forecasting functions for the type distribution and aggregate capital stock in the next period.

A household with good credit history ($h=0$) that files for bankruptcy ($h'=1$) solves the following problem:

$$V_1(z,K,m,x,0,a) = \max_{c,\ell} \left\{ u(c,1-\ell) + \beta\pi\gamma \sum_{z',x'} \Gamma^z_{z,z'} \Gamma^x_{x,x'|z'} V(z',K',m',x',1,0) \right\}, \quad (8)$$

$$\text{subject to} \qquad c = e\ p\ t\ \ell\ w(z,K,L,m)\ (1-\xi), \quad (9)$$

and conditions (5-7). Upon filing for bankruptcy, the household cannot save during the current period, and there is garnishment of labor income by creditors, which is a proportion ξ to the household's current labor income. Moreover, the household starts the following period with a bad credit history ($h' = 1$) and no assets ($a' = 0$).

The household optimally chooses whether to file for bankruptcy. Formally, $V(z,K,m,x,0,a)$ satisfies

$$V(z,K,m,x,0,a) = \max\ \{V_0(z,K,m,x,0,a), V_1(z,K,m,x,0,a)\}. \quad (10)$$

A household with a bad credit history ($h = 1$) cannot file for bankruptcy, and its problem is

$$V(z,K,m,x,1,a) = \max_{c,\ell,a'} \left\{ u(c,1-\ell) + \beta\pi\gamma \sum_{z',x',h'} \Gamma^z_{z,z'} \Gamma^x_{x,x'|z'} \Gamma^h_{h'} V(z',K',m',x',h',a') \right\}, \quad (11)$$

$$\text{subject to} \qquad c + a'\pi = a[1 + r(z,K,L,m)] + e\ p\ t\ \ell\ w(z,K,L,m), \quad (12)$$

$$a' \geq 0, \quad (13)$$

and conditions (5-7). One notable difference from the previous problems is the existence of $\sum_{h'} \Gamma^h_{h'}$. In particular, $\Gamma^h_1 = (1-\lambda)$ (credit history remains bad) and $\Gamma^h_0 = \lambda$ (credit history becomes good). The budget constraint (12) is simpler, since the household cannot borrow with a bad credit history as reflected in condition (13).

Decision rules are denoted $c = g^c(z,K,m,x,h,a)$ for consumption, $a' = g^a(z,K,m,x,h,a)$ for borrowing or saving, $\ell = g^{\ell}(z, K, m, x, h, a)$ for hours worked, and $h' = g^{h}(z, K, m, x, a)$ for defaulting. The latter function does not have an argument h because there is no bankruptcy decision for a household with a bad credit history. Using these decision rules, the probability that a household of type $x = (\gamma, e, p, t)$ with a good credit history ($h = 0$) and amount of debt a' in an aggregate state (z, K, m) files for a bankruptcy in the next period is given by

$$d(z, K, m, x, a') = \sum_{z', x'} \Gamma^{z}_{z,z'} \; \Gamma^{x}_{x,x'|z'} \quad \mathbb{1}_{g^{h}(z', \phi_K(z,K,m), \phi_m(z,z',K,m), x', a')=1}. \tag{14}$$

3.8. Unsecured Credit Industry

The financial intermediaries that make unsecured loans are owned by mutual funds (explained in the next section) and operate with zero costs, and the industry has free entry. The opportunity cost for the mutual fund is the return on the alternative investment, real capital. Ignoring the subtle issues associated with the lack of perfect correlation between the rates of return on both assets, we simply assume that equilibrium requires that both assets yield the same expected rate of return. The return on a loan depends on the interest rate charged, on the probability that the loan is repaid, and, in the case of default, on how much income can be garnished. Although whether a loan is repaid depends on the realization of idiosyncratic shocks, lenders can eliminate the idiosyncratic risk by lending to a positive mass of the same type of borrowers and exploiting the law of large numbers, making profits linear in the measure of each type of loan. At the same time, because of free entry, the expected profit of each firm that lends to a certain type of household with a certain amount of debt is driven to zero. Although in the steady state actual profits are zero, with aggregate uncertainty, realized profits are typically nonzero; hence, the role of the mutual funds that own lending firms to absorb profits and losses. In sum, the expected zero profit condition of a firm in the credit industry that makes one loan of amount a' to type x households when the aggregate state is (z, K, m) is

$$[1 + r(z, K, L, m)]\ q(z, K, m, x, a')\ (-a') \\ = \sum_{z', x'} \Gamma^z_{z,z'} \Gamma^x_{x,x'|z'} \left[\mathbb{1}_{g'^h=1}\ \xi\ e'\ p'\ t'\ g'^\ell\ w(z', K', L', m') + \mathbb{1}_{g'^h=0}(-a') \right], \quad (15)$$

using conditions (5-7). We use g'^h and g'^ℓ as shorthand for $g^h(z', K', m', x', a')$ and $g^\ell(z', K', m', x', h', a')$, respectively. The first term on the right-hand side represents the income garnishment in case the borrower defaults, and the second term represents the amount of the loan repaid. This expected zero profit condition implies the following formula for $q(.)$:

$$q(z, K, m, x, a') = \sum_{z', x'} \Gamma^z_{z,z'}\ \Gamma^x_{x,x'|z'}\ \frac{\mathbb{1}_{g'^h=1}\ \xi\ e'\ p'\ t'\ g'^\ell\ w(z', K', L', m') + \mathbb{1}_{g'^h=0}(-a')}{[1 + r(z', K', L', m')]\ (-a')}. \quad (16)$$

Notice that, when for some (z, K, m, x, a') no borrowers default, we have

$$q(z, K, m, x, a') = \sum_{z'} \Gamma^z_{z,z'} \frac{1}{[1 + r(z', K', L', m')]}. \quad (17)$$

The inverse of the discount rate of financial assets is the expected interest rate of savings. Finally, for $a' \geq 0$, $q(z, K, m, x, a') = 1$.

3.9. Factor Prices and the Mutual Fund

We assume a constant returns to scale production technology where the price of an efficient unit of labor is the standard marginal condition

$$w(z, K, L, m) = z\ F_L(K, L). \quad (18)$$

In aggregate state (z, K, m), total labor input in efficiency units, L, is given by

$$L = \phi_L(z, K, m) = \int g^{\ell}(z, K, m, x, h, a) \; e \; p \; t \; dm(x, h, a). \tag{19}$$

The aggregate amount loaned today, D, is

$$D = -\int \mathbb{1}_{g^a<0} \; \pi \; g^a \; q(z, K, m, x, g^a) \; dm(x, h, a), \tag{20}$$

where g^a is shorthand for $g^a(z, K, m, x, h, a)$. The aggregate capital stock in the next period, K', is total wealth tomorrow net of loans made today:

$$K' = \int \pi \; g^a \; q(z, K, m, x, g^a) \; dm(x, h, a) \; - D. \tag{21}$$

Capital earns the marginal rate of return r_K according to the production technology, and the return on loans, r_D, is a weighted average of the returns of each type of loan. We assume that there is a representative risk-neutral mutual fund and that all savers hold their wealth in this mutual fund. Because of risk neutrality, although the return on the loans r_D can be different from the rate of return on capital r_K *ex-post*, they have to be the same *ex-ante*. Specifically, the return on the mutual fund, $r(z, K, L, m)$, is

$$r(z, K, L, m) = \frac{K}{K+D} r_K(z, K, L, m) + \frac{D}{K+D} r_D(z, K, L, m), \tag{22}$$

$$r_K(z, K, L, m) = z \; F_K(K, L) \; - \; \delta, \tag{23}$$

$$r_D(z, K, L, m) = \frac{\int \mathbb{1}_{a<0}[\mathbb{1}_{g^h=1}\xi \; e \; p \; t \; g^{\ell} \; w(z, K, L, m) + \mathbb{1}_{g^h=0}(-a)] \; dm}{D} - 1. \tag{24}$$

Equation (22) defines the return on the mutual fund as the weighted average of the return from capital (r_K) and the return from loans (r_D). Equation (23) is the standard marginal condition for capital, and equation (24) characterizes the *ex-post* return of loans — basically the total income from loans divided by the amount loaned.

