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THE NEOCLASSICAL MODEL OF LOCATION

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ABSTRACT

We analyze a neoclassical general-equilibrium model to explain cross-metro variation in population, density, and land supply based on three amenity types: quality-of-life, productivity in tradables, and productivity in non-tradables. We develop a new method to estimate elasticities of housing and land supply, and local-productivity estimates, from cross-sectional density and land-area data. From wage and housing-cost indices, the model explains half of U.S. density and total population variation, and finds that quality of life determines locations more than employment opportunities. We show how changing quality of life, relaxing land-use regulations, or neutralizing federal taxes can redistribute populations massively.

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1 Introduction

Academics and policy makers have long sought to understand the household location decisions that shape human geography, such as the decline of the Rust Belt and the rise of the Sunbelt (e.g., Blanchard and Katz, 1992; Glaeser, Ponzetto, and Tobio, 2014). For decades, economists have used the neoclassical model pioneered by Rosen (1979) and Roback (1982) to understand how amenities — broadly defined — determine wages and housing rents across locations.[1] Despite its widespread application, no one has yet to use it to explain how amenities may determine precise population *levels* and *densities* city by city, nor the possibile inferences such predictions open up.

Below, we develop the neoclassical model analytically and empirically in its full generality. We find it explains location choices across metropolitan areas rather well based on amenities inferred from local wages and rents. Moreover, we find that quality-of-life amenities — originating mainly from climate and geography (Albouy, 2008) — explain location decisions more than employment opportunities. Housing production possibilities may play an even larger role. Furthermore, we develop novel methods to estimate local heterogeneity in housing and land supply — separately — from level differences in population and land area. This method passes specification tests and produces plausible results. Accounting for local heterogeneity, we then infer how urban populations could change dramatically with shifts in amenities, or reforms in regulatory or federal tax policies.

The theory models a system of cities with three inputs — mobile labor and capital, and immobile land — and two outputs — a good tradable across cities, and a home good that is not. Local amenities vary in three dimensions: quality-of-life for households, and trade-productivity and home-productivity for firms. The first two concern the classic problem of whether jobs follow people or people follow jobs, while the third addresses whether both jobs and people follow housing or other non-traded goods (e.g., Glaeser, Gyourko, and Saks 2006, Saiz 2010). Our cross-sectional method assesses the importance of these dimensions, without the timing assumptions critical to time-series studies (e.g., Carlino and Mills 1987; Hoogstra, Florax, and Dijk 2005).

[1] We name this the "neoclassical model" of urban location because of its standard modeling apparatus and its particular resemblance to the two-sector models of Hecksher (1919) and Ohlin (1924) on trade, Uzawa (1961) and Stiglitz (1967) on growth, and Harberger (1962) on tax incidence.

In section 2, we derive the structural relationships between prices and quantities, such as population, and the three amenity types. Our analytical expressions clarify how these relationships depend on cost and expenditure shares, tax rates, land supply, and — most notably — substitution responses in consumption and production. We go beyond work by Roback (1982), Glaeser and Gottlieb (2009) and Albouy (2016), to show how to use population levels, in addition to wages and rents, to identify home-productivity, improve estimates of trade-productivity, and infer land values.

Parametrizing the model to reflect the U.S. economy, section 3 demonstrates that density — population holding land area constant — is more sensitive to amenities than are prices. This is consistent with how density varies by an order of magnitude more than wages and rents across metros. The analysis also makes clear how little population can change with local conditions without flexible production of housing and other non-traded sectors.

In section 4, we map commonly estimated reduced-form elasticities – e.g., of local labor or housing supply from Bartik (1991) – to underlying structural parameters. We obtain large elasticities that resemble estimates from the literature, implying that the level differences in population we model may be consistent with long-run changes over time. This suggests the model may be used credibly to simulate relationships which have yet to be estimated.

In two parts, Section 5 examines the relationship between population densities of 276 U.S. metro areas with amenities. First — assuming home-productivity is constant — we find the parametrized model explains half of the observed variation in population density using wage and rent data, without estimating a single parameter.[2] We then demonstrate visually how differences in home-productivity may be used to explain the remaining lack of fit. Together, our three amenity measures provide a full accounting for why people live where they do.

Second, we develop a non-linear regression model that uses variation in land-use regulation and geography to estimate city-specific heterogeneity in productivity and factor substitution in the non-traded sector. These estimates, identified from level data, conform to predictions that regulations and rugged terrain impede efficiency and reduce substitution. Our approach builds on interesting

[2] Albouy (2016) discusses how to use wage and rent data to infer quality-of-life and trade-productivity.

work by Saks (2008) and Saiz (2010), but differs by focusing on cross-sectional variation nested in a general-equilibrium model

Section 6 supplements the housing supply equation in the neoclassical model with a novel land supply equation. This relaxes its problematic assumption of fixed land, and helps to explain total population differences across metros, rather than density alone. Estimates imply that land endowments are determined by local geography, and that the price elasticity of land supply is near one but falls with local regulations and rugged terrain. The land-supply model explains half the variation in total population levels using wages and rents alone.

Using these estimates, section 7 conducts simple counterfactual simulations. Quality of life explains location choices more than trade-productivity, implying jobs follow people more than the opposite. City-by-city, the implications are provocative: for example, if Chicago had the same quality-of-life amenities as San Diego, the population of its urban area would quadruple. Finally, we demonstrate how the model can be used to perform general policy experiments, such as relaxing land-use constraints or neutralizing the geographic impact of federal taxes. These two reforms would produce mutually reinforcing effects: people would move to larger cities in droves, particularly in the West and Northeast, raising real income and quality of life.

Based on our understanding of the previous literature, we are the first to derive, analyze, and assess predictions of a flexible neoclassical model for both total population and density *in levels* across *specific* metros. Given its prominence, generality, and orthodoxy, the model is a natural benchmark. Importantly, its assumptions are transparent and pre-determined: they are all contained in Roback (1982) except for federal taxes, from Albouy (2009), which provides our pre-set parametrization. Our work here is more of an examination of an established model than an endorsement of it. We make no ad-hoc changes. Our supplementary land-supply equation merely generalzes it to handle the often-neglected fact that cities vary in land area.

Our analytical presentation helps to assess the role of core urban forces — regarding jobs, quality of life, and housing — that may themselves depend on deeper causes. It abstracts from complexities arising from less orthodox elements such as moving costs, search frictions, trade costs, and path dependence. Yet a strength of the model is that it is easily amended to handle deficiencies, which can sometimes be intuited city by city. For instance, Albouy et al. (2015) show how to do add heterogeneous skills and preferences, based on observed and unobserved types.

The general neoclassical framework that we develop contributes in several ways to previous work. First, our model relaxes a number of limiting restrictions, e.g., that productivity in both home and and traded-production are identical; or, that housing supply elasticities are identical, greater than 2, or exactly 2. It restricts neither input markets (e.g., labor is used in non-traded production), nor elasticities of substitution in production and consumption (e.g., as opposed to a Cobb-Douglas economy). Relaxing these restrictions leads to a deeper understanding of urban forces and affects the model's quantitative predictions. Second, we show how to use density data rather than restrict it to be uniform or to depend strictly on the ratio of wages to rents. Third, the data we use are widely available, and are not imputed coarsely from other sources. Fourth, we consider population levels for specific cities — not broad distributions, such as Zipf's Law — taking amenity estimates for each city seriously. Fifth, our identification is relatively transparent and does not rely on non-linearities, which are often impossible to specify using economic theory alone. Sixth, the neoclassical model does not rely on unobservable and inherently untestable differences in tastes. While taste heterogeneity may explain frictions to mobility, it is a weak explanation for why so many live in Dallas as opposed to Dothan.[3]

[3]Here we list a few examples; see Appendix G for more details. We do not argue that these papers are unjustified in making various simplifying assumptions. However, to assess the explanatory power of the baseline model, and to understand the importance of common simplifying assumptions, it is necessary to consider a general model without these modifications.

Haughwout and Inman (2001) has no local production and is used for a one-city simulation. Rappaport (2008a, 2008b) assumes equal productivity, fixes land supply, and engages in only a two-city simulation. Glaeser and Gottlieb (2009) assume unitary elasticities of substitution (and thus uniform housing supply elasticities), fix separate land supplies in home and traded-production, and consider only a single amenity. Lee and Li (2013) have a similar model with multiple amenities to explain Zipf's Law. Saiz (2010) and Desmet and Rossi-Hansberg (2013) use monocentric city models with constant density, no land in trade-production, no labor in home-production, and inelastic housing demand. Desmet and Rossi-Hansberg have an elasticity of housing supply of exactly 2, while Saiz's are merely constrained to always be above 2. Most of these models conflict with the majority of Saiz's empirical estimates being heterogeneous and below 2. Ahlfeldt et al. (2015), who focus on within-city location choices, constrain elasticities of substitution in demand and traded production to be one. Suárez Serrato and Zidar (2014) and Hsieh and Moretti (2015) assume unitary elasticities of substitution, and exclude labor from non-traded production. Diamond (2016) fixes land supply, has no land in traded-production, no labor in home-production, and fixes housing demand. Allen and Arkolakis (2014), Bartelme (2015), Caliendo et al. (2015), and Fajgelbaum et al. (2015), consider trade costs and monopolistic competition in models that start from, yet restrict, the neoclassical benchmark in ways already mentioned.

2 The Neoclassical Model of Location

2.1 System of Cities with Consumption and Production

The national economy contains many cities, indexed by j, which trade with each other and share a homogeneous population of mobile households. Cities differ in three attributes, each of which is an index summarizing the value of amenities; quality-of-life Q^j raises household utility, trade-productivity A_X^j lowers costs in the traded sector, and home-productivity A_Y^j lowers costs in the non-traded sector. Households supply a single unit of labor in their city of residence, earning local wage w^j. They consume a numeraire traded good x and a non-traded "home" good y with local price p^j. All input and output markets are perfectly competitive, and all prices and per-capita quantities are homogeneous within cities.

Firms produce traded and home goods out of land, capital, and labor. Land, L^j, is heterogeneous across cities, immobile, and receives a city-specific price r^j. Each city's land supply $L_0^j \tilde{L}^j(r^j)$ depends on an exogenous endowment L_0^j and a supply function $\tilde{L}^j(r^j)$. The supply of capital in each city K^j is perfectly elastic at the price $\bar{\imath}$. Labor, N^j, is supplied by households who have identical size, tastes, and own diversified portfolios of land and capital, which pay an income $R = \sum_j r^j L^j / N_{TOT}$ from land and $I = \sum_j \bar{\imath} K^j / N_{TOT}$ from capital, where $N_{TOT} = \sum_j N^j$ is the total population. Total income $m^j = w^j + R + I$ varies across cities only as wages vary. Out of this income households pay a linear federal income tax τm^j, which is redistributed in uniform lump-sum payments T.[4] Household preferences are modeled by a utility function $U(x, y; Q^j)$ which is quasi-concave over x, y, and Q^j. The expenditure function for a household in city j is $e(p^j, u; Q^j) \equiv \min_{x,y}\{x + p^j y : U(x, y; Q^j) \geq u\}$. Quality-of-life Q enters neutrally into the utility function and is normalized so that $e(p^j, u; Q^j) = e(p^j, u)/Q^j$, where $e(p^j, u) \equiv e(p^j, u; 1)$.

Firms produce traded and home goods according to the function $X^j = A_X^j F_X(L_X^j, N_X^j, K_X^j)$ and $Y^j = A_Y^j F_Y(L_Y^j, N_Y^j, K_Y^j)$, where F_X and F_Y are weakly concave and exhibit constant returns to scale, with Hicks-neutral productivity. Unit cost in the traded good sector is $c_X(r^j, w^j, \bar{\imath}; A_X^j) \equiv$

[4]The model can be generalized to allow nonlinear income taxes. Our application adjusts for state taxes and tax benefits to owner-occupied housing.

$\min_{L,N,K}\{r^j L + w^j N + \bar{\imath} K : A_X^j F(L, N, K) = 1\}$. Let $c_X(r^j, w^j, \bar{\imath}; A_X^j) = c_X(r^j, w^j, \bar{\imath})/A_X^j$, where $c_X(r^j, w^j, \bar{\imath}) \equiv c_X(r^j, w^j, \bar{\imath}; 1)$ is the uniform unit cost function. A symmetric definition holds for unit cost in the home good sector c_Y.

2.2 Equilibrium of Prices, Quantities, and Amenities

Each city is described by a block-recursive system of sixteen equations in sixteen endogenous variables: three prices p^j, w^j, r^j, two per-capita consumption quantities, x^j, y^j, and eleven city-level production quantities X^j, Y^j, N^j, N_X^j, N_Y^j, L^j, L_X^j, L_Y^j, K^j, K_X^j, K_Y^j. The endogenous variables depend on three exogenous attributes Q^j, A_X^j, A_Y^j and the land endowment L_0^j. As in the Hecksher-Ohlin model, the system first determines prices — where most researchers stop — then, per-capita consumption quantities and city-level production quantities. The recursive structure vanishes if amenities depend endogenously on quantities, as described below. We adopt a "small open city" assumption and take nationally determined variables $\bar{u}, \bar{\imath}, I, R, T$ as given.

We log-linearize the system, as in Jones (1965), to obtain a model that can be solved analytically with linear methods. The full nonlinear system is explained in Appendix A. In Appendix B, we verify that the log-linearized model generally offers satisfying approximations. The non-linear model is too costly to compute with hundreds of cities, for benefits we are not confident of.

The log-linearized model involves several economic parameters, evaluated at the national average. For households, denote the shares of gross expenditures spent on the traded and home good as $s_x \equiv x/m$ and $s_y \equiv py/m$; the shares of income received from land, labor, and capital income as $s_R \equiv R/m$, $s_w \equiv w/m$, and $s_I \equiv I/m$. For firms, denote the cost shares of land, labor, and capital in the traded good sector as $\theta_L \equiv rL_X/X$, $\theta_N \equiv wN_X/X$, and $\theta_K \equiv \bar{\imath}K_X/X$; the equivalents in the home good sector as ϕ_L, ϕ_N, and ϕ_K. Finally, denote the shares of land, labor, and capital used to produce traded goods as $\lambda_L \equiv L_X/L$, $\lambda_N \equiv N_X/N$, and $\lambda_K \equiv K_X/K$. To fix ideas, assume the home good is more cost-intensive in land relative to labor than the traded good, both absolutely, $\phi_L \geq \theta_L$, and relatively, $\phi_L/\phi_N \geq \theta_L/\theta_N$, implying $\lambda_L \leq \lambda_N$. For any variable z, we denote the log differential by $\hat{z}^j \equiv \ln z^j - \ln \bar{z} \cong (z^j - \bar{z})/\bar{z}$, where $\bar{z}$ is the national average.

2.2.1 Equilibrium Price Conditions for Households and Firms

Since households are fully mobile, they receive the same utility $\bar{u}$ across all inhabited cities. Firms earn zero profits in equilibrium. These conditions imply

$$-s_w(1-\tau)\hat{w}^j + s_y\hat{p}^j = \hat{Q}^j \tag{1}$$

$$\theta_L\hat{r}^j + \theta_N\hat{w}^j = \hat{A}_X^j \tag{2}$$

$$\phi_L\hat{r}^j + \phi_N\hat{w}^j - \hat{p}^j = \hat{A}_Y^j. \tag{3}$$

Equations (1) - (3) simultaneously determine the city-level prices $\hat{p}^j$, $\hat{r}^j$, and $\hat{w}^j$ as functions of the three attributes $\hat{Q}^j$, $\hat{A}_X^j$, and $\hat{A}_Y^j$ plus cost and expenditure shares and the marginal tax rate. These conditions provide a one-to-one mapping between unobservable city attributes and potentially observable prices. Households pay more for housing and get paid less in nicer areas. Firms pay more to their factors in more trade-productive areas, and they do the same relative to output prices in more home-productive areas.[5]

2.2.2 Consumption Conditions for Households

In their consumption $\hat{x}^j$ and $\hat{y}^j$, households face a budget constraint and obey a tangency condition:

$$s_x\hat{x}^j + s_y\left(\hat{p}^j + \hat{y}^j\right) = (1-\tau)s_w\hat{w}^j \tag{4}$$

$$\hat{x}^j - \hat{y}^j = \sigma_D\hat{p}^j \tag{5}$$

where $\hat{w}^j$ and $\hat{p}^j$ are determined by the price conditions. Equation (5) depends on the elasticity of substitution in consumption, $\sigma_D \equiv -e\cdot(\partial^2 e/\partial p^2)/[\partial e/\partial p\cdot(e-p\cdot\partial e/\partial p)] = -\partial\ln(y/x)/\partial\ln p$. Substituting equation (1) into equations (4) and (5) produces the consumption solutions $\hat{x}^j = s_y\sigma_D\hat{p}^j - \hat{Q}^j$ and $\hat{y}^j = -s_x\sigma_D\hat{p}^j - \hat{Q}^j$. Because of homothetic preferences, in areas where Q^j is higher, but p^j is the same, households consume less of x and y in equal proportions, so the ratio

[5] Albouy (2009, 2016) examines these conditions in detail.

y/x remains constant — similar to an income effect. Holding Q^j constant, areas with higher p^j induce households to reduce the ratio y/x through a substitution effect.

Higher values of σ_D approximate a more general model with greater taste heterogeneity for home goods. In such a model, households with stronger tastes for y sort to areas with a lower p.[6]

2.2.3 Production Conditions for Traded and Home-Good Sectors

Given prices and per-capita consumption, output $\hat{X}^j, \hat{Y}^j$, employment $\hat{N}^j, \hat{N}^j_X, \hat{N}^j_Y$, capital $\hat{K}^j, \hat{K}^j_X, \hat{K}^j_Y$, and land $\hat{L}^j, \hat{L}^j_X, \hat{L}^j_Y$ are determined by eleven equations describing production and market clearing. The first six are conditional factor demands describing how input demands depend on output, productivity, and relative input prices:

$$\hat{N}^j_X = \hat{X}^j - \hat{A}^j_X + \theta_L \sigma^{LN}_X \left(\hat{r}^j - \hat{w}^j\right) - \theta_K \sigma^{NK}_X \hat{w}^j \quad (6)$$

$$\hat{L}^j_X = \hat{X}^j - \hat{A}^j_X + \theta_N \sigma^{LN}_X (\hat{w}^j - \hat{r}^j) - \theta_K \sigma^{KL}_X \hat{r}^j \quad (7)$$

$$\hat{K}^j_X = \hat{X}^j - \hat{A}^j_X + \theta_L \sigma^{KL}_X \hat{r}^j + \theta_N \sigma^{NK}_X \hat{w}^j \quad (8)$$

$$\hat{N}^j_Y = \hat{Y}^j - \hat{A}^j_Y + \phi_L \sigma^{LN}_Y (\hat{r}^j - \hat{w}^j) - \phi_K \sigma^{NK}_Y \hat{w}^j \quad (9)$$

$$\hat{L}^j_Y = \hat{Y}^j - \hat{A}^j_Y + \phi_N \sigma^{LN}_Y (\hat{w}^j - \hat{r}^j) - \phi_K \sigma^{KL}_Y \hat{r}^j \quad (10)$$

$$\hat{K}^j_Y = \hat{Y}^j - \hat{A}^j_Y + \phi_L \sigma^{KL}_Y \hat{r}^j + \phi_N \sigma^{NK}_Y \hat{w}^j \quad (11)$$

The dependence on input prices is determined by partial (Allen-Uzawa) elasticities of substitution in each sector for each pair of factors, e.g., $\sigma^{LN}_X \equiv c_X \cdot (\partial^2 c_X/\partial w \partial r) \,/\, (\partial c_X/\partial w \cdot \partial c_X/\partial r)$. Our baseline model assumes that production technology does not differ across cities, implying constant elasticities; we relax this assumption for the housing sector below. To simplify, we also assume that partial elasticities within each sector are the same, i.e., $\sigma^{NK}_X = \sigma^{KL}_X = \sigma^{LN}_X \equiv \sigma_X$, and similarly for σ_Y, as with a constant elasticity of substitution (CES) production function.

