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AGGREGATE EXTERNAL FINANCING AND SAVINGS WAVES

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ABSTRACT

US data display aggregate external financing and savings waves. Firms can allocate costly external finance to productive capital, or to liquid assets with low physical returns. If firms raise costly external finance and accumulate liquidity, either the cost of external finance is relatively low, or the total return to liquidity accumulation, including its shadow value as future internal funds, is particularly high. We formalize this intuition by estimating a dynamic model of firms' financing and savings decisions, and use our model along with firm level data to construct an empirical estimate of the average cost of external finance from 1980-2014.

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US corporate sector data display a strong, positive correlation between aggregate external financing and savings activity. As a result, we observe aggregate external financing and savings waves in US data 1980-2014. Aggregating all but the very largest firms, the correlation between external finance raised and liquidity accumulated is a statistically significant 0.57. This high correlation is not due to some firms raising external finance, and other firms saving. Conditioning on firms that raise external finance, the aggregate correlation increases to 0.80. We also document that the time series of the percentage of firms raising external finance, and, more importantly, the time series of the cross-sectional correlation between external finance and savings, are highly correlated with traditional proxies for the cost of external finance such as the default spread, index of lending standards, and sentiment index.

Motivated by the stylized facts we document, we propose and implement a method for using data on firms' sources and uses of funds in the cross-section in order to make inferences about the aggregate level of the cost of external finance. The basic intuition is as follows: Firms which raise costly external finance can invest the issuance proceeds in productive capital assets, or in liquid financial assets with a low physical rate of return. If firms raise costly external finance and allocate some of the funds to liquid assets, either the cost of external finance is relatively low at that time, or the total return to liquidity accumulation, including its shadow value as future internal funds, is particularly high.

To formalize this intuition, we construct and estimate a quantitative, dynamic model economy consisting of a panel of heterogeneous firms in partial equilibrium. Firms have identical risk-neutral objective functions, and maximize the present value of their net payouts. They are heterogeneous ex-post due to variation in their idiosyncratic productivity realizations. Firms choose external finance, investment, and savings as a function of their size, savings balance, and idiosyncratic productivity shock, and the aggregate level of productivity and of the cost of external finance. We then use variation in firms' investment returns and the economy's implied cross-sectional moments in order to quantitatively measure variation in the aggregate cost of external finance. In the model, as in the data, firms typically finance investment with operating cash flows. However, when the cost of external finance is low, firms in our model raise external finance, invest some of the proceeds, and save the remainder in liquid assets. As a result, when the aggregate cost of external finance is low, firms are more likely to both raise external finance, and to accumulate liquid assets. Consistent with this idea, we show that in the model and in the data, cross-sectional moments describing the incidence of firms raising external finance, and importantly, the co-incidence of firms raising external finance and saving the proceeds, are informative about the aggregate level of the cost of external finance. Our study is aimed at providing an estimate of the aggregate cost of external finance in the US time series from 1980-2014 based on this intuition. We do so formally using the quantitative implications of our structurally estimated model.

We estimate the key parameters of our model using Simulated Method of Moments. We show that in the model, variation in the aggregate average cost of external finance raised can be almost fully explained by variation in two key moments describing firms' financing and savings decisions. These two moments are the fraction of firms raising external finance, and the cross-sectional correlation between aggregate net external finance raised and liquidity accumulation, which we call "excess external finance". Accordingly, we construct a continuous series for this average cost using a regression based cost-of-external finance index. Specifically, we compute the index weights by running a time series regression using model data of the cost of external finance at each date on the cross-sectional moments describing firms' external financing and savings decisions on that date. We then use the model implied weights, along with the empirical moments from Compustat data, to construct an estimate of the empirical cost of external finance in US data. Figure 1 graphs the resulting series, which exhibits substantial and intuitively appealing variation over our sample. The implied average cost of external finance is 2.3% per dollar raised, which is close to existing empirical estimates. Moreover, it has an economically reasonable standard deviation of 1.6% and an autocorrelation of 0.63, which is close to that of the Baa-Aaa default spread.

We discuss the model's additional implications and conduct robustness checks. In particular, we provide a formal statistical test our model featuring aggregate shocks to both productivity and cost of external finance shocks against a nested alternative model with a constant cost of external finance. The model with constant costs can be rejected statistically, and also performs poorly on key empirical moments. In fact, the constant cost model at estimated parameters does not generate aggregate external financing and savings waves. The key distinction between our model and earlier business cycle models with costly external finance, such as Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), and Gomes *et al.* (2003) is the addition of shocks to the cost of external finance. These shocks break the otherwise tight relationship between TFP and the demand for external finance, and thereby allow for aggregate issuance and savings waves as well as a significantly better overall statistical fit.

1. Related Literature

This paper contributes to the growing literature at the intersection of finance and macroeconomics which studies the interaction between firm financing, savings, and investment decisions, and the macroeconomy. Two recent prominent papers document the cyclical behavior of firm financing. Covas and Den Haan (2011), and Jermann and Quadrini (2012) both document that debt issuances are highly procyclical, and Covas and Den Haan also report procyclical equity issuance.[1] Distinct from existing papers, we use both cross-sectional and aggregate information not only about the source of firm financing, but also about its uses, to empirically uncover shocks to the cost of external finance.[2] We argue that looking at the joint dynamics of liquidity accumulation and external finance is useful for examining the role of shocks to the cost of external finance, since how firms use funds may help to disentangle financing shocks from shocks that drive investment opportunities.

Several recent papers develop theoretical models which employ a shock which originates in the financial sector to better match business cycle facts.[3] We extend this line of research by using our model to estimate the cost of external finance in the US time series. Thus, our paper is most closely related to the important contribution of Jermann and Quadrini (2012), with a few key differences. First, Jermann and Quadrini (2012) focus on the distinction between debt vs. equity in their estimation, and estimate a debt financing cost shock. Instead, in our model, the key friction occurs between firm insiders and outsiders. We abstract from the distinction between debt and equity in order to focus on what is new in our paper. Whereas other studies have focused on variation in firms' *sources* of funds, our study focuses on firms' *uses* of the external finance they raise. Jermann and Quadrini (2012) estimate a time series for financial shocks using a clever relationship between recovery rates and financing needs which allows them to solve for their shocks in closed form. While we cannot solve our model for the cost of external finance shock in closed form, we think that the use of cross-sectional moments to identify a hidden aggregate state is a complementary methodology to existing methods,

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[1] Choe *et al.* (1993), and Korajczyk and Levy (2003) also study issuance over the business cycle. Both find that equity issuance is procyclical. Korajczyk and Levy (2003) report countercyclical debt issuance. Huang and Ritter (2009) provides evidence that active external financing decisions are driven by the relative cost of equity vs. debt.

[2] Eisfeldt and Rampini (2009) builds an aggregate model of internal and external finance to study the implications of corporate liquidity demand for the observed low return on liquid assets, but does not consider shocks to the cost of external finance. Covas and Den Haan (2011) focus on debt and equity issuance, but they do note that, empirically, firms tend to both accumulate financial assets and invest when they issue external finance.

[3] For example, Jermann and Quadrini (2012), Covas and Den Haan (2012), Khan and Thomas (2013), and Hugonnier *et al.* (forthcoming) build on the seminal contributions of Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Carlstrom and Fuerst (1997) on the role of financial market conditions on firm investment and business cycle dynamics.

with other potential uses.[4] For example, one could use the information in cross-sectional moments describing consumer decisions in order to uncover aggregate household financial constraints.

Despite renewed interest, the fact that financial constraints, or shocks originating in the financial sector, are important for either firm level investment, or business cycle dynamics, is still not a foregone conclusion amongst economists. Gomes *et al.* (2003) point out that the shadow cost of external finance is procyclical in a standard business cycle model with agency costs of external finance and no financial shocks.[5] Our study contributes to our understanding of the role of financial frictions by formally statistically testing a nested alternative without time varying frictions.

Our paper also builds on papers which develop models of corporate saving.[6] The main difference is in focus; these papers are focused on understanding *firm level* dynamics or making inferences about firm level of financial constraints. In contrast, our paper, which is focused on understanding the dynamics and the effects of the *aggregate* component of the cost of external finance connects ideas from this corporate finance literature to the macroeconomic literature which studies business cycles with financial frictions.

2. Stylized Facts

Our main data set consists of annual firm level data from Compustat covering 1980-2014. Our sample selection follows that in Covas and Den Haan (2011). The Online Appendix gives a detailed description of the sources and construction of our data, as well as a discussion of alternative definitions and normalizations.[7] We use firm level cash flow statements to track corporate flows. We define net external finance raised as the negative of the sum of net flows to debt and net flows to equity. We define flows to debt as debt reduction plus changes in current debt plus interest paid, less debt issuances, and flows to equity as purchases of common stock plus dividends less sale of common stock. We define liquidity accumulation as changes in cash and cash equivalents. We define investment (in

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[4] Building on our work, Belo *et al.* (2014) use a closely related measure that exploits the cross-section of firm policies to study the asset pricing implications of a model of costly external finance.