The use of this *risk-neutral* mutual fund allows us to circumvent the problem of how to determine the portfolio of savers. This convention has another property that we want to emphasize: that the lending institutions are not leveraged at all. They are like banks with 100 percent reserves. As such, any losses that they may suffer during large recessions — and they do suffer them — are absorbed by all savers in what is simply a proportional reduction to their wealth. Consequently, our economies have none of the problems that can be thought of as associated with the systemic risk of large banks.

3.10. Equilibrium

Definition 1 *A recursive equilibrium is a value function, $V(z, K, m, x, h, a)$, associated decision rules, $g^c(z, K, m, x, h, a)$, $g^a(z, K, m, x, h, a)$, $g^\ell(z, K, m, x, h, a)$, $g^h(z, K, m, x, a)$, functions for prices, $r(z, K, L, m)$, $r_K(z, K, L, m)$, $r_D(z, K, L, m)$, $w(z, K, L, m)$, and $q(z, K, m, x, a')$, for aggregate labor $\phi_L(z, K, m)$, and laws of motion, $\phi_m(z, z', K, m)$ and $\phi_K(z, K, m)$, such that*

1. **Household optimization.** *Given pricing and aggregate labor functions and laws of motion, $V(z, K, m, x, h, a)$ solves the household's problem characterized in Section 3.7, and $g^c(z, K, m, x, h, a)$, $g^a(z, K, m, x, h, a)$, $g^\ell(z, K, m, x, h, a)$, and $g^h(z, K, m, x, a)$ are associated decision rules.*
2. **Aggregate labor is the result of households' choices.** *Function $\phi_L(z, K, m)$ satisfies (19).*
3. **Expected zero profit condition for unsecured credit industry.** *Given pricing functions and laws of motion, $q(z, K, m, x, a')$ satisfies the expected zero profit condition* (16).
4. **Competitive factor prices.** *$r_K(z, K, L, m)$ and $w(z, K, L, m)$ satisfy* (23) *and* (18)*, respectively.*
5. **Mutual fund's indifference of allocations of investment.** *$r(z, K, L, m)$ satisfies* (22).
6. **Consistency/market clearing.** *$\phi_m(z, z', K, m)$ is consistent with the consumer's optimal decision rules and the law of motion for exogenous shocks, and $\phi_K(z, K, m)$ satisfies condition* (21).

4. Mapping the Model to Data

We specify the model period to be one year. Table 2 summarizes the calibration and contains three panels. Panel A includes the parameters that can be pinned down without solving the model. Panel B includes six parameters that are pinned down by solving a system of six equations that target steady-state values for the number of bankruptcies, the wealth-to-income ratio, the proportion of disposable time spent working, the number of borrowers, the size of their debt, and the cross-sectional coefficient of variation of earnings. Panel C consists of the parameters associated with business cycles. We have not targeted the wealth distribution explicitly, because the computational demands already loom large. This paper is about how the fear of being at the bottom of the income and wealth distribution shapes the behavior of the economy with various credit arrangements; therefore, we are concerned with the left tail of the distribution, not the right tail.

4.1. Parameters Set Ex-Ante

Demographics The survival probability, $\pi = 0.98$, implies that an average household's adult life is 50 years.

Preferences We use a per-period Cobb-Douglas utility function, $u(c) = \frac{[c^{\alpha}(1-\ell)^{1-\alpha}]^{1-\sigma}}{1-\sigma}$, where σ is set to 3.72 so that the coefficient of risk aversion for consumption is 2. The process for preference shocks is independent and identically distributed (i.i.d.) with two values; one of them, $\gamma_1 = 1$, is a normalization. We think of γ_1 as the normal state, whereas γ_2 captures the state in which the marginal utility from consumption is high. This state captures various occasions that often lead to defaulting, such as large uninsured medical expenditures, a divorce, an unwanted/unplanned pregnancy, and so on.

The discount rate, β, the share parameter of consumption, α, the value of the impatient state, γ_2, and its probability, Γ^{γ}_2, are determined jointly to target various aggregates and are described in Section 4.2.

Table 2: Parameters and Calibration Strategy

Parameter	Value	Description	Calibration Strategy
		A. Parameters Determined Ex-Ante	
		Aggregate Parameters	
λ	0.1000	Prob. of default history erased	Avg. punishment of default is 10 years
π	0.9800	Survival probability	Average life of 50 years
σ	3.7167	Curvature of utility function	Coefficient of RRA = 2
γ_1	1.0000	Good preference shock	Normalization
θ	0.3600	Curvature of production function	Labor share is 0.64
δ	0.0800	Depreciation rate	Depreciation rate is 0.08
		Parameters for Average Earnings Risk	
σ_e	0.4400	S.D. of permanent shock	Storesletten et al. (2004)
ρ_p	0.9630	Persistence of productivity shock	Storesletten et al. (2004)
σ_p	0.1300	S.D. of persistent shock (acyclical)	Storesletten et al. (2004)
σ_t	0.3500	S.D. of transitory shock	Storesletten et al. (2004)
		B. Parameters that Require Solving the Model	
ξ	0.3395	Income garnishment rate	Bankruptcies = 0.84% per year
β	1.0011	Discount factor	K/Y=3.0
α	0.3681	Avg. hours worked	33% disposable time
Γ_2^{γ}	0.0310	Prob. of bad preference shock	8.4% are in debt
γ_2	0.0000	Bad preference shock	Avg. debt over income is 20%
η	0.7500	Adjustment factor for productivity shock	Earnings coefficient of variation is 0.815
		C. Parameters Related to Business Cycles	
$\sigma_{p\|1}$	0.0880	S.D. of persistent shock in expansions	Storesletten et al. (2004)
$\sigma_{p\|2}$	0.1620	S.D. of persistent shock in recessions	Storesletten et al. (2004)
$\nu_1 = \nu_2$	0.0134	Size of TFP shock (normal)	S.D. of output = 1.2%
ν_3	0.0267	Size of TFP shock (disaster)	TFP drops twice as much in disaster
$\gamma_{1,1}^z$	0.6667	Persistence of good TFP shock	Avg. duration of expansion = 3 years
$\gamma_{2,2}^z$	0.6667	Persistence of bad TFP shock	Avg. duration of recession = 3 years
$\gamma_{3,3}^z$	0.3333	Persistence of disastrous TFP shock	Avg. duration of disaster = 1.5 years
γ_3^z	0.0200	Frequency of disastrous TFP shock	Avg. frequency of disaster is 50 years

Technology We assume a Cobb-Douglas production function, $Y = zF(K, L) = zK^{\theta}L^{1-\theta}$, where the capital share, $\theta = 0.36$, is calibrated to match the average capital share of income in the United States (the contribution of interest income to capital share is very small). The depreciation rate is set to $\delta = 0.08$.

