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ABSTRACT

We propose a new framework for understanding the effectiveness of central bank announcements when firms have heterogeneous inflation expectations. Expectations are updated through social dynamics and, with heterogeneity, not all firms choose to operate, putting downward pressure on realized inflation. Our model rationalizes why countries stuck at the zero lower bound have had a hard time increasing inflation without being aggressive. The same model also predicts that announcing an abrupt target to disinflate will cause inflation to undershoot the target whereas announcing gradual targets will not. We present new empirical evidence that corroborates this prediction.

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1 Introduction

Communication has become an important part of the monetary policy toolkit. Central banks operating in a zero lower bound environment use communication about the future to influence expectations and stimulate the economy (e.g., Bernanke (2004) and Yellen (2013)). Flexible inflation targeters have also relied on transparent communication to anchor inflation expectations over various horizons (e.g., Carney (2012)) and transparency is now a more general trend in central banking (e.g., Woodford (2005) and Blinder et al. (2008)). However, heterogeneity in inflation expectations exists despite transparency (e.g. Mankiw et al. (2004)) and prominent policy-makers have called for new models of the expectation formation process to better understand the transmission from communication to economic outcomes (e.g. King (2005), Boivin (2011), and Kroszner (2012)). Our paper takes a step in this direction and proposes a new framework for understanding the effectiveness of central bank announcements when heterogeneity is present.

We construct a model of inflation determination that is tractable and preserves its original non-linearities. A continuum of monopolistically competitive firms make pricing and production decisions before the aggregate price level is known, introducing a need for inflation forecasts. We consider two forecasting rules. One rule (Random Walking) is based on last period's inflation. The other rule (Fed Following) is based on central bank announcements about the current period. We also allow for heterogeneity within each rule. In other words, heterogeneity is present both within and across rules.

We discipline our model so that each rule is individually mean rational – that is, each rule yields unbiased forecasts of inflation when adopted by all firms. If all firms are Fed Followers, then firm decisions should be such that the central bank's announcement is in fact realized. If all firms are Random Walkers, then firm decisions should be such that last period's inflation persists. Such discipline helps rationalize why either rule could be viable in the long-run. However, as we show, the mean rational property is not trivial when there is heterogeneity within rules: the dispersion of inflation expectations introduces pricing non-linearities which put upward pressure on realized inflation relative to the mean expectation. We establish that a fixed cost of production – which generates exit among firms who expect to be unprofitable net of this cost – offsets the pricing non-linearities and supports two individually mean rational rules without assuming away in-rule heterogeneity. We then establish that this exit channel causes realized inflation to undershoot a target announced by the central bank if there is a sufficiently interior mixture of the Fed Following and Random Walking rules when the mean expectation reaches the target. In contrast, if the vast majority of firms use the Fed Following rule, the mean rational property implies convergence to the target.

The fraction of Fed Followers is thus a crucial variable in our model. It also summarizes the credibility of the central bank. Credibility is earned, not endowed, so we must endogenize how this happens. We use social dynamics to model the evolution of credibility. In particular, once inflation has been realized, firms can meet and potentially switch forecasting rules based on relative performance. A small and/or temporary divergence of realized inflation from the central bank's announcements may not have enough momentum to significantly affect credibility. However, prolonged divergence may convince many firms to abandon the central bank's cues in favor of more successful forecasting rules, limiting the extent to which future announcements will be realized.

Combining our model of inflation determination with social dynamics, we investigate how announcements can be tailored to limit divergence and build credibility.[1] We show that a period of gradual announcements helps build credibility when social dynamics are at play. Neither this result nor the result that the central bank needs to be highly credible to avoid undershooting its announcements depends on whether the central bank wants to increase or decrease inflation.

In contrast, the direction that the central bank would like to move in does affect how useful a period of aggressive announcements will be. The downward pressure from the exit channel can make increasing inflation difficult. This result is consistent with the observation

[1]Notice that we are defining the central bank's problem in terms of announcements, not conventional policy tools such as a short-term nominal interest rate. This has precedent in Reis (2011).

that countries stuck at the zero lower bound since the recent financial crisis have had a hard time increasing inflation. Our model suggests that it may be useful to pursue an aggressive strategy, wherein the central bank sets short-run targets above the long-run goal, before moving to a period of gradual announcements. With high short-run targets, many Fed Followers will set high prices because they expect high prices, leading to higher inflation.

On the other hand, aggressive strategies do not work as well when trying to decrease inflation. If the central bank's target is much lower than current inflation, then Fed Followers will tend to exit rather than set low prices and, once the majority of them have exited, there are no gains to an even lower target. Our model thus supports gradualism during disinflations. We present new empirical evidence that corroborates our predictions. In particular, we establish that countries which disinflated with gradual targets rather than abrupt targets were more successful in the sense of not undershooting their long-run goals.

To the best of our knowledge, our paper is the first to (i) empirically connect undershooting with abrupt disinflation targets and (ii) show that a new model with social dynamics rationalizes the empirics while still prescribing aggressive announcements at the zero lower bound. In-rule heterogeneity, social dynamics, and the economic model all play important roles in getting these results. Without in-rule heterogeneity, all Fed Followers and all Random Walkers would have the same expectation once inflation reaches the central bank's goal. There would be no exit channel and hence no undershooting. Social dynamics provides a natural way to simulate rule selection when performance is not homogeneous within rules. Moreover, the benefit of developing our own economic model rather than using a workhorse (e.g., New Keynesian) model is that we can introduce new non-linearities in a tractable way and see how they interact with social dynamics. Our exit channel and its implications for undershooting are examples of this interaction.

The rest of the paper proceeds as follows: Subsection 1.1 discusses related literature; Section 2 explains the evolution of credibility through social dynamics in our paper; Section 3 builds a model of inflation determination and introduces the exit channel; Section 4 presents the simulation results for increasing inflation and discusses how our framework can be extended to think about communications surrounding major initiatives such as Quantitative Easing; Section 5 presents the simulation results for decreasing inflation along with new empirical evidence that links undershooting with abruptness; Section 6 discusses the key roles played by exit and social dynamics in our results; Section 7 establishes the robustness of our results to alternative parameterizations; and Section 8 concludes. All proofs are collected in the Appendix.

1.1 Related Literature

Our paper is related to several literatures. First is the literature on central bank communication. Many of the insights in this literature depend on information asymmetries between the central bank and the public (e.g. Cukierman and Meltzer (1986), Stein (1989), Eggertsson and Pugsley (2006), Melosi (2012)) and/or representative learning by the public to overcome such asymmetries (e.g., Orphanides and Williams (2005), Berardi and Duffy (2007), Eusepi and Preston (2010), Branch and Evans (2011)). In contrast, our paper studies what happens when central bank announcements are fully transparent but their credibility must be established through social networks. In this regard, we also differ from Arifovic et al. (2010) who allow the central bank to choose both the inflation announcement and the realized value of inflation in a cheap talk economy with social learning.

Our paper is also more broadly related to work on heterogeneous agents and behavioral expectations. This is in contrast to workhorse models of monetary policy where expectations are assumed to be homogeneous and rational.[2] A common building block in the behavioral literature is the use of rule-based agents.[3] Applied to expectations formation, agents select from a discrete number of forecasting rules and the popularity of each rule is determined by a heuristic. There are two main approaches to modeling the selection heuristic in the literature. The first approach follows Brock and Hommes (1997) in that the popularity of a

[2]See, for example, Clarida et al. (1999), Woodford (2003), Smets and Wouters (2003), and Christiano et al. (2005).

[3]Ellison and Fudenberg (1993) show that even naive rules-of-thumb can achieve fairly efficient outcomes so rule-based agents have become a convenient way to bridge tractability and realism.

given rule is performance-based and there is no heterogeneity within each rule. Recent papers that use this approach include Anufriev et al. (2013b), Cornea et al. (2013), Hommes (2013), and Hommes et al. (2015). See also Brazier et al. (2008) and DeGrauwe (2010) for agents that select specifically between a constant inflation target and past inflation. The second approach is social dynamics. Agents can be heterogeneous within rules and the popularity of a given rule is determined by genetic algorithms (e.g., mutation and tournament selection) at the individual level. Tournaments can be either performance-based or epidemiological and the structure of meetings can be varied to analyze different modes of interaction. Versions of the approach appear in Carroll (2003a), Acemoglu et al. (2010), Arifovic et al. (2010), and Burnside et al. (2013) among others.[4] For an application of social dynamics to the New Keynesian context, see Arifovic et al. (2013).

Our paper also connects to work on heterogeneous expectations as a source of macroeconomic persistence (e.g., Milani (2007), Fuhrer (2015a), Fuhrer (2015b)). In our framework, inflation is persistent in the sense that it does not immediately converge to newly-announced central bank targets. This is because credibility has to be established via social dynamics. The more stubborn are beliefs, the longer it will take for credibility to accumulate and the slower convergence will be. Our framework allows us to explore how the central bank can exploit social dynamics and build credibility by varying announcements over time.[5]

Finally, some of our results are relevant to the literature on positive expectations feedback. In Haltiwanger and Waldman (1985) and Heemeijer et al. (2009), strategic complementarities increase the impact of irrational expectations on realized inflation and impede convergence. See also Anufriev et al. (2013a) and Pfajfar and Zakelj (forthcoming). Our model is set up so that each forecasting rule is individually mean rational. The exit channel plays an important role in offsetting other non-linearities and getting the mean rational property to co-exist with in-rule heterogeneity. As a result, neither rule is destabilizing or intrinsically

[4]For more general discussions of agent-based models, see LeBaron (2001), Judd and Tesfatsion (2006), Colander et al. (2008), Ashraf and Howitt (2008), and Page (2012).

[5]This focus also clearly differs from the large literature on the extent to which macroeconomic persistence is driven by costly price adjustment at the microeconomic level (see Caballero and Engel (2007) for a survey).

more irrational than the other.[6]

2 Expectation Formation via Social Dynamics

Consider a continuum of agents $i \in [0,1]$. At the beginning of date t, agent i expects an inflation rate $\widehat{\pi}_t^i$. This expectation is drawn from a forecasting rule and social dynamics is the process through which agents select their rules. To illustrate how social dynamics work, we will consider two individually mean rational rules (i.e., the mean expectation is realized whenever all agents use the same forecasting rule, regardless of which rule it is). In the short-run, selection between competing alternatives is the core of social dynamics so having at least two rules is important. In the long-run, selection algorithms typically converge to one alternative so focusing on mean rationality ensures convergence to an unbiased rule.

2.1 Forecasting Rules

Central bank announcements and random walk forecasts are natural candidates for our two forecasting rules: Atkeson and Ohanian (2001) find that random walk forecasts of inflation perform very well against more sophisticated statistical models, Faust and Wright (2012) find that the Fed's Greenbook forecasts are difficult to beat, and Gurkaynak et al. (2005) and Campbell et al. (2012) find that markets do indeed view FOMC statements as a source of new and reliable information about future economic conditions. While the viability of each rule will be determined endogenously, emergence of these rules in the real world motivates their inclusion in the option set.

Our two rules also find support in the Survey of Professional Forecasters (SPF). A special question on the 2012Q2 survey asked respondents whether their point forecasts were consistent with the Fed's inflation target. Forecasters who self-identified as consistent were distributed around the Fed's target. The remaining forecasters were instead distributed

[6] For another context where a new channel introduces non-monotonicity into an environment that could otherwise be prone to the problems of strategic complementarities, see Hachem (2014).

around prevailing inflation, consistent with a random walk model. The distribution of the remaining forecasters was also wider than the distribution of the Fed-consistent group, suggesting more heterogeneity within random walk forecasts.[7]

To formalize our two forecasting rules, let $\overline{\pi}_t$ denote the central bank's date t announcement and let π^*_{t-1} denote realized inflation at the end of date $t-1$. The rules are then:

Definition 1 *Agent i is a Fed Follower (FF) at date t if* $\widehat{\pi}^i_t \sim N\left(\overline{\pi}_t, \sigma^2_F\right)$

Definition 2 *Agent i is a Random Walker (RW) at date t if* $\widehat{\pi}^i_t \sim N\left(\pi^*_{t-1}, \sigma^2_R\right)$

where the standard deviations satisfy $\sigma_F < \sigma_R$ and will later be parameterized to make each rule individually mean rational. In our definitions, $N\left(\overline{\pi}_t, \sigma^2_F\right)$ and $N\left(\pi^*_{t-1}, \sigma^2_R\right)$ are not priors. Instead, each agent i has a point forecast $\widehat{\pi}^i_t$. The universe of FF forecasts then looks normal with mean $\overline{\pi}_t$ and variance σ^2_F while the universe of RW forecasts looks normal with mean π^*_{t-1} and variance σ^2_R. Notice that $\sigma_F < \sigma_R$ incorporates the SPF feature of lower heterogeneity among Fed-consistent forecasts. The rationale is two-fold. First, there is *some* pass-through from professional forecasts to individual expectations (e.g., Carroll (2003b)).[8] Second, our random walking rule is a stand-in for statistical models more generally. Agents who use such models can disagree about which variables to include, leading to more varied forecasts. In contrast, Fed Followers are the set of agents who agree that the most important variable is the central bank's announcement.

Let $\xi_t \in [0,1]$ denote the fraction of Fed Followers. If $\xi_t = 1$, then everyone adopts the FF rule in Definition 1 and mean rationality implies $\pi^*_t = \overline{\pi}_t$. In other words, the central bank can achieve inflation goals using only communication. We thus interpret ξ_t as a measure of central bank credibility. To endogenize the evolution of ξ_t, we posit that agents whose forecasts are consistently outperformed by their peers will want to change how they forecast. Social dynamics then provides a useful approach to modeling how an agent discovers he is

[7] Raw data is available at www.phil.frb.org/research-and-data/real-time-center/survey-of-professional-forecasters/2012/survq212.cfm

[8] We emphasize the partial nature of this pass-through because firms and households do more than just outsource their inflation expectations (e.g., Bryan et al. (2014) and Coibion and Gorodnichenko (2015)). The existence of professional forecasts thus does not negate the need for a model of expectation formation.

being outperformed and how much of this outperformance he attributes to one-time shocks rather than fundamentals. Initializing $\xi_1 = 0$, we let ξ_{t+1} evolve via tournament selection and mutation, as described next.

2.2 Social Dynamics and the Evolution of Forecasts

Tournament selection simulates information transmission in a complex world. Our tournaments favor agents who are endogenously more successful. This is in line with Arifovic et al. (2013). Our tournaments will also permit success to be judged over multiple observations.

To be more precise, agents meet in pairs and compare forecast errors after the realization of π_t^*. Consider a meeting between agents i and j. Agent i counts one strike against his forecasting rule if j used a different rule and $\left|\widehat{\pi}_t^j - \pi_t^*\right| < \left|\widehat{\pi}_t^i - \pi_t^*\right|$. When ξ_t is sufficiently high, π_t^* will be close to $\overline{\pi}_t$ so $|\overline{\pi}_t - \pi_{t-1}^*| \gg 0$ would allow Fed Followers to outperform Random Walkers in many meetings. Strikes would thus tend to be counted against the RW rule, suggesting $\xi_{t+1} \geq \xi_t$. In contrast, when ξ_t is sufficiently low, π_t^* will be close to π_{t-1}^* so $|\overline{\pi}_t - \pi_{t-1}^*| \gg 0$ would instead allow Random Walkers to outperform Fed Followers in many meetings. Strikes would thus tend to be counted against the FF rule, suggesting $\xi_{t+1} \leq \xi_t$. This is the sense in which success is endogenous.

