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INVESTMENT HANGOVER AND THE GREAT RECESSION

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ABSTRACT

We present a model of investment hangover motivated by the Great Recession. In our model, overbuilding of residential capital requires a reallocation of productive resources to nonresidential sectors, which is facilitated by a reduction in the real interest rate. If the fall in the interest rate is limited by the zero lower bound and nominal rigidities, then the economy enters a liquidity trap with limited reallocation and low output. The drop in output reduces nonresidential investment through a mechanism similar to the acceleration principle of investment. The burst in nonresidential investment is followed by an even greater boom due to low interest rates during the liquidity trap. The boom in nonresidential investment induces a partial and asymmetric recovery in which the residential sector is left behind, consistent with the broad trends of the Great Recession.

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1 Introduction

Since 2008, the US economy has been going through the worst macroeconomic slump since the Great Depression. Real GDP per capita declined from more than $49,000 in 2007 (in 2009 dollars) to less than $47,000 in 2009, and surpassed its pre-recession level only in 2013. The civilian employment ratio, which stood at about 63% in January 2008, fell below 58% by January 2010, and remains below 59% in June 2014.

Recent macroeconomic views emphasize the burst of the housing bubble—and its effects on financial institutions, firms, and households—as the main culprit for these developments. The collapse of home prices arguably affected the economy through at least two principal channels. First, it triggered the *financial crisis*, which led financial institutions that suffered losses related to the housing market to cut back their lending to firms and households (Brunnermeier (2008), Gertler and Kiyotaki (2010)). Second, the reduction in home prices also generated a *household deleveraging crisis*, in which homeowners that suffered leveraged losses from their housing equity cut back their consumption so as to reduce their outstanding leverage (Guerrieri and Lorenzoni (2011), Eggertsson and Krugman (2012), Mian and Sufi (2014)). Both crises reduced aggregate demand, plunging the economy into a Keynesian recession. The recession was exacerbated by the zero lower bound on the nominal interest rate, also known as *the liquidity trap*, which restricted the ability of monetary policy to counter these demand shocks (Hall (2011), Christiano, Eichenbaum, Trabandt (2014)).

A growing body of empirical evidence shows that these views are at least partially correct: the financial and the household crises both appear to have played a part in the Great Recession.[1] But these views also face a challenge in explaining the

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[1]Several recent papers, such as Campello, Graham, and Harvey (2010) and Chodorow-Reich (2014), provide some evidence that financial crisis affected firms' investment before 2010. Mian, Rao, Sufi (2013) and Mian and Sufi (2012) provide evidence that household deleveraging reduced

nature of the recovery after the Great Recession. As Figure 1 illustrates, the recovery has been quite asymmetric across components of aggregate private spending. Non-residential investment and consumption—measured as a fraction of output—reached or exceeded their pre-recession levels by 2013, while residential investment remained depressed. One explanation for this pattern is that households are unable to buy houses due to ongoing deleveraging. But the left panel of Figure 1 casts doubt on this explanation: sales of durable consumption goods such as cars—which should also be affected by household deleveraging—rebounded strongly in recent years while residential investment has lagged behind.

In this paper, we supplement the two accounts of the Great Recession with a third channel, which we refer to as the *investment hangover*, which could help explain the asymmetric recovery. Our key observation is that the housing bubble was an *investment bubble* as much as an asset price bubble. Overbuilding during the bubble years created excess supply of housing capital by 2007, especially certain types of capital such as owner occupied housing. Between 1996 and 2006, the share of US households living in their own homes rose from about 65% to about 69%. The homeownership rate fell back below 65% in 2014, suggesting that the housing capital might have been in excess for many years after 2007. The excess housing capital lowers residential investment and slows down economic activity.

Our argument so far is similar to the Austrian theory of the business cycle, in which recessions are times at which excess capital built during boom years is liquidated (Hayek (1931)). The Hayekian view, however, faces a challenge in explaining how low investment in the liquidating sector reduces aggregate output and employment. As noted by Krugman (1998), the economy has a natural adjustment mechanism

household consumption and employment between 2007 and 2009.
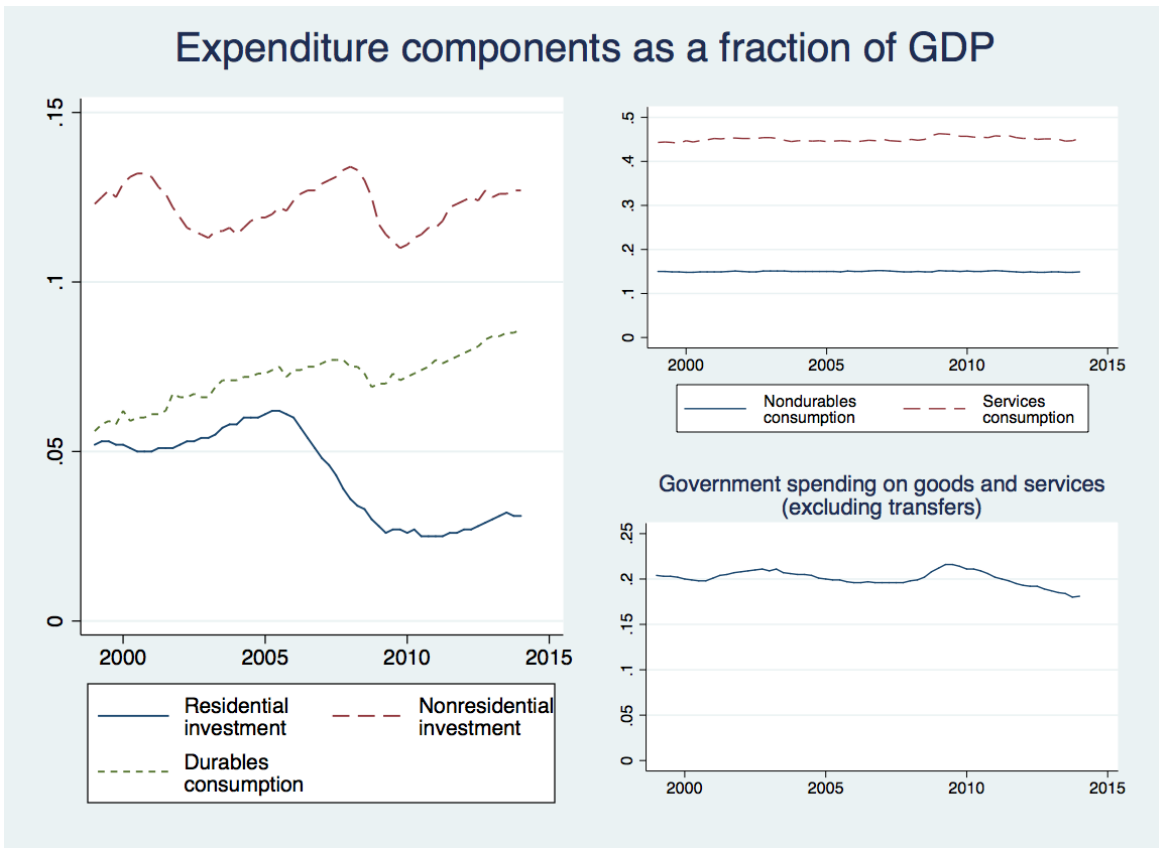
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Figure 1: The plots illustrate different components of aggregate demand as a fraction of GDP between 1999 and 2004. The data is quarterly and reported as the seasonally adjusted annual rate. Source: St. Louis Fed.

that facilitates the reallocation of labor (and other productive resources) from the liquidating sector to other sectors. As the interest rate falls during the liquidation phase due to low aggregate demand, other sectors expand and keep employment from falling. This reallocation process can be associated with some increase in frictional unemployment. But it is unclear in the Austrian theory how employment can fall in *both* the liquidating and the nonliquidating sectors, which seems to be the case for major recessions such as the Great Recession. To fit that evidence, an additional— Keynesian—aggregate demand mechanism is needed.

Accordingly, we depart from the Hayekian view by emphasizing that, during the Great Recession, the aggregate reallocation mechanism was undermined by the zero lower bound constraint on monetary policy. If the initial overbuilding is sufficiently large, then the interest rate hits a lower bound and the economy enters a liquidity trap. As this happens, low investment in the residential sector cannot be countered by the expansion of other sectors. Instead, low investment reduces aggregate demand and output, contributing to the Keynesian slump.

We also illustrate how overbuilding of residential capital can actually *reduce* nonresidential investment and consumption through two channels. First, the Keynesian slump reduces the return to nonresidential capital such as business equipment. We show that this can generate an initial reduction in nonresidential investment and capital, despite the low interest rate and the low cost of capital. The nonresidential investment response in turn aggravates the recession as emphasized by the previous literature on the acceleration principle of investment (see Samuelson (1939)). Second, the Keynesian slump also reduces the income of many individuals—for instance, those who work in the residential sector. If those individuals have relatively high marginal propensities to consume (MPC) out of income, then overbuilding also reduces

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aggregate consumption. The consumption response further aggravates the recession through a Keynesian income multiplier.

Our model explains the asymmetric recovery depicted in Figure 1. As the economy liquidates the excess residential capital, nonresidential investment gradually recovers in anticipation of a recovery in output. In fact, the initial burst in nonresidential investment is followed by an even greater boom due to low interest rates, leaving the economy with a high level of capital at the end of the liquidity trap episode. It follows that, from the lens of our model, the recession can be roughly divided into two phases. In the first phase, both types of investment as well as consumption decline, generating a severe slump. In the second phase, residential investment remains low but nonresidential investment is high. The increase in nonresidential investment also mitigates the slump and induces a partial recovery in output and consumption. Hence, the residential sector is left behind in the recovery, as in Figure 1.

Our paper is part of a large macroeconomic literature that attempts to identify the mechanisms of the Great Recession.[2] Two features differentiate our analysis from recent accounts that also emphasize demand shocks and the liquidity trap. First, motivated by the asymmetric recovery depicted in Figure 1, we emphasize overbuilding of residential capital as a key driving factor of the recession. In contrast, Eggertsson and Krugman (2012) emphasize a consumption shock due to household deleveraging, and Christiano, Eichenbaum, Trabandt (2014) emphasize a nonresidential investment shock due to firms' financial frictions (as well as a consumption shock). Second, we

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[2] In addition to the above mentioned papers, see Gertler and Karadi (2011), Jermann and Quadrini (2012), He and Krishnamurthy (2014), Midrigan and Philippon (2011) for quantitative dynamic macroeconomic models that emphasize either banks', firms', or households' financial frictions during the Great Recession. There is also a vast theoretical literature that analyzes the amplification mechanisms that could have exacerbated the financial crisis (see Brunnermeier, Eisenbach, Sannikov (2013) for a survey). Another theoretical literature investigates the liquidity trap and its policy implications (see Korinek and Simsek (2014) and the references therein).

illustrate how the problems in the residential investment sector can spread to non-residential investment and consumption, even without financial shocks to firms or households. We do not claim that financial shocks were unimportant during the Great Recession. Rather, our point is that overbuilding was also an important contributing factor, with implications that confound financial shocks, especially earlier in the recession. These confounding effects should be taken into account by empirical analyses of the Great Recession.