[5] Chari *et al.* (2007) use business cycle accounting to argue that shocks to the return on capital, are only of tertiary importance for explaining US fluctuations. However, papers such as Justiniano *et al.* (2010), Christiano *et al.* (2010), Hall (2011), Shourideh and Zetlin-Jones (2012), and Gilchrist and Zakrajsek (2012b), find that financial shocks explain a large fraction of business cycle fluctuations. Baker and Wurgler (2002) attributes rare capital structure changes to firms market timing the variation in the external finance, and Zhang (2013) studies the role of external finance in firm entry and exit.

[6] See, for example, Kim *et al.* (1998), Gamba and Triantis (2008), Almeida *et al.* (2004), Riddick and Whited (2009), Bolton *et al.* (2011), and Bolton *et al.* (2013). Warusawitharana and Whited (2015) is a contemporaneous paper focused on the effect of equity misvaluation on firm level policies.

[7] We show that our results are robust to excluding employee stock issuances. We also show there that using Flow of Funds data to construct aggregate external financing and savings moments yields similar results, leading to an aggretate correlation of external finance and savings of 0.55.

physical capital) as capital expenditures. To compute most aggregate and firm level moments, we normalize firm level variables by current total book assets. To compute aggregate correlations, we instead normalize by the lag of book assets, to avoid inducing spurious correlations.[8] We use the Hodrick and Prescott (1997) filter to remove any remaining low frequency trends when computing aggregate correlations, since, for example, cash holdings have trended upwards as a share of assets over our sample (see Bates *et al.* (2009)). The HP filter ensures that the empirical series are stationary, consistent with the stationary business cycle model we study, and with our focus on the business cycle dynamics of the cost of external finance.

As in Covas and Den Haan (2011), our main analysis drops the top 10% of firms by asset size. There are several reasons to do this. First, as Covas and Den Haan (2011) point out, the timing of external finance for the largest firms is not representative of the rest of the sample. They advocate dropping the top firms because they behave anomalously. This may be because the very largest firms face little or no financial constraints.[9] Moreover, the very largest firms present unique measurement problems. A significant fraction of the investment for these firms falls under the accounting category "other investments", which are typically comprised by long term receivables from unconsolidated subsidiaries. Thus, we are not able to identify accumulated liquidity vs. physical investment. Cash accumulation for large firms, which have larger foreign earnings, is also more influenced by tax motives and repatriation timing. Finally, and perhaps most importantly, we note that our stationary model *can replicate the pattern across size categories* for the correlation between external finance and liquidity accumulation. However, the *distribution* of firm sizes is much less skewed than that in the data.[10] Accordingly, we show that weighting Compustat data by the distribution of firm sizes in our model reduces the impact of the largest firms in a way that is similar to removing the largest firms. However, restricting the sample imposes a less structural restriction and matches with the prior literature.

US data from 1980 to 2014 display aggregate external financing and savings waves. Figure 2 plots the cyclical component of aggregate net liquidity accumulation vs. the cyclical component of aggregate net external finance and clearly illustrates this stylized fact. Table 1 presents corresponding correlations for different groups of firms. For all but the largest 10% of Compustat firms, the aggregate correlation between liquidity accumulation and external finance is 0.57 and is statistically significant at

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[8] Book assets are slow moving and fairly acyclical and thus shouldn't induce any cyclical variation, however our results are robust to alternative normalizations, such as aggregate output or aggregate gross-value added from the corporate sector.

[9] See Eberly *et al.* (2012).

[10] The bottom panel of Table 3 displays this correlation in the model and Compustat data across size bins.

the 5% level.[11] When one includes the largest firms, the correlation drops to 0.08 because, empirically, large firms' size implies weights much larger than those in the model. Accordingly, the equally weighted correlation is 0.50, and weighting by our model's size distribution the correlation is 0.52.

The aggregate correlation is higher (0.80) if one conditions on firms that are currently raising external finance. Thus, the positive aggregate correlation does not seem to be driven by some firms saving, and other firms issuing external finance, nor is it driven by the behavior of payouts.[12] The aggregate correlation is also higher when one excludes more of the largest firms. For the smallest half of firms, the correlation between aggregate external finance raised and liquidity accumulated is 0.82. This is higher than the correlation conditioning on other measures of financial constraints, such as whether a firm pays no dividends, or has no credit rating, in which case we find correlations close to that for the larger sample (0.58 and 0.56 respectively). This could be due to the importance of fixed costs in accessing external financial markets, or it could be that size is simply a better proxy for financial constraints.

The first column in Table 3 presents additional aggregate moments for comparison. We note in particular that savings are more correlated with external finance than investment is. By contrast, investment is more correlated with GDP than savings are. We argue that this is indicative of the fact that information about firms' uses of funds, and in particular firms' external financing and savings activity, can help to disentangle shocks to the cost of external finance from TFP shocks.

3. Model

A. Model Description:

We study a continuum of risk neutral firms in partial equilibrium. To model firm behavior, we use a parsimonious version of the model featured in Hennessy and Whited (2005), Hennessy and Whited (2007), and Riddick and Whited (2009), which is based on the important early structural contribution by Gomes (2001). All firms have access to the same production and financing technologies, and are subject to common aggregate shocks. However, firms experience different idiosyncratic productivity realizations and as a result choose heterogeneous stocks for their physical capital and liquid assets. Thus, we study a panel of heterogeneous firms which experience both idiosyncratic and aggregate shocks. We begin by describing the model, and the implied investment returns, and then turn to its

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[11] This result robust to alternative measures of external finance that remove employee stock options from gross equity issuance.

[12] See the Online Appendix for the analogous correlations for debt and equity separately.

estimation and policy function analysis.

Firms produce output or cash flows using physical capital k according to:

$$y = zk^{\theta}$$

where z is the level of the firm's productivity and $\theta \in (0, 1)$. Each firm's productivity z is the product of an idiosyncratic shock z_i, and an aggregate shock z_{agg}. The aggregate productivity level, and each idiosyncratic productivity level, follow AR(1) processes in logs with identical persistence parameters, however, we allow for the idiosyncratic and aggregate processes to have different volatilities. These assumptions allow us to construct each firm's productivity shock $z = z_i z_{agg}$ as follows:

$$\begin{aligned} \ln(z_i') &= \rho \ln(z_i) + \epsilon_i' && (1) \\ \ln(z_{agg}') &= \rho \ln(z_{agg}) + \epsilon_{agg}' && (2) \\ \ln(z') &= \rho \ln(z) + \epsilon_i' + \epsilon_{agg}'. && (3) \end{aligned}$$

Both ϵ shocks are normally distributed with zero mean and standard deviations given by σ_i and σ_{agg}, respectively. Capital evolves according to the standard law of motion $k' = (1 - \delta)k + i_k$, where i_k is investment and $\delta \in (0, 1)$ is the depreciation rate. Investment in physical capital is subject to convex adjustment costs $\phi_i(i_k, k)$ given by:

$$\phi_i(i_k, k) = \frac{a}{2} \left(\frac{i_k}{k} \right)^2 k. \tag{4}$$

where a determines the slope of the marginal adjustment cost. Liquid assets l evolve according to $l' = (l + i_l)(1 + r(1 - \tau))$ where i_l is investment in liquid assets and r is the risk free rate. Thus, taken literally, corporate payouts are motivated by the tax wedge, $\tau > 0$. However, because actual payout policy is also driven by agency and information considerations, τ is effectively a stand-in for all of these payout motivations.

Because financing costs will be paid only if payouts gross of financing costs are negative, it is convenient to define a firm's pre-financing payout e as internal cash flows minus investment in physical capital and liquidity accumulation, less investment adjustment costs:

$$e \equiv zk^{\theta}(1 - \tau) - i_l - i_k - \phi_i(i_k, k). \tag{5}$$

If $e > 0$ the firm is paying out funds and if $e < 0$ the firm is raising external finance. Intuitively, the firm raises external finance if after tax operating profits do not cover the firms' total investment in physical and liquid assets, net of physical adjustment costs. Firms maximize this payout, net of financing costs. Following Gomes (2001), and in order to facilitate estimation, we parameterize the cost of external finance exogenously as follows:

$$\phi_e(e, \xi) = \mathbb{1}_{\{e<0\}}(\lambda_1 + \frac{\xi}{2}\lambda_2 e^2), \tag{6}$$

where $\lambda_1, \lambda_2 > 0$, $\mathbb{1}_{\{e<0\}}$ is an indicator that takes the value 1 when $e < 0$ and 0 otherwise, and $\xi > 0$ denotes the aggregate level of external financing costs. Note that even though the shock ξ only affects the marginal cost of external finance, it will affect firms' propensity to pay the fixed cost because a high marginal cost lowers the value of paying the fixed cost. See Figure 3 in Gomes (2001) for a discussion of interpreting fixed and variable costs in the context of microfounded financing costs. Our preferred specification, in which only the marginal cost varies with the ξ shock is, perhaps, closest to what is implied by existing models of the microfoundations of time varying costs of external finance, although no exogenous specification captures the endogenous effects in these models perfectly.[13] We discuss and compare results for alternative, more complex, specifications in Section 4.B., and find a very high degree of robustness across specifications. The aggregate state of external financing costs, ξ, follows an AR(1) in logs.

$$\ln(\xi') = c + \gamma \ln(\xi) + \eta' \tag{7}$$

where η' is a normal i.i.d. shock with mean zero and standard deviation σ_η. We choose the value c so that ξ, is on average equal to one. The average level of the marginal cost of external finance is then given by λ_2.

