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STRATEGIC TRADING IN INFORMATIONALLY COMPLEX ENVIRONMENTS

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**ABSTRACT**

We study trading behavior and the properties of prices in informationally complex markets. Our model is based on the single-period version of the linear-normal framework of Kyle (1985). We allow for essentially arbitrary correlations among the random variables involved in the model: the value of the traded asset, the signals of strategic traders and competitive market makers, and the demand from liquidity traders. We show that there always exists a unique linear equilibrium, characterize it analytically, and illustrate its properties in a series of examples. We then use this characterization to study the informational efficiency of prices as the number of strategic traders becomes large. If liquidity demand is positively correlated (or uncorrelated) with the asset value, then prices in large markets aggregate all available information. If liquidity demand is negatively correlated with the asset value, then prices in large markets aggregate all information except that contained in liquidity demand.

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An Online Appendix is available at  
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# 1 Introduction

Whether and how dispersed information enters into market prices is one of the central questions of information economics. A key obstacle to full information revelation and aggregation in markets is the strategic behavior of informed traders. A trader who has private information about the value of an asset has an incentive to trade in the direction of that information. However, the more he trades, the more he reveals his information, and the more he moves the prices closer to the true value of an asset. Thus, to maximize his profits, an informed trader may stop short of fully revealing his information, and so the informational efficiency of market prices may fail.

Thus, an important and natural question is when we should expect market prices to in fact reflect all information available to market participants. One stream of literature considers trading in dynamic environments, with informed traders having multiple opportunities for trading.<sup>1</sup> In these settings, in each period, traders may have an incentive to withhold some of their information in order not to eliminate their profits. However, over time, traders will gradually reveal all of their information, and in many (although not all) cases, by the end of trading, market prices will in fact aggregate all available information.

One issue with the case of dynamic trading is that while at the end, market prices accurately reflect all available information, that is generally not the case during most of the time the market is in operation—and thus much of the trading may happen at prices that are far away from the ones that would prevail if all private information was publicly available to all market participants. Therefore, another important stream of research abstracts away from the time dimension and repeated trading in markets, and considers instead an alternative intuition for when market prices may accurately reflect information: when the number of informed traders is large, and each one of them is informationally small. In that case, each of the informed traders has limited impact on market prices, but their aggregate behavior does in fact reflect the aggregate information available in the market. As a result, market prices are close to those that would prevail if all private information were publicly available, and all trades happen at those prices.

Non-strategic explorations of this intuition go back to Hayek (1945), Grossman (1976), and Radner (1979).<sup>2</sup> Subsequently, a line of research (which we discuss in more detail in Section 2) has considered strategic foundations for this intuition, studying strategic behavior of informed agents in finite markets, and then considering the properties of prices as the number of these agents becomes large. This stream of work, however, imposes very strict assumptions on how information is distributed among the agents, typically assuming that the signals of informed agents are symmetrically

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<sup>1</sup>See, e.g., Hellwig (1982), Kyle (1985), Dubey et al. (1987), Wolinsky (1990), Foster and Viswanathan (1996), Back et al. (2000), Ostrovsky (2012), and Golosov et al. (2014), among others.

<sup>2</sup>Other foundational papers in the rich literature on Rational Expectations Equilibrium and related non-strategic solution concepts include Kreps (1977), Hellwig (1980), Allen (1981), and Anderson and Sonnenschein (1982); for surveys of the literature, see Jordan and Radner (1982), Allen and Jordan (1998), and Glycopantis and Yannelis (2005). These papers focus on equilibrium existence and the amount of information aggregated and communicated by prices in equilibrium, but only consider environments with either an infinite number of infinitesimally small traders, or with a finite number of traders who essentially ignore the impact they have on market prices and behave non-strategically.

distributed, or satisfy other related restrictions so that in equilibrium, the strategies of all informed traders are identical (see Section 2). In practice, however, the distribution of information in the economy can be much more complex. Some agents may be strictly more informed than others. Groups of agents may have access to different sources of information, so that the correlations of signals within a group are very different from correlations across groups (and the sizes of the groups may be different, and the correlations of signals between different groups may be different as well). Some agents may be informed about the fundamental value of the security, while others may be uninformed about the fundamentals but possess some “technical” information about the market or other traders. And of course all such possibilities may be present in a market at the same time.

Our paper makes two main contributions.

First, we present an analytically tractable framework that makes it possible to study trading in such informationally complex environments. Our model is based on the single-period version of the model of Kyle (1985). As in that paper, an important assumption that makes our model analytically tractable is the assumption of joint normality of random variables involved in the setting: the true value of the traded asset, the signals of strategic traders, the signals of competitive market makers, and the demand coming from liquidity traders. Beyond that assumption, however, we impose essentially no restrictions on the joint distribution of these variables, making it possible to model informationally rich situations such as those described above. In this framework, we show that there always exists a unique linear equilibrium, which can be computed analytically.

Second, we explore the informational properties of equilibrium prices as the number of informed agents becomes large. We assume that there are several types of agents, with each agent of a given type receiving the same information, and fix the matrix of correlations of signals across the types (and other random variables in the model). We then allow the numbers of agents of every type to grow (without restricting the rates of growth in any way; e.g., the number of agents of one type may grow much faster than the number of agents of another type). We find that the informational properties of prices in these large markets depend on the informativeness of the demand from liquidity traders. If the demand from liquidity traders is uncorrelated with the true value of the asset or is positively correlated with it (conditional on other signals), then prices in large markets aggregate all available information. If, however, the demand from liquidity traders is negatively correlated with the true value of the asset, then prices in large markets aggregate all available information except that contained in liquidity demand.

We also illustrate our model with several applications. One example shows that under fairly simple (but, crucially, asymmetric) information structures, an informed trader may choose to trade “against” his information, i.e., sell the asset when his signal implies that the expected value of the asset is positive, and vice versa. Two examples explore the profitability of “technical” trading, and show that a trader may be able to make substantial positive expected profit even if he has no information about the value of the asset, provided there is at least one other (“fundamental”) trader who does, and provided that the technical trader has information about the demand from liquidity traders or about the mistakes of the fundamental trader. Our last set of examples shows

how equilibrium trading and outcomes depend on the amount of private information available to the market maker (beyond the aggregate market demand), and in particular shows that having a market maker observe a particular signal is not equivalent to having that signal observed publicly.

This distinction plays an important role for the last result of the paper, which characterizes the informational properties of prices in a “hybrid” case: some information is available only to a small number of traders (“scarce” information), while some other information is available to a large number of traders (“abundant” information). As the number of traders having access to “abundant” information becomes large, the equilibrium converges to the one that would obtain if these traders were not present in the market at all, and instead their information was observed by the market maker (but not by the remaining strategic traders, who continue to observe “scarce” information).

The remainder of this paper is organized as follows. In Section 2, we discuss related literature. In Section 3, we present the model. In Section 4, we state and prove our first main result, on the existence and uniqueness of linear equilibrium, and characterize this equilibrium analytically. In Section 5, we illustrate our result with several applications in informationally complex settings. In Section 6, we present our second main result, on information aggregation in large markets. In Section 7, we explore the “hybrid” case in which some information is “scarce” and some is “abundant.” Section 8 concludes.

## 2 Related Literature

The literature on strategic foundations of information aggregation and revelation in markets goes back to Wilson (1977), who considers an auction-based model in which multiple partially informed agents bid on a single object. Other work in this tradition includes Milgrom (1981), Pesendorfer and Swinkels (1997), Kremer (2002), and Reny and Perry (2006). These papers find that under various suitable conditions, information does get aggregated (and revealed in winning bids) when the number of bidders becomes large. However, these results depend critically on strong symmetry assumptions on the bidders’ signals and strategies.

Another related stream of literature, going back to Kyle (1989), considers equilibria in demand and supply functions, where bidders specify how many units of an asset they demand or supply for each possible price level, and then the market maker picks the price that clears the market.<sup>3</sup> The papers in this tradition also assume a very high degree of symmetry among the trading agents, typically assuming that these agents are ex ante identical, receive symmetrically distributed information, and employ identical strategies in equilibrium. A recent paper by Rostek and Weretka (2012) partially relaxes this symmetry assumption, and replaces it with a somewhat weaker “equicommonality” assumption on the matrix of correlations among the agents’ values. This assumption states that the sum of correlations in each column (or, equivalently, each row) of the correlation matrix is the same, and that the variances of all traders’ values are also the same. While

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<sup>3</sup>See Vives (2008) for a textbook treatment of that literature.

this assumption is more general than full symmetry among the agents, it is still quite restrictive: for example, the equilibrium in this model is still symmetric, with all traders using identical strategies.

Finally, a closely related stream of literature is the work building on Kyle's (1985) model. In that literature, as in our paper, one or more strategic traders, fully or partially informed about the value of the traded asset, are present in the market. These strategic traders submit market orders to centralized market makers. There are also liquidity traders who submit exogenously determined market orders. The market makers set the price of the asset equal to their Bayesian estimate of its value, given their prior information, the knowledge of strategic traders' strategies, and the observed order flow. Our paper borrows much of its analytical framework from this literature. The key difference is that while many of the papers in this area consider both static and dynamic models of trading but place restrictive assumptions on the information structure, our paper places virtually no restrictions on the information structure (beyond joint normality), and focuses on the one-period model of trading and on the informational properties of prices as the number of strategic traders becomes large.

In the original model of Kyle (1985), there is only one informed trader, who knows the value of the asset. Holden and Subrahmanyam (1992) study a generalization with multiple fully informed traders. Foster and Viswanathan (1996) further extend the model by allowing these traders to observe imperfect signals about the value of the asset. Different traders may observe different signals, but the distribution of these signals across the traders has to be symmetric, as are the traders' strategies. Back et al. (2000) consider a continuous-time analog of the model of Foster and Viswanathan (1996). Caballé and Krishnan (1994) and Pasquariello (2007) consider multi-asset versions of the one-period model with multiple traders, but still maintain the assumption of symmetry of information among the traders.

Several papers go beyond the fully symmetric case. Foster and Viswanathan (1994) consider a model with two strategic traders in which one trader is strictly more informed than the other. Colla and Mele (2010) consider a model in which informed traders are located on a circle, with the correlations of signals being stronger for traders who are closer to each other (in this model, as in the Rostek and Weretka (2012) model discussed above, all traders use identical strategies in equilibrium). Bernhardt and Miao (2004) consider a model with a very general information structure, allowing, as our paper does, for an asymmetric covariance matrix of traders' signals.<sup>4</sup> However, while Bernhardt and Miao (2004) characterize necessary and sufficient conditions for linear equilibria, and use these conditions to study the properties of such equilibria analytically and numerically in some specific examples, they do not provide any general results on equilibrium existence or uniqueness and do not provide general closed-form equilibrium characterizations.

Finally, there are several papers building on the Kyle (1985) framework in which the information

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<sup>4</sup>There are several differences between the models. Unlike Bernhardt and Miao (2004), we allow liquidity demand to be correlated with the value of the asset and/or the signals of informed traders. We also allow the market maker to observe signals correlated with the value of the security, the demand from noise traders, and/or the signals of informed traders. Finally, we do not impose any special structure on how the informed traders' signals are related to the value of the asset (and other random variables in the model), beyond joint normality. On the other hand, Bernhardt and Miao (2004) consider a model with multiple trading periods, while we restrict attention to one period.

structure is not limited to strategic traders observing signals about the value of the asset. In Jain and Mirman (1999), the market maker receives a separate informative signal about the value of the asset, in addition to simply observing the aggregate order flow. In Rochet and Vila (1994) and Foucault and Lescourret (2003), some of the strategic traders observe informative signals about the amount of liquidity demand.<sup>5</sup> These features of the information structure are naturally incorporated in our general model. Hence, our equilibrium existence and uniqueness result, as well as the characterization we derive, provide a unified approach with closed-form solutions to various models that include these features. In Section 5, we provide a number of applications illustrating the flexibility of our general model, and its ability to naturally incorporate such features as the market maker receiving a signal about the value of the asset and the strategic traders observing signals about liquidity demand, among others.

### 3 Model

There is a security traded in the market, whose value  $v$  is not initially known to market participants. There are  $n$  strategic traders,  $i = 1, \dots, n$ . Prior to trading, each strategic trader  $i$  (he) privately observes a multidimensional signal  $\theta_i \in \mathbb{R}^{k_i}$ , where  $k_i \geq 1$  is the dimensionality of the signal. For convenience, we will denote by  $\theta = (\theta_1; \theta_2; \dots; \theta_n)$  the vector<sup>6</sup> summarizing the signals of all strategic traders. The dimensionality of vector  $\theta$  is  $K = \sum_{i=1}^n k_i$ . There is also a market maker (she), who privately observes signal  $\theta_M \in \mathbb{R}^{k_M}$ ,  $k_M \geq 0$  (when  $k_M = 0$ , the market maker does not receive any signals, as in the standard Kyle (1985) model).<sup>7,8</sup> Finally, there are liquidity traders, whose exogenously given random demand, denoted by  $u$ , is in general not directly observed by either the strategic traders or the market maker.

The key assumption that makes the model analytically tractable is that all of the random variables mentioned above— $v$ ,  $\theta$ ,  $\theta_M$ , and  $u$ —are jointly normally distributed. Specifically, we assume that the vector  $\mu = (v; \theta; \theta_M; u)$  is drawn randomly from the multivariate normal distribution with expected value 0 and variance-covariance matrix  $\Omega$ . The assumption that the expected value of vector  $\mu$  is equal to zero is simply a normalization that allows us to simplify the notation. We will also assume that every variance-covariance matrix for signal  $\theta_i$  of strategic trader  $i$  and the variance-covariance matrix of the market maker's signal  $\theta_M$  are full rank. This assumption is without loss of generality; it simply eliminates redundancies in each trader's signals. Note that we do not place a full rank restriction on matrix  $\Omega$  itself: for instance, two different strategic traders are

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<sup>5</sup>Röell (1990), Sarkar (1995), and Madrigal (1996) also consider related models in which some agents observe signals about liquidity demand.

<sup>6</sup>We denote the row vector with elements  $x_1, \dots, x_k$  by  $(x_1, \dots, x_k)$ , and the column vector with the same elements by  $(x_1; \dots; x_k)$ . All vectors are column vectors unless specified otherwise.

<sup>7</sup>Strictly speaking,  $\theta_i$  and  $\theta_M$  are random variables whose realizations are in  $\mathbb{R}^{k_i}$  and  $\mathbb{R}^{k_M}$ .

<sup>8</sup>The multidimensionality of the traders' and the market maker's signals allows our model to incorporate complex relationships among their information sets: for example, one trader can observe strictly more information than another trader; one trader can observe the union of two other traders' signals; some information may be common to several players while some other information is not; and so forth. We illustrate the richness of the model with several applications in Section 5.

allowed to have perfectly correlated signals. The only substantive restrictions that we place on matrix  $\Omega$  are as follows.

**Assumption 1** At least one strategic trader receives at least some information about the value of the security, beyond that contained in the market maker's signal. Formally:

$$\text{Cov}(v, \theta | \theta_M) \neq 0. \quad (1)$$

**Assumption 2** The market maker does not perfectly observe the demand from liquidity traders. Formally:

$$\text{Var}(u | \theta_M) > 0. \quad (2)$$

### 3.1 Trading and Payoffs

After observing his signal  $\theta_i$ , each strategic trader  $i$  submits his demand  $d_i(\theta_i)$  to the market. In addition, the realized demand from liquidity traders,  $u$ , is also submitted to the market. The market maker observes her signal  $\theta_M$  and the total demand  $D = \sum_{i=1}^n d_i(\theta_i) + u$ , and subsequently sets the price of the security,  $P(\theta_M, D)$ , based on these observations. Securities are traded at this price  $P(\theta_M, D)$  (with each strategic trader getting his demand  $d_i(\theta_i)$ , liquidity traders getting  $u$ , and the market maker taking the position of size  $-D$  to clear the market). At a later time, the true value of the security is realized, and each strategic trader  $i$  obtains profit  $\pi_i = d_i(\theta_i) \cdot (v - P(\theta_M, D))$ .

