CAPITAL DEPRECIATION AND LABOR SHARES AROUND THE WORLD:
MEASUREMENT AND IMPLICATIONS

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ABSTRACT

The labor share is typically measured as compensation to labor relative to gross value added ("gross labor share"), in part because gross value added is more directly measured than net value added. Labor compensation relative to net value added ("net labor share") may be more important in some settings, however, because depreciation is not consumed. In this paper we make three contributions. First, we document that gross and net labor shares generally declined together in most countries around the world over the past four decades. Second, we use a simple economic environment to show that declines in the price of capital necessarily cause gross and net labor shares to move in the same direction, whereas other shocks such as a decline in the real interest rate may cause the net labor share to rise when the gross labor share falls. Third, we illustrate that whether the gross or the net labor share is a more useful proxy for inequality during an economy's transition depends sensitively on the nature of the underlying shocks that hit the economy.

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1 Introduction

A recent wave of empirical work has invigorated interest in theories of factor shares, capital accumulation, growth, and inequality. In Karabarbounis and Neiman (2014), we documented a pervasive decline in the labor share of income since 1975 and highlighted its comovement with the decreasing relative price of investment goods. Piketty (2014) and Piketty and Zucman (2014) similarly characterized this factor share movement and related it to increases in the capital-output ratio. The decline in the labor share and hypotheses about its causes have spurred a debate among economists and policymakers as they carry implications for the determinants and future evolution of inequality and growth.

Capital depreciation, often treated as an accounting construct of limited importance in macroeconomics, plays a critical role in this discussion.[1] Depreciation cannot be consumed, and therefore labor's share of income net of depreciation ("net labor share") may be more closely related to inequality than labor's share of income inclusive of depreciation ("gross labor share"). Most economic theory and empirical work, however, are based on gross concepts, in part because depreciation is not directly measured in national accounts.

In this paper, we make three contributions. First, we document that gross and net labor shares generally declined together in most countries around the world over the past four decades. Second, we use a simple economic environment to show how their joint movement is informative about the causes of their trends. Declines in the price of capital cause gross and net labor shares to move in the same direction, whereas declines in the real interest rate may cause the net labor share to rise when the gross labor share falls. Third, we illustrate that it is not obvious whether the gross or the net labor share is a more useful proxy for inequality during an economy's transition. Instead, the relationship between factors shares and inequality during the transition depends crucially on the nature and timing of the underlying shocks that hit the

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[1]Summers (2014) points out that the distinction between the marginal product of capital net of depreciation and its gross counterpart is important for the behavior of factor shares. Krusell and Smith (2014) argue that the exclusion of depreciation significantly changes Piketty's quantitative predictions of how a growth slowdown would impact the capital-output ratio, a point also recognized by Jones (2014). Additionally, Bridgman (2014) and Rognlie (2014) question the extent to which the observed declines in gross labor shares were accompanied by declines in net labor shares.

economy.

We begin our analysis by documenting the time series and cross country patterns of labor share trends. We use the dataset introduced in Karabarbounis and Neiman (2014) to examine trends in four labor share measures that differ in their sector coverage (total economy vs. corporate sector) and treatment of depreciation (gross vs. net). We focus particularly on estimates from the corporate sector as they offer several key advantages. Corporate labor share measures are less affected by the existence of unincorporated enterprises and proprietors whose reported income combines payments to both labor and capital, a well-known difficulty in the measurement of the labor share (Gollin, 2002). Further, gross and net corporate labor shares are largely insensitive to the measurement and economic interpretation of residential housing, a controversial topic in studies of the economy-wide labor share (Bonnet, Bono, Chapelle, and Wasmer, 2014; Jones, 2014; Rognlie, 2014; Acemoglu and Robinson, 2014).

All four measures of the labor share have decreased meaningfully since 1975. For the average country, gross and net labor share declines were similar in the corporate sector, and the net decline was even larger in the total economy. In addition, the cross-country variation in net labor share declines closely mirrors that in gross labor share declines. The pattern of declining labor shares in the corporate sector is somewhat different when we weight countries by their size, predominantly due to the influence of the United States. In the U.S. corporate sector, the value of depreciation has increased as a share of value added since 1975. As a result, its net labor share declined by 2.6 percentage points, compared to the 4.7 percentage point decline in its gross labor share. Finally, we confirm that both the time series and cross sectional patterns that we document are quantitatively similar in the newly released data from the Penn World Tables (PWT 8.0) that allows for measurement of depreciation for a larger sample of countries.

Next, we analyze the joint dynamics of depreciation and factor shares within a simple variant of the neoclassical growth model. Production combines labor with two types of capital, one that depreciates slowly and another that depreciates rapidly. Consistent with measurement practices in national accounts, depreciation as a share of gross value added in our model fluctuates in

response to shifts in the composition of capital and to changes in the aggregate capital-output ratio. An increase in depreciation as a share of gross value added causes the net labor share to increase relative to the gross labor share.

We use the model to derive simple analytical relationships that link steady state changes in gross and net labor shares to each other and to the underlying shocks. We start by considering a decline in the real interest rate, a case emphasized in important recent work by Rognlie (2014). Rognlie highlights a mismatch between Piketty's explanation for the behavior of net labor shares and estimates of the elasticity of substitution between capital and labor that typically come from studies, including our own, of gross labor shares. He demonstrates that, in response to declines in the real interest rate, the elasticity of substitution in net production is significantly smaller than that in gross production. Summers (2014), in his book review of Piketty, makes this point most forcefully:

> Piketty argues that the economic literature supports his assumption that returns diminish slowly (in technical parlance, that the elasticity of substitution is greater than 1), and so capital's share rises with capital accumulation. But I think he misreads the literature by conflating gross and net returns to capital. It is plausible that as the capital stock grows, the increment of output produced declines slowly, but there can be no question that depreciation increases proportionally. And it is the return net of depreciation that is relevant for capital accumulation. I know of no study suggesting that measuring output in net terms, the elasticity of substitution is greater than 1, and I know of quite a few suggesting the contrary.

We confirm in our environment the hypothesis of Summers (2014) and reproduce the finding of Rognlie (2014). A decline in the real interest rate causes the capital-output ratio to rise, which increases depreciation as a share of gross value added. For reasonable parameterizations, reductions in the real interest rate cause the net labor share to increase despite a decrease in the gross labor share. This happens when the elasticity of substitution in net production is less than one while the elasticity of substitution in gross production exceeds one.

By contrast, we show that gross and net labor shares always move in the same direction in response to technology-driven changes in the relative price of investment goods.[2] Declines in the

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[2]The decline in the relative price of investment can also explain other macroeconomic developments over the

price of capital mute the increase in depreciation's share of gross value added that arises from the higher real capital-output ratio. Whereas Summers (2014) and Rognlie (2014) emphasize that the real interest rate affects the net rental rate proportionately more than the gross rental rate, we emphasize that declines in the relative price of investment affect both gross and net returns to capital proportionally in steady state. As a result, if labor share movements are driven by declines in the relative price of investment, the elasticities of substitution in gross and net production are both on the same side of (or equal to) one.

Our empirical findings demonstrated that very few countries experienced opposite movements in gross and net labor shares over the past four decades. Our theoretical analysis points out that gross and net labor shares always move in the same direction in response to changes in the relative price of investment, but not necessarily in response to changes in the real interest rate. Collectively, our results can reconcile declines in the price of investment when the elasticity of substitution in gross production exceeds one, as highlighted by Karabarbounis and Neiman (2014), with the narrative of Piketty (2014) that rests on a high elasticity in net production.

Finally, we address the question of which labor share concept – gross or net – one should use. We clarify that, depending on data availability and the economic application, both concepts can be useful and complementary. Since depreciation is imputed by national statistical agencies, a preference by researchers for direct measurement argues for the use of gross concepts. On the other hand, unlike the rest of gross income, depreciation cannot be consumed by households. A body of work since at least Weitzman (1976), therefore, has argued that net concepts may be more closely associated with welfare and inequality than their gross counterparts. We point out that this logic most naturally applies in an economy's steady state. It is not obvious whether gross or net concepts are most useful for thinking about welfare and inequality during the economy's transition.

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past decades. As Greenwood, Hercowitz, and Krusell (1997) argue, technology-driven changes in the relative price of investment goods constitute a major driver of economic growth. Krusell, Ohanian, Rios-Rull, and Violante (2000) show that the increase in capital equipment is a key force for understanding the increase in the skill premium.

To illustrate this point, we introduce into our model two types of agents, workers and capitalists. Workers cannot save. The dynamics of consumption inequality between these two groups are governed by the assumption that capitalists, in contrast to workers, are forward looking and have a positive saving rate. The net labor share perfectly summarizes inequality between workers and capitalists in the steady state of our model as, in each period, workers consume their labor earnings and capitalists consume their capital income net of depreciation expenses. This simple relationship, however, ceases to hold along the transition. Intuitively, the net labor share only captures the net income position of workers relative to capitalists in a specific time period, whereas welfare-based measures of inequality take into account the entire future paths of consumption for workers and capitalists. We use several examples to compare gross and net labor share measures to welfare-based measures of inequality. A preference for the gross or net labor share as a simple proxy for inequality during an economy's transition depends sensitively on the nature and timing of the underlying shocks that hit the economy.

2 Depreciation and the Labor Share

The gross labor share is more commonly used than the net labor share, in part because it is better and more directly measured. The net labor share may be more appropriate in some settings, however, because depreciation is not consumed. In this section, we first discuss how depreciation is measured in the national accounts. We then demonstrate that most countries have exhibited meaningful and broadly similar declines since 1975 in both their gross and net labor shares.

2.1 Measurement of Depreciation

Depreciation, typically referred to as consumption of fixed capital in the national accounts, is the decline in the value of fixed capital due to tear, damage, obsolescence, and aging. It is the implicit income that is consumed by the use of fixed capital and therefore it is included in gross domestic product (GDP), the measure of gross value added for the aggregate economy. This inclusion of the value of depreciation in GDP does not, in general, require any explicit

measurement of depreciation.[3] The cost of consuming fixed capital during production is reflected in final goods prices, in the value of produced goods, and in factor prices. Depreciation will therefore be automatically incorporated when GDP is measured using the expenditure, production, or income approaches.

To measure net domestic product (NDP), which equals GDP less depreciation, national accountants combine estimates of depreciation rates with estimates of the value of the capital stock. In the United States, the Bureau of Economic Analysis (BEA) estimates depreciation rates since 1996 for more than 100 different types of capital, typically using the geometric method that imposes a constant rate of depreciation every period. Estimates of the geometric rate of decline in the economic value of assets come from studies of the prices of used assets in resale markets, as described in Fraumeni (1997). These asset-specific depreciation rates are then used to derive the capital stock for each type of asset and for the aggregate economy, along with the associated values of depreciation (Bureau of Economic Analysis, 2003).[4] Note that the depreciation rate for each type of capital is assumed to be constant. Changes in the depreciation rate of the aggregate capital stock purely reflect compositional changes in the types of capital used by the economy.

The 1993 System of National Accounts (SNA) recommends measurement of depreciation in a way that is intended to be theoretically appropriate and comparable across countries. The international comparability of measured depreciation depends on the coverage of fixed assets and the depreciation rates used. As OECD (2014) recognizes, depreciation practices differ across countries with some countries using geometric profiles and others using straight-line profiles (corresponding to a value of depreciation for each asset and each period that is constant in levels terms rather than in percentage terms). For example, the national accounts in

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[3]There are some exceptions to this. For example, estimates of depreciation are used to calculate the value added from non-market producers such as the government, which also contribute to GDP.

