NBER WORKING PAPER SERIES

BEQUESTS AND HETEROGENEITY IN RETIREMENT WEALTH

Mariacristina De Nardi
Fang Yang

Working Paper 20058
http://www.nber.org/papers/w20058

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 2014

We thank Michele Boldrin, John Boyd, V. V. Chari, Betty Daniel, Zvi Eckstein, Helen Koshy, Ayse Imrohoroglu, John B. Jones, Larry Jones, an assistant editor, two referees, and seminar participants at various seminars and conferences for helpful comments and suggestions. Yang acknowledges support by the Center for Retirement Research at Boston College pursuant to grants from the U.S. Social Security Administration (SSA), funded as part of the Retirement Research Consortium (RRC). The findings and conclusions expressed are solely those of the authors and do not represent the views of SSA, any agency of the Federal Government, the Federal Reserve Bank of Chicago, the RRC, Boston College, or the National Bureau of Economic Research.

NBER working papers are circulated for discussion and comment purposes. They have not been peer-reviewed or been subject to the review by the NBER Board of Directors that accompanies official NBER publications.

© 2014 by Mariacristina De Nardi and Fang Yang. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Bequests and Heterogeneity in Retirement Wealth
Mariacristina De Nardi and Fang Yang
NBER Working Paper No. 20058
April 2014
JEL No. E21,J14

ABSTRACT

Households hold vastly heterogenous amounts of wealth when they reach retirement, and differences in lifetime earnings explain only part of this variation. This paper studies the role of intergenerational transmission of ability, voluntary bequest motives, and the recipiency of accidental and intended bequests (both in terms of timing and size), in generating wealth dispersion at retirement, in the context of a rich quantitative model. Modeling voluntary bequests, and realistically calibrating them, not only generates more wealth dispersion at retirement and reduces the correlation between retirement wealth and lifetime income, but also generates a skewed bequest distribution that is close to the one in the observed data.

Mariacristina De Nardi
Federal Reserve Bank of Chicago
230 South LaSalle St.
Chicago, IL 60604
and University College London
and also NBER
denardim@nber.org

Fang Yang
Louisiana State University
Department of Economics, 2322
Business Education Complex,
Nicholson Extension
Baton Rouge, LA 70803
Email: fyang@lsu.edu
fyang@albany.edu

1 Introduction

Why do U.S. households reach retirement with vastly different wealth levels, even when we condition on realized lifetime income? Can we construct a model that explains this fact? What are the important features that help generate this heterogeneity?

The answers to these questions help improve our understanding of the factors affecting savings, and thus help inform many kinds of policy reforms, including taxation and social insurance reforms. As Heckman, Lochner, and Taber (1998) wrote, "It is potentially very dangerous to 'solve' problems whose origins are not well understood."

Our main contribution is to study the role of intergenerational links, in the form of intergenerational transmission of ability and both accidental and voluntary bequests, in shaping wealth dispersion at retirement and its correlation with lifetime income. The aggregate importance of wealth transmitted across generations calls for an investigation of the impact of these transfers on individual savings, wealth inequality, and its interaction with earnings shocks over the life cycle. To do so, we study these elements in the context of a rich, quantitative, overlapping generations model with incomplete markets, which features earnings risks and incorporates the key institutional policies providing public insurance for these risks. The interaction of risks and insurance, in fact, affects saving motives and, thus, observed wealth accumulation in an interesting and non-trivial way.

Besides voluntary and accidental bequests, and transmission of ability from parents to children, other factors have been shown to be important to understanding saving behavior and cross-sectional wealth inequality in the U.S. However, we know little about the effects of these elements on wealth holdings at retirement. These institutional factors include a government-provided minimum consumption (Hubbard et al. (1995)), defined benefit pensions, and a more realistic modeling of Social Security rules (Scholz et al. (2006)). We explicitly model all of these factors, as they might have important effects directly, but might also interact with the effects of intergenerational links on savings at retirement, thus providing a misleading picture of the effects of intergenerational links, should these important programs not be accounted for.

In our framework, after controlling for lifetime earnings, wealth differences at retirement arise for the following two reasons. First, when borrowing constraints prevent households from smoothing consumption intertemporally, households that differ in the timing of earnings over the life cycle will have very different wealth levels at retirement. Second, inheritances add another source of wealth heterogeneity among households with similar lifetime earnings. Our calibrated bequest motive implies that bequests are a luxury good, and thus increases the desire to leave bequests for households receiving large bequests, thereby increasing their saving rate and leading to more wealth inequality and less correlation of wealth to lifetime earnings.

Our main finding is that the model with intergenerational links matches the data well. More specifically, it generates an average of the Gini coefficients for wealth, after controlling for lifetime income, of 0.53, compared with .52 in the data; and a correlation coefficient between lifetime earnings and retirement wealth of 0.75, which is a bit higher than in the data (0.62), but lower than in the version of the model that does not allow for intergenerational links. Adding a modest amount of measurement error in earnings and wealth further reduces this correlation to 0.71.

Removing the voluntary bequest motive, even the in presence of different assumptions about the receipt of involuntary bequests, increases the gap between the model and the data. To make this point, we compare the benchmark model with all intergenerational links, including voluntary bequests, with two models without voluntary bequest motives, which differ in the timing and size of accidental inheritances receipt. First, accidental bequests of different sizes are received at age 50. Second, accidental bequests of different sizes are randomly received at realistic ages between 35 and 55. The versions of the model without voluntary bequest motives generate average Gini coefficients for wealth that, controlling for lifetime income, are 0.46 and 0.46, respectively, which are lower than the .52 observed in the data. They also imply a tighter relationship between lifetime earnings and retirement wealth, and more specifically, the correlation coefficients between these variables are 0.81 and 0.80, respectively, compared with 0.62 in the data.

In contrast, as one might have expected, the intergenerational transmission of earnings ability from parents to children tends to increase the correlation between wealth and lifetime earnings at retirement. In fact, the model with bequest motives, but without intergenerational transfer of productivity, implies a correlation of 0.73, which is lower than the 0.75 in the benchmark calibration. Without intergenerational transmission of earnings ability, the distribution of inheritance received is independent of households earnings; this implies that poorer people tend to receive comparatively larger inheritances, which helps reduce the correlation coefficient between lifetime earnings and retirement wealth.

We also investigate the distributional effects of means-tested minimum consumption programs in old age, Social Security, and pensions. Eliminating the realistically calibrated consumption floor from our benchmark calibration results in the income-poor households having more of an incentive to save to insure against bad income shocks; this tends to decrease wealth inequality and generates a lower average Gini coefficient, after controlling for lifetime income, of 0.49, compared with 0.53 in the benchmark model, while the correlation of wealth at retirement and permanent income barely changes.

Removing private pensions raises the average wealth of the households in the high earnings deciles, as they save much more to smooth consumption over their lifetime. As a result, the Gini coefficients decrease a lot at the higher earnings deciles, and the average of the Gini coefficients across income deciles drops to 0.49. For the same reason, the correlation coefficient between retirement wealth and lifetime income increases to 0.79. If, on top of removing private pensions, one were to switch from a history-dependent Social Security system, the wealth Gini coefficients for the lower earnings deciles would increase, while the ones for the highest earners would decrease. On net, the average of the Gini coefficients across income deciles would barely change, while the correlation coefficient between wealth and lifetime earnings would increase to 0.82.