Our paper is also related to a macroeconomic literature that investigates the role of reallocation shocks relative to aggregate shocks in generating unemployment fluctuations (see Lilien (1982), Abraham and Katz (1986), Blanchard and Diamond (1989), Davis and Haltiwanger (1990)). Our paper shows how reallocation shocks can *endogenously* turn into aggregate shocks. In our model, expanding sectors are constrained due to nominal rigidities and constrained monetary policy, which restricts reallocation and triggers a Keynesian recession. Caballero and Hammour (1996) alternatively emphasize a supply-side channel by which reallocation is restricted because the expanding sectors are constrained due to a hold-up problem.

As we have noted, our paper makes contact with the Austrian (or Hayekian) theory of the business cycle. As DeLong (1990) discusses, liquidationist views along these lines were quite popular before and during the Great Depression, but were relegated to the sidelines with the Keynesian revolution in macroeconomics. Our paper illustrates how Hayekian and Keynesian mechanisms can come together to generate a recession. The Hayekian mechanism finds another modern formulation in the recent literature on news-driven business cycles. A strand of this literature argues that positive news about future productivity can generate investment booms, occasionally followed by liquidations if the news is not realized (see Beaudry and Portier (2013) for a review).

This literature typically generates business cycles from supply side considerations (see, for instance, Beaudry and Portier (2004), Jaimovich and Rebelo (2009)), whereas we emphasize demand shortages as the key mechanism by which liquidations trigger a recession.

In recent work, Beaudry, Galizia, Portier (BGP, 2014) also investigate channels by which overbuilding can induce a recession driven by demand shortages. Their paper is complementary to ours in the sense that they use different ingredients and emphasize a different mechanism. In BGP, aggregate demand affects employment due to a matching friction in the labor market, whereas we obtain demand effects through nominal rigidities. In addition, BGP emphasize how overbuilding increases the (uninsurable) unemployment risk, which exacerbates the recession due to households' precautionary savings motive. In contrast, we emphasize how overbuilding reduces the return to other (nonresidential) types of capital, which exacerbates the recession due to the endogenous investment response. We also apply our model to explain the asymmetric recovery from the Great Recession.

Our paper is also related to the literature on the acceleration principle of investment (see Clark (1917)). This principle posits that the target level of capital is proportional to output, so that investment is driven by *changes* in output. Samuelson's (1939) famous multiplier-accelerator analysis shows that this principle can also aggravate business cycles driven by demand shocks. The acceleration principle fell out of fashion partly because it relies on mechanical relations between investment and output, without considering changes in the cost of capital (see Caballero (1999)). In our model, a version of the acceleration principle emerges *endogenously* from agents' optimizing behavior. Intuitively, the liquidity trap keeps the cost of capital constant, resuscitating the acceleration principle and some of its macroeconomic implications.

The rest of the paper is organized as follows. Section 2 describes the basic environment, defines the equilibrium, and establishes the properties of equilibrium that facilitate subsequent analysis. The remaining sections characterize the dynamic equilibrium starting with excess residential capital. Section 3 presents our main result that excessive overbuilding induces a recession, and establishes conditions under which this outcome is more likely. Section 4 investigates the nonresidential investment response and discusses the relationship of our model with the acceleration principle. Section 5 investigates the consumption response in a version of the model in which some agents have high MPCs out of income. Section 6 concludes.

2 Basic environment and equilibrium

The economy is set in infinite discrete time $t \in \{0, 1, ...\}$ with a single consumption good, and three factors of production: residential capital, nonresidential capital, and labor. For brevity, we also refer to nonresidential capital as "capital." A neoclassical production function $F(k_t, l_t)$ converts k_t units of capital and l_t units of labor into $F(k_t, l_t)$ units of consumption good. Residential capital produces housing services according to a separate neoclassical production function $G(h_t)$.

One unit of the consumption good can be converted into one unit of residential or nonresidential capital without any adjustment costs. Thus, the two types of capital evolve according to

$$h_{t+1} = h_t\left(1 - \delta^h\right) + i_t^h \tag{1}$$
$$\text{and } k_{t+1} = k_t\left(1 - \delta^k\right) + i_t^k.$$

Here, i_t^h (resp. i_t^k) denote residential (resp. nonresidential) investment, and δ^h (resp.

δ^k) denotes the depreciation rate for residential (resp. nonresidential) capital.

As we will see, absent shocks, the economy will be at a neoclassical steady state in which the two types of capital are kept at fixed levels denoted by h^* and k^*. We will analyze situations in which the economy starts with excess residential capital, $h_0 > h^*$ (see Eq. (12) below). This assumption can be thought of as capturing an unmodeled overbuilding episode that took place before the start of our model. We are agnostic about the reason for overbuilding, which could be driven, among other things, by optimistic beliefs (or news) in the past that were ultimately corrected. Our focus will be on understanding how the economy decumulates the overbuilt capital.

The demand side is captured by a representative household that makes residential investment, saving, and labor supply decisions. The household invests in residential capital directly (for simplicity), and invests in physical capital indirectly by holding financial claims on competitive investment firms. In particular, the real interest rate between dates t and $t+1$ is given by

$$r_{t+1} = R_{t+1} - \delta^k, \tag{2}$$

where R_{t+1} denotes the rental rate of capital.

The household has a utility function over consumption, labor, and housing services $U(\tilde{c}_t, l_t, G(h_t))$. She takes the interest rate r_{t+1} and the wage level w_t at any date t as given, and solves:

$$\max_{\{\tilde{c}_t, k_t, h_t, l_t\}_t} \sum_{t=0}^{\infty} \beta^t U(\tilde{c}_t, l_t, G(h_t)) \tag{3}$$
$$\text{s.t. } h_{t+1} = h_t(1 - \delta^h) + i_t^h \text{ and}$$
$$\tilde{c}_t + k_{t+1} + i_t^h = l_t w_t + k_t(1 + r_t) + \Pi_t \text{ for each } t.$$

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Here, Π_t denotes profits from firms that are described below.

We adopt the following specific functional form for the household's utility function

$$U\left(\tilde{c}_t, l_t, h_t\right) = u\left(\tilde{c}_t - v\left(l_t\right)\right) + u^h \mathbf{1}\left[G\left(h_t\right) \geq G\left(h^*\right)\right]. \tag{4}$$

Here, the function $u\left(\cdot\right)$ is strictly increasing and concave. The expression $\mathbf{1}\left[G\left(h_t\right) \geq G\left(h^*\right)\right]$ is equal to 1 if $G\left(h_t\right) \geq G\left(h^*\right)$ and zero otherwise, and u^h is a large constant.

The specification in (4) entails two simplifying assumptions. First, the utility term for housing services implies that, starting with any h_0, the household accumulates or decumulates its residential capital as quickly as possible so as to reach and stay at the level h^*. Hence, we refer to h^* as *the target level of residential capital.* This assumption considerably simplifies the residential investment part of the model, and enables us to focus on the effect of overbuilding on the rest of the equilibrium allocations.

The second simplification in (4) is the functional form $u\left(\tilde{c}_t - v\left(l_t\right)\right)$, which implies that the household's labor supply decision does not depend on its consumption (see Greenwood, Hercowitz and Huffman (1988)). In particular, the efficient supply of labor and output depend only on the capital stock, which is predetermined within a period. This specification enables us to isolate the demand effects as deviations from the efficient supply, which facilitates the exposition.

The supply side of the model is Keynesian, and uses the equilibrium concept introduced in Korinek and Simsek (2014). The key ingredient is that there is a lower bound on the real interest rate, which we normalize to 0 to simplify the notation:

$$r_{t+1} \geq 0 \text{ for all } t. \tag{5}$$

The lower bound on the real rate follows from two assumptions. First, we assume the nominal interest rate is bounded from below by 0%, because households can guarantee 0% return by simply hoarding cash. Second, we assume that nominal prices are completely sticky, which turns the bound on the nominal rate into the bound in (5). Since December 2008, the 3-month nominal interest rate in the US has been constant at 0% and the core inflation has been relatively stable between 1.5% and 2%, providing some empirical support for our assumptions.

When the constraint in (5) binds, the real interest rate is too high relative to its market clearing level. Since the interest rate is the price of current consumption good (in terms of the future consumption good), an elevated interest rate leads to a demand shortage for current goods and a rationing of supply. We capture the possibility of rationing via a competitive final goods sector that solves

$$\Pi_t = \max_{k_t, l_t} F(k_t, l_t) - R_t k_t - w_t l_t \tag{6}$$

$$\text{s.t.} \begin{cases} k_t, l_t \geq 0 & \text{if } r_{t+1} > 0 \\ k_t, l_t \geq 0 \text{ and } F(k_t, l_t) \leq \tilde{c}_t + i_t^k + i_t^h & \text{if } r_{t+1} = 0 \end{cases}.$$

When the real interest rate is above the lower bound, the sector optimizes as usual. When the interest rate is at its lower bound, $r_{t+1} = 0$, the sector is subject to an additional constraint that supply cannot exceed the aggregate demand for goods. In this case, the equilibrium output is determined by aggregate demand at the bounded interest rate, $r_{t+1} = 0$.

Definition 1. *The equilibrium is a path of allocations, $\left\{ \tilde{c}_t, k_t, l_t, i_t^h, i_t^k, e_t \right\}_t$, and real prices and profits, $\left\{ w_t, R_t, \Pi_t, r_{t+1} \right\}_t$, such that the household allocations solve problem (3), the final good sector solves problem (6), investment sector optimizes so that Eq.*

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(2) *holds, residential and nonresidential capital evolve according to* (1) *and markets clear.*

Our equilibrium notion is similar to the rationing equilibria analyzed by a strand of the Keynesian macroeconomics literature, e.g., Barro and Grossman (1971), Malinvaud (1977). We focus on the special case in which there is rationing in the goods market if $r_{t+1} = 0$, but no rationing in the labor market. We put the rationing in the goods market not because we think it is more realistic, but because it features the minimally required departure from a Walrasian equilibrium to capture a liquidity trap. Adding wage rigidities and rationing to the labor market could exacerbate the outcomes, but it would not change our qualitative conclusions. As described in Korinek and Simsek (2014), our equilibrium concept is also very similar to a New-Keynesian model with monopolistic competition in which firms' prices are completely sticky. The rationing equilibrium captures the real effects of a liquidity trap without introducing nominal variables and monopolistic competition, thereby simplifying the exposition.

