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ABSTRACT

This paper studies optimal dynamic tax policy under the threat of political reform. A policy will be reformed ex post if a large enough political coalition supports reform; thus, credible policies are those that will continue to attract enough political support in the future. If the only credible reform threat is to fully equalize consumption, we find that optimal marginal capital taxes are U-shaped, so that savings are subsidized for the middle class but are taxed for the poor and rich. If ex post the government may strategically propose a reform other than full equalization in order to secure additional political support, then optimal capital taxes are instead progressive throughout the income distribution.

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1 Introduction

In a democratic society, the credibility of a course of public policy depends on the level of political support it can be expected to enjoy in the future. This level of support is degenerate in representative-agent economies: a policy is credible if and only if it is time-consistent from the representative agent's perspective (Kydland and Prescott, 1977). If, on the other hand, society consists of diverse individuals, a democratic government needs to take the resulting heterogeneity of political preferences into account when formulating policy, and in particular must realize that a policy is credible only if it will continue to receive the support of a large enough coalition of citizens in the future.

This paper adopts the above perspective on the credibility of public policy to reexamine the classical capital taxation problem in a heterogeneous society. We consider a simple two-period model where individuals produce in the first period only but consume in both periods. A utilitarian government has access to arbitrary nonlinear labor and capital taxes. The key feature of the model is that the government is able to reform its original policy in period 2 *if there is sufficient political support for a reform*. Thus, the government's ability to commit to intertemporal tax policy is limited and determined by politics.

In the simplest version of the model (the "equalizing reforms" version), the only credible reform involves full redistribution of capital: since the government tolerates inequality only if this enhances the incentives for production, it prefers to fully equalize period-2 consumption once production has occurred. In this case, poor voters tend to support reform, rich voters tend to oppose it, and middle-class voters tend to be close to indifferent and thus pivotal. Therefore, the government can make its original policy credible by making the status quo as appealing as possible to the middle class, relative to full equalization. It does this both by subsidizing saving for middle class voters (so they have high period-2 consumption under the status quo) and by taxing capital for the poor and rich (so consumption under an equalizing reform is low). That is, the socially optimal manner for the government to forestall reform is to impose a U-shaped marginal tax on capital. As we argue in more detail in Section 5.3, this novel prediction of U-shaped marginal capital taxes resonates well with policy in many advanced economies, once the savings incentives implied by means-tested government benefits are taken into account.

The prediction of U-shaped marginal capital taxes crucially depends on the fact that full equalization is the only credible reform in this version of the model. If other reforms are also credible, the government may have a strong incentive to propose reforms other than full equalization in order to secure more political support; for example, it may want to expropriate the richest 1% in period 2 to sweeten the reform for the remaining 99%. To

allow for this possibility, we next consider the polar case where the government is able to credibly propose *any* reform in period 2, with the reform again being implemented if it has sufficient political support relative to the status quo (the “strategic reforms” version of the model). In this case, we show that the government’s planning problem may be written as a standard welfare maximization problem with an additional “no-reform” constraint, where a marginal increase in a given individual’s period-2 consumption under the status quo relaxes the no-reform constraint on net if and only if her period-2 consumption is higher under the reform than under the status quo. Furthermore, the reform is always more egalitarian than the status quo, so it is the poor who have higher period-2 consumption under the reform, and hence they face lower capital taxes under the government’s optimal policy. Thus, when the government may strategically propose reforms other than full equalization, the prediction of U-shaped marginal capital taxes is replaced with the prediction of progressive marginal capital taxes throughout the income distribution.

Putting our results together, we find that when the government’s credibility is limited by the possibility of future political reform, optimal capital taxes may be either U-shaped or progressive, depending on the government’s ability to credibly tailor reforms to popular demand. We formally consider the two polar versions of the model—where only fully equalizing reforms are credible, and where all reforms are—and reality is likely to lie somewhere in between. However, we can roughly map the two versions of our model to more representative and more direct versions of democratic government, respectively: in representative democracies, politicians retain the final say on fiscal policy, and are thus unlikely to be able to commit to reforms that they have a strong incentive to modify ex post; while in direct democracies, politicians may have the “agenda-setting” power to propose a specific policy, while being unable to modify it after it is approved by the voters. With this interpretation, our model predicts that capital taxes are more likely to be U-shaped when fiscal policy is ultimately determined by representatives, and more likely to be progressive when fiscal policy is determined by direct referendum.^{1,2}

This paper lies at the intersection of the public finance literature on dynamic taxation

¹This prediction is hard to test directly for two reasons. First, effective marginal capital taxes are difficult to measure, as discussed below. Second, obvious ways of measuring whether a democracy is “direct” or “representative” are likely to pick up effects that are outside the model but that could also affect fiscal policy. For example, the recent empirical paper by Hinnerich and Pettersson-Lidbom (2013) argues that representative democracies are more redistributive than direct democracies because they are less easily captured by elites.

²An alternative interpretation is that the two versions of the model differ in the government’s degree of sophistication. In the strategic reforms version, the government is able to design (and commit to) sophisticated vote-buying strategies at the reform stage. In the equalizing reforms version, the government simply pursues its most-preferred (fully equalizing) reform. This can capture naivety on the part of the government, as well as an inability to commit at the reform stage.

with limited commitment and the political economy literature on endogenous coalition formation. Relative to the public finance literature, we introduce a new model of limited commitment based on political coalition formation: our perspective is that a policy is credible if it retains the support of a large enough coalition to block reform. The most classical branch of the literature on limited commitment assumes a representative agent (Fischer, 1980, Klein, Krusell and Rios-Rull, 2008), which makes the coalition formation problem degenerate; this remains true in models where “reputation” can mitigate the government’s time-inconsistency problem (Kotlikoff, Persson, and Svensson, 1988, Chari and Kehoe, 1990, Benhabib and Rustichini, 1997, Phelan and Stacchetti, 2001). Hassler et al. (2005) consider a two-type model, which again precludes non-trivial coalitions.³

More closely related are the few papers where the extent of commitment is explicitly determined by political economy factors. One such paper is due to Acemoglu, Golosov, and Tsyvinski (2010), who analyze an infinite-horizon Mirrlees model with self-interested politicians and study whether the resulting distortions eventually vanish. A particularly closely related paper that we build on is Farhi et al. (2012). They consider a model similar to ours, but assume that the government is always able to reform its original policy (at a cost) in period 2, without the need for political support in the population. This leads to progressive capital taxes: the government is tempted to fully equalize period-2 consumption, so it reduces this temptation by partially equalizing period-2 consumption under the status quo—relative to the Atkinson-Stiglitz (1976) benchmark of zero capital taxes—which corresponds to taxing saving for the rich and subsidizing it for the poor. The strategic reforms version of our model also predicts progressive capital taxes, but the logic is quite different. In particular, all reforms in Farhi et al.’s two-period model involve full equalization—indeed, we show below that the *equalizing* reforms version of our model nests their model as a special case—but with equalizing reforms we predict U-shaped capital taxes.

There is also an influential positive political economy literature on capital taxation with heterogeneous voters and linear taxes (Bertola, 1993, Alesina and Rodrik, 1994, Persson and Tabellini, 1994a). One paper in this literature that emphasizes time-inconsistency is Persson and Tabellini (1994b), who consider a two-period model with linear taxes and show that voters may want to elect a government that is biased against taxing capital.

³Similar commitment problems can arise in moral hazard models rather than the Mirrleesian adverse selection model considered here. See, for instance, Fudenberg and Tirole (1990) and Netzer and Scheuer (2010) for two-period models where, *ex ante*, the principal optimally offers incomplete insurance to a risk-averse agent in order to provide incentives for efficient effort, but *ex post*, once effort is sunk, prefers to provide full insurance. Our approach to modeling limited commitment could also be applied to moral hazard models like these.

Restricting to linear taxes leads to a median voter theorem, which again rules out many of the coalition formation issues that underlie our approach.

Relative to the coalition formation literature, we sidestep the indeterminacy inherent in most such models by viewing coalition formation as a constraint in a planning problem: that is, we ask what coalition will be formed by a utilitarian government that needs a certain level of political support. While we assume that the government is utilitarian—rather than inherently representing the interests of certain groups in society—this could easily be endogenized as resulting from political competition in a probabilistic voting model with uniform taste shocks (Cox and McCubbins, 1986, Lindbeck and Weibull, 1987). We also relax the assumption that the government is utilitarian in Appendix D, and show that if the government is biased toward certain groups in society then our results on the shape of optimal capital taxes continue to hold within each group. The alternative modeling approach of studying competition among political parties offering nonlinear tax schedules—while interesting—would lead to a Colonel Blotto (or “divide-the-dollar”) game, which is notoriously difficult to analyze even in the simplest redistributive settings (Myerson, 1993, Lizzeri and Persico, 2001; see also Aumann and Kurz, 1977, for a cooperative game theory approach). Our approach is thus both more tractable and closer to the public finance literature on limited commitment.

The result in the coalition formation literature that is most closely related to our approach is Director’s Law (Stigler, 1970), which observes that public redistribution tends to benefit the middle class rather than the poor. The literature on Director’s law typically considers static settings (Lindbeck and Weibull, 1987; Dixit and Londregan 1996, 1998); to the best of our knowledge, our paper is the first to ask how such coalition formation concerns affect optimal dynamic tax policy. Another difference with this literature is that Director’s Law is usually interpreted as predicting a coalition of the poor and middle-class against the rich: for example, in Stigler (1970) this happens because ganging up to rob the rich is more profitable than ganging up to rob the poor. In contrast, the problem in our model is how to form a coalition to forestall an equalizing reform, which naturally leads to a coalition of the middle-class and the rich. It should also be emphasized that this “coalition” exists only in terms of opposition to reform; the government remains utilitarian, and labor taxes redistribute toward the poor as in standard Mirrlees models.

Finally, our results about the shape of the nonlinear capital tax schedule mirror an extensive literature on the shape of optimal income taxes in static Mirrlees models. In particular, many authors have found U-shaped marginal income taxes to be optimal (see e.g. Diamond, 1998, and Saez, 2001). However, whereas this property crucially depends on the shape of the underlying skill distribution (specifically on an unbounded Pareto-tail at

the top), our results about U-shaped or progressive marginal capital taxes are completely independent of the form of the skill distribution.

The paper proceeds as follows. Section 2 introduces our basic framework, which is a standard two-period Mirrlees model as discussed above. Section 3 analyzes the version of the model where only fully equalizing reforms are credible. Section 4 considers the version where the government can credibly propose more sophisticated strategic reforms. Section 5 offers a quantitative illustration of our results based on a calibrated version of the model and discusses various extensions. Section 6 concludes. Omitted proofs, as well as technical details on several extensions of the model, are presented in the appendix.

2 Model

We consider a standard Mirrlees model with two periods, $t = 1, 2$. There is a continuum of individuals indexed by their ability $\theta \in \Theta$. Assume that Θ is an open subset of \mathbb{R} and that θ has cumulative distribution function F with positive density f on Θ .

Individuals produce in period 1 only and consume in both periods. A type θ individual has utility function

$$u(c_1(\theta)) + \beta u(c_2(\theta)) - h(y(\theta), \theta),$$

where $c_1(\theta)$ and $c_2(\theta)$ are consumption in periods 1 and 2; u is a strictly increasing, strictly concave, and twice-differentiable consumption utility function satisfying the Inada conditions $\lim_{c \rightarrow 0} u'(c) = \infty$ and $\lim_{c \rightarrow \infty} u'(c) = 0$; $\beta > 0$ is the discount factor; $y(\theta)$ is production in period 1; and h is a continuous function, strictly increasing and convex in y and with strictly decreasing differences, that measures the cost of production.

There is a linear saving technology, so the economy faces aggregate resource constraints in $t = 1, 2$ given by

$$\begin{aligned} \int c_1(\theta) dF + K &\leq \int y(\theta) dF, \\ \int c_2(\theta) dF &\leq RK, \end{aligned}$$

where K is aggregate capital and $R > 0$ is its gross rate of return. These may be combined to form a single intertemporal resource constraint

$$\int \left(c_1(\theta) + \frac{1}{R} c_2(\theta) \right) dF \leq \int y(\theta) dF. \quad (1)$$

In addition to the continuum of individuals, there is a government, assumed for now to be utilitarian (we relax this in Appendix D).⁴ As in Mirrlees (1971), the government cannot observe ability. Therefore, the revelation principle implies that the government's problem when it can fully commit to an intertemporal allocation is

$$\max_{c_1, c_2, y} \int (u(c_1(\theta)) + \beta u(c_2(\theta)) - h(y(\theta), \theta)) dF$$

subject to the intertemporal resource constraint (1) and a standard incentive compatibility constraint

$$u(c_1(\theta)) + \beta u(c_2(\theta)) - h(y(\theta), \theta) \geq u(c_1(\theta')) + \beta u(c_2(\theta')) - h(y(\theta'), \theta) \text{ for all } \theta, \theta', \quad (2)$$

where c_1 , c_2 , and y are arbitrary measurable functions from Θ to \mathbb{R}_+ .

Most of our results will concern the implicit marginal capital tax, defined by

$$\tau_k(\theta) \equiv 1 - \frac{u'(c_1(\theta))}{\beta R u'(c_2(\theta))} < 1. \quad (3)$$

This “wedge” is well-defined in any allocation, and in addition can be interpreted as the actual marginal capital tax rate faced by agents of type θ in a nonlinear tax implementation of the optimal allocation, as we will discuss in Section 5.2.⁵ At a solution to the above full-commitment problem, Atkinson and Stiglitz's (1976) uniform taxation result implies that $\tau_k(\theta) = 0$ for all θ .⁶ As we will see, this result does not continue to hold when the government's credibility is limited by the possibility of reform in period 2.

We will consider two versions of limited commitment on the part of the government, which differ in the government's ability in period 2 to commit to the details of a proposed reform. In both versions, the timing of the model is as follows.

1. The government proposes consumption and production schedules (c_1, c_2, y) .
2. Production and period-1 consumption occurs.
3. The period-2 consumption schedule c_2 may be reformed to an alternative schedule \hat{c}_2 .

⁴As is standard, this could be interpreted as the outcome of political competition between two parties in a probabilistic voting model with uniform taste shocks. In the equilibrium of such a model, both parties offer the same platform, which maximizes a utilitarian welfare function.

⁵For results on the intratemporal labor wedge, see also Section 5.1.

⁶Throughout, we omit caveats regarding measure-0 sets when stating results. We address this issue in various proofs where it may cause confusion.

The two versions of the model differ in how the “reform” consumption schedule \hat{c}_2 is specified; the details of this are described below. In both versions, the reform is defeated if and only if

$$\int H(c_2(\theta), \hat{c}_2(\theta)) dF \geq \alpha, \quad (4)$$

for some bounded function H of status quo consumption c_2 and reform consumption \hat{c}_2 and some constant $\alpha \in [0, 1]$. The interpretation is that the status quo is supported by fraction $H(c_2(\theta), \hat{c}_2(\theta)) \in [0, 1]$ of those individuals with status quo consumption $c_2(\theta)$ and reform consumption $\hat{c}_2(\theta)$, and that the status quo prevails if it is supported by at least fraction α of the total population.⁷ In particular, one can interpret H as the cumulative distribution function of taste shocks in a probabilistic voting model (e.g., Cox and McCubbins, 1986, Lindbeck and Weibull, 1987). An interesting extreme case obtains when these taste shocks are all zero, so that H is a step function with

$$H(a, b) = \begin{cases} 1 & \text{if } a \geq b \\ 0 & \text{if } a < b. \end{cases}$$

In this case, the “no-reform” constraint (4) simply requires that the fraction of the population who receive higher consumption (or utility) under the status quo than under the reform exceeds the critical threshold α . For most of the analysis, we make the technically convenient assumption that H is continuously differentiable in each argument, and thus admit step functions only as a limiting case. However, step functions themselves are also easy to work with—and indeed yield particularly sharp results—so we sometimes consider them separately. Another natural specification is where $H(c_2(\theta), \hat{c}_2(\theta))$ depends only on the difference $c_2(\theta) - \hat{c}_2(\theta)$ or the difference $u(c_2(\theta)) - u(\hat{c}_2(\theta))$, and its corresponding density is single-peaked at zero (so H is S-shaped in the difference), which is a common assumption in the probabilistic voting literature. This captures a situation where taste shocks can be non-zero but are concentrated around zero, for instance following a normal distribution.

Constraint (4) can capture formal political rules according to which previously announced policies can be reformed only if a sufficiently large number of voters are in favor, for instance a (super-)majority rule (indexed by α) in parliament or in a referendum. Alternatively, it can be interpreted as an informal stability requirement, where the government needs to maintain sufficient support for its policies to prevent major unrest or a coup in the future. Even if such revolts do not occur along the equilibrium path, their

⁷The assumption that the range of H is $[0, 1]$ is natural and is required for this interpretation, but is not technically required for the analysis.

possibility will induce the government to offer platforms that appeal to a large enough share of the population at any point in time, as captured by (4).

In Appendix C, we formulate the model as a game between the individuals and the government. The timing of the game is that the government first sets labor and capital tax schedules; individuals then choose how much to produce and consume in period 1; and finally the capital tax schedule may be reformed, and individuals consume their final capital in period 2. We then prove a version of the revelation principle, which says that any optimal (for the government) equilibrium allocation (c_1, c_2, y) satisfies (1), (2) and (4) when viewed as a direct mechanism. Thus, as far as optimal allocations are concerned, it is without loss of generality to restrict attention to feasible, incentive compatible direct mechanisms that satisfy the no-reform constraint. We thus restrict attention to such mechanisms throughout. The revelation principle proved in Appendix C does however require some mild assumptions on H , which differ in the two versions of the model. These assumptions are discussed in the relevant sections below.

3 Equalizing Reforms

In this section, we consider the version of the model where the only credible reform consumption schedule \hat{c}_2 involves full equalization in period 2, that is

$$\hat{c}_2(\theta) = \int c_2(\theta) dF - \kappa \text{ for all } \theta, \quad (5)$$

where $\kappa \geq 0$ is an exogenous cost of implementing a reform, which can equal 0.⁸ This assumption is appropriate if the government (rather than the voters) always has the final say on fiscal policy, as the (utilitarian) government's most-preferred reform is always full equalization.⁹ In other words, the timing of a possible reform is as follows.

1. Individuals vote on whether to allow a reform to c_2 .
2. If a reform is allowed, the government chooses the reform consumption schedule \hat{c}_2 .

We also impose the following mild assumption on H throughout this section, which says that a reform that makes everyone weakly worse-off is always defeated.

⁸ κ may include both physical and "reputational" costs, as discussed in Section 5.5.

⁹See Section 5.4 and Appendix D for how this would change if the government uses more general welfare weights. It is also worth noting that "ratchet effects," where the government could exploit information about individual types revealed in period 1, are not an issue here, because no information is required to implement the reform most preferred by the utilitarian government.

Assumption 1 If $c_2(\theta) \geq \hat{c}_2(\theta)$ for all θ , then (4) holds.

As $\kappa \geq 0$, Assumption 1 implies that constant period-2 consumption schedules are never reformed. This in turn implies that any equilibrium allocation satisfies the no-reform constraint when viewed as a direct mechanism, and thus that the revelation principle applies (see Appendix C). The government's problem is therefore

$$\max_{c_1, c_2, y} \int (u(c_1(\theta)) + \beta u(c_2(\theta)) - h(y(\theta), \theta)) dF$$

subject to (1), (2), and (4), where in (4) \hat{c}_2 is given by (5).

