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ABSTRACT

We develop a tractable model of banks' liquidity management and the credit channel of monetary policy. Banks finance loans by issuing demand deposits. Loans are illiquid, and transfers of deposits across banks must be settled with reserves in a frictional over the counter market. To mitigate the risk of large withdrawals of deposits, banks hold a precautionary buffer of reserves and government bonds. We show how monetary policy affects the banking system by altering the tradeoff between profiting from lending and incurring greater liquidity risk. We consider two applications of the theory, one involving the connection between the implementation of monetary policy and pass-through to loan rates, and another considering a quantitative decomposition behind the collapse in bank lending during the 2008 financial crisis. Our analysis underscores the importance of liquidity frictions and the functioning of interbank markets for the conduct of monetary policy.

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1 Introduction

The transmission and implementation of monetary policy operates through the banking system. In practice, central banks set a target for the interbank market rate and implement that target via open market operations and standing facilities. The ultimate goal is to affect the amount of credit and thus overall economic activity. It is therefore of paramount importance to understand how monetary policy affects the interbank market and, in turn, how the interbank market affects the real economy.

The leading macroeconomic framework used for monetary policy analysis, the New Keynesian model, abstracts from the implementation and transmission of monetary policy through the interbank market. In the New Monetarist framework, interactions between money and credit are explicit, but the interbank market and its impact on credit have received little attention.[1] Moreover, for the most part, the focus of analysis has been on a sole policy instrument, either a nominal interest rate or the growth in the monetary base. During the 2008 financial crisis, and more recently during the Covid-19 crisis, disruptions in interbank markets were met with a broad set of policy responses designed to inject liquidity into the financial system and mitigate contractions in credit. These events call for a model that can be used to analyze the effects of frictions in the interbank market and the transmission of monetary policy through the banking system.

This paper provides a tractable general equilibrium model with a banking system that articulates a notion of the credit channel of monetary policy. At the heart of the theory lies a liquidity management problem that emerges from frictions in the interbank market. Liquidity management concerns the trade-off between holding high-yield illiquid loans and holding low-yield liquid assets. It is by influencing this trade-off that monetary policy affects the supply of credit and gives rise to a credit channel. In support of this transmission channel, we document a strong empirical relationship between measures of disturbances in the interbank market and liquidity premia. We put this framework to work in two quantitative applications that showcase the importance of examining the transmission of monetary policy through the banking system.

In the theory, banks are competitive. Their portfolio is composed of deposits, loans, government bonds, and reserves. When a bank grants a loan and simultaneously issues deposits, it gains intermediation profits. However, deposits circulate in an unpredictable way, and thus banks face deposit withdrawal shocks. When a deposit is transferred out of a bank, another bank absorbs that liability. As occurs in practice, that transfer is settled with reserves.[2] If a

[1] For a textbook treatment of the NK model, see Woodford (2004) or Galí (2015). New Monetarist models are surveyed in Williamson and Wright (2010), and Lagos, Rocheteau, and Wright (2017).

[2] The Federal Reserve Wire Network (Fedwire) is the real-time gross settlement (RTGS) funds transfer system that electronically settles funds between any of the United States banks registered in the Federal Reserve System, totalling approximately 10,000 banks. The amount of funds transferred daily is approximately USD 3.30 trillion.

deposit withdrawal is too large, the bank will end short of reserves. The bank can sell bonds in exchange for reserves, but this may not be enough. At that point, the bank must incur in expensive borrowing of reserves, either from the discount window at a penalty rate, or from the interbank market.The interbank market is over the counter (OTC). The probability of finding a counterpart in the interbank market depends on the scarcity of reserves: when few banks have reserve surpluses, the interbank market rate is high—and hence a shortage is expensive. Thus, the efficiency and tightness of the interbank market affect the degree of liquidity risk. By holding a large buffer of liquid assets composed of bonds and reserves, a bank reduces its exposure to liquidity risk, at the expense of intermediation profits. Tilting this trade-off, monetary policy affects the supply of bank credit by affecting liquidity premia.

From a methodological standpoint, a contribution of the paper is to integrate an OTC interbank market into a dynamic general equilibrium model of the banking system. The interbank market here is modeled after Afonso and Lagos (2015), who study the Federal funds market in a repeated OTC setting and deliver predictions for the intraday volume of interbank market loans and the distribution of interbank rates. That model takes the distribution of reserve balances as a primitive. Here, the distribution of balances is endogenous, as it results from banks' optimally chosen portfolios, which are in turn influenced by monetary policy. We furthermore show how to encode the effects of the liquidity frictions into a single object: a liquidity yield function. Despite the non-linear nature of the liquidity yield function, the bank's problem features aggregation, and thus the economy features a representative bank. Being analytically tractable, this makes the analysis transparent and amenable to various applications, both theoretical and quantitative.

The analysis of the transmission of monetary policy through the banking system reveals several insights. In contrast to models in which the demand for reserves is exogenously determined by reserve requirements, monetary policy here affects the risk-return trade-off between holding reserves and loans. The central bank alters this trade-off through open market operations, both conventional and unconventional, and by setting the rate of interest on reserves and discount window. We show that although the composition between bonds and reserves is indeterminate for an individual bank, the composition matters at the macro level. This result has two implications. First, it implies that the total holdings of liquid assets, and not only the holdings of reserves, is the correct measure of the precautionary liquidity demand. Second, a policy that swaps government bonds for reserves has aggregate effects by altering the interbank market tightness and the liquidity premium. Moreover, by absorbing illiquid assets into the central bank's balance sheet, unconventional open market operations have even more potent effects. At the limit, when the interbank bank market shuts down entirely, only unconventional open market operations remain effective.

A central insight of the paper is to show how the implementation of monetary policy matters for macroeconomic outcomes. We first study how the pass-through from the interest on reserves

to credit is potentially non-monotone and depends critically on the interaction with capital requirements. When the interest on reserves is low, deposits are in effect more costly and capital requirements do not bind. As the interest on reserves increases, banks expand deposits, reserves, and potentially credit. Once capital requirements bind, further increases in the interest on reserves necessarily contract lending. The analysis reveals that reserves can be complements to or substitutes for bank lending, depending on whether capital requirement constraints bind. We then study how the interest on reserves and the size of the central bank's balance sheet are *independent* instruments, and we show how different configurations that achieve the same interbank market rate target affect the level of credit and the pass-through. In particular, configurations with a larger balance sheet induce a larger supply of credit, as well as a higher pass-through from the interbank market rate to the loan rate.

A final contribution is to employ the framework to quantitatively examine the credit crunch during the US financial crisis. In particular, we examine the role of aggravated liquidity conditions, as evidenced by the severe collapse in the interbank market and the increase in discount window borrowing. We devise a procedure to reverse engineer the shocks that are required to match the data, and then feed the model with alternative shock sequences to produce counterfactuals. We find that disruptions to the matching efficiency of the interbank market and to the volatility of funding played a substantial role around the time of the Lehman Brothers bankruptcy. By 2010, loan demand becomes the dominant factor. Turning to the role of policy, we study the contribution of conventional and unconventional open market operations to mitigating the credit crunch. We find that conventional operations had a negligible effect, while unconventional ones had a sizable impact. In line with the theoretical insights highlighted above, the quantitative analysis suggests that the move toward unconventional open market operations during the crisis was critical for the attenuation of the credit crunch.

Related Literature. Our paper relates to several branches of the literature in monetary economics, banking, and macroeconomics. One branch studies monetary policy implementation through banks' reserve management in partial equilibrium. Seminal papers in this area are Poole (1968) and Frost (1971), and many applications are found in Bindseil (2014). Building on these works, several influential studies have analyzed recent proposed changes in monetary policy frameworks (Ennis and Weinberg, 2007; Keister, Martin, and McAndrews, 2008; Keister and McAndrews, 2009; Ennis and Keister, 2008; Martin et al., 2013; Bech and Keister, 2017). There are two important differences in our approach. First, the interbank market here is OTC, enabling us to study disruptions the interbank market that cannot be analyzed in a Walrasian setup. Second, we embed the interbank market into a general equilibrium model. This feature allows us to analyze credit, prices, and interest rates in a unified framework.

The paper also builds on the banking literature. Important examples include Diamond (1984);

Diamond and Dybvig (1983); Boyd and Prescott (1986); Allen and Gale (1998); Holmstrom and Tirole (1998); and Gu et al. (2013). With some exceptions, for the most part, these theories have evolved separately from macroeconomics. Gertler and Karadi (2011) and Curdia and Woodford (2009) incorporate a banking sector into quantitative monetary models. Following these models, a growing literature now studies how shocks to bank equity or leverage constraints disrupt financial intermediation.[3] A distinct approach is taken by Corbae and D'Erasmo (2013, 2018), who provide a model with heterogenous banks and analyze the role of bank concentration and how it interacts with capital requirements. The present paper emphasizes interbank market frictions and the transmission of monetary policy through the liquidity premium.

The OTC nature of the interbank market builds on monetary search theory. Seminal contributions in this literature are Kiyotaki and Wright (1989) and Lagos and Wright (2005). The interbank market here is a version of the OTC model developed by Afonso and Lagos (2015).[4] Related studies also include Freeman (1996) and Smith (2002), who study environments where inside money is used as a medium of exchange, as a result of spatial frictions. Williamson (2012) studies an environment in which assets of different maturity have different properties as mediums of exchange. Relative to this earlier work, we have little to say about the foundations that bring about a banking architecture. Here, the focus is on the effects of trading frictions in the interbank market and the transmission of monetary policy through the credit channel.

Finally, following the framework developed in this paper, several recent studies have analyzed the link between liquidity management and monetary policy, focusing on the effects on asset prices (Piazzesi and Schneider, 2018), collateral values (De Fiore, Hoerova, and Uhlig 2018), shadow banking (Chen, Ren, and Zha 2017), nominal rigidities (Arce, Nuño, Thaler, and Thomas 2019; Piazzesi, Rogers, and Schneider 2019), insurance and productive efficiency (Bigio and Sannikov, 2019), and exchange rates (Bianchi, Bigio, and Engel 2019).

Outline. The paper is organized as follows. Section 2 presents the model, and Section 3 provides theoretical results. Section 4 presents evidence on the correlation between interbank-market spreads and the liquidity premium. Section 5 presents the calibration of the model and the applications. Section 6 concludes. All proofs are in the appendix.

[3] Gertler and Kiyotaki (2010) and Gertler, Kiyotaki, and Prestipino (2016) present benchmark models and references to many other papers in the literature. This literature also builds to a large extent on a broader literature on financial frictions in firms and how they affect macroeconomic fluctuations. Notable examples include Bernanke and Gertler (1989) and Kiyotaki and Moore (1997)—and the literature's related microfoundations in Townsend (1979) and Williamson (1987).

[4] See Weill (2020) for a survey of the recent OTC literature and Rocheteau and Nosal (2017) for a textbook on monetary search theory.

2 The Model

We present a dynamic general equilibrium model of the banking system featuring an OTC interbank market and analyze the transmission of monetary policy. The presentation of the model begins with the liquidity management problem of an individual bank, followed by the description of the interbank market. We then introduce the non-financial block of the model, describing households and firms, and analyze the policies of the central bank, which we refer to as the *Fed*. After characterizing the problems of all agents, we define the general equilibrium.

2.1 Banks: Preferences and Budgets

Preferences. There is a unit-mass continuum of heterogeneous banks indexed by j and a final consumption good. Banks' preferences over a stochastic stream of dividend payments $\{c_t^j\}$ are given by

$$\mathbb{E}_0 \sum_{t \geq 0} \beta^t u(c_t^j), \tag{1}$$

where $\beta < 1$ is the time discount factor, and $u(c) \equiv \frac{c^{1-\gamma}-1}{1-\gamma}$ is the utility function over the consumption good with $\gamma \geq 0$.[5]

Timing. Time is discrete, indexed by t, and of infinite horizon. Each period is divided into two stages: a lending (l) and a balancing (b) stage. At the lending stage, banks make portfolio decisions. At the balancing stage, banks experience random idiosyncratic withdrawals of deposits. A deposit withdrawn from one bank is transferred to another bank. That transaction must be settled with reserves. If banks lack reserves to settle that transaction, they can sell government bonds, borrow reserves from other banks, or borrow reserves at a penalty rate from the Fed. We describe next the two stages—a summary of the timeline of events is found in Figure 8 in Appendix B.

Lending stage. Banks enter the lending stage with a portfolio of assets/liabilities and collect/make associated interest payments. Among assets, banks hold loans, b_t, and liquid assets in the form of reserves, m_t, or government bonds, g_t. On the liability side, banks issue demand deposits, d_t, discount window loans, w_t, and net interbank loans, f_t (if the bank has borrowed funds, f_t is positive, and vice versa). All assets are nominal (denominated in units of reserves).[6] Reserves are the numeraire and P_t is the price level.

[5] Concave preferences generate slow-moving bank equity, as observed in practice. A rationale for endowing banks with preferences is undiversified ownership or frictions that limit equity funding.

[6] Absent aggregate shocks, having assets denominated in units of reserves or denominated in units of consumption are equivalent.

During the lending stage, banks choose real dividends, c_t, and a portfolio. The portfolio is a choice $\{\tilde{b}^j_{t+1}, \tilde{m}^j_{t+1}, \tilde{g}^j_{t+1}, \tilde{d}^j_{t+1}\}$, which corresponds to holdings of loans, reserves, government bonds, and deposits, respectively. We use $\tilde{x}_{t+1}$ to denote a portfolio variable chosen in the lending stage and x_{t+1} to denote the end-of-period portfolio variable in the balancing stage (and the beginning-of-period portfolio variable for $t+1$). Aggregate holdings are denoted in uppercase letters, for example, $B_{t+1} \equiv \int_j b^j_{t+1} dj$. The bank's budget constraint in the lending stage is

$$P_t c^j_t + \tilde{b}^j_{t+1} + \tilde{m}^j_{t+1} - \tilde{d}^j_{t+1} + \tilde{g}^j_{t+1} = \\ (1 + i^b_t) b^j_t + (1 + i^m_t) m^j_t + (1 + i^g_t) g^j_t - (1 + i^d_t) d^j_t - (1 + \bar{i}^f_t) f^j_t - (1 + i^w_t)\, w^j_t - P_t T^j_t, \quad (2)$$

where i^b_t, i^g_t and i^d_t, denote the nominal returns on loans, government bonds, and deposits, respectively. The policy rates i^m_t and i^w_t are interest on reserves and discount window loans set by the Fed. These rates must satisfy $i^w_t \geq i^m_t$; otherwise, there is a pure arbitrage to the detriment of the Fed. Outstanding interbank market loans earn $\bar{i}^f_t$, which we label the *fed funds rate*. The fed funds rate is the average rate at which banks borrow in the interbank market, a market described below. All rates indexed with t are accrued between period $t-1$ and t. The term T^j_t are taxes proportional to bank equity.

Banks are subject to a capital requirement constraint

$$\tilde{d}^j_{t+1} \leq \kappa \left(\tilde{b}^j_{t+1} + \tilde{g}^j_{t+1} + \tilde{m}^j_{t+1} - \tilde{d}^j_{t+1} \right). \quad (3)$$

The upper bound κ on leverage can be motivated by regulation or agency frictions.

The problem of the bank in the lending stage is to choose the portfolio and dividend payments, subject to the budget constraint (2) and the capital requirement (3).

Balancing stage. Banks enter the balancing stage with $\{\tilde{b}^j_{t+1}, \tilde{m}^j_{t+1}, \tilde{g}^j_{t+1}, \tilde{d}^j_{t+1}\}$. At the start of the balancing stage, banks experience an idiosyncratic withdrawal shock ω^j_t. The shock induces a random inflow/withdrawal of deposits $\omega^j_t \tilde{d}^j_{t+1}$. Given this shock, the end-of-balancing-stage deposits, d^j_{t+1}, are

$$d^j_{t+1} = \tilde{d}^j_{t+1}(1 + \omega^j_t). \quad (4)$$

When ω^j_t is positive, the bank receives (net) deposits from other banks. When ω is negative, the bank loses deposits. The ω shock has a cumulative distribution $\Phi(\cdot)$ common to all banks. The support of Φ is $[\omega_{\min}, \infty)$, where $\omega_{\min} \geq -1$ and $\int_{\omega_{\min}}^{\infty} \omega_t \Phi_t(\omega) = 0$, $\forall t$. This distribution implies that deposits are reshuffled but preserved within banks.

The randomness of ω captures the unpredictability and complexity of the payment system. The circulation of deposits is a fundamental feature of the payments system, because it enables banks to facilitate transactions between third parties: When a bank issues a loan, a borrower is

credited with deposits. As the borrower makes payments to third parties, deposits are transferred to other banks. The outflow of a deposit from one bank is an inflow to another. Because the receptor bank absorbs a liability, an asset also must be transferred to settle the transaction. As it occurs in practice, reserve balances at the Fed are the settlement instrument.[7] We adopt the convention that the bank that issues deposits pays for the interest on those deposits, and thus a transfer of one unit of deposits is settled with $\left(1+i_{t+1}^{d}\right)/\left(1+i_{t+1}^{m}\right)$ reserves. This guarantees that the bank that receives the deposit is compensated for the interest on the absorbed deposits.

By the end of the balancing stage, banks must maintain a minimum reserve balance,

$$m_{t+1}^{j} \geq \rho d_{t+1}^{j}, \quad \rho \in [0,1] \tag{5}$$

If a bank receives a large withdrawal, it must raise reserves to be able to satisfy (5).[8] While loans are assumed to be illiquid, banks can exchange government bonds for reserves in a Walrasian market at the beginning of the balancing stage.[9] After trade takes place in that market, if banks are still in deficit, they borrow reserves in an OTC interbank market or from the discount window.[10] By the opening of the interbank market, the surplus (or deficit) of reserves is:

$$s_t^j \equiv \underbrace{\left(\tilde{m}_{t+1}^{j}+\left(\frac{1+i_{t+1}^{d}}{1+i_{t+1}^{m}}\right)\omega_t^j \tilde{d}_{t+1}^{j}\right)}_{\text{reserve balance after } \omega \text{ shock}} - \underbrace{\rho\tilde{d}_{t+1}^{j}(1+\omega_t^j)}_{\text{required reserves after } \omega \text{ shock}} + \underbrace{(\tilde{g}_{t+1}^{j}-g_{t+1}^{j})}_{\text{sales of gov. bonds}} \tag{6}$$

The excess balance of reserves at the Fed, (6), is determined by the portfolio chosen in the lending stage, the withdrawal shock ω_t^j, and the trading of government bonds . The first term is the end-of period reserve position brought from the lending stage plus/minus the reserves transferred after the withdrawal. The second term is the required reserves. The third term is the change in reserves accounted for by the trade in government bonds.[11] The reserves with which the bank ends the period are given by the reserves left after the withdrawal shocks, the sales of government

[7] We do not explicitly model why reserves are used as the settlement instrument . For possible microfoundations, we refer the reader to Cavalcanti, Erosa, and Temzelides (1999) or Lester, Postlewaite, and Wright (2012).

[8] A requirement that reserves have to be positive is non-essential. What matters is that there is a lower bound. An alternative regulation, a liquidity coverage ratio, imposes a minimum amount of liquid assets relative to illiquid assets.

[9] An earlier version of the paper considered a framework with a single liquid asset. That framework is nested in the current one if we set the supply of government bonds to zero, or if we assume that government bonds can also be used for settlements and pay the same return as reserves. In the latter, a conventional open market operation would be irrelevant as in Wallace (1981).

[10] The lack of a liquid market for loans during the balancing stage can be explained by informational frictions. Reserves are instead special assets issued by the central bank and are fully liquid. The market for government bonds is assumed to be Walrasian because it is a market for a homogeneous asset and easy to enforce. The OTC nature of the interbank market is the empirically relevant assumption, as advocated by Ashcraft and Duffie (2007), and in line with the bilateral and unsecured nature of the transaction. We take these features as given and do not provide explicit microfoundations.

[11] Implicit in the accounting is a result that shows that the price of government bonds must equal unity in the balancing stage in equilibrium.

bonds, and the loans obtained in the interbank market and from the discount window.

$$m_{t+1}^j = \tilde{m}_{t+1}^j + \left(\frac{1+i_{t+1}^d}{1+i_{t+1}^m}\right)\omega_t^j \tilde{d}_{t+1}^j + (\tilde{g}_{t+1}^j - g_{t+1}^j) \ + f_{t+1}^j \ + w_{t+1}^j. \tag{7}$$

The rate at which banks trade in the interbank market will be key to determine banks' portfolios in the lending stage. Below, we analyze how the rate and the volume in the interbank market are determined.

Interbank market. Withdrawal shocks generate a distribution of reserve surpluses and deficits across banks. When the interbank market opens, banks with a surplus want to lend, and banks with deficit want to borrow. Because of the matching frictions, banks on either side of the market may be unable to lend/borrow all of their balances. If a bank in deficit cannot obtain enough funds in the interbank market, it must borrow the remainder from the discount window. If a bank in surplus is unable to lend all of its surplus, it deposits the balance at the Fed and earns the interest on reserves.[12] In equilibrium, because interbank rates lie between the interest rates on reserves and discount loans, banks will seek to trade in the interbank market before trading with the Fed. All loans are repaid by the next lending stage.

The interbank market is an OTC search market. We follow closely the basic formulation in Afonso and Lagos (2015) but make some departures that render analytic solutions and allows us to embed this friction into the dynamic model. A complete description of the interbank market is intensive in notation, so here we provide only a brief description. Details can be found in the companion paper (Bianchi and Bigio, 2017).

The interbank market operates sequentially. At the beginning of the trading session, each bank gives an order to a continuum of traders. If $s_t^j > 0$ ($s_t^j < 0$), the bank gives an order to lend (borrow). Each trader must close an infinitesimal position, as in Atkeson, Eisfeldt, and Weill (2015). This "large family" assumption simplifies the solution of the bargaining problem by making the marginal value of the interbank loan depend only on the sign of the balance, and not on the scale. Absent this assumption, it becomes necessary to keep track of the identity of matching banks in their bargaining problems. The resulting combinatorial problem of determining the distribution of matches is intractable. Our approach, which combines Afonso and Lagos (2015) and Atkeson et al. (2015) allows us to obtain analytic expressions for all of the objects.

There are N trading rounds. The probability of a match at a given round depends on the aggregate amount of surplus and deficit positions that remain open at each round. We consider a Leontief matching function with efficiency parameter λ. When traders meet, they bargain over

[12] Notice that we assume that the interest on required reserves is equal to the interest on excess reserves. It is possible, however, to allow for different rates. For the applications, we will set $\rho = 0$, and as a result, only the interest on excess reserves will be the relevant one.

the rate and split the surplus according to Nash bargaining. Key for the determination of the interbank market rate at any given round, are the rates and probabilities of finding a match in future rounds. Let us define the interbank market tightness at the opening of the interbank market as

$$\theta_t \equiv S_t^- / S_t^+$$

where $S_t^+ \equiv \int_0^1 \max\left\{s_t^j, 0\right\} dj$ and $S_t^- \equiv -\int_0^1 \min\left\{s_t^j, 0\right\} dj$ denote the aggregate surplus and deficit, respectively.[13] The following proposition characterizes the split between interbank market and discount window loans, $\{f^j, w^j\}$, as a function of θ as we take the limit of N rounds to infinity (while keeping the overall number of matches at a given balancing stage constant):

Proposition 1. *Given θ, the amount of interbank market loans and discount window loans for a bank of surplus s_t^j is*

$$(f_{t+1}^j, w_{t+1}^j) = \begin{cases} -s_t^j \cdot (\Psi_t^-(\theta), 1 - \Psi_t^-(\theta)) & \text{for } s_t^j < 0 \\ -s_t^j \cdot (\Psi_t^+(\theta), 0) & \text{for } s_t^j \geq 0 \end{cases}, \tag{8}$$

and the average interbank market rate is $\bar{i}_t^f(\theta) = i_t^m + (1 - \phi_t(\theta))(i_t^w - i_t^m)$. Analytic expressions for $\{\Psi_t^+(\theta), \Psi_t^-(\theta), \phi_t(\theta)\}$ are found in Appendix A.

A bank short of reserves ($s_t^j < 0$) patches a fraction Ψ_t^- of its deficit with interbank loans and the remaining $1 - \Psi_t^-$ with a discount window loan from the Fed. Similarly, a bank with surplus lends a fraction Ψ_t^+ in the interbank market and keeps the remaining balance, $1 - \Psi_t^+$, at the Fed. These fractions are endogenous objects that depend on market tightness. If many banks are in deficit (surplus), the probability that a deficit bank finds a match is low (high). Market clearing in the interbank market requires $\Psi_t^+(\theta_t) S_t^+ = -\Psi_t^-(\theta_t) S_t^-$. We say that the interbank market is active if $\Psi_t^+(\theta_t) S_t^+ > 0$ and inactive otherwise.

Proposition 1 also characterizes the mean interbank market, $\bar{i}_t^f(\theta)$, as a function of the market tightness. The fed funds rate is a weighted average of the corridor rates i_t^m and i_t^w. The weight, given by $\phi_t(\theta)$, is an endogenous bargaining power, as in Afonso and Lagos (2015). If many banks are in deficit, the fed funds rate is closer to i^w, because this lowers the outside option and the bargaining power of banks in deficit. Conversely, the fed funds rate is closer to i^m if more banks are in surplus.[14]

[13] Notice that market tightness vary within the balancing stage, as trading rounds are carried out.

[14] The interbank market rate has actually traded below the interest on reserves for a large part of the post-crisis period. This suggests a violation of arbitrage: a depository institution, in principle, could borrow in the interbank market and lend to the Fed at a higher rate. An explanation for this pattern is related to the presence of non-depositosry institutions and costs from leverage and deposit insurance premiums (Williamson, 2019; Martin et al., 2013; Armenter and Lester, 2017). To keep the model parsimonious, we abstract from these issues for now, but we address this in Section 5.3 when we turn to calibrate the model.

As shown in Appendix A, the functional forms for $\phi(\theta_t)$ and $\{\Psi^-(\theta_t), \Psi^+(\theta_t)\}$, which determine the volume and the fed funds rate, depend on two structural parameters: the matching efficiency λ and the bargaining power, η. In particular, for given θ, a higher efficiency leads to higher fractions of matches $\{\Psi^-, \Psi^+\}$, and a higher η increases the effective bargaining power of banks in deficit, lowering the fed funds rate.

A single function, which we call liquidity yield function, encodes all the activity in the interbank market and simplifies the recursive bank problem.

Definition 1. The liquidity yield function is

$$\begin{aligned} \chi_t(s) &= \begin{cases} \chi_t^+ s & \text{if } s \geq 0, \\ \chi_t^- s & \text{if } s < 0, \end{cases} \qquad (9) \\ \text{where } \chi_t^- &= \Psi_t^-(\bar{i}_t^f - i_t^m) + \left(1 - \Psi_t^-\right)(i_t^w - i_t^m), \text{ and } \chi_t^+ = \Psi_t^+\left(\bar{i}_t^f - i_t^m\right). \end{aligned}$$

Here, χ_t represents the yield earned on a reserve surplus (when $s \geq 0$) and paid on a reserve deficit (when $s < 0$). The kink on χ generates a positive wedge between the marginal cost of reserve deficits and the marginal benefit of surpluses. Notice that a bank that borrows from the interbank market or from the discount window can hold reserves at the Fed and earn an interest. As a result, the net cost of borrowing will be given by the difference between the borrowing rate and the interest on reserves, as reflected in the formula for χ^-. Leveraging on Proposition 1, the liquidity yield function admits a closed-form solution, which allows us to conveniently embed it in the dynamic recursive problem of the bank.

It is useful to define the following real returns. We let $R_t^x \equiv \left(1 + i_{t+1}^x\right)/(1 + \pi_{t+1})$ be the gross returns on asset $x \in \{w, m, g, b, d\}$ and let $1 + \pi_{t+1} \equiv P_{t+1}/P_t$ be the gross inflation rate. We also define the liquidity cost function in real terms as a function of the portfolio as $\bar{\chi}_{t+1}(\tilde{m}, \tilde{g}, \tilde{d}, \omega) \equiv \chi_{t+1}(s_t)/(1 + \pi_{t+1})$.

2.2 Non-Financial Sector and General Equilibrium

The non-financial block is presented in detail in Appendix C. This block is composed of households that supply labor and save in deposits, currency, and government bonds. Firms produce the final consumption good using labor and are subject to a working capital constraints. This block delivers endogenous demand schedules for working capital loans, and households' holdings of deposits, government bonds, and currency. These household schedules emerge from asset-in-advance constraints, as in Lucas and Stokey (1983). We purposefully work with quasi-linear preferences, as in Lagos and Wright (2005), so that these schedules are not forward-looking. The schedules asset-demand system for the non-financial block are derived in Appendix and summarized by the proposition below:

Proposition 2. *Given the non-financial sector block presented in Appendix C, we have that:(i) The firm loan demand is* $\frac{B^f_{t+1}}{P_t} = \Theta^b_t \left(R^b_{t+1}\right)^{\epsilon^b}$ and *output is* $y_{t+1} = \Theta^y_t \left(R^b_{t+1}\right)^{\epsilon^y}$*; (ii) Households deposits and government bond holdings are given by*

$$\frac{X^h_{t+1}}{P_t} = \begin{cases} \Theta \left(R^x_{t+1}\right)^{\epsilon^x} & \text{if } R^x_{t+1} \leq 1/\beta^h, \\ [\bar{X}_t, \infty) & \text{if } R^x_{t+1} = 1/\beta^h \quad \text{for } x \in \{d, g\}, \\ \infty & \text{otherwise.} \end{cases}$$

(iii) Households currency demand is

$$\frac{M^h_{t+1}}{P_t} = \begin{cases} \Theta^m_t \left((1+\pi_{t+1})^{-1}\right)^{\epsilon^x} & \text{if } (1+\pi_{t+1})^{-1} \leq 1/\beta^h, \\ [\bar{M}_t, \infty) & \text{if } (1+\pi_{t+1})^{-1} = 1/\beta^h, \\ \infty & \text{otherwise.} \end{cases}$$

The term β^h is the household's discount factor. The household demand schedules are iso-elastic up to some the point where quantities reach scale parameters, $\bar{X}_t$ for $x \in \{m, d, g\}$, which correspond to asset-satiation points. The parameters ϵ^x are elasticity coefficients, and Θ^x_t scale coefficients, which depend on structural parameters regarding technology and households' preferences (see Table 4 in Appendix C for the conversion from the structural to the reduced form parameters).

A convenient property of the model is that once we solve for the equilibrium real rates, equating the asset supply and demand scheduled derived from banks and the reduced form schedules obtained from non-financial sector, we can obtain the equilibrium output, employment, and consumption of different goods. It is therefore by affecting the lending rate that monetary policy affects output and employment in the model. For the rest of the paper, we do not make further references to non-financial block and work directly with the iso-elastic portion of these schedules—there always exists a β^h that guarantees that this is the case.

2.3 Monetary and Fiscal Authority

The Fed's policy tools are the discount window rate, the interest on reserves and open market operations (conventional and unconventional). $\tilde{M}^{Fed}$ and M^{Fed} denote the money supply issued by the Fed during the lending and balancing stages, respectively. On the asset side, the Fed holds discount window loans, W^{Fed}, private loans, B^{Fed}, and government bonds, G^{Fed}. Government bonds are issued by the fiscal authority, which we denote by G^{FA}. The supply of Fed liabilities can be held as currency by households or as bank reserves—$M^{Fed} = M + M^h$.

Here, we present the consolidated government budget constraint. Appendix D.1 presents the

corresponding constraints of the Fed and the fiscal authority. Consolidating the revenues/expenses of the Fed and the fiscal authority, we have that

$$(1+i_t^m)M_t + M_t^h + B_{t+1}^{Fed} - \left(G_{t+1}^{FA} - G_{t+1}^{Fed}\right) + W_{t+1}^{Fed} = \\ M_{t+1}^{Fed} + (1+i_t^b)B_t^{Fed} - (1+i_t^g)\left(G_t^{FA} - G_t^{Fed}\right) + (1+i_t^w)W_t^{Fed} + P_t(T_t + T_t^h). \quad (10)$$

The equation captures that the consolidated government generates operating profits/losses by paying interest on government bonds (net of Fed holdings), reserves (but not on currency) and collecting interest on discount window loans and private sector loans. Given these net revenues and the evolution of its balance sheet, the government sets taxes on households and banks to balance the budget constraint.

We adopt the following protocol for taxes on banks:

$$T_t = (i_t^m - \pi_t)\frac{M_t}{P_t} + (i_t^g - \pi_t)\frac{G_t}{P_t} - \left(i_t^b - \pi_t\right)\frac{B_t^{Fed}}{P_t} - (i_t^w - i_t^m)\frac{W_t}{P_t}. \quad (11)$$

That is, the Fed taxes banks to finance the real interest on their holdings of reserves and government bonds and rebates the real interest income on its loans holdings and its operating revenues from discount window. With this tax protocol, as we will see, the law of motion for aggregate bank equity will depend exclusively on total loans and deposits, and their rates of return, allowing us to isolate the credit channel. To balance the budget, taxes on households T_t^h are set as a residual, in the spirit of passive fiscal policy.

2.4 Competitive Equilibrium

We adopt the convention of denoting aggregate bank variables by their corresponding capital letter—e.g. the aggregate holdings of loans is denoted by $B_t = \int b_t^j dj$. The competitive equilibrium is defined as follows:[15]

Definition 2. Given a distribution $\{d_0^j, m_0^j, b_0^j, g_0^j, f_0^j, w_0^j\}$ and a deterministic sequence of government policies $\left\{G_t^{Fed}, G_t^{FA}, B_t^{Fed}, M_t^{Fed}, W_t^{Fed}, T_t, T_t^h, i_t^m, i_t^w, i_t^g\right\}_{t\geq 0}$, a **competitive equilibrium** is a deterministic path for aggregates $\left\{D_{t+1}, B_{t+1}, W_{t+1}, M_{t+1}, G_{t+1}, G_{t+1}^h, D_{t+1}^h M_{t+1}^h\right\}$, a stochastic sequence of bank policies $\left\{\tilde{g}_{t+1}^j, \tilde{m}_{t+1}^j, \tilde{d}_{t+1}^j, b_{t+1}^j, c_t^j, f_{t+1}^j, w_{t+1}^j, m_{t+1}^j\right\}_{t\geq 0}$, a deterministic sequence of interest rates $\left\{i_t^b, i_t^d, \bar{i}_t^f\right\}_{t\geq 0}$, a deterministic sequence for the price level $\{P_t\}$, and a deterministic sequence of matching probabilities $\{\Psi_t^+, \Psi_t^-\}$, such that

[15] As noted earlier, we do not make reference to the household sector only indirectly through the demand for loans and supply of deposits. Appendix C covers the equilibrium conditions that follow from firms' and households' problems, which in equilibrium give rise to the loan demand and deposit supply schedules. The mathematical representation of the market clearing conditions is presented in full detail in Appendix B, together with a summary of the equilibrium conditions of the model.

(i) bank policies solve the banks' optimization problems, and $\{f_{t+1}^j, w_{t+1}^j\}_{t\geq 0}$ are given by Proposition 1;

(ii) the government's budget constraint (10) is satisfied and the tax on banks follow (11);

(iii) households and firms are on their supply/demand schedules, as given by Proposition 2;

(iv) markets for deposits, loans and government assets (M and G) clear $\forall t \geq 0$;

(v) the matching probabilities $\left\{\Psi_t^+, \Psi_t^-\right\}_{t\geq 0}$ and the fed funds rate $\bar{i}_t^f$ are consistent with the surplus and deficit masses S_t^- and S_t^+, as given by Proposition 1.

We refer to a competitive equilibrium where the value of all nominal assets and the price level P_t grow at a constant rate as a *stationary equilibrium*. A steady-state equilibrium is a stationary competitive equilibrium in which the growth rate is zero.

3 Theoretical Analysis

Next, we proceed with the analysis. We first show that the bank's portfolio problem aggregates and can be reduced to two choices: leverage and liquidity. We then provide an aggregation result and study the monetary-policy transmission.

3.1 Recursive Bank Problems

Denote by V_t^l and V_t^b the bank value functions during the lending and balancing stages, respectively. To keep track of aggregate states, which follow a deterministic path, we index the policy and value functions by t. To ease notation, we omit the individual superscript j and suppress the time subscripts inside the Bellman equations.

At the beginning of each lending stage, the individual states are $\{g, b, m, d, f, w\}$. Choices in the lending stage are consumption, c, and portfolio variables $\{\tilde{b}, \tilde{g}, \tilde{m}, \tilde{d}\}$. These portfolio variables together with the idiosyncratic shock, ω become the initial states in the balancing stage. The continuation value is the expected value of the balancing stage V_t^b under the probability distribution of ω. We have the following bank problem in the lending stage

Problem 1 (Lending-Stage Bank Problem)**.**

$$V_t^l(g, b, m, d, f, w) = \max_{\left\{c, \tilde{b}, \tilde{d}, \tilde{m}, \tilde{g}\right\}\geq 0} u(c) + \mathbb{E}\left[V_t^b(\tilde{g}, \tilde{b}, \tilde{m}, \tilde{d}, \omega)\right] \tag{12}$$

subject to budget constraint (2) and capital requirement (3).

In turn, the balancing-stage problem is

Problem 2 (Balancing Stage Bank Problem)**.**

$$V_t^b(\tilde{g},\tilde{b},\tilde{m},\tilde{d},\omega) = \max_{g'\geq 0} \beta V_t^l(g',b',m',d',f',w') \tag{13}$$

$$b' = \tilde{b} \qquad \text{(Evolution of Loans)}$$

$$d' = \tilde{d} + \omega\tilde{d} \qquad \text{(Evolution of Deposits)}$$

$$m' = \tilde{m} + \left(\frac{1+i_{t+1}^d}{1+i_{t+1}^m}\right)\omega\tilde{d} + f' + w' + \tilde{g} - g' \qquad \text{(Evolution of Reserves)}$$

$$s = \tilde{m} + \left(\frac{1+i_{t+1}^d}{1+i_{t+1}^m}\right)\omega\tilde{d} - (1+\omega)\,\rho\tilde{d} + (\tilde{g} - g') \qquad \text{(Reserve surplus)}$$

$$(f',w') = \begin{cases} -s(\Psi_t^-, 1-\Psi_t^-) & \text{for } s<0 \\ -s(\Psi_t^+, 0) & \text{for } s\geq 0. \end{cases} \qquad \text{(Interbank Market Transactions)}$$

In Problem 2, the bank chooses its purchase (sales) of government bonds after the withdrawal shock, which in turn induces the balance position at the interbank market, s. In equilibrium, we can show that as long as the supply of government bonds held by banks in deficit does not exceed the surplus of reserves by banks in surplus, banks in deficit will sell their entire holdings of government bonds to banks in surplus. When this is the case, an interbank market remains active, as $S^+ > 0$ and $S^- > 0$. Because the government bond market is Walrasian, this implies that in equilibrium $R^g = R^m + \chi^+$. If the return on government bonds were higher, banks in surplus would bid up the price until the return falls. If the return on government bonds were lower, banks would rather preserve their reserves and lend it in the interbank market rather than purchase government bonds. For the rest of the paper, we consider only this case, but develop results in greater generality in the appendix.[16]

Toward a characterization, let us define a bank's real equity as

$$e_t \equiv \frac{(1+i_t^m)m_t + \left(1+i_t^b\right)b_t - \left(1+i_t^d\right)d_t + (1+i_t^g)\,g_t - (1+\bar{i}_t^f)f_t - (1+i_t^w)\,w_t}{P_t}(1-\tau_t), \tag{14}$$

where τ is a linear tax on bank equity tax rate (i.e., $T^j = \tau \cdot e_t^j$). Next, we present a characterization of the bank's problem.

Proposition 3 (Homogeneity and Portfolio Separation)**.** *The bank's problem has the following features:*

[16] In Appendix E , we also examine the case in which the initial amount of reserve surplus is not enough to purchase the bonds of banks in deficit. In that case, case banks in deficit do not sell their entire stock of bonds. Specific conditions for either case in terms of initial asset holdings are presented in the appendix.

(i) Problems 1 and 2 can be, combined into a single Bellman equation with equity as the only individual state variable, and the holdings of government bonds and reserves can be consolidated into a single liquid asset $\tilde{a} \equiv \tilde{m} + \tilde{g}$,

$$V_t(e) = \max_{\left\{c,\tilde{a},\tilde{b},\tilde{d}\right\}\geq 0,\tilde{d}\in[0,\kappa]} u(c) + \beta \mathbb{E}\left[V_{t+1}(e')\right], \text{ subject to} \tag{15}$$

$$\frac{\tilde{a}}{P_t} + \frac{\tilde{b}}{P_t} - \frac{\tilde{d}}{P_t} + c = e$$

$$e' = \left[R^b_{t+1}\frac{\tilde{b}}{P_t} + R^m_{t+1}\frac{\tilde{a}}{P_t} - R^d_{t+1}\frac{\tilde{d}}{P_t} + \bar{\chi}_{t+1}\left(\frac{\tilde{a}}{P_t}, \frac{\tilde{d}}{P_t}, \omega\right)\right](1-\tau_{t+1}). \tag{16}$$

(ii) The optimal portfolio in (15) is given by the solution to

$$\Omega \equiv (1-\tau_t) \max_{\left\{\bar{b},\bar{a},\bar{d}\right\}\geq 0} \left\{\mathbb{E}\left[R^b_{t+1}\bar{b} + R^m_{t+1}\bar{a} - R^d_{t+1}\bar{d} + \bar{\chi}_{t+1}(\bar{a},\bar{d},\omega)\right]^{1-\gamma}\right\}^{\frac{1}{1-\gamma}}, \tag{17}$$

$$\bar{b} + \bar{a} - \bar{d} = 1, \text{ and } \bar{d} \leq \kappa.$$

(iii) The optimal bank dividend–equity ratio $\bar{c} \equiv c/e$ *is*

$$\bar{c}_t = \frac{1}{1 + \left[\beta(1-\gamma)v_{t+1}{\Omega_t}^{1-\gamma}\right]^{1/\gamma}} \text{ where } v_t = \frac{1}{1-\gamma}\left[1 + \left(\beta(1-\gamma)^{1-\gamma}\Omega_t^{1-\gamma}v_{t+1}\right)^{\frac{1}{\gamma}}\right]^{\gamma}, \tag{18}$$

and the value of (15) is $V_t(e) = v_t\,(e)^{1-\gamma} - 1/(1-\beta)(1-\gamma)$.

(iv) Policy functions $\tilde{x} \in \{\tilde{b}, \tilde{a}, \tilde{d}\}$ *from (15) can be recovered from the optimal portfolio weights* $\tilde{x} \in \left\{\bar{b}, \bar{a}, \bar{d}\right\}$ *obtained in (17) and consumption decisions* $\{\bar{c}\}$ *obtained in (18) via the relationship* $\tilde{x}_{t+1}(e_t) = \bar{x}_t(1-\bar{c}_t)P_t e_t$. *The individual holdings* $\{\tilde{m}_{t+1}, \tilde{g}_{t+1}\}$ *satisfy* $\tilde{m}_{t+1} + \tilde{g}_{t+1} = \tilde{a}_{t+1}$.

There are four elements in Proposition 3. Item (i) shows that we can synthesize the value functions in (12) and (13) into a single Bellman equation with real equity as a single state. The liquidity yield function, χ, shows in this Bellman equation summarizing parsimoniously all the trades in the interbank market. Equation (15) is, in effect, a portfolio savings problem with a leverage constraint: the bank starts with equity, e, which is distributed between dividends or portfolio investments. Investments can be allocated into loans, $\tilde{b}$, and liquid assets, $\tilde{a}$—the sum of reserves and government bonds is determined, but not the individual composition. The bank can lever by issuing deposits $\tilde{d}$. The continuation value of the bank depends on next period equity e', which in turn depends on the portfolio return. The proposition establishes that although there is a distribution of bank equity, all banks are replicas of a representative bank: item (ii) indicates that banks choose the same portfolio weights, which solves an auxiliary problem; item (iii) shows

that all banks feature the same dividend rate; item (iv) shows that banks' portfolio investments are linear in equity.

We highlight two key takeaways. First, the model aggregates. While aggregation is known to hold under linear budget constraints and homothetic preferences, a contribution here is to show that aggregation also holds despite a kink in the return function. This showcases how to integrate search frictions into a standard dynamic model with a representative agent.

As shown in Appendix D.2, this aggregation result allows us to express the real aggregate equity law of motion as

$$E_{t+1} = \left(1 + \left(R^b_{t+1} - 1\right)\left(\bar{b}_t + \bar{b}^{Fed}_t\right) - \left(R^d_{t+1} - 1\right)\bar{d}_t\right) E_t(1 - \bar{c}_t), \tag{19}$$

where $\bar{b}^{Fed}_t \equiv B^{Fed}_{t+1} / \left(P_t \cdot (1 - \bar{c}_t) E_t\right)$. This equation says that next-period aggregate equity is given by the current aggregate equity net of dividend payments times the aggregate return. The aggregate return simply depends on the portfolio weights on loans and deposits chosen by the representative bank and their respective returns. Implicit in this result is that (i) the returns on interbank market loans cancel out on aggregate; (ii) operating Fed profits that originate from the discount window or holding loans, and interest on government bonds are compensated by taxes.

The second takeaway from Proposition 3 is that at the individual level, the composition between reserves and government bonds is indeterminate. Key to this result is that there is a Walrasian market between reserves and government bonds that allows banks to freely reverse any portfolio mix between reserves and government bonds once they face a withdrawal shock. This is different for loans, which stay with the bank, and therefore the portfolio mix matters.

Despite the fact that banks are individually indifferent regarding the composition of their liquid assets, the composition of liquid assets matters at the aggregate level because it affects the interbank market tightness θ_t. To obtain the equilibrium composition between reserves and government bonds, we obtain $\bar{g}$ as the residual of the market clearing condition,

$$P_t \bar{g}_t (1 - \bar{c}_t) E_t = G^{FA}_{t+1} - G^{Fed}_{t+1} - G^h_{t+1}. \tag{20}$$

The weight on reserves $\bar{m}$ is obtained via $\bar{m} = \bar{a} - \bar{g}$. A corollary of this result is that the cutoff for the withdrawal shock that determines whether banks are in deficit or surplus depends on their ratio of total liquid assets to deposits:

$$\omega^* \equiv -\left(\bar{a}/\bar{d} - \rho\right) / \left(R^d_{t+1}/R^m_{t+1} - \rho\right). \tag{21}$$

With this expression, we can solve for market tightness and all the relevant objects in the interbank-market. As shown in Proposition E.6 given in Appendix E, the interbank market

tightness can be expressed as

$$\theta_t = \frac{\int_{-1}^{\omega^*} \left(\bar{a} + \left(\frac{R^d_{t+1}}{R^m_{t+1}} \right) \omega \bar{d} - \rho \bar{d}(1+\omega) \right) f(\omega)\, d\omega}{\int_{\omega^*}^{1} \left(\bar{a} + \left(\frac{R^d_{t+1}}{R^m_{t+1}} \right) \omega \bar{d} - \rho \bar{d}(1+\omega) \right) f(\omega)\, d\omega - \bar{g}}. \tag{22}$$

Hence, for fixed liquid asset holdings $\bar{a}$, tightness increases with a greater portion of $\bar{g}$. This result is an important lesson. It tells us that whereas reserves and government bonds are perfect substitutes at the individual bank level, at a macroeconomic level, the composition of liquid assets matters.

Discussion on aggregation property. Thanks to this aggregation property, the model provides a sharp characterization of the bank liquidity management. This renders a transparent analysis of the transmission of monetary policy. In addition, a notable advantage is that the model is straightforward to compute, as aggregate equity is the single state, and this makes the model amenable to different extensions. The model, however, is not designed to address three important features that emerge in quantitative models with an endogenous size distribution, as in notable work by Corbae and D'Erasmo (2018). First, the model is not suited to analyze heterogeneous responses to monetary policy, a feature documented in Kashyap and Stein (2000). Second, the model is not suitable to study the effects of size-dependent policies such as capital requirements or changes in policies that give rise to changes in concentration. Third, the model does not speak to market power and changes in bank concentration. While these are important aspects, we think there is much to gain from a benchmark banking model that aggregates into a representative bank while featuring a rich liquidity management problem, in addition to being modular and easy to compute.

3.2 Liquidity Management and Liquidity Premia

As outlined in Proposition 3 (item ii), a bank's portfolio problem is to choose portfolio weights on loans $\bar{b}$, total liquid assets $\bar{a}$, and deposits $\bar{d}$ to maximize the certainty equivalent of the bank's return on equity:

$$R^e \equiv R^b \bar{b} + \ R^m \bar{a} - R^d \bar{d} + \bar{\chi}(\bar{a}, \bar{d}, \omega).$$

Using the first-order conditions, we obtain the following relationship between the returns of all assets and liabilities.