We next establish basic properties of equilibrium that will be useful in subsequent analysis. In view of the specification in (4), we work with *net consumption* $c_t = \tilde{c}_t - v(l_t)$, that is, consumption net of the disutility of labor. We also define the *net output* as

$$Y_t = c_t + k_{t+1} + i_t^h, \text{ where } k_{t+1} = k_t\left(1 - \delta^k\right) + i_t^k. \tag{7}$$

Note that net output includes not only investment but also nondepreciated capital, which simplifies the notation. It is also useful to define *the maximum supply* as the

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maximum level of net output the economy can obtain,

$$S\left(k_{t}\right) = \max_{l_{t}} F\left(k_{t}, l_{t}\right) - v\left(l_{t}\right) + \left(1 - \delta^{k}\right) k_{t}, \tag{8}$$

and *the efficient labor supply* l_{t}^{*} as the solution to problem (8). The economy is then subject to the resource constraints

$$Y_{t} \leq S\left(k_{t}\right) \text{ and } Y_{t} \geq \left(1 - \delta^{k}\right) k_{t}. \tag{9}$$

The following lemma describes the possibilities for equilibrium within a period.

Lemma 1. *(i) If $r_{t+1} > 0$, then $Y_{t} = S\left(k_{t}\right)$ and $l_{t} = l_{t}^{*}$.*

(ii) If $r_{t+1} = 0$, then Y_{t} satisfies (9). The labor supply is the unique solution to

$$Y_{t} = F\left(k_{t}, l_{t}\right) - v\left(l_{t}\right) + \left(1 - \delta^{k}\right) k_{t}, \text{ over the range } l_{t} \in \left[0, l_{t}^{*}\right]. \tag{10}$$

The first part shows that, if the interest rate is unconstrained, then the economy utilizes its resources efficiently. Net output is maximized and labor supply is at its efficient level.

The second part describes *the liquidity trap* scenario in which the interest rate is at its lower bound. In this case, net output satisfies the resource constraints in (9) but it is otherwise unrestricted. The actual level of net output is determined by the aggregate demand at date t as illustrated by Eq. (7). Given Y_{t}, the level of employment is found as the solution to (10) and satisfies $l_{t} \leq l_{t}^{*}$. Intuitively, the economy features a demand-driven recession with potentially low output and employment. For future reference, it is also useful to characterize the gross return to capital.

Lemma 2. *The gross return to capital is given by*

$$1 + r_t = 1 + R_t - \delta^k = \begin{cases} S'\left(k_t\right) & \text{if } r_{t+1} > 0, \\ s\left(k_t, Y_t\right) \leq S'\left(k_t\right) & \text{if } r_{t+1} = 0, \end{cases}$$

where the function $s\left(k_t, Y_t\right)$ is strictly decreasing in k_t and strictly increasing in Y_t.

Absent a liquidity trap, capital earns its marginal contribution to supply, $S'\left(k_t\right)$. In a liquidity trap, capital's return is lower and given by a function $s\left(k_t, Y_t\right)$. Intuitively, the shortage of demand reduces factor returns. Higher Y_t increases the return to capital due to higher demand, while higher k_t reduces it due to diminishing returns.

Combining Lemma 2 with the lower bound in (5) also implies a *maximum level of capital $\bar{k}$*, defined as the solution to

$$S'\left(\bar{k}\right) = 1. \tag{11}$$

In particular, starting with $k_0 \leq \bar{k}$, capital in this economy cannot reach above $\bar{k}$ since this would lead to a gross real return below 1. It is also useful to define *the steady state level of capital k^** as the solution to

$$\beta S'\left(k^*\right) = 1.$$

We next turn to the characterization of the dynamic equilibrium.

3 Investment hangover and the Keynesian recession

We characterize the equilibrium under the assumption that the economy starts with too much residential capital

$$h_0 = (1 + b_0) h^*, \text{ where } b_0 > 0. \tag{12}$$

This assumption can be thought of as capturing an unmodeled investment bubble that took place before the start of our model. The parameter b_0 measures the degree of overbuilding as a fraction of the steady-state stock of residential capital h^*. Our main result, which we present in this section, shows that this type of overbuilding can induce a recession.

Given (4), the residential investment level at date 0 is

$$i_0^h = h^* - \left(1 - \delta^h\right) h_0 = \left(\delta^h - b_0 \left(1 - \delta^h\right)\right) h^*. \tag{13}$$

Note that the residential investment is below the level required to maintain the target residential capital $i_0^h < \delta^h h^*$. Hence, overbuilding represents a negative shock to the residential investment demand relative to a steady state. The equilibrium depends on how the remaining components of aggregate demand—nonresidential investment and consumption—respond to this shock.

To characterize this response, we solve the equilibrium backwards. Suppose the economy reaches date 1 with $h_1 = h^*$ and some capital level $k_1 \leq \overline{k}$. Consider the continuation equilibrium. Residential investment is given by $i_t^h = \delta^h h^*$. Since there are no further demand shocks, the equilibrium does not feature a liquidity trap,

that is, $r_{t+1} > 0$ for each $t \geq 1$. Labor and output are then at their efficient levels, respectively given by l_t^* and $S(k_t)$. The equilibrium path $\{c_t, k_{t+1}\}_{t=1}^{\infty}$ is characterized as the solution to the neoclassical system

$$
\begin{aligned}
c_t + k_{t+1} + \delta^h h^* &= S(k_t) \qquad\qquad (14)\\
u'(c_t) &= \beta S'(k_t) u'(c_{t+1}),
\end{aligned}
$$

along with a transversality condition. For the rest of the analysis, we make the following assumption, which ensures that the economy is able to afford the required residential investment at the initial period as well as the steady state.

Assumption 1. $\min(S(k_0), S(k^*)) > k^* + \delta^h h^*$, and

Under this assumption, there is a unique solution to the system in (14) that converges to a steady state (c^*, k^*) characterized in the appendix. The initial consumption can be written as $c_0 = C(k_T)$, where $C(\cdot)$ is an increasing function.

Next consider the equilibrium at date 0. The key observation is that both nonresidential investment and consumption are bounded from above due to the lower bound on the interest rate. Recall that capital cannot exceed its maximum level, $k_1 \leq \bar{k}$ [cf. Eq. (11)]. This also implies a bound on nonresidential investment

$$
i_0^k \leq \bar{k} - (1 - \delta^k) k_0. \qquad\qquad (15)
$$

Intuitively, there are only so many projects that can be undertaken without violating the lower bound on the interest rate. Consumption is similarly bounded. Combining the inequality $c_1 \leq C(\bar{k})$ with the lower bound on the interest rate implies

$$
c_0 \leq \bar{c}_0, \text{ where } u'(\bar{c}_0) = \beta u'(C(\bar{k})). \qquad\qquad (16)
$$

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Intuitively, the household can be incentivized to consume only so much without violating the interest rate bound.

Combining the bounds in (15) and (16) with the demand shock in (13), the aggregate demand (and output) at date 0 is also bounded from above, that is

$$Y_0 \leq \overline{Y}_0 \equiv \overline{k} + \overline{c}_0 + \left(\delta^h - b_0 \left(1 - \delta^h \right) \right) h^*. \tag{17}$$

The equilibrium depends on the comparison between the maximum demand and the maximum supply, i.e., whether $\overline{Y}_0 < S(k_0)$. This in turn depends on whether the amount of overbuilding b_0 exceeds a threshold level,

$$\overline{b}_0 \equiv \frac{\delta^h h^* + \overline{k} + \overline{c}_0 - S(k_0)}{\left(1 - \delta^h \right) h^*}. \tag{18}$$

Proposition 1 (Overbuilding and the Liquidity Trap). *Consider the model with $b_0 > 0$ (and thus $h_0 > h^*$). Suppose Assumption 1 holds.*

(i) Suppose $b_0 \leq \overline{b}_0$. Then, the date 0 equilibrium features

$$r_1 \geq 0, Y_0 = S(k_0) \ \ and \ l_0 = l_0^*.$$

(ii) Suppose $b_0 > \overline{b}_0$. Then, the date 0 equilibrium features a liquidity trap with

$$r_1 = 0, k_1 = \overline{k}, Y_0 = \overline{Y}_0 < S(k_0) \ \ and \ l_0 < l_0^*.$$

Moreover, output Y_0 and labor supply l_0 are decreasing in the amount of overbuilding b_0.

In either case, starting date 1, the economy converges to the steady state (k^*, c^*) according to the system in (14).
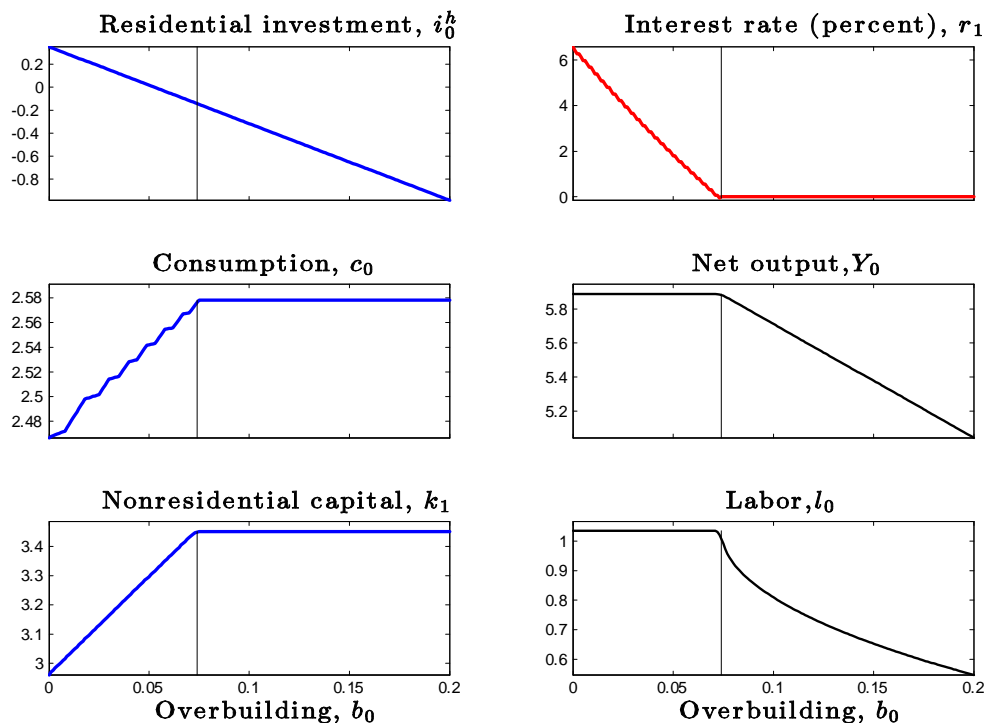
Figure 2: Date 0 equilibrium variables as a function of the initial overbuilding b_0 (measured as a fraction of the target residential capital stock, h^*), starting with $k_0 = k^*$.