We note at the outset that the government's problem is typically not concave, because $H(c_2(\theta), \hat{c}_2(\theta))$ is typically not concave in $c_2(\theta)$; for example, step functions are not concave. This is not a "technical" problem (although it will lead to some mathematical complications), but rather a central economic ingredient of the model. In particular, to design a credible policy, the government must in effect select a coalition of voters that will support this policy against a potential future reform. This coalition-formation problem is non-concave under natural assumptions, as it is natural to assume that the voters who are closest to indifferent between the status quo and the reform are the ones who are most sensitive to slight changes in these policies. For example, if $H(c_2(\theta), \hat{c}_2(\theta))$ is a cumulative distribution function that depends only on the difference $c_2(\theta) - \hat{c}_2(\theta)$, then it is natural to assume that H is S-shaped in the difference, while it is not possible for H to be concave in the difference over the entire real line.

While the government's problem is not concave, it is still true that it must be solved by any solution to the dual problem

$$\min_{c_1, c_2, y} \int \left(c_1(\theta) + \frac{1}{R} c_2(\theta) - y(\theta) \right) dF$$

subject to

$$\int (u(c_1(\theta)) + \beta u(c_2(\theta)) - h(y(\theta), \theta)) dF \geq V, \quad (6)$$

(2), and (4), where V is the value of the primal.¹⁰ Note that in the constraints (2) and (6), $c_1(\theta)$ and $c_2(\theta)$ only enter through total consumption utility $U(\theta)$. Hence, any solution

¹⁰If not, then one could take a solution to the dual and vary c_1 so as to increase $u(c_1(\theta))$ to $u(c_1(\theta)) + \varepsilon$ for all θ . This variation would increase the objective while leaving (2) and (4) unaffected, and would not violate (1) for small enough ε .

must solve the subproblem

$$\min_{c_1, c_2, K} \int \left(c_1(\theta) + \frac{1}{R} c_2(\theta) \right) dF$$

subject to

$$u(c_1(\theta)) + \beta u(c_2(\theta)) = U(\theta), \quad (7)$$

$$\int H(c_2(\theta), RK - \kappa) dF \geq \alpha, \quad (8)$$

$$\int c_2(\theta) dF \leq RK. \quad (9)$$

The first-order (necessary) conditions of this program deliver the following characterization.

Lemma 1. *In any solution to the government's problem, the intertemporal wedge $\tau_k(\theta)$ satisfies*

$$\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} = -R\eta [\bar{H}_2(c_2, RK - \kappa) + H_1(c_2(\theta), RK - \kappa)], \quad (10)$$

where $\eta \geq 0$ is the multiplier on (8), subscripts on H denote partial derivatives, and

$$\bar{H}_2(c_2, RK - \kappa) \equiv \int H_2(c_2(\theta), RK - \kappa) dF.$$

Proof. Substituting out for $c_1(\theta)$ using (7), and letting $\eta \geq 0$ and $\mu \geq 0$ be the multipliers on (8) and (9), respectively, we form the Lagrangian

$$\int \left(u^{-1}(U(\theta) - \beta u(c_2(\theta))) + \frac{1}{R} c_2(\theta) \right) dF - \eta \int H(c_2(\theta), RK - \kappa) dF - \mu \left(RK - \int c_2(\theta) dF \right).$$

The first-order (necessary) condition with respect to K yields

$$\frac{\mu}{\eta} = -\bar{H}_2(c_2, RK - \kappa).$$

Next, rewrite the Lagrangian as

$$\int \left(u^{-1}(U(\theta) - \beta u(c_2(\theta))) + \frac{1}{R} c_2(\theta) - \eta \left(H(c_2(\theta), RK - \kappa) - \frac{\mu}{\eta} c_2(\theta) \right) \right) dF - \mu RK,$$

and differentiate under the integral with respect to $c_2(\theta)$ to obtain the necessary condition

$$u'(c_1(\theta)) = \frac{\beta R u'(c_2(\theta))}{1 - R\eta [\bar{H}_2(c_2, RK - \kappa) + H_1(c_2(\theta), RK - \kappa)]}.$$

Finally, use (3) to rewrite this condition as (10). ■

Equation (10) is the key tool for drawing conclusions about the progressivity of optimal capital taxes. In order to use it, however, we need to know the shape of $c_2(\theta)$. While it is quite intuitive that $c_2(\theta)$ should be non-decreasing, this is not immediate in the current context because the problem is not concave. The following mathematical lemma will let us conclude that $c_2(\theta)$ is non-decreasing in some solution to the government's problem. We will make use of this lemma repeatedly in different contexts throughout the paper, so we state it in general language. Note that the lemma concerns randomized consumption schedules, while the model allows only deterministic consumption schedules. The strategy is thus to show that monotone, deterministic consumption schedules are optimal in the class of all deterministic schedules by showing that they are optimal in the larger class of all randomized schedules. A side benefit of this approach is that it shows that our results would not change if we allowed randomized schedules in the model, as such schedules would not be optimal.

Lemma 2. *Let P be drawn from the set of right-continuous functions from an open set $\Theta \subseteq \mathbb{R}$ to $\Delta(\mathbb{R})$, the set of Borel distributions over real-valued allocations a .¹¹ Let X be an arbitrary index set, let F be a cumulative distribution function on Θ with positive density f , and consider the program*

$$W = \sup_P \int \int w(a, t(\theta)) dPdF$$

subject to one of the following constraints

$$\int \int y_x(a, A(P)) dPdF \leq 0 \text{ for all } x \in X, \text{ where } A(P) = \int \int a dPdF, \quad (C)$$

or

$$\int \int y_x(a) dPdF \leq 0 \text{ for all } x \in X.¹² \quad (C')$$

Assume that w is continuous and has strictly increasing differences in a and t , y_x is continuous for all $x \in X$, and t is right-continuous. Then

(i) In any solution P , if $t(\theta') < t(\theta'')$ then $a(\theta') \leq a(\theta'')$ for all $a(\theta') \in \text{supp } P(\theta')$ and $a(\theta'') \in \text{supp } P(\theta'')$.

(i') If the constraint takes the more restrictive form of constraint C' , then for any solutions P'

¹¹Continuity here is with respect to the weak topology. That is, we require that if $\theta' \downarrow \theta$ then $P(\theta')$ converges in distribution to $P(\theta)$.

¹²That is, constraint C is a more general version of constraint C' that allows the functions y_x to depend on the aggregate allocation $A(P)$. Note also that constraint C may equivalently be written as

$$\sup_{x \in X} \int \int y_x(a, A(P)) dPdF \leq 0,$$

and similarly for constraint C'.

and P'' , if $t(\theta') < t(\theta'')$ then $a(\theta') \leq a(\theta'')$ for all $a(\theta') \in \text{supp } P'(\theta')$ and $a(\theta'') \in \text{supp } P''(\theta'')$.

(ii) If a solution exists and t is non-decreasing, then there exists a deterministic solution in which a is non-decreasing.

Proof. See Appendix A.1. ■

To see the intuition for Lemma 2, restrict attention to deterministic allocations and assume that t is the identity, X is a singleton, and the constraint takes the simpler form of constraint C' . The program in the lemma is then

$$\sup_{a: \Theta \rightarrow \mathbb{R}} \int w(a(\theta), \theta) dF$$

subject to

$$\int y(a(\theta)) dF \leq 0.$$

The corresponding Lagrangian is

$$\int (w(a(\theta), \theta) - \lambda y(a(\theta))) dF.$$

Standard monotone comparative statics results (e.g., Theorem 4' of Milgrom and Shannon, 1994) imply that any (right-continuous) function a^* that maximizes the Lagrangian is monotone in the sense of (i), even if y is not convex. However, if y is not convex, then a saddle point of the Lagrangian may not exist, so it does not immediately follow that any solution to the constrained optimization problem is monotone. The proof of Lemma 2 uses an explicit variational argument to show that this must nonetheless be the case. Lemma 2 is the key monotone comparative statics tool of this paper—and seems like it could also be useful in other contexts—but it does require somewhat carefully chosen assumptions; in Appendix D, we point out both possible generalizations and limitations of the lemma.

We use Lemma 2 to establish the desired result about the shape of $c_2(\theta)$.

Lemma 3. *There exists a solution to the government's problem in which $c_2(\theta)$ is non-decreasing.*

Proof. See Appendix A.2. ■

The remaining results in this section concern such monotone solutions to the government's problem; by Lemma 2, the only possible loss of generality involved in this restriction is that other solutions may involve non-monotonicity of $c_2(\theta)$ in θ over intervals where both $U(\theta)$ and $y(\theta)$ are constant (i.e. for types that are pooled).

Our first main result is the following. Note that $H_1(c_2(\theta), RK - \kappa)$ being single-peaked in $c_2(\theta)$ corresponds to $H(c_2(\theta), RK - \kappa)$ being S-shaped in $c_2(\theta)$, which we have argued is a natural assumption.

Proposition 1. *If $H_1(c_2(\theta), RK - \kappa)$ is single-peaked in $c_2(\theta)$, then optimal marginal capital taxes are U-shaped in θ .*

Proof. By (10), $\tau_k(\theta)$ is non-increasing in $H_1(c_2(\theta), RK - \kappa)$. As $c_2(\theta)$ is non-decreasing in θ , if $H_1(c_2(\theta), RK - \kappa)$ is single-peaked in $c_2(\theta)$ then $H_1(c_2(\theta), RK - \kappa)$ is single-peaked in θ , and therefore $\tau_k(\theta)$ is U-shaped in θ . ■

Proposition 1 says that when a proposed policy is credible only if the period-2 allocation is preferred to full redistribution by a large enough share of the population, optimal marginal capital taxes are U-shaped. Although the derivation of this result was complicated by the non-concavity on the government's problem, the intuition is quite simple. Ex post, most poor agents will support a fully equalizing reform, most rich agents will oppose it, and middle-class agents will tend to be pivotal; this feature that those in the middle are most sensitive to the details of their allocation under the status quo relative to the reform is captured by the assumption that H_1 is single-peaked. Thus, in order to make the status quo credible, the government ensures that middle-class agents' period-2 consumption is high under the status quo and low under the reform. Considering variations in the timing of consumption that leave fixed total consumption utility $U(\theta) = u(c_1(\theta)) + \beta u(c_2(\theta))$ (and thus do not affect the incentive compatibility constraint), this may be achieved by subsidizing capital (i.e., backloading consumption) for the middle class (which increases middle-class period-2 consumption under the status quo) and taxing capital (i.e., frontloading consumption) for the poor and rich (which decreases middle-class period-2 consumption under the reform). Note that which agents end up being "poor," "rich," and "middle-class" in terms of period-2 consumption is endogenous to government policy, and one of the key results behind Proposition 1 is Lemma 3, which shows that it is indeed optimal for agents' rankings in terms of their period-2 consumption to match their rankings in terms of ability.

We now consider the implications of Proposition 1 for some leading specifications of H . These stronger functional form assumptions will also let us sign optimal marginal capital taxes.

Definition 1. *H depends on consumption differences if $H(c_2(\theta), \hat{c}_2(\theta)) = \tilde{H}(c_2(\theta) - \hat{c}_2(\theta))$ for some \tilde{H} . H depends on utility differences if $H(c_2(\theta), \hat{c}_2(\theta)) = \tilde{H}(u(c_2(\theta)) - u(\hat{c}_2(\theta)))$ for some \tilde{H} .*

H will depend on consumption (utility) differences if it is the probability of supporting the status quo in a probabilistic voting model with additive consumption (taste) shocks.

Whether H depends on consumption or utility differences, it is natural to assume that \tilde{H} is S-shaped, so that \tilde{H}' is single-peaked. Under this assumption, we obtain the following corollary of Proposition 1.¹³

Corollary 1. *If H depends on consumption (utility) differences and \tilde{H}' is single-peaked ($\tilde{H}'u'$ is single-peaked in $c_2(\theta)$ for all $\hat{c}_2(\theta)$), then optimal marginal capital taxes are U-shaped in θ . In addition, if the allocation (c_1, c_2, y) is non-constant and \tilde{H}' is strictly single-peaked ($\tilde{H}'u'$ is strictly single-peaked in $c_2(\theta)$ for all $\hat{c}_2(\theta)$), then optimal marginal capital taxes are negative for individuals with intermediate θ , and are positive for individuals with low and/or high θ .*

Proof. See Appendix A.3. ■

To interpret the condition that $\tilde{H}'u'$ is single-peaked, note that if H depends on utility differences then (10) becomes

$$\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} = R\eta \left[\int \tilde{H}'(c_2(\theta') - RK + \kappa) u'(RK - \kappa) dF - \tilde{H}'(c_2(\theta) - RK + \kappa) u'(c_2(\theta)) \right].$$

When \tilde{H} is uniform—so that \tilde{H}' is constant—we recover precisely the optimal capital tax formula of Farhi et al. (2012), which prescribes increasing marginal capital taxes. If \tilde{H}' is single-peaked, we then obtain a U-shaped adjustment to their progressive tax schedule, with taxes being U-shaped overall if and only if $\tilde{H}'u'$ is single-peaked.

Indeed, the version of the model studied in this section is a strict generalization of Farhi et al.'s model whenever consumption utility can be bounded. To see this, let $\underline{u} = \inf_{c \in C} u(c)$ and let $\bar{u} = \sup_{c \in C} u(c)$, where C is some (large) set of relevant consumption levels. Suppose that

$$H(c_2(\theta), \hat{c}_2(\theta)) = \tilde{H}(u(c_2(\theta)) - u(\hat{c}_2(\theta))) = \frac{1}{2} + \frac{1}{2} \frac{u(c_2(\theta)) - u(\hat{c}_2(\theta))}{\bar{u} - \underline{u}},$$

so that \tilde{H} is a uniform cumulative distribution function that is symmetric around zero in $u(c_2(\theta)) - u(\hat{c}_2(\theta))$. In addition, suppose $\alpha = 1/2$. Then the no-reform constraint in the current model, (4), becomes

$$\int \left(\frac{1}{2} + \frac{1}{2} \frac{u(c_2(\theta)) - u(RK - \kappa)}{\bar{u} - \underline{u}} \right) dF \geq \frac{1}{2},$$

¹³In what follows, a function $f : \mathbb{R} \rightarrow \mathbb{R}$ is *strictly single-peaked* if there exists $x^* \in \mathbb{R}$ such that f is strictly increasing on $\{x : x < x^*\}$ and strictly decreasing on $\{x : x > x^*\}$.

or equivalently

$$\int u(c_2(\theta)) dF \geq u(RK - \kappa),$$

which is precisely the no-reform constraint in Farhi et al. (2012). To understand this coincidence, recall that the no-reform constraint in their model requires that a utilitarian government does not wish to equalize consumption at resource cost κ , and observe that this is the case if and only if a simple majority of voters does not wish to equalize consumption at resource cost κ in a probabilistic voting model with uniform taste shocks. From this perspective, the results of this section may be viewed as a generalization of Farhi et al.'s analysis to the case where voters' probabilities of supporting reform are not all equally sensitive to marginal policy changes. This in turn is exactly the case where political coalition formation matters.

We close this section by noting that the finding that optimal capital taxes are U-shaped holds in a particularly sharp way when H is a step function.

Proposition 2. *If H is a step function, then in every monotone solution to the government's problem there is an interval of types $[\theta_l, \theta_h]$ such that*

1. *Marginal capital taxes are positive and constant for $\theta < \theta_l$ and $\theta > \theta_h$, and in particular are given by*

$$\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} = R\mu \text{ for all } \theta < \theta_l, \theta > \theta_h$$

where $\mu \geq 0$ is the multiplier on (9).

2. *Period 2 consumption $c_2(\theta)$ equals $RK - \kappa$ for all $\theta \in [\theta_l, \theta_h]$. In addition, marginal capital taxes are non-decreasing on the interval $[\theta_l, \theta_h]$, and if c_2 is non-constant then*

$$\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} \leq R\mu \text{ for all } \theta \in [\theta_l, \theta_h].$$

Proof. See Appendix A.4. ■

Figure 1 illustrates the pattern derived in Proposition 2. To sustain support from a large enough fraction α of the population, the government raises the consumption of the "middle class" (types between θ_l and θ_h) to $RK - \kappa$, making them just indifferent to a reform. To achieve this, the government imposes a flat savings tax on the "poor" and "rich" (to depress consumption under a reform) and an increasing, lower tax (or a subsidy) on the middle class (to raise their period-2 consumption under the status quo). As in the case where H is smooth, this leads to a U-shaped marginal capital tax schedule.

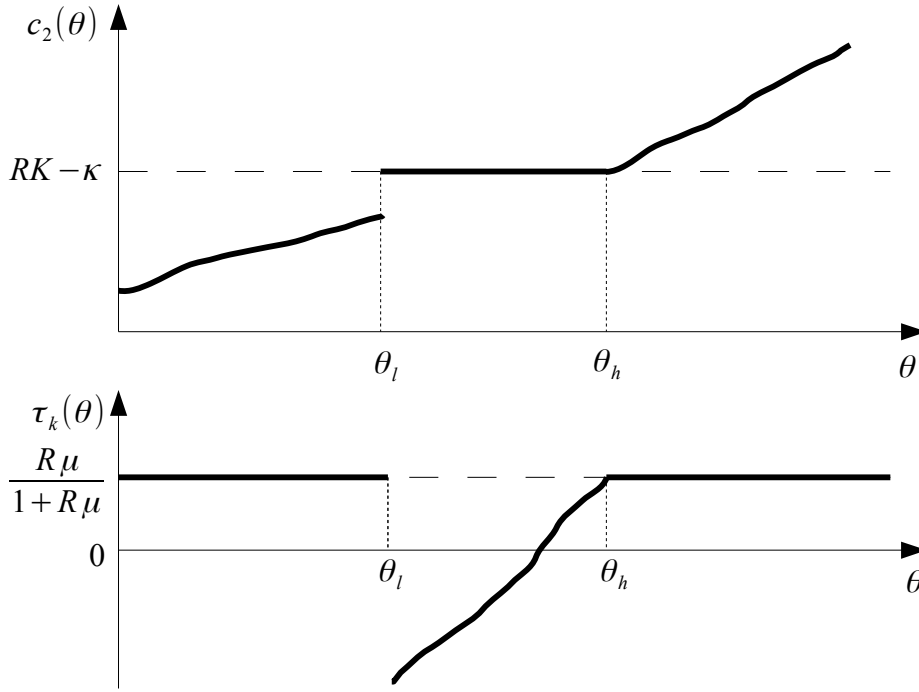


Figure 1: Equalizing reforms when H is a step function

4 Strategic Reforms

In this section, we consider an alternative version of the model where the government can credibly propose an arbitrary reform in period 2. While fully equalizing consumption remains the government's most-preferred reform, in period 2 it may wish to propose a reform other than full equalization in order to secure additional political support. As we assume that such "vote-buying" proposals are credible, the implicit timing of the reform stage in this section is

1. The government proposes a reform consumption schedule \hat{c}_2 .
2. Individuals vote on whether to implement the status quo c_2 or the reform \hat{c}_2 .

This reversed timing compared to Section 3 captures situations where voters, rather than the government, have the final say on fiscal policy, for instance through a referendum. We will establish the unexpected result that in this version of the model, optimal capital taxes are progressive throughout the skill distribution, even if H is strictly single-peaked. Indeed, a weak form of progressivity holds completely independently of the shape of H .

Before stating our results formally, we impose the assumption, maintained throughout this section, that the government would rather implement the optimal policy that forestalls reform than pay the resource cost κ to implement the full-commitment solution. This assumption ensures that a reform does not actually occur in an optimal equilibrium for the government, as the government's payoff in an equilibrium in which reform occurs is at most its full-commitment payoff minus κ ; in Appendix C, it is shown that this lets one restrict attention to feasible, incentive compatible direct mechanisms that satisfy the no-reform constraint. The assumption always holds if, for example, $\kappa > 0$ and α is sufficiently small, as in that case the full-commitment solution itself forestalls reform.¹⁴

Assumption 2 The value of the government's problem described below is greater than the value of the government's full-commitment solution minus κ .