Proposition 4 (Liquidity Premiums)**.** *Let* $\{\bar{a}, \bar{d}, \bar{b}\} > 0$ *be a solution to the problem in item (ii) of Proposition 3. Then, we obtain the following equilibrium expressions for the liquidity premia*

(LP) on loans, government bonds and deposits:

$$R^b - R^m = \bar{\chi}^+ + \left(\bar{\chi}^- - \bar{\chi}^+\right) \cdot \underbrace{F\left(\omega^*\right)}_{\text{deficit prob}} \cdot \underbrace{\frac{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma} | \omega < \omega^*\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]}}_{\text{risk-aversion correction}} \tag{Loan LP}$$

$$R^g - R^m = \bar{\chi}^+ \tag{Gov. Bond LP}$$

$$R^d - R^m = (1+\rho)\left(R^b - R^m\right) + \left(\frac{R^d}{R^m} - \rho\right) \underbrace{\frac{\mathbb{COV}_\omega\left[\left(R^e\right)^{-\gamma} \cdot \bar{\chi}, \omega\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]}}_{\text{risk premium}} - \mu \tag{Deposit LP}$$

where $\mu \cdot \left(\kappa - \bar{d}\right) = 0$. *Furthermore,* $R^w \geq R^b \geq R^g \geq R^m$. *The inequalities become equalities only if either* $\theta_t = 0$ *or* $R^w = R^m$.

Proposition 4 displays the LP of each asset relative to reserves when the portfolio solution is interior—when $\bar{a} > 0$.[17] Consider the loan LP. Loans command a higher direct return than reserves, $R^b \geq R^m$, because reserves also yield an expected return in the interbank market: if the bank ends in surplus, a marginal reserve is lent out at an average of $\bar{\chi}^+$; in deficit, the marginal reserve has a additional value of $\chi^- - \chi^+$, the additional borrowing cost. The risk-adjusted probability of a deficit state is the physical probability $F\left(\omega^*\right)$ times an appropriate correction for risk-aversion. The LP of loans is thus the risk-adjusted interbank market return.

The gov. bond LP is also positive but lower than the premium on loans.[18] In a deficit state, a bank that holds a government bond sells it and saves the spread χ^-. The bank therefore obtains $R^m + \chi^-$ the next period, which is the same as the return in a deficit state. By arbitrage, it follows that the return on a surplus state must be equalized. In a surplus state, the bank is indifferent about trading government bonds: the government bond yields R^g, and reserves yield $R^m + \chi^+$. Arbitrage requires $R^g = R^m + \chi^+$. This positive premium reflects how payments clear with reserves but not with government bonds.[19]

We say that banks *are satiated* if the loan and bond liquidity premia are zero. The Fed can induce satiation in two ways: by eliminating the spread in its policy rates, $i_t^w = i_t^m$, or supplying reserves such that all banks end in surplus after any shock, $s_t > 0$. In either case, there is no role for the interbank market and we have $\bar{i_t}^f = i_t^m$.

[17] By convention, we use the expectation operator $\mathbb{E}_\omega$ in this condition excludes the zero-measure point where $\partial\bar{\chi}\left(\bar{m}, \bar{d}, \omega\right)/\partial\bar{d}$ is not defined, the point $\omega = \omega^*$. Since $\omega = \omega^*$ has zero measure, we obtain can take first-order condition by partitioning the bank's objective into the regions $(\omega < \omega^*)$ and $(\omega > \omega^*)$. Within those regions, the bank's objective is smooth.

[18] In a generalization of Proposition 4 in Appendix E, it can occur that $R^g < R^m$ and banks are at a corner with $\tilde{g} = 0$.

[19] The model can be extended to include assets with intermediate liquidity. For example, we can introduce assets for which only a fraction of their value can be traded at the balancing stage. It is straightforward to show that the LP of these semi-liquid asset would be a weighted average of the LP of government bonds and the LP of loans.

Finally, Proposition 3 displays the deposit LP, which can be of either sign. The deposit LP has three terms: The first term captures the expected change in the surplus, considering the reserve requirement—the effect is proportional to the LP of loans because withdrawals are mean zero, and is therefore positive. The second term is a liquidity-risk premium, which captures that an increase in deposits raises liquidity risk. The risk premium is present even if banks are risk neutral because the concavity in $\bar{\chi}$ produces endogenous risk-aversion. The final term, μ, is a non-negative Lagrange multiplier on the capital requirement constraint.

Role of OTC frictions. The decomposition of the LP of loans, bonds, and deposit clarifies the fundamental role of OTC frictions for the transmission of monetary policy. As we take the efficiency parameter $\lambda \to \infty$, we recover a Walrasian interbank market along the lines of the Poole (1968) model.[20] To understand the differences, Figure 1 presents a comparison for the coefficients of χ_t and the fed funds rate as a function of the log inverse of market tightness θ, for the frictional OTC market (left panel) and the Walrasian limit (right panel). In a Walrasian market, the costs of deficits equals the benefits of a surplus, $\chi_t^+ = \chi_t^-$, and thus $\bar{i}_t^f = i_t^m + \chi_t^+$. Furthermore, in Walrasian market, there are only two relevant regimes: either the banking system has excess reserves and $\bar{i}_t^f = i_t^m$ or a deficit of reserves and $\bar{i}_t^f = i_t^w$.

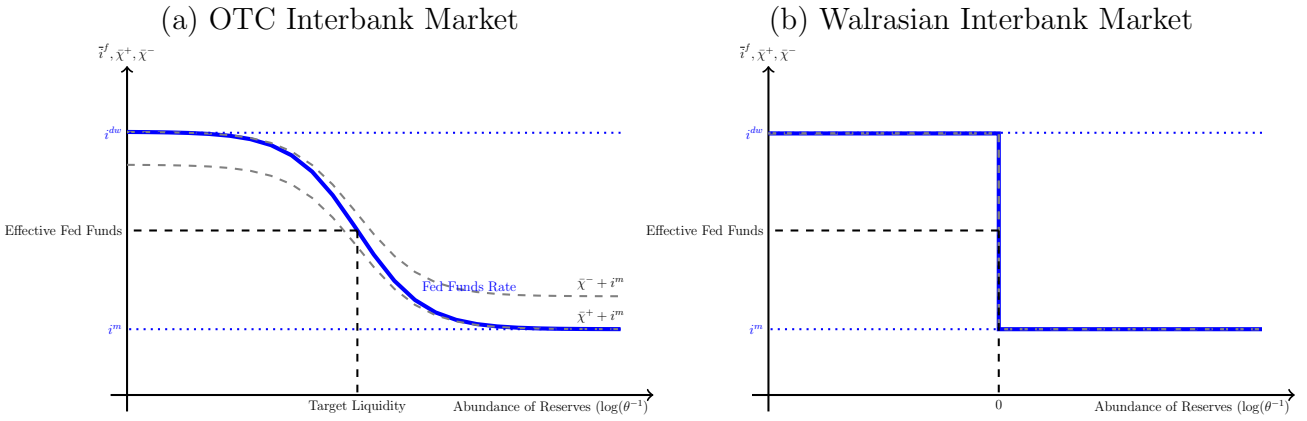

Figure 1: OTC versus Walrasian Markets

These observations showcase that a Walrasian interbank market produces counterfactual implications: First, it implies a fed funds rate that trades either at the floor or the ceiling of the corridor—unless the aggregate reserve balance is exactly zero, in which case the rate on reserves would be indeterminate.[21] Second, it produces the same liquidity premium for loans and government bonds, which would be either zero or $i^w - i^m$ depending on the sign of aggregate excess

[20] See the companion paper Bianchi and Bigio (2017) for a derivation of the Walrasian limit ($\lambda \to \infty$) of the interbank market.

[21] Burdett, Shi, and Wright (2001) also emphasize the importance of search frictions in smoothing market outcomes.

reserves. This means that open market operations operations, or changes in withdrawal risk, would not have any effects unless aggregate excess reserves change sign. Indeed, there would be no effects in economies with zero reserve requirements, $\rho = 0$, since in fact, excess aggregate reserves are always positive. Notwithstanding, central banks that do not impose reserve requirements, still alter interbank market rates by conducting open market operations. In section 4, we present further evidence on the macroeconomic impact of an OTC interbank market.

3.3 Policy Analysis

This section analyzes the effects of monetary policy. The main insight is that Fed policies can alter the liquidity premium and induce real effects, a formalization of the credit channel. Let us first discuss the price level determination.

Price-Level determination. The price level is determined through a quantity-theory equation expressed in terms of liquid assets:

$$P_t \underbrace{\bar{a}_t \cdot (1 - \bar{c}_t) \cdot E_t}_{\text{real liquidity (demand)}} = \underbrace{\tilde{M}^{Fed}_{t+1} - M^h_{t+1}}_{\text{nominal reserves (supply)}} + \underbrace{G^{FA}_{t+1} - G^{Fed}_{t+1} - G^h_{t+1}}_{\text{nominal bond (supply)}}. \tag{23}$$

Given E_t, and a set of real rates, the portfolio demand for total real liquid assets is determined. The price level must be such that at equilibrium real rates, the real supply of liquid assets equals the real liquidity demand. Once we substitute the clearing condition for government bonds, (20) and use $\bar{m} = \bar{a} - \bar{g}$, we obtain a quantity equation but now expressed in a more familiar way, in terms of monetary balances:

$$\underbrace{M^h_{t+1} + P_t \cdot \bar{m}_t \cdot (1 - \bar{c}_t) \cdot E_t}_{\text{money demand}} = \underbrace{\tilde{M}^{Fed}_{t+1}}_{\text{money supply}}. \tag{24}$$

Although the demand for reserves is not determined at the individual level, the aggregate amount is. As a result, the price level can be determined from the aggregate demand for reserves, based on equation (24).[22]

We note that the price level remains determined, even if banks are satiated with reserves. In this regard, our paper relates to Ennis (2014), who analyzes the link between money and prices in a perfect-foresight model with a static banking system. He shows that when capital requirements are slack, a policy of paying interest on reserves equal to the market return of the risk-free asset leads to an indeterminacy result, but when the capital requirement constraint binds, the real

[22] As in much of the literature, we abstract away from the possibility of speculative hyperinflations and focus on equilibria that transition toward stationary equilibria. Cochrane (2019, ch. 17) presents a detailed discussion on conditions that allow to rule out speculative hyperinflations.

demand for reserves is determined, and hence the price level. One difference in our setup is that the presence of equity constraints in our framework implies that the price level is determined even absent binding capital requirements. In addition, the price level is determined through a quantity theory equation involving both government bonds and reserves.

Classical monetary properties. The model features classic long-run neutrality: any increase in the scale of $\left\{M, G^{FA}, G^{Fed}, B^{Fed}\right\}$ leads to a proportional increase in the price level without any changes in real allocations. On the other hand, changes in the permanent growth rate of the Fed's the balance sheet do have real effects, unless all nominal policy rates are adjusted by inflation to keep real rates constant—and when the demand for real currency balances is perfectly inelastic. Both results are proven in Appendix G.

OMO. Policies that produce real effects operate through the liquidity premium. We define conventional (unconventional) OMO as a swap between reserves for government bonds (loans). Next, we consider OMO in which the Fed exchanges reserves for loans or government bonds at time-zero, and reverses the operation the following period. The following proposition characterizes the effects of such OMO.

Proposition 5 (Real Effects of OMO)**.** *Consider an original policy sequence with a Fed balance sheet* $\left\{M_{o,t}^{Fed}, G_{o,t}^{Fed}, B_{o,t}^{Fed}\right\}_{t\geq 0}$ *and an OMO at* $t = 0$*. That is, consider an alternative policy sequence that differs from the original one only in that* $B_{s,1}^{Fed} = B_{o,1}^{Fed} + \Delta B^{Fed}$*,* $G_{s,1}^{Fed} = G_{o,1}^{Fed} + \Delta G^{Fed}$*, and* $M_{s,1}^{Fed} = M_{o,1}^{Fed} + \Delta M^{Fed}$*, for* $\Delta M^{Fed} = \Delta G^{Fed} + \Delta B^{Fed} \geq 0$*. We have the following two cases:*

i) Functioning interbank market: If $\lambda > 0$*, then the OMO has effects on prices and aggregate asset allocations if and only if banks are not satiated with reserves at* $t = 0$ *under the original allocation.*

ii) Shut down interbank market: If $\lambda = 0$*, and the operation is conventional,* $\Delta B^{Fed} = 0$*, the OMO induces the same sequence of prices and real asset allocations; If the operation is unconventional,* $\Delta B^{Fed} > 0$*, then the OMO has effects on prices and aggregate asset allocations if and only if banks are not satiated with reserves at* $t = 0$ *under the original allocation.*

The proposition establishes that when banks are satiated with reserves, open market operations are irrelevant, as in Wallace (1981). In effect, when banks are satiated, all assets are perfect substitutes. As a result, for every unit of loans (government bonds) that the Fed purchases, banks reduce holdings loans (government bonds) by one unit and increase reserves in the same amount. In effect, there are no changes in the real returns. Moreover, there are no changes

in the price level.[23] Away from satiation, however, the operations alter the liquidity premium and induce a change in the total amount of loans. When the Fed swaps government bonds or loans for reserves, this increases the relative abundance of reserves and reduces the costs from being short of reserves for an individual bank. As a result, for a given level of bank equity, this contributes to reduce the liquidity premium. Ultimately, this increases the supply of bank lending.

Moreover, the swap of government bonds or loans for reserves leads to an increase in the price level, but not one for one. Notice that for a given price level, a conventional OMO keeps constant the total amount of liquid assets. At the same time, since the composition is tilted toward reserves, market tightness θ falls, leading to a lower demand for total liquid assets. It then follows from (23) that the price level increases, but less than proportional to the increase in M^{Fed}.

Finally, an important result is that standard operations are irrelevant if the interbank market is shut down ($\lambda = 0$). When the interbank market is shut down, the benefits of holding liquid assets are independent of the abundance of reserves on the aggregate, because reserves cannot be lent to other banks. In particular, we have $\chi^+ = 0, \chi^- = i^w - i^m$. As a result, a swap of reserves for government bonds for reserves simply changes the composition of liquid assets without any real effects. This result shows that in an extreme event of an interbank market shutdown, the Fed should conduct *unconventional* OMO if it aims at reducing the liquidity premium and stimulate credit.

Bounds loans rates and the Friedman rule. This section describes the set of rates that can be induced by the Fed in a stationary equilibrium in connection with a banking version of the Friedman rule. We define this version of the Friedman rule as monetary policy where the Fed lends at the discount window without penalty, that is, when the discount window rate equals the rate on reserves:

Definition 3 (Friedman Rule)**.** Monetary policy is consistent with a Friedman rule if $R_t^m = R_t^w$.

Under this rule, $\chi^+ = \chi^- = 0$. Hence, banks are satiated, not through large holdings of liquid assets but through free borrowing from the discount window. As a result, there are no liquidity premia. This rule is in the same vein as the common version of the Friedman rule, under which the nominal interest rate on government bonds is zero and there is no opportunity cost of holding currency. Likewise, in this banking version, there is no cost of being short of reserves. Moreover,

[23] In the case of a conventional OMO, the fact that the price level remains constant can be clearly seen from (23). In this case, the OMO changes the supply of only one liquid asset without a change in the total amount of liquid assets or the market tightness, which is already at zero. The same logic applies to unconventional OMO because banks' real holdings adjust in response to the operations without causing any changes in returns.

with strictly positive liquid assets, there is also no opportunity cost of holding reserves, since $R_t^b = R_t^m$.

Notice that as defined here, there are many values of R_t^m consistent with this Friedman rule, and as we will show, there is a different loan rate associated with each value of R_t^m.[24] We denote by $R^{b,FR}$ the stationary loan rate that prevails if the monetary authority follows a Friedman rule associated with a fixed stationary interest on reserves R^m. The following proposition characterizes this stationary loan rate, focusing on the case with $G_{ss} = 0$ and $B_{ss}^{Fed} = 0$.[25]

Proposition 6 (Stationary loan rate under Friedman Rule)**.** *Assume that $G_{ss} = 0$ and $B_{ss}^{Fed} = 0$. Consider the following parameter condition:*

$$\Theta^b \left(1/\beta\right)^{\epsilon^b} \geq \left(1 + \kappa^{-1}\right) \Theta^d \left(1/\beta\right)^{\epsilon^d}. \tag{25}$$

Also, let $\bar{R}^d$ be the unique solution to

$$(1+\kappa)\left(\frac{\Theta^d}{\Theta^b}\left(1+\kappa^{-1}\right)\right)^{1/\epsilon^b}\left(\bar{R}^d\right)^{\epsilon^d/\epsilon^b} = \frac{1}{\beta} + \kappa\bar{R}^d.$$

and

$$\bar{R}^b = \frac{1}{\beta}\frac{1+\kappa\beta\bar{R}^d}{1+\kappa}.$$

We have that the following two cases:

i) Slack Capital Requirements: If (25) holds, then capital requirements are slack and

$$R^{b,FR} = \begin{cases} 1/\beta & \text{if } R^m < 1/\beta, \\ R^m & \text{if } R^m \geq 1/\beta. \end{cases} \tag{26}$$

Moreover, if $R^m \leq 1/\beta$, then $\bar{a} = 0$ (with $a \geq 0$ binding strictly if $R^m < 1/\beta$). In all cases, the deposit rate equals $R^{b,FR}$.

ii) Binding Capital Requirements: If (25) does not hold, capital requirements are binding and

$$R^{b,FR} = \begin{cases} \bar{R}^b & \text{if } R^m < \bar{R}^b, \\ R^m & \text{if } R^m \geq \bar{R}^b, \end{cases} \tag{27}$$

where $\bar{R}^b < 1/\beta$. Moreover, if $R^m \leq \bar{R}^b$, then $\bar{a} = 0$ (with $a \bar{\geq} 0$ strictly binding if $R^m < \bar{R}^b$).

To characterize the stationary loans rate, Proposition 6 exploits the fact that in any stationary equilibrium, the return on bank equity equals $1/\beta$. There are two cases to consider depending

[24] This contrasts with the common version of the Friedman rule, the non-banking version under which the real rate is pinned down by the discount factor.

[25] Recall that under the Friedman rule, unconventional and conventional open-market operations are neutral.

on whether capital requirements bind, as determined by (25). Consider first the case of slack capital requirements. In this case, we know that the deposit rate must equal the loan rate. We also have that if $R^m < 1/\beta$, banks are at a corner of liquid assets and $R^{b,FR} = 1/\beta$. Instead, if $R^m > 1/\beta$, banks hold liquid assets in equilibrium, in which case $R^{b,FR} = R^m$. Notice that because in general equilibrium the after-tax return of liquid assets is zero, a loan rate $R^{b,FR} > 1/\beta$ guarantees stationarity. When the capital requirement constraint binds, the characterization is similar except that there is a spread between the loan rate and the deposit rate. As a result, we have that $R^{b,FR}$ becomes equal to R^m for lower values of R^m compared to the case with slack capital requirements.[26]

Observe that $R^{b,FR}$ can be raised to any arbitrary level simply by raising R^m. Intuitively, there is no upper bound on the loans rate because the Fed has the ability to crowd out loans by paying a higher interest rate on reserves (financed with bank taxes). On the flip side, by lowering the rate on reserves, the Fed lowers the loan rate, but only to the point where reserves are no longer held in equilibrium. Once banks are at a corner with zero reserves, further declines in R^m have no effects.

Proposition 6 applies to stationary equilibria induced by the Friedman rule. Next, we discuss how the characterization of $R^{b,FR}$ allows us to obtain bounds on the loan rate that can be induced by policies away from the Friedman rule.

Corollary 1. *Consider any stationary policy sequence such that* $G_{ss} = 0$ *and* $B^{Fed}_{ss} = 0$ *and let* $R^m_{ss} \geq \min\left\{1/\beta, \bar{R}^b\right\}$. *Then, the stationary loan rate satisfies* $R^b_{ss} \geq R^{b,FR} = R^m$.

The corollary says that if we consider any policy such that $R^m_{ss} \geq \min\left\{1/\beta, \bar{R}^b\right\}$, then the loans rate induced by the Friedman rule constitutes a lower bound on the loans rate. The qualification $R^m_{ss} \geq \min\left\{1/\beta, \bar{R}^b\right\}$ is important, as it ensures that banks hold positive liquid assets is in equilibrium. The idea is that considering equilibrium with strictly positive liquid assets, a policy that raises the liquidity premium necessarily raises the loan rate above the one that would prevail under the Friedman rule.[27]

The Friedman rule is not only useful for understanding the set of rates that can be induced by policies but also for characterizing efficiency. The following proposition establishes the Friedman

[26] If condition (25) is violated, then capital requirements bind. If in addition, $\bar{a} = 0$, then $R^{b,FR} = \bar{R}^b < 1/\beta$. The spread between $R^{b,FR}$ and $R^{d,FR}$, is such that the total return on bank equity is $1/\beta$, despite the return on loans being less than $1/\beta$.

[27] For a low R^m such that banks find it strictly optimal to hold zero liquid assets, an increase in R^w may induce a greater spread between R^b_{ss} and R^d_{ss}, in which case R^b_{ss} must fall to guarantee stationarity. In this case, the Fed can induce a lower rate by raising R^w.

rule is necessary to achieve efficiency when capital requirements do not bind.[28]

Proposition 7. *If (25) holds, and households have the same discount factor as banks $\beta^h = \beta$, the stationary equilibrium is efficient if the Fed follows a Friedman rule policy where $R^m_{ss} = R^w_{ss} = 1/\beta$.*

Discussion on normative issues. The results here regard positive analysis. Having established that a version of the Friedman rule achieves efficiency, it is important to discuss what frictions outside the model could motivate a deviation from the Friedman rule. First, because of macroprudential concerns the Fed may want to reduce the amount of bank credit and use monetary policy for such an objective, as advocated by Stein (2012). Another concern relates to the costs of eliminating liquidity premia. For example, eliminating the LP may require the Fed to hold a large balance sheet, exposing it to credit risk or interest-rate risk, features outside of this model. Finally, there is a moral hazard consideration when lending reserves freely. There is, in fact, a tradition that sees a penalizing discount rate as a disciplining device to sustain the circulation of deposits (see Cavalcanti, Erosa, and Temzelides, 1999; Hoerova and Monnet, 2016).We leave for future work the assessment of the tradeoffs that emerge in the face of these considerations. However, we believe our model provides a useful setup to study these normative aspects.

4 Empirical Evidence

Over the last decade, a large empirical literature has developed conveying evidence that liquidity frictions play an important role in financial markets. The goal of this section is twofold. First, we provide new evidence that specifically point toward the importance of the interbank market. Second, we discuss other available empirical evidence that support our key mechanism.

A central prediction of the theory is that frictions in the interbank markets are translated, at the macro level, into a premium for liquid assets. To examine whether this relationship is present in the data, one needs measures both of the frictions in the interbank market and of asset liquidity premia. As a measure of frictions in the interbank market, we use the dispersion in interbank market rates. More precisely, using the daily distribution of the Fed Fund rates provided by the NY Fed, we construct a monthly time series, labeled FF Range, as the monthly

[28] Appendix H defines the set of Pareto optimal allocations. The Pareto optimal allocations feature equalization of marginal utilities across agents and across goods and a non-binding working capital constraint for firms. When households have a value for currency, this efficiency condition requires deflation so that the efficient real rate is consistent with zero nominal rates on reserves. Appendix J proves conditions for monotone convergence toward a stationary equilibrium under this specific Friedman rule. As long as deviations are not large from this Friedman rule, we expect similar properties to hold in any stationary equilibrium.

average of the daily spread between the maximum and the minimum Fed funds range.[29] In terms of the liquidity premia, we use two measures constructed in Nagel (2016): the spreads between the generalized collateral Repo rate (GC) and the certificate deposit (CD) with respect to the three month T-Bill rate.[30] It is worth noting that the liquidity premium is large, reaching 4% percent around 2008. This is direct evidence of the quantitative relevance of the precautionary demand for liquid assets. These large premia show that banks are willing to forgo large returns to hold assets that are easy to sell in certain dates. In the theory, withdrawal risk and matching efficiency in the interbank market produce greater dispersion in interbank rates. Equipped with these measures, we proceed to test the relationship between the two.

Panel (c) of Figure 2 presents the monthly-series for the GC and CD spreads and the FF Range, from June 2000 and December 2011 whereas panels (a) and (b) present the scatter points of the GC and CD against the FF Range series, respectively. Table 1 reports results from an ordinary least squares regression. The positive correlation between the FF Range and the two measures of liquidity premia is striking. Columns (1) and (4) present the results for the baseline univariate regressions. Columns (2) and (5) show that the sign of the regression coefficients are unchanged after the average Fed funds rate is included, an indication that dispersion in rates captures information not contained in the policy target. Similarly, the correlation remains even when we include the VIX index, which suggests that dispersion in rates is picking up uncertainty inherent to the interbank market. The standard deviation of FF Range series is 60bps, so the average impact on liquidity premia are 16bps and 36bps on the GC and CD spreads, respectively. This average impact may seem small. However, the FF Range series is highly skewed (Hamilton, 1996, as pointed out earlier in). The FF Range series is above 200bps in 5 percent of the sample, and these events produce an impact of 50bps and 120bps on the GC and CD spreads, respectively. Recent independent work by Altavilla et al. (2019) provide similar evidence for the European bank market and obtains similar effects in magnitude. Appendix L presents additional robustness checks which similar correlations results for the correlations.

These results should not come as a surprise in light of other available evidence. The scale of the interbank market is large: banks in the US clear about 3.3 trillion USD transactions daily. Moreover, according to Afonso and Lagos (2014), between 2005-2008, the annual dollar traded volume of Fed funds was about twice the annual US GDP. Also, there is ample evidence of trading

[29] The NY Fed provides historical data on the daily distribution of the fed fund rates: the data include the max and min, 99, 75, 50, 25 and 1st quantiles, and the standard deviation of daily Fed funds rate for the years 2000 through 2012.

[30] As a measure of liquidity premia, Nagel (2016) works with two time series for spreads, the spread between the generalized collateral Repo rate and the T-Bill rate (the GC spread) and the spread between certificate deposit and the T-Bill rate. The GC spread is an ideal counterpart for the spread between loans and government bonds in the model because the GC has the same risk-profile as the T-Bill but according to that paper, "The GC repo term loan is illiquid, as the money lent is locked in for three months and the bid-ask spreads between lending and borrowing rates are relatively wide compared with government bonds." Similarly, the CD to T-Bill rate is a counterpart for the spread between deposits and the T-Bill in the model.

frictions in the Fed funds market (e.g. Ashcraft and Duffie, 2007). Bindseil and Jablecki (2011) show that an increase in 100bps in the width of the policy corridor leads to greater volatility in interbank rates for a cross-section of countries. At a narrative level, the August 2019 Senior Financial Officer Survey reports that the primary reason why banks currently hold reserves is meeting deposit outflows. In fact, 72% of the respondents regard as very important holding reserves to meet deposit outflows (compared with 10% who regard as very important to earn the interest on reserves).[31]

Another prediction of the theory is that monetary policy affects the supply of loans through the liquidity management of banks. In an early contribution, Bernanke and Blinder (1988) provides evidence that expansionary monetary policy stimulates lending and deposits. Using micro data, Kashyap and Stein (2000) found evidence that liquidity plays a key role in bank lending while Jiménez et al. (2012; 2014) further corroborated this evidence using micro data on bank-loan pairs. Longstaff, Mithal, and Neis (2005) and Krishnamurthy and Vissing-Jørgensen (2012) furthermore show that the supply of government bonds impact liquidity premia.

To conclude, a large body of empirical evidence points to the importance of liquidity frictions. Adding to the existing literature, we provide new evidence that point specifically toward the importance of frictions in the interbank market to determine liquidity premia. In the next section, we calibrate our model and show how these frictions matter for the monetary transmission.

Table 1: Liquidity Premia and Interbank Spreads

	(1) GC Spread	(2) GC Spread	(3) GC Spread	(4) CD Spread	(5) CD Spread	(6) CD Spread
FF Range	0.208***	0.175***	0.159***	0.672***	0.721***	0.587***
	(12.57)	(11.08)	(9.75)	(10.17)	(10.32)	(8.95)
FF Rate		0.0291***	0.0374***		-0.0428*	0.0232
		(5.95)	(6.87)		(-1.98)	(1.06)
VIX			0.0857**			0.687***
			(3.10)			(6.17)
Constant	0.0395*	-0.00523	-0.272**	0.0330	0.0988	-2.038***
	(2.53)	(-0.33)	(-3.11)	(0.53)	(1.41)	(-5.79)
Observations	138	138	138	138	138	138

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

[31] The complete ranking is as follows: meeting potential deposit outflows (73%), meeting routine intraday payment flows (57%), satisfying internal liquidity stress metrics (63%), satisfying the bank's reserve requirements (50%), managing liquidity portfolio (30%) seeking to earn IOR rate (10%). See the Senior Financial Officer Survey.

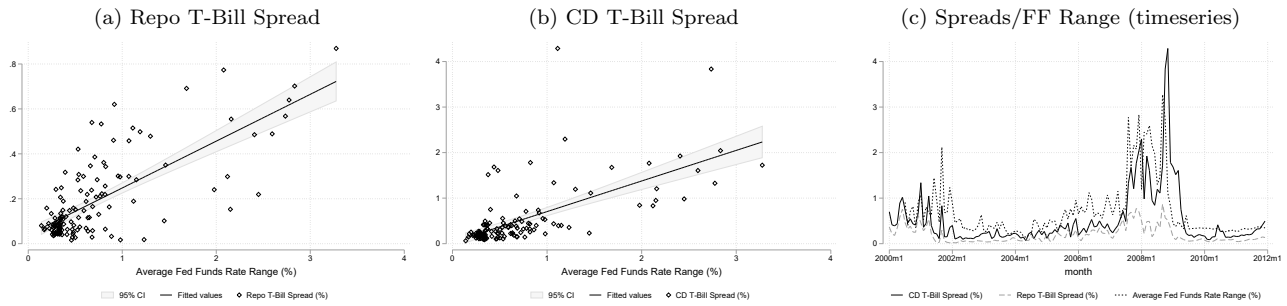

Figure 2: Liquidity Premia and Fed Funds Range

Note this one: Panels (a) and (b) presents a scatter plot corresponding to the data points in Panel (c). Appendix K.1 provides details of the data series.

5 Applications

We now provide two applications of our model to address key questions at the intersection of monetary policy and banking. We use a version of the model calibrated to the US banking system, as we explain next.

5.1 Calibration

We calibrate the steady state of the model using data from 2006 as a reference period. In Section 5.3, we then extend the calibration analysis here to the crisis and post-crisis period. Appendix K.1 provides the details of data measurements and sources.

Model period. We define the time period to be a month and use annualized rates to describe the calibration. The choice of a month is guided by several factors. On the one hand, the federal funds market operates daily, and reserve requirements are computed based on a two-week window average over end-of-day balances. On the other hand, bank portfolio decisions and loan sales typically take longer than two weeks to materialize.[32] In addition, shocks and overall positions in the interbank market are likely to be persistent, whereas they are not in the model. Capturing these institutional details would require a more complex model, one with multiple balancing stages, and additional state variables to keep track of lagged reserve requirements. We

[32]Given that the period length of the model is one month, one can think about the structure of the model such that the sale of a loan takes one month to materialize. Stigum and Crescenzi (2007) provides a clear account of the securitization process.

view a monthly model as a parsimonious compromise between the daily nature of the federal funds market, the bi-weekly nature of regulation, and the lower frequency of bank portfolio adjustments. The choice of a monthly model is also practical. Once we turn to the application in Section 5.3, most data are available monthly. We do not expect that the quantitative results of the calibrated model would change significantly with a different length of the model period.

Additional features. We extend the environment with two additional features to enrich the quantitative applications. These features only modify the portfolio problem (17) without altering any other condition in the model. First, we allow for Epstein-Zin preferences. We assume a unit intertemporal elasticity of substitution (IES), which implies that the dividend rate is $\bar{c}_t = 1 - \beta$, and set the risk aversion, denoted by γ to 10. Second, we introduce credit risk. In particular, we assume that the return of loans is given by $(1 + \delta)R^b$, where δ follows a log-normal distribution with standard deviation σ^δ and zero mean. The shock δ is distributed identically across banks and is independent of ω. By the law of large numbers, the average return across banks is R^b—hence the law of motion for aggregate equity remains the same. We introduce this second feature because it allows us to devise a procedure to match key moments in the data and to provide an exact decompositions of the decline in credit in Section 5.3. The volatility that we need to replicate the asset portfolio is small. In scale, it is about 6% of the liquidity premium.

Distribution of withdrawal shocks. For the distribution of withdrawal shocks, Φ, we assume it is a log normal distribution with standard deviation σ^ω and zero mean. A log-normal distribution approximates well the empirical distribution of excess reserves.

External calibration We set $\left\{\rho, \beta, \gamma, \epsilon^b, \epsilon^d, \epsilon^g, i^m, i^w, \pi, B^{FED}, G^{FED}, G^{FA}\right\}$ externally. We list their values in Table 2. Regarding preferences, we set the IES equal to one, as mentioned above, and the risk aversion to 10, both common values in macroeconomics. With a unit IES, stationarity of aggregate bank equity implies $R^e_{ss} = 1/\beta$. As a target for the return on equity, R^e_{ss}, based on Atkeson et al. (2018, Table 2,), reports a return on equity for banks with the highest asset quality ratings of 8 percent. Accordingly, we set β equal to $(1/1.08)^{1/12} = 0.993$.[33]

Regarding regulatory policies, we set $\rho = 0$. While regulatory reserve requirements were about 10 in the reference period, the use of sweep accounts has implied that the most relevant constraint is that reserves cannot go negative. For that reason, we calibrate the effective requirement to

[33] Notice that while the return on equity in the data has implicitly risk premia, our model does not. With risk premia, however, we would have the same stationarity condition, except that the steady state return on equity would be replaced by the mean of return on equity over time.

zero.[34]

In line with the pre-crisis landscape, i^m and B^{Fed} are set to zero as baseline values, but we then vary these variables as we analyze policies. The relevant value for the discount window rate incorporates the well-documented stigma associated with discount loans. We deduce the stigma by considering the difference between the highest interbank market rate observed and the statutory discount window rate. This approach is reasonable because the fact that many banks borrow at interbank rates above the discount rate implies there are non-pecuniary costs associated with the discount window. Accordingly, we construct a time series for the maximum observed interbank market rate and average out the differences with respect to the statutory discount window rates. The procedure yields a stigma of 5% amounting to a de facto discount window rate of 11%.

We set the consolidated government bonds to be consistent with the holdings of government bonds by banks and households. In particular, based on Call Report data, we have that holdings of government bonds represent about 10 percent of banks' assets, whereas households hold about 56 percent of total holdings (Krishnamurthy and Vissing-Jørgensen, 2012). The growth rate of money balances is set to be consistent with a steady state annual inflation of 2 percent per year.

Finally, we set loan demand elasticity with respect to the annualized loan rate to 2.5, which is in the range of empirical studies (see, e.g., Gilchrist et al., 2009), and use the same value for the elasticity of the supply of deposits and the household's government bonds demand.[35] Neither elasticities matter for the stationary equilibrium, they only matter for transitional dynamics.

Deduced parameter. The remaining set of parameters is $\left\{\lambda, \sigma^{\omega}, \eta, \sigma^{\delta}, \kappa, \Theta^{d}, \Theta^{b}, \Theta^{g}\right\}$. This set is obtained by targeting a set of moments from the data. The data that we employ are the size of interbank loans relative to deposits, F/D; an average Federal Funds rate; the discount window loans relative to deposits, W/D; a measure of the loans liquidity premium, LP; and banks' loan, bond, and reserve portfolio holdings, $\left\{\bar{b}, \bar{g}, \bar{m}\right\}$. Our procedure allows us to sequentially determine each of these parameters.

A summary of the procedure to obtain these parameters is as follows; details are relegated to Appendix K.3. We use $\hat{x}$ to refer to parameter or variable x deduced from the equilibrium equations in the model. If a variable enters without that symbol, it is measured directly from

[34] For banks with net transactions over USD 48.3 million as of 2006, the reserve requirement is 10 percent (see Federal Reserve Bulletin, Table 1.15). No corresponding sources in references. However, since the introduction of sweep accounts in the US, banks are able to circumvent reserve requirements by transferring funds overnight to accounts not subject to requirements. All the results are quantitatively similar for small levels of reserve requirements, for example, 2.5%.

[35] The parameter ϵ is a semi-elasticity: in terms of the monthly calibration, this implies $\epsilon^{b} = 35$. Notice that we do not need to specify the elasticity of currency to solve for all the allocations and loan returns. This is because of quasi-linearity; namely, households preferences are linear in the good that does not require cash or deposits to be consumed.

Table 2: Calibration

External Parameters	Value	Reference
Discount factor	$\beta = 0.993$	Return on Equity
Risk aversion	$\gamma = 10$	Standard
Interest on reserves	$i^m = 0$	Observed
Discount window rate	$i^w = 11\%$	Measured Stigma
Steady state inflation	$\pi = 2\%$	Inflation Target
Fed holdings of loans	$B^{FED} = 0$	Observed
Government bonds	$G^{FA} - G^{Fed} = (1+\bar{d})0.1 \times (1 + G^h/G)$	Observed
Reserve requirement	$\rho = 0$	Observed
Elasticities	$\epsilon^b = -\epsilon^d = -\epsilon^g = 35$	Literature
Deduced Parameters	**Value**	**Target**
Matching friction	$\hat{\lambda} = 7.9$	$W/(W+F) = 0.035\%$
Volatility of withdrawals	$\hat{\sigma}^\omega = 0.12$	$W/(D+E) \ = 0.0011\%$
Bargaining powers	$\hat{\eta} = 0.15$	$\bar{i}^f = 4.4\%$
Credit Risk	$\hat{\sigma}^\delta = 6\% \times LP$	$\bar{b}/(\bar{b}+\bar{a}) = 97.5\%$
Capital requirement	$\kappa = 8.8$	Bank Leverage
Deposit supply intercept	$\hat{\Theta}^d = 9.4$	$R^d = 2\%$
Loan demand intercept	$\hat{\Theta}^b = 10.9$	Loan LP$= 50bps$
Bond demand intercept	$\hat{\Theta}^g = 0.275$	$G^h/G = 0.56$

the data.

The first step is to obtain a matching efficiency, $\hat{\lambda}$, deduced from observed activity in the interbank market relative to discount window loans. We first infer the probability that a reserve deficit position is matched in the interbank market, using $\hat{\Psi}^- = F/(W+F)$. When the model's implied market tightness is $\hat{\theta} < 1$, we obtain:

$$\hat{\lambda} = \log\left(1 - \hat{\Psi}^-\right)^{-1}.$$

This relationship follows from by inverting (A.1) under the assumption that $\theta < 1$.[36] The condition $\theta < 1$ is verified in a later step.

The second step is to obtain the volatility of withdrawals, inferred from observed banks's liquidity holdings and activity in the discount window. To do so, we first deduce the cutoff $\hat{\omega}^*$ from the definition (21). Then, we use that $W = (1 - \hat{\Psi}^-)S^-$ to deduce $\hat{\sigma}^\omega$ as the implicit solution to

$$\frac{W}{A} = \left(1 - \hat{\Psi}^-\right)\Phi\left(\hat{\omega}^*; \hat{\sigma}^\omega\right)\left(\frac{\bar{m}+\bar{g}}{\kappa+1} + \frac{R^d}{R^m}\mathbb{E}\left[\omega|\omega < \hat{\omega}^*; \hat{\sigma}\right]\frac{\kappa}{\kappa+1}\right).$$

[36] Under a Leontief matching technology, if deficit positions are less than surplus positions, i.e. when $\theta < 1$, then $\hat{\Psi}^- = 1 - \exp(-\lambda)$.

The third step is to obtain the bargaining power. We infer η from the interbank market rate, taking into consideration the matching efficiency and aggregate liquidity holdings . That is, we obtain $\hat{\eta}$ using

$$i^f = i^m + (i^w - i^m)\,\phi\left(\hat{\theta}; \hat{\lambda}, \hat{\eta}\right).$$

This step uses the effective bargaining power $\phi(\hat{\theta}; \hat{\lambda}, \hat{\eta})$ defined in Appendix A and a measurement of the market tightness consistent with the previous steps $\hat{\theta}$.[37]

The fourth step is to obtain a value for credit risk, $\hat{\sigma}^\delta$, which we infer by rationalizing the banks portfolios given the returns of assets and liabilities, and the liquidity yield function χ. Given all the objects we have so far, we can compute directly $\bar{\chi}^+$and $\bar{\chi}^-$.[38] The return on loans is deduced using the equilibrium condition $R^b = R^m + \hat{\bar{\chi}}^+ + LP$, where LP is observed in the data as constructed by Del Negro et al. (2017).

We can then deduce the parameter controlling credit risk, σ^δ, and a leverage requirement κ, such that the bank optimization problem delivers the observed portfolios in the data

$$\left(\bar{b}, \bar{d}\right) = \underset{\bar{b},\bar{d}\leq\kappa,\bar{a}+\bar{b}-\bar{d}=1}{\operatorname{argmax}} \left\{\mathbb{E}\left[(1-\delta)\,R^b\bar{b} + \; R^m\bar{a} - \; R^d\bar{d} + \bar{\chi}(\bar{a},\bar{d},\omega)|\hat{\sigma}^\delta, \hat{\sigma}^\omega\right]^{1-\gamma}\right\}^{\frac{1}{1-\gamma}}.$$

where the expectation $\mathbb{E}$ is over δ and ω.[39]

Finally, given total credit supply in the model, the value for Θ^b is chosen to guarantee that R^b is the equilibrium return using $\frac{B^f_{t+1}}{P_t} = \Theta^b_t \left(R^b_{t+1}\right)^{\epsilon^b}$. We proceed analogously for $\left\{\Theta^d, R^d\right\}$ and for $\left\{\Theta^g, R^g\right\}$ using the targets for deposits and the fraction of government bonds owned by households.

5.2 Implementation of Monetary Policy and Pass-Through

In the first application, we examine the implementation of monetary policy and the pass-through from policy rates to lending rates. We address the following questions: What are the effects of varying the interest on reserves (IOR) on bank credit? What are the different policy configurations that can implement a target for the Federal funds rate? What are the implications of these different configurations for the lending rate and pass-through of interest rates?

[37] By definition,

$$\hat{\theta} = \left[\Phi\left(\hat{\omega}^*; \hat{\sigma}^\omega\right)\left(\bar{m} + \bar{g} + \frac{R^d}{R^m}\mathbb{E}\left[\omega|\omega < \hat{\omega}^*; \hat{\sigma}^\omega\right]\bar{d}\right)\right] / \left[\left(1 - \Phi\left(\hat{\omega}^*; \hat{\sigma}^\omega\right)\right)\left(\bar{m} + \bar{g} + \frac{R^d}{R^m}\mathbb{E}\left[\omega|\omega > \hat{\omega}^*; \hat{\sigma}^\omega\right]\bar{d}\right) - \bar{g}\right].$$

[38] We use $\hat{\Psi}^+ = \hat{\Psi}^- \cdot \hat{\theta}$, and deduce $\hat{\bar{\chi}}^+ = \hat{\Psi}^+ \cdot (R^f - R^m)$ and $\hat{\bar{\chi}}^- = \hat{\Psi}^- \cdot (R^f - R^m) + \left(1 - \hat{\Psi}^-\right) \cdot (R^w - R^m)$.

[39] The procedure leverages upon the feature the capital requirement binds in the model for any κ lower or equal than the observed leverage.

IOR and capital requirements. We first examine the effects of changes in the IOR. In the US, interest on reserves was introduced in October 2008. Since then, it has generated many policy discussions along different fronts, specifically on whether it contracts or expands bank lending.[40] The following analysis shows that the effects on bank lending may be non-monotonic. In particular, whether credit increases or decreases with the IOR depends on whether capital requirements bind.

We study how the stationary equilibrium changes as we vary the steady state IOR, while keeping all other policies and model parameters constant. Figure 3 presents the results.[41] In panel (a), we display the steady state lending rate as a function of the IOR. The figure shows a non-monotone pass-through. For low IOR, increases in the IOR lead to a slight decline in the lending rate and stimulate credit. For high IOR, increases in the IOR lead to a sharp increase in the lending rate and depress credit. In panel (b), we also display the portfolio weights as we change the IOR. As the figure shows, the change in the sign of the pass-through from the IOR to the lending rate occurs at exactly the point in which the deposit portfolio share becomes constant and the capital requirements begins to bind.

To understand the intuition behind this non-monotonicity consider the Loan LP. If we let $\tilde{\mathbb{E}}$ be the bank's risk-adjusted expectation, this premium can be written as

$$R^b = R^m + \tilde{\mathbb{E}}\left[\bar{\chi}\left(\bar{a}, \bar{d}\right)\right]. \tag{28}$$

One can see from equation (28) that an increase in R^m has a direct one-for-one change in R^b, given portfolio weights $(\bar{a}, \bar{d})$. Notice also that capital requirements bind when the IOR is high. This is because in effect, a high IOR makes it less costly to issue deposits. When capital requirements bind, $\bar{d} = \kappa$ and is therefore invariant to the IOR, but $\bar{a}$ increases with R^m. The increase in the liquidity ratio lowers the liquidity premium, but only partially offsets the direct effect of the increase in R^m. This means that when capital requirements bind, reserves and loans are substitutes and an increase in the IOR is necessarily contractionary.

When the IOR is low, by contrast, the capital requirements are non binding. Capital requirements do not bind for low IOR because a low IOR increases the costs of insuring against deposit withdrawals, hence making deposits in effect more costly. Starting from a point where capital requirements do not bind, an increase in the IOR increases the incentives to issue deposits. The

[40] See a fascinating discussion between George Selgin and John Taylor, one one side and Robert Eisenbeis, and Todd Keister on the opposite side in a testimony on the House Financial Services Hearing on the Fed's Balance Sheet and Interest on Reserves, May 17, 2016, https://www.sifma.org/resources/general/house-financial-services-hearing-on-the-fed-s-balance-sheet-and-interest-on-reserves/.

[41] We follow the baseline calibration with two modifications to better illustrate the results. First, we set κ so that the capital requirement constraint holds with equality but does not bind in the stationary equilibrium with the baseline values. This allows us to better highlight the importance of the capital requirement for the sign of the pass-through. Second, we mute the response of the interest rate on deposits by considering a perfectly elastic supply of deposits.

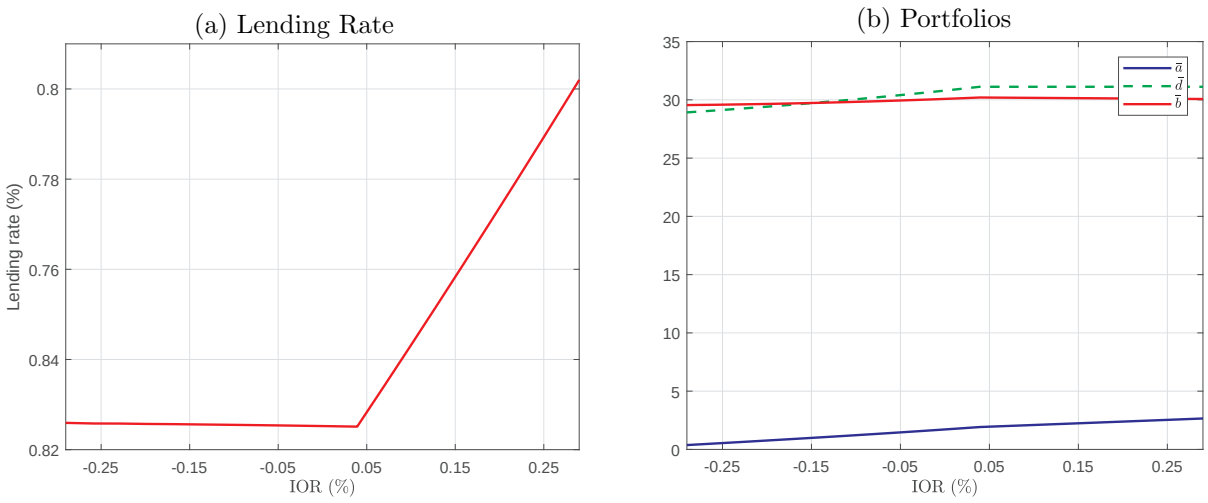

Figure 3: Stationary equilibrium lending rate and portfolios as a function of IOR

Note: We use the the benchmark calibration, except that we set $\kappa = 31$, implying that the capital requirement holds with equality but it does not bind for $i^m = 0$. We also use a perfectly elastic supply of deposits to mute the effects on the interest rate on deposits.

increase in the IOR also stimulates banks to hold more liquidity, but if the deposit increase is greater, the increase in the IOR will stimulate lending, as occurs in Figure 3. This showcases that when capital requirements do not bind, reserves are potentially complements to lending. Proposition 8 below formalizes the non-monotonicity that appears in Figure 3. Namely, the proposition shows that when capital requirements bind, the effect of an increase in the IOR is necessarily contractionary, under any parameter configuration. When capital requirements do not bind, the effect of the lending rate is generically ambiguous.