How quickly do strikes lead to $\xi_{t+1} \neq \xi_t$? Experimental evidence suggests that agents are very reluctant to contradict their own information, even when Bayesian updating suggests they should (e.g., Weizsäcker (2010) and Andreoni and Mylovanov (2012)). We thus allow agents to accumulate several strikes before deciding to switch forecasting rules. This is the sense in which success is judged over multiple observations.

We use S to denote the number of strikes needed for a switch: after S strikes, agent i switches forecasting rules and begins counting strikes against his new rule. Strikes do not have to be consecutive to trigger a switch.

The accumulation of strikes occurs across meetings and periods so we must also specify how pairwise meetings come about. In our baseline specification, pairs are drawn randomly with replacement from the entire population. Drawing with replacement means that each agent can have zero to many meetings in a given period. Drawing from the full population ensures that even agents who do not participate in economic activity are represented in tournaments. This is appealing since participation decisions are driven by expectations.

Going forward, we will also refer to S as stubbornness, with higher S implying more stubborn beliefs. To gauge the importance of stubbornness and better understand what it might represent, the robustness analysis in Section 7 will consider different values of S along with different specifications for how firms meet and accumulate strikes. As we will see, higher values of S can be interpreted as a stand-in for more localized interactions.[9]

Finally, to capture the fact that some changes may not be performance-driven, we incorporate mutations: at the beginning of date $t+1$, a very small fraction $\theta \in (0,1)$ of agents randomly switches rules regardless of strikes.

The timing of our social forces can now be summarized as follows: (i) mutation turns the fraction of FFs into $\widetilde{\xi}_t = (1-\theta)\,\xi_t + \theta\,(1-\xi_t)$ if $t \geq 2$; (ii) each agent i draws expectation $\widehat{\pi}_t^i$ from his forecasting rule; (iii) the set of expectations $\left\{\widehat{\pi}_t^i \mid i \in [0,1]\right\}$ determines π_t^* as per the model developed next in Section 3; (iv) tournament selection transforms ξ_t into ξ_{t+1} if $t=1$ and $\widetilde{\xi}_t$ into ξ_{t+1} if $t \geq 2$.

3 An Expectations-Based Model of Inflation

We now present a formal model to map inflation expectations into realized inflation. Interpret the continuum of agents in Section 2 as a continuum of firms, each producing a different perishable good $i \in [0,1]$. The demand for good i at date t is of the form $D_{it} = \left(\gamma_t \frac{P_t}{p_{it}}\right)^{\frac{1}{1-\rho}}$, where p_{it} is the price charged by firm i, P_t is an aggregate price level, $\gamma_t \in [1-\varepsilon, 1+\varepsilon]$ is an exogenous and independently distributed aggregate taste shock, and $\rho \in (0,1)$ is a constant.

The demand for good i is decreasing in the relative price $\frac{p_{it}}{P_t}$ rather than just the absolute

[9] Regardless of whether our meetings are random or local, the probability that any one agent connects with any other agent (possibly through multiple links) is positive. The assumptions in Acemoglu et al. (2010) for a strongly connected graph are therefore satisfied.

price p_{it}. This creates a role for inflation expectations. In particular, firms have to make pricing and production decisions before P_t and γ_t are realized – that is, before they know the actual demand for their products. At the beginning of date t, each firm i forecasts an aggregate price level of $\widehat{P}_t^i \equiv \exp(\widehat{\pi}_t^i)P_{t-1}^*$, where P_{t-1}^* is last period's realized price level and $\widehat{\pi}_t^i$ is the firm's inflation expectation for the current period. Firm i will ultimately draw $\widehat{\pi}_t^i$ from one of the two forecasting rules described in Section 2 but, for now, we can imagine a generic set of inflation expectations. To simplify the exposition and remain focused on inflation expectations, assume that all firms also forecast a taste shock of one. In other words, the forecasting rule for γ_t is simply the mean of the shock.[10]

The supply of good i is given by the production function $F(\ell_{it}) = \ell_{it}^\alpha$, where $\alpha \in (0,1)$ is a constant and ℓ_{it} is the labor input used by firm i. The aggregate stock of labor is normalized to one and inelastically available at unit wage w_t. In order to produce at date t, firm i must also pay a fixed cost. The fixed cost has a constant real value $U > 0$ which is paid in nominal terms $(P_t^* U)$ at the end of the period. A firm is said to operate at date t if and only if it expects profit net of the nominal fixed cost to be positive. When expectations are heterogeneous, the fixed cost gives rise to an exit channel which, as we will see later, plays an important role in our paper.

3.1 Equilibrium

Individual Decisions Conditional on its forecast $\widehat{P}_t^i$ and the prevailing wage w_t, firm i chooses p_{it} and ℓ_{it} to solve a static profit maximization problem. Charging p_{it} for good i yields an expected demand of $\left(\frac{\widehat{P}_t^i}{p_{it}}\right)^{\frac{1}{1-\rho}}$ which in turn necessitates $\left(\frac{\widehat{P}_t^i}{p_{it}}\right)^{\frac{1}{\alpha(1-\rho)}}$ units of labor. Firm i's problem is thus:

$$\max\left\{\max_{p_{it}}\left[p_{it}\left(\frac{\widehat{P}_t^i}{p_{it}}\right)^{\frac{1}{1-\rho}} - w_t\left(\frac{\widehat{P}_t^i}{p_{it}}\right)^{\frac{1}{\alpha(1-\rho)}} - \widehat{P}_t^i U\right], 0\right\}$$

[10] The role of γ_t is purely technical (as we will explain in more detail at the end of Subsection 3.1) so the forecasting rule for γ_t is just a formality.

From the inner maximization problem, the pricing decision of an operating firm is:

$$p\left(w_t; \widehat{P}_t^i\right) = \left(\frac{w_t}{\alpha\rho}\right)^{\frac{\alpha(1-\rho)}{1-\alpha\rho}} \left(\widehat{P}_t^i\right)^{\frac{1-\alpha}{1-\alpha\rho}} \tag{1}$$

From the outer maximization problem, the set of operating firms is then:

$$O_t\left(w_t\right) = \left\{ i \mid \widehat{P}_t^i \geq \frac{1}{\alpha\rho}\left(\frac{U}{1-\alpha\rho}\right)^{\frac{1-\alpha\rho}{\alpha\rho}} w_t \right\} \tag{2}$$

If $i \notin O_t\left(w_t\right)$, then the firm's labor demand is $\ell\left(w_t; \widehat{P}_t^i\right) = 0$. Otherwise, the first order condition from the inner problem yields:

$$\ell\left(w_t; \widehat{P}_t^i\right) = \left(\alpha\rho\frac{\widehat{P}_t^i}{w_t}\right)^{\frac{1}{1-\alpha\rho}} \tag{3}$$

Notice that operating firms with higher price expectations charge higher prices. They also hire more labor, resulting in more output. For any given wage, firms with higher expectations are also more likely to operate. The higher the wage though, the smaller the set of operating firms, the lower the output of each operating firm, and the higher the prices charged.

Wage Determination Given the individual decisions above, the wage is set to clear the labor market. More precisely, an auctioneer chooses w_t^* to solve $\int_{O_t(w_t^*)} \ell\left(w_t^*; \widehat{P}_t^i\right) di = 1$, with $O_t\left(w_t\right)$ and $\ell\left(w_t; \widehat{P}_t^i\right)$ as per equations (2) and (3) respectively. This yields:

$$w_t^* = \alpha\rho\left[\int_{O_t(w_t^*)} \left(\widehat{P}_t^i\right)^{\frac{1}{1-\alpha\rho}} di\right]^{1-\alpha\rho} \tag{4}$$

Following the determination of w_t^*, each firm $i \in O_t\left(w_t^*\right)$ posts price $p_{it}^* \equiv p\left(w_t^*; \widehat{P}_t^i\right)$ and hires labor to produce its expected demand $q_{it}^* \equiv \left(\frac{\widehat{P}_t^i}{p_{it}^*}\right)^{\frac{1}{1-\rho}}$. Price expectations are not updated based on w_t^* so, in this sense, the model deviates from rational expectations (RE). While some updating can certainly be accommodated, there must be residual heterogeneity for social dynamics to operate at the end of the period.

Realized Inflation After p_{it}^* and q_{it}^* have been set, the taste shock γ_t is realized and the aggregate price level is computed as a consumption-weighted average of individual prices. At price level P_t, the realized demand for good i is $\left(\frac{\gamma_t P_t}{p_{it}^*}\right)^{\frac{1}{1-\rho}}$ which may differ from the available supply q_{it}^*. Consumption is thus the minimum of demand and supply so the auctioneer computes P_t^* to solve $P_t^* = \int \frac{c_{it}}{\int c_{jt} dj} p_{it}^* di$ and $c_{it} = \min\left\{q_{it}^*, \left(\frac{\gamma_t P_t^*}{p_{it}^*}\right)^{\frac{1}{1-\rho}}\right\}$. The result is:

$$P_t^* \int_{O_t(w_t^*)} \left(\frac{\min\left\{\widehat{P}_t^i, \gamma_t P_t^*\right\}}{p\left(w_t^*; \widehat{P}_t^i\right)}\right)^{\frac{1}{1-\rho}} di = \int_{O_t(w_t^*)} \left(\frac{\min\left\{\widehat{P}_t^i, \gamma_t P_t^*\right\}}{p\left(w_t^*; \widehat{P}_t^i\right)}\right)^{\frac{1}{1-\rho}} p\left(w_t^*; \widehat{P}_t^i\right) di \tag{5}$$

with $p(\cdot)$, $O_t(\cdot)$, and w_t^* as per equations (1), (2), and (4) respectively. We now have a mapping from a set of price expectations $\left\{\widehat{P}_t^i \mid i \in [0,1]\right\}$ to the realized price level P_t^*. Recalling $\widehat{P}_t^i \equiv \exp(\widehat{\pi}_t^i) P_{t-1}^*$ and invoking $P_t^* \equiv \exp(\pi_t^*) P_{t-1}^*$, the mapping from a set of inflation expectations $\left\{\widehat{\pi}_t^i \mid i \in [0,1]\right\}$ to realized inflation π_t^* is straightforward.[11]

Flow of Funds There is a household in the background who supplies workers and buys goods. Workers remit their wages to the household. Fixed costs and profits of operating firms are also remitted to the household. The household's total income thus equals its total expenditure on goods.

Discussion We now elaborate on some of the modeling elements used above, namely the role of γ_t and the choice of $\alpha < 1$. As per Woodford (2013), "it is appealing to assume that people's beliefs should be rational, in the ordinary-language sense, though there is a large step from this to the RE hypothesis." The principle of ordinary-language rationality motivates the timing of our taste shock. Without γ_t in equation (5), the auctioneer could compute P_t^* at the same time as w_t^*. While our firms deviate from the RE hypothesis by not updating $\widehat{P}_t^i$ based on w_t^*, they would also be deviating from rationality in the ordinary-

[11] As equations (4) and (5) show, our model is one where actual inflation is determined entirely by expected inflation. In other words, the central bank can only change inflation by changing expectations. While our abstraction from conventional policy tools is done to isolate the effect of communication, work by Campbell (2013) demonstrates that it may in fact be optimal for policymakers to rely on open mouth operations, even when open market operations are available.

language sense if they did not update $\widehat{P}_t^i$ based on P_t^*. The choice of $\alpha < 1$ is similarly motivated. Notice from equation (1) that $\alpha = 1$ prompts all firms to set the same price, namely a constant mark-up over the wage, regardless of expectations. On one hand, this isolates the effect of expectations on realized inflation through just the labor market. On the other hand, it eliminates the lag between w_t^* and P_t^* in the auctioneer's problem. We can thus use $\alpha = 1$ to build some intuition but our full model will employ $\alpha < 1$. With $\alpha < 1$, prices are not simple mark-ups so γ_t enters equation (5) and the lag between w_t^* and P_t^* is restored. Importantly, this restoration stems from production being non-linear in labor, not from production being decreasing returns to scale.[12]

3.2 Results with One Forecasting Rule

Before combining the model of Subsection 3.1 with the two forecasting rules in Section 2, it is instructive to establish how the model works with one forecasting rule since that covers the two limits: 100% Random Walking and 100% Fed Following.

Proposition 1 *If $\widehat{\pi}_t^i \sim N(\mu, \sigma^2)$ for all i, then $\pi_t^* = \mu + f(\sigma; \Omega)$, where Ω is the set of all parameters excluding μ and σ. To conserve on notation, $f(\sigma) \equiv f(\sigma; \Omega)$ henceforth.*

Proposition 1 says that excess inflation, $\pi_t^* - \mu$, does not depend on the mean expectation μ. It does, however, depend on the extent of expectations heterogeneity σ which is a dependence we want to emphasize. In particular, our model is sufficiently non-linear that realized inflation may not equal the mean expectation when expectations are heterogeneous. Naturally though, realized inflation should line up with the mean expectation in the long-run: if everyone draws inflation forecasts from $N(\mu, \sigma^2)$ but realized inflation ends up being $\pi_t^* \neq \mu$, then $N(\mu, \sigma^2)$ is a biased forecasting rule and its survival into the long-run would seem at odds with ordinary-language rationality. We therefore seek parameter conditions under which heterogeneity ($\sigma > 0$) and mean rationality ($\pi_t^* = \mu$) co-exist. The rest of

[12] In a previous version, we showed that the key properties of our model hold when production is instead given by $F(\ell_{it}, z_{it}) = \ell_{it}^{\alpha} z_{it}^{1-\alpha}$, where z_{it} is firm effort and the real disutility of such effort is $\frac{z_{it}^{\lambda}}{\lambda}$ with $\lambda \in (1, \infty]$. Notice that $F(\ell_{it}) = \ell_{it}^{\alpha}$ is just the limiting case of $\lambda \to \infty$.

this subsection will build up to these parameter conditions. Unless otherwise specified, all lemmas and propositions assume $\widehat{\pi}_t^i \sim N(\mu, \sigma^2)$ for all i as in Proposition 1.

To fix ideas, start with the limiting case of $\alpha = 1$. As per Subsection 3.1, prices will be a constant mark-up over the wage so realized inflation will be determined in the labor market. We can thus focus on equation (4). Absent a fixed cost, all firms will operate so $O_t(w_t) = [0,1]$ for any w_t. In other words, the operating set will be independent of the wage. The following lemma reveals that realized inflation will exceed the mean expectation:

Lemma 1 *If $\alpha = 1$ and $U = 0$, then $f(\sigma) = \frac{\sigma^2}{2(1-\rho)}$.*

The excess inflation in Lemma 1, $\frac{\sigma^2}{2(1-\rho)}$, is a Jensen's inequality term which arises because normal inflation expectations are compounded into log-normal price expectations and aggregated. The Jensen's term is exacerbated by more heterogeneity in inflation expectations (i.e., higher σ) and more substitutability between goods (i.e., higher ρ). As σ increases, the compounding process skews the distribution of price expectations further right. The highest expectation firms thus drive wages up more than the lowest expectation firms drive them down. The effect is strongest when goods are more substitutable because high expectation firms foresee a huge increase in sales by undercutting the aggregate price level and thus participate more actively in the labor market.