We also maintain the following assumption on H throughout this section.

Assumption 3 H either depends on consumption differences or utility differences, and \tilde{H}' is single-peaked at 0.

The following proposition summarizes our findings in this section, proven in the subsequent Propositions 4 and 5 in this section and Propositions 7 and 8 in Appendix B. In what follows, a type θ is *non-pooled* if $c_2(\theta) \neq c_2(\theta')$ for all $\theta' \neq \theta$.

Proposition 3. *Suppose Assumptions 2 and 3 are satisfied. Then in any monotone solution there is a threshold type θ^* such that capital is subsidized for all types $\theta < \theta^*$ and taxed for all non-pooled types $\theta > \theta^*$. In addition, $\tau_\kappa(\theta)$ is non-decreasing for all $\theta < \theta^*$ and all non-pooled $\theta > \theta^*$ if*

- (i) H depends on consumption differences and $u'''(c) \geq 0$, or
- (ii) H depends on utility differences and $-u''(c) / u'(c)^2$ is non-increasing.

The most delicate issue in this result concerns the qualifications regarding non-pooled types. As we will see, these qualifications can be completely dispensed with in several leading cases, including when H is a step function. In addition, note that the progressivity result in (i) goes through under the additional assumption that $u''' \geq 0$, which is a necessary condition for non-increasing absolute or relative risk aversion. The assumption in (ii) that $-u''(c) / u'(c)^2$ is non-increasing is somewhat stronger than the assumption of non-increasing absolute risk-aversion, which says that $-u''(c) / u'(c)$ is non-increasing. It is satisfied by CRRA utility with coefficient less than or equal to one, for example.

¹⁴It can also be easily shown that for any $\kappa > 0$, there exists an $\alpha > 0$ for which the assumption holds but the full-commitment solution does not forestall reform, so that the no-reform constraint is binding.

We analyze the case where H depends on consumption differences in the text, and defer the case where H depends on utility differences to Appendix B. The analysis in the two cases is similar.

By the revelation principle (see Appendix C), the government's problem is exactly same as in Section 3, except that the credibility constraint is no longer

$$\int H(c_2(\theta), RK - \kappa) dF \geq \alpha,$$

but rather the condition that there does not exist any consumption schedule $\hat{c}_2 : \Theta \rightarrow \mathbb{R}$ such that

$$\begin{aligned} \int u(\hat{c}_2(\theta)) dF &> \int u(c_2(\theta)) dF, \\ \int \hat{c}_2(\theta) dF &\leq \int c_2(\theta) dF - \kappa, \end{aligned} \quad (11)$$

$$\int \tilde{H}(c_2(\theta) - \hat{c}_2(\theta)) dF \leq \alpha. \quad (12)$$

In words, this requires that there is no reform that the government prefers to the status quo, that is resource feasible and that would defeat the status quo in terms of popular support.¹⁵ This constraint is clearly equivalent to the value of the following deviation program (DP), which we denote by $V_D(c_2)$, being less than $\int u(c_2(\theta)) dF$.

$$\max_{\hat{c}_2} \int u(\hat{c}_2(\theta)) dF$$

subject to (11) and (12).

As with the government's problem in Section 3, non-convexity is an unavoidable feature of the deviation program under natural specifications of H , and hence its solution $\hat{c}_2(\theta)$ may not be unique. Letting $x(\theta) \equiv \hat{c}_2(\theta) - c_2(\theta)$, it is useful to rewrite (DP) as

$$\max_x \int u(c_2(\theta) + x(\theta)) dF \quad (14)$$

subject to

¹⁵Given the convention from the previous section that the status quo is implemented when (12) is satisfied with equality, the correct constraint here would involve a strict inequality in (12). However, whenever there exists some \hat{c}_2 that satisfies (11) and

$$\int H(c_2(\theta), \hat{c}_2(\theta)) dF < \alpha, \quad (13)$$

then $\sup_{\hat{c}_2} \int u(\hat{c}_2(\theta)) dF$ s.t. (11) and (13) equals $V_D(c_2)$ by continuity of F and \tilde{H} , so using the weak inequality in (12) does not make a difference. If on the other hand there does not exist any \hat{c}_2 that satisfies (11) and (13), then the no-reform constraint is not binding and the full commitment solution applies.

$$\int x(\theta) dF \leq -\kappa, \quad (15)$$

$$\int \tilde{H}(-x(\theta)) dF \leq \alpha. \quad (16)$$

Observe that (DP) does not depend directly on heterogeneity in θ , but only on the distribution of c_2 in the population.¹⁶ Let

$$X(\theta) \equiv \{x(\theta) : x \text{ is a solution to (DP)}\}.$$

Using our general Lemma 2, we can collect the following results about solutions to (DP).

Lemma 4. (i) A solution to (DP) exists.

(ii) If $c_2(\theta) < c_2(\theta')$ then $\inf X(\theta) \geq \sup X(\theta')$.

(iii) For any c_2 -schedule and any $\bar{c}_2 \in \mathbb{R}$, let $\Theta_{\bar{c}_2} \equiv \{\theta : c_2(\theta) = \bar{c}_2\}$. Then for almost all \bar{c}_2 , $\bigcup_{\theta \in \Theta_{\bar{c}_2}} X(\theta)$ is a singleton.

(iv) For every $\bar{c}_2 \in \mathbb{R}$, there exists at most one $s > 0$ such that $s \in \bigcup_{\theta \in \Theta_{\bar{c}_2}} X(\theta)$.

Proof. See Appendix A.5. ■

In words, property (ii) shows that any solution x must be non-increasing in c_2 , formalizing the intuition that, even though reforms are no longer necessarily fully equalizing, they still reduce the inequality implied by c_2 (since reform consumption is $\hat{c}_2 = c_2 + x$). Properties (iii) and (iv) concern the uniqueness of x in terms of c_2 , which are important later on for establishing differentiability of $V_D(c_2)$. Properties (i) to (iii) hold independently from the shape of \tilde{H} . If \tilde{H} is single-peaked at 0 (as we assume), then in addition property (iv) shows that there can be at most one strictly positive value of x for any c_2 .

With these preliminary observations about the deviation program in hand, we consider the following dual formulation of the government's planning problem.¹⁷

$$\min_{c_1, c_2, y} \int \left(c_1(\theta) + \frac{1}{R} c_2(\theta) - y(\theta) \right) dF$$

subject to

$$\int u(c_2(\theta)) dF \geq V_D(c_2),$$

(2) and (6). Using (7) to substitute out for $c_1(\theta)$, the planning problem becomes

$$\min_{U, c_2, y} \int \left(u^{-1}(U(\theta) - \beta u(c_2(\theta))) + \frac{1}{R} c_2(\theta) - y(\theta) \right) dF \quad \text{s.t.} \quad \int u(c_2(\theta)) dF \geq V_D(c_2),$$

¹⁶Consequently, ratchet effects again do not arise here, as c_2 is observable and the government cannot gain from information about θ revealed in period 1.

¹⁷The same variation as in footnote 3 shows that a solution to the primal must also solve the dual.

(2) and (6). As (2) and (6) depend only on U and y , a necessary condition for optimality is that c_2 solves the subproblem

$$\min_{c_2} \int \left(u^{-1} (U(\theta) - \beta u(c_2(\theta))) + \frac{1}{R} c_2(\theta) \right) dF \quad \text{s.t.} \quad \int u(c_2(\theta)) dF \geq V_D(c_2). \quad (17)$$

Let $\eta \geq 0$ denote the multiplier on the constraint, so that, as in Section 3, η denotes the multiplier on the relevant no-reform constraint.

The following result characterizes optimal marginal capital taxes. It is the basis of our main results on the progressivity of optimal capital taxes with strategic reforms.

Lemma 5. (i) *In any solution c_2 , for almost all θ where $X(\theta)$ is single-valued, we have*

$$\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} = R\eta [u'(c_2(\theta) + x(\theta)) - u'(c_2(\theta))], \quad \text{where } X(\theta) = \{x(\theta)\}. \quad (18)$$

(ii) *For almost all non-pooled types θ , $X(\theta)$ is single-valued, so (18) holds.*

Proof. (i) Suppose toward a contradiction that (18) fails on a positive measure set of types with single-valued $X(\theta)$ for some optimal consumption schedule c_2 . Then there exists either a positive measure set Θ' on which $\tau_k(\theta)/(1 - \tau_k(\theta))$ is less than the right-hand side of (18) (and $X(\theta)$ is single-valued for all $\theta \in \Theta'$), or a positive measure set Θ'' on which $\tau_k(\theta)/(1 - \tau_k(\theta))$ exceeds it (and $X(\theta)$ is single-valued for all $\theta \in \Theta''$). Assume the first case applies; the argument for the second case is symmetric.

Consider the variant consumption schedule \tilde{c}_2 given by

$$\tilde{c}_2(\theta) = \begin{cases} c_2(\theta) & \text{for } \theta \notin \Theta', \\ c_2(\theta) + t & \text{for } \theta \in \Theta', \end{cases}$$

for $t \in \mathbb{R}$. Slightly abusing notation, for variants \tilde{c}_2 of this form, let $V_D(t)$ be the corresponding value function in (DP). Letting $V_D'(0-)$ and $V_D'(0+)$ denote the left- and right-derivative of $V_D(t)$ at $t = 0$, respectively, Corollary 4 of Milgrom and Segal (2002) implies that $V_D'(0-)$ and $V_D'(0+)$ exist and satisfy

$$\begin{aligned} V_D'(0-) &= \inf_{x \text{ that solve (DP)}} \int_{\Theta'} u'(c_2(\theta) + x(\theta)) dF \\ V_D'(0+) &= \sup_{x \text{ that solve (DP)}} \int_{\Theta'} u'(c_2(\theta) + x(\theta)) dF. \end{aligned}$$

Since $X(\theta)$ is single-valued for all $\theta \in \Theta'$ by hypothesis, the supremum and infimum must coincide, so $V_D(t)$ is differentiable at $t = 0$ with

$$V_D'(0) = \int_{\Theta'} u'(c_2(\theta) + x(\theta)) dF.$$

Therefore, a necessary condition for optimality of c_2 is that $t = 0$ is a stationary point (over $t \in \mathbb{R}$) of the

Lagrangian

$$\int_{\Theta'} \left(u^{-1} (U(\theta) - \beta u(c_2(\theta) + t)) + \frac{1}{R} (c_2(\theta) + t) - \eta (u(c_2(\theta) + t) - u(c_2(\theta) + t + x(\theta))) \right) dF.$$

The corresponding first-order condition is

$$\int_{\Theta'} \left(-\frac{\beta u'(c_2(\theta))}{u'(c_1(\theta))} + \frac{1}{R} - \eta [u'(c_2(\theta)) - u'(c_2(\theta) + x(\theta))] \right) dF = 0.$$

Multiplying through by R and using the definition (3), this implies

$$\int_{\Theta'} \left(-\frac{1}{1 - \tau_k(\theta)} + 1 - R\eta [u'(c_2(\theta)) - u'(c_2(\theta) + x(\theta))] \right) dF = 0.$$

Together with the fact that Θ' has positive measure, this contradicts the initial hypothesis that

$$\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} < R\eta [u'(c_2(\theta) + x(\theta)) - u'(c_2(\theta))]$$

for all $\theta \in \Theta'$. Hence, no such set Θ' can exist.

(ii) If (18) fails on a positive measure set of non-pooled agents for some optimal consumption schedule c_2 , there exists either a positive measure set Θ' on which $\tau_k(\theta)/(1 - \tau_k(\theta))$ is less than the right-hand side of (18) (and $c_2(\theta)$ is strictly increasing in $\theta \in \Theta'$), or a positive measure set Θ'' on which $\tau_k(\theta)/(1 - \tau_k(\theta))$ exceeds it (and $c_2(\theta)$ is strictly increasing in $\theta \in \Theta''$). Consider again the first case and recall that $\cup_{\theta: c_2(\theta) = \bar{c}_2} X(\theta)$ is single-valued for almost all \bar{c}_2 , by Lemma 4 (iii). Since $c_2(\theta)$ is strictly increasing on Θ' , this implies that $X(\theta)$ is single-valued for almost all $\theta \in \Theta'$, so the argument from (i) can be applied. ■

We discuss both the intuition for formula (18) and the requirement that $X(\theta)$ is single-valued below. Before doing so, however, we apply Lemma 2 to show the existence of a monotone solution (as in Section 3).

Lemma 6. *There exists a solution to the government's problem in which $c_2(\theta)$ is non-decreasing.*

Proof. See Appendix A.6. ■

In what follows, we restrict attention to such monotone solutions. They feature progressive marginal capital taxation in the following sense.

Proposition 4. *In any monotone solution, there exists a threshold type θ^* such that capital is subsidized ($\tau_k(\theta) < 0$) for all types with $\theta < \theta^*$ and capital is taxed ($\tau_k(\theta) \geq 0$) for non-pooled types with $\theta > \theta^*$.*

Proof. Let $c_2^* = \inf \{c_2 : \exists \theta \text{ such that } c_2(\theta) = c_2 \text{ and } x(\theta) \leq 0 \text{ for some } x(\theta) \in X(\theta)\}$, and let $\theta^* = \inf \{\theta : c_2(\theta) \geq c_2^*\}$. Note that if θ satisfies equation (18) then

$$\text{sign}(\tau_k(\theta)) = \text{sign}(-x(\theta)).$$

We will show that θ satisfies (18) for (almost) all $\theta < \theta^*$ and non-pooled $\theta > \theta^*$, and that $x(\theta) > 0$ for $\theta < \theta^*$ while $x(\theta) \leq 0$ for non-pooled $\theta > \theta^*$. This will complete the proof.

If $\theta < \theta^*$ then $c_2(\theta) < c_2^*$ and hence, by definition of c_2^* , $x(\theta) > 0$ for all $x(\theta) \in X(\theta)$. By Lemma 4 (iv), for such types $X(\theta)$ is single-valued if \tilde{H}' is single-peaked at 0. Hence by Lemma 5, such types satisfy (18). In sum, types with $\theta < \theta^*$ satisfy (18) and $x(\theta) > 0$.

If $\theta > \theta^*$ is non-pooled then $c_2(\theta) > c_2^*$ and therefore $x(\theta) \leq 0$ for some $x(\theta) \in X(\theta)$, by definition of c_2^* and Lemma 4 (ii). By Lemma 5 (ii), for almost all non-pooled types $X(\theta)$ is a singleton and satisfies (18). In sum, non-pooled types with $\theta > \theta^*$ satisfy (18) and $x(\theta) \leq 0$. ■

The intuition for Proposition 4 relies on the fact that the capital tax, given by equation (18), is designed to make individuals of each type θ internalize the effect of an additional unit of their saving on the no-reform constraint. This involves comparing the effect on period-2 welfare under the most tempting reform, given by $u'(c_2(\theta) + x(\theta))$, with the effect on welfare under the status quo, $u'(c_2(\theta))$. Since by Lemma 4 a reform will equalize period-2 consumption relative to the status quo (i.e., $x(\theta)$ is decreasing), low θ types face $x(\theta) > 0$, so their saving relaxes the no-reform constraint (as $u'(c_2(\theta) + x(\theta)) < u'(c_2(\theta))$ when $x(\theta) > 0$), motivating the capital subsidy. In contrast, high θ types face $x(\theta) < 0$, so their saving tightens the no-reform constraint, which makes it optimal for them to face a capital tax.

Note that this logic is independent from the shape of the function \tilde{H} ; notably, it does not depend on whether \tilde{H}' is single-peaked or not.¹⁸ This is in contrast to our results in Section 3, where the shape of \tilde{H} is crucial for the U-shaped pattern of the intertemporal wedge. The reason for this difference is that, in Section 3, the key comparison for determining the capital tax is between $c_2(\theta)$ and $RK - \kappa$, which implies that agents with intermediate $c_2(\theta)$ are “pivotal” when \tilde{H}' is single-peaked, and are therefore subsidized. In contrast, in the current section the key comparison is between $u'(c_2(\theta))$ and $u'(c_2(\theta) + x(\theta))$, so that agents with $x(\theta) > 0$ are more sensitive to their period-2 consumption under the status quo than under the reform—regardless of the shape of \tilde{H} —which leads them to be subsidized (and $x(\theta)$ is decreasing regardless of the shape of \tilde{H} , as for any status quo schedule c_2 , the most tempting reform schedule is more egalitarian than c_2).

In Appendix B, we show that a similar tradeoff determines the shape of τ_k when \tilde{H} depends on differences in utility rather than consumption. In this case, both the government’s planning problem and deviation program can most conveniently be written in dual utility space, where period 2 consumption utility $u_2(\theta) \equiv u(c_2(\theta))$ is chosen for each

¹⁸The only place where single-peakedness of \tilde{H} matters in this section so far is part (iv) of Lemma 4, which says that $X(\theta)$ can have at most one positive element for almost all θ . This in turn implies that formula (18) applies for almost all $\theta < \theta^*$ without restricting to non-pooled types. Without this assumption, Proposition 4 would still hold for non-pooled types.

Table 1: Comparison of key tradeoffs and tax results

Model	Tradeoff	Shape of τ_k
Equalizing reforms	H_1 vs. \bar{H}_2	U-shaped
Strategic reforms – consumption differences	$u'(c_2 + x)$ vs. $u'(c_2)$	progressive
Strategic reforms – utility differences	$\Phi'(u_2 + x)$ vs. $\Phi'(u_2)$	progressive

individual. The no-reform constraint can then be framed as requiring that the optimal deviation must be more costly in terms of resources than the status quo when achieving at least the same level of welfare. Hence, the key tradeoff becomes $\Phi'(u_2(\theta) + x(\theta))$ versus $\Phi'(u_2(\theta))$, where $\Phi = u^{-1}$ and $x(\theta) = \hat{u}_2(\theta) - u_2(\theta)$ (so that now $x(\theta)$ is reform minus status quo utility, rather than consumption). Since $x(\theta)$ will again be decreasing, the same qualitative results apply in this model. Table 1 summarizes these key tradeoffs and results.

We also have the following stronger result about tax progressivity, which completes the proof of Proposition 3 for the case where \tilde{H} depends on consumption differences.

Proposition 5. *In any monotone solution, the following hold.*

- (i) *If $u''' \geq 0$, then $\tau_k(\theta)$ is non-decreasing on $\{\theta : \theta < \theta^*\}$.*
- (ii) *$\tau_k(\theta)$ is non-decreasing on $\{\theta : \theta \text{ is non-pooled and } \theta > \theta^*\}$.*

Proof. See Appendix A.7. ■

To understand Proposition 5, note that, when formula (18) applies, $\tau_k(\theta)$ is non-decreasing in $u'(c_2(\theta) + x(\theta)) - u'(c_2(\theta))$. Supposing for now that c_2 and x are differentiable, and omitting θ arguments, the derivative of this difference with respect to θ may be written as either

$$(u''(c_2 + x) - u''(c_2))c'_2 + u''(c_2 + x)x' \quad (19)$$

or

$$u''(\hat{c}_2)\hat{c}'_2 - u''(c_2)c'_2. \quad (20)$$

If $u''' \geq 0$, then the first way of writing the derivative shows that $\tau_k(\theta)$ is non-decreasing over types for which $x(\theta) \geq 0$, or equivalently types $\theta < \theta^*$. The proof of Proposition 5 also shows that, if \tilde{H} is single-peaked, then $\hat{c}_2(\theta)$ is non-increasing over non-pooled types $\theta > \theta^*$ (in fact, \hat{c}_2 is single-peaked overall), so the second way of writing the derivative—combined with Lemma 6—shows that $\tau_k(\theta)$ is also non-decreasing over these types.