Endowments We use the estimates of Storesletten et al. (2004) to specify the process of individual shocks to the efficiency units of labor, but we adjust the parameter values to take into account that the estimates of Storesletten et al. (2004) are for *earnings*, whereas our shocks are for *labor productivity*. In particular, individual productivity i takes the following form:

$$\log i = \log e + \log p + \log t, \tag{25}$$

where the permanent shock e is drawn at birth from $N(0, (\eta\sigma_e)^2)$, and the transitory shock t is drawn each period from $N(0, (\eta\sigma_t)^2)$. The persistent shock p follows the following AR(1) process:

$$p' = \rho_p p + \epsilon_p \qquad \epsilon_p \sim N(0, (\eta\sigma_{p|z})^2), \tag{26}$$

where η is an adjustment factor that adjusts the variance of all individual productivity shocks equally. This adjustment is necessary because the estimates of Storesletten et al. (2004) are for earnings, and in this model earnings are the product of the shocks and the choice of hours worked. We set this factor so that the implied coefficient of variation of earnings matches its data counterpart. This adjustment requires that we solve the model to determine its size. We come back to the determination of η in Section 4.2. We set the standard deviation of the innovation for the AR(1) process, σ_p, to 0.13, the average values reported by Storesletten et al. (2004).

In computing the model, we determine that the permanent shock to individual productivity e is approximated by a two-state distribution using the method developed by Adda and Cooper (2003). Similarly, the transitory shock t is approximated by a three-state distribution. As for the persistent shock p, we apply the method of Tauchen (1986) with 15 grid points.

Bankruptcy Filings The parameter that regulates the length of the borrowing exclusion, $\lambda = 0.10$, implies that a history of past bankruptcy filing remains on the credit history for 10 years on average. This is consistent with the Fair Credit Reporting Act, which prohibits keeping a record of past defaults for more than 10 years. See Chatterjee et al. (2007) for a more detailed discussion about how to map the legal environment of the United States to the model. The fraction of labor income that can be garnished, ξ, is determined in the second stage of the calibration, which is described next in Section 4.2.

4.2. Calibration of the Residual Parameters

The six parameters that are calibrated via solving a system of equations that require model moments to satisfy certain targets are the discount rate, β, the share parameter of consumption, α, the value of the impatient state, γ_2, its probability, Γ^{γ}_{2}, the adjustment factor for individual productivity shocks, η, and the fraction of labor income that can be garnished, ξ, upon a bankruptcy filing. The targets and parameter values are shown in panel B of Table 2, but they merit a discussion. We target a capital-output ratio of 3.0 in the steady state. Although the wealth-to-output ratio can be argued to be slightly higher, we are very interested in getting the wealth of the poor right. Consequently, if we obtain both too little wealth as well as too few ultra-wealthy people, this seems to be a reasonable way to obtain the right wealth holdings of the poor while still having a reasonable aggregate amount of capital.[3] As for the proportion of time spent working, we target one-third. We target the proportion of consumers in debt in the steady state to be 8.4 percent, and the average amount of debt per debtor to be 20 percent of the average income. Both targets are obtained by using households with a head between ages 20 and 65 in the Survey of Consumer Finances (SCF) in 2004. The proportion of consumers filing for bankruptcy each year is 0.84 percent of the population (as computed by Livshits et al. (2007)). In our attempts to target these moments with these parameters, γ_2 was pushed toward (and beyond) its natural limit of zero. Consequently, we set it to zero. Notice that the only relevant implication for a consumer that has $\gamma = \gamma_2 = 0$ is that it consumes as much as it can in the current period. With

[3] Table A2 in the appendix reports the implied distributional statistics in some detail.

Table 3: Calibration Results

Target Statistics	Data	Model
Capital-to-output ratio	3.0000	3.0004
Proportion of hours spent working	0.3300	0.3301
Proportion of bankruptcy filers	0.0084	0.0086
Proportion in debt	0.0840	0.0860
Debt-to-income ratio	0.1986	0.2016
Earnings coefficient of variation	0.8148	0.8194

this limit, we were close to obtaining the targets that we wanted. Finally, the adjustment factor for individual productivity shocks η has a value of 0.75. This implies that the shocks to productivity are 75 percent of the measures of the shocks from Storesletten et al. (2004) if we want the endogenous choice of hours to imply an earnings coefficient of variation of 81.5 percent.[4] Table 3 shows the precision with which we attain the targets.

Our estimates imply that $\xi = 0.340$. Considering that the federal garnishment limit for defaulters is 25 percent of income, and the garnishment parameter also includes the attempts by the debtors to repay after deciding to default, we find $\xi = 0.340$ to be within the reasonable range. Our estimate for β is 1.001. This estimate may seem large, but notice first that β has no real meaning independent of the model, that there is early mortality (even though a perfect annuity market is available), and that the effective β is actually the product of β and the probability of not being impatient ($\gamma = \gamma_1$), which yields 0.970 given that our estimate of Γ_2^γ is 0.031.

4.3. Specification of Aggregate Shocks

We assume three states for aggregate productivity shocks. The three states correspond to expansion, recession, and disaster. The latter allows us to explicitly consider sample paths in which an event like the Great Recession occurs, and the other two states represent normal business cycles. We assume the

[4] We use the coefficient of variation instead of, for example, the cross-sectional variance of log earnings because some households in the model choose not to work, thereby making their earnings zero.

following four parameters for the transition matrix of z:

$$\Gamma^z_{z,z'} = \begin{pmatrix} (1-\gamma^z_3)\gamma^z_{1,1} & (1-\gamma^z_3)(1-\gamma^z_{1,1}) & \gamma^z_3 \\ (1-\gamma^z_3)(1-\gamma^z_{2,2}) & (1-\gamma^z_3)\gamma^z_{2,2} & \gamma^z_3 \\ (1-\gamma^z_{3,3})/2 & (1-\gamma^z_{3,3})/2 & \gamma^z_{3,3} \end{pmatrix}. \qquad (27)$$

Seven parameters are to be determined: the four parameters above, plus the three values of the shock, which we write as $\{1+\nu_1, 1-\nu_2, 1-\nu_3\}$. Our choices are to have a disaster that happens every 50 years on average ($\gamma^z_3 = 0.02$). We also normalize the values of the shock by setting $\nu_1 = \nu_2$. We also want a disaster that is twice as bad as a normal recession, $\nu_3 = 2\nu_2$. We set the average duration of normal expansions and recessions, conditional on a disaster not occurring, to be three years ($\gamma^z_{1,1} = \gamma^z_{2,2} = 2/3$).[5] We assume the average duration of a disaster to be one-and-a-half years ($\gamma^z_{3,3} = 1/3$). The final parameter, $\nu_1 = 0.013$, is calibrated such that the standard deviation of H-P filtered log output in the model matches the empirical counterpart, which is 1.2 percent. We employ the model with occasional disasters as our baseline, and in Section 7.2, we discuss briefly whether modeling disasters matters for cyclical properties of the model (it does not).

With respect to the countercyclical earnings risk, Storesletten et al. (2004) obtain that the standard deviation of the individual earnings shock is 0.088 in expansions, whereas it is 0.162 in recessions. On the other hand, the skewness of one year log earnings growth calculated by Guvenen et al. (2012) has an average over the 1979-2010 period of -0.31, with a standard deviation of 19.5 percent, and a correlation with output of 0.63. To implement these features, we set in our economy $\sigma_{p|z} = \eta\, 0.088$ in expansions ($z = 1$) and $\sigma_{p|z} = \eta\, 0.162$ in recessions ($z = 2$) and disasters ($z = 3$). Notice that we use the same adjustment factor η already calibrated. This choice yields the same properties reported by Storesletten et al. (2004). With respect to the skewness, we obtain a much lower average (-0.05) and standard deviation (1.2%) than Guvenen et al. (2012). We replicate, however, its cyclical behavior, obtaining a very large correlation of skewness and output (0.89).