### 3.2 Linear Equilibrium

Our solution concept is essentially the same as that in Kyle (1985). We say that a profile of demand functions  $d_i(\cdot)$  and pricing rule  $P(\cdot, \cdot)$  form an *equilibrium* if

- (i) on the equilibrium path, the price  $P$  set by the market maker is equal to the expected value of the security conditional on  $\theta_M$  and  $D$ , given the primitives and the demand functions  $d_i(\cdot)$ ; and
- (ii) for every player  $i$ , for every realization of signal  $\theta_i$ , the expected payoff from submitting demand  $d_i(\theta_i)$  is at least as high as the expected payoff from submitting any alternative demand  $d'_i$ , given the realization of signal  $\theta_i$ , the pricing rule  $P(\cdot, \cdot)$  and the profile of strategies of other players  $(d_j(\cdot))_{j \neq i}$ .<sup>9</sup>

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<sup>9</sup>Note that condition (i) is required to hold only on the equilibrium path. In the standard Kyle (1985) model and many of its generalizations, every observation of the market maker can be rationalized as being on the equilibrium path, and thus this qualifier is not needed. In our case, it is in general possible that for some strategy profiles  $d_i(\cdot)$ , only some realizations of aggregate demand  $D$  can be observed by the market maker if the strategic traders follow those strategies. In such cases, by analogy with perfect Bayesian equilibrium, our definition restricts the beliefs of the market maker on the equilibrium path, where they are pinned down by Bayes rule, and does not restrict them off the equilibrium path. For an example in which not all realizations of aggregate demand are observed in equilibrium, consider the following market. Value  $v \sim N(0, 1)$ . There is one strategic trader with signal  $\theta_1$  who observes the value perfectly:  $\theta_1 = v$ . The demand of liquidity traders is  $u = -v$ . Then in the unique linear equilibrium, the demand of the strategic trader is equal to the value of the security, and the aggregate demand is thus always equal to zero.



The equilibrium is *linear* if functions  $d_i$  (for all  $i$ ) and pricing rule  $P$  are linear functions of their arguments, i.e.,  $d_i(\theta_i) = \alpha_i^T \theta_i$  for some  $\alpha_i \in \mathbb{R}^{k_i}$  and  $P(\theta_M, D) = \beta_M^T \theta_M + \beta_D D$  for some  $\beta_M \in \mathbb{R}^{k_M}$  and  $\beta_D \in \mathbb{R}$ .<sup>10</sup>

## 4 Equilibrium Existence and Uniqueness

We can now state and prove our first main result.

**Theorem 1** *There exists a unique linear equilibrium.*

The proof of Theorem 1 is in Appendix A. The notation used in the proof, as well as in some of the subsequent sections, is given in Section 4.1 below.

The proof consists of five steps. The first two steps are fairly standard, and are essentially the same as in the earlier literature on linear-normal equilibria: they show that if all strategic traders follow linear strategies, then the pricing rule resulting from Bayesian updating is also linear; and that if all strategic traders other than trader  $i$  follow linear strategies, and the market maker is also using a linear pricing rule (with a positive coefficient  $\beta_D$  on aggregate demand  $D$ ), then the best response of trader  $i$  is also linear and is uniquely determined by the other traders' strategies and the pricing rule. The substantively novel parts of the proof are the next three steps. First, we show that the conditions derived in the first two steps allow us to express all parameters of the pricing rule and the traders' strategies as functions of "market depth"  $\gamma = 1/\beta_D$ . Next, using that derivation, we show that the entire system of equations from the first two steps collapses into one quadratic equation in  $\gamma$ . Finally, we prove that this quadratic equation has exactly one positive root, which concludes the proof.

### 4.1 Notation

We decompose the covariance matrix  $\Omega$  of the vector  $(v; \theta_1; \dots; \theta_n; \theta_M; u)$  as follows:

$$\begin{pmatrix} \sigma_{vv} & \Sigma_{v1} & \cdots & \Sigma_{vn} & \Sigma_{vM} & \sigma_{vu} \\ \Sigma_{1v} & \Sigma_{11} & \cdots & \Sigma_{1n} & \Sigma_{1M} & \Sigma_{1u} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \Sigma_{nv} & \Sigma_{n1} & \cdots & \Sigma_{nn} & \Sigma_{nM} & \Sigma_{nu} \\ \Sigma_{Mv} & \Sigma_{M1} & \cdots & \Sigma_{Mn} & \Sigma_{MM} & \Sigma_{Mu} \\ \sigma_{uv} & \Sigma_{u1} & \cdots & \Sigma_{un} & \Sigma_{uM} & \sigma_{uu} \end{pmatrix}.$$

In this matrix, every  $\sigma$  represents a (scalar) variance or covariance of the asset value and/or the demand of liquidity traders, and every  $\Sigma$  represents a (generally non-scalar) covariance matrix of an element of vector  $(v; \theta_1; \dots; \theta_n; \theta_M; u)$  with another element. We also introduce notation for the

<sup>10</sup>In principle, we could consider a more general definition of linear equilibrium and allow the strategies and the pricing rule to potentially have nonzero intercepts. However, one can show that in our setting, linear equilibria with nonzero intercepts do not exist. The proof of this statement is available upon request.

covariance matrices of the entire vector of strategic traders' signals,  $\theta = (\theta_1; \dots; \theta_n)$ , with itself and with other elements of vector  $\mu$ . Specifically:

$$\Sigma_{\theta\theta} = Var(\theta) = \begin{pmatrix} \Sigma_{11} & \cdots & \Sigma_{1n} \\ \vdots & \ddots & \vdots \\ \Sigma_{n1} & \cdots & \Sigma_{nn} \end{pmatrix}, \quad \Sigma_{\theta M} = Cov(\theta, \theta_M) = \begin{pmatrix} \Sigma_{1M} \\ \vdots \\ \Sigma_{nM} \end{pmatrix},$$

$$\Sigma_{\theta v} = Cov(\theta, v) = \begin{pmatrix} \Sigma_{1v} \\ \vdots \\ \Sigma_{nv} \end{pmatrix}, \quad \Sigma_{\theta u} = Cov(\theta, u) = \begin{pmatrix} \Sigma_{1u} \\ \vdots \\ \Sigma_{nu} \end{pmatrix}.$$

In addition, we will use the following matrices:

$$\Sigma_{diag} = \begin{pmatrix} \Sigma_{11} & 0 & 0 & 0 \\ 0 & \Sigma_{22} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \Sigma_{nn} \end{pmatrix},$$

$$\Lambda = \Sigma_{diag} + \Sigma_{\theta\theta} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{\theta M}^T,$$

$$A_u = \Lambda^{-1}(\Sigma_{\theta u} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{M u}),$$

$$A_v = \Lambda^{-1}(\Sigma_{\theta v} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{M v}).$$

(We will show in the proof of Theorem 1 that matrix  $\Lambda$  is invertible).

## 4.2 Closed-Form Solution

The proof of Theorem 1 is constructive, producing the following expressions for the parameters of interest.

Depth  $\gamma = -\left(b + \sqrt{b^2 - 4ac}\right)/2a$ , where

$$a = -A_v^T \Sigma_{diag} A_v,$$

$$b = A_v^T (2\Sigma_{diag} + \Lambda) A_u + \Sigma_{uM} \Sigma_{MM}^{-1} \Sigma_{M v} - \sigma_{uv},$$

$$c = Var(A_u^T \theta - u | \theta_M).$$

(The proof shows that  $a < 0$ ,  $c > 0$ , and thus  $\gamma > 0$ .) Equilibrium pricing rule and strategies are then as follows:

$$\beta_D = \frac{1}{\gamma},$$

$$\beta_M = \Sigma_{MM}^{-1} (\Sigma_{M v} - \Sigma_{\theta M}^T A_v) - \beta_D \Sigma_{MM}^{-1} (\Sigma_{M u} - \Sigma_{\theta M}^T A_u);$$

$$\alpha = \frac{1}{\beta_D} A_v - A_u.$$

These expressions are simplified in the case  $k_M = 0$ , when the market maker does not observe

any private signals (other than the aggregate demand  $D$ ).<sup>11</sup> In that case,

$$\begin{aligned} a &= -A_v^T \Sigma_{diag} A_v, \\ b &= A_v^T (2\Sigma_{diag} + \Lambda) A_u - \sigma_{uv}, \\ c &= Var(A_u^T \theta - u), \end{aligned}$$

where

$$\begin{aligned} \Lambda &= \Sigma_{\theta\theta} + \Sigma_{diag}, \\ A_u &= \Lambda^{-1} \Sigma_{\theta u}, \\ A_v &= \Lambda^{-1} \Sigma_{\theta v}. \end{aligned}$$

These expressions are further simplified if, in addition, the demand from liquidity traders,  $u$ , is uncorrelated with the other random variables in the model. Then  $b = 0$  and  $\gamma = \sqrt{\frac{\sigma_{uu}}{A_v^T \Sigma_{diag} A_v}}$ , and so

$$\beta_D = \sqrt{\frac{A_v^T \Sigma_{diag} A_v}{\sigma_{uu}}} \quad \text{and} \quad \alpha = \sqrt{\frac{\sigma_{uu}}{A_v^T \Sigma_{diag} A_v}} A_v.$$

## 5 Applications and Examples

In this section, we illustrate the general framework presented above with several specific applications. We first present a simple yet seemingly counterintuitive example in which a trader informed about the value of the security trades in the direction opposite to that value. Next, we study what happens when one of the strategic traders is informed about the demand of liquidity traders. We conclude by analyzing several examples in which the market maker possesses private information about the value of the security and study how this information gets incorporated into the price of the security and how it affects equilibrium trading strategies and the sensitivity of equilibrium prices to market demand.

### 5.1 Trading “Against” Own Signal

In this section, we present an example of information structure under which a trader who receives a signal about the value of the security trades in the opposite direction: i.e., when based on his information the value of the security is positive, he shorts the security, and when it is negative, he buys it. Note that since our model is a one-shot game, there cannot be any incentives to do that of the form “I will try to mislead others first, and then take advantage of the mispricing.”

**Example 1** *The value of the security is distributed as  $v \sim N(0, 1)$ . There are two strategic traders. Trader 1 observes a noisy estimate of  $v$ :  $\theta_1 = v + \rho_1 \xi$ , where  $\xi \sim N(0, 1)$  is a random variable*

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<sup>11</sup>Strictly speaking, our proof does not apply directly to the case  $k_M = 0$  since, for example, it uses the inverse of the covariance matrix of  $\theta_M$ . However, one can drop all terms related to  $\theta_M$  from the proof and immediately obtain the proof for that case. Alternatively, one can consider a model in which the market maker observes a signal that is independent of all other random variables. The equilibrium in that model will be equivalent to one in which  $k_M = 0$ .

independent of  $v$ , and  $\rho_1$  is a parameter that determines how accurate trader 1's signal is (e.g., if  $\rho_1 = 0$ , then trader 1 observes  $v$  exactly, and if  $\rho_1$  is very large, then trader 1's signal is not very accurate). Trader 2 also observes a noisy estimate of  $v$ :  $\theta_2 = v + \rho_2\xi$ , with the same "driver" of noise,  $\xi$ , as in trader 1's signal, but with a potentially different magnitude of noise,  $\rho_2$ . Finally, there is demand from liquidity traders,  $u \sim N(0,1)$ , which is independent of all other random variables. Formally, the resulting correlation matrix is

$$\Omega = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 + \rho_1^2 & 1 + \rho_1\rho_2 & 0 \\ 1 & 1 + \rho_1\rho_2 & 1 + \rho_2^2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

From the analysis and closed-form characterization in the preceding section, we know that in the unique linear equilibrium the pricing rule is characterized by some  $\beta_D > 0$ , and the strategies of traders 1 and 2 are characterized by:

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \frac{1}{\beta_D} \Lambda^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad (3)$$

where  $\Lambda = \begin{pmatrix} 2 + 2\rho_1^2 & 1 + \rho_1\rho_2 \\ 1 + \rho_1\rho_2 & 2 + 2\rho_2^2 \end{pmatrix}$ .

Using the matrix inversion formula and setting  $\delta = \frac{1}{\beta_D \cdot \det(\Lambda)}$  (which is positive, since  $\Lambda$  is positive definite), we get

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \delta \begin{pmatrix} 2 + 2\rho_2^2 & -1 - \rho_1\rho_2 \\ -1 - \rho_1\rho_2 & 2 + 2\rho_1^2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \delta \begin{pmatrix} 1 + 2\rho_2^2 - \rho_1\rho_2 \\ 1 + 2\rho_1^2 - \rho_1\rho_2 \end{pmatrix}. \quad (4)$$

Thus, if  $\rho_1 = 2\rho_2 + \frac{1}{\rho_2}$ , trader 1 *never trades*, despite  $\theta_1$  being informative about the value of the security, and for  $\rho_1 > 2\rho_2 + \frac{1}{\rho_2} > 0$ , trader 1 *always trades in the direction opposite to his signal*  $\theta_1$ , despite  $\theta_1$  being positively correlated with the value of the security,  $v$ . Similarly, if  $\rho_2$  is equal to or greater than  $2\rho_1 + \frac{1}{\rho_2}$ , then trader 2 does not trade or trades in the direction opposite to his signal.

To get the intuition behind this seemingly puzzling behavior, consider a slight variation of Example 1.

**Example 2** *The value of the security is  $v \sim N(0,1)$ . There are two strategic traders. Trader 1 observes a noisy estimate of  $v$ :  $\theta_1 = v + \xi$ , where  $\xi \sim N(0,1)$ , independent of  $v$ . Trader 2 observes  $\xi$ :  $\theta_2 = \xi$ . The demand from liquidity traders,  $u \sim N(0,1)$ , is independent of all other random variables. The resulting correlation matrix is*

$$\Omega = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

In this case,  $\Lambda = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix}$  and

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \frac{1}{\beta_D} \Lambda^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \delta \begin{pmatrix} 2 & -1 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \delta \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad (5)$$

for some  $\delta > 0$ , and thus trader 2 trades in the direction opposite to his signal. Note that in this example, trader 2 is not informed about the value of the security: his signal  $\xi$  is independent of  $v$ . However, he is informed about the bias in trader 1's signal, and thus knows in which direction trader 1 is likely to "err" when submitting his demand. Thus, trader 2, by partly "undoing" this error (i.e., trading against it), can in expectation make a positive profit, despite not having any direct information about the value of the security. In a sense, while trader 1 trades on "fundamental" information, trader 2 trades on "technical" information: trader 1 would be able to make money even without having trader 2 around, but trader 2's ability to make a profit depends critically on having trader 1 around and on exploiting that trader's mistake.

In Example 1, the intuition is similar. If  $\rho_2$  is large relative to  $2\rho_1 + \frac{1}{\rho_1}$ , then the main "chunk" of trader 2's information is about the mistake that trader 1 makes, and not about the fundamental value of the security. This causes trader 2 to want to "undo" that mistake and trade "against" his signal, while trader 1 continues to trade in a natural direction. When  $\rho_2 = 2\rho_1 + \frac{1}{\rho_1}$ , the incentives of trader 2 to trade on "fundamental" information (the positive correlation of his signal with the value of the security) and on the "technical" information (the positive correlation of his signal with the mistake of trader 1) cancel out, and trader 2 ends up not trading.