Eliminating excess inflation without eliminating heterogeneity requires overcoming the Jensen's inequality term. Intuitively, this requires giving low expectation firms more pull to overcome the pull that compounding gives high expectation firms. Allowing low expectation firms to not produce by introducing a fixed cost is one way to achieve this, motivating our use of $U > 0$.[13] With fixed costs, $O_t(w_t) \subset [0,1]$ and, as shown next, $|O_t(w_t)|$ decreases when expectations become more heterogeneous:

Lemma 2 *If $\alpha \leq 1$ and $U > 0$, then the fraction of firms that operate is decreasing in σ.*

[13] In principle, one could instead assume normality (rather than log-normality) of price expectations. In practice though, prices cannot be negative so the appropriate assumption would be truncated normality which, like log-normality, is not symmetric.

The intuition for Lemma 2 is as follows. Higher σ amplifies the disproportionate effect that high expectation firms have on wage determination. Since a higher wage cuts into expected profit, the presence of a positive fixed cost means more firms will choose not to operate.

What we have just described is an exit channel which will put downward pressure on the wage and thus potentially offset the Jensen's inequality term. Can the offset be complete in the sense of restoring mean rationality ($\pi_t^* = \mu$) without eliminating heterogeneity ($\sigma > 0$)? We first saw the Jensen's term in Lemma 1 so let us start with that context:

Lemma 3 *If $\alpha = 1$ and $U \geq 1 - \rho$, then $f(\sigma_0) = 0$ for a unique $\sigma_0 > 0$. Moreover, $f'(\sigma_0) > 0$.*

In words, Lemma 3 says that a sufficiently high fixed cost introduces a point of mean rationality – that is, a point $\sigma_0 > 0$ with no excess inflation – when $\alpha = 1$. The solid gray line in Figure 1(a) provides a graphical representation of this result.[14]

We will now show that the model with $\alpha < 1$ can have two points of mean rationality. We want the main results to be derived under $\alpha < 1$ for the reasons discussed at the end of Subsection 3.1. We also want two points of mean rationality so that the Fed Following and Random Walking rules in Definitions 1 and 2 are not restricted to having the same standard deviation. Proposition 2 below identifies parameter conditions under which our model has two mean rational points. We will denote these points as σ_F and σ_R and use them later as the standard deviations in Definitions 1 and 2 respectively. Proposition 2 also establishes a region of negative excess inflation in between these two points.

Proposition 2 *There exist values $\underline{\alpha} \in (0,1)$ and $\overline{U} > \underline{U} > 0$ such that $\alpha \in (\underline{\alpha}, 1)$ and $U \in \left(\underline{U}, \overline{U}\right)$ yield $f(\sigma_F) = f(\sigma_R) = 0$ for $\sigma_R > \sigma_F > 0$. Moreover, $f'(\sigma_F) < 0$ and $f'(\sigma_R) > 0$ so $f(\sigma) < 0$ for $\sigma \in \Sigma \subseteq (\sigma_F, \sigma_R)$, where Σ is a non-empty set.*

The parameter conditions identified in Proposition 2 are qualitative in nature so the blue area in Figure 1(b) provides a numerical example when $\rho = 0.9$. For any combination of α

[14] The ability of exit to counter Jensen's inequality does not hinge on inflation expectations being single draws from a normal distribution: if each firm were to draw a full prior as its forecast, the proof of Lemma 3 shows that exit can be restored with two different forecasts and appropriate bounds on U.

and U in this blue area, our model has two mean rational points with a region of negative excess inflation in between. It also turns out that the region of negative excess inflation is the entire region between the two mean rational points. The blue line in Figure 1(a) provides a graphical representation.

To understand why excess inflation is negative between the two mean rational points, recall the competing effects of higher σ on wages in equation (4). As σ increases, the compounding of inflation expectations into price expectations skews the distribution of price expectations right and puts upward pressure on the wage through the labor demands of high expectation firms. As the wage increases though, low expectation firms find production unprofitable and the resulting decline in operation puts downward pressure on the wage. For lower values of σ, the exit of low expectation firms dominates and dampens the wage but, when σ becomes sufficiently large, the labor demands of high expectation firms take over. The dependence of w_t^* on σ is thus U-shaped. We know from equation (1) that individual prices respond positively to wages so, all else constant, the shape of w_t^* feeds into P_t^*. Notice, however, that $\alpha < 1$ implies additional upward pressure on P_t^* at the price aggregation stage. Since only firms with sufficiently high expectations produce, the individual prices aggregated by equation (5) are $p\left(w_t^*; \widehat{P}_t^i\right)$ with $\widehat{P}_t^i$ high. Therefore, the pass-through from w_t^* to P_t^* varies across σ but, for $\alpha \in (\underline{\alpha}, 1)$ and $U \in \left(\underline{U}, \overline{U}\right)$, it is enough to generate two mean rational points with a U-shaped pattern in between. If the fixed cost is too high (i.e., $U > \overline{U}$) or the returns to labor are too low (i.e., $\alpha < \underline{\alpha}$), then exit is too strong relative to labor demand and we get only one mean rational point. If the fixed cost is too low (i.e., $U < \underline{U}$), then exit is weak and we get no mean rational points.

3.3 Two Forecasting Rules with Common Means

We now move to an environment with two forecasting rules. Consider the two rules given in Definitions 1 and 2 but with $\overline{\pi}_t = \pi_{t-1}^* = \mu$. Understanding how the model works under this restriction will help us understand what happens when inflation approaches the central bank's target in our simulation results. The simulations in Sections 4 and 5 will then allow for $\overline{\pi}_t \neq \pi^*_{t-1}$ to see how inflation can be made to approach the target in the first place.

Proposition 3 *Suppose $\widehat{\pi}^i_t \sim N\left(\mu, \sigma^2_F\right)$ for a group of size ξ_t and $\widehat{\pi}^i_t \sim N\left(\mu, \sigma^2_R\right)$ for the rest, where $f\left(\sigma_F\right) = f\left(\sigma_R\right) = 0$. If $\alpha = 1$, then $\pi^*_t = \mu$ for all $\xi_t \in (0, 1)$.*

Under $\alpha = 1$, Proposition 3 says that the entire population is mean rational when each subpopulation is individually mean rational. We know from equation (1) that all firms set the same price when $\alpha = 1$ so any heterogeneity in expectations only affects the economy through labor market clearing, namely equation (4). The latter aggregates linearly across subpopulations so, if the component distributions are each parameterized to deliver $\pi^*_t = \mu$, then their mixture will also deliver $\pi^*_t = \mu$.

Now consider the more interesting case of $\alpha < 1$ which introduces heterogeneity into the price aggregation of equation (5). Recall that any combination of α and U from the blue area in Figure 1(b) produces two mean rational distributions when $\rho = 0.9$. Call this the feasible parameter space for $\rho = 0.9$. The blue area in Figure 2(a) plots the feasible parameters for which any mixture of the two mean rational distributions produces negative excess inflation. To provide a concrete example of what this means, Figure 2(b) plots excess inflation, $y_t \equiv \pi^*_t - \mu$, as a function of the degree of mixing, ξ_t, for one combination of α and U drawn from the blue area in Figure 2(a). Any other combination will deliver the same general shape. Note that we are using y_t to denote excess inflation when there are two forecasting rules rather than $f\left(\cdot\right)$ which denoted excess inflation when there was only one forecasting rule. We verify that the blue area in Figure 2(a) overlaps the entire feasible space in Figure 1(b) so combinations of α and U which generate two distinct mean rational distributions also generate negative excess inflation for any mixture of these distributions when $\rho = 0.9$. Similar results are verified for $\rho = 0.85$ and $\rho = 0.95$.

Notice that the shape of y_t over $\xi_t \in [0, 1]$ in Figure 2(b) resembles the shape of $f\left(\cdot\right)$ over $\sigma \in [\sigma_F, \sigma_R]$. This is useful as it permits interpretation of our results vis-à-vis Figure 1(a): if one uses $N\left(\mu, \sigma^2_x\right)$ with some $\sigma_x \in (\sigma_F, \sigma_R)$ to approximate the aggregate distribution generated by $\xi_t \in (0,1)$ and $\overline{\pi}_t = \pi^*_{t-1} = \mu$, then $\pi^*_t < \mu$ follows for the reasons discussed in Subsection 3.2.

Before proceeding, let us comment on how the feasible parameter space changes with ρ. As mentioned above, for each value of ρ listed in Figure 2(a), the shaded area exactly overlaps the feasible region for that ρ. Notice that the shaded area shifts up as ρ decreases. In other words, the feasible region involves higher values of U as ρ decreases. Lower ρ means goods are less substitutable so the demand for good i falls by less when the price of good i rises. Firms can achieve higher profits in this environment so a higher fixed cost is needed to create the exit channel. Also notice that lower values of ρ admit a larger feasible region. With less substitutability between goods, firms are less responsive to each other and this helps generate similar equilibrium properties across a wider range of parameter choices.

4 Announcements to Increase Inflation

We now combine the economic model in Section 3 with the social dynamics in Section 2 to study how central bank announcements can be used to increase inflation. Our model abstracts from conventional policy tools such as short-term interest rates, isolating the effect of communication and mimicking the situation faced by a central bank at the zero lower bound. We first consider announcements about the level of inflation. We then consider announcements about major initiatives (e.g., Quantitative Easing) which have the potential to skew the entire distribution of inflation expectations.

4.1 Inflation Targets

Recall that announcements affect the forecasting rule in Definition 1 but this rule only matters for inflation if used by enough firms. The economic model of Section 3 allows us to calculate π^*_t conditional on ξ_t and the forecasting rules. Using social dynamics as per Section 2, we can then determine ξ_{t+1} conditional on π^*_t, ξ_t, and the forecasting rules.

Suppose inflation starts at $\pi_0 = -2\%$. A less extreme starting point (e.g., $\pi_0 = 1\%$)

would not change the nature of the results and, if anything, would be easier to overcome. With $\xi_1 = 0$, all firms are Random Walkers who forecast according to $\widehat{\pi}_1^i \sim N(-2\%, \sigma_R^2)$. The mean rational property thus yields $\pi_1^* = -2\%$. Now suppose the central bank tries to increase inflation by announcing $\overline{\pi}_t = 2\%$ for all $t \geq 1$. This announcement introduces a new forecasting rule, Fed Following, which a small fraction of firms mutate towards. We set the mutation parameter to $\theta = 0.02$ which is conservative compared to the social dynamics literature.[15] This ensures that Fed Following will only become popular if it performs well in tournaments.

In our baseline social dynamics, randomly matched firms compare forecasting performance and switch rules after being outperformed eight times ($S = 8$). We use $N = 1000$ firms and randomly draw N pairs with replacement each period to simulate the meetings. Alternative specifications – for example, different strike accumulation rules, different values of θ and S, and more/less meetings per period – are discussed in Section 7. The model parameters are set to $\rho = 0.9$, $\alpha = 0.9$, and $U = 0.18$. The choice of ρ captures high but imperfect substitutability between goods. The choices of α and U come from the blue region in Figure 1(b) so that we have two individually mean rational subpopulations. Any parameter combination from this region will deliver qualitatively similar results.[16] Lastly, we assume that the taste shock is uniformly distributed according to $\gamma_t \sim \mathcal{U}[0.99, 1.01]$. These parameter choices deliver $\sigma_F = 0.0036$ and $\sigma_R = 0.0643$ as the solutions to $f(\cdot) = 0$.

Figure 3(a) summarizes the results. Notice that inflation converges to 2%, albeit slowly. Slow convergence reflects two competing forces. Recall from Subsection 3.1 that firms with higher price expectations demand more labor and set higher prices. FFs start as the high expectation firms so their initial impact is to increase inflation via wage and price-setting. However, also recall from Subsection 3.1 that firms with low expectations (relative to their peers) are less likely to operate. RWs start as the low expectation firms so their initial impact is to decrease inflation by not fully participating in the labor market.

[15] Arifovic et al. (2013), for example, use a mutation parameter of 0.1.
[16] Recall from Subsection 3.3 that the blue regions in Figures 1(b) and 2(a) coincide.

If the upward pressure from FF price-setting outweighs the downward pressure from RW exit, then the central bank may want to make even more aggressive announcements. The path we consider is one where the central bank announces short-term targets above its long-run goal of 2%. The idea that there could be differences between short-run announcements and long-run goals is not unprecedented. For example, as part of the "Evans Rule" adopted between December 2012 and March 2014, the Federal Reserve was willing to tolerate one- to two-year ahead inflation of up to 2.5% despite a 2% long-run target. The aggressive announcements we have in mind take this a step further and target, rather than simply tolerate, higher short-term inflation.

As shown in Figure 3(b), aggressive short-term targets, 3% in this numerical example, induce any FFs to set very high prices, boosting inflation. At the same time though, the big gap between realized and targeted inflation early on does nothing to help the central bank accumulate more FFs. Therefore, dropping the target to 2% right when 2% is reached will lead to less than 2% inflation in subsequent periods. This is an application of Subsection 3.3: when the FF and RW rules are centered around similar points, having a mix of FFs and RWs will lead to inflation below the common means. Basically, the mixture of rules involves an amount of dispersion that leads the exit channel to dominate. We will refer to the "inflation below the common means" phenomenon as undershooting of the central bank's long-run target when the common means equal this target. Recall that the FF and RW rules are individually mean rational so, if the vast majority of firms used the same rule, the exit channel would not dominate the pricing non-linearities in our model and inflation would not undershoot. It would thus help the central bank to have ξ_t close to 1 when the 2% announcement takes hold. Figure 3(c) shows that gradually lowering the aggressive target towards 2% gives FFs more chances to succeed in tournaments and achieves the desired result.

4.2 Additional Anti-Deflation Announcements

In Subsection 4.1, the direct effect of central bank announcements was limited to the mean of the FF distribution. We now consider more potent announcements which can directly affect the skewness of the economy-wide distribution. A practical example is what Krishnamurthy and Vissing-Jorgensen (2011) dub the inflation channel of QE: curbing deflationary expectations through the Fed's widely publicized large scale asset purchases.[17]

We introduce this channel into our model via redraws. More precisely, some firms with deflationary expectations redraw their expectations ($\widehat{\pi}_t^i$) after hearing that the central bank will take a proactive approach to stimulating the economy. We will refer to the announcements that convey this proactive approach as anti-deflation announcements. Each redraw comes from the same distribution as the original draw so not all deflationary expectations will be eliminated. However, redraws do have the effect of skewing the RW and FF distributions so that more mass exists in the positive region compared to the original normal distribution. Since very few FFs actually expect deflation, the skew is stronger for RWs but, either way, the effect of anti-deflation announcements is to increase expectations, reduce dispersion, enlarge the set of operating firms, and put upward pressure on inflation.

We consider two dimensions of anti-deflation announcements: rounds and intensity. In our context, rounds means the number of periods with media coverage about an anti-deflation policy like QE and, therefore, the number of periods that have redraws. Intensity means the fraction of deflationary firms that are exposed to this coverage and, therefore, the fraction that redraw in a given period. The central bank again faces $\pi_0 = -2\%$ but now it would like to return to 2% without using short-term targets that differ from its long-run goal. Figure 4 illustrates how this can be achieved by varying rounds and intensity.