The paper is organized as follows. Section 2 frames our contribution in the context of the literature. Section 3 presents the model and the calibration of the model. Section 4 shows the quantitative results of the benchmark model and investigates the role of various features of the model. Section 5 concludes.

2 Related Literature

Many authors document that households with similar characteristics, such as lifetime income, age, and family structure, hold vastly different amounts of wealth at retirement (see Hurst et al. (1998), and Grafova et al. (2006)). Hendricks (2007a) shows that the correlation coefficient between lifetime earnings and wealth at retirement (0.61) is much less than unity, and that substantial wealth differences remain after controlling for lifetime earnings and age. In fact, the average of the Gini coefficients in wealth at retirement across lifetime earnings deciles is 0.54, compared with 0.62 in the full sample.

Several economists (for example Bernheim et al. (2001) and Hendricks (2007a)) argue that these features of the data are inconsistent with most life-cycle models of consumption-saving behavior, and thus constitute a challenge to such theories and their policy implications. Our goal is to see how close we can get to the observed data once we allow for a rich model with earnings risk and many features of public insurance, which also includes intergenerational transmission of physical and human capital across generations.

An extensive literature, both empirical and theoretical, shows that the transmission of physical and human capital from parents to children is a very important determinant of households' wealth in the aggregate economy (see Kotlikoff and Summers (1981) and Gale and Scholz (1994)), and of both wealth and earnings ability over the household's life cycle (see Hurd and Smith (1999) and Becker and Tomes (1986)). As a result, it is also a prime candidate to investigate retirement savings.

This paper builds on a large literature that studies the cross-sectional wealth inequality at all ages (see Huggett (1996), Quadrini (2000), Casteneda et al. (2003), De Nardi (2004), and Cagetti and De Nardi (2006)). It is also related to studies on retirement saving. Engen et al. (1999), Engen et al. (2004), and Scholz et al. (2006) study the adequacy of household retirement saving. Those papers abstract from the intergenerational links of bequests and earnings ability. Gokhale et al. (2001) abstract from voluntary bequest motives.

We build on work by Hendricks (2007a) by introducing transmission of both physical and human capital across generations. These features, once realistically calibrated, imply that the households receive an inheritance at a random time, whose timing is consistent with their parent's life expectancy, and whose size is consistent both with their parents' estate and with the distributions of estates left in the U.S. economy. In our framework, therefore, households expect to receive and do receive different amounts of inheritances at different times. In addition, due to the luxury voluntary bequest motive coming out of our calibration, people who either receive high labor incomes or large bequests hold onto more wealth due to a voluntary bequest motive, which is stronger for richer people.

3 The Model

The model is a discrete-time, incomplete markets, overlapping generations economy with an infinitely lived government.

3.1 Government

The government taxes capital and labor income at rates τ_a and τ_l, respectively, to finance government spending and government transfers. It also taxes earnings at rate τ_{ss} to finance Social Security. These two government budgets are balanced during each period. The Social Security benefits that agents receive, $P(\widetilde{y})$, are linked to their realized average annual earnings $\widetilde{y}$.

3.2 Firm, Technology, and Defined Benefit Plan

There is one representative firm producing goods, which also maintains a defined benefit plan, that is financed by contributions on each worker's behalf at the rate τ_{DB}. The defined benefit pension budget is balanced at each period.

Retired households receive pensions from the defined benefit plan each period until they die. These pension benefits, $DB(\widetilde{y})$, are linked to each individual's realized average annual earnings.

3.3 Demographics, Preferences, and Labor Productivity

Each model period lasts five years. Agents start their economic life at the age of 20 ($t = 1$). By age 35, ($t = 4$), the agents' children are born. The agents retire at age 65 ($t = 10$). From that period on, each household faces a positive probability of dying, given by $(1 - p_t)$, which only depends on age.[1] The maximum life span is age 90 ($T = 14$), and the population grows at a constant rate n. Figure 1 displays the structure of the overlapping generations model.

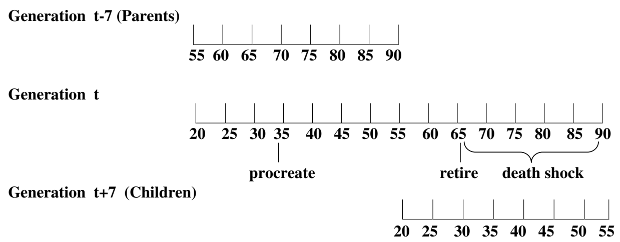


Figure 1: Model Demographics

Preferences are time separable, with a constant discount factor β. The period utility function from consumption is given by $U(c) = (c^{1-\gamma} - 1)/(1-\gamma)$.

Agents also derive utility from the bequests transferred to their children upon death. This form of 'impure' bequest motive implies that an individual cares about total bequests left to his/her children, but not about the consumption of his/her children.

Following De Nardi (2004), the utility from bequests b is denoted by $\phi(b) = \phi_1[(b + \phi_2)^{1-\gamma} - 1]$. The term ϕ_1 measures the strength of bequest motives,

[1] We make the assumption that people do not die before age 65 to reduce computational time. This assumption does not affect the results since in the U.S., the number of adults dying before age 65 is small.

while ϕ_2 reflects the extent to which bequests are luxury goods; if $\phi_2 > 0$, the marginal utility of small bequests is bounded, while the marginal utility of large bequests declines more slowly than the marginal utility of consumption.

Total labor productivity of worker i at age t is given by $y_t^i = e^{z_t^i + \epsilon_t}$, in which ϵ_t is the deterministic age-efficiency profile. The process for the stochastic productivity shock z_t^i is: $z_t^i = \rho_z z_{t-1}^i + \mu_t^i,\ \mu_t^i \sim N(0, \sigma_\mu^2)$.

To capture the intergenerational correlation of earnings, we assume that the productivity of worker i at age 55 is transmitted to children j at age 20 as follows: $z_1^j = \rho_h z_8^i + \nu^j,\ \nu^j \sim N(0, \sigma_h^2)$, as parents are 35 years (seven model periods) older than their children.

3.4 The Household's Recursive Problem

We assume that children have full information about their parents' state variables and infer the size of the bequests that they are likely to receive based on this information. The household's state variables are given by $x = (t, a, z, \widetilde{y}, S_p)$, where a denotes the agent's financial assets carried from the previous period. The last term, S_p, denotes the parent's state variables.