We close this section with a brief discussion of the caveats regarding pooled types in Proposition 3, while providing the details in Appendix E. These caveats become irrelevant whenever (i) the set of solutions to the deviation program $X(\theta)$ is single-valued for

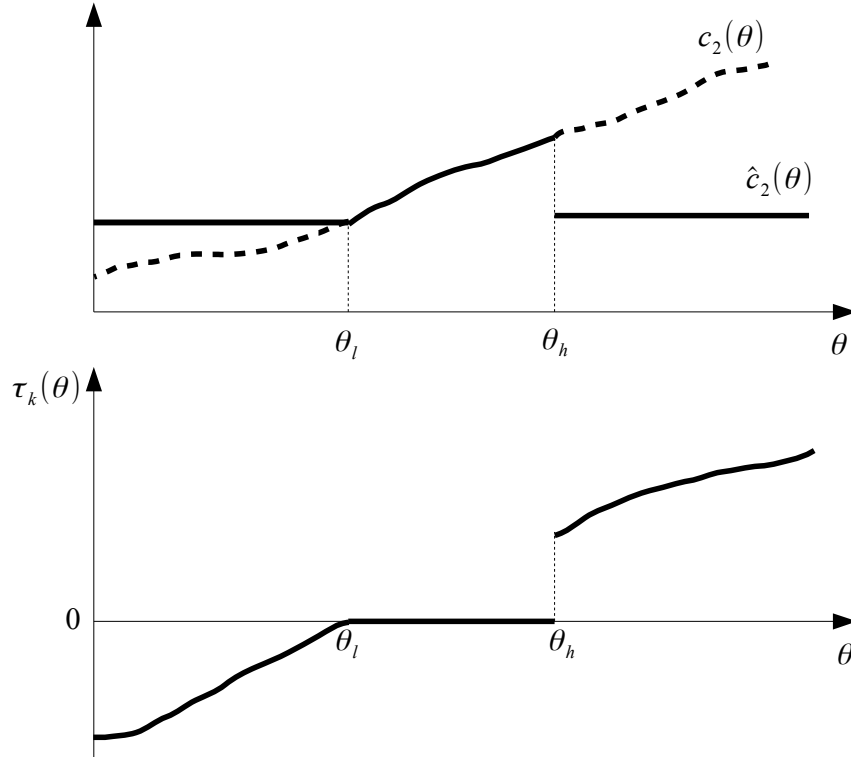


Figure 2: Strategic reforms when H is a step function

almost all θ , or (ii) the government's problem (17) is convex. As the following proposition shows, these sufficient conditions are indeed satisfied in some natural cases in terms of model fundamentals.

Proposition 6. *Propositions 4 and 5 hold without the caveats regarding pooled types if either (i) \tilde{H} is a step function, or (ii) u is quadratic.*

A sketch of the proof is provided in Appendix E.¹⁹ For example, if \tilde{H} is a step function, then it may be shown that $X(\theta)$ is single-valued almost everywhere, and in particular that it is flat at $x[\theta] = 0$ for all θ in an interval (θ_l, θ_h) (corresponding to those types that are indifferent between the status quo and the reform), and is decreasing on the intervals $(-\infty, \theta_l)$ and (θ_h, ∞) (in order to fully equalize final consumption under the reform for types in those intervals).²⁰ Figure 2 depicts the resulting shape of \hat{c}_2 compared to c_2 as well as the intertemporal wedge τ_k . Even if the conditions in Proposition 6 are not satisfied,

¹⁹One might think that if u is quadratic then the comparative statics in Proposition 5 (i) would never hold strictly, as that result assumes $u''' \geq 0$. However, inspecting the proof reveals that there is slack in this sufficient condition, and that in fact the comparative static holds strictly at any type θ where $x(\theta)$ is differentiable and non-constant, even if $u''' = 0$.

²⁰Unlike in the equalizing reforms section, we do not need a separate result to cover the step function case here, as the optimal tax formulas derived in this section do not assume that \tilde{H} is differentiable.

however, we argue more generally in Appendix E that the caveats regarding pooled types may be viewed as a technicality rather than an important feature of the model.

5 Discussion

In this section, we discuss how to numerically solve for the entire allocation in a quantitative example, how to implement the optimal allocation with taxes, how the predicted patterns of capital taxes relate to effective capital taxes in practice, how our results extend to more general versions of the government's objective function, and how to move from two periods to overlapping generations.

5.1 Labor Wedge and Numerical Illustration

This paper focuses on qualitative properties of the implicit marginal capital tax (3). We consider it a strength of the model that it allows for sharp results on this margin, despite the complexity of characterizing the entire optimal allocation (which is a standard feature of Mirrlees models). Nonetheless, it is interesting to compute the full optimal allocation numerically in an example. This gives some feel for the quantitative implications of the political economy constraints we consider, and also demonstrates that adding these constraints does not make the model intractable numerically.

We first show how to compute the intratemporal labor wedge, defined as

$$\tau_l(\theta) \equiv 1 - \frac{h_y(y(\theta), \theta)}{u'(c_1(\theta))}, \quad (21)$$

where h_y denotes the partial derivative of the disutility function h with respect to y . In particular, and specializing to the standard case with $h(y, \theta) = h(y/\theta)$, it is straightforward to show that—both in the models with equalizing reforms from Section 3 and with strategic reforms in Section 4—the implicit labor income tax (21) has to satisfy

$$\frac{\tau_l(\theta)}{1 - \tau_l(\theta)} = \left(1 + \frac{1}{\varepsilon(\theta)}\right) \frac{u'(c_1(\theta))}{\theta f(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left(\frac{1}{u'(c_1(s))} - \frac{1}{\lambda}\right) dF \quad (22)$$

whenever there is no bunching, where $\varepsilon(\theta)$ is the Frisch elasticity of labor supply at θ , $\underline{\theta}$ and $\bar{\theta}$ are the lower and upper bounds of the skill distribution, and

$$\lambda = \left(\int \frac{1}{u'(c_1(s))} dF\right)^{-1}$$

is the multiplier on the resource constraint (1).²¹

The key observation here is that formula (22) is exactly the same as in a static Mirrlees model (see Mirrlees, 1971, or Saez, 2001, for standard interpretations), as well as in our two-period model when there is full commitment (since (22) is completely independent of the form of the political support constraint). In this sense, the labor tax schedule is affected by our political economy constraint only indirectly through the $c_1(\theta)$ -schedule on the right-hand side of (22). This justifies our focus on the intertemporal wedge (3) as the key margin of interest, and demonstrates that introducing political economy constraints does not make the model less tractable along the intratemporal dimension.

Formula (22), together with our characterization of the capital wedge, also allows us to numerically compute the entire optimal allocation $(c_1(\theta), c_2(\theta), y(\theta))$ for a calibrated version of our model. We illustrate this using the equalizing reforms model from Section 3. We consider iso-elastic preferences of the form $u(c) = c^{1-\sigma}/(1-\sigma)$ and $h(l) = \psi l^{1+1/\varepsilon}/(1+1/\varepsilon)$ where $l = y/\theta$, so that the Frisch elasticity of labor supply is constant and given by ε . σ is set to be 0.9, which is close to the standard log-specification and ensures that utility is bounded below, and we set $\varepsilon = 1$, consistent with the evidence in Kimball and Shapiro (2010) and Erosa, Fuster and Kambourov (2011). We interpret a model period as $T = 10$ years and accordingly set $\beta = 0.95^{10}$ (so the annual discount factor is 0.95) and $R = 1/\beta$ (so the optimum under full commitment involves consumption smoothing with $c_1(\theta) = c_2(\theta)$).

For the skill distribution F , we follow Mankiw, Weinzierl and Yagan (2009) who fit a lognormal distribution to the empirical wage distribution from the 2007 Current Population Survey and append a Pareto distribution for the upper tail of wages to obtain asymptotic marginal tax rates as in Saez (2001). We extend their numerical procedure for a static Mirrlees model to our dynamic setting in order to compute both $\tau_l(\theta)$ and $\tau_k(\theta)$ and follow them in setting the scaling factor for the disutility of labor $\psi = 2.55$. We perform our simulations for $\kappa = 0$ and assume that the distribution of taste shocks H depends on consumption differences and is given by a normal cdf with mean 0 and standard deviation 2.5 (corresponding to just under 20% of mean consumption).

The left panel in Figure 3 shows the resulting optimal labor income tax rates (for wages up to \$100/hour) both under full commitment ($\alpha = 0$) and when the political economy

²¹Bunching does not occur whenever the (global) incentive constraints (2) can be replaced by the local incentive constraints $V'(\theta) = h_\theta(y(\theta), \theta) \forall \theta$, where

$$V(\theta) \equiv \max_{\theta'} u(c_1(\theta')) + \beta u(c_2(\theta')) - h(y(\theta'), \theta),$$

and the monotonicity constraint—requiring that $y(\theta)$ is non-decreasing—is not binding. This can be easily checked numerically as in the example we provide below.

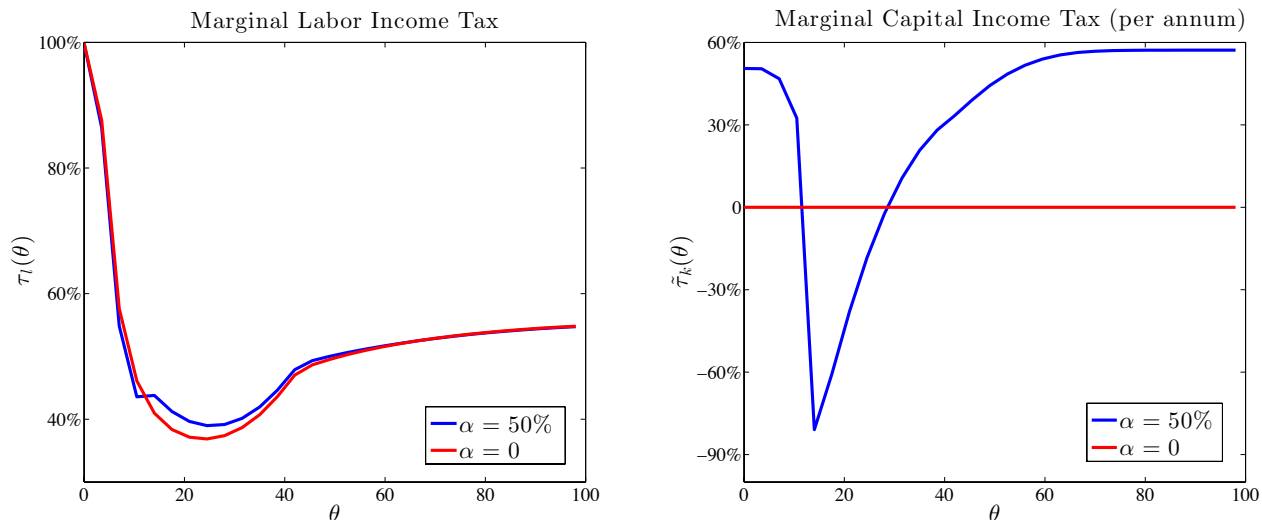


Figure 3: Marginal labor and capital taxes with and without political constraints

constraint (4) binds (in which case we assume majority voting with $\alpha = 50\%$). The right panel shows the optimal annualized marginal capital tax rate $\tilde{\tau}_k(\theta)$ on the *net* return to saving for the case where the political support constraint binds with $\alpha = 50\%$ (the optimal capital tax under full commitment is zero).²² As can be seen from the graphs, the labor tax schedules are very similar under full commitment and limited commitment, confirming that the key effects of the political economy constraint are on the intertemporal margin. Both schedules exhibit the typical U-shaped pattern emphasized in Diamond (1998) and Saez (2001), which is driven by the phase-out of the lump-sum transfer for low wages and by convergence to the asymptotic marginal tax rate due to the Pareto tail for high wages. The right panel demonstrates the U-shaped pattern for the capital tax rate emphasized here and predicted by Corollary 1. In particular, capital tax rates are negative for intermediate wages between \$15 and \$30 per hour and positive otherwise (and of sizable absolute amounts).

The intuition for this pattern is illustrated in Figure 4. The left panel shows consumption in both periods. While c_1 and c_2 coincide under full commitment (as shown by the red line for $\alpha = 0$), they are distorted when the threat of reform is binding. In particular, the capital subsidy increases c_2 (the green line) for intermediate wages in order to increase political support for the status quo to 50% of the population (from 39% under the

²²Formally, $\tilde{\tau}_k(\theta)$ is defined such that

$$1 + (1 - \tilde{\tau}_k(\theta)) \left(R^{1/T} - 1 \right) \equiv [R(1 - \tau_k(\theta))]^{1/T}.$$

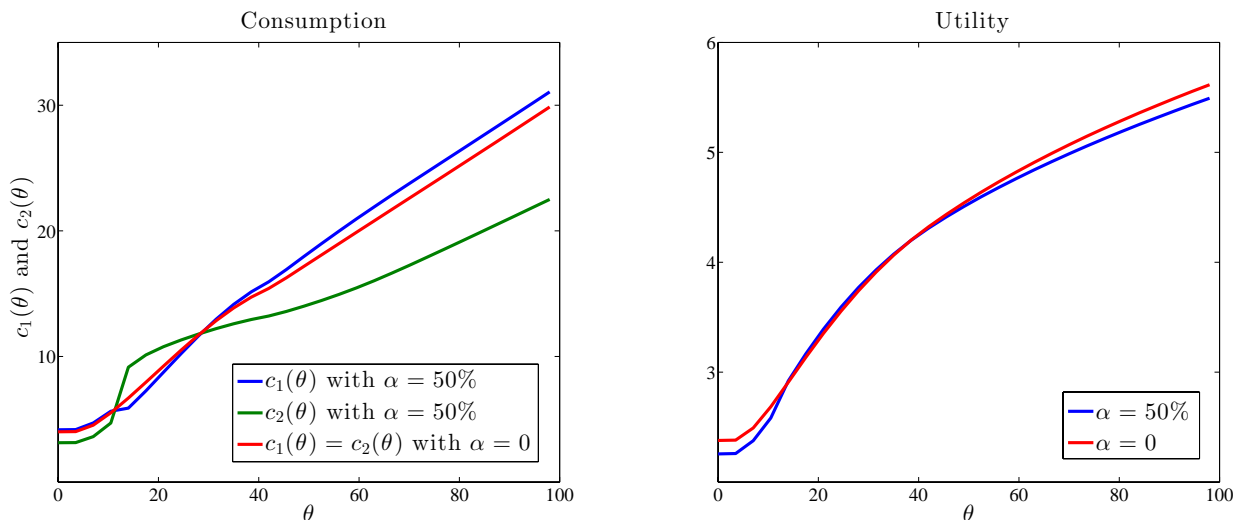


Figure 4: Consumption and utility with and without political constraints

full commitment solution). Of course, this must lead to an aggregate welfare loss, which here is equivalent to a 1.5% consumption drop for everyone in both periods compared to the full commitment solution. However, there is considerable heterogeneity in how this welfare loss is distributed across the population. In fact, as shown in the right panel, intermediate types with hourly wages between between \$15 and \$40 benefit from the presence of the political support constraint, whereas all other types lose. This illustrates how our model can generate a pattern of redistribution where the middle class actually benefits from political economy constraints—rather than just having their consumption backloaded—consistent with Director’s law.

5.2 Tax Implementation

Our analysis so far has implicitly considered direct mechanisms, where the government allocates $c_1(\theta)$, $c_2(\theta)$ and $y(\theta)$ conditional on individual reports about θ , taking into account technological, incentive compatibility and political credibility constraints. It is straightforward to show that these allocations can alternatively, and more realistically, be implemented through a tax system where each individual is confronted with the same budget set and picks her preferred allocation within this set. In particular, with a non-linear labor income tax T_y and a non-linear capital income tax T_k , individuals are faced with the budget constraint $c_1 + k \leq y - T_y(y)$ in period 1 and $c_2 \leq Rk - T_k(Rk)$ in period 2 and choose c_1, c_2, y, k to maximize $u(c_1) + \beta u(c_2) - h(y, \theta)$ subject to these two constraints.

By Proposition 3 in Farhi et al. (2012), any incentive compatible allocation $(c_1(\theta), c_2(\theta))$,

$y(\theta)$) that is non-decreasing in θ can be implemented using such a tax system. Since we show that $c_2(\theta)$ is always non-decreasing in an optimal allocation and $y(\theta)$ is non-decreasing by incentive compatibility, their result can be applied to our framework.²³ Moreover, the first-order conditions from the above utility-maximization problem imply

$$u'(c_1(\theta)) = \beta R(1 - T'_k(Rk(\theta)))u'(c_2(\theta))$$

for all θ whenever T_k is differentiable, so the wedge $\tau_k(\theta)$ defined in (3) and characterized throughout this paper coincides with the actual marginal capital income tax rate $T'_k(Rk(\theta))$ faced by individuals of type θ in this implementation.²⁴

5.3 Relation to Capital Taxes in Practice

Our results make clear-cut predictions about the shape of marginal capital taxes depending on the ability of governments to commit and the dynamics of political constraints. While our model is admittedly stylized and the level and progressivity of effective capital taxes in advanced economies are hard to measure, it is worth pointing out the consistency between our results and the patterns observed in practice.

First, as also noted by Farhi et al. (2012), policies such as income, estate, and wealth taxes, as well as the tax treatment of retirement accounts and subsidies to savings and education by the poor and middle class, contribute to the progressivity of capital taxation. However, it is plausible that overall capital taxes are U-shaped in many countries once means-tested government benefits are accounted for. For example, in the US, only individuals with sufficiently few assets qualify for Medicaid or Federal Student Aid, and only individuals with sufficiently low investment income qualify for the Earned Income Tax Credit. These asset tests can lead to very high effective savings distortions for the poor. More generally, the contribution to capital tax progressivity of subsidies to savings and education by the poor are at least partially offset by the phase-out of these subsidies, unless eligibility is solely determined by labor income.

In addition, some of these programs are targeted more directly at middle-class voters than the very poor. Examples include subsidies to college education, many retirement savings programs (where the subsidy is increasing in the marginal income tax rate up to some caps) and the mortgage interest deduction, which subsidizes the accumulation of

²³The statement of Proposition 3 in Farhi et al. (2012) also requires $c_1(\theta)$ to be non-decreasing, which holds at the optimum in their framework but may or may not hold in our model. However, inspecting their proof reveals that this condition is in fact not needed for the result.

²⁴If $T_k(Rk(\theta))$ is not differentiable because there is pooling at consumption level $c_2(\theta)$ (so T_k has a convex kink), $\tau_k(\theta)$ is still bounded between the (well-defined) left- and right-derivatives of T_k .

housing wealth. Finally, Doepke and Schneider (2006) show that the inflation tax effectively redistributes from rich, bond-holding households to middle class households with fixed-rate mortgage debt, consistent with the prediction of U-shaped capital taxes.

5.4 General Objective Function

Our maintained assumption that the government is utilitarian can be relaxed substantially. This is shown in Appendix D, where we relax the utilitarian assumption in two ways. First, we let the government use general Pareto weights G to evaluate welfare, which can differ from the population distribution F . Second, we let individuals differ along a second dimension ρ in addition to θ , where ρ is observable to the government and enters into the government's Pareto weights, but is otherwise payoff-irrelevant. This second dimension of heterogeneity allows for a government that is "non-benevolent," in that it favors certain groups in society over others: for example, ρ could capture an individual's race, ethnicity, or other minority status, her age, her geographic region of origin, or any other observable marker of membership in some group that the government may favor or disfavor. We show that our main results go through within each group ρ if the government's redistributive preferences are at least as inequity averse as the utilitarian criterion would imply. Formally, if $g(\theta, \rho)$ is the density corresponding to the Pareto weights G over the two dimensions of heterogeneity, then our main results go through whenever $g(\theta, \rho)/f(\theta, \rho)$ is decreasing in θ , holding ρ fixed, where f is the joint density corresponding to F .