Proposition 8. *Consider the set of stationary equilibria. If capital requirements bind, then* $\frac{dr^b}{dr^m} \in [0, 1]$ *and* $\frac{dr^b}{dr^m} = 1$ *when banks are satiated with reserves. If capital requirements do not bind and the deposit supply is perfectly elastic at* r^d*, the pass-through is ambiguous.*

These results highlight how the interaction between capital requirements and liquidity frictions plays a key role for the transmission of monetary policy. We next explore how the corridor rates and the balance sheet can be jointly managed to achieve monetary policy objectives.[42]

Fed balance sheet and policy pass-through. A central feature of the model is that, away from satiation, the interest on reserves and the size of the balance sheet of the monetary authority are *independent* instruments. Namely, the monetary authority can target a given interbank rate (FFR) via different configurations of the IOR and a size of its balance sheet. We argue next

[42]For recent interesting related analysis with a focus on negative interest rates, see Brunnermeier and Koby (2019), Wang (2019), and Eggertsson et al. (2019).

that how the FFR is actually implemented matters for the level of the lending rate and for the pass-through.

We consider stationary equilibrium, in which we fix a corridor spread, $\Delta \equiv i^m - i^w$, and then construct menus of $\{i^m, B^{Fed}\}$ that implement a given target for the FFR.[43] We label this menu the "iso-Fed Funds curve." Panel (a) of Figure 4 displays the iso-Fed Funds rate curve for two different Fed funds targets; each point in the straight-red curve is consistent with a target of 2.5%, whereas the dashed-blue is consistent with a target of 2.75%. We display B^{fed} in the x-axis and i^m is in the y-axis. Since bank equity is normalized to 1, at steady state, B^{Fed} should be interpreted as Fed holdings of loans relative to bank equity.

Panel (a) shows that the iso-Fed funds curve is upward sloping. This positive relationship emerges because the FFR is increasing in the IOR and decreasing in the balance sheet. To see why, recall from Proposition 1 that the Fed funds can be expressed as

$$\bar{i}^f = i^m + \Delta \cdot \phi(\theta), \tag{29}$$

where ϕ is an endogenous weight that increases with θ. From this expression, we observe that an increase i^m has a direct one-for-one effect on $\bar{i}^f$. This effect is coupled with an indirect effect that partly mitigates the direct effect: the increase in the IOR generates more abundant reserves, leading to a lower θ and hence a lower ϕ. In other words, as reserves become more abundant, the FFR moves to the floor of the corridor, but because the floor increases, the FFR also increases. In terms of the iso-Fed Funds curve, an increase in B^{Fed} is warranted to keep the FFR at a target. Indeed, a higher B^{Fed} generates a decline in θ, as the monetary authority absorbs a larger fraction of the illiquid assets. It also interesting to note that as B^{Fed} increases, the iso-Fed funds curve eventually becomes horizontal. This reflects that in a satiation regime, the size of the Fed's balance sheet has no effect on the liquidity premium, and the iso-Fed funds is flat at $\bar{i}^f$.

What are the implications for credit of these different configurations? Panel (b) shows that as we move along the iso-Fed funds—by increasing B^{Fed} and i^m to keep the FFR constant—the lending rate falls (and credit expands). The logic can be explained through a reformulation of the Loan LP:

$$i^b = i^m + \chi^+ + \left(\chi^- - \chi^+\right) \cdot F(\omega^*) \cdot \frac{\mathbb{E}_\omega\left[(R^e)^{-\gamma} \,|\omega < \omega^*\right]}{\mathbb{E}_\omega\left[(R^e)^{-\gamma}\right]}. \tag{30}$$

Notice that because we are moving across stationary equilibria with the same inflation, the real lending rate moves one-to-one with i^b. Equation (30) highlights that the reduction in liquidity premia can offset the increase in the IOR, and hence configurations with higher IOR and balance sheet may stimulate lending.

[43] We keep the rest of the parameters at the baseline values, listed in Table 2. Notice the difference with the exercise above in which we changed only the IOR.

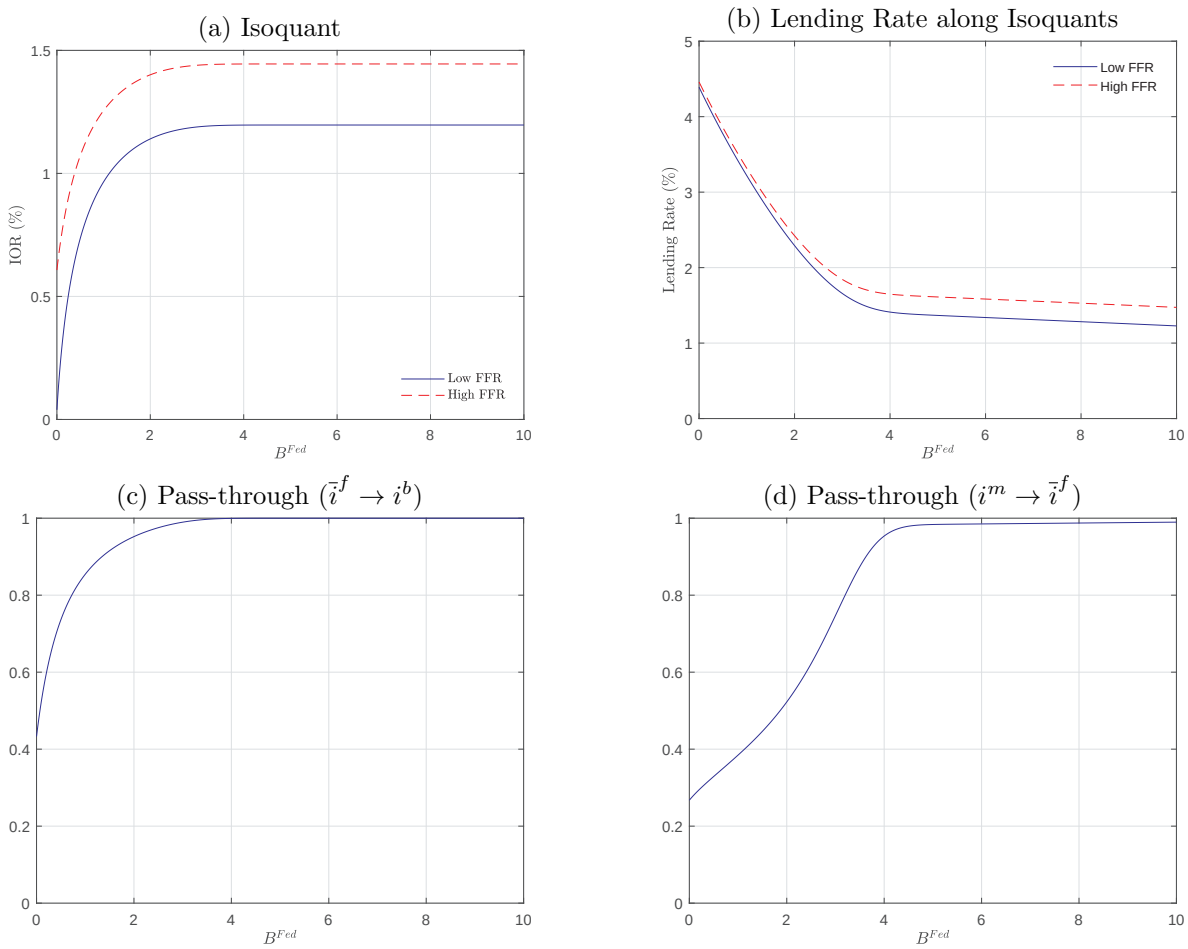

Figure 4: Interest on Reserves and Balance Sheet as Independent Instruments

Note: the figure is constructed with the parameters obtained from the baseline calibration.

In Panels (c) and (d), we turn to analyze pass-through. Specifically, we change the IOR to achieve a 25bps increase in the FFR and show how the lending rate and the FFR vary depending on the level of the Fed balance sheet. In the figure, we measure the pass-through as the changes in the lending rate and the FFR relative to the increase in the IOR. As the figure shows, both the pass-through for the FFR and the lending rate are increasing in the size of the balance sheet. Moreover, as the balance sheet reaches a level close to satiation, the pass-through becomes close to one, as anticipated in Proposition 8.

Discussion. This section's analysis is useful for framing ongoing policy discussions on the reform of the monetary policy frameworks in the US and Europe.[44] Following the large-scale asset purchases that started in 2008, policy circles (see e.g. Potter, 2017; Logan, 2019) have been discussing whether to operate in a system where the Fed Funds rate trades slightly above the interest on reserves or to return to the pre-2008 corridor system, where it traded closer

[44]See the Fed's Review of Monetary Policy Strategy Tools and Communication.

to the discount window. These central banks are also re-evaluating whether to maintain their large balance sheets, or to unwind their large asset holdings. Importantly, these discussions have taken place in the context of a change in the regulatory landscape, including increases in capital requirements and the liquidity ratios. As important disruptions occurred first in October 2019, with the repo market freeze, and more recently, a renovation of large-scale asset purchases by these central banks in the context of the Covid-19 crisis the design of their operating frameworks will likely continue to be a macroeconomic priority in the coming years.

These discussions cannot be framed in the context of the New Keynesian model, where once a policy target is set, there is a unique balance sheet consistent with that target and the pass-through from policy to credit rates is always one. By contrast, the model here provides a unified framework to assess these issues. A key takeaway from this section is that the same interbank target can result from multiple configurations of balance sheet size and interest on reserves. For example, we can obtain the same interbank rate with a lower interest on reserves and a lean balance sheet (in a corridor system) or with a higher interest on reserves and a large balance sheet (nearer to satiation in a floor system). We highlight that these configurations have different implications for bank credit. A floor system produces lower lending rates, increases bank credit, and results in a higher policy pass-through than a corridor system that implements the same interbank target. Furthermore, both systems interact differently with capital requirements: higher interest on reserves can actually expand credit in a corridor system with lax capital requirements—although it always contracts credit near satiation.

In practice, central banks have to deal with many policy objectives, including price, macroeconomic, and financial stability. Our model suggests that perhaps a monetary policy should not tie its hands to either a floor or a corridor system. Doing so inhibits the ability to balance multiple objectives.

5.3 Inspecting the Decline in Lending during the Great Recession

We now examine the sources of the credit crunch that occurred during the 2008 financial crisis. Motivated by the severe collapse of the interbank market and the rise in discount window facilities, we ask: What was the contribution of liquidity factors to the lending decline? What was the contribution of unconventional open market operations in helping to mitigate the credit crunch?

Additional institutional features. In order to map the model to the data in the period of study, it is important to take into account two additional institutional features of the interbank market. First, many participants in the Fed funds market (i.e., "non-depository institutions") did not have access to interest on reserves at the Federal Reserve. As has been well observed, this feature has a created a "leak" in the corridor system (i.e., the Fed funds rate was below

the IOR) once the Fed started paying interest on reserves in October 2008.[45] Considering that the Fed funds rate is actually an average of all interbank market rates, what this data pattern reveals is that trades have been dominated by non-depository institutions lending below the IOR. Basic arbitrage, however, indicates that the remaining trades that are performed between banks still trade above the IOR. In order to have a data analogue to the FFR in the model, we therefore need to reconstruct a FFR series that excludes transactions with non-banks. A second related feature is that government bonds provide collateral for many trades within the repo market where depositary and non-depositary institutions participate. As a result, the rate on government bonds has often traded below the interest on reserves. While we abstract from these practical features in our baseline model, mapping the model to the data for some of the post-crisis period requires taking these features into account. In Appendix K.3, we present an extension of the model with non-depository institutions and a collateral value for government bonds and show how the calibration can be adjusted to incorporate these features.

Measurement procedure. We present an estimation procedure to infer the sequence of the underlying structural parameters. The estimation procedure is in the spirit of the business cycle accounting methodology in Chari, Kehoe, and McGrattan (2007), but here we seek to account for the source of the credit decline. We take 2006.1-2014.12 as a sample period. The procedure follows the basic approach we used for the calibration of the steady state in Section 5.1, which we now repeat by feeding in the data inputs for each point in time. In addition, we need to incorporate three factors concerning dynamics. First, we account for the fact that equity may be away from steady state and that equity growth is not necessarily zero. To capture these dynamics, we feed in the path of real bank equity growth obtained from the data, then compute a residual between the observed equity growth and the one predicted by the model, which we denote by ξ_t.[46] Second, we feed the path for the nominal quantity of reserves, as well as the other changes in the Fed balance sheet resulting from conventional and unconventional open market operations.[47] Third, inflation may also be away from the steady state value. To determine the real demand for assets, one-period ahead inflation expectations are needed. (Notice that thanks to a unitary IES, the dividend rate is a constant fraction of equity, and future bank values do not affect the real demand for assets). In the data, we observe only the nominal rates and the ex-post real rates. Since inflation expectations were anchored around a target, we assume a constant

[45] This so-called leak held for most of the post-crisis period, and after March 2019, the FFR again started trading strictly above the IOR.

[46] At each point in time, the procedure generates a bank equity return, given banks' portfolios and returns. The residual equity growth equation is $\hat{\xi}_t \equiv \beta \left(1 + \left(\hat{R}_t^b - 1\right)\left(1 + \bar{d}_t + \bar{b}_t^{Fed} - \bar{a}_t\right) - \left(\hat{R}_t^d - 1\right)\kappa_t\right) - E_{t+1}/E_t$.

[47] Notice that to compute the steady state we do not have to specify the level of nominal balances to determine allocations since the model has a long-neutrality property; (i.e., the nominal balances only matter for the price level).

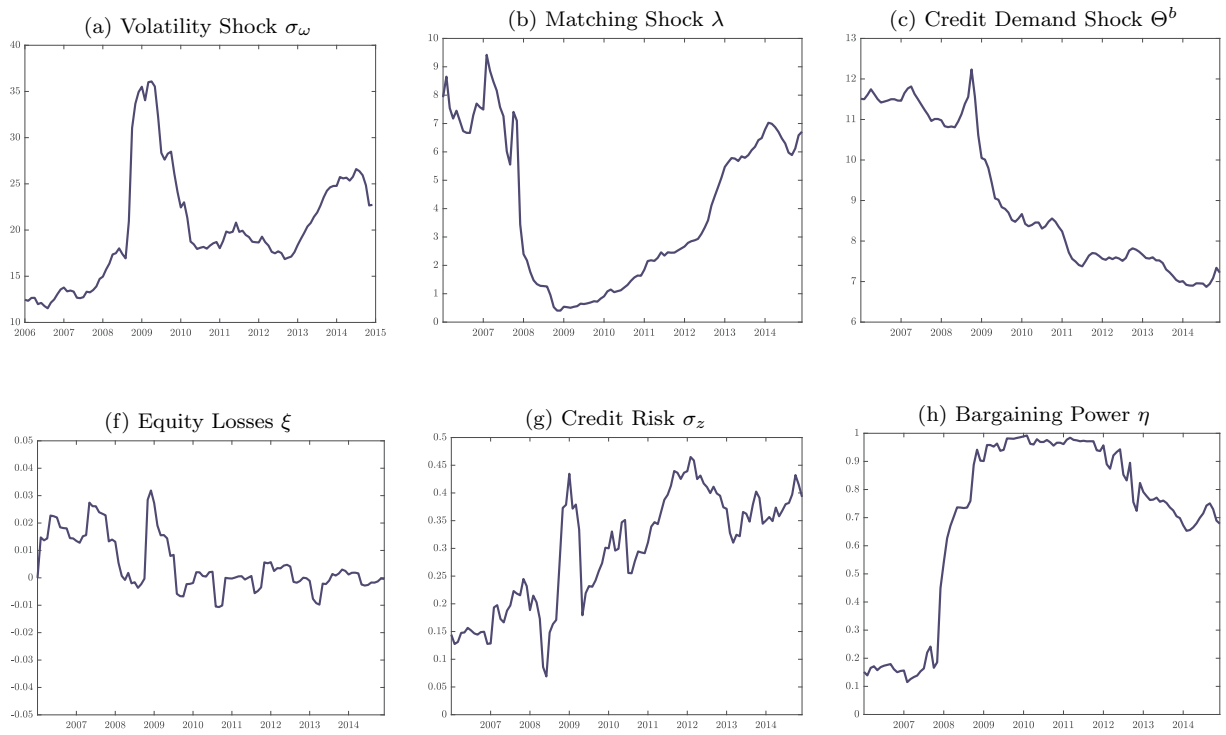

Figure 5: Deduced shocks

Note: This figure presents the shocks that generate the simulations in the model that replicate the data counterparts as described in the text.

expectation of inflation equal to 2 percent in the simulations.[48] Importantly, this *does not mean* that the model mechanically produces a constant inflation. The price level in the model is still determined endogenously based on (23).

Deduced shocks. In Figure 5, we report the key deduced shocks that fit the data. The series for these shocks are stable until the financial crisis: Around Lehman, we see both a sharp decline in the matching efficiency and an abnormally high volatility of withdrawals. These shocks gradually return to their pre-crisis levels, with matching technology coming back at a slower pace compared to volatility. On the other hand, credit demand rises in the run-up to the crisis, and begins to fall in mid 2009, experiencing a substantial decline that continues for years. Credit risk also begins at a low level, and experiences an upward trend, with spikes around Lehman. The evolution of η points to a rise in the bargaining power of borrowers: this possibly captures changes in the outside options which the simple bargaining problem does not capture explicitly (see Afonso and Lagos, 2015). Likewise, the accounting produces moderate equity losses at the pre-Lehman phase, which also spike around that period.

[48] Results would be similar if we consider expectations of inflation equal to the realized inflation.

Price level. It is important to highlight that we are matching exactly the path of the price level. In fact, we are matching the banks' portfolios in the data for real reserves while we also feed the nominal amount of reserves to the model. Considering that the amount of nominal reserves increased by more 50 times in the data and that the price level was fairly constant, this implies an increase in the nominal holdings of reserves of around 50 times.[49] Overall, holdings of liquid assets had a much more modest increase. Thus, the model rationalizes the fairly constant price level partly through an increase in the real demand for liquid assets and partly through an increase in the share of liquid assets held as reserves.[50]

Lending Decline Decomposition. Equipped with the estimated shocks, we can feed in different combinations of shocks and recompute the model. In particular, we proceed to shut down a subset of shocks at a time. Because our baseline parameters exactly match the data by construction, the difference in a given simulation relative to the baseline is a measure of the partial contribution of each shock to the observed time-series. The counterfactuals are generated as follows. We take the estimated parameters for 2006.1 as the starting point.[51] For each date, we input the bank equity and the deduced parameter values. We then ask, What would be the equilibrium outcome if a particular subset of shocks did not occur (i.e., if the value of the parameter for that subset were the same as the 2006.1 value)?[52] Figure 6 reports the results. We present four variables: credit (panel a), the liquidity premium (panel b), discount window loans (panel c) and interbank market (panel d). We consider three counterfactual scenarios: (i) no liquidity shocks (i.e., no shocks to σ or λ), (ii) no credit demand shocks (no change to Θ^b), and (iii) no equity losses and no credit risk ($\xi = 0$ and no change in σ^δ).

The importance of allowing for interbank market shocks can be seen from panels (c) and (d). Absent the matching shock, one would have observed an increase in trade in interbank market loans around the Lehman episode. Similarly, the model would predict very little activity in the discount window absent the volatility shock. Panel (a) shows that these interbank markets shocks indeed played a role in reducing credit. In the peak of the crisis, credit would have been about 5 percent higher without liquidity shocks. After 2011, the effects of interbank market frictions become very small, consistent with the reduction in liquidity premia and in response to the Fed

[49] The 50-fold increase is measured using Total Reserves of Depository Institutions (Totressns), which includes vault cash. Excluding vault cash, the jump increases by more than 100 times, reflecting that the Fed remuneration of reserves does not apply to vault cash.

[50] Recall also that the model predicts that conventional open market operations have no effects on quantities or prices either when the interbank market shuts down or when banks are satiated. The period around Lehman and the period post 2010, respectively, come close to those two scenarios.

[51] Alternatively, we could take an average of pre-crisis values and obtain very similar effects.

[52] To solve for the counterfactual equilibrium outcome, we can obtain, for given structural parameters, the beginning of period equity and policies $\{R^m, R^w, B^{fed}, G^{fed}\}$, the values for (R_t^b, θ_t) consistent with the market clearing for loans and market tightness in the interbank market. We do this fall for every period in the simulating sample. Notice that if we use the original parameters estimated, we recover the observed data series.

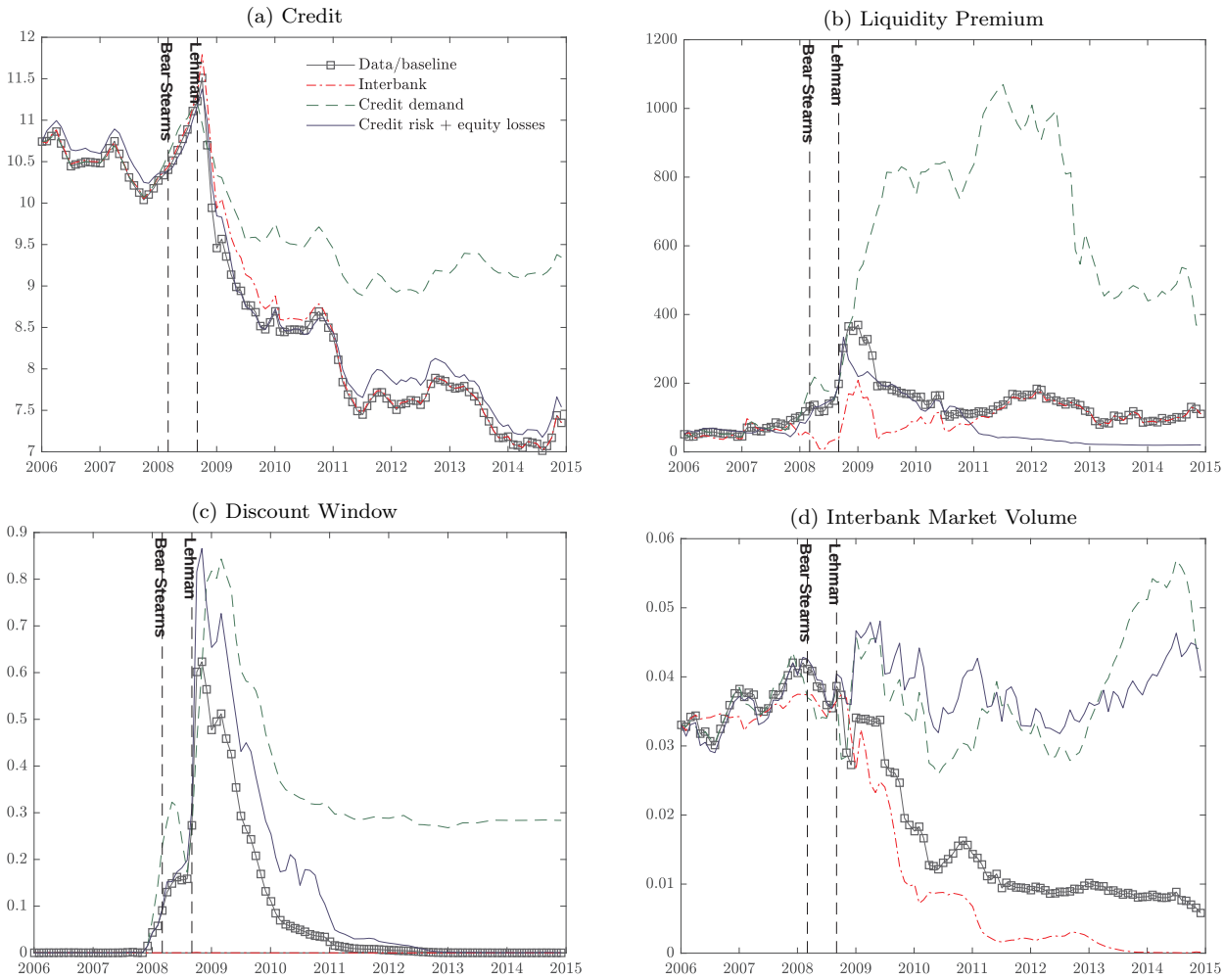

Figure 6: Counterfactuals

Note: This figure presents the simulations of the model for the following four scenarios: (i) baseline simulations with all shocks that replicate the data; (ii) simulations without interbank market shocks; (iii) simulations without credit demand shocks; (iv) simulations without equity losses and credit risk.

policies which alleviated liquidity risk.

Loan demand plays a modest role in explaining the decline in credit in the early stages of the crisis. However, after 2010, it becomes the dominant factor in explaining a persistent reduction in the level of bank credit. Finally, the combination of credit risk and equity losses has a relatively moderate impact around the crisis, and its importance is reduced gradually through 2011-2012.

From conventional to unconventional OMO. Next, we investigate the quantitative role of unconventional open market operations. We ask two questions: First, what would have been the decline in total credit absent loan/MBS purchases by the Fed?[53] Second, we ask what would have been the decline had the Fed conducted purchases of government bonds instead of MBS?

[53]Technically, the Fed purchased MBS among other assets, which take to be analogous to loans in the model.

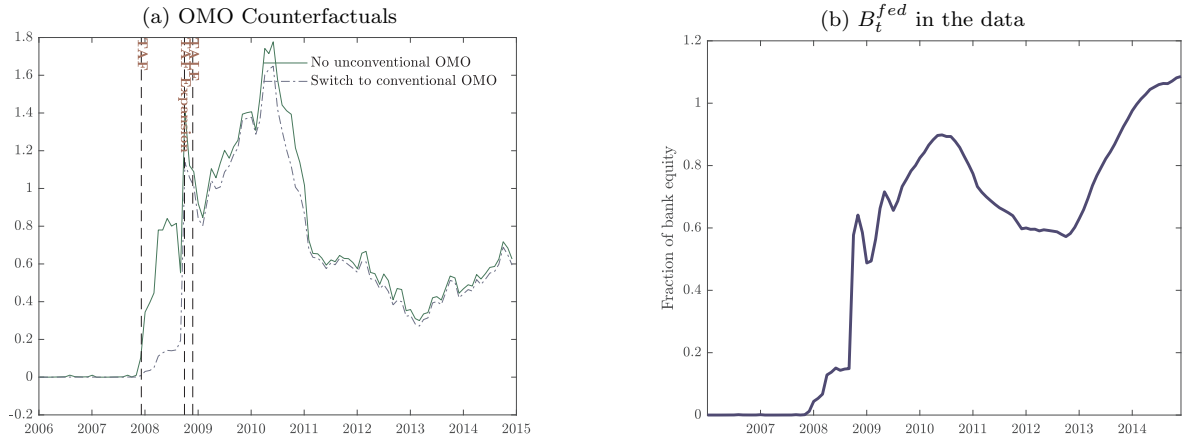

Figure 7: Role of unconventional open market operations

Note: Panel (a) presents the declines in credit for the benchmark simulations if the Fed had not carried unconventional open market operations (i) and if the Fed had used conventional open market operations instead of unconventional ones. Panel (b) presents the data counterpart for B^{fed}.

Figure 7 shows that around mid-2010 the drop in lending would have been 1.8 percent larger should the Fed have not engaged in unconventional OMO. This result showcases that open market operations were important to mitigate the collapse in total credit, notwithstanding the crowding out effect—notice that the amount of loans purchased by the Fed reaches about 10 percent of the stock. It is also interesting to note that while the size of the operations continues to increase after 2010, the overall effect is smaller. In fact, the interventions contribute to expand credit by reducing the liquidity risk of banks. Once, the interbank market shocks return to more normal conditions, these operations have a modest impact.[54]

Figure 7 also shows that if the Fed had purchased government bonds instead of loans, the decline in total credit would have been about the same as if the Fed had not conducted open market operations at all. In other words, it was key that the Fed engaged in unconventional open market operations to mitigate the decline in credit. Essentially, through unconventional open market operations, the Fed absorbs more illiquid assets in its balance sheet, which is especially stimulating when interbank market frictions are severe. On the other hand, conventional open market operations exchange assets of similar degrees of liquidity and have more modest effects.

Taking stock. An important quantitative lesson from the analysis is that liquidity shocks can indeed be important determinants of credit supply. In our model, these shocks manifest as a more severe matching friction between banks and by a larger volatility in deposit withdrawals. These shocks do not have to be interpreted literally: in practice, they can be associated with

[54]The importance of interbank market frictions resonated again in the recent "repo" crisis of October of 2019 and amid the COVID-19 crisis. Even in a regime with large excess reserves, increases in liquidity demand triggered interbank rates to hit the ceiling of the corridor rate, until the Fed activated a program of large-scale OMO.

an increase in counterparty risk, resulting from, for example, from imperfect information on risk exposure. It is also important to note that while we treat these shocks as independent, they could have a common source. For example, the liquidity shocks that we uncover in the estimation could have been triggered by equity losses. Our analysis reveals that that while equity losses per se may have had modest impact on lending during the crisis, there were potentially major indirect effects through the amplification of liquidity frictions. Similarly, the large decline in credit demand is suggestive of a deeper phenomenon by which an initial contraction in the the level of credit eventually translates into a decline in the loan demand. More research is needed to shed light on these interactions. A key takeaway for policy is the importance of unconventional open market operations for tackling instability in the interbank market. Failure to address such instability may lead liquidity frictions to spread to the rest of the financial system and ultimately to the real economy.

6 Conclusion

Historically, the topics of money and banking have been studied and taught together. Despite this historical connection, modern monetary models developed, to a large extent, independently from banking. The financial crises of the last decades in the United States, Europe, and Japan, however, have revealed the need for a unified framework.

This paper presents a new tractable framework for studying money and banking within a unified setup. Frictions in the interbank market give rise to a bank liquidity management problem and a credit channel of monetary policy. In the model, banks engage in maturity transformation, which exposes them to liquidity risk. To insure against unexpected deposit withdrawals, banks hold reserves as a precautionary buffer. Banks that face large withdrawals deplete their reserves and resort to a frictional OTC interbank market and discount window borrowing. Monetary policy has the power to alter the liquidity premium and, in that way, to affect real economic activity.

We consider two applications of the model. In one application, we use the model to study aspects related to monetary policy pass-through and the implementation of monetary policy. In the second one, we study the contribution of liquidity factors to the decline in credit in the 2008 financial crises. There are other possible applications, ranging from historical episodes like the Friedman and Schwartz (2008)'s hypothesis of the liquidity contraction of the Great Depression to modern policy questions regarding interactions between monetary policy and financial regulation.

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Online Appendix "Banks, Liquidity Management, and Monetary Policy"

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Contents

Table 3: Model Variable List

Interest rates	
$\bar{i}^f$	average interbank market rate
i^b	nominal interest rate on loans
i^d	nominal interest rate on deposits
i^g	nominal interest rate on bonds
Individual bank variables	
Portfolios	
$\tilde{b}$	bank loans in lending stage
$\tilde{m}$	reserves held by banks at end of lending stage
$\tilde{d}$	bank deposits at end of lending stage
$\tilde{g}$	government bonds at end of lending stage
b	loans at beginning of lending stage
m	reserves held at beginning of lending stage
d	deposits owed at beginning of lending stage
g	government bonds at beginning of lending stage
$\bar{b}$	portfolio share in loans
$\bar{m}$	portfolio share in reserves
$\bar{d}$	portfolio share in deposits
$\bar{g}$	portfolio share in bonds
Others	
c	bank consumption
Ω	risk-adjusted value of bank equity
R^e	return on equity
e	real equity
μ	Lagrange multiplier on capital requirement constraint
V^l .V^b	value of the bank at lending/balancing stage
V	value of the bank as a function of bank equity
Interbank market	
ω	withdrawal shock
s	surplus at beginning of balancing stage after shock ω
θ	market tightness in interbank market
f	interbank market loans
w	discount window loans
Ψ^+	probability that a bank with surplus finds a match
Ψ^-	probability that a bank with deficit finds a match
Aggregates	
E	aggregate reak bank equity
Θ_t^x	intercept demand $x \in \{g, b, d, m\}$
ϵ^x	elasticity of demand $x \in \{g, b, d, m\}$
B,B^f	bank loan supply / firm demand
D, D^h	bank / household deposits
M, M^h	reserve / currency holdings
G, G^h	bank / household gov. bond holdings
P	price level
π	inflation
Government and Fed Policies	
i^m	nominal interest rate on reserves
i^w	nominal interest rate on discount window loans
G^{FA}	Total Supply of government bonds
G^{Fed}	Fed holding of government bonds
M^{Fed}	supply of reserves
B^{Fed}	Fed holdings of private loans
W^{Fed}	discount window loans

A Expressions for $\{\Psi^+, \Psi^-, \phi, \bar{i}^f, \chi^+, \chi^-\}$ in Proposition 1

Here we reproduce formulas derived from Proposition 1 in the companion paper, Bianchi and Bigio (2017). The companion paper includes the market structure that delivers these functional forms. This proposition gives us the formulas for the liquidity yield function and the matching probabilities as functions of the tightness of the interbank market. The formulas are the following.

Given θ, the market tightness after the federal funds trading session is

$$\bar{\theta} = \begin{cases} 1 + (\theta - 1)\exp(\lambda) & \text{if } \theta > 1 \\ 1 & \text{if } \theta = 1 \\ (1 + (\theta^{-1} - 1)\exp(\lambda))^{-1} & \text{if } \theta < 1 \end{cases}.$$

Trading probabilities are given by

$$\Psi^+ = \begin{cases} 1 - e^{-\lambda} & \text{if } \theta \geq 1 \\ \theta\left(1 - e^{-\lambda}\right) & \text{if } \theta < 1 \end{cases}, \qquad \Psi^- = \begin{cases} \left(1 - e^{-\lambda}\right)\theta^{-1} & \text{if } \theta > 1 \\ 1 - e^{-\lambda} & \text{if } \theta \leq 1 \end{cases}. \tag{A.1}$$

The reduced-form bargaining parameter is

$$\phi \equiv \begin{cases} \frac{\theta}{\theta - 1}\left(\left(\frac{\bar{\theta}}{\theta}\right)^{\eta} - 1\right)(\exp(\lambda) - 1)^{-1} & \text{if } \theta > 1 \\ \eta & \text{if } \theta = 1 \\ \frac{\theta(1-\bar{\theta})-\bar{\theta}}{\bar{\theta}(1-\theta)}\left(\left(\frac{\bar{\theta}}{\theta}\right)^{\eta} - 1\right)(\exp(\lambda) - 1)^{-1} & \text{if } \theta < 1 \end{cases},$$

and $\bar{i}^f = (1 - \phi)i^w + \phi i^m$. The slopes of the liquidity yield function are given by

$$\chi^+ = (i^w - i^m)\left(\frac{\bar{\theta}}{\theta}\right)^{\eta}\left(\frac{\theta^{\eta}\bar{\theta}^{1-\eta} - \theta}{\bar{\theta} - 1}\right) \text{ and } \chi^- = (i^w - i^m)\left(\frac{\bar{\theta}}{\theta}\right)^{\eta}\left(\frac{\theta^{\eta}\bar{\theta}^{1-\eta} - 1}{\bar{\theta} - 1}\right). \tag{A.2}$$

B Compendium: Equilibrium Conditions

B.1 Transitional Dynamics

Here we present the set of equilibrium conditions of the model. Given a sequence of government policy $\left\{i_t^m, i_t^w, i_t^g, B_t^{Fed}, M_t^{Fed}, G_t^{FA}, G_t^{Fed}, T_t^h, T_t\right\}$ that satisfies the Fed's and fiscal authorities budget constraint, the system that characterizes equilibrium yields a solution for individual bank variables,$\left\{\bar{b}_t, \bar{a}_t, \bar{d}_t, \bar{c}_t, \Omega_t, v_t\right\}$, aggregate variables, $\{B_t, M_t, D_t, G_t, E_t\}$, and a system of prices and real returns $\{P_t, R_t^b, R_t^m, R_t^g, R_t^d, \bar{\chi}_t{}^+, \bar{\chi}_t{}^-\}$. The system features 18 unknowns to be determined for all t. There is only one endogenous aggregate state variable, E_t from which the entire equilibrium is solved for.

Individual Bank Variables. The portfolio solution to $\left\{\bar{b}_t, \bar{a}_t, \bar{d}_t\right\}$ and the values of $\{\Omega_t, v_t\}$ are the solutions and value of the following problem:

$$\begin{aligned}
&\Omega_t \equiv \max_{\left\{\bar{b},\bar{a},\bar{d}\right\}\geq 0} \left\{\mathbb{E}_\omega \left[R_t^b \bar{b} + \ R_t^m \bar{a} - R_t^d \bar{d} + \bar{\chi}_t(\bar{a}, \bar{d}, \omega)\right]^{1-\gamma}\right\}^{\frac{1}{1-\gamma}}, \qquad \text{(B.1.1)}\\
&\bar{b} + \bar{a} - \bar{d} = 1,\\
&\bar{d} \leq \kappa_t.
\end{aligned}$$

The value of the bank's problem is

$$v_t = \frac{1}{1-\gamma}\left[1 + \left(\beta(1-\gamma)\Omega_t^{1-\gamma} v_{t+1}\right)^{\frac{1}{\gamma}}\right]^{\gamma}. \qquad \text{(B.1.2)}$$

Dividends depend on $\{\Omega_t, v_t\}$ via

$$\bar{c}_t \ = \frac{1}{1 + \left[\beta(1-\gamma) v_{t+1} {\Omega_{t+1}}^{1-\gamma}\right]^{1/\gamma}}. \qquad \text{(B.1.3)}$$

This block of equations yields the equations needed to obtain $\{\bar{b}_t, \bar{a}_t, \bar{d}_t, \bar{c}_t, \Omega_t, v_t\}$ for a given path for real rates $\left\{R_t^b, R_t^m, R_t^d, \bar{\chi}_t\right\}$.

Aggregate Banking Variables. Next, homogeneity in policy functions gives us the aggregate bank portfolio:

$$\begin{aligned}
B_{t+1} &= P_t \bar{b}_t (1 - \bar{c}_t) E_t \qquad \text{(B.1.4)}\\
D_{t+1} &= P_t \bar{d}_t (1 - \bar{c}_t) E_t \qquad \text{(B.1.5)}\\
A_{t+1} &= P_t \bar{a}_t (1 - \bar{c}_t) E_t. \qquad \text{(B.1.6)}
\end{aligned}$$

Real aggregate equity evolves according to

$$E_{t+1} = \frac{P_t\left((1 + i_{t+1}^b)\bar{b}_t + (1 + i_{t+1}^m)\bar{m}_t + (1 + i_{t+1}^g)\bar{g}_t - (1 + i_{t+1}^d)\bar{d}_t\right)(1 - \bar{c}_t)\, E_t - (1 + i_{t+1}^w) W_{t+1} - P_t T_t}{P_{t+1}}. \qquad \text{(B.1.7)}$$

This block of equations determines $\{B_t, G_t, D_t, E_t\}$ given a path for inflation and nominal rates—which together determine real rates—and transfers.

Market Clearing Conditions. The real rates and the path for prices follow from the market clearing conditions in all the asset markets:

$$\frac{B_{t+1} + B_{t+1}^{FED}}{P_t} = \Theta_t^b \left(R_t^b\right)^{\epsilon_b}, \tag{B.1.8}$$

$$\frac{D_{t+1}}{P_t} = \Theta_t^d \left(R_t^d\right)^{\epsilon_d}, \tag{B.1.9}$$

$$M_{t+1}^{Fed} = M_{t+1} + P_t \Theta_t^m \left(R_t^m\right)^{\epsilon_m}, \tag{B.1.10}$$

$$G_{t+1} = G_{t+1}^{FA} - \left(G_{t+1}^{Fed} + P_t \Theta_t^g \left(R_t^g\right)^{\epsilon_g}\right) \tag{B.1.11}$$

$$R_t^m = \frac{1 + i_t^m}{P_{t+1}/P_t}. \tag{B.1.12}$$

Using these market clearing conditions, we can determine $\{\bar{m}, \bar{a}\}$

$$P_t \bar{a}_t \cdot (1 - \bar{c}_t) \cdot E_t = \tilde{M}_{t+1}^{Fed} - M_{t+1}^h + G_{t+1} - G_{t+1}^{Fed} - G_{t+1}^h. \tag{B.1.13}$$

$$M_{t+1}^h + P_t \cdot \bar{m}_t (1 - \bar{c}_t) E_t \;\; = \;\; \tilde{M}_{t+1}^{Fed}. \tag{B.1.14}$$

The last term is the definition of R_t^m. This block determines $\{P_t, R_t^b, R_t^m, R_t^d\}$ given aggregate bank variables. The return for the government bond comes from the clearing of government bonds at the balancing stage. This condition is:

$$R_t^g = \begin{cases} R_t^m + \chi_t^+ & \text{if } P_t \Theta_t^g \left(R_t^m + \chi_t^+\right)^{\epsilon_g} \leq G_t^s - G_t^{Fed} \\ \left[\frac{G_t^s - G_t^{Fed}}{P_t \Theta_t^g}\right]^{1/\epsilon_g} & \text{otherwise} \end{cases}$$

To close the system, we need the equations that determine χ_t.

Interbank Market Block. We need to determine $\bar{\chi}_t$.This follows from the conditions obtained from Proposition 1:

$$\tilde{S}_t^- = (1 - \bar{c}_t) E_t \cdot \int_1^{\omega_t^*} \tilde{s}(\omega) d\Phi \text{ and } \tilde{S}_t^+ = (1 - \bar{c}_t) E_t \cdot \int_{\omega_t^*}^{\infty} \tilde{s}(\omega) d\Phi$$

complete set of where we employ the definition of reserve balances prior to the exchange of government bonds:

$$\tilde{s}(\omega) = \bar{a}_t + \left(\frac{1 + i_{t+1}^d}{1 + i_{t+1}^m}\right) \omega \bar{d}_t - \rho \bar{d}_t (1 + \omega).$$

The market tightness is defined as

$$\theta_t = \frac{\tilde{S}_t^-}{\tilde{S}_t^+ - \bar{g}_t},$$

and the deposit threshold:

$$\omega_t^* \equiv -\frac{\frac{\bar{a}_t}{\bar{d}_t} - \rho}{\frac{R_{t+1}^d}{R_{t+1}^m} - \rho}.$$

From here, discount-window loans are

$$W_t = (1 - \Psi^-(\theta_t)) \left(\tilde{S}_t^- - \bar{g}_t^- \right), \tag{B.1.15}$$

and the average interbank market rate, $\bar{i}_t^f$, is

$$\bar{i}_t^f = \phi(\theta_t) i_t^m + (1 - \phi(\theta_t)) i_t^w.$$

This system of equations gives us

$$\chi_t^- = \Psi_t^- \left(\bar{i}_t^f - i_t^m \right) + \left(1 - \Psi_t^- \right) \left(i_t^w - i_t^m \right) \text{ and } \chi_t^+ = \Psi_t^+ \left(\bar{i}_t^f - i_t^m \right). \tag{B.1.16}$$

Note that here we take the probabilities Ψ_t^- and Ψ_t^+ as given functions of market tightness, as in the main text. This block determines $\bar{\chi}_t$ and the amount of discount window loans, W_t. Note that so far, we have provided enough equations to solve for $\left\{ \bar{b}_t, \bar{a}_t, \bar{d}_t, \bar{c}_t, \Omega_t, v_t \right\}$, $\{B_t, M_t, D_t, G_t, E_t\}$, and $\{P_t, R_t^b, R_t^m, R_t^d, R_t^g, \bar{\chi}_t\}$. The value of W_t enters in the Fed's budget constraint.

Law of Motion for Aggregate Equity. A useful expression is obtained combining the individual laws of motion with the Fed's budget constraint:

$$E_{t+1} = \left(1 + \left(R_{t+1}^b - 1 \right) \left(\bar{b}_t + \bar{b}_t^{Fed} \right) - \left(R_{t+1}^d - 1 \right) \bar{d}_t \right) (1 - \bar{c}_t) E_t \tag{B.1.17}$$

where $\bar{b}_t^{Fed} \equiv B_{t+1}^{Fed} / \left(P_t \cdot (1 - \bar{c}_t) E_t \right)$. Equation (B.1.17) shows that portfolio choices, market returns, and next-period Fed policies and price level determine next-period aggregate real equity.

Consolidated Government Budget Constraint. The government's budget policy sequence $\left\{ i^m, i_t^w, W_t, B_t^{Fed}, M_t^{Fed}, T, T^h \right\}$ satisfies the following constraint:

$$\begin{aligned} (1 + i_t^m) M_t + M_t^h + B_{t+1}^{Fed} - \left(G_{t+1}^{FA} - G_{t+1}^{Fed} \right) + W_{t+1}^{Fed} = \\ M_{t+1}^{Fed} + (1 + i_t^b) B_t^{Fed} - (1 + i_t^g) \left(G_t^{FA} - G_t^{Fed} \right) + (1 + i_t^w) W_t^{Fed} + P_t (T_t + T_t^h). \end{aligned} \tag{B.1.18}$$

and the tax on banks satisfy

$$T_t = (i^m - \pi) \beta \frac{M_t}{P_t} + (i_t^g - \pi_t) \frac{G_t}{P_t} - \left(i_t^b - \pi_t \right) \frac{B_t^{Fed}}{P_t} - (i_t^w - i_t^m) \frac{W_t}{P_t}. \tag{B.1.19}$$

B.2 Stationary Equilibrium

Consider now the equilibrium conditions for a stationary equilibrium. Inflation is constant. These are summarized by replacing time subscripts for steady state subscripts ss. **Individual**

Bank Variables. For the individual bank variables, we have

$$
\begin{aligned}
c_{ss} &= 1-\beta^{\frac{1}{\gamma}}\Omega_{ss}^{1/\gamma-1} && \text{(B.2.1)}\\
v^{ss} &= \frac{1}{1-\gamma}\left(\frac{1}{1-\left(\beta\Omega_{ss}^{1-\gamma}\right)^{\frac{1}{\gamma}}}\right)^{\gamma} && \text{(B.2.2)}\\
\Omega_{ss} &\equiv (1-\tau^{ss})\max_{\left\{\bar{b},\bar{a},\bar{d}\right\}\geq 0}\left\{\mathbb{E}_{\omega}\left[R_{ss}^{b}\bar{b}+R_{ss}^{m}\bar{a}-R_{ss}^{d}\bar{d}+\bar{\chi}(\bar{a},\bar{d})\right]^{1-\gamma}\right\}^{\frac{1}{1-\gamma}} && \text{(B.2.3)}\\
\bar{b}+\bar{a}-\bar{d} &= 1 && \text{(B.2.4)}\\
\bar{d} &\leq \kappa && \text{(B.2.5)}
\end{aligned}
$$

where $\{\bar{b}_{ss},\bar{a}_{ss},\bar{d}_{ss}\}$ are the optimal choices of $\{\bar{b},\bar{a},\bar{d}\}$ in the problem above.

Market Clearing Conditions. The real rates and the path for prices follow from the market clearing conditions in all the asset markets:

$$
\begin{aligned}
\frac{B_{t+1}+B_{t+1}^{FED}}{P_t} &= \Theta_t^b\left(R_t^b\right)^{\epsilon_b}, && \text{(B.2.6)}\\
\frac{D_{t+1}}{P_t} &= \Theta_t^d\left(R_t^d\right)^{\epsilon_d}, && \text{(B.2.7)}\\
M_{t+1}^{Fed} &= M_{t+1}+P_t\Theta_t^m\left(R_t^m\right)^{\epsilon_m}, && \text{(B.2.8)}\\
G_{t+1} &= G_{t+1}^{FA}-\left(G_{t+1}^{Fed}+P_t\Theta_t^g\left(R_t^g\right)^{\epsilon_g}\right) && \text{(B.2.9)}\\
R_t^m &= \frac{1+i_t^m}{P_{t+1}/P_t}. && \text{(B.2.10)}
\end{aligned}
$$

The last term is the definition of R_{ss}^m. This block determines $\{P_t,R_t^b,R_t^m,R_t^d\}$ given aggregate bank variables. The return for the government bond comes from the clearing of government bonds at the balancing stage. This condition is:

$$
R_{ss}^g=\begin{cases}R_{ss}^m+\chi_{ss}^+ & \text{if } \Theta_{ss}^g\left(R_{ss}^m+\chi_{ss}^+\right)^{\epsilon_g}\leq P_t\left(G_{ss}^{FA}-G_{ss}^{Fed}\right)\\ \left[\frac{G_{ss}^{FA}-G_{ss}^{Fed}}{P_t\Theta_{ss}^g}\right]^{1/\epsilon_g} & \text{otherwise}\end{cases}
$$

Notice that in a stationary equilibrium, the price level is pinned down by M_{t+1}^{Fed}, using the demand for reserves and currency. This is because, reserves are obtained as a residual, given the indifference. To close the system, we need the equations that determine χ_t.