Part (i) describes the equilibrium for the case in which the initial overbuilding is not too large. In this case, the economy does not fall into a liquidity trap. Residential disinvestment is offset by a reduction in the interest rate and an increase in nonresidential investment and consumption, leaving the output and employment determined by productivity. The left part of the panels in Figure 2 (the range corresponding to $b_0 \leq \bar{b}_0$) illustrate this outcome. This is the Austrian case.

Part (ii) of Proposition 1, our main result, characterizes the case in which the initial overbuilding is sufficiently large. In this case, the demand shock associated with residential disinvestment is large enough to plunge the economy into a liquidity trap. The lower bound on the interest rate prevents the nonresidential investment

and consumption sectors from expanding sufficiently to pick up the slack aggregate demand. As a consequence, the initial shock translates into a Keynesian recession with low output and employment. Figure 2 illustrates this result. A greater initial residential shock—driven by greater overbuilding—triggers a deeper recession. This is the Keynesian case of our model.

3.1 Comparative statics of the liquidity trap

We next investigate the conditions under which a given amount of overbuilding b_0 triggers a liquidity trap. As illustrated by Eq. (18), factors that reduce aggregate demand at date 0, such as a higher discount factor β (that lowers $\bar{c}_0$), increase the incidence of the liquidity trap in our setting. More generally, other frictions that reduce aggregate demand during the decumulation phase, such as household deleveraging or the financial crisis, are also complementary to our mechanism.

Perhaps less obviously, Eq. (18) illustrates that a higher initial level of nonresidential capital stock k_0 also increases the incidence of a liquidity trap. A higher k_0 affects the equilibrium at date 0 through two main channels. First, it increases output $F(k_0, l_0)$ for any given amount of labor, which makes it more likely that aggregate demand will fall short of the maximum supply. Second, a higher k_0 also reduces nonresidential investment at date 0, which in turn lowers aggregate demand. Hence, overbuilding of the two types of capital is complementary in terms of triggering a liquidity trap.

A distinguishing feature of residential capital is its high durability relative to other types of capital. A natural question is whether high durability is conducive to triggering a liquidity trap in our setting. Our model so far is not well suited to address this question, since changing the depreciation rate δ^h creates general equilibrium effects

that are orthogonal to the question (for instance, it changes the steady-state compo-sition of output between consumption, residential, and nonresidential investment).

To isolate the effect of durability, consider a slight variant of the model in which there are two types of residential capital denoted by h^d and h^n, each of which has a target level $h^*/2$. The key difference is their depreciation rate, which are respectively given by δ^{h^d} and δ^{h^n}, with $\delta^{h^d} < \delta^{h^n}$. Thus, type d (durable) residential capital has a lower depreciation rate than type n (nondurable) residential capital. Suppose $\left(\delta^{h^d} + \delta^{h^n}\right)/2 = \delta^h$ so that the average depreciation rate is the same as before. Let $h_0^d = \left(1 + b_0^d\right)(h^*/2)$ and $h_0^n = \left(1 + b_0^n\right)(h^*/2)$, so that b_0^d and b_0^n capture the overbuilding in respectively durable and nondurable capital relative to their target levels. Suppose also that $\left(b_0^d + b_0^n\right)/2 = b_0$ so that the total amount of overbuilding is the same as before. The case with symmetric overbuilding, $b_0^d = b_0^n = b_0$, results in the same equilibrium as in the earlier model. Our next result investigates the effect of overbuilding one type of capital more than the other.

Proposition 2 (Role of Durability). *Consider the model with two types of res-idential capital with different depreciation rates. Given the average overbuilding $b_0 = \left(b_0^d + b_0^n\right)/2$, the incidence of a liquidity trap $1\left[l_0 < l_0^*\right]$ is increasing in over-building of the more durable residential capital b_0^d.*

To provide an intuition, consider the aggregate demand at date 0, which can be written as

$$\overline{Y}_0 \equiv \overline{k} + \overline{c}_0 + \delta^h h^* - b_0^d \left(1 - \delta^{h^d}\right)\frac{h^*}{2} - b_0^n \left(1 - \delta^{h^n}\right)\frac{h^*}{2}. \tag{19}$$

Note that $1 - \delta^{h^d} > 1 - \delta^{h^n}$, and thus, overbuilding of the durable residential capital (relative to the nondurable capital) induces a greater reduction in aggregate demand
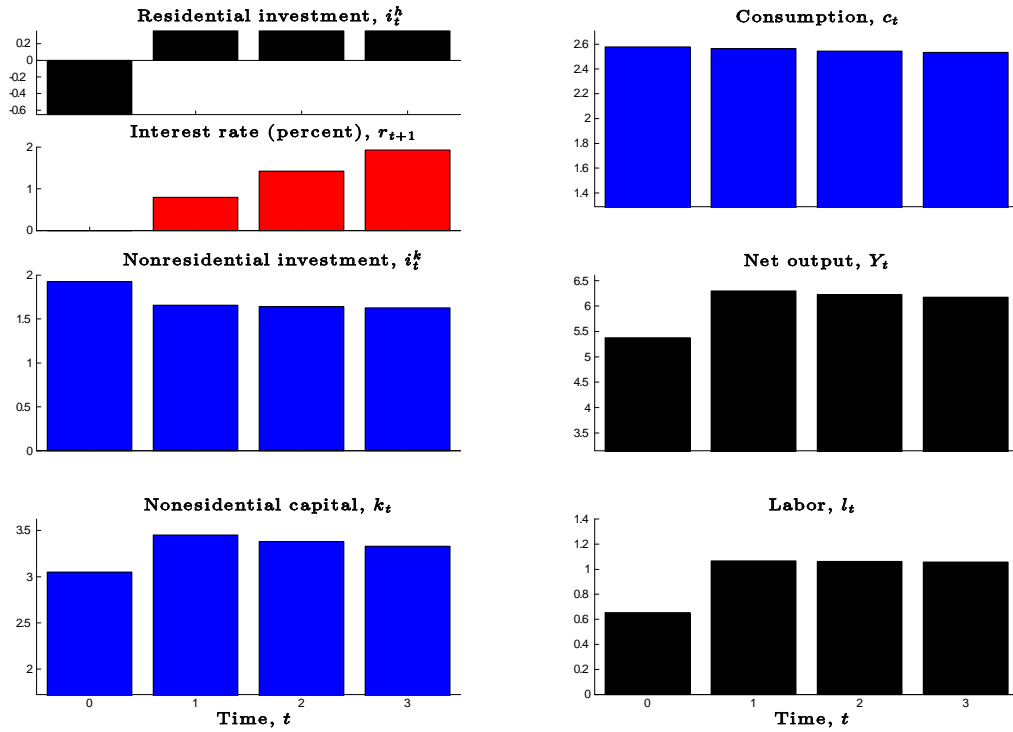
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Figure 3: The evolution of equilibrium variables over time, starting with $b_0 > \bar{b}_0$ and $k_0 = k^*$.

at date 0. Intuitively, depreciation helps to "erase" the overbuilt capital naturally, thereby inducing a smaller reduction investment. When the capital is more durable, there is less natural erasing. This in turn leads to lower investment and aggregate demand, and makes a liquidity trap more likely. This result suggests that overbuilding is more of a concern when it hits durable capital such as residential investment, structures, or infrastructure (e.g., railroads), as opposed to less durable capital such as equipment or machinery.

3.2   Aftermath of the recession

We next investigate the equilibrium behavior in the aftermath of the liquidity trap. Figure 3 plots the full dynamic equilibrium in the original model with single residential capital (and in the liquidity trap scenario). The initial shock generates a temporary recession, followed by a neoclassical adjustment after the recession.

The interest rate gradually increases during the aftermath of the recession, and might remain below its steady-state level for several periods. This is because the economy accumulates capital during the liquidity trap thanks to low interest rates. The economy decumulates this capital only gradually over time, which leaves the rate of return low after the recession. These low rates are reminiscent of the secular stagnation hypothesis, which was recently revived by Summers (2013). According to this hypothesis, the economy could permanently remain depressed with low interest rates due to a chronic demand shortage (see Eggertsson and Mehrota (2014) for a formalization). In our model, the economy eventually recovers. But the low rates in the aftermath suggest that the economy remains fragile to another demand shock, even though it does not feature secular stagnation.

Figure 3 illustrates further that, while there is a recession at date 0, several components of aggregate demand—especially nonresidential investment—actually expand. The recession is confined to the residential investment sector in which the shock originates. This feature is inconsistent with facts in major recessions, such as the Great Recession, in which all components of aggregate demand decline simultaneously. To resolve this puzzle, we next analyze the investment and consumption responses in more detail.
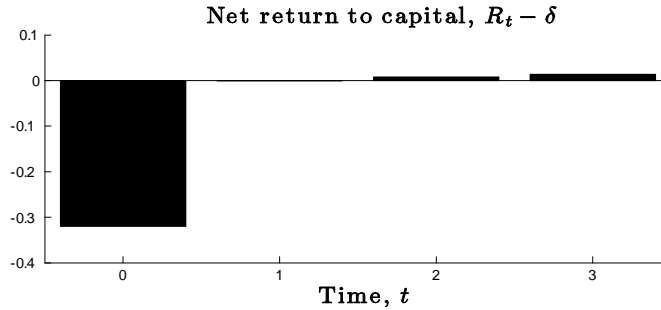
Figure 4: The evolution of net return to capital over time, starting with $h_0 > \overline{h}_0$ and $k_0 = k^*$.

4 Nonresidential investment response and the acceleration principle

This section investigates the investment response in a slight variant of the model in which the liquidity trap persists over multiple periods. We show that overbuilding of residential capital can induce an initial burst in nonresidential investment followed by a boom. We also discuss the relationship of our model to the acceleration principle of investment.