## 5.2 Information about Liquidity Demand

In this section, we study what happens when one of the strategic traders does not know anything about the value of the security, but is informed about the amount of liquidity trading, and compare the equilibrium to that of the standard model without such a trader.

**Example 3** *The value of the security is distributed as  $v \sim N(0, \sigma_{vv})$ , and the demand from liquidity traders is distributed as  $u \sim N(0, \sigma_{uu})$ , independently of  $v$ . There are two strategic traders. Trader 1's signal is equal to  $v$ :  $\theta_1 = v$ . He is fully informed about the value of the security, just like in the standard Kyle model. Trader 2 is uninformed about the value of the security, but has insider information about the demand from liquidity traders:  $\theta_2 = u$ . Formally, the correlation matrix is*

$$\Omega = \begin{pmatrix} \sigma_{vv} & \sigma_{vv} & 0 & 0 \\ \sigma_{vv} & \sigma_{vv} & 0 & 0 \\ 0 & 0 & \sigma_{uu} & \sigma_{uu} \\ 0 & 0 & \sigma_{uu} & \sigma_{uu} \end{pmatrix}.$$

The auxiliary matrices in this example are:

$$\Lambda = \begin{pmatrix} 2\sigma_{vv} & 0 \\ 0 & 2\sigma_{uu} \end{pmatrix}, \quad A_u = \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}, \quad \text{and} \quad A_v = \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}.$$

Coefficient  $b$  in the quadratic equation is equal to zero, and therefore

$$\begin{aligned}\gamma &= \sqrt{-\frac{c}{a}} = \sqrt{\frac{\sigma_{uu}}{\sigma_{vv}}}; \\ \alpha_1 &= \frac{1}{2}\gamma = \frac{1}{2}\sqrt{\frac{\sigma_{uu}}{\sigma_{vv}}}; \\ \alpha_2 &= -\frac{1}{2}.\end{aligned}$$

For comparison, if the second strategic trader was not present, the model would reduce to the standard model of Kyle (1985), and the equilibrium would be characterized by

$$\begin{aligned}\gamma &= 2\sqrt{\frac{\sigma_{uu}}{\sigma_{vv}}}; \\ \alpha_1 &= \sqrt{\frac{\sigma_{uu}}{\sigma_{vv}}}.\end{aligned}$$

In other words, when the second strategic trader (who is informed about the demand from liquidity traders) is present in the market, that trader “takes away” one half of that “liquidity” demand. As a result, the first strategic trader, who knows the value of the security, trades half as much as he would in the absence of that second trader, and the market maker’s pricing rule is twice as sensitive. Therefore, for any realization of  $v$  and  $u$ , the price in the market with the second strategic trader will be exactly the same as that in the market without that trader—and thus the informativeness of prices is not affected in either direction by whether there is a trader in that market who observes the trading flow from liquidity traders. Likewise, the expected loss of liquidity traders is also unaffected by the presence of a trader who observes their demand. Since, by construction, the market maker in expectation breaks even, it has to be the case that the profit of the second strategic trader comes out of the first trader’s pocket. In fact, the second trader takes away exactly one half of the first trader’s profit.<sup>12</sup> Also, as in Example 2, the second trader is trading on “technical” information, and is only able to make a profit because a “fundamental” trader is also present in the market.

### 5.3 Informed Market Maker

In the preceding examples, as in much of the literature, the market maker does not receive any information other than the aggregate demand coming from strategic and liquidity traders. In this subsection, we turn to examples in which the market maker does possess some additional information. We show how this information affects the strategies of other traders and illustrate the interplay between the weight the market maker places on this additional information and the weight she places on market demand.

Our first two examples show that the equilibrium obtained when the market maker has private information is generally not the same as when that information is publicly available (i.e., known both to the market maker and to all strategic traders). This difference will turn out to be important

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<sup>12</sup>To see this, note that the prices in the two markets are always the same, realization by realization, while the demand of the first strategic trader, in the presence of the second one, is exactly one half of what it would be in the absence of that trader.

later in the paper, in Section 7, where we study the informativeness of prices as the sizes of some (but not all) groups of strategic traders become large.

**Example 4** *The value of the security is  $v \sim N(0, 1)$ . There is one strategic trader, who observes signal  $\theta = v + \epsilon_1$ . The market maker observes signal  $\theta_M = v + \epsilon_2$ . Variables  $\epsilon_1$  and  $\epsilon_2$  are distributed normally with mean 0 and variance 1, independently of each other and of all other variables. The demand from liquidity traders is also independently distributed as  $u \sim N(0, 1)$ . Formally, the covariance matrix that describes this information structure is*

$$\Omega = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Applying the formulas derived in Section 4, we get  $\Sigma_{diag} = \Sigma_{\theta\theta} = \Sigma_{MM} = 2$  and  $\Sigma_{\theta M} = \Sigma_{\theta v} = \Sigma_{Mv} = 1$ . Thus,  $\Lambda = 2 + 2 - 1/2 = 7/2$ ,  $A_u = 0$ , and  $A_v = 2/7(1 - 1/2) = 1/7$ . The coefficients in the quadratic equation for  $\gamma$  are  $a = -2/49$ ,  $b = 0$ , and  $c = 1$ , and thus

$$\beta_D = \frac{1}{\gamma} = \frac{\sqrt{2}}{7}.$$

Hence, the strategic trader's behavior is given by

$$\alpha = \frac{1}{\beta_D} A_v = \frac{1}{2} \sqrt{2},$$

and the market maker's sensitivity to her own signal is

$$\beta_M = \Sigma_{MM}^{-1} (\Sigma_{Mv} - \Sigma_{\theta M}^T A_v) = \frac{3}{7}.$$

Consider now a variation of Example 4, in which the market maker's signal is public information (i.e., known to both the market maker and the strategic trader).

**Example 5** *The value of the security is  $v \sim N(0, 1)$ . The market maker observes signal  $\theta_M = v + \epsilon_2$ . The strategic trader now observes two signals,  $\theta^1 = v + \epsilon_1$  and  $\theta^2 = v + \epsilon_2$ . Both  $\epsilon_1$  and  $\epsilon_2$  are normally distributed with mean 0 and variance 1, independently of each other and of all other variables. The demand from liquidity traders is independently distributed as  $u \sim N(0, 1)$ . The covariance matrix that describes this information structure is now*

$$\Omega = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 2 & 2 & 0 \\ 1 & 1 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

We now have  $\Sigma_{diag} = \Sigma_{\theta\theta} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ ,  $\Sigma_{MM} = 2$ ,  $\Sigma_{\theta M} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ,  $\Sigma_{\theta v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ , and  $\Sigma_{Mv} = 1$ .

Thus,  $\Lambda = \begin{pmatrix} 7/2 & 1 \\ 1 & 2 \end{pmatrix}$ ,  $\Lambda^{-1} = 1/6 \begin{pmatrix} 2 & -1 \\ -1 & 7/2 \end{pmatrix}$ ,  $A_u = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ , and  $A_v = \Lambda^{-1} \left( \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot 1/2 \right) = 1/6 \begin{pmatrix} 1 \\ -1/2 \end{pmatrix}$ .

The coefficients of the quadratic equation on  $\gamma$  are now  $a = -1/36 \begin{pmatrix} 1 \\ -1/2 \end{pmatrix}^T \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ -1/2 \end{pmatrix} = -1/24$ ,  $b = 0$ , and  $c = 1$ , and thus

$$\beta_D = \frac{1}{\gamma} = \frac{\sqrt{6}}{12},$$

the strategic trader's behavior is given by

$$\alpha = \frac{1}{\beta_D} A_v = \begin{pmatrix} \frac{\sqrt{6}}{3} \\ -\frac{\sqrt{6}}{6} \end{pmatrix},$$

and the market maker's sensitivity to her own signal is now given by

$$\beta_M = \Sigma_{MM}^{-1} (\Sigma_{Mv} - \Sigma_{\theta M}^T A_v) = \frac{1}{2}.$$

The equilibria in these two examples are substantively different: the sensitivities of the market maker to the aggregate demand and to her own signal are different, and the sensitivity of the strategic trader's demand to signal  $\theta^1$  is different as well. We can also compute the expected profits that the strategic trader makes in these two markets (and thus the losses of liquidity traders): in the first example, the expected profit is  $\sqrt{2}/7$ , while in the second one it is greater:  $\sqrt{6}/12$ . These differences illustrate the point that having the market maker observe a signal is substantively different from having that signal observed publicly.

Our next example considers the case in which a strategic trader's information is strictly worse than the information available to the market maker.

**Example 6** *Let  $\nu_1$ ,  $\nu_2$ ,  $\epsilon_1$ ,  $\epsilon_2$ , and  $u$  be independent random variables, each distributed normally with mean 0 and variance 1. The value of the security is  $v = \nu_1 + \nu_2$ . The demand from liquidity traders is  $u$ . There are two partially informed strategic traders and a partially informed market maker. Trader 1's signal is  $\theta_1 = \nu_1 + \epsilon_1$ . Trader 2's signal is  $\theta_2 = \nu_2 + \epsilon_2$ . Market maker's signal is  $\theta_M = \nu_2$ . Note that while trader 1 possesses some "exclusive" information about the value of the security, trader 2 does not (because  $\nu_2$  is observed by the market maker, and  $\epsilon_2$  is pure noise). Formally, the correlation matrix is*

$$\Omega = \begin{pmatrix} 2 & 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 1 & 0 & 2 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$



The auxiliary matrices in this example are:

$$\Lambda = \begin{pmatrix} 4 & 0 \\ 0 & 3 \end{pmatrix}, \quad A_u = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \text{and} \quad A_v = \begin{pmatrix} \frac{1}{4} \\ 0 \end{pmatrix}.$$

Therefore, in this case, we have

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \frac{1}{\beta_D} \begin{pmatrix} \frac{1}{4} \\ 0 \end{pmatrix},$$

and so  $\alpha_2 = 0$ . Thus, trader 2 does not trade in equilibrium. This illustrates a more general phenomenon: in equilibrium, a strategic trader cannot make a positive profit (and does not trade) if his information is the same as or worse than (in the information-theoretic sense) that of the market maker.<sup>13</sup>

Our final example considers a sequence of markets, indexed by the number of strategic traders,  $m$ . All traders receive the same information, which is imperfectly correlated with both the value of the asset and the market maker's information.

**Example 7** *The value of the security,  $v$ , the demand from liquidity traders,  $u$ , and two information shocks,  $\epsilon_1$  and  $\epsilon_2$ , are all distributed normally with mean 0 and variance 1, independently of each other. There are  $m$  identically informed strategic traders and a partially informed market maker. Each strategic trader observes a signal  $\theta_1 = v + \epsilon_1$ . The market maker observes a signal  $\theta_M = v + \epsilon_2$ . Formally (indexing all matrices by the number of strategic traders in the market,  $m$ ), the correlation matrix is*

$$\Omega^m = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 & 1 & 0 \\ 1 & 2 & 2 & \cdots & 2 & 1 & 0 \\ 1 & 2 & 2 & \cdots & 2 & 1 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 1 & 2 & 2 & \cdots & 2 & 1 & 0 \\ 1 & 1 & 1 & \cdots & 1 & 2 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{pmatrix}.$$

The auxiliary matrices are:

$$\Lambda^m = \begin{pmatrix} 3\frac{1}{2} & 1\frac{1}{2} & \cdots & 1\frac{1}{2} & 1\frac{1}{2} \\ 1\frac{1}{2} & 3\frac{1}{2} & \cdots & 1\frac{1}{2} & 1\frac{1}{2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1\frac{1}{2} & 1\frac{1}{2} & \cdots & 3\frac{1}{2} & 1\frac{1}{2} \\ 1\frac{1}{2} & 1\frac{1}{2} & \cdots & 1\frac{1}{2} & 3\frac{1}{2} \end{pmatrix}, \quad \text{so that} \quad (\Lambda^m)^{-1} = \begin{pmatrix} \frac{3m+1}{6m+8} & \frac{-3}{6m+8} & \cdots & \frac{-3}{6m+8} & \frac{-3}{6m+8} \\ \frac{-3}{6m+8} & \frac{3m+1}{6m+8} & \cdots & \frac{-3}{6m+8} & \frac{-3}{6m+8} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{-3}{6m+8} & \frac{-3}{6m+8} & \cdots & \frac{3m+1}{6m+8} & \frac{-3}{6m+8} \\ \frac{-3}{6m+8} & \frac{-3}{6m+8} & \cdots & \frac{-3}{6m+8} & \frac{3m+1}{6m+8} \end{pmatrix};$$

$$A_u^m = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}; \quad \text{and} \quad A_v^m = \begin{pmatrix} \frac{1}{3m+4} \\ \vdots \\ \frac{1}{3m+4} \end{pmatrix}.$$

<sup>13</sup>We omit the proof of this statement; it is available upon request.

Coefficient  $b$  in the quadratic equation is equal to zero, and so

$$\begin{aligned}\gamma^m &= \sqrt{-\frac{c}{a}} = \frac{3m+4}{\sqrt{2m}}; \\ \alpha_i^m &= \gamma^m A_{vi}^m = \frac{1}{\sqrt{2m}}; \\ \beta_M^m &= \frac{1}{2} \left(1 - \frac{m}{3m+4}\right) = \frac{2m+4}{6m+8} = \frac{m+2}{3m+4}.\end{aligned}$$

Note that the weight  $\beta_M$  that the market maker places on her own signal is not constant in  $m$ . If there were no strategic traders at all, and only noise traders, it would be equal to  $\frac{1}{2} = \frac{Cov(v, \theta_M)}{Var(\theta_M)}$ . As  $m$  grows, this weight is monotonically decreasing (converging to  $\frac{1}{3}$  in the limit). Thus, it is not the case that the market maker simply combines the information contained in her own signal and the additional information contained in the aggregate demand  $D$  “additively”—the interplay between the two sources of information is more intricate, and the weight that the market maker places on her own signal depends on the overall information structure.

The second observation concerns the informativeness of prices. Take any  $m$ , and consider a realization of  $\theta_1$ ,  $\theta_M$ , and  $u$ . In this realization, demand  $D$  is equal to  $m\alpha_i^m\theta_1 + u = \frac{m}{\sqrt{2m}}\theta_1 + u$ , and the market price  $P$  set by the market maker is equal to  $\beta_D^m D + \beta_M^m \theta_M = \frac{m}{3m+4}\theta_1 + \frac{m+2}{3m+4}\theta_M + \frac{\sqrt{2m}}{3m+4}u$ . Now, fix the realization of random variables, and let the number of strategic traders,  $m$ , grow to infinity. Then price  $P$  converges to  $\frac{1}{3}\theta_1 + \frac{1}{3}\theta_M$ . But notice that this expression is precisely the expected value of the asset,  $v$ , conditional on the information available in the market:  $u$  is uninformative, because it is independent of all other random variables, and

$$\begin{aligned}E[v|\theta_1, \theta_M] &= Cov\left(v, \begin{pmatrix} \theta_1 \\ \theta_M \end{pmatrix}\right)^T Var\left(\begin{pmatrix} \theta_1 \\ \theta_M \end{pmatrix}\right)^{-1} \begin{pmatrix} \theta_1 \\ \theta_M \end{pmatrix} \\ &= \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} \theta_1 \\ \theta_M \end{pmatrix} \\ &= \frac{1}{3}\theta_1 + \frac{1}{3}\theta_M.\end{aligned}$$

Hence, as the number of strategic traders becomes large, their information and the information of the market maker get incorporated into the market price with precisely the weights that a Bayesian observer with access to *all* information available in the market would assign. In other words, as the number of strategic traders becomes large, all information available in the market is aggregated and revealed by the market price. In the next section, we show that this is not a coincidence: the information aggregation result holds very generally.