[4]For a given type of capital j, let $K_{t,i}^j$ be the real stock at the end of period t contributed from investment undertaken during period i. Assuming that real investment X_i^j is purchased in the middle of year i, we have $K_{t,i}^j = (1 - \delta^j/2)(1 - \delta^j)^{t-i} X_i^j$, where δ^j is the depreciation rate. The total real stock net of depreciation from all vintages of investment is $K_t^j = \sum_{i=1}^t K_{t,i}^j$. Finally, the nominal value of depreciation for each asset j is then measured as $\xi_t^j (K_{t-1}^j + X_t^j - K_t^j)$, where ξ_t^j is the price index used to deflate nominal investment in asset j and $K_{t-1}^j + X_t^j - K_t^j$ represents the real value of depreciation of asset j.

the United Kingdom apply the straight-line method to derive estimates of depreciation and the net capital stock (Dey-Chowdhury, 2008). Some international databases such as the PWT and KLEMS attempt to make depreciation patterns more comparable across countries by applying the depreciation rates used by the BEA to each country.

Some analyses may call for a theoretical focus on the net labor share in addition to, or instead of, the gross labor share. As an empirical matter, however, the difficulty in imputing depreciation likely makes net labor share measures less precise than gross labor share measures. This same observation underlies the pervasive use of GDP rather than NDP in discussions of aggregate economic performance. As described in the 1993 SNA, "In general, the gross figure is obviously the easier to estimate and may, therefore, be more reliable...".

2.2 Results from National Accounts Data

Our primary analyses use the national accounts dataset introduced in Karabarbounis and Neiman (2014), which combines a number of sources including country-specific web pages and digital files and physical books published by the United Nations (UN) and Organization for Economic Cooperation and Development (OECD).[5] Our data are annual, start in 1975, and generally conform with SNA standards, though there are certainly some differences in conventions across countries and over time. Importantly for our purposes, we make use of the "Detailed National Accounts," which allow us to distinguish between government, household, and corporate flows.

We now introduce four different concepts of the labor share of income s_L. All four labor shares express a measure of compensation of employees as a fraction of a measure of income.[6]

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[5]We refer the reader to Karabarbounis and Neiman (2014) for details on the construction of these data. The analyses presented in this paper use the "KN Merged" version of our dataset. Our qualitative conclusions do not change if we instead use the "OECD and UN" version, which is based only on data digitally available and readily downloaded from OECD and UN web pages with minimal additional manipulations.

[6]Typical labor share measures include taxes on production in income but not in the compensation of employees. Therefore, this practice effectively treats all taxes as capital income. A reasonable alternative treatment of these taxes is to augment compensation of employees with a proportion of taxes that equals the share of non-tax value added belonging to compensation. We document in the Online Appendix that the tax share of value added has not changed significantly either in the United States or globally since 1975. An adjustment for taxes, therefore, does not matter quantitatively for the decline in the labor share. This evidence argues against the claim made in Bridgman (2014) that taxes on production meaningfully alter the declining trend in the labor share.

They differ in their treatment of depreciation as well as their sectoral coverage. The four labor share measures are:

1. "Total Gross Labor Share":

$$s_L^{TG} = \frac{\text{Total Compensation of Employees}}{\text{Gross Domestic Product}}.$$

2. "Total Net Labor Share":

$$s_L^{TN} = \frac{\text{Total Compensation of Employees}}{\text{Gross Domestic Product} - \text{Total Depreciation}}.$$

3. "Corporate Gross Labor Share":

$$s_L^{CG} = \frac{\text{Corporate Compensation of Employees}}{\text{Corporate Gross Value Added}}.$$

4. "Corporate Net Labor Share":

$$s_L^{CN} = \frac{\text{Corporate Compensation of Employees}}{\text{Corporate Gross Value Added} - \text{Corporate Depreciation}}.$$

We index these four labor share measures with superscript $j \in \{\text{TG}, \text{TN}, \text{CG}, \text{CN}\}$ and denote the labor share of type j in country i at year t by $s_{L,i,t}^j$.

Across all notions of the labor share, compensation of employees includes wages and salaries in cash or in kind. Compensation also includes various supplements to wages such as employer contributions for sickness, pensions, health insurance, and social insurance. It also tries to capture the estimated value of stock option grants (Lequiller, 2002).

The first two labor share measures, $j \in \{TG, TN\}$, are taken from the overall economy. The "Total Gross Labor Share" is the most commonly used measure and simply divides compensation of employees by GDP. The "Total Net Labor Share" has the same numerator, but it instead subtracts depreciation in the economy from GDP and uses that net income concept, equal to NDP, in the denominator.[7]

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[7]In U.S. data, "Total Compensation of Employees" equals Line 2 from the U.S. Bureau of Economic Analysis National Income and Product Accounts (NIPA) Table 1.12. "Gross Domestic Product," can be found in Line 1 of NIPA Table 1.1.5. "Total Depreciation" is typically referred to in the national accounts as consumption of fixed capital and can be found for the U.S. economy as Line 5 in NIPA Table 1.7.5. Equivalently, "Net Domestic Product" can be used directly as the denominator instead of GDP minus depreciation and is found in Line 1 of NIPA Table 1.9.5.

The second two labor share measures, $j \in \{CG, CN\}$, use the same corresponding concepts in the numerators and denominators, but with the measures taken only from the corporate sector, which includes both financial and non-financial corporations. "Corporate Gross Value Added", for example, is the contribution of the corporate sector to total GDP (and in fact is often referred to as Corporate GDP).[8] The precise definition of the corporate sector varies somewhat across countries. The sector generally includes most market producers located in a given country that are not controlled by the government and excludes most unincorporated enterprises and sole proprietors.[9] In both the United States and the world more generally, the corporate sector accounts for roughly 60 percent of GDP. This share has remained largely stable over our sample period.

2.2.1 Time Series Patterns Using Raw Data

We start our analysis of the time series patterns of these labor share measures at the global level by running regressions of the form:

$$s_{L,i,t}^{j} = \gamma_t^j + \gamma_i^j + \epsilon_{i,t}^j. \tag{1}$$

Each regression only includes countries with at least 15 observations of the corresponding labor share measure and excludes the few outliers with labor shares above 0.99 or below 0.01. We include country fixed effects γ_i^j because we do not have a balanced panel. Country fixed effects eliminate the influence of entry and exit of countries with different labor share levels on our inference about the time series behavior of the global labor share.

To show the time series behavior of the global labor share in the total economy, Figure 1 plots the year dummies γ_t^j from four regressions, all normalized to equal the average level of the corresponding labor share measure in 1975. The solid line, corresponding to the "Gross

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[8]In U.S. data, the required information on the corporate sector is found in NIPA Table 1.14. "Corporate Compensation of Employees" equals Line 4, "Corporate Gross Value Added" equals Line 1, "Corporate Depreciation" equals Line 2, and "Corporate Net Value Added" equals Line 3.

[9]Most unincorporated enterprises are included in the household sector and "can range from single persons working as street traders or shoe cleaners with virtually no capital or premises of their own through to large manufacturing, construction or service enterprises with many employees." (United Nations Statistical Commission, 2008). A small share of unincorporated enterprises may be included in the corporate sector as something called quasi-corporations if their business accounts are sufficiently detailed.
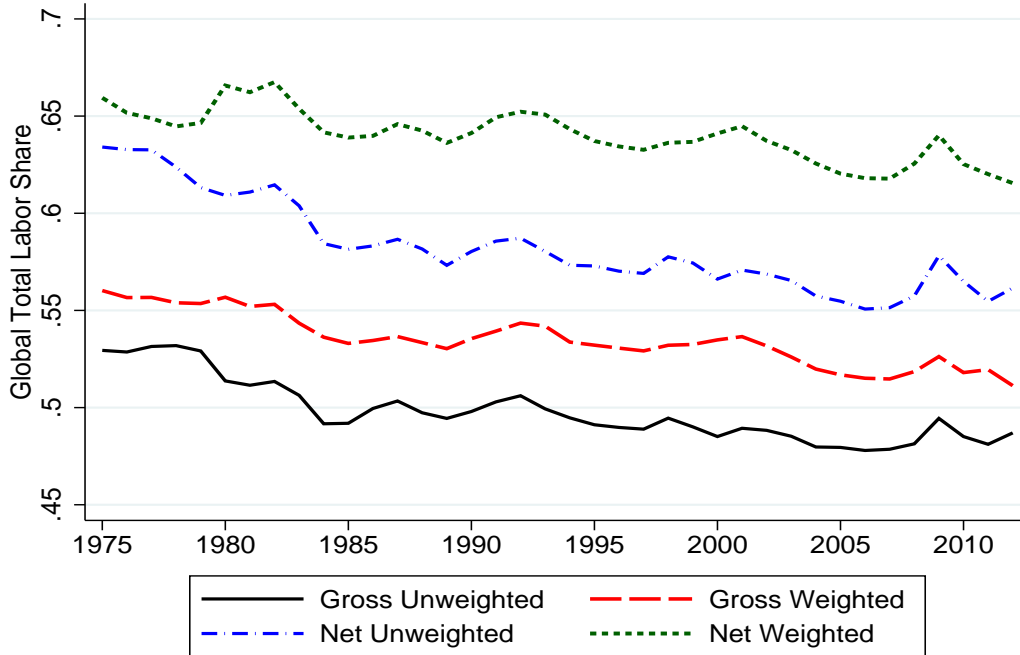
Figure 1: The Evolution of Total Global Labor Share Measures in KN (2014) Data

Unweighted" measure, plots the gross labor share of the total economy for the average country in our sample. The long-dashed line, corresponding to the "Gross Weighted" measure, plots the year dummies when observations in regression (1) are weighted by GDP, after translating GDPs into current U.S. dollars using market exchange rates. Because regressions are weighted, movements in this measure of the labor share disproportionately reflect movements in the labor shares of the largest economies. Fitting these two lines with a linear trend implies that the global gross labor share for the total economy has declined globally by 4.6 percentage points (unweighted) and 4.0 percentage points (weighted) between 1975 and 2012.

The two other lines plot the unweighted and weighted labor shares net of depreciation. The net labor share measures differ from their corresponding gross measures for three reasons. First, the net labor share is by definition greater than the gross labor share because it excludes depreciation from the denominator. Second, for the weighted regressions, differences in depreciation across countries render weights based on GDP different from weights based on NDP. Third, the set of countries included is different. We only include countries with at least 15 observations in each regression. For some countries and years we do not observe depreciation

Table 1: Global Labor Share Trends

		Percentage Points		Percent		
Dataset	Labor Share	Unweighed	Weighted	Unweighed	Weighted	Countries
KN (2014)	Total Gross	-4.6	-4.0	-9.1	-7.5	70
KN (2014)	Total Net	-7.0	-3.6	-11.9	-5.6	59
KN (2014)	Corporate Gross	-9.2	-5.4	-14.5	-8.8	40
KN (2014)	Corporate Net	-9.8	-3.7	-13.4	-5.1	29
PWT 8.0	Adjusted Gross	-8.7	-6.4	-13.7	-10.6	72
PWT 8.0	Adjusted Net	-9.3	-5.8	-13.0	-8.7	68

and therefore cannot calculate net labor shares. Of the 70 countries with data sufficient to calculate 15 years of total gross labor shares, only 59 have data sufficient to do the same for net labor shares.[10] Fitting the net labor share series with a linear trend implies a decline of 7.0 percentage points for the unweighted regressions and a decline of 3.6 percentage points for the weighted regressions.

Table 1 summarizes the results from our global time series analyses. As seen in the top two rows, the decline in the net labor share is larger than the decline in the gross labor share in percentage points for the average country in our sample (corresponding to the unweighted regressions). The net labor share declines by slightly less in percentage points relative to the gross when we weight countries by their GDP. Since net labor shares are higher in levels, the table also reports these declines in percent terms.