Interbank Market Block. We need to determine $\bar{\chi}_{ss}$.This follows from the conditions obtained from Proposition 1:

$$
\tilde{S}_{ss}^{-}=(1-c_{ss})E_{ss}\cdot\int_1^{\omega_{ss}^*}\tilde{s}(\omega)d\Phi \text{ and } S_{ss}^{+}=(1-c_{ss})E_{ss}\cdot\int_{\omega_{ss}^*}^{\infty}\tilde{s}(\omega)d\Phi
$$

where we employ the definition:

$$s(\omega) = \bar{a}_{ss} + \left(\frac{1+i^d_{ss}}{1+i^m_{ss}}\right)\omega\bar{d}_{ss} - \rho\bar{d}_{ss}(1+\omega). \tag{B.2.11}$$

The market tightness is defined as

$$\theta_{ss} = \frac{\tilde{S}^-_{ss}}{\tilde{S}^+_{ss} - \bar{g}_{ss}}. \tag{B.2.12}$$

and the deposit threshold:

$$\omega^*_{ss} \equiv -\frac{\frac{\bar{a}_{ss}}{\bar{d}_{ss}} - \rho}{\left(\frac{R^d_{ss}}{R^m_{ss}} - \rho\right)}. \tag{B.2.13}$$

From here, discount window loans are

$$W_{ss} = (1 - \Psi^-(\theta_{ss}))\tilde{S}^-_{ss}, \tag{B.2.14}$$

and the average interbank market rate

$$\bar{i}^f_{ss} = \phi(\theta_{ss})i^m_{ss} + (1-\phi(\theta_{ss}))i^w_{ss}.$$

This system of equations gives us

$$\chi^-_{ss} = \Psi^-_{ss}\left(\bar{i}^f_{ss} - i^m_{ss}\right) + \left(1 - \Psi^-_{ss}\right)\left(i^w_{ss} - i^m_{ss}\right) \text{ and } \chi^+_{ss} = \Psi^+_{ss}\left(\bar{i}^f_{ss} - i^m_{ss}\right). \tag{B.2.15}$$

Note that here we take the probabilities Ψ^-_{ss} and Ψ^+_{ss} as given functions of market tightness, as in the main text. This block determines $\bar{\chi_{ss}}$ and the amount of discount window loans, W_{ss}.

Law of Motion for Aggregate Equity. The steady state condition for the law of motion of bank equity is:

$$1/\beta = \left(1 + \left(R^b_{ss} - 1\right)\left(\bar{b}_{ss} + \bar{b}^{Fed}_{ss}\right) - \left(R^d_{ss} - 1\right)\bar{d_{ss}}\right) \tag{B.2.16}$$

where $\bar{b}^{Fed}_{ss} \equiv B^{Fed}_{t+1}/\left(P_t \cdot (1-\bar{c}_{ss})E_{ss}\right)$.

Consolidated government budget constraint. The policy sequence satisfies the following consolidated budget constraint:

$$T_{ss}+T^h_{ss} = \left[\left(i^m_{ss} - \pi_{ss}\right)\frac{M_{t+1}}{P_t} + \left(i^g_{ss} - \pi_{ss}\right)\frac{G^{FA}_{t+1} - G^{Fed}_{t+1}}{P_t} - \left(i^b_{ss} - \pi_{ss}\right)\frac{B^{Fed}}{P_t} - \left(i^w_{ss} - i^m_{ss}\right)\frac{W_{t+1}}{P_t}\right] \tag{B.2.17}$$

and the tax on banks, replacing market clearing, satisfies

$$T_{ss} = E_{ss}(1-\bar{c}_{ss})\left[\left(i^m_{ss} - \pi_{ss}\right)\bar{m}_{ss} + \left(i^g_{ss} - \pi_{ss}\right)\bar{g}_{ss} - \left(i^b_{ss} - \pi_{ss}\right)b^{Fed}_{ss} - \left(i^w_{ss} - i^m_{ss}\right)\bar{w}_{ss}\right]. \tag{B.2.18}$$

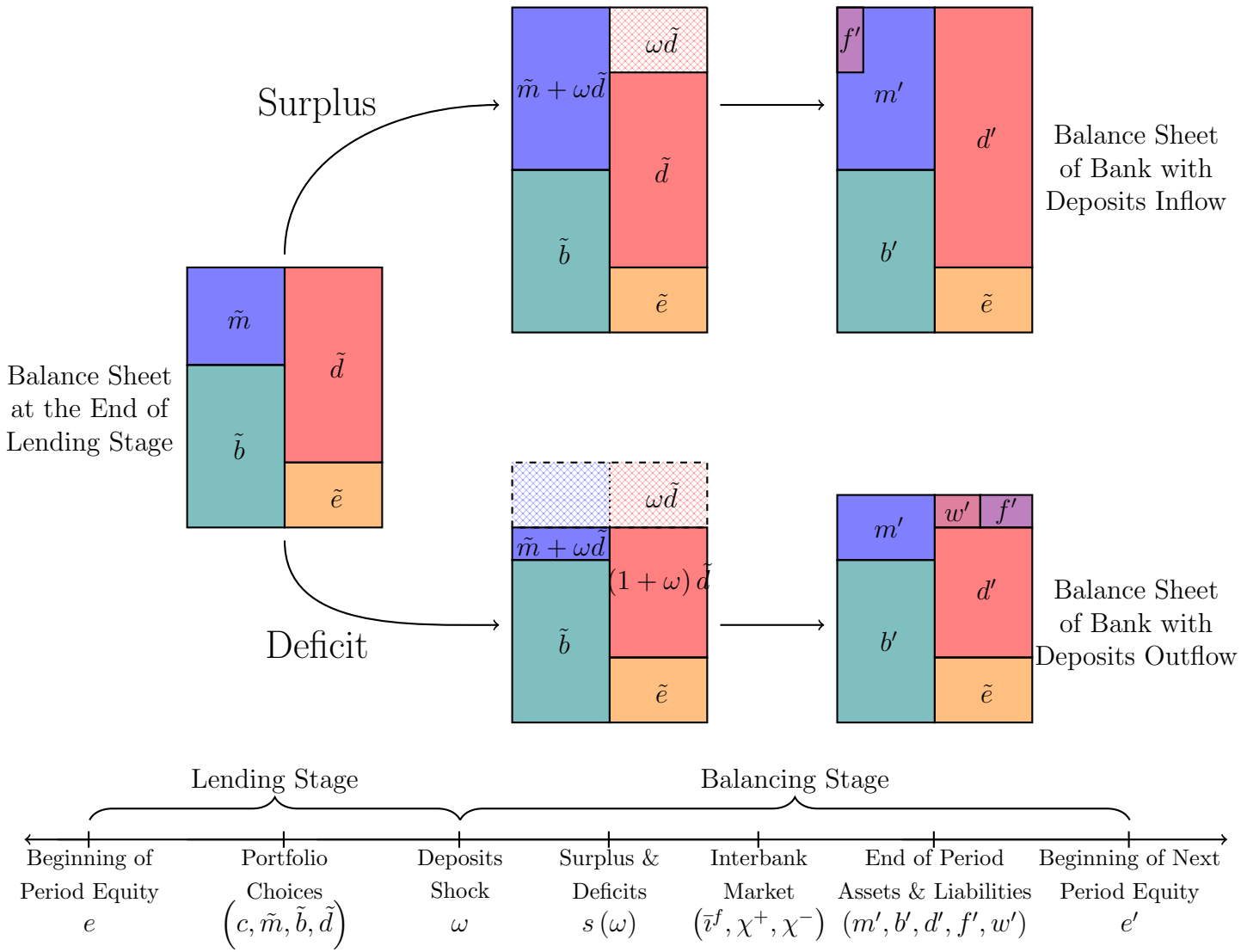

Figure 8: Timeline diagram and banks' balance sheet. For illustration purposes, it is assumed that banks do not accumulate tovernment bonds $g'=0$ and that $(\tilde{m}=\rho\tilde{d})$

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C Non-Financial Sector (Proof of Proposition 2)

This Appendix describes the non-financial sector of the model, which closes the general equilibrium. The non-financial sector is composed of a representative household that supplies labor and stores wealth in deposits, government bonds, currency, and owns shares a representative firm. The firm uses labor for production and is subject to a working capital constraint. This block delivers an endogenous demand schedule for loans, a supply for deposits, and a demand for government bonds. Preference and technology assumptions are such that the equilibrium has no feedback from future state variables to the asset demands at period t. The assumptions make all the schedules static and autonomous. This formulation has two virtues: First, we can solve the equilibrium allocations by solving the equilibrium in the deposit market and loan markets, by solving the bank's problem that takes these schedules as given. From then, since quantities are consistent with an equilibrium demand equation from the non-financial sector, we know it is satisfying market clearing in the labor market. If all asset markets clear, the goods market also clear. The formulation is convenient because it allows us to focus on the banking system, as we can effectively treat these schedules as exogenous functions with exogenous shocks to their intercepts. We exploit this feature in the application.

The non–financial sector is populated by a representative household that saves in deposits, currency, government bonds, and own shares of a productive firm. Assets are special, because different goods are bought with different assets. Similar assumptions are common in new-monetarist models (Lagos et al., 2017). We see this formulation as a convenient way to obtain asset demands. The firm is subject to a working capital constraint, that delivers a demand for loans. The household's Bellman equation is:

$$V_t^h\left(G,M,D,\Upsilon\right)=\max_{\{c^x,X',\Upsilon',h\}}\sum_{x\in\{d,g,m\}}U^x\left(c^x\right)+c^h-\frac{h^{1+\nu}}{1+\nu}+\beta^h V_{t+1}^h\left(G',M',D',\Upsilon'\right)$$

subject to the budget constraint:

$$P_t\left(\sum_{x\in\{d,g,m\}}c^x+c^h\right)+\sum_{X\in\{G,M,D\}}X'+q_t\Upsilon'=\sum_{X\in\{G,M,D\}}\left(1+i_t^X\right)X+\left(q_t+P_tr_t^h\right)\Upsilon+z_th-P_tT_t^h, \tag{C.1}$$

and the following payment constraints:

$$P_tc^d\ \leq\ \left(1+i_t^d\right)D^h, P_tc^g\leq\left(1+i_t^g\right)G^h,\ P_tc^m\leq M. \tag{C.2}$$

In the problem, the household supplies h hours and consumes four types of goods: c^d are goods subject to a deposits in advance constraints, (C.2): c^g goods subject to a bond-in-advance constraint, c^m goods subject to a currency-in-advance constraint. The c^h goods that are not subject to any constraint and yield linear utility. The quasi-linearity in c^h is key to produce the static nature of demand schedules, because it allows to fix marginal utility to one in any Euler equation. Labor supply is h and has an inverse Frisch elasticity of ν, the key parameter for the effect of the loans rate on output. Also, note that β^h is the household's discount factor, that can differ from the bankers discount factor. Equation (C.1) is the household's nominal budget constraint. The right hand side includes the value of the households portfolio of assets,

$G, M,$ and D. These assets earn nominal interest rates paid by banks, the government—currency has no interest. The term Υ are firm shares, which can be normalized to 1.The nominal price of the firm is q_t, firm profits (in real terms) are r_t^h. The wage is z_t is earned on hours worked. Finally, households pay a lump-sum tax T_t^h.

The portfolio $G, M,$ and D matters because each asset is a store of wealth in the budget constraint (C.1), but also, because each asset is a special medium of exchange in the corresponding deposit, bond and currency goods markets. The preference specification (quasi-linear preferences) is identical to the one in Lagos and Wright (2005). Furthermore, the fact that some goods must be bought with specific assets, the transaction technology, is akin to the transaction technology in new monetarist models (Lagos et al., 2017), but the trading protocol stemming from random search is replaced by a Walrasian market. We employ the following utility specification for each good:

$$U^d \equiv \left(\bar{D}_t\right)^{\gamma^d} \frac{\left(c^d\right)^{1-\gamma^d}}{1-\gamma^d}, U^m \equiv \left(\bar{M}_t\right)^{\gamma^m} \frac{\left(c^m\right)^{1-\gamma^m}}{1-\gamma^m}, \text{ and } U^g \equiv \left(\bar{G}_t\right)^{\gamma^g} \frac{\left(c^g\right)^{1-\gamma^g}}{1-\gamma^g},$$

where $\left\{\gamma^d, \gamma^g, \gamma^m\right\} > 0$. This specification delivers an iso-elastic asset demand with $\left\{\bar{D}_t, \bar{M}_t, \bar{G}_t\right\}$ as demand shifters. Notice that if $c^d = \bar{D}_t$,we have $\partial U^d / \partial c^d = 1$. The presence of the linear term c^h in the utility function implies that at the household optimum, we must have $c^d \leq \bar{D}_t$. This bound will be achieved, in effect, when the household is satiated in deposits. The same holds for c^m and c^g.

Next, we present the firm's problem. The firm has access to a production technology that uses h_t^d units of labor that are transformed into $t+1$ output via a production function $y_{t+1} = A_{t+1} h_t^\alpha$. Production is scaled by A_{t+1}, a productivity shock that works as a loan demand shifter. The term A_{t+1} is known at t. The firms uses bank loans to pay workers in the first period to maximize shareholder value:

Problem 3 (Firm's problem)**.**

$$P_{t+1} r_t^h = \max_{\left\{B_{t+1}^d, h_t\right\} \geq 0} P_{t+1} y_{t+1} - \left(1 + i_{t+1}^b\right) B_{t+1}^d + \left(1 + i_{t+1}^d\right) \left(B_{t+1}^d - z_t h_t\right)$$

subject to the working capital constraint, $z_t h_t \leq B_{t+1}^d$.

In the firm's problem, the firm maximize, profits the sum of sales minus financial expenses. The firm borrows B_{t+1}^d from banks and uses these funds to finance payroll, $z_t h_t$. What the firm does not spend, is saved as deposits. Notice that in equilibrium, the firm does not save. The next proposition is a generalized version of Proposition 2.[55] It is more general because it describes the equilibrium solution to the asset demands when asset markets for the household are not necessarily satiated:

Proposition C.1 *The household demand for loans, deposits, and government bonds are given by,*

$$\frac{X_{t+1}^h}{P_t} \begin{cases} = \Theta_t^x \left(R_{t+1}^x\right)^{\epsilon^x} & R_{t+1}^x \leq 1/\beta^h \\ \geq \bar{X}_t & R_{t+1}^x = 1/\beta^h \\ = \infty & \textit{otherwise.} \end{cases} \textit{ for } x \in \{m, d, g\}.$$

[55] We use the superscript h to indicate the aggregate household holdings of a specific asset.

Reduced	Θ^x	ϵ^x	Θ^b	ϵ^b
Structural	$\bar{X}_t \left(\beta^h\right)^{1/\gamma^x}$	$\frac{1}{\gamma^x} - 1$	$(\alpha A_{t+1})^{-\left(\frac{\nu+1}{\alpha-(\nu+1)}\right)}$	$\left(\frac{\nu+1}{\alpha-(\nu+1)}\right)$

Table 4: Structural to Reduced form Parameters

The firm's loan demand is:

$$\frac{B^d_{t+1}}{P_t} = \Theta^b_t \left(R^b_{t+1}\right)^{\epsilon^b}$$

Output and hours are given by:

$$y_{t+1} = \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-(\nu+1)}} A_{t+1}^{\frac{(\nu+1)}{\nu+1-\alpha}} \left(R^b_{t+1}\right)^{\frac{\alpha}{\alpha-(\nu+1)}} \text{ and } h_t = \left(\frac{1}{\alpha A_{t+1}}\right)^{\frac{1}{\alpha-(\nu+1)}} \left(R^b_{t+1}\right)^{\frac{1}{\alpha-(\nu+1)}},$$

and profits and the value of the firm are given by:

$$r^h_{t+1} = A_{t+1}^{\frac{(\nu+1)}{\nu+1-\alpha}} \left(\alpha^{-\frac{\alpha}{\alpha-(\nu+1)}} - \alpha^{-\frac{\nu+1}{\alpha-(\nu+1)}}\right) \cdot \left(R^b_{t+1}\right)^{\frac{\alpha}{\alpha-(\nu+1)}} \text{ and } q_t = \sum_{s\geq 0} \left(\beta^h\right)^s r^h_s.$$

One important note is that R^x_{t+1} for $x \in \{m, d, g\}$ refers to the real return of each asset correspondingly. In the context of the household, R^m_{t+1} is the inverse inflation, not the real rate on reserves. Table 4 is the conversion table from structural parameters to the reduced form parameters of the non-financial sector demand functions.

The rest of the appendix proceeds with the proof.

Proof of Proposition 2.

Derivation of household deposit, bond and currency demands To ease the notation, we remove the h superscripts from Problem C. Define the household's net worth $e^h = \left(1 + i^d_t\right) D + (1 + i^m_t) M + (1 + i^g_t) G + (q_t + r_t) \Upsilon - T^h_t$, as the right-hand side of its budget constraint, excluding labor income. Then, substitute c^h from the budget constraint and employ the definition e^h. We obtain the following value function:

$$\begin{aligned} V^h_t (G, M, D, \Upsilon) &= \max_{\{c^d, c^g, h, G', D', \Upsilon'\}} U^d\left(c^d\right) + U^g\left(c^g\right) + U^m\left(c^m\right) - \frac{h^{1+\nu}}{1+\nu} + e^h \\ &+ \frac{z_t h - \left(P_t c^g + P_t c^d + D' + G' + M' + q_t \Upsilon'\right)}{P_t} \\ &+ \beta^h V^h_{t+1}\left(G', M', D', \Upsilon'\right) \end{aligned} \tag{C.3}$$

subject to the payment in advance constraints in (C.2).

Step 1 - derivation of the deposit, currency, and bond-goods demand. The step is to take the first-order conditions for $\left\{c^d, c^g, c^m\right\}$. Since $\{G, D, M\}$ enter symmetrically into the problem, we express the formulas in terms of $x \in \{d, g, m\}$, an index that corresponds to each

asset. From the first-order conditions in the objective of (C.3) with respect to $\frac{D}{P_t}$, $\frac{G}{P_t}$ and $\frac{M}{P_t}$, we obtain that:

$$(U^x_{c^x}) = 1 + \mu^x_t$$

where $\mu^x_t \geq 0$ are associated multipliers payment in advance constraints in (C.2). The multiplier is activated when $U^x_{c^x} \leq 1$, and thus, $c^x \leq R^x_t \cdot \frac{X}{P_{t-1}}$. Solving for the multiplier to $\mu^x_t = \max\left\{U^x_{c^x}\left(R^x_t \cdot \frac{X}{P_{t-1}}\right) - 1, 0\right\}$. Combining this multiplier yields:

$$c^x(X,t) \;=\; \min\left\{(U^x_{c^x})^{-1}(1), R^x_t \cdot \frac{X}{P_{t-1}}\right\} \text{ for } x \in \{d,g,m\}. \tag{C.4}$$

The expression shows that the deposit- and bond-in-advance constraints bind if the marginal utility associated with their consumption is less than one. Note that

$$U^x_{c^x}\left(\bar{X}\right) = \left(\bar{X}\right)^{\gamma^x} x^{-\gamma^x} \text{ for } x \in \{d,g,m\}, \tag{C.5}$$

marginal utility is above 1, for $X/P_t < \bar{X}$. Then, the marginal consumption as a function of real balances is:

$$\frac{\partial c^x}{\partial\left(X'/P_t\right)} = \begin{cases} R^x_t & X/P_t < \bar{X} \\ 0 & \text{otherwise} \end{cases} \text{ for } x \in \{d,g,m\}.$$

We return to this conditions below to derive the demand for deposits and bonds by the non-financial sector.

Step 2 - labor supply. The first-order condition with respect to labor supply yields a labor supply that only depends on the real wage:

$$h^\nu_t = z_t/P_t. \tag{C.6}$$

Step 3 - deposit and bond demand. Next, we the derive household demand for deposits, government bond and currency. By taking first-order conditions with respect to D'/P_t, G'/P_t, and M'/P_t the real balances of deposits, bonds and currency:

$$1 \;=\; \beta^h \frac{\partial V^h_{t+1}}{\partial\left(X'/P_t\right)} = \beta^h\left[\frac{\partial U^x}{\partial c^x}\cdot\frac{\partial c^x}{\partial\left(X'/P_t\right)} + \frac{\partial U^h}{\partial c^h}\cdot\frac{\partial c^h}{\partial\left(X'/P_t\right)}\right] \text{ for } x \in \{d,g,m\}.$$

The first equality follows directly from the first-order condition and the second uses the envelope Theorem and the solution for the optimal consumption rule. If we shift the period in (C.4), by one period, the first-order condition then becomes:

$$\frac{1}{\beta^h} = \begin{cases} \frac{\partial U^x}{\partial c^x} R^x_t & X/P_t < \bar{X} \\ R^x_t & \text{otherwise} \end{cases} \text{ for } \text{ for } x \in \{d,g,m\}.$$

Finally, once we employ the definition of marginal utility, we obtain:

$$\frac{1}{\beta^h} = \begin{cases} \left(\bar{X}\right)^{\gamma^x}\left(R^x_t X/P_t\right)^{-\gamma^x} R^x_t & X/P_t < \bar{X} \\ R^x_t & \text{otherwise} \end{cases} \text{ for } \text{ for } x \in \{d,g,m\}.$$

Inverting the condition yields:

$$X/P_t = \begin{cases} \bar{X}\left(\beta^h\right)^{1/\gamma^x}\left(R_t^x\right)^{\frac{1}{\gamma^x}-1} & R_t^x < 1/\beta^h \\ [\bar{X},\infty) & R_t^x = 1/\beta^h \\ \infty & R_t^x > 1/\beta^h \end{cases} \text{ for } x \in \{d,g,m\}.$$

Thus, we have that

$$\Theta_t^x = \bar{X}_t\left(\beta^h\right)^{1/\gamma^x} \text{ and } \epsilon^x = \frac{1}{\gamma^x} - 1 \text{ for } x \in \{d,g\}.$$

This verifies the functional form for the household demand schedules. Next, we move to the firm's problem to obtain the demand for loans.

Firm Problem. In this appendix, we allow the firm to save in deposits whatever it doesn't spend in wages. From firm's problem, if we substitute the production function into the objective we obtain:

$$P_{t+1}r_{t+1}^h = \max_{B_{t+1}^d \geq 0, x_{t+1}, h_t \geq 0} P_{t+1}A_{t+1}h_t^\alpha - \left(1+i_{t+1}^b\right)B_{t+1}^d + \left(1+i_{t+1}^d\right)\left(B_{t+1}^d - z_t h_t\right)$$

subject to $z_t h_t \leq B_{t+1}^d$. Observe that

$$\begin{aligned} P_{t+1}A_{t+1}h_t^\alpha - \left(1+i_{t+1}^b\right)B_{t+1}^d + \left(1+i_{t+1}^d\right)\left(B_{t+1}^d - z_t h_t\right) \\ = P_{t+1}A_{t+1}h_t^\alpha - z_t h_t - \left(i_{t+1}^b - i_{t+1}^d\right)\left(B_{t+1}^d + z_t h_t\right). \end{aligned}$$

Step 4 - loans demand. Since $i_{t+1}^b \geq i_{t+1}^d$, then it is without without loss of generality, that the working capital constraint is binding, $z_t h_t = B_{t+1}^d$. Thus, the objective is

$$P_{t+1}A_{t+1}h_t^\alpha - \left(1+i_{t+1}^b\right)z_t h_t.$$

The first-order condition in h_t yields

$$P_{t+1}\alpha A_{t+1}h_t^\alpha = \left(1+i_{t+1}^b\right)z_t h_t.$$

Dividing both sides by P_t, we obtain

$$\frac{P_{t+1}}{P_t}\alpha A_{t+1}h_t^\alpha = \left(1+i_{t+1}^b\right)\frac{z_t}{P_t}h_t.$$

Next, we use the labor supply function (C.6), to obtain the labor demand as a function of the loans rate:

$$\frac{P_{t+1}}{P_t}\alpha A_{t+1}h_t^\alpha = \left(1+i_{t+1}^b\right)h_t^{\nu+1} \rightarrow R_t^b = \frac{\alpha A_{t+1}h_t^\alpha}{h_t^{\nu+1}}. \tag{C.7}$$

Once we have the wage bill, and the fact that the working capital constraint is biding,

$$\frac{B_{t+1}^d}{P_t} = h_t \frac{z_t h_t}{P_t} = h_t^{\nu+1} \rightarrow h_t = \left(\frac{B_{t+1}^d}{P_t}\right)^{\frac{1}{\nu+1}}. \tag{C.8}$$

We combine (C.7) and (C.8) to obtain the demand for loans:

$$R_t^b = \alpha A_{t+1} \left(\frac{B_{t+1}^d}{P_t}\right)^{-1} \left(\frac{B_{t+1}^d}{P_t}\right)^{\frac{\alpha}{\nu+1}} \rightarrow \frac{B_{t+1}^d}{P_t} = \Theta_t \left(R_{t+1}^b\right)^{\epsilon^b} \tag{C.9}$$

Thus, the coefficients of the loans demand are

$$\Theta_t^b = \left(\alpha A_{t+1}\right)^{-\epsilon^b} \text{ and } \epsilon^b = \left(\frac{\nu+1}{\alpha-(\nu+1)}\right).$$

This concludes the elements of the proposition. Next, we present the formulas for hours, output and the market price of shares.

Step 5 - Equilibrium Output and Hours. We replace the loans demand (C.9) into (C.8), to obtain the labor market equilibrium:

$$h_t = \left(\frac{1}{\alpha A_{t+1}}\right)^{\frac{1}{\alpha-(\nu+1)}} \left(R_{t+1}^b\right)^{\frac{1}{\alpha-(\nu+1)}}.$$

We replace (C.8) into the production function to obtain:

$$y_{t+1} = A_{t+1} \left(\frac{1}{\alpha A_{t+1}}\right)^{\frac{\alpha}{\alpha-(\nu+1)}} \left(R_{t+1}^b\right)^{\frac{\alpha}{\alpha-(\nu+1)}} \rightarrow y_{t+1} = \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-(\nu+1)}} A_{t+1}^{\frac{(\nu+1)}{\nu+1-\alpha}} \left(R_{t+1}^b\right)^{\frac{\alpha}{\alpha-(\nu+1)}}.$$

The profit of the firm is given by:

$$r_{t+1}^h = y_{t+1} - R_{t+1}^b B_{t+1} \rightarrow r_{t+1}^h = A_{t+1}^{\frac{(\nu+1)}{\nu+1-\alpha}} \left(\alpha^{-\frac{\alpha}{\alpha-(\nu+1)}} - \alpha^{-\frac{\nu+1}{\alpha-(\nu+1)}}\right) \cdot \left(R_{t+1}^b\right)^{\frac{\alpha}{\alpha-(\nu+1)}}.$$

Step 6 - Market Price. The asset price q_t then is determine as:

$$q_t = \sum_{s \geq 1} \left(\beta^h\right)^s r_s^h.$$

With this, we conclude that output, hours and the firm price are decreasing in current (and future) loans rate. Throughout the proof we use the labor market clearing condition, so this market clears independently of other markets. Thus, once we compute equilibria taking the schedules as exogenous in the bank's problem, it is possible to obtain output, hours, and household consumption from the equilibrium rates. By Walras's law, if asset markets clear, so does the goods market.

D Law of Motion for Aggregate Equity and Transfers

D.1 Disaggregate and Consolidate Government Budget Constraints

We present here the budget constraint of the monetary and fiscal authority separately and show how the consolidation leads to (10).

The Fed's budget constraint during the lending stage of period t is given by:

$$\left(\tilde{M}_t^{Fed} - M_t^h\right)(1+i_t^m) + B_{t+1}^{Fed} + G_{t+1}^{Fed} = \tilde{M}_{t+1}^{Fed} + W_t^{Fed}(i_t^w - i_t^m) + B_t^{Fed}(1+i_t^b) + G_t^{Fed}(1+i_t^g) + P_t T_t - INT_t. \quad \text{(D.1)}$$

The left-hand side are the uses of funds. The Fed used funds to pay for the interest on the holdings of reserves, which equals the money supply $\tilde{M}_t^{Fed}$ minus the currency holdings M_t^h by households, to buy new loans B_{t+1}^{Fed}, and to buy government bonds G_{t+1}^{Fed}. The Fed obtains funds by issue reserves $\tilde{M}_{t+1}^{Fed}$,through the income flow generated by the discount window, $W_t^{Fed}(i_t^w - i_t^m)$, the value of the current portfolio of loans and government bonds, B_t^{Fed} and G_t^{Fed}, plus taxes on banks $P_t T_t$ and internal transfers from the Fed to the fiscal authority, INT_t.

During the balancing stage of period t, the budget constraint is:

$$M_{t+1}^{Fed} = \tilde{M}_{t+1}^{Fed} + W_{t+1}^{Fed}. \quad \text{(D.2)}$$

This budget constraint simply tracks the increase in reserves that results from discount loans.

We can combining these two constraints, (D.1) and (D.2), and also use $t-1$ period version of (D.2), to obtain the balance sheet of the Fed from one lending stage to the other:

$$(1+i_t^m)\left(M_t^{Fed} - M_t^h\right) + B_{t+1}^{Fed} + G_{t+1}^{Fed} + W_{t+1}^{Fed} = M_{t+1}^{Fed} + (1+i_t^w)W_t^{Fed} + (1+i_t^b)B_t^{Fed} + (1+i_t^g)G_t^{Fed} + P_t T_t - INT_t. \quad \text{(D.3)}$$

The fiscal authority's budget constraint at t is given by,

$$(1+i_t^g)\, G_t^{FA} = G_{t+1}^{FA} + P_t T_t + INT_t. \quad \text{(D.4)}$$

In this expression, the left-hand side is the value of government bonds inclusive of the interest rate. For the right hand side, G_{t+1}^{FA} are new issuances of government bonds, T_t^h are transfers to households, which we derive in Section D.3 of this appendix.

We can substitute INT_t from the this budget constraint into (D.3) to obtain a consolidated version of the budget constraint:

$$(1+i_t^m)\left(M_t^{Fed} - M_t^h\right) + (1+i_t^g)G_t^{Gov} + B_{t+1}^{Fed} + W_{t+1}^{Fed} = M_{t+1}^{Fed} + G_{t+1}^{Gov} + (1+i_t^b)B_t^{Fed} + (1+i_t^w)W_t^{Fed} + P_t\left(T_t + T_t^h\right). \quad \text{(D.5)}$$

where $G_t^{Gov} \equiv G_t^{FA} - G_t^{Fed}$,is the issuances of government bonds, net of Fed holdings.

D.2 Law of Motion for Bank Equity

Law of Motion of Real Aggregate Bank Equity. The law of motion of real aggregate bank equity depends on assumptions made on the amount of transfers to banks. We choose transfers to keep to isolate the effects.

On the bank's side, replace $m = \tilde{m} + f + w$, on the individual equity (14) to obtain:

$$e_t^j = \left(\frac{\tilde{m}_t^j(1+i_t^m) + \tilde{g}_t^j(1+i_t^g) + \tilde{b}_t^j(1+i_t^b) - \tilde{d}_t^j(1+i_t^d) - w_t^j(i_t^w - i_t^m) - f_t^j(\bar{i}_t^f - i_t^m) - P_t T_t^j}{P_t} \right). \tag{D.1}$$

We iterate this law of motion forward one period, and integrate across banks, using the market clearing conditions to obtain:

$$E_{t+1} = \frac{\left(\tilde{M}_{t+1} - M_{t+1}^h\right)(1+i_{t+1}^{ior}) + \tilde{G}_{t+1}(1+i_{t+1}^g) + \tilde{B}_{t+1}(1+i_{t+1}^b)}{P_{t+1}} \\ \frac{-\tilde{D}_{t+1}(1+i_{t+1}^d) - W_{t+1}(i_{t+1}^w - i_{t+1}^{ior}) - P_{t+1}\int T_{t+1}^j dj}{P_{t+1}}.$$

Multiplying and dividing by P_t on each term on the right hand side, we obtain:

$$E_{t+1} = R_{t+1}^m \left(\tilde{M}_{t+1} - M_{t+1}^h\right) + R_{t+1}^g \tilde{G}_{t+1} + R_{t+1}^b \tilde{B}_{t+1} - R_{t+1}^d \tilde{D}_{t+1} - (R_{t+1}^w - R_{t+1}^m)\frac{W_{t+1}}{P_t} - \int T_{t+1}^j dj.$$

Now substitute the definition of portfolio shares from Proposition 3, to obtain

$$\begin{aligned} E_{t+1} &= (R_{t+1}^b \bar{b}_t + R_{t+1}^m \bar{m}_t + R_{t+1}^g \bar{g}_t - R_{t+1}^d \bar{d}_t) E_t(1-\bar{c}_t) - (R_{t+1}^w - R_{t+1}^m)\frac{W_{t+1}}{P_t} - \int T_{t+1}^j dj \\ &= (\bar{a}_t + R_{t+1}^b \bar{b}_t - R_{t+1}^d \bar{d}_t) E_t(1-\bar{c}_t) + \left(R_{t+1}^m - 1\right)\bar{m}_t + \left(R_{t+1}^g - 1\right)\bar{g}_t \\ &\quad -(R_{t+1}^w - R_{t+1}^m)\frac{W_{t+1}}{P_t} - \int T_{t+1}^j dj. \end{aligned} \tag{D.2}$$

Next, we demonstrate how two different tax rates for banks, lead to different laws of motions for aggregate bank equity.

Tax Protocol. Consider a tax scheme that returns the nominal interest minus and the arbitrage income earned on banks, equation (11) shifted one period forward. We have that:

$$T_t = \int T_{t+1}^j dj = \left(i_{t+1}^{ior} - \pi_{t+1}\right) \frac{\tilde{M}_{t+1}}{P_{t+1}} + \left(i_{t+1}^{g} - \pi_{t+1}\right) \frac{\tilde{G}_{t+1}}{P_{t+1}} \dots$$
$$- \left(i_{t+1}^{b} - \pi_{t+1}\right) \frac{B_{t+1}^{Fed}}{P_{t+1}} - (i_{t+1}^{w} - i_{t+1}^{ior}) \frac{W_{t+1}}{P_{t+1}}.$$

Observe the equality,

$$i_{t+1}^{x} - \pi_{t+1} = 1 + i_{t+1}^{x} - 1 - \pi_{t+1} = \left(R_{t+1}^{x} - 1\right) \frac{P_{t+1}}{P_t}.$$

The tax is rearranged to the following expression:

$$\int T_{t+1}^j dj = \left(R_{t+1}^{m} - 1\right) \cdot \frac{P_{t+1}}{P_t} \frac{\tilde{M}_{t+1}}{P_{t+1}} + \left(R_{t+1}^{g} - 1\right) \frac{P_{t+1}}{P_t} \frac{\tilde{G}_{t+1}}{P_{t+1}} \dots$$
$$- \left(R_{t+1}^{b} - 1\right) \frac{P_{t+1}}{P_t} \frac{B_{t+1}^{Fed}}{P_{t+1}} - (R_{t+1}^{w} - R_{t+1}^{m}) \frac{W_{t+1}}{P_t}. \quad \text{(D.3)}$$

Equilibrium law of motion. Then, substitute the tax (D.3) inside the law of motion (D.2) to obtain:

$$\begin{aligned} E_{t+1} &= (\bar{m}_t + \bar{g}_t + R_{t+1}^{b} \bar{b}_t - R_{t+1}^{d} \bar{d}_t) E_t(1 - \bar{c}_t) + \left(R_{t+1}^{b} - 1\right) \bar{b}_t^{Fed} E_t(1 - \bar{c}_t) \\ &= \left(1 + \left(R_{t+1}^{b} - 1\right) \bar{b}_t - \left(R_{t+1}^{d} - 1\right) \bar{d}_t\right) E_t(1 - \bar{c}_t) + \left(R_{t+1}^{b} - 1\right) \bar{b}_t^{Fed} E_t(1 - \bar{c}_t) \\ &= \left(1 + \left(R_{t+1}^{b} - 1\right) \left(\bar{b}_t + \bar{b}_t^{Fed}\right) - \left(R_{t+1}^{d} - 1\right) \bar{d}_t\right) E_t(1 - \bar{c}_t), \end{aligned} \quad \text{(D.4)}$$

where in the second line we use $\bar{b}_t + \bar{m}_t = 1 + \bar{d}_t$. We also use the definition where $\bar{b}_t^{Fed} \equiv B_{t+1}^{Fed} / \left(P_t(1 - \bar{c}_t) E_t\right)$. This is the law of motion for aggregate equity that appears in the body of the text, equation (19).

D.3 Household Transfers

Next, recall that clearing in the market for money is given by:

$$\tilde{M}_t^{Fed} = \tilde{M}_t + M_t^h.$$

and similarly for government bonds:

$$G_t^{FA} = G_t^{Fed} + G_t + G_t^h.$$

Recall that:

$$\int T_t^j dj = i_t^m \frac{\tilde{M}_t}{P_t} + i_t^g \frac{\tilde{G}_t}{P_t} + i_t^b \frac{\tilde{B}_t^{Fed}}{P_t} - \frac{W_t^{Fed}(i_t^w - i_t^m)}{P_t}.$$

To derive the law of motion for aggregate equity, we combine the consolidate the Fed's budget constraint with the budget constraint of the fiscal authority. Then, we net out the holdings of household bonds. From the resulting budget, we combine this with the bank's budget constraint

and use the market clearing conditions. Next, take the budget constraint of the consolidated government. Similarly, we can add the fiscal authority's budget constraint into (10) to obtain:

$$P_t\left(T_t^h+T_t\right)=\tilde{M}_t^{Fed}(1+i_t^m)+G_t^{Gov}(1+i_t^g)+B_{t+1}^{Fed}-\tilde{M}_{t+1}^{Fed}-G_{t+1}^{Gov}-W_t^{Fed}\left(i_t^w-i_t^m\right)-B_t^{Fed}(1+i_t^b),$$

and using the clearing conditions and expressions for government debt we obtain:

$$T_t^h=\frac{G_t^h(1+i_t^g)-G_{t+1}^h+\Delta B_t^{Fed}-\Delta\tilde{M}_t^{Fed}-\Delta\tilde{G}_t}{P}.$$

E Proof of Proposition 3

E.1 Proof of Item (i)

E.1.1 Steps in the Proof of Item (i) of Proposition 3

The proof of item (i) is carried out in four steps. Along the proof, we prove results in greater generality, for the case where the supply of government bonds is enough to satisfy all payment needs by banks, as assumed in the body of the text. We specialize the results to the case considered in the paper. The sequence of steps are the following:

Step 1. We first show that there exists a function $V_t(e)$ with a single state variable, e, such that $V_t(e)=V_t^l(g,b,m,d,f,w)$ where e is defined in terms of end-of-balancing stage variables:

$$e\equiv\frac{(1+i_t^m)m+\left(1+i_t^b\right)b-\left(1+i_t^d\right)d+(1+i_t^g)\,g-(1+\bar{i}_t^f)f-(1+i_t^w)\,w}{P_t}(1-\tau_t).$$

Step 2. The second step is to , we show that $V_t^b(\tilde{g},\tilde{b},\tilde{m},\tilde{d},\omega)=\beta V_t(e')$ is given by:

$$e'=\left(R_{t+1}^b\frac{\tilde{b}}{P_t}+R_{t+1}^m\frac{(\tilde{m}+\tilde{g})}{P_t}-R_{t+1}^d\frac{\tilde{d}}{P_t}+Z(\tilde{s})\right)(1-\tau_{t+1})$$

where

$$Z(\tilde{s})=\max_{g'\geq 0}\left(R_{t+1}^g-R_{t+1}^m\right)g'+\bar{\chi}_{t+1}(\tilde{s}-g')).$$

In tandem with Step 1, this shows that there's a recursive representation for the bank's problem, with a Bellman equation that depends exclusively on equity $V_t(e)$.

Step 3. The third step is to characterize the equilibrium in the government bonds market during the balancing stage. Two special cases of that characterization are considered in the paper. If banks hold government bonds at the lending stage, and there is a surplus of reserves after the government bond market opens at the balancing stage, then we can show that $R_{t+1}^g=R_{t+1}^m+\bar{\chi}_{t+1}^+$. The other case occurs when $R_t^g<R_t^m+\chi_t$, and in that case, we show that banks cannot hold government bonds during the lending stage—only households hold them.

Step 4. Then final step is to show that if $R^g_{t+1} \leq R^m_{t+1} + \bar{\chi}^+_{t+1}$, future equity can be written in terms of end-of-lending stage variables:

$$e' = \left[R^b_{t+1} \frac{\tilde{b}}{P_t} + R^m_{t+1} \left(\frac{\tilde{m}}{P_t} + \frac{\tilde{g}}{P_t} \right) - R^d_{t+1} \frac{\tilde{d}}{P_t} + \bar{\chi}_{t+1} \left(\frac{\tilde{g}}{P_t} + \frac{\tilde{m}}{P_t}, \frac{\tilde{d}}{P_t}, \omega \right) \right] (1 - \tau_{t+1}).$$

This last step is key to show that banks are indifferent between the composition of their liquid assets. We conclude the proof with a set of sufficient conditions that guarantee that $R^g_{t+1} \leq R^m_{t+1} + \bar{\chi}^+_{t+1}$ in equilibrium. If the condition is not satisfied, equity can be written recursively, but the function $\bar{\chi}_{t+1}$ is slightly modified. We proceed with the formal steps of the proof by establishing the following propositions:

Proposition E.1 *For any t, there exists a function $V_t(e)$ that yields the value of the bank's problem at the lending stage. In particular, $V_t(e) = V^l_t(g, b, m, d, f, w)$ for*

$$e \equiv \frac{(1 + i^m_t)m + \left(1 + i^b_t\right) b - \left(1 + i^d_t\right) d + (1 + i^g_t)\, g - (1 + \bar{i}^f_t) f - (1 + i^w_t)\, w}{P_t} (1 - \tau_t).$$

This proposition shows that we can define the value at the lending stage through a value function V_t that depends on a notion of bank equity, regardless of the composition of the banks' balance sheet. Once we obtain this result, we solve the problem at the balancing stage, and obtain a recursive expression for V_t. We need to define the balance of reserves that each bank starts with during the balancing stage, considering the value of treasury bills at the lending stage, prior to the trade of government bonds at the balancing stage.[56] This is balance is defined as

$$\tilde{s}(\tilde{g}, \tilde{m}, \tilde{d}, \omega) \equiv \tilde{g} + \tilde{m} + \left(\frac{R^d_{t+1}}{R^m_{t+1}} \right) \omega \tilde{d} - \rho \tilde{d}(1 + \omega).$$

The following proposition is an intermediate step toward characterizing the value during the balancing stage, exclusively in terms of variables chosen at the lending stage. The goal is to find a single Bellman equation for V_t without reference to the transactions that occur during the balancing stage.

Proposition E.2 *For any t, the value at the balancing stage satisfies $V^b_t(\tilde{g}, \tilde{b}, \tilde{m}, \tilde{d}, \omega) = \beta V_t(e')$ where*

$$e' = \left(R^b_{t+1} \frac{\tilde{b}}{P_t} + R^m_{t+1} \frac{(\tilde{m} + \tilde{g})}{P_t} - R^d_{t+1} \frac{\tilde{d}}{P_t} + Z\left(\tilde{s}(\tilde{g}, \tilde{m}, \tilde{d}, \omega) \right) \right) (1 - \tau_{t+1})$$

and

$$Z(\tilde{s}) = \max_{g' \geq 0} \left(R^g_{t+1} - R^m_{t+1} \right) g' + \bar{\chi}_{t+1}(\tilde{s} - g')). \tag{E.1}$$

Furthermore, the solution to g' in $Z(\tilde{s})$ is the solution to g' in V^b_t. As a result, we can express

[56] Notice that this balance is not the balance with which they end the lending stage, nor the balance with with they end the balancing stage. Rather, its the balance computed as if all banks would sell all their government bonds. The policy functions during the lending stage are characterized by $\tilde{s}$.

V_t *recursively,*

$$\begin{aligned} V_t(e) &= \max_{\{c,\tilde{a},\tilde{b},\tilde{d}\}\geq 0,\tilde{d}\in[0,\kappa]} u(c)+\beta\mathbb{E}\left[V_{t+1}(e')\right], \text{ subject to} \qquad \text{(E.2)} \\ \tfrac{\tilde{a}}{P_t} &+\frac{\tilde{b}}{P_t}-\frac{\tilde{d}}{P_t}+c=e \\ e' &= \left[R^b_{t+1}\frac{\tilde{b}}{P_t}+R^m_{t+1}\frac{(\tilde{m}+\tilde{g})}{P_t}-R^d_{t+1}\frac{\tilde{d}}{P_t}+Z\left(\tilde{s}\right)\right](1-\tau_{t+1}). \qquad \text{(E.3)} \end{aligned}$$

Proposition E.2 uses that the value function during the balancing stage equals $\beta V_t\left(e'\right)$ and shows that e' can be written in terms of the state $\left(\tilde{g},\tilde{b},\tilde{m},\tilde{d},\omega\right)$ and the value of the auxiliary problem in $Z\left(\tilde{s}\right)$. The auxiliary problem is the optimal choice of g' in the balancing stage that maximizes future equity. Since the objective at the balancing stage is to maximize the value at the lending stage, but we showed that the value at the lending stage can be written only in terms of equity, the solution to the auxiliary problem is the solution to the problem at the balancing stage. Next, Proposition E.3 characterizes the solution $Z\left(\tilde{s}\right)$. The optimal choice of g' depends on the liquidity premium of the government bond:

Proposition E.3 *The solution to* $Z\left(\tilde{s}\right)$ *in Proposition E.2 is given by:*

Region 1. If $R^g_t > R^m_t+\bar{\chi}^-_t$*, then*

$$g'=\infty \text{ for any } \tilde{s}.$$

Region 2. If $R^g_t < R^m_t+\bar{\chi}^+_t$ *then,*

$$g'=0 \text{ for any } \tilde{s}.$$

Region 3. If $R^g_t = R^m_t+\bar{\chi}^-_t$

$$g'=\begin{cases}\tilde{s} & \tilde{s}\geq 0\\ [0,\infty] & \tilde{s}<0,\end{cases} \text{ and } Z\left(\tilde{s}\right)=\begin{cases}\bar{\chi}^-_t\tilde{s} & \tilde{s}\geq 0\\ \bar{\chi}^-_t\tilde{s} & \tilde{s}<0.\end{cases}$$

Region 4. If $R^g_t = R^m_t+\bar{\chi}^*_t\in\left(R^m_t+\bar{\chi}^+_t, R^m_t+\bar{\chi}^-_t\right)$

$$g'=\begin{cases}\tilde{s} & \tilde{s}\geq 0\\ 0 & \tilde{s}<0,\end{cases} \text{ and } Z\left(\tilde{s}\right)=\begin{cases}\bar{\chi}^*_t\tilde{s} & \tilde{s}\geq 0\\ \bar{\chi}^-_t\tilde{s} & \tilde{s}<0.\end{cases}$$

Region 5. If $R^g_t = R^m_t+\bar{\chi}^+_t$

$$g'=\begin{cases}[0,\tilde{s}] & \tilde{s}\geq 0\\ 0 & \tilde{s}<0,\end{cases} \text{ and } Z\left(\tilde{s}\right)=\begin{cases}\bar{\chi}^+_t\tilde{s} & \tilde{s}\geq 0\\ \bar{\chi}^-_t\tilde{s} & \tilde{s}<0.\end{cases}$$

Proposition E.3 characterizes the solution and value of the individual bank's problem of choosing g'. Next, we use the policy functions obtained in E.3 to find the possible range of equilibrium government bond rates. It is useful to define the threshold shock that produces a deficit consider-

ing the sales of government bonds, ω^*, as $\omega^* = -\left(\left(\tilde{m}+\tilde{g}\right)/\tilde{d}-\rho\right)/\left(R^d_{t+1}/R^m_{t+1}-\rho\right)$. Clearing in the government bond market during the balancing stage requires the following equation to hold:

$$\tilde{G}_t = \int_{-1}^{\infty} g'(\tilde{g}^j,\tilde{m}^j,\tilde{d}^j,\omega^j)dF\left(\omega^j\right) \tag{E.4}$$

$$= \int_{\omega^*}^{\infty} g'(\tilde{g}^j,\tilde{m}^j,\tilde{d}^j,\omega^j)dF\left(\omega^j\right) + \int_{-1}^{\omega^*} g'(\tilde{g}^j,\tilde{m}^j,\tilde{d}^j,\omega^j)dF\left(\omega\right). \tag{E.5}$$

We make the following remarks:

Corollary E.1 *In any equilibrium, $R^g_t \leq R^m_t + \bar{\chi}^-_t$. Furthermore, if $R^g_t < R^m_t + \bar{\chi}^+_t$ then, $\tilde{g} = 0$.*

The proof follows directly from Proposition E.3: If $R^g_t > R^m_t + \bar{\chi}^-_t$ we are in Region 1 in Proposition E.3, but since the supply of government bonds is finite, this case cannot occur in equilibrium and satisfy E.4. If if $R^g_t < R^m_t$, we are in Region 2 in Proposition E.3. Thus, it must be that $\tilde{g} = 0$ during the lending stage. We are left with the characterization of the market equilibrium when $R^g_t \in \left[R^m_t + \bar{\chi}^+_t, R^m_t + \bar{\chi}^-_t\right]$.