For example, this implies that—perhaps somewhat surprisingly—the U-shaped pattern of marginal capital taxes from the first version of our model emerges even if the government puts very high weight on low- θ types, as with a Rawlsian objective.

5.5 Overlapping Generations

It is straightforward to extend the two-period model here to an infinite-horizon overlapping generations (OLG) setting, as for instance in Farhi et al. (2012). This would allow us to endogenize the cost of reform κ as a "reputational" cost borne by the government when it deviates by reforming its proposed policy. In particular, consider an OLG-version of our model where individuals of each generation live for two periods. Suppose that whenever a reform occurs, play reverts to the worst continuation equilibrium for the government. This worst continuation equilibrium is the one where no further production takes place, and the government fully equalizes future consumption, spending down the remaining

capital stock optimally. In such “grim trigger” equilibria, the results from our two-period model, both from Sections 3 and 4, would continue to hold.

One technicality here is that, unlike in Farhi et al. (2012), grim trigger equilibria would not necessarily be the best equilibria overall from the perspective of the government in an OLG version of our model. To see this, note that an optimal equilibrium for the government is one in which it is punished as harshly as possible for any deviation. In grim trigger equilibria, the government gets its lowest possible continuation payoff (corresponding to full equalization) starting in period $t + 1$ after a deviation in period t , but in period t voters approve the proposed reform if it is myopically optimal for them to do so. In general, it might be possible to punish the government more harshly by specifying that continuation play after a deviant reform proposal that is myopically appealing to voters is such that the reform is not approved, while continuation play is something other than full equalization (for example, continuation play might specify that the government rewards “pivotal” voters in the future if they do not support a deviant reform today).²⁵ Note that this issue would go away if, for example, we assumed that only the old generation votes, which is a relatively common assumption in OLG models of political economy (Glomm and Ravikumar, 1992, Saint-Paul and Verdier, 1997, Benabou, 2000).

6 Conclusion

This paper has studied dynamic non-linear taxation under the assumption that a policy is credible if it maintains the support of a large enough political coalition over time. Optimal taxes in this setting differ starkly from those in settings where the government can fully commit to policy, or where it can always change policy at a fixed cost. Rather than predicting zero capital taxes (as in the full commitment case) or purely progressive capital taxes (as in Farhi et al., 2012), the simplest version of our model (which is still a generalization of both the full commitment case and the baseline model of Farhi et al.) predicts U-shaped capital taxes, so that saving is subsidized for the middle class but taxed for the poor and rich, recalling Director’s law of redistribution (Stigler, 1970). In a more complicated version of the model where the government can engage in sophisticated vote-buying schemes, we instead find that purely progressive capital taxation is optimal. These versions of the model can be interpreted as capturing varying degrees of government commitment at the reform stage, as for instance resulting from more direct

²⁵Such a punishment might be harsher than grim trigger because it gives a lower instantaneous payoff for the government, even though it also gives a higher continuation payoff (note that there is no contradiction with the results of Abreu (1988) and others here, as the current model is not a repeated game).

versus indirect forms of democracy. More generally, our analysis suggests that the nature of potential political reforms is an important determinant of the progressivity and middle-class bias of capital taxes, and of redistribution more broadly.

References

- ABREU, D. (1988): "On the Theory of Infinitely Repeated Games with Discounting," *Econometrica*, pp. 383–396.
- ACEMOGLU, D., M. GOLOSOV, AND A. TSYVINSKI (2010): "Dynamic Mirrlees Taxation under Political Economy Constraints," *Review of Economic Studies*, 77, 841–881.
- ALESINA, A., AND D. RODRIK (1994): "Distributive Politics and Economic Growth," *The Quarterly Journal of Economics*, 109(2), 465–490.
- ATKINSON, A. B., AND J. E. STIGLITZ (1976): "The design of tax structure: Direct versus indirect taxation," *Journal of Public Economics*, 6(1), 55–75.
- AUMANN, R., AND M. KURZ (1977): "Power and Taxes," *Econometrica*, 5, 1137–1161.
- BENABOU, R. (2000): "Unequal Societies: Income Distribution and the Social Contract," *American Economic Review*, pp. 96–129.
- BENHABIB, J., AND A. RUSTICHINI (1997): "Optimal taxes without commitment," *Journal of Economic Theory*, 77(2), 231–259.
- BERTOLA, G. (1993): "Factor Shares and Savings in Endogenous Growth," *American Economic Review*, 83, 1184–1199.
- CHARI, V., AND P. KEHOE (1990): "Sustainable Plans," *Journal of Political Economy*, 98, 783–802.
- DIAMOND, P. (1998): "Optimal Income Taxation: An Example with a U-shaped Pattern of Optimal Tax Rates," *American Economic Review*, 88, 83–95.
- DIXIT, A., AND J. LONDREGAN (1996): "The Determinants of Success of Special Interests in Redistributive Politics," *Journal of Politics*, 58, 1132–1155.
- (1998): "Ideology, Tactics, and Efficiency in Redistributive Politics," *The Quarterly Journal of Economics*, 113(2), 497–529.

- DOEPKE, M., AND M. SCHNEIDER (2006): "Inflation and the redistribution of nominal wealth," *Journal of Political Economy*, 114(6), 1069–1097.
- EROSA, A., L. FUSTER, AND G. KAMBOUROV (2011): "Towards a Micro-Founded Theory of Aggregate Labor Supply," *Mimeo*.
- FARHI, E., C. SLEET, I. WERNING, AND S. YELTEKIN (2012): "Non-linear Capital Taxation Without Commitment," *Review of Economic Studies*, 79(4), 1469–1493.
- FISCHER, S. (1980): "Dynamic inconsistency, cooperation and the benevolent dissembling government," *Journal of Economic Dynamics and Control*, 2, 93–107.
- FUDENBERG, D., AND J. TIROLE (1990): "Moral Hazard and Renegotiation in Agency Contracts," *Econometrica*, 58, 1279–1320.
- GLOMM, G., AND B. RAVIKUMAR (1992): "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality," *Journal of Political Economy*, pp. 818–834.
- HASSLER, J., P. KRUSELL, K. STORESLETTEN, AND F. ZILIBOTTI (2005): "The Dynamics of Government," *Journal of Monetary Economics*, 52(7), 1331–1358.
- HINNERICH, B. T., AND P. PETTERSSON-LIDBOM (2013): "Democracy, redistribution, and political participation: Evidence from Sweden 1919-1950," *Econometrica*, forthcoming.
- KIMBALL, M., AND M. SHAPIRO (2010): "Labor Supply: Are the Income and Substitution Effects Both Large or Both Small?," *Mimeo, University of Michigan*.
- KLEIN, P., P. KRUSELL, AND J.-V. RIOS-RULL (2008): "Time-consistent public policy," *Review of Economic Studies*, 75(3), 789–808.
- KOTLIKOFF, L., T. PERSSON, AND L. SVENSSON (1988): "Social Contracts as Assets: A Possible Solution to the Time-Consistency Problem," *American Economic Review*, 78, 662–677.
- KYDLAND, F., AND E. PRESCOTT (1977): "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy*, 85, 473–492.
- LINDBECK, A., AND J. W. WEIBULL (1987): "Balanced-budget redistribution as the outcome of political competition," *Public Choice*, 52(3), 273–297.
- LIZZERI, A., AND N. PERSICO (2001): "The Provision of Public Goods under Alternative Electoral Incentives," *American Economic Review*, pp. 225–239.

- MANKIW, G., M. WEINZIERL, AND D. YAGAN (2009): “Optimal Taxes in Theory and Practice,” *Journal of Economic Perspectives*, 23, 147–174.
- MILGROM, P., AND I. SEGAL (2002): “Envelope Theorems for Arbitrary Choice Sets,” *Econometrica*, 70, 583–601.
- MILGROM, P., AND C. SHANNON (1994): “Monotone Comparative Statics,” *Econometrica*, pp. 157–180.
- MIRRLEES, J. (1971): “An Exploration in the Theory of Optimum Income Taxation,” *Review of Economic Studies*, 38, 175–208.
- MYERSON, R. B. (1993): “Incentives to Cultivate Favored Minorities under Alternative Electoral Systems,” *American Political Science Review*, pp. 856–869.
- NETZER, N., AND F. SCHEUER (2010): “Competitive Markets Without Commitment,” *Journal of Political Economy*, 118, 1079–1109.
- PERSSON, T., AND G. TABELLINI (1994a): “Is Inequality Harmful for Growth?,” *American Economic Review*, 84(3), 600–621.
- (1994b): “Representative Democracy and Capital Taxation,” *Journal of Public Economics*, 55(1), 53–70.
- PHELAN, C., AND E. STACCHETTI (2001): “Sequential Equilibria in a Ramsey Tax Model,” *Econometrica*, 69, 1491–1518.
- SAEZ, E. (2001): “Using Elasticities to Derive Optimal Tax Rates,” *Review of Economic Studies*, 68, 205–229.
- SAINT PAUL, G., AND T. VERDIER (1997): “Power, Distributive Conflicts, and Multiple Growth Paths,” *Journal of Economic Growth*, 2(2), 155–168.
- STIGLER, G. J. (1970): “Director’s law of public income redistribution,” *Journal of Law and Economics*, 13(1), 1–10.

A Appendix: Omitted Proofs

A.1 Proof of Lemma 2

Part (i). To obtain a contradiction, suppose that there exist $\theta', \theta'' \in \Theta$ such that $t(\theta') < t(\theta'')$ and yet $a(\theta') > a(\theta'')$ for some $a(\theta') \in \text{supp } P(\theta'), a(\theta'') \in \text{supp } P(\theta'')$. Since t and P are right-continuous and Θ

is open, there exist disjoint closed intervals of positive length $\Theta' \subseteq \Theta$ and $\Theta'' \subseteq \Theta$ such that $t(\theta') < t(\theta'')$ and $a(\theta') > a(\theta'')$ for some $a(\theta') \in \text{supp } P(\theta'), a(\theta'') \in \text{supp } P(\theta'')$ for all $\theta' \in \Theta', \theta'' \in \Theta''$. Let $\bar{a}(\theta) = \sup \{\text{supp } P(\theta')\}, \underline{a}(\theta) = \inf \{\text{supp } P(\theta')\}$, and

$$\nu \equiv \inf_{\theta' \in \Theta', \theta'' \in \Theta''} \bar{a}(\theta') - \underline{a}(\theta'') > 0.$$

Without loss of generality, let the lengths of Θ' and Θ'' be equal. Define $\phi : \Theta' \rightarrow \Theta''$ by $\phi(\theta) = \theta + \underline{a} - \underline{\theta}'$, so that in particular ϕ is an invertible bijection. Given a distribution $P(\theta)$, let $\bar{P}(\theta)$ and $\underline{P}(\theta)$ denote the truncation of $P(\theta)$ on $[\bar{a}(\theta) - \nu/4, \bar{a}(\theta)]$ and $[\underline{a}(\theta), \underline{a}(\theta) + \nu/4]$, respectively. Define a new randomized schedule \hat{P} by

$$\hat{P}(\theta) \equiv \begin{cases} P(\theta) + \frac{\varepsilon}{f(\theta)} \left(\bar{\gamma}(\theta) \underline{P}(\phi(\theta)) - \underline{\gamma}(\phi(\theta)) \bar{P}(\theta) \right) & \text{if } \theta \in \Theta' \\ P(\theta) + \frac{\varepsilon}{f(\theta)} \left(\underline{\gamma}(\theta) \bar{P}(\phi^{-1}(\theta)) - \bar{\gamma}(\phi^{-1}(\theta)) \underline{P}(\theta) \right) & \text{if } \theta \in \Theta'' \\ P(\theta) & \text{if } \theta \notin \Theta' \cup \Theta'', \end{cases}$$

where the factors

$$\bar{\gamma}(\theta) \equiv \int d\bar{P}(\theta) \text{ and } \underline{\gamma}(\theta) \equiv \int d\underline{P}(\theta)$$

ensure that $\hat{P}(\theta)$ integrates to one for each θ , and we fix some $\varepsilon > 0$ such that $\varepsilon < \inf_{\theta \in \Theta' \cup \Theta''} f(\theta)$, which (together with $\bar{\gamma}(\theta), \underline{\gamma}(\theta) \leq 1$ for all θ) ensures that $\hat{P}(\theta)$ is a well-defined probability distribution for all θ .

The variation \hat{P} is constructed such that, for any $x \in X$, we have

$$\int \int y_x(a) d\hat{P}dF = \int \int y_x(a) dPdF,$$

because

$$\begin{aligned} & \int \int y_x(a) d\hat{P}dF - \int \int y_x(a) dPdF \\ &= \varepsilon \int_{\Theta'} \int y_x(a) \left(\bar{\gamma}(\theta) d\underline{P}(\phi(\theta)) - \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) \right) d\theta + \varepsilon \int_{\Theta''} \int y_x(a) \left(\underline{\gamma}(\theta) d\bar{P}(\phi^{-1}(\theta)) - \bar{\gamma}(\phi^{-1}(\theta)) d\underline{P}(\theta) \right) d\theta \\ &= \varepsilon \int_{\Theta'} \int y_x(a) \left(\bar{\gamma}(\theta) d\underline{P}(\phi(\theta)) - \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) + \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) - \bar{\gamma}(\theta) d\underline{P}(\phi(\theta)) \right) d\theta = 0. \end{aligned}$$

In particular, this implies that $A(\hat{P}) = A(P)$ and therefore

$$\int \int y_x(a, A(\hat{P})) d\hat{P}dF = \int \int y_x(a, A(P)) dPdF$$

for all $x \in X$; that is, if P satisfies constraint C or C', then so does \hat{P} . In addition,

$$\begin{aligned} & \int \int w(a, t(\theta)) d\hat{P}dF - \int \int w(a, t(\theta)) dPdF \\ &= \varepsilon \int_{\Theta'} \left[\int w(a, t(\theta)) \bar{\gamma}(\theta) d\underline{P}(\phi(\theta)) - \int w(a, t(\theta)) \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) \right] d\theta \\ & \quad + \varepsilon \int_{\Theta''} \left[\int w(a, t(\theta)) \underline{\gamma}(\theta) d\bar{P}(\phi^{-1}(\theta)) - \int w(a, t(\theta)) \bar{\gamma}(\phi^{-1}(\theta)) d\underline{P}(\theta) \right] d\theta \\ &= \varepsilon \int_{\Theta'} \left[\int [w(a, t(\theta)) - w(a, t(\phi(\theta)))] \bar{\gamma}(\theta) d\underline{P}(\phi(\theta)) - \int [w(a, t(\theta)) - w(a, t(\phi(\theta)))] \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) \right] d\theta. \end{aligned}$$

For each $\theta \in \Theta'$, $t(\theta) < t(\phi(\theta))$, and because $w(a, t)$ has increasing differences, $w(a, t(\theta)) - w(a, t(\phi(\theta)))$ is decreasing in a . Therefore,

$$\int [w(a, t(\theta)) - w(a, t(\phi(\theta)))] \bar{\gamma}(\theta) d\underline{P}(\phi(\theta)) > [w(\underline{a}(\phi(\theta)) + \nu/4, t(\theta)) - w(\underline{a}(\phi(\theta)) + \nu/4, t(\phi(\theta)))] \bar{\gamma}(\theta) \underline{\gamma}(\phi(\theta)),$$

where we used $\int d\underline{P}(\phi(\theta)) = \underline{\gamma}(\phi(\theta))$. Similarly, for each $\theta \in \Theta'$,

$$\int [w(a, t(\theta)) - w(a, t(\phi(\theta)))] \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) < [w(\bar{a}(\theta) - \nu/4, t(\theta)) - w(\bar{a}(\theta) - \nu/4, t(\phi(\theta)))] \underline{\gamma}(\phi(\theta)) \bar{\gamma}(\theta).$$

Hence,

$$\begin{aligned} & \varepsilon \int_{\Theta'} \left[\int [w(a, t(\theta)) - w(a, t(\phi(\theta)))] \bar{\gamma}(\theta) d\underline{P}(\phi(\theta)) - \int [w(a, t(\theta)) - w(a, t(\phi(\theta)))] \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) \right] d\theta \\ & > \varepsilon \int_{\Theta'} \left[\begin{array}{l} w(\underline{a}(\phi(\theta)) + \nu/4, t(\theta)) - w(\underline{a}(\phi(\theta)) + \nu/4, t(\phi(\theta))) \\ - [w(\bar{a}(\theta) - \nu/4, t(\theta)) - w(\bar{a}(\theta) - \nu/4, t(\phi(\theta)))] \end{array} \right] \underline{\gamma}(\phi(\theta)) \bar{\gamma}(\theta) d\theta \\ & > 0, \end{aligned}$$

where the last inequality follows because $\underline{a}(\phi(\theta)) + \nu/4 < \bar{a}(\theta) - \nu/4$ for all $\theta \in \Theta'$ and $w(a, t(\theta)) - w(a, t(\phi(\theta)))$ is decreasing in a (as $w(a, t)$ has increasing differences). Therefore, \hat{P} achieves a strictly higher value of the objective than P , so P cannot be a solution.

Part (i'). Under Constraint C' , if P' and P'' are both solutions then so is the function $\frac{1}{2}P' + \frac{1}{2}P''$ given by $(\frac{1}{2}P' + \frac{1}{2}P'')(\theta) = \frac{1}{2}P'(\theta) + \frac{1}{2}P''(\theta)$ for all θ . Noting that $\text{supp } P'(\theta') \subseteq \text{supp}(\frac{1}{2}P' + \frac{1}{2}P'')(\theta')$ and $\text{supp } P''(\theta'') \subseteq \text{supp}(\frac{1}{2}P' + \frac{1}{2}P'')(\theta'')$, the result follows from applying (i) to $\frac{1}{2}P' + \frac{1}{2}P''$.

Part (ii). Let P be a solution. Taking $\theta'' \downarrow \theta'$ and recalling that $P(\theta)$ is right-continuous, (i) implies that P is already deterministic and monotone over every interval $\Theta' \subseteq \Theta$ on which t is strictly increasing. It remains only to show that P may be replaced by a deterministic and monotone allocation on those intervals Θ' on which t is constant. To see that this is possible, fix such an interval $\Theta' = [\underline{\theta}', \bar{\theta}']$, and let

$$\underline{a} = \inf \{a_0 : a_0 \in \text{supp } P(\theta), \theta \in \Theta'\}.$$

Now define the deterministic and monotone allocation $a : \Theta' \rightarrow \mathbb{R}$ by

$$a(\theta) = \inf \left\{ a_0 : \int_{\Theta'} \mathbb{I}\{a \in [\underline{a}, a_0]\} dP dF \geq F(\theta) - F(\underline{\theta}') \right\},$$

where $\mathbb{I}\{\cdot\}$ denotes the indicator function. It follows that for every interval of allocations $A = [\underline{a}, a_0]$,

$$\int_{\Theta'} \mathbb{I}\{a(\theta) \in A\} dF = \int_{\Theta'} \mathbb{I}\{a \in A\} dP dF,$$

and therefore that the same holds for every measurable set of allocations $A \subseteq \mathbb{R}$. Since t is constant on Θ' , this implies that replacing P with a on Θ' does not affect the objective or the constraints of the program. Therefore, performing this replacement on all intervals on which t is constant yields a deterministic and monotone solution.

A.2 Proof of Lemma 3

We relax the government's problem by allowing randomized consumption schedules, and show that there exists a deterministic solution to the relaxed problem with $c_2(\theta)$ non-decreasing. This implies that there exists a solution to the original problem with $c_2(\theta)$ non-decreasing.