We next characterize movements in the labor shares of the corporate sector. As in our previous work, we emphasize corporate labor share measures because they are less affected by the existence of unincorporated enterprises and proprietors whose income combines payments to both labor and capital. This issue, most notably highlighted by Gollin (2002), is quite

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[10]The largest countries without sufficient data on depreciation include Argentina, Brazil, China, Colombia, and Saudi Arabia. Throughout our analyses we have verified that our results do not change meaningfully when we require that a country have both measures in order to be included in the regression.
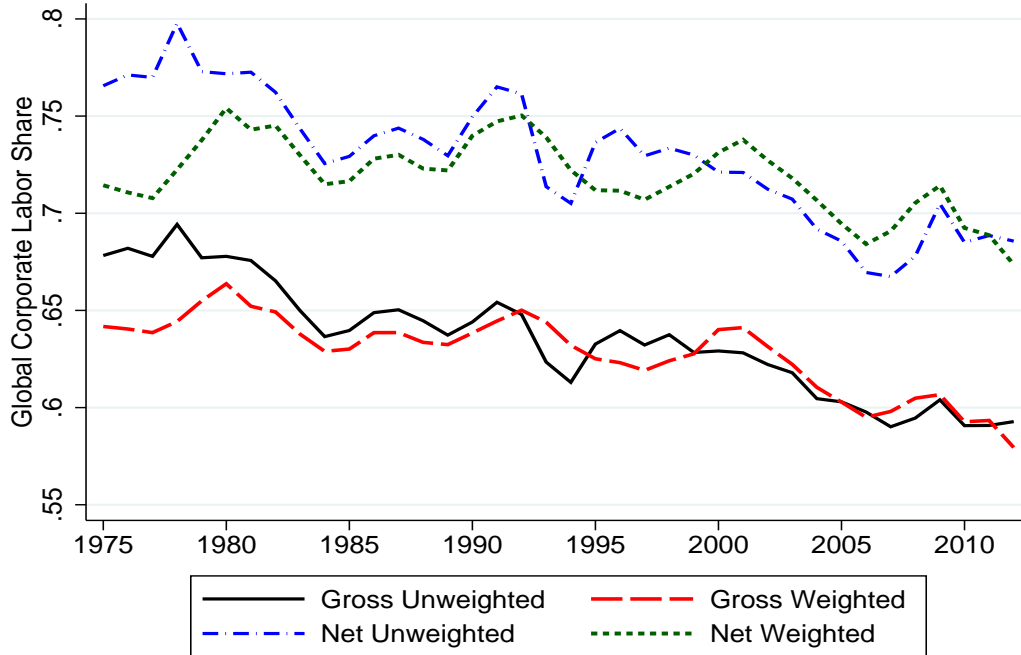
Figure 2: The Evolution of Corporate Labor Share Measures in KN (2014) Data

problematic for the consistent measurement of the labor share across countries and over time because the share of these enterprises likely varies across countries and over time.

In the context of this paper, a focus on measures of the labor share taken from the corporate sector offers two additional benefits. First, it removes any influence of residential housing on the dynamics of depreciation and, therefore, on the gap between gross and net labor shares. This is in line with key dynamics in recent papers such as Bridgman (2014) and Rognlie (2014), which instead emphasize the changing composition of the corporate capital stock. Second, papers such as Bonnet, Bono, Chapelle, and Wasmer (2014), Jones (2014), Rognlie (2014), and Acemoglu and Robinson (2014) have noted that elasticity estimates derived from the relationship between labor shares and the capital-output ratio are sensitive to the inclusion of, and method of deflating, housing capital. Reasonable arguments can be made as to whether housing capital should or should not be excluded from analyses aiming to estimate the shape of the production function. This issue applies less to the corporate sector, where nearly all structures are offices, factories, and commercial stores used for productive purposes.

Figure 2 plots four times series corresponding to unweighted and weighted gross and net

labor shares in the corporate sector. The corporate labor share is in general larger than the labor share in the total economy shown in Figure 1 for two reasons. First, GDP includes certain taxes on products that are excluded from the equivalent denominator in the corporate sector.[11] Second, the income of owners of unincorporated enterprises that are part of the household sector is included in value added but is not included in compensation of employees. Implicitly, the economic activity of those proprietors is treated as having a zero labor share. This downward force impacts labor share measures for the total economy but it impacts less measures for the corporate sector.

We summarize our results in Table 1. When we do not weight our regressions, the corporate gross and net labor shares decline by 9.2 and 9.7 percentage points, or by 13.4 and 14.5 percent respectively. These declines are so similar because, for the average country in our sample, corporate depreciation as a share of corporate gross value added has not changed meaningfully over time. When we weight observations in our regressions with value added, we observe a 5.4 percentage point decline in the corporate gross labor share and a 3.7 percentage point decline in the corporate net labor share. The smaller decline in the weighted net measure largely reflects the increase in depreciation relative to value added in the U.S. corporate sector.

2.2.2 Time Series Patterns Imputing Net Labor Shares

The above analyses only included labor share measures for countries with raw data sufficient to directly construct these measures. To examine the sensitivity of our results when a more representative global aggregate is examined, we now make imputations that rely on proportionality assumptions when a country has sufficient data to get close to, but not all the way to, a given labor share measure. More formally, let Ω^j for $j = \{TG, TN, CG, CN\}$ denote the set of countries i that contain at least 15 years of data on $s^j_{L,i,t}$. Let $\eta^{j,j'}_t$ denote the year fixed effects from regressions of the form:

$$\frac{s^j_{L,i,t}}{s^{j'}_{L,i,t}} = \eta^{j,j'}_t + \eta^{j,j'}_i + \epsilon^{j,j'}_{i,t}. \tag{2}$$

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[11]This statement holds for most countries' national accounts but there are several exceptions, including the United States.
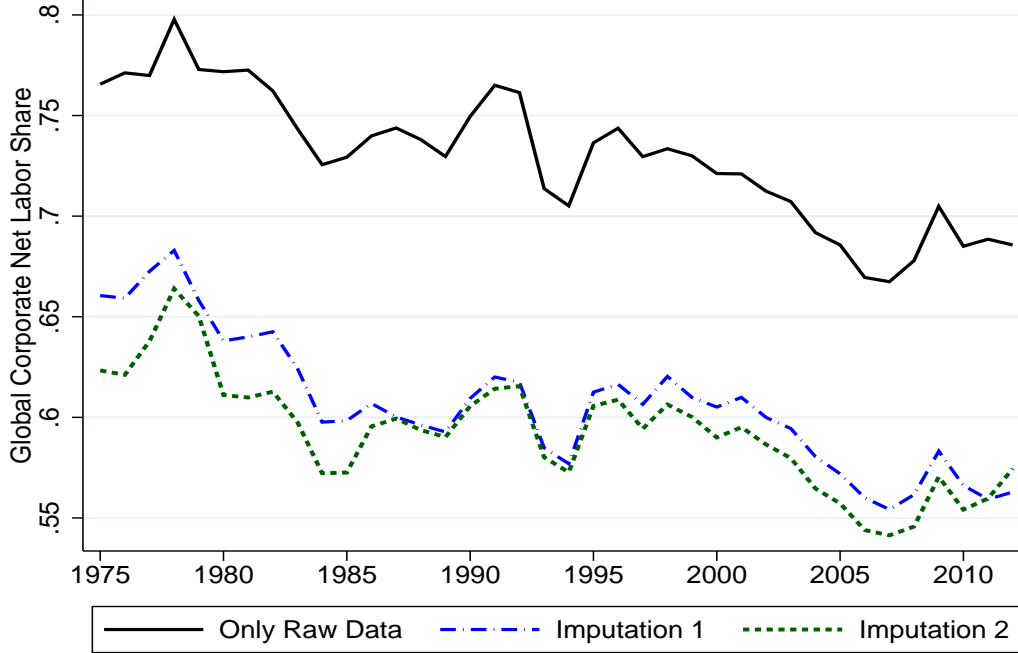
Figure 3: Comparison of Corporate Net Labor Shares by Method

As before, we absorb country fixed effects to account for entry and exit into the sample and we normalize the level of the time dummies to equal the average of the left-hand side variable in 1975. Note that only countries in both sets Ω^j and $\Omega^{j'}$ will be included in this regression.

Our imputation applies to a country that has a measure of $s_{L,i,t}^{j'}$ but lacks $s_{L,i,t}^{j}$. In that case we impute the latter as $s_{L,i,t}^{j'} \times \eta_t^{j,j'}$. For example, New Zealand has data on its total net labor share $s_{L,i,t}^{TN}$ but not on its corporate net labor share $s_{L,i,t}^{CN}$. But many other countries have both of these measures. We use those other countries to calculate the appropriate ratio of the two measures in each year, $\eta_t^{CN,TN}$, and then impute $s_{L,i,t}^{CN}$ for New Zealand as $s_{L,i,t}^{CN} = s_{L,i,t}^{TN} \times \eta_t^{CN,TN}$. Note that this implies that while the inclusion of the imputed New Zealand corporate net labor share can impact the global trends of gross and net labor shares, it cannot by construction influence their relative movement. Similarly, Tunisia has data on its corporate gross labor share $s_{L,i,t}^{CG}$ but lacks data on its corporate net labor share $s_{L,i,t}^{CN}$. We might therefore wish to use $\eta_t^{CN,CG}$ to impute its corporate net labor share.[12]

Figure 3 plots the unweighted corporate net labor share under three alternative methods.

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[12]For some of our analyses we also use this procedure to impute the corporate gross labor share using $\eta_t^{CG,TG}$. This imputation also includes some large developed countries such as Australia and Japan.

The solid line shows the path of corporate net labor shares without any imputation (i.e. using "Only Raw Data") and therefore is identical to the path of the unweighted corporate net labor share shown in Figure 2. The short-dashed line, labeled "Imputation 1," imputes the corporate net labor share when it is missing from the raw data using a particular sequence of steps. First, if a country's corporate gross labor share is available, it imputes the corporate net labor share using $\eta_t^{CN,CG}$. Next, if the corporate gross labor share is unavailable but the total net labor share is available, this method imputes corporate net labor share using $\eta_t^{CN,TN}$. Finally, if only the total gross labor share is available, this method imputes the corporate net labor share using the product of these factors, $\eta_t^{CN,CG} \times \eta_t^{CG,TG}$. The long-dashed line, labeled "Imputation 2," instead imputes the corporate net labor share directly from the total gross labor share using $\eta_t^{CN,TG}$.[13] As can be seen in Figure 3, all methods for calculating the global corporate net labor share reveal similar patterns of decline.[14]

2.2.3 Cross Country Patterns

Above we documented that both gross and net labor shares exhibit declines. This is true both for the average country and for the world as a whole. We now document that the cross country pattern of corporate gross labor share declines largely mirrors that of the net labor share declines. Figure 4 compares plots of these two trends for each country, measured in percentage points per 10 years. The plot includes all countries with 15 years of raw data on both measures. Most countries, such as Mexico, lay below the plotted 45-degree line, indicating that the corporate net labor share declined by more than the gross. Others, such as the United States (difficult to visualize as it is close to the origin), are above the 45-degree line, and therefore had a smaller decline in the net labor share.

Overall, the cross sectional variation in the two trends is quite similar. Of the 24 countries with declines in their gross labor shares, 22 experienced declines in their net labor shares. Of the 20 countries with statistically significant at the 5 percent level declines in their gross labor

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[13]We have confirmed the robustness of our results to reversing the order of the first two steps in "Imputation 1." It makes little difference if we first try to impute $s_{L,i,t}^{CN}$ using $\eta_t^{CN,TN} \times s_{L,i,t}^{TN}$ before trying to use $\eta_t^{CN,CG} \times s_{L,i,t}^{CG}$.