We denote the firm's value function by V and write the firm's problem in recursive form, using the constraints and laws of motion given in equations (1)-(7). Each firm's state is given by its size, k, its liquid asset balance, l, its productivity z (the product of an aggregate and idiosyncratic shock), and the aggregate level of the marginal cost of external finance, ξ. We define $s \equiv \{k, l, z, \xi\}$ to summarize an individual firm's state vector. We use the standard "prime" notation to denote next period values

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[13] Microfoundations of a time varying marginal cost include agency frictions that vary over time, along the lines of Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997), collateral constraints which vary with asset values as in Kiyotaki and Moore (1997), and endogenously time varying adverse selection problems as in Eisfeldt (2004), Kurlat (2013), and Bigio (2015). Note that bankruptcy costs would also make firms averse to low payout states.

for the state, and a subscript s to denote expectations conditional on the current state.

Each firm then solves the Bellman Equation:

$$V(s) \quad = \quad \max_{k' \in [0,\infty), l' \in [0,\infty)} \left\{ e - \mathbb{1}_{\{e<0\}} (\lambda_1 + \frac{\xi}{2} \lambda_2 e^2) + \frac{1}{1+r} E_s \left[V(s') \right] \right\} \tag{8}$$

subject to:

$$e \quad = \quad (1-\tau) z k^{\theta} - \frac{l'}{1+(1-\tau)r} + l - k' + (1-\delta)k - \frac{a}{2} \left(\frac{k' - (1-\delta)k}{k} \right)^2 k.$$

B. Firm Policies

In our partial equilibrium dynamic model, firms adjust their policy functions for investment and liquidity accumulation, and the implied policy for external finance (or payouts), in order to set the marginal returns to investment and liquidity accumulation, and the marginal cost of external finance, equal to the gross interest rate. We provide an analytical solution to a simple two-date model in the Online Appendix, to provide additional intuition for our dynamic model.

For the dynamic model, we provide analytical intuition by extending the production returns from Cochrane (1991) and Cochrane (1996) to include the effects of financial constraints, closely following Gomes *et al.* (2003) and Gomes *et al.* (2006), while also adding stochastic financing costs. We additionally account for firms' investment in more than one asset type, in order to provide intuition for our use of excess external finance as a measure of financing costs. Details on computing the returns are collected in the Online Appendix.

The Euler equations which set the discounted expected return on investment in physical capital and liquid assets equal to the value of a dollar payout today are, respectively:

$$1+r = E_s \left[\frac{(1+\psi'_e)}{(1+\psi_e)} \quad \frac{\left(\theta(1-\tau) z' k'^{\theta-1} + (1-\delta) + \frac{a i'_k}{k'} \left(\frac{1}{2} \frac{i'_k}{k'} + (1-\delta) \right) \right)}{\left(1 + \frac{a i_k}{k} \right)} \right] \text{ and} \tag{9}$$

$$1+r = E_s \left[\frac{1+\psi'_e+\psi_l}{1+\psi_e} \quad (1+(1-\tau)r) \right]. \tag{10}$$

where ψ_e and ψ_l account for financing costs and the binding non-negativity constraint on liquidity accumulation, respectively. These Euler equations highlight how the costs of external finance changes the firm's discounting, and capture the variation in the shadow value of internal funds over time.

Moreover, by comparing the two equations it is clear that financing costs can be quantitatively very important for liquidity accumulation, since the productivity shocks enter only indirectly, whereas the ξ shocks affect liquidity accumulation directly through ψ_e.

Next, we describe the policy functions graphically, using the numerical solution to the model at estimated parameters, in order to provide intuition for our results and to describe firms' decisions. Section 4.A. describes the model estimation and the parameters used, but results for policy functions are similar for a wide range of parameter values. Firms construct policies as a function of their four state variables: size k, liquid assets i_l, productivity z and external finance cost level ξ. In the Online Appendix we present complete graphs of the policy functions as a function of the state variables. In Figure 4, we condition on the state for the level of the cost of external finance (left panel is low cost, right panel is high cost), and graph policies as a function of either productivity, z, or size k. Thus, to present firm policies in two dimensions, we average over the other state variables, and this eliminates some of the heterogeneity which leads to the variation in the cross-section we exploit empirically. It is important to keep this averaging in mind, for example when interpreting the levels of the variables plotted in Figure 4. However, the main intuition can be illustrated in two dimensions, which are easier to view.

The top panel of Figure 4 plots the percentage of firms raising external finance as a function of firm level productivity from low to high across the six model states, averaging in each state over size and liquid assets. In both the low external finance cost state (left panel) and high external finance cost state (right panel), the fraction of firms raising external finance exhibits a hump-shaped pattern in productivity. The intuition is as follows. Low productivity firms have poor investment opportunities, both currently and likely in the near future due to persistent productivity, so they do not need additional sources of funds. By contrast, high productivity firms potentially have good current and future investment opportunities, however they will also tend to have abundant internal funds from high operating cash flows and are likely to be large and therefore experiencing greater decreasing returns to scale. Medium productivity firms, depending on their histories, can be growing and can thus have good investment opportunities relative to their supply of internal funds. In fact the "high medium productivity" firms are the most likely to raise external finance, and, as can be seen by comparing the left and right panels, medium productivity firms' decisions to raise external finance are also the most sensitive to the cost of external finance. The variation in the marginal cost of external finance as ξ varies has the most important effect on the value of paying the fixed cost of raising funds for these firms. Moreover, these firms are important in the economy in terms of probability mass. Because productivity shocks are normally distributed, most firm year observations will have realizations near the mean level.

Comparing the left hand panel to the right hand panel illustrates why $\%raise$ serves as a useful proxy for the level of the cost of external finance, since the area under the curve in the left panel is clearly larger than that in the right panel. When the cost of external finance is low, the cutoff productivity above which firms raise external finance is lower than when the cost of external finance is high, leading to a larger fraction of firms raising. We note that while the decision to raise external finance exhibits important variation with productivity, as shown in the bottom panel of Figure 4, most of the variation in productivity in our model, as well as in the data, comes from idiosyncratic, firm level shocks rather than aggregate fluctuations in TFP. That said, the cross-sectional moment describing $\%raise$ is positively correlated with aggregate TFP to a similar extent in both the model and the data, as we will show and discuss below. Higher aggregate TFP, all else equal, implies that firm level TFP will be above average and hence more firms will raise external finance. However, in the model and in the data, and consistent with the top panel of Figure 4, $\%raise$ is even more positively correlated with the aggregate state for the level of the cost of external finance.

The middle panel of Figure 4 graphs liquidity accumulation and net external finance ($-e$), also as a function of firm level productivity.[14] The policies for the level of net external finance raised illustrate two things: First, $-e$ is negative on average meaning that firms in the model, as in the data, are on average making payouts. However, as the top panel of Figure 4 shows, the average masks important heterogeneity and many firms do raise funds. Second, the shape for the amount of net external finance raised in the middle panel mimics that for $\%raise$ in the top panel, and has the same intuition. Comparing the left and right panels shows that the average net external finance raised is higher when the cost of external finance is low. This shift arises due to an increase in external finance raised rather than a reduction in payouts, since payouts fall, all else equal, in the high cost state when funds are worth more internally than in the low cost state. The policy for liquidity accumulation shows that on average, firms tend to invest in physical capital, with its higher rate of return, although the averages again mask individual variation. However, when the cost of external finance is low,

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[14] A plot of only external financed raised that does not include payouts looks similar in shape but is shifted upwards. We choose to plot the net measure as this variable is most comparable to the main moments we analyze in the data.

high-medium productivity firms raise external finance, and on average save some of the proceeds in liquid assets. These medium productivity firms are also important in terms of their contribution to the overall distribution of firms. Finally, we note that in the low cost of external finance state, the cross-sectional pattern across all productivity states for external finance and liquidity accumulation mimic each other closely. In contrast, in the right panel this correlation is low. This tight relationship is why $\rho^{xs}_{i_l,e}$ serves as a sharp proxy for the cost of external finance.

The bottom panel of Figure 4 plots investment for high and low productivity firms, defined as having above and below the average level of productivity at each date, as a function of firm size for the two external finance cost states. Comparing the left and right panels, high productivity firms always invest more than low productivity firms so that the red line is positive, but this difference is larger when the cost of external finance is low and investment is relatively unconstrained. Intuitively, if costs of external finance were zero, there would be no wedge between the shadow values of internal and external funds, so investment would be determined only by the marginal product of capital. In this case differences in investment would be large as productivity varied. In contrast, the more costly external finance is, the more constrained investment will be for high productivity firms and this will tend to narrow the difference in investment. In particular, smaller firms with high marginal products of capital invest up to twice as much in the low cost of external finance state relative to the high cost state. These results motivate an additional test of our model and our method for identifying the hidden cost of external finance state. The model makes the prediction that the difference in differences in investment will be both positive on average and the effect will be largest for smaller firms who tend to have a higher marginal product of capital. Both of these results are supported in the data, as we show in the Online Appendix. Thus, our model explains why investment dispersion increases as financing costs decrease.