## 6 Information Aggregation in Large Markets

We now turn to the second main result of our paper: the aggregation of dispersed information when the number of traders becomes large.

Consider a sequence of markets, indexed by  $m = 1, 2, \dots$ . Every market is in the general

framework of Section 3. In every market, there are  $n$  groups of strategic traders, with at least one trader in each group. Index  $i$ ,  $1 \leq i \leq n$ , now denotes a group of traders. The size of group  $i$  in market  $m$  is denoted by  $\ell_i^{(m)}$ . All traders in the same group  $i$  receive the same signal  $\theta_i \in \mathbb{R}^{k_i}$ . The notation from the preceding sections carries over, except that  $\theta_i$  now denotes the signal common to all the traders in group  $i$ .

The covariance matrix,  $\Omega$ , of vector  $\mu = (v; \theta; \theta_M; u)$  is the same for all  $m$ . The number of traders in each group, however, changes with  $m$ : specifically, we assume that for every  $i$ ,  $\lim_{m \rightarrow \infty} \ell_i^{(m)} = \infty$ , i.e., all groups become large as  $m$  becomes large. We do not impose any restrictions on the rates of growth of those groups: e.g., the sizes of some groups may grow much faster than those of other groups.

We slightly strengthen one of the two conditions on matrix  $\Omega$  made in Section 3, replacing Assumption 2 with the following:<sup>14</sup>

**Assumption 2L**  $Var(u|\theta, \theta_M) > 0$ .

It follows from Theorem 1 that for each  $m$ , there exists a unique linear equilibrium in the corresponding market. Let  $p^{(m)}$  denote the random variable that is equal to the resulting price in the unique linear equilibrium of market  $m$ .

We can now state and prove our main result on information aggregation in large markets. If the demand from liquidity traders is positively correlated with the true value of the asset (conditional on other signals), then prices in large markets aggregate all available information:  $p^{(m)}$  converges to  $E[v|\theta, \theta_M, u]$ . If the demand from liquidity traders is negatively correlated with the true value of the asset, then prices in large markets aggregate all available information *except* that contained in liquidity demand:  $p^{(m)}$  converges to  $E[v|\theta, \theta_M]$ . If the demand from liquidity traders is uncorrelated with the true value of the asset, then both statements are true:  $p^{(m)}$  converges to  $E[v|\theta, \theta_M, u] = E[v|\theta, \theta_M]$ .

## Theorem 2

- If  $Cov(u, v|\theta, \theta_M) \geq 0$ , then  $\lim_{m \rightarrow \infty} E \left[ (p^{(m)} - E[v|\theta, \theta_M, u])^2 \right] = 0$ .
- If  $Cov(u, v|\theta, \theta_M) \leq 0$ , then  $\lim_{m \rightarrow \infty} E \left[ (p^{(m)} - E[v|\theta, \theta_M])^2 \right] = 0$ .

In Appendix B, we prove Theorem 2 in the special case when the covariance matrix of random vector  $(\theta; \theta_M; u)$  is full rank. This additional assumption guarantees that certain matrices remain invertible in the limit as  $m$  becomes large, which in turn allows us to give a direct proof of the theorem without technical complications. However, this special case rules out some interesting

<sup>14</sup>Under the original Assumptions 1 and 2, the information aggregation result may not hold. To see that, consider a modification of the example introduced in footnote 9. Value  $v \sim N(0, 1)$ . There are  $m$  strategic traders with the same signal  $\theta_1 = v$ . The demand of liquidity traders is  $u = -v$ . Then in the unique linear equilibrium, the demand of each strategic trader is equal to  $\theta_1/m$ , the aggregate demand of all strategic trader is equal to  $\theta_1 = v = -u$ , the aggregate demand of all traders is equal to zero, and thus the equilibrium price is also always equal to zero, for any  $m$ . Thus, there is no information aggregation of any kind in the limit as  $m$  becomes large.

possibilities (e.g., one type of traders knowing strictly more than another type of traders), so in the Online Appendix we provide the full proof of Theorem 2, which does not rely on this simplifying assumption.

The intuition for the information aggregation result is that when the number of informed traders of each type is large, the information of each strategic “type” has to be (almost) fully incorporated into the market price, since otherwise each trader of that type would be able to make a non-negligible profit, which cannot happen in equilibrium. The signal of the market maker gets incorporated into the market price by construction. Finally, with liquidity demand, the situation is more subtle. When liquidity demand is positively correlated with the asset value ( $Cov(u, v|\theta, \theta_M) > 0$ ), equilibrium strategies and market depth adjust precisely in a way that makes liquidity demand get incorporated into the market price “correctly,” i.e., with the same weight as it would be incorporated into the market price by a Bayesian observer who was fully informed about all the random variables in the model (except value  $v$ ). As a result, price  $p^{(m)}$  converges to  $E[v|\theta, \theta_M, u]$ , and so all information available in the market is incorporated into the market price. However, when liquidity demand is negatively correlated with the value of the asset ( $Cov(u, v|\theta, \theta_M) < 0$ ), this cannot happen. In equilibrium, aggregate demand always enters the market price with a positive sign (sensitivity  $\beta_D$  is positive). Thus, liquidity demand also enters the market price with a positive sign. However, a fully informed Bayesian observer would put a *negative* weight on liquidity demand—which cannot happen in any linear equilibrium, for any parameter values. So what happens instead as  $m$  becomes large is that the variance of the aggregate demand from informed traders grows to infinity (in contrast to the case  $Cov(u, v|\theta, \theta_M) > 0$ , in which it converges to a finite value). And thus as  $m$  grows, liquidity demand  $u$  has less and less impact on the market price, and in the limit it has no impact at all: price  $p^{(m)}$  converges to  $E[v|\theta, \theta_M]$ . The same happens in the case  $Cov(u, v|\theta, \theta_M) = 0$ , for the same reason, but in that case  $E[v|\theta, \theta_M]$  is equal to  $E[v|\theta, \theta_M, u]$ , and so price  $p^{(m)}$  does converge to the expected value of the asset given all the information available in the market.

## 7 Information in “Hybrid” Markets

In many situations, some “scarce” information about the value of a security is known by only a small number of traders, perhaps just one, while some other information, while not publicly available, may be more “abundant,” and may be observed by a large number of traders. In this section, we explore how these two types of information get incorporated into market prices in equilibrium.

It is intuitive that due to market impact and the resulting strategic considerations, “scarce” information will not be fully incorporated into market prices, and the traders possessing this information will make positive profits, while “abundant” information will be almost fully incorporated into market prices (and the traders possessing it will make vanishingly small profits). What is less immediate is the interplay between these two types of information, and how they get combined with the information observed directly by the market maker and the information contained

in liquidity demand. In particular, a seemingly natural conjecture is that “abundant” information will enter the price essentially as a public signal, observed by everyone in the economy. Our last result shows that this is not the case: instead, “abundant” information, in the limit, enters into market prices in the same way as if it were directly observed by the market maker—but not by the strategic traders observing “scarce” information. As Examples 4 and 5 in Section 5 illustrate, this is substantively different from the case in which “abundant” information is observed by all the agents in the economy.

Formally, using the notation introduced in Section 6, suppose that for some  $s \geq 1$ , the sizes of the groups  $i = 1, \dots, s < n$  remain constant as  $m$  varies, i.e.,  $\ell_i^{(m)} = \ell_i$  for some  $\ell_i$ , while for  $i = s + 1, \dots, n$ , the size of group  $i$  grows to infinity, i.e.,  $\ell_i^{(m)} \rightarrow \infty$ . We will refer to groups  $i = 1, \dots, s$  as “small groups,” and to groups  $i = s + 1, \dots, n$  as “large groups.”

Let  $\theta_S$  be the vector of signals of the small groups, i.e.,  $\theta_S = (\theta_1; \dots; \theta_s)$ , and let  $\theta_L$  be the vector of signals of the large groups, i.e.,  $\theta_L = (\theta_{s+1}; \dots; \theta_n)$ . We make two assumptions:

**Assumption 1H**  $Cov(v, \theta_S | \theta_L, \theta_M) \neq 0$ .

**Assumption 2H** Matrix  $Var((\theta_S; \theta_L; \theta_M; u))$  is positive definite.

The first assumption states that at least one of the small groups has some information about the value of the asset that is not included in the information of the large groups or in the information observed by the market maker. This assumption is substantive: if the information available to the small groups was fully subsumed by the information available to the large groups and the market maker, then instead of the result below, the information aggregation results of the Section 6 would hold. The second assumption is for simplicity of exposition; it is analogous to the simplifying assumption made in the special case of Theorem 2 in Appendix B, and serves the same purpose.

Our last result shows that under Assumptions 1H and 2H, equilibrium prices in the above sequence of markets converge to the equilibrium price that would obtain in an alternative market, in which only the small groups of traders are present (with the same information as in the original markets,  $\theta_S$ ), and in which the market maker observes both her original signal  $\theta_M$  and the signals observed by the large groups of traders in the original markets,  $\theta_L$ . Let  $p^{(alt)}$  denote the random variable that corresponds to the equilibrium price obtained in this alternative market.

**Theorem 3**  $\lim_{m \rightarrow \infty} E \left[ \left( p^{(m)}((\theta_S; \theta_L), \theta_M, u) - p^{(alt)}(\theta_S, (\theta_M; \theta_L), u) \right)^2 \right] = 0$ .

The proof of Theorem 3 is in Appendix C. The techniques used in the proof are similar to those used in the proofs of Theorem 2 in the special and general cases, except that the presence of small groups requires a separate treatment, since for the traders in those groups, strategic incentives do not vanish in the limit. Also, note that unlike in Theorem 2, the statement of Theorem 3 does not depend on the sign of the covariance of liquidity demand with the other random variables in the model.

We conclude this section with a final observation. As we saw in Examples 4 and 5 in Section 5, the expected profit of an informed agent is higher when he also observes the signal of the market maker than when he does not. Since in the case of “hybrid” markets, equilibria converge to those that would obtain if the information of “large” groups was observed by the market maker, but not publicly, and since individual agents in these “large” groups make vanishingly small profits, there will be strong incentives for trading information: by buying information from one of the agents in a large group (say,  $j$ ), a trader in a small group (say,  $i$ ) can substantively increase his expected profit, while the resulting decrease in the profit of agent  $j$  is vanishingly small, simply because his original, “pre-trade” profit was small. Thus, “abundant” information may in fact end up being essentially publicly observed, but only via “external” trade in information rather than via the trading mechanism itself. We leave the formal analysis of this intuition to future research.

## 8 Conclusion

This paper studies trading behavior and the properties of prices in informationally complex markets. Our framework generalizes the single-period version of the linear-normal model of Kyle (1985), allowing for multiple differentially informed strategic traders and for essentially arbitrary correlations among the random variables involved in the model: the value of the traded asset, the signals of the strategic traders and competitive market makers, and the demand from liquidity traders.

In this framework, we establish two main results.

First, we show that there always exists a unique linear equilibrium. We characterize the equilibrium analytically, with the agents’ equilibrium behavior expressed in closed form. This characterization makes the framework very convenient for modeling various applied issues in informationally complex settings. We illustrate the result and the equilibrium characterization in a series of examples.

Second, we explore the informational properties of equilibrium prices as the number of informed agents becomes large. Our general framework allows us to avoid imposing the usual symmetry restrictions that are typically made in the literature on the strategic foundations of information aggregation in large markets. Instead, we assume that there are several distinct types of agents, with agents of the same type receiving the same information, and fix the matrix of correlations of signals across the types (and other random variables in the model), imposing essentially no restrictions on this matrix. We then allow the numbers of agents of every type to grow (without restricting the rates of growth in any way). We find that the informational properties of prices in these large markets depend on the informativeness of the demand from liquidity traders. If the demand from liquidity traders is uncorrelated with the true value of the asset or is positively correlated with it (conditional on other signals), then prices in large markets aggregate all available information. If, however, the demand from liquidity traders is negatively correlated with the true value of the asset, then prices in large markets aggregate all available information except that contained in liquidity demand. Finally, in the “hybrid” case, in which only some groups of traders become large, the

information of these large groups also gets fully incorporated into market prices: in the limit, the equilibrium in the “hybrid” case is equivalent to the equilibrium of an alternative market in which there are no traders of these large types, and instead all their information is observed directly by the market maker.

## Appendix A: Proof of Theorem 1

The proof of Theorem 1 is constructive. For convenience, it is broken into several steps. Step 1 expresses the linear relationship implied by condition (i) of the definition of equilibrium, that price must be equal to the expected value of the security conditional on the information available to the market maker. Step 2 derives the best response of a strategic trader to a linear pricing rule and linear strategies of other strategic traders, and shows that this best response is linear. It also establishes that in equilibrium, coefficient  $\beta_D$  has to be positive. Step 3 summarizes the equations in Steps 1 and 2 and reorganizes them in a system of three “almost” linear equations (they are all linear if one scalar variable,  $\gamma = 1/\beta_D$ , is fixed). Step 4 reduces this system of equations to one quadratic equation in  $\gamma$ . Step 5 shows that this quadratic equation has exactly one positive root, thus completing the proof.

**Step 1.** Let  $\alpha = (\alpha_1; \dots; \alpha_n)$  be a profile of linear strategies for the strategic traders. Each  $\alpha_i$  in this profile is a vector  $(\alpha_i^1; \dots; \alpha_i^{k_i}) \in \mathbb{R}^{k_i}$ , corresponding to linear strategy

$$d_i(\theta_i) = \alpha_i^1 \theta_i^1 + \dots + \alpha_i^{k_i} \theta_i^{k_i},$$

where  $\theta_i^1, \dots, \theta_i^{k_i}$  are the elements of vector  $\theta_i \in \mathbb{R}^{k_i}$ .

Take any linear pricing rule  $(\beta_M; \beta_D)$ ,  $\beta_M \in \mathbb{R}^{k_M}$ ,  $\beta_D \in \mathbb{R}$ . For convenience, let vector  $\beta = (\beta_M; \beta_D)$  summarize the pricing rule and let random vector  $\eta = (\theta_M; D = \alpha^T \theta + u)$  denote the information available to the market maker when she sets the price. Then for this pricing rule to be consistent with profile  $\alpha$ , condition (i) of the definition of equilibrium requires that

$$\beta^T \eta = E[v|\eta],$$

which is equivalent to the following condition:<sup>15</sup>

$$Cov(v, \eta) = \beta^T Var(\eta).$$

Expressing  $Cov(v, \eta)$  and  $Var(\eta)$  using the notation introduced in Section 4.1, we thus get the following equivalent characterization of condition (i) of the definition of equilibrium:

$$(\beta_M^T, \beta_D) \begin{pmatrix} \Sigma_{MM} & \Sigma_{\theta M}^T \alpha + \Sigma_{Mu} \\ \alpha^T \Sigma_{\theta M} + \Sigma_{Mu}^T & \alpha^T \Sigma_{\theta \theta} \alpha + 2 \Sigma_{\theta u}^T \alpha + \sigma_{uu} \end{pmatrix} = (\Sigma_{vM}, \Sigma_{\theta v}^T \alpha + \sigma_{vu}). \quad (6)$$

<sup>15</sup>To see the equivalence, note first that  $\beta^T \eta = E[v|\eta] \implies Cov(v, \eta) = Cov(E[v|\eta], \eta) = Cov(\beta^T \eta, \eta) = \beta^T Var(\eta)$ . To go in the opposite direction, note that  $Cov(v, \eta) = \beta^T Var(\eta) = Cov(\beta^T \eta, \eta) \implies Cov(v - \beta^T \eta, \eta) = 0$ . Since variables  $v - \beta^T \eta$  and  $\eta$  are jointly normal,  $Cov(v - \beta^T \eta, \eta) = 0$  implies that they are independent, and thus for every realization  $\tilde{\eta}$  of random variable  $\eta$ ,  $E[v - \beta^T \eta | \eta = \tilde{\eta}] = E[v - \beta^T \eta] = 0$ , which implies that for every realization  $\tilde{\eta}$ ,  $E[v | \eta = \tilde{\eta}] = E[\beta^T \eta | \eta = \tilde{\eta}] = \beta^T \tilde{\eta}$ .