[14]Countries lacking the corporate net labor share have systematically lower corporate gross labor share and total net labor share values, causing the two imputed lines to start at a level significantly lower than the raw series.
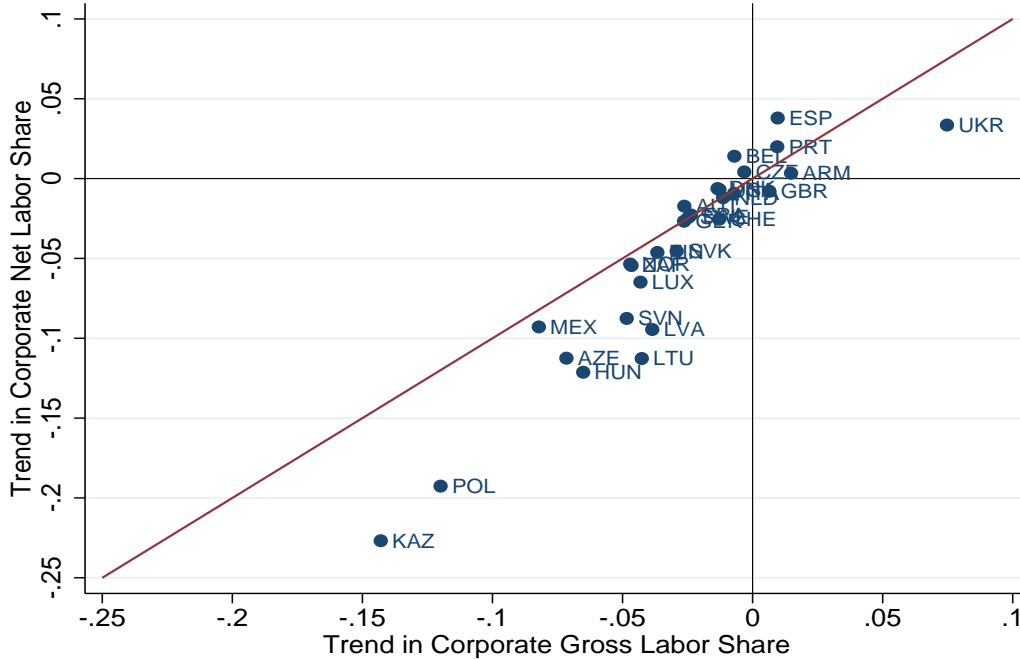
Figure 4: Comparison of Trends in Corporate Gross and Net Labor Shares

shares, 17 also had statistically significant declines in their net labor shares.

2.3   Results from the Penn World Tables

The methodology and quality of depreciation measurements in the national accounts is heterogeneous across countries. The recently released PWT 8.0 (Feenstra, Inklaar, and Timmer, 2013) contains new measures of the capital stocks, labor shares, and depreciation. The key advantage of using the PWT is that it harmonizes measurement of depreciation across countries. It does not allow us to focus on the corporate sector as we can in the Karabarbounis and Neiman (2014) dataset, but depreciation is measured for a broader set of countries than in our baseline analysis.

The PWT 8.0 starts with depreciation rates for six types of capital (structures, transport equipment, computers, communication equipment, software, and other machinery). These asset-specific depreciation rates are assumed to be constant across countries and years and equal the official BEA depreciation rates (given in Fraumeni, 1997). The aggregate depreciation rate for each country equals the weighted average of asset-specific depreciation rates, with weights
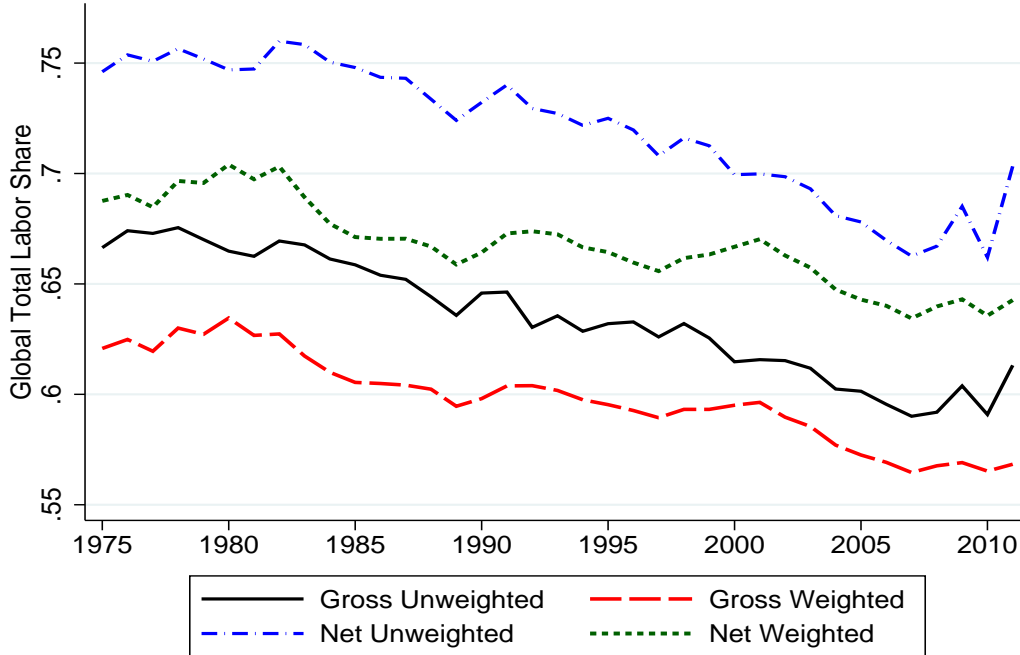
Figure 5: Global Labor Shares in the PWT

given by the value of each asset in the total capital stock. As a result, cross-country differences in the overall depreciation rate exist purely due to cross-country differences in the composition of their capital stocks. We refer the reader to Inklaar and Timmer (2013) for more details on the measurement of the labor share, capital, and depreciation in PWT 8.0.

PWT 8.0 offers four underlying series of the labor share for the total economy. Three of the four series build upon the adjustments proposed by Gollin (2002) to incorporate data on the self-employed in calculating the labor share for the total economy. The fourth series adds the value added in agriculture to labor compensation of employees, with the rationale being that agriculture is labor intensive and it employs a large fraction of the self-employed in poor countries. The PWT constructs a "best estimate" labor share from these four series for each country and we use this measure in our analysis.[15]

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[15]The first method attributes all the self-employed's "mixed income" – a term referring to the fact that their income likely pools compensation for labor and capital – to labor. The second method assumes that the self-employed's income is split between labor and capital in the same proportion as in the rest of the economy. The third method assumes that self-employed earn the same wage as other workers and uses data on the number of self-employed to impute their labor earnings. For most countries, and especially the developed ones, the best estimate labor share uses the second adjustment. We drop those observations where the labor share is assumed to be constant within a country or is interpolated by the PWT.
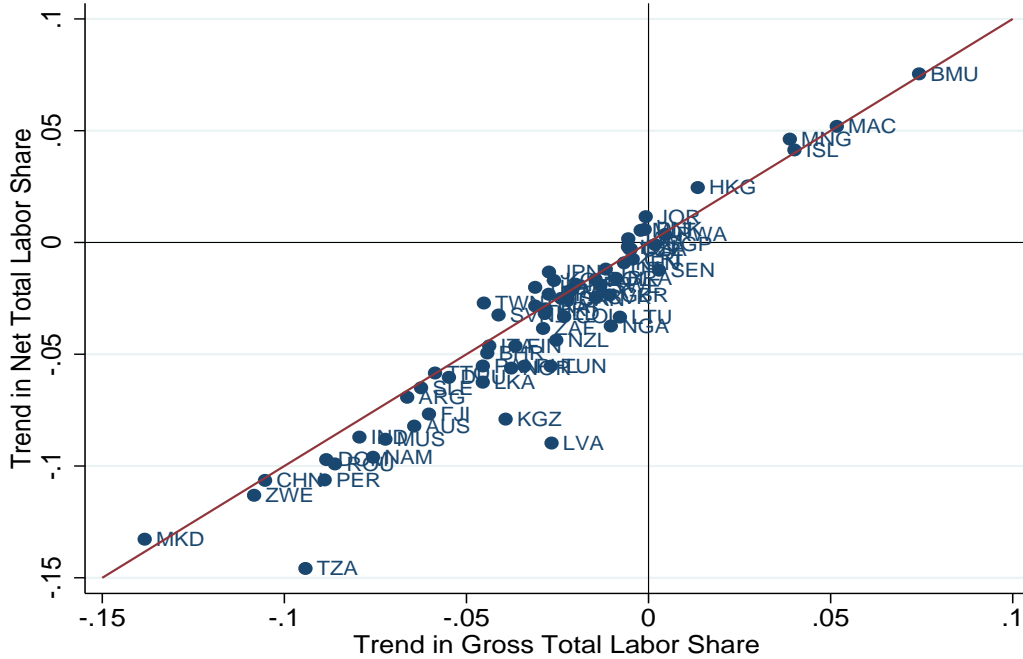
Figure 6: Gross vs. Net Labor Shares in the PWT

As in our previous analyses, we run unweighted and weighted regressions of the form shown in equation (1). Figure 5 plots the year fixed effects for the gross and the net labor shares. The bottom rows of Table 1 summarize the percentage point and percent declines in the labor shares. Similarly to the facts documented in the Karabarbounis and Neiman (2014) dataset, the PWT data shows comparable declines in both gross and net labor shares. This reflects the fact that the value of depreciation as a fraction of gross value added did not increase significantly over time.[16]

Figure 6 plots the country trends in gross labor shares against the country trends in net labor shares, after removing an outlier (Azerbaijan) to improve the figure's presentation. Most of the points lay below, but close to, the plotted 45 degree line, which indicates that the cross-country variation in gross labor share trends closely tracks the variation in net labor share trends. Of the 60 countries with declines in their gross labor shares, 56 also experience declines in their net labor shares. Of the 41 countries with statistically significant declines in their gross labor shares, 40 also experienced statistically significant declines in their net labor shares.

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[16]The depreciation rate as estimated by the PWT shows a mild increase from roughly 4 percent to roughly 4.2 percent over the sample period for the average country.

3 A Model of Gross and Net Labor Shares

Having demonstrated a robust pattern of declining gross and net labor shares globally, we now use a variant of the neoclassical growth model to understand the joint dynamics of depreciation, factor shares, and inequality. We recognize that the model abstracts from various subtleties that affect both factor shares and inequality. Despite its simplicity, the model serves two purposes. First, we use the model to characterize analytically how the relative movement of gross and net labor shares across steady states depend on the nature of the underlying shocks and the structure of production. We demonstrate conditions under which gross and net labor shares necessarily move in the same direction. Second, we work through some examples to illustrate that, even in this simple model, the relationship between factor shares and inequality is complicated when an economy is not in steady state.

We consider an infinite horizon deterministic economy with periods $t = 0, 1, 2, ...$, populated by two types of agents indexed by $i \in \{N, K\}$. Workers ($i = N$) simply consume labor earnings and do not save. Capitalists ($i = K$) decide how to allocate capital income between consumption and investment. All inequality in this economy therefore reflects the "between" component in comparing these two groups.

There are two types of capital indexed by $j \in \{L, H\}$ that are produced with different technologies and that depreciate at different rates. Changes in the composition of the aggregate capital stock generate changes in the aggregate depreciation rate. We think of capital K_t^H as a high depreciation asset (for instance, capital related to information and communication technologies) and K_t^L as a lower depreciation asset (for instance, structures and transportation equipment).