The next Proposition characterizes the equilibrium market equilibrium as function of the aggregate portfolio holdings during the lending stage $\left\{\tilde{G},\tilde{M},\tilde{D}\right\}$ for the cases where $R^g_t \in \left[R^m_t + \bar{\chi}^+_t, R^m_t + \bar{\chi}^-_t\right]$. The prevailing equilibrium return of government bonds depends on whether there is a large enough surplus of government bonds relative aggregate reserve-balance deficit in the interbank market. To simplify the calculations in the characterization, without loss of generality, we use the portfolio of the representative bank. Item (iv) of Proposition 3 indeed verifies that the model has a representative bank. We summarize this condition excess function of government bonds is captured by:

$$\Gamma\left(\tilde{G},\tilde{M},\tilde{D}\right) \equiv \int_{\max\{\omega^*,-1\}}^{\infty} \underbrace{\left(\tilde{s}(\tilde{G},\tilde{M},\tilde{D},\omega)-\tilde{G}\right)}_{\text{Surplus of Reserves}} dF(\omega) - \underbrace{\tilde{G}F\left(\max\left\{\omega^*,-1\right\}\right)}_{\text{Gov. bonds held by Banks in Deficit}} \tag{E.6}$$

$$= \int_{\max\{\omega^*,-1\}}^{\infty} \tilde{s}(\tilde{G},\tilde{M},\tilde{D},\omega)dF(\omega) - \tilde{G}. \tag{E.7}$$

The following Lemma is used to show that the equilibrium prices must be unique given aggregate portfolio holdings.

Lemma E.1 *Γ is decreasing and convex in $\tilde{G}$, and has the following limits: $\Gamma(0) > 0$ and $\lim_{\tilde{g}\to\infty}\Gamma(\tilde{g}) = \tilde{m} - \rho\tilde{d}$.*

We obtain the following characterization.

Proposition E.4 *The equilibrium rates are given by:*

Case 1: If $\Gamma\left(\tilde{G},\tilde{M},\tilde{D}\right) < 0$ then $S^+_t = 0,\ S^-_t > 0$ and,

$$R^g_t = R^m_t + \bar{\chi}^-_t, \bar{\chi}^+_t = \left(R^w_t - R^m_t\right)\left(1 - e^{-\bar{\lambda}(1-\eta)}\right), \bar{\chi}^-_t = \left(R^w_t - R^m_t\right).$$

Case 2: If $\Gamma\left(\tilde{G},\tilde{M},\tilde{D}\right)=0$ *then* $S_t^+=0,\ S_t^->0$ *and,*

$$R_t^g\in\left[R_t^m+\bar{\chi}_t^+,R_t^m+\bar{\chi}_t^-\right],\bar{\chi}_t^+=\left(R_t^w-R_t^m\right)\left(1-e^{-\bar{\lambda}(1-\eta)}\right),\bar{\chi}_t^-=\left(R_t^w-R_t^m\right).$$

Case 3: If $0<\Gamma\left(\tilde{G},\tilde{M},\tilde{D}\right)$ *and* $\omega^*>-1$*, then* $S_t^+>0,\ S_t^->0$ *and,*

$$R_t^g=R_t^m+\bar{\chi}_t^+,\bar{\chi}_t^+<\left(R_t^w-R_t^m\right)\left(1-e^{-\bar{\lambda}(1-\eta)}\right),\bar{\chi}_t^-<\left(R_t^w-R_t^m\right).$$

Case 4: If $0<\Gamma\left(\tilde{G},\tilde{M},\tilde{D}\right)$ *and* $\omega^*\leq-1$*, then* $S_t^+>0,\ S_t^-=0$ *and*

$$R_t^g=R_t^m,\bar{\chi}_t^+=0,\bar{\chi}_t^-<\left(R_t^w-R_t^m\right)e^{-\bar{\lambda}\eta}.$$

Proposition E.4 establishes four possible scenarios for the equilibrium spread between bonds and reserves, depending on the aggregate holdings of bonds, money, and deposits. The first two scenarios (Cases 1 and 2) are characterized by a large supply of government bonds. In fact, the holding of government bonds is so that all the trade in the interbank market occurs in the government bond market, prior to the trade in the interbank market. By contrast, in cases 3 and 4 the government bond supply is not that large enough to absorb all of the excess of government bonds. Case 4 corresponds to a regime with reserve satiation, in which no bank ends in deficits—a case we also study in detail.

The next proposition establishes two key results in the paper: that banks are indifferent between their holdings of government bonds and reserves and that the value function has a single state variable.

Proposition E.5 *If* $R_t^g\leq R_t^m+\bar{\chi}_t^+$*, the law of motion of bank net worth can be written as:*

$$e'=\left[R_{t+1}^b\frac{\tilde{b}}{P_t}+R_{t+1}^m\frac{\tilde{a}}{P_t}-R_{t+1}^d\frac{\tilde{d}}{P_t}+\bar{\chi}_{t+1}\left(\frac{\tilde{a}}{P_t},\frac{\tilde{d}}{P_t},\omega\right)\right](1-\tau_{t+1}).$$

where $\tilde{a}\equiv\tilde{m}+\tilde{g}$*. As a result, we can express* V_t *recursively,*

$$\begin{aligned}V_t(e) &= \max_{\left\{c,\tilde{a},\tilde{b},\tilde{d}\right\}\geq 0,\tilde{d}\in[0,\kappa]} u(c)+\beta\mathbb{E}\left[V_{t+1}(e')\right],\ \textit{subject to} &\text{(E.8)}\\ \tfrac{\tilde{a}}{P_t} &\quad +\frac{\tilde{b}}{P_t}-\frac{\tilde{d}}{P_t}+c=e\\ e' &= \left[R_{t+1}^b\frac{\tilde{b}}{P_t}+R_{t+1}^m\frac{\tilde{a}}{P_t}-R_{t+1}^d\frac{\tilde{d}}{P_t}+\bar{\chi}_{t+1}\left(\frac{\tilde{a}}{P_t},\frac{\tilde{d}}{P_t},\omega\right)\right](1-\tau_{t+1}). &\text{(E.9)}\end{aligned}$$

Else, if $R_t^g\in(R_t^m+\bar{\chi}_t^+,R_t^m+\bar{\chi}_t^-)$*, then* $\bar{\chi}_t^+$ *in the definition of the function* $\bar{\chi}_{t+1}$ *is replaced by some* $\bar{\chi}_t^*\in\left(\bar{\chi}_t^+,\bar{\chi}_t^-\right)$*. If* $R_t^g=R_t^m+\bar{\chi}_t^-$*, then* $\bar{\chi}_t^+$ *is replaced by* $\bar{\chi}_t^-$*.*

The proof to this proposition is immediate after we replace $\tilde{Z}(s)$ in Proposition E.3 into Proposition E.2, for the case in which $R_t^g \leq R_t^m + \bar{\chi}_t^+$. When $R_t^g = R_t^m + \bar{\chi}_t^-$, then $\bar{\chi}_{t+1}$ in the proposition is replaced by a linear function with slope $\bar{\chi}_t^-$.

Problems 1 and 2 can be combined into a single Bellman equation as presented in (E.4). This concludes the proof of Item (i) in Proposition 3. As stated in the body of the paper, we focus on the cases (3 and 4) where there the supply of government bonds is not that large, and$R_t^g \leq R_t^m + \bar{\chi}_t^+$. This concludes the proof of Item (i) in the proposition.

Naturally, given an aggregate portfolio during the lending stage, $\left\{\tilde{G}, \tilde{M}, \tilde{D}\right\}$, we must fall in one of the 4 possible cases. Naturally, if $\tilde{G} = 0$, we are in case 2. If at $\lim_{\tilde{G}\to 0^+} \Gamma < 0$, then by Lemma $\Gamma < 0$ for any combination of parameters, and thus $R_t^g = R_t^m + \bar{\chi}_t^+$. If at $\lim_{\tilde{G}\to 0^+} \Gamma \geq 0$, then, $R_t^g = R_t^m + \bar{\chi}_t^-$ for any value of $\tilde{G}$ such that $\Gamma > 0$, and after that point R_t^g falls to $R_t^m + \bar{\chi}_t^+$. This suggests that as long as the supply of government bonds is not too large, government bonds will not deplete a surplus of reserves.

Next, we present two sufficient conditions that guarantee that $R_t^g \leq R_t^m + \bar{\chi}_t^+$, the case presented in the paper.

Corollary E.2 *If $\tilde{m} \geq \rho\tilde{d}$, then the bond premium falls in cases (3) or (4) of Proposition E.4.*

As special case which we consider in the paper is when $\rho = 0$.

Corollary E.3 *Assume that $\rho = 0$, then $R_t^g = R_t^m + \bar{\chi}_t^+$ without loss of generality.*

Finally, notice that when $R_t^g \leq R_t^m + \bar{\chi}_t^+$ the value function depends on the sign of $\tilde{s}$. Banks with deficit of $\tilde{s}$ sell all their government bonds. Their reserve deficit after selling government bonds is given by:

$$S_t^- = -\int_{-1}^{\omega^*} \tilde{s}(\tilde{g}, \tilde{m}, \tilde{d}, \omega) F(\omega).$$

Bank above the threshold ω^* end with a surplus of:

$$S_t^+ = \underbrace{\int_{\omega^*}^{\infty} \left(\tilde{s}(\tilde{g}, \tilde{m}, \tilde{d}, \omega) - \tilde{g}\right) dF(\omega)}_{\text{Initial Reserve Surplus}} - \underbrace{\tilde{g}F(\omega^*)}_{\text{Purchases of governmentbonds}}.$$

We then have that,

$$\begin{aligned} S_t^+ &= \int_{\omega^*}^{\infty} \left(\tilde{s}(\tilde{g}, \tilde{m}, \tilde{d}, \omega) - \tilde{g}\right) dF(\omega) + \tilde{g}\left(1 - F(\omega^*)\right) - \left(1 - F(\omega^*)\right) - \tilde{g}F(\omega^*) \\ &= \int_{\omega^*}^{\infty} \tilde{s}(\tilde{g}, \tilde{m}, \tilde{d}, \omega) dF(\omega) - \tilde{g}F(\omega^*). \end{aligned}$$

If we combine these features, we establish the following proposition.

Proposition E.6 *Let $R_t^g = R_t^m + \bar{\chi}_t^+$, then market tightness of the interbank market can be expressed in terms of lending stage variables as follows:*

$$\theta_t \equiv \frac{S_t^-}{S_t^+} = -\frac{\int_{-1}^{\omega^*} \tilde{s}(\tilde{g}, \tilde{m}, \tilde{d}, \omega) F(\omega)}{\int_{\omega^*}^{\infty} \tilde{s}(\tilde{g}, \tilde{m}, \tilde{d}, \omega) dF(\omega) - \tilde{g}}.$$

E.2 Proofs of Lemma E.1 and Propositions E.1-E.5

Proof of Lemma E.1. Observe that

$$\Gamma_{\tilde{G}}\left(\tilde{G},\tilde{M},\tilde{D}\right)=-F(\omega^*)\leq 0$$

where we used Leibniz's rule and the fact that $\tilde{s}(\tilde{g},\tilde{m},\tilde{d},\omega^*)=0$. This shows that as we increase the number of government bonds, relative to reserves, moves the condition closer to cases 1 and 2. Then,

$$\Gamma_{\tilde{G}\tilde{G}}\left(\tilde{G}\right)=-f(\omega^*)\frac{\partial\omega^*}{\partial\tilde{g}}\geq 0.$$

Hence, we know that the surplus function is decreasing and convex. Furthermore,

$$\Gamma\left(0,\tilde{M},\tilde{D}\right)=\int_{\omega^*}^{\infty}\tilde{s}(0,\tilde{m},\tilde{d},\omega)dF(\omega)=\left(\tilde{m}-\rho\tilde{d}+\left(\frac{R^d_{t+1}}{R^m_{t+1}}-\rho\right)\mathbb{E}\left[\omega|\omega>\omega^*\right]\tilde{d}\right)F(\omega^*)>0$$

and

$$\lim_{\tilde{G}\to\infty}\Gamma\left(0,\tilde{M},\tilde{D}\right)=\tilde{m}-\rho\tilde{d}.$$

This property shows that there will be a surplus in the government bond market if there is a surplus of reserves, and furthermore, that even if there are infinite government bonds, there will be banks in deficit if there is an aggregate deficit of reserves. **QED.**

Proof of Proposition E.1. We have to show that the recursive problem of banks during the lending stage, $V^l_t(g,b,m,d,f,w)$, has a value that can be summarized by $V_t(e)$ where e is a single state variable. To show this result, we define the after-tax real value of equity at the start of a lending stage:

$$e_t\equiv\frac{(1+i^b_t)b_t+(1+i^m_t)m_t-(1+i^d_t)d_t+(1+i^g_t)\,g_t-\left(1+i^f_t\right)f_t-(1+i^w_t)\,w_t-P_tT^j_t}{P_t}.$$

This term is the right-hand side of equation (12) in Problem 1 over the price level. If we use this definition, the budget constraint of a given bank satisfies,

$$c_t\ +\frac{\tilde{b}_t+\tilde{m}_t-\tilde{d}_t}{P_t}=e_t. \tag{E.1}$$

The choice of $\left\{\tilde{g}_t,\tilde{b}_t,\tilde{m}_t,\tilde{d}_t\right\}$ is constrained by the capital requirement and the budget constraint is independent of the composition of real equity. Hence, the value $V^l_t(b,m,d,f,w)$ must depend on e, but not its composition. Therefore, we can define $V_t(e)\equiv V^l_t(b,m,d,f,w)$. **QED**.

Proof of Proposition E.2. Define e as in the body of the paper. Consider the value at the lending stage, V^l. The value function is increasing in e, since it increases the budget constraint. Since U is strictly increasing, the policy functions that solve the problem at the balancing stage, must also maximize e'. Thus, the choice at the balancing stage must be given by:

$$
\begin{aligned}
e' &= (1-\tau_{t+1})\left[\max_{g'\geq 0} R^b_{t+1}\tilde{b} + R^m_{t+1}\left(\tilde{m}-(g'-\tilde{g})\right) + R^g_{t+1}g' - R^d_{t+1}\tilde{d} + \bar{\chi}_{t+1}(\tilde{s}-g')\right] \\
&= (1-\tau_{t+1})\left[R^b_{t+1}\tilde{b} + R^m_{t+1}\left(\tilde{m}+\tilde{g}\right) - R^d_{t+1}\tilde{d} + \max_{g'\geq 0}\left(R^g_{t+1}-R^m_{t+1}\right)g' + \bar{\chi}_{t+1}(\tilde{s}-g')\right].
\end{aligned}
$$

The second line factors out predetermined variables from the objective. Therefore, we write:

$$
e' = \left(R^b_{t+1}\tilde{b} + R^m_{t+1}\left(\tilde{m}+\tilde{g}\right) - R^d_{t+1}\tilde{d} + Z\left(\tilde{s}\right)\right)(1-\tau_{t+1}).
$$

where

$$
\begin{aligned}
Z\left(\tilde{s}\right) = \quad & \max_{g'\geq 0}\left(R^g_{t+1}-R^m_{t+1}\right)g' + \bar{\chi}_{t+1}(\tilde{s}-g') && \text{(E.2)} \\
& s.t. \\
& g' \geq 0. && \text{(E.3)}
\end{aligned}
$$

This concludes the proof of Proposition E.2. **QED.**

Proof of Proposition E.3. The objective is piece-wise linear and concave. The constraint set is linear. Standard textbooks in linear programming show that piece wise linear programs can be written as linear programs. Hence we have the following conditions for the choice of g'. The derivative of the objective function with respect to g' is given by:

$$
R^g_{t+1} - R^m_{t+1} - \chi^+_{t+1} \text{ if } \tilde{s} > g' \tag{E.4}
$$

and

$$
R^g_{t+1} - R^m_{t+1} - \chi^-_{t+1} \textbf{ if } \tilde{s} < g'. \tag{E.5}
$$

By Proposition 2 in Bianchi and Bigio (2017), we have that $\chi^+ < \chi^-$, for any market tightness. Hence, we obtain

$$
R^g_{t+1} - R^m_{t+1} - \chi^+_{t+1} \leq 0 \Rightarrow R^g_{t+1} - R^m_{t+1} - \chi^-_{t+1} < 0
$$

and also the converse:

$$
R^g_{t+1} - R^m_{t+1} - \chi^-_{t+1} \geq 0 \Rightarrow R^g_{t+1} - R^m_{t+1} - \chi^-_{t+1} > 0.
$$

Next, we characterize, $g'(\tilde{s})$, the optimal policy of an agent with surplus $\tilde{s}$. The solution depends on the value R^g_t as follows:

Case 1. Assume $R^g_t > R^m_t + \bar{\chi}^-_t$. Then, the objective in $Z\left(\tilde{s}\right)$ is increasing everywhere in g'. Thus, the maximizer of $Z\left(\tilde{s}\right)$ is $g' = \infty$ for any $\tilde{s}$.

Case 2. Assume $R^g_t < R^m_t + \bar{\chi}^+_t$. Then, the objective in $Z\left(\tilde{s}\right)$ is decreasing everywhere in g'. Thus, the maximizer of $Z\left(\tilde{s}\right)$ is $g' = 0$ for any $\tilde{s}$. In this case, $Z\left(\tilde{s}\right) = \bar{\chi}_t\left(\tilde{s}\right)$.

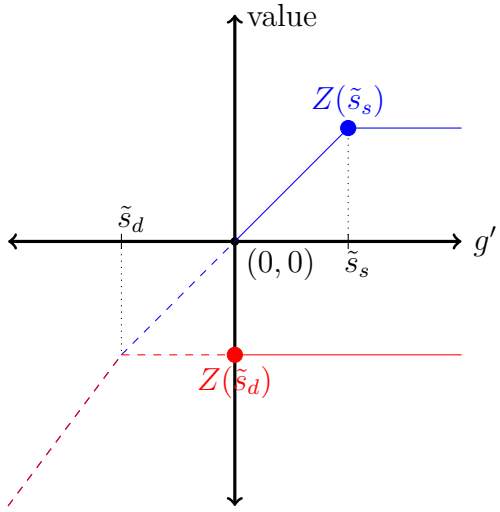

Figure 9: Values of Objective in Z as functions of g' (Case 3: $R_t^g = R_t^m + \bar{\chi}_t^-$). Note: The figure considers two values for the reserve balance $-\tilde{s}_d = \tilde{s}_s > 0$. The red and blue lines correspond to the objective of banks that starts with $\tilde{s}_d$ (deficit) and $\tilde{s}_s$ (surplus), respectively. Dashed lines represent values outside the constraint set ($g' < 0$). The figure shows how banks must get rid of their excess reserves. Banks in deficit are indifferent between increasing their deficits or not.

Case 3. Assume $R_t^g = R_t^m + \bar{\chi}_t^-$. If a bank starts with $\tilde{s} > 0$, the objective in $Z(\tilde{s})$ is increasing in $g' \in [0, \tilde{s}]$. Because $R_{t+1}^g > R_{t+1}^m + \chi_{t+1}^+$, as long as the bank remains in surplus, it is better of selling government bonds in exchange for reserves. At then point where $g' \geq \tilde{s}$, the objective is flat, because the bank becomes a deficit bank after that point. Thus, after entering a deficit, the bank in indifferent between buying government bonds and widening its deficit. Thus, a bank, banks with an initial surplus, end with $g' \geq \tilde{s} \geq 0$. Since one particular solution is $g' = \tilde{s}$, the value of the objective for a bank with an initial surplus is $Z(\tilde{s}) = \left(R_{t+1}^g - R_{t+1}^m\right)\tilde{s} = \bar{\chi}_{t+1}^- \tilde{s}$.

Now consider a bank in deficit. If the bank buys government bonds, it widens the deficit. The marginal return of a government bond is R_t^g and the cost of a unit deficit of reserves is $R^m + \bar{\chi}_{t+1}^-$, hence the bank is indifferent. Thus, for any bank that starts in deficit, $\tilde{s} \leq 0$ any choice of $g' \geq 0$ is a solution—the bank necessarily ends in deficit. One particular feasible solution is $g' = 0$, and thus the value for banks in deficit is, $Z(\tilde{s}) = \bar{\chi}_{t+1}^- \tilde{s}$. Combining these observations

$$g' = \begin{cases} [\tilde{s}, \infty] & \tilde{s} \geq 0 \\ [0, \infty] & \tilde{s} < 0, \end{cases} \text{ and } Z(\tilde{s}) = \bar{\chi}_{t+1}^- \tilde{s}.$$

Figure 9 presents a graphical representation of the objective function in Z, for two banks, one that starts in deficit and another with surplus. It shows how a bank in surplus must get rid of any excess balance whereas a bank in deficit is indifferent.

Case 4. Consider now the region where $R_t^g = R_t^m + \bar{\chi}_t^*$ for some $\bar{\chi}_t^* \in \left(\bar{\chi}_t^+, \bar{\chi}_t^-\right)$. This means that the objective in $Z(\tilde{s})$ is decreasing in g' as long a s bank has a deficit, but increasing as long as a bank has a surplus. Since a bank with $\tilde{s} < 0$ cannot cover its deficit, it will set $g' = 0$ in order not to increase its deficit, i.e. $g' = 0$. Conversely, a bank in surplus will sell all of its surplus $g' = \tilde{s}$, but will not purchase government bonds beyond that point. If we replace this

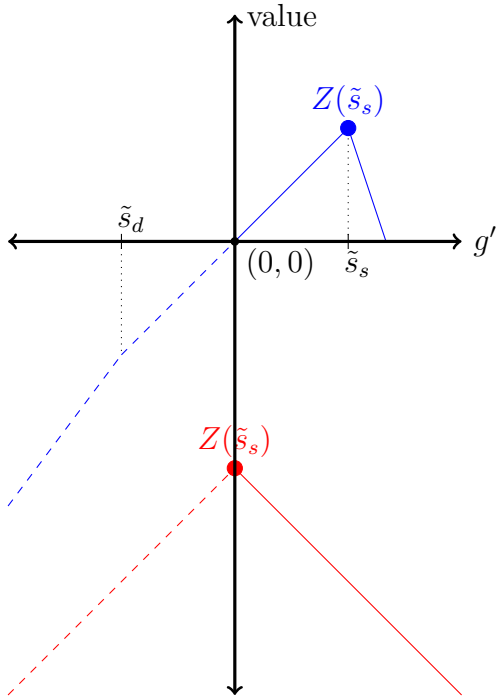

Figure 10: Values of Objective in Z as functions of g' (Case 4: $R_t^g \in \left(R_t^m + \bar{\chi}_t^+, R_t^m + \bar{\chi}_t^-\right)$ for some $\bar{\chi}_t^*$). Values of Objective in Z as functions of g' (Case 3: $R_t^g = R_t^m + \bar{\chi}_t^-$). Note: The figure considers two values for the reserve balance $-\tilde{s}_d = \tilde{s}_s > 0$. The red and blue lines correspond to the objective of banks that starts with $\tilde{s}_d$ (deficit) and $\tilde{s}_s$ (surplus), respectively. Dashed lines represent values outside the constraint set ($g' < 0$). The figure shows how banks with an initial surplus get rid of their excess balances. Banks in deficit do not increase their deficits.

conditions into objective in $Z(\tilde{s})$ we obtain:

$$g' = \begin{cases} \tilde{s} & \tilde{s} \geq 0 \\ 0 & \tilde{s} < 0, \end{cases} \text{ and } Z(\tilde{s}) = \begin{cases} \bar{\chi}_t^* \tilde{s} & \tilde{s} \geq 0 \\ \bar{\chi}_t^- \tilde{s} & \tilde{s} < 0. \end{cases}$$

Figure 10 presents a graphical representation of the objective function in Z, for two banks, one that starts in deficit and another with surplus; in the context of Case 4 now. It shows how a bank in surplus must get rid of any excess balance, but not end in deficit. A bank in deficit will not increase its deficit.

Case 5. Assume $R_t^g = R_t^m + \bar{\chi}_t^+$. In this case, the objective in Z is decreasing in g' as long as the bank in is deficit. Consider a bank that starts in deficit. Then, any choice of $g' > 0$ increases its deficit and thus, reduces future equity. Thus, banks that start in deficit always remain in deficit and must set $g' = 0$. Thus, $Z(\tilde{s}) = \bar{\chi}_t^- \tilde{s}$. By contrasts, the objective is constant as long as $0 \leq g' \leq \tilde{s}$. Hence, banks that begin with a surplus are indifferent between selling any amount in $[0, \tilde{s}]$. One particular solution is $g' = 0$ which yields a value $Z(\tilde{s}) = \bar{\chi}_t^+ \tilde{s}$. Summing up, we have:

$$g' = \begin{cases} [0, \tilde{s}] & \tilde{s} \geq 0 \\ 0 & \tilde{s} < 0, \end{cases} \text{ and } W(\tilde{s}) = \begin{cases} \bar{\chi}_t^+ \tilde{s} & \tilde{s} \geq 0 \\ \bar{\chi}_t^- \tilde{s} & \tilde{s} < 0. \end{cases}$$

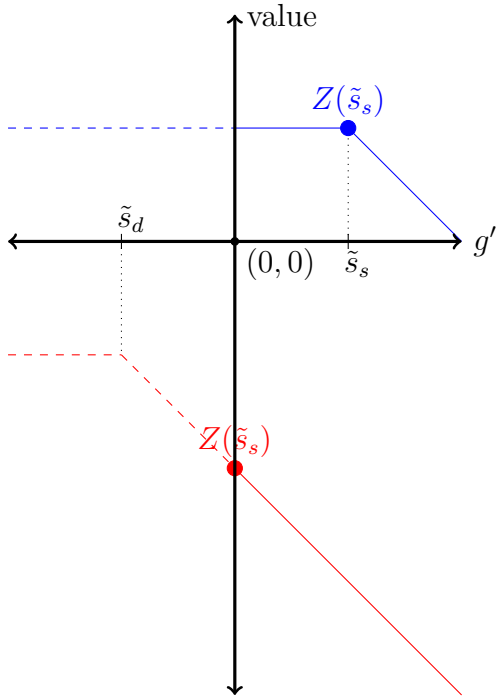

Figure 11: Values of Objective in Z as functions of g' (Case 5: $R_t^g = R_t^m + \bar{\chi}_t^+$). Note: The figure considers two values for the reserve balance $-\tilde{s}_d = \tilde{s}_s > 0$. The red and blue lines correspond to the objective of banks that starts with $\tilde{s}_d$ (deficit) and $\tilde{s}_s$ (surplus), respectively. Dashed lines represent values outside the constraint set ($g' < 0$). The figure shows how in deficit set $g' = 0$, implying that they sell all their initial holdings of government bonds. Banks in surplus are indifferent between reducing their surpluses, as long as they don't enter into deficit.

Figure 11 presents a graphical representation of the objective function in Z, for two banks, one that starts in deficit and another with surplus, but now for Case 5. It shows how a bank in surplus is indifferent between buying any amount of government bonds as long as it doesn't become a deficit bank. A bank in deficit sets $g' = 0$, and thus sells all of its initial balance $\tilde{g}$. This concludes the proof of Proposition E.3. **QED.**

Proof of Corollary E.2. We now consider the market clearing conditions in the market of government bonds. The goal is to find conditions on the quantities of reserves and government bonds determined at the lending stage, such that given the returns on government bonds, reserves and the interbank market, deliver market clearing conditions in the government bond market. We break the analysis into the cases studied in Proposition E.3.

Case 1. Assume that $R_t^g > R_t^m + \bar{\chi}_t^-$. By Proposition E.3 we have that $g' = \infty$ for all banks. However, since the stock of government bonds is finite, clearing in the government bond market, (E.4), cannot hold.

Case 2. Assume that $R_t^g < R_t^m + \bar{\chi}_t^+$. By Proposition E.3 we have that $g' = 0$ for all banks. In this case, this price can only clear the government bond market, (E.4), if $\tilde{g} = 0$.

Case 3. Assume that $R_t^g = R_t^m + \bar{\chi}_t^-$. We can rewrite (E.4) as:

$$\tilde{g}F(\omega^*) = \int_{\omega^*}^{\infty} \left(g'(\tilde{g}, \tilde{m}, \tilde{d}, \omega) - \tilde{g}\right) dF(\omega) + \int_{-1}^{\omega^*} g'(\tilde{g}, \tilde{m}, \tilde{d}, \omega) dF(\omega).$$

By Proposition E.3, we also know that $g' \geq \tilde{s}$ for $\tilde{s} \geq 0$ or, equivalently for $\omega \geq \omega^*$. Thus, we can replace the optimal policy into (E.4)

$$\tilde{g}F(\omega^*) = \int_{\omega^*}^{\infty} \left(\tilde{s}(\tilde{g}, \tilde{m}, \tilde{d}, \omega) - \tilde{g}\right) dF(\omega) + \int_{-1}^{\omega^*} g'(\tilde{g}, \tilde{m}, \tilde{d}, \omega) dF(\omega).$$

Since we know also by Proposition E.3 that $g' \geq 0$ for $\omega < \omega^*$,

$$\tilde{g}F(\omega^*) \geq \int_{\omega^*}^{\infty} \left(\tilde{s}(\tilde{g}, \tilde{m}, \tilde{d}, \omega) - \tilde{g}\right) dF(\omega),$$

or simply $\Gamma(\tilde{g}) \leq 0$. Thus, if $R_t^g = R_t^m + \bar{\chi}_t^-$ then, $\Gamma(\tilde{g}) \geq 0$. Furthermore, since we know that $g' = \tilde{s}$ for banks with $\omega > \omega^*$, but that banks in deficit end in deficit, there is no surplus left in the interbank market.

Case 4. Assume that $R_t^g = R_t^m + \bar{\chi}_t^*$ for some $\bar{\chi}_t^* \in \left(\bar{\chi}_t^+, \bar{\chi}_t^-\right)$. Then, following the same steps, as in the previous region, but now setting $g'(\tilde{g}, \tilde{m}, \tilde{d}, \omega) = 0$ for banks with $\omega < \omega^*$, we obtain:

$$\tilde{g}F(\omega^*) = \int_{\omega^*}^{\infty} \left(\tilde{s}(\tilde{g}, \tilde{m}, \tilde{d}, \omega) - \tilde{g}\right) dF(\omega)$$

or simply put, $\Gamma(\tilde{g}) = 0$. Furthermore, since we know that $g' = \tilde{s}$ for banks with $\omega > \omega^*$, then there is no surplus available in the Fed funds market.

Case 5. Assume that $R_t^g = R_t^m + \bar{\chi}_t^+$. We now have that $g' = 0$ for $\omega < \omega^*$. Thus, if we substitute this result in (E.4), we obtain,

$$\tilde{g}F(\omega^*) = \int_{\omega^*}^{\infty} \left(g'(\tilde{g}, \tilde{m}, \tilde{d}, \omega) - \tilde{g}\right) dF(\omega).$$

Now, since by Proposition E.3 we have that $g' \geq \tilde{s}$ for banks in surplus, we have that :

$$\tilde{g}F(\omega^*) \leq \int_{\omega^*}^{\infty} \left(\tilde{s}(\tilde{g}, \tilde{m}, \tilde{d}, \omega) - \tilde{g}\right) dF(\omega),$$

or namely $0 \geq \Gamma(\tilde{g})$. Furthermore, if the condition holds with equality, it must be that $g' = \tilde{s}$ for banks with $\omega > \omega^*$, and hence, there is no surplus available in the Fed funds market. However, if the condition is strict, then, there must be a positive mass of banks with surplus (the supply of government bonds by deficit banks $\tilde{g}F(\omega^*)$ does not exceed the holdings of reserves of banks in surplus $\int_{\omega^*}^{\infty} \left(\tilde{s}(\tilde{g}, \tilde{m}, \tilde{d}, \omega) - \tilde{g}\right)$. Consider the special cases where $\omega^* \leq -1$, no bank has an initial deficit. Thus, all banks must end with a surplus, and this means we are in case 5, since this is the only case where this is possible. **QED.**

E.3 Proof of Corollaries E.2 and E.3

Proof of Corollary E.2. Assume that there is an aggregate deficit of reserves. Then, assume by contradiction that

$$\begin{aligned} & \tilde{g}F\left(\omega^*\right) && > \int_{\omega^*}^{\infty}\left(\tilde{s}(\tilde{g},\tilde{m},\tilde{d},\omega)-\tilde{g}\right)dF(\omega) \\ \Rightarrow\quad & \tilde{g}F\left(\omega^*\right)+\textstyle\int_{-1}^{\omega^*}\left(\tilde{s}(\tilde{g},\tilde{m},\tilde{d},\omega)-\tilde{g}\right)dF(\omega) && > \int_{-1}^{\infty}\left(\tilde{s}(\tilde{g},\tilde{m},\tilde{d},\omega)-\tilde{g}\right)dF(\omega) \\ \Rightarrow\quad & \textstyle\int_{-1}^{\omega^*}\tilde{s}(\tilde{g},\tilde{m},\tilde{d},\omega)dF(\omega) && > \tilde{m}-\rho\tilde{d}. \end{aligned} \tag{E.1}$$

Now observe by definition, $\tilde{s}(\tilde{g},\tilde{m},\tilde{d},\omega)\leq 0$, hence a contradiction. This rules out case 1. Now assume that the condition holds with equality. The only possibility is that $\omega^*<-1$ and $\tilde{m}=\rho\tilde{d}$. This case rules out 2 since $F\left(-1\right)=0$ and F is not degenerate. Hence, the only two scenarios are cases (3) or (4). **QED.**

Proof of Corollary E.3. The result is immediate after we set $\rho=0$ in the statement of Corollary E.2. **QED.**

E.4 Proof of Items (ii)-(iv)

Auxiliary Lemmas The proofs of items (ii)-(iv) of proposition 3 make use of the following two lemmas.

Lemma E.2 *The function $\bar{\chi}_t$ is homogeneous of degree 1 in (m,d).*

Proof. We need to show $\bar{\chi}_t\left(km,kd,\omega\right)=k\bar{\chi}_t\left(m,d,\omega\right)$ for any $k>0$. By definition:

$$\begin{aligned} \bar{\chi}_t(km,kd,\omega) &= \begin{cases} \chi_t^+ s & if\ s\geq 0 \\ \chi_t^- s & if\ s<0 \end{cases}, \\ s &= km+k\omega d\frac{1+i_{t+1}^d}{1+i_{t+1}^{ior}}-\rho kd\left(1+\omega\right), \end{aligned} \tag{E.1}$$

where χ_t^- and χ_t^+ are functions of $\left\{\Psi_t^-,\Psi_t^+,\bar{i}_t^f,\theta_t\right\}$ and independent of m and d. We can factor the constant k from the right-hand side of (E.1) and obtain

$$s=k\left(m+\omega d\frac{1+i_{t+1}^d}{1+i_{t+1}^{ior}}-\rho d\left(1+\omega\right)\right).$$

Define the position without the scaling factor k as $\tilde{s}$ given by

$$\tilde{s}=\left(m+\omega d\frac{1+i_{t+1}^d}{1+i_{t+1}^{ior}}-\rho d\left(1+\omega\right)\right).$$

Observe that $(s>0)\longleftrightarrow(\tilde{s}>0)$, $(s<0)\longleftrightarrow(\tilde{s}<0)$ and $(s=0)\longleftrightarrow(\tilde{s}=0)$. Thus,

$$\bar{\chi}_t(am,ad,\omega) = \begin{cases} \chi_t^+ s & if\ s\geq 0 \\ \chi_t^- s & if\ s<0 \end{cases} = \begin{cases} \chi_t^+ k\tilde{s} & if\ s\geq 0 \\ \chi_t^- k\tilde{s} & if\ s<0 \end{cases} = k\begin{cases} \chi_t^+ \tilde{s} & if\ \tilde{s}\geq 0 \\ \chi_t^- \tilde{s} & if\ \tilde{s}<0 \end{cases} = k\bar{\chi}_t(m,d,\omega).$$

The last line verifies that χ is homogeneous of first degree. **QED.**

The next lemma establishes that an increase in the (gross) nominal policy rates by a constant scales χ_t by that constant. We use this lemma in the policy analysis results when we discuss the neutrality of inflation.

Lemma E.3 *Let χ_t be given by two policy rates, $\{i_t^m, i_t^w\}$, given θ_t. Consider alternative rates $\left\{i_{a,t}^m, i_{a,t}^w\right\}$ such that they satisfy $\left(1+i_{a,t}^m\right)\equiv k\left(1+i_t^m\right)$ and $\left(1+i_{a,t}^w\right)\equiv k\left(1+i_t^w\right)$ for some k. Then, the $\bar{\chi}_{a,t}$ associated with $\left\{i_{a,t}^m, i_{a,t}^w\right\}$ for the same θ_t satisfy $\bar{\chi}_{a,t}=k\bar{\chi}_t$.*

Proof. Observe that χ_t in Definition 1 (which follows from Proposition 1) is a function scaled by the width of the corridor system $(i_t^w - i_t^m)$. Then,

$$i_{a,t}^w - i_{a,t}^{ior} = (1+i_{a,t}^w) - (1+i_{a,t}^{ior}) = k((1+i_t^w)-(1+i_t^m)) = k(i_t^w - i_t^m).$$

Then the result follows immediately from the functional form of χ_t in Proposition 1. **QED.**

Proofs of items (ii)-(iv) of Proposition 3 This section presents a proof of items (ii)-(iv) in Proposition 3. Item (ii) establishes that the single state representation satisfies homogeneity. We follow the guess-and-verify approach, common to dynamic programming. Our guess is that the value function satisfies $V_t(e) = v_t\frac{e^{1-\gamma}}{1-\gamma} - 1/((1-\beta)(1-\gamma))$, where v_t is a time-varying scaling factor in the value function, common to all banks. From item (i), the bank's problem is summarized by

$$\begin{aligned} V_t(e) = \max_{c,\tilde{a},\tilde{b},\tilde{d}} & u(c) + \beta\mathbb{E}_t\left[V_{t+1}(e')\right], \\ & \text{subject to} \\ & c + \frac{\tilde{b}+\tilde{a}-\tilde{d}}{P_t} = e, \\ & \tilde{d} \leq \kappa\left(\tilde{b}+\tilde{a}-\tilde{d}\right) \\ & e' = \left((1+i_{t+1}^b)\tilde{b} + (1+i_{t+1}^{ior})\tilde{a} - (1+i_{t+1}^d)\tilde{d} + \bar{\chi}_{t+1}\left(\tilde{a},\tilde{d},\omega\right)\right)\frac{(1-\tau_{t+1})}{P_{t+1}}. \end{aligned}$$

Note that multiplying and dividing by P_t, we have that e' can also be written as

$$e' = \frac{\left(\tilde{b}(1+i_{t+1}^b) + \tilde{a}(1+i_{t+1}^{ior}) - \tilde{d}(1+i_{t+1}^d) + \bar{\chi}_{t+1}\left(\tilde{a},\tilde{d},\omega\right)\right)}{P_t}\frac{(1-\tau_{t+1})}{(1+\pi_{t+1})}, \tag{E.2}$$

where $(1+\pi_{t+1}) = P_{t+1}/P_t$.

If the conjecture for the value function is correct, then this condition satisfies

$$
\begin{aligned}
v_t e^{1-\gamma} - \frac{1}{(1-\beta)(1-\gamma)} = \max_{c,\tilde{a},\tilde{b},\tilde{d}} & \frac{c^{1-\gamma}-1}{1-\gamma} + \beta \mathbb{E}_t \left[v_{t+1} \left(e'\right)^{1-\gamma} - \frac{1}{(1-\gamma)(1-\beta)} \right], \\
& \text{subject to} \\
& c + \frac{\tilde{b} + \tilde{a} - \tilde{d}}{P_t} = e, \\
& \tilde{d} \leq \kappa \left(\tilde{b} + \tilde{a} - \tilde{d} \right) \\
& e' = \frac{\left(\tilde{b}(1+i^b_{t+1}) + \tilde{a}(1+i^{ior}_{t+1}) - \tilde{d}(1+i^d_{t+1}) + \bar{\chi}_{t+1}\left(\tilde{a},\tilde{d},\omega\right) \right)}{P_t} \frac{(1-\tau_{t+1})}{(1+\pi_{t+1})}.
\end{aligned}
$$

Observe that we can factor out constants from the objective:

$$
\begin{aligned}
& \frac{c^{1-\gamma}-1}{1-\gamma} + \beta \mathbb{E}_t \left[v_{t+1} \left(e'\right)^{1-\gamma} - \frac{1}{(1-\gamma)(1-\beta)} \right] ... \\
= \quad & \frac{c^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}_t \left[v_{t+1} \left(e'\right)^{1-\gamma} \right] - \frac{1}{(1-\beta)(1-\gamma)}.
\end{aligned}
$$

Then, if we substitute the evolution of e' in (E.2), we obtain

$$
\begin{aligned}
v_t e^{1-\gamma} = \max_{c,\tilde{a},\tilde{b},\tilde{d}} & \frac{c^{1-\gamma}}{1-\gamma} + \qquad (E.3) \\
& \beta \mathbb{E}_\omega \left[v_{t+1} \left(\frac{\left(\tilde{b}(1+i^b_{t+1}) + \tilde{a}(1+i^{ior}_{t+1}) - \tilde{d}(1+i^d_{t+1}) + \bar{\chi}_{t+1}\left(\tilde{a},\tilde{d},\omega\right) \right)}{P_t} \frac{(1-\tau_{t+1})}{(1+\pi_{t+1})} \right)^{1-\gamma} \right] \\
& \text{subject to} \\
& e = \frac{\tilde{b} + \tilde{a} - \tilde{d}}{P_t} + c \\
& \tilde{d} \leq \kappa \left(\tilde{b} + \tilde{a} - \tilde{d} \right).
\end{aligned}
$$

Let us define variables in terms of equity, $\bar{c} = c/e$. Also, define $\bar{b} = \tilde{b}/\left((1-\bar{c}\,)eP_t\right)$, $\bar{a} = \tilde{a}/\left((1-\bar{c}\,)eP_t\right)$, and $\bar{d} = \tilde{d}/\left((1-\bar{c}\,)eP_t\right)$, as in the statement of Proposition 3. By Lemma E.2, we can factor constants $(1-\bar{c}\,)eP_t$ from $\bar{\chi}_t$ and express it as

$$
(1-\bar{c}\,)eP_t\bar{\chi}_t \left(\frac{\tilde{a}}{P_t(1-\bar{c}\,)e}, \frac{\tilde{d}}{P_t(1-\bar{c}\,)e}, \omega \right) = P_t(1-\bar{c})e\bar{\chi}_t \left(\bar{a},\bar{d},\omega\right).
$$

Using this observation, we can replace $\bar{c}$ in the value function to obtain

$$v_t e^{1-\gamma} = \max_{c,\tilde{a},\tilde{b},\tilde{d}} e^{1-\gamma} \frac{\bar{c}^{1-\gamma}}{(1-\gamma)} + \beta v_{t+1} \left((1-\bar{c})\, e\right)^{1-\gamma} \mathbb{E}_\omega ... \tag{E.4}$$

$$\left(\left[\frac{\tilde{b}(1+i^b_{t+1})/P_t}{(1-\bar{c})\, e} + \frac{\tilde{a}(1+i^{ior}_{t+1})/P_t}{(1-\bar{c})\, e} - \frac{\tilde{d}(1+i^d_{t+1})/P_t}{(1-\bar{c})\, e} + \bar{\chi}_{t+1}\left(\bar{a}, \bar{d}, \omega\right) \right] \frac{(1-\tau_{t+1})}{(1+\pi_t)} \right)^{1-\gamma}$$

subject to:

$$\frac{\tilde{b} + \tilde{a} - \tilde{d}}{(1-\bar{c})\, e P_t} = 1$$

$$\frac{\tilde{d}/P_t}{(1-\bar{c})\, e} \leq \kappa \left(\frac{\tilde{b}/P_t}{(1-\bar{c})\, e} + \frac{\tilde{a}/P_t}{(1-\bar{c})\, e} - \frac{\tilde{d}/P_t}{(1-\bar{c})\, e} \right)$$

From this expression, we can cancel out $e^{1-\gamma}$ from both sides of (E.4), which verifies that the objective is scaled by $e^{1-\gamma}$. Thus, we verify the guess that $V_t(e) = v_t e^{1-\gamma} - ((1-\beta)(1-\gamma))^{-1}$.

Next, we derive the policies that attain $V_t(e)$ and the value of v_t. If the conjecture is correct, using the definition of $\bar{b}$, $\bar{a}$, and $\bar{d}$, we obtain

$$v_t = \max_{\left\{\bar{c},\bar{b},\bar{a},\bar{d}\right\} \geq 0} \frac{\bar{c}^{1-\gamma}}{(1-\gamma)} + \beta v_{t+1} (1-\bar{c})^{1-\gamma} ... \tag{E.5}$$

$$\mathbb{E}_\omega \left(\left[(1+i^b_{t+1})\bar{b} + (1+i^{ior}_{t+1})\bar{a} - (1+i^d_{t+1})\bar{d} + \bar{\chi}_{t+1}\left(\bar{a}, \bar{d}, \omega\right) \right] \frac{(1-\tau_{t+1})}{(1+\pi_{t+1})} \right)^{1-\gamma}$$

subject to

$$\bar{b} + \bar{a} - \bar{d} = 1$$

$$\bar{d} \leq \kappa$$

Thus, any solution to $V_t(e)$ must be consistent with the solution of v_t if the conjecture is correct. Define real return on equity as follows:

$$R^E_{t+1}(\bar{b}, \bar{a}, \bar{d}, \omega) \equiv \left(R^b_{t+1}\bar{b} + R^m_{t+1}\bar{a} - R^d_{t+1}\bar{d} + \bar{\chi_{t+1}}(\bar{d}, \bar{a}, \omega) \right)(1-\tau_{t+1}).$$

Then, the value function can be written as

$$v_t = \max_{\left\{\bar{c},\bar{b},\bar{a},\bar{d}\right\}} \frac{\bar{c}^{1-\gamma}}{(1-\gamma)} + \beta v_{t+1} (1-\bar{c})^{1-\gamma} \mathbb{E}_\omega \left[\left(R^E_{t+1}(\bar{b}, \bar{a}, \bar{d}, \omega) \right)^{1-\gamma} \right].$$

We now use the principle of optimality. Let Ω_t be the certainty equivalent of the bank's optimal portfolio problem, that is,

$$\Omega_t \equiv \max_{\left\{\bar{b},\bar{a},\bar{d}\right\}} \left[\mathbb{E}_\omega \left[\left(R^E_{t+1}(\bar{b}, \bar{a}, \bar{d}, \omega) \right)^{1-\gamma} \right] \right]^{\frac{1}{1-\gamma}}$$

subject to $\bar{b}+\bar{a}-\bar{d} = 1$ and $\bar{d} \leq \kappa$. Assume $\bar{c}$ is optimal. If $\gamma < 1$, the solution that attains v_t must maximize $\mathbb{E}_\omega \left[R^E_{t+1}(\bar{b}, \bar{a}, \bar{d}, \omega)^{1-\gamma} \right]$ if v_{t+1} is *positive*. If $\gamma > 1$, the solution that attains v_t must minimize $\mathbb{E}_\omega \left[R^E_{t+1}(\bar{b}, \bar{a}, \bar{d}, \omega)^{1-\gamma} \right]$ if v_{t+1} is *negative*. We guess and verify that when $\gamma < 1$, the

term v_{t+1} is positive and v_{t+1} is negative when $\gamma > 1$. Under this assumption, if $\gamma < 1$, we have that $v_{t+1} > 0$, so $1-\gamma > 0$. Thus, by maximizing Ω_t, we are effectively maximizing the right-hand side of v_t. Instead, when $\gamma > 1$, we have that $v_{t+1} < 0$, so $1-\gamma < 0$. Thus, by maximizing Ω_t, we are minimizing $\Omega_t^{1-\gamma}$, which multiplied by a negative number—v_{t+1}—maximizes the right-hand side of v_t.