The analysis is motivated by Figure 4, which illustrates the evolution of the net return to capital $R_t - \delta^k$ corresponding to the equilibrium plotted in Figure 3. The near-zero return during the recovery phase reflects the high level of capital (and low interest rates). The figure illustrates that the net return at date 0 is even lower—in fact, in negative territory—even though the initial capital level is not high (we use $k_0 = k^*$ in simulations). Intuitively, the recession at date 0 lowers not only the output but also factor returns, including the return to capital (see Lemma 2). This suggests that, if nonresidential investment could respond to the shock during period 0, it could also fall.

To investigate this possibility, we modify the model so that the residential disin-

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vestment is spread over many periods. One way to ensure this is to assume that there is a lower bound on housing investment at every period.

Assumption 2. $i_t^h \geq i^h$ for each t, for some $i^h < \delta^h h^*$.

For instance, the special case $i^h = 0$ captures the idea that housing investment is irreversible. More generally, the lower bound provides a tractable model of adjustment costs. To simplify the exposition, we also assume that the initial overbuilding b_0 (and thus $h_0 = h^*\left(1 + b_0\right)$) is such that the economy adjusts to the target level in exactly $T \geq 1$ periods.

Assumption 3. $\delta^h h^* = \left(\delta^h h_0\right)\left(1 - \delta^h\right)^T + i^h\left(1 - \left(1 - \delta^h\right)^T\right)$ for an integer $T \geq 1$.

With these assumptions, the residential investment path is given by

$$
i_t^h = \begin{cases} i^h < \delta^h h^* & \text{if } t \in \{0, ..., T-1\} \\ \delta^h h^* & \text{if } t \geq T \end{cases}.
\tag{20}
$$

For future reference, note that the parameter i^h also provides an (inverse) measure of the severity of the residential investment shock.

As before, we characterize the equilibrium backwards. The economy reaches date T with residential capital $h_T = h^*$ and some $k_T \leq \overline{k}$. The continuation equilibrium is characterized by the same conditions as before (see Eq. (14)). In particular, consumption is given by $c_T = C\left(k_T\right)$, where recall that $C\left(\cdot\right)$ is an increasing function.

Next consider the equilibrium during the decumulation phase, $t \in \{0, .., T-1\}$. We conjecture that—under appropriate assumptions—there is an equilibrium that features a liquidity trap at all of these dates, that is, $r_{t+1} = 0$ for each $t \in \{0, .., T-1\}$. In this equilibrium, the economy reaches date T with the maximum level of capital, $k_T = \overline{k}$. Consumption is also equal to its maximum level, that is, $c_t = \overline{c}_t$ for each t,

where

$$u'\left(\bar{c}_t\right) = \beta u'\left(\bar{c}_{t+1}\right) \text{ for each } t \in \{0, 1, .., T-1\}.$$

We still need to characterize the path of the capital stock $\{k_t\}_{t=1}^{T-1}$ during the decumulation phase.

To this end, consider the investment decision at some date $t-1$, which determines the capital stock at date t. The gross return from this investment is given by $1+R_t-\delta^k$, which is determined by the function $s\left(k_t, Y_t\right)$ (cf. Lemma 2). Since $r_t = 0$, the gross cost of investment is given by $1+r_t = 1$. The economy invests at date $t-1$ up to the point at which the gross benefit is equal to the gross cost, which gives a break-even condition

$$s\left(k_t, Y_t\right) = 1 \text{ for each } t \in \{1, .., T-1\}. \tag{21}$$

Recall that the gross return function $s\left(\cdot\right)$ is decreasing in the capital stock k_t and increasing in net output Y_t. Hence, Eq. (21) says that, if the (expected) output at date t is large, then the economy invests more at date $t-1$ and ends up with greater capital stock at date t.

The level of output is in turn determined by the aggregate demand at date t:

$$Y_t = \bar{c}_t + k_{t+1} + i^h \text{ for each } t \in \{0, ..., T-1\}. \tag{22}$$

Eqs. (21) and (22) represent a difference equation that can be solved backwards starting with $k_T = \bar{k}$. The resulting path corresponds to an equilibrium as long as $S\left(k_0\right) > Y_0$, so that there is a liquidity trap in the first period as we have conjectured. The next result establishes that this is the case if the shock is sufficiently severe, as captured by low i^h, and characterizes the behavior of nonresidential capital in

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equilibrium.[3] The result requires Assumption 4, which is a regularity condition on shocks and parameters that ensures an interior liquidity trap equilibrium at date 0 with positive output. This assumption is satisfied for all of our numerical simulations and is relegated to the appendix for brevity.

Proposition 3 (Nonresidential Investment Response). *Consider the model with the adjustment length $T \geq 1$. Suppose Assumptions 1-3 and Assumption 4 in the appendix hold.*

(i) There exists i_1^h such that if $i^h < i_1^h$, then there is a unique equilibrium path $\{k_t, Y_{t-1}\}_{t=1}^T$, which solves Eqs. $(21) - (22)$ along with $k_T = \overline{k}$. The equilibrium features a liquidity trap at each date $t \in \{0, .., T-1\}$ with $r_{t+1} = 0$ and $Y_t < S(k_t)$.

(ii) There exists i_2^h such that, if $i^h < i_2^h$, then the nonresidential capital declines at date 1, and then increases before date T:

$$k_0 > k_1 \ and \ k_1 < k_T = \overline{k}.$$

The main result of this section is the second part, which establishes conditions under which the nonresidential capital (and investment) follow a non-monotone path during the recession: falling initially, but eventually increasing.

To understand the drop in investment, note that a negative shock to residential investment reduces aggregate demand and output. This in turn lowers nonresidential investment as captured by the break-even condition (21). When the shock is sufficiently severe, the aggregate demand at date 1 is sufficiently low that capital declines. Intuitively, the economy is optimally responding to the low return to capital depicted in Figure 4.

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[3]If the condition $i^h < i_1^h$ is violated, then there is an alternative equilibrium in which there is a partial liquidity trap at dates $t \in \{T_b - 1, .., T-1\}$ for some $T_b \geq 2$. We omit the characterization of these equilibria for brevity.
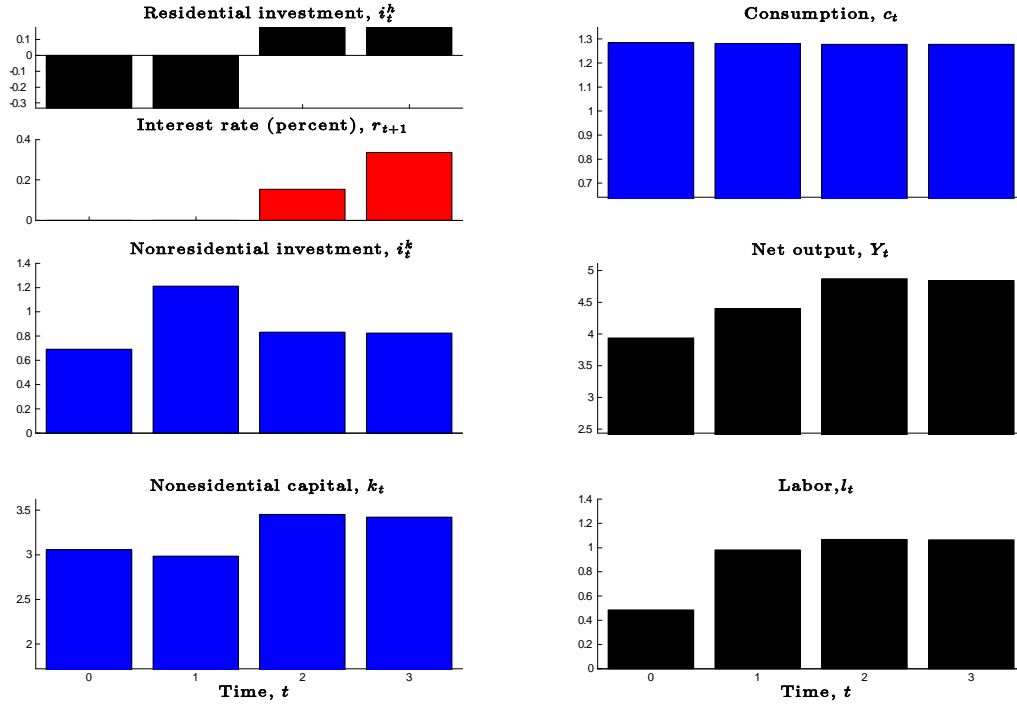
Figure 5: The evolution of equilibrium variables over time, given the length of decumulation $T = 2$.

In later periods, aggregate demand and output gradually increase in anticipation of the eventual recovery. As this happens, the low interest rate—or the low cost of capital—becomes the dominant factor for nonresidential investment. Consequently, the economy starts reaccumulating capital, and in fact—exits the liquidity trap with the maximum level of capital $\overline{k}$ as in the earlier model.

Figure 5 illustrates the dynamic evolution of the equilibrium variables for the case $T = 2$. The parameters are chosen so that the figure can be compared to Figure 3 after replacing a single period with two periods. The lower panels on the left illustrate the non-monotonic response of capital and investment identified in Proposition 3. The figure illustrates further that the recession can be roughly divided into two phases. In the first phase, captured by date 0, both types of investment fall. This in-

duces a particularly severe recession with low output and employment. In the second phase, captured by date 1 in the figure (and dates $t \in \{1, .., T-1\}$ more generally), residential investment remains low whereas the nonresidential investment gradually recovers and eventually booms. The nonresidential investment response also raises aggregate demand and creates a partial recovery in output and employment.

4.1   Relationship to the acceleration principle

Our analysis of nonresidential investment is related to the accelerator theory of investment (see Clark (1917)). To illustrate this, let us linearize Eq. (21) around $(k, Y) \simeq \left(\overline{k}, S\left(\overline{k}\right)\right)$, to obtain the approximation

$$k_t \simeq \alpha + \beta E_{t-1}\left[Y_t\right] \text{ for each } t \in \{1, .., T-1\}, \tag{23}$$

where $\beta = -s_Y/s_k > 0, \alpha = \overline{k} - \beta S\left(\overline{k}\right)$, and $E_{t-1}\left[Y_t\right] = Y_t$. We introduce the (redundant) expectations operator to contrast our rational expectations approach with the previous literature. Taking the first differences of Eq. (23), and assuming that the depreciation rate is small, $\delta^k \simeq 0$, we further obtain

$$i_t^k \simeq k_{t+1} - k_t \simeq \beta\left(Y_{t+1} - Y_t\right) \text{ for each } t \in \{0, .., T-2\}.$$

Our model thus implies a version of the acceleration principle, which says that investment is proportional to changes in output (see Eckaus (1953) for a review). Note, however, that the relationship in (23) is mechanically assumed in the accelerator literature, whereas Eq. (21) emerges in our setting from the optimal investment behavior of firms.