Higher values of σ_X correspond to more flexible production of the traded good, as firms can

[6] At equilibrium utility levels, an envelope of the mobility conditions for each type forms that of a representative household, with greater preference heterogeneity reflected as more flexible substitution. Roback (1980) discusses this generalization as well as the below generalizations in production.

vary the proportion of inputs they employ. In a generalization with multiple traded goods sold at fixed prices, firms specialize in producing goods for which their input costs are relatively low.[7]

A related argument exists for home goods. A higher value of σ_Y means that housing producers can better combine labor and capital to build taller buildings in areas with expensive land. For non-housing home goods, retailers may use taller shelves and restaurants would hire extra servers to make better use of space.[8]

Three conditions express the local resource constraints for labor, land, and capital under the assumption that factors are fully employed:

$$\hat{N}^j = \lambda_N \hat{N}_X^j + (1 - \lambda_N)\hat{N}_Y^j \tag{12}$$

$$\hat{L}^j = \lambda_L \hat{L}_X^j + (1 - \lambda_L)\hat{L}_Y^j \tag{13}$$

$$\hat{K}^j = \lambda_K \hat{K}_X^j + (1 - \lambda_K)\hat{K}_Y^j. \tag{14}$$

Equations (12)-(14) imply that sector-specific factor changes affect overall changes in proportion to the factor share. Local land is determined by the supply function in log differences

$$\hat{L}^j = \hat{L}_0^j + \varepsilon_{L,r}^j \hat{r}^j \tag{15}$$

with the endowment differential $\hat{L}_0^j$ and the land supply elasticity $\varepsilon_{L,r}^j \equiv (\partial \tilde{L}^j / \partial r) \cdot (r^j / \tilde{L}^j)$.

Finally, the market clearing condition for home goods that demand equals supply is

$$\hat{N}^j + \hat{y}^j = \hat{Y}^j. \tag{16}$$

Walras' Law makes redundant the market clearing equation for traded output, which includes per-capita net transfers from the federal government.

[7]For example, areas with high land costs and low labor costs would produce goods that use labor intensively. A representative zero-profit condition is formed by an envelope of the zero-profit conditions for each good, with a greater variety of goods reflected in greater substitution possibilities.

[8]If home goods are perfect substitutes, then an envelope of zero-profit conditions would form a representative zero-profit condition. An alternative sufficient condition, which holds when considering traded goods, is that relative prices of types of home goods do not vary across cities.

2.3 Total Population, Density, and Land

The log-linearized model readily separates intensive population differences holding land supply constant, i.e. density, from extensive differences driven by land supply. If we define population density as $N_*^j \equiv N^j/L^j$, then the total population differential is a linear function of differentials in density, the land endowment, and land supply determined by rent:

$$\hat{N}^j = \hat{N}_*^j + \hat{L}_0^j + \varepsilon_{L,r}^j \hat{r}^j \tag{17}$$

where $\hat{N}_*^j$ and $\hat{r}^j$ depend on amenities $\hat{Q}^j$, $\hat{A}_X^j$, $\hat{A}_Y^j$ but the land endowment $\hat{L}_0^j$ does not.[9]

2.4 Solving the Model for Relative Quantity Differences

We express solutions for the endogenous variables in terms of the amenity differentials $\hat{Q}^j$, $\hat{A}_X^j$, and $\hat{A}_Y^j$. Only equations (1) - (3) are needed to solve the price differentials.

$$\hat{r}^j = \frac{1}{s_R}\frac{\lambda_N}{\lambda_N - \tau\lambda_L}\left[\hat{Q}^j + \left(1 - \frac{\tau}{\lambda_N}\right) s_x \hat{A}_X^j + s_y \hat{A}_Y^j\right] \tag{18a}$$

$$\hat{w}^j = \frac{1}{s_w}\frac{1}{\lambda_N - \tau\lambda_L}\left[-\lambda_L \hat{Q}^j + (1-\lambda_L) s_x \hat{A}_X^j - \lambda_L s_y \hat{A}_Y^j\right] \tag{18b}$$

$$\hat{p}^j = \frac{1}{s_y}\frac{1}{\lambda_N - \tau\lambda_L}\left[(\lambda_N - \lambda_L)\hat{Q}^j + (1-\tau)(1-\lambda_L) s_x \hat{A}_X^j - (1-\tau)\lambda_L s_y \hat{A}_Y^j\right] \tag{18c}$$

Higher quality-of-life leads to higher land and home good prices but lower wages. Higher trade-productivity increases all three prices, while higher home-productivity increases land prices but decreases wages and the home good price.

[9] In principle, land supply can vary on two different margins. At the extensive margin, an increase in land supply corresponds to a growing city boundary. Extensive margin differences can be driven by the land endowment $\hat{L}_0^j$ or the supply function $\varepsilon_{L,r}^j \hat{r}^j$. At the intensive margin, an increase in land supply takes the form of employing previously unused land within a city's border. The assumption of full utilization in (13) and (15), rules out unmeasured intensive changes.

Putting solution (18c) in equations (4) and (5) yields the per-capita consumption differentials

$$\hat{x}^j = \frac{\sigma_D(1-\tau)}{\lambda_N - \tau\lambda_L}\left[\frac{\sigma_D(\lambda_N-\lambda_L)-(\lambda_N-\tau\lambda_L)}{\sigma_D(1-\tau)}\hat{Q}^j + (1-\lambda_L)s_x\hat{A}_X^j - \lambda_L s_y \hat{A}_Y^j\right]$$
$$\hat{y}^j = -\frac{s_x}{s_y}\frac{\sigma_D(1-\tau)}{\lambda_N - \tau\lambda_L}\left[\frac{s_x\sigma_D(\lambda_N-\lambda_L)+s_y(\lambda_N-\tau\lambda_L)}{s_x\sigma_D(1-\tau)}\hat{Q}^j + (1-\lambda_L)s_x\hat{A}_X^j - \lambda_L s_y \hat{A}_Y^j\right]$$

Households in home-productive areas substitute towards home goods and away from traded goods, while households in trade-productive areas do the opposite. In nicer (high Q) areas, households consume fewer home goods; whether they consume fewer traded goods is ambiguous: the substitution effect is positive, and the income effect is negative.

Solutions for the other quantities, which rely on equations (6) - (16), are more complicated and harder to intuit. To simplify notation, we express the change in each quantity with respect to amenities using three reduced-form elasticities, each composed of structural parameters. For our central example, the population differential is written

$$\hat{N}^j = \varepsilon_{N,Q}\hat{Q}^j + \varepsilon_{N,A_X}\hat{A}_X^j + \varepsilon_{N,A_Y}\hat{A}_Y^j + \hat{L}_0^j, \tag{20}$$

where $\varepsilon_{N,Q}$ is the elasticity of population with respect to quality-of-life; ε_{N,A_X} and ε_{N,A_Y} are defined similarly. In terms of structural parameters, the first reduced-form elasticity, $\varepsilon_{N,Q}$, is

$$\begin{aligned}\varepsilon_{N,Q} = {} & \frac{\lambda_N-\lambda_L}{\lambda_N} + \sigma_D\left[\frac{s_x(\lambda_N-\lambda_L)^2}{s_y\lambda_N(\lambda_N-\lambda_L\tau)}\right] + \sigma_X\left[\frac{\lambda_L}{\lambda_N-\lambda_L\tau}\left(\frac{\lambda_L}{s_w}+\frac{\lambda_N}{s_R}\right)\right] \\ & + \sigma_Y\left[\frac{1}{\lambda_N-\lambda_L\tau}\left(\frac{\lambda_L^2(1-\lambda_N)}{s_w\lambda_N}+\frac{\lambda_N(1-\lambda_L)}{s_R}-\frac{(\lambda_N-\lambda_L)^2}{s_y\lambda_N}\right)\right] \\ & + \varepsilon_{L,r}\left[\frac{\lambda_N}{s_R(\lambda_N-\tau\lambda_L)}\right]\end{aligned} \tag{21}$$

We provide similar expressions for ε_{N,A_X} and ε_{N,A_Y} in Appendix C. The full structural solution to (20) is obtained by substituting in these expressions.

Collecting terms for each structural elasticity in (21) highlights that nicer areas can have higher population via five behavioral responses. The first term reflects how households consume fewer goods from the income effect, and thus require less land per capita, e.g. by crowding into existing housing. The second term, with σ_D, captures how households substitute away from land-intensive goods, accepting additional crowding. The third, with σ_X, expresses how firms in the traded sector substitute away from land towards labor and capital, freeing up space for households. The fourth, with σ_Y, reflects how home goods become less land intensive, e.g., buildings get taller. The fifth, with $\varepsilon_{L,r}$, provides the population gain on the extensive margin from more land being used.

Each reduced-form elasticity between a quantity and amenity-type has up to five similar structural effects. Unlike the price solutions, (18a-18c), the quantity solutions require more epistemically demanding knowledge of substitution elasticities, i.e., of behavioral responses to prices.

Below we initially focus on quantity differences holding geography constant, i.e., focusing on density. This case sets $\hat{L}^j = 0$. In section 6, we consider how to estimate $\varepsilon^j_{L,r}$ and $\hat{L}^j_0$.

2.5 Endogenous Amenities

The above set-up readily admits simple forms of endogenous amenities.[10] We consider two common forms: positive economies of scale in traded production (or "agglomeration"), and negative economies in quality-of-life (or "congestion"). For simplicity, we assume that both processes follow a conventional power law and depend on density alone: $A^j_X = A^j_{X0}(N^j_*)^\alpha$ and $Q^j = Q^j_0(N^j_*)^{-\gamma}$, where A^j_{X0} and Q^j_0 represent "natural advantages," and $\alpha \geq 0$ and $\gamma \geq 0$ are reduced-form elasticities. Natural advantages may be determined by local geography or policies. Economies of scale in productivity may be due to non-rival input sharing, improved matching in labor markets, or knowledge spillovers (e.g., Jaffe et al. 1993, Glaeser 1999, Arzaghi and Henderson 2008, Davis and Dingel 2012, Baum-Snow 2013); diseconomies in quality-of-life may be due to congestion, pollution, or crime.

[10] Our model incorporates aspects of both locational fundamentals and increasing returns; see Davis and Weinstein (2002). Its unique predictions make it less capable of representing historical path dependence (e.g., Bleakley and Lin 2012, 2015). However, mobility frictions discussed in appendix C.5 can help conserve it since population levels may depend on past amenity levels levels that differ from current ones. The greater the frictions, the more populations may depend on past amenities, or differences in how amenities were valued relative to now.

The feedback effects on density are easily expressed using the reduced-form notation:

$$\begin{aligned}\hat{N}_*^j &= \varepsilon_{N_*,Q}(\hat{Q}_0^j - \gamma\hat{N}_*^j) + \varepsilon_{N_*,A_X}(\hat{A}_{X0}^j + \alpha\hat{N}_*^j) + \varepsilon_{N_*,A_Y}\hat{A}_{Y0}^j \\ &= \frac{1}{1+\gamma\varepsilon_{N_*,Q} - \alpha\varepsilon_{N_*,A_X}}\left(\varepsilon_{N_*,Q}\hat{Q}_0^j + \varepsilon_{N_*,A_X}\hat{A}_{X0}^j + \varepsilon_{N_*,A_Y}\hat{A}_{Y0}^j\right) \\ &\equiv \tilde{\varepsilon}_{N_*,Q}\hat{Q}_0^j + \tilde{\varepsilon}_{N_*,A_X}\hat{A}_{X0}^j + \tilde{\varepsilon}_{N_*,A_Y}\hat{A}_{Y0}^j, \end{aligned} \tag{22}$$

where $\varepsilon_{N_*,Q}$ is the reduced-form elasticity of density with respect to quality-of-life, and $A_Y^j = A_{Y0}^j$ is fixed. Equation (22) simply modifies the reduced-form elasticities to incorporate the multiplier $(1+\gamma\varepsilon_{N_*,Q} - \alpha\varepsilon_{N_*,A_X})^{-1}$, which determines whether the impacts of natural advantages are magnified by positive economies or dampened by negative ones.

This framework could be used to study more complicated forms of endogenous amenities, although these typically require more complicated solutions. Interesting extensions which deserve attention in future work include accounting for spillovers across cities and examining the implications of a city's internal structure. Appendix C.5 discusses an extension to the model with imperfect mobility and preference heterogeneity. This reveals that decreasing willingness-to-pay for a marginal resident to live in a city operates like — and may be confused for — congestion costs.

2.6 Identification of Production Amenities and Land Values

While cross-metro data on wages and housing rents (which proxy for home-good prices) are readily available, land values are not. As a result, we cannot identify trade and home-productivity from (2) and (3).[11] Our proposed solution is to use widely available data on population density as a replacement for land values. Consider combining equations (2) and (3) to eliminate $\hat{r}^j$:

$$\text{Inferred costs}^j = \frac{\theta_L}{\phi_L}\hat{p}^j + \left(\theta_N - \phi_N\frac{\theta_L}{\phi_L}\right)\hat{w}^j = \hat{A}_X^j - \frac{\theta_L}{\phi_L}\hat{A}_Y^j. \tag{23}$$

[11] Albouy, Ehrlich, and Shun (2016) estimate $\hat{r}^j$ using transaction purchase data, which is only available for recent years. Their analysis discusses several conceptual and empirical challenges from this approach. Moreover, land-value data is generally not available in most years in most countries.

The left hand side of (23) equals traded producer costs inferred from wages and home good prices. Trade-productivity raises these inferred costs, while home-productivity lowers them. Albouy (2016) assumes that home-productivity is constant, $\hat{A}_Y^j = 0$, so that land values may be inferred from (3), and $\hat{A}_X^j$ equals the inferred costs. The ensuing estimates are biased downwards in home-productive areas, although $\hat{A}_X$ is only slightly biased if $\theta_L << \phi_L$.

Combining equations (1) and the analog of equation (20) for density yields the following expression, which says that "excess density" not explained by quality-of-life, on the left, must be explained by either trade or home-productivity, on the right:

$$\text{Excess density}^j = \hat{N}_*^j - \varepsilon_{N_*,Q}[\underbrace{s_y\hat{p}^j - s_w(1-\tau)\hat{w}^j}_{\hat{Q}^j}] = \varepsilon_{N_*,A_X}\hat{A}_X^j + \varepsilon_{N_*,A_Y}\hat{A}_Y^j. \tag{24}$$

Equations (23) and (24) are exactly identified: the inferred amenities *perfectly predict* density. Solving these equations identifies each productivity from observable differentials $\hat{N}_*^j$, $\hat{w}^j$, and $\hat{p}^j$:

$$\hat{A}_X^j = \frac{\theta_L[\hat{N}_*^j - \varepsilon_{N_*,Q}(s_y p^j - s_w(1-\tau)w^j)] + \phi_L\varepsilon_{N_*,A_Y}[\frac{\theta_L}{\phi_L}p^j + (\theta_N - \phi_N\frac{\theta_L}{\phi_L})w^j]}{\theta_L\varepsilon_{N_*,A_X} + \phi_L\varepsilon_{N_*,AY}} \tag{25a}$$

$$\hat{A}_Y^j = \frac{\phi_L[\hat{N}_*^j - \varepsilon_{N_*,Q}(s_y p^j - s_w(1-\tau)w^j)] - \phi_L\varepsilon_{N_*,A_X}[\frac{\theta_L}{\phi_L}p^j + (\theta_N - \phi_N\frac{\theta_L}{\phi_L})w^j]}{\theta_L\varepsilon_{N_*,A_X} + \phi_L\varepsilon_{N_*,AY}} \tag{25b}$$

High excess density and high inferred costs imply high trade-productivity. Low inferred costs and high excess density imply high home-productivity, with the latter effect stronger as $\phi_L > \theta_L$. We solve for the value of land by substituting the above solutions into (2) or (3).

$$\hat{r}^j = \frac{\hat{N}_*^j - \varepsilon_{N_*,Q}(s_y\hat{p}^j - s_w(1-\tau)\hat{w}^j) - \varepsilon_{N_*,A_X}\theta_N\hat{w}^j - \varepsilon_{N_*,A_Y}(\phi_N\hat{w}^j - \hat{p}^j)}{\theta_L\varepsilon_{N_*,A_X} + \phi_L\varepsilon_{N_*,AY}} \tag{25c}$$

As seen in the numerator of (25c), this rent measure depends on density not explained either by quality-of-life or productivity differences inferred from non-land prices.

The critical step underlying this approach is use of an observed quantity, population density, in place of unobserved land rents. In principle, we could use data on population and land instead of density, but our results would depend on the value of the land supply elasticity $\varepsilon_{L,r}$. There is no consensus on the appropriate value of this parameter, although we attempt to estimate it below.

3 Parameter Choices and Reduced-Form Elasticities

3.1 Parameter Choices

The main parametrization we use, shown in Table 1, was set in Albouy (2009), who based it on a literature review, without referring to density or population data. We focus on the substitution elasticities, set to $\sigma_D = \sigma_X = \sigma_Y = 0.667$. This is consistent with higher housing expenditures in high-rent areas and a higher cost-share of land for housing in high-value areas. We choose $\alpha = 0.06$ for agglomeration economies in trade-productivity and $\gamma = 0.015$ for congestion effect on quality-of-life, which are large for illustration purposes. Appendix D contains additional details on the parametrization. Given the number of parameters, an exhaustive sensitivity analysis is not feasible; we focus on sensitivity to substitution elasticities as they are the least-known and most relevant.

3.2 Parametrized Reduced-Form Elasticities

Panel A of Table 2 demonstrates how the three reduced-form elasticities for population depend on the structural elasticities, ignoring feedback effects. For example, the five ways that quality-of-life increases population from (21) are given by: $\varepsilon_{N,Q} \approx 0.77 + 1.14\sigma_D + 1.95\sigma_X + 8.00\sigma_Y + 11.84\varepsilon_{L,r}$. Substitution in the housing sector stands out as the most important dimension for the response of population density to amenities. The intuition is straightforward: increasing population density without building densely strains other substitution margins: higher densities are accommodated solely by increasing the occupancy of existing structures or releasing land from the traded-good sector. When $\sigma_D = \sigma_X = 0.667$ and $\hat{L}^j = 0$, density and amenities are related through σ_Y as:

$$\hat{N}_*^j \approx (2.84 + 8.00\sigma_Y)\hat{Q}^j + (0.79 + 2.06\sigma_Y)\hat{A}_X^j + (1.15 + 2.61\sigma_Y)\hat{A}_Y^j. \tag{26}$$

Setting $\sigma_Y = 0.667$ produces $\hat{N}_*^j \approx 8.17\hat{Q}^j + 2.16\hat{A}_X^j + 2.88\hat{A}_Y^j$. The elasticity of substitution in non-traded production accounts for about two-thirds of the reduced-form elasticities.