Final Goods. Competitive firms produce gross final output Y_t with a constant elasticity of substitution (CES) technology:

$$Y_t = \left(\alpha \left(A_{K,t} K_t \right)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) \left(A_{N,t} N_t \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{3}$$

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where K_t denotes the aggregate capital stock, N_t denotes aggregate labor supply, $\sigma > 0$ denotes the elasticity of substitution between labor and capital, $\alpha \in (0,1)$ is a distribution parameter, $A_{K,t}$ denotes capital-augmenting technology, and $A_{N,t}$ denotes labor-augmenting technology. Final goods producers rent labor and capital at prices W_t and R_t and sell final output at a price of one.

Denoting by C_t^N workers' consumption and by C_t^K capitalists' consumption, final output Y_t in the economy is allocated between consumptions and intermediate inputs used in the production of investment goods:

$$Y_t = C_t^N + C_t^K + I_t^L + I_t^H. \tag{4}$$

Investment Goods. Firms in investment sector $j = \{L, H\}$ purchase inputs I_t^j from final goods producers and produce output $X_t^j = \frac{1}{\xi_t^j} I_t^j$ that is used to augment the capital stock. Investment firms are perfectly competitive and sell their output at price $p_t^j = \xi_t^j$.[17]

Workers. There is a measure χ of identical workers. Each worker exogenously supplies n_t units of labor, so aggregate labor supply is $N_t = \chi n_t$. Workers cannot save and simply consume their labor earnings in each period, $c_t^N = W_t n_t = C_t^N / \chi$. Instantaneous utility from consumption is given by $U\left(c_t^N\right)$.

Capitalists. There is a measure $1 - \chi$ of identical capitalists. We denote the consumption of each capitalist by $c_t^K = C_t^K / (1 - \chi)$ and their investment in the two capital stocks by x_t^L and x_t^H. Capitalists choose capital stocks k_{t+1}^L and k_{t+1}^H and bonds d_{t+1} to maximize the discounted present value of utility flows:

$$V_0 = \max_{\{k_{t+1}^L, k_{t+1}^H, d_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U\left(c_t^K\right), \tag{5}$$

subject to the budget constraint:

$$c_t^K + \xi_t^L x_t^L + \xi_t^H x_t^H + (1 + r_t)d_t = R_t^L k_t^L + R_t^H k_t^H + d_{t+1}, \tag{6}$$

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[17]We note that, without adjustment costs, the price of each unit of type-j investment equals the price of each unit of the type-j capital stock. Therefore, we use the terms "price of investment" and "price of capital" interchangeably and apply the price ξ_t^j to translate capital into units of the numeraire good.

and the capital accumulation equations for $j = L, H$:

$$k_{t+1}^j = (1 - \delta^j)k_t^j + x_t^j. \tag{7}$$

The first-order conditions for utility maximization are:

$$U'\left(c_t^K\right) = \beta(1 + r_{t+1})U'\left(c_{t+1}^K\right), \tag{8}$$

$$R_t^j = \xi_{t-1}^j\left(1 + r_t\right) - \xi_t^j(1 - \delta^j). \tag{9}$$

Equation (8) is the standard Euler equation for consumption. Equation (9) characterizes the optimal investment decision of capitalists. The rental rate R_t^j for one unit of capital j that was purchased in period $t-1$ equals the gross return investors would have earned had they invested this unit in bonds, $\xi_{t-1}^j(1 + r_t)$, less today's resale value of undepreciated capital, $\xi_t^j(1 - \delta^j)$. Finally, bonds are in zero net supply, $(1 - \chi)d_t = 0$.

Capital Intermediaries. Capital intermediaries produce competitively the capital bundle K_t using the two types of capital $K_t^j = (1 - \chi)k_t^j$, rented from capitalists at prices R_t^j. The aggregate capital K_t is then rented at price R_t to final goods producers. The production technology is:

$$K_t = \left(\left(K_t^L\right)^{\frac{\theta-1}{\theta}} + \left(K_t^H\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}, \tag{10}$$

where $\theta > 0$ denotes the elasticity of substitution between the two types of capital. Cost minimization implies:

$$K_t^j = \left(\frac{R_t^j}{R_t}\right)^{-\theta} K_t \quad \text{with} \quad R_t = \left(\left(R_t^L\right)^{1-\theta} + \left(R_t^H\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}, \tag{11}$$

where R_t denotes the rental rate of the aggregate capital stock K_t.

Equilibrium. An equilibrium for this economy consists of quantities and prices such that capitalists maximize their utility, firms maximize their profits, and all capital and labor markets clear. Goods market clearing and zero profits in the production of final goods imply:

$$Y_t = C_t^N + C_t^K + \xi_t^L X_t^L + \xi_t^H X_t^H = W_t N_t + R_t K_t. \tag{12}$$

The first part of equation (12) says that final gross output is allocated between consumption of workers, consumption of capitalists, and investment in the two types of capital. The second part of this equation shows how total gross income is distributed between labor and capital. From zero profits in the production of the aggregate capital bundle, gross capital income equals $R_t K_t = R_t^L K_t^L + R_t^H K_t^H$.

Capital Prices, Depreciation, and Factor Shares. The price of the aggregate capital stock ξ_t is a weighted average of the price of each type of capital:

$$\xi_t = \frac{K_t^L}{K_t}\xi_t^L + \frac{K_t^H}{K_t}\xi_t^H, \tag{13}$$

and the economy-wide depreciation rate δ_t is a weighted average of each type of capital's depreciation rate:

$$\delta_t = \frac{\xi_t^L K_t^L}{\xi_t K_t}\delta^L + \frac{\xi_t^H K_t^H}{\xi_t K_t}\delta^H. \tag{14}$$

Consistent with national accounting practices, depreciation rates for each type of capital are treated as constants δ^j. The aggregate depreciation rate δ_t varies over time only because of changes in the composition of the capital stock. The share of depreciation in gross income can be written as:

$$\psi_t = \frac{\delta_t \xi_t K_t}{Y_t}, \tag{15}$$

which highlights that it varies over time with the aggregate depreciation rate δ_t and the aggregate capital-output ratio $\xi_t K_t / Y_t$.

We define the gross and net labor shares as:

$$s_{L,t}^G = \frac{W_t N_t}{Y_t} \quad \text{and} \quad s_{L,t}^N = \frac{W_t N_t}{Y_t - \delta_t \xi_t K_t} = s_{L,t}^G \frac{1}{1 - \psi_t}, \tag{16}$$

where Y_t is gross value added, $Y_t - \delta_t \xi_t K_t$ is net value added, and ψ_t is the share of depreciation in gross value added. We can relate the aggregate rental rate of capital to the gross labor share using the first-order condition of the production function (3) with respect to capital:

$$\alpha A_{K,t}^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{K_t}\right)^{\frac{1}{\sigma}} = R_t \quad \implies \quad s_{L,t}^G = 1 - \alpha^\sigma R_t^{1-\sigma} A_{K,t}^{\sigma-1}. \tag{17}$$

When the elasticity of substitution in the aggregate production function exceeds one, $\sigma > 1$, the gross labor share $s_{L,t}^G$ decreases when the aggregate rental rate of capital R_t decreases or when capital-augmenting technology $A_{K,t}$ increases. When $\sigma = 1$, the case of Cobb-Douglas production, the gross labor share is always constant, $s_{L,t}^G = 1 - \alpha$.

4 Labor Shares in Steady State: Two-Sector Model

To derive simple analytical expressions that summarize the relationship between gross and net labor shares, we now focus our analysis on changes across steady states of the model. We drop time subscripts of variables to denote steady state values. A key input in our derivations is the rental rate of aggregate capital. Using the first-order conditions for investment, equation (9), together with the definitions of the price of aggregate capital in equation (13) and the aggregate depreciation rate in equation (14), we write the rental rate in steady state as:

$$R = \xi(r + \delta). \tag{18}$$

We will use equation (18) to express s_L^G and s_L^N as a function of capital-augmenting technology A_K, the real interest rate r (that in steady state simply equals $1/\beta - 1$), the depreciation rate of aggregate capital δ, and the price of aggregate capital ξ.

We emphasize that δ and ξ are endogenous variables in our three-sector model and will in general vary as a function of the exogenous shocks β, A_N, A_K, ξ^H, and ξ^L. We summarize the implications for labor shares of these shocks in terms of the aggregates δ and ξ as this allows us to compare our model with two-sector models that feature exogenous (or no) movements in δ and ξ. In Section 5 we examine how endogenous changes in δ and ξ impact our conclusions.

4.1 Relative Changes in Gross and Net Labor Shares

Gross and net labor shares are linked through the share of depreciation in gross value added ψ. Substituting equation (18) and the definition of the gross labor share $s_L^G = 1 - RK/Y$ into the definition of ψ in equation (15), we write ψ in steady state as:

$$\psi = \frac{\delta \xi K}{Y} = \frac{\delta \xi}{R} \frac{RK}{Y} = \left(\frac{\delta}{r + \delta} \right) (1 - s_L^G). \tag{19}$$

Depreciation as a share of gross value added is decreasing in s_L^G, increasing in δ, and decreasing in r.

Substituting equation (19) into the equation $s_L^G = s_L^N(1 - \psi)$ and totally differentiating the resulting expression, we can relate the log change of the net labor share to the log changes of the gross labor share, of the aggregate depreciation rate, and of the real interest rate:

$$\mathrm{d} \log \left(s_L^N\right) = \left(\frac{1 - s_L^N}{1 - s_L^G}\right) \mathrm{d} \log \left(s_L^G\right) + \left(\frac{s_L^N - s_L^G}{s_L^N} \frac{1 - s_L^N}{1 - s_L^G}\right) \left[\mathrm{d} \log \left(\delta\right) - \mathrm{d} \log \left(r\right)\right]. \qquad (20)$$

Consider the case when shocks either do not change the real interest rate and the aggregate depreciation rate or change these variables by the same rate, $\mathrm{d} \log \left(\delta\right) = \mathrm{d} \log \left(r\right)$. Examples of such shocks include capital-augmenting technological progress A_K and exogenous movements in the relative price of capital ξ in a two-sector model. In such a case, the last term of equation (20) equals zero and gross and net labor shares necessarily move in the same direction. If we set the labor shares to their average values observed in our sample, $s_L^G = 0.64$ and $s_L^N = 0.73$, we obtain that the log change in the net labor share equals 75 percent of the log change in the gross labor share.

More broadly, if some combination of shocks causes the aggregate depreciation rate to grow proportionately more than the real interest rate, $\mathrm{d} \log \left(\delta\right) > \mathrm{d} \log \left(r\right)$, this will create a force that increases the growth rate of the net labor share relative to that of the gross labor share. Changes in δ or r, unlike changes in ξ or A_K, can potentially cause gross and net labor shares to move in different directions across steady states.

4.2   Gross and Net Elasticities

In Karabarbounis and Neiman (2014), we related declines across countries and industries in the relative price of investment ξ to declines in the gross labor share s_L^G since 1975. This comovement underlies our estimate that the gross elasticity of substitution σ, which relates movements in the real capital to output ratio and the gross rental rate of capital, exceeds one. Rognlie (2014) correctly notes that one should be careful in importing estimates of σ to analyses of movements in the net labor share such as those in Piketty (2014) and Piketty and

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Zucman (2014). Instead, Rognlie highlights that the "net elasticity of substitution" ϵ, which relates movements in the real capital to net output ratio and the net rental rate of capital, is the key elasticity governing the response of the net labor share. He demonstrates that $\epsilon < \sigma$ and, therefore, evidence that $\sigma > 1$ does not imply that $\epsilon > 1$ and is insufficient to understand the directional response of the net labor share. In fact, he argues that most estimates of the gross elasticity, including those that modestly exceed one, imply that the net elasticity is below one.