Figures 4(a) and 4(b) fix intensity at 1, meaning that all firms with deflationary expectations redraw. Figure 4(a) shows that one round of anti-deflation announcements helps increase inflation but more time is needed to reach 2%. Figure 4(b) shows that two rounds

[17] Much of the existing QE literature abstracts from the inflation channel and focuses on yield curves (e.g., Gagnon et al. (2010), D'Amico and King (2010), Williams (2011), Hamilton and Wu (2012), Wu and Xia (2016), Wu and Zhang (2016), etc).

of anti-deflation announcements push the economy above 2% for a short time before inflation dips back below 2% for several periods.

Figure 4(c) then demonstrates that two rounds with less than full intensity can bring the economy to 2% quickly and without any time above target. However, once the rounds run out, the economy again dips back below 2% for several periods. Just as redraws skew the distributions and increase operation, the end of redraws unskews the distributions and decreases operation. Therefore, if there is a sufficiently interior mix of FFs and RWs when the redraws stop, exit among RWs returns the undershooting problem of Figure 3(b). This is the same problem we are seeing in Figures 4(b) and 4(c).

Figure 4(d) illustrates the outcome of many rounds and low intensity. Notice that increasing the number of rounds makes it possible to find an intensity that returns inflation to 2% monotonically. With more rounds, the central bank has more time to accumulate FFs before anti-deflation announcements stop. This will help eliminate the undershooting problem discussed above. At the same time, more rounds must be combined with a sufficiently low intensity to avoid exceeding 2% as in Figure 4(b).

5 Disinflation and Empirical Evidence

The discussion so far has focused on *increasing* inflation. Regardless of whether the central bank uses short-term inflation targets or announcements about anti-deflation policies like QE, we showed that the ultimate goal will be undershot when there is a clear mix of FFs and RWs. This is triggered by the "inflation below the common means" phenomenon mentioned earlier and reflects the existence of an exit channel. We also showed that announcements can be designed to help FFs accumulate through tournament selection. In other words, undershooting is avoided when central bank announcements are such that inflation approaches the target with a high fraction of FFs. The aftermath of the recent global financial crisis has not yet produced enough observations to allow testing this idea in a low inflation climate. However, as we will now describe, a similar idea is relevant for *decreasing* inflation. This is useful because a number of countries have gone through disinflations with the use of targets. We can exploit cross-country differences in how these targets were introduced to test our model.

5.1 Disinflation in the Model

Suppose initial inflation is high. We consider two scenarios which are guided by the data that will be presented in Subsection 5.2. In the first (abrupt) scenario, initial inflation is $\pi_0 = 8\%$ and the central bank announces $\overline{\pi}_t = 4\%$ for all $t \geq 1$. In the second (gradual) scenario, initial inflation is $\pi_0 = 15\%$ and the central bank announces a downward path which interpolates between 15% and a long-run target of 4%. Figure 5 shows how realized inflation in the model differs across the two scenarios. The critical difference between these two scenarios is the degree of abruptness, not the level of initial inflation. The same qualitative results obtain if initial inflation is assumed to be the same in the two scenarios.

Notice from Figure 5 that the central bank can convert more firms into Fed Followers by achieving interim targets along the gradual path. This leads to most firms forecasting according to $\widehat{\pi}_t^i \sim N\left(2\%, \sigma_F^2\right)$ when 2% is reached, preventing undershooting because of the mean rational property. In contrast, the abrupt scenario involves large gaps between realized and announced inflation early on. These gaps impede the accumulation of FFs via tournament selection. It is only once realized inflation is in the vicinity of the 2% target that FFs begin to accumulate more quickly. However, with stubborn beliefs, this accumulation is not quick enough to avoid having a clear mix of RWs and FFs when 2% is reached, opening the door for undershooting. Our model thus predicts that undershooting is more likely when the central bank tries to disinflate by abruptly introducing low inflation targets.[18]

[18] To be sure, conventional policy tools (e.g., short-term nominal interest rates) can be used together with announcements in a disinflation. Our abstraction from conventional tools allows us to isolate the effect of the announcements.

5.2 Disinflation in the Data

As just described, a testable implication of our model is that abrupt targets during a disinflation lead to temporary undershooting while gradual targets do not. We now demonstrate that this prediction is borne out in the data.

We start with the set of 29 inflation targeters in Roger (2009) and Svensson (2010). The sample for each country starts in the quarter in which it formally adopted inflation targets and ends in 2013Q3. The earliest observation is thus New Zealand in 1990Q1. We collect data on the path of inflation targets from individual central bank websites. Our data on actual inflation then come from the IMF's International Financial Statistics database, as per Mishkin and Schmidt-Hebbel (2007). The number of countries that begin with above-target inflation is much greater than the number that begin with below-target inflation so we focus on the former. Excluding countries that eventually abandoned inflation targeting, we are left with 19 countries. These countries are listed in the first column of Table 1. The rest of the table provides qualitative indicators of abruptness and undershooting for each country using the metrics described next.

We first construct some intuitive dummies for abruptness in a given country. The first, `abrupt1`, equals one if the targeted path is flat. This is the least subjective measure of abruptness so we adopt it as the benchmark. The second dummy, `abrupt2`, equals one if the absolute value of the net change in inflation targets between the time of introduction and the end of our sample is less than 40% of the absolute value of the net change in realized inflation over the same period. The third dummy, `abrupt3`, equals one if the standard error of the inflation targets that make up the targeted path is smaller than the final target. The left panel in Table 1 shows the division of countries between abrupt and gradual according to each measure. By definition, any country with `abrupt1` $= 1$ will also have `abrupt2` $= 1$ and `abrupt3` $= 1$.

To help visualize the data, Figure 6 compares the average inflation rates and targeted paths for abrupt versus gradual targeters. The top panel averages over countries with abrupt1 = 1 while the bottom panel averages over countries with abrupt1 = 0. Averages are for each point in time, with time 0 denoting the introduction of inflation targets. For both abrupt and gradual targeters, the average long-run inflation target is around 4%. Notice from Figure 6 that undershooting is more characteristic of abrupt inflation targeters, consistent with the model predictions in Figure 5.

For more formal empirical evidence on abruptness and undershooting, we construct some undershooting dummies then run simple qualitative cross-country regressions. The benefit of this approach is that it is model-free and scale-free: the results are less susceptible to outliers and thus more robust for small samples. To construct a dummy variable for undershooting, let t_1 denote the first quarter in which actual inflation hits the targeted path from above and let t_2, where $t_2 > t_1$, denote the first quarter in which actual inflation hits the final target from below.[19] Our main dependent variable, under1, is a dummy that equals one if average inflation is less than the average target over the period t_1 to t_2. As an alternative, we also define under2 which equals one if average inflation is less than the average target for the period t_1 to 2013Q3. The right panel in Table 1 shows the division of countries according to each undershooting measure.

The first two panels in Table 2 report the results of our qualitative regressions. The first three columns in the left panel regress under1 on abrupt1, abrupt2, and abrupt3 respectively. The middle panel repeats the exercise using under2 as the dependent variable. The intercepts in our regressions are generally small and statistically insignificant whereas the abruptness coefficients are generally large and significant. On the whole, the range of intercepts suggests that the probability of undershooting is 0-29% for gradual targeters while the range of intercepts plus slopes suggests that this probability is 46-100% for abrupt targeters. The difference is economically and statistically significant.

To check the robustness of our empirical results, we also run regressions that control for the distance between initial and desired inflation (d2target). Comparing the first and

[19] If actual inflation never hit the final target from below, we set t_2 equal to the last quarter in our sample (i.e., 2013Q3). If actual inflation also never hit the targeted path from above, we set t_1 equal to the first quarter in which the final target became effective.

fourth columns in each panel of Table 2 reveals that the abruptness coefficient is largely unchanged, both quantitatively and qualitatively. In contrast, the coefficient on `d2target` is always small and insignificant. Therefore, even controlling for initial conditions, there is evidence that undershooting is more characteristic of abrupt targeters.

As a complement to the qualitative results, the last panel in Table 2 presents some quantitative cross-country regressions. Our dependent variable, `under_num`, is defined as the average difference between actual and targeted inflation over the period t_1 to t_2. Notice that `under1` $= 1$ if and only if `under_num` < 0 so undershooting will now be indicated by negative regression coefficients. The intercepts in the first three columns of this last panel measure the average undershooting among gradual targeters whereas the sum of the intercepts and abruptness coefficients measure the average undershooting among abrupt targeters. The intercepts are positive but generally insignificant whereas the abruptness coefficients are negative and generally significant. Classifying abruptness according to `abrupt1`, the first column says that actual inflation will average 0.97 percentage points below an abrupt target. The last column then says this result is robust to initial condition controls. The quantitative results thus align with the qualitative ones.

6 Discussion of Key Channels

There are two recurring themes in our results: the possibility of temporarily undershooting the central bank's inflation target and the ultimate convergence of inflation to this target. The exit channel introduced in Section 3 provides the economic mechanism for undershooting. In what follows, we elaborate on the asymmetries implied by the exit channel and the relation of these asymmetries to our undershooting results. We also comment on the role of social dynamics in limiting undershooting to a temporary phenomenon and generating convergence.

6.1 Exit and Asymmetry

Undershooting can occur regardless of whether inflation hits the central bank's target from above or below. See Figures 3(b), 4(b), 4(c), and 5(a). Also notice that we did not obtain any results where inflation overshoots a constant target, except in the case of anti-deflation announcements which involved extra tools. The dynamics of our model are therefore asymmetric. The intuition goes back to our discussion of Figures 1 and 2. Parameters which support two individually mean rational forecasting rules also generate negative excess inflation (relative to a common mean) when expectations are a mixture of these two rules. This is because the exit channel dominates the Jensen's inequality term for such mixtures.

The downward pressure imparted by exit can also apply if the mixture involves different means. Consider, for example, Figures 3(a) and 5(a). The central bank is using constant targets to change inflation by 4 percentage points, with Figure 3(a) illustrating a 4 percentage point increase and Figure 5(a) illustrating a 4 percentage point decrease. In both cases, realized inflation falls below the mean expectation once there is a clear mix of forecasting rules. This is another instance of negative excess inflation but without the restriction of common means. There is an important implication here for how sluggish (if at all) the inflation adjustment will be. In particular, notice that it takes longer for inflation to rise by 4 percentage points in Figure 3(a) than it does for inflation to fall by 4 percentage points in Figure 5(a).[20] If realized inflation equaled the mean expectation along the entire transition path, then the time to target would be the same regardless of whether the central bank was trying to increase or decrease inflation. The exit channel, by creating negative excess inflation during the transition, accelerates what the central bank is trying to achieve in Figure 5(a) but makes it more sluggish in Figure 3(a).

The above discussion suggests that, with constant targets, increasing inflation is harder than decreasing inflation. The opposite is true with aggressive targets and the reason is again the exit channel. To be more precise, recall that the central bank in Subsection 4.1 increased

[20] What matters is the direction of the 4 percentage point change, not the starting point. Similar results would be obtained if Figure 5(a) went from 2% to −2% instead of 8% to 4%. We have omitted this figure for brevity.

inflation by announcing a short-term target above its long-run goal (an aggressive strategy) whereas the central bank in Subsection 5.1 decreased inflation using a gradual path. Why not decrease inflation by simply announcing a short-term target below the long-run goal? The logic stems from an asymmetry in how Fed Followers help the central bank achieve its goals. When the central bank announces a high target to increase inflation, FFs help by setting high prices. When the central bank announces a low target to decrease inflation, FFs help either by setting low prices or by exiting. Recall that exit puts downward pressure on prices via the market-clearing wage. If the attempted disinflation is sufficiently large and abrupt, then FFs will tend to exit and, once the majority of them have exited, they are unavailable to help any further. There would be no gains to setting an even lower target in this case.[21] Therefore, the exit channel implies that the gains to being aggressive deteriorate faster when trying to decrease inflation than when trying to increase it.

The asymmetries generated by the exit channel help reconcile two seemingly contradictory observations about the real world. On one hand, the new empirical evidence in Subsection 5.2 established that countries which disinflated with gradual targets rather than abrupt targets were more successful in the sense of not undershooting their long-run goals. On the other hand, countries stuck at the zero lower bound since the recent financial crisis have only mustered sluggish increases in inflation, arguably because they have not been aggressive enough. The exit channel in our paper helps reconcile these two observations. More precisely, our exit channel (i) opens the door for negative excess inflation when expectations are a mixture and (ii) implies asymmetric gains to being aggressive when trying to increase rather than decrease inflation. These asymmetric gains do not negate the fact that a period of gradual targets helps build credibility regardless of whether the central bank wants to push inflation up or down. However, they do make a case for preceding gradualism with aggressive announcements when trying to increase inflation.

[21] If anything, an even lower target would be bad for credibility-building because it would just widen the gap between realized and targeted inflation.

6.2 The Role of Social Dynamics

Our paper has used social dynamics to endogenize the mix of FFs and RWs. To better appreciate the contribution of social dynamics to our results, we now consider what happens if the mix of FFs and RWs is set according to some simple benchmarks.

Figure 7(a) shows what happens if we fix $\xi_t = \bar{\xi}$ for all t, where $\bar{\xi} \in \{0, 0.25, 0.5, 0.75, 1\}$. Initial inflation and the targeted path are both as in Figure 5(a). The black line labeled "baseline" in Figure 7(a) replicates the blue line labeled "realized" in Figure 5(a). The colored lines in Figure 7(a) then show the results for the different values of $\bar{\xi}$. If $\bar{\xi} = 0$, then no one uses central bank announcements as a basis for forecasting. Inflation stays at its initial level and announcements are never effective. If $\bar{\xi} = 1$, then everyone uses the central bank's announcements and inflation immediately falls to the long-run target. If $\bar{\xi} \in \{0.25, 0.5, 0.75\}$, then the mix of FFs and RWs is sufficiently interior that inflation settles noticeably below the target: with a constant mix, ξ_t is independent of how well different rules perform so there is no mechanism (e.g., tournament selection) to accumulate FFs and eliminate undershooting once it happens.

The role of tournaments is also evidenced in Figure 7(b) which shows what happens if social dynamics involve only mutation. The black line labeled "baseline" again replicates the blue line labeled "realized" in Figure 5(a). Recall that the baseline specification starts at $\xi_1 = 0$ then lets ξ_{t+1} evolve via mutation and tournament selection. The colored lines in Figure 7(b) correspond to different starting points $\xi_1 \in \{0, 0.25, 0.5, 0.75, 1\}$, with ξ_{t+1} evolving via mutation only. With just mutation, $\xi_{t+1} = (1 - \theta)\,\xi_t + \theta\,(1 - \xi_t)$ for all t so the fraction of FFs converges smoothly to 0.5.

Notice from the right panel of Figure 7(b) that, for the first 15 or so periods, the fraction of FFs in the baseline is very similar to the fraction of FFs in the mutation-only specification with $\xi_1 = 0$. When an abrupt target is first introduced, FF forecasts tend to be outperformed by RWs so FFs do not accumulate, even when tournaments exist. Around the target, however, the tables turn and many RW forecasts are outperformed by FFs. Tournaments will now accelerate the accumulation of FFs which, given mean rationality of our forecasting rules, eliminates undershooting. In contrast, all of the mutation-only specifications settle well below the target because, as in the fixed proportions case, there is no mechanism to accumulate FFs and eliminate undershooting once it happens.