Hence, the Bellman equation becomes

$$v_t = \max_{\left\{\bar{c},\bar{b},\bar{a},\bar{d}\right\}\geq 0} \frac{\bar{c}^{1-\gamma}}{(1-\gamma)} + \beta v_{t+1} \left(1-\bar{c}\right)^{1-\gamma} \Omega_t^{1-\gamma}.$$

This yields the statements in items (i) and (ii), provided that v_t inherits the sign of $(1-\gamma)$.

To prove item (iii), we take the first-order conditions with respect to $\bar{c}$, and raising both sides to the $-\frac{1}{\gamma}$ power, we obtain

$$\bar{c} = (\beta v_{t+1})^{-1/\gamma} \Omega_t^{-(1-\gamma)/\gamma} \left(1-\bar{c}\right) (1-\gamma)^{-\frac{1}{\gamma}}.$$

We can rearrange terms to obtain

$$\bar{c} = \frac{1}{1 + \left[\beta v_{t+1}(1-\gamma)\Omega_t^{1-\gamma}\right]^{1/\gamma}}.$$

Define $\xi_t = (1-\gamma)\beta v_{t+1}\Omega_t^{1-\gamma}$. Under the conjectured sign of v_t, the term ξ_t is always positive. Substituting this expression for dividends, we obtain a functional equation for the value function

$$v_t = \frac{\left(1+\xi_t^{1/\gamma}\right)^{-(1-\gamma)}}{(1-\gamma)} + \beta v_{t+1}\Omega_t^{1-\gamma}\left[\frac{\xi_t^{1/\gamma}}{1+\xi_t^{1/\gamma}}\right]^{(1-\gamma)} = \frac{\left(1+\xi_t^{1/\gamma}\right)^{-(1-\gamma)}}{(1-\gamma)} + \frac{\xi_t}{(1-\gamma)}\left[\frac{\xi_t^{1/\gamma}}{1+\xi_t^{1/\gamma}}\right]^{(1-\gamma)}$$

and finally,

$$= \frac{1}{(1-\gamma)}\left[\left(1+\xi_t^{1/\gamma}\right)^{-(1-\gamma)} + \xi_t\left[\frac{\xi_t^{1/\gamma}}{1+\xi_t^{1/\gamma}}\right]^{(1-\gamma)}\right].$$

Thus, we obtain

$$\begin{aligned} v_t &= \frac{1}{(1-\gamma)}\left[\frac{1}{\left(1+\xi_t^{1/\gamma}\right)^{(1-\gamma)}} + \frac{\xi_t^{1/\gamma}}{\left(1+\xi_t^{1/\gamma}\right)^{(1-\gamma)}}\right] \\ &= \frac{1}{(1-\gamma)}\frac{1+\xi_t^{1/\gamma}}{\left(1+\xi_t^{1/\gamma}\right)^{(1-\gamma)}} = \frac{1}{(1-\gamma)}\left(1+\xi_t^{1/\gamma}\right)^{\gamma}. \end{aligned}$$

This verifies that v_t inherits the sign of $(1-\gamma)$. Thus, we can use Ω^* directly in the value

function. Furthermore, v_t satisfies the following difference equation:

$$v_t = \frac{1}{1-\gamma}\left[1 + \left(\beta(1-\gamma)\Omega_t^{1-\gamma} v_{t+1}\right)^{\frac{1}{\gamma}}\right]^{\gamma}. \tag{E.6}$$

We can treat the right-hand side of this functional equation, solved independently of consumption. If we solve for this equation independently of the banker's consumption, we can obtain a solution to the banker's consumption policy via

$$\bar{c} = \frac{1}{1 + \left[\beta v_{t+1}(1-\gamma)\Omega_t^{1-\gamma}\right]^{1/\gamma}}.$$

This concludes the proof of items (i)-(iv), for all cases except $\gamma \to 1$. We work out that case next.

Log-Case. Observe that as $\gamma \to 1$, then v_t in (E.6) explodes. However, we can guess and verify that

$$\lim_{\gamma \to 1} v_t(1-\gamma) = \frac{1}{1-\beta}.$$

This assumption can be verified in equation (E.6). In this case,

$$\lim_{\gamma \to 1} (1-\gamma)\, v_t = \lim_{\gamma \to 1}\left[1 + \left(\beta(1-\gamma)\Omega_t^{*1-\gamma} v_t\right)^{\frac{1}{\gamma}}\right]^{\gamma} = 1 + \beta/(1-\beta) = 1/(1-\beta).$$

Thus, as $\gamma \to 1$, we have that $c = (1-\beta)$. Thus,

$$\Omega_t \equiv \max_{\{\bar{b},\bar{a},\bar{d}\}} \exp\left(\mathbb{E}_\omega\left[\log\left(R_t^E(\bar{b},\bar{a},\bar{d},\omega)\right)\right]\right).$$

This final step completes the proof of aggregation. **QED.**

F Derivation of Liquidity Premia (Proof of Proposition 4)

In this section, we suppress time subscripts and study the liquidity premia that emerge from portfolio problem (17). The calculations here proof Proposition 4. We derive the premia in the case where government bonds are not large enough to eliminate the surplus of reserves, as in the paper. A more general statement follows simply by substituting $\{\bar{\chi}^-, \bar{\chi}^+\}$ for the coefficients given in Proposition E.5. Everything else remains the same. The starting point is to replace the budget constraint $\bar{b} + \bar{a} = 1 + \bar{d}$, into the objective in (17), we obtain:

$$\Omega_t \equiv (1-\tau) \max_{\left\{\bar{b},\bar{a}\right\}\geq 0, \bar{d}\in[0,\kappa]} \left\{ \mathbb{E}_\omega \left[\left(R^m - R^b\right)\bar{a} - \left(R^b - R^d\right)\bar{d} + \bar{\chi}\left(\bar{a},\bar{d},\omega\right)\right]^{1-\gamma} \right\}^{\frac{1}{1-\gamma}}.$$

Let ω^* be the threshold shock that makes $\tilde{s} < 0$. Partition the expectation inside the objective into two terms:

$$\int_{-1}^{\omega^*\left(\bar{a},\bar{d},\omega\right)} \left[R^b + \left(R^m - R^b\right)\bar{a} + \left(R^b - R^d\right)\bar{d} + \bar{\chi}^-(\bar{a},\bar{d},\omega)\right]^{1-\gamma} f(\omega)\, d\omega +$$
$$\int_{\omega^*\left(\bar{a},\bar{d},\omega\right)}^{\infty} \left[R^b + \left(R^m - R^b\right)\bar{a} + \left(R^b - R^d\right)\bar{d} + \bar{\chi}^+(\bar{a},\bar{d},\omega)\right]^{1-\gamma} f(\omega)\, d\omega.$$

Derivatives of the Liquidity Cost Function. For the rest of the proof we use the following calculations:

$$s = \bar{a} - \rho\bar{d} + \bar{d}\left(\frac{R^d}{R^m} - \rho\right)\omega.$$

Hence, we have that

$$\frac{\partial\bar{\chi}\left(\bar{a},\bar{d},\omega\right)}{\partial\bar{a}} = \frac{1}{1+\pi}\begin{cases}\chi^+ & if\ \omega > \omega^* \\ \chi^- & if\ \omega < \omega^*\end{cases}$$

and

$$\frac{\partial\bar{\chi}\left(\bar{a},\bar{d},\omega\right)}{\partial\bar{d}} = \frac{1}{1+\pi}\begin{cases}\chi^+\left(-\rho + \left(\frac{R^d}{R^m} - \rho\right)\omega\right) & if\ \omega > \omega^* \\ \chi^-\left(-\rho + \left(\frac{R^d}{R^m} - \rho\right)\omega\right) & if\ \omega < \omega^*\end{cases}.$$

Derivation of the Loan Liquidity Premium. Assuming the solution is interior in $\bar{m}$, we can take the derivative with respect to $\bar{m}$ to obtain:

$$\mathbb{E}_\omega\left[\left(R^e_\omega\right)^{-\gamma}\right]\left(R^m - R^b\right) + \int_{-1}^{\omega^*}\left(R^e_\omega\right)^{-\gamma}\bar{\chi}^-\frac{\partial\tilde{s}}{\partial\bar{a}}f(\omega)\,d\omega + \int_{\omega^*}^{\infty}\left(R^e_\omega\right)^{-\gamma}\bar{\chi}^+\frac{\partial\tilde{s}}{\partial\bar{a}}f(\omega)\,d\omega + \quad \ldots$$
$$+\left(R^e_\omega\right)^{-\gamma}\bar{\chi}^-\tilde{s}f(\omega)\big|_{\omega=\omega^*\left(\bar{g},\bar{m},\bar{d}\right)}\cdot\frac{\partial\omega^*}{\partial\bar{m}} - \left(R^e_\omega\right)^{-\gamma}\bar{\chi}^+\tilde{s}f(\omega)\,f(\omega)\big|_{\omega=\omega^*\left(\bar{g},\bar{m},\bar{d}\right)}\cdot\frac{\partial\omega^*}{\partial\bar{m}} \quad = \quad 0$$

Since $\tilde{s}\left(\bar{a},\bar{d},\omega^*\right) = 0$, the second line in the expression vanishes. Importantly, the expectation operator in $\mathbb{E}_\omega\left[\left(R^e_\omega\right)^{-\gamma}\right]$ excludes the point $\omega = \omega^*$—since this is a zero probability event, we simply exclude the point where the derivative is not included in the notation. We rearrange

terms to express the condition as::

$$R^b - R^m = \bar{\chi}^+ \frac{\mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \middle| \omega > \omega^* \right]}{\mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \right]} (1 - F(\omega^*)) + \bar{\chi}^- \frac{\mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \middle| \omega < \omega^* \right]}{\mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \right]} F(\omega^*).$$

This expression uses the definition of conditional expectation. Furthermore, we use the decomposition of an unconditional into two conditional expectations to obtain:

$$\mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \middle| \omega > \omega^* \right] (1 - F(\omega^*)) = \mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \right] - \mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \middle| \omega < \omega^* \right] F(\omega^*).$$

We thus express the loans premium as:

$$R^b - R^m = \bar{\chi}^+ + \left(\bar{\chi}^- - \bar{\chi}^+ \right) \frac{\mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \middle| \omega < \omega^* \right]}{\mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \right]} F(\omega^*).$$

Clearly, since $\bar{\chi}^- > \bar{\chi}^+ > 0$ and marginal utility is positive, the *loan premium is positive*. Finally, since we know that $(R^e_\omega)^{-\gamma} > 0$, we have that $0 < \mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \middle| \omega < \omega^* \right] F(\omega^*) < \mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \right]$. This condition implies that $R^w \geq R^b \geq R^m$.

Derivation of the Bond Liquidity Premium. Earlier, in the proof of of Proposition 3, item (i), where Proposition E.4 shows in the cases where $S^+>0$, we have that $R^g \leq R^m + \chi^+$. If the equality is strict, we also showed that $\tilde{g} = 0$. Observe again that since $0 < \mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \middle| \omega < \omega^* \right] F(\omega^*) < \mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \right]$, we have that $R^w > R^b \geq R^g \geq R^m$. The inequalities are strict if and only if, the Fed eliminates the spread in its corridor rates, $R^w = R^m$, or if banks are satiated with reserves $F(\omega^*) = 0$.

Derivation of the External Financing Premium and the Deposit Liquidity Premium. The derivation of the external financing premium follows the same steps as the loans premium. However, the presence of the capital requirement constraint implies that

$$R^b - R^d = \frac{\mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \frac{\partial \bar{\chi}(\bar{a}, \bar{d}, \omega)}{\partial \bar{d}} \right]}{\mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \right]} + \mu, \tag{F.1}$$

where μ is a Kuhn-Tucker multiplier associated with the capital requirement condition. We can subtract the loans liquidity premium to obtain:

$$R^m - R^d = -\frac{\mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \left[\frac{\partial \bar{\chi}(\bar{a}, \bar{d}, \omega)}{\partial \bar{a}} - \frac{\partial \bar{\chi}(\bar{a}, \bar{d}, \omega)}{\partial \bar{d}} \right] \right]}{\mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \right]} + \mu.$$

Thus, the liquidity premium on deposits is the difference between the marginal benefit of additional reserves minus the benefit

The expression inside the the right-hand side is given by:

$$\left[\frac{\partial \bar{\chi}(\bar{a}, \bar{d}, \omega)}{\partial \bar{a}} - \frac{\partial \bar{\chi}(\bar{a}, \bar{d}, \omega)}{\partial \bar{d}} \right] = \begin{cases} \bar{\chi}^+ \left(1 + \rho - \left(\frac{R^d}{R^m} - \rho \right) \omega \right) & \text{if } \omega > \omega^* \\ \bar{\chi}^- \left(1 + \rho - \left(\frac{R^d}{R^m} - \rho \right) \omega \right) & \text{if } \omega < \omega^* \end{cases}.$$

Then, it is convenient to partition the expectation:

$$\begin{aligned}\frac{\mathbb{E}_\omega\left[\left(R^e_\omega\right)^{-\gamma}\left[\frac{\partial\bar{\chi}(\bar{m},\bar{d},\omega)}{\partial\bar{m}}-\frac{\partial\bar{\chi}(\bar{m},\bar{d},\omega)}{\partial\bar{d}}\right]\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]} &= \bar{\chi}^+\frac{\mathbb{E}_\omega\left[\left(R^e_\omega\right)^{-\gamma}\left(\left(1+\rho\right)-\left(\frac{R^d}{R^m}-\rho\right)\omega\right)\middle|\,\omega>\omega^*\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]}\left(1-F\left(\omega^*\right)\right)\\ &+ \bar{\chi}^-\frac{\mathbb{E}_\omega\left[\left(R^e_\omega\right)^{-\gamma}\left(\left(1+\rho\right)-\left(\frac{R^d}{R^m}-\rho\right)\omega\right)\middle|\,\omega<\omega^*\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]}F\left(\omega^*\right).\end{aligned}$$

Thus, liquidity premium of deposits as:

$$\begin{aligned}\frac{\mathbb{E}_\omega\left[\left(R^e_\omega\right)^{-\gamma}\left[\frac{\partial\bar{\chi}(\bar{m},\bar{d},\omega)}{\partial\bar{m}}-\frac{\partial\bar{\chi}(\bar{m},\bar{d},\omega)}{\partial\bar{d}}\right]\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]} &= \left(1+\rho\right)\left[\bar{\chi}^+ + \left(\bar{\chi}^- - \bar{\chi}^+\right)\frac{\mathbb{E}_\omega\left[\left(R^e_\omega\right)^{-\gamma}\middle|\,\omega<\omega^*\right]}{\mathbb{E}_\omega\left[\left(R^e_\omega\right)^{-\gamma}\right]}F\left(\omega^*\right)\right]...\\ &- \left(\frac{R^d}{R^m}-\rho\right)\bar{\chi}^+\frac{\mathbb{E}_\omega\left[\left(R^e_\omega\right)^{-\gamma}\omega\middle|\,\omega>\omega^*\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]}\left(1-F\left(\omega^*\right)\right)....\\ &- \left(\frac{R^d}{R^m}-\rho\right)\bar{\chi}^-\frac{\mathbb{E}_\omega\left[\left(R^e_\omega\right)^{-\gamma}\omega\middle|\,\omega<\omega^*\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]}F\left(\omega^*\right)\end{aligned}$$

Using the the same decomposition, we have:

$$\begin{aligned}\frac{\mathbb{E}_\omega\left[\left(R^e_\omega\right)^{-\gamma}\left[\frac{\partial\bar{\chi}(\bar{m},\bar{d},\omega)}{\partial\bar{m}}-\frac{\partial\bar{\chi}(\bar{m},\bar{d},\omega)}{\partial\bar{d}}\right]\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]} &= \left(1+\rho\right)\left[\bar{\chi}^+ + \left(\bar{\chi}^- - \bar{\chi}^+\right)\frac{\mathbb{E}_\omega\left[\left(R^e_\omega\right)^{-\gamma}\middle|\,\omega<\omega^*\right]}{\mathbb{E}_\omega\left[\left(R^e_\omega\right)^{-\gamma}\right]}F\left(\omega^*\right)\right]...\\ &- \left(\frac{R^d}{R^m}-\rho\right)\left[\bar{\chi}^+ + \left(\bar{\chi}^- - \bar{\chi}^+\right)\frac{\mathbb{E}_\omega\left[\left(R^e_\omega\right)^{-\gamma}\omega\middle|\,\omega<\omega^*\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]}F\left(\omega^*\right)\right].\end{aligned}$$

We combine these expressions to obtain:

$$R^m - R^d = -\left(1+\rho\right)\left(R^b - R^m\right) + \left(\rho - \frac{R^d}{R^m}\right)DRP + \mu,$$

where DRP_t stands for a deposit risk premium:

$$DRP \equiv \left[\bar{\chi}^+ + \left(\bar{\chi}^- - \bar{\chi}^+\right)\frac{\mathbb{E}_\omega\left[\left(R^e_\omega\right)^{-\gamma}\omega\middle|\,\omega<\omega^*\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]}F\left(\omega^*\right)\right].$$

We re-arrange the expression, to obtain:

$$R^d - R^m = \left(1+\rho\right)\left(R^b - R^m\right) + \left(\frac{R^d}{R^m} - \rho\right)DRP - \mu.$$

Next, we show that the deposit risk premium, the second term, is also positive.

Next, we demonstrate the sign of DRP. Since, $\mathbb{E}_\omega\left[\omega\right] = 0$ marginal utility is decreasing, the

risk-weighted expectations operator carries a premium over a fair bet:

$$\frac{\mathbb{E}_\omega\left[\left(R_\omega^e\right)^{-\gamma}\omega\middle|\,\omega<\omega^*\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]}\leq\mathbb{E}_\omega\left[\frac{\left(R_\omega^e\right)^{-\gamma}}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]}\omega\right]\leq 0.$$

Therefore:

$$\begin{aligned}DRP_t&=\bar{\chi}^+\frac{\mathbb{E}_\omega\left[\left(R_\omega^e\right)^{-\gamma}\omega\middle|\,\omega>\omega^*\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]}\left(1-F\left(\omega^*\right)\right)+\bar{\chi}^-\frac{\mathbb{E}_\omega\left[\left(R_\omega^e\right)^{-\gamma}\omega\middle|\,\omega<\omega^*\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]}F\left(\omega^*\right)\\&\leq\bar{\chi}^+\frac{\mathbb{E}_\omega\left[\left(R_\omega^e\right)^{-\gamma}\omega\middle|\,\omega>\omega^*\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]}\left(1-F\left(\omega^*\right)\right)+\bar{\chi}^+\frac{\mathbb{E}_\omega\left[\left(R_\omega^e\right)^{-\gamma}\omega\middle|\,\omega<\omega^*\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]}F\left(\omega^*\right)\\&=\bar{\chi}^+\frac{\mathbb{E}_\omega\left[\left(R_\omega^e\right)^{-\gamma}\omega\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]}\leq 0.\end{aligned}$$

A necessary condition for the deposit liquidity premium to be negative, $\rho>\frac{R^d}{R^m}$. However, if the premium is negative, $R^d>R^m$, but since $\rho<1$, the necessary condition is violated. Thus, the DRP is negative. This concludes the proof of Proposition 4 and the claims about their sign. **QED.**

G Proofs of Policy Analysis (Section 3)

To present formal proofs, we define two important concepts: reserve satiation and neutrality.

Definition 4 (Satiation)**.** Banks are **satiated** with reserves at period t if the liquidity premium is zero, that is, if $R_t^b = R_t^m$.

The following Lemma states that banks are satiated with reserves under two conditions.

Lemma G.1 *Banks are satiated with reserves if and only if either* ***(case 1)*** $i_t^w = i_t^m$ *or* ***(case 2)*** *a bank is in surplus for* $\omega = \omega_{min}$.

To discuss policy effects, we compare an original policy sequence—with sub-index o—with an alternative (shocked) policy—sub-index s in all of the exercises. We mean that a policy is neutral relative to the other in the following sense.

Definition 5 (Neutrality)**.** Consider original and alternative policy sequences:

$$\left\{\rho_{o,t}, B_{o,t}^{Fed}, G_{o,t}^{Fed}, G_{o,t}^{FA}, M_{o,t}, W_{o,t}, T_{o,t}, \kappa_{o,t}, i_{o,t}^{ior}, i_{o,t}^{w}\right\} \text{ and}$$

$$\left\{\rho_{s,t}, B_{s,t}^{Fed}, G_{s,t}^{Fed}, G_{s,t}^{FA}, M_{s,t}, W_{s,t}, T_{s,t}, \kappa_{s,t}, i_{s,t}^{ior}, i_{s,t}^{w}\right\}.$$

Policy s is **neutral**—relative to o—if the induced equilibria satisfies

$$\left\{E_{o,t}, c_{o,t}, \bar{b}_{o,t}, \bar{d}_{o,t}, \bar{m}_{o,t}, \bar{g}_{o,t}\right\} = \left\{E_{s,t}, c_{s,t}, \bar{b}_{s,t}, \bar{d}_{s,t}, \bar{m}_{s,t}, \bar{g}_{s,t}\right\} \text{ for all } t \geq 0.$$

When the condition holds, real aggregate loans and deposits are also determined, and identical to those of the original allocation—and also for currency and holdings of government bonds and currency. The rest of this appendix shows the proofs for the classic exercises in monetary policy analysis that we studied in the main text. We begin by establishing to classic results:

Proposition G.1 *Consider an equilibrium sequence induced policy* $\{M_{t+1}, W_{t+1}, B_{t+1}^{Fed}, G_{t+1}^{Fed}, \}$ *that grows at rate* k_t *and a sequence of nominal policy rates* $\{i_t^w, i_t^m\}$*. Then,*

i) Consider a policy sequence that induces a stationary equilibrium and another policy sequence where Fed balance sheet is scaled by a constant $K > 0$*. Then, the alternative policy induces another stationary equilibrium where the price level is scaled by* K*, but all real variables are the same as in the original stationary equilibrium.*

ii) Consider an increase the balance sheet of the Fed by k_t *for some* t*, has no real effects if and only if the demand for currency is inelastic (or zero) and the Fed alters its nominal policy rates to keep* $\left\{\frac{1+i_t^m}{1+k_t}, \frac{1+i_t^w}{1+k_t}\right\}$ *constant.*

Part i) establishes long-run money neutrality. This result applies only to the stationary equilibrium because assets are nominal. Thus, changes at any point in time, by changing the price level have redistributive consequences. Even if the policy is anticipated, if policy rates are not adjusted, the policy change induces a different equilibrium. In the long run, however, a change in the scale of the Fed's balance sheet leads to a scaled stationary equilibrium. Part ii) is a condition for super-neutrality; the condition that changes in the inflation rate of the economy is neutral. The result says that if the Fed increases the growth rate of its nominal balance sheet, by a scalar, and adjusts its nominal policy rates to keep real rates constant, variations in the

growth rate of its nominal balance sheet only translates into changes in the unit of account, and inflation generates no changes. It is important to note that a qualification for this result is that the demand for real balances of currency are inelastic. Otherwise, changes in inflation produce a change in the money demand by households, and inflation adjusts differently in that context. Part ii) can be also interpreted as an approximation, for mild inflation rates, as long as the currency demand is close to inelastic, changes in inflation will be neutral.

G.1 Proof of Lemma G.1 (Conditions for Satiation)

By definition of satiation, the right-hand side of (Loan LP-Deposit LP) must equal zero under satiation, and thus:

$$0 = \bar{\chi}^{+} + \left(\bar{\chi}^{-} - \bar{\chi}^{+}\right) \cdot F\left(\omega^{*}\right) \cdot \frac{\mathbb{E}_{\omega}\left[\left(R^{e}\right)^{-\gamma} \omega < \omega^{*}\right]}{\mathbb{E}_{\omega}\left[\left(R^{e}\right)^{-\gamma}\right]},$$

and

$$0 = \bar{\chi}^{+}.$$

This expression equals zero in two cases:

Case 1. If $i_t^w = i_t^m$, then the condition holds immediately since $\chi^- = \chi^+ = 0$. This case is condition (i) in the proposition.

Case 2. If $i_t^w > i_t^m$, then since $\chi^- > \chi^+$ for any θ,we must have that: $F\left(\omega^{*}\right) = 0$ and $\bar{\chi}^{+} = 0$. This occurs only if $\omega^{*} \leq \omega_{min}$.

Under condition (ii) of the proposition, no bank is in deficit even for the worst shock. **QED.**

G.2 Proof of Proposition G.1 Item (i)

Consider a policy sequence $\{o\}$ and an alternative policy $\{s\}$ such that

1. $X_{s,t} = kX_{o,t}$ for some $k > 0$ for the balance sheet variables $X \in \left\{B^{Fed}, G^{Fed}, G^{FA}, M, W\right\}$,
2. policies are identical for non-balance-sheet variables $\left\{\rho_{o,t}, \kappa_{o,t}, i_{o,t}^{m}, i_{o,t}^{w}\right\} = \left\{\rho_{s,t}, \kappa_{s,t}, i_{s,t}^{m}, i_{s,t}^{w}\right\}$.

The proposition states that the stationary equilibrium induced by either policy features identical real asset positions and price levels that satisfy $P_{s,t} = kP_{o,t}$.

The proof is by construction and requires us to verify that the equilibrium conditions that determine $\{\bar{b}_{ss}, \bar{a}_{ss}, \bar{d}_{ss}, \bar{c}_{ss}, E_{ss}\}$ in Section B.2 are satisfied by any pair of policy sequences

$$\left\{B_{o,t}^{Fed}, G_{o,t}^{Fed}, G_{o,t}^{FA}, M_{o,t}, W_{o,t}\right\}_{t \geq 0}$$

and

$$\left\{B_{s,t}^{Fed}, G_{s,t}^{Fed}, G_{s,t}^{FA}, M_{s,t}, W_{s,t}\right\}_{t \geq 0}$$

that satisfies the relationship above. We proceed to check that $\left\{\bar{b}_{ss}, \bar{a}_{ss}, \bar{d}_{ss}, \bar{c}_{ss}, E_{ss}\right\}$ solves the set of equilibrium equations in Section B.2 in both cases.

Consider the original and alternative policies. We are considering stationary equilibria, so by hypothesis these satisfy

$$X_{a,t} = X_{a,t-1}(1+\pi_{ss}), \quad \text{for some } \pi_{ss} \text{ and } a \in \{o,s\} \text{ and } \left\{B^{Fed}, G^{Fed}, G^{FA}, M, W\right\}.$$

By hypothesis also, inflation and nominal rates are equal under both policies. Thus, the real interest rate on reserves is equal under both policies. We check the equilibrium conditions in the order in which they appear in Section B.

First, we guess and verify that the real returns on loans and deposits are also equal under both policies. If both policies yield the same real rates, the solution for bank portfolios (the solution for Ω_t) must also be equal in both equilibria:

$$\left\{\bar{b}_{o,ss}, \bar{a}_{o,ss}, \bar{d}_{o,ss}, \bar{c}_{o,ss}\right\} = \left\{\bar{b}_{s,ss}, \bar{a}_{s,ss}, \bar{d}_{s,ss}, \bar{c}_{s,ss}\right\}.$$

Consider now the aggregate supply of loans and reserves under either policy:

$$(1-c_{ss})\bar{b}_{ss}E_{ss} \ = \Theta^b \left(R^b_{ss}\right)^{\epsilon} - B^{Fed}_{t+1}/P_t.$$

That equation can be satisfied under both policies because $B^{Fed}_{o,t+1}/P_{o,t} = (1+g)B^{Fed}_{o,t+1}/(1+g)P_{o,t} = B^{Fed}_{s,t+1}/P_{s,t}$. This verifies that the real rate on loans is equal under both policies.

The same steps verify that R^d_{ss} is the same under both policies. Similarly, the demand for reserves and currency can be satisfied in both equations because

$$(1- \ c_{ss})\bar{m}_{ss}E_{ss} \ = M_{o,t}/P_{o,t} = M_{a,t}/P_{a,t}.$$

A similar argument holds for the holdings of government bonds. This verifies market clearing for reserves.

Now, the ratio of surpluses to deficits is also equal under both policies:

$$\theta_{ss} \equiv S^-_{a,t}/S^+_{a,t} \text{ for } a \in \{o,s\}.$$

Because θ and policy rates are equal, the liquidity cost function χ is also equal under both policies. Observe that χ is a function of θ only. With equal inflation under both policies, the liquidity return R^{χ} must also be equal. This verifies that all the real rates in both equilibria are the same under both policies. Since rates are the same, both policies satisfy the same law of motion for equity (19). It is immediate to verify that the consolidated government budget constraint is satisfied under both policies, once all portfolios from the private sector are identical in real terms. **QED.**

G.3 Proof of Proposition G.1 Item (ii)

This statement of the proposition regards superneutrality and non-superneutrality. The proof closely follows the proof of Proposition G.1, item (i). The main difference is that we prove neutrality along an equilibrium sequence, not only in stationary equilibrium. The proof is again by construction and only requires that we verify that the equilibrium conditions that determine

$\{\bar{b}_t, \bar{a}_t, \bar{d}_t, \bar{c}_t, E_t\}$ in Section B lead to the same values under both policies. Let

$$\left\{M_{o,t}, G^{Fed}_{a,t}, G^{FA}_{a,t}, B^{Fed}_{o,t}, W_{o,t}\right\}_{t\geq 0}$$

and

$$\left\{M_{a,t}, G^{Fed}_{a,t}, G^{FA}_{a,t}, B^{Fed}_{a,t}, W_{a,t}\right\}_{t\geq 0}$$

be two policy sequences. Again, to ease notation, we follow the order of the equations in Section B.

Consider the original and alternative policies. By the hypothesis of stationary equilibrium, both equilibria satisfy

$$X_{a,t} = X^{Fed}_{a,t-1}(1+k_a),\ B^{Fed}_{a,t} = \text{ for some } k_a \text{ and for } a \in \{o,s\} \text{ and } X \in \left\{M, G^{Fed}, G^{FA}, B^{Fed}, W\right\}.$$

Also, let the initial conditions be the same, $X_{s,0} = X_{o,0}$.

Then, the condition for the consolidated government implies that

$$\begin{aligned} X_{s,t+1} &= (1+k_s)^t X_{s,0} = (1+k_s)^t X_{o,0}, \\ X_{o,t+1} &= (1+k_o)^t X_{o,0}, \end{aligned}$$

for $X \in \left\{M, G^{Fed}, G^{FA}, B^{Fed}, W\right\}$. Thus, we can relate both government policy paths via:

$$X_{s,t+1} = \left(1 + \frac{k_s - k_o}{1+k_o}\right)^t X_{o,t+1}.$$

Through the proof, we guess and verify the following:

A.1 $\left\{R^b_{o,t}, R^d_{o,t}, R^m_{o,t}, R^g_{o,t}, R^{\bar{\chi}}_{o,t}\right\} = \left\{R^b_{s,t}, R^d_{s,t}, R^m_{s,t}, R^g_{o,t}, R^{\bar{\chi}}_{s,t}\right\}$.

A.2 $P_{o,0} = P_{s,0} = P_0$.

A.3 $(1+\pi_{s,t}) = (1+\pi_{o,t})\left(1 + \frac{k_s - k_o}{1+k_o}\right)$.

First, we verify (A.1). Under the conjecture that real returns are the same along a sequence, we have that

$$\left\{\bar{b}_{o,t}, \bar{a}_{o,t}, \bar{d}_{o,t}, \bar{c}_{o,t}\right\} = \left\{\bar{b}_{s,t}, \bar{a}_{s,t}, \bar{d}_{s,t}, \bar{c}_{s,t}\right\},$$

so the optimality conditions are satisfied in both cases.

Next, consider the aggregate supply of loans and reserve demand. Equilibrium in the loans market requires

$$(1-c_t)\bar{b}_t E_t = \Theta^b \left(R^b_t\right)^\epsilon - B^{Fed}_{t+1}/P_t.$$

If the equation is satisfied under both policies, then we must verify that $B^{Fed}_{o,t+1}/P_{o,t} = B^{Fed}_{s,t+1}/P_{s,t}$. To see that this condition holds, recall that

$$B^{Fed}_{s,t+1} = \left(1 + \frac{k_s - k_o}{1+k_o}\right)^t B^{Fed}_{s,0}.$$

Now, if $\pi_{s,t} - \pi_{o,t} = (k_s - k_o)/(1+k_o)$, by (A.2) we have that

$$P_{a,t} = \prod_{\tau=1}^{t} (1 + \pi_{a,\tau}) P_0 \text{ for } a \in \{o, s\}.$$

Combined with the guess (A.3) above, we obtain

$$P_{s,t} = \prod_{\tau=1}^{t} (1 + \pi_{o,t}) \left(1 + \frac{k_s - k_o}{1 + k_o}\right) P_0 = P_{o,t} \left(1 + \frac{k_s - k_o}{1 + k_o}\right)^t.$$

Therefore,

$$B^{Fed}_{s,t+1}/P_{s,t} = \left(1 + \frac{k_s - k_o}{1 + k_o}\right)^t B^{Fed}_{s,0}/P_{s,t} = B^{Fed}_{0,t+1}/P_{o,t},$$

which shows that the real holdings of loans under both policies are equal. The arguments are identical for the equilibrium in the deposit and government bond market are identical, but the market for Fed assets works differently.

We needed to verify that under our guess, $\left\{R^b_t, R^g_t, R^d_t\right\}$ is the same under both policies. Note that R^m_t is the same under both policies:

$$R^m_{o,t} = \left(1 + i^{ior}_{o,t+1}\right) / (1 + \pi_{o,t+1}) = \left(1 + i^{ior}_{s,t+1}\right) \left(1 + \frac{k_s - k_o}{1 + g_o}\right) / (1 + \pi_{o,t+1}),$$

and by assumption (A.3), the condition is also equal:

$$\left(1 + i^{ior}_{s,t+1}\right) \frac{(1 + \pi_{o,t+1})}{(1 + \pi_{s,t+1})} / (1 + \pi_{o,t+1}) = R^m_{s,t}.$$

Next, consider the condition for an equilibrium for Fed liabilities:

$$(1 - c_t)\bar{m}_t E_t \ = M_{o,t}/P_{o,t} - M^h_{o,t}/P_{o,t} = M_{s,t}/P_{s,t} - M^h_{s,t}/P_{s,t}.$$

It is important that the demand for real balances of currency is inelastic—possibly zero. Otherwise, since currency earns no interest rate, its rate of return does change with the rate of inflation. Consider it is, then bank reserve demand must be the same: the condition is used to verify our guess (A.3). The condition above requires

$$\frac{P_{s,t+1}}{P_{o,t+1}} = \frac{M_{s,t+1}}{M_{o,t+1}} = \left(1 + \frac{k_s - k_o}{1 + k_o}\right)^t \frac{M_{o,t+1}}{M_{o,t+1}} = \left(1 + \frac{k_s - k_o}{1 + k_o}\right)^t.$$

Then, since by Assumption (A.2), initial prices are the same, we have that

$$\frac{P_{s,t+1}}{P_{o,t+1}} = \frac{\prod_{\tau=1}^{t} (1 + \pi_{s,t}) P_0}{\prod_{\tau=1}^{t} (1 + \pi_{o,t}) P_0} = \left(1 + \frac{k_s - k_o}{1 + k_o}\right)^t \Rightarrow \prod_{\tau=1}^{t} (1 + \pi_{s,t}) = \prod_{\tau=1}^{t} (1 + \pi_{o,t}) \left(\frac{1 + k_s}{1 + k_o}\right).$$

Since the condition holds for all t, then A.3 is deduced from the quantity equation of reserves.

The next step is to verify that $R_t^{\bar{\chi}}$ is constant under both policies. For that, observe that the interbank market tightness is the same under both economies. To see that, simply note that the ratio of reserves to deposits is the same under both policies, and that this is enough to guarantee that θ_t is equal under both policies. By Lemma E.3 and the condition for policy rates in the proposition—$\left(1+i_{o,t}^{x}\right)=\left(1+i_{s,t}^{x}\right)\left(\frac{1+k_s}{1+k_o}\right)$ for $x\in\{w,m\}$—in states away from satiation,

$$\chi\left(.;i_{s,t}^{w},i_{s,t}^{ior}\right)=\left(\frac{1+k_s}{1+k_o}\right)\chi\left(.;i_{o,t}^{w},i_{o,t}^{ior}\right).$$

Therefore, we have that

$$R_{o,t}^{\chi}=\frac{\chi\left(.;i_{o,t}^{w},i_{o,t}^{ior}\right)}{1+\pi_{o,t}}=\frac{\left(\frac{1+k_s}{1+k_o}\right)\chi\left(.;i_{o,t}^{w},i_{o,t}^{ior}\right)}{\left(\frac{1+k_s}{1+k_o}\right)\left(1+\pi_{o,t}\right)}=\frac{\chi\left(.;i_{s,t}^{w},i_{s,t}^{ior}\right)}{\left(1+\pi_{s,t}\right)}=R_{s,t}^{\chi}.$$

This step verifies that $R_{o,t}^{\bar{\chi}}=R_{s,t}^{\bar{\chi}}$. So far, we have checked the consistency of assumptions (A.1) and (A.3), and that the policy rules for $\left\{\bar{b}_t,\bar{a}_t,\bar{d}_t,\bar{c}_t\right\}$ and the real rates are the same under both equilibria. We still need to show that the sequences for E_t are the same under both policies, that the initial price level is the same, and that the Fed's budget constraint is satisfied under both policies. This follows immediately from the law of motion of bank equity:

$$E_{t+1}=\left(1+\left(R_{t+1}^{b}-1\right)\bar{b}_t-\left(R_{t+1}^{d}-1\right)\bar{d}_t\right)\left(1-\bar{c}_t\right)E_t,$$

which as noted, must be the same. We have already verified that $B_{s,t+1}^{Fed}/P_{s,t}=B_{s,0}^{Fed}/P_{o,t}$. Following the same steps, we can show that real reserves M_t^{Fed}/P_t, government bonds G_t^{Fed}/P_t and G_t^{FA}/P_t and discount loans W_t^{Fed}/P_t are identical under both policies. Away from satiation, $R_{o,t}^{\chi}=R_{s,t}^{\chi}$, so that means that real income from the discount window, $\frac{W_t^{Fed}}{P_t}\left(\frac{1+i_t^w}{1+\pi_t}\right)$, is constant under both policies—τ_t is identical under both policies. Consider now $\left(B_0,D_0,M_0,G_0,W_0\right)$, the initial condition under both policies. If P_0 is same initial price under both policies, $E_{o,0}=E_{s,0}$. This is precisely the initial conditions that we need to confirm our guess $E_{o,0}=E_{s,0}$ and $P_{o,0}=P_{s,0}$. **QED.**

G.4 Proof of Proposition 5

Consider two policies, o and s, and let the alternative policy feature a mix of conventional and unconventional open-market operations performed at $t=0$ and reverted at $t=1$ in the sense that

1. $B_{s,1}^{Fed}=B_{o,1}^{Fed}+\Delta B^{Fed}$, $G_{s,1}^{Fed}=G_{o,0}^{Fed}+\Delta G^{Fed}$, and $M_{s,1}^{Fed}=M_{o,1}^{Fed}+\Delta M^{Fed}$, such that, $\Delta M^{Fed}=\Delta G^{Fed}+\Delta B^{Fed}$ and $\Delta M^{Fed},\Delta G^{Fed},\Delta B^{Fed}\geq 0$.

2. For all $t\geq 0$, we have

$$\left\{i_{o,t}^{m},i_{o,t}^{w},G_{o,t}^{FA},W_{o,t}\right\}=\left\{i_{s,t}^{m},i_{s,t}^{w},G_{s,t}^{FA},W_{s,t}\right\}.$$

3. For all $t \neq 1$, we have

$$\left\{i^m_{o,t}, i^w_{o,t}, M^{Fed}_{o,t}, G^{Fed}_{o,t}, G^{FA}_{o,t}, B^{Fed}_{o,t}, W_{o,t}\right\} = \left\{i^m_{s,t}, i^w_{s,t}, M^{Fed}_{s,t}, G^{Fed}_{s,t}, G^{FA}_{s,t}, B^{Fed}_{s,t}, W_{s,t}\right\}.$$

The statement of the proposition is that for $\lambda > 0$, the operation is neutral if and only if banks are satiated with reserves at time zero under both policies. If $\lambda \to 0$ and the economy is away from satiation, then a conventional policy, i.e. $\Delta B = 0$, is neutral but an unconventional policy $\Delta B > 0$ is not. We refer to neutrality as a situation where, as we compare across both policy sequences, the total outstanding amount of loans, deposits, and bonds remains unchanged in real terms.

The proof requires an intermediate step: First, we show that if two policies induce identical real aggregate loans deposits and bond holdings, the equilibrium prices $P_{o,0} = P_{s,0}$ must be equal. Then, we show for positive λ that if the price is constant, the open-market operation must have real effects away from satiation. Then, we show that if banks are satiated, the policy has no effects. Finally, we show that if $\lambda = 0$, the stated results holds.

Auxiliary Lemma. First, we prove the following auxiliary lemma corresponding to the first step of the proof.

Lemma G.2 *Consider two arbitrary policy sequences o and s, as described above. If total real loans, deposits, dividends, reserves, government bonds, and bank equity are equal across equilibria for all $t \geq 0$, then $P_{o,0} = P_{s,0}$.*

Proof. Without loss of generality, normalize the price in the original equilibrium to $P_{o,0} = 1$, but not the price of the alternative sequence—we can always re-scale the original sequence to obtain a price of one. The idea of the proof is to start from the quantity equation in one equilibrium, and use real market clearing conditions to express obtain a relationship using quantities of the second equilibrium. Using the quantity equation of the second equilibrium, the result must follow.

Consider now a given bank. By hypothesis, real equity is equal in both equilibria, $E_{s,0} = E_{o,0}$ and $\bar{c}_{o,0} = \bar{c}_{s,0}$. Also, recall that

$$\left\{B^{Fed}_{o,1}, B_{o,1}, G^{Fed}_{o,1}, G_{o,1}, M^{Fed}_{o,1}, M_{o,1}\right\}$$

and

$$\left\{B^{Fed}_{s,1}, B_{s,1}, G^{Fed}_{s,1}, G_{s,1}, M^{Fed}_{s,1}, M_{s,1}\right\}$$

are the nominal loans, government bonds, and reserves of the Fed and the representative bank, respectively—under the original and alternative policies.

We use the following relationships: Since equity, dividends, and real deposits are constant, from the bank's budget constraints, we obtain

$$B_{o,1} - B_{s,1}/P_{s,1} = A_s/P_{s,1} - A_o. \tag{G.1}$$

Also, we know that since market clearing must hold in the loans market under both equilib-

rium sequences,

$$\begin{aligned}\Theta^b \left(R^b_{o,1}\right)^{\epsilon^b} &\equiv B^{Fed}_{o,1} + B_{o,1} \\ &= \left(B^{Fed}_{s,1} + B_{s,1}\right)/P_{s,1},\end{aligned} \tag{G.2}$$

—recall that $P_{o,0} = 1$.

We now exploit the quantity equations of both equilibria, through the following relationship:

$$\begin{aligned}M^{Fed}_{o,t} + G^{FA}_{o,t} - G^{Fed}_{o,t} - G^h_{o,t} + \Delta M - \Delta G &= M^{Fed}_{1,t} + G^{FA}_{1,t} - G^{Fed}_{1,t} - G^h_{1,t} \\ &= P_{s,0}\bar{a}_{s,0}\left(1-\bar{c}_{s,0}\right)E_{s,0} \\ &= P_{s,0}\left(\bar{b}_{o,0} + \bar{a}_{o,0} - \bar{b}_{s,0}\right)\left(1-\bar{c}_{s,0}\right)E_{s,0} \\ &= P_{s,0}\left(\bar{b}_{o,0} + \bar{a}_{o,0}\right)\left(1-\bar{c}_{s,0}\right)E_{s,0}\ldots \\ &\quad - \left(\Theta^b\left(R^b_{o,1}\right)^{\epsilon^b} - B^{Fed}_{s,1}\right).\end{aligned} \tag{G.3}$$

The first equality just uses the relationship between both policies, the second equality follows from the quantity equation (23) which holds under the alternative equilibrium, the third equality uses (G.1) expressed in terms of the banks portfolio, and fourth follows from (G.2). Substituting out $B^{Fed}_{s,1}$ from the last term into (G.3), we obtain

$$\begin{aligned}M^{Fed}_{o,t} + G^{FA}_{o,t} - G^{Fed}_{o,t} - G^h_{o,t} + \Delta M - \Delta G = \ldots \\ P_{s,0}\left(\bar{b}_{o,0} + \bar{a}_{o,0}\right)\left(1-\bar{c}_{s,0}\right)E_{s,0} - \left(\Theta^b\left(R^b_{o,1}\right)^{\epsilon^b} - \left(B^{Fed}_{o,1} + \Delta B\right)\right).\end{aligned}$$

Then, using that $\Delta M = \Delta B + \Delta G$, the equation simplifies to

$$M^{Fed}_{o,t} + G^{FA}_{o,t} - G^{Fed}_{o,t} - G^h_{o,t} = P_{s,0}\left(\bar{b}_{o,0} + \bar{a}_{o,0}\right)\left(1-\bar{c}_{s,0}\right)E_{s,0} - \left(\Theta^b\left(R^b_{o,1}\right)^{\epsilon^b} - B^{Fed}_{o,1}\right).$$

Using the loans clearing condition $\Theta^b\left(R^b_{o,1}\right)^{\epsilon^b} - B^{Fed}_{o,1} = \bar{b}_{o,0}\left(1-\bar{c}_{s,0}\right)E_{s,0}$, we obtain:

$$\begin{aligned}M^{Fed}_{o,t} + G^{FA}_{o,t} - G^{Fed}_{o,t} - G^h_{o,t} &= P_{s,0}\left(\bar{b}_{o,0} + \bar{a}_{o,0}\right)\left(1-\bar{c}_{s,0}\right)E_{s,0} - \bar{b}_{o,0}\left(1-\bar{c}_{s,0}\right)E_{s,0} \\ &= \left(P_{s,0}-1\right)\bar{b}_{o,0}\left(1-\bar{c}_{s,0}\right)E_{s,0} + P_{s,0}\bar{a}_{o,0}\left(1-\bar{c}_{s,0}\right)E_{s,0}\end{aligned}$$

Finally, using the quantity equation (23) applied to the first equilibrium, $M^{Fed}_{o,t} + G^{FA}_{o,t} - G^{Fed}_{o,t} - G^h_{o,t} = \bar{a}_{o,0}\left(1-\bar{c}_{s,0}\right)E_{s,0}$ we obtain:

$$0 = \left(P_{s,0}-1\right)\bar{b}_{o,0}\left(1-\bar{c}_{o,0}\right)E_{o,0} + \left(P_{o,0}-1\right)\bar{a}_{o,0}\left(1-\bar{c}_{o,0}\right)E_{o,0}.$$

Since this equation is independent of ΔM, and $\bar{b}_{o,0}, \bar{c}_{o,0}, E_{o,0}, \bar{a}_{o,0}$ are all positive numbers, and any price is positive, it must be that $P_o = P_s = 1$. **QED.**

Next, we establish the main results.

Item 1: Non-neutrality away from satiation and $\lambda > 0$. First, we argue that if the policy change is neutral away from satiation, we reach a contradiction. Assume that the policy is indeed neutral. If policy s is neutral with respect to policy o, real assets, real asset returns, dividends,

and bank equity must be equal across both equilibria. Consider Loan LP. Since real loans are the same, R^b must be the same in both equilibria. Also R^m must be the same. This is the case because by $t = 1$, the policy is reverses and thus the equilibrium and the price must be the same. Since by assumption, the policy is neutral, also, the $t = 0$ price must be equal, as shown in Lemma G.2. Hence, since i^m is constant across both policies, then, R^m must the same. However, since under one equilibrium liquid assets are lower, but deposits are the by assumption same, the liquidity premium cannot be the same. A contradiction.