**Step 2.** We now consider the optimization problem of a strategic trader  $i$ . Suppose he observes signal realization  $\tilde{\theta}_i$  of signal  $\theta_i$ , and subsequently submits demand  $d$ . Assuming that other traders  $j \neq i$  follow linear strategies  $\alpha_j$ , and that the market maker follows a linear pricing rule  $(\beta_M; \beta_D)$ , the expected profit of trader  $i$  from submitting demand  $d$  when observing realization  $\tilde{\theta}_i$  is equal to

$$E \left[ d \left( v - \beta_M^T \theta_M - \beta_D \left( d + \sum_{j \neq i} \alpha_j^T \theta_j + u \right) \right) \middle| \theta_i = \tilde{\theta}_i \right]. \quad (7)$$

Using the fact that  $d$  is a choice variable, and thus  $d$  and  $d^2$  are constants from the point of view of taking expectations, we can rewrite equation (7) as

$$\tilde{d} \cdot E \left[ v - \beta_M^T \theta_M - \beta_D \left( \sum_{j \neq i} \hat{\alpha}_j^T \theta_j + u \right) \middle| \theta_i = \tilde{\theta}_i \right] - d^2 \cdot \beta_D. \quad (8)$$

Now, if  $\beta_D < 0$ , trader  $i$  can make an arbitrarily large expected profit, and no single  $d$  maximizes it—hence,  $\beta_D$  cannot be negative in equilibrium.

If  $\beta_D = 0$ , and  $E \left[ v - \beta_M^T \theta_M | \theta_i = \tilde{\theta}_i \right] \neq 0$ , then again trader  $i$  can make an arbitrarily large expected profit, and no single  $d$  maximizes it. But it follows from Assumption 1 in the model<sup>16</sup> that for at least one trader  $i$ , for at least some (in fact, for almost all) realizations  $\tilde{\theta}_i$ , we have  $E \left[ v - \beta_M^T \theta_M | \theta_i = \tilde{\theta}_i \right] \neq 0$ —hence,  $\beta_D$  cannot be equal to zero in equilibrium.

Finally, if  $\beta_D > 0$ , then there is a unique  $d$  maximizing the expected profit:

$$d^* = \frac{1}{2\beta_D} E \left[ v - \beta_M^T \theta_M - \beta_D \left( \sum_{j \neq i} \alpha_j^T \theta_j + u \right) \middle| \theta_i = \tilde{\theta}_i \right] \quad (9)$$

$$= \frac{1}{2\beta_D} \left( \Sigma_{iv}^T - \beta_M^T \Sigma_{iM}^T - \beta_D \left( \sum_{j \neq i} \alpha_j^T \Sigma_{ij}^T + \Sigma_{iu}^T \right) \right) \Sigma_{ii}^{-1} \tilde{\theta}_i, \quad (10)$$

where equation (10) is the standard projection/signal extraction formula, which can be used because of the joint normality of the relevant variables. Note that  $d^*$  is a linear function of  $\tilde{\theta}_i$ , and vector  $\alpha_i$  is uniquely determined by pricing rule  $(\beta_M; \beta_D)$  and strategies  $\alpha_j$  for  $j \neq i$ .

**Step 3.** It therefore follows from the arguments in Steps 1 and 2 that profile of strategies  $\alpha$  and pricing rule  $(\beta_M; \beta_D)$  form a linear equilibrium if and only if  $\beta_D > 0$  and the following two conditions hold:

- (i)  $(\beta_M^T, \beta_D) \begin{pmatrix} \Sigma_{MM} & \Sigma_{\theta M}^T \alpha + \Sigma_{Mu} \\ \alpha^T \Sigma_{\theta M} + \Sigma_{Mu}^T & \alpha^T \Sigma_{\theta \theta} \alpha + 2\Sigma_{\theta u}^T \alpha + \sigma_{uu} \end{pmatrix} = (\Sigma_{vM}, \Sigma_{\theta v}^T \alpha + \sigma_{vu})$ ;
- (ii) for all  $i$ ,  $\alpha_i = \frac{1}{2\beta_D} \left( \Sigma_{iv}^T - \beta_M^T \Sigma_{iM}^T - \beta_D \left( \sum_{j \neq i} \alpha_j^T \Sigma_{ij}^T + \Sigma_{iu}^T \right) \right) \Sigma_{ii}^{-1}$ .

We will now show that there is a unique profile  $(\alpha, \beta)$  satisfying these conditions, thus proving the existence and uniqueness of linear equilibrium.

<sup>16</sup> Assumption 1 says that at least one strategic trader  $i$  has some useful information beyond that contained in the market maker's signal:  $Cov(v, \theta | \theta_M) \neq 0$ .



First, we re-write condition (ii), for all  $i$ , as:

$$2\Sigma_{ii}\alpha_i = \frac{1}{\beta_D}(\Sigma_{iv} - \Sigma_{iM}\beta_M) - \sum_{j \neq i} \Sigma_{ij}\alpha_j - \Sigma_{iu}$$

or equivalently

$$\Sigma_{ii}\alpha_i + \sum_j \Sigma_{ij}\alpha_j = \frac{1}{\beta_D}(\Sigma_{iv} - \Sigma_{iM}\beta_M) - \Sigma_{iu}. \quad (11)$$

“Stacking” equations (11) for all  $i$  one under another, and rewriting the resulting system of equations in matrix form using the notation defined in Section 4.1, we obtain the following condition (equivalent to condition (ii)):

$$(\Sigma_{diag} + \Sigma_{\theta\theta})\alpha = \gamma\Sigma_{\theta v} - \Sigma_{\theta M}\beta'_M - \Sigma_{\theta u}, \quad (12)$$

where for convenience we define  $\gamma = 1/\beta_D$ ,  $\beta'_M = \beta_M/\beta_D$ .

Next, using this notation, and transposing the matrix equation in condition (i), that condition can be written as a system of two equations:

$$\Sigma_{MM}\beta'_M + \Sigma_{\theta M}^T\alpha + \Sigma_{Mu} = \gamma\Sigma_{Mv}, \quad (13)$$

$$\alpha^T\Sigma_{\theta M}\beta'_M + \Sigma_{uM}\beta'_M + \alpha^T\Sigma_{\theta\theta}\alpha + 2\Sigma_{\theta u}^T\alpha + \sigma_{uu} = \gamma(\Sigma_{\theta v}^T\alpha + \sigma_{vu}). \quad (14)$$

**Step 4.** We will now solve the system of equations (12), (13), and (14). Equation (13) allows us to express  $\beta'_M$  as a function of  $\alpha$  and  $\gamma$ :

$$\beta'_M = \Sigma_{MM}^{-1}(\gamma\Sigma_{Mv} - \Sigma_{\theta M}^T\alpha - \Sigma_{Mu}). \quad (15)$$

We then plug this expression of  $\beta'_M$  into equation (12):

$$(\Sigma_{diag} + \Sigma_{\theta\theta})\alpha = \gamma\Sigma_{\theta v} - \Sigma_{\theta M}\Sigma_{MM}^{-1}(\gamma\Sigma_{Mv} - \Sigma_{\theta M}^T\alpha - \Sigma_{Mu}) - \Sigma_{\theta u},$$

or, isolating  $\alpha$  on the left-hand side and collecting the terms with  $\gamma$ ,

$$(\Sigma_{diag} + \Sigma_{\theta\theta} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{\theta M}^T)\alpha = (\Sigma_{\theta v} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{Mv})\gamma - (\Sigma_{\theta u} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{Mu}).$$

Note that

$$\begin{aligned} \Sigma_{\theta\theta} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{\theta M}^T &= \text{Var}(\theta) - \text{Cov}(\theta, \theta_M)\text{Var}(\theta_M)^{-1}\text{Cov}(\theta_M, \theta) \\ &= \text{Var}(\theta|\theta_M), \end{aligned}$$

where the last equation follows from the standard projection formula for multivariate normal distributions. Thus, matrix  $\Sigma_{\theta\theta} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{\theta M}^T$  is positive semidefinite, and matrix  $\Sigma_{diag} + \Sigma_{\theta\theta} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{\theta M}^T$  is positive definite (and thus invertible). Letting

$$\begin{aligned} \Lambda &= \Sigma_{diag} + \Sigma_{\theta\theta} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{\theta M}^T, \\ A_u &= \Lambda^{-1}(\Sigma_{\theta u} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{Mu}), \\ A_v &= \Lambda^{-1}(\Sigma_{\theta v} - \Sigma_{\theta M}\Sigma_{MM}^{-1}\Sigma_{Mv}), \end{aligned}$$

we can express  $\alpha$  as a linear function of  $\gamma$ :

$$\alpha = \gamma A_v - A_u.$$

Plugging this expression into equation (15), we can also express  $\beta'_M$  as a linear function of  $\gamma$ :

$$\begin{aligned}\beta'_M &= \Sigma_{MM}^{-1} (\gamma \Sigma_{Mv} - \Sigma_{\theta M}^T (\gamma A_v - A_u) - \Sigma_{Mu}) \\ &= \gamma \Sigma_{MM}^{-1} (\Sigma_{Mv} - \Sigma_{\theta M}^T A_v) - \Sigma_{MM}^{-1} (\Sigma_{Mu} - \Sigma_{\theta M}^T A_u).\end{aligned}$$

Using these expressions, we can now rewrite equation (14) as a quadratic equation of just one scalar variable,  $\gamma$ :

$$a\gamma^2 + b\gamma + c = 0, \quad (16)$$

where

$$\begin{aligned}a &= A_v^T \Sigma_{\theta M} \Sigma_{MM}^{-1} (\Sigma_{Mv} - \Sigma_{\theta M}^T A_v) + A_v^T \Sigma_{\theta\theta} A_v - \Sigma_{\theta v}^T A_v, \\ b &= -A_v^T \Sigma_{\theta M} \Sigma_{MM}^{-1} (\Sigma_{Mu} - \Sigma_{\theta M}^T A_u) - A_u^T \Sigma_{\theta M} \Sigma_{MM}^{-1} (\Sigma_{Mv} - \Sigma_{\theta M}^T A_v) \\ &\quad + \Sigma_{uM} \Sigma_{MM}^{-1} (\Sigma_{Mv} - \Sigma_{\theta M}^T A_v) - 2A_v^T \Sigma_{\theta\theta} A_u + 2\Sigma_{\theta u}^T A_v + \Sigma_{\theta v}^T A_u - \sigma_{vu}, \\ c &= A_u^T \Sigma_{\theta M} \Sigma_{MM}^{-1} (\Sigma_{Mu} - \Sigma_{\theta M}^T A_u) - \Sigma_{uM} \Sigma_{MM}^{-1} (\Sigma_{Mu} - \Sigma_{\theta M}^T A_u) \\ &\quad + A_u^T \Sigma_{\theta\theta} A_u - 2\Sigma_{\theta u}^T A_u + \sigma_{uu}.\end{aligned}$$

Therefore, finding a linear equilibrium is equivalent to finding a positive root of equation (16). To prove that this equation has a unique such root, we first simplify the expressions for  $a$ ,  $b$ , and  $c$ . (For the proof, it is sufficient to simplify  $a$  and  $c$ , but getting a simplified expression for  $b$  is useful for deriving an explicit analytic characterization of the equilibrium.) Starting with  $a$ :

$$\begin{aligned}a &= A_v^T \Sigma_{\theta M} \Sigma_{MM}^{-1} (\Sigma_{Mv} - \Sigma_{\theta M}^T A_v) + A_v^T \Sigma_{\theta\theta} A_v - \Sigma_{\theta v}^T A_v, \\ &= A_v^T [(\Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{Mv} - \Sigma_{\theta v}) + (\Sigma_{\theta\theta} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{\theta M}^T) A_v] \\ &= A_v^T [(-\Lambda A_v) + (\Lambda - \Sigma_{diag}) A_v] \\ &= -A_v^T \Sigma_{diag} A_v.\end{aligned}$$

Next,

$$\begin{aligned}b &= -A_v^T \Sigma_{\theta M} \Sigma_{MM}^{-1} (\Sigma_{Mu} - \Sigma_{\theta M}^T A_u) - A_u^T \Sigma_{\theta M} \Sigma_{MM}^{-1} (\Sigma_{Mv} - \Sigma_{\theta M}^T A_v) \\ &\quad + \Sigma_{uM} \Sigma_{MM}^{-1} (\Sigma_{Mv} - \Sigma_{\theta M}^T A_v) - 2A_v^T \Sigma_{\theta\theta} A_u + 2\Sigma_{\theta u}^T A_v + \Sigma_{\theta v}^T A_u - \sigma_{vu}, \\ &= 2A_v^T (\Sigma_{\theta u} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{Mu}) + A_u^T (\Sigma_{\theta v} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{Mv}) \\ &\quad + 2A_v^T (\Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{\theta M}^T - \Sigma_{\theta\theta}) A_u + \Sigma_{uM} \Sigma_{MM}^{-1} \Sigma_{Mv} - \sigma_{uv} \\ &= 2A_v^T \Lambda A_u + A_u^T \Lambda A_v \\ &\quad + 2A_v^T (\Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{\theta M}^T - \Sigma_{\theta\theta}) A_u + \Sigma_{uM} \Sigma_{MM}^{-1} \Sigma_{Mv} - \sigma_{uv} \\ &= A_v^T (2\Sigma_{diag} + \Lambda) A_u + \Sigma_{uM} \Sigma_{MM}^{-1} \Sigma_{Mv} - \sigma_{uv}.\end{aligned}$$

Finally,

$$\begin{aligned}
c &= A_u^T \Sigma_{\theta M} \Sigma_{MM}^{-1} (\Sigma_{Mu} - \Sigma_{\theta M}^T A_u) - \Sigma_{uM} \Sigma_{MM}^{-1} (\Sigma_{Mu} - \Sigma_{\theta M}^T A_u) \\
&\quad + A_u^T \Sigma_{\theta\theta} A_u - 2 \Sigma_{\theta u}^T A_u + \sigma_{uu} \\
&= -(\Sigma_{uM} - A_u^T \Sigma_{\theta M})^T \Sigma_{MM}^{-1} (\Sigma_{Mu} - \Sigma_{\theta M}^T A_u) \\
&\quad + A_u^T \Sigma_{\theta\theta} A_u - 2 \Sigma_{\theta u}^T A_u + \sigma_{uu} \\
&= \begin{pmatrix} A_u \\ -1 \end{pmatrix}^T C \begin{pmatrix} A_u \\ -1 \end{pmatrix},
\end{aligned}$$

where

$$\begin{aligned}
C &= \begin{pmatrix} \Sigma_{\theta\theta} & \Sigma_{\theta u} \\ \Sigma_{\theta u}^T & \sigma_{uu} \end{pmatrix} - \begin{pmatrix} \Sigma_{\theta M} \\ \Sigma_{uM} \end{pmatrix} \Sigma_{MM}^{-1} \begin{pmatrix} \Sigma_{\theta M} \\ \Sigma_{uM} \end{pmatrix}^T \\
&= \text{Var}((\theta; u)) - \text{Cov}((\theta; u), \theta_M) \text{Var}(\theta_M)^{-1} \text{Cov}(\theta_M, (\theta; u)) \\
&= \text{Var}((\theta; u) | \theta_M).
\end{aligned}$$

Thus,

$$c = \text{Var}(A_u^T \theta - u | \theta_M).$$

**Step 5.** We will now determine the signs of coefficients  $a$  and  $c$ .

Matrix  $\Sigma_{diag}$  is positive definite, by construction. Vector  $A_v$  is not equal to zero: matrix  $\Lambda^{-1}$  is positive definite, and vector  $\Sigma_{\theta v} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{Mv} = \text{Cov}(\theta, v | \theta_M)$  is not equal to zero (by Assumption 1 of the model). Thus,  $a = -A_v^T \Sigma_{diag} A_v < 0$ .