Formally, allow the government to choose, for each $\theta \in \Theta$, a distribution $P(\theta)$ over consumption levels $(c_1(\theta), c_2(\theta))$ such that $P(\theta)$ is right-continuous in the weak topology and $U(\theta) = u(c_1(\theta)) + \beta u(c_2(\theta))$ is constant for all $(c_1(\theta), c_2(\theta)) \in \text{supp } P(\theta)$. Rewrite the dual problem as

$$\min_{U, P, y} \int \int \left(u^{-1}(U(\theta) - \beta u(c_2)) + \frac{1}{R}c_2 - y(\theta) \right) dPdF$$

subject to

$$\begin{aligned} U(\theta) - h(y(\theta), \theta) &\geq U(\theta') - h(y(\theta'), \theta) \text{ for all } \theta, \theta', \\ \int (U(\theta) - h(y(\theta), \theta)) dF &\geq V, \\ \int \int H\left(c_2, \int \int c_2 dPdF - \kappa\right) dPdF &\geq \alpha. \end{aligned}$$

Our assumptions ensure that a solution to this problem exists because the objective is continuous and the constraint set is closed and can be bounded using the Inada conditions on u . Moreover, observe that at any solution, P must solve the subproblem

$$\min_P \int \int \left(u^{-1}(U(\theta) - \beta u(c_2)) + \frac{1}{R}c_2 \right) dPdF$$

subject to

$$\int \int H\left(c_2, \int \int c_2 dPdF - \kappa\right) dPdF \geq \alpha.$$

Note that $u^{-1}(U(\theta) - \beta u(c_2(\theta)))$ has strictly decreasing differences in $U(\theta)$ and $c_2(\theta)$ by strict concavity of u , and that $U(\theta)$ is right-continuous and is non-decreasing by the incentive-compatibility constraint (2). The result then follows from Lemma 2 (ii).

A.3 Proof of Corollary 1

If H depends on consumption differences, then

$$H_1(c_2(\theta), RK - \kappa) = \tilde{H}'(c_2(\theta) - RK + \kappa)$$

and

$$\bar{H}_2(c_2, RK - \kappa) = - \int \tilde{H}'(c_2(\theta') - RK + \kappa) dF,$$

so (10) becomes

$$\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} = R\eta \left[\int \tilde{H}'(c_2(\theta') - RK + \kappa) dF - \tilde{H}'(c_2(\theta) - RK + \kappa) \right].$$

It follows immediately that optimal marginal capital taxes are U-shaped in θ in any monotone solution. In addition, if the allocation is non-constant and \tilde{H} is strictly single-peaked then $\tilde{H}'(c_2(\theta) - RK + \kappa)$ must be greater than $\int \tilde{H}'(c_2(\theta') - RK + \kappa) dF$ for some values of θ and less than $\int \tilde{H}'(c_2(\theta') - RK + \kappa) dF$ for others,²⁶ so $\tau_k(\theta)$ is negative for some individuals (who must be those with intermediate θ) and positive for others.

The argument when H depends on utility differences is identical, except that now

$$H_1(c_2(\theta), RK - \kappa) = \tilde{H}'(u(c_2(\theta)) - u(RK - \kappa)) u'(c_2(\theta))$$

and

$$\tilde{H}_2(c_2, RK - \kappa) = - \int \tilde{H}'(u(c_2(\theta')) - u(RK - \kappa)) u'(RK - \kappa) dF,$$

so the relevant derivative is $\tilde{H}'u'$ rather than \tilde{H}' .

A.4 Proof of Proposition 2

When H is a step function, the government's dual problem is

$$\min_{c_1, c_2, y} \int \left(c_1(\theta) + \frac{1}{R} c_2(\theta) - y(\theta) \right) dF$$

subject to (2), (9), (6) and the no-reform constraint

$$\int \mathbb{I}\{c_2(\theta) \geq RK - \kappa\} dF \geq \alpha, \quad (23)$$

where $\mathbb{I}\{\cdot\}$ is the indicator function. As in the case where H is differentiable, any solution must solve the subproblem

$$\min_{c_1, c_2, K} \int \left(c_1(\theta) + \frac{1}{R} c_2(\theta) \right) dF$$

subject to (9), (7) and (23). Substituting out for $c_1(\theta)$ using (7) and letting $\mu \geq 0$ and $\phi \geq 0$ be the multipliers on (9) and (23), respectively, the Lagrangian for this problem is

$$\int \left(u^{-1}(U(\theta) - \beta u(c_2(\theta))) + \left(\frac{1}{R} + \mu \right) c_2(\theta) - \phi \mathbb{I}\{c_2(\theta) \geq RK - \kappa\} \right) dF. \quad (24)$$

If $c_2(\theta) \neq RK - \kappa$, then differentiating under the integral with respect to $c_2(\theta)$ yields first-order condition

$$\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} = R\mu. \quad (25)$$

Hence, in any solution either (25) holds or $c_2(\theta) = RK - \kappa$. Furthermore, in any monotone solution, the set of types θ with $c_2(\theta) = RK - \kappa$ forms an interval $[\theta_l, \theta_h]$, so it remains only to show that marginal capital taxes are non-decreasing on $[\theta_l, \theta_h]$ and (if c_2 is non-constant) satisfy $\tau_k(\theta) / (1 - \tau_k(\theta)) \leq R\mu$ on $[\theta_l, \theta_h]$. The former statement follows immediately from the fact that $U(\theta)$ is non-decreasing on $[\theta_l, \theta_h]$ and (3). For the latter statement, note that the restriction of any solution (c_1^*, c_2^*, y^*) to $\Theta' = \{\theta : c_2^*(\theta) \geq RK - \kappa\}$ must

²⁶This follows because if the allocation is non-constant then $U(\theta)$ must be non-constant by (2), and if $U(\theta)$ is non-constant then $c_2(\theta)$ must be non-constant by (10).

solve the subproblem

$$\min_{c_1, c_2} \left(c_1(\theta) + \frac{1}{R} c_2(\theta) \right) dF$$

subject to (7),

$$\int_{\Theta'} c_2(\theta) dF \leq \int_{\Theta'} c_2^*(\theta) dF, \quad (26)$$

and

$$c_2(\theta) \geq RK - \kappa \text{ for all } \theta \in \Theta'. \quad (27)$$

Letting $\tilde{\mu}$ be the multiplier on (26) and letting $\psi(\theta) f(\theta) \geq 0$ be the multiplier on (27), the Lagrangian for this subproblem is

$$\int \left(u^{-1}(U(\theta) - \beta u(c_2(\theta))) + \left(\frac{1}{R} + \tilde{\mu} - \psi(\theta) \right) c_2(\theta) \right) dF.$$

The first order condition for $c_2(\theta)$ and the fact that $\psi(\theta) \geq 0$ immediately imply that $\tau_k(\theta) / (1 - \tau_k(\theta)) \leq R\tilde{\mu}$ for all $\theta \in \Theta'$, and hence for all θ such that $c_2(\theta) = RK - \kappa$. Finally, since $c_2 = c_2^*$ at a solution to the subproblem and $\psi(\theta) = 0$ for all θ such that $c_2(\theta) > RK - \kappa$, we have

$$\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} = R\tilde{\mu} = R\mu \text{ for all } \theta \text{ such that } c_2(\theta) > RK - \kappa.$$

As such a type θ exists whenever c_2 is non-constant (by (9) and $\kappa \geq 0$), we may conclude that $\tilde{\mu} = \mu$ and hence that $\tau_k(\theta) / (1 - \tau_k(\theta)) \leq R\mu$ for all θ such that $c_2(\theta) = RK - \kappa$.

A.5 Proof of Lemma 4

(i) follows from the fact that the objective (14) is continuous and the constraint set defined by (15) and (16) is closed (by continuity of \tilde{H} and F) and can be bounded by the Inada conditions on u . (ii) follows from Lemma 2 (i') because $u(c_2(\theta) + x)$ has strictly decreasing differences in $c_2(\theta)$ and x , due to the concavity of u . For (iii), let $\bar{x}(\bar{c}_2) = \sup \bigcup_{\theta \in \Theta_{\bar{c}_2}} X(\theta)$ and $\underline{x}(\bar{c}_2) = \inf \bigcup_{\theta \in \Theta_{\bar{c}_2}} X(\theta)$. It follows from (ii) that \bar{x} and \underline{x} are monotone, and thus continuous for almost all \bar{c}_2 , and also that $\bar{x}(\bar{c}_2) = \underline{x}(\bar{c}_2)$ whenever \bar{x} and \underline{x} are continuous at \bar{c}_2 . Hence, $\bar{x}(\bar{c}_2) = \underline{x}(\bar{c}_2)$ for almost all \bar{c}_2 , so $\bigcup_{\theta \in \Theta_{\bar{c}_2}} X(\theta)$ is single-valued for almost all \bar{c}_2 . Finally, to see (iv), note that a necessary condition for $x(\theta) \in X(\theta)$ is that

$$-u'(c_2(\theta) + x(\theta)) + \lambda = \mu \tilde{H}'(-x(\theta)), \quad (28)$$

where $\lambda \geq 0$ and $\mu \geq 0$ are the multipliers on (15) and (16). Since the left-hand side of (28) is strictly increasing in $x(\theta)$ whereas the right-hand side is weakly decreasing in $x(\theta)$ for $x(\theta) > 0$ when \tilde{H}' is single-peaked at 0, there can be at most one solution with $x(\theta) > 0$.

A.6 Proof of Lemma 6

Note that the constraint $\int u(c_2(\theta)) dF \geq V_D(c_2)$ can be written as $\int u(c_2(\theta)) dF \geq \int u(c_2(\theta) + x(\theta)) dF$ for all $x \in X$, where X is the set of all x -schedules that satisfy (15) and (16). The constraint set therefore takes the same form as (C') in Lemma 2, so the result follows from Lemma 2 (ii) by exactly the same argument as in Lemma 3.

A.7 Proof of Proposition 5

Note that if (almost) all θ in a set $\Theta' \subseteq \Theta$ satisfy (18), then $\tau_k(\theta)$ is (almost everywhere) non-decreasing on Θ' if and only if $u'(\hat{c}_2(\theta)) - u'(c_2(\theta))$ is (almost everywhere) non-decreasing.

For the first part, recall that types $\theta < \theta^*$ satisfy (18) as well as $x(\theta) > 0$. If $u'(c_2(\theta) + x(\theta)) - u'(c_2(\theta))$ is differentiable at θ , then (omitting the θ -arguments) its derivative equals (19). Note that $c_2'(\theta) \geq 0$ by Lemma 6 and $x'(\theta) \leq 0$ by Lemma 4 (i), so if $x(\theta) > 0$ and $u''' \geq 0$ then (19) is non-negative. In addition, if $u'(c_2(\theta) + x(\theta)) - u'(c_2(\theta))$ is discontinuous at θ , then either $c_2(\theta)$ jumps up or $x(\theta)$ jumps down, and when $x(\theta) > 0$ and $u''' \geq 0$ either of these jumps increases $u'(c_2(\theta) + x(\theta)) - u'(c_2(\theta))$. Hence, $u'(c_2(\theta) + x(\theta)) - u'(c_2(\theta))$ is non-decreasing on $\{\theta : \theta < \theta^*\}$.

For the second part, we first claim that $\hat{c}_2(\theta)$ is non-increasing on $\Theta' = \{\theta : c_2(\theta) > c_2^* \text{ and } \theta > \theta^*\}$ (which is a superset of $\{\theta : \theta \text{ is non-pooled and } \theta > \theta^*\}$). To see this, write (DP) as

$$\min_{\hat{c}_2} \int \tilde{H}(c_2(\theta) - \hat{c}_2(\theta)) dF$$

subject to

$$\begin{aligned} \int \hat{c}_2(\theta) dF &\leq RK - \kappa, \\ \int u(\hat{c}_2(\theta)) dF &\geq \int u(c_2(\theta)) dF.^{27} \end{aligned}$$

Fix a solution $\hat{c}_2^* : \Theta \rightarrow \mathbb{R}$. Let

$$\begin{aligned} \bar{K} &= \int_{\theta \leq \theta^*} \hat{c}_2^*(\theta) dF, \\ \bar{V} &= \int_{\theta \leq \theta^*} u(\hat{c}_2^*(\theta)) dF. \end{aligned}$$

Then a necessary condition for optimality is that the restriction of \hat{c}_2^* to Θ' solves the subproblem

$$\min_{\hat{c}_2 : \Theta' \rightarrow \mathbb{R}} \int_{\Theta'} \tilde{H}(c_2(\theta) - \hat{c}_2(\theta)) dF$$

subject to

$$\begin{aligned} \bar{K} + \int_{\Theta'} \hat{c}_2(\theta) dF &\leq RK - \kappa, \\ \bar{V} + \int_{\Theta'} u(\hat{c}_2(\theta)) dF &\geq \int u(c_2(\theta)) dF. \end{aligned}$$

If $c_2(\theta) > c_2^*$ then $c_2(\theta) \geq \hat{c}_2^*(\theta)$ for every solution to (DP), by the definition of c_2^* and Lemma 4 (ii). Hence, a necessary condition for optimality is that \hat{c}_2^* still solves the above subproblem when \hat{c}_2 is restricted to satisfy $c_2(\theta) \geq \hat{c}_2(\theta)$ for all $\theta \in \Theta'$. Now, since \tilde{H}' is single-peaked at 0, the objective in this subproblem has strictly increasing differences in $c_2(\theta)$ and $\hat{c}_2(\theta)$ over this range, while $c_2(\theta)$ does not enter in the

²⁷This dual approach to (DP) is valid whenever the no-reform constraint in the government's problem is binding: if the value of this dual program is less than α , then varying \hat{c}_2 toward more equal consumption will increase $\int u(\hat{c}_2(\theta)) dF$ without violating the resource constraint and will still receive enough support relative to the status quo.

constraints except through the constant $\int u(c_2(\theta))dF$, so Lemma 2 (i) implies that at every solution $\hat{c}_2(\theta)$ is non-increasing in $c_2(\theta)$, and hence in θ .

Finally, note that if $u'(\hat{c}_2(\theta)) - u'(c_2(\theta))$ is differentiable at θ , then (omitting the θ -arguments) its derivative equals (20). Thus, if c_2 is non-decreasing and \hat{c}_2 is non-increasing, (20) is non-negative. If $u'(\hat{c}_2(\theta)) - u'(c_2(\theta))$ is discontinuous at θ , then either $c_2(\theta)$ jumps up or $\hat{c}_2(\theta)$ jumps down, and either or these jumps increases $u'(\hat{c}_2(\theta)) - u'(c_2(\theta))$. Hence, $u'(\hat{c}_2(\theta)) - u'(c_2(\theta))$ is non-decreasing on $\{\theta : c_2(\theta) > c_2^* \text{ and } \theta > \theta^*\}$. Finally, non-pooled types satisfy (18), so $\tau_k(\theta)$ is non-decreasing on $\{\theta : \theta \text{ is non-pooled, } c_2(\theta) > c_2^*, \text{ and } \theta > \theta^*\} = \{\theta : \theta \text{ is non-pooled and } \theta > \theta^*\}$.

B Appendix: Dependence on Utility Differences

This appendix shows that results very similar to Propositions 4 and 5 hold when \tilde{H} depends on utility differences in the model of Section 4. The no-reform constraint is now that there does not exist a scheme \hat{c}_2 such that

$$\begin{aligned} \int u(\hat{c}_2(\theta)) dF &> \int u(c_2(\theta)) dF, \\ \int \hat{c}_2(\theta) dF &\leq \int c_2(\theta) dF - \kappa, \end{aligned} \tag{29}$$

$$\int \tilde{H}(u(\hat{c}_2(\theta)) - u(c_2(\theta))) dF \leq \alpha. \tag{30}$$

This constraint is equivalent to the value of the following deviation program being less than $\int u(c_2(\theta)) dF$.

$$\max_{\hat{c}_2} \int u(\hat{c}_2(\theta)) dF$$

subject to (29) and (30). Letting $x(\theta) = u(\hat{c}_2(\theta)) - u(c_2(\theta))$, $\Phi = u^{-1}$, and $u_t(\theta) = u(c_t(\theta))$, this is in turn equivalent to the value of the following dual program, which we denote by $R_D(u_2)$, being greater than $\int \Phi(u_2(\theta)) dF - \kappa$.

$$\min_{x(\theta)} \int \Phi(u_2(\theta) + x(\theta)) dF$$

subject to

$$\begin{aligned} \int x(\theta) dF &\geq 0, \\ \int \tilde{H}(-x(\theta)) dF &\leq \alpha. \end{aligned}$$

This deviation program—which we denote by (DP')—is framed in utility space, while the deviation program (DP) in the text is framed in consumption space. The key feature which makes (DP') tractable is that the status quo utility schedule u_2 does not enter the constraints, just as the status quo consumption schedule c_2 does not enter the constraints in (DP).

Denote the set of possible solutions to (DP') at each θ by

$$X(\theta) \equiv \{x(\theta) : x \text{ is a solution to (DP')}\}.$$

The next lemma, which parallels Lemma 4 in the text, again collects properties of these solutions:

Lemma 7. (i) A solution exists.

(ii) If $u_2(\theta) < u_2(\theta')$ then $\inf X(\theta) \geq \sup X(\theta')$.

(iii) For any u_2 -schedule and any \bar{u}_2 , let $\Theta_{\bar{u}_2} \equiv \{\theta : u_2(\theta) = \bar{u}_2\}$. Then for almost all \bar{u}_2 , $\bigcup_{\theta \in \Theta_{\bar{u}_2}} X(\theta)$ is singleton.

(iv) For every $\bar{u}_2 \in \mathbb{R}$, there exists at most one $s > 0$ such that $s \in \bigcup_{\theta \in \Theta_{\bar{u}_2}} X(\theta)$.

Proof. Noting that the objective has strictly increasing differences in u_2 and x by convexity of Φ , the proof is analogous to the proof of Lemma 4. ■

We can now write the government's planning problem as follows.

$$\min_{U, u_2, y} \int \left(\Phi(U(\theta) - \beta u_2(\theta)) + \frac{1}{R} \Phi(u_2(\theta)) - y(\theta) \right) dF \quad \text{s.t.} \quad R_D(u_2) \geq \int \Phi(u_2(\theta)) dF - \kappa,$$

subject to

$$\int (U(\theta) - h(y(\theta), \theta)) dF \geq V,$$

$$U(\theta) - h(y(\theta), \theta) \geq U(\theta') - h(y(\theta'), \theta).$$

Since the constraints of this program only depend on U and y , u_2 must solve

$$\min_{u_2} \int (\Phi(U(\theta) - \beta u_2(\theta))) dF \quad \text{s.t.} \quad R_D(u_2) \geq \int \Phi(u_2(\theta)) dF - \kappa. \quad (31)$$

Using this, we can reproduce Lemma 5, denoting by $\eta \geq 0$ the multiplier on the constraint in (31).

Lemma 8. (i) In any solution u_2 , for almost all θ where $X(\theta)$ is single-valued, we have

$$\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} = R\eta [1 - \Phi'(u_2(\theta) + x(\theta)) / \Phi'(u_2(\theta))] \quad \text{where} \quad X(\theta) = \{x(\theta)\}. \quad (32)$$

(ii) For almost all non-pooled θ (i.e. θ such that $u_2(\theta) \neq u_2(\theta')$ for all $\theta' \neq \theta$), $X(\theta)$ is single-valued, so (32) holds.

The proof is analogous to the proof of Lemma 5. Next, note that Lemma 6 goes through because $\Phi(U(\theta) - \beta u_2(\theta))$ has strictly decreasing differences in $U(\theta)$ and $u_2(\theta)$, so there exists a solution in which u_2 is non-decreasing. This immediately leads to the following result reproducing Proposition 4.