From $t = 1$ to $t = 9$ (from 20 to 60 years of age), the agent survives for sure to the next period. Let $V(t, a, z, \widetilde{y}, S_p)$ and $V^I(t, a, z, \widetilde{y})$ denote the value function of an agent whose parent is alive and dead, respectively, where I stands for "inherited." In the former case, the household's parents are still alive, and might die with probability p_{t+7}, in which case the value function for the orphan household applies, and assets are augmented of inheritances in per-capita terms. That is,

$$(1) \qquad V(t, a, z, \widetilde{y}, S_p) = \max_{c,a'} \Big\{ U(c) + \beta p_{t+7} E\big[V(t+1, a', z', \widetilde{y}', S_p') \\ + \beta(1 - p_{t+7}) E\big[V^I(t+1, a' + a_p'/N, z', \widetilde{y}')\big] \Big\},$$

subject to

$$
\begin{aligned}
&(2) \qquad & c + a' &= (1 - \tau_l - \tau_{ss})wy + [1 + r(1 - \tau_a)]a, \\
&(3) \qquad & a' &\geq 0, \\
&(4) \qquad & \widetilde{y}' &= \big[(t-1)\widetilde{y} + wy/5\big]/t, \\
&(5) \qquad & \widetilde{y}_p' &= \left\{ \begin{array}{ll} \big[(t+6)\widetilde{y}_p + wy_p/5\big]/(t+7) & \text{if } t < 3 \\ \widetilde{y}_p & \text{otherwise} \end{array} \right\} \\
&(6) \qquad & a_p' &= a_p'(S_p),
\end{aligned}
$$

where N is the average number of kids determined by the growth rate of the population. The expected values of the value functions are taken with respect to (z', z_p'), conditional on (z, z_p). The agent's resources depend on labor endowment y and asset holdings a.

To compute defined benefits and Social Security payments, we keep track of yearly lifetime average labor income $\widetilde{y}$. Since current income y refers to a five-year period, current income is divided by five when the yearly lifetime average labor income ($\widetilde{y}$) is updated. Average yearly earnings for children and parents thus evolve according to equations (4) and (5), respectively. Equation (6) is the law of motion of assets for the parents, which uses their optimal decision rule.

The value function of an agent whose parent is dead is

$$
(7) \qquad V^I(t, a, z, \widetilde{y}) = \max_{c, a'} \left\{ U(c) + \beta E\big[V^I(t+1, a', z', \widetilde{y}')\big] \right\},
$$

subject to (2), (3), and (4).

From 65 to 85 years of age ($t = 10$ to $t = 14$), the agent does not work any more and receives Social Security benefits and pensions from the defined benefit plan. He also faces a positive probability of dying, in which case he derives utility from bequeathing the remaining assets. Following Hubbard et al. (1995), we assume that government transfers Tr provide a consumption floor $\underline{c}$ as specified in equation (10).

$$V(t,a,\widetilde{y}) = \max_{c,a'} \left\{ U(c) + \beta p_t V(t+1,a',\widetilde{y}) + (1-p_t)\phi(a') \right\}, \tag{8}$$

subject to (3) and

$$c + a' = [1 + r(1-\tau_a)]a + (1-\tau_l)[P(\widetilde{y}) + DB(\widetilde{y})] + Tr, \tag{9}$$

$$Tr = \max\{0, \underline{c} - [1 + r(1-\tau_a)]a - (1-\tau_l)[P(\widetilde{y}) + DB(\widetilde{y})]\}. \tag{10}$$

3.5 Calibration

	Parameters		Value
Demographics	n	annual population growth	1.2%
	p_t	survival probability	see text
Preference	γ	risk aversion coefficient	1.5
Labor productivity	ϵ_t	age-efficiency profile	see text
	ρ_z	AR(1) coef. of 5-year prod. process	0.85
	σ_μ^2	innovation of 5-year prod. process	0.30
	ρ_h	AR(1) coef. of prod. inheritance process	0.40
	σ_h^2	innovation of prod. inheritance process	0.37
Government policy	$\underline{c}$	minimum consumption	0.21
	τ_a	tax on capital income	20%
	τ_l	tax on labor income	21.7%
	$P(\widetilde{y})$	Social Security benefit	see text
	τ_{ss}	Social Security tax	10.0%
Firm	$DB(\widetilde{y})$	pension from defined benefit plan	see text
	τ_{DB}	contribution rate to defined benefit plan	3.9%
	r	interest rate	5%

Table 1: Exogenous parameters used in the benchmark model

Unless stated otherwise, we report parameters at an annual frequency. Table 1 lists the parameters that are either taken from other studies or can be solved independently of the endogenous outcomes of the model. The tax rate on labor income and the contribution rate to the defined benefit plan, belong to the latter group because, due to the assumption of exogenous labor supply and retirement decisions, they only depend on the earnings shocks and the population demographics, which are exogenous to the model.

We set the population growth rate, n, to the average value of population growth from 1950 to 1997 from the Council of Economic Advisors (1998). The p_t's are the vectors of conditional survival probabilities for people older than 65 and are set to the survival probabilities for people born in 1965 (Bell et al. (1992)). We take the risk aversion coefficient, γ, to be 1.5.

The deterministic age-profile of labor productivity ϵ_t has been estimated by Hansen (1993). Since we impose mandatory retirement at the age of 65, we set $\epsilon_t = 0$ for $t > 9$. The labor productivity persistence ρ_z and variance σ_μ^2 were estimated from the Panel Study of Income Dynamics (PSID) data using five-year periods by Altonji and Villanueva (2002).[2] We take the persistence ρ_h of the productivity inheritance process from Zimmerman (1992) and Solon (1992) and the variance σ_h^2 from De Nardi (2004).

The minimum consumption floor $\underline{c}$ is 0.21 of average household income, because in 1992 the consumption floor was \$8,159 (Scholz et al. (2006)), while the average income was \$38,840 (Census Bureau).

The capital income tax rate τ_a is set at 20% as recommended by Kotlikoff et al. (1999). The tax rate on labor τ_l is set at 21.7%, so that the ratio of government spending to output is 0.18 (Council of Economic Advisors (1998)).

The Social Security benefit $P(\widetilde{y})$ mimics the Old Age and Survivor Insurance component of the Social Security system and is set as

$$P(\widetilde{y}) = 0.9\min(\widetilde{y}, 0.2) + 0.32\max(0, \min(\widetilde{y}, 1.24) - 0.2) + \\ 0.15\max(0, \min(\widetilde{y}, 2.47), 1.24).$$

In this formula, the bend points are expressed in terms of average earnings and the marginal rates of Social Security benefits are taken from Huggett and Ventura (2000). More specifically, their formula applies to an economy with average earnings of one. The bend points are multiplied by average earnings in our model economy to make the formula consistent with our model economy. The tax rate on labor income τ_{ss} is set at 10.0% to balance the Social Security budget.

[2]See De Nardi (2004) for details on the estimated parameters.

The defined benefit formula is given by

$$\begin{aligned} DB(\widetilde{y}) = 0.49\max[0,\min(\widetilde{y},0.72)-0.58] + 0.36\max[0,\min(\widetilde{y},0.88)-0.72] \\ +0.28\max[0,\min(\widetilde{y},1.06)-0.88] + 0.20\max[0,\min(\widetilde{y},1.28)-1.06] \\ +0.16\max[0,\min(\widetilde{y},1.59)-1.28] + 0.28\max[0,\min(\widetilde{y},2.12)-1.59] \\ +0.02\max[0,\min(\widetilde{y},3.23)-2.12]. \end{aligned}$$

These bend points are also expressed in terms of average earnings, while the marginal rates are chosen to match the holding of defined benefit wealth relative to Social Security wealth by lifetime earning deciles from the Health and Retirement Study (HRS) data set, as reported in Scholz et al. (2006). Appendix A reports the details of these computations. According to this formula and consistent with the data, households in the first three lifetime earnings categories barely have any defined benefit wealth.