   Intuitively, the liquidity trap ensures that the interest rate and the cost of capital

is constant. Consequently, the return on capital becomes the main determinant of investment. In our model (and in many settings), the return on capital is increasing in output, which yields a positive relationship between capital and output. In his review of the accelerator theory, Caballero (1999) notes: "the absence of prices (the cost of capital, in particular) from the right-hand side of the flexible accelerator equation has earned it disrespect despite its empirical success." In our analysis, the liquidity trap keeps the cost of capital constant, reviving the acceleration principle.

Our model has several distinct features as compared to the accelerator theory. First, our acceleration principle applies only temporarily during the liquidity trap. From time T onwards, investment is driven by neoclassical forces [cf. (14)]. Second, our acceleration principle captures a nonlinear relationship [cf. Eq. (21)], whereas the accelerator theory often uses a linear form as in (23). Third, our investing firms hold rational expectations, whereas the macroeconomic applications of the accelerator theory often use Eq. (23) with adaptive expectations, for instance, $E_{t-1}[Y_t] = Y_{t-1}$. We show that, even with rational expectations, the acceleration principle exacerbates the earlier phase of the recession similar to Samuelson (1939). However, our model does not feature the periodic oscillations of output emphasized in Samuelson (1939) or Metzler (1941), which seem to be driven by adaptive expectations.

5 Consumption response and the Keynesian multiplier

This section investigates the consumption response in a version of the model that features households with high marginal propensities to consume (MPCs) out of income. In this context, we illustrate how the overbuilding of residential capital can reduce

consumption, and not just nonresidential investment, and how the consumption response aggravates the recession through a Keynesian income multiplier.

The models described so far feature a representative household whose consumption satisfies the Euler equation. However, the Euler equation—and the permanent income hypothesis that it implies—cannot fully capture the behavior of consumption in response to income changes in the data. After reviewing the vast empirical literature on this topic, Jappelli and Pistaferri (2010) note "there is by now considerable evidence that consumption appears to respond to anticipated income increases, over and above by what is implied by standard models of consumption smoothing."

To make consumption more responsive to income, we introduce households with high MPCs out of income. Suppose, in addition to the representative household analyzed earlier, there is an additional mass l^{tr} of households which we refer to as *income-trackers*. These agents are excluded from financial markets so that they consume all of their income, that is, their MPC is equal to 1 (for simplicity). Each income-tracker inelastically supplies 1 unit of labor in a competitive market for a wage level w_t^{tr}, which provides her only source of income. Consequently, total consumption is now given by $c_t + w_t^{tr} l^{tr}$, where c_t is the consumption of the representative household and $w_t^{tr} l^{tr}$ denotes the consumption of income-trackers.

The aggregate production function can generally be written as $\tilde{F}(k_t, l_t, l^{tr})$, where l_t is the labor supply by the representative household and l^{tr} is the total labor supply by income-trackers. To simplify the analysis, we focus on the special case

$$\tilde{F}(k_t, l_t, l^{tr}) = F(k_t, l_t) + \eta^{tr} l^{tr},$$

where F is a neoclassical production function and $\eta^{tr} > 0$ is a scalar. We continue to use the notation $Y_t = F(k_t, l_t)$ to refer to the output excluding the supply of income-

trackers. Total output is given by $Y_t + \eta^{tr} l^{tr}$. The rest of the model is the same as in the previous section.

In view of these assumptions, the economy is subject to the resource constraint

$$c_t + k_{t+1} + i_t^h + w_t^{tr} l^{tr} = Y_t + \eta^{tr} l^{tr} \leq S(k_t) + \eta^{tr} l^{tr}. \tag{24}$$

In Eq. (24), the equality says that total demand equals total output, whereas the inequality says that total output is below the maximum total supply. Lemma 3 in the appendix characterizes the income-trackers' wage level as

$$w_t^{tr} = \psi(k_t, Y_t) \eta^{tr}. \tag{25}$$

Here, $\psi(k_t, Y_t) \in [0, 1]$ is a measure of efficient resource utilization (more specifically, $\psi = 1 - \tau$ where τ is the labor wedge). Absent a liquidity trap, $\psi = 1$ and output is at its efficient level, in which case the income-trackers also earn their marginal product η^{tr}. In a liquidity trap, $\psi \leq 1$ and output is below its efficient level due to the demand shortage. In this case, the income-trackers' wage is also below their marginal product, $w_t^{tr} \leq \eta^{tr}$. Moreover, their wage is increasing in Y_t, since greater demand increases factor returns.

Combining Eqs. (24) and (25) implies

$$Y_t = c_t + k_{t+1} + i_t^h + (\psi(k_t, Y_t) - 1) \eta^{tr} l^{tr} \text{ for each } t \in \{0, 1, .., T-1\}. \tag{26}$$

This expression illustrates a *Keynesian cross* in our setting. Total demand depends on net output Y_t through income-trackers' income and consumption. The equilibrium obtains when the actual and demanded net outputs are equal.

Next consider a residential investment shock that lasts T periods as in the previous section. We conjecture an equilibrium with a liquidity trap for all dates $t \in \{0, 1, .., T-1\}$. As before, the optimality of investment implies the break-even condition (21). Eqs. (26) and (21) can then be solved backwards starting with $k_T = \overline{k}$. The next result establishes conditions under which the solution exists and corresponds to an equilibrium, and characterizes the behavior of consumption in equilibrium.

Proposition 4 (Consumption Response). *Consider the model with mass l^{tr} of income trackers and the adjustment length $T \geq 1$. Suppose Assumptions 1-3 and Assumption 4^{tr} in the appendix hold.*

(i) There exists i_1^h such that if $i^h < i_1^h$, then there is an equilibrium path $\{k_t, Y_{t-1}\}_{t=1}^T$, which solves Eqs. (21) and (26) along with $k_T = \overline{k}$. Any equilibrium features a liquidity trap at each date $t \in \{0, .., T-1\}$ with $r_{t+1} = 0$ and $Y_t < S(k_t)$.

(ii) There exists l_1^{tr} such that if $l^{tr} > l_1^{tr}$, then total consumption at date 0 (in any equilibrium) is below its steady-state level, that is

$$c_0 + w_0^{tr} l^{tr} < c^* + \eta^{tr} l^{tr}.$$

The main result of this section is the second part, which establishes conditions under which overbuilding also lowers total consumption at date 0 in any equilibrium.[4] When the economy is in a liquidity trap, output Y_t falls due to the demand shortage. As illustrated by Eq. (25), the drop in output also lowers income-trackers' income and consumption. With sufficiently many income trackers, this also reduces total consumption in contrast to the previous sections.

Figure 6 illustrates the equilibrium using the same parameters as before (except

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[4]The equilibrium is unique in all of our numerical simulations. However, there could in principle be multiple equilibria because Eq. (26) represents an intersection of two increasing curves in Y_t.
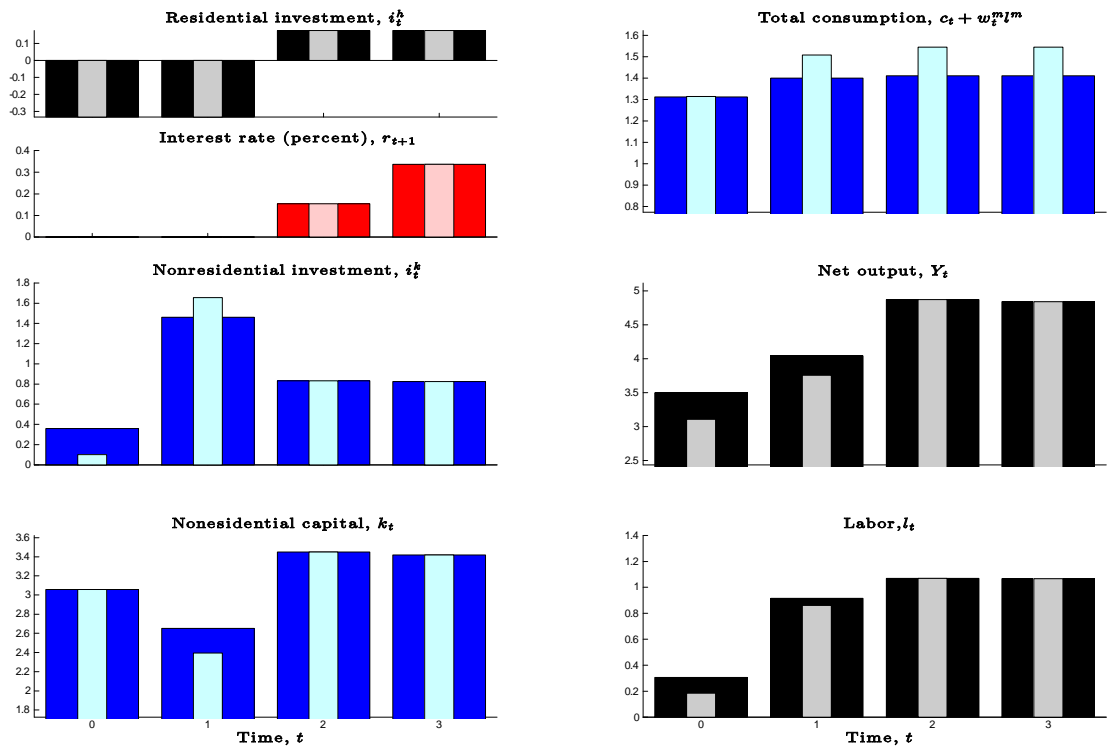
Figure 6: The evolution of equilibrium variables with additional households whose consumption tracks their income. The light bars illustrate the effect of increasing the mass of income-trackers.

for the new parameters $\eta^{tr}, l^{tr} > 0$). The darker bars illustrate the case with relatively few income-trackers, and the lighter bars illustrate the effect of having more income-trackers. In each case the initial consumption declines, illustrating Proposition 4.

The figure illustrates further that increasing the mass of income-trackers aggravates the recession. More specifically, greater l^{tr} leads to a greater drop in output and employment. The intuition is provided by the Keynesian cross equation (26), which implies a Keynesian multiplier. As income-trackers' consumption falls, aggregate demand falls even further. This induces a second round reduction in net output Y_t and income-trackers' income, which further reduces income-trackers' consumption and aggregate demand, and so on. Hence, income-trackers' consumption behavior multiplies the effect of the initial demand shock. Note that greater l^{tr} also leads to a more severe drop in investment at date 0 followed by a stronger recovery at date 1. Thus, income-trackers exacerbate the non-monotonic response of investment identified in Proposition 3.