A one-point increase in $\hat{Q}^j$ has the value of a one-point increase in income, while one-point increases in $\hat{A}_X^j$ and $\hat{A}_Y^j$ have values of s_x and s_y of income due to their sector sizes. To compare the effects of the three attributes, we normalize them to have equal value:

$$\hat{N}_*^j \approx 8.17\hat{Q}^j + 3.38s_x\hat{A}_X^j + 8.01s_y\hat{A}_Y^j. \tag{27}$$

Quality-of-life and home-productivity have large impacts on local population density: increasing their value by one-percent of income results in a density increase of eight percentage points. Trade-productivity's impact is less than half as large. As a result, funds spent to attract households directly may be more effective at boosting density than funds spent to attract firms.

Setting the marginal tax rate τ to zero reveals that taxes cause much of these asymmetries: $\hat{N}_*^j \approx 6.32\hat{Q}^j + 5.81s_x\hat{A}_X^j + 7.55s_y\hat{A}_Y^j$. Taxes push workers away from from trade-productive areas towards high quality-of-life and home-productive areas (Albouy 2009). Remaining asymmetries arise mainly from the income effect from quality-of-life, crowding individuals into existing space, and an output effect from home-productivity, providing additional residential space.

In a Cobb-Douglas economy, $\sigma_D = \sigma_X = \sigma_Y = 1$, the implied elasticities are 35-50 percent higher than if $\sigma = 0.667$. If substitution margins are inelastic, then assuming a Cobb-Douglas economy – as many do – may inflate quantity predictions and associated welfare calculations.

Parametrizing the multiplier in (22) reveals the effects of agglomeration feedback:

$$(1 + \gamma\varepsilon_{N_*,Q} - \alpha\varepsilon_{N_*,A_X})^{-1} \approx (1 + (0.015)(8.17) - (0.06)(2.16))^{-1} \approx 1.01.$$

In this case, the positive and negative economies are small and largely offset each other, and so biases from ignoring agglomeration feedback appear to be modest.

Table 3 displays the reduced-form elasticities for all endogenous prices and quantities: Panel A for the baseline parametrization, and Panel B with geographically neutral federal taxes. Appendix Table A.1 contains results with agglomeration effects. While we focus on population and density here, many other quantities — such as capital stocks — deserve investigation. A key challenge for these other quantities is that accurate data on them are generally unavailable across metro areas.

4 General Equilibrium Elasticities and Existing Estimates

Elasticities characterizing how population and housing respond to changes in prices are commonly estimated and are often predicated on simpler models. The general equilibrium model here analyzes consumption and labor markets simultaneously, complementing empirical work in two distinct ways. First, it clarifies restrictions used to identify estimates. Second, it may simulate long-run effects that cannot be credibly estimated. The comparative statics of the neoclassical model requires adjustments that may take decades, including adjustments in the housing stock, the amortization of moving costs, and adaptation to local conditions.

4.1 Local Labor Supply and Demand

In partial equilibrium, increasing demand traces out a local labor supply curve. The immediate analogy of an increase in labor demand here is an increase in trade-productivity; the following ratio provides a general equilibrium elasticity of labor supply:

$$\left.\frac{\partial \hat{N}_*}{\partial \hat{w}}\right|_{\hat{Q},\hat{A}_Y} = \frac{\partial \hat{N}_*/\partial \hat{A}_X}{\partial \hat{w}/\partial \hat{A}_X} \approx 0.66\sigma_D + 0.43\sigma_X + 1.88\sigma_Y \approx 1.98. \tag{28}$$

The resulting labor supply curve slopes upwards as higher density raises demand for home goods and their prices, requiring higher wage compensation. A ceteris paribus increase in the wage, holding home-good prices constant, does not identify a labor supply elasticity in this model. Since trade-productivity increases home-good prices, a constant home-good price requires either a simultaneous decrease in quality-of-life, shifting in labor supply, or an increase in home-productivity, shifting out housing supply.

Labor supply elasticity estimates in Bartik (1991), Blanchard and Katz (1992), and Notowidigdo (2012) are in the range of 2 to 4, close to the values predicted in (28), especially if substitution elasticities are higher. Empirical estimates may be biased upwards if higher demand (A_X) is positively correlated with higher supply (Q).[12]

Increasing supply traces out a local labor demand curve. The closest analogy to a shift in supply is an increase in quality-of-life. The resulting labor demand curve slopes downward: holding productivity (and agglomeration economies) constant, a larger work force pushes down wages, as firms complement labor with ever scarcer land. The parametrized elasticity of labor demand is

$$\left.\frac{\partial \hat{N}_*}{\partial \hat{w}}\right|_{\hat{A}_X,\hat{A}_Y} = \frac{\partial \hat{N}_*/\partial \hat{Q}}{\partial \hat{w}/\partial \hat{Q}} \approx -2.15 - 3.18\sigma_D - 5.44\sigma_X - 22.31\sigma_Y \approx -22.78$$

This prediction might be seen as consistent with the weak effects on relative wages of immigration-induced changes in relative labor supply, predicted by immigrant enclaves (e.g., Bartel 1989, Card 2001). Relative wages at the city level are fairly unresponsive to increases in relative labor supply, broadly consistent with the large elasticity above.[13]

[12]Estimates in Notowidigdo (2012) reveal an increase in housing costs, along with higher wages, that are consistent with a slight decrease in quality-of-life. As explained in Appendix C.5, heterogeneity in worker tastes would increase the slope of the supply curve, as higher wages attract those with weaker tastes for the location, although the heterogeneity parameter, "ψ", cannot be readily identified separately from the congestion parameter γ.

[13]If demand for the traded good is not perfectly elastic, as in a model with heterogeneous traded output, then the elasticity of labor demand will be lower. To illustrate this in a partial equilibrium setting, let demand for the local traded good be $\hat{X}^j = -\eta\hat{p}_X^j$ where p_X^j is its price, formerly fixed. Let land supply for traded-good firms be provided in a segmented market by $\hat{L}_X^j = \hat{L}_{0X}^j + \varepsilon_X\hat{r}_X^j$. We may then derive a general form of Marshall's Rule for labor demand in the trade sector, that includes trade-productivity and the land endowment:

$$\hat{N}_X^j = \frac{-\left[\sigma_X\left(\eta+\varepsilon_X\right)+\theta_N\varepsilon_X\left(\eta-\sigma_X\right)-\theta_K\left(\eta-\sigma_X\right)\sigma_X\right]\hat{w}^j+\left(\eta-1\right)\left(\sigma_X+\varepsilon_X\right)\hat{A}_X^j+\left(\eta-\sigma_X\right)\theta_L\hat{L}_{0X}^j}{\eta\theta_L+\sigma_X\left(1-\theta_L\right)+\varepsilon_X}$$

The coefficient on wages increases with η, meaning labor demand is more elastic when product demand is elastic. If we take $\varepsilon_X = 1$, then a value of $\eta = 4$ produces a labor demand elasticity of -3.2, while $\eta = \infty$ produces a an elasticity of -22. We also see that wages here rise with the endowment of land, comparable to a fixed capital, as in Glaeser and Gottlieb (2008). A number of papers estimate the relationship between immigration-induced (total)

Panel A of Figure 1 illustrates how general equilibrium elasticities of labor supply and demand vary with elasticities of substitution in consumption and production, assumed to be equal ($\sigma_D = \sigma_X = \sigma_Y \equiv \sigma$). When substitution responses are shut down, $\sigma = 0$, labor supply is perfectly inelastic, and labor demand has an elasticity of -2.15, due only to income effects. The structural substitution elasticities have large impacts on the demand and supply elasticities.

4.2 Local Housing Supply and Demand

A city's housing stock is closely tied to population and density, with the difference due to substitution and income effects in consumption:

$$\hat{Y}^j = \hat{N}^j - s_x \sigma_D \hat{p}^j - \hat{Q}^j = 6.19\hat{Q}^j + 2.41 s_x \hat{A}_X^j + 8.20 s_y \hat{A}_Y^j \tag{29}$$

Relative to population, housing responds less to quality-of-life and trade-productivity and more to home-productivity. The same relationship holds when considering housing and population for a given supply of land.

Two potential demand shifts may trace out a housing supply curve. The elasticity generally is greater if quality-of-life rather than trade-productivity shifts demand:

$$\left.\frac{\partial \hat{Y}}{\partial \hat{p}}\right|_{\hat{A}_X, \hat{A}_Y} = \frac{\partial \hat{Y}/\partial \hat{Q}}{\partial \hat{p}/\partial \hat{Q}} \approx -0.09 - 0.13\sigma_D + 0.77\sigma_X + 3.15\sigma_Y + 4.66\varepsilon_{L,r} \tag{30a}$$

$$\left.\frac{\partial \hat{Y}}{\partial \hat{p}}\right|_{\hat{Q}, \hat{A}_Y} = \frac{\partial \hat{Y}/\partial \hat{A}_X}{\partial \hat{p}/\partial \hat{A}_X} \approx -0.13\sigma_D + 0.29\sigma_X + 1.28\sigma_Y + 2.50\varepsilon_{L,r} \tag{30b}$$

These equal 2.44 and 0.96 when $\varepsilon_{L,r} = 0$. These numbers reflect a standard production response through σ_Y. Supply also expands from land growth on the extensive margin, through $\varepsilon_{L,r}$, as more land is incorporated into the city, and on the intensive margin through σ_X, with land released from the traded-good sector. The elasticities also incorporate reductions in household demand, seen

labor supply changes and (average) wage changes. In fact, this is the closest empirical analog to $(\partial \hat{N}/\partial \hat{w})|_{\hat{A}_X, \hat{A}_Y}$. However, results from such regressions vary widely, as discussed by Borjas (1999).

in the negative constant and coefficient on σ_D. Assuming σ_Y or $\varepsilon_{L,r}$ vary considerably due to geography or land restrictions, these general-equilibrium elasticity formulae are consistent with the range of estimates seen in Malpezzi et al. (2005) and Saiz (2010), for different cities. The two formulae also point out that source of the demand shock can greatly affect the measured elasticity, as the underlying parameters differ.[14]

Shifts in supply due to home-productivity arguably identify metro-level housing demand curves. Higher home-productivity greatly increases the amount of housing, while lowering prices slightly:

$$\left.\frac{\partial \hat{Y}}{\partial \hat{p}}\right|_{\hat{Q},\hat{A}_X} = \frac{\partial \hat{Y}/\partial \hat{A}_Y}{\partial \hat{p}/\partial \hat{A}_Y} \approx -4.48 - 0.13\sigma_D - 3.69\sigma_X - 15.12\sigma_Y - 22.36\varepsilon_{L,r}$$

When $\varepsilon_{L,r} = 0$, this is -17.11. Increasing the supply of housing stock requires a greater number of workers to build, maintain, and refresh this stock, which increases the demand for land and housing. This suggests that improvements to housing productivity, such as from reducing regulations, will be seen much more in quantities than prices.

Panel B of Figure 1 illustrates how general equilibrium elasticities of housing supply and demand vary with elasticities of substitution in consumption and production. As elasticities of substitution increase, the difference between housing supply elasticities identified by quality-of-life and trade-productivity grows.

[14] Consider again a partial equilibrium setting with fixed wages and a segmented land market with $\hat{L}_Y^j = \hat{L}_{0Y}^j + \varepsilon_Y \hat{r}_Y^j$. Then the supply of housing is increasing in prices and productivity and land endowments, and falling in wages:

$$\hat{Y}^j = \frac{\sigma_X (1-\phi_L) + \varepsilon_Y}{\phi_L}\hat{p}^j + \left[1 + \frac{\sigma_X (1-\phi_L) + \varepsilon_Y}{\phi_L}\right]\hat{A}_Y^j - (\varepsilon_Y + \sigma_Y)\frac{\phi_N}{\phi_L}\hat{w}^j + \hat{L}_{0Y}^j$$

Parametrized, $\hat{Y} = (2.2 + 4.3\varepsilon_Y)\,\hat{p}^j - (1.8 + 2.6\varepsilon_Y)\,\hat{w}^j + (3.2 + 4.3\varepsilon_Y)\,\hat{A}_Y^j$ The base coefficient of 2.2 is similar to many estimates, however, the formula highlights the role of land supply in ε_Y and $\hat{L}_{0Y}^j$, productivity in $\hat{A}_Y^j$, as well as local costs in $\hat{w}^j$ in determining supply. Local wages play a particular role as $\hat{Q}^j$ and $\hat{A}_Y^j$ lower wages, while $\hat{A}_X^j$ raises them. As covered in Appendix C.5, with heterogeneous preferences, the total elasticity, net of demand, is lower (Aura and Davidoff, 2008).

5 The Relationship between Density, Prices, and Amenities

5.1 Data

We define cities at the Metropolitan Statistical Area (MSA) level using 1999 Office of Management and Budget consolidated definitions (e.g., San Francisco is combined with Oakland and San Jose), of which there are 276. We use the 5-percent sample of the 2000 United States Census from Ruggles et al. (2004) to calculate wage and housing price differentials, controlling for relevant covariates (see Appendix E for details). Population density is calculated from the 2000 Census Summary Tract Files. For each census tract, we take the ratio of population to land area, and then population average these densities to form metro-level densities, shown in figure 2. We use MSA population weights throughout.

Figure 3 displays estimated densities of wage, housing price, and density series across MSAs. Here we see that population density varies by an order of magnitude more than wages and prices.

5.2 Predicting and Explaining Population Density

We first consider how well the model predicts population density using price information alone. As in Albouy (2016), we use estimates of $\hat{Q}^j$ and $\hat{A}_X^j$ based on $\hat{w}^j$ and $\hat{p}^j$, from equations (1) and (23) assuming $\hat{A}_Y^j = 0$. With the parametrized reduced-form elasticities, predicted population density is simply $\varepsilon_{N_*,Q}\hat{Q}^j + \varepsilon_{N_*,A_X}\hat{A}_X^j$. We denote the specification error $\xi^j = \hat{N}_*^j - \varepsilon_{N_*,Q}\hat{Q}^j - \varepsilon_{N_*,A_X}\hat{A}_X^j$.

Figure 4 plots actual and predicted density for the 276 MSAs along with a 45 degree line. Overall, 49 percent of density variation is explained by the restricted neoclassical model without fitting a single parameter.[15] The restricted model underpredicts density for a number of large, relatively old cities — such as New York, Chicago, and Philadelphia — as well as large Texan metros — including Houston, Dallas, and Austin. The model overpredicts density for a number of metros in California and Florida, including San Francisco and Naples. Figure 3 shows that the

[15] We assess model fit by reporting the square of a linear correlation coefficient, from a linear fit with an imposed slope of one.

restricted model underestimates density in the tails of the distribution.

To see if other elasticities of substitution fit the data better, we consider how well combinations of σ_D, σ_X, and σ_Y predict density. Figure 5 graphs the variance of the prediction error, $Var(\xi^j)$, as a function of these elasticities. If we restrict $\sigma_D = \sigma_X = \sigma_Y = \sigma$, $Var(\xi^j)$ is minimized at $\sigma = 0.710$, very close to the value of 0.667 from the pre-set parametrization. The common Cobb-Douglas case $\sigma = 1$ fits notably worse. Fixing $\sigma_X = 0.667$ reduces $Var(\xi^j)$ for all other values of $\sigma_D = \sigma_Y$. Fixing both $\sigma_D = \sigma_X = 0.667$, as in the lowest curve, reduces $Var(\xi^j)$ by roughly the same amount. The greatest reduction comes from setting $\sigma_Y = 0.667$, underlining the importance of housing in accommodating population responses to differences in amenities.

5.3 Using Density to Estimate Trade and Home Productivity

We next relax the restriction $\hat{A}_Y^j = 0$ by using density data to separately identify trade and home-productivity, as described in Section 2.6. Panel A of Figure 6 displays estimated measures of inferred cost and excess density (relative to quality-of-life) for MSAs from the left hand sides of equations (23) and (24) under the parametrization with $\sigma_D = \sigma_X = \sigma_Y = 0.667$. The figure includes iso-productivity lines for both traded and home sectors.

To understand the estimates, consider the downward-sloping iso-trade-productivity line, along which cities have average trade-productivity. Above and to the right of this line, cities have higher excess density or inferred costs, indicating above-average trade-productivity. Above and to the left of the upward-sloping iso-home-productivity line, cities have high excess density or low inferred costs, indicating high home-productivity. Vertical deviations from this line equal what we called specification error ξ^j in section 5.2. Since the first line is almost vertical, and the second almost horizontal, excess density, or specification error for N_*^j, has a small impact on trade-productivity measures and a large impact on home-productivity measures. The slopes increase with the structural substitution elasticities, as the effects of either productivity on density increases.

Panel B of Figure 6 graphs trade and home-productivity directly, through a change in coordinates of Panel A. Examining each quadrant in turn, Chicago and Philadelphia have high levels of both trade and home-productivity, while New York is the most productive overall. San Francisco has the highest trade-productivity, but low home-productivity. San Antonio has low trade-productivity and high home-productivity. Santa Fe and Myrtle Beach are unproductive in both sectors.[16]

Home-productivity estimates deserve several comments. First, they strongly reflect density measures and weakly reflect prices.[17] Second, the relative dependence of the home-productivity estimate on inferred costs relative to excess density increases with σ_Y. Third, home-productivity is strongest in large, older cities. While this may be specification error, the core of these cities were largely built prior to World War I, when most land-use regulations were absent. Thus, their high densities may have grandfathered in high home-productivities of a former time.[18]

To summarize the data and findings, Table 4 contains estimates of population density, wages, housing costs, inferred land values, and attribute differentials for a selected sample of metropolitan areas. Table A.2 contains a full list of metropolitan and non-metropolitan areas and compares inferred costs with trade-productivity estimates.

5.4 City-Specific Elasticities of Substitution

Because of heterogeneous geographic and regulatory environments, the ability of housing producers to substitute between land, labor and capital may vary considerably across cities. This

[16]Panel B of Figure 6 also includes isoclines for excess density and inferred costs, which correspond to the axes in Panel A. Holding quality-of-life constant, trade-productivity and home-productivity must move in opposite directions to keep population density constant. Holding quality-of-life constant, home-productivity must rise faster than trade-productivity to keep inferred costs constant.

[17]According to the parametrization, $\hat{A}_Y^j \approx 0.32\hat{N}_*^j + 0.73\hat{w}^j - 0.93\hat{p}^j$, which largely reflects density since density varies so greatly and prices and wages are positively correlated. Trade-productivity is $\hat{A}_X^j \approx 0.03\hat{N}_*^j + 0.84\hat{w}^j + 0.01\hat{p}^j$. Quality-of-life depends only on the price measures: $\hat{Q}^j \approx -0.48\hat{w}^j + 0.33\hat{p}^j$. Land values reflect all three measures positively, $\hat{r}^j \approx 1.37\hat{N}_*^j + 0.49\hat{w}^j + 0.32\hat{p}^j$, although density is key. See Appendix Table A.3.