The contribution rate τ_{DB} to the defined benefit plan that balances the pension budget is 3.9%. The annual interest rate is set at 5%.

Moment		Data	Model
Wealth-earnings ratio		6.90	6.90
Bequest-wealth ratio		0.0088	0.0089
90th percentile of bequest distribution		4.53	5.19
Parameters		Value	
β	discount factor	0.96	
ϕ_1	weight of bequests in utility function	−107.6	
ϕ_2	shifter of bequests in utility function	16.5	

Table 2: Parameters calibrated using the benchmark model

Table 2 lists the parameters we use to calibrate the model. We choose the parameters β, ϕ_1, ϕ_2 to match the ratio of wealth to after-tax earnings (Hendricks (2007a)), the bequest-wealth ratio (Gale and Scholz (1994)), and the 90th percentile of bequest distribution normalized by income (Hurd and Smith (2002)). We use the distribution for single decedents instead of the one for all decedents. As is argued in De Nardi (2004), typically a surviving spouse inherits a large share of the estate, consumes part of it, and only leaves the remainder to the couple's children.

The discount factor affects saving and average wealth in the economy. The term ϕ_1 measures the strength of bequest motives, thus we choose the aggregate bequest as a moment. The term ϕ_2 reflects the extent to which bequests are luxury goods, affecting the bequest distribution, especially the high end of it.

To better gauge the quantitative implications of the model, we begin by evaluating the lifetime earnings implications of the exogenous earnings process that we feed into the model. Table 3 first reports the percentage of total lifetime earnings earned at selected percentiles as a fraction of total lifetime earnings generated by the model and then displays the corresponding figures computed from the PSID observed data. The model earnings process produces lifetime earnings earned by each lifetime earnings decile that are very close to those from the PSID data.

		Percentile (%)							
	Gini	0-20	20-40	40-60	60-80	80-90	90-95	95-99	99-100
All models	0.37	6.3	11.0	16.0	23.4	16.7	11.1	11.7	3.9
PSID[a]	0.32	6.6	12.3	18.0	24.5	15.5	9.1	9.5	4.4

[a]From Hendricks (2007a), page 439, Table 7, row 1. Compared to that table, we flipped the numbers from top to bottom for easier comparison with our graphs

Table 3: Percentage of total lifetime earnings held at selected percentiles

4 Numerical Results

We present our numerical results as follows.[3]

1. The benchmark model and its comparison to the actual data.
2. The role of bequest motives.
3. The role of the distribution of bequests.
4. The role of intergenerational transmission of ability.
5. The role of government-provided minimum consumption.

[3]Details about computing algorithm are provided in Appendix B.

6. The role of pensions and Social Security.

7. The role of measurement error in both earnings and wealth in the observed data.

In each run, we solve for the dynamic programming problem and impose budget balance for the government. We then simulate 100,000 households starting from age 20, drawn from the initial distribution of each model. We define retirement wealth to be the wealth at age 65 and lifetime earnings to be the total earnings from ages 20 to 60, discounted to age 65.

The contribution rate τ_{DB} is set to zero when pensions are removed. The payroll tax τ_{ss} is set to zero when Social Security is removed. When Social Security payments are equalized, the payroll taxes τ_{ss} are kept as in the benchmark model, while the payments are set to balance Social Security budget.

4.1 The Benchmark Model

Overall, the benchmark model fits the data well. The first subsection regarding the benchmark model discusses its fit of the wealth distribution at retirement and of wealth distribution across all ages. The second subsection focuses on its implied correlation between wealth at retirement and lifetime income.

4.1.1 Wealth Inequality in the Benchmark Model

			Percentile (%)					
			40-	60-	80-	90-	95-	99-
Age		Gini	60	80	90	95	99	100
65	PSID	0.62	10.2	18.8	17.0	13.8	18.3	15.7
	Benchmark model	0.64	9.3	20.4	19.6	16.7	21.1	8.9
All	PSID	0.76	6.4	16.3	16.8	14.4	22.5	22.8
	Benchmark model	0.76	4.8	16.0	18.8	17.5	27.4	14.8

Table 4: Wealth percentiles at retirement time (top panel) and in the whole population (bottom panel), data and benchmark model

The top panel of Table 4 reports wealth percentiles at retirement. The first line of the top panel refers to data from the PSID[4] and shows that, in the data, wealth at age 65 is highly unevenly distributed. The richest 1% of people at retirement time hold 16% of total retirement wealth, while the richest 5% hold 34% of total net worth at retirement time. The second line of the top panel reports the corresponding numbers for the benchmark model with intergenerational links and bequest motives. That model succeeds in generating a skewed retirement wealth distribution that is comparable with the data, with the exception of the top 1% of the wealth holdings.

The bottom panel of Table 4 reports values of the wealth distribution for the whole economy. Wealth for the whole economy is more unevenly distributed than wealth at retirement, both in the data and in all models. This indicates that a large amount of wealth dispersion in the economy is due to differences in age. Here, too, the model fits the data well.

4.1.2 Wealth and Lifetime Earnings in the Benchmark Model

The benchmark model generates a correlation between lifetime earnings and retirement wealth of 0.75. While this is a bit higher than the 0.61 in the PSID data, the model does generate a good deal of wealth inequality even after controlling for lifetime earnings and age.

	Earnings decile										
	1	2	3	4	5	6	7	8	9	10	mean
PSID	0.66	0.67	0.62	0.55	0.57	0.45	0.43	0.42	0.50	0.55	0.54
Model	0.91	0.73	0.60	0.54	0.51	0.48	0.46	0.42	0.34	0.32	0.53

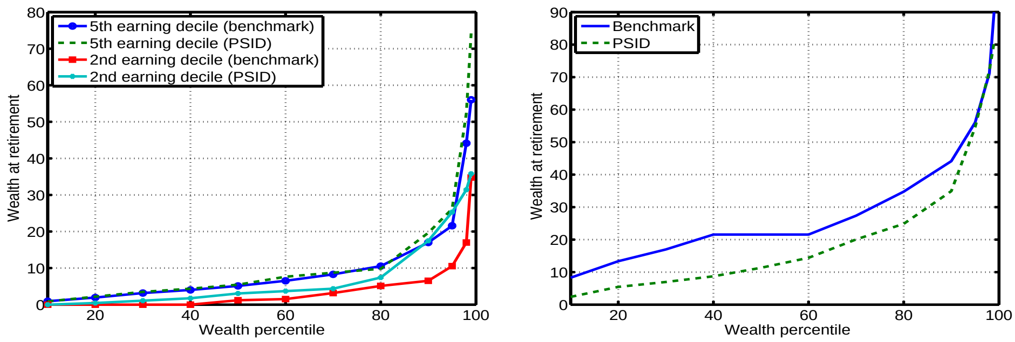
Table 5: Gini coefficient of retirement wealth by lifetime earnings decile

Table 5 illustrates the Gini coefficients of retirement wealth for each lifetime earnings decile. We notice two important features. First, after controlling for age and lifetime earnings, there is still large wealth inequality in the benchmark model: All the Gini coefficients are above 0.32. Second, the degree of

[4]The PSID data computations in Section 4 are all from Hendricks (2001) and Hendricks (2007a).

wealth inequality declines as lifetime earnings increases as is observed in the data. Third, the model does generate significant wealth inequality conditional on permanent income. In fact, the average of the Gini coefficients across permanent income quantiles turns out to be 0.53 in the model and 0.54 in the data.