This argument seems at odds, however, with equation (20) which shows that gross and net labor shares always move together if the percent change in the depreciation rate equals that of the real interest rate. The results appear at first to be inconsistent because the expression in Rognlie (2014) for the net elasticity ϵ assumes that all movements in the rental rate of capital are due to changes in the real interest rate. As Rognlie notes, his results need not hold if changes in the relative price of capital also affect movements in the rental rate of capital.

To illustrate this key difference, we define the gross and net elasticities of substitution as:

$$\sigma = \frac{1}{1 - \frac{\mathrm{d}\log(1-s_L^G)}{\mathrm{d}\log(K/Y)}} = -\frac{\mathrm{d}\log\left(\frac{K}{Y}\right)}{\mathrm{d}\log(R)} \quad \text{and} \quad \epsilon = \frac{1}{1 - \frac{\mathrm{d}\log(1-s_L^N)}{\mathrm{d}\log(K/Y(1-\psi))}} = -\frac{\mathrm{d}\log\left(\frac{K}{Y(1-\psi)}\right)}{\mathrm{d}\log(R - \xi\delta)}. \quad (21)$$

Dividing the two elasticities, we get:

$$\frac{\epsilon}{\sigma} = \left[\frac{\mathrm{d}\log\left(\frac{K}{Y(1-\psi)}\right)}{\mathrm{d}\log\left(\frac{K}{Y}\right)}\right]\left[\frac{\mathrm{d}\log(R)}{\mathrm{d}\log(R - \xi\delta)}\right]. \quad (22)$$

We can solve for the each term in brackets in equation (22) under alternative assumptions on which shock generates movement in the system from one steady state to another. To make our point analytically and in the simplest way, we compare cases in which only one shock operates at a time.[18]

We start by considering movements in labor shares caused by changes in β as this case most closely corresponds to the analysis of Rognlie (2014). Changes in β cause movements in the real

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[18]The first bracket can be written as $1 - \mathrm{d}\log(1 - \psi)/\mathrm{d}\log(K/Y)$. Using the definition of ψ in equation (19) we take $\mathrm{d}\log(1 - \psi)/\mathrm{d}\log(K/Y) = [K/Y(1 - \psi)]\left[-\delta\xi - \delta(K/Y)\mathrm{d}\xi/\mathrm{d}(K/Y)\right]$. For the second bracket, we use $\mathrm{d}\log(R) = ((r + \delta)\mathrm{d}\xi + \xi\mathrm{d}r)/R$ and $\mathrm{d}\log(R - \xi\delta) = (r\mathrm{d}\xi + \xi\mathrm{d}r)/(R - \xi\delta)$.

interest rate, $dr \neq 0$, while all other shocks are assumed to equal zero, $d\xi = d\delta = dA_K = 0$.[19]
Evaluating equation (21) under these assumptions yields:

$$\frac{\epsilon}{\sigma} = \left[\frac{1}{1-\psi}\right]\left[\frac{r}{r+\delta}\right] = \frac{1-s_L^N}{1-s_L^G} < 1. \tag{23}$$

Since $s_L^N > s_L^G$, the net elasticity is always smaller the gross elasticity of substitution. As explained by Rognlie, a given change in r affects the net rental rate proportionately more than the gross rental rate (i.e. the second bracket in equation (22) is smaller than one), which mutes the response of the net labor share. For example, using the estimated elasticity $\sigma = 1.25$ from Karabarbounis and Neiman (2014) together with the average labor share values in our sample, $s_L^G = 0.64$ and $s_L^N = 0.73$, equation (23) implies that the net elasticity $\epsilon = 0.94$. This calculation accords with the claim of Summers (2014) that "I know of no study suggesting that measuring output in net terms, the elasticity of substitution is greater than 1...".

Next, we consider movements in labor shares caused by changes in ξ as this case most closely corresponds to the evidence in Karabarbounis and Neiman (2014). All other shocks are assumed to equal zero, $dr = d\delta = dA_K = 0$. Evaluating equation (21) under these assumptions yields:

$$\frac{\epsilon}{\sigma} = \left[\frac{1}{1-\psi}\left(1-\frac{\psi}{\sigma}\right)\right]\left[1\right] \implies (\epsilon-1) = \frac{s_L^N}{s_L^G}(\sigma-1). \tag{24}$$

Equations (23) and (24) differ because shocks to the relative price of investment, unlike those to the real interest rate, cause proportional changes in the net and the gross rental rate (i.e. the second bracket of equation (22) now equals one). Equation (24) shows that if the only shocks are to ξ and $\sigma = 1$, then $\epsilon = 1$.[20] Since $s_L^N > s_L^G$, the net elasticity exceeds the gross elasticity whenever the gross elasticity exceeds one, i.e. $\epsilon > \sigma > 1$, and the net elasticity is smaller than the gross elasticity whenever the gross elasticity is smaller than one, i.e. $\epsilon < \sigma < 1$. Under the assumption that declines in the rental rate of capital are caused only by declines in the relative

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[19]The real interest rate r in our model is the net return to an investment other than physical capital accumulation. As such, r should be interpreted broadly as an after-tax return on some alternative asset. Our analysis of the impact of r on labor shares would similarly hold for changes in any components of the gross return on physical capital R (such as those due to capital taxes) other than changes to ξ and δ.

[20]That is, both labor shares are constant. Recall that s_L^G is constant when $\sigma = 1$ and $s_L^N = s_L^G/(1-\psi)$. Equation (19) shows that ψ is also constant in this case because $d\delta = dr = 0$.

price of investment, $\sigma > 1$ is a sufficient condition to generate declines in both the gross labor share and the net labor share. Using the values $s_L^G = 0.64$, $s_L^N = 0.73$, and $\sigma = 1.25$ we obtain $\epsilon = 1.29$.

To summarize, our theoretical analysis shows that declines in the gross labor share caused by declines in the real interest rate are likely to be associated with increases in the net labor share. By contrast, declines in the gross labor share caused by declines in the relative price of investment necessarily also imply declines in the net labor share. Our empirical analysis documented that both gross and net labor shares declined globally since 1975. Therefore, under $\sigma > 1$, the decline in the relative price of investment is a more plausible explanation of the decline in the labor share than a decline in the real interest rate.[21]

5   Labor Shares in Steady State: Three-Sector Model

Equations (17) and (20) express steady state labor shares as functions of the depreciation rate of aggregate capital δ and the price of aggregate capital ξ. In Section 4, we took changes in δ and ξ as given, but in our three-sector model these two variables endogenously respond to the underlying shocks. We now turn to numerical solutions across steady states which allow us to understand how various shocks and parameters affect the response of both labor shares, allowing δ and ξ to change endogenously. The key parameters we focus on are the elasticity of substitution between capital and labor σ and the elasticity of substitution between the two types of capital θ.[22]

In Figure 7, we show responses across steady states to a 2/3 decline in the price of the high-depreciation capital ξ^H. All four panels plot the elasticity of substitution between capital types θ on the x-axis and contain three lines. The solid horizontal line shows the pre-shock value in the initial steady state, the dashed line shows the post-shock steady state value for the

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[21]We can plug values from the PWT for $\mathrm{d}\log\left(s_L^N\right)$, $\mathrm{d}\log\left(s_L^G\right)$, $\mathrm{d}\log\left(\delta\right)$, s_L^N, and s_L^G into equation (20) to back out the implied $\mathrm{d}\log\left(r\right)$. With weighting, the results imply that the real interest rate barely changed. The unweighted results actually imply an increase in the real interest rate.

[22]For our numerical solutions, we set $A_N = N = 1$ and $\alpha = 0.40$. We choose the low depreciation rate $\delta^L = 0.02$ and the high depreciation rate $\delta^H = 0.20$. As we vary θ and σ, we pick β, ξ^H, ξ^L, and A_K to always target the same initial steady state values of $s_L^G = 0.64$, $s_L^N = 0.73$, $R = 0.10$, and $\delta = 0.04$.
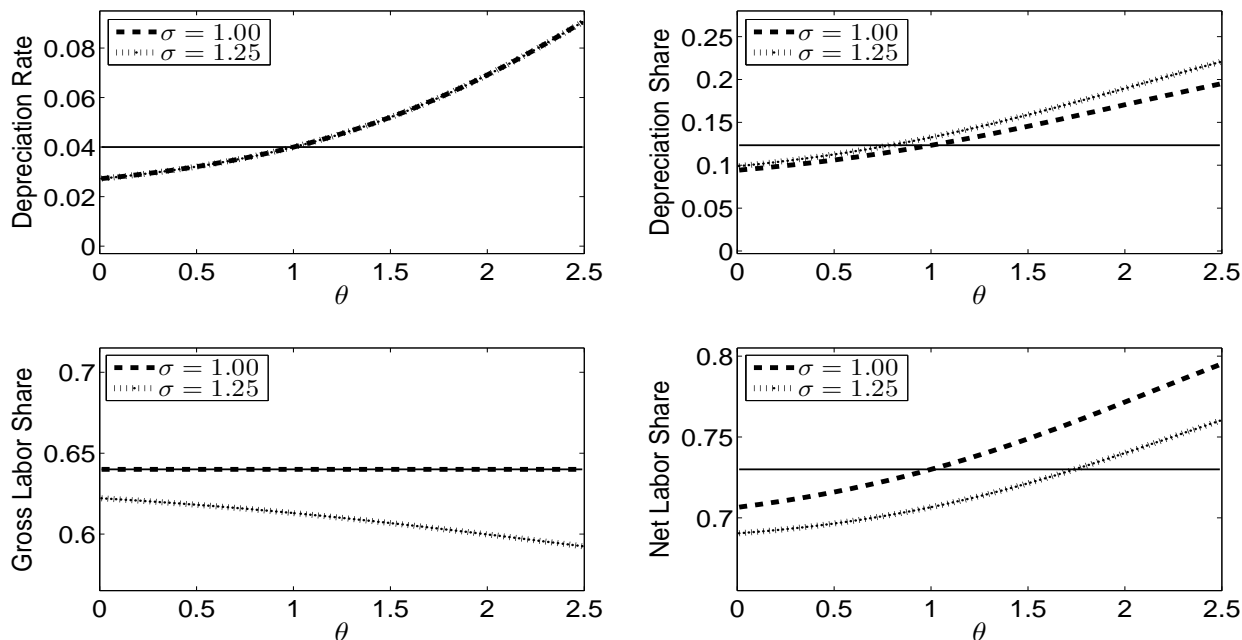
Figure 7: Decline in Price of High Depreciation Capital

Cobb-Douglas case with $\sigma = 1$, and the dotted line shows the post-shock steady state value when $\sigma = 1.25$.