[5] Whether we include the term $(1-\gamma^z_3)$ or not does not matter, since σ^z_3 is tiny.

5. Computation

We solve the model numerically, since there is no analytical solution. In particular, our solution method is based on the *approximate aggregation approach* developed by Krusell and Smith Jr. (1997) and Krusell and Smith Jr. (1998). Our computational requirements are more involved than theirs. In their 1997 paper, the computation of equilibrium requires the determination of factor prices or the risk-free interest rate, which requires the joint determination of a price and quantity (either labor and wages, or bond holdings and the risk-free interest rate) and the forecast of capital. In our model, we not only have to forecast capital and jointly determine wages and the quantity of labor, but we also have to forecast the prices of discount bonds for all types of borrowers and all sizes of debt.

In the same spirit as Krusell and Smith Jr. (1997), we pose agents with bounded rationality in the size of the state space, taking advantage of the approximate aggregation feature of this type of environment. Specifically, we assume that agents summarize the aggregate state of the economy by the aggregate productivity shock, capital stock, and average individual labor productivity (necessary because of the countercyclical variance of individual productivity) (z, K, S), instead of using the proper state space (z, K, m). Unlike Krusell and Smith Jr. (1997), in our model, agents use forecasted rather than actual factor prices and risk-free interest rates, as has recently become the norm (e.g., Krusell, Mukoyama, and Şahin (2010) or Krusell, Mukoyama, Rogerson, and Şahin (2012)). As long as the rate of return of the mutual fund, r, and aggregate capital stock in the next period, K', are well forecasted by (z, K, S), and average individual labor productivity in the next period, S', is well forecasted by (z, z', K, S), this drawback is not a serious one.[6] We will show that (z, K, S) or (z, z', K, S) has sufficiently high predictive power over the important variables to be forecasted. The details are in Appendix A.

[6] Notice that in the case of our baseline model with countercyclical variance of individual productivity, it is necessary that the next period TFP shock z' be known to forecast average individual labor productivity in the next period S', because the distribution of individual productivity in the next period depends on the realization of the TFP shock in the following period, z'.

Table 4: Main Cyclical Properties of the Baseline Economy and the U.S. Data

	Baseline Economy		U.S. Data 1980-2013	
	St Dev %	Correl w Output	St Dev %	Correl w Output
Output	1.20	1.00	1.22	1.00
Consumption	0.47	0.98	0.97	0.91
Investment	3.52	1.00	5.77	0.93
Hours	0.63	1.00	1.79	0.90
Credit	1.28	0.80	3.81	0.15
Bankruptcy filings	19.98	−0.90	21.02	−0.34

Note: Logs of the data are H-P filtered using a smoothing parameter of 6.25.

6. Result: Business Cycle Properties

Table 4 displays the main findings of our analysis. The top panel shows the aggregate statistics. Our choice of the size of the productivity shock was to match output volatility, so we do so. This is accomplished by having a larger variance of the shock than that of the Solow residual of the U.S. economy and a large Frisch elasticity of labor (an implication of Cobb-Douglas preferences). The aggregates agree with the data in the main features: Both consumption and investment are strongly correlated with output, and the latter is a few times more volatile than the former.

6.1. Cyclical Properties of Credit and Bankruptcy

The bottom panel of Table 4 shows the cyclical properties of credit and bankruptcy. Credit is highly procyclical, even more than it is in the data. Recall our statement that this behavior may be counter-intuitive: An expansion is characterized by more credit, so credit is not used mostly to accommodate temporary bad times, as intuition may tell us. Credit is also quite volatile — more so than output, yet not as much as in the data. Bankruptcy filings are extremely volatile, as they are in the data. They are also countercyclical, even more than they are in the data.

Table 5: Additional Cyclical Properties of the Baseline Economy

	Standard Deviation %	Relative to S.D. of Output	Correlation with Output
Mutual fund return $(1+r)$	0.16	0.13	0.99
Capital return $(1+r)$	0.16	0.13	0.99
Loan return $(1+r)$	1.35	1.12	0.95
Avg. loan rate[a] $(1+r)$	0.91	0.76	0.16
Loan risk premium (expected)	7.19	5.99	0.17
Loan risk premium (realized)	6.79	5.65	0.01

Note: Logs of the data are H-P filtered using a smoothing parameter of 6.25.
[a] Weighted by loan amount.

The procyclicality of credit in the model deserves further study, since it goes against the intuition that it is during times of distress, which are more prevalent during recessions, when households use credit to smooth their consumption. The key to the explanation is in Table 5. Although the rate of return on savings (the mutual fund rate, first row) is perfectly procyclical and highly volatile, the average interest rate charged on loans (fourth row) is only slightly procyclical and even more volatile. This result arises from the extreme volatility and weak cyclicality of the loan premium (the last two rows): During recessions, lenders raise the default premium, and thus the interest rate of loans relative to capital, because they expect defaults to be higher. When the interest rate of loans is higher during recessions, households borrow less, which offsets the rise in the average loan rate during recessions. This is why the average loan rate turns out to be relatively acyclical.

The second and third rows of Table 5 show that loans play a very small role in the performance of the mutual fund. For the mutual fund, what matters is the return on physical capital (in our baseline model, aggregate capital stock is 300 percent of output on average, and the total amount of debt is 1.3 percent). Moreover, the correlation between both components of the mutual fund is almost one, thereby easing any concern that the mutual fund may allocate its funds differently in search of some form of insurance.

We want to reemphasize at this point that we should think of lenders in this economy as banks with

100 percent reserves, since not only do poorly performing loans have low volatility, but also any possible losses are absorbed by the mutual fund without reducing the capacity to make additional loans.

At this point, we have answered the first question addressed in this paper. The standard theory of unsecured credit aligns with the cyclical behavior of credit and bankruptcy in the U.S. data. We will see in Section 7.1 how the cyclicality of the variance of earnings qualifies this answer.

Still, even though the model economy has the right properties of credit and bankruptcies relative to the data and the right volatility of bankruptcy filings, the volatility of credit is only about one-third that of the U.S. data. Moreover, although the correlations in the model have the right sign relative to the data, they are too large. One possible explanation for this disparity is that the credit variable in the data is gross revolving consumer credit, whereas in the model we use negative net asset positions, which are not exactly the same. Another clear possibility is that additional shocks may affect the credit channel directly.

6.2. The Contribution of Access to Credit to Business Cycle Fluctuations

We now turn to answering the second question addressed in this paper: Does access to credit contribute to fluctuations in the business cycle? We pose the question by comparing the business cycle properties of the baseline economy in which households have access to unsecured credit that is characteristic of that in the U.S. economy (i.e., borrowers have the right to file for bankruptcy unilaterally — a right that many people exercise) with the properties of otherwise identical economies in which households either do not have access to credit or do have access to credit but without the possibility of filing for bankruptcy.[7] Notice that we use the same shocks that generate aggregate fluctuations. Table 6 displays the findings.

As Table 6 shows, the volatility of output is highly similar across all economies. The economy with

[7] Table A3 in the appendix reports the steady-state properties of these economies.