Tournaments thus play an important role in generating convergence to the long-run target. For similar reasons, tournaments also play an important role in explaining why gradual targets can achieve convergence without any undershooting. In Figure 5(b), for example, the achievement of interim targets along the gradual path allowed tournaments to accelerate the accumulation of FFs before the long-run target was reached. Introducing a gradual path into the fixed proportions or mutation-only specifications would not eliminate undershooting or achieve convergence because FF accumulation cannot accelerate.

7 Robustness Analysis

A key insight from our model is that inflation undershoots its long-run target when prior gaps between realized and targeted inflation prevent FFs from accumulating quickly enough. We explained how undershooting is related to the exit channel when discussing Figures 1(a) and 2(b). We also saw undershooting materialize when: inflation was increased with aggressive targets in Figure 3(b); inflation was increased with high intensity anti-deflation announcements in Figures 4(b) and 4(c); and inflation was decreased with abrupt targets in Figure 5(a) which we argued is our model's counterpart to the data in Figure 6(a). We now establish that our undershooting result is robust to alternative parameterizations, taking Figure 5(a) – disinflation with an abrupt target – as the baseline.

To facilitate comparison, the first row of Table 3 presents six quantitative statistics that summarize the properties of realized inflation in Figure 5(a). The first three statistics are related to undershooting. We calculate hitting time as the number of periods it takes inflation to hit the 4% target from above (first column of Table 3). This corresponds to t_1 in Subsection 5.2. We also calculate the fraction of FFs at hitting time (second column) and the lowest value of inflation realized along the transition path (third column). In the event of undershooting, the lowest value of inflation will be noticeably below the 4% target. The next three statistics in Table 3 are related to convergence. We calculate convergence time as the number of periods until 90% of firms use the FF rule (fourth column). With a sufficiently high fraction of FFs, mean rationality of the FF rule implies that realized inflation will be very close to targeted inflation so we can use the fraction of FFs to assess convergence. We also report realized inflation at convergence to make sure it is effectively 4% (fifth column). Note that the difference between convergence time in the fourth column and hitting time in the second column captures the amount of time spent undershooting. Finally, we report the standard deviation of realized inflation at convergence across simulations to verify that the frequency of convergence is effectively 100% (sixth column).

The key properties of Figure 5(a) are salient in the first row of Table 3. Around 21% of firms are using the FF rule when inflation first hits the target so inflation undershoots to 3.3% before converging to 3.9% once sufficiently many FFs (90%) have been accumulated.

The second and third rows of Table 3 show the results for different values of the stubbornness parameter S. Decreasing stubbornness increases the fraction of FFs at the first hit. With a high fraction of FFs when the target is first hit, both the depth of undershooting and the amount of time spent undershooting are lower than in the baseline. Increasing stubbornness has the opposite effect: more undershooting relative to the baseline and a longer time to convergence after the first hit. These results confirm the insight that undershooting only occurs when inflation hits the target with insufficiently many FFs.

The fourth row of Table 3 considers a different mode of interaction. Instead of tournaments between randomly matched firms, suppose tournaments occur locally: firms are arranged in a circle and each firm meets its right and left neighbors every period. With interactions set up as such, the same firms always meet each other. This will create clusters of firms that use the same forecasting rule. Firms at the center of a cluster are thus more likely to meet other firms using the same rule, increasing their *effective* stubbornness for any value of S. The fourth row of Table 3 shows that local interactions increase the time to convergence relative to the baseline. This is consistent with local interactions generating more stubbornness.

The fifth row of Table 3 returns to random interactions but allows for negative strikes. In particular, suppose randomly matched firms still switch after $S = 8$ strikes and still add a strike when outperformed by a different forecasting rule but now they also subtract a strike when their rule is the outperformer. The idea that agents respond to both good and bad performance (rather than just bad performance as in the baseline) is consistent with Brock and Hommes (1997), although we are still using social dynamics as in our baseline. The fifth row of Table 3 shows that the baseline results are robust to allowing for negative strikes.

The sixth and seventh rows of Table 3 show that the main messages about undershooting and convergence in the baseline are robust to different values of the mutation parameter θ. The eighth and ninth rows show that these messages are also robust to different starting points for initial inflation π_0. The tenth row shows that widening the support for the taste shock γ_t also does not have a material impact on the baseline results.

The last two rows of Table 3 vary the number of random meetings per period in the baseline to assess the effect of more or less information transmission between firms. With more meetings, we find that more periods are needed to hit the central bank's target for the first time but, once there, undershooting is minimal and convergence is quick. This is intuitive: increasing the number of meetings without increasing the number of strikes needed for a rule switch is akin to decreasing the effective stubbornness of firms with the under-performing rule. RWs (FFs) tend to have the under-performing rule when realized inflation is close to (far from) the central bank's target. With lower effective stubbornness for RWs around the target, FFs start accumulating very quickly and undershooting is minimal. In contrast, when we decrease the number of meetings per period, undershooting is salient and convergence takes longer than in the baseline.

8 Conclusion

This paper has investigated the effectiveness of central bank announcements when firms have heterogeneous inflation expectations. We constructed a model of inflation determination that is tractable and preserves its original non-linearities. We introduced the possibility of exit by some firms and showed that this exit channel (i) accommodates two individually mean rational forecasting rules with in-rule heterogeneity and (ii) puts downward pressure on realized inflation when the mix of rules is sufficiently interior. We used social dynamics to model the updating of inflation expectations and endogenize the fraction of firms using each rule. Our model rationalizes why countries stuck at the zero lower bound have had a hard time increasing inflation without being aggressive. The same model also predicts that announcing an abrupt target to disinflate will cause inflation to undershoot the target whereas announcing gradual targets will not. We presented new empirical evidence that corroborates this prediction.

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Table 1: Data

Country	abrupt1	abrupt2	abrupt3	under1	under2
Canada	0	1	1	1	1
Chile	0	0	0	0	0
Colombia	0	0	0	0	0
Czech Republic	0	1	1	1	1
Ghana	1	1	1	1	1
Guatemala	0	1	1	0	0
Hungary	0	0	1	0	0
Iceland	1	1	1	1	0
Indonesia	0	0	1	0	0
Israel	0	0	0	0	0
Mexico	0	0	0	0	0
New Zealand	0	1	1	0	0
Norway	1	1	1	1	1
Peru	0	0	0	1	0
Romania	0	0	1	0	0
South Africa	1	1	1	1	0
Sweden	1	1	1	1	1
Turkey	0	0	0	0	0
United Kingdom	0	0	1	1	1

Notes: The left panel classifies countries as abrupt versus gradual targeters based on the abruptness measures defined in Subsection 5.2. The right panel codes whether or not the targeted path was undershot based on the undershooting measures in the same subsection.

Table 2: Regressions

	under1				under2				under_num			
intercept	0.29**	0.20	0.17	0.33**	0.21*	0.10	0.00	0.32*	0.39	0.66**	0.40	0.15
	(0.02)	(0.16)	(0.41)	(0.05)	(0.10)	(0.47)	(1.00)	(0.07)	(0.15)	(0.04)	(0.40)	(0.67)
abrupt1	0.71***			0.68**	0.39			0.28	-1.36**			-1.14*
	(0.00)			(0.01)	(0.12)			(0.29)	(0.02)			(0.05)
abrupt2		0.58***				0.46**				-1.33***		
		(0.01)				(0.03)				(0.01)		
abrupt3			0.45*				0.46**				-0.53	
			(0.08)				(0.05)				(0.35)	
d2target				0.00				-0.01				0.02
				(0.70)				(0.33)				(0.33)

Notes: Regression coefficients with p-values in brackets. * denotes rejection at the 10% level, ** at the 5% level, and *** at the 1% level. All variables are as defined in Subsection 5.2.

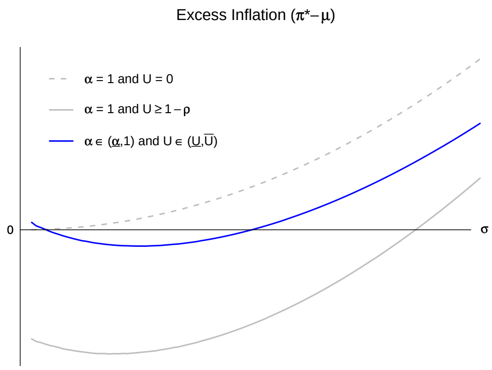
Table 3: Robustness

		Undershooting			Convergence		
		hitting time	%FF at first hit	minimum inflation	convergence time	inflation at convergence	stdev at convergence
1)	baseline	14	21	0.033	30	0.039	0.0007
2)	S=1	11	68	0.038	12	0.039	0.0006
3)	S=10	14	20	0.032	47	0.039	0.0007
4)	local	13	24	0.033	61	0.039	0.0007
5)	positive/negative	14	20	0.033	33	0.039	0.0007
6)	$\theta = 0.01$	19	14	0.032	35	0.039	0.0006
7)	$\theta = 0.05$	10	30	0.033	32	0.038	0.0010
8)	$\pi_0 = 0.06$	11	17	0.032	28	0.039	0.0007
9)	$\pi_0 = 0.10$	16	21	0.033	31	0.039	0.0006
10)	$\gamma_t \in [0.95, 1.05]$	14	20	0.033	30	0.039	0.0007
11)	$5 \times N$ meetings	16	67	0.038	17	0.039	0.0005
12)	$0.8 \times N$ meetings	14	20	0.032	48	0.039	0.0007

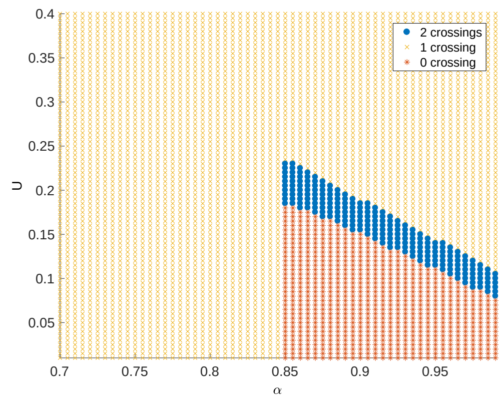
Notes: All inflation rates in this table are reported as fractions. The *baseline* row summarizes the properties of realized inflation in Figure 5(a). It is based on $N = 1000$ firms, N random meetings drawn with replacement each period, firms only counting positive strikes, stubbornness $S = 8$, mutation $\theta = 0.02$, initial inflation $\pi_0 = 0.08$, inflation target $\overline{\pi}_t = 0.04$ for all t, and taste shock $\gamma_t \in [0.99, 1.01]$. The rest of the rows vary one parameter at a time as indicated. The results in the columns are based on 100 simulations. The first five columns report averages and are defined as follows: *hitting time* (in periods) is the first time inflation hits the inflation target from above; *%FF at first hit* is the percentage of Fed Followers at the hitting time; *minimum inflation* is the lowest value of realized inflation along the transition path; *convergence time* (in periods) is the first time the percentage of Fed Followers exceeds 90% (80% for the row with $\theta = 0.05$); *inflation at convergence* is realized inflation at the convergence time. The last column is the standard deviation of realized inflation at the convergence time.

Figure 1: One Forecasting Rule

(a) Getting Mean Rationality



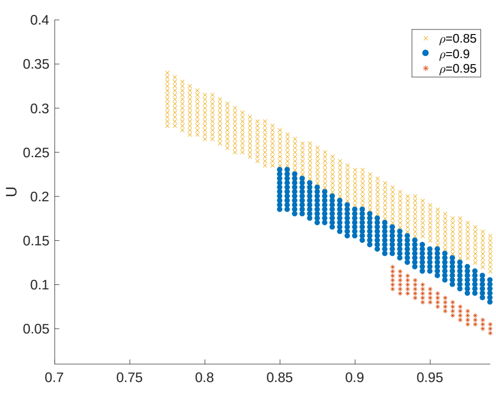
(b) Parameter Space Example



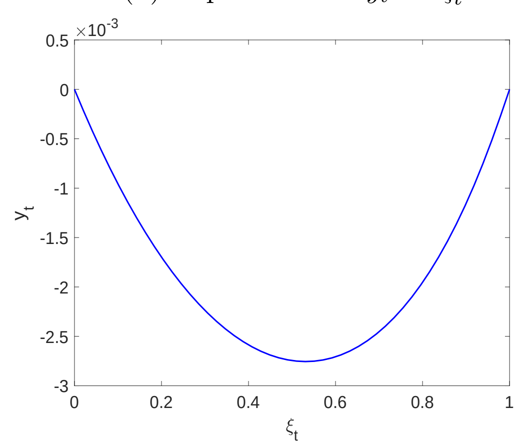
Notes: The left panel illustrates key properties of $f(\sigma)$ under different assumptions. The dashed and solid gray lines correspond to Lemmas 1 and 3 respectively. The solid blue line is consistent with Proposition 2. An intersection with the horizontal axis is a mean rational point. The red, yellow, and blue areas in the right panel demarcate combinations of $\alpha < 1$ and $U > 0$ which produce zero, one, and two mean rational points respectively when $\rho = 0.9$.

Figure 2: Two Forecasting Rules with Common Mean

(a) Parameter Space Example



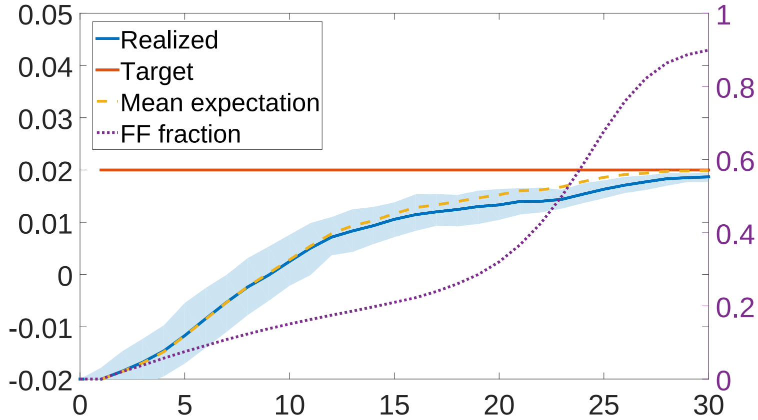
(b) Dependence of y_t on ξ_t



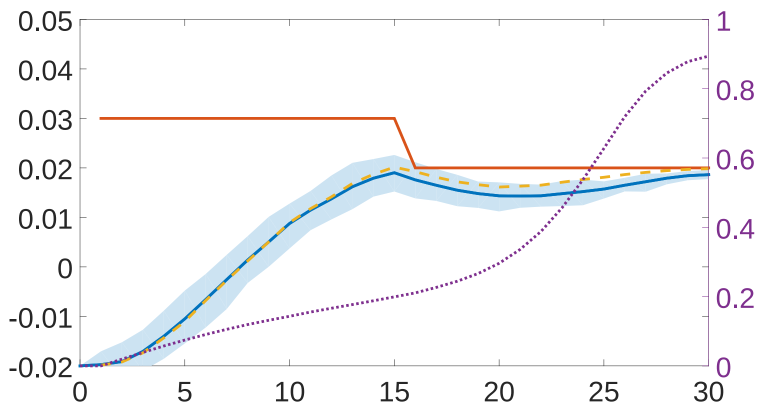
Notes: The left panel illustrates combinations of $\alpha < 1$ and $U > 0$ which (i) produce two mean rational points $\sigma_F > 0$ and $\sigma_R > 0$ and (ii) generate negative excess inflation when expectations are a mixture of $N(\mu, \sigma_F^2)$ and $N(\mu, \sigma_R^2)$. The right panel plots excess inflation as a function of the fraction of expectations drawn from $N(\mu, \sigma_F^2)$. The parameters used to plot the right panel come from the shaded area in the left panel.