Finally, Figure 6 illustrates that our model can explain the asymmetric recovery from the Great Recession depicted in Figure 1. Similar to the previous section, the recession can be roughly divided into two phases. In the first phase, all components of aggregate demand—including consumption—simultaneously fall, triggering a deep recession. In the second phase, nonresidential investment booms, which also increases output, employment, as well as consumption. Hence, the second phase of the recession in our model represents a partial and asymmetric recovery in which the residential sector is left behind, as in Figure 1.

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6 Conclusion

We have presented a model of investment hangover in the Great Recession that combines both Austrian and Keynesian features. On the Austrian side, the recession is precipitated by overbuilding in the residential sector, which necessitates a reallocation of resources to other sectors. The reallocation problem is exacerbated by the durability of residential capital, which prevents depreciation from naturally erasing the overbuilt capital. On the Keynesian side, a lower bound on interest rates slows down reallocation and creates an aggregate demand shortage. The demand shortage can also reduce consumption and investment in sectors that are not overbuilt, leading to a severe recession. Eventually, consumption and investment recover, but the slump in the residential sector continues for a long time. The broad trends of the Great Recession on GDP, consumption, residential investment, and other types of investment are consistent with the predictions of this model.

Although we have focused on the Great Recession, the model is more widely applicable. Perhaps the most straightforward extension is to overbuilding in sectors other than housing. In the 1930s, when both Hayek and Keynes wrote, speculative overbuilding was seen as a critical impetus to recessions, but the focus was more on railroads and perhaps industrial plant than on housing. In our model, such extensions would require only a relabeling of variables.

Less obvious is the extension to other forms of restrictions on interest rates, such as currency unions, which also slow down the Austrian reallocation of resources from the overbuilt sector to others. In the recent European context, such restrictions may have played a critical role, and generated Keynesian aggregate demand effects along the lines suggested by our model. We leave an elaboration of these mechanisms to future work.

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A   Appendix: Extensions and omitted proofs

A.1   Omitted proofs

Proof of Lemma 1. First consider the case $r_{t+1} > 0$. In this case, the first order conditions for the firm's problem (6) implies

$$F_k (k_t, l_t) = R_t \text{ and } F_l (k_t, l_t) = w_t = v' (l_t), \tag{A.1}$$

where the latter equality also uses the first order condition for the household problem (3). The first order condition for problem (8) is also given by $F_l (k_t, l_t) = v' (l_t)$. It follows that $Y_t = S (k_t)$ and $l_t = l_t^*$, proving the first part.

Next consider the case $r_{t+1} = 0$. In this case, combining the first order conditions for problems (6) and (3) imply $F_l (k_t, l_t) \geq v' (l_t)$ [see Eq. $(A.2)$ below]. This in turn implies that $l_t \in [0, l_t^*]$. By feasibility, net output satisfies

$$Y_t = c_t + k_{t+1} + i^h = F (k_t, l_t) - v (l_t) + \left(1 - \delta^k\right) k_t.$$

This expression is strictly increasing in l_t over the range $[0, l_t^*]$. The minimum and the maximum are respectively given by $\left(1 - \delta^k\right) k_t$ and $S (k_t)$, establishing the constraints (9). Moreover, given Y_t that satisfies these resource constraints, there is a unique solution to problem (10), which we denote by $L (k_t, Y_t)$, completing the proof. $\qquad\square$

Proof of Lemma 2. The first order conditions for problem (6) can be generally written as

$$(1 - \tau_t) F_k (k_t, l_t) = R_t \text{ and } (1 - \tau_t) F_l (k_t, l_t) = w_t = v' (l_t), \tag{A.2}$$

where $\tau_t \geq 0$ is the Lagrange multiplier on the demand constraint, also known as the labor wedge. Combining these expressions, the gross return to capital is given by

$$\begin{aligned}
1 + R_t - \delta^k &= \frac{v' (l_t)}{F_l (k_t, l_t)} F_k (k_t, l_t) + 1 - \delta^k \\
&= \frac{v' (L (k_t, Y_t))}{F_l (k_t, L (k_t, Y_t))} F_k (k_t, L (k_t, Y_t)) + 1 - \delta^k, \\
&= s (k_t, Y_t), \tag{A.3}
\end{aligned}$$

where the last line defines the function $s\left(\cdot\right)$. If $r_{t+1} > 0$, then the labor wedge is zero and $F_l = v'$ [cf. Eq. $(A.1)$]. Thus, in this case, we have

$$s\left(k_t, Y_t\right) = F_k\left(k_t, L\left(k_t, Y_t\right)\right) + 1 - \delta^k = S'\left(k_t\right),$$

where the last line uses the envelope theorem. If instead $r_{t+1} = 0$, then the labor wedge is nonnegative and $F_l \geq v'$. In this case, Eq. $(A.3)$ implies $s\left(k_t, Y_t\right) \leq S'\left(k_t\right)$. It can also be checked that $s_Y > 0$ and $s_k < 0$, completing the proof. $\qquad\square$

Proof of Proposition 1. For each $r_0 \geq 0$, define the function $K_1\left(r_0\right)$ as the solution to

$$S'\left(K_1\left(r_0\right)\right) = 1 + r_0.$$

Note that $K_1\left(r_0\right)$ is decreasing in the interest rate, with $K_1\left(0\right) = \overline{k}$ and $\lim_{r_0 \to \infty} K_1\left(r_0\right) = 0$. Similarly, define the function $C_0\left(r_0\right)$ as the solution to the Euler equation

$$u'\left(C_0\left(r_0\right)\right) = \beta\left(1 + r_0\right)u'\left(C\left(K_1\left(r_0\right)\right)\right).$$

Note that $C_0\left(r_0\right)$ is decreasing in the interest rate, with $C_0\left(0\right) = \overline{c}_0$ and $\lim_{r_0 \to \infty} C_0\left(r_0\right) = 0$. Finally, define the aggregate demand function

$$Y_0\left(r_0\right) = C_0\left(r_0\right) + K_1\left(r_0\right) + i^h.$$

Note that $Y_0\left(r_0\right)$ is also decreasing in the interest rate, with

$$Y_0\left(0\right) = \overline{Y}_0 \text{ and } \lim_{r_0 \to \infty} Y_0\left(r_0\right) = i^h < \delta^h h^*.$$

Next consider the time 0 equilibrium for the case $b_0 \leq \overline{b}_0$, which implies $S\left(k_0\right) \leq \overline{Y}_0$. Assumption 1 implies $S\left(k_0\right) \geq k_0 + \delta^h h^* > i^h$. It follows that there is a unique equilibrium interest rate $r_0 \in [0, \infty)$ such that $Y_0\left(r_0\right) = S\left(k_0\right)$. The equilibrium consumption and investment are determined by $c_0 = C_0\left(r_0\right)$ and $K_1\left(r_0\right) = k_1$, and the equilibrium output and labor supply satisfy $Y_0 = S\left(k_0\right)$ and $l_0 = l_0^*$.

Next consider the date 0 equilibrium for the case $b_0 > \overline{b}_0$. In this case, $Y_0\left(r_0\right) < S\left(k_0\right)$ for each $r_0 \geq 0$. Thus, the unique equilibrium features $r_0 = 0$ and $Y_0 = \overline{Y}_0 < S\left(k_0\right)$. Consumption and investment are given by $c_0 = \overline{c}_0$ and $k_1 = \overline{k}_1$. Labor supply l_0 is determined as the unique solution to (10) over the range $l_0 \in \left(0, l_0^*\right)$. Finally, Eq.

(17) implies the equilibrium output, $Y_0 = \overline{Y}_0$, is declining in the initial overbuilding b_0.

In either case, it can also be checked that the economy reaches time 1 with capital stock $k_1 \geq \min(k_0, k^*)$. Under Assumption 1, the system in (14) corresponds to a standard neoclassical model. Using the standard steps, there is a unique equilibrium path $\{c_t, k_{t+1}\}_{t=1}^{\infty}$, which converges to the steady state (c^*, k^*) characterized by the equations

$$\beta S'(k^*) = 1 \text{ and } c^* = S(k^*) - k^* - \delta^h h^*,$$

completing the proof. $\qquad\square$

Proof of Proposition 2. Note that the recession is triggered if $\overline{Y}_0 < S(k_0)$, where $\overline{Y}_0$ is given by Eq. (19). Since $1 - \delta^{h^d} > 1 - \delta^{h^n}$, increasing b_0^d (while keeping $b_0 = (b_0^d + b_0^n)/2$ constant) reduces $\overline{Y}_0$, proving the result. $\qquad\square$

To prove Proposition **3**, we also make the following assumption.

Assumption 4. (i) $i^h \in [-\overline{c}_T, S(\overline{k}) - \overline{k} - \overline{c}_0)$ and (ii) $s(k_0, \overline{c}_0 - \overline{c}_T + \overline{k}) < 1$.

Part (i) ensures that i^h is not too low to induce zero aggregate demand in any period, but also not too high so that a liquidity trap at date 0 is possible. Part (ii) ensures that the worst possible shock $i^h = -\overline{c}_T$ is sufficient to induce a liquidity trap at date 0.

Proof of Proposition 3. We first claim that the solution to Eq. (21) can be written as $k_t = K(Y_t)$, where $K(\cdot)$ is an increasing function over $(0, S(\overline{k}))$. To this end, consider some $Y \in (0, S(\overline{k}))$. Let $\tilde{k} < \overline{k}$ denote the unique capital level such that $Y = S(\tilde{k})$. Note that

$$s(\tilde{k}, Y) = S'(\tilde{k}) > 1 \text{ and } s(\overline{k}, Y) < S'(\overline{k}) = 1,$$

where the latter inequality follows from Lemma 2 since $Y < S(\overline{k})$. Since $s_k < 0$, there exists a unique $K(Y) \in (\tilde{k}, \overline{k})$ such that $s(K(Y), Y) = 1$. Thus, the function $K(\cdot)$ is well defined. Note also that $K(\cdot)$ is continuous and strictly increasing. Note also that $\lim_{Y \to 0} K(Y) = 0$.