Table 6: Cyclical Properties of the Baseline Economy with Varying Types of Credit

	Credit and Bankruptcy		No Credit		Credit No Bankruptcy	
	Stand Dev %	Corr w Output	Stand Dev %	Corr w Output	Stand Dev %	Corr w Output
Output	1.20	1.00	1.19	1.00	1.19	1.00
Consumption	0.47	0.98	0.40	0.98	0.38	0.98
Investment	3.52	1.00	3.71	1.00	3.81	1.00
Hours	0.63	1.00	0.61	1.00	0.61	1.00
Credit	1.28	0.80	.	.	0.73	−0.80
Bankruptcy	19.98	−0.90	.	.	.	.
Loan premium	7.19	0.17	.	.	.	.
Loan return	1.35	0.95	.	.	0.03	0.35
Mutual fund return	0.16	0.99	0.15	0.99	0.16	0.98
Capital return	0.16	0.99	0.15	0.99	0.15	0.98
Debtors	1.11	0.68	.	.	0.65	−0.79

access to credit and bankruptcy is slightly more volatile, due to the slightly higher volatility of hours. The volatility of the different components of output varies more across economies. In the baseline economy (the one in which households have access to credit and can file for bankruptcy), consumption is much more volatile — about 15-20 percent more volatile — than in the other two economies. As a consequence, investment is less volatile. The economy in which households have access to credit but no possibility of filing bankruptcy has less volatile consumption — not only less volatile than that in the baseline, but also less volatile than in the economy with no access to credit.

The rest of the cyclical properties are almost identical between the economies in which households have no possibility of defaulting. As is evident, the intuition that access to credit helps households to smooth their consumption in times of financial distress is applicable only when there is no possibility of defaulting and hence no interest rate premium when borrowing. But even in this case, the differences between that economy and the economy with no access to credit are small.

We conclude that the combination of access to credit and the possibility of filing for bankruptcy makes consumption more, not less, volatile, and that the role of the interest rate premium is fundamental in shaping the cyclical properties of credit.

Table 7: Cyclical Properties of Economies without Countercyclical Earnings Variance

	Credit and Bankruptcy		No Credit		Credit No Bankruptcy	
	Stand Dev %	Corr w Output	Stand Dev %	Corr w Output	Stand Dev %	Corr w Output
Output	1.20	1.00	1.20	1.00	1.20	1.00
Consumption	0.38	0.95	0.37	0.97	0.34	0.98
Investment	3.81	1.00	3.82	1.00	3.99	1.00
Hours	0.56	0.99	0.54	1.00	0.56	1.00
Credit	0.85	−0.63	.	.	0.89	−0.88
Bankruptcy	9.25	−0.35	.	.	.	.
Loan premium	6.25	0.04	.	.	.	.
Loan return	0.43	0.72	.	.	0.04	0.18
Mutual fund return	0.16	0.98	0.15	0.98	0.16	0.98
Capital return	0.16	0.98	0.15	0.98	0.16	0.98
Debtors	0.93	−0.59	.	.	0.80	−0.84

7. Analysis of Nonstandard Features

Besides credit and bankruptcy, our baseline model economy has two relatively nonstandard features. One feature is the countercyclical earnings variance documented by Storesletten et al. (2004). The other feature is the explicit distinction between large and small recessions, which allows us to analyze the impact of the Great Recession.

7.1. No Countercyclical Earnings Variance

To explore its importance, we have recalibrated the baseline to eliminate the dependence of the variance of the idiosyncratic risk on the aggregate state of the economy.[8] The most important change is a higher variance of the process for the aggregate shock, which is orchestrated in order to achieve the same volatility of output in the version of the economy with access to credit and the possibility of filing for bankruptcy. Table 7 displays the properties of the three versions of this economy that differ with respect to the type of credit available.

[8] Tables A3 and A4 in the appendix show the details of how the economies are recalibrated.

Without the countercyclical earnings variance, the details of the credit markets matter less. Still, access to credit and bankruptcy yields a higher variance of consumption than the other two types of credit arrangements, but only about 2 and 10 percent more, respectively.

In comparing the baseline economy with access to credit and bankruptcy and a countercyclical variance of earnings with the economy with the same credit arrangements but no countercyclical variance of earnings, we see some important differences. In the baseline economy, credit is highly volatile and *procyclical*, as it is in the U.S. data. In the economy with no countercyclical variance of earnings, however, credit is *countercyclical* and slightly less volatile than output. This difference is, in part, the result of the lower loan premium arising from fewer bankruptcies in the economy without a countercyclical earnings variance. We believe that this feature, along with the slightly higher volatility of hours displayed in the baseline economy, points to the need for modeling recessions not as situations in which all households fare slightly worse than normal, but as situations in which more households than normal fare very poorly.

The answer to our second question, however, is not affected by the details of how we model aggregate fluctuations. In all cases, access to credit and default that is characteristic of the U.S. economy increases the volatility of consumption and hours, although the role of that access is more important in a credit arrangement with countercyclical earnings variance.

We have also explored the robustness of these findings to recalibrating the various model economies when we change the extent of access to credit to ensure that total wealth is the same. We conduct this exercise because different forms of access to credit imply different amounts of precautionary savings in the economy. Table A5 in the appendix shows the business cycle properties of versions of the baseline economy with no credit and with credit but no option of default, calibrated to have the same aggregate capital as in the baseline model economy. The cyclical properties of these economies are identical to those in Table 6. Whether to recalibrate the discount factor (β) so that the model economies have the same amount of capital does not matter in terms of business cycle properties.

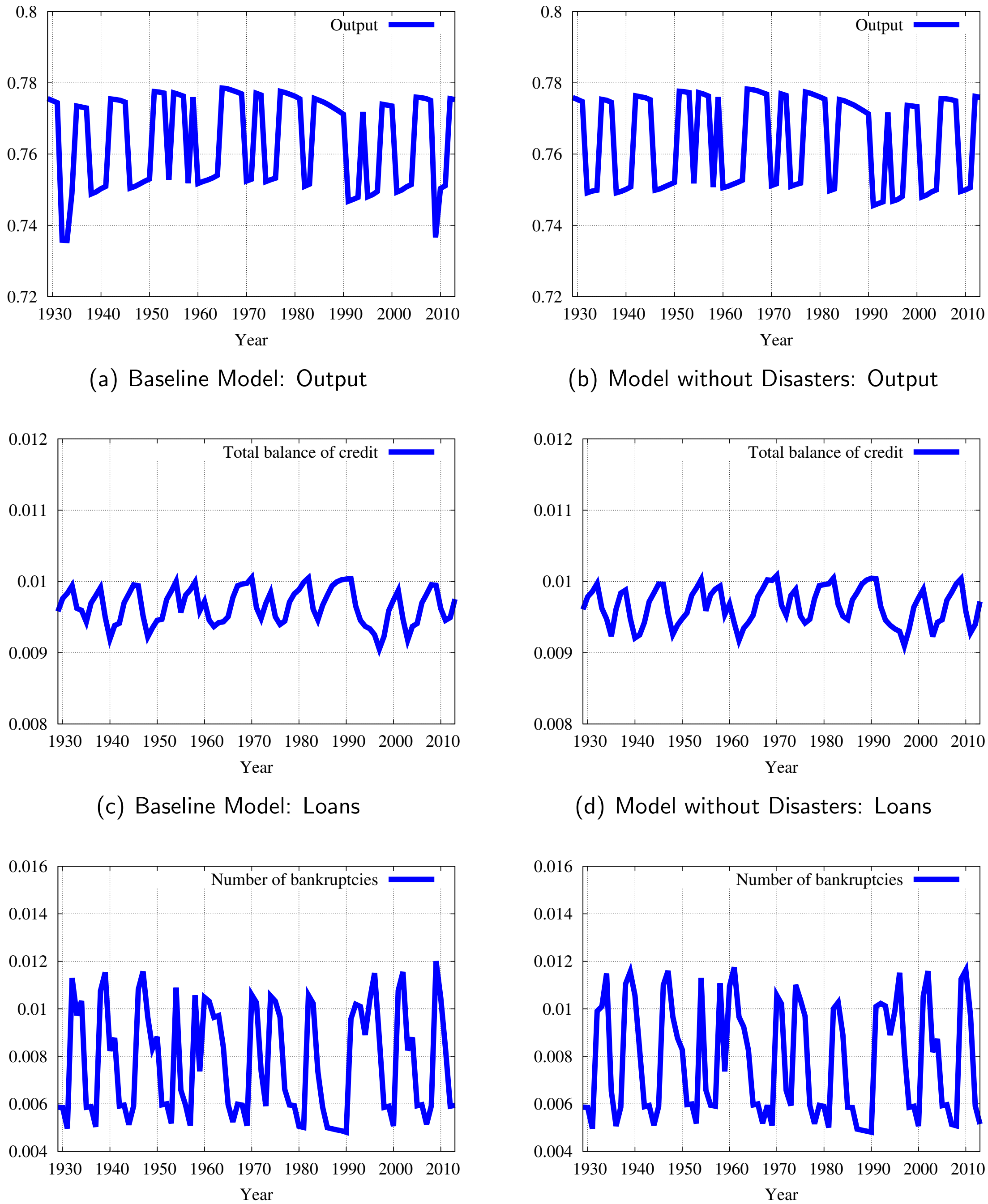
7.2. Credit, Default, and Great Recessions