Figure 3: Announcements to Increase Inflation

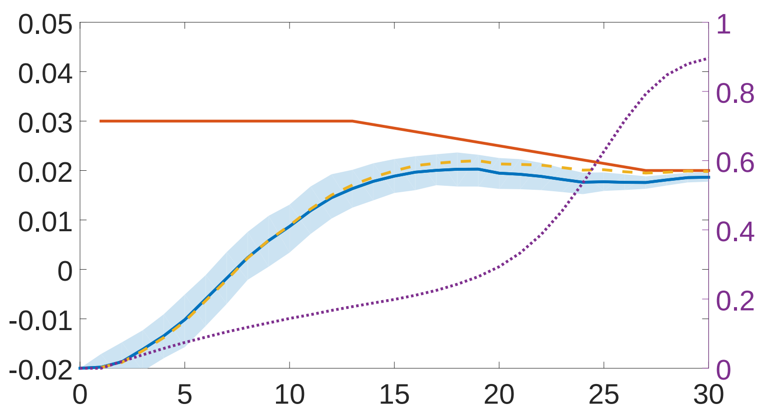
(a) Constant Target



(b) Aggressive Short-Term Target with Abrupt Drop



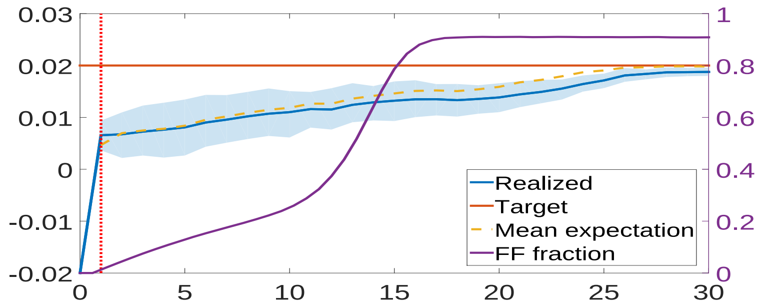
(c) Aggressive Short-Term Target with Gradual Drop



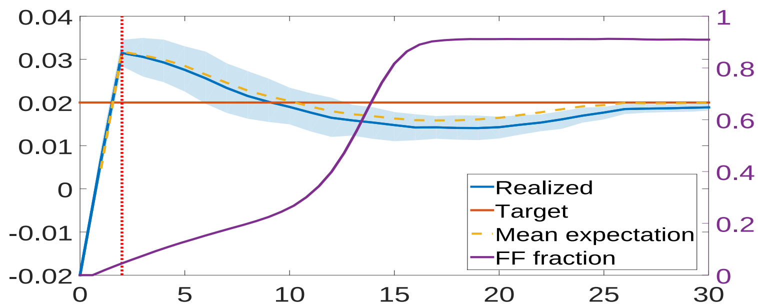
Notes: Panels plot results for different target paths. Lines average over 100 simulations. Shaded areas are [10%, 90%] confidence intervals for realized inflation. The x-axis is time in periods. The y-axis reports fractions, with inflation (realized, target, and mean expectation) on the left scale and the fraction of Fed Followers on the right scale.

Figure 4: Anti-Deflation Announcements

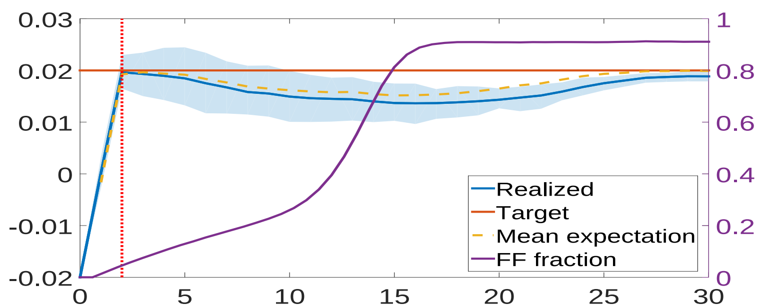
(a) Rounds=1, Intensity=1



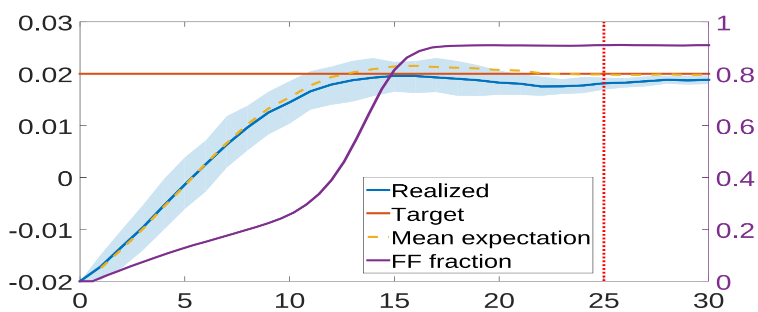
(b) Rounds=2, Intensity=1



(c) Rounds=2, Intensity=0.75



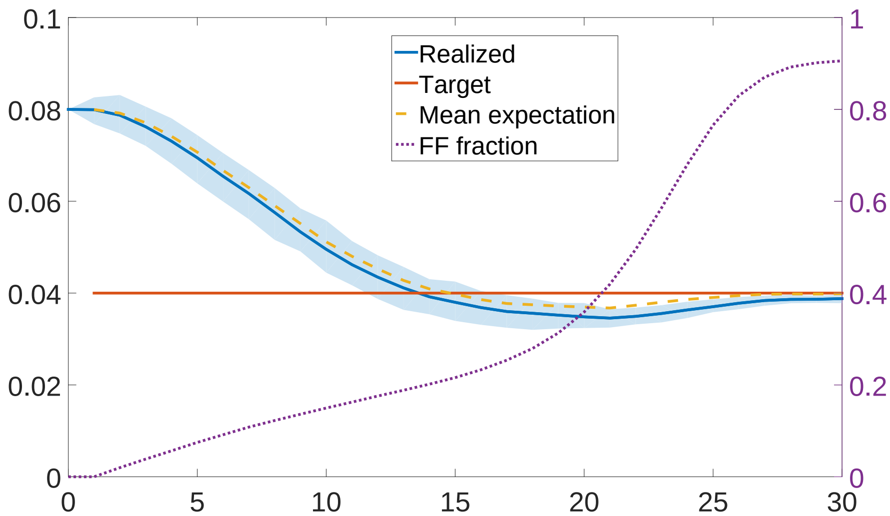
(d) Rounds=25, Intensity=0.1



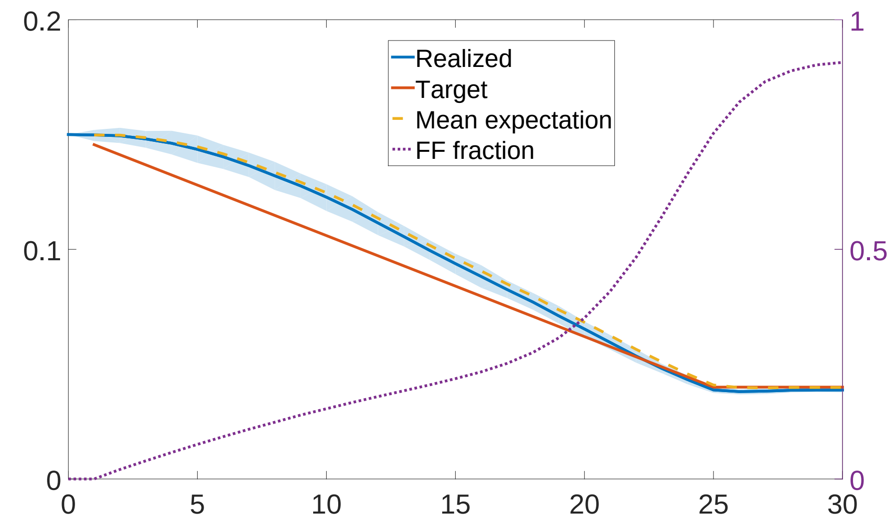
Notes: Panels plot results for different combinations of rounds and intensity. The x-axis is time in periods. The red vertical bar indicates when the anti-deflation announcements stop. The y-axis reports fractions, with inflation (realized, target, and mean expectation) on the left scale and the fraction of Fed Followers on the right scale.

Figure 5: Abrupt vs Gradual Targeters in the Model

(a) Abrupt



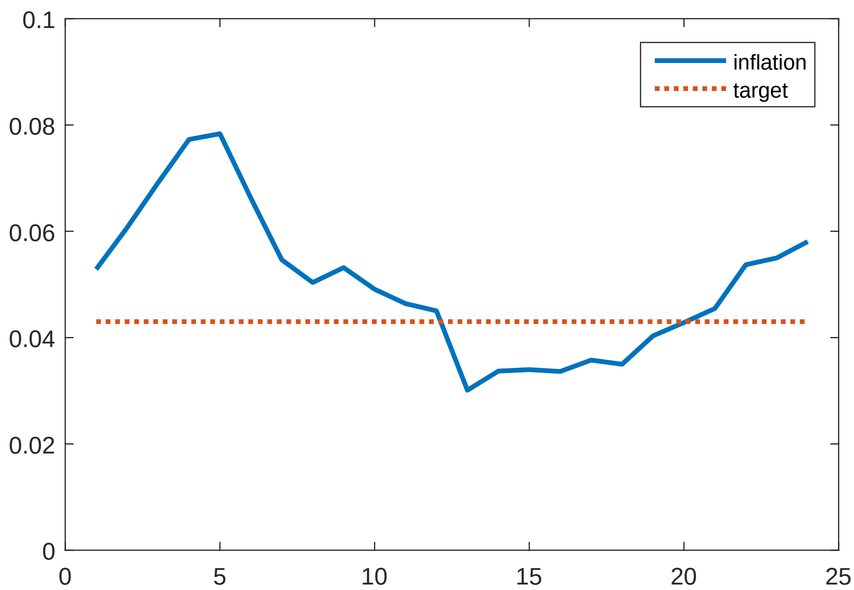
(b) Gradual



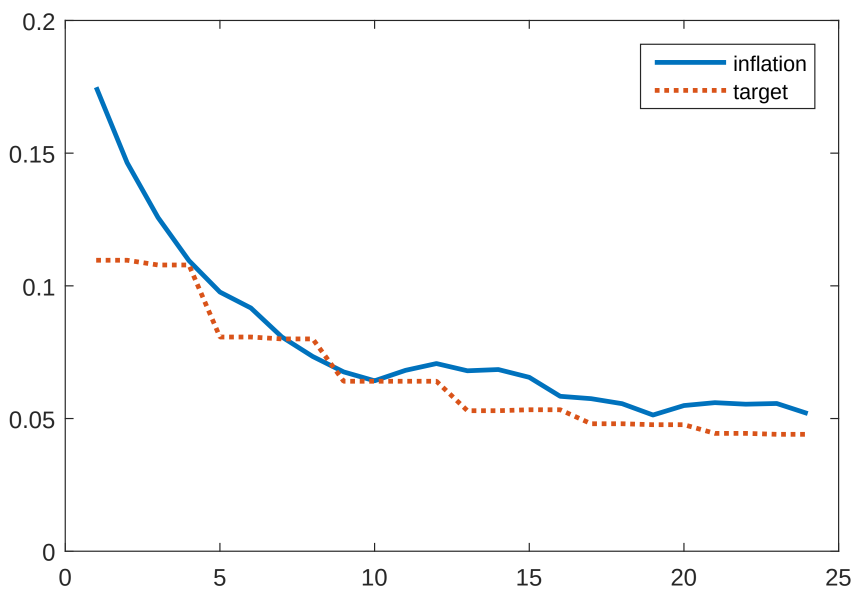
Notes: The x-axis is time in periods. The y-axis reports fractions, with inflation (realized, target, and mean expectation) on the left scale and the fraction of Fed Followers on the right scale.

Figure 6: Abrupt vs Gradual Targeters in the Data

(a) Abrupt



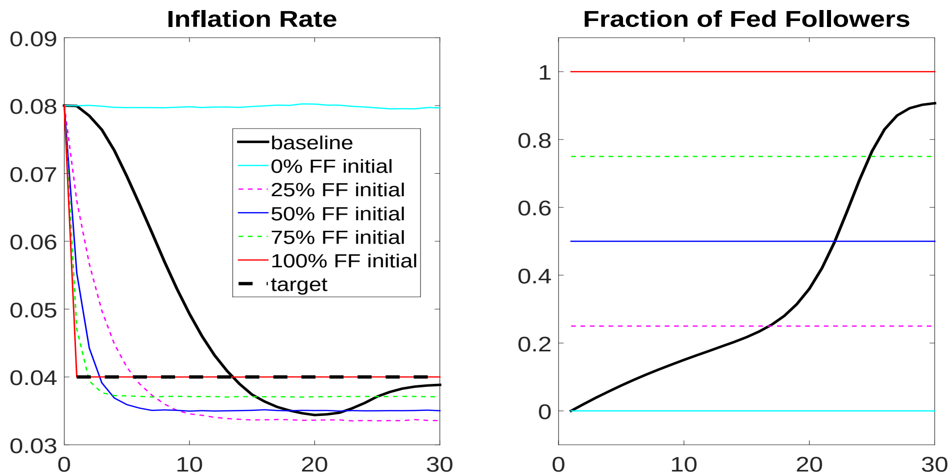
(b) Gradual



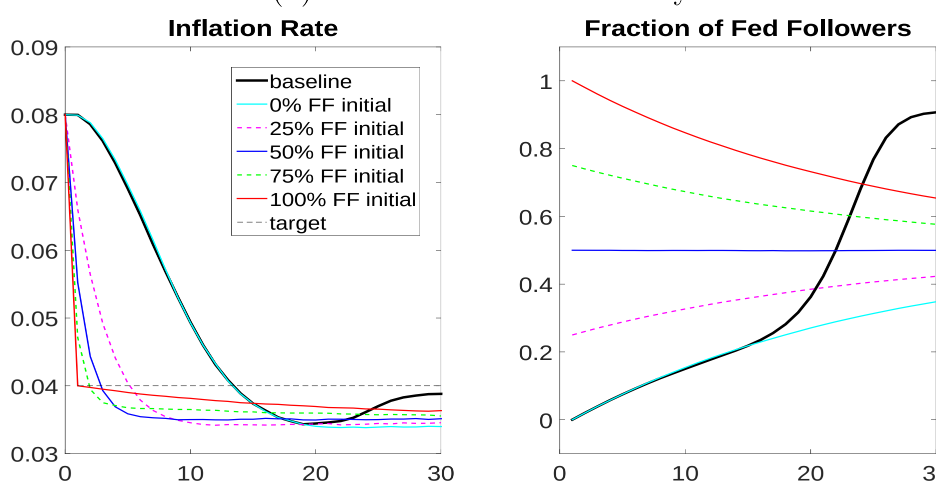
Notes: Average inflation targets and average realized inflation. The horizontal axis is time in quarters since IT adoption. The top panel averages over abrupt targeters (`abrupt1` $= 1$) while the bottom panel averages over gradual targeters (`abrupt1` $= 0$).