Given the function $K(\cdot)$, the path of capital can be written as the solution to the

system,

$$k_t = K_t(Y_t), \text{ where } Y_t = \bar{c}_t + k_{t+1} + i^h \qquad \text{(A.4)}$$

for each $t \in \{1, .., T-1\}$, starting with $k_T = \bar{k}$. To solve this system by induction, consider some $k_{t+1} \in (0, \bar{k}]$. Consider the corresponding aggregate demand Y_t. Part (i) of Assumption 4 implies $Y_t > 0$ (using $\bar{c}_t \geq \bar{c}_T$) and $Y_t < S(\bar{k})$ (using $\bar{c}_t \leq \bar{c}_0$ and $k_{t+1} \leq \bar{k}$). We thus have $Y_t \in \left(0, S\left(\bar{k}\right)\right)$. Since $K(\cdot)$ is a strictly increasing function, there is a unique solution to $(A.4)$ which also satisfies $k_t \in \left(0, \bar{k}\right)$. By induction, we obtain a unique path for capital $\{k_t\}_{t=1}^{T-1}$. Combining the path of capital with Eq. (22) also implies a unique path of output $\{Y_{t-1}\}_{t=0}^{T-1}$. Since $k_t < \bar{k}$ and $s(k_t, Y_t) = 1$, we also have $Y_t < S(k_t)$ for each $t \in \{1, .., T-1\}$.

It remains to show that there is a liquidity trap also at date 0 with $Y_0 < S(k_0)$, verifying our conjecture. We first claim this is the case for the worst allowed shock, $i^h = -\bar{c}_T$. We then establish it also for any shock below a threshold level.

Consider the worst allowed shock $i^h = -\bar{c}_T$. Note that $K(Y_0) \in \left(0, \bar{k}\right)$ is well defined, and describes the capital level at date 0 that would generate a gross return of 1 given the demand Y_0. The demand at date 0 is in turn given by

$$Y_0 = \bar{c}_0 - \bar{c}_T + k_1 \leq \bar{c}_0 - \bar{c}_T + \bar{k}.$$

Combining this with Part (ii) of Assumption 4, we obtain $s(k_0, Y_0) < 1$. This implies $K(Y_0) < k_0$, which further implies

$$Y_0 < S(K(Y_0)) < S(k_0).$$

Here, the first inequality from the definition of $K(Y_0)$ and the second inequality follows since $K(Y_0) < k_0$. We thus have $Y_0 < S(k_0)$, proving the claim that the worst allowed shock induces a liquidity trap.

Next note from Eq. $(A.4)$ that, for any k_{t+1}, the implied k_t is strictly increasing in i^h. Consequently, k_1 and Y_0 are also strictly increasing in i^h. Since $i^h = -\bar{c}_T$ induces $Y_0 < S(k_0)$, there exists $i_1^h > -\bar{c}_T$ such that $Y_0 = S(k_0)$. It follows that there is a liquidity trap at date 0 with $Y_0 < S(k_0)$ whenever $i^h < i_1^h$, proving the first part.

Similarly, we claim that the worst allowed shock $i^h = -\bar{c}_T$ induces $k_1 < k_0$. To

see this, consider the aggregate demand at date 1 given by

$$Y_1 = \bar{c}_1 - \bar{c}_T + k_2 \le \bar{c}_0 - \bar{c}_T + \overline{k}.$$

Combining this with Part (ii) of Assumption 4, we obtain $s(k_0, Y_1) < 1$. This in turn implies $k_1 = K(Y_1) < k_0$, proving the claim. Since k_1 is strictly increasing in i^h, there exists $i_2^h > -\bar{c}_T$ such that $k_1 = k_0$. It follows that $k_1 < k_0$ whenever $i^h < i_2^h$, completing the proof. $\qquad\square$

Lemma 3. *The income-trackers' wage level is given by Eq. (25) for some function $\psi(k_t, Y_t)$, which has the following properties:*

(i) $\psi(k_t, Y_t) = 1 - \tau_t = \frac{v'(l_t)}{F_l(k_t, l_t)}$,

(ii) $\psi(k_t, Y_t) = 1$ if $r_{t+1} > 0$ and $\psi(k_t, Y_t) \in [0, 1]$ if $r_{t+1} = 0$,

(iii) $\psi(k_t, Y_t)$ is strictly decrasing in k_t, and strictly increasing in Y_t.

Proof. As in the proof of Lemma 2, let $L(k, Y)$ denote the labor supply corresponding to capital level $k \le \overline{k}$ and output $Y \in \left[\left(1 - \delta^k\right)k, S(k)\right]$. Next consider the analogue of Problem (6) that also includes firms' demand for hand-to-mouth labor. The firm's optimization in this case implies

$$w^{tr}(k_t, Y_t) = (1 - \tau_t)\eta^{tr},$$

where $\tau_t \ge 0$ is the Lagrange multiplier on the demand constraint. As before, the same problem also implies that τ_t is equal to the labor wedge, that is:

$$1 - \tau_t = \frac{v'(L(k_t, Y_t))}{F_l(k_t, L(k_t, Y_t))} \equiv \psi(k_t, Y_t).$$

Here, the last line defines the function $\psi(k_t, Y_t)$. Combining these expressions proves the first part. Recall that the labor wedge satisfies $\tau_t = 0$ if $r_{t+1} = 0$, and $\tau_t \in [0, 1]$ if $r_{t+1} > 0$, proving the second part. It can also be checked that $\psi_k < 0$ and $\psi_Y > 0$, completing the proof. $\qquad\square$

To prove Proposition **4**, we strengthen Assumption 4 as follows.

Assumption 4$^{\mathrm{tr}}$. (i) $i^h \in \left[-\left(\bar{c}_T - \eta^{tr}l^{tr}\right), S\left(\overline{k}\right) - \overline{k} - \bar{c}_0\right)$, (ii) $s\left(k_0, \bar{c}_0 - \bar{c}_T + \overline{k}\right) < 1$.

Proof of Proposition 4. Let $K(Y)$ denote the function defined in the proof of Proposition 3 that describes the break-even capital level $k_t = K(Y_t)$ given aggregate demand Y_t. Eqs. (21) and (26) can then be written as

$$Y_t = f(Y_t) \equiv \bar{c}_t + k_{t+1} + i^h + (\psi(K(Y_t), Y_t) - 1)\eta^{tr}l^{tr}, \qquad (A.5)$$

for each $t \geq 1$. The output at date 0 is separately characterized as the solution to Eq. (26) with the initial k_0 (as opposed to $K(Y_0)$).

We next claim that, given $k_{t+1} \in (0, \bar{k}]$, there exists a solution to $(A.5)$ over the range $Y_t \in (0, S(\bar{k}))$. To see this, note that

$$\lim_{Y_t \to 0} f(Y_t) > \bar{c}_T + i^h - \eta^{tr}l^{tr} \geq 0,$$

where the first inequality uses $\bar{c}_t \geq \bar{c}_T$, $k_{t+1} > 0$ and $\psi \geq 0$, and the second inequality uses Part (i) of Assumption 4^{tr}. Next note that

$$f(S(\bar{k})) \leq \bar{c}_0 + \bar{k} + i^h < S(\bar{k}),$$

where the first inequality uses $\bar{c}_t \leq \bar{c}_0$, $k_{t+1} \leq \bar{k}$ and $\psi \leq 1$, and the second inequality reuses Part (i) of Assumption 4^{tr}. Combining the last two inequalities implies the existence of a solution $Y_t \in (0, S(\bar{k}))$. This also implies a capital stock $k_t = K(Y_t) \in (0, \bar{k})$. Applying the same argument recursively, we obtain the path $\{k_t, Y_t\}_{t=1}^{T-1}$. By the same argument, there exists Y_0 that solves Eq. (26) with the initial k_0. Note that the solution satisfies $Y_t < S(k_t)$ for each $t \in \{1, .., T-1\}$ as in the proof of Proposition 3.

Note that there could be multiple solutions to Eq. $(A.5)$ [and Eq. (26) for date 0], which could generate multiple equilibria. We establish the desired results for the "best" equilibrium that has the highest capital and net output, which also implies the results for any other equilibrium. To this end, let Y_t^b denote the supremum over all Y_t's that solve Eq. $(A.5)$ [and Eq. (26) for date 0] given k_{t+1}^b. Then let $k_t^b = K(Y_t^b)$. By induction, we obtain a particular solution to Eq. $(A.5)$ [and Eq. (26) for date 0]. It is easy to show that this is the "best" solution in the sense that $k_t^b \geq k_t$ and $Y_t^b \geq Y_t$ for each t for any other solution.

We next claim that, given the worst allowed shock $i^h = -(\bar{c}_T - \eta^{tr}l^{tr})$, the best solution results in a liquidity trap at date 0 with $Y_0^b < S(k_0)$. To see this, note that

the aggregate demand at date 0 satisfies

$$Y_0^b = \bar{c}_0 - \bar{c}_T + k_1^b + (\psi - 1)\eta^{tr}l^{tr} \leq \bar{c}_0 - \bar{c}_T + \bar{k}.$$

Combining this with Part (ii) of Assumption 4^{tr}, we obtain $s\left(k_0, Y_0^b\right) < 1$. As in the proof of Proposition 3, this implies $K\left(Y_0^b\right) < k_0$, which in turn implies $Y_0^b < S\left(k_0\right)$. Using $Y_0 \leq Y_0^b$, this further implies that any solution features a liquidity trap at date 0 with $Y_0 < S\left(k_0\right)$, proving the first part.

To prove the second part, first note that $Y_t^b < S\left(k_t^b\right)$ also implies $\psi_t\left(k_t^b, Y_t^b\right) < 1$ for each $t \in \{0, .., T-1\}$. Eqs. $(A.5)$ and (26) then imply that Y_t^b is strictly decreasing in l^{tr} for each $t \in \{0, .., T-1\}$. Next note that the required inequality can be rewritten as

$$\bar{c}_0 - c^* < (1 - \psi(k_0, Y_0))\eta^{tr}l^{tr}. \tag{A.6}$$

Since Y_0^b is strictly decreasing in l^{tr}, so is the expression $\psi\left(k_0, Y_0^b\right)$. Thus, there exists l_1^{tr} such that $(A.6)$ holds for the "best" equilibrium $\left\{k_t^b, Y_{t-1}^b\right\}_{t=0}^{T-1}$ if and only if $l^{tr} > l_1^{tr}$. Note also that any other equilibrium features $Y_0 \leq Y_0^b$, and thus $\psi(k_0, Y_0) \leq \psi\left(k_0, Y_0^b\right)$. It follows that, if $l^{tr} > l_1^{tr}$, then the inequality in $(A.6)$ holds for any equilibrium, completing the proof. $\qquad\square$