To see whether our assumption of having one state in which the recession is larger than usual makes any difference to our findings, we compared two versions of the model economy. In the first, there are two types of recessions (large and small — the baseline economy). In the second, all recessions are of the same (normal) type. The quantitative properties of these two economies are found to be virtually identical. Figure 1 shows the sample paths of the two economies: In one, large recessions or disasters occur in 1929, 1930, and 2008 (left panels); in the other economy, the recessions at these dates are of normal size (right panels).

The two economies have very similar paths for all variables except for output, which is exogenously set to differ at those dates. Remember that, in the model with disasters, TFP in disasters declines twice as much as in normal recessions. The amounts of loans and the number of bankruptcies are very similar in both economies. We conclude that the efforts to model a situation resembling the Great Recession do not change the answers that we obtain — at least to the extent that disasters are simply large reductions in average productivity and do not further increase (beyond the fraction typical in a normal recession) the fraction of households that are faring very poorly.

8. Conclusion

We have explored how access to credit affects the nature of business cycles, and we have asked two questions. First, does the standard theory of unsecured credit account for the high volatility and procyclicality of credit as well as the high volatility and countercyclicality of bankruptcy filings found in U.S. data? We find that it does, but only if we explicitly model recessions as displaying countercyclical earnings variance. That is, rather than having all households fare slightly worse than normal during recessions, we ensure that more households than normal fare very poorly. The second question asks whether explicitly modeling access to credit smooths aggregate consumption or aggregate hours worked.



(a) Baseline Model: Output

(b) Model without Disasters: Output

(c) Baseline Model: Loans

(d) Model without Disasters: Loans

(e) Baseline Model: Bankruptcies

(f) Model without Disasters: Bankruptcies

Figure 1: Simulation Results: Model Economies with Disasters (Left) and without (Right)

We have found that it does not. The crucial mechanism at work here is that the interest premium in recessions increases because the high risk of bankruptcy discourages households from using credit in these situations. This finding contradicts the intuition that access to credit helps households to smooth their consumption. Such intuition is valid only in environments that do not include the right to default — a right enshrined in the U.S. Constitution.

Some questions remain. The volatility of credit in our model economies is still lower than in the data, pointing to the possibility that credit availability may itself be a source of business cycle fluctuations. In addition, bankruptcy filings are too procyclical, indicating that we have yet to understand some of the subtleties involved in these economies.

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Appendix

A. Computation Appendix

This section provides an overview of the algorithm that solves the baseline model with countercyclical variance of individual productivity. As we discussed in Section 5, we assume that (z, K, S) rather than (z, K, m) is the set of aggregate state variables. We first define the approximate equilibrium with (z, K, S) as the state variables. We then describe the solution algorithm of the approximate equilibrium. At the end of this section, we argue that future aggregate state variables are precisely forecasted by (z, K, S) or (z, z', K, S), which justifies the use of approximation.

A.1. Definition of Approximate Equilibrium

Definition 2 *An approximate recursive equilibrium is a value function,* $V(z, K, S, x, h, a)$*, associated decision rules,* $g^c(z, K, S, x, h, a)$*,* $g^a(z, K, S, x, h, a)$*,* $g^\ell(z, K, S, x, h, a)$*, and* $g^h(z, K, S, x, a)$*, pricing functions,* $r_K(z, K, L, S)$*,* $w(z, K, L, S)$*, and* $q(z, K, S, x, a')$*, and forecasting functions,* $\phi_r(z, K, S)$*,* $\phi_L(z, K, S)$*,* $\phi_K(z, K, S)$*, and* $\phi_S(z, z', K, S)$*, such that*

1. **Household optimization.** *Given pricing functions and forecasting functions,* $V(z, K, S, x, h, a)$ *is a solution of the household's optimization problem, and* $g^c(z, K, S, x, h, a)$*,* $g^a(z, K, S, x, h, a)$*,* $g^\ell(z, K, S, x, h, a)$*, and* $g^h(z, K, S, x, a)$ *are associated decision rules.*
2. **Forecasted aggregate labor.** *Aggregate labor in efficiency units, which is computed by condition (19), is forecasted by* $\phi_L(z, K, S)$*.*
3. **Expected zero profit condition for unsecured credit industry.** *Given pricing and forecasting functions,* $q(z, K, S, x, a')$ *satisfies the expected zero profit condition* (16)*.*
4. **Competitive factor prices.** $r_K(z, K, L, S)$ *and* $w(z, K, L, S)$ *satisfy* (23) *and* (18)*, respectively.*
5. **Forecasted mutual fund return.** *The rate of return on mutual funds, which is computed by* (22)*, is forecasted by* $\phi_r(z, K, S)$*.*
6. **Consistency/market clearing.** *Average individual labor productivity in the next period is forecasted by* $\phi_S(z, z', K, S)$*. Aggregate capital in the next period, which is computed by condition* (21)*, is forecasted by* $\phi_K(z, K, S)$*.*

A.2. Computation Algorithm

We specify the forecasting functions in the following log-linear functional form:

$$\log K' = \log \phi_K(z, K, S) = \phi^0_{K,z} \log K + \phi^1_{K,z} \log S, \tag{A1}$$

$$\log L = \log \phi_L(z, K, S) = \phi^0_{L,z} \log K + \phi^1_{L,z} \log S, \tag{A2}$$

$$\log r = \log \phi_r(z, K, S) = \phi^0_{r,z} \log K + \phi^1_{r,z} \log S, \tag{A3}$$

$$\log S'(z') = \log \phi_S(z, z', K, S) = \phi^0_{S,z,z'} \log K + \phi^1_{S,z',z'} \log S. \tag{A4}$$

Notice that average labor productivity in the next period, S', depends on the current aggregate state (z, K, S) and on the realization of the TFP shock in the next period z'. We are looking for parameters $\Phi = \{\phi^0_{K,z}, \phi^1_{K,z}, \phi^0_{L,z}, \phi^1_{L,z}, \phi^0_{r,z}, \phi^1_{r,z}, \phi^0_{S,z,z'}, \phi^1_{S,z,z'}\}$ for $\forall z, z'$ that satisfy condition (6) of the definition of approximate equilibrium.