[18]Albouy and Ehrlich (2016) use data on land values to infer productivity in the housing sector, which comprises most of the non-traded sector. While the two approaches generally agree on which large areas have high home-productivity, the land values approach suggests that larger, denser cities generally have lower, rather than higher housing productivity. This apparent contradiction actually highlights what the two methodologies infer differently. Productivity measures based on current land values provide a better insight into the marginal cost of increasing the housing supply, by essentially inferring the replacement cost. Productivity measures based on density are more strongly related to the average cost of the housing supply, thereby reflecting the whole history of building in a city. The distinction matters particularly for older cities where older housing was built on the easiest terrain, and in decades prior strict residential land-use regulations, which typically grandfather pre-existing buildings.

heterogeneity is of direct interest and can impact the model's ability to explain location decisions. To proceed, we assume that σ_Y^j is a linear function of the Wharton Residential Land Use Regulatory Index (WRLURI) from Gyourko et al. (2006), denoted by I^j, the average slope of land from Albouy et al. (2016), denoted by S^j, and a residual: $\sigma_Y^j = \sigma_{Y0} + \sigma_{YI} I^j + \sigma_{YS} S^j + v^j$. We normalize I^j and S^j to have mean zero and standard deviation one. We assume that home-productivity is also a linear function of these observed variables and a residual: $\hat{A}_Y^j = a_I I^j + a_S S^j + u^j$. As shown in Appendix F, these assumptions yield the following equation:

$$\begin{aligned}\hat{N}_e^j =& \sigma_{Y0}\hat{G}^j + \sigma_{YI} I^j \hat{G}^j + \sigma_{YS} S^j \hat{G}^j + a^I(k_1 + \sigma_{Y0}k_2) I^j + a_S(k_1 + \sigma_{Y0}k_2) S^j \\ &+ \sigma_{YI} a_I k_2 (I^j)^2 + \sigma_{YS} a_S k_2 (S^j)^2 + (\sigma_{YI} a_S + \sigma_{YS} a_I) k_2 I^j S^j + e^j, \qquad (31)\end{aligned}$$

where $\hat{N}_e^j \equiv \hat{N}_*^j - 1.00\hat{p}^j + 0.77\hat{w}^j$ is density explained by all but σ_Y^j and $\hat{A}_Y^j$, $\hat{G}^j \equiv 2.82\hat{p}^j - 2.37\hat{w}^j$ captures observable demand shifts from $\hat{Q}^j$ and $\hat{A}_X^j$, k_1 and k_2 are known positive constants, and e^j is a residual. We can consistently estimate the parameters of equation (31) using non-linear least squares under orthogonality conditions for u^j and v^j discussed in Appendix F.

The estimator here differs from competing estimates of labor and housing supply in several ways. First, it is identified from level differences in population, not changes. Second, as implied by (30a) and (30b), it handles demand shifts asymmetrically, putting more weight on quality of life than trade-productivity. Third, it accounts for *all* demand shifts, absent specification error. This eliminates the need for instrumenting demand shifts (e.g. with January temperature or Bartik employment shares) that are ostensibly exogenous to supply. The only remaining concern is that S and I are correlated with unobserved supply shifters in v^j or w^j. As a way of testing the specification, the model provides three over-identifying restrictions. The linear reduced-form equation of (31) has eight terms $\{\hat{G}^j, I^j, S^j, I^j\hat{G}^j, S^j\hat{G}^j, I^jS^j, (I^j)^2, (S^j)^2\}$, with coefficients that depend non-linearly on the five structural parameters, $\{\sigma_{Y0}, \sigma_{YI}, \sigma_{YS}, a_I, a_S\}$.

We do not reject the implied structural restrictions of the model ($p = 0.13$), providing support for our estimates of (31), shown in table 5. In column 2, $\hat{A}_Y^j = 0$, but σ_Y^j varies and is negatively related to both regulations and average slope — corresponding to intuition – with a one standard-deviation increase in either measure reducing the elasticity by roughly 0.3. The predicted elasticities σ_Y^j have a mean of 0.93, higher than without the interactions, with a standard deviation of 0.51. This model explains 62 percent of density variation, an improvement over the 49 percent explained with uniform σ_Y. Column 3 holds σ_Y^j constant and lets $\hat{A}_Y^j$ vary, finding it falls by about 7 percent with a one standard deviation increase in slope. Column 4 presents the full model and produces results consistent with columns 2 and 3.

Estimates of σ_Y^j imply city-specific elasticities of housing supply according to the formulae from section 4. For comparision, we calculate these assuming demand variation from trade-productivity and constant geography ($\varepsilon_{L,r} = 0$), with $\sigma_D = \sigma_X = 0.667$ and σ_Y^j as the predicted value from column 3 of Table 5. A regression of the supply elasticities from Saiz (2010) on our elasticities yields a slope of 0.95 (s.e. 0.15) and an intercept of 0.34 (s.e. 0.21), with a correlation coefficient of 0.52. The slope is indistinguishable from one, and the intercept is close to the value predicted in (29) from the consumption response, $s_x\sigma_D = 0.43$, due to Saiz using data on population, N, rather than housing, Y.[19] The similarity is remarkable as given how different his estimation strategy is from ours.

6 Land Area and the Total Population of Cities

The neoclassical model does a fairly good job of explaining density. Yet, to explain a metro's full population, it must also model land area, which varies tremendously across metro areas. The model as delineated by Rosen and Roback takes land as homogeneous — abstracting away from the internal structure of cities — and supply as exogenous. We add a simple land supply function from equation (15), which depends on an unknown, and possibly heterogeneous, land endowment,

[19]Saiz's empirical strategy examines temporal variation using industrial composition, immigrant enclaves, and sunshine as sources of exogenous variation in demand. By combining quality-of-life and productivity shifters, the estimates may not be directly comparable, although we suspect that productivity shifters are more important in his analysis. Saks (2008) also estimates lower elasticities in more regulated markets, focusing on labor supply, although her results are not as comparable.

$\hat{L}_0^j$, and supply elasticity, $\varepsilon_{L,r}^j$. To our knowledge, we are the first to try to estimate both intensive (density) and extensive (land) margins of urban growth separately. The neoclassical model allows this type of estimation because of its separable structure with homogeneous land.

For estimation, we model these as linear functions of covariates X^j, with $\hat{L}_0^j = X^j\beta_{L_0} + u^j$ and $\varepsilon_{L,r}^j = \bar{\varepsilon} + X^j\beta_{\varepsilon} + v^j$. X^j includes I^j, S^j, and also the log land share (i.e., the share which is not water) from Saiz (2010). We measure land using the number of square miles in the Census urban area; metropolitan areas, defined by counties, contain a considerable amount of land for non-urban use, which we exclude. Panel A of Figure 7 plots land area against the land rent inferred from (3) when $\hat{A}_Y^j = 0$, i.e., $\hat{r}^j = (\hat{p}^j - \phi_N\hat{w}^j)/\phi_L$. Since cities are small and open to mobile labor and capital, the demand for land is perfectly elastic at each city's price $\hat{r}^j$. The slope of the regression line then provides a supply elasticity, given here by $\bar{\varepsilon} = 0.82$ with no other covariates.

Table 6 reports results from the full specification (summary statistics are in appendix table A.4). A one standard deviation increase in slope lowers the land endowment by almost a half, while a one standard deviation in land not covered by water increases it by almost a quarter. In the fully interacted model, the average elasticity of land is 1.4 but is reduced by about 0.25 from a one standard deviation increase in slope and regulation. While these results are not as well identified as those in table 5, they do accord with intuition, suggesting that the land measure and inferred land rents do contain valuable information.

To examine how well the model explains cross-metro population differences, we use equation (17) to predict the total population differential as the sum of the predicted land differential $\hat{L}^j$, conditional on $\hat{r}^j$, I^j, and S^j, from column 2, and the simple predicted density differential, conditional on $\hat{p}^j$ and $\hat{w}^j$. This prediction explains 53 percent of cross-metro population variation, without using data on either density or population. For the neoclassical model built on price theory to value amenities, this seems rather successful.

7 Population Determinants and Counterfactual Exercises

7.1 Why Do People Live Where They Do?

To answer the question of whether people follow jobs or jobs follow people, we use simple variance decompositions to measure the relative importance of quality-of-life, trade-productivity, and home-productivity in explaining cross-metro differences in density and population. Column 1 of Table 7 considers the restricted model of population density with constant home-productivity $\hat{A}_Y = 0$, and uniform substitution elasticities $\sigma = 0.667$, to keep the accounting parsimonious. Quality-of-life accounts for nearly half of the explained variance, dominating trade-productivity (i.e., inferred costs), even though the latter shows greater cross-sectional variation in value (see Appendix Figure A.2). Quality-of-life and trade-productivity are positively correlated.

In the model allowing A_Y to vary across metros, column 2 decomposes the variance of observed (which now equals predicted) population density across all three attributes. As before, quality-of-life dominates trade-productivity, yet both are dominated by home-productivity. While all three attributes are important in explaining density, it appears that people and jobs follow housing more than anything else. Given the residual nature of the home-productivity measure, this conclusion should be treated with caution, but it complements the finding that heterogeneous substitution in housing production is key to explaining the responsiveness of population to amenities.

The decompositions in columns 3 and 4 bring in land supply to account for total population. To keep the accounting tractable, we use the specification from column 2 of table 6, with a uniform price elasticity of 1.12 for land, and allow base land endowments to vary. In column 3, we see quality-of-life continues to dominate trade-productivity, while both dominate the land endowment. Finally, column 4 considers the full model for population. As in column 2, home-productivity dominates quality-of-life and trade-productivity. The largest interaction is the positive one between home and trade-productivity.

One of the more stimulating results from this table is that quality of life is negatively related to land and home productivity. It seems rather unfortunate that many of the most attractive areas of the United States are difficult to build on. This appears to stem mainly from two causes. First, coastlines and rugged terrain are associated with higher quality of life (Albouy 2008) but lower land supply and ability to build densely, as shown above. Second, higher quality of life areas tend to have more land-use restrictions although these restrictions do not actually improve quality of life by very much (Albouy and Ehrlich 2016). Of course, the equilibrium model ignores that people are gradually moving to areas with nicer weather (Rappaport 2007, Glaeser and Tobio 2008).

Appendix Table A.6 explores how the results are affected by endogenous amenity feedback and non-neutral federal taxation. Feedback reinforces the role of natural advantages in quality-of-life, as the observed values are reduced through congestion. On the other hand, natural advantages in trade-productivity are less important, as they are created partly from other amenities that cause agglomeration. On the policy side, if federal taxes were made neutral, trade-productivity would determine locations more than quality of life; people would follow jobs more than the opposite.[20]

7.2 What if Chicago was as Nice as San Diego?

As quality of life is so important in determining where people live, we consider what would happen if the city with the largest growth potential, Chicago, were given the quality of life of one of America's nicest cities, San Diego. In this counter-factual, Chicago receives none of the other determinants that lower San Diego's population. According to the estimates seen in Tables 5 and 8, Chicago has a very elastic home-good sector, with $\sigma_Y = 1.39$, which from (26), implies $\hat{N}_* = 13.96\hat{Q}^j$. The difference in quality of life between San Diego and Chicago is 0.11, explained entirely by climate and geography (Albouy 2008), making it "exogenous" in a sense. Therefore, the model predicts that the population of Chicago would expand by a factor $e^{13.96(0.11)} - 1 = 3.64$ times. Based on the 2000 numbers, this implies a population of 43 million, double that of New York

[20]In particular, we use our amenity estimates and parametrized model to predict prices and quantities (including population density) for each city in the absence of location-distorting federal income taxes. Because we estimate amenities using observed density, wage, and housing price data, we cannot estimate amenities in the absence of distortionary federal taxes.

City![21] A sunny and beautiful "City of Broad Shoulders" would be full of gleaming skyscrapers, packed with residents.

On the other hand, if we were to reduce San Diego's quality of life to that of Chicago's, the long-run effect would be much less dramatic, given how unresponsive its home-good sector is. With $\sigma_Y = 0.13$, we have $\hat{N}_* = 3.88\hat{Q}^j$, so that its population would fall by 37 percent, from 2.8 to 1.8 million.

7.3 The Effect of Relaxing Land-Use Regulations and Neutralizing Taxes

The parametrized model readily permits nationwide counter-factual policy exercises. Below, we consider two possibilities. One is to lower land-use regulations in cities for inhabitants with above-average regulation. This is somewhat similar to Hsieh and Moretti (2015) — who lower regulations more dramatically — although we examine levels instead of growth.[22] The second is to neutralize tax differences — similar to Albouy (2009), but with heterogeneous home-good supply according to estimated σ_Y^j. Most interestingly, we combine the two reforms to envision what more "ideal" American cities would look like based on their amenities.

Table 8 presents results from these counter-factual exercises. Column 2 shows the estimated elasticity of substitution in housing, and column 3 shows the predicted elasticity when lowering land-use regulations in cities with above-average regulation. Column 5 shows the impact of lowering land-use regulations on population (the resulting population is the product of columns 1 and 5). The elasticities in several coastal cities, notably San Francisco, Los Angeles, and San Diego grow substantially, permitting many more people to take advantage of their amenities. Because the population must balance, less attractive cities, such as Detroit, Atlanta, and Dallas, lose population regardless of changes in their elasticity. As seen in Panels B and C, the West would gain population from the South and Midwest, and people would live in more amenable and productive places.

[21] A change of this kind would increase the welfare of the country, which would lower the population increase by about 10 percent.

[22] We use the regulation experienced by a median *inhabitant*, who lives in a metro of 2.6 million. Hsieh and Moretti (2015) lower regulation to that of the median *city*, half a standard deviation lower (in our data), corresponding to a metro with 0.8 million.

Column 4 shows the federal tax differential paid by residents of each city, which is driven by above-average wages. As seen in column 6, neutralizing federal taxes increases population levels in cities with both high federal tax burdens and elastic home supply. New York, Detroit, and Chicago are the biggest gainers from this reform. This reform would draw the population towards the Northeast, and in the most productive cities more generally.

Making both reforms would dramatically alter the urban landscape, as seen in columns 7 and 8. San Francisco would more than double in size and surpass Chicago as the third largest metro. New York would eclipse Tokyo as the largest city in the world. In general, many of the largest cities, amenable to both households and firms — such as Boston and Los Angeles — would grow substantially. Most minor cities, such as St. Louis and Cleveland would shrink, although the changes depend on more than just city size. In all, this would cause the Northeast and West to gain population, and the South to lose.

8 Conclusion

The neoclassical model provides intuitive micro-foundations for explaining urban population, using a familiar framework based on price theory. The off-the shelf parametrized model fits the data surprisingly well, revealing that location choices are driven more by quality of life than by jobs, although both are only possible with housing. Agglomeration effects, as modeled here, reinforce this conclusion. However we show that in this class of model, they lack the magnitude for multiple equilibria or path dependence.

Our econometric estimates provide even more compelling evidence that the model produces meaningful predictions, which link together cross-sectional population differences with large estimated elasticities of labor and housing supply. Using simple data and insights from general-equilibrium modeling, we find that geography and regulations influence both the intensive, density margin of urbanization, as well as the extensive, land margin. This both reinforces and clarifies existing work. Moreover, our city-specific analysis gives us better insight into meanings of particular numbers, and allows us to make precise predictions about urban population. A small change in quality of life can lead to a large change in population, especially in cities with permissive building environments, such as Chicago. Our counter-factual predictions of city size with neutral federal taxation and moderated land-use restrictions, imply that Boston, San Francisco, and other amenable cities should be considerably larger, while many other could be effectively abandoned.

The neoclassical framework accounts for the most basic factors that affect urban life. It is remarkably versatile for adding features, such as agglomeration, multiple types and preference heterogeneity. At the same time, the flexible form we examine, which shies away from simplifications — such as unit or zero elasticities, or iso-elastic housing supplies — may reveal alternative models are under-identified without non-linearities. We hope this paper will help to unify the disparate literature on urban population and amenities and help push it forward.

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Table 1: U.S. Constant Geography Parametrization

Parameter Name	Notation	Value
Cost and Expenditure Shares		
Home good expenditure share	s_y	0.36
Income share to land	s_R	0.10
Income share to labor	s_w	0.75
Traded good cost share of land	θ_L	0.025
Traded good cost share of labor	θ_N	0.825
Home good cost share of land	ϕ_L	0.233
Home good cost share of labor	ϕ_N	0.617
Share of land used in traded good	λ_L	0.17
Share of labor used in traded good	λ_N	0.70
Tax Parameters		
Average marginal tax rate	τ	0.361
Average deduction level	δ	0.291
Structural Elasticities		
Elasticity of substitution in consumption	σ_D	0.667
Elasticity of traded good production	σ_X	0.667
Elasticity of home good production	σ_Y	0.667
Elasticity of land supply	$\varepsilon_{L,r}$	0.0

Parametrization pre-set in Albouy (2009). See Appendix D for details.

Table 2: Relationship between Reduced-Form and Structural Elasticities, Population and Housing

A: Reduced-Form Population Elasticity with Respect to:			
	Quality of Life	trade-productivity	Home Productivity
	$\varepsilon_{N,Q}$	ε_{N,A_X}	ε_{N,A_Y}
σ_D	1.141	0.719	-0.077
σ_X	1.951	0.468	0.636
σ_Y	8.004	2.056	2.607
$\varepsilon_{L,r}$	11.837	4.016	3.856
Constant	0.773	0.000	0.773
B: Reduced-Form Housing Elasticity with Respect to:			
	Quality of Life	trade-productivity	Home Productivity
	$\varepsilon_{Y,Q}$	ε_{Y,A_X}	ε_{Y,A_Y}
σ_D	-0.336	-0.212	0.023
σ_X	1.951	0.468	0.636
σ_Y	8.004	2.056	2.607
$\varepsilon_{L,r}$	11.837	4.016	3.856
Constant	-0.227	0.000	0.773

Table 2 decomposes reduced-form elasticities into substitution elasticities in consumption (σ_D), traded good production (σ_X), home good production (σ_Y), and the elasticity of land supply ($\varepsilon_{L,r}$). For example, the reduced-form elasticity of population with respect to quality-of-life is $\varepsilon_{N,Q} = 0.773 + 1.141\sigma_D + 1.951\sigma_X + 8.004\sigma_Y + 11.837\varepsilon_{L,r}$.

Table 3: Parametrized Relationship between Amenities, Prices, and Quantities

		A: With Taxes (current regime)			B: Neutral Taxes (counterfactual)		
		Quality of Life	Trade Productivity	Home Productivity	Quality of Life	Trade Productivity	Home Productivity
Price/quantity	Notation	$\hat{Q}$	$\hat{A}_X$	$\hat{A}_Y$	$\hat{Q}$	$\hat{A}_X$	$\hat{A}_Y$
Land value	$\hat{r}$	11.837	4.016	3.856	10.001	6.400	3.600
Wage	$\hat{w}$	-0.359	1.090	-0.117	-0.303	1.018	-0.109
Home price	$\hat{p}$	2.540	1.609	-0.172	2.146	2.121	-0.227
Trade consumption	$\hat{x}$	-0.446	0.349	-0.037	-0.916	-0.905	0.097
Home consumption	$\hat{y}$	-1.985	-0.621	0.067	0.515	0.509	-0.055
Population density	$\hat{N}$	8.175	2.164	2.884	6.319	3.721	2.718
Capital	$\hat{K}$	7.931	2.866	2.779	6.182	4.385	2.616
Land	$\hat{L}$	0.000	0.000	0.000	0.000	0.000	0.000
Trade production	$\hat{X}$	7.957	3.339	2.934	5.815	4.805	2.777
Home production	$\hat{Y}$	6.189	1.543	2.951	5.402	2.816	2.815
Trade labor	$\hat{N}_X$	8.196	2.279	3.012	6.017	3.793	2.850
Home labor	$\hat{N}_Y$	8.123	1.889	2.581	7.036	3.551	2.403
Trade capital	$\hat{K}_X$	7.957	3.006	2.934	5.815	4.472	2.777
Home capital	$\hat{K}_Y$	7.884	2.616	2.503	6.834	4.230	2.330
Trade land	$\hat{L}_X$	0.061	0.328	0.362	-0.856	0.203	0.376
Home land	$\hat{L}_Y$	-0.012	-0.062	-0.069	0.163	-0.039	-0.072

Each value in Table 3 represents the partial effect that a one-point increase in each amenity has on each price or quantity, e.g., $\hat{N}^j = 8.175\hat{Q}^j + 2.164\hat{A}_X^j + 2.884\hat{A}_Y^j$ under the current U.S. tax regime. Values in panel A are derived using the parameters in Table 1. Values in panel B are derived using geographically neutral taxes. All variables are measured in log differences from the national average.