The intuition in our model for why firms' use of the external finance they raise provides information about the cost of those funds is as follows: The marginal benefit of investing in physical capital is high at low levels, but decreases with the level of investment. Liquidity accumulation displays less decreasing returns to scale, but has a marginal benefit that is on average lower than the marginal benefit of investment in physical capital. As a result, firms will only invest in liquid assets once they have invested enough in physical capital to push the marginal return below that on liquid assets, but will typically invest a positive amount in physical capital. Importantly, the lower the cost of external finance is, the more likely it is that the marginal return to physical capital investment will be pushed down to the marginal return to cash because a lower cost of funds increases investment. This is simple to show analytically in the simple, two-date model in the Online Appendix, and the dynamic intuition is similar. In our dynamic model, physical capital investment has decreasing marginal returns due to both decreasing returns to scale, as well as to convex adjustment costs. The physical return to liquid assets is fixed at a rate *lower* than the discount rate, and thus positive liquidity accumulation only arises when the total return is endogenously pushed above the discount rate due to the benefit of liquid assets' use as future internal funds for investment.

4. Estimation and Results

A. Estimation

We simultaneously estimate as many of the key structural parameters of the model we found to be computationally feasible using Simulated Method of Moments (SMM). The remainder of parameters are either estimated independently, or specified based on estimates from the literature. SMM estimation, like the Generalized Method of Moments, estimates parameters by choosing values which minimize the weighted sum of squared errors between model moments and their empirical counterparts.[15] SMM is well suited to a model like ours which features substantial non-linearities and for which moments cannot be solved in closed form.

Table 2 displays our estimated and calibrated parameters. We set the risk free rate to four percent as in the real business cycle (RBC) literature.[16] We also use a standard RBC value for depreciation, 8%. We note that 8% also matches the average rate of investment we observe in the Compustat data, and can thus be thought of as having been estimated independently. For the production function curvature parameter, we specify 0.65, which is consistent with the estimate in Cooper and Haltiwanger (2006). We found that higher curvature parameters, i.e. production functions closer to linear, imply too large investment volatilities and disinvestment frequencies. We calibrate the persistence of both idiosyncratic and aggregate productivity shocks to be 0.66, which allows us to consolidate idiosyncratic and aggregate productivity into a single state variable. Khan and Thomas (2008), page 407, contains a detailed discussion of the disagreement in the literature about this parameter, however based on our

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[15] See Lee and Ingram (1991), Hansen (1982), and Hansen and Singleton (1982).

[16] The Online Appendix provides robustness checks for alternative values of the risk free rate and discusses having a stochastic interest rate. We find that our main results are not substantially affected for values of the risk free rate between 2-6%, with the biggest effect being larger average cash balances for lower levels of the risk free rate.

reading 0.66 is a modal value from prior studies. Moreover, we also estimated the average industry level persistence in the data from Basu *et al.* (2006) to be 0.65. Similarly, we estimate an aggregate persistence of about 0.62, using two trend breaks as advocated in Fernald (2007) and using the data from Fernald (2014). We set the total volatility of firm level productivity equal to the firm level estimate from Hennessy and Whited (2007), which is also near the value used by Khan and Thomas (2013). We then specify that aggregate volatility is about one fourth of total volatility as in Khan and Thomas (2008) and Cooper and Haltiwanger (2006). There are six productivity states, which govern both idiosyncratic and aggregate productivity, and two aggregate states for ξ. We assume that the stochastic cost is uncorrelated with the aggregate productivity shock, an assumption consistent with our empirical estimates. The empirical correlation between innovations in the default spread and TFP shocks is -0.2 and is not statistically significantly different from zero. Similarly, the empirical correlation between innovations to the expected default adjusted excess bond spread from Gilchrist and Zakrajsek (2012a) and TFP shocks is -0.1 and is again not statistically significantly different from zero. This is not so surprising considering that financial markets and real activity are not highly correlated in general. Moreover, because lower financing costs result in more efficient allocations, the cost and TFP shocks do not have to be correlated for financing costs to be correlated with real activity, a fact also highlighted in Khan and Thomas (2013). Finally, we set τ, the tax rate to 10%. In our model, liquid assets are only accumulated in order to ultimately hedge investment opportunities in physical capital. Moreover, firms can simply over-accumulate physical capital and hedge via the additional cash flows that capital produces. Thus, if the tax rate is too high, liquidity can become a dominated asset, in particular in the nested model we describe below in which the cost of external finance is constant. Furthermore, in our model, and others like it, the only force for payouts is the tax wedge, whereas in practice monitoring and information considerations will also influence payout policies. Taking into account these considerations, we choose a rate of 10%, which leads to positive liquidity accumulation in both the full and restricted models we estimate.

Given these calibrated parameters, we estimate the remaining, key, parameters simultaneously using SMM. Specifically, we estimate: the fixed cost of raising external finance λ_1, the quadratic cost λ_2, the persistence of the stochastic cost γ, the volatility of the cost shocks σ_η, and the marginal investment adjustment cost parameter a, via Simulated Method of Moments (SMM). The SMM algorithm solves the model and constructs data for a large set of parameter combinations, and efficiently searches for the set of parameters which imply moments with the lowest weighted sum of squared errors relative to their empirical counterparts. We use a bootstrapping technique to construct standard errors by computing the distributions over the estimates which result from estimating the parameters on samples of limited size.

We use the following seven moments to estimate the vector of parameters b: the pairwise correlations and the volatilities of investment, liquid asset accumulation, and external finance, and the average level of liquid assets to total assets. We target these correlations because our main focus is the co-movement between corporate external financing, savings and investment. Targeting both correlations and volatilities also implies that co-variances will be close to their empirical counterparts.

Identification of our estimation parameters requires that the moments respond to changes in parameter values. In the Online Appendix, we discuss which moments are primarily affected by which parameters by performing comparative statics on the model moments for changes in each of the estimated parameters. We find that the fixed cost of external finance is important in matching the level of cash holdings, and the quadratic cost is helpful in matching the volatility of external finance. The persistence and volatility of the stochastic cost help to match the correlations between liquidity accumulation and external finance as well as between investment and external finance. Without the stochastic cost, the correlation between external finance and investment is very large for *any* reasonable parameter values because firms only raise external finance when there are good productivity shocks and hence good investment opportunities. The stochastic cost breaks this high correlation and simultaneously generates the strong correlation between external finance and liquid assets through market timing by firms.

We report the empirical moments and their counterparts in model data at estimated parameter values in Table 3. We also report a large number of additional moments which our estimation procedure was not explicitly targeted to match. We report parameter estimates and standard errors in the middle panel of Table 2. The standard errors show that all parameter values are significant. Intuitively, the fairly small standard errors mean that the moments chosen are highly sensitive to changes in parameter values, so that the moments could not easily be generated by significantly different parameter values. Our estimation implies an average percentage external financing cost of 2.3%, which is well within empirical estimates.[17] Our estimated parameters support an important role for the fixed cost of

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[17] For example, Asquith and Mullins (1986) find that abnormal stock returns around secondary equity offerings are about 3% and Hennessy and Whited (2007) estimate that firms face an external financing cost of 8.3% on the first million dollars

raising external funds. The importance of a fixed financing cost is consistent with the evidence regarding the importance of the variation in the extensive margin of external finance over the business cycle presented in Figure 3.[18] Finally, we find an average investment adjustment cost paid of 1.4% per dollar of investment.

We estimate the volatility of the ξ shock to be around 2. Recall that, given this volatility, we choose the mean of the ξ shock ($\exp\left(\mu - \frac{\sigma^2}{2}\right)$) to generate an average credit shock of one. This is without loss of generality, but makes the cost parameter λ_2 more easily interpretable as the average marginal cost, and enables comparison to estimates in prior work. We estimate the persistence of the stochastic cost to be 0.22. The annual persistence of the Baa-Aaa default spread and innovations to the expected default adjusted excess bond spread from Gilchrist and Zakrajsek (2012a), not targeted in our estimation, are 0.4 and 0.35, respectively.

B. Results

Table 3 presents a comparison of aggregate external financing, savings, and investment moments in the data and in the model. We report firm level moments in the model and data in the Online Appendix. The top panel presents a comparison of the moments targeted in the estimation. The model generates a strong, positive correlation between liquidity accumulation and external finance, with an implied correlation of 0.70, similar to the empirical correlation of 0.57. Moreover, the correlation between liquidity accumulation and external finance is higher, as it is in the data, when one conditions on firms raising external finance. The model also does fairly well on matching the other cross correlations, however it overshoots on the correlation between external finance and investment. This is because the only ultimate use for funds is for investment. Still, we note that the ranking of the cross-correlations matches the empirical one. Finally, the estimated model produces realistic levels of volatility for investment, savings, and external finance, though the volatility of investment is slightly higher than in the data. The model undershoots on average liquidity accumulation, a moment which does not receive a high weight in the estimation since it is a mean, whereas the other moments are related to variances. In general, because we use the efficient weighting matrix in our estimation, the weights are based on the statistical precision with which the empirical moments are estimated. These weights

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raised.