Algorithm 1 (Solution Algorithm for the Approximate Recursive Equilibrium)

1. *Guess parameters of forecasting functions Φ^0. The initial guess can be obtained by running the complete market version of the model and running regressions with simulated data. With Φ^0, $(K', L, r, S'(z'))$ can be forecasted. Furthermore, using forecasted L, wage w can be computed using marginal conditions.*

2. *Guess the bond price function $q^0(z, K, S, x, a')$.*

3. *Guess the value function $V^0(z, K, S, x, h, a)$.*

4. *Using the Bellman equations described in Section 3.7, obtain the updated value function $V^1(z, K, S, x, h, a)$.*

5. *Check convergence of the value function. If the distance between $V^0(z, K, S, x, h, a)$ and $V^1(z, K, S, x, h, a)$ is smaller than a predetermined tolerance criterion, convergence is achieved. Go to the next step. Otherwise, update the value function using $V^1(z, K, S, x, h, a) = V^1(z, K, S, x, h, a)$ and go back to step 3.*

6. *Record the optimal decision rules $g^c(z, K, S, x, h, a)$, $g^a(z, K, S, x, h, a)$, $g^\ell(z, K, S, x, h, a)$, and $g^h(z, K, S, x, a)$ associated with the converged value function.*

7. *Using the optimal decision rules obtained in the previous step and equation (16), compute the updated bond price function $q^1(z, K, S, x, a')$.*

8. *Check convergence of the bond price function. If the distance between $q^0(z, K, S, x, a')$ and $q^1(z, K, S, x, a')$ is smaller than a predetermined tolerance criterion, convergence is achieved. Go to the next step. Otherwise, update the bond price function using the following formula (where ψ_q controls the speed of updating) and go back to step 2.*

$$q^{0,new}(z, K, S, x, a') = (1 - \psi_q)\ q^{0,old}(z, K, S, x, a') + \psi_q\ q^1(z, K, S, x, a').$$

9. *Simulate the model for $N_0 + N_1$ periods. Simulating the model gives the sequence of aggregate variables $\{z_t, K_t, S_t(z'), L_t, r_t\}_{t=1}^{N_0+N_1}$.*

   (a) *Use the type distribution in the steady state as the type distribution in period 1.*

   (b) *At the beginning of each period, (z, K, S) are known. The type distribution of households, m, is also available. Using the forecasting functions, $(\widetilde{K}', \widetilde{L}, \widetilde{r}, \widetilde{S}'(z'))$ are forecasted.*

   (c) *Wage w can be computed using $(z, K, \widetilde{L})$.*

   (d) *Furthermore, by solving the optimization problem of households in the next period, conditional on z', the exact bond prices $q(x, a')$ in the current period can be computed.*

   (e) *Solve the household's optimization problem to obtain optimal decision rules.*

   (f) *Using the type distribution of households and optimal decision rules, true $(K', L, r, S'(z'))$ can be computed. The updated distribution m' can also be computed.*

   (g) *Once z' is drawn, the next period aggregate state (z', K', S') is obtained.*

10. *Using the sequence of aggregate variables $\{z_t, K_t, S_t(z'), L_t, r_t\}_{t=N_0+1}^{N_1}$, update the forecasting function. Notice that the first N_0 periods are dropped. Use ordinary least squares (OLS) regression. Regressions then give updated parameters of forecasting functions Φ^1.*

11. *Check convergence of the forecasting functions. If the distance between Φ^0 and Φ^1 is smaller than a predetermined tolerance criterion, an approximate recursive equilibrium is obtained. Otherwise, update the forecasting functions using the following formula (where ψ_ϕ controls the speed of updating) and go back to step 1.*

$$\Phi^{0,new} = (1 - \psi_\phi)\ \Phi^{0,old} + \psi_\phi\ \Phi^1.$$

A.3. Predictive Power of Forecasting Functions

Table A1 summarizes the predictive power of the forecasting function in the baseline model economy with countercyclical variance of individual labor productivity represented by the adjusted R^2. We make three remarks. First, perhaps not surprisingly, the adjusted R^2 is extremely high for S'. Even though the distribution of individual productivity is exogenous, there is a loss in predictive power by not keeping track of the exact type distribution. Second, the adjusted R^2 for L and r when a disaster state $z = z_3$ hits the economy is slightly lower than in the other aggregate states, because a disaster shock affects the type distribution of households in a relatively significant manner. Third, our assessment of the performance of the forecasting functions is more than satisfactory for our purposes.

B. Additional Tables

This appendix contains additional details of the models discussed in the main text. Table A2 reports the details of the implied earnings, income, and wealth distribution.

Table A1: Predictive Power of Forecasting Functions

	Aggregate TFP		
	z_1	z_2	z_3
K'	0.99998	0.99998	0.99996
L	0.99975	0.99960	0.99614
r	0.99979	0.99924	0.99505
$S'(z' = z_1)$	1.00000	1.00000	1.00000
$S'(z' = z_2)$	1.00000	1.00000	1.00000
$S'(z' = z_3)$	1.00000	1.00000	1.00000

Note: Adjusted R^2 of the forecasting functions in the baseline model economy.

Table A2: Distributional Statistics

Statistics	Earnings	Income	Wealth
Coefficient of Variation	0.819	0.730	1.460
Gini index	0.417	0.370	0.649
1st Quintile	0.040	0.059	-0.002
2nd Quintile	0.102	0.111	0.033
3rd Quintile	0.160	0.163	0.093
4th Quintile	0.241	0.236	0.216
5th Quintile	0.458	0.431	0.660
Top 5%	0.170	0.158	0.290
Top 1%	0.048	0.044	0.089

Table A3 compares the steady-state statistics of the baseline model, the model without credit, the model without bankruptcy (but with credit), and the model without countercyclical earnings risk. Cyclical properties of the first three models are compared in Table 6. In the model without credit, all the parameters are the same as in the baseline model economy with the borrowing constraint, which is now exogenous, set to zero. In the model without bankruptcy, again, all the parameters are the same as in the baseline model economy. The exogenous borrowing constraint is set such that the total amount of debt is twice as large as in the baseline model economy.

Table A4 compares the parameters of the baseline model economy and those of the model economy without countercyclical variance of individual earnings. Since the two model economies are the same in the steady state, the parameters calibrated using the steady-state version of the model (top panel of the table) are the same. The only difference is the size of the TFP shock. In the model economy without countercyclical earnings variance, the size of the TFP shock is recalibrated so that the standard deviation of log detrended output is 1.2 percent, which is its empirical value. Since the average individual labor productivity in the baseline model moves countercyclically, the size of the TFP shock is smaller without countercyclical earnings variance (1.24 percent versus 1.34 percent).

Table A5 corresponds to Table 6, but here we recalibrate β in the economies without credit or

Table A3: Comparison of Steady-State Statistics

Model	Baseline	No default[a]	No loans[b]	No CER[c]
Capital	2.2882	2.2737	2.3322	2.2882
Debt	0.0098	0.0197	–	0.0098
Hour	0.3301	0.3314	0.3295	0.3301
Output	0.7626	0.7619	0.7674	0.7626
Return of capital	0.0400	0.0406	0.0385	0.0400
Wage	1.1874	1.1838	1.1959	1.1874
Total income	0.5800	0.5808	0.5809	0.5800
Proportion in debt	0.0839	0.1409	–	0.0839
Proportion defaulting	0.0086	–	–	0.0086
Capital/Output	3.0004	2.9842	3.0389	3.0004
Debt/Output	0.0129	0.0259	–	0.0129
Average debt	0.1169	0.1402	–	0.1169
Debt/Income	0.2016	0.2413	–	0.2016
Coef. of var. of earnings	0.8194	0.8183	0.8180	0.8194

[a] Baseline economy parameters except for the borrowing limit, which is set at twice the average debt of the baseline model.

[b] Baseline economy parameters except for the borrowing limit, which is set at zero.