Table 4: List of Selected Metropolitan Areas, Ranked by Population Density

Name of Metropolitan Area	Population Density $\hat{N}^j$	Wage $\hat{w}^j$	Home Price $\hat{p}^j$	Land Value $\hat{r}^j$	Quality of Life $\hat{Q}^j$	Trade Productivity $\hat{A}_X^j$	Home Productivity $\hat{A}_Y^j$
New York, Northern New Jersey, Long Island, NY-NJ-CT-PA	2.294	0.217	0.430	3.405	0.031	0.272	0.504
Honolulu, HI	1.302	-0.012	0.614	1.953	0.208	0.039	-0.166
Los Angeles-Riverside-Orange County, CA	1.258	0.134	0.452	1.946	0.080	0.163	0.088
San Francisco-Oakland-San Jose, CA	1.218	0.259	0.815	2.050	0.137	0.273	-0.171
Chicago-Gary-Kenosha, IL-IN-WI	1.200	0.134	0.227	1.789	0.007	0.160	0.276
Miami-Fort Lauderdale, FL	0.972	0.010	0.124	1.372	0.036	0.043	0.202
Philadelphia-Wilmington-Atlantic City, PA-NJ-DE-MD	0.967	0.115	0.059	1.409	-0.038	0.134	0.343
San Diego, CA	0.881	0.061	0.483	1.439	0.122	0.088	-0.108
Salinas (Monterey-Carmel), CA	0.847	0.102	0.600	1.443	0.141	0.123	-0.198
Boston-Worcester-Lawrence, MA-NH-ME-CT	0.806	0.121	0.341	1.278	0.050	0.136	0.035
...							
Myrtle Beach, SC	-1.393	-0.188	-0.128	-2.026	0.050	-0.212	-0.466
Florence, SC	-1.397	-0.140	-0.339	-2.115	-0.039	-0.173	-0.244
Johnson City-Kingsport-Bristol, TN-VA	-1.409	-0.190	-0.354	-2.149	-0.020	-0.217	-0.269
Gadsden, AL	-1.437	-0.136	-0.424	-2.194	-0.068	-0.172	-0.175
Goldsboro, NC	-1.500	-0.197	-0.291	-2.229	0.002	-0.225	-0.356
Dothan, AL	-1.524	-0.189	-0.406	-2.323	-0.037	-0.220	-0.257
Anniston, AL	-1.570	-0.202	-0.428	-2.399	-0.038	-0.234	-0.262
Ocala, FL	-1.573	-0.170	-0.298	-2.364	-0.010	-0.205	-0.363
Hickory-Morganton-Lenoir, NC	-1.615	-0.135	-0.222	-2.358	-0.005	-0.175	-0.414
Rocky Mount, NC	-1.631	-0.122	-0.243	-2.386	-0.018	-0.165	-0.392
Standard Deviation	0.870	0.116	0.283	1.322	0.052	0.129	0.200

Table 4 includes the top and bottom ten metropolitan areas ranked by population density. The first three columns are estimated from Census data, while the last four columns come from the parametrized model. See text for estimation procedure. Standard deviations are calculated among the 276 metropolitan areas using metro population weights. All variables are measured in log differences from the national average.

Table 5: The Determinants of Substitution Possibilities and Productivity in the Home Sector

Dependent variable: Population density not explained by home sector		(1)	(2)	(3)	(4)
Elasticity of Substitution in Home Sector					
Baseline	σ_{Y0}	0.693***	0.934***	0.861***	1.068***
		(0.247)	(0.261)	(0.327)	(0.342)
Wharton Land-Use Regulatory Index (s.d.)	σ_I		-0.309***		-0.289**
			(0.0855)		(0.129)
Average slope of land (s.d.)	σ_S		-0.335*		-0.279
			(0.189)		(0.177)
Housing Productivity					
Wharton Land-Use Regulatory Index (s.d.)	a_I			0.0163	-0.00362
				(0.0380)	(0.0165)
Average slope of land (s.d.)	a_S			-0.0715***	-0.0527***
				(0.0178)	(0.0188)
Observations		274	274	274	274

Table 5 presents results of estimating equation (31) by nonlinear least squares. All explanatory variables are normalized to have mean zero and standard deviation one. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 6: The Determinants of Land Supply

Dependent variable: Log urban area, square miles	(1)	(2)	(3)
Inferred land rent	0.816***	1.123***	1.397***
	(0.185)	(0.219)	(0.151)
Wharton Land-Use Regulatory Index (s.d.)		0.134	-0.0469
		(0.0990)	(0.0929)
Average slope of land (s.d.)		-0.641***	-0.563***
		(0.110)	(0.0850)
Log land share (s.d.)		0.223**	0.261***
		(0.106)	(0.0833)
Interaction between inferred land rent and			
Wharton Land-Use Regulatory Index (s.d.)			-0.248***
			(0.0922)
Average slope of land (s.d.)			-0.234**
			(0.0936)
Log land share (s.d.)			0.104
			(0.116)
Constant	6.634***	6.702***	6.965***
	(0.145)	(0.109)	(0.10)
Observations	276	227	227
R-squared	0.353	0.522	0.596

Inferred land rent is constructed without using density data. All explanatory variables are normalized to have mean zero and standard deviation one. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 7: Fraction of Density and Population Explained by Quality of Life, Trade Productivity, Home Productivity, and Land

Variance/Covariance Component	Notation	Density		Population	
		(1)	(2)	(3)	(4)
Quality-of-life	Var$(\varepsilon_{N,Q}\hat{Q})$	0.501	0.238	0.577	0.225
Trade-productivity	Var$(\varepsilon_{N,A_X}\hat{A}_X)$	0.183	0.103	0.295	0.136
Home-productivity	Var$(\varepsilon_{N,A_Y}\hat{A}_Y)$	-	0.439	-	0.390
Land	Var$(\hat{L}_0)$	-	-	0.164	0.064
Quality-of-life and trade-productivity	Cov$(\varepsilon_{N,Q}\hat{Q}, \varepsilon_{N,A_X}\hat{A}_X)$	0.315	0.137	0.447	0.161
Quality-of-life and home-productivity	Cov$(\varepsilon_{N,Q}\hat{Q}, \varepsilon_{N,A_Y}\hat{A}_Y)$	-	-0.153	-	-0.135
Quality-of-life and land	Cov$(\varepsilon_{N,Q}\hat{Q}, \hat{L}_0)$	-	-	-0.389	-0.152
Trade and home-productivity	Cov$(\varepsilon_{N,A_X}\hat{A}_X, \varepsilon_{N,A_Y}\hat{A}_Y)$	-	0.236	-	0.254
Trade-productivity and land	Cov$(\varepsilon_{N,A_X}\hat{A}_X, \hat{L}_0)$	-	-	-0.095	-0.029
Home-productivity and land	Cov$(\varepsilon_{N,A_Y}\hat{A}_Y, \hat{L}_0)$	-	-	-	0.084
Total variance of prediction		0.359	0.757	2.075	5.322
Data used to construct attributes					
Wages and housing prices		Yes	Yes	Yes	Yes
Density		No	Yes	No	Yes
Predicted land intercept		No	No	Yes	Yes

Predicted land intercepts come from column 3 of table 6 and do not include the interactions between inferred land rent and explanatory variables.

Table 8: Changes in Population from Relaxing Land-Use Regulations and Neutralizing Federal Taxes, Allowing Both Density and Land Supply to Change

Panel A: Metro-Level	Pop. in	Home Subs. Elastic. σ_Y		Fed.	Relative Pop. with Reform			Pop. Under
	2000 Mill. (1)	Esti-mated (2)	Lower Regul. (3)	Tax Diff (4)	Lower Regul. (5)	Neut. Tax (6)	Both Refs (7)	Both Reforms (8)
Main city in MSA								
San Francisco	7.0	0.13	0.41	0.05	1.57	1.41	2.29	16.1
New York	21.2	0.95	1.03	0.05	1.03	1.66	1.69	35.9
Los Angeles	16.4	0.33	0.58	0.02	1.29	1.10	1.40	22.9
Detroit	5.5	1.23	1.23	0.03	0.93	1.50	1.36	7.4
Boston	5.8	0.52	1.02	0.02	1.15	1.15	1.35	7.9
Philadelphia	6.2	0.81	1.11	0.03	1.02	1.27	1.30	8.1
Chicago	9.2	1.39	1.39	0.03	0.93	1.42	1.30	11.9
Washington-Baltimore	7.6	0.85	0.99	0.03	0.94	1.32	1.24	9.4
San Diego	2.8	0.12	0.47	0.00	1.46	0.85	1.18	3.3
Houston	4.7	1.33	1.33	0.02	0.93	1.21	1.10	5.2
Atlanta	4.1	0.99	1.04	0.02	0.90	1.22	1.08	4.4
Minneapolis	3.0	1.06	1.12	0.02	0.92	1.20	1.08	3.2
Dallas	5.2	1.31	1.31	0.02	0.93	1.12	1.02	5.3
Seattle	3.5	0.00	0.28	0.01	1.05	0.97	1.00	3.5
Denver	2.6	0.27	0.77	0.01	1.06	0.94	0.97	2.5
Phoenix	3.3	0.54	0.88	0.01	0.95	0.90	0.82	2.7
St. Louis	2.6	1.57	1.57	0.01	0.93	0.89	0.81	2.1
Cleveland	3.0	1.21	1.21	0.01	0.93	0.88	0.80	2.4
Miami	3.9	1.03	1.30	0.00	0.99	0.81	0.76	3.0

Panel B: Effect on Regional Distribution

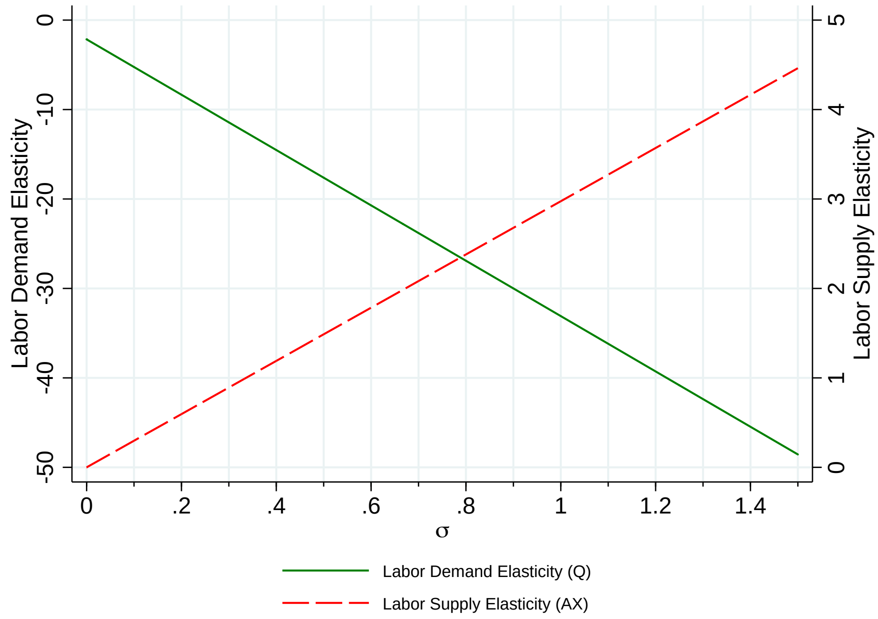
	Relative Pop.		
Census Region	Lower Regul. (1)	Neut. Tax (2)	Comb. Both (3)
Northeast	1. 01	1.28	1.20
Midwest	0.93	1.02	0.92
South	0.92	0.81	0.72
West	1.16	1.01	1.20

Panel C: Change in Amenity Dist.

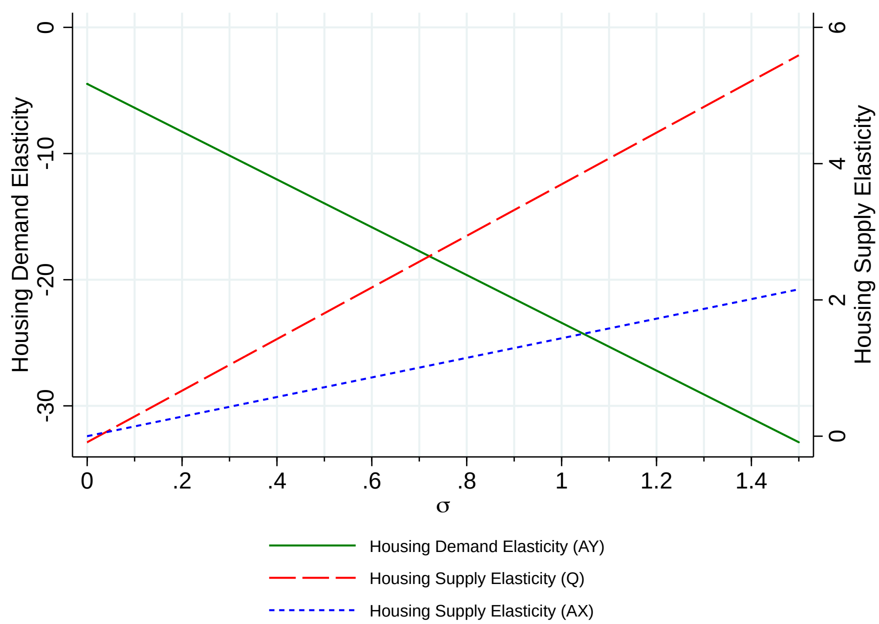
	Amenity Change		
Amenity Type	Lower Regul. (1)	Neut. Tax (2)	Comb. Both (3)
Quality of Life	0.006	0.004	0.011
Trade-Product.	0.013	0.043	0.057
Home-Product.	0.011	0.030	0.042
Total Value	0.018	0.042	0.063

Estimated home substitution elasticity from column 2 of Table 5. Lower WRLURI reduces those with WRLURI above the average to the population-weighted mean. Federal tax differential from Albouy (2009) determined by wage level times marginal tax rate, minus discounts for owner-occupied houisng. Elasticity of land supply given 0.77 from Table 6. The first counterfactual exercise raises the home substitution elasticity in high WRLURI cities. The second counterfactual exercise neutralizes the effect of federal taxes.

Figure 1: Relationship between Labor and Housing General Equilibrium Elasticities and Substitution Possibilities



(a) Labor



(b) Housing

Panel (a) displays $\partial \hat{N}/\partial \hat{w}$, where the change in both density and wages is due to a change in the indicated amenity, as a function of the substitution elasticity $\sigma_D = \sigma_X = \sigma_Y \equiv \sigma$. Panel (b) displays similar results for the elasticity of housing with respect to housing prices.

Figure 2: Metropolitan Population Density, Thousands per Square Mile, 2000

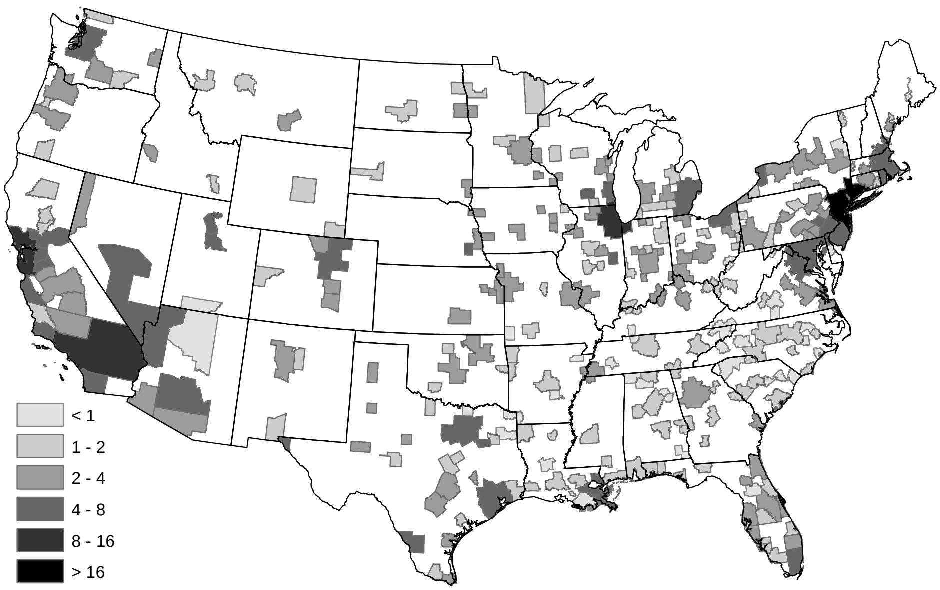
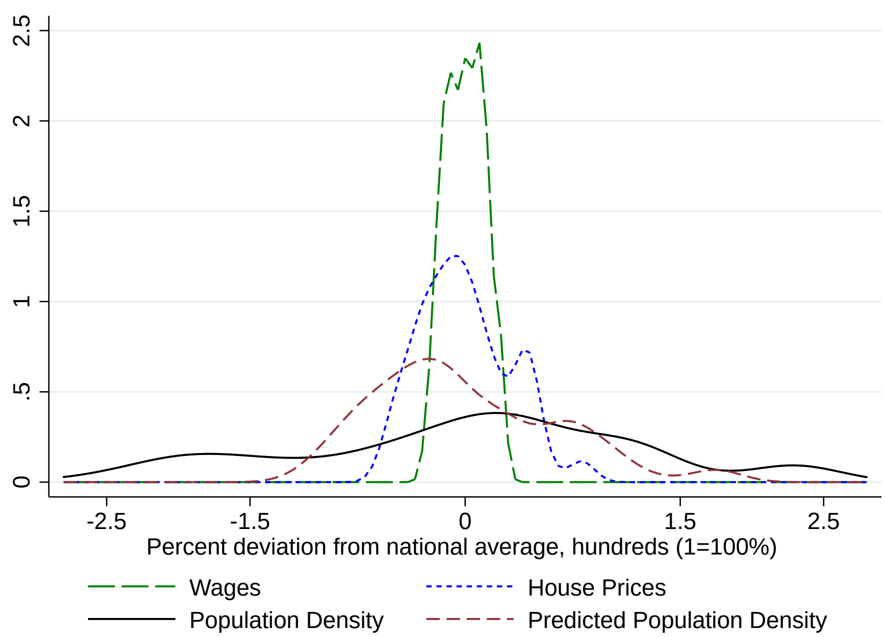
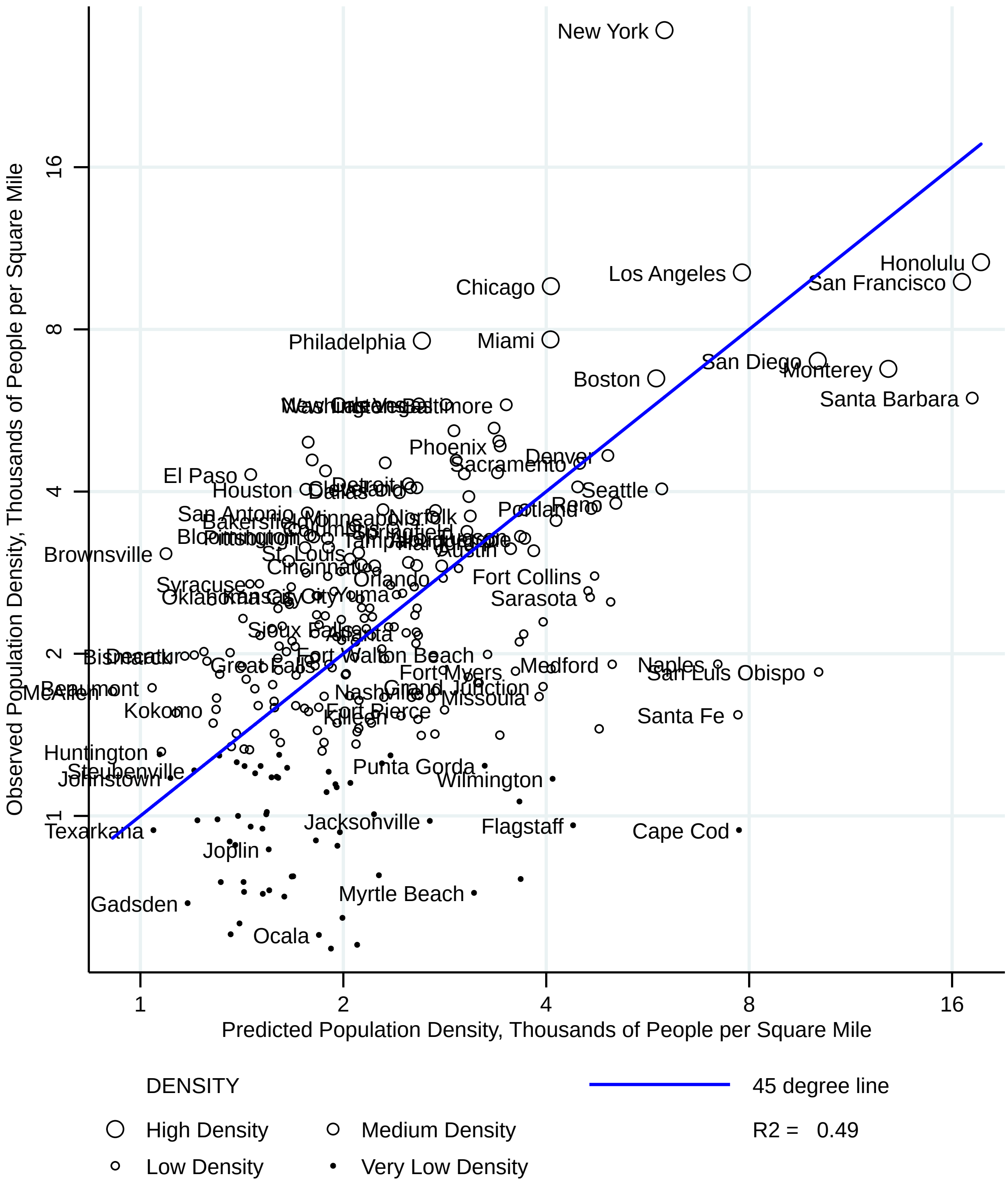


Figure 3: Distribution of Wages, House Prices, and Population Density, 2000



Predicted population density, calculated under the assumption of equal home-productivity across metros, depends only on wages and housing prices.