To determine the sign of coefficient  $c$ , recall that we have shown in Step 4 that  $c = \text{Var}(A_u^T \theta - u | \theta_M)$ . So if we show that  $c \neq 0$ , it will immediately follow that  $c > 0$ .

If  $A_u = 0$ , then  $c \neq 0$  follows from Assumption 2 of the model (which says that the market maker does not perfectly observe liquidity demand:  $\text{Var}(u | \theta_M) > 0$ ).

Suppose  $A_u \neq 0$ . It is convenient to introduce an auxiliary random variable,  $\phi$ , drawn randomly from the normal distribution with mean zero and covariance matrix  $\Sigma_{diag}$ , independent of all other random variables in the model. Note that matrix  $A_u$  now has a simple interpretation:

$$A_u = \text{Var}(\theta + \phi | \theta_M)^{-1} \text{Cov}(\theta, u | \theta_M) = \text{Var}(\theta + \phi | \theta_M)^{-1} \text{Cov}(\theta + \phi, u | \theta_M).$$

Let  $\epsilon = u - A_u^T (\theta + \phi)$ . Then  $c = \text{Var}(\epsilon + A_u^T \phi | \theta_M)$ . To show that  $c \neq 0$ , it is thus sufficient to show that  $\epsilon + A_u^T \phi$  is not constant, conditional on  $\theta_M$ . To show that, consider  $\text{Cov}(\epsilon + A_u^T \phi, A_u^T (\theta + \phi) | \theta_M) = \text{Cov}(\epsilon, A_u^T (\theta + \phi) | \theta_M) + \text{Cov}(A_u^T \phi, A_u^T (\theta + \phi) | \theta_M)$ .

First,  $\text{Cov}(\epsilon, A_u^T (\theta + \phi) | \theta_M) = \text{Cov}(u - A_u^T (\theta + \phi), A_u^T (\theta + \phi) | \theta_M) = \text{Cov}(u, \theta + \phi | \theta_M) A_u - A_u^T \text{Var}(\theta + \phi | \theta_M) A_u = 0$ .

Second,  $\text{Cov}(A_u^T \phi, A_u^T (\theta + \phi) | \theta_M) = \text{Var}(A_u^T \phi | \theta_M) = A_u^T \Sigma_{diag} A_u$ , which is not equal to zero, because  $A_u \neq 0$  and  $\Sigma_{diag}$  is positive definite. Therefore,  $\text{Cov}(\epsilon + A_u^T \phi, A_u^T (\theta + \phi) | \theta_M) \neq 0$ , and thus  $\epsilon + A_u^T \phi$  is not constant conditional on  $\theta_M$ , and so  $c > 0$ .

Thus,  $a < 0$ ,  $c > 0$ , and hence equation (16) has exactly one positive root. Therefore, there

exists a unique linear equilibrium.

## Appendix B: Proof of Theorem 2 (Special Case)

In this Appendix, we prove Theorem 2 in the special case in which the covariance matrix of random vector  $(\theta; \theta_M; u)$  is full rank. In the Online Appendix, we provide the full proof of Theorem 2, without making this simplifying assumption.

**Step 1.** Consider first a specific market  $m$ , and, for convenience, drop superscript  $(m)$ . We know there exists a unique linear equilibrium. It then has to be the case that in this equilibrium, any two strategic traders in the same group have the same linear strategy (otherwise, by swapping the strategies of these two traders, we would be able to obtain a different linear equilibrium). Denote by  $\alpha_i$  the aggregate demand multiplier, in equilibrium, of group  $i$ ; i.e., given signal  $\theta_i$ , each trader in the group submits demand  $\frac{1}{\ell_i} \alpha_i^T \theta_i$ .

With this notation, note that the expression for condition (i) in Step 3 of the proof of Theorem 1—the market maker’s inference given her information—remains unchanged, and equations (13) and (14) remain unchanged as well. The expression for condition (ii)—the best response of a strategic trader—is now slightly different. In this new notation, it becomes: for all  $i$ ,

$$\frac{1}{\ell_i} \alpha_i = \frac{1}{2\beta_D} \left( \Sigma_{iv}^T - \beta_M^T \Sigma_{iM}^T - \beta_D \left( \sum_{j \neq i} \alpha_j^T \Sigma_{ij}^T + \frac{\ell_i - 1}{\ell_i} \alpha_i \Sigma_{ii}^T + \Sigma_{iu}^T \right) \right) \Sigma_{ii}^{-1}.$$

As in equation (12) in the proof of Theorem 1, this condition can be rewritten as

$$(\widehat{\Sigma}_{diag} + \Sigma_{\theta\theta})\alpha = \gamma \Sigma_{\theta v} - \Sigma_{\theta M} \beta_M' - \Sigma_{\theta u}, \quad (17)$$

where  $\gamma$  and  $\beta_M'$  are defined as before, and instead of  $\Sigma_{diag}$  we now have

$$\widehat{\Sigma}_{diag} = \begin{pmatrix} \frac{1}{\ell_1} \Sigma_{11} & 0 & \dots & \dots \\ 0 & \frac{1}{\ell_2} \Sigma_{22} & 0 & \dots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & \frac{1}{\ell_n} \Sigma_{nn} \end{pmatrix}.$$

Next, again by analogy with the proof of Theorem 1, we define

$$\begin{aligned} \widehat{\Lambda} &= \widehat{\Sigma}_{diag} + \Sigma_{\theta\theta} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{\theta M}^T, \\ \widehat{A}_u &= \widehat{\Lambda}^{-1} (\Sigma_{\theta u} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{Mu}), \\ \widehat{A}_v &= \widehat{\Lambda}^{-1} (\Sigma_{\theta v} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{Mv}), \end{aligned}$$

and then finding a linear equilibrium is equivalent to solving the quadratic equation

$$a\gamma^2 + b\gamma + c = 0,$$

where

$$\begin{aligned}
a &= -\widehat{A}_v^T \widehat{\Sigma}_{diag} \widehat{A}_v, \\
b &= \widehat{A}_v^T \left( 2\widehat{\Sigma}_{diag} + \widehat{\Lambda} \right) \widehat{A}_u + \Sigma_{uM} \Sigma_{MM}^{-1} \Sigma_{Mv} - \sigma_{uv}, \\
c &= Var(\widehat{A}_u^T \theta - u | \theta_M).
\end{aligned}$$

Since by definition  $\gamma = 1/\beta_D$ , solving the above quadratic equation is equivalent to solving the quadratic equation

$$c\beta_D^2 + b\beta_D + a = 0,$$

which turns out to be a more convenient characterization that we will proceed with. Similarly to the proof of Theorem 1, we also have a simple expression for the vector of strategies  $\alpha$ :

$$\alpha = \widehat{A}_v / \beta_D - \widehat{A}_u.$$

**Step 2.** Let us now consider the entire sequence of markets, and restore superscript  $(m)$  for the variables. From the simplifying assumption that  $Var(\theta; \theta_M; u)$  is full rank, it follows that both  $Var(\theta | \theta_M)$  and  $Var(\theta_M | \theta)$  are full rank, and thus invertible.

As  $m \rightarrow \infty$ ,  $\widehat{\Sigma}_{diag}^{(m)} \rightarrow 0$ . Thus,

$$\begin{aligned}
\widehat{\Lambda}^{(m)} &\rightarrow \Sigma_{\theta\theta} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{\theta M}^T = Var(\theta | \theta_M), \\
\widehat{A}_u^{(m)} &\rightarrow Var(\theta | \theta_M)^{-1} (\Sigma_{\theta u} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{Mu}) = Var(\theta | \theta_M)^{-1} Cov(\theta, u | \theta_M), \\
\widehat{A}_v^{(m)} &\rightarrow Var(\theta | \theta_M)^{-1} (\Sigma_{\theta v} - \Sigma_{\theta M} \Sigma_{MM}^{-1} \Sigma_{Mv}) = Var(\theta | \theta_M)^{-1} Cov(\theta, v | \theta_M).
\end{aligned}$$

Therefore,

$$\begin{aligned}
a^{(m)} &\rightarrow 0, \\
b^{(m)} &\rightarrow Cov(v, \theta | \theta_M) Var(\theta | \theta_M)^{-1} Var(\theta | \theta_M) Var(\theta | \theta_M)^{-1} Cov(\theta, u | \theta_M) + \Sigma_{uM} \Sigma_{MM}^{-1} \Sigma_{Mv} - \sigma_{uv} \\
&= Cov(v, \theta | \theta_M) Var(\theta | \theta_M)^{-1} Cov(\theta, u | \theta_M) - Cov(u, v | \theta_M) \\
&= -Cov(u, v | \theta, \theta_M), \\
c^{(m)} &\rightarrow Var(Cov(u, \theta | \theta_M) Var(\theta | \theta_M)^{-1} \theta - u | \theta_M) \\
&= Var(E[u | \theta, \theta_M] - u | \theta_M) \\
&= Var(u | \theta, \theta_M).
\end{aligned}$$

Note that these convergence results imply that  $\beta_D^{(m)}$  converges to some finite value, since  $\lim_{m \rightarrow \infty} c^{(m)} = Var(u | \theta, \theta_M) > 0$  (where the last inequality is due to Assumption 2L). If  $Cov(u, v | \theta, \theta_M) > 0$ , then  $\lim_{m \rightarrow \infty} \beta_D^{(m)} = Var(u | \theta, \theta_M)^{-1} Cov(u, v | \theta, \theta_M)$ . If  $Cov(u, v | \theta, \theta_M) \leq 0$ , then  $\lim_{m \rightarrow \infty} \beta_D^{(m)} = 0$ . We now consider the limiting behavior of price  $p^{(m)}$  in these two cases separately.

**Step 3, Case  $Cov(u, v|\theta, \theta_M) > \mathbf{0}$ .** Note first that

$$\begin{aligned}
E[v|\theta, \theta_M, u] &= E[v|\theta_M] \\
&\quad + Cov(v, \theta|\theta_M)Var(\theta|\theta_M)^{-1}(\theta - E[\theta|\theta_M]) \\
&\quad + Cov(v, u|\theta, \theta_M)Var(u|\theta, \theta_M)^{-1}(u - E[u|\theta, \theta_M]) \\
&= E[v|\theta_M] \\
&\quad + Cov(v, \theta|\theta_M)Var(\theta|\theta_M)^{-1}(\theta - E[\theta|\theta_M]) \\
&\quad + Cov(v, u|\theta, \theta_M)Var(u|\theta, \theta_M)^{-1} \\
&\quad \quad \times (u - E[u|\theta_M] - Cov(u, \theta|\theta_M)Var(\theta|\theta_M)^{-1}(\theta - E[\theta|\theta_M])),
\end{aligned}$$

where the second equality follows from  $E[u|\theta, \theta_M] = E[u|\theta_M] + Cov(u, \theta|\theta_M)Var(\theta|\theta_M)^{-1}(\theta - E[\theta|\theta_M])$ . Thus,  $E[v|\theta, \theta_M, u]$  is a linear function of  $\theta$ ,  $\theta_M$ , and  $u$ :

$$E[v|\theta, \theta_M, u] = w_M^T \theta_M + w_\theta^T \theta + w_u u,$$

where weights  $w$  are as follows:

$$\begin{aligned}
w_M^T &= Cov(v, \theta_M)Var(\theta_M)^{-1} \\
&\quad - Cov(v, \theta|\theta_M)Var(\theta|\theta_M)^{-1}Cov(\theta, \theta_M)Var(\theta_M)^{-1} \\
&\quad - Cov(v, u|\theta, \theta_M)Var(u|\theta, \theta_M)^{-1}Cov(u, \theta_M)Var(\theta_M)^{-1} \\
&\quad + Cov(v, u|\theta, \theta_M)Var(u|\theta, \theta_M)^{-1}Cov(u, \theta|\theta_M)Var(\theta|\theta_M)^{-1}Cov(\theta, \theta_M)Var(\theta_M)^{-1}; \\
w_\theta^T &= Cov(v, \theta|\theta_M)Var(\theta|\theta_M)^{-1} \\
&\quad - Cov(v, u|\theta, \theta_M)Var(u|\theta, \theta_M)^{-1}Cov(u, \theta|\theta_M)Var(\theta|\theta_M)^{-1}; \\
w_u &= Cov(v, u|\theta, \theta_M)Var(u|\theta, \theta_M)^{-1}.
\end{aligned}$$

Next, price  $p^{(m)}(\theta, \theta_M, u)$  in market  $m$  can be expressed as

$$\begin{aligned}
p^{(m)}(\theta, \theta_M, u) &= \beta_M^{(m)T} \theta_M + \beta_D^{(m)} \left( \alpha^{(m)T} \theta + u \right) \\
&= \beta_M^{(m)T} \theta_M + \beta_D^{(m)} \alpha^{(m)T} \theta + \beta_D^{(m)} u.
\end{aligned}$$

To prove the statement of the theorem for this case, note that

$$E \left[ \left( p^{(m)}(\theta, \theta_M, u) - E[v|\theta, \theta_M, u] \right)^2 \right] = \begin{pmatrix} \beta_M^{(m)} - w_M \\ \beta_D^{(m)} \alpha^{(m)} - w_\theta \\ \beta_D^{(m)} - w_u \end{pmatrix}^T Var \left( \begin{pmatrix} \theta_M \\ \theta \\ u \end{pmatrix} \right) \begin{pmatrix} \beta_M^{(m)} - w_M \\ \beta_D^{(m)} \alpha^{(m)} - w_\theta \\ \beta_D^{(m)} - w_u \end{pmatrix}.$$

Thus, it is sufficient to show that as  $m$  grows,  $\beta_D^{(m)} \rightarrow w_u$ ,  $\beta_D^{(m)} \alpha^{(m)} \rightarrow w_\theta$ , and  $\beta_M^{(m)} \rightarrow w_M$ .