[c] Exactly the same in the steady-state economy.

bankruptcy, in order to keep the total amount of savings the same across different models. In Section 6, we keep the value of β and thus do not control the total amount of savings in the alternative model economies. By comparing Table A5 and Table 6, it is easy to see that recalibrating β is not important in shaping the cyclical properties of the model.

Table A6 is the expanded version of Table 6. The table also compares the cyclical properties of the baseline model economy with the alternative model economies without credit and without bankruptcy, but it contains more variables than Table 6.

Table A4: Comparison of Model Parameters

	Description	Baseline	No CER
β	Discount factor	1.0011	1.0011
$\mathbb{E}\beta$	Average discount factor[a]	0.9701	0.9701
σ	Curvature of utility function	2.0000	2.0000
η	Cobb-Douglas parameter for leisure	0.3681	0.3681
π	Survival probability	0.0200	0.0200
b_1	Good preference shock (normalization)	1.0000	1.0000
b_2	Bad preference shock	0.0000	0.0000
Γ_2^b	Prob of bad preference shock	0.0310	0.0310
θ	Curvature of production function	0.3600	0.3600
δ	Depreciation rate	0.0800	0.0800
σ_e	S.D. of permanent shock	0.4400	0.4400
ρ_p	Persistence of productivity shock	0.9630	0.9630
$\sigma_{p\|1}$	S.D. of persistent shock in expansions	0.0880	0.0880
$\sigma_{p\|2}$	S.D. of persistent shock in recessions	0.1620	0.1620
σ_t	S.D. of transitory shock	0.3500	0.3500
χ	Adjustment factor for productivity shocks	0.7500	0.7500
λ	Prob of default history erased	0.1000	0.1000
ξ	Income garnishment rate	0.3395	0.3395
$\nu_1 = \nu_2$	Size of TFP shock (normal)	0.0134	0.0124
ν_3	Size of TFP shock (disaster)	0.0267	0.0247
$\gamma_{1,1}^z$	Persistence of good TFP shock	0.6667	0.6667
$\gamma_{2,2}^z$	Persistence of bad TFP shock	0.6667	0.6667
$\gamma_{3,3}^z$	Persistence of disastrous TFP shock	0.3333	0.3333
γ_3^z	Frequency of disastrous TFP shock	0.0200	0.0200
$\underline{a}$	Borrowing limit[b]	–0.2340	–0.2340
$\underline{a}$	Borrowing limit[c]	0.0000	0.0000

[a] Take into account the shock to discount factor (b).
[b] For the economy without default.
[c] For the economy without loans.

Table A5: Main Cyclical Properties of the Baseline Economy with and without Access to Credit (β Recalibrated)

	Credit and Bankruptcy		No Credit		Credit No Bankruptcy	
	Stand Dev %	Corr w Output	Stand Dev %	Corr w Output	Stand Dev %	Corr w Output
Output	1.20	1.00	1.19	1.00	1.19	1.00
Consumption	0.47	0.98	0.40	0.98	0.38	0.98
Investment	3.52	1.00	3.75	1.00	3.80	1.00
Hours	0.63	1.00	0.61	1.00	0.61	1.00
Credit	1.28	0.80	.	.	0.73	−0.80
Bankruptcy	19.98	−0.90	.	.	.	.
Loan premium	7.19	0.17	.	.	.	.
Loan return	1.35	0.95	.	.	0.03	0.35
Mutual fund return	0.16	0.99	0.15	0.99	0.15	0.98
Capital return	0.16	0.99	0.15	0.99	0.15	0.98
Debtors	1.11	0.68	.	.	0.66	−0.79

Table A6: Cyclical Properties of the Baseline Economy (Expanded)

Variable	SD%	Relative SD%	Auto-corr	Cross-Correlation of Output with x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}
Model with Credit and Default								
Output	1.20	1.00	−0.11	−0.28	−0.11	1.00	−0.11	−0.28
Consumption	0.47	0.39	−0.06	−0.33	−0.20	0.98	0.02	−0.22
Investment	3.52	2.93	−0.12	−0.26	−0.07	1.00	−0.16	−0.30
Capital	0.26	0.22	0.30	−0.45	−0.49	0.68	0.46	0.08
Credit (excl interests)	1.28	1.06	0.15	−0.31	−0.26	0.80	0.27	−0.14
Credit (incl interests)	0.94	0.79	−0.05	−0.19	−0.19	0.38	0.34	−0.02
Hours	0.63	0.52	−0.10	−0.31	−0.16	1.00	−0.05	−0.25
Wage	0.83	0.69	−0.07	−0.34	−0.22	0.98	0.03	−0.21
Capital return	0.16	0.13	−0.11	−0.22	−0.01	0.99	−0.23	−0.34
Loan return	1.35	1.12	−0.30	−0.20	−0.03	0.95	−0.38	−0.23
Mutual fund return	0.16	0.13	−0.12	−0.22	−0.01	0.99	−0.23	−0.34
Defaults	19.98	16.64	−0.04	0.26	0.15	−0.90	−0.03	0.28
Debtors	1.11	0.93	0.20	−0.26	−0.21	0.68	0.29	−0.12
Average debt	0.41	0.34	−0.11	−0.27	−0.23	0.65	0.07	−0.12
Loan premium (expected)	7.19	5.99	−0.10	0.11	0.25	0.17	−0.76	−0.02
Loan premium (realized)	6.79	5.65	−0.02	0.16	0.26	0.01	−0.73	0.02
Model without Credit								
Output	1.19	1.00	−0.11	−0.28	−0.11	1.00	−0.11	−0.28
Consumption	0.40	0.33	−0.06	−0.34	−0.21	0.98	0.03	−0.21
Investment	3.71	3.11	−0.11	−0.26	−0.07	1.00	−0.15	−0.30
Capital	0.26	0.22	0.30	−0.45	−0.49	0.68	0.46	0.08
Hours	0.61	0.51	−0.10	−0.31	−0.16	1.00	−0.05	−0.25
Wage	0.84	0.70	−0.07	−0.35	−0.22	0.98	0.03	−0.20
Capital return	0.15	0.13	−0.11	−0.22	−0.01	0.99	−0.23	−0.34
Mutual fund return	0.15	0.13	−0.11	−0.22	−0.01	0.99	−0.23	−0.34
Model with Credit but without Default								
Output	1.19	1.00	−0.10	−0.28	−0.10	1.00	−0.10	−0.28
Consumption	0.38	0.32	−0.06	−0.34	−0.21	0.98	0.03	−0.21
Investment	3.81	3.20	−0.11	−0.26	−0.07	1.00	−0.15	−0.30
Capital	0.27	0.22	0.30	−0.45	−0.49	0.68	0.46	0.08
Credit (excl interests)	0.73	0.61	0.16	0.44	0.46	−0.80	−0.36	0.03
Credit (incl interests)	0.72	0.61	0.19	0.44	0.48	−0.77	−0.39	0.01
Hours	0.61	0.51	−0.10	−0.31	−0.16	1.00	−0.05	−0.25
Wage	0.83	0.70	−0.07	−0.34	−0.22	0.98	0.03	−0.21
Capital return	0.16	0.13	−0.11	−0.22	−0.00	0.98	−0.23	−0.34
Loan return	0.03	0.03	0.16	−0.09	0.06	0.35	0.70	−0.58
Mutual fund return	0.15	0.13	−0.11	−0.22	−0.00	0.98	−0.23	−0.34
Debtors	0.65	0.55	0.09	0.43	0.46	−0.79	−0.28	0.02
Average debt	0.18	0.15	0.07	0.21	0.19	−0.36	−0.43	0.06

Note: Logs of the data are filtered using the H-P filter with a smoothing parameter of 6.25.