Figure 4: Actual and Predicted Population Density, 2000



See text for estimation details. High density metros have population density which exceeds the national average by 80 percent, medium density metros are between the national average and 80 percent. Low density and very low density metros are defined symmetrically.

Figure 5: Variance of Error in Fitting Population Density using Quality of Life and Trade-productivity, as Function of Substitution Elasticities

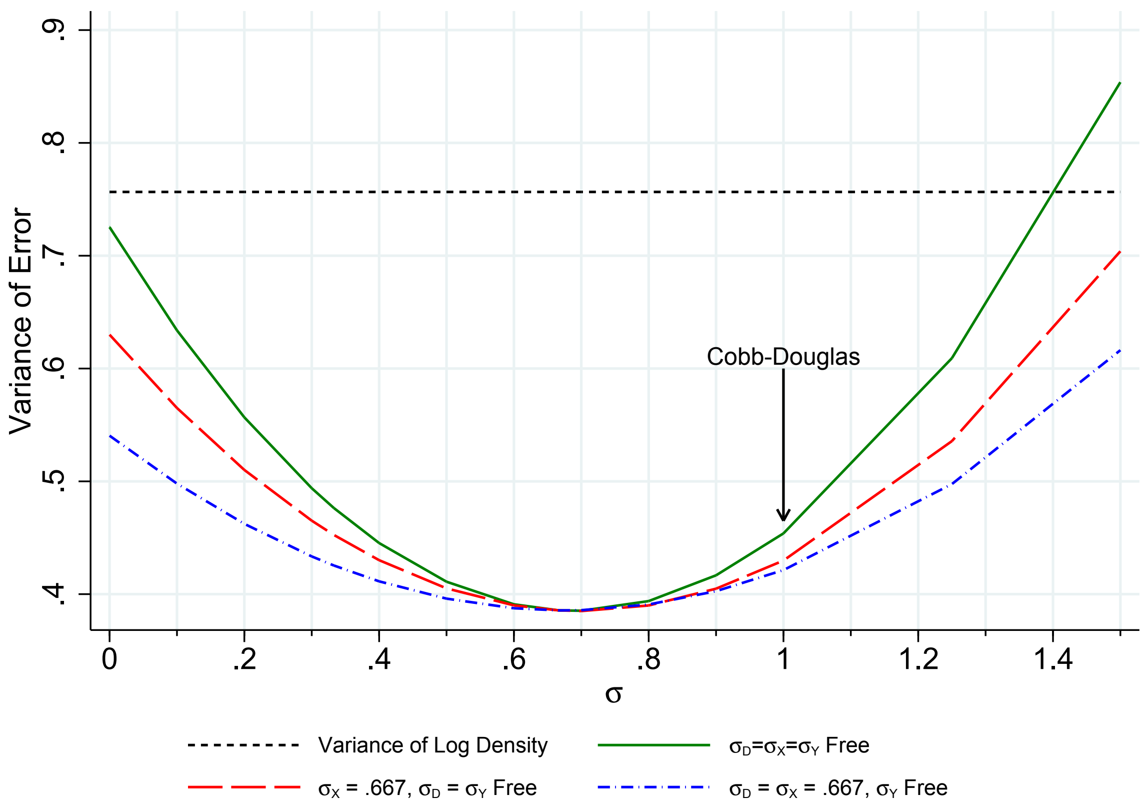
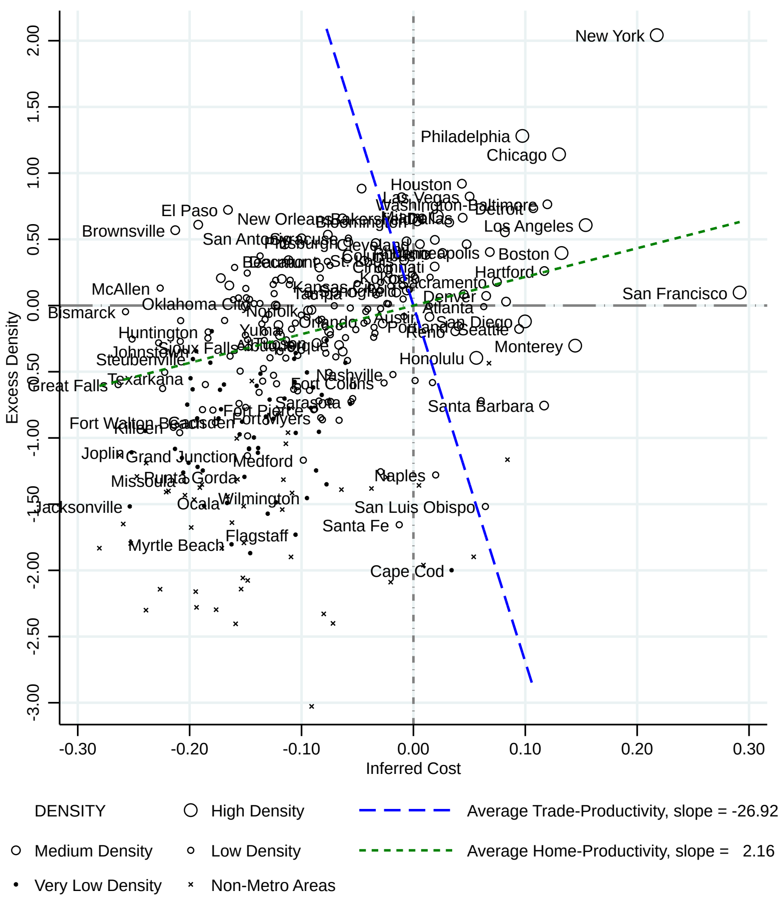
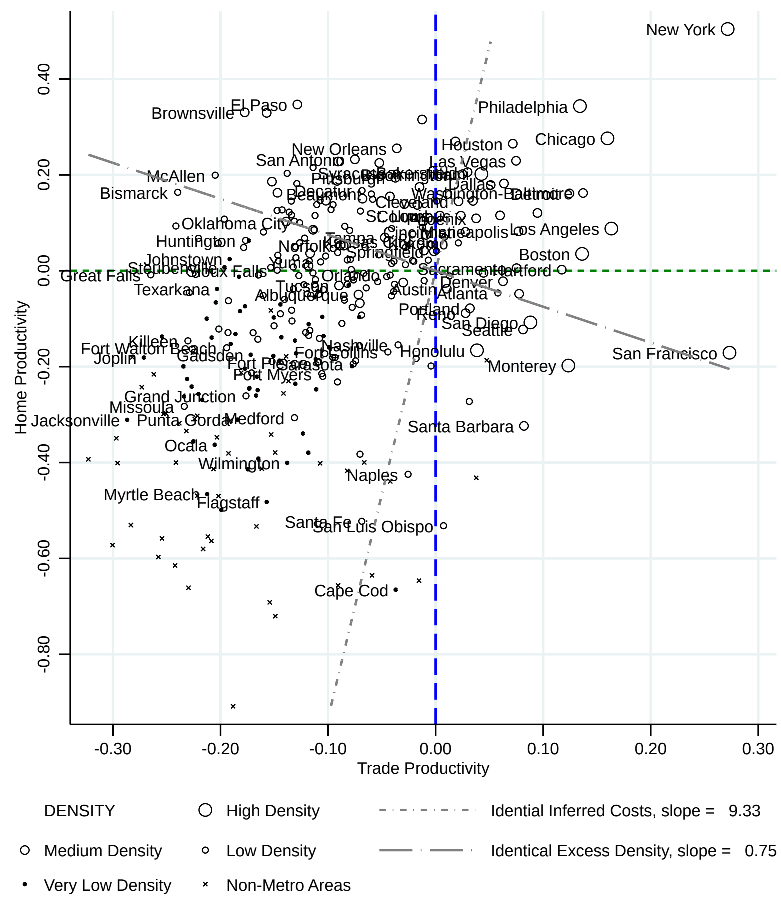


Figure 6: Results of Parametrized Model, 2000



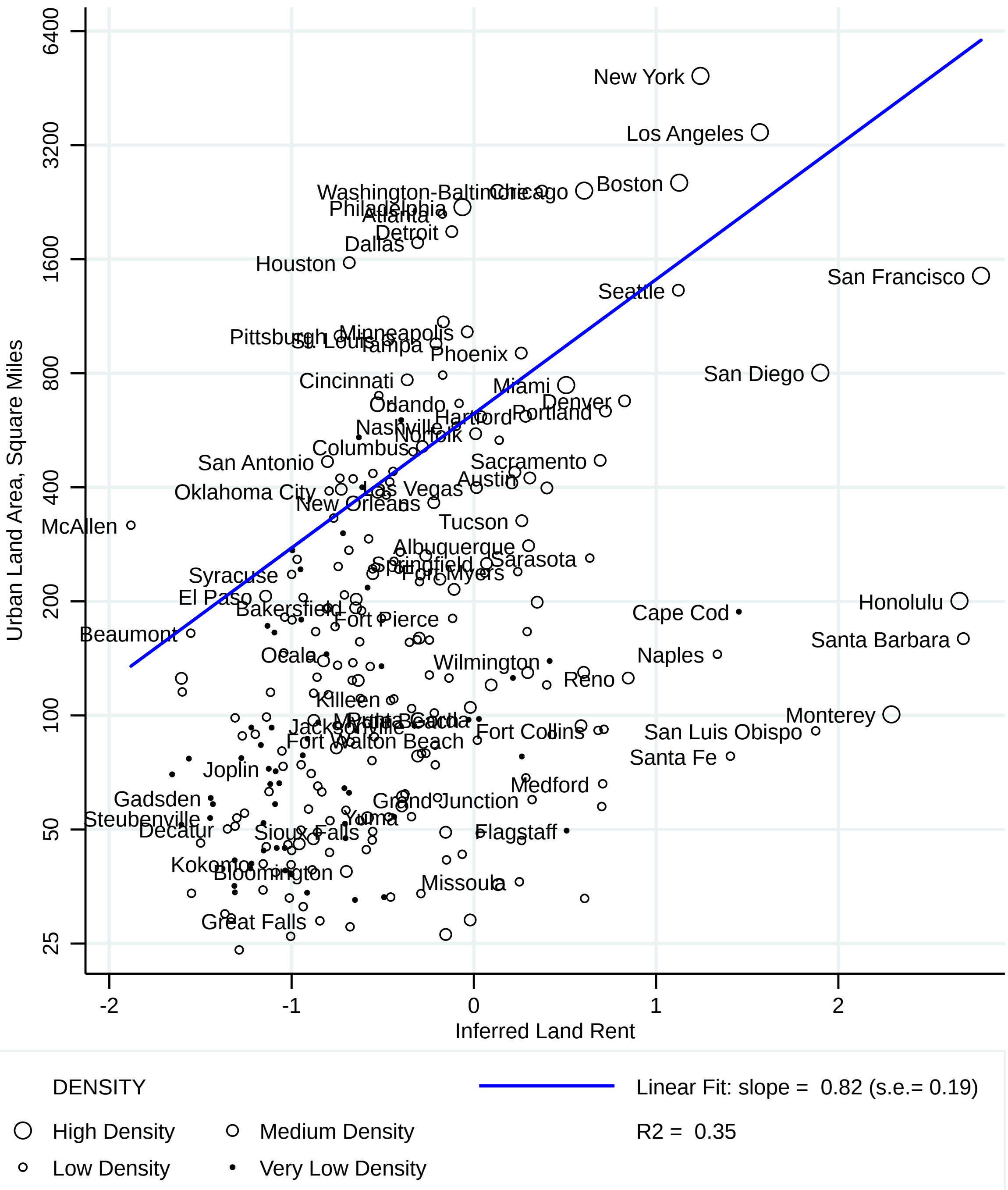
(a) Excess Density and Inferred Cost Estimates



(b) Trade and Home Productivity Estimates

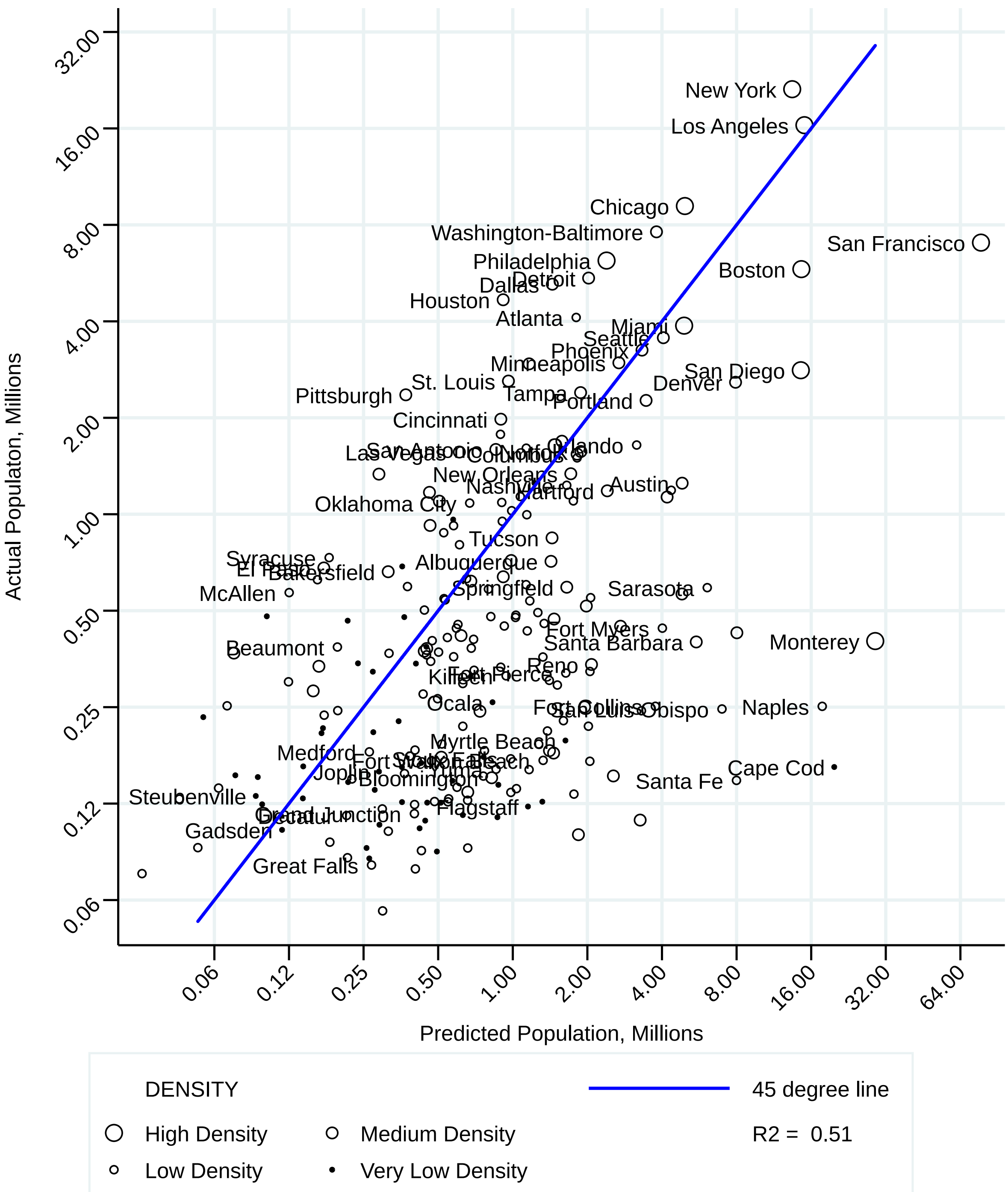
See note to figure 4 for metro density definitions.

Figure 7: Urban Land Area and Inferred Land Rents



See note to figure 4 for metro density definitions.

Figure 8: Actual and Predicted Population



See note to figure 4 for metro density definitions.

Appendix - For Online Publication

A Full Nonlinear Model

This appendix lists the 16 nonlinear equilibrium conditions used to drive the log-linearized conditions discussed in the text.