To better gauge the amount of wealth dispersion at retirement generated by the benchmark model, Figure 2 compares the retirement wealth distributions for the 2nd, 5th, and 9th lifetime earnings deciles in the model with those in the PSID. The model successfully replicates the fact that households with similar lifetime earnings hold diverse amounts of wealth. At each lifetime earnings decile, households in the lower wealth deciles hold very little wealth, while households in the higher wealth deciles hold much more wealth. Compared with the data, the model matches especially well for the households in the middle deciles of the income distribution, while it overstates the high wealth percentiles at the 2nd earnings decile but overstates low wealth percentiles at the 9th earnings decile.



(a) 2nd and 5th lifetime earnings deciles (b) 9th lifetime earnings decile

Figure 2: Cumulative distribution of wealth (normalized by average household after-tax earnings)

4.2 The Role of Bequest Motives

In the benchmark model, retirement wealth inequality arises among households with similar lifetime earnings because households differ in the timing of earnings over the life cycle and in the amount and timing of inheritances received. Let us now turn to understanding the effects of bequest and inheritance heterogeneity on retirement wealth heterogeneity.

The model with intergenerational links of bequests and earnings ability endogenously generates differences in the timing and amount of inheritances. To see how large the variation in inheritances is, Table 6 reports values for the discounted lifetime inheritance distribution. In the PSID, inheritances are highly unevenly distributed with a Gini coefficient of 0.89, and 50% of the households receive very little or no inheritance. The top 1% of the households receive 35% of all the inheritances.

The model generates a skewed inheritance distribution that is comparable with the one in the data, with the exception of its top 1%. Treating bequests as luxury goods and modeling the transmission of earnings ability across generations are essential to match the observed skewness in the inheritance distribution. Several modeling choices and aspects of the calibration are important in generating this result. First, the marginal utility of leaving a bequest is finite at zero bequests, which helps to generate a large fraction of households receiving no inheritances. Second, some large inheritances are transmitted across generations because of the voluntary bequest motive to hold onto assets if alive at a very old age. Because the marginal utility of bequests declines more slowly than the marginal utility of consumption, the richest households have strong bequest motives to save to leave some assets to their children even when they are very old. Moreover, in the presence of a positive correlation between parents' and children's earnings, children are more likely to be earnings-rich and to receive a large inheritance and thus tend to save to leave more wealth to their offspring, hence generating a skewed inheritance distribution.

		Percentile (%)						
	Gini	0-50	50-70	70-80	80-90	90-95	95-99	99-100
PSID	0.89	0.0	2.3	4.9	12.5	14.1	30.9	35.3
Benchmark model	0.88	0.0	1.0	4.9	13.8	17.1	40.0	23.3

Table 6: Percentiles of the inheritance distribution, data and benchmark model

Table 7 shows the fraction of lifetime inheritance received by households in each lifetime earnings decile. In the PSID, there is a positive correlation between lifetime inheritance and lifetime earnings. The benchmark model generates an increasing relation between inheritance and lifetime earnings. Modeling the transmission of earnings ability across generations and a highly correlated lifetime earnings process is key in generating this pattern. The monotonicity relation is weaker in the data. This might due to the fact that in reality, individuals differ by numbers of siblings and marital status and each married couple might receive inheritances multiple times. More inheritances at the lowest earnings decile might help the model to generate less correlation between retirement wealth and earnings. Interestingly, assuming that the parent's productivity and his child's initial productivity are perfectly correlated increases the discrepancies between the fraction of inheritances received by each lifetime earnings decile in the model and the data. The reason is that higher intergenerational persistence in productivity generates a stronger positive correlation between lifetime inheritance and lifetime earnings. As is shown in Table 7, this implies that the fraction of inheritances received by the lowest two earnings deciles is only 4.7%, compared with 14% in the data and 10% in the benchmark model.

	Earnings decile				
	1-2	3-4	5-6	7-8	9-10
PSID	13.9	6.0	22.2	16.6	41.3
Benchmark model	9.6	14.4	19.5	23.7	32.9
$\rho_h = 1$	4.7	8.5	14.2	24.0	48.6

Table 7: Fraction of inheritance received by lifetime earnings decile (%), benchmark model, and model with perfect correlation of initial earnings ability

We now turn to discussing the effects of inheritance heterogeneity on retirement wealth in the benchmark model. The upper panel in Table 8 shows extra wealth holding at selected percentiles among those who did inherit, compared with those who did not. At each earnings decile, those who never inherited hold less wealth than those who have inherited, and the difference increases as the wealth percentile increases. The reason is, with operative bequest motives, those who have inherited hold a large part of the inheritances at retirement.

Model	Earnings	Percentile						
	Decile	20	40	60	80	90	95	99
Benchmark	2nd	0.0	1.2	1.6	2.5	4.0	8.7	60.6
	4th	1.0	1.5	1.4	4.0	6.5	14.0	54.1
	6th	1.5	1.4	2.2	3.6	10.4	22.6	62.9
	8th	1.7	2.8	3.6	5.8	16.7	21.3	46.1
	All	2.5	3.4	6.9	10.4	28.6	27.0	24.3
Random inher. at age 50	All	1.6	1.4	2.8	5.8	9.4	0.0	19.1

Table 8: Extra wealth holding at selected percentiles among those who have inherited (normalized by average household after-tax earnings)

4.3 The Role of Inheritance Distribution

To isolate the role of the distribution of inheritances from voluntary bequest motives, we also compute two versions of the model without bequest motives. In both cases, the inheritance distribution is exogenously taken from the data, See Appendix B for more detail.

1. All 50 year olds receive an inheritance of random size (taken from the data), whose size is uncorrelated with their earnings at age 50, as in Hendricks (2007a).

2. The timing of inheritance receipt is random and consistent with the data.

Table 9 reports several measures of inequality for the PSID data and the model-generated data in various versions of the model.

While the benchmark model that intergenerational links and bequest motives generates a skewed wealth distribution that is comparable with the one in the data, the two models without intergenerational links and bequest motives generate much less concentration than in the observed data.