The first convergence result is immediate:

$$\lim_{m \rightarrow \infty} \beta_D^{(m)} = Var(u|\theta, \theta_M)^{-1}Cov(u, v|\theta, \theta_M) = w_u.$$

Next:

$$\begin{aligned}
\lim_{m \rightarrow \infty} \beta_D^{(m)} \alpha^{(m)} &= \lim_{m \rightarrow \infty} \widehat{A}_v - \beta_D^{(m)} \widehat{A}_u \\
&= \text{Var}(\theta|\theta_M)^{-1} \text{Cov}(\theta, v|\theta_M) \\
&\quad - \text{Var}(u|\theta, \theta_M)^{-1} \text{Cov}(u, v|\theta, \theta_M) \text{Var}(\theta|\theta_M)^{-1} \text{Cov}(\theta, u|\theta_M) \\
&= w_\theta.
\end{aligned}$$

Finally:

$$\begin{aligned}
\lim_{m \rightarrow \infty} \beta_M^{(m)} &= \lim_{m \rightarrow \infty} \Sigma_{MM}^{-1} \left( \Sigma_{Mv} - \Sigma_{\theta M}^T \widehat{A}_v^{(m)} \right) - \beta_D^{(m)} \Sigma_{MM}^{-1} \left( \Sigma_{Mu} - \Sigma_{\theta M}^T \widehat{A}_u^{(m)} \right) \\
&= \Sigma_{MM}^{-1} \left( \Sigma_{Mv} - \Sigma_{\theta M}^T \text{Var}(\theta|\theta_M)^{-1} \text{Cov}(\theta, v|\theta_M) \right) \\
&\quad - \text{Var}(u|\theta, \theta_M)^{-1} \text{Cov}(u, v|\theta, \theta_M) \Sigma_{MM}^{-1} \left( \Sigma_{Mu} - \Sigma_{\theta M}^T \text{Var}(\theta|\theta_M)^{-1} \text{Cov}(\theta, u|\theta_M) \right) \\
&= \text{Var}(\theta_M)^{-1} \text{Cov}(\theta_M, v) \\
&\quad - \text{Var}(\theta_M)^{-1} \text{Cov}(\theta, \theta_M)^T \text{Var}(\theta|\theta_M)^{-1} \text{Cov}(\theta, v|\theta_M) \\
&\quad - \text{Var}(u|\theta, \theta_M)^{-1} \text{Cov}(u, v|\theta, \theta_M) \text{Var}(\theta_M)^{-1} \text{Cov}(\theta_M, u) \\
&\quad + \text{Var}(u|\theta, \theta_M)^{-1} \text{Cov}(u, v|\theta, \theta_M) \text{Var}(\theta_M)^{-1} \text{Cov}(\theta, \theta_M)^T \text{Var}(\theta|\theta_M)^{-1} \text{Cov}(\theta, u|\theta_M) \\
&= w_M.
\end{aligned}$$

**Step 3, Case  $\text{Cov}(u, v|\theta, \theta_M) \leq 0$ .** In this case, note that

$$\begin{aligned}
E[v|\theta, \theta_M] &= E[v|\theta_M] \\
&\quad + \text{Cov}(v, \theta|\theta_M) \text{Var}(\theta|\theta_M)^{-1} (\theta - E[\theta|\theta_M]).
\end{aligned}$$

Thus,  $E[v|\theta, \theta_M]$  is a linear function of  $\theta$  and  $\theta_M$ :

$$E[v|\theta, \theta_M] = w_M^T \theta_M + w_\theta^T \theta,$$

where weights  $w$  are as follows:

$$\begin{aligned}
w_M^T &= \text{Cov}(v, \theta_M) \text{Var}(\theta_M)^{-1} \\
&\quad - \text{Cov}(v, \theta|\theta_M) \text{Var}(\theta|\theta_M)^{-1} \text{Cov}(\theta, \theta_M) \text{Var}(\theta_M)^{-1}; \\
w_\theta^T &= \text{Cov}(v, \theta|\theta_M) \text{Var}(\theta|\theta_M)^{-1}.
\end{aligned}$$

As before, price  $p^{(m)}(\theta, \theta_M, u)$  in market  $m$  can be expressed as

$$\begin{aligned}
p^{(m)}(\theta, \theta_M, u) &= \beta_M^{(m)T} \theta_M + \beta_D^{(m)} \left( \alpha^{(m)T} \theta + u \right) \\
&= \beta_M^{(m)T} \theta_M + \beta_D^{(m)} \alpha^{(m)T} \theta + \beta_D^{(m)} u.
\end{aligned}$$

To prove the statement of the theorem for this case, it is thus sufficient to show that as  $m$  grows,  $\beta_D^{(m)} \rightarrow 0$ ,  $\beta_D^{(m)} \alpha^{(m)} \rightarrow w_\theta$ , and  $\beta_M^{(m)} \rightarrow w_M$ .

The first convergence result,  $\beta_D^{(m)} \rightarrow 0$ , was proven at the end of Step 2 above.

Next,

$$\begin{aligned}
\lim_{m \rightarrow \infty} \beta_D^{(m)} \alpha^{(m)} &= \lim_{m \rightarrow \infty} \widehat{A}_v - \beta_D^{(m)} \widehat{A}_u \\
&= \text{Var}(\theta | \theta_M)^{-1} \text{Cov}(\theta, v | \theta_M) \\
&\quad - \left[ \lim_{m \rightarrow \infty} \beta_D^{(m)} \right] \text{Var}(\theta | \theta_M)^{-1} \text{Cov}(\theta, u | \theta_M) \\
&= \text{Var}(\theta | \theta_M)^{-1} \text{Cov}(\theta, v | \theta_M) \\
&= w_\theta.
\end{aligned}$$

Finally,

$$\begin{aligned}
\lim_{m \rightarrow \infty} \beta_M^{(m)} &= \lim_{m \rightarrow \infty} \Sigma_{MM}^{-1} \left( \Sigma_{Mv} - \Sigma_{\theta M}^T \widehat{A}_v^{(m)} \right) - \beta_D^{(m)} \Sigma_{MM}^{-1} \left( \Sigma_{Mu} - \Sigma_{\theta M}^T \widehat{A}_u^{(m)} \right) \\
&= \lim_{m \rightarrow \infty} \Sigma_{MM}^{-1} \left( \Sigma_{Mv} - \Sigma_{\theta M}^T \widehat{A}_v^{(m)} \right) \\
&= \text{Var}(\theta_M)^{-1} \text{Cov}(\theta_M, v) \\
&\quad - \text{Var}(\theta_M)^{-1} \text{Cov}(\theta, \theta_M)^T \text{Var}(\theta | \theta_M)^{-1} \text{Cov}(\theta, v | \theta_M) \\
&= w_M.
\end{aligned}$$

## Appendix C: Proof of Theorem 3

**Step 1.** In addition to the markets indexed  $m = 1, 2, \dots$ , we consider the alternative market which includes  $s$  groups of traders  $i = 1, \dots, s$ . The size of group  $i$  is  $\ell_i$  and each trader of group  $i$  receives signal  $\theta_i$ . In this alternative market, the market maker receives signal  $(\theta_L; \theta_M)$ . We use superscript  $(m)$  to refer to the variables in the market  $(m)$ , and superscript  $(alt)$  for the variables in the alternative market. We know from Theorem 1 that unique linear equilibria exist, both in the the sequence of markets, and in the alternative market.

As before,

$$\widehat{\Sigma}_{diag}^{(alt)} = \begin{pmatrix} \frac{1}{\ell_1} \Sigma_{11} & 0 & \cdots & 0 \\ 0 & \frac{1}{\ell_2} \Sigma_{22} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\ell_s} \Sigma_{ss} \end{pmatrix}$$

and

$$\widehat{\Sigma}_{diag}^{(m)} = \begin{pmatrix} \frac{1}{\ell_1^{(m)}} \Sigma_{11} & 0 & \cdots & 0 \\ 0 & \frac{1}{\ell_2^{(m)}} \Sigma_{22} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\ell_n^{(m)}} \Sigma_{nn} \end{pmatrix}.$$

Note that

$$\widehat{\Sigma}_{diag}^{(m)} \rightarrow \widehat{\Sigma}_{diag}^{(\infty)} := \begin{pmatrix} \widehat{\Sigma}_{diag}^{(alt)} & 0 \\ 0 & 0 \end{pmatrix}.$$



We could proceed by showing various convergence results directly, by matrix manipulation, as in the proof of the special case of Theorem 2 in Appendix B. However, it turns out that the proof becomes simpler and more intuitive if instead we follow the methodology of the proof of the general case of Theorem 2 in the online appendix, introduce auxiliary random variables, and interpret various matrices in the proof as covariance matrices of various combinations of these auxiliary random variables and the random variables in the model.

Specifically, for each market  $m$ , we introduce a random vector  $\widehat{\theta}^{(m)}$ , which is independent of the other random variables in the model, and is distributed normally with mean 0 and covariance matrix  $\widehat{\Sigma}_{diag}^{(m)}$ . We also introduce a random vector  $\widehat{\theta}_S$ , which is independent of the other random variables in the model, and is distributed normally with mean 0 and covariance matrix  $\widehat{\Sigma}_{diag}^{(alt)}$ . Finally, we introduce a random vector  $\widehat{\theta}^{(\infty)}$ , which is defined as  $\widehat{\theta}^{(\infty)} = (\widehat{\theta}_S; 0)$ , and is therefore distributed normally with mean 0 and covariance matrix  $\widehat{\Sigma}_{diag}^{(\infty)}$ .

First let us focus on the linear equilibrium in market  $(m)$ . We have

$$\begin{aligned}\widehat{\Lambda}^{(m)} &= \widehat{\Sigma}_{diag}^{(m)} + Var(\theta|\theta_M) = Var(\theta + \widehat{\theta}^{(m)}|\theta_M), \\ \widehat{A}_u^{(m)} &= (\widehat{\Lambda}^{(m)})^{-1}Cov(\theta, u|\theta_M), \\ \widehat{A}_v^{(m)} &= (\widehat{\Lambda}^{(m)})^{-1}Cov(\theta, v|\theta_M).\end{aligned}$$

Finding the linear equilibrium is equivalent to solving the quadratic equation

$$c^{(m)}(\beta_D^{(m)})^2 + b^{(m)}\beta_D^{(m)} + a^{(m)} = 0,$$

where

$$\begin{aligned}a^{(m)} &= -(\widehat{A}_v^{(m)})^T \widehat{\Sigma}_{diag}^{(m)} \widehat{A}_v^{(m)}, \\ b^{(m)} &= (\widehat{A}_v^{(m)})^T \left( 2\widehat{\Sigma}_{diag}^{(m)} + \widehat{\Lambda}^{(m)} \right) \widehat{A}_u^{(m)} - Cov(u, v|\theta_M), \\ c^{(m)} &= Var((\widehat{A}_v^{(m)})^T \theta - u|\theta_M).\end{aligned}$$

Similarly, there exists a unique linear equilibrium of the alternative market. Let

$$\begin{aligned}\widehat{\Lambda}^{(alt)} &= \widehat{\Sigma}_{diag}^{(alt)} + Var(\theta_S|\theta_M, \theta_L) = Var(\theta_S + \widehat{\theta}_S|\theta_M, \theta_L), \\ \widehat{A}_u^{(alt)} &= (\widehat{\Lambda}^{(alt)})^{-1}Cov(\theta_S, u|\theta_M, \theta_L), \\ \widehat{A}_v^{(alt)} &= (\widehat{\Lambda}^{(alt)})^{-1}Cov(\theta_S, v|\theta_M, \theta_L).\end{aligned}$$

Finding the linear equilibrium is equivalent to solving the quadratic equation

$$c^{(alt)}(\beta_D^{(alt)})^2 + b^{(alt)}\beta_D^{(alt)} + a^{(alt)} = 0,$$

where

$$\begin{aligned}a^{(alt)} &= -(\widehat{A}_v^{(alt)})^T \widehat{\Sigma}_{diag}^{(alt)} \widehat{A}_v^{(alt)}, \\ b^{(alt)} &= (\widehat{A}_v^{(alt)})^T \left( 2\widehat{\Sigma}_{diag}^{(alt)} + \widehat{\Lambda}^{(alt)} \right) \widehat{A}_u^{(alt)} - Cov(u, v|\theta_M, \theta_L), \\ c^{(alt)} &= Var((\widehat{A}_v^{(alt)})^T \theta - u|\theta_M, \theta_L).\end{aligned}$$

The equilibrium price in the market ( $m$ ) is

$$\begin{aligned} p^{(m)} &= (\beta_M^{(m)})^T \theta_M + \beta_D^{(m)} \left( (\alpha^{(m)})^T \theta^{(m)} + u \right) \\ &= (\beta_M^{(m)})^T \theta_M + \beta_D^{(m)} \left( (\alpha_S^{(m)})^T \theta_S^{(m)} + (\alpha_L^{(m)})^T \theta_L^{(m)} + u \right), \end{aligned}$$

where we “decompose” the vector of coefficients  $\alpha^{(m)}$  as  $\alpha^{(m)} = \left( \alpha_S^{(m)}; \alpha_L^{(m)} \right)$ .

The equilibrium price in the alternative market is

$$\begin{aligned} p^{(alt)} &= (\beta_M^{(alt)})^T \theta_M^{(alt)} + \beta_D^{(alt)} \left( (\alpha^{(alt)})^T \theta_S + u \right) \\ &= (\beta_{M,M}^{(alt)})^T \theta_M + (\beta_{M,L}^{(alt)})^T \theta_L + \beta_D^{(alt)} \left( (\alpha^{(alt)})^T \theta_S + u \right), \end{aligned}$$

where  $\theta_M^{(alt)} = (\theta_M; \theta_L)$  and  $\beta_M^{(alt)}$  is “decomposed” as  $\beta_M^{(alt)} = (\beta_{M,M}^{(alt)}; \beta_{M,L}^{(alt)})$ .

We will show in Step 2 that  $\beta_D^{(m)} \rightarrow \beta_D^{(alt)}$ , and then in Step 3 we will show that  $\beta_M^{(m)} \rightarrow \beta_{M,M}^{(alt)}$ ,  $\beta_D^{(m)} \alpha_L^{(m)} \rightarrow \beta_{M,L}^{(alt)}$ , and  $\beta_D^{(m)} \alpha_S^{(m)} \rightarrow \beta_D^{(alt)} \alpha^{(alt)}$ . By the same argument as in Step 3 of the proof of the special case of Theorem 2 in Appendix B, showing these four convergence results is sufficient to prove the statement of Theorem 3.

**Step 2.** First, we show that the coefficients of the quadratic equation that  $\beta_D^{(m)}$  satisfies converge to those of the quadratic equation that  $\beta_D^{(alt)}$  satisfies. As the coefficient on  $(\beta_D^{(alt)})^2$  in the latter equation is positive (as shown in Step 5 on the proof of Theorem 1 in Appendix A), this convergence implies that  $\beta_D^{(m)}$  converges to  $\beta_D^{(alt)}$ .