[18] Bazdresch (2013) and Cummins and Nyman (2004) both emphasize importance of lumpy external finance, and provide complementary evidence to ours regarding the importance of fixed external financing costs. Finally, strong empirical evidence for the important role of fixed financing costs is also provided by Welch (2004) and Leary and Roberts (2005), which document a very high level of persistence in corporate capital structures, and emphasize rare active capital structure changes, although Baker and Wurgler (2002) attribute such rare changes to market timing.

assist in a good statistical fit of the model overall.

The bottom four panels of Table 3 present a comparison of the estimated model's implied moments relative to their empirical counterparts for thirteen additional aggregate moments. Considering that none of these additional moments were targeted in the estimation, the model does a good job of matching them. In particular, the model matches the empirical autocorrelation of aggregate investment of 0.40, with an implied estimate of 0.34. This autocorrelation is sensitive to the convex adjustment cost parameter, and the match between these moments supports the ability of our model to match the persistence of aggregate investment. In the model, as in the data, the correlation of liquidity accumulation and external finance increases when we condition only on firms raising external finance, highlighting that the correlation isn't driven by some firms saving while others are raising. Investment and liquidity accumulation also become more correlated when conditioning on raising firms. The model generates business cycle correlations which match the ranking from Compustat data: investment tracks the business cycle most closely, followed by external finance, and then liquidity accumulation. The magnitudes of the correlations are, not surprisingly, higher in the model as it features only two shocks. Importantly, the model also generates the observed, highly positive, correlations between ξ, excess external finance $\rho^{xs}_{i_l,e}$, and the percentage of firms raising external finance. As illustrated by the firm policy functions in Figure 4, only when the cost of external finance is low do firms find it optimal to issue costly external finance and save the proceeds. We also note that the correlation between TFP and these cross-sectional moments describing financing and savings by firms are lower than the correlations with ξ, as is true empirically. When computing TFP in the model, we estimate TFP with a log linear production function, treating financial and investment costs as deadweight costs, which matches the empirical estimation of TFP used in the data. In the model, the correlation between $\rho^{xs}_{i_l,e}$ and TFP is slightly positive due to the positive effect of better resource allocation when ξ is low.

As discussed above, we present results for several richer alternative specifications of the cost of external finance in Table 4, including those with a stochastic fixed cost, a linear financing cost, and a stochastic discount factor which is correlated with TFP and financing shocks.[19]

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[19] Additional results appear in the Online Appendix. In particular, we show that changing the firm's discounting to account for systematic risk reduces the required volatility and magnitude of ξ, without significantly altering other results. This supports our partial equilibrium approach.

5. Estimating The Cost of External Finance

In this section, we use the relationship in the model between financing and savings activity in the cross-section, and the aggregate cost of external finance, to estimate the average cost of external finance per dollar raised in the US time series 1980-2014. Specifically, we construct an aggregate cost of external finance index in the model, in which the cost at each date is predicted to be a weighted average of cross-sectional moments describing firms' external financing and savings on that date. We then use the coefficients from this index, along with the empirical cross-sectional moments at each date, to construct our estimates for the US time series. When the cost of external finance is low, firms are both more likely to raise external finance, and also to have large, positive, deviations from the average amount that they raise and the amount that they save. Figure 4 shows these changes in policy functions in the dynamic model as a function of the aggregate cost. Thus, we use the following two main cross-sectional moments to estimate the aggregate cost of external finance at each date: the fraction of firms raising external finance, and the cross-sectional correlation between liquidity accumulation and external finance. These two moments capture the extensive margin of the external financing decision, and the intensive margin of what firms do with the finance they raise. Accordingly, in our model, they are able to explain about ninety-four percent of the variation in the cost of external finance. As can be seen by comparing Figures 2 and 3, other moments such as the aggregate amount of finance raised, and liquidity accumulated, contain similar information to these moments but do not add to the variation explained once controlling for them.

In the model, we can express the average cost of a unit of external finance per dollar raised as $\frac{E[\phi_e(e,\xi)]}{E[e|e<0]}$. This is the average cost of external finance paid across firms at any given date, divided by the average amount raised. We compute this expression at each point in time in the model. The statistical properties of this series appear in Table 5. The average cost appears to have quite reasonable properties, with an average value of 2.3%, a standard deviation of 1.6%, and an autocorrelation of 0.63. Similarly, the average marginal cost is 0.9% with a standard deviation of 0.8%. We use the average cost of a unit of external finance per dollar raised series as the dependent variable in the following time series regression estimated on model data:

$$\frac{E[\phi_e(e,\xi)]}{E[e|e<0]}_t = \alpha + \beta_1 \% raise_t + \beta_2 \rho^{xs}_{i_l,e,t} + \varepsilon_t,$$

where we normalize the independent variables, $\%raise_t$ and $\rho^{xs}_{i_l,e,t}$, to have mean zero and unit standard deviation. We report the regression results in Table 5. We find that both of the coefficients, β_1, β_2 are negative, as expected. The coefficient on $\rho^{xs}_{i_l,e,t}$ is -1.3% in magnitude. This implies that a standard deviation increase in $\rho^{xs}_{i_l,e,t}$ is associated with a 1% decrease in the average cost paid per dollar raised. The coefficient on $\%raise_t$ is smaller at -0.3%. The constant term, α, is 2.3% and represents the average cost of external finance paid per dollar raised in the model. Thus, each of these two cross-sectional moments is economically important for explaining variation in external financing costs in the model data. Moreover, the R^2 in this regression is around 94%, and thus just these two variables alone do a very good job explaining variation in the cost paid by firms in the model economy. In Table 5, we also document that these variables also effectively capture variation in ξ_t, the stochastic cost of external finance "level" shock series, in the model.

We then apply the estimated regression coefficients, $\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2$, to the analogous empirical moments to form a time series of estimates of the cost of external finance paid per dollar raised for each year from 1980 to 2014. We plot this series in the top panel of Figure 1. Our estimated cost of external finance series has very intuitively appealing properties. The estimated cost is high in the deep recession of the early 1980's, the early 1990's credit crunch and Savings and Loan Crisis, around the 2001 dot com stock market crash and recession, and in the recent financial crisis and Great Recession. Moreover, each spike in this cost is associated with a recession. Economically, the cost per dollar raised varies from essentially zero in the mid to late 1990's, up to over 4% during the recent financial crisis. This is a full 75% larger than the overall average cost of 2.3%. This value seems economically large too, considering the impact of the implied average 170 basis point increase on the required return for a firm debating whether to take on an investment project for which it must raise external finance. Note that the variation in our estimated cost is strongly robust to the alternative cost specifications discussed above. The last row of Table 4 shows that our estimated cost series is highly correlated (0.96-0.99) across all alternative specifications that feature a stochastic external financing cost.

Table 6 gives correlations of our estimated cost with three proxies for the cost of external finance: the default spread, the index of lending standards, and the consumer sentiment survey. The default spread provides a market based measure of debt financing costs, while the index of lending standards picks up a measure of banks' willingness to lend, and the sentiment index is interesting if one thinks the cost is potentially driven by mispricing. We find that our estimated cost has a correlation of 0.66

with the default spread, 0.48 with the index of tightening lending standards, 0.69 with sentiment, and 0.71 with an equal weighted average of the three (the average is taken after standardizing each series to have zero mean and unit variance). Thus the estimated cost clearly picks up important common variation in these measures. Finally, it is worth noting that the estimated cost is typically more correlated with these variables than either $\%raise_t$ or $\rho^{xs}_{i_l,e,t}$ alone, meaning that, consistent with the model, combining the information from both moments appears to improve our measure's ability to capture the relevant variation in the cost of external finance. We also note that the existing measures are each aimed at capturing the either the cost of equity or debt finance, but not the overall cost of external, vs. internal, funds. If firms can minimize overall financing costs by optimally using debt and equity, then it is this minimized overall cost which matters for external funds.

Figure 3 illustrates the strong relationship between $\rho^{xs}_{i_l,e}$, as well as the percentage of firms raising external finance, with these traditional proxies for the cost of external finance by plotting the time series for $\rho^{xs}_{i_l,e}$ and the percent of firms raising external finance (top panel) along with the negative of the default spread, lending standards, and consumer sentiment (bottom panel). Although all the series are highly correlated, there is independent information in $\rho^{xs}_{i_l,e}$. For example, the high $\rho^{xs}_{i_l,e}$ indicates a low cost of external finance in the boom of 1986, however the default spread was not particularly low then. The tech bust of 2001 is also more apparent in the drop in $\rho^{xs}_{i_l,e}$ than it is in the relatively small increase in the default spread, potentially suggesting that firms faced an increase in the cost of equity external financing at that time which is not captured by the default spread.