In the model with random bequests that are received at age 50, the correlation coefficient between lifetime earnings and retirement wealth and the mean Gini coefficient are farther away from the data compared with the benchmark model, thus pointing out that a bequest motive is needed for those who have

Age		Gini	Percentile (%)					
			40-60	60-80	80-90	90-95	95-99	99-100
65	PSID	0.62	10.2	18.8	17.0	13.8	18.3	15.7
	Benchmark model	0.64	9.3	20.4	19.6	16.7	21.1	8.9
	Change bequest motives and inheritance							
	Random inher. at age 50	0.58	11.3	23.0	19.8	15.7	18.0	6.1
	Random inher. at ages 35-55	0.58	11.2	22.9	19.8	15.6	18.2	6.2
	Change intergenerational persistence in earnings							
	$\rho_h = 0$	0.63	9.8	20.9	19.3	16.5	20.3	8.6
	$\rho_h = 1$	0.66	8.9	20.4	20.5	16.7	21.2	9.1
All	PSID	0.76	6.4	16.3	16.8	14.4	22.5	22.8
	Benchmark model	0.76	4.8	16.0	18.8	17.5	27.4	14.8
	Change bequest motives and inheritance							
	Random inher. at age 50	0.71	6.6	19.0	20.3	17.6	24.1	11.1
	Random inher. at ages 35-55	0.71	6.7	19.0	20.1	17.6	24.1	11.2
	Change intergenerational persistence in earnings							
	$\rho_h = 0$	0.74	5.5	16.7	18.9	17.2	26.7	13.9
	$\rho_h = 1$	0.78	3.6	15.2	19.7	18.5	27.6	15.2

Table 9: Wealth percentiles at retirement time (top panel) and in the whole cross-section of the population (bottom panel) in the data and several different versions of the model

received large inheritances to hold a significant portion of them by the time retirement comes around. In the benchmark model, in fact, households hold onto more wealth due to operative bequest motives, and this generates more heterogeneity in retirement wealth for given lifetime earnings. As is shown in Table 8, extra wealth holdings among those who have inherited are much smaller in the random bequest model than in the benchmark model with bequest motives.

Making the timing of bequest receipt more realistic further weakens the relationship between lifetime earnings and retirement wealth: A borrowing constrained household that receives an inheritance earlier consumes more of it and holds onto less wealth at retirement than an otherwise identical household that receives an inheritance later. However, the quantitative effect of this factor is very small.

	Corr(W, E)[a]	Mean Gini[b]
PSID	0.61	0.54
Benchmark	0.75 (0.14)	0.53 (-0.01)
Change bequest motives and inheritance		
Random inher. at age 50	0.81 (0.20)	0.46 (-0.08)
Random inher. at ages 35-55	0.80 (0.19)	0.46 (-0.08)
Change intergenerational persistence in earnings		
$\rho_h = 0$	0.73 (0.11)	0.53 (0.00)
$\rho_h = 1$	0.77 (0.16)	0.54 (0.00)

[a]The correlation coefficient between lifetime earnings and retirement wealth
[b]The average of the Gini coefficients of retirement wealth within lifetime earnings deciles

Table 10: Relationship between retirement wealth and lifetime earnings (the numbers in the parentheses are deviations from the data)

4.4 The Role of Intergenerational Transmission of Ability

To study the interaction of bequest motives and intergenerational transmission of ability, we keep the bequest motive, but we change the intergenerational persistence in earnings, which in turn endogenously changes the distribution of inheritance. In particular, if there is no intergenerational link of earnings, inheritances are evenly distributed by lifetime earnings decile.

Higher intergenerational persistence of earnings ability leads to more wealth accumulation across generations and increases wealth inequality.[5]

In case of no intergenerational transfer of productivity ($\rho_h = 0$), the correlation between retirement wealth and lifetime earnings is reduced to 0.73, down from 0.75 in the benchmark economy. There are several reasons for this drop. The first one is that without the intergenerational productivity link, inheritances are evenly distributed by lifetime earnings decile, which is in contrast with the data (see Table 7). In this version of the model, some households

[5]De Nardi (2004) provides a detailed discussion of the effects of intergenerational links on the cross-sectional wealth distribution.

in the low lifetime earnings deciles receive unrealistically large amounts of inheritances, weakening the correlation between lifetime earnings and retirement wealth in an unplausible way. The second reason is that, as in the benchmark model, with an operative bequest motive, those who inherit large estates from their parents will consume only a small part of their inheritances by the age of 65, thus helping to increase wealth heterogeneity at retirement.

However, the modification that parents' and children's productivity are perfectly correlated ($\rho_h = 1$) brings the correlation coefficient further away from the data. The reason is that, higher intergenerational persistence in productivity generates stronger positive correlation between lifetime inheritance and lifetime earnings. As is shown in Table 7, the amount of inheritance received by the lowest two earnings deciles is only 4.7%.

	Corr(W, E)[a]	Mean Gini[b]
PSID	0.61	0.54
Benchmark	0.75	0.53
No consumption floor	0.75	0.49
No pension	0.79	0.49
No pension, no Social Security	0.80	0.49
No pension, equal Social Security	0.82	0.49

[a]The correlation coefficient between lifetime earnings and retirement wealth
[b]The average of the Gini coefficients of retirement wealth within lifetime earnings deciles

Table 11: Relationship between retirement wealth and lifetime earnings

4.5 The Role of the Government-Provided Consumption Floor

Table 11 shows that setting the government-provided consumption floor to zero reduces the mean Gini by 0.04 to 0.49, because a means-tested minimum consumption guarantee provides strong incentives for low-income individuals not to save. Due to the persistence of the earnings process, those low-income households are more concentrated in low lifetime-earnings deciles. Without a consumption floor, the Gini coefficients decrease a lot at the lowest three earnings deciles, since there are fewer poor households in those deciles. The Gini coefficients at the higher deciles barely change, since households in those deciles receive high pensions and Social Security income and do not qualify for a transfer.

	Earnings decile									
	1	2	3	4	5	6	7	8	9	10
Benchmark	0.91	0.73	0.60	0.54	0.51	0.48	0.46	0.42	0.34	0.32
no $\underline{c}$	0.68	0.60	0.54	0.52	0.51	0.48	0.46	0.41	0.34	0.32
no DB	0.89	0.70	0.59	0.51	0.45	0.40	0.39	0.36	0.29	0.28
no DB, no SS	0.93	0.80	0.68	0.57	0.46	0.37	0.31	0.28	0.23	0.24
no DB, equal SS	0.86	0.72	0.61	0.55	0.47	0.41	0.37	0.32	0.26	0.27

Table 12: Gini coefficient of retirement wealth by lifetime earnings decile

4.6 The Role of Pensions and Social Security

Eliminating pension benefits raises the correlation coefficient between lifetime income and retirement wealth substantially because it raises average wealth for households in the high earnings deciles. Compared with the benchmark model with pensions, the Gini coefficients decrease a lot at the highest five earnings deciles, since they now increase saving for retirement.

Also, eliminating any kind of Social Security payments (in addition to removing pension benefits) increases the Gini coefficients at the lower earnings deciles: Wealthy households increase savings, while poor households do not increase their saving to be able to qualify for the government-provided consumption floor. The Gini coefficients at the higher earnings deciles decrease since poor households, who are not likely to qualify for the government-provided consumption floor, increase saving relatively more than rich households.