Equilibrium Price Conditions:

$$e(p^j, \bar{u})/Q^j = (1-\tau)(w^j + R + I) + T \tag{1*}$$
$$c_X(r^j, w^j, \bar{\imath})/A_X^j = 1 \tag{2*}$$
$$c_Y(r^j, w^j, \bar{\imath})/A_Y^j = p^j \tag{3*}$$

Consumption Conditions:

$$x^j + p^j y^j = (1-\tau)(w^j + R + I) + T \tag{4*}$$
$$(\partial U/\partial y) \,/\, (\partial U/\partial x) = p^j \tag{5*}$$

Production Conditions:

$$\partial c_X/\partial w = A_X^j N_X^j / X^j \tag{6*}$$
$$\partial c_X/\partial r = A_X^j L_X^j / X^j \tag{7*}$$
$$\partial c_X/\partial i = A_X^j K_X^j / X^j \tag{8*}$$
$$\partial c_Y/\partial w = A_Y^j N_Y^j / Y^j \tag{9*}$$
$$\partial c_Y/\partial r = A_Y^j L_Y^j / Y^j \tag{10*}$$
$$\partial c_Y/\partial i = A_Y^j K_Y^j / Y^j \tag{11*}$$

Local Resource Constraints:

$$N^j = N_X^j + N_Y^j \tag{12*}$$
$$L^j = L_X^j + L_Y^j \tag{13*}$$
$$K^j = K_X^j + K_Y^j \tag{14*}$$

Land Supply:

$$L^j = L_0^j \tilde{L}(r^j) \tag{15*}$$

Home Market Clearing:

$$Y^j = N^j y^j \tag{16*}$$

B Comparison of Nonlinear and Log-linear Models

To assess the error introduced by log-linearizing the model, we employ a two-step simulation method to solve a nonlinear version of the model.[23] We assume that utility and production functions display constant elasticity of substitution,

$$U(x,y;Q) = Q(\eta_x x^\alpha + (1-\eta_x)y^\alpha)^{1/\alpha}$$
$$F_X(L_X,N_X,K_X;A_X) = A_X(\gamma_L L^\beta + \gamma_N N^\beta + (1-\gamma_L-\gamma_N)K^\beta)^{1/\beta}$$
$$F_Y(L_Y,N_Y,K_Y;A_Y) = A_Y(\rho_L L^\chi + \rho_N N^\chi + (1-\rho_L-\rho_N)K^\chi)^{1/\chi}$$

where

$$\alpha \equiv \frac{\sigma_D - 1}{\sigma_D}$$
$$\beta \equiv \frac{\sigma_X - 1}{\sigma_X}$$
$$\chi \equiv \frac{\sigma_Y - 1}{\sigma_Y}$$

Throughout, we assume that $\sigma_D = \sigma_X = \sigma_Y = 0.667$. We first consider a "large" city with attribute values normalized so that $Q = A_X = A_Y = 1$. We fix land supply, population, and the rental price of capital $\bar{\iota}$. We then solve a nonlinear system of fifteen equations, corresponding to equations (1*)-(14*) and (16*), for fifteen unknown variables: $(\bar{u}, w, r, p, x, y, X, Y, N_X, N_Y, L_X, L_Y, K_X, K_Y, K)$. We simultaneously choose values of $(\eta_x, \gamma_L, \gamma_N, \rho_L, \rho_N)$ so that the model matches values of $(s_y, \theta_L, \theta_N, \phi_L, \phi_N)$ in Table 1. The large city solution also yields values for (R, I, T).[24]

We then consider a "small" city, which we endow with land equal to one one-millionth of the large city's land.[25] The population for the small city is endogenous, and the reference utility level $\bar{u}$ is exogenous. The baseline attribute values of the small city are $Q = A_X = A_Y = 1$. While holding two attributes fixed at the baseline, we solve the model after setting the third attribute to be somewhere between 0.8 and 1.2. We solve the same system as for the large city, but now solve for $(w, r, p, x, y, X, Y, N_X, N_Y, L_X, L_Y, K_X, K_Y, N, K)$.

For comparison, we simulate a one-city log-linear model using parameter values from Table 1, but set the marginal tax rate $\tau = 0$ and deduction level $\delta = 0$. The baseline attribute differences are $\hat{Q} = \hat{A}_X = \hat{A}_Y = 0$. As with the nonlinear model, we vary a single attribute while holding the other amenities at their baseline value. We can express the entire log-linear system of equations (1)-(16):

[23] Rappaport (2008a, 2008b) follows a similar procedure.

[24] To simulate the model, we solve a mathematical program with equilibrium constraints, as described in Su and Judd (2012).

[25] We do this to avoid any feedback effects from the small city to the large one. In particular, this permits use of values of $\bar{u}, \bar{\iota}, R, I$, and T from the large city calibration, which simplifies the procedure considerably.

$$
\left[\begin{array}{cccccccccccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -\theta_L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -\phi_L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & s_x & s_y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 1 & 0 & -\theta_L\sigma_X & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & (1-\theta_L)\sigma_X & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -\theta_L\sigma_X & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -\phi_L\sigma_Y & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & (1-\phi_L)\sigma_Y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & -\phi_L\sigma_Y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & \lambda_N & 0 & 0 & 0 & 1-\lambda_N & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_L & 0 & 0 & 0 & 1-\lambda_L & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_K & 0 & 0 & 0 & 1-\lambda_K & 0 & 0 & -1 \\
0 & 0 & 0 & \varepsilon_{L,r} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{array}\right]
\left[\begin{array}{c}
\hat{Q} \\ \hat{A}_X \\ \hat{A}_Y \\ \hat{r} \\ \hat{x} \\ \hat{y} \\ \hat{N}_X \\ \hat{L}_X \\ \hat{K}_X \\ \hat{X} \\ \hat{N}_Y \\ \hat{L}_X \\ \hat{K}_X \\ \hat{Y} \\ \hat{L} \\ \hat{K}
\end{array}\right]
=
\left[\begin{array}{c}
-s_w(1-\tau)\hat{w} + s_y\hat{p} \\
\theta_N\hat{w} \\
\phi_N\hat{w} - \hat{p} \\
s_w(1-\tau)\hat{w} - s_y\hat{p} \\
\sigma_D\hat{p} \\
-(1-\theta_N)\sigma_X\hat{w} \\
\theta_N\sigma_X\hat{w} \\
\theta_N\sigma_X\hat{w} \\
-(1-\phi_N)\sigma_Y\hat{w} \\
\phi_N\sigma_Y\hat{w} \\
\phi_N\sigma_Y\hat{w} \\
\hat{N} \\
0 \\
0 \\
0 \\
\hat{N}
\end{array}\right]
$$

or, in matrix form, as $\mathbf{Av} = \mathbf{C}$. The first three rows of $\mathbf{A}$ correspond to price equations, the second two to consumption conditions, the next six to factor demand equations, and the final five to market clearing conditions. The above form demonstrates that, given a parametrization and data on wages, home prices, and population, the matrices $\mathbf{A}$ and $\mathbf{C}$ are known, so we can solve the above system for the unknown parameters $\mathbf{v}$. In our simulation, we use a slightly different formulation, where the right hand side vector consists only of known attribute differentials. Figure A.3 presents results of both models in terms of reduced-form population elasticities with respect to each amenity.[26] The log-linear model does quite well in approximating density responses to trade and home-productivity differences of up to 20-percent, and approximates responses to quality-of-life quite well for differences of up to 5-percent, the relevant range of estimates for U.S. data in Figure A.2.

C Additional Theoretical Details

C.1 Reduced-Form Elasticities

The analytic solutions for reduced-form elasticities of population with respect to amenities are given below.

$$
\begin{aligned}
\varepsilon_{N,Q} = {} & \left[\frac{\lambda_N - \lambda_L}{\lambda_N}\right] + \sigma_D\left[\frac{s_x(\lambda_N-\lambda_L)^2}{s_y\lambda_N(\lambda_N-\lambda_L\tau)}\right] + \sigma_X\left[\frac{\lambda_L^2}{s_w(\lambda_N-\lambda_L\tau)} + \frac{\lambda_L\lambda_N}{s_R(\lambda_N-\lambda_L\tau)}\right] \\
& + \sigma_Y\left[\frac{\lambda_L^2(1-\lambda_N)}{s_w\lambda_N(\lambda_N-\lambda_L\tau)} + \frac{\lambda_N(1-\lambda_L)}{s_R(\lambda_N-\lambda_L\tau)} - \frac{(\lambda_N-\lambda_L)^2}{s_y\lambda_N(\lambda_N-\lambda_L\tau)}\right] \\
& + \varepsilon_{L,r}\left[\frac{\lambda_N}{s_R(\lambda_N-\lambda_L\tau)}\right]
\end{aligned}
$$

[26] We normalize the elasticities in Figure A.3 for trade and home-productivity by s_x and s_y.

$$\varepsilon_{N,A_X} = \sigma_D \left[\frac{s_x^2(\lambda_N - \lambda_L)(1-\lambda_L)(1-\tau)}{s_y \lambda_N(\lambda_N - \lambda_L \tau)}\right] + \sigma_X \left[\frac{s_x \lambda_L(\lambda_N - \tau)}{s_R(\lambda_N - \lambda_L \tau)} - \frac{s_x \lambda_L (1-\lambda_L)}{s_w(\lambda_N - \lambda_L \tau)}\right] +$$
$$\sigma_Y \left[\frac{s_x(1-\lambda_L)(\lambda_N - \tau)}{s_R(\lambda_N - \lambda_L \tau)} - \frac{s_x \lambda_L (1-\lambda_L)(1-\lambda_N)}{s_w \lambda_N(\lambda_N - \lambda_L \tau)} - \frac{s_x(1-\lambda_L)(\lambda_N - \lambda_L)(1-\tau)}{s_y \lambda_N(\lambda_N - \lambda_L \tau)}\right]$$
$$+ \varepsilon_{L,r}\left[\frac{s_x(\lambda_N - \tau)}{s_R(\lambda_N - \lambda_L \tau)}\right]$$

$$\varepsilon_{N,A_Y} = \left[\frac{\lambda_N - \lambda_L}{\lambda_N}\right] + \sigma_D \left[\frac{-s_x \lambda_L(\lambda_N - \lambda_L)(1-\tau)}{\lambda_N(\lambda_N - \lambda_L \tau)}\right] + \sigma_X \left[\frac{s_y \lambda_N \lambda_L}{s_R(\lambda_N - \lambda_L \tau)} + \frac{s_y \lambda_L^2}{s_w(\lambda_N - \lambda_L \tau)}\right]$$
$$+ \sigma_Y \left[-\left(\frac{\lambda_N - \lambda_L}{\lambda_N}\right) + \frac{s_y \lambda_L^2(1-\lambda_N)}{s_w \lambda_N(\lambda_N - \lambda_L \tau)} + \frac{s_y \lambda_N(1-\lambda_L)}{s_R(\lambda_N - \lambda_L \tau)} + \frac{\lambda_L(\lambda_N - \lambda_L)(1-\tau)}{\lambda_N(\lambda_N - \lambda_L \tau)}\right]$$
$$+ \varepsilon_{L,r}\left[\frac{s_y \lambda_N}{s_R(\lambda_N - \lambda_L \tau)}\right]$$

C.2 Special Case: Fixed Per-Capita Housing Consumption

Consider the case in which per-capita housing consumption is fixed, $\hat{y}^j = 0$. The model then yields $\hat{N}^j = \tilde{\varepsilon}_{N,Q}\hat{Q}^j + \tilde{\varepsilon}_{N,A_X}\hat{A}_X^j + \tilde{\varepsilon}_{N,A_Y}\hat{A}_Y^j$, where the coefficients are defined as:

$$\tilde{\varepsilon}_{N,Q} = \sigma_X \left[\frac{\lambda_L^2}{s_w(\lambda_N - \lambda_L \tau)} + \frac{\lambda_L \lambda_N}{s_R(\lambda_N - \lambda_L \tau)}\right] + \varepsilon_{L,r}\left[\frac{\lambda_N}{s_R(\lambda_N - \lambda_L \tau)}\right]$$
$$+ \sigma_Y \left[\frac{\lambda_L^2(1-\lambda_N)}{s_w \lambda_N(\lambda_N - \lambda_L \tau)} + \frac{\lambda_N(1-\lambda_L)}{s_R(\lambda_N - \lambda_L \tau)} - \frac{(\lambda_N - \lambda_L)^2}{s_y \lambda_N(\lambda_N - \lambda_L \tau)}\right]$$
$$\tilde{\varepsilon}_{N,A_X} = \sigma_X \left[\frac{s_x \lambda_L(\lambda_N - \tau)}{s_R(\lambda_N - \lambda_L \tau)} - \frac{s_x \lambda_L(1-\lambda_L)}{s_w(\lambda_N - \lambda_L \tau)}\right] + \varepsilon_{L,r}\left[\frac{s_x(\lambda_N - \tau)}{s_R(\lambda_N - \lambda_L \tau)}\right]$$
$$+ \sigma_Y \left[\frac{s_x(1-\lambda_L)(\lambda_N - \tau)}{s_R(\lambda_N - \lambda_L \tau)} - \frac{s_x \lambda_L(1-\lambda_L)(1-\lambda_N)}{s_w \lambda_N(\lambda_N - \lambda_L \tau)} - \frac{s_x(1-\lambda_L)(\lambda_N - \lambda_L)(1-\tau)}{s_y \lambda_N(\lambda_N - \lambda_L \tau)}\right]$$
$$\tilde{\varepsilon}_{N,A_Y} = \sigma_X \left[\frac{s_y \lambda_N \lambda_L}{s_R(\lambda_N - \lambda_L \tau)} + \frac{s_y \lambda_L^2}{s_w(\lambda_N - \lambda_L \tau)}\right] + \varepsilon_{L,r}\left[\frac{s_y \lambda_N}{s_R(\lambda_N - \lambda_L \tau)}\right]$$
$$+ \sigma_Y \left[\frac{s_y \lambda_L^2(1-\lambda_N)}{s_w \lambda_N(\lambda_N - \lambda_L \tau)} + \frac{s_y \lambda_N(1-\lambda_L)}{s_R(\lambda_N - \lambda_L \tau)} + \frac{\lambda_L(\lambda_n - \lambda_L)(1-\tau)}{\lambda_N(\lambda_N - \lambda_L \tau)}\right]$$

These reduced-form elasticities no longer depend on the elasticity of substitution in consumption σ_D. In addition, above-average quality-of-life and/or home-productivity no longer lead to higher population independently of the substitution elasticities, as seen by the term $(\lambda_N - \lambda_L)/\lambda_N$ dropping out of the elasticities.

C.3 Deduction

Tax deductions are applied to the consumption of home goods at the rate $\delta \in [0, 1]$, so that the tax payment is given by $\tau(m - \delta py)$. With the deduction, the mobility condition becomes

$$\begin{aligned}\hat{Q}^j &= (1 - \delta\tau')s_y\hat{p}^j - (1 - \tau')s_w\hat{w}^j \\ &= s_y\hat{p}^j - s_w\hat{w}^j + \frac{d\tau^j}{m}\end{aligned}$$

where the tax differential is given by $d\tau^j/m = \tau'(s_w\hat{w}^j - \delta s_y p^j)$. This differential can be solved by noting

$$\begin{aligned}s_w\hat{w}^j &= s_w\hat{w}_0^j + \frac{\lambda_L}{\lambda_N}\frac{d\tau^j}{m} \\ s_y\hat{p}^j &= s_y\hat{p}_0^j - \left(1 - \frac{\lambda_L}{\lambda_N}\right)\frac{d\tau^j}{m}\end{aligned}$$

and substituting them into the tax differential formula, and solving recursively,

$$\begin{aligned}\frac{d\tau^j}{m} &= \tau' s_w\hat{w}_0^j - \delta\tau' s_y\hat{p}_0^j + \tau'\left[\delta + (1-\delta)\frac{\lambda_L}{\lambda_N}\right] \\ &= \tau'\frac{s_w\hat{w}_0^j - \delta s_y\hat{p}_0^j}{1 - \tau'\left[\delta + (1-\delta)\lambda_L/\lambda_N\right]}\end{aligned}$$

We can then solve for the tax differential in terms of amenities:

$$\frac{d\tau^j}{m} = \tau'\frac{1}{1 - \tau'\left[\delta + (1-\delta)\lambda_L/\lambda_N\right]}\left[(1-\delta)\left(\frac{1-\lambda_L}{\lambda_N}s_x\hat{A}_X^j - \frac{\lambda_L}{\lambda_N}s_y A_Y^j\right) - \frac{(1-\delta)\lambda_L + \delta\lambda_N}{\lambda_N}\hat{Q}^j\right]$$

This equation demonstrates that the deduction reduces the dependence of taxes on productivity and increases the implicit subsidy for quality-of-life.

C.4 State Taxes

The tax differential with state taxes is computed by including an additional component based on wages and prices relative to the state average, as if state tax revenues are redistributed lump-sum to households within the state. This produces the augmented formula

$$\frac{d\tau^j}{m} = \tau'\left(s_w\hat{w}^j - \delta\tau' s_y\hat{p}^j\right) + \tau_S'[s_w(\hat{w}^j - \hat{w}^S) - \delta_S s_y(\hat{p}^j - \hat{p}^S)] \tag{A.1}$$

where τ_S' and δ_S are are marginal tax and deduction rates at the state-level, net of federal deductions, and $\hat{w}^S$ and $\hat{p}^S$ are the differentials for state S as a whole relative to the entire country.

C.5 Imperfect Mobility from Preference Heterogeneity

The model most accurately depicts a long-run equilibrium, for which idiosyncratic preferences or imperfect mobility seem less important. Yet, the model may be appended to include such features, which could be used to rationalize path dependence. Suppose that quality-of-life for household i in metro j equals the product of a common term and a household-specific term, $Q_i^j = \underline{Q}^j \xi_i^j$. In addition, assume that ζ_i^j comes from a Pareto distribution with parameter $1/\psi > 0$, common across metros, and distribution function $F(\zeta_i^j) = 1 - (\underline{\zeta}/\zeta_i^j)^{1/\psi}, \zeta_i^j \geq \underline{\zeta}$. A larger value of ψ corresponds to greater preference heterogeneity; $\psi = 0$ is the baseline value.

For each populated metro, there ezetasts a marginal household, denoted by k, such that

$$\frac{e(p^j, \bar{u})}{\underline{Q}^j \zeta_k^j} = (1-\tau)(w^j + R + I) + T. \tag{A.2}$$

For some fixed constant $N_{\max}^j$, population density in each metro can be written $N^j = N_{\max}^j \Pr[\zeta_i^j \geq \zeta_k^j] = N_{\max}^j (\underline{\zeta}/\zeta_k^j)^{1/\psi}$. Log-linearizing this condition yields $\psi \hat{N}^j = -\hat{\zeta}_k^j$. The larger is ψ, and the greater the population shift $\hat{N}^j$, the greater the preference gap in between supra- and infra-marginal residents. Log-linearizing the definition of Q_k^j yields $\hat{Q}_k^j = \hat{\underline{Q}}^j + \hat{\zeta}_k^j = \hat{Q}_k^j = \hat{\underline{Q}}^j - \psi \hat{N}^j$. Ignoring agglomeration, the relationship between population density and amenities with is now lower

$$\hat{N}^j = \frac{1}{1 + \psi \varepsilon_{N,Q}} \left(\varepsilon_{N,Q} \hat{\underline{Q}}^j + \varepsilon_{N,A_X} \hat{A}_X^j + \varepsilon_{N,A_Y} \hat{A}_Y^j \right)$$

This dampening effect occurs because firms in a city need to be paid incoming migrants an increasing schedule in after-tax real wages to have them overcome their taste differences. With a value of ψ, we may adjust all of the predictions. The comparative statics with imperfect mobility are indistinguishable from congestion effects: ψ and γ are interchangeable. The welfare implications are different as infra-marginal residents share the value of local amenities with land-owners.[27]

[27] Note that log-linearizing equation (A.2) yields $s_w(1-\tau)\hat{w}^j - s_y \hat{p}^j = \psi \hat{N}^j - \hat{\underline{Q}}^j$. It is straightforward to show that the rent elasticities in (18a) is equal to $1/(1 + \psi \varepsilon_{N,Q}) \leq 1$ its previous value. The increase in real income is given by $s_w(1-\tau) d\hat{w}^j - s_y d\hat{p}^j = \psi \hat{N}^j = -s_R d\hat{r}^j$, where "d" denotes price changes between actual and full mobility. The main challenge in operationalizing imperfect mobility is specifying the baseline level of population that deviations $\hat{N}^j$ are taken from, as a baseline of equal density may not be appropriate. Differences in baseline population together with the frictions modeled here, may provide a way of introducing historical path-dependence in the model.