In the Online Appendix, we also describe and implement a rolling SMM procedure to provide a structural estimate of the two-state ξ time series which best matches the time series for the moments included in our regression index. This procedure yields roughly similar results to the regression based measure shown in the main text. Both of our estimation procedures appear to pick up events that our priors suggest should be associated with costly external finance, such as the recession of the early 80's, the crash of the tech boom in 2001, and the recent financial crisis. We take this finding to be additional evidence which supports using moments implied by our model to identify times when the cost of external finance is high or low. Thus, we argue that our model and moment implied cost estimates provide a structurally motivated measure of the cost of external finance which is complementary to the existing list of empirical proxies. Moreover, our strategy of using cross-sectional moments along with a structural model to uncover hidden aggregate states may have other useful applications.

6. Statistical Rejection of the Constant Cost Model

In this section, we show that a nested, restricted version of the model, with constant costs, is rejected in favor of the full model with stochastic costs. In the Online Appendix, we also show that a unique additional prediction of the stochastic cost model which distinguishes it from the constant cost model is supported by empirical evidence. Specifically, we exploit the heterogeneous effect of a high cost of external finance on high productivity vs. low productivity firms. We show that the difference in investment between high productivity firms and low productivity firms will be smaller when the cost of external finance is high, a fact we show is strongly supported empirically. Finally, we highlight again the high correlation between our estimated series and traditional measures of the cost of external finance, despite the fact that we did not target any of these traditional proxies in our estimation. By contrast, we show in the Online Appendix that the correlation between an analogous cost series estimated using a nested model with constant external financing costs, and the default spread, SLOOS lending stanadard series, and consumer sentiment index are only 0.33, 0.11, and 0.58, respectively.

We show that stochastic costs are statistically justified by the improvement in model fit, relative to this nested alternative model in which the cost of external finance is constant and the only shock firms face is to their (aggregate and idiosyncratic) productivity. Specifically, we test the improvement that the full model offers over a restricted version in which the volatility of the stochastic cost shock is restricted to be zero (in which case the persistence parameter is no longer identified). The difference in the minimized objective function for the unrestricted version of the model will clearly be smaller, but the J-test evaluates whether the difference in model errors more than justifies the addition of the cost persistence and volatility parameters. The difference between the objective functions, $J_{restr} - J_{unrestr}$, is distributed χ^2 with degrees of freedom equal to 2 (the number of restrictions). We are able to reject the the null hypothesis that the improvement in fit by the full model, relative to the restricted model, does not justify the addition of two additional parameters, at well below the 1% level. Thus, the data strongly prefer the full model with stochastic costs. We present a full comparison of the parameter estimates and implied moments from the full and restricted models in the Online Appendix. Finally, we also note that the estimate of the volatility of the shock to the cost of external finance in the full model is highly significant.

We can also analyze the individual moments from the nested, restricted model in order to understand its failures economically. The last column of Table 4 shows that a model with constant costs has several important shortcomings. The first is that it has difficulty generating positive cash balances, with liquid assets only making up 1% of total assets. The second is that the only time the firm will want to raise external finance is when it also has a good shock to investment opportunities. Thus, the correlation between investment and external finance in this model is essentially 1, whereas it is 0.46 in the data. Indeed, the relatively low empirical correlation between external finance and investment provides some model-independent support that these two flows are partially driven by different shocks. Finally, and importantly, at estimated parameter values the constant cost model does not generate aggregate external financing and savings waves in line with the data. The estimation procedure has trouble matching the empirical moments with the restricted model, and is forced to compromise amongst all the moments it aims to match. As a result, at estimated parameters, the correlation between liquidity accumulation and external finance is only weakly positive, with an implied value of 0.28. The model also implies a volatility for liquidity accumulation near zero. In short, in the restricted model liquid assets play very little role as they are low on average and don't vary meaningfully. This means that investment is overly correlated with external finance.

7. Conclusion

We document the empirical regularity of aggregate external financing and savings waves, and show that cross-sectional moments describing firms' external financing and savings behavior are informative about the aggregate cost of external finance. We document that, empirically, the time series of the cross-sectional correlation between external finance and liquidity accumulation, or "excess external finance", and the time series of the percentage of firms raising external finance, are highly correlated with standard measures of the aggregate state of the cost of external finance, such as the default spread and the fraction of banks tightening lending standards.

We argue that both "excess external finance", or the observed realization of the correlation between external finance and liquidity accumulation in the cross-section, and our model implied estimate of the level of the cost of external finance in the US time series 1980-2014, are useful measures of the state of the aggregate level of the cost of external finance. Using firms' actual decisions about how much external finance to raise and how they use the proceeds from external finance provides a method of making inferences about the true cost of external finance which exploits firms' relative investment returns and financing costs. A quantitative measure which is based on data describing corporate financing, saving, and investment decisions seems useful for measurement, and for policy guidance.

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Figure 1: Estimated average cost of external finance paid per dollar of external finance raised using cross-sectional moments at each date.

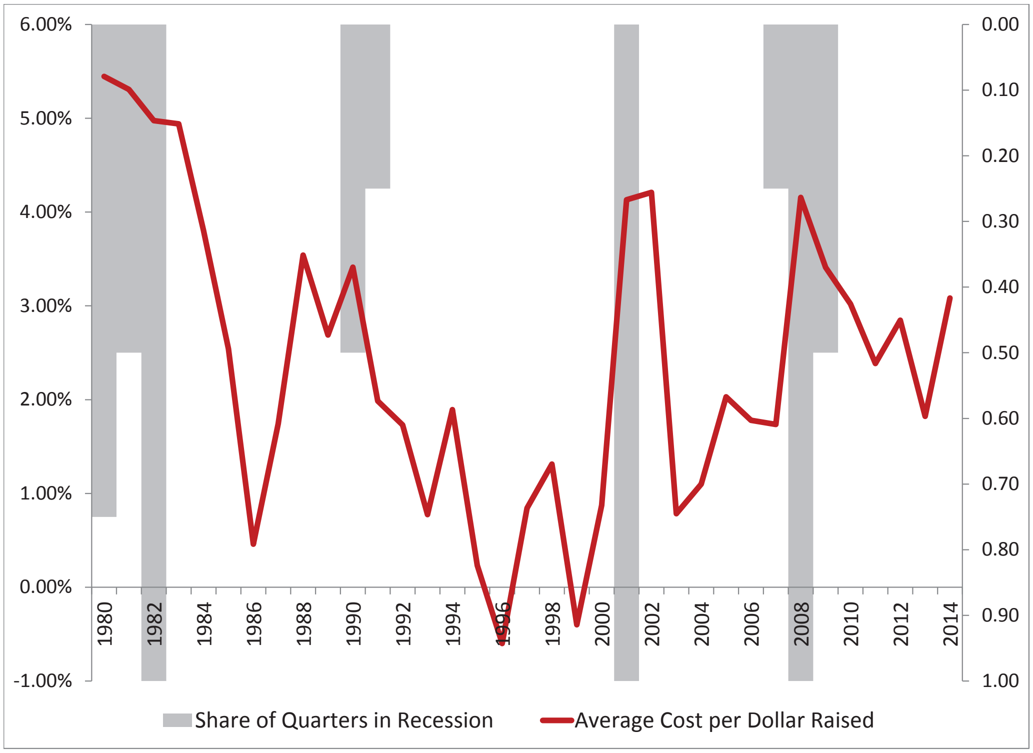

Figure 2: Aggregate accumulation of liquid assets against aggregate external finance. Plotted data are normalized by lagged assets, HP-filtered, and then scaled to have unit variance. Gray bars indicate fraction of quarters economy is in a recession in the given year (right axis). The correlation between liquidity accumulation and external finance is 0.6. A univariate regression of liquidity accumulation on external finance yields an R^2 of 0.36 and a regression coefficient estimate of 0.19.