As an additional test, after eliminating Social Security payments, we give the same Social Security payment to all retirees, in addition to removing private pensions. As a result of this change, the resulting Social Security payment to all retirees, determined from the government budget balance, is large enough that no retiree qualifies for minimum consumption floor transfer anymore. This experiment decreases the Gini coefficients at the lower earnings deciles because rich households reduce saving while poor households, holding no wealth before the experiment, do not reduce saving. The Gini coefficients at the higher earnings deciles increase because the poor households decrease saving relatively more than rich households.

4.7 The Role of Measurement Error in Income and Wealth

In the data, both earnings and wealth are likely measured with error. To evaluate the magnitude of this element, we add random noise to the sample simulated from the benchmark model. We assume that the log of observed earnings for household i at age t, $\hat{e}_t^i$, follows

$\hat{e}_t^i = e_t^i + v_t^i$, in which e_t^i is the log of true earnings and v_t^i is the random error that follows $v_t^i = \rho_v e_t^i + \iota_t^i$, $\iota_t^i \sim N(0, \sigma_\iota^2)$.

The observed log wealth at age 65 follows

$\hat{W}^i = W^i + \eta^i$, and $\eta^i \sim N(0, \sigma_\eta^2)$, where η^i and ι_t^i are uncorrelated with each other.

We use $\rho_v = -0.104$, and $\sigma_\iota^2 = 0.138$ (Bound et al. (1989)). Using 1982 PSID earnings, Bound et al. (1989) reported that $\sigma_v^2/(\sigma_v^2 + \sigma_e^2) = 0.15$, and $\rho_v = -0.104$. In the model, σ_e^2 is 0.831, which gives $\sigma_v^2 = 0.147$ and $\sigma_\iota^2 = 0.138$.

The measurement error of wealth may be substantial as well. To be conservative, we choose a small variance $\sigma_\eta^2 = 0.01$ as a lower bound.

Adding a small amount of measurement error reduces the correlation coefficient from 0.75 in the benchmark model to 0.71. The mean Gini coefficient of retirement wealth is slightly higher (0.55) than that in the benchmark (0.53). The reason is that, adding random noise in earnings affects Gini coefficients only when enough agents are regrouped into other deciles. Given the wide range of earnings in each decile, not many agents are regrouped.

5 Conclusions

Empirical studies using micro data find that there is large heterogeneity in retirement wealth among households with similar lifetime earnings and raise doubts about the ability of a standard life-cycle model of saving behavior to reproduce the observed facts.

We use an incomplete-market life-cycle model with intergenerational links of bequests and earnings ability, government-provided minimum consumption, a history-dependent Social Security system, and a defined benefit pension. We show that this model with earnings heterogeneity and inheritance heterogeneity generates a substantial amount of heterogeneity in retirement wealth for given lifetime earnings. This suggests that a properly specified life-cycle model with bequest motives, consumption floor, and Social Security pensions captures the fundamental determinants of households saving and wealth accumulation. This framework might shed light on the effects of policy reforms that affect saving. We show that government-provided minimum consumption, pensions, and Social Security have very different distributional effects. In a separate paper, Yang (2013) studies the consequences of eliminating Social Security in a similar environment to the one constructed in this paper and finds that the presence of bequest motives reduces life-cycle saving and thus reduces the gains from Social Security reform.

We have assumed that households are ex-ante identical. Interesting directions for future work include allowing for heterogeneity in wealth holdings by education (Hubbard et al. (1995) and Cagetti (2003)), and by marital status (Cubeddu and Rios-Rull (2003) and Guner and Knowles (2007)). Other interesting extensions include allowing for heterogeneity in preferences (Krusell and Smith (1998), Samwick (1998), and Hendricks (2007b)) and in self-control (Ameriks et al. (2007)), in number of children (Scholz and Seshadri (2007)), in rates of return (Guvenen (2006)), and in health expenditures (De Nardi et al. (2010)).

References

[1] J.G. Altonji, and E. Villanueva, 2002. "The Effect of Parental Income on Wealth and Bequests," NBER Working Paper 9811.

[2] J. Ameriks, A. Caplin, J. Leahy, and T. Tyler, 2007. "Measuring Self-Control Problems," *American Economic Review*, 97, 966–972.

[3] G. S. Becker, and N. Tomes, 1979. "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility," *Journal of Political Economy*, 87, 1153–1189.

[4] F.C. Bell, A.H. Wade, and S.C. Goss, 1992. "Life Tables for the United States Social Security Area 1900-2080," Actuarial Study No. 107, Social Security Administration, Office of the Actuary.

[5] D. Bernheim, J. Skinner, and S. Weinberg, 2001. "What Accounts for the Variation in Retirement Wealth Among U.S. Households?" *American Economic Review*, 91, 832–857.

[6] J. Bound, C. Brown, G. Duncan, and W. Rodgers, 1989. "Measurement Error in Cross-sectional and Longitudinal Labor Market Surveys: Validation Study Evidence," NBER working paper 2884.

[7] M. Cagetti, 2003. "Wealth Accumulation over the Life Cycle and Precautionary Savings," *Journal of Business and Economic Statistics*, 21(3), 339–353.

[8] M. Cagetti, and M. De Nardi, 2006. "Entrepreneurship, Frictions, and Wealth," *Journal of Political Economy*, 114(5), 835–870.

[9] A. Castaneda, J. Diaz-Gimenez, and J. Rios-Rull, 2003. "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, 111(4), 818–857.

[10] T.F. Cooley, and E. Prescott, "Economic Growth and Business Cycles," In Thomas F. Cooley eds., *Frontiers of Business Cycle Research* (Princeton: Princeton University Press, 1995), 1–38.

[11] Council of Economic Advisors, 1998. Economic Report of the President. United States Government Printing Office, Washington.

[12] L.M. Cubeddu, and J. Rios-Rull, 2003. "Families as Shocks," *Journal of the European Economic Association*, 1, 671–682.

[13] M. De Nardi, 2004, "Wealth Inequality and Intergenerational Links," *Review of Economic Studies*, 71, 734–768.

[14] M. De Nardi, E. French, and J. Jones, 2010, "Why do the Elderly Save? The Role of Medical Expenses," *Journal of Political Economy*, 118, 39–75.

[15] E.M. Engen, W.G. Gale, C. Carroll, and D.I. Laibson, 1999. "The Adequacy of Retirement Saving," *Brookings Papers on Economic Activity*, 1999, 65–187.

[16] E.M. Engen, W.G. Gale, and C.E. Uccello, 2005. "Lifetime Earnings, Social Security Benefits, and the Adequacy of Retirement Wealth Accumulation," *Social Security Bulletin*, 66(1), 38-57.

[17] W. Gale, and J.K. Scholz, 1994. "Intergenerational Transfers and the Accumulation of Wealth," *Journal of Economic Perspectives*, 8, 145–160.