Proposition 7. In any monotone solution, there exists a threshold type θ^* such that capital is subsidized for agents with $\theta < \theta^*$ and capital is taxed for all non-pooled agents with $\theta > \theta^*$.

Proof. Analogous to the proof of Proposition 4, using Lemmas 7 and 8 in place of Lemmas 4 and 5. The basic idea is that, as in the case where \tilde{H} depends on consumption differences, $\text{sign}(\tau_k(\theta)) = \text{sign}(-x(\theta))$ whenever θ satisfies (32), and $x(\theta)$ is non-increasing whenever $X(\theta)$ is a singleton. ■

We can also reproduce Proposition 5 if the condition that $u''' \geq 0$ is strengthened to non-increasing $-u''(c) / u'(c)^2$.

Proposition 8. In a monotone solution, the following hold.

(i) If $-u''(c) / u'(c)^2$ is non-increasing, then $\tau_k(\theta)$ is non-decreasing on $\{\theta : \theta < \theta^*\}$.

(ii) $\tau_k(\theta)$ is non-decreasing on $\{\theta : \theta \text{ is non-pooled and } \theta > \theta^*\}$.

Proof. For simplicity, we prove the proposition under the additional hypothesis that the functions u_2 and x are differentiable. This hypothesis can be dispensed with as in the proof of Proposition 5.

Note that $\tau_k(\theta)$ is non-decreasing if (32) is satisfied and $\Phi'(u_2(\theta) + x(\theta)) / \Phi'(u_2(\theta))$ is non-increasing. For the first part, this holds whenever (omitting θ -arguments)

$$\Phi'(u_2) \Phi''(u_2 + x) (u_2' + x') - \Phi'(u_2 + x) \Phi''(u_2) u_2' \leq 0, \quad (33)$$

or

$$\frac{\Phi''(u_2 + x)}{\Phi'(u_2 + x)} (u_2' + x') \leq \frac{\Phi''(u_2)}{\Phi'(u_2)} u_2'.$$

As $x' \leq 0$ by Lemma 7, a sufficient condition for this is

$$\frac{\Phi''(u_2 + x)}{\Phi'(u_2 + x)} \leq \frac{\Phi''(u_2)}{\Phi'(u_2)},$$

or, since $x \geq 0$ for $\theta < \theta^*$, $\Phi''(u_2) / \Phi'(u_2)$ non-increasing. This is easily seen to be equivalent to $-u''(c) / u'(c)^2$ being non-increasing.

For the second part, we first argue that $\hat{u}_2(\theta) = u_2(\theta) + x(\theta)$ is non-increasing for $\theta > \theta^*$. To see this, write the deviation program (DP') as

$$\min_{\hat{u}_2} \int \tilde{H}(u_2(\theta) - \hat{u}_2(\theta)) dF$$

subject to

$$\begin{aligned} \int \Phi(\hat{u}_2(\theta)) dF &\leq RK - \kappa, \\ \int \hat{u}_2(\theta) dF &\geq \int u_2(\theta) dF, \end{aligned}$$

and apply the same argument (using Lemma 2) as in the proof of Proposition 5, which implies that every solution $\hat{u}_2(\theta)$ is non-increasing in $u_2(\theta)$, and hence in θ , for $\theta > \theta^*$. Finally, rewrite (33) as

$$\Phi'(u_2) \Phi''(\hat{u}_2) \hat{u}_2' - \Phi'(\hat{u}_2) \Phi''(u_2) u_2' \leq 0.$$

Since $\Phi' \geq 0$, $\Phi'' \geq 0$, $\hat{u}_2' \leq 0$ and $u_2' \geq 0$, this is satisfied. ■

C Appendix: Revelation Principle

This section formulates the model as a game between the individuals and the government, and establishes the relevant version of the revelation principle. Individuals' and the government's preferences are as in the text. The game is as follows.

First, the government proposes a tax schedule (T_y, T_k) . A tax schedule is required to satisfy the resource constraint whatever production decisions individuals make. Formally, let \mathcal{G} be the set of probability distributions on \mathbb{R}_+ , corresponding to possible distributions of output or capital. A labor tax schedule T_y is a map from $\mathbb{R}_+ \times \mathcal{G} \rightarrow \mathbb{R}$ such that

$$\int_{y \in \mathbb{R}_+} T_y(y, G) dG \geq 0$$

for all $G \in \mathcal{G}$. A capital tax schedule T_k is a map from $\mathbb{R}_+ \times \mathcal{G} \rightarrow \mathbb{R}$ such that

$$\int_{k \in \mathbb{R}_+} T_k(Rk, G) dG \geq 0$$

for all $G \in \mathcal{G}$.

Second, individuals produce, pay labor taxes, and consume in period 1. More specifically, this part of the game resolves as follows:

1. Individuals simultaneously choose production y .
2. Given the resulting distribution of output G^y , an individual who produced y pays labor tax $T_y(y, G^y)$.
3. An individual with after-tax income $y - T_y(y, G^y)$ chooses period-1 consumption $c_1 \in [0, y - T_y(y, G^y)]$. This leaves her with capital $k \equiv y - T_y(y, G^y) - c_1 \geq 0$. Denote the resulting distribution of capital by G^k .

Third, the government either implements the status quo capital tax schedule $T_k(\cdot, G^k)$ or implements a reform capital tax schedule $\tilde{T}_k : \mathbb{R}_+ \rightarrow \mathbb{R}$ such that

$$\int_{k \in \mathbb{R}_+} \tilde{T}_k(Rk) dG^k \geq \kappa, \quad (34)$$

where $\kappa \geq 0$ is the resource cost of a reform. (Note that G^k is determined before the reform schedule, so there is no need to let \tilde{T}_k depend on the distribution of capital.) If the status quo tax schedule is implemented, an individual with capital k receives period-2 consumption $c_2 = Rk - T_k(Rk, G^k)$. If reform schedule \tilde{T}_k is implemented, an individual with capital k receives period-2 consumption $c_2 = Rk - \tilde{T}_k(Rk)$.

To complete the description of the game, it remains only to determine which capital tax schedule is implemented. This depends on the version of the model under consideration.

In the *equalizing reforms* model, the reform schedule is given by

$$\tilde{T}_k(Rk) = Rk - RK + \kappa,$$

where

$$K = \int_k k dG^k.$$

(Note that under this schedule, an individual with capital k receives period-2 consumption $c_2 = Rk - (Rk - RK + \kappa) = RK - \kappa$.) The status quo schedule is implemented if

$$\int_k H(Rk - T_k(Rk, G^k), RK - \kappa) dG^k \geq \alpha.$$

Otherwise, the reform schedule is implemented.

In the *strategic reforms* model, define the *deviation program* by

$$\max_{\tilde{T}_k : \mathbb{R}_+ \rightarrow \mathbb{R}} \int_k u(Rk - \tilde{T}_k(Rk)) dG^k$$

subject to (34) and

$$\int_k H(Rk - T_k(Rk, G^k), Rk - \tilde{T}_k(Rk)) dG^k \leq \alpha.$$

If the value of the deviation program is not more than

$$\int_k u \left(Rk - T_k \left(Rk, G^k \right) \right) dG^k,$$

then the status quo schedule is implemented. Otherwise, an arbitrary reform schedule is implemented among those that solve the deviation program.

A (symmetric, pure strategy, subgame perfect) *equilibrium* then consists of a proposed tax schedule (T_y, T_k) and production and consumption strategies

$$\begin{aligned} Y &: \mathcal{T}_y \times \mathcal{T}_k \times \Theta \rightarrow y, \\ C &: \mathcal{T}_y \times \mathcal{T}_k \times \Theta \rightarrow c_1, \end{aligned}$$

(where \mathcal{T}_y and \mathcal{T}_k are the sets of possible tax schedules T_y and T_k , respectively), such that:

1. (T_y, T_k) maximizes the government's payoff given (Y, C) .
2. (Y, C) maximizes the utility of each type θ given (T_y, T_k) and given that other individuals follow (Y, C) .
3. $C(T_y, T_k, \theta) \leq Y(T_y, T_k, \theta) - T_y(Y(T_y, T_k, \theta), G^{Y(T_y, T_k)})$ for all $T_y \in \mathcal{T}_y, T_k \in \mathcal{T}_k, \theta \in \Theta$ (i.e., individuals do not consume more than their after-tax incomes in period 1).

An *allocation* $(c_1 : \Theta \rightarrow \mathbb{R}_+, c_2 : \Theta \rightarrow \mathbb{R}_+, y : \Theta \rightarrow \mathbb{R}_+)$ is a mapping from types to period-1 consumption, period-2 consumption, and production. An allocation is *feasible* if it satisfies the intertemporal resource constraint

$$\int \left(c_1(\theta) + \frac{1}{R} c_2(\theta) \right) dF \leq \int y(\theta) dF.$$

An allocation is *implementable* if there exists an equilibrium (T_y, T_k, Y, C) such that

$$\begin{aligned} Y(T_y, T_k, \theta) &= y(\theta) \forall \theta \in \Theta, \\ C(T_y, T_k, \theta) &= c_1(\theta) \forall \theta \in \Theta, \end{aligned}$$

and

$$D(T_y, T_k, Y, C, \theta) = c_2(\theta) \forall \theta \in \Theta,$$

where $D(T_y, T_k, Y, C, \theta)$ is the period-2 consumption of a type θ individual in equilibrium (T_y, T_k, Y, C) . An implementable allocation is *optimal* if it maximizes the government's payoff over all implementable allocations.

In the text, attention is restricted to feasible, incentive-compatible direct mechanisms that satisfy the appropriate no-reform constraint. This approach is justified by the following result. Recall that the no-reform constraint in the equalizing reforms model is

$$\int_{\theta} H(c_2(\theta), RK - \kappa) dF \geq \alpha,$$

while the no-reform constraint in the strategic reforms model is that the value of the deviation program defined in the text does not exceed the value of the status quo.

Proposition 9 (Revelation Principle). *In the equalizing reforms model, under Assumption 1, every implementable allocation is feasible, incentive-compatible and satisfies the no-reform constraint. In the strategic reforms model, under Assumption 2, every optimal allocation is feasible, incentive-compatible and satisfies the no-reform constraint.*

Note that the converse also holds for monotone allocations, as shown in Section 5.2 (building on Proposition 3 of Farhi et al., 2012).

Proof. Showing that any implementable allocation is feasible is a simple accounting exercise. Any implementable allocation is incentive-compatible as a direct mechanism, by the usual revelation principle argument (whether or not the allocation is optimal, and whether or not it is implemented in an equilibrium in which a reform occurs): a unilateral deviation does not affect G^y or G^k , and hence does not affect the tax schedules faced by the deviator, so if $y(\theta)$ and $c_1(\theta)$ are the optimal production and period-1 consumption choices of a type θ individual given others' behavior, then in particular she prefers $(c_1(\theta), c_2(\theta), y(\theta))$ to $(c_1(\theta'), c_2(\theta'), y(\theta'))$ for all $\theta' \in \Theta$. Thus, it suffices to show that every implementable (resp., optimal) allocation satisfies the no-reform constraint when viewed as a direct mechanism in the equalizing reforms model under Assumption 1 (resp., in the strategic reforms model under Assumption 2).

For the equalizing reforms case, if an allocation is implemented in an equilibrium in which no reform occurs, then it satisfies the no-reform constraint when viewed as a direct mechanism, as the condition for no reform to occur in equilibrium is precisely the no-reform constraint for the corresponding direct mechanism. In addition, if an allocation (c_1, c_2, y) is implemented in an equilibrium in which reform does occur, then c_2 is constant. Therefore, (c_1, c_2, y) satisfies the no-reform constraint when viewed as a direct mechanism, as

$$\int_{\theta} H(RK, RK - \kappa) dF \geq \alpha$$

for all $K \geq 0$, by Assumption 1.

For the strategic reforms case, let $(c_1(\theta), c_2(\theta), y(\theta))$ be a monotone solution to the government's problem, which exists by Lemma 5 in the text. As is shown in Section 5.2 (using Proposition 3 of Farhi et al., 2012), $(c_1(\theta), c_2(\theta), y(\theta))$ is implementable in an equilibrium in which no reform occurs. In addition, by Assumption 2, the government's payoff under $(c_1(\theta), c_2(\theta), y(\theta))$ is greater than its payoff under any allocation that is implementable in an equilibrium in which a reform occurs. As every optimal allocation must give the government at least as high a payoff as does $(c_1(\theta), c_2(\theta), y(\theta))$ (since $(c_1(\theta), c_2(\theta), y(\theta))$ is implementable), it follows that every optimal allocation is implementable in an equilibrium in which no reform occurs. Finally, as we have seen, such an allocation necessarily satisfies the no-reform constraint when viewed as a direct mechanism. ■

D Appendix: General Welfare Weights

In this appendix, we discuss situations where the government uses general Pareto weights G to evaluate welfare, which could be different from the population distribution F . Moreover, we allow the government's Pareto weights to depend not only on θ , but also on a second dimension of heterogeneity ρ , which is assumed to be payoff-irrelevant and observable to the government. As discussed in the text, this second dimension of heterogeneity allows for a government that is "non-benevolent." We let $F(\theta, \rho)$ be the joint cdf over θ and ρ and let $G(\theta, \rho)$ be the corresponding cdf of the government's welfare weights. We assume that F and G are absolutely continuous with respect to each other, so that the government puts positive weight

on the welfare of all groups in society (although these weights may be arbitrarily close to 0).²⁸ We will show that all of our main results continue to hold *within each group* ρ , with little or no modification, under this more general specification of the government's objective, so long as $g(\theta, \rho) / f(\theta, \rho)$ is non-increasing in θ for each ρ : that is, we require that, within each group, the government is at least as redistributive as a utilitarian would be. This shows that our results are not overly sensitive to either the assumption that the government has utilitarian preferences for redistribution, or that the government cares equally about all groups in society.

D.1 Non-Strategic Reforms

We begin with the case where the government cannot commit to the details of a reform. This corresponds to the "equalizing reforms" model of Section 3, except that the government's most-preferred reform is no longer full equalization of consumption due to the Pareto weights $G(\theta, \rho)$. Instead, the dual problem for c_1 , c_2 , and K becomes

$$\min_{c_1, c_2, K} \int \left(c_1(\theta, \rho) + \frac{1}{R} c_2(\theta, \rho) \right) dF \quad (35)$$

subject to

$$u(c_1(\theta, \rho)) + \beta u(c_2(\theta, \rho)) = U(\theta, \rho), \quad (36)$$

$$\int c_2(\theta, \rho) dF \geq RK, \quad (37)$$

and

$$\int H(c_2(\theta, \rho), \hat{c}_2(\theta, \rho)) dF \geq \alpha, \quad (38)$$

where reform consumption \hat{c}_2 solves

$$\max_{\hat{c}_2} \int u(\hat{c}_2(\theta, \rho)) dG \quad \text{s.t.} \quad \int \hat{c}_2(\theta, \rho) dF \leq RK - \kappa.$$

It should be pointed out that in this model abstracting from ratcheting is no longer without loss of generality. This is because the government in period 2 allocates consumption using the information about θ revealed in period 1 (as a consequence, the deviation program does not involve incentive constraints). In principle, the government may therefore do better by offering an allocation in period 1 that reveals less information about types. It would thereby tie its hands and prevent itself from offering perfectly θ -targeted consumption at the reform stage, which may relax the no-reform constraint. Nonetheless, we feel that the most direct way to study how our main results change with a non-utilitarian government is to continue to assume that the government employs a fully revealing direct mechanism, so we maintain this assumption throughout.

This leads to the following result about the implicit marginal capital tax.

Lemma 9. *With Pareto weights $G(\theta, \rho)$, the intertemporal wedge at any solution to the government's problem satis-*

²⁸The absolute continuity also rules out the possibility that the government may put non-zero weight on the welfare of a measure-zero set of agents, such as for example the leader of the government herself. This differentiates the model from one with a self-interested politician, such as Acemoglu, Golosov, and Tsyvinski (2010).

fies

$$\frac{\tau_k(\theta, \rho)}{1 - \tau_k(\theta, \rho)} = -R\eta [\bar{H}_2(c_2, \hat{c}_2) - H_1(c_2(\theta, \rho), \hat{c}_2(\theta, \rho))], \quad (39)$$

where $\eta \geq 0$ is the multiplier on (38) and

$$\bar{H}_2(c_2, \hat{c}_2) \equiv \int H_2(c_2(\theta, \rho), \hat{c}_2(\theta, \rho))m(\theta, \rho)dF$$

with

$$m(\theta, \rho) \equiv \frac{(u''(\hat{c}_2(\theta, \rho))g(\theta, \rho)/f(\theta, \rho))^{-1}}{\int (u''(\hat{c}_2(\theta', \rho'))g(\theta', \rho')/f(\theta', \rho'))^{-1}dF}.$$

Proof. Note that the reform consumption \hat{c}_2 depends on the choice variables c_1, c_2, K only through K , so we may write $\hat{c}_2(\theta, \rho, K)$ in the following. $\hat{c}_2(\theta, \rho, K)$ must satisfy the first order condition

$$u'(\hat{c}_2(\theta, \rho, K)) = \lambda(K)f(\theta, \rho)/g(\theta, \rho), \quad (40)$$

where $\lambda(K)$ is the multiplier on the post-reform resource constraint as a function of K . We can use (40) to solve for $\hat{c}_2(\theta, \rho, K) = u'^{-1}(\lambda(K)f(\theta, \rho)/g(\theta, \rho))$, so $\lambda(K)$ is implicitly determined by

$$\int u'^{-1}(\lambda(K)f(\theta, \rho)/g(\theta, \rho))dF = RK - \kappa.$$

Implicitly differentiating this equation and (40) yields

$$\frac{\partial \hat{c}_2(\theta, \rho, K)}{\partial K} = \frac{\partial \hat{c}_2(\theta, \rho)}{\partial \lambda} \frac{\partial \lambda}{\partial K} = R \frac{(u''(\hat{c}_2(\theta, \rho))g(\theta, \rho)/f(\theta, \rho))^{-1}}{\int (u''(\hat{c}_2(\theta', \rho'))g(\theta', \rho')/f(\theta', \rho'))^{-1}dF} \equiv Rm(\theta, \rho).$$

Using this in the first order condition for K corresponding to the planning problem (35) to (38) and combining it with the first order conditions for $c_1(\theta, \rho)$ and $c_2(\theta, \rho)$ delivers the result. ■

Observe that, with utilitarian welfare weights $G = F$, (40) implies full equalization of post-reform consumption \hat{c}_2 , so $m(\theta, \rho) = 1$ for all ρ, θ , and (39) collapses back to the standard formula from Section 3. However, even if $G \neq F$, we will see that the results from Section 3 will continue to hold within each group ρ . For instance, if H depends on consumption differences with $H(c_2, \hat{c}_2) = \tilde{H}(c_2 - \hat{c}_2)$ and \tilde{H}' is single-peaked, marginal capital taxes are U-shaped in θ for any given ρ whenever $c_2(\theta, \rho) - \hat{c}_2(\theta, \rho)$ is monotone in θ (a similar result holds when H depends on utility differences and $\tilde{H}'u'$ is single-peaked). This will be established in the following proposition:

Proposition 10. *Suppose $H(c_2, \hat{c}_2) = \tilde{H}(c_2 - \hat{c}_2)$ depends on consumption differences and \tilde{H}' is single-peaked. Then for any given ρ , marginal capital taxes are U-shaped in θ if $g(\theta, \rho)/f(\theta, \rho)$ is non-increasing in θ given ρ .*

Proof. Fix a solution c_1, c_2, K to (35)-(38), with reform consumption schedule \hat{c}_2 . Consider any given ρ and let $c_2^\rho \equiv c_2(\cdot, \rho)$, $F^\rho \equiv F(\cdot, \rho)$ and analogously for G^ρ , and let

$$\bar{K}^\rho \equiv \int c_2(\theta, \rho)dF^\rho / R,$$

$$\bar{\alpha}^\rho \equiv \int H(c_2(\theta, \rho), \hat{c}_2(\theta, \rho, K))dF^\rho$$

(i.e., we fix ρ and only integrate in the θ -dimension). Using (36) to substitute $c_1(\theta, \rho) = u^{-1}(U(\theta, \rho) - \beta u(c_2(\theta, \rho)))$, we see that c_2^ρ has to solve the subproblem

$$\min_{c_2^\rho} \int \left(u^{-1}(U(\theta, \rho) - \beta u(c_2(\theta, \rho))) + \frac{1}{R} c_2(\theta, \rho) \right) dF^\rho$$

s.t.

$$\int c_2(\theta, \rho) dF^\rho \leq R\bar{K}^\rho,$$

$$\int H(c_2(\theta, \rho), \hat{c}_2(\theta, \rho, K)) dF^\rho \geq \bar{\alpha}^\rho.$$

Since \hat{c}_2^ρ does not depend on c_2^ρ (holding K constant), we can apply Lemma 2, which implies that $c_2(\theta, \rho)$ is non-decreasing in $U(\theta, \rho)$ and hence θ . A sufficient condition for $c_2(\theta, \rho) - \hat{c}_2(\theta, \rho, K)$ to be non-decreasing in θ is therefore that $\hat{c}_2(\theta, \rho, K)$ is non-increasing in θ . By (40), $\hat{c}_2(\theta, \rho, K)$ is non-increasing in θ if and only if $g(\theta, \rho)/f(\theta, \rho)$ is non-increasing in θ , for any given ρ and K . ■

Proposition 10 shows that, unsurprisingly, we can think of the problem for each group ρ separately, and thus, for any given ρ , the marginal tax rate is U-shaped in θ whenever the relative social welfare weights $g(\theta, \rho)/f(\theta, \rho)$ are non-increasing in θ . This is because these relative welfare weights determine the shape of post-reform consumption. With utilitarian weights $G = F$, they are constant and equal to one, leading to fully equalized \hat{c}_2 . Whenever $g(\theta, \rho)/f(\theta, \rho)$ is decreasing, $\hat{c}_2(\theta, \rho)$ will be decreasing in θ for any given ρ . This is because the government values low θ -types relatively more, and therefore allocates them higher consumption. Since $c_2(\theta, \rho)$ is still weakly increasing in θ by Lemma 2, this makes sure that $c_2(\theta, \rho) - \hat{c}_2(\theta, \rho)$ is also weakly increasing. The case where \tilde{H} depends on utility differences may be treated analogously.