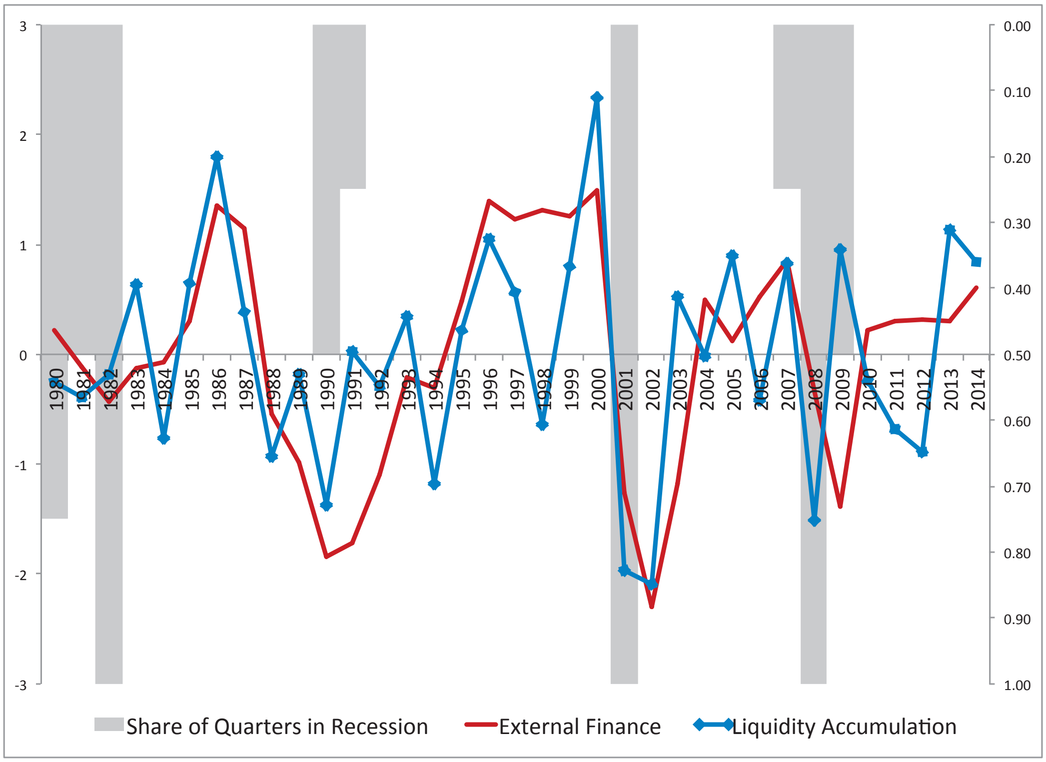

Figure 3: We plot the the cross-sectional correlation between liquidity accumulation and external finance and % of firms raising external finance in the top panel. These moments are highly correlated with the shock to the cost of external finance in our quantitative model. The bottom panel plots the negative of the default spread, the net % of banks tightening lending standards, and the sentiment index. Gray bars indicate fraction of quarters economy is in a recession in the given year (right axis). All series are standardized to have mean zero and unit variance.

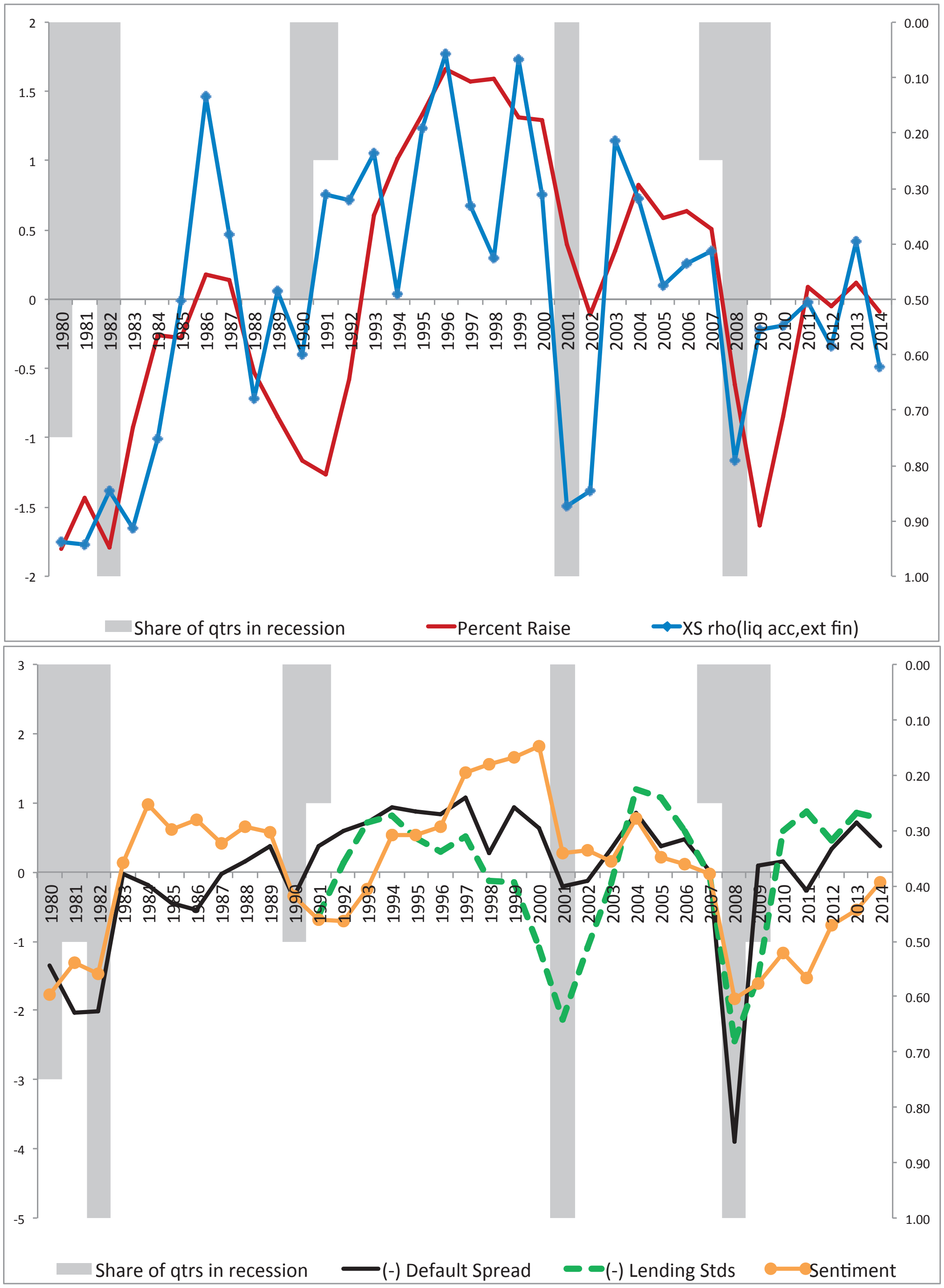

Figure 4: This figure plots firm policy functions across low vs. high cost of external finance states in the left and right panels, respectively. The top panel plots the percentage of firms raising external finance (or the probability of raising for a given firm) conditional on firm level productivity. The middle panel plots external finance (blue line) and investment in liquid assets (green line) as a function of productivity. The bottom panel plots investment in physical capital as a function of both productivity and firm size. The blue line plots investment for high productivity firms as a function of firm size while the green plots investment for low productivity firms. The red triangle line indicates the difference in investment across high and low productivity firms.

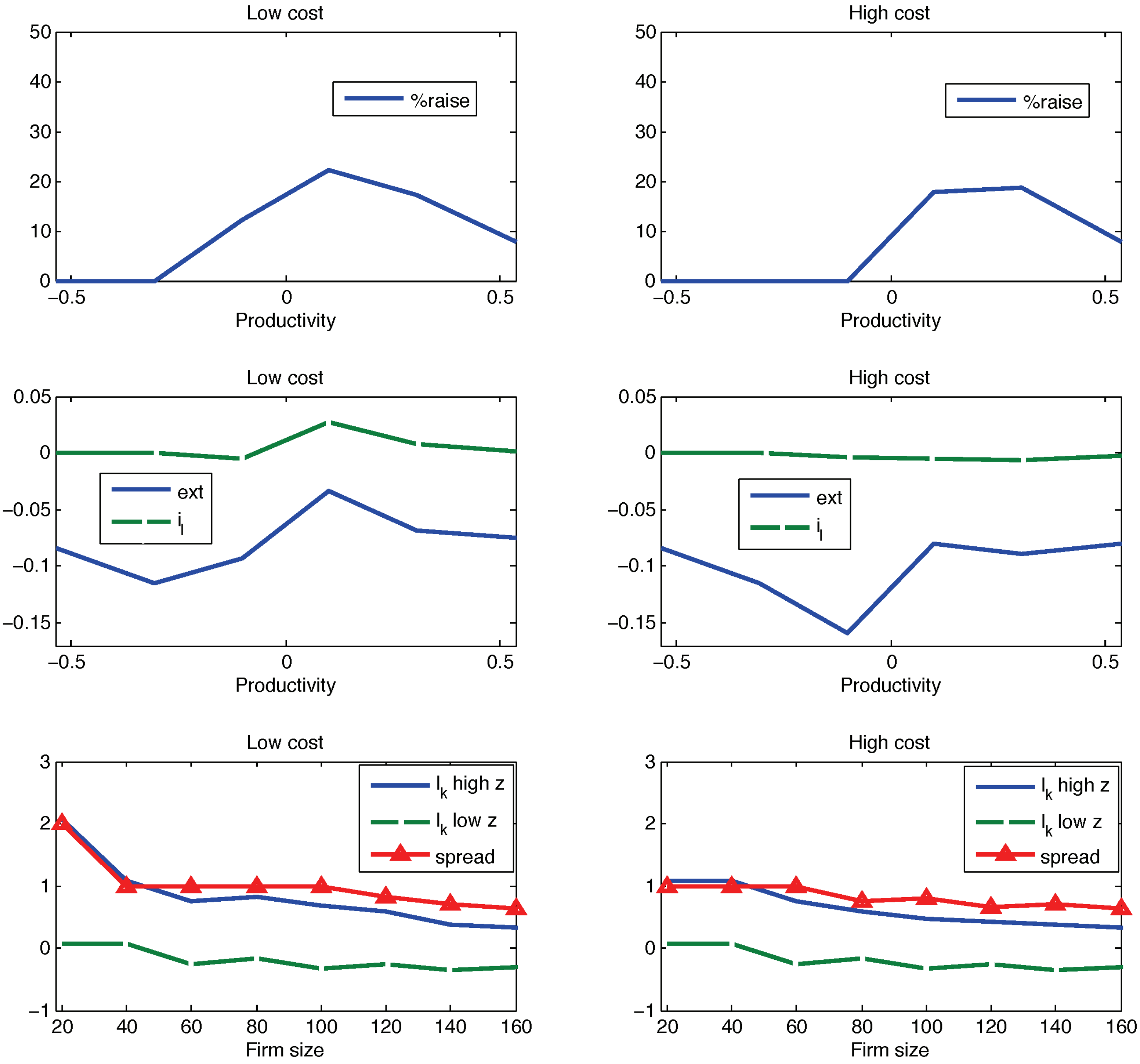

Table 1: This table describes aggregate external finance and savings waves. Specifically, we document the correlation between external finance and liquidity accumulation for different levels of aggregation and different firm categories. * indicates significance at 5% level. $[0,x]\%$ represents aggregation of firms at or below the x^{th} percentile based on total assets. Equal Weighted uses all firms but equal weights them so as not to only emphasize the largest firms. Weighted with Model Size Distribution uses simulated data from our model (described later) to value weight firms based on their total assets. No Dividends and No Rating represent firms which do not pay dividends and which do not have a credit rating, respectively.