[18] J. Gokhale, L.J. Kotlikoff, J. Sefton, and M. Weale, 2001. "Simulating the Transmission of Wealth Inequality via Bequests", *The Journal of Public Economics*, 79, 93–128.

[19] I. Grafova, K. McGonagle, and F. Stafford, "Functioning and Well-Being in the Third Age", in Jacquelyn B. James and Paul Wink, ed., *The Crown of Life: Dynamics of the Early Post Retirement Years* (Springer Publishing Company, New York, 2006).

[20] N. Guner, and J. Knowles, 2007. "Marital Instability and the Distribution of Wealth,"Working Paper.

[21] M.F. Guvenen, 2006. "Reconciling Conficting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective,"*Journal of Monetary Economics*, 53(7), 1451–1472.

[22] G.D. Hansen, 1993. "The Cyclical and Secular Behavior of the Labor Input: Comparing Efficiency Units and Hours Worked," *Journal of Applied Econometrics*, 8, 71–80.

[23] J.J. Heckman, Lance Lochner, and Christopher Taber, 1998. "Explaining Rising Wage Inequality: Explorations with a Dynamic General Equilibrium Model of Labor Earnings with Heterogeneous Agents," *Review of Economic Dynamics,*, 1(1), 1–58.

[24] L. Hendricks, 2001. "Bequests and Retirement Wealth in the United States," Working Paper.

[25] L. Hendricks, 2007a. "Retirement Wealth and Lifetime Earnings," *International Economic Review*, 48(2), 421–456.

[26] L. Hendricks, 2007b. "How Important Is Preference Heterogeneity for Wealth Inequality?" *Journal of Economic Dynamics and Control*, 31(9), 3042–3068.

[27] R.G. Hubbard, J. Skinner, and S.P. Zeldes, 1995. "Precautionary Saving and Social Insurance," *Journal of Political Economy*, 103, 360–399.

[28] M. Huggett, 1996. "Wealth Distribution in Life-cycle Economies,"*Journal of Monetary Economics*, 38(3), 469–494.

[29] M. Huggett, and G. Ventura, 2000. "Understanding Why High Income Households Save More Than Low Income Households,"*Journal of Monetary Economics*, 45(2), 361–397.

[30] M. Hurd, and J.P. Smith, 2002. "Anticipated and Actual Bequests "NBER Working Paper 7380.

[31] M. Hurd, and J.P. Smith, 2002. "Expected Bequest and Their Distribution,"NBER Working Paper 9142.

[32] E. Hurst, M. Luoh, and F. Stafford, 1998. "The Wealth Dynamics of American Families, 1984-1994,"*Brookings Papers on Economic Activity,* 267–337.

[33] F. T. Juster, J. P. Smith, and F. Stafford, 1999. "The Measurement and Structure of Household Wealth,"*Labour Economics,* 6, 253–275.

[34] L.J. Kotlikoff, K. Smetters, and J. Walliser, 1999. "Privatizing Social Security in the United States: Comparing the Options,"*Review of Economic Dynamics*, 2, 532–574.

[35] L.J. Kotlikoff, K. Smetters, and L. H. Summers, 1981. "The Role of Intergenerational Transfers in Aggregate Capital Accumulation,"*Journal of Political Economy*, 89, 706–732.

[36] P. Krusell, and A.A. Smith, 1998. "Income and Wealth Heterogeneity in the Macroeconomy,"*Journal of Political Economy,* 5, 867–896.

[37] E.C. Prescott, 1986. "Theory Ahead of Business Cycle Measurement,"*Federal Reserve Bank of Minneapolis Quarterly Review*, 10, 9–22.

[38] V. Quadrini, 2000. "Entrepreneurship, Saving and Social Mobility,"*Review of Economic Dynamics*, 3(1), 1–40.

[39] A.A. Samwick, 1998. "Discount Rate Heterogeneity and Social Security Reform,"*Journal of Development Economics*, 57, 117–146.

[40] J. Scholz, A. Seshadri, and S. Khitatrakun, 2006. "Are Americans Saving "Optimally" for Retirement?" *Journal of Political Economy*, 114(4), 607–643.

[41] J. Scholz, and A. Seshadri, 2007. "Children and Household Wealth," Working Paper.

[42] G. Solon, 1992. "Intergenerational Income Mobility in the United States," *American Economic Review*, 82, 393-408.

[43] S.F. Venti, and D.A. Wise, 2000. “Choice, Chance and Wealth Dispersion at Retirement,” NBER Working Paper 7521.

[44] F. Yang, 2013. “Social Security Reform with Impure Intergenerational Altruism,” *Journal of Economic Dynamics and Control*, 37, 52-67.

[45] D.J. Zimmerman, 1992. “Regression Toward Mediocrity in Economic Stature,” *American Economic Review*, 82, 409–429.

Appendix A: Computation of Social Security and Defined Benefit Payments

The bend points in the Social Security and defined benefit formulas are expressed in terms of average earnings, and the marginal rates at the bend points of Social Security benefits are taken from Huggett and Ventura (2000). More specifically, their formula applies to an economy with an average earnings of one. The bend points are multiplied by average earnings to make the formula consistent with our model economy.

Here is more detail about the computation of defined benefit:

1. Discretize average yearly lifetime earnings $\widetilde{y}$ and calculate its distribution at age 65.

2. Find the median of average yearly lifetime earnings in each decile.

3. Use the formula of Social Security payment to calculate the median Social Security payment in each decile.

4. Scholz et al. (2006, page 625) report median defined-benefit pension wealth, Social Security wealth, and net worth (excluding defined-benefit pensions) by lifetime earnings decile. We use these numbers to calculate the ratio of DB/Social Security wealth in the data.

5. Using steps 3 and 4, back out the DB payment at each median of average yearly lifetime earnings in each decile in our model, so that it matches the same ratio as in Scholz et al. (2006).

6. Use the median average yearly lifetime earnings from the 4th decile to the 10th decile as bend points and calculate the slope in each piecewise linear function of that formula. The first to third deciles are not used because the median DB in the data is zero.

Appendix B: Computation Algorithm

For a given set of parameters, we solve for the steady-state equilibrium as follows:

1. Solve the optimal consumption and saving plans recursively.

2. Guess an initial joint distribution of parents and children at the beginning of the life cycle; compute the associated stationary distribution of households using the policy functions.

3. Check whether the implied joint distribution of parents and children at the beginning of the life cycle is consistent with the initial guess. If so, an equilibrium is found. Otherwise, go back to step 2 with an updated initial guess.

The above algorithm applies to models with voluntary bequests. For models without voluntary bequests, we skip step 3 because the initial distribution at the beginning of the life cycle is exogenously given. In the random bequest model, the probabilities of inheritances are given by (0.50, 0.20, 0.10, 0.10, 0.05, 0.04, 0.01), while the corresponding bequest amounts are (0.0, 0.3, 1.2, 3.0, 6.8, 18.5, 84.6), expressed in terms of average after-tax earnings. Those numbers are taken from Table 8 in Hendricks (2007a), who calculates the distribution of the present value of lifetime inheritances received by the household head and wife in PSID.