Item 2: Neutrality under satiation. Next, we verify that under satiation, the policy change has no effects. The key to verify the result is to show that if the economy is under satiation under both policies, if the bank's portfolio changes exactly in the opposite direction as the Fed's portfolio, the policy is neutral—that is, we guess that there is no crowding in or crowding out effects. Thus, we guess that:

$$B_{o,1} - \Delta B = B_{s,1}, G_{o,1} - \Delta G = G_{s,1}, \text{ and } M_{o,1} + \Delta M = M_{s,1}.$$

If the allocation is the same in real terms, then by Lemma G.2, $t = 0$ and $t = 1$ prices are the same, and $P_{o,1} = P_{s,1}$. Under satiation, we also know that $R^b = R^m = R^g$. Hence, the aggregate quantity of loans and bonds be equal under both policies. Hence, clearing in the loans market implies:

$$\Theta^b \left(R^b_{o,1}\right)^{\epsilon^b} = \Theta^b \left(R^m_{o,1}\right)^{\epsilon^b} = \frac{B_{o,1} + B^{Fed}_{o,1}}{P_{o,1}} = \frac{B_{o,1} - \Delta B + B^{Fed}_{o,1} + \Delta B}{P_{o,1}} \dots$$
$$= \frac{B_{s,1} + B^{Fed}_{s,1}}{P_{s,1}} = \Theta^b \left(R^b_{s,1}\right)^{\epsilon^b}.$$

and in the bond market,

$$\Theta^g \left(R^g_{o,1}\right)^{\epsilon^g} = \Theta^g \left(R^m_{o,1}\right)^{\epsilon^g} = \frac{G^{FA}_1 - G_{o,1} + G^{Fed}_{o,1}}{P_{o,1}} = \frac{G^{FA}_1 - G_{o,1} - \Delta G + G^{Fed}_{o,1} + \Delta G}{P_{o,1}} \dots$$
$$= \frac{G^{FA}_1 - G_{s,1} + G^{Fed}_{s,1}}{P_{s,1}} = \Theta^g \left(R^g_{s,1}\right)^{\epsilon^g}.$$

and in the deposit market,

$$\frac{D_{o,1}}{P_{o,1}} = \frac{B_{o,1} + G_{o,1} + M_{o,1}}{P_{o,1}} + E_{o,1} = \frac{B_{o,1} + G_{o,1} + M_{o,1} - \Delta B - \Delta G + \Delta M}{P_{o,1}} + E_{o,1} \dots$$
$$= \frac{B_{s,1} + G_{s,1} + M_{s,1}}{P_{o,1}} + E_{s,1} = \frac{D_{s,1}}{P_{s,1}}.$$

Thus, all market clearing conditions are satisfied. Since, if banks start under satiation, and the increase in total liquid assets is positive, then banks are satiated under both policies. Thus, under both policies, banks are indifferent between loan, bond, and reserve holdings, the guess is consistent with bank equilibrium choices. Since the law of motion of equity is unchanged across both equilibria, the path of dividends is also the same, which verifies the guess that the price

level is the same in both cases.

Item 3: Limit case as $\lambda \to 0$. Finally, we verify that if $\lambda \to 0$, conventional policies are neutral, but unconventional policies are not. Recall, a conventional policy is one where $\Delta B = 0, \Delta G > 0$ and an unconventional is one where $\Delta B > 0$. Also, recall that if $\lambda \to 0$, then $\bar{\chi}^{+}_{o,1} = 0$ and $\left(\bar{\chi}^{-}_{o,1} - \bar{\chi}^{+}_{o,1}\right) = R^{w}_{o,1}$, for any interbank market tightness. Thus, we have that the Loan LP becomes:

$$R^b - R^m = R^w_{o,1} \cdot F\left(\omega^*\right) \cdot \frac{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma} | \omega < \omega^*\right]}{\mathbb{E}_\omega\left[\left(R^e\right)^{-\gamma}\right]},$$

where $\omega^* \equiv -\left(\bar{a}/\bar{d} - \rho\right) / \left(R^d_{t+1}/R^m_{t+1} - \rho\right)$. In turn, the bond return satisfies:

$$R^g = R^m.$$

First, we verify that the conventional policy is neutral: Suppose it is. By Lemma G.2, $t = 0$ and $t = 1$ prices are the same, $P_{o,1} = P_{s,1}$. Thus, $R^m_{o,1} = R^m_{s,1}$. Since the price level is the same, and the policy is exclusively a conventional policy, $\Delta M = \Delta G$. Under this guess, if the policy does not crowd out household bonds, $G_{o,1} + M_{o,1} = G_{o,1} + M_{o,1} + \Delta M - \Delta G = G_{s,1} + M_{s,1}$. This in turn, implies that $\bar{a}_{o,0} = \bar{a}_{s,0}$, and thus, $\omega^*_{o,0} = \omega^*_{s,0}$. Since the threshold remains unchanged, and tightness does not affect the loans nor the bond premium. The guess is therefore verified.

To close the proposition, we verify that the unconventional policy has an effect. Suppose not. Then, prices do not change, again by Lemma G.2. We know that:

$$G_{o,1} + M_{o,1} = G_{o,1} + M_{o,1} + \Delta M - \Delta G - \Delta B = G_{s,1} + M_{s,1} - \Delta B.$$

Thus, since $\Delta B \neq 0$, the real value of liquid assets under the original and alternative policies differ. Hence, we have a contradiction: either the threshold ω^*differs across both policies or the deposits adjust, or both. In either case, the liquidity premium must be different across both policies. The result follows. **QED.**

G.5 Proof of Proposition 6 and Corollary 1

We first demonstrate Proposition 6 and then proof the bound in Corollary 1. For the rest of this proof, we avoid time subscripts under the understanding that the condition applies to stationary equilibria. A stationary equilibrium satisfies four equilibrium conditions under any Friedman rule. We have that:

1. A stationarity condition:

$$1/\beta = \left(\bar{a} + R^b\left(1 + \bar{d} - \bar{a}\right) - R^d\bar{d}\right). \tag{G.1}$$

2. Two stationary clearing conditions:

$$B = \Theta^b\left(\bar{R}\right)^{\epsilon^b} \text{ and } D = \Theta^d\left(\bar{R}\right)^{\epsilon^d}.$$

3. An aggregate budget balance:

$$B = D + \left(1 - \bar{a}\right)\tilde{E}. \tag{G.2}$$

Here $\tilde{E}$ is the steady-state equity after dividends. These conditions hold regardless of the stationary dividend.

Proof of Proposition 6 There are four possible outcomes: either capital requirements bind or not and either $\bar{a} > 0$ or $a = 0$. We develop observations for each case. We first observe that under the Friedman rule, $R^b \geq R^m$, with equality if $a > 0$. We first investigate the cases where $a = 0$.

Case I: $\bar{a} = 0$ ***and capital requirements do not bind.*** If the capital requirement does not bind and $\bar{a} = 0$, then we know that $R^b \geq R^m$ and $R^b = R^d$. Because the Friedman rule eliminates the liquidity premium, the stationary condition (G.1) requires:

$$\frac{1}{\beta} = \left(\bar{a} + R^b\left(1 - \bar{a} + \bar{d}\right) - R^d\bar{d}\right) = R^b,$$

where the first equality is the definition of equity returns and the second equality uses $R^b = R^d$ and $a = 0$. Thus, we have that $R^b = R^d = 1/\beta$ and $R^m \leq 1/\beta$.

In this case, the stationary equilibrium loans and deposits are given by

$$B = \Theta^b \left(1/\beta\right)^{\epsilon^b} \text{ and } D = \Theta^d \left(1/\beta\right)^{\epsilon^d}.$$

Thus, (G.2), becomes

$$\tilde{E} = \Theta^b \left(1/\beta\right)^{\epsilon^b} - \Theta^d \left(1/\beta\right)^{\epsilon^d}. \tag{G.3}$$

If capital requirements are indeed satisfied, it must be that:

$$\tilde{E} \geq \frac{1}{\kappa}\Theta^d \left(1/\beta\right)^{\epsilon^d}. \tag{G.4}$$

Combining, (G.3-G.4) yields:

$$\Theta^b \left(1/\beta\right)^{\epsilon^b} \geq \frac{\kappa + 1}{\kappa}\Theta^d \left(1/\beta\right)^{\epsilon^d}. \tag{G.5}$$

If the condition is not satisfied, then, it is not possible to have a stationary equilibrium under the Friedman rule with $a = 0$ and where capital requirements do not bind. We summarize this case with the following observation:

Remark 1. If $\bar{a} = 0$ and capital requirements do not bind, then $R^b = R^d = 1/\beta$ and $R^m \leq 1/\beta$ and condition (25) must hold.

Case II: capital requirements binds and $\bar{a} = 0$**.** If the capital requirement binds and $\bar{a} = 0$, we know that $R^b > R^d$ and $R^b > R^m$. Since capital requirements bind, after dividend equity must equal, $\tilde{E} = \frac{1}{\kappa}\Theta^d \left(R^d\right)^{\epsilon^d}$. Again, because the Friedman rule eliminates the liquidity premium, the stationary condition (G.1) to bank equity is $1/\beta$:

$$1/\beta = R^b + \left(R^b - R^d\right)\kappa.$$

Rewriting this expression yields a relationship between R^d and R^b,

$$R^b = \frac{\frac{1}{\beta} + \kappa R^d}{(1+\kappa)}. \tag{G.6}$$

We have the following observation, which we proof consequently:

Remark 2. If the capital requirement binds and $\bar{a} = 0$, we have that $R^d < 1/\beta$.

Suppose the contrary, then

$$R^b = \frac{\frac{1}{\beta} + \kappa R^d}{(1+\kappa)} > \frac{\frac{1}{\beta} + \kappa \frac{1}{\beta}}{(1+\kappa)} = \frac{1}{\beta}.$$

But if then, $R^b > 1/\beta$, and $R^b > R^d$, implies that the return on equity is above $1/\beta$. Clearly a contradiction. Hence, Remark 2 must hold.

Substituting the equilibrium conditions into the aggregate budget constraint yields:

$$\Theta^b \left(R^b\right)^{\epsilon^b} = \Theta^d \left(R^d\right)^{\epsilon^d} + \frac{1}{\kappa}\Theta^d \left(R^d\right)^{\epsilon^d} = \Theta^d \left(R^d\right)^{\epsilon^d} \left(\frac{\kappa+1}{\kappa}\right).$$

Substituting (G.6) and re-arranging produces:

$$(1+\kappa)\left(\frac{\Theta^d}{\Theta^b}\frac{1+\kappa}{\kappa}\right)^{1/\epsilon^b} \left(R^d\right)^{\epsilon^d/\epsilon^b} = \frac{1}{\beta} + \kappa R^d. \tag{G.7}$$

This is the same expression for $\bar{R}^d$ in Proposition 6. We have the following property:

Remark 3. Equation (G.7) has a unique solution.

To see this, note that the left-hand side is decreasing in R^d and the right increasing in R^d. For $R^d = 0$, the left hand side is above $1/\beta$, so the solution must be unique. With this, and using (G.6) we obtain $\bar{R}^b$ as in the Proposition.

Next, we must verify that indeed the unique solution holds for $R^d < 1/\beta$, as needed—see Remark 2. Since the right hand side of (G.7) is increasing and the left decreasing, $\bar{R}^d < 1/\beta$ if and only if:

$$(1+\kappa)\left(\frac{\Theta^d}{\Theta^b}\frac{1+\kappa}{\kappa}\right)^{1/\epsilon^b} (1/\beta)^{\epsilon^d/\epsilon^b} < \frac{1}{\beta} + \kappa 1/\beta.$$

Rearranging the terms, leads to:

$$\Theta^b (1/\beta)^{\epsilon^b} < \frac{\kappa+1}{\kappa}\Theta^d (1/\beta)^{\epsilon^d}.$$

This is enough to conclude that:

Remark 4. If $\bar{a} = 0$ and capital requirements bind, then $R^m \leq \bar{R}^b \leq 1/\beta$ and condition (25) must be violated.

We can combine this result and remark 1, to obtain the following:

Remark 5. Whether condition (25) holds or not, if $\bar{a} = 0$, then $R^m \leq \min\left\{1/\beta, \frac{\frac{1}{\beta}+\kappa\bar{R}^d}{(1+\kappa)}\right\}$ for $(1+\kappa)\left(\frac{\Theta^d}{\Theta^b}\frac{1+\kappa}{\kappa}\right)^{1/\epsilon^b}\left(\bar{R}^d\right)^{\epsilon^d/\epsilon^b} = \frac{1}{\beta} + \kappa\bar{R}^d$.

Next, we move to the cases where $\bar{a} > 0$.

Case III: capital requirements do not bind and $\bar{a} > 0$**.** In this case, we know that $R^b = R^m = R^d$. The (G.1) becomes:

$$1/\beta = \left(\bar{a} + R^m\left(1 + \bar{d} - \bar{a}\right) - R^m\bar{d}\right) = \left(R^m - \bar{a}\left(R^m - 1\right)\right).$$

Hence, we have that:

$$\bar{a} = \frac{R^m - 1/\beta}{R^m - 1}.$$

This implies that that $R^m > 1/\beta$. We have the following remark:

Remark 6. If capital requirements do no bind and $\bar{a} > 0$, then $R^m > 1/\beta$.

Case IV: capital requirements bind and $\bar{a} > 0$**.** So far we have shown conditions for stationary equilibria in which $a = 0$,which specify hold only if $R^m \leq 1/\beta$—with an exact threshold given in Remark 5. Remark 6 shows that if capital requirements do not bind and if $R^m > 1/\beta$, then $\bar{a} > 0$. To complete the statement of the proposition, we need to show if condition (25) is not satisfied and $R^m \leq \bar{R}^b = \frac{1}{\beta}\frac{1+\kappa\beta\bar{R}^d}{1+\kappa}$ where $\bar{R}^d$ solves:

$$(1+\kappa)\left(\frac{\Theta^d}{\Theta^b}\left(1+\kappa^{-1}\right)\right)^{1/\epsilon^b}\left(\bar{R}^d\right)^{\epsilon^d/\epsilon^b} = \frac{1}{\beta} + \kappa\bar{R}^d,$$

then $\bar{a} = 0$ and the capital requirement binds. Hence, any $R^m > \bar{R}^b$ must feature $\bar{a} > 0$ establishing that $R^b = R^m$.

To prove this, we assume by contradiction that $\bar{a} > 0$. By assumption,

$$R^b = R^m \leq \bar{R}^b = \frac{1}{\beta}\frac{1+\kappa\beta\bar{R}^d}{1+\kappa}.$$

and $a > 0$. Under the stated assumptions, (G.1) becomes:

$$1/\beta = \left(R^m - \left(R^m - 1\right)\bar{a} + \left(R^m - R^d\right)\kappa\right). \tag{G.8}$$

However, we also know that:

$$1/\beta = \left(\bar{R}^b + \left(\bar{R}^b - \bar{R}^d\right)\kappa\right). \tag{G.9}$$

Hence, we have the following condition:

Remark 7. If condition (25) is not satisfied, capital requirements bind, $\bar{a} > 0$, and $R^m \leq \bar{R}^b$, then it would be the case that $R^d < \bar{R}^d$.

The remark can be shown to hold simply by noticing that $R^m < \bar{R}^b$ and $\bar{a} > 0$, in then it must be that $R^d < \bar{R}^d$, by comparing (G.8) and (G.9).

Next, solving for $\bar{a}$ from (G.8) yields:

$$\bar{a} = \frac{R^m - 1/\beta + \left(R^m - R^d\right)\kappa}{R^m - 1}.$$

Observe that by monotonicity:

$$\Theta^b \left(R^m\right)^{\epsilon^b} > \Theta^b \left(\bar{R}^b\right)^{\epsilon^b}$$

and also

$$\Theta^d \left(R^d\right)^{\epsilon^d} \left(1+\kappa^{-1}\left(1-\bar{a}\right)\right) < \Theta^d \left(\bar{R}^d\right)^{\epsilon^d} \left(1+\kappa^{-1}\left(1-\bar{a}\right)\right) < \Theta^d \left(\bar{R}^d\right)^{\epsilon^d}.$$

Then, if we substitute real rates into (G.2), we obtain:

$$\Theta^b \left(R^m\right)^{\epsilon^b} = \Theta^d \left(R^d\right)^{\epsilon^d} + \frac{1-\bar{a}}{\kappa} \cdot \Theta^d \left(R^d\right)^{\epsilon^d} = \Theta^d \left(R^d\right)^{\epsilon^d} \left(\frac{\kappa+(1-\bar{a})}{\kappa}\right).$$

Substituting the inequalities above:

$$\Theta^b \left(\bar{R}^b\right)^{\epsilon^b} < \Theta^b \left(R^m\right)^{\epsilon^b} = \Theta^d \left(R^d\right)^{\epsilon^d} \left(\frac{\kappa+(1-\bar{a})}{\kappa}\right) < \Theta^d \left(\bar{R}^d\right)^{\epsilon^d} \left(\frac{\kappa+1}{\kappa}\right).$$

However, this contradicts the definition of $\left\{\bar{R}^b, \bar{R}^d\right\}$. Hence, we conclude that:

Remark 8. If condition (25) is not satisfied and $R^m \leq \bar{R}^b$, then we have that $\bar{a} = 0$ and capital requirements binds.

Collecting the results. Consider the cases where condition (25) holds. Combining remarks (1) and (6) we have that:

Remark 9. If (25) holds, then the loans rate associated with the Friedman rule is:

$$R^{b,FR} = \begin{cases} 1/\beta & \text{if } R^m < 1/\beta, \\ R^m & \text{if } R^m \geq 1/\beta. \end{cases} \tag{G.10}$$

Moreover, $\bar{a} = 0$ and the capital requirement does not bind if and only if $R^m \leq 1/\beta$. If $R^m \leq 1/\beta$, the stationary deposit rate is also $1/\beta$.

Now consider the cases where condition (25) does not hold. Combining remarks (5) and (8) we have that:

Remark 10. If (25) does not hold

$$R^{b,FR} = \begin{cases} \bar{R}^b & \text{if } R^m < \bar{R}^b, \\ R^m & \text{if } R^m \geq \bar{R}^b, \end{cases} \tag{G.11}$$

where

$$\bar{R}^b = \frac{1}{\beta}\frac{1+\kappa\beta\bar{R}^d}{1+\kappa} \leq \frac{1}{\beta},$$

where $\bar{R}^d$ the stationary deposit rate which is the unique solution to

$$(1+\kappa)\left(\frac{\Theta^d}{\Theta^b}\left(1+\kappa^{-1}\right)\right)^{1/\epsilon^b} \left(\bar{R}^d\right)^{\epsilon^d/\epsilon^b} = \frac{1}{\beta} + \kappa\bar{R}^d.$$

Moreover, $\bar{a} = 0$ if and only if $R^m \leq \bar{R}^b$. In this case, the capital requirement binds.

Combining remarks (9) and (10) we obtain the statement of Proposition 6. **QED.**

Proof of Corollary 1 The proof is immediate. First, observe that if (25) holds, then we had showed that the $\bar{R}^d$ that solves

$$(1+\kappa)\left(\frac{\Theta^d}{\Theta^b}\left(1+\kappa^{-1}\right)\right)^{1/\epsilon^b}\left(\bar{R}^d\right)^{\epsilon^d/\epsilon^b}=\frac{1}{\beta}+\kappa\bar{R}^d,$$

is above $\frac{1}{\beta}$. Thus, $\bar{R}^b > 1/\beta$. Otherwise, if (25) does not hold, the solution is less than $1/\beta$ and $\bar{R}^b < 1/\beta$. This implies that a compact way to write $R^{b,FR}$ is:

$$R^{b,FR}=\begin{cases}\min\left\{\bar{R}^b, 1/\beta\right\} & \text{if } R^m_{ss} < \min\left\{\bar{R}^b, 1/\beta\right\},\\ R^m & \text{if } R^m_{ss} \geq \min\left\{\bar{R}^b, 1/\beta\right\}\end{cases}$$

Then, since by assumption of the corollary $R^m \geq \min\left\{\bar{R}^b, 1/\beta\right\} = R^{b,FR}$ and $R^b \geq R^m$ in any equilibrium, the bound follows. **QED.**

H Efficient Allocations and Proof of Proposition 7

In this Appendix, we derive the efficient allocations under the assumption that $\beta^h = \beta$. We also show that a version of the Friedman rule, with the appropriate choice of R^m, can implement the first-best allocation, provided that capital requirements are sufficiently ample. We let a planner maximize a weighted average of households and bankers' utility subject to the resource constraint for goods and labor. The planner's problem is:

Problem 4 (Planner's Problem)**.** The unconstrained planner's problem is given by:

$$\max_{\left\{c_t, c_t^d, c_t^g, c_t^m, h_t\right\}_{t=0}^{\infty}} \sum_{t=0,1..} \beta^t \left[(1-\varpi) \left(\sum_{x \in \{d,g,m\}} U^x\left(c_t^x\right) + c_t^h - \frac{h_t^{1+\nu}}{1+\nu} \right) + \varpi \cdot u(c_t) \right]$$

subject to the resource constraint:

$$\sum_{x \in \{d,g,m\}} c_t^x + c_t^h + c_t = y_t, \tag{H.1}$$

and the technological constraint:

$$y_t = A_t h_{t-1}^{\alpha}. \tag{H.2}$$

The initial labor input h_{-1} is given.

Here, we use ϖ for the Pareto weight on the banker's consumption. The next Proposition characterizes an optimal allocation.

Proposition H.1 *The optimality conditions of the unconstrained planner problem are:*

$$U_{c^x}^x\left(\bar{X}\right) = 1 = \frac{\varpi}{(1-\varpi)} \phi u'(c_t) \text{ for } x \in \{d, g, m\} \tag{H.3}$$

and

$$\beta \alpha A_{t+1} h_t^{\alpha-1} = h_t^{\nu}. \tag{H.4}$$

The proposition states that in the first best allocation, the planner equalizes the labor wedge to zero and equalizes all the marginal utility across goods to one. The latter is optimal because the marginal rate of transformation is one across all goods. Notice also that the planner's solution is characterized by a sequence of static problems, so there are no dynamic trade-offs in the allocation. We say an allocation is efficient if it coincides with the planner's solution for some ϖ.

Next, we state a detailed version of Proposition 7.

Proposition H.2 *Consider a competitive equilibrium and a version of the Friedman rule where the Fed sets $R_t^m = R_t^w = 1/\beta$, (and $\pi_t = \beta - 1$) and adjusts M_t^{Fed}, and G_t^{Fed} such that $R^g = 1/\beta$. A necessary condition for this Friedman rule to induce an efficient allocation in its stationary equilibrium is that:*

$$\Theta^b \left(1/\beta\right)^{\epsilon^b} \geq \left(\frac{1+\kappa}{\kappa} \right) \Theta^d \left(1/\beta\right)^{1/\epsilon^d}.$$

If the condition is violated, the first-best is not attainable.

We proceed with a proof.

H.1 Decentralization and the Friedman Rule (Proof of Propositions H.1 and H.2)

We begin with the proof of Proposition H.1.

Proof. Substituting out c^h from the resource constraint into the objective, replacing y_t from the technological constraint yields a modified objective function:

$$\max_{\left\{c_t,c_t^d,c_t^g,c_t^m,h_t\right\}} \sum_{t=0,1..} \beta^t \left[(1-\varpi) \left(\sum_{x\in\{d,g,m\}} U^x\left(c_t^x\right) + A_t h_{t-1}^{\alpha} - \sum_{x\in\{d,g,m\}} c^x + c_t - \frac{h_t^{1+\nu}}{1+\nu} \right) + \varpi \cdot u(c_t) \right].$$

The conditions are verified by taking first-order conditions with respect to $\left\{c_t, c_t^d, c_t^g, c_t^m, h_t\right\}$. **QED.**

We now move to proof Proposition H.2.

Proof. Note that a necessary condition for efficiency is that $R^d = R^g = 1/\beta$ and $\frac{1}{1+\pi} = 1/\beta$. This follows directly from the household's optimality condition in (C.4)—for the case where each asset-in-advance constraint is slack. In that case, the household's allocation across goods coincides with the planner problem's unconstrained condition, (H.3). Similarly, for the firm's problem, the optimality condition (C.7) coincides (H.4) for $R^b = 1/\beta$. Also, notice that it is only possible to $R^b = R^d = R^g = R^m = 1/\beta$ if there is no liquidity premium.

Assume that the efficiency condition holds at every period. Then loans and deposits are given by:

$$B_t = \Theta^b \left(1/\beta\right)^{\epsilon^b} \text{ and } D_t = \Theta^d \left(1/\beta\right)^{1/\epsilon^d}.$$

From the bank's budget constraint, and portfolio constraints, using these quantities, we have that:

$$A_t + \Theta^b \left(1/\beta\right)^{\epsilon^b} = \Theta^d \left(1/\beta\right)^{1/\epsilon^d} + E_t \tag{H.1}$$

$$E_t \geq \frac{1}{\kappa} \Theta^d \left(1/\beta\right)^{1/\epsilon^d} \tag{H.2}$$

$$A_t \geq 0. \tag{H.3}$$

A condition for stationarity is:

$$1/\beta = \left(\bar{a}_{ss} + R^b_{ss}\left(1 + \bar{d} - \bar{a}\right) - R^d_{ss}\bar{d}\right), \tag{H.4}$$

and replacing the efficiency condition, $R^b = R^d = 1/\beta$, yields:

$$1/\beta = \left(1/\beta - 1/\beta\left(1-\bar{a}\right)\right).$$

This condition implies that $A_t \geq 0$.

Combining (H.1) and (H.3) we obtain:

$$E_{ss} = \Theta^b \left(1/\beta\right)^{\epsilon^b} - \Theta^d \left(1/\beta\right)^{1/\epsilon^d}.$$

Substituting this result into the capital requirement condition yields:

$$\Theta^b \left(1/\beta\right)^{\epsilon^b} \geq \left(\frac{1+\kappa}{\kappa}\right) \Theta^d \left(1/\beta\right)^{1/\epsilon^d}.$$

Hence, the necessary conditions for efficiency in Proposition 7. **QED.**

I Proof of Proposition 8

Here we present the proof of Proposition 8 in Section 5.2 regarding the pass-through of monetary policy. In this appendix we present a more general version of the Proposition in the text. In particular, we derive the comparative statics with respect to changes in the the interest on reserves under two scenarios: (i) keeping the discount window rate constant, and (ii) keeping the spread in both policy rates constant. For ease of exposition we restrict to the cases where, $\bar{b}^{fed} = 0$, but the result can be extended along that dimension without difficulty. Let $\mathcal{LP}_y^x$ denote the derivative of the liquidity premium of asset x with respect to portfolio holdings of asset y. The general version of of Proposition 8 is as follows.

Proposition I.1 *Consider stationary equilibria. Consider an increase from a stationary level of r^m that leaves the stationary level of r^w constant or leaves the corridor spread, $\Delta = r^w - r^m$, constant. If capital requirements are binding, then the increase in r^m unambiguously increases r^b. Then, if capital requirements bind, in the region where capital requirements bind:*

$$\frac{dr^b}{dr^m} = \frac{1 + \epsilon^d \cdot \frac{\left(\bar{b}+\bar{b}^{Fed}\right)}{\kappa} \frac{r^b}{R^d}}{1 + \epsilon^d \cdot \frac{\left(\bar{b}+\bar{b}^{Fed}\right)}{\kappa} \frac{r^b}{R^d} - \mathcal{LP}_a^b \frac{\left(\bar{b}+\bar{b}^{Fed}\right)}{R^b} \left(\epsilon^d \cdot \frac{\bar{b}}{\kappa} - \epsilon^b\right)} \left(1 - \frac{\mathbb{E}_\omega \left[\chi\left(\theta\right)\right]}{\Delta} \cdot \mathbb{I}\left[dr^w = 0\right]\right) \in [0, 1],$$

and $\frac{dr^b}{dr^m} = 1$, when banks are satiated with reserves. If capital requirements do not bind and the deposit supply is perfectly elastic at r^d, the pass-through is ambiguous and given by:

$$\frac{dr^b}{dr^m} = \frac{\left(\left(\mathcal{LP}_d^b + \mathcal{LP}_d^d\right)\right) r^b + \left(\mathcal{LP}_b^b + \mathcal{LP}_b^d\right) r^d}{\mathcal{LP}_b^b r^d + \mathcal{LP}_d^d r^b + \left(\mathcal{LP}_b^b \left(\mathcal{LP}_b^d + \mathcal{LP}_d^d\right) - \mathcal{LP}_b^d \left(\mathcal{LP}_b^b + \mathcal{LP}_d^b\right)\right) \bar{b}} \left(1 - \frac{\mathbb{E}_\omega \left[\chi\left(\theta\right)\right]}{\Delta} \cdot \mathbb{I}\left[dr^w = 0\right]\right).$$

Proof. We first prove the result for the case with a perfectly elastic deposit supply schedule and binding capital requirements. We then relax one assumption at a time for the general result. First, recall that the the slopes of the liquidity yield function are given by

$$\chi^+ = \left(i^w - i^m\right) \left(\frac{\bar{\theta}}{\theta}\right)^\eta \left(\frac{\theta^\eta \bar{\theta}^{1-\eta} - \theta}{\bar{\theta} - 1}\right) \text{ and } \chi^- = \left(i^w - i^m\right) \left(\frac{\bar{\theta}}{\theta}\right)^\eta \left(\frac{\theta^\eta \bar{\theta}^{1-\eta} - 1}{\bar{\theta} - 1}\right). \tag{I.1}$$

Thus, we can write them as be written as:

$$\chi^+ = \Delta q^+\left(\theta\right) \text{ and } \chi^- = \Delta q^-\left(\theta\right).$$

Clearly, $\{q^+, q^-\} \in [0, 1]^2$. We proof the results for the case where $dr^w > 0$, but the steps are the same to obtain the general result above. **QED.**

Case #1: Infinitely Elastic Deposit Supply and Binding Capital Requirements. The gist of the proof is to perform a comparative statics analysis with respect to r^m on the following sub-system of equilibrium equations:

$$1 = \beta \left(1 + r^b \left(\bar{b} + \bar{b}^{Fed}\right) - r^d \bar{d}\right), \tag{I.2}$$

and

$$r^b = r^m + \mathbb{E}_\omega\left[\bar{\chi}\right], \tag{I.3}$$

where

$$\mathbb{E}_\omega\left[\bar{\chi}\right] = \int_{-1}^{\omega^*} \bar{\chi}^- f\left(\omega\right) d\omega + \int_{\omega^*}^{\infty} \bar{\chi}^+ f\left(\omega\right) d\omega.$$

This subsystem is the loans premium and the stationarity condition for equity.

Then, taking total differentials with respect to r^m on (I.2) and (I.3) we respectively obtain:

$$\left(\bar{b} + \bar{b}^{Fed}\right) \frac{dr^b}{dr^m} + r^b \frac{d\bar{b}}{dr^m} = 0. \tag{I.4}$$

and

$$\frac{dr^b}{dr^m} = 1 + \frac{d\left[\mathbb{E}\left[\bar{\chi}\right]\right]}{dr^m}. \tag{I.5}$$

Then, we have that

$$\frac{d\left[\mathbb{E}\left[\bar{\chi}\right]\right]}{dr^m} = -\mathbb{E}_\omega\left[q\left(\theta\right)\right] + \Delta \frac{d\bar{b}}{dr^m} \tag{I.6}$$

where

$$\mathcal{LP}_b^b \equiv \left[\mathbb{E}\left[\left(\bar{\chi}^- - \bar{\chi}^+\right) f\left(\omega^*\right) \frac{d\omega^*}{d\bar{b}} + \mathbb{E}_\omega\left[\bar{\chi}_\theta f\left(\omega\right) d\omega\right] \frac{d\theta^*}{d\bar{b}}\right]\right] > 0.$$

We employed Leibnitz's rule. Thus, substituting the expressions we obtain:

$$\frac{dr^b}{dr^m} - \mathcal{LP}_b^b \frac{d\bar{b}}{dr^m} = 1 - \mathbb{E}_\omega\left[q\left(\theta\right)\right] > 0.$$

The system (I.4) and (I.5) in matrix form is represented as:

$$\begin{bmatrix} \bar{b} + \bar{b}^{Fed} & r^b \\ 1 & -\mathcal{LP}_b^b \end{bmatrix} \cdot \begin{bmatrix} \frac{dr^b}{dr^m} \\ \frac{d\bar{b}}{dr^m} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - \mathbb{E}_\omega\left[q\left(\theta\right)\right] \end{bmatrix}. \tag{I.7}$$

Inverting the matrix in the left yields the solution to the local comparative statics of (I.2) and (I.3):

$$\begin{bmatrix} \frac{dr^b}{dr^m} \\ \frac{d\bar{b}}{dr^m} \end{bmatrix} = \begin{bmatrix} \bar{b} + \bar{b}^{Fed} & r^b \\ 1 & -\mathcal{LP}_b^b \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 - \mathbb{E}_\omega\left[q\left(\theta\right)\right] \end{bmatrix}.$$

To compute the solution, we need only the upper right element of the inverse matrix. That term is by construction:

$$\frac{dr^b}{dr^m} = \frac{r^b}{\mathcal{LP}_b^b\left(\bar{b} + \bar{b}^{Fed}\right) + r^b}\left(1 - \mathbb{E}_\omega\left[q\left(\theta\right)\right]\right) > 0.$$

Similarly, we can also sign the portfolio share:

$$\frac{d\bar{b}}{dr^m} = -\frac{\left(\bar{b} + \bar{b}^{Fed}\right)}{\mathcal{LP}_b^b\left(\bar{b} + \bar{b}^{Fed}\right) + r^b}\left(\left(1 - \mathbb{E}_\omega\left[q\left(\theta\right)\right]\right)\right) < 0.$$

Observation 1. Notice that under satiation $q(\theta) = \mathcal{LP}_b^b = 0$. Thus, the passthrough is one for one. Away from satiation the pass-through is less than one because $\mathcal{LP}_b^b \bar{b} > 0$.

Observation 2. Notice that for fixed Δ, the result goes through since $(1 - \mathbb{E}_\omega [q(\theta)])$ is replaced by 1.

Case #2: Finetely Elastic Deposit Supply and Binding Capital Requirements. We now move to a more general result, with an elastic deposit supply schedule. Equilibrium in the loan supply and deposit

$$\begin{aligned} \left(\bar{b} + \bar{b}^{fed}\right) \cdot \beta \cdot E_{ss} &= \left(\Theta^b\right)^{-1} \cdot \left(r^b + 1\right)^{\epsilon^b}, \\ \kappa \cdot \beta \cdot E_{ss} &= \left(\Theta^d\right)^{-1} \left(r^d + 1\right)^{\epsilon^d} \end{aligned}$$

Combining both conditions yields a single equilibruum condition that we append to the equilibrium system (I.2) and (I.3).

$$\frac{\left(\bar{b} + \bar{b}^{fed}\right)}{\kappa} = \frac{\Theta^d}{\Theta^b} \cdot \frac{\left(r^b + 1\right)^{\epsilon^b}}{\left(r^d + 1\right)^{\epsilon^d}}. \tag{I.8}$$

We write (I.8) in differential form:

$$\frac{1}{\kappa}\frac{d\bar{b}}{dr^m} - \epsilon^b \frac{\Theta^b}{\Theta^d} \cdot \frac{\left(r^b + 1\right)^{\epsilon^b - 1}}{\left(r^d + 1\right)^{\epsilon^d}} \frac{dr^b}{dr^m} + \epsilon^d \frac{\Theta^d}{\Theta^d} \cdot \frac{\left(r^b + 1\right)^{\epsilon^b}}{\left(r^d + 1\right)^{\epsilon^d + 1}} \frac{dr^d}{dr^m} = 0.$$

Substituting I.8, this expression is written as:

$$\frac{1}{\kappa}\frac{d\bar{b}}{dr^m} - \epsilon^b \frac{\left(\bar{b} + \bar{b}^{fed}\right)}{\kappa} \frac{1}{R^b} \frac{dr^b}{dr^m} + \epsilon^d \cdot \frac{\left(\bar{b} + \bar{b}^{fed}\right)}{\kappa} \frac{1}{R^d} \frac{dr^d}{dr^m} = 0.$$

In addition the differential form of (I.2) is now

$$\left(\bar{b} + \bar{b}^{Fed}\right) \frac{dr^b}{dr^m} + r^b \frac{d\bar{b}}{dr^m} - \kappa \frac{dr^d}{dr^m} = 0, \tag{I.9}$$

which replaces (I.4).

Hence, in matrix form, the local comparative statics is given by:

$$\begin{bmatrix} \frac{dr^b}{dr^m} \\ \frac{d\bar{b}}{dr^m} \\ \frac{dr^d}{dr^m} \end{bmatrix} = \begin{bmatrix} \left(\bar{b} + \bar{b}^{Fed}\right) & r^b & -\kappa \\ 1 & -\mathcal{LP}_b^b & 0 \\ -\epsilon^b \frac{\left(\bar{b} + \bar{b}^{fed}\right)}{\kappa} \frac{1}{R^b} & \frac{1}{\kappa} & \epsilon^d \cdot \frac{\left(\bar{b} + \bar{b}^{fed}\right)}{\kappa} \frac{1}{R^d} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ (1 - \mathbb{E}_\omega [q(\theta)]) \\ 0 \end{bmatrix}.$$

We can use standard linear algebra tools to obtain the solution to the pass-through to the credit

rate. In this case:

$$\frac{dr^b}{dr^m} = -\frac{\left|\begin{bmatrix} r^b & -\kappa \\ \frac{1}{\kappa} & \epsilon^d \cdot \frac{\left(\bar{b}+\bar{b}^{Fed}\right)}{\kappa}\frac{1}{R^d} \end{bmatrix}\right|}{\left|\begin{bmatrix} \left(\bar{b}+\bar{b}^{Fed}\right) & r^b & -\kappa \\ 1 & -\mathcal{LP}_b^b & 0 \\ -\epsilon^b\frac{\left(\bar{b}+\bar{b}^{fed}\right)}{\kappa}\frac{1}{R^b} & \frac{1}{\kappa} & \epsilon^d \cdot \frac{\left(\bar{b}+\bar{b}^{fed}\right)}{\kappa}\frac{1}{R^d} \end{bmatrix}\right|}\left(1-\mathbb{E}_\omega\left[q\left(\theta\right)\right]\right) \tag{I.10}$$

$$= -\frac{\epsilon^d \cdot \frac{\left(\bar{b}+\bar{b}^{fed}\right)}{\kappa}\frac{r^b}{R^d}+1}{-\mathcal{LP}_b^b\epsilon^d \cdot \frac{\left(\bar{b}+\bar{b}^{fed}\right)}{\kappa}\frac{\bar{b}}{R^d}-1+\epsilon^b\frac{\left(\bar{b}+\bar{b}^{fed}\right)}{\kappa}\frac{1}{R^b}\kappa\mathcal{LP}_b^b-\epsilon^d \cdot \frac{\left(\bar{b}+\bar{b}^{fed}\right)}{\kappa}\frac{r^b}{R^d}}\left(1-\mathbb{E}_\omega\left[q\left(\theta\right)\right]\right) \tag{I.11}$$

$$= \frac{1+\epsilon^d \cdot \frac{\left(\bar{b}+\bar{b}^{Fed}\right)}{\kappa}\frac{1}{R^d}r^b}{1+\epsilon^d \cdot \frac{\left(\bar{b}+\bar{b}^{fed}\right)}{\kappa}\frac{r^b}{R^d}+\mathcal{LP}_b^b\epsilon^d \cdot \frac{\left(\bar{b}+\bar{b}^{fed}\right)}{\kappa}\frac{\left(\bar{b}+\bar{b}^{Fed}\right)}{R^d}-\epsilon^b\frac{\left(\bar{b}+\bar{b}^{fed}\right)}{\kappa}\frac{1}{R^b}\kappa\mathcal{LP}_b^b}\left(1-\mathbb{E}_\omega\left[q\left(\theta\right)\right]\right) \tag{I.12}$$

and since all terms are positive, the solution holds ($\epsilon^b < 0$), this step proves the first statement of the Proposition. Note simply that $\mathcal{LP}_a^b = -\mathcal{LP}_b^b$.

Observation 3. Notice that under satiation $q\left(\theta\right) = \vartheta = \mathcal{LP}_b^b$. Thus, the pass-through is one for one. Away from satiation, the pass-through is less than one because $\mathcal{LP}_b^b\epsilon^d \cdot \frac{\left(\bar{b}+\bar{b}^{Fed}\right)}{\kappa}\frac{\left(\bar{b}+\bar{b}^{Fed}\right)}{R^d} - \epsilon^b\frac{\left(\bar{b}+\bar{b}^{Fed}\right)}{\kappa}\frac{1}{R^b}\kappa\mathcal{LP}_b^b > 0$.

Case #3: Infinitely Elastic Deposit Supply and Non-Binding Capital Requirements. In this case, the equilibrium system is given by (I.2) and (I.3), but now we also include the deposit liquidity premium. In this case:

$$r^d = r^m + \mathbb{E}_\omega\left[\bar{\chi}\right] - \mathbb{E}_\omega\left[\bar{\chi}\cdot\omega\right]. \tag{I.13}$$

Once the deposit share is free to move, the differential form of (I.2) is

$$\left(\bar{b}+\bar{b}^{Fed}\right)\frac{dr^b}{dr^m} + r^b\frac{d\bar{b}}{dr^m} - r^d\frac{d\bar{d}}{dr^m} = 0, \tag{I.14}$$

which replaces (I.4).

From the loan LP

$$\frac{dr^b}{dr^m} = 1 + \frac{d\left[\mathbb{E}\left[\bar{\chi}\right]\right]}{dr^m},$$

where

$$\frac{d\left[\mathbb{E}\left[\bar{\chi}\right]\right]}{dr^m} = \mathcal{LP}_b^b\frac{d\bar{b}}{dr^m} + \mathcal{LP}_d^b\frac{d\bar{d}}{dr^m}.$$

Following the same notation as before:

$$\mathcal{LP}_b^b \equiv \left[\left[\left(\bar{\chi}^- - \bar{\chi}^+\right)f\left(\omega^*\right)\frac{d\omega^*}{d\bar{b}} + \mathbb{E}\left[\bar{\chi}_\theta f\left(\omega\right)d\omega\right]\frac{d\theta^*}{d\bar{b}}\right]\right] > 0$$

and

$$\mathcal{LP}_d^b \equiv \left[\left[\left(\bar{\chi}^- - \bar{\chi}^+\right) f\left(\omega^*\right) \frac{d\omega^*}{d\bar{d}} + \mathbb{E}\left[\bar{\chi}_\theta f\left(\omega\right) d\omega\right] \frac{d\theta^*}{d\bar{d}}\right]\right] < 0$$

The differential form of (I.13):

$$0 = 1 + \frac{d\left[\mathbb{E}_\omega\left[\bar{\chi}\right]\right]}{dr^m} + \frac{d\left[\mathbb{E}_\omega\left[\bar{\chi}\cdot\omega\right]\right]}{dr^m}$$

where

$$\frac{d\left[\mathbb{E}_\omega\left[\bar{\chi}\cdot\omega\right]\right]}{dr^m} = \Delta_b^d \frac{d\bar{b}}{dr^m} + \Delta_d^d \frac{d\bar{d}}{dr^m}.$$

$$\mathcal{LP}_b^d \equiv -\left[\left[\left(\bar{\chi}^- - \bar{\chi}^+\right) \omega^* f\left(\omega^*\right) \frac{d\omega^*}{d\bar{b}} + \mathbb{E}\left[\omega\bar{\chi}_\theta f\left(\omega\right) d\omega\right] \frac{d\theta^*}{d\bar{b}}\right]\right]$$

and

$$\mathcal{LP}_d^d \equiv -\left[\left[\left(\bar{\chi}^- - \bar{\chi}^+\right) \omega^* f\left(\omega^*\right) \frac{d\omega^*}{d\bar{d}} + \mathbb{E}\left[\omega\bar{\chi}_\theta f\left(\omega\right) d\omega\right] \frac{d\theta^*}{d\bar{d}}\right]\right].$$

Where we also have that $\mathcal{LP}_b^d > 0, \mathcal{LP}_d^d > 0$.

As in the previous two examples, we construct the matrix representation of the comparative statics.

$$\begin{bmatrix} \frac{dr^b}{dr^m} \\ \frac{d\bar{b}}{dr^m} \\ \frac{d\bar{d}}{dr^m} \end{bmatrix} = \begin{bmatrix} \left(\bar{b} + \bar{b}^{Fed}\right) & r^b & -r^d \\ 1 & -\mathcal{LP}_b^b & -\mathcal{LP}_b^d \\ 1 & -\left(\mathcal{LP}_b^b + \mathcal{LP}_b^d\right) & -\left(\mathcal{LP}_d^b + \mathcal{LP}_d^d\right) \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \left(1 - \mathbb{E}_\omega\left[q\left(\theta\right)\right]\right) \\ 0 \end{bmatrix}.$$

To obtain the solution to the pass-through, we do the same calculation as in the earlier step.

$$\frac{dr^b}{dr^m} = -\frac{\left|\begin{bmatrix} r^b & -r^d \\ -\left(\mathcal{LP}_b^b + \mathcal{LP}_b^d\right) & -\left(\left(\mathcal{LP}_d^b + \mathcal{LP}_d^d\right)\right) \end{bmatrix}\right|}{\begin{vmatrix} \left(\bar{b} + \bar{b}^{Fed}\right) & r^b & -r^d \\ 1 & -\mathcal{LP}_b^b & -\mathcal{LP}_b^d \\ 1 & -\left(\mathcal{LP}_b^b + \mathcal{LP}_d^b\right) & -\left(\mathcal{LP}_b^d + \mathcal{LP}_d^d\right) \end{vmatrix}} \tag{I.15}$$

$$= \frac{\left(\left(\mathcal{LP}_d^b + \mathcal{LP}_d^d\right)\right) r^b + \left(\mathcal{LP}_b^b + \mathcal{LP}_b^d\right) r^d}{\mathcal{LP}_b^b r^d + \mathcal{LP}_d^d r^b + \left(\mathcal{LP}_b^b\left(\mathcal{LP}_b^d + \mathcal{LP}_d^d\right) - \mathcal{LP}_b^d\left(\mathcal{LP}_b^b + \mathcal{LP}_d^b\right)\right)\bar{b}}. \tag{I.16}$$

Using that $\mathcal{LP}_b^b = -\mathcal{LP}_a^b, \mathcal{LP}_b^d = -\mathcal{LP}_a^d$ we obtain:

$$\frac{\left(\left(\mathcal{LP}_d^b + \mathcal{LP}_d^d\right)\right) r^b - \left(\mathcal{LP}_a^b + \mathcal{LP}_a^d\right) r^d}{-\mathcal{LP}_a^b r^d + \mathcal{LP}_d^d r^b - \left(\mathcal{LP}_a^b\left(\mathcal{LP}_b^d + \mathcal{LP}_d^d\right) + \mathcal{LP}_a^d\left(\mathcal{LP}_d^b - \mathcal{LP}_a^b\right)\right)\bar{b}}.$$

Observation 4. In this case, the sign is ambiguous and depends on the sign of:

$$\mathcal{LP}_b^b r^d + \mathcal{LP}_d^d r^b + \mathcal{LP}_b^b\left(\mathcal{LP}_b^d + \mathcal{LP}_d^d\right)\bar{b} \geq \mathcal{LP}_b^d\left(\mathcal{LP}_b^b + \mathcal{LP}_d^b\right)\bar{b}.$$

This concludes the proof of Proposition 8 . **QED.**

J Existence, Uniqueness, and Convergence under Friedman Rule

This Appendix characterizes the existence and uniqueness of a stationary equilibria when the bank has log preferences ($\gamma = 1$), the Fed eliminates all distortions $R^m = R^w$ and sets R^m low enough so that banks do not hold liquid assets. We can treat these results as holding for an approximation where bank dividends are close to constant and interbank market distortions are not too large.

J.1 Dynamical Properties

In this section, we study the dynamical properties of the model. We fully characterize these dynamics when banks have log utility and the Fed carries out a policy of no distortions in the interbank market. Both assumptions simplify the analysis. Although the results are not general, for small deviations around that policy, the dynamic properties should be similar.