Figure 7: Benchmarks

(a) Baseline vs Fixed Proportions



(b) Baseline vs Mutation Only



Notes: The x-axis is time in periods. The y-axis in the left panels is realized inflation in fractions, where *baseline* replicates the blue line labeled *realized* in Figure 5(a). The y-axis in the right panels is the fraction of Fed Followers, where *baseline* replicates the purple line labeled *FF fraction* in Figure 5(a).

Appendix A - Proofs

Proof of Proposition 1

Let $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard normal PDF and CDF respectively. From Pezzey and Sharples (2007), the moment generating function of a truncated normal random variable with mean 0 and variance σ^2 is:

$$\int\limits_{x\geq c} \exp(rx)\,\phi\left(x,\sigma^2\right)dx = \exp\left(\tfrac{r^2\sigma^2}{2}\right)\Phi\left(r\sigma - \tfrac{c}{\sigma}\right) \tag{6}$$

Using $\widehat{P}_t^i = \exp\left(\widehat{\pi}_t^i\right)P_{t-1}$ and $\widehat{\pi}_t^i = \mu + \varepsilon_{it}$ with $\varepsilon_{it} \sim N\left(0,\sigma^2\right)$ in equation (2), we can rewrite the operation constraint as:

$$\varepsilon_{it} \geq \ln\left(\tfrac{1}{\alpha\rho}\left(\tfrac{U}{1-\alpha\rho}\right)^{\frac{1-\alpha\rho}{\alpha\rho}}\tfrac{w_t^*}{P_{t-1}^*}\right) - \mu \equiv X \tag{7}$$

Combining equations (6) and (7) with the wage equation in (4) then yields an implicit definition of X which is independent of μ:

$$X = \tfrac{1-\alpha\rho}{\alpha\rho}\ln\left(\tfrac{U}{1-\alpha\rho}\right) + \tfrac{\sigma^2}{2(1-\alpha\rho)} + (1-\alpha\rho)\ln\Phi\left(\tfrac{\sigma}{1-\alpha\rho} - \tfrac{X}{\sigma}\right) \tag{8}$$

Turn now to inflation. Substitute the firm pricing equation (1) into the price aggregator (5) and simplify to get:

$$\pi_t^* = \tfrac{1-\rho}{\rho}\ln\left(\tfrac{1-\alpha\rho}{U}\right) + \tfrac{\alpha(1-\rho)(X+\mu)}{1-\alpha\rho} + \ln\left(\frac{\int\limits_{\varepsilon_{it}\geq X}\exp\left(\min\left\{\frac{\widehat{\pi}_t^i}{1-\rho},\frac{\pi_t^*+\ln(\gamma_t)}{1-\rho}\right\} - \frac{\rho(1-\alpha)\widehat{\pi}_t^i}{(1-\rho)(1-\alpha\rho)}\right)di}{\int\limits_{\varepsilon_{it}\geq X}\exp\left(\min\left\{\frac{\widehat{\pi}_t^i}{1-\rho},\frac{\pi_t^*+\ln(\gamma_t)}{1-\rho}\right\} - \frac{(1-\alpha)\widehat{\pi}_t^i}{(1-\rho)(1-\alpha\rho)}\right)di}\right) \tag{9}$$

Combining equations (8) and (9) then yields:

$$\begin{aligned}\pi_t^* \;=\; & \tfrac{\alpha(1-\rho)}{1-\alpha\rho}\left(\mu + \tfrac{\sigma^2}{2(1-\alpha\rho)}\right) + \alpha(1-\rho)\ln\Phi\left(\tfrac{\sigma}{1-\alpha\rho} - \tfrac{X}{\sigma}\right) \\ & + \ln\left(\frac{\int\limits_{\varepsilon_{it}\geq X}\exp\left(\min\left\{\frac{\widehat{\pi}_t^i}{1-\rho},\frac{\pi_t^*+\ln(\gamma_t)}{1-\rho}\right\} - \frac{\rho(1-\alpha)\widehat{\pi}_t^i}{(1-\rho)(1-\alpha\rho)}\right)di}{\int\limits_{\varepsilon_{it}\geq X}\exp\left(\min\left\{\frac{\widehat{\pi}_t^i}{1-\rho},\frac{\pi_t^*+\ln(\gamma_t)}{1-\rho}\right\} - \frac{(1-\alpha)\widehat{\pi}_t^i}{(1-\rho)(1-\alpha\rho)}\right)di}\right)\end{aligned} \tag{10}$$

Now use $\widehat{\pi}_t^i = \mu + \varepsilon_{it}$ and $\varepsilon_{it} \sim N\left(0,\sigma^2\right)$ with (6) to simplify (10). It will help to define:

$$\Upsilon(X,\sigma) \equiv \frac{\Phi\left(-\frac{(1-\alpha)\sigma}{(1-\rho)(1-\alpha\rho)} - \frac{X}{\sigma}\right)\left[\Phi\left(\frac{\sigma}{1-\alpha\rho} - \frac{X}{\sigma}\right)\exp\left(\frac{[2-\alpha(1+\rho)]\sigma^2}{2(1-\rho)(1-\alpha\rho)^2}\right)\right]^{1-\alpha}\left(\frac{U}{1-\alpha\rho}\right)^{\frac{1-\alpha\rho}{\alpha\rho}}}{\Phi\left(-\frac{\rho(1-\alpha)\sigma}{(1-\rho)(1-\alpha\rho)} - \frac{X}{\sigma}\right)} \tag{11}$$

If $\gamma_t \leq \Upsilon(X,\sigma)$, then:

$$\pi_t^* - \mu = \frac{(\alpha-\rho)(1-\alpha\rho)-(1-\alpha)^2}{(1-\rho)(1-\alpha\rho)^2}\frac{\sigma^2}{2} + \alpha(1-\rho)\ln\Phi\left(\frac{\sigma}{1-\alpha\rho} - \frac{X}{\sigma}\right) + \ln\left(\frac{\Phi\left(-\frac{\rho(1-\alpha)\sigma}{(1-\rho)(1-\alpha\rho)} - \frac{X}{\sigma}\right)}{\Phi\left(-\frac{(1-\alpha)\sigma}{(1-\rho)(1-\alpha\rho)} - \frac{X}{\sigma}\right)}\right) \quad (12)$$

Otherwise, $\pi_t^* - \mu$ solves:

$$\begin{aligned}\pi_t^* - \mu \quad = \quad & \frac{1-\alpha^2+\alpha(1-\rho)}{(1-\alpha\rho)^2}\frac{\sigma^2}{2} + \alpha(1-\rho)\ln\Phi\left(\frac{\sigma}{1-\alpha\rho} - \frac{X}{\sigma}\right) \quad (13)\\ & + \ln\left(\frac{\Phi\left(\frac{\sigma}{1-\alpha\rho} - \frac{X}{\sigma}\right) - \Phi\left(\frac{\sigma}{1-\alpha\rho} - \frac{\pi_t^*-\mu+\ln(\gamma_t)}{\sigma}\right) + \exp\left(\frac{\pi_t^*-\mu+\ln(\gamma_t)}{1-\rho} - \frac{(1+\alpha\rho-2\rho)\sigma^2}{2(1-\rho)^2(1-\alpha\rho)}\right)\Phi\left(-\frac{\rho(1-\alpha)\sigma}{(1-\rho)(1-\alpha\rho)} - \frac{\pi_t^*-\mu+\ln(\gamma_t)}{\sigma}\right)}{\Phi\left(\frac{\alpha\sigma}{1-\alpha\rho} - \frac{X}{\sigma}\right) - \Phi\left(\frac{\alpha\sigma}{1-\alpha\rho} - \frac{\pi_t^*-\mu+\ln(\gamma_t)}{\sigma}\right) + \exp\left(\frac{\pi_t^*-\mu+\ln(\gamma_t)}{1-\rho} + \frac{(1+\alpha\rho-2\alpha)\sigma^2}{2(1-\rho)^2(1-\alpha\rho)}\right)\Phi\left(-\frac{(1-\alpha)\sigma}{(1-\rho)(1-\alpha\rho)} - \frac{\pi_t^*-\mu+\ln(\gamma_t)}{\sigma}\right)}\right)\end{aligned}$$

Either way, we have a definition of $\pi_t^* - \mu$ which is independent of μ. ■

Proof of Lemma 1

Impose $\alpha = 1$ on equations (1), (2), and (3) to get $p\left(w_t; \widehat{P}_t^i\right) = \frac{w_t}{\rho}$, $O_t(w_t) = [0,1]$, and $\ell\left(w_t; \widehat{P}_t^i\right) = \left(\frac{\rho\widehat{P}_t^i}{w_t}\right)^{\frac{1}{1-\rho}}$. Substituting $p\left(w_t^*; \widehat{P}_t^i\right)$ into equation (5) gives $P_t^* = \frac{w_t^*}{\rho}$ and substituting $\ell\left(w_t^*; \widehat{P}_t^i\right)$ into equation (4) gives $\frac{w_t^*}{\rho} = \left[\int\left(\widehat{P}_t^i\right)^{\frac{1}{1-\rho}} di\right]^{1-\rho}$. Combining these two expressions and using the definitions of $\widehat{\pi}_t^i$ and π_t^* then yields $\pi_t^* = (1-\rho)\ln\left(\int\exp\left(\frac{\widehat{\pi}_t^i}{1-\rho}\right)di\right)$. With $\widehat{\pi}_t^i \sim N(\mu, \sigma^2)$, we can use the moment generating function of the normal distribution to simplify the preceding integral. The integral is taken over the entire set so the moment generating function just yields $\pi_t^* = \mu + \frac{\sigma^2}{2(1-\rho)}$. ■

Proof of Lemma 2

The fraction of firms not operating is $\Delta \equiv \Phi\left(\frac{X}{\sigma}\right)$. Taking derivatives yields $\frac{d\Delta}{d\sigma} \propto \frac{dX}{d\sigma} - \frac{X}{\sigma}$ so what we want to show is $\frac{dX}{d\sigma} > \frac{X}{\sigma}$. Using equation (8) from the proof of Proposition 1 produces:

$$\frac{dX}{d\sigma} = \frac{\sigma}{1-\alpha\rho} + \frac{\frac{X}{\sigma}}{1 + \frac{\Phi\left(\frac{\sigma}{1-\alpha\rho} - \frac{X}{\sigma}\right)}{\phi\left(\frac{\sigma}{1-\alpha\rho} - \frac{X}{\sigma}\right)}\frac{\sigma}{1-\alpha\rho}} \quad (14)$$

The desired inequality is thus $\left(\frac{\sigma}{1-\alpha\rho} - \frac{X}{\sigma}\right)\frac{\Phi\left(\frac{\sigma}{1-\alpha\rho} - \frac{X}{\sigma}\right)}{\phi\left(\frac{\sigma}{1-\alpha\rho} - \frac{X}{\sigma}\right)} > -1$. Showing $x\Phi(x) > -\phi(x)$ completes the proof: $x\Phi(x) = x\int_{-\infty}^{x}\phi(t)\,dt > \int_{-\infty}^{x}t\phi(t)\,dt = -\int_{-\infty}^{x}\phi'(t)\,dt = -\phi(x)$. ■

Proof of Lemma 3

If $\alpha = 1$, then the equations in the proof of Proposition 1 reduce to:

$$X = \tfrac{1-\rho}{\rho} \ln\left(\tfrac{U}{1-\rho}\right) + \tfrac{\sigma^2}{2(1-\rho)} + (1-\rho)\ln\Phi\left(\tfrac{\sigma}{1-\rho} - \tfrac{X}{\sigma}\right) \tag{15}$$

$$\pi_t^* - \mu = \tfrac{\sigma^2}{2(1-\rho)} + (1-\rho)\ln\Phi\left(\tfrac{\sigma}{1-\rho} - \tfrac{X}{\sigma}\right) \tag{16}$$

We can thus write $f(\sigma) = X - \frac{1-\rho}{\rho}\ln\left(\frac{U}{1-\rho}\right)$ with X dependent on σ as per (15). To make this dependency explicit, we further write $X(\sigma)$ in place of just X. Consider any $\sigma_0 > 0$ satisfying $f(\sigma_0) = 0$. That is, consider any $\sigma_0 > 0$ satisfying $X(\sigma_0) = \frac{1-\rho}{\rho}\ln\left(\frac{U}{1-\rho}\right)$. If $U \geq 1-\rho$, then $X(\sigma_0) \geq 0$ which, given $\frac{dX}{d\sigma} > \frac{X}{\sigma}$ from the proof of Lemma 2, implies $X'(\sigma_0) > 0$. Notice $f'(\cdot) = X'(\cdot)$. This means that, if $U \geq 1-\rho$, then any $\sigma_0 > 0$ satisfying $f(\sigma_0) = 0$ must also satisfy $f'(\sigma_0) > 0$. There is thus at most one $\sigma_0 > 0$ such that $f(\sigma_0) = 0$. To show exactly one such $\sigma_0 > 0$, it will suffice to show $\lim\limits_{\sigma\to 0^+} f(\sigma) < 0$ and $\lim\limits_{\sigma\to\infty} f(\sigma) > 0$. Equation (15) yields $X(0) \equiv \lim\limits_{\sigma\to 0^+} X(\sigma) = (1-\rho)\left[\frac{1}{\rho}\ln\left(\frac{U}{1-\rho}\right) + \ln\Phi\left(\lim\limits_{\sigma\to 0^+}\frac{-X(\sigma)}{\sigma}\right)\right]$. Notice that $X(0) > 0$ is impossible while $X(0) < 0$ is only possible if $U < 1-\rho$. Therefore, $U \geq 1-\rho$ implies $X(0) = 0$ and thus $\lim\limits_{\sigma\to 0^+} f(\sigma) = -\frac{1-\rho}{\rho}\ln\left(\frac{U}{1-\rho}\right) < 0$. Equation (15) also yields $\lim\limits_{\sigma\to\infty} X(\sigma) = \infty$ and thus $\lim\limits_{\sigma\to\infty} f(\sigma) = \infty$. Putting everything together, we can now conclude that there is exactly one $\sigma_0 > 0$ such that $f(\sigma_0) = 0$. Moreover, $f'(\sigma_0) > 0$.