The top left panel shows the behavior of the aggregate depreciation rate δ, which equals 0.04 in the initial steady state. We note that the response of δ does not depend on the value σ, which is why the two lines are indistinguishable in that plot. Changes in δ purely reflect changes in the composition of the capital stock, which is governed by the elasticity θ. When $\theta = 1$, the share of each type of capital in total capital is constant, and therefore δ does not change in response to the ξ^H shock. When $\theta < 1$, the two types of capital are complements. In response to a lowering in ξ^H, the share of the low depreciation capital in total capital increases and δ decreases. The opposite happens when $\theta > 1$.[23]

The bottom left panel shows the response of the gross labor share s_L^G following the decline

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[23]These intuitions can be formalized by deriving the response of δ as a function of changes in ξ^H, ξ^L, and r:

$$\mathrm{d}\delta = (1-\theta)(r+\delta)\left(\left(\chi^L + \zeta^L\right)\mathrm{d}\log\left(\xi^L\right) + \left(\chi^H + \zeta^H\right)\mathrm{d}\log\left(\xi^H\right)\right) + \theta\left(1 + \frac{(\zeta^L)^2}{\chi^L} + \frac{(\zeta^H)^2}{\chi^H}\right)\mathrm{d}r,$$

where $\chi^j = R^j K^j/(RK)$ and $\zeta^j = \chi^j(r+\delta)/(r+\delta^j)$. A decline in ξ^H implies that δ decreases when $\theta < 1$, stays constant when $\theta = 1$, and increases when $\theta > 1$. We also note that δ is increasing in r for any value of θ.
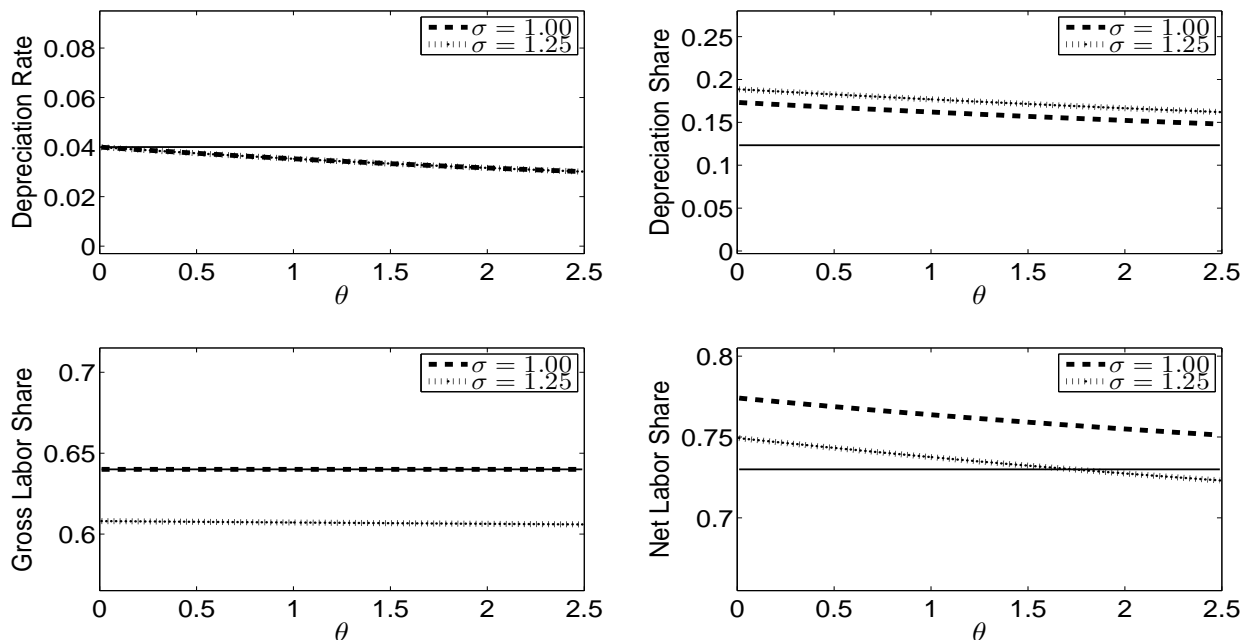
Figure 8: Increase in Discount Factor

in ξ^H. When $\sigma = 1$, s_L^G is constant. For $\sigma > 1$, the gross labor share declines because the aggregate rental rate of capital $R = \xi(r+\delta)$ declines. As θ increases, the decline in the aggregate price of capital ξ becomes stronger and dominates the increase in δ, causing the gross labor share to decline by even more.

The top right panel shows the response of the depreciation share in gross value added ψ. It is useful to recall equation (19) that shows that $\psi = (1 - s_L^G)\delta/(r + \delta)$ in steady state. When $\sigma = 1$, the gross labor share is constant. Additionally, with $\theta = 1$, δ does not change, and therefore ψ does not change. With $\sigma = 1$, but $\theta > 1$, δ increases and so ψ increases relative to the pre-shock steady state value. With $\sigma = 1.25$, the gross labor share declines which causes an upward shift of the ψ response relative to $\sigma = 1$ for any given θ. Comparing the top panels for the $\sigma = 1.25$ case illustrates that δ and ψ need not move in the same direction.

Finally, the bottom right panel shows the change in the net labor share following the decline in ξ^H. In proximity to $\theta = 1$, where δ does not change, the net and the gross labor shares move in the same direction as dictated by equation (20). Since the gross labor share is constant for the $\sigma = 1$ case, the net labor share is also constant at $\theta = 1$. Since the gross labor share

declines for the $\sigma > 1$ case, the net labor share also declines at $\theta = 1$. As we increase the value of θ above one, the wedge between the growth of gross and net labor shares increases because of the rise in δ (and ψ).

Figure 8 shows steady state changes in response to a lowering of r (due to a 3 percentage point increase in β). Unlike the case of the ξ^H shock, in response to a decline in the real interest rate, δ always declines. For sufficiently low values of θ, ψ increases. Therefore, while the gross labor share declines when $\sigma > 1$, the directional response of the net labor share depends on the elasticities σ and θ. For relatively lower values of θ and σ, the increase in depreciation as a share of gross value added ψ is sufficiently strong to actually cause an increase in s_L^N.

6   Transitional Dynamics of Labor Shares and Inequality

Our analysis thus far has shown that both gross and net labor shares are informative for welfare and macroeconomic dynamics. For example, their relative movement can teach us about the source of shocks hitting the economy as well as about the structure of production. An additional source of interest in labor share movements stems from their relevance for inequality, which we now turn to in this section.

It is helpful to start with the relationship between the ratio of consumption and the net labor share in steady state:

$$\frac{C^K}{C^N} = \frac{(R - \delta\xi) K}{W N} = \frac{1 - s_L^N}{s_L^N}. \tag{25}$$

Shocks that decrease the net labor share in steady state are directly mapped into increases in the consumption of capitalists relative to the consumption of workers. Since the consumption streams for each type of agent are constant in steady state, this consumption ratio is a welfare-relevant measure of inequality. In this sense, changes in the net capital share from one steady state to another are perfectly informative about inequality. Motivated in part by this argument, we demonstrated empirically that both gross and net labor shares have in fact declined globally since 1975 and we demonstrated theoretically that a shock to the relative price of capital will cause both labor share measures to move together across steady states.
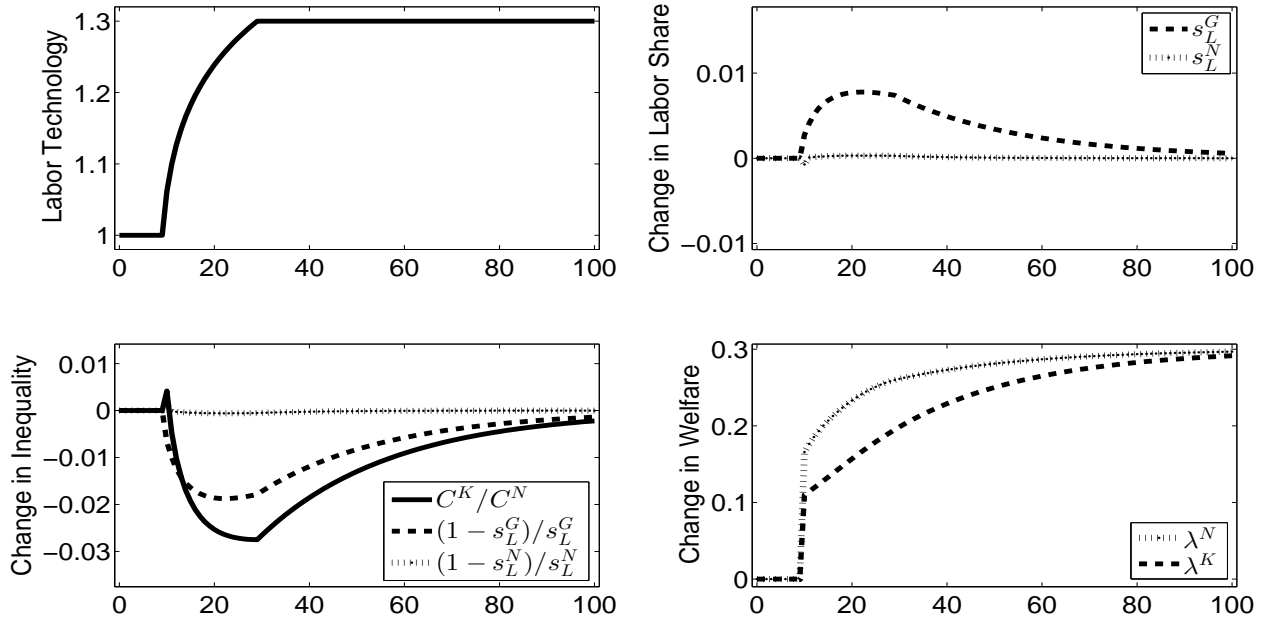
Figure 9: Increase in Labor Augmenting Technology

We stress, however, that this direct mapping of the net labor share to inequality is a limiting result that applies only in steady state. We now analyze some experiments that relate labor shares to inequality over the transition in response to various shocks. In these experiments, we treat a model period as a year and assume the period utility is given by $U = (1 - 1/\rho)^{-1} c^{1-1/\rho}$ with $\rho = 0.5$. We set $\sigma = 1.25$, $\theta = 1.10$, and use the other parameter values given in Section 5.[24]

We start by considering a labor-augmenting productivity shock A_N. This is a interesting case because A_N does not affect the gross or net labor shares in steady state.[25] The relationship between labor shares and inequality is therefore non-trivial only during the transition.

The top left panel in Figure 9 shows the path of the shock. Desired investment increases in response to the increase in $A_{N,t}$, pushing up the real interest rate r_t and the rental rate for aggregate capital R_t. Since $\sigma > 1$, the increase in R_t causes an increase in the gross labor share

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[24]The value $\sigma = 1.25$ comes from Karabarbounis and Neiman (2014). We choose $\theta = 1.10$ because with this value the depreciation rate δ changes from 4.0 to 4.2 percent in response to the ξ^H shock shown in Figure 7. This change is consistent with the change in δ observed for the average country in our sample.

[25]Changes in A_N across steady states do not impact ξ, r, δ, and the rental rate $R = \xi(r + \delta)$. As a result, we see in equations (17) and (20) that neither labor share measure changes in response to changes in A_N.

$s_{L,t}^G$, shown in the top right panel of Figure 9.

Though the increase in r_t differentially impacts the returns on high and low depreciation capital, their absolute and relative movements are small. As a result, there is only a minor reallocation of capital between the two types, and the depreciation rate δ_t and aggregate capital price ξ_t remain relatively constant. Because the capital-output ratio decreases, the share of depreciation in gross income $\psi_t = \delta_t \xi_t K_t / Y_t$ declines and is always below its pre-shock steady state value. As a result, the net labor share always lies below the gross labor share during the transition. In fact, in this particular example, the net labor share initially falls below its pre-shock steady state value and then quickly reverts to a value that roughly equals its pre-shock steady state value.