Stationary Equilibrium and Policy Effects with Satiation. We begin describing the transitional dynamics of the model when the Fed carries out a policy that satiates the market with reserves via $i_t^w = i_t^m$ by setting a sufficiently low value for i_t^m. For simplicity, we set the supply of government bonds to zero and assume the Fed does not purchase loans. . By inducing satiation, and maintaining an equal amount of reserves as Fed loans, the Fed eliminates the liquidity premium of loans. Thus, a spread between loans and deposits only results from capital requirements. This characterization is useful because it describes the dynamics of the model in absence of any distortions.

For this section, it is useful to define the inverse demand elasticity of loans and supply elasticity of deposits, $\bar{\epsilon}^x \equiv (\epsilon^x)^{-1}$ for $x \in \{d, b\}$, respectively. Also, intercept of the inverse demand for loans and supply of deposits are $\bar{\Theta}^x \equiv (\Theta^x)^{-1/\epsilon^x}$ for $x \in \{d, b\}$. We obtain the following characterization:

Proposition J.1 *[Transitions under Friedman Rule] Consider a policy sequence such that $i_t^w = i_t^m$, $B_t^{Fed} = G_t^{Fed} = 0$, and $M_t = G_t = 0$. Then:*

a) Real aggregate bank equity follows:

$$E_{t+1} = \left(R_t^b + \kappa \min\left\{\left(R_t^b - R_t^d\right), 0\right\}\right)\beta E_t, \text{ with } E_0 > 0 \text{ given.}$$

The dynamics are given by a critical threshold

$$E_\kappa \equiv \frac{1}{\beta}\left[\frac{\bar{\Theta}^b/\bar{\Theta}^d}{(1+\kappa)^{-\bar{\epsilon}^b}\,\kappa^{\bar{\epsilon}^d}}\right]^{\frac{1}{\bar{\epsilon}^d - \bar{\epsilon}^b}},$$

If $E_t > E_\kappa$, then $\left\{R_t^b, R_t^d, \bar{d}_t\right\}$ solve:

$$R_t^b = \bar{\Theta}^b\left(\beta E_t\left(1+\bar{d}_t\right)\right)^{\bar{\epsilon}^b} = \bar{\Theta}^d\left(\beta E_t \bar{d}_t\right)^{\bar{\epsilon}^d} = R_t^d.$$

Otherwise, $\bar{d}_t = \kappa$ *and*

$$R_t^b = \bar{\Theta}^b \left(\beta E_t \left(1+\kappa\right)\right)^{\bar{\epsilon}^b}, \bar{\Theta}^d \left(\beta E_t \kappa\right)^{\bar{\epsilon}^d} = R_t^d$$

b) There $\exists!$ *steady state level of* $E_{ss} > 0$*. The steady state features binding capital requirements if and only if*

$$\Theta^b \left(1/\beta\right)^{\epsilon^b} < \Theta^d \left(1/\beta\right)^{\epsilon^d} \left(1+\kappa^{-1}\right). \tag{J.1}$$

If capital requirements do not-bind at steady state, then E_{ss} *solves:*

$$E_{ss} = \frac{\Theta^b \left(1/\beta\right)^{\epsilon^b} - \Theta^d \left(1/\beta\right)^{\epsilon^d}}{\beta}.$$

Otherwise, E_{ss} *solves:*

$$1/\beta = \bar{\Theta}^b \left(\beta E_{ss} \left(1+\kappa\right)\right)^{\bar{\epsilon}^b} \left(\kappa+1\right) - \kappa \bar{\Theta}^d \left(\beta E_{ss} \kappa\right)^{\bar{\epsilon}^d}.$$

c) If $\frac{\left(1-1/\epsilon^b\right)}{\left(1+1/\epsilon^d\right)} \geq \frac{\kappa}{(1+\kappa)}$*, and capital requirements bind at steady state, then* E_t *converges to* E_{ss} *monotonically.*

In the paper, the calibration satisfies these parameter restrictions.

J.2 Proof of Proposition J.1

The proof of the proposition is presented in three steps. First, we derive a threshold equity level where capital requirements are binding. Second, we prove that there can be at most one steady state. Third, we provide conditions such that the equilibrium features binding reserve requirements. Finally, we derive the sufficient condition for monotone convergence. We then establish the result for the rate of inflation and the determination of the price level.

Part 1 - Law of Motion of Bank Equity. As shown in the Proof of Proposition 3, under log utility $\bar{c}_t = (1-\beta)$. Then, the law of motion in (19) becomes

$$E_{t+1} = \left(R_t^b + \kappa \min\left\{\left(R_t^b - R_t^d\right), 0\right\}\right) \beta E_t. \tag{J.1}$$

This follow directly by substituting $\bar{b} = 1 + \bar{d}$ and noticing that the equity constraint binds if $\left(R_t^b - R_t^d\right)$ and if not, deposits don't affect equity. This is enough to show that the law of motion of bank equity satisfies the difference equation in the proposition. Thus, we have obtained a law of motion for bank equity in real terms. We use this to establish convergence. Consider now the condition such that capital requirements are binding for a given $E_t = E$. For that we need that $R_t^b > R_t^d$. Using the inverse of the loan demand function, R_t^b can be written in terms of the supply of loans using the market clearing condition:

$$R_t^b = \bar{\Theta}^b \left(\bar{b} \beta E_t\right)^{\bar{\epsilon}^b}.$$

If the capital requirement constraint binds,

$$R_t^b = \bar{\Theta}^b \left(\beta E_t \left(1+\kappa\right)\right)^{\bar{\epsilon}^b}.$$

Using the result that capital requirements are binding when $R_t^b > R_t^d$, we obtain

$$\bar{\Theta}^b \left(\beta E_t \left(1+\kappa\right)\right)^{\bar{\epsilon}^b} \geq \bar{\Theta}^d \left(\beta E_t \kappa\right)^{\bar{\epsilon}^d}.$$

Clearing E at equality delivers a threshold,

$$E_\kappa \equiv \frac{1}{\beta}\left[\frac{\bar{\Theta}^b/\bar{\Theta}^d}{\left(1+\kappa\right)^{-\bar{\epsilon}^b}\kappa^{\bar{\epsilon}^d}}\right]^{\frac{1}{\bar{\epsilon}^d-\bar{\epsilon}^b}},$$

such that for any $E < E_\kappa$, capital requirements are binding in a transition. Thus, the law of motion of capital is broken into a law of motion for the binding and non-binding capital requirements regions.

We obtain

$$E_{t+1} = \bar{\Theta}^b \left(\beta E_t \left(1+\kappa\right)\right)^{1+\bar{\epsilon}^b} - \bar{\Theta}^d \left(\beta E_t \kappa\right)^{1+\bar{\epsilon}^d} \text{ for } E_t \leq E_\kappa$$

and

$$E_{t+1} = \bar{\Theta}^b \left(\left(1+d_t\right)\beta E_t\right)^{\bar{\epsilon}^b} \beta E_t \text{ for } E_t > E_\kappa.$$

Here, we substituted $\bar{d} = \kappa$ in (J.1) for the law of motion in the constrained region and $\bar{d}_t \left(R_t^b - R_t^d\right) = 0$ in the second region.

Part 2 - Uniqueness of Steady State. Here we show that there cannot be more than one steady state level of real bank equity. We prove this in a couple of steps. First, we ask whether there can be more than one steady state in each region—in the binding and non-binding regions. We show that there can be only one steady state in each region. Then, we ask if two steady states can co-exists, given that they must lie in separate regions. The answer is no.

To see this, define

$$\Gamma\left(E\right) \equiv \bar{\Theta}^b \left(\beta\left(1+\kappa\right)\right)^{1+\bar{\epsilon}^b} E^{\bar{\epsilon}^b} - \bar{\Theta}^d \left(\beta\left(1+\kappa\right)\right)^{1+\bar{\epsilon}^d} E^{\bar{\epsilon}^d}.$$

If a steady state exists in the binding region, it must satisfy the following condition:

$$1 = \Gamma\left(E_{ss}\right) \text{ and } E_{ss} \leq E_\kappa.$$

It is straightforward to verify that

$$\Gamma'\left(E\right) < 0, \lim_{E\to 0} \Gamma\left(E\right) \to \infty, \text{ and } \lim_{E\to\infty} \Gamma\left(E\right) \to -\infty.$$

Since the function is decreasing and starts at infinity, and the function ends at minus infinity, there can be at most one steady state—with positive E—in the constrained region, $E_{ss} < E_\kappa$.

In the unconstrained region, $E_{ss} \geq E_\kappa$ a steady state is occurs only when

$$1 = R_t^b \beta.$$

We need to find the level of equity that satisfies that condition. Also, we know that $R^d = R^b$ in the unconstrained region. Thus, the supply of loans in the unconstrained region is given by

$$\beta E_t + \bar{\Theta}^d \left(R^b\right)^{\bar{\epsilon}^d},$$

the sum of real bank equity plus real deposits. Thus, we can define the equilibrium rate on loans through the implicit map, $\tilde{R}^b(E)$, that solves

$$\tilde{R}^b(E) \equiv \left\{ \tilde{R} | \tilde{R} = \bar{\Theta}^b \left(\beta E_t + \bar{\Theta}^d \left(\tilde{R} \right)^{\bar{\epsilon}^d} \right)^{\bar{\epsilon}^b} \right\}.$$

If we can show that $\tilde{R}^b(E)$ is a function and $\tilde{R}^b(E) = \beta^{-1}$ for only one E, then we know that there can be at most one steady state in the unconstrained region. To show that $\tilde{R}^b(E)$ is a function, we must show that there is a unique value of $\tilde{R}^b$ for any E. Note that $\tilde{R}^b(E) = \tilde{R}$ for $\tilde{R}$ that solves

$$\bar{\Theta}^b \left(\tilde{R} \right)^{\bar{\epsilon}^b} - \bar{\Theta}^d \left(\tilde{R} \right)^{\bar{\epsilon}^d} = \beta E.$$

Since the first term on the left is decreasing and the second increasing, this function is monotone, and thus, its inverse is a function, i.e., $\tilde{R}^b(E)$ is a function. Observe that

$$\lim_{\tilde{R} \to 0} \bar{\Theta}^b \left(\tilde{R} \right)^{\bar{\epsilon}^b} - \bar{\Theta}^d \left(\tilde{R} \right)^{\bar{\epsilon}^d} = \infty, \text{ and } \lim_{\tilde{R} \to \infty} \bar{\Theta}^b \left(\tilde{R} \right)^{\bar{\epsilon}^b} - \bar{\Theta}^d \left(\tilde{R} \right)^{\bar{\epsilon}^d} = -\infty,$$

so $\tilde{R}^b(E)$ exists for any positive E. Since $\tilde{R}^b$ is decreasing in E and defined everywhere, there exists at most one value for E such that $\tilde{R}^b(E) = (\beta)^{-1}$. This shows that there exists at most one steady state in the unconstrained region.

Next, we need to show that if there exists a steady state where $E_{ss} \leq E_\kappa$, there cannot exist another steady state where $E_{ss} \geq E_\kappa$. To see this, suppose that there $\exists$ a steady state in the unconstrained region. Thus, there exists some value $E_u > E_\kappa$ such that

$$\tilde{R}^b(E_u) = 1/\beta.$$

Since $\tilde{R}^b$ is decreasing and $E_u > E_\kappa$, by assumption we obtain that

$$1/\beta < \tilde{R}^b(E_\kappa) = R^b\left(\beta E_\kappa (1+\kappa)\right), \tag{J.2}$$

where the equality follows from the definition of E_κ.

As a false hypothesis, suppose that there is another steady state where $E_c < E_\kappa$. Then, using the law of motion for equity in the constrained region,

$$\begin{aligned} R^b\left(\beta E_c\left(1+\kappa\right)\right) &= 1/\beta - \kappa\left(R^b\left(\beta E_c\left(1+\kappa\right)\right) - R^d\left(\beta E_c \kappa\right)\right) \\ R^b\left(\beta E_c\left(1+\kappa\right)\right) &< 1/\beta, \end{aligned} \tag{J.3}$$

where the second line follows from $R^b > R^d$ for any $E_c < E_\kappa$. Thus,

$$R^b\left(\beta E_\kappa\left(1+\kappa\right)\right) < R^b\left(\beta E_c\left(1+\kappa\right)\right) < \beta^{-1}$$

because R^b is decreasing. However, (J.3) and (J.2) cannot hold at the same time. Thus, there $\exists!$ steady state with positive real equity.

Part 3 - Conditions for Capital Requirements Binding at steady state. We have shown in Appendix G.5 that a condition for a steady state with slack capital requirements is:

$$\Theta^b\left(1/\beta\right)^{\epsilon^b} \geq \Theta^d\left(1/\beta\right)^{\epsilon^d}\left(1+\kappa^{-1}\right).$$

Then, the steady state level of equity is:

$$E_{ss} = \frac{\Theta^b\left(1/\beta\right)^{\epsilon^b} - \Theta^d\left(1/\beta\right)^{\epsilon^d}}{\beta}.$$

If the condition is violated, we use the stationarity condtion:

$$1/\beta = R^b\left(\beta E\left(1+\kappa\right)\right) + \left(R^b\left(\beta E\left(1+\kappa\right)\right) - R^d\left(\beta E\kappa\right)\right)\kappa.$$

This allows equity to grow at the point where the constraint begins to bind.

Part 4 - Conditions for monotone convergence. Assume that parameters satisfy the conditions for a steady state with binding capital requirements. Observe that if $E_t > E_\kappa$, then $E_{t+1} < E_t$ since $R_t^b < \left(\beta\right)^{-1}$ for all $E > E_\kappa$. Thus, any sequence that starts from $E_0 > E_\kappa$ eventually abandons the region. Thus, without loss of generality, we only need to establish monotone convergence within the $E < E_\kappa$ region.

Now consider $E_t < E_{ss}$. We must show that E_{t+1} also satisfies $E_{t+1} < E_{ss}$ if that is the case. Employing the law of motion of equity in the constrained region, notice that

$$E_{t+1} - E_{ss} = \bar{\Theta}^b\left(\beta E_t\left(1+\kappa\right)\right)^{1+\bar{\epsilon}^b} - \bar{\Theta}^d\left(\beta E_t \kappa\right)^{1+\bar{\epsilon}^d} - E_{ss}.$$

Define $g\left(E\right) \equiv \Gamma\left(E\right)E$. Thus,

$$\begin{aligned} E_{t+1} - E_{ss} &= \Gamma\left(E_t\right)E_t - E_{ss} \\ &= -\int_{E_t}^{E_{ss}} g'\left(e\right)de. \end{aligned}$$

It is enough to show that $g'(e) > 0$ for any e. We verify that under the parameter assumptions, this is indeed the case. Note that

$$\begin{aligned} g'(e) &= \left(1+\bar{\epsilon}^b\right)\bar{\Theta}^b\left(\beta\left(1+\kappa\right)\right)^{1+\bar{\epsilon}^b}e^{\bar{\epsilon}^b} - \left(1+\bar{\epsilon}^d\right)\bar{\Theta}^d\left(\beta\kappa\right)^{1+\bar{\epsilon}^d}e^{\bar{\epsilon}^d} \\ &= \left(1+\bar{\epsilon}^b\right)R^b\left(\beta\left(1+\kappa\right)e\right)\beta\left(1+\kappa\right) - \left(1+\bar{\epsilon}^d\right)R^d\left(\beta\kappa e\right)\beta\kappa, \end{aligned}$$

where the second line follows from the definition of R^b and R^d and the result that capital requirements are binding in $E < E_{ss}$. Furthermore, since in this region, $R^b > R^d$ for all $E < E_\kappa$, then a sufficient condition for $g'(E) > 0$ is to have:

$$\left(1+\bar{\epsilon}^b\right)\beta\left(1+\kappa\right) \geq \left(1+\bar{\epsilon}^d\right)\beta\kappa.$$

Thus, a sufficient condition for monotone convergence is

$$\frac{1+1/\epsilon^b}{1+1/\epsilon^d} \geq \frac{\kappa}{1+\kappa}.$$

K Calibration

K.1 Data Sources

Most data series are obtained from the Federal Reserve Bank of St. Louis Economic Research Database (FRED©) and are available at the FRED©website. The original data sources for each series are collected by the Board of Governors of the Federal Reserve System (US). We use the following series corresponding to:

Aggregate Variables. For aggregate variables we use the following:

- total reserves
  - Total Reserves of Depository Institutions, Billions of Dollars, Monthly, Not Seasonally Adjusted

    https://fred.stlouisfed.org/series/TOTRESNS

- bank equity
  - Total Equity Capital for Commercial Banks in United States, Thousands of Dollars, Not Seasonally Adjusted (USTEQC)

    https://fred.stlouisfed.org/series/USTEQC
  - This data is available only at a quarterly frequency. We interpolate the series linearly from quarter to quarter to obtain the monthly series
- the volume of interbank market loans:
  - Board of Governors of the Federal Reserve System (US), Interbank Loans, All Commercial Banks [IBLACBW027NBOG], H.8 Assets and Liabilities of Commercial Banks in the United States,

    https://fred.stlouisfed.org/series/IBLACBW027NBOG
- the volume of discount window loans:
  - Discount Window Borrowings of Depository Institutions from the Federal Reserve [DISCBORR], H.3 Aggregate Reserves of Depository Institutions and the Monetary Base,

    https://fred.stlouisfed.org/series/DISCBORR
- bank deposits:
  - Board of Governors of the Federal Reserve System (US), Deposits, All Commercial Banks [DPSACBM027NBOG],

https://fred.stlouisfed.org/series/DPSACBM027NBOG

- bank credit:
  - Board of Governors of the Federal Reserve System (US), Commercial and Industrial Loans, All Commercial Banks [BUSLOANS],

    https://fred.stlouisfed.org/series/BUSLOANS

Series for interest rates. For series on interest rates we use the following:

- the interest on discount window loans:
  - Board of Governors of the Federal Reserve System (US), Primary Credit Rate [DPCREDIT],

    https://fred.stlouisfed.org/series/DPCREDIT

- the interest on reserves:
  - Board of Governors of the Federal Reserve System (US), Interest Rate on Excess Reserves [IOER],

    https://fred.stlouisfed.org/series/IOER

- Interest rate on deposits:
  - We use the series used in Drechsler et al. (2017),

    academic.oup.com/qje/article-abstract/132/4/1819/3857743.

- the government bond rate is:
  - Board of Governors of the Federal Reserve System (US), 3-Month Treasury bill: Secondary Market Rate [TB3MS],

    https://fred.stlouisfed.org/series/TB3MS

Open Market Operations. The series that corresponds to open-market operations is the ratio of a measure of the Fed's assets, normalized by total bank credit. During the crisis, the Fed's balance sheet grows for multiple factors: some of these factors include Swaps to foreign governments and direct loans to institutions such as American International Group (AIG). We consider the purchase of government bonds as the equivalent of conventional OMO. For unconventional OMO we consider the sum of mortgage-backed securities and Federal agency securities. These series are weekly and aggregated to the monthly level. The references for these series are:

- total bank credit

  - Board of Governors of the Federal Reserve System (US), Bank Credit of All Commercial Banks [TOTBKCR],

https://fred.stlouisfed.org/series/TOTBKCR

- Treasury bills
  - Board of Governors of the Federal Reserve System (US), Assets: Securities Held Outright: U.S. Treasury Securities [WSHOTS],

https://fred.stlouisfed.org/series/WSHOTS

- Federal agency paper
  - Board of Governors of the Federal Reserve System (US), Assets: Securities Held Outright: Federal Agency Debt Securities [WSHOFDSL]

https://fred.stlouisfed.org/series/WSHOFDSL

- Mortgage-Backed Securities
  - Board of Governors of the Federal Reserve System (US), Assets: Securities Held Outright: Mortgage-Backed Securities: Wednesday Level [WSHOMCB]

https://fred.stlouisfed.org/series/WSHOMCB

Ratio Series. The series for ratios are derived as follows.

- Portfolio shares $\bar{g}, \bar{d}$
  - The series for the data analogues of $\{\bar{g}, \bar{d}\}$ are constructed using the micro data from commercial banks from Phillip Schnabl. We take the series for Treasury securities that mature in less than three months, and between three months and one year—typically government bonds are thought of as Treasury securities with maturity below a year. The series for the data analogue of $\bar{d}$ is obtained as the sum of total liablities and divide by equity. Then we aggregate across banks and divide by the equity series. The raw data is available in Philip Schnabl's website:

http://pages.stern.nyu.edu/∼pschnabl/data.html

- Portfolio shares $\bar{m}$:
  - We take the series for cash assets for all commercial banks. The series includes vault cash and reserves held by banks. The series is available at the Board of Governors of the Federal Reserve System (US), (Cash Assets, All Commercial Banks/Total Assets, All Commercial Banks)*100,

https://fred.stlouisfed.org/graph/?g=IW4

  - We divide the series by difference between all commercial bank assets minus liabilities.

- Liquidity Premium

  - The liquidity premium that is used to construct the return on loans is obtained from Del Negro et al. (2017),

https://www.aeaweb.org/articles?id=10.1257/aer.20121660

Federal Funds Interest Rate Distribution.

- The series for the dispersion in the Fed funds rates are obtained from the New York Federal Reserve Bank. The NY Fed provides two data sets, one for the daily min and max and another where it includes quantiles. We use both data sets. In section 4 we use the max-min spread because the length of the data is longer.

  - To construct the series FF Range, we construct the monthly average of the daily distance between the max and the min of the Fed funds distribution, and average over the month. The original series are found here:https://apps.newyorkfed.org/markets/autorates/fed-funds-search-page

- When we perform the financial crisis counterfactuals in Section 5.3, we reconstruct the Fed funds rate among banks. We also use that data in the robustness checks to section 4 in Appendix L, w. For that, we use the quantiles of the Fed Funds.

  - The data available from the NY Fed also includes the max and min, 99, 75, 50, 25 and 1st quantiles, and the standard deviation of daily Fed funds rate. The data is available here:

https://www.newyorkfed.org/medialibrary/media/markets/ff-volumes-oct2006-feb2016.xlsx

K.2 Construction of Bank and Non-bank Fed Funds

Mapping the model to the data after October 2008 requires to account for two additional features: First, in this period, the average Fed Funds rate fell below the the interest on reserves, the analogue of R^m in the model. Second, the 1-month T-Bill rate, the analogue of R^g in the model, also traded below the interest on reserves. One important consideration in accounting for both features in the model is that many in trades in the Fed funds market are transactions between banks, which were eligible to receive interest on reserves and other institutions that were not eligible. Thus, after 2008, the average Fed funds rate reflects in part transactions that occur due to that regulatory arbitrage, with an interest rate below the rate on reserves. We argue that to execute the with non-banks, banks need to use government bonds as collateral. Thus, these trades generate an additional value of holding government bonds. Next, we explain we describe how we add these features into the model, to address these issues and reconstruct series of Fed Funds rate corresponding to trades only among banks.

Non-Bank Fed Funds Participants. We introduce a set of non-banks (nb) that hold of reserves, participate in the Fed funds market, but do not receive interest on reserves. In particular, we assume that by the end of the balancing stage, banks with a surplus of government bonds will match (in one round) with non-banks—banks in deficit, by that stage, have already sold all of their government bonds. We assume that all banks are matched, with a non-bank, on a per-bond basis. Once a match occurs, banks and non-banks trade 1 unit of government bonds for one unit of reserves—the position is reversed by the end of the period, but the interest is paid to the agent that holds the asset overnight. We call this transaction a Repo.

When a bank meets a non-bank, they solve the following Nash bargaining problem:

$$R^{f,nb} = \arg\max_{R} \left(R - 1/\left(1+\pi\right)\right)^{1-\eta^b} \left(R^m - R\right)^{\eta^b}.$$

The first term in parenthesis is the surplus minus the outside option for the non-bank: the non-bank earns R instead of storing the reserve at no interest. The second parenthesis, is the surplus for the bank: the bank earns R^m, but pays R. The bank's bargaining power is η^b. Solution to the rate in a Repo transaction is:

$$R^{f,nb} = 1 + \left(1-\eta^b\right)\left(R^m - 1\right). \tag{K.1}$$

Now, a bank that ends in surplus, not only earns R^m, but also the gains from the regulatory arbitrage, $R^m - R^{f,nb}$. Therefore, (Gov. Bond LP) is now modified to obtain:

$$R^g + \left(R^m - R^{f,nb}\right) = R^m + \chi^+. \tag{K.2}$$

Equations (K.1) and (K.2) account for **government bond** and Fed funds rates that trade below R^m.

Construction of Fed Funds analogue. Next, we explain how we approximate $\hat{R}^{f,nb}$ and, $\hat{R}^{f,ib}$, the average Fed funds rate among bank trades using data on the distribution of Fed funds rates. Conceptually, the average Fed funds rate is the average Fed funds rates between interbank and non-interbank transactions:

$$R^f = \left(1-\nu^{nb}\right)\cdot R^{f,ib} + \nu^{nb}\cdot R^{f,nb}, \tag{K.3}$$

where ν^{nb} is the fraction of Fed fund trades that occur among banks and non-banks. From the data, we observe R^f at a given point in time. Thus with a data counterpart for $\hat{\nu}^{nb}$ and $\hat{R}^{f,nb}$ we could reconstruct $\hat{R}^{f,ib}$. We observe data on 1-st, 25-th, 75-th and 99-th percentiles of the Federal Funds distribution, respectively $\left\{R^f_{t,1}, R^f_{t,25}, R^f_{t,75}, R^f_{t,99}\right\}$, at a given date t. To reconstruct the data analogue $\hat{\nu}^{nb}_t$, for each t, we find the pair of contiguous percentiles $\left\{R^f_{t,x}, R^f_{t,y}\right\}$ such that the interest rate on reserves fell within that interval, i.e., $R^m_t \in \left[R^f_{t,x}, R^f_{t,y}\right]$. Naturally, we attribute all trades executed below $\hat{R}^m_t$ to trades among bank and non banks. Thus, the mass of trades with non-bank trades will be $F\left(R^f_{t,x}\right)$ plus a fraction of trades that fell within the $\left[R^f_{t,x}, R^m\right]$ interval. We approximate the date using assuming a uniform distribution among the trades

within that interval. Hence, the data analogue of ν^{nb} is:

$$\hat{\nu}_t^{nb} = F\left(R_{t,x}^f\right) + \frac{R^m - R_{t,x}^f}{R_{t,y}^f - R_{t,x}^f} \cdot \left[F\left(R_{t,y}^f\right) - F\left(R_{t,x}^f\right)\right]. \tag{K.4}$$

For the analogue of $\hat{R}^{f,nb}$, we reconstruct it using the same approximation to the distribution of rates, that is:

$$\begin{aligned}\hat{R}^{f,nb} &= \frac{1}{2}\left(\sum_{\{x,\,y\}\,\in[\ldots]}\left(R_{t,y}^f - R_{t,x}^f\right)\cdot\left[F\left(R_{t,y}^f\right) - F\left(R_{t,x}^f\right)\right]\right)\\ &+ \frac{1}{2}\left(\left(R^m - R_{t,x}^f\right)\frac{R^m - R_{t,x}^f}{R_{t,y}^f - R_{t,x}^f}\cdot\left[F\left(R_{t,y}^f\right) - F\left(R_{t,x}^f\right)\right]\right).\end{aligned}$$

We use this construction to backout $\hat{R}^{f,ib}$ and $\hat{R}^g$ using (K.3) and (K.2) respectively. We use the month moving average series for $\hat{R}^{f,ib}$ and $\hat{R}^g$ in the procedure that follows.

K.3 Calibration Procedure

In the procedure, we infer steady state parameters using data analogues to for the volume of interbank loans, discount window loans, $\{W, F\}$, interest rates $\left\{R_{ss}^d, R_{ss}^m, R_{ss}^f, R_{ss}^m, R_{ss}^w\right\}$, an analogue for the loan liquidity premium $\{LP_{ss}\}$ and bank portfolio shares on reserve and government bond holdings, am well as the capital requirement, $\{\bar{m}_{ss}, \bar{g}_{ss}, \kappa\}$.

Here we explain how we deduce $\left\{\hat{\sigma}_{ss}, \hat{\lambda}_{ss}, \hat{\eta}_{ss}, \hat{\sigma}_{ss}^{\delta}, \hat{\Theta}_{ss}^b, \hat{\Theta}_{ss}^d\right\}$ in section 5.1:

1. Obtain $\hat{\Psi}_{ss}^-$ from
$$\hat{\Psi}_{ss}^- = \frac{\hat{\Psi}_{ss}^- S_{ss}^-}{S_{ss}^-} = \frac{F_{ss}}{W_{ss} + F_{ss}}.$$

2. Obtai
$$\hat{\lambda}_{ss} = \log\left(\frac{1}{1 - \hat{\Psi}_{ss}^-}\right),$$
by inverting (A.1) under the assumption that $\theta_{ss} < 1$.

3. Deduce $\hat{\omega}_{ss}^*$ from the definition (B.2.13) substituting $\rho = 0$ and $\left\{\bar{a}, \bar{d}\right\} = \{\bar{m}_{ss}, \bar{g}_{ss}\}$ and obtain:
$$\hat{\omega}_{ss}^* = -\frac{\bar{m}_{ss} + \bar{g}_{ss}}{\kappa} / \frac{R_{ss}^d}{R_{ss}^m}.$$

4. Deduce $\hat{\sigma}_{ss}$ as the solution $\hat{\sigma}$ that solves:
$$\frac{W}{A} = \left(1 - \hat{\Psi}_{ss}^-\right)\Phi\left(\hat{\omega}_{ss}^*; \hat{\sigma}\right)\left(\frac{\bar{m}_{ss} + \bar{g}_{ss}}{\kappa + 1} + \frac{R_{ss}^d}{R_{ss}^m}\mathbb{E}\left[\omega | \omega < \hat{\omega}_{ss}^*; \hat{\sigma}\right]\frac{\kappa}{\kappa + 1}\right). \tag{K.1}$$
This step uses (B.2.14), where S_{ss}^- is obtained by integrating (B.2.11) among all $\omega < \hat{\omega}_{ss}^*$, dividing by all assets

5. We deduce a value for $\hat{\theta}_{ss}$ from:

$$\hat{\theta}_{ss} = \frac{\Phi\left(\hat{\omega}^*_{ss};\hat{\sigma}\right)\left(\bar{m}_{ss} + \bar{g}_{ss} + \frac{R^d_{ss}}{R^m_{ss}}\mathbb{E}\left[\omega|\omega < \hat{\omega}^*_{ss};\hat{\sigma}\right]\kappa\right)}{\left(1-\Phi\left(\hat{\omega}^*_{ss};\hat{\sigma}\right)\right)\left(\bar{m}_{ss} + \bar{g}_{ss} + \frac{R^d_{ss}}{R^m_{ss}}\mathbb{E}\left[\omega|\omega > \hat{\omega}^*_{ss};\hat{\sigma}\right]\kappa\right) - \bar{g}_{ss}}. \tag{K.2}$$

This step uses S^-_{ss} obtained by integrating (B.2.11) among all $\{\omega < \hat{\omega}^*_{ss}\}$, and S^+_{ss} obtained by integrating (B.2.11) among all $\omega > \hat{\omega}^*_{ss}$, and applying the formula (B.2.12).

6. We deduce $\hat{\Psi}^+_{ss}$ from the clearing condition in the Fed funds market $\hat{\Psi}^+_{ss} = \hat{\Psi}^-_{ss} \cdot \hat{\theta}_{ss}$

7. We deduce $\hat{\bar{\chi}}^+_{ss}$ using (B.2.15) and thus, $\hat{\bar{\chi}}^+_{ss} = \hat{\Psi}^+_{ss} \cdot (R^f - R^m)$.

8. We deduce $\hat{\eta}_{ss}$ that solves:

$$\hat{\bar{\chi}}^+_{ss} = (R^w - R^m)\left(\frac{\hat{\bar{\theta}}_{ss}}{\hat{\theta}_{ss}}\right)^{\hat{\eta}_{ss}}\left(\frac{\hat{\theta}^{\eta}_{ss}\hat{\bar{\theta}}^{1-\hat{\eta}_{ss}}_{ss} - \hat{\theta}_{ss}}{\hat{\bar{\theta}}_{ss} - 1}\right) \tag{K.3}$$

where

$$\hat{\bar{\theta}}_{ss} = \begin{cases} 1 + (\hat{\theta}_{ss} - 1)\exp(\hat{\lambda}_{ss}) & \text{if } \hat{\theta}_{ss} \leq 1 \\ \\ 1 + ((\hat{\theta}_{ss})^{-1} - 1)\exp(\hat{\lambda}_{ss}))^{-1} & \text{if} \hat{\theta}_{ss} > 1. \end{cases}$$

which direct from (A.2).

9. We also deduce $\hat{\bar{\chi}}^-_{ss}$ using

$$\hat{\bar{\chi}}^-_{ss} = (R^w_{ss} - R^m_{ss})\left(\frac{\hat{\bar{\theta}}_{ss}}{\hat{\theta}_{ss}}\right)^{\hat{\eta}_{ss}}\left(\frac{\hat{\theta}^{\hat{\eta}_{ss}}_{ss}\left(\hat{\bar{\theta}}_{ss}\right)^{1-\hat{\eta}_{ss}} - 1}{\hat{\bar{\theta}}_{ss} - 1}\right)$$

which is also obtained from (A.2).

10. We deduce a volatility of defaults, $\hat{\sigma}^{\delta}_{ss}$, and κ from the solution to

$$(\bar{b}_{ss}, 1 + \kappa - (\bar{b}_{ss}, \kappa) \equiv \underset{\bar{b}\leq\kappa, \bar{a}+\bar{b}-\kappa=1}{\text{argmax}} \left\{\mathbb{E}\left[(1-\delta)\, R^b_{ss}\bar{b} + \; R^m_{ss}\bar{a} - \; R^d_{ss}\bar{d} + \bar{\chi}_{ss}(\bar{a},\bar{d},\omega)\right]^{1-\gamma}\right\}^{\frac{1}{1-\gamma}}$$

where the expectation $\mathbb{E}$ is over δ and ω.

11. We obtain $\hat{\Theta}^b_{ss}$ by inverting (**??**)

$$\hat{\Theta}^b_{ss} = \left(\bar{b}_{ss} + \bar{b}^{Fed}_{ss}\right) \cdot \hat{\beta}_{ss} \cdot E_{ss} \cdot (R_{ss})^{\epsilon^b}.$$

12. We obtain $\hat{\Theta}^d_{ss}$ by inverting (**??**)

$$\hat{\Theta}^d_{ss} = \bar{d} \cdot \hat{\beta}_{ss} \cdot E_{ss} \cdot \left(R^d_{ss}\right)^{\epsilon^d}.$$

L Appendix to Section 4: Robustness Analysis

The next sections present additional corroborating evidence as the one presented in Section 4.

Estimates with other Liquidity Measures. A first robustness check runs the same regressions as in Table 1, but where using two other measures of liquidity premia. The first measure is a classic measure advocated by Stock and Watson (1989) and Friedman and Kuttner (1993), the three month spread between the AAA commercial paper and the 3 month T-Bill. The second measure is the TED spread, the difference between the three-month Treasury bill and the three-month US dollar LIBOR. In this case, the data availability is longer, from July 2000 to February 2016, when the NY Fed stops reporting the daily max and min values of the Fed Funds market. Table 5 reports the results. The pattern is consistent with the earlier estimation in Table 1, in significance in magnitude.

The next robustness check compares the results with two popular measures introduced by Gilchrist and Zakrajšek (2012), the GZ Spread and the GZ excess-bond premium (EBP). The authors construct these measures by taking individual fixed-income securities and discount their promised cash-flows according to zero-coupon US Treasury yields. This delivers a spread for each security. The "GZ excess bond premium" of each security is constructed as the portion of the overall credit spread that cannot be accounted for by individual predictors of default, nor bond-specific characteristics. Specifically, the authors regress their credits spreads on a firm-specific measure of expected default and a vector of bond-specific characteristics; the residual of this regression is the GZ excess bond premium.

Table 1 reports the results. The pattern is resembles the earlier estimation in Table 1, but the regressions lose power once we control for the VIX, a measure of dispersion, that remains significant. This feature indicated that bonds can have premia liqudity premia that correlate with the cycle, but are independent of the liquidity premia among near-money assets.

Other Spreads and Placebos. Table 7 reports two addition sets of robustness checks. Regressions (1-3) in Table 7 are the same regressions as regression (3) in Table 1, except that the measure FF Range is replaced by three alternative measures of interbank market dispersion. The first two measures are FF 99-1 which corresponds to the monthly average of the daily spread between the 99th and 1st quantiles of the Fed Funds distribution and FF 75-25 which corresponds to the 75th and 25th quantiles. Finally, FF std is the monthly average daily standard deviation of the Fed funds rates. The time series for quantiles are shorter than FF range, ranging only from January 2006 through December 2018. The overall fit is similar, and in particular, the magnitude of the coefficient of FF 99-1 series is very similar to FF Range, not surprisingly. The coefficient for the FF 75-25 series is larger, which is unsurprising since the standard deviation of this series is larger. The FF std series, is also significantly correlated, and the coefficient is even larger.

Regressions (4-6) in Table 7 employ other measures of liquidity premia in Nagel (2016), which are interpreted as placebo tests. These are the series for the 10y AAA to T-Bill corporate bond spread, the Note T-Bill spread and the spread between on the run and off the run bonds. In neither case, is the FF Range variable significant.

Subsamples. As final robustness check, we re-run the regressions in Table 8, but this time, we limit the sample to the pre-crisis period, from July 2000 through December 2007. Again, the pattern is the same. The FF Range variable remains a significantly correlated variable with other measures of spreads. By contrast, the VIX index is no longer significantly correlated with measures of spreads.

Table 5: Liquidity Premia and Interbank Spreads - Robustness Checks

	(1) CP Spread	(2) CP Spread	(3) CP Spread	(4) TD Spread	(5) TD Spread	(6) TD Spread
FF Range	0.352***	0.320***	0.293***	0.587***	0.609***	0.534***
	(17.72)	(14.71)	(13.05)	(16.86)	(15.63)	(13.92)
FFR		0.0206**	0.0267***		-0.0137	0.00430
		(3.26)	(4.20)		(-1.23)	(0.40)
VIX			0.115***			0.315***
			(3.53)			(5.70)
Constant	0.00714	-0.00882	-0.345***	0.0893**	0.101***	-0.816***
	(0.44)	(-0.53)	(-3.57)	(3.13)	(3.36)	(-5.00)
Observations	184	184	184	188	188	188
Adjusted R^2	0.631	0.649	0.670	0.602	0.603	0.661

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 6: Liquidity Premia and Interbank Spreads - Robustness Checks

	(1) GZ Spread	(2) GZ Spread	(3) GZ Spread	(4) GZ ebp	(5) GZ ebp	(6) GZ ebp
FF Range	0.536***	0.769***	0.150	0.463***	0.526***	0.112
	(4.01)	(5.29)	(1.87)	(4.91)	(4.98)	(1.62)
FFR		-0.147***	0.00114		-0.0395	0.0598**
		(-3.55)	(0.05)		(-1.31)	(3.07)
VIX			2.598***			1.738***
			(22.50)			(17.39)
Constant	2.302***	2.422***	-5.139***	-0.127	-0.0947	-5.152***
	(21.00)	(21.70)	(-15.07)	(-1.64)	(-1.17)	(-17.46)
Observations	188	188	188	188	188	188
Adjusted R^2	0.075	0.129	0.767	0.110	0.113	0.663

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 7: Liquidity Premia and Interbank Spreads - Robustness Checks

	(1) GC Spread	(2) GC Spread	(3) GC Spread	(4) 10y AAA Spr	(5) Note Spr	(6) OF Spr
FF 99-1	0.207***					
	(7.26)					
FFR	0.0555***	0.0774***	0.0429***	-0.0912***	-0.00383	0.00384***
	(6.14)	(9.66)	(8.36)	(-8.33)	(-1.84)	(4.28)
VIX	0.138**	0.200***	0.0584*	0.851***	0.0943***	0.00462
	(3.05)	(4.57)	(2.07)	(15.12)	(8.95)	(1.01)
FF 75-25		0.658***				
		(6.48)				
FF std			1.051***			
			(10.08)			
FF Range				-0.0383	0.00375	-0.00146
				(-0.98)	(0.60)	(-0.54)
Constant	-0.411**	-0.585***	-0.184*	-0.696***	-0.239***	-0.0165
	(-2.84)	(-4.11)	(-2.08)	(-4.19)	(-7.17)	(-1.14)
Observations	63	63	138	188	138	138
Adjusted R^2	0.791	0.768	0.660	0.685	0.486	0.121

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 8: Liquidity Premia and Interbank Spreads - Robustness Checks

	(1) GC Spread	(2) GC Spread	(3) GC Spread	(4) CD Spread	(5) CD Spread	(6) CD Spread
FF Range	0.238***	0.162***	0.157***	0.581***	0.529***	0.513***
	(8.92)	(6.60)	(6.19)	(13.09)	(10.83)	(10.23)
FFR		0.0465***	0.0482***		0.0322*	0.0377*
		(6.72)	(6.63)		(2.34)	(2.62)
VIX			0.0281			0.0886
			(0.80)			(1.28)
Constant	0.0308	-0.0691**	-0.153	-0.0110	-0.0802	-0.345
	(1.35)	(-2.90)	(-1.42)	(-0.29)	(-1.70)	(-1.62)
Observations	90	90	90	90	90	90
Adjusted R^2	0.469	0.646	0.645	0.657	0.674	0.676

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

M Algorithms

This appendix presents the numerical algorithms that we use to solve the model. We first present the algorithm to solve the stationary equilibrium. We then present the algorithm to solve for transitional dynamics.

M.1 Stationary Equilibrium

The stationary equilibrium of the model can be conveniently reduced to solving a system of two-non linear equations in two unknowns (R^b_{ss}, θ_{ss}). In a stationary equilibrium, all nominal variables grow at a constant rate (in this case, zero) and real variables are constant. To simplify the presentation, we assume that the intertemporal elasticity of substitution equals one, which gives rise to a constant dividend-to-equity ratio, a zero nominal growth of the nominal balance sheet of the monetary authority, and $B^{Fed} = 0$. We also set a value for R^d_{ss} based on the calibration target and infer the intercept term Θ^d that is consistent with that value.

1. Guess a stationary value for (R^b_{ss}, θ_{ss}), the real return on loans and market tightness.

2. Given market tightness, nominal policy rates, the given long-run inflation, and R^d_{ss}, compute the liquidity yield function $\bar{\chi}$ using (9).

3. Solve banks' optimization problem for the portfolio weights $\{\bar{b}, \bar{d}, \bar{a}\}$:

$$\begin{aligned} &\max_{\{\bar{b},\bar{a},\bar{d}\}\geq 0} \left\{ \mathbb{E}\left[R^b \bar{b} + \ R^m \bar{a} - \ R^d \bar{d} + \bar{\chi}(\bar{a}, \bar{d}, \omega) \right]^{1-\gamma} \right\}^{\frac{1}{1-\gamma}}, \\ &\bar{b} + \bar{a} - \bar{d} = 1, \text{ and } \bar{d} \leq \kappa. \end{aligned} \tag{M.1}$$

4. Check whether banks' policies are consistent with steady state:

5. Compute aggregate gross equity growth as

$$E'/E = \left(1 + \left(R^b - 1\right)\bar{b} - \left(R^d - 1\right)\bar{d}\right)(1 - \bar{c}),$$

6. Compute implied market tightness:

$$S^- = \int_1^{\frac{\bar{a}/\bar{d}-\rho}{(1-\rho)}} s(\omega) d\Phi \text{ and } S^+ = \int_{\frac{\bar{a}/\bar{d}-\rho}{(1-\rho)}}^{\infty} s(\omega) d\Phi.$$

   where we used that market tightness is defined as

$$\tilde{\theta} = S^-/S^+.$$

7. If $E'/E = 1$ and $\tilde{\theta} = \theta_{ss}$, move to step 7. Otherwise, adjust the guess for R^b_{ss} and θ_{ss} and go to step 3.

8. Compute household demand for government bonds, using $R^g = R^m + \chi^+$

$$\frac{G^h}{P} = \Theta^g \left(R^g\right)^{\epsilon^g}$$

9. and banks portfolio weights on government bonds by using market clearing condition

$$\frac{G}{P} - \frac{G^h}{P} = E\bar{g}(1-\bar{c})$$

10. Compute the nominal amount of reserves and the intercepts of the loan demand and deposit supply functions using that real equity and the initial price level are normalized to one (i.e., $P = 1, E = 1$) and

$$\begin{aligned} \tilde{M}^{Fed} &= (1-\bar{c})(\bar{a}-\bar{g})EP, \\ \Theta^b \left(\frac{1}{R^b}\right)^{\epsilon} &= E\bar{b}(1-\bar{c}), \\ \Theta^d \left(\frac{1}{R^d}\right)^{-\varsigma} &= E\bar{d}(1-\bar{c}). \end{aligned}$$

11. Compute nominal returns using definitions of real returns and transfers T^{Fed} from the Fed budget constraint:

$$\tau = (1-\bar{c})\left[(i^m - \pi)\,\bar{m} + (i^g - \pi)\,\bar{g} - (i^w - i^m)\bar{w}\right].$$

where

$$\bar{w} = (1 - \Psi^-(\theta))S^-.$$

Let us comment on some details from the computations. To solve for the pair (R^b_{ss}, θ_{ss}), we use the fsolve command in Matlab. To solve for the portfolio problem, we use the first-order conditions, which we solve, again using fsolve. Notice that if the capital requirement binds, there is only one portfolio variable to solve for.

To compute expectations, we use a Newton-Cotes quadrature method. Specifically, we apply the trapezoid rule with a grid of 2,000 equidistant points. To specify the lower and upper boundaries of the grid, we take the shock values that guarantee 10^{-5} mass in the tails of the distribution.

M.2 Transitional Dynamics

The basic procedure to solve for transitional dynamics is to start by conjecturing an initial price level P_0, solve for all sequences of prices and quantities using market clearing conditions and bank problems, and finally that the price converges to the path of the price level in the stationary equilibrium. Essentially, the solution can be reduced to one-equation and one-unknown.

To simplify the presentation, we assume that the intertemporal elasticity of substitution equals one, which gives rise to a constant dividend-to-equity ratio, a zero nominal growth of the nominal balance sheet of the monetary authority, $B^{Fed}_t = 0$, and an inelastic demand for government bonds by households.

1. Establish a finite period $T \in \mathbb{N}$ for convergence to steady state, a convergence criterion ε, and an initial value for aggregate real equity E_0

2. Guess an initial price level P_0.

3. Set $t = 0$.

4. Given E_0, P_0, and level of nominal reserves set by the monetary authority $\tilde{M}^{Fed}$, we can obtain an implied level of real reserve holdings:

$$\tilde{m_0} \equiv \frac{\tilde{M}^{Fed}}{\beta P_0 E_0}.$$

5. Compute

$$\tilde{g}_0 = \frac{\frac{G}{P} - \frac{G^h}{P}}{E_0(1-\bar{c})}$$

6. Denote $\tilde{a_0} = \tilde{m} + \tilde{g}$

7. Find (R_1^m, R_1^b) that solves

$$\begin{aligned} \bar{a}_0 - \tilde{a_0} &= 0 \\ \beta E_0(1 + \bar{d}_0 - (\tilde{m_0} + \tilde{g_0})) &= \Theta_0^b \left(\frac{1}{1 + r_{t+1}^b}\right)^{\epsilon}, \end{aligned}$$

where $\bar{a}_0, \bar{d}_0$, satisfy

$$\left(\bar{a}_0, \bar{d}_0\right) = \arg \max_{\bar{b}, \bar{a}, \bar{d} \leq \kappa} \left\{ \mathbb{E}\left[R_1^b(1 + \bar{d}_0 - \bar{a}) + R_1^m \bar{a} - R_1^d \bar{d} + \bar{\chi}(\bar{a}, \bar{d}, \omega) \right]^{1-\gamma} \right\}^{\frac{1}{1-\gamma}}$$

and $\bar{\chi}$ follows (9).

Given $i^m$and R_1^m, compute inflation between period 0 and 1 as

$$\pi_1 = \left(\frac{1 + i^m}{R_1^m}\right) - 1.$$

8. Given π_1 and $P_{0.}$, compute next-period price $P_1 = (1 + \pi_1) P_0$.

9. Compute next-period equity using the law of motion

$$E_1 = \left(1 + \left(R^b - 1\right) \bar{b} - \left(R^d - 1\right) \bar{d}\right) (1 - \bar{c}) E_0,$$

10. Repeat steps 4-9 for $t = 1.....T$

11. Compute criteria for convergence of $z = P_{T+1} - P_0$. Notice that if there is steady state inflation, this condition for convergence is replaced by $z = P_{T+1} - P_0(1 + \pi_{ss})^T$.

12. If $|z| < \varepsilon$, exit algorithm. Otherwise, adjust P_0 and go to step 4.