The exit channel behind this result depends on heterogeneity, not on $\widehat{\pi}_t^i$ being a point expectation. To see this, consider two types of firms $j \in \{1, 2\}$. The fraction of type 1 firms is τ and the fraction of type 2 firms is $1-\tau$. A type j firm takes inflation expectations over the entire distribution $N\left(\mu, \sigma_j^2\right)$, with the resulting CDF for its price expectations denoted by $F_j(\cdot)$. The price-setting problem with $\alpha = 1$ still yields $P_t^* = \frac{w_t^*}{\rho}$ but the labor demand of a type j firm is now $\left(\frac{\rho}{w_t^*}\right)^{\frac{1}{1-\rho}} \int \widehat{P}_t^{\frac{1}{1-\rho}} dF_j\left(\widehat{P}_t\right)$ and operation requires:

$$\tfrac{w_t^*}{\rho} \leq \left[\tfrac{1-\rho}{U} \int \widehat{P}_t^{\frac{\rho}{1-\rho}} dF_j\left(\widehat{P}_t\right)\right]^{\frac{1-\rho}{\rho}} \tag{17}$$

If only one type operates (say $j = 1$), then labor market clearing yields:

$$\tfrac{w_t^*}{\rho} = \left[\tau \int \widehat{P}_t^{\frac{1}{1-\rho}} dF_1\left(\widehat{P}_t\right)\right]^{1-\rho} \tag{18}$$

To ensure that only type 1 firms operate, we need (17) with $\frac{w_t^*}{\rho}$ as per (18) to hold at $j = 1$

but not at $j = 2$. Stated otherwise, we need:

$$\frac{1-\rho}{\tau^{\rho}\exp\left(\frac{\rho\left(\sigma_1^2-\rho\sigma_2^2\right)}{2(1-\rho)^2}\right)} < U \leq \frac{1-\rho}{\tau^{\rho}\exp\left(\frac{\rho\sigma_1^2}{2(1-\rho)}\right)}$$

A necessary condition is $\frac{\rho\left(\sigma_1^2-\rho\sigma_2^2\right)}{2(1-\rho)^2} > \frac{\rho\sigma_1^2}{2(1-\rho)}$ or, equivalently, $\sigma_1 > \sigma_2$.[22] Combining $P_t^* = \frac{w_t^*}{\rho}$ with (18), we can now write:

$$\pi_t^* = \mu + \frac{\sigma_1^2}{2(1-\rho)} + (1-\rho)\ln\tau$$

If $\tau = 1$, then all firms use the same prior so everyone operates and we again have $\pi_t^* > \mu$. If $\tau \in (0,1)$, then only type 1 firms operate and $\pi_t^* = \mu$ provided $\sigma_1 = (1-\rho)\sqrt{2\ln\left(\frac{1}{\tau}\right)}$. ■

Proof of Proposition 2

Define $\overline{U} \equiv 1 - \alpha\rho$. It will suffice to establish the result for some subset of $\left(0, \overline{U}\right)$. At $\sigma = 0$, equations (12) and (13) both reduce to:

$$f(0) \equiv \lim_{\sigma\to 0^+} f(\sigma) = \alpha(1-\rho)\ln\Phi\left(\lim_{\sigma\to 0^+}\frac{-X(\sigma)}{\sigma}\right) \tag{19}$$

If $\lim_{\sigma\to 0^+}\frac{-X(\sigma)}{\sigma} = \infty$, then $f(0) = 0$. Moreover, (12) and (13) will both also produce:

$$f'(0) \equiv \lim_{\sigma\to 0^+} f'(\sigma) = \alpha(1-\rho)\lim_{\sigma\to 0^+}\frac{\phi\left(-\frac{X(\sigma)}{\sigma}\right)}{\Phi\left(-\frac{X(\sigma)}{\sigma}\right)}\frac{1}{\sigma}\left(\frac{X(\sigma)}{\sigma} - X'(\sigma)\right) = \frac{\alpha(1-\rho)}{1-\alpha\rho}\lim_{\sigma\to 0^+} X'(\sigma) \tag{20}$$

where the last equality follows from using equation (14). Turn now to $X(\cdot)$. At $\sigma = 0$, equation (8) yields:

$$X(0) \equiv \lim_{\sigma\to 0^+} X(\sigma) = \overline{U}\left[\frac{1}{\alpha\rho}\ln\left(\frac{U}{\overline{U}}\right) + \ln\Phi\left(\lim_{\sigma\to 0^+}\frac{-X(\sigma)}{\sigma}\right)\right] \tag{21}$$

Notice from (21) that $X(0) > 0$ is impossible while $X(0) < 0$ is only possible if $U < \overline{U}$. Therefore, $U = \overline{U}$ implies $X(0) = 0$ and thus $\lim_{\sigma\to 0^+}\frac{-X(\sigma)}{\sigma} = \infty$. Since $X'(0) \equiv \lim_{\sigma\to 0^+} X'(\sigma) \cong \frac{X(h)-X(0)}{h-0} = \frac{X(h)}{h} \xrightarrow{h\to 0^+} -\infty$, it now follows that $f(0) = 0$ and $f'(0) < 0$ when $U = \overline{U}$. Taken together, $f(0) = 0$ and $f'(0) < 0$ imply existence of a $\sigma > 0$ such that $f(\sigma) < 0$. Combined with $\lim_{\sigma\to\infty} f(\sigma) = \infty$, this then implies existence of a $\sigma_R > 0$ satisfying $f(\sigma_R) = 0$ and $f'(\sigma_R) > 0$. For $f(\cdot)$ continuous in U, we can thus find an $\epsilon > 0$ such that there also exists a $\sigma_R > 0$ satisfying $f(\sigma_R) = 0$ and $f'(\sigma_R) > 0$ when $U \in \left(\overline{U} - \epsilon, \overline{U}\right)$. To show existence

[22] Recall that this derivation assumes $\alpha = 1$. With lower values of α, it is possible to get $\sigma_1 < \sigma_2$.

of a $\sigma_F \in (0, \sigma_R)$ satisfying $f(\sigma_F) = 0$ and $f'(\sigma_F) < 0$, it will suffice to show $f(0) = 0$ and $f'(0) > 0$ when $U \in \left(\overline{U} - \epsilon, \overline{U}\right)$. If $f'(0) = 0$, then it will suffice to show $f(0) = 0$ and $f''(0) \equiv \lim_{\sigma \to 0^+} f''(\sigma) > 0$. For any $U \in \left(0, \overline{U}\right)$, we have $X(0) \in (-\infty, 0)$ from (21) and thus $f(0) = 0$ from (19). We also have $X'(0) = \frac{1-\alpha\rho}{X(0)} \lim_{\sigma \to 0^+} \left(\frac{X(\sigma)}{\sigma}\right)^2 \phi\left(-\frac{X(\sigma)}{\sigma}\right) = 0$ from (14) and the property $\lim_{z \to -\infty} z^2 \phi(-z) = 0$. Therefore, equation (20) yields $f'(0) = 0$ and it remains to show $f''(0) > 0$. With $\sigma = 0$ and $\lim_{\sigma \to 0^+} \frac{-X(\sigma)}{\sigma} = \infty$, equation (11) implies $\Upsilon(X(0), 0) = \left(\frac{U}{\overline{\overline{U}}}\right)^{\frac{1-\alpha\rho}{\alpha\rho}} < 1$ so, for small taste shocks, the behavior of $f''(\cdot)$ around zero is dictated by equation (13). After some algebra (available upon request), we obtain $f''(0) = \frac{(1-\alpha)(\alpha-\rho)+\alpha(1-\rho)^2}{(1-\alpha\rho)^2(1-\rho)}$ which is positive for $\alpha \in \left(1 - \frac{\rho(1-\rho)}{2} - \sqrt{\left(1 - \frac{\rho(1-\rho)}{2}\right)^2 - \rho}, 1\right) \equiv (\underline{\alpha}, 1)$. Therefore, there must exist a $\sigma_F \in (0, \sigma_R)$ satisfying $f(\sigma_F) = 0$ and $f'(\sigma_F) < 0$ when $U \in \left(\overline{U} - \epsilon, \overline{U}\right)$ and $\alpha \in (\underline{\alpha}, 1)$. ■

Proof of Proposition 3

We first characterize π_t^* for any $\alpha \leq 1$. We then establish the properties under $\alpha = 1$.

General Characterization To simplify notation, define:

$$y_t \equiv \frac{\pi_t^* - \mu}{1-\alpha\rho},\ x_t \equiv \frac{\ln\left(\frac{1}{\alpha\rho}\left(\frac{U}{1-\alpha\rho}\right)^{\frac{1-\alpha\rho}{\alpha\rho}} \frac{w_t^*}{P_{t-1}^*}\right) - \mu}{1-\alpha\rho},\ \text{and } \upsilon_j \equiv \frac{\sigma_j}{1-\alpha\rho}$$

The variable y_t is just excess inflation scaled up by a constant. The variable x_t provides a more compact way to express the operation condition in equation (2). In particular, if the difference between a firm's inflation forecast and the mean forecast is greater than or equal to x_t, then the firm operates. With a mixture of normal expectations, equations (2) and (4) yield x_t implicitly defined by:[23]

$$x_t = \frac{1}{\alpha\rho} \ln\left(\frac{U}{1-\alpha\rho}\right) + \ln\left[\xi_t \exp\left(\frac{\upsilon_F^2}{2}\right) \Phi\left(\upsilon_F - \frac{x_t}{\upsilon_F}\right) + (1-\xi_t) \exp\left(\frac{\upsilon_R^2}{2}\right) \Phi\left(\upsilon_R - \frac{x_t}{\upsilon_R}\right)\right]$$

The expression for excess inflation then comes from equation (5). If the realized taste shock is small enough to support $\gamma_t P_t^* \leq \widehat{P}_t^i$ for all operating firms, then $\gamma_t P_t^*$ will drop out of the consumption weights, leaving P_t^* and y_t^* explicitly defined. Otherwise, P_t^* and y_t^* will be

[23] The derivations that follow parallel those in the proof of Proposition 1 and are thus omitted. The only difference is the use of a mixture of normals rather than a single normal when evaluating any integrals.

implicitly defined. The threshold γ_t works out to:

$$\Upsilon(x_t,\xi_t) \equiv \frac{\left[\xi_t \exp\left(\frac{\upsilon_F^2}{2}\right)\Phi\left(\upsilon_F-\frac{x_t}{\upsilon_F}\right)+(1-\xi_t)\exp\left(\frac{\upsilon_R^2}{2}\right)\Phi\left(\upsilon_R-\frac{x_t}{\upsilon_R}\right)\right]^{1-\alpha}\left(\frac{U}{1-\alpha\rho}\right)^{\frac{1-\alpha\rho}{\alpha\rho}}}{\frac{\xi_t \exp\left(\frac{\left(\frac{\rho(1-\alpha)\upsilon_F}{1-\rho}\right)^2}{2}\right)\Phi\left(-\frac{\rho(1-\alpha)\upsilon_F}{1-\rho}-\frac{x_t}{\upsilon_F}\right)+(1-\xi_t)\exp\left(\frac{\left(\frac{\rho(1-\alpha)\upsilon_R}{1-\rho}\right)^2}{2}\right)\Phi\left(-\frac{\rho(1-\alpha)\upsilon_R}{(1-\rho)}-\frac{x_t}{\upsilon_R}\right)}{\xi_t \exp\left(\frac{\left(\frac{(1-\alpha)\upsilon_F}{1-\rho}\right)^2}{2}\right)\Phi\left(-\frac{(1-\alpha)\upsilon_F}{1-\rho}-\frac{x_t}{\upsilon_F}\right)+(1-\xi_t)\exp\left(\frac{\left(\frac{(1-\alpha)\upsilon_R}{1-\rho}\right)^2}{2}\right)\Phi\left(-\frac{(1-\alpha)\upsilon_R}{1-\rho}-\frac{x_t}{\upsilon_R}\right)}}$$

If $\gamma_t \leq \Upsilon(x_t,\xi_t)$, then $y_t = x_t - \frac{\ln \Upsilon(x_t,\xi_t)}{1-\alpha\rho}$. Otherwise, y_t solves:

$$y_t = \frac{\alpha(1-\rho)}{1-\alpha\rho}\left[x_t - \frac{1}{\alpha\rho}\ln\left(\frac{U}{1-\alpha\rho}\right)\right] + \frac{1}{1-\alpha\rho}\ln\left(\frac{\xi_t h\left(x_t,y_t,\gamma_t,\upsilon_F,1,\frac{\rho(1-\alpha)}{1-\rho}\right)+(1-\xi_t)h\left(x_t,y_t,\gamma_t,\upsilon_R,1,\frac{\rho(1-\alpha)}{1-\rho}\right)}{\xi_t h\left(x_t,y_t,\gamma_t,\upsilon_F,\alpha,\frac{1-\alpha}{1-\rho}\right)+(1-\xi_t)h\left(x_t,y_t,\gamma_t,\upsilon_R,\alpha,\frac{1-\alpha}{1-\rho}\right)}\right)$$

where

$$\begin{aligned} h(x_t,y_t,\gamma_t,\upsilon,\beta,\delta) \equiv{}& \exp\left(\frac{(\beta\upsilon)^2}{2}\right)\left[\Phi\left(\beta\upsilon-\frac{x_t}{\upsilon}\right)-\Phi\left(\beta\upsilon-\frac{y_t+\frac{\ln(\gamma_t)}{1-\alpha\rho}}{\upsilon}\right)\right] \\ &+\exp\left(\frac{(\delta\upsilon)^2}{2}+\frac{(1-\alpha\rho)\left(y_t+\frac{\ln(\gamma_t)}{1-\alpha\rho}\right)}{1-\rho}\right)\Phi\left(-\delta\upsilon-\frac{y_t+\frac{\ln(\gamma_t)}{1-\alpha\rho}}{\upsilon}\right) \end{aligned}$$

Notice that the limiting cases of $\xi_t = 0$ and $\xi_t = 1$ return the model with expectations characterized by a single normal distribution.

Special Case Suppose $\alpha = 1$. Defining $\lambda \equiv \frac{1}{\rho}\ln\left(\frac{U}{1-\rho}\right)$, the mixture equations derived above reduce to:

$$\exp(y_t) = \xi_t \exp\left(\frac{\upsilon_F^2}{2}\right)\Phi\left(\upsilon_F - \frac{y_t+\lambda}{\upsilon_F}\right) + (1-\xi_t)\exp\left(\frac{\upsilon_R^2}{2}\right)\Phi\left(\upsilon_R - \frac{y_t+\lambda}{\upsilon_R}\right) \tag{22}$$

Under $\xi_t = 0$, equation (22) yields $\exp(y_t) = \exp\left(\frac{\upsilon_R^2}{2}\right)\Phi\left(\upsilon_R - \frac{y_t+\lambda}{\upsilon_R}\right)$. Under $\xi_t = 1$, it yields $\exp(y_t) = \exp\left(\frac{\upsilon_F^2}{2}\right)\Phi\left(\upsilon_F - \frac{y_t+\lambda}{\upsilon_F}\right)$. Since $y_t = f(\sigma_R) = 0$ at $\xi_t = 0$ and $y_t = f(\sigma_F) = 0$ at $\xi_t = 1$, it follows that $\exp\left(\frac{\upsilon_F^2}{2}\right)\Phi\left(\upsilon_F - \frac{\lambda}{\upsilon_F}\right) = 1$ and $\exp\left(\frac{\upsilon_R^2}{2}\right)\Phi\left(\upsilon_R - \frac{\lambda}{\upsilon_R}\right) = 1$. Consider now $\xi_t \in (0,1)$. If $y_t < 0$, then $\exp\left(\frac{\upsilon_i^2}{2}\right)\Phi\left(\upsilon_i - \frac{y_t+\lambda}{\upsilon_i}\right) > \exp\left(\frac{\upsilon_i^2}{2}\right)\Phi\left(\upsilon_i - \frac{\lambda}{\upsilon_i}\right) = 1$ for $i \in \{F,R\}$ so equation (22) implies $y_t > 0$ which is a contradiction. If $y_t > 0$, then $\exp\left(\frac{\upsilon_i^2}{2}\right)\Phi\left(\upsilon_i - \frac{y_t+\lambda}{\upsilon_i}\right) < \exp\left(\frac{\upsilon_i^2}{2}\right)\Phi\left(\upsilon_i - \frac{\lambda}{\upsilon_i}\right) = 1$ for $i \in \{F,R\}$ so equation (22) implies $y_t < 0$ which is a contradiction. Therefore, $y_t = 0$ for $\xi_t \in (0,1)$. ■