**Step 2(a).** We first show that  $a^{(m)} \rightarrow a^{(alt)}$ . We have

$$\widehat{\Sigma}_{diag}^{(m)} \rightarrow \widehat{\Sigma}_{diag}^{(\infty)} := Var((\widehat{\theta}_S; 0)),$$

thus

$$\widehat{\Lambda}^{(m)} \rightarrow \widehat{\Lambda}^{(\infty)} := Var((\theta_S + \widehat{\theta}_S; \theta_L) | \theta_M),$$

and, as  $\widehat{\Lambda}^{(\infty)}$  is positive definite (which follows from Assumption 2H),

$$\begin{aligned} \widehat{A}_v^{(m)} \rightarrow \widehat{A}_v^{(\infty)} &:= (\widehat{\Lambda}^{(\infty)})^{-1} Cov(\theta, v | \theta_M) \\ &= Var((\theta_S + \widehat{\theta}_S; \theta_L) | \theta_M)^{-1} Cov((\theta_S + \widehat{\theta}_S; \theta_L), v | \theta_M). \end{aligned}$$

This identity implies that for any (fixed) vectors  $\widetilde{\theta}_S$  (of the same dimension as random vectors  $\theta_S$  and  $\widehat{\theta}_S$ ) and  $\widetilde{\theta}_L$  (of the same dimension as random vector  $\theta_L$ ), we have

$$(\widehat{A}_v^{(\infty)})^T (\widetilde{\theta}_S; \widetilde{\theta}_L) = E[v | \theta_M = 0, \theta_S + \widehat{\theta}_S = \widetilde{\theta}_S, \theta_L = \widetilde{\theta}_L]. \quad (18)$$

Now, note that

$$\begin{aligned} a^{(m)} \rightarrow a^{(\infty)} &:= -(\widehat{A}_v^{(\infty)})^T \widehat{\Sigma}_{diag}^{(\infty)} \widehat{A}_v^{(\infty)} \\ &= -(\widehat{A}_v^{(\infty)})^T Var((\widehat{\theta}_S; 0)) \widehat{A}_v^{(\infty)} \\ &= -Var\left( (\widehat{A}_v^{(\infty)})^T (\widehat{\theta}_S; 0) \right). \end{aligned}$$

Likewise, for any (fixed) vector  $\tilde{\theta}_S$  (of the same dimension as  $\theta_S$  and  $\hat{\theta}_S$ ), we have

$$(\hat{A}_v^{(alt)})^T \tilde{\theta}_S = E[v | \theta_M = 0, \theta_S + \hat{\theta}_S = \tilde{\theta}_S, \theta_L = 0]. \quad (19)$$

Also,

$$\begin{aligned} a^{(alt)} &= -(\hat{A}_v^{(alt)})^T \hat{\Sigma}_{diag}^{(alt)} \hat{A}_v^{(alt)} \\ &= -(\hat{A}_v^{(alt)})^T Var(\hat{\theta}_S) \hat{A}_v^{(alt)} \\ &= -Var\left((\hat{A}_v^{(alt)})^T \hat{\theta}_S\right). \end{aligned}$$

Equations (18) and (19) imply that for every realization  $\tilde{\theta}_S$  of random vector  $\hat{\theta}_S$ ,

$$\begin{aligned} (\hat{A}_v^{(\infty)})^T(\tilde{\theta}_S; 0) &= E[v | \theta_M = 0, \theta_S + \hat{\theta}_S = \tilde{\theta}_S, \theta_L = 0] \\ &= (\hat{A}_v^{(alt)})^T \tilde{\theta}_S, \end{aligned}$$

and thus

$$Var\left((\hat{A}_v^{(\infty)})^T(\tilde{\theta}_S; 0)\right) = Var\left((\hat{A}_v^{(alt)})^T \tilde{\theta}_S\right)$$

and so  $a^{(m)} \rightarrow a^{(\infty)} = a^{(alt)}$ .

**Step 2(b).** Next, we show that  $b^{(m)} \rightarrow b^{(alt)}$ . In the limit,

$$b^{(m)} \rightarrow b^{(\infty)} := (\hat{A}_v^{(\infty)})^T \left(2\hat{\Sigma}_{diag}^{(\infty)} + \hat{\Lambda}^{(\infty)}\right) \hat{A}_u^{(\infty)} - Cov(u, v | \theta_M),$$

where

$$\hat{A}_u^{(\infty)} := \lim_{m \rightarrow \infty} \hat{A}_u^{(m)} = (\hat{\Lambda}^{(\infty)})^{-1} Cov(\theta, u | \theta_M).$$

Similarly to equations (18) and (19) above, for any fixed vectors  $\tilde{\theta}_S$  and  $\tilde{\theta}_L$ , we have

$$(\hat{A}_u^{(\infty)})^T(\tilde{\theta}_S; \tilde{\theta}_L) = E[u | \theta_M = 0, \theta_S + \hat{\theta}_S = \tilde{\theta}_S, \theta_L = \tilde{\theta}_L]; \quad (20)$$

$$(\hat{A}_u^{(alt)})^T \tilde{\theta}_S = E[u | \theta_M = 0, \theta_S + \hat{\theta}_S = \tilde{\theta}_S, \theta_L = 0]. \quad (21)$$

Note that

$$\begin{aligned} (\hat{A}_v^{(\infty)})^T \hat{\Sigma}_{diag}^{(\infty)} \hat{A}_u^{(\infty)} &= (\hat{A}_v^{(\infty)})^T Var((\hat{\theta}_S; 0)) \hat{A}_u^{(\infty)} \\ &= Cov\left((\hat{A}_v^{(\infty)})^T(\tilde{\theta}_S; 0), (\hat{A}_u^{(\infty)})^T(\tilde{\theta}_S; 0)\right) \end{aligned}$$

and

$$\begin{aligned} (\hat{A}_v^{(alt)})^T \hat{\Sigma}_{diag}^{(alt)} \hat{A}_u^{(alt)} &= (\hat{A}_v^{(alt)})^T Var(\hat{\theta}_S) \hat{A}_u^{(alt)} \\ &= Cov\left((\hat{A}_v^{(alt)})^T \tilde{\theta}_S, (\hat{A}_u^{(alt)})^T \tilde{\theta}_S\right). \end{aligned}$$

By equations (18)–(21), for any realization  $\tilde{\theta}_S$  of random vector  $\hat{\theta}_S$ ,

$$\begin{aligned} (\hat{A}_v^{(\infty)})^T(\tilde{\theta}_S; 0) &= (\hat{A}_v^{(alt)})^T \tilde{\theta}_S \quad \text{and} \\ (\hat{A}_u^{(\infty)})^T(\tilde{\theta}_S; 0) &= (\hat{A}_u^{(alt)})^T \tilde{\theta}_S, \end{aligned}$$

and so

$$(\widehat{A}_v^{(\infty)})^T \widehat{\Sigma}_{diag}^{(\infty)} \widehat{A}_u^{(\infty)} = (\widehat{A}_v^{(alt)})^T \widehat{\Sigma}_{diag}^{(alt)} \widehat{A}_u^{(alt)}.$$

Next, note that

$$(\widehat{A}_v^{(\infty)})^T \widehat{\Lambda}^{(\infty)} \widehat{A}_u^{(\infty)} = Cov(v, (\theta_S + \widehat{\theta}_S; \theta_L) | \theta_M) [Var((\theta_S + \widehat{\theta}_S; \theta_L) | \theta_M)]^{-1} Cov((\theta_S + \widehat{\theta}_S; \theta_L), u | \theta_M)$$

and so

$$(\widehat{A}_v^{(\infty)})^T \widehat{\Lambda}^{(\infty)} \widehat{A}_u^{(\infty)} - Cov(u, v | \theta_M) = -Cov(u, v | \theta_M, \theta_L, \theta_S + \widehat{\theta}_S).$$

Similarly,

$$(\widehat{A}_v^{(alt)})^T \widehat{\Lambda}^{(alt)} \widehat{A}_u^{(alt)} = Cov(v, \theta_S + \widehat{\theta}_S | \theta_M, \theta_L) [Var(\theta_S + \widehat{\theta}_S | \theta_M, \theta_L)]^{-1} Cov(\theta_S + \widehat{\theta}_S, u | \theta_M, \theta_L),$$

and so

$$(\widehat{A}_v^{(alt)})^T \widehat{\Lambda}^{(alt)} \widehat{A}_u^{(alt)} - Cov(u, v | \theta_M, \theta_L) = -Cov(u, v | \theta_M, \theta_L, \theta_S + \widehat{\theta}_S).$$

Therefore, we have

$$\begin{aligned} b^{(m)} \rightarrow b^{(\infty)} &= 2(\widehat{A}_v^{(\infty)})^T \widehat{\Sigma}_{diag}^{(\infty)} \widehat{A}_u^{(\infty)} + \left( (\widehat{A}_v^{(\infty)})^T \widehat{\Lambda}^{(\infty)} \widehat{A}_u^{(\infty)} - Cov(u, v | \theta_M) \right) \\ &= 2(\widehat{A}_v^{(alt)})^T \widehat{\Sigma}_{diag}^{(alt)} \widehat{A}_u^{(alt)} - Cov(u, v | \theta_M, \theta_L, \theta_S + \widehat{\theta}_S) \\ &= b^{(alt)}. \end{aligned}$$

**Step 2(c).** Finally, we show that  $c^{(m)} \rightarrow c^{(alt)}$ . We have

$$c^{(m)} \rightarrow c^{(\infty)} := Var((\widehat{A}_u^{(\infty)})^T \theta - u | \theta_M)$$

and

$$c^{(alt)} = Var((\widehat{A}_u^{(alt)})^T \theta_S - u | \theta_M, \theta_L).$$

Let random variable  $\chi$  be the residual from the projection of  $u$  on  $(\theta_S + \widehat{\theta}_S; \theta_L; \theta_M)$ . By construction,  $\chi$  is orthogonal to  $\theta_L$  and  $\theta_M$  and thus, by the properties of the normal distribution, is independent of those two random variables. Recall that  $\widehat{\theta}_S$  was also chosen to be independent of  $\theta_L$  and  $\theta_M$ .

Next,

$$\begin{aligned} Var((\widehat{A}_u^{(\infty)})^T \theta - u | \theta_M) &= Var\left(u - (\widehat{A}_u^{(\infty)})^T (\theta_S + \widehat{\theta}_S; \theta_L) + (\widehat{A}_u^{(\infty)})^T (\widehat{\theta}_S; 0) | \theta_M\right) \\ &= Var\left(\chi + (\widehat{A}_u^{(\infty)})^T (\widehat{\theta}_S; 0) | \theta_M\right) \end{aligned}$$

and

$$\begin{aligned} Var((\widehat{A}_u^{(alt)})^T \theta_S - u | \theta_M, \theta_L) &= Var\left(u - (\widehat{A}_u^{(alt)})^T (\theta_S + \widehat{\theta}_S) + (\widehat{A}_u^{(alt)})^T \widehat{\theta}_S | \theta_M, \theta_L\right) \\ &= Var\left(\chi + (\widehat{A}_u^{(alt)})^T \widehat{\theta}_S | \theta_M, \theta_L\right). \end{aligned}$$

Since  $\chi$  and  $\widehat{\theta}_S$  are both independent of  $\theta_M$  and  $\theta_L$ , we have

$$\text{Var} \left( \chi + (\widehat{A}_u^{(\infty)})^T (\widehat{\theta}_S; 0) | \theta_M \right) = \text{Var} \left( \chi + (\widehat{A}_u^{(\infty)})^T (\widehat{\theta}_S; 0) \right)$$

and

$$\text{Var} \left( \chi + (\widehat{A}_u^{(alt)})^T \widehat{\theta}_S | \theta_M, \theta_L \right) = \text{Var} \left( \chi + (\widehat{A}_u^{(alt)})^T \widehat{\theta}_S \right).$$

Take any realizations  $\widetilde{\chi}$  and  $\widetilde{\theta}_S$  of random variables  $\chi$  and  $\widehat{\theta}_S$ . From equations (20) and (21) in Step 2(b), we have

$$\begin{aligned} \widetilde{\chi} + (\widehat{A}_u^{(\infty)})^T (\widetilde{\theta}_S; 0) &= \widetilde{\chi} + E[u | \theta_M = 0, \theta_S + \widehat{\theta}_S = \widetilde{\theta}_S, \theta_L = 0] \\ &= \widetilde{\chi} + (\widehat{A}_u^{(alt)})^T \widetilde{\theta}_S, \end{aligned}$$

and so

$$\text{Var} \left( \chi + (\widehat{A}_u^{(\infty)})^T (\widehat{\theta}_S; 0) \right) = \text{Var} \left( \chi + (\widehat{A}_u^{(alt)})^T \widehat{\theta}_S \right)$$

and thus

$$c^{(m)} \rightarrow c^{(\infty)} = c^{(alt)}.$$

**Step 3.** We now show that  $\beta_M^{(m)} \rightarrow \beta_{M,M}^{(alt)}$ ,  $\beta_D^{(m)} \alpha_L^{(m)} \rightarrow \beta_{M,L}^{(alt)}$ , and  $\beta_D^{(m)} \alpha_S^{(m)} \rightarrow \beta_D^{(alt)} \alpha^{(alt)}$ . The arguments below rely on Assumption 2H, which implies that various conditional expectations that we compute below are guaranteed to be well-defined. They also rely on the result we showed in the previous step,  $\beta_D^{(\infty)} = \beta_D^{(alt)}$ .

First, note that for any  $\widetilde{\theta}_S$ ,  $(\alpha_S^{(m)})^T \widetilde{\theta}_S = (\alpha^{(m)})^T (\widetilde{\theta}_S; 0)$ , and so

$$\begin{aligned} \lim_{m \rightarrow \infty} \beta_D^{(m)} (\alpha^{(m)})^T (\widetilde{\theta}_S; 0) &= \beta_D^{(\infty)} \left( (\widehat{A}_v^{(\infty)})^T / \beta_D^{(\infty)} - (\widehat{A}_u^{(\infty)})^T \right) (\widetilde{\theta}_S; 0) \\ &= E[v - \beta_D^{(\infty)} u | \theta_M = 0, \theta_S + \widehat{\theta}_S = \widetilde{\theta}_S, \theta_L = 0] \\ &= \beta_D^{(alt)} (\alpha_S^{(alt)})^T \widetilde{\theta}_S. \end{aligned}$$

Thus,  $\beta_D^{(m)} \alpha_S^{(m)} \rightarrow \beta_D^{(alt)} \alpha^{(alt)}$ .

Next, we have

$$\beta_M^{(m)} \rightarrow \beta_M^{(\infty)} := \Sigma_{MM}^{-1} \left( \Sigma_{Mv} - \Sigma_{\theta M}^T A_v^{(\infty)} \right) - \beta_D^{(\infty)} \Sigma_{MM}^{-1} \left( \Sigma_{Mu} - \Sigma_{\theta M}^T A_u^{(\infty)} \right),$$

and so for any  $\widetilde{\theta}_M$ , we have

$$(\beta_M^{(\infty)})^T \widetilde{\theta}_M = E[v - \beta_D^{(\infty)} u | \theta_M = \widetilde{\theta}_M, \theta_S + \widehat{\theta}_S = 0, \theta_L = 0].$$

Also, similarly to the above expression for  $\beta_D^{(\infty)} \alpha_S^{(\infty)}$ , for any  $\widetilde{\theta}_L$ , we have

$$\beta_D^{(\infty)} (\alpha_L^{(\infty)})^T \widetilde{\theta}_L = E[v - \beta_D^{(\infty)} u | \theta_M = 0, \theta_S + \widehat{\theta}_S = 0, \theta_L = \widetilde{\theta}_L].$$

Thus,

$$(\beta_M^{(\infty)}; \beta_D^{(\infty)} \alpha_L^{(\infty)})^T (\widetilde{\theta}_M; \widetilde{\theta}_L) = E[v - \beta_D^{(\infty)} u | \theta_M = \widetilde{\theta}_M, \theta_S + \widehat{\theta}_S = 0, \theta_L = \widetilde{\theta}_L].$$

Analogously to the expression for  $(\beta_M^{(\infty)})^T \tilde{\theta}_M$ , we also have

$$\begin{aligned} (\beta_M^{(alt)})^T (\tilde{\theta}_M; \tilde{\theta}_L) &= E[v - \beta_D^{(alt)} u | \theta_S + \hat{\theta}_S = 0, (\theta_M; \theta_L) = (\tilde{\theta}_M; \tilde{\theta}_L)]. \\ &= (\beta_M^{(\infty)}; \beta_D^{(\infty)} \alpha_L^{(\infty)})^T (\tilde{\theta}_M; \tilde{\theta}_L). \end{aligned}$$

Thus,  $\beta_M^{(m)} \rightarrow \beta_{M,M}^{(alt)}$  and  $\beta_D^{(m)} \alpha_L^{(m)} \rightarrow \beta_{M,L}^{(alt)}$ , and combining all the convergence results above and using the same argument as in Step 3 of the proof of the special case of Theorem 2 in Appendix B, we conclude the proof of Theorem 3.

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