Aggregate External Financing and Savings

$\rho\left(\frac{\Sigma(-e)_t}{\Sigma TA_{t-1}}, \frac{\Sigma i_{l,t}}{\Sigma TA_{t-1}}\right)$	
[0,90]%	**0.57***
[0,50]%	0.82*
[0,100]%	0.08
Equal Weighted	0.52*
Weighted with Model Size Distribution	0.50*
Conditional on Raising funds: e<0	0.80*
No Dividends	0.58*
No Rating	0.56*

Table 2: Calibrated and estimated parameters. The label e.c.f. denotes external cost of finance. The lower panel gives the implied average percentage costs of external financing and investment firms pay with the given parameters.

Calibrated and Separately Estimated Parameters

Symbol	Description	Value	
τ	tax rate	0.10	
δ	depreciation	0.08	
θ	curvature	0.65	
ρ	persistence	0.66	
σ	total vol of prod	0.121	
σ_i	idiosyncratic vol	0.11	
σ_{agg}	aggregate vol	0.03	
r	risk-free rate	0.04	
	Estimated Parameters		
Symbol	Description	Estimate	Standard Error
a	invest adj cost	0.26	0.02
λ_1	e.c.f. fixed	0.16	0.01
λ_2	e.c.f. quadratic	0.0004	0.00006
σ_η	vol cost of funds	1.97	0.22
γ	persistence	0.22	0.06
	Implied Average Costs Paid		
Symbol	Description	Model	
$E\left[\frac{\phi_e(e)}{e}\right]$	implied average external financing cost	0.023	
$E\left[\frac{\phi_i(i_k,k)}{i_k}\right]$	implied average investment cost	0.014	

Table 3: This table displays aggregate moments from the model using a simulated panel of firms. We compare these moments with those from annual Compustat data, 1980-2014. For correlations, we normalize each series by lagged assets and apply the hp-filter. All other series are normalized by current assets. $\rho^{xs}_{i_l,e,t}$ is $\rho_t\left(\frac{-e_i}{TA_i}, \frac{i_{l,i}}{TA_i}\right)$. TFP are TFP level shocks. We use the average of the Baa-Aaa default spread, the index of lending standards, and the sentiment index as an empirical proxy for ξ after first normalizing each of these series to have zero mean and unit variance. * indicates significance at 5% level. We use notation from the model: $-e$ represents external finance (negative of payouts), i_l liquidity accumulation, i_k investment in physical capital, and l liquid balances. We normalize each series by total assets except investment which is normalized by physical capital k.

Estimation Moments

Moment	Data	Model
$E[l]$	0.11	0.03
$\sigma(i_l)$	0.01	0.01
$\sigma(i_k)$	0.02	0.02
$\sigma(-e)$	0.03	0.03
$\rho(i_l, -e)$	0.57*	0.70
$\rho(i_l, i_k)$	0.16	0.18
$\rho(-e, i_k)$	0.42*	0.84

Additional Moments

Moment	Data	Model
$E[i_k]$	0.07	0.08
$\rho(i_{k,t}, i_{k,t-1})$	0.40*	0.34
$E[-e]$	-0.01	-0.10
$prob(-e > 0)$	0.04	0.02

Moments Conditional on e<0

Moment	Data	Model
$\rho\left(i_l, -e\right)$	0.80*	0.96
$\rho\left(i_l, i_k\right)$	0.32*	0.39

Business Cycle Correlations

Moment	Data	Model
$\rho(i_l, gdp)$	0.06	0.06
$\rho(-e, gdp)$	0.27*	0.71
$\rho(i_k, gdp)$	0.47*	0.94

TFP vs. ξ Moments

	Data	Model
$\rho(\rho^{XS}_{i_l,e}, \xi)$	0.64*	0.92
$\rho(\%raise, \xi)$	0.71*	0.47
$\rho(\rho^{XS}_{i_l,e}, TFP)$	0.45*	0.04
$\rho(\%raise, TFP)$	0.24	0.32

$\rho\left(\frac{\Sigma(-e)_t}{\Sigma TA_{t-1}}, \frac{\Sigma i_{l,t}}{\Sigma TA_{t-1}}\right)$ by Size Bin

Size Bin	Data	Model
$[0, 50]$	0.86*	0.85
$[0, 75]$	0.78*	0.71
$[0, 90]$	0.57*	0.70
$[90, 100]$	-0.09	0.38

Table 4: Robustness of alternative model choices. We provide robustness to different model choices in each column. Stochastic fixed costs applies the cost shock ξ to the fixed cost as well as the quadratic cost. The linear cost model adds a linear cost $\lambda = 0.09$ where the value and parameterization is from Hennessy and Whited (2007). The model with lower tax rate, $\tau = 0.012$ is chosen to match average liquid balances, l. The last column, SDF, uses an exogenous SDF following the specification in Belo, Lin, and Yang (2014). We compare our main moments across these specifications. Additional details appear in the appendix

Estimation Moments

Moment	Data	Model	Stochastic Fixed	Linear Cost	Lower τ	SDF	Constant Cost
$E[l]$	0.11	0.03	0.06	0.05	0.12	0.03	0.01
$\sigma(i_l)$	0.01	0.01	0.02	0.004	0.01	0.02	0.00
$\sigma(i_k)$	0.02	0.02	0.03	0.02	0.03	0.03	0.03
$\sigma(-e)$	0.03	0.03	0.04	0.02	0.03	0.04	0.03
$\rho(i_l, -e)$	0.57*	0.70	0.81	0.47	0.50	0.61	0.28
$\rho(i_l, i_k)$	0.16	0.18	0.32	0.29	0.11	0.22	0.34
$\rho(-e, i_k)$	0.42*	0.84	0.81	0.96	0.90	0.90	0.99

Correlation of Regression Implied Cost of External Finance Across Models

	Model	Stochastic Fixed	Linear Cost	Lower τ	SDF	Constant Cost
	1	0.99	0.96	0.98	0.97	0.37

Table 5: Estimates of the cost of external finance in the model using regressions of cost measures on $\rho^{xs}_{il,e}$ and $\%raise$. We measure the cost of external finance as the average cost paid per dollar of external finance raised, $E_N\left[\frac{\phi_e}{e}\right]$, and as the level of the stochastic cost series ξ. We also report the average marginal cost per dollar.

Infering the Cost of External Finance via Regression Index

y	Constant	$\rho^{xs}_{il,e}$	$\%raise$	R^2
$E_N\left[\frac{\phi_e}{e}\right]$	0.023^{***}	-0.013^{***}	-0.004^{***}	93.6%
ξ	0.00	-0.98^{***}	-0.24^{***}	94.1%

Properties of cost measures

Measure	Mean	Std	Auto Corr
Avg Cost	2.3%	1.6%	0.63
Marg Cost	0.9%	0.8%	0.55
$\rho^{xs}_{il,e}$	0.15	0.07	0.50
$\%raise$	0.47	0.08	0.85

Table 6: Correlations of our estimated cost of external finance estimated using the cross-section of firm policies, ξ, with existing empirical proxies for the cost of external finance. The final row in each panel gives the correlation with an average of the three proxies after normalizing each series to have zero mean and unit variance.

Correlation of estimated cost with:

Moody's BaaAaa Default spread	0.66^{*}
SLOOS Index of lending standards, C&I	0.48^{*}
Sentiment index	0.69^{*}
Average of three	0.71^{*}

Correlation of $\rho^{xs}_{il,e}$ with:

Default spread	0.63^{*}
SLOOS Index of lending standards, C&I	0.46^{*}
Sentiment index	0.48^{*}
Average of three	0.64^{*}

Correlation of % raise with:

Default spread	0.58^{*}
SLOOS Index of lending standards, C&I	0.38^{*}
Sentiment index	0.74^{*}
Average of three	0.71^{*}