Hence, our main result about the shape of the marginal capital tax from Section 3 continue to hold within each group ρ whenever the government is utilitarian or more redistributive than utilitarian within each group, in the sense of putting higher Pareto weights on low-skill types. What does change is the average level of the distortion, due to the additional weighting factor $m(\theta, \rho)$ in $\bar{H}_2(c_2, \hat{c}_2)$ in equation (39). Intuitively, $m(\theta, \rho)$ measures the effect of a marginal increase in period 2 aggregate resources RK on post-reform consumption $\hat{c}_2(\theta, \rho)$ of an individual of type (θ, ρ) . The overall effect of an additional unit of RK on political support for the status quo is then given by the weighted average $\int H_2(c_2(\theta, \rho), \hat{c}_2(\theta, \rho)) m(\theta, \rho) dF$, which includes both the effect of RK on post-reform consumption (through $m(\theta, \rho)$) and the effect of a change in post-reform consumption on the probability of supporting the status quo (through H_2). For instance, if $g(\theta, \rho)/f(\theta, \rho)$ is decreasing in θ and $u''' \geq 0$, then $m(\theta, \rho)$ is increasing in θ , so more weight is put on the political responsiveness of high- θ individuals (those whose post-reform consumption moves more in response to aggregate savings).

D.2 Strategic Reforms

We now turn to the case where the government can commit to the details of a reform, corresponding to Section 4 of the text. We will see that most of our results in that section can also be extended to allow for general Pareto weights, although the analysis is somewhat more complicated. In this appendix, we restrict attention to the case where H depends on consumption differences; the case where H depends on utility differences can again be treated similarly.

With general Pareto weights, the planning subproblem (17) becomes

$$\min_{c_2} \int \left(u^{-1}(U(\theta, \rho) - \beta u(c_2(\theta, \rho))) + \frac{1}{R} c_2(\theta, \rho) \right) dF \quad \text{s.t.} \quad \int u(c_2(\theta, \rho)) dG \geq V_D(c_2)$$

where

$$\begin{aligned} V_D(c_2) &\equiv \max_x \int u(c_2(\theta, \rho) + x(\theta, \rho)) dG \\ \text{s.t.} \quad &\int x(\theta, \rho) dF \leq -\kappa \\ &\int \tilde{H}(-x(\theta, \rho)) dF \leq \alpha. \end{aligned}$$

The following result shows that Lemma 4 extends to this more general framework for any given ρ if $g(\theta, \rho)/f(\theta, \rho)$ is non-increasing in θ :

Lemma 10. *If $g(\theta, \rho)/f(\theta, \rho)$ is non-increasing in θ for a given ρ , then $x(\theta, \rho) \equiv \hat{c}_2(\theta, \rho) - c_2(\theta, \rho)$ is non-increasing in $c_2(\theta, \rho)$, for any selection $x(\theta, \rho) \in X(\theta, \rho)$, where $X(\theta, \rho)$ is the set of solutions to the deviation program.*

Proof. We first show that, as in the non-strategic reforms case, the subproblem for each ρ can be considered separately. To see this, fix a solution c_2 with implied x and let

$$\begin{aligned} \bar{\kappa}^\rho &\equiv - \int x(\theta, \rho) dF^\rho \\ \bar{\alpha}^\rho &\equiv \int \tilde{H}(-x(\theta, \rho)) dF^\rho. \end{aligned}$$

Then $x^\rho \equiv x(\cdot, \rho)$ must solve

$$\begin{aligned} \max_{x^\rho} \bar{V}_D^\rho(c_2^\rho) &\equiv \int u(c_2(\theta, \rho) + x(\theta, \rho)) dG^\rho \\ \text{s.t.} \quad &\int x(\theta, \rho) dF^\rho \leq -\bar{\kappa}^\rho \\ &\int \tilde{H}(-x(\theta, \rho)) dF^\rho \leq \bar{\alpha}^\rho. \end{aligned} \tag{41}$$

Hence, the deviation program (41) has the same structure, for each given ρ , as (DP) in Section 4, *except* for the fact that the integrals in the objective and the constraints involve different weights G and F . To extend the comparative statics result from Lemma 4, we therefore require a generalized version of our technical Lemma 2, which is as follows:

Lemma 11. *Consider the same setting as in Lemma 2 but with the modified program*

$$W \equiv \sup_P \int \int w(a, t(\theta)) dPdG$$

subject to

$$\int \int y_x(a) dPdF \leq 0 \text{ for all } x \in X. \tag{42}$$

Assume that w is continuous and has strictly increasing differences in a and t , y_x is continuous for all $x \in X$, and t is right-continuous. Suppose that either

(i) $w(a, t)$ is non-decreasing in a and $g(\theta)/f(\theta)$ is non-decreasing in θ , or

(ii) $w(a, t)$ is non-increasing in a and $g(\theta)/f(\theta)$ is non-increasing in θ .

Then for any solutions P' and P'' , if $t(\theta') < t(\theta'')$ then $a(\theta') \leq a(\theta'')$ for all $a(\theta') \in \text{supp } P'(\theta')$ and $a(\theta'') \in \text{supp } P''(\theta'')$. Moreover, if a solution exists, there exists a deterministic solution in which a is non-decreasing in t .

Proof. The proof follows the same variational argument as the proof of Lemma 2, where the perturbed randomized schedule \hat{P} is defined as before. As before, \hat{P} satisfies the constraints. The difference appears when comparing the value of the objective under \hat{P} and P , which becomes

$$\begin{aligned}
& \int \int w(a, t(\theta)) d\hat{P} dG - \int \int w(a, t(\theta)) dP dG \\
&= \varepsilon \int_{\Theta'} \left[\int w(a, t(\theta)) \bar{\gamma}(\theta) d\underline{P}(\phi(\theta)) - \int w(a, t(\theta)) \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) \right] \frac{g(\theta)}{f(\theta)} d\theta \\
& \quad + \varepsilon \int_{\Theta''} \left[\int w(a, t(\theta)) \underline{\gamma}(\theta) d\bar{P}(\phi^{-1}(\theta)) - \int w(a, t(\theta)) \bar{\gamma}(\phi^{-1}(\theta)) d\underline{P}(\theta) \right] \frac{g(\theta)}{f(\theta)} d\theta \\
&= \varepsilon \int_{\Theta'} \left[\int \left[w(a, t(\theta)) \frac{g(\theta)}{f(\theta)} - w(a, t(\phi(\theta))) \frac{g(\phi(\theta))}{f(\phi(\theta))} \right] \bar{\gamma}(\theta) d\underline{P}(\phi(\theta)) \right. \\
& \quad \left. - \int \left[w(a, t(\theta)) \frac{g(\theta)}{f(\theta)} - w(a, t(\phi(\theta))) \frac{g(\phi(\theta))}{f(\phi(\theta))} \right] \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) \right] d\theta.
\end{aligned}$$

Note that

$$\begin{aligned}
& w(a, t(\theta)) \frac{g(\theta)}{f(\theta)} - w(a, t(\phi(\theta))) \frac{g(\phi(\theta))}{f(\phi(\theta))} \\
&= [w(a, t(\theta)) - w(a, t(\phi(\theta)))] \frac{g(\theta)}{f(\theta)} + w(a, t(\phi(\theta))) \left[\frac{g(\theta)}{f(\theta)} - \frac{g(\phi(\theta))}{f(\phi(\theta))} \right].
\end{aligned}$$

For each $\theta \in \Theta'$, $t(\theta) < t(\phi(\theta))$, and because w has increasing differences, the first term in the second line is non-increasing in a as before. Moreover, the second term is also non-increasing in a if either condition (i) or (ii) in the lemma is satisfied. This ensures that the entire expression in the first line is decreasing in a for all $\theta \in \Theta'$, so the rest of the proof of Lemma 2 goes through. ■

Applying Lemma 11(ii) to the modified deviation program (41) with $a = -x^\rho(\theta)$, $t(\theta) = c_2^\rho(\theta)$ and $w(a, t) = u(c_2 - x)$ immediately implies that, for any given ρ , $x^\rho(\theta)$ is non-decreasing in $c^\rho(\theta)$ when $g(\theta, \rho)/f(\theta, \rho)$ is non-decreasing in θ , which is the desired result. ■

We can also reproduce Lemma 5 as follows:

Lemma 12. *In any solution, for almost all non-pooled θ ,*

$$\frac{\tau_k(\theta, \rho)}{1 - \tau_k(\theta, \rho)} = R\eta [u'(c_2(\theta, \rho) + x(\theta, \rho)) - u'(c_2(\theta, \rho))] g(\theta, \rho) / f(\theta, \rho). \quad (43)$$

Proof. Observe first that, for each given ρ , c_2^ρ must solve

$$\min_{c_2^\rho} \int \left(u^{-1}(U(\theta, \rho) - \beta u(c_2(\theta, \rho))) + \frac{1}{R} c_2(\theta, \rho) \right) dF^\rho \quad \text{s.t.} \quad \int u(c_2(\theta, \rho)) dG^\rho \geq \bar{V}_D^\rho(c_2^\rho), \quad (44)$$

where $\bar{V}_D^\rho(c_2^\rho)$ is defined in (41). This is the same as (17) except that the constraint involves the weights G rather than F . Accounting for this when following the same steps as in the proof of Lemma 5 yields the result. ■

Since Lemma 10 has shown that $x(\theta, \rho)$ is decreasing in $c_2(\theta, \rho)$ for any given ρ , Lemma 12 immediately yields the following analogue of Proposition 4:

Proposition 11. *Suppose $g(\theta, \rho)/f(\theta, \rho)$ is non-increasing in θ for each ρ . In any solution, for each ρ , there exists a threshold level of period 2 consumption $c_2^{\rho*}$ such for almost all non-pooled agents, capital is subsidized for agents with $c_2(\theta, \rho) < c_2^{\rho*}$ and taxed for those with $c_2(\theta, \rho) > c_2^{\rho*}$.*

Proof. Analogous to the proofs of Propositions 4 and 7, now using Lemmas 10 and 12. ■

By inspection of the formula in (43), Proposition 5 also goes through if $c_2(\theta, \rho)$ is increasing and $g(\theta, \rho)/f(\theta, \rho)$ is decreasing in θ given ρ . In words, the marginal capital tax is increasing in θ in this case for non-pooled agents, as it was in Section 4.

The one major way in which the results in this appendix are weaker than those in Section 4 is that we cannot conclude that $c_2(\theta, \rho)$ is non-decreasing in θ , even when $g(\theta, \rho)/f(\theta, \rho)$ is decreasing in θ . The reason is that our generalized monotonicity lemma, Lemma 11, is still not general enough to deliver comparative statics with respect to $c_2(\theta, \rho)$. This is because Lemma 11 requires that $w(a, t)$ is monotone in a . Yet, the objective in (44), while satisfying increasing differences, is not monotone in c_2 . The monotonicity requirement on the objective can be dropped only when the relative weights $g(\theta, \rho)/f(\theta, \rho)$ are constant in θ . Thus, the full set of comparative statics from Section 4 can be guaranteed only when the government is utilitarian within each group ρ .²⁹

E Appendix: Pooling

In this appendix, we discuss the caveats regarding pooled types in Proposition 3. It is first worth recalling that $\tau_k(\theta)$ is non-decreasing at all types θ for which formula (18) applies, which by Lemma 5 holds whenever there is a unique solution to the deviation program for type θ , or, even if this not the case, type θ is not pooled with other types in terms of c_2 . The reason why we need one of these two assumptions is that we use an envelope theorem in the proof of Lemma 5, and the deviation value function $V_D(c_2)$ may not be differentiable if $X(\theta)$ is not single-valued (the left- and right-derivatives are always well-defined, but they may not coincide).

However, by Lemma 4 (iii), we know that the solution to (DP) is unique for almost all c_2 -levels (recall that (DP) does not directly depend on θ but only through the implied c_2 -schedule). Hence, potential non-differentiability of V_D can occur almost nowhere in terms of c_2 , and problems could only arise when a non-unique solution to (DP) happens to occur at a c_2 -level at which there is a strictly positive mass of agents, i.e. where there is pooling. This is illustrated in Figure 5, where types in $[\theta_1, \theta_2]$ are pooled at \tilde{c}_2 , and X happens to be non-singleton at \tilde{c}_2 (note that, by Lemma 4 (iv), such non-uniqueness can only occur for negative x -values, i.e. for types $\theta > \theta^*$). In this case, the marginal capital tax could be non-monotone overall, although it is still increasing among the non-pooled types. There is no obvious reason, however, to expect that non-unique solutions to (DP) are more or less likely to emerge at c_2 -levels at which there is pooling, so we view this as a technicality that needs to be accounted for but not as a particularly important economic feature of our model.

²⁹Of course, the conditions on $g(\theta, \rho)/f(\theta, \rho)$ in Lemma 11 are only sufficient, so it is certainly possible that our full set of results goes through even when the government is not utilitarian.

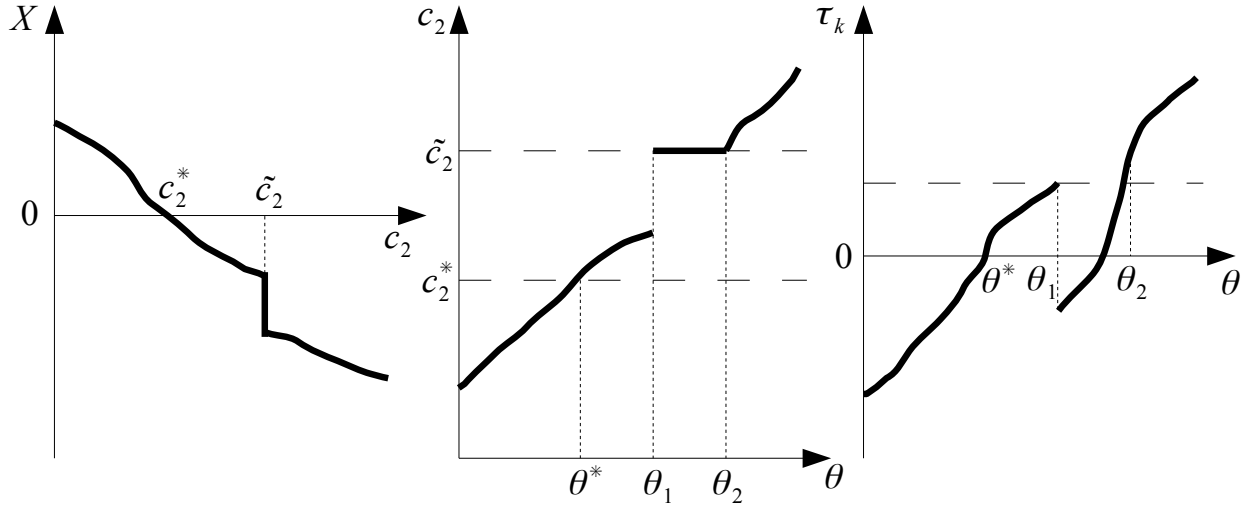


Figure 5: Potential non-monotonicity of τ_k with pooled individuals

In fact, these caveats can be completely dispensed with if for every period-2 consumption schedule c_2 , one of the following two conditions holds.

1. The set of solutions to the deviation program $X(\theta)$ is single-valued for almost all θ .
2. $\int u(c_2(\theta)) dF - V_D(c_2)$ is convex in c_2 (which implies the government's problem is convex).

Both of these points require some explanation. For the first one, note that if $X(\theta)$ is (almost) always single-valued, then the argument of Lemma 5 (i) applies—and therefore (18) holds—for (almost) all types θ . Simpler versions of the proofs of Propositions 4 and 5 then imply that both of these results will hold without the qualifications concerning pooled types. Furthermore, $X(\theta)$ is indeed single-valued almost everywhere in some natural cases. For example, if \tilde{H} is a step function, then it can be shown that $X(\theta)$ is single-valued almost everywhere, as discussed in the text.

For the second point, observe that the government's problem is convex if $\int u(c_2(\theta)) dF - V_D(c_2)$ is convex. The intuition is that, unlike the government's problem in the model with equalizing reforms or the deviation problem in the current model, the government's planning problem in the current model does not directly depend on the non-concave political support function \tilde{H} (though of course it depends indirectly on \tilde{H} through V_D), so there is some hope that this problem might actually be convex. If it is convex, then an explicit variational approach can be taken to minimizing the Lagrangian (19) in the proof of Lemma 5 (as now an extremum of the government's problem must also be an extremum of the Lagrangian), and it may be shown that the following version of (18) holds for almost all types θ (whether or not θ is pooled), where $\bar{x}(\theta) = \sup X(\theta)$ and $\underline{x}(\theta) = \inf X(\theta)$:

$$\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} \in [R\eta [u'(c_2(\theta) + \bar{x}(\theta)) - u'(c_2(\theta))], R\eta [u'(c_2(\theta) + \underline{x}(\theta)) - u'(c_2(\theta))]].$$

Lemma 4 may then be used to show that Propositions 4 and 5 hold without the qualifications about pooled types. It is worth pointing out that the condition that $\int u(c_2(\theta)) dF - V_D(c_2)$ is convex is not overly strong. For example, it is straightforward to check that it holds if u is quadratic. Proposition 6 in the text summarizes this discussion.