A simplistic analysis might observe the relative stability of the net labor share and conclude that the A_N shock did not affect inequality. Such a conclusion would not hold, however, for various notions of inequality during the transition. In the bottom left panel of Figure 9 we plot the change in the ratio of consumption C_t^K / C_t^N relative to its initial steady state value together with changes in two transformations of gross and net labor shares, $(1 - s_{L,t}^G)/s_{L,t}^G$ and $(1 - s_{L,t}^N)/s_{L,t}^N$. Recall that in equation (25), $(1 - s_{L,t}^N)/s_{L,t}^N$ equals C_t^K / C_t^N in steady state. However, for the largest part of the transition the ratio of consumption C_t^K / C_t^N and the transformation of the gross labor share $(1 - s_{L,t}^G)/s_{L,t}^G$ lie below their pre-shock value, whereas $(1 - s_{L,t}^N)/s_{L,t}^N$ is roughly stable at its pre-shock value. If one wanted to argue that workers are receiving a "larger share of the pie," the behavior of the gross labor share would have been more informative than the behavior of the net labor share in making this argument.[26]

A similar result is obtained if we look at welfare-based measures of consumption inequality that take into account the full consumption path. For each type of agent $i = \{N, K\}$, we define the time-varying consumption-equivalent change in welfare as the solution λ_t^i of the equation:

$$\left(\frac{1}{1 - \beta}\right) U\left(C^i(1 + \lambda_t^i)\right) = U\left(C_t^i\right) + \beta V_{t+1}. \tag{26}$$

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[26]The ratio of consumption goes up in the first few periods after the shock reflecting the fact that capitalists' consumption responds to changes in permanent income whereas the workers' consumption is simply pinned down by contemporaneous labor earnings.

In this equation, $V_t = \sum_{j=t}^{\infty} \beta^{j-t} U\left(C_j^i\right)$ denotes the discounted present value of utility flows for the agent from period t onwards when the shock takes place and $U\left(C^i\right)/(1-\beta)$ denotes the discounted present value of utility flows had the shock not taken place (and hence the economy had remained in the initial steady state). The variable λ_t^i is the percent increase in consumption (relative to the initial steady state) required to make the agents indifferent between receiving and not receiving the path with the shock.

The bottom right panel of Figure 9 shows that both workers' and capitalists' welfare increases immediately with the positive $A_{N,t}$ shock. However, and in parallel to the dynamics of relative consumption, workers gain relatively more than capitalists throughout the transition. The dynamics of the gross labor share more closely capture this decrease in welfare-based inequality than the dynamics of the net labor share.

While this example is admittedly simple, it does allow us to draw some general lessons for the informativeness of labor shares for inequality outside of steady state. In response to shocks that increase desired investment, capitalists may optimally decide to postpone consumption early on in return for higher consumption later on. The net labor share only captures the net income position of workers relative to capitalists in a specific time period. Net income inequality does not necessarily translate into consumption inequality when some agents are forward looking and can use savings and borrowing to achieve an optimal allocation of resources across time.

Table 2 summarizes quantitatively the dynamics of factor shares and inequality for various shocks.[27] For each shock, the rows of the table show the percent change from the pre-shock steady state in four measures of inequality, $(1 + \lambda_t^K)/(1 + \lambda_t^N)$, C_t^K/C_t^N, $(1 - s_t^N)/s_t^N$, and $(1 - s_t^G)/s_t^G$. The first three columns show these changes 10, 20, and 50 periods after the shock, and the fourth column $(t \to \infty)$ shows the changes when the system reaches the new steady state. For example, consider the response 20 periods after an increase in $A_{N,t}$. The table shows that $(1 + \lambda_t^K)/(1 + \lambda_t^N)$ is 5.2 percent below its pre-shock steady state value, C_t^K/C_t^N is 7.4

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[27]Across all four experiments we standardize the path of the shocks in the following way. Let x_0 denote the pre-shock steady state value and x_T denote the value of the shock in the final steady state. T is the first period in which the shock converges to its final value. We then generate the path of the shock as $x_t = (t+1)^{\nu}$ with $\nu = \log(x_T/x_0)/\log(T+1)$. In all experiments we set $T = 20$. We set $A_{N,T}/A_{N,0} = 1.30$, $A_{K,T}/A_{K,0} = 1.30$, $\beta_T/\beta_0 = 1.0323$, $\xi_T^H/\xi_0^H = 1/3$.

Table 2: Inequality and Factor Shares

		Change From Initial Steady State				$t \in [1,50]$
Shock	Inequality Measure	$t = 10$	$t = 20$	$t = 50$	$t \to \infty$	% Same Sign
$\uparrow A_N$	$(1 + \lambda_t^K)/(1 + \lambda_t^N)$	-0.062	-0.052	-0.018	0.000	–
	C_t^K/C_t^N	-0.067	-0.074	-0.026	0.000	98
	$(1 - s_t^N)/s_t^N$	-0.002	-0.001	0.000	0.000	98
	$(1 - s_t^G)/s_t^G$	-0.033	-0.032	-0.011	0.000	100
$\uparrow A_N = A_K$	$(1 + \lambda_t^K)/(1 + \lambda_t^N)$	-0.026	0.002	0.072	0.110	–
	C_t^K/C_t^N	-0.049	-0.043	0.056	0.110	76
	$(1 - s_t^N)/s_t^N$	0.078	0.102	0.109	0.110	62
	$(1 - s_t^G)/s_t^G$	0.016	0.041	0.087	0.110	64
$\uparrow \beta$	$(1 + \lambda_t^K)/(1 + \lambda_t^N)$	-0.176	-0.145	-0.087	-0.033	–
	C_t^K/C_t^N	-0.237	-0.202	-0.115	-0.033	100
	$(1 - s_t^N)/s_t^N$	-0.001	-0.005	-0.016	-0.033	100
	$(1 - s_t^G)/s_t^G$	0.030	0.058	0.107	0.151	2
$\downarrow \xi^H$	$(1 + \lambda_t^K)/(1 + \lambda_t^N)$	0.013	0.033	0.082	0.109	–
	C_t^K/C_t^N	-0.007	0.005	0.070	0.109	76
	$(1 - s_t^N)/s_t^N$	0.105	0.119	0.108	0.109	100
	$(1 - s_t^G)/s_t^G$	0.045	0.076	0.111	0.128	98

percent below its pre-shock value, $(1 - s_t^N)/s_t^N$ is essentially unchanged relative to its pre-shock value, and $(1 - s_t^G)/s_t^G$ is 3.2 percent below its pre-shock value. At that particular point in time and for that particular shock, consistent with Figure 9, the gross labor share appears a better proxy for inequality than the net labor share.

Figure 10 shows the response of the economy to a factor-neutral shock (i.e. when $A_{N,t} = A_{K,t}$ for all t), plotted in the top left panel. A key difference relative to a labor-augmenting tech-
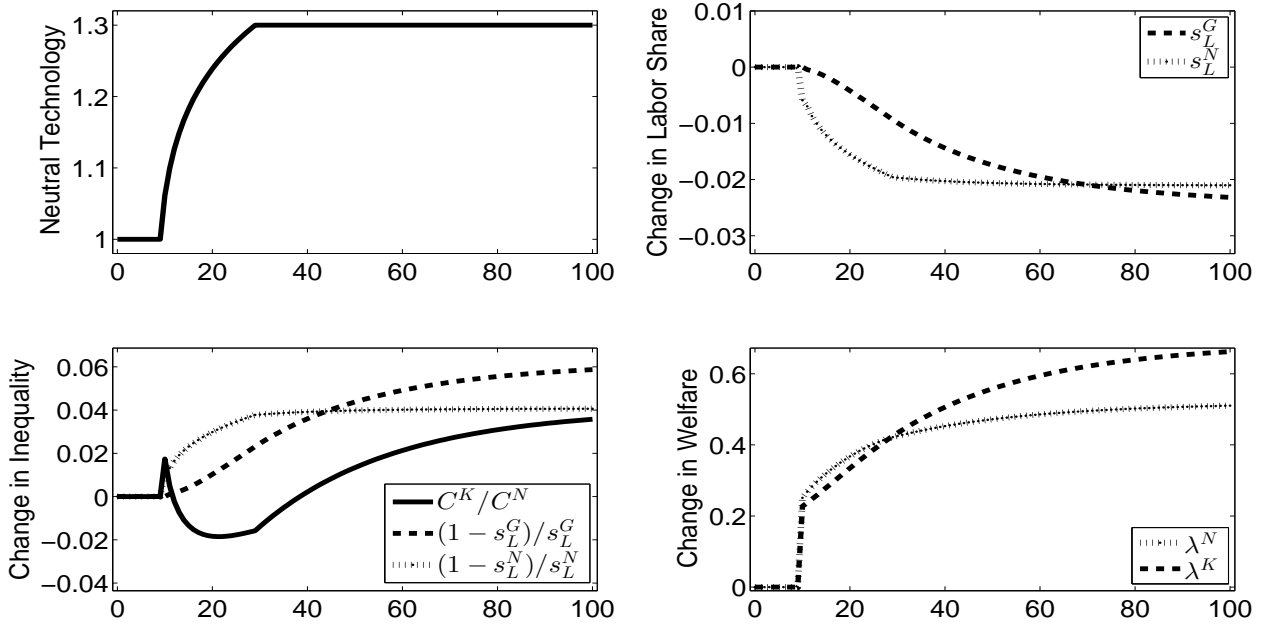
Figure 10: Increase in Factor-Neutral Technology

nology shock is that this shock changes the steady state distribution of consumption between the capitalists and workers. As shown in Table 2, both labor shares are unable to capture the dynamics of inequality in the first few periods. In the initial 10 periods after the shock, both labor share transformations increase whereas both the relative consumption and the welfare-based measures of inequality decrease. Capitalists ultimately benefit more than workers in this case, with the bulk of the relative gains realized only 30 or more periods after the shock (corresponding to $t > 40$ in the figure). The dynamics of the gross labor share appear more closely linked to the behavior of inequality during most of the transition.

The inequality measure $(1 + \lambda_t^K)/(1 + \lambda_t^N)$ captures relative changes in the workers' and capitalists' welfare. Across steady states, its percent change exactly equals the percent change in relative consumption. We report in the final column of Table 2 the share of the first 50 transition periods in which the remaining three inequality measures have the same sign (i.e. have increased or decreased) as $(1 + \lambda_t^K)/(1 + \lambda_t^N)$. We think of this as one summary statistic for the degree to which the labor share measures teach us about inequality during the transition. In the cases of shocks to A_N and ξ^H, we find that both labor shares move together with

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$(1 + \lambda_t^K)/(1 + \lambda_t^N)$ during nearly all of the first 50 periods and therefore are useful proxies for the directional change in inequality. This is not the case for the neutral technology shock $A_N = A_K$, for which both labor share measures do not capture the directional change in inequality in more than one-third of the first 50 periods. In the case of a β shock, the net labor share unambiguously outperforms the gross labor share as a proxy for inequality during the transition.

7 Conclusion

Gross and net labor shares have declined around the world over the past several decades. These factor shares are subject to different measurement concerns, but both have the potential to offer insights into the structure of production, the shocks that hit the economy, and the implications of these shocks for macroeconomic dynamics. We demonstrate that, in fact, the joint behavior of gross and net labor shares is informative for these issues, particularly in a multi-sector environment with rich dynamics in the price of capital and the economy-wide depreciation rate. Declines in the relative price of investment goods, unlike declines in the real interest rate, are consistent with the observed declines in both labor shares.

The finding of declining global labor shares has generated significant attention in part due to their association with increasing inequality. We think this is a useful starting point and a reasonable basis for interest in factor share dynamics. But significant further work is needed. In a simple model we demonstrated the inadequacy of either labor share measure to capture inequality in a simple way outside of steady state, even when the economy is exposed only to basic shocks. The environment imposed a stark, unrealistic, and unchanging split between hand-to-mouth workers and dynamically optimizing capitalists. It abstracted from critical features for inequality such as capital-skill complementarity, redistributive taxation, and life-cycle considerations. We hope that future research can enrich the empirical and theoretical relationship between factor shares and inequality.

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