D Parametrization Details

TABLE 1: MODEL PARAMETERS AND CHOSEN VALUES

Parameter	Notation	Parametrized Value
Home-goods share	s_y	0.36
Income share to land	s_R	0.10
Income share to labor	s_w	0.75
Traded-good cost-share of land	θ_L	0.025
Traded-good cost-share of labor	θ_N	0.825
Home-good cost-share of land	ϕ_L	0.233
Home-good cost-share of labor	ϕ_N	0.617
Share of land used in traded good (derived)	λ_L	0.17
Share of labor used in traded good (derived)	λ_N	0.70
Elast. of subs. in consumption	σ_D	0.667
Elast. of subs. in traded production	σ_X	0.667
Elast. of subs. in home production	σ_Y	0.667
Average marginal tax rate	τ'	0.361
Deduction rate for home-goods	δ	0.291
Agglomeration parameter	α	0.060
Congestion parameter	γ	0.015

All but the agglomeration and congestion parameter are chose in Albouy (2009); all but the elasticities of substitution reappear in Albouy (2016).

Ciccone and Hall (1996) estimate an elasticity of labor productivity with respect to population density of 0.06. Rosenthal and Strange (2004) argue that a one percent increase in population leads to no more than a 0.03-0.08 percent increase in productivity. For γ we combine estimated costs of commuting and pollution. First, we estimate an elasticity of transit time with respect to population density of 0.10 (unreported results available by request). Assuming that the elasticity of monetary and after-tax time costs of commuting as a fraction of income is 9 percent, commuting contributes $(0.09)(0.010) \approx 0.009$ to our estimate of γ. Second, Chay and Greenstone (2005) estimate that the elasticity of housing values with respect to total suspended particulates, a measure of air quality, lies between -0.2 and -0.35; we take a middle estimate of -0.3. The Consumer Expenditure Survey reports the gross share of income spent on shelter alone (no utilities) is roughly 0.13. We estimate an elasticity of particulates with respect to population density of 0.15 (unreported results available by request). Together, this implies that the contribution of air quality is $|(0.13)(-0.6)(0.15)| \approx 0.006$. Population density affects quality-of-life through more than commuting and air quality, but if we assume these effects cancel out, then a plausible value of estimate of $\gamma = 0.009 + 0.006 = 0.015$. Estimates from Combes et al. (2012), using data on French cities, suggest a larger value of $\gamma = 0.041$, but their emphasis is on population, not density. See Rosenthal and Strange (2004) and Glaeser and Gottlieb (2008) for recent discussions of issues in estimating agglomeration elasticities.

E Data and Estimation

The wage and housing cost parameters are from the 2000 Integrated Public-Use Microdata Series (IPUMS), from Ruggles et al. (2004), to calculate wage and housing price differentials. They are identical to hose in Albouy (2016). They depend on the logarithm of hourly wages on worker characteristics (education, experience, race, immigrant status, etc.) and indicator variables for each metro area. The population-demeaned coefficients on the indicator variables are taken as the city wage differentials. The regression for housing costs is analagous, controlling for housing characteristics (type and age of structure, number of rooms, etc.) combining gross rents with imputed rents from owner-occupied units. Imputed rents are the sum of utility costs and a user-cost imputed from housing values.

F City-Specific Estimates of Home-Productivity and Substitution

This section derives the equation used to estimate city-specific elasticities of substitution in the housing sector. Recall the parametrized relationship between density and attributes from equation (26) with variable σ_Y in the text. We generalize it here to allow for alternative parametrizations and the specification error ξ^j

$$\hat{N}_*^j = (\varepsilon_{N_*,Q}^0 + d_{Y,Q}\sigma_Y^j)\hat{Q}^j + (\varepsilon_{N_*,A_X}^0 + d_{Y,A_X}\sigma_Y^j)\hat{A}_X^j + (\varepsilon_{N_*,A_X}^0 + d_{Y,A_Y}\sigma_Y^j)\hat{A}_Y^j + \xi^j. \quad \text{(A.3)}$$

$\varepsilon_{N,Q}^0$ is the density elasticity component common across cities, with $\sigma_Y^j = 0$, while $d_{Y,Q}$ is the coefficient on σ_Y^j, parametrized in the third row of Panel A, in Table 2. The remaining notation is similar.

Substituting in equations (1) and (23) we create an equation in terms of the observable $\hat{w}^j$ and $\hat{p}^j$. This involves collecting on the right all terms involving σ_Y^j or $\hat{A}_Y^j$, while on the left we create an alternate measure of excess density based on known parameters,

$$\hat{N}_e^j = \hat{G}^j\sigma_Y^j + (k_1 + k_2\sigma_Y^j)\hat{A}_Y^j + \xi^j, \quad \text{(A.4)}$$

where we define the generalized excess density measure as

$$\hat{N}_e^j \equiv \hat{N}_*^j - \left[\varepsilon_{N_*,Q}^0 s_y + \varepsilon_{N_*,A_X}^0 \frac{\theta_L}{\phi_L}\right]\hat{p}^j - \left[\varepsilon_{N_*,A_X}^0\left(\theta_N - \phi_L\frac{\theta_L}{\phi_L}\right) - \varepsilon_{N_*,Q}^0(1-\tau)s_w\right]\hat{w}^j; \quad \text{(A.5)}$$

the demand shifter, which depends on quality of life and observable trade-productivity, is

$$\hat{G}^j \equiv \left[d_{Y,Q}s_y + d_{Y,A_X}\frac{\theta_L}{\phi_L}\right]\hat{p}^j + \left[d_{Y,A_X}\left(\theta_N - \phi_L\frac{\theta_L}{\phi_L}\right) - d_{Y,Q}(1-\tau)s_w\right]\hat{w}^j; \quad \text{(A.6)}$$

and the two constants for the level of A_Y^j and its interaction with σ_Y^j are:

$$k_1 \equiv \varepsilon^0_{N,A_Y} + \varepsilon^0_{N,A_X}\frac{\theta_L}{\phi_L},$$
$$k_2 \equiv d_{Y,A_Y} + d_{Y,A_X}\frac{\theta_L}{\phi_L}.$$

To identify heterogeneity in either σ_Y^j or A_Y^j, we need observable variables that change them. Here, we consider a two variable model (which can easily be extended) to account for regulatory and geographic variables. First, assume that the elasticity of substitution in the home good sector is given by the linear function of I^j and S^j:

$$\sigma_Y^j = \sigma_{Y0} + \sigma_{YI}I^j + \sigma_{YS}S^j + v^j. \tag{A.7}$$

Second, assume that differences in home-productivity are also a linear function of the same two variables:

$$\hat{A}_Y^j = a_I I^j + a_S S^j + u^j. \tag{A.8}$$

Substituting equations (A.7) and (A.8) into (A.4) and simplifying yields an equation with several quadratic interactions,

$$\begin{aligned}\hat{N}_e^j = {} & \sigma_{Y0}\hat{G}^j + \sigma_{YI}\hat{G}^j I^j + \sigma_{YS}\hat{G}^j S^j + (k_1 + k_2\sigma_{Y0})a_I I^j + (k_1 + k_2\sigma_{Y0})a_S S^j \\ & + k_2\sigma_{YI}a_I(I^j)^2 + k_2\sigma_{YS}a_S(S^j)^2 + k_2(\sigma_{Y1}a_S + \sigma_{YS}a_I)S^j I^j + e^j\end{aligned} \tag{A.9}$$

and the heteroskedastic error term:

$$e^j \equiv v^j[G^j + k_2(a_I I^j + a_S S^j)] + u^j[k_1 + k_2(\sigma_{Y0} + \sigma_{YI}I^j + \sigma_{YS}S^j)] + k_2 v^j u^j + \xi^j \tag{A.10}$$

The following orthogonality conditions permit consistent estimation of the parameters in equation (A.9) using standard non-linear least squares:

$$E[v^j|\hat{G}^j, \hat{G}^j I^j, \hat{G}^j S^j, I^j, S^j, (I^j)^2, (S^j)^2, I^j S^j, u^j] = 0 \tag{A.11}$$
$$E[u^j|\hat{G}^j, \hat{G}^j I^j, \hat{G}^j S^j, I^j, S^j, (I^j)^2, (S^j)^2, I^j S^j, v^j] = 0 \tag{A.12}$$
$$E[\xi^j|\hat{G}^j, \hat{G}^j I^j, \hat{G}^j S^j, I^j, S^j, (I^j)^2, (S^j)^2, I^j S^j] = 0 \tag{A.13}$$

We do not use higher-order moments from the heteroskedastic error term to estimate the model.

The 8-parameter reduced-form specification of equation (A.9) is given by

$$\begin{aligned}\hat{N}_e^j = {} & \pi_1\hat{G}^j + \pi_2\hat{G}^j I^j + \pi_3\hat{G}^j S^j + \pi_4 I^j + \\ & + \pi_5 S^j + \pi_6(I^j)^2 + \pi_7(S^j)^2 + \pi_8 S^j I^j + e^j\end{aligned} \tag{A.14}$$

It may be used to test the model by checking if these three constraints hold:

$$\pi_6(k_1 + k_2\pi_1) = k_2\pi_2\pi_4,$$
$$\pi_7(k_1 + k_2\pi_1) = k_2\pi_3\pi_5,$$
$$\pi_8(k_1 + k_2\pi_1) = k_2(\pi_2\pi_5 + \pi_3\pi_4).$$

We can consider more restricted models. The first set of models assumes that $A_Y^j = 0$. These correspond to the first two regressions in table. In that case, the estimates of A_X^j remain accurate. The error e^j is due either to specification error, ξ^j, or unobserved determinants of σ_Y^j. This is the model we use to assess the predictive power of the model.

The second set of models allows for variable A_Y^j. The initial version of this model, with fixed σ_Y, also assumes $\xi^j = 0$, applying all deviations to A_Y^j. σ_Y^j to vary, (25b) still applies so long as $v^j = \xi^j = 0$. If not, an alternative is to assume $u^j = 0$, and infer A_Y^j from what is predicted in (A.8). In either case, estimates of A_X^j should be updated using (23).

G Models in the Literature

G.1 Rappaport (2008a, 2008b)

Rappaport's model most resembles our own. Most importantly it imposes the restriction that $\hat{A}_X^j = \hat{A}_Y^j$. It also imposes the restriction that i) traded production is Cobb-Douglas, $\sigma_X = 1$, and ii) home production is a nested CES.

G.2 Glaeser and Gottlieb (2009)

The differences between Glaeser and Gottlieb (2009) and the general neoclassical model include that the former imposes i) unit elasticities of substitution, i.e., $\sigma_D = \sigma_X = \sigma_Y = 1$ and ii) separate land markets in the traded and non-traded sector, implying separate prices r_X and r_Y^j. In addition, Glaeser and Gottlieb impose that all non-labor income is taken by absentee landlords, $s_w = 1$ and that federal taxes are zero $\tau = 0$.

Following the order of our own system, this simplifies the first mobility condition to $-\hat{w}^j + s_y\hat{p}^j = \hat{Q}^j$, and leaves the other two unaltered. The simplified budget constraint and tangency condition implies $\hat{x}^j = \hat{w}^j = s_y\hat{p}^j - \hat{Q}^j$, and $\hat{y}^j = \hat{w}^j - \hat{p}^j = -(1-s_y)\hat{p}^j - \hat{Q}^j$.

Production in the traded sector is simplified. With zero profits, the factor demands are just $\hat{N}_X^j = \hat{X}^j - \hat{w}^j, \hat{K}_X^j = \hat{X}^j$. Land in traded production is fixed so that $\hat{r}_X^j = \hat{X}^j = \hat{N}_X^j + \hat{w}^j$. Substituting this rent solution into the zero-profit condition, $\hat{A}_X = \theta_L\hat{N}_X^j + (\theta_L + \theta_N)\hat{w}^j$.

In the non-traded or housing sector, factor demands under zero profits are $\hat{N}_Y^j = \hat{Y}^j + \hat{p}^j - \hat{w}^j, K_Y^j = \hat{Y}^j + \hat{p}^j - w^j$. Since land supply is exogenous, it makes sense to rearrange the demand as $Y^j = \hat{r}_Y^j + \hat{L}^j - \hat{p}^j$. Using the zero profit condition to infer land rents and substituting it in provides the relevant housing supply function:

$$r_Y^j = \frac{\hat{p}^j + \hat{A}_Y^j - \phi_N w^j}{\phi_L} \Rightarrow Y^j = \frac{1-\phi_L}{\phi_L}\hat{p}^j + \frac{1}{\phi_L}\hat{A}_Y^j - \frac{\phi_N}{\phi_L}\hat{w}^j + \hat{L}_Y^j$$

The partial-equilibrium elasticity of housing supply is $\eta = (1-\phi_L)/\phi_L$, with additional terms for productivity, local wages, and land supply, which all matter in general equilibrium.

The market clearing condition for housing is that $\hat{N}^j + \hat{y}^j = Y^j$. Substituting in the conditions for household demand and the demand for labor, this means $\hat{N}_Y^j + \hat{w}^j - \hat{p}^j = \hat{N}^j + \hat{w}^j - \hat{p}^j$, or just $\hat{N}_Y^j = \hat{N}^j$. Through the resource constraint from labor this also means that $\hat{N}_X^j = \hat{N}^j$. This simplification follows from $s_w = 1$.[28]. Combined these conditions imply $\hat{X}^j = \hat{K}^j = \hat{K}_X^j = \hat{K}_Y^j = \hat{N}^j + \hat{w}^j = \hat{Y}^j + \hat{p}^j = \hat{r}_X^j = \hat{r}_Y + \hat{L}_Y$. The last part of the expression implies how the wages are different. The resulting inference measures are

$$\begin{aligned}
\hat{Q}^j &= s_y \hat{p}^j - \hat{w}^j \\
\hat{A}_X^j &= \theta_L \hat{N}^j + (\theta_L + \theta_N)\hat{w}^j \\
\hat{A}_Y^j &= \phi_L \hat{N}^j + (\phi_L + \phi_N)\hat{w}^j - \hat{p}^j - \phi_L \hat{L}_Y^j
\end{aligned}$$

The last equation is now under-identified unless we use density, in which case it is

$$\hat{A}_Y = \phi_L(\hat{N} - \hat{L}) + (\phi_L + \phi_N)\hat{w} - \hat{p}$$

which is much different than what we have derived before. From the solutions we have

$$\begin{aligned}
\hat{N} &= \frac{[s\phi_K + (1-s)]\hat{A}_X^j + (1-\theta_K)\hat{Q}^j + s(1-\theta_K)(\hat{A}_Y^j + \phi_L \hat{L}_Y^j)}{\theta_L + s(\theta_N\phi_L - \theta_L\phi_N)} \\
\hat{w} &= \frac{s\phi_L \hat{A}_X^j - \theta_L \hat{Q}^j - s\theta_L(\hat{A}_Y^j + \phi_N \hat{L}_Y^j)}{\theta_L + s(\theta_N\phi_L - \theta_L\phi_N)} \\
\hat{p} &= \frac{\phi_L \hat{A}_X^j + (\theta_N\phi_L - \theta_L\phi_N)\hat{Q}^j - \theta_L(\hat{A}_Y^j + \phi_N \hat{L}_Y^j)}{\theta_L + s(\theta_N\phi_L - \theta_L\phi_N)}
\end{aligned}$$

Note that there is no difference between A_Y and L for those numbers. The prediction for density is that it falls with land supply at a different rate

$$\hat{N} - \hat{L} = \frac{[s\phi_K + (1-s)]\hat{A}_X^j + (1-\theta_K)\hat{Q}^j + s(1-\theta_K)\hat{A}_Y^j - \theta_L[1 - s(\phi_L + \phi_N)]\hat{L}^j}{\theta_L + s(\theta_N\phi_L - \theta_L\phi_N)}$$

The density is restriction is more easily examined using the data arranged in the housing supply function:

$$\hat{N} - \hat{L} = \frac{1}{\phi_L}\hat{p} - \frac{\phi_L + \phi_N}{\phi_L}\hat{w} + \frac{1}{\phi_L}\hat{A}_Y$$

In the data the unadjusted RMSE error is 0.87021. The fit is poor in the fully restricted model in

[28]Note that the authors state that $N_Y = s_y \phi_N N$, which implies that absentee income is paid entirely in the traded good. More generally, $(1-s_w)\hat{w}^j = \lambda_L \left(\hat{N}_X^j - N_Y^j\right)$, so that the proportion of labor in the traded sector rises with the wage.

this paper's parametrization, but does better with Glaeser and Gottlieb's

ϕ_L	β_w	$RMSE$	R^2
0.223	−3.64	0.7255	0.305
0.333	−2.33	0.5761	0.562
1.125	−0.62	0.5607	0.937

The last row corresponds to a one restriction restricted fit that the coefficient on $\hat{w}$ has to be $-(1-\phi_K)$ that on $\hat{p}$. It does produces an estimate for ϕ_L greater than one.

For heterogeneous housing supply, take the density equation and substitute in $1/\phi_L = 1+\eta$.

$$\hat{N} - \hat{L} = (1+\eta)\hat{p}^j - (1 + (1+\eta)\phi_N)\,\hat{w}^j + (1+\eta)\hat{A}_Y^j$$

Restricting A_Y^j to be uniform, we get

$$\eta^j = \frac{(\hat{N}^j - \hat{L}^j) + \hat{w}^j}{\hat{p}^j - \phi_N \hat{w}^j} - 1$$

In the data this produces a large number of negative supply elasticities. This suggests the model is not appropriate for inferring heterogeneous supply elasticities. Incidentally, the model handles agglomeration economies with $\hat{A}_X + \gamma\hat{N} = (1-\theta_K)\hat{w} + (1-\theta_K-\theta_N)\hat{N}$.

G.3 Lee and Li (2013)

The authors' iso-elastic housing production function is equivalent to the case with $\phi_N = 0$, $\sigma_Y = 1$, and uniform $A_Y = 1$, but with variable land endowments, L.

$$\hat{Y} = \frac{1-\phi_L}{\phi_L}\hat{p} + \hat{L}$$

Although the model does not address land directly, it implies $\hat{p}^j = \phi_L \hat{r}^j$. Furthermore, $\hat{N}^j = \hat{N}_X^j$.

Households have Cobb-Douglas utility, $\sigma_D = 1$. There are no taxes $\tau = 0$, and non-labor income is given to absentee landlords. Thus $\hat{Q}^j = s_y\hat{p}^j - \hat{w}^j$, $\hat{x}^j = \hat{w}^j = s_y\hat{p}^j - \hat{Q}^j$, and $\hat{y}^j = \hat{w}^j - \hat{p}^j = -(1-s_y)\hat{p}^j - \hat{Q}^j$.

The greatest departure from the Roback model is that firms purchase housing directly, so that traded production: $X = A_X N_X^{\theta_N} Y_X^{1-\theta_N}$. In this Cobb-Douglas economy, this simplification is easy to untangle, since we can think of $Y_X^{1-\theta_N} = K_X^{(1-\theta_N)(1-\phi_N)} L_X^{(1-\theta_N)\phi_L}$. Therefore, the main imposition is that $\theta_K = (1-\phi_N)(1-\theta_N)$, and $\theta_L = \phi_L(1-\theta_N)$, or that factor proportions in the traded-sector should mirror those in the housing sector with $\theta_L/\theta_K = \phi_L/(1-\phi_L)$, which is not self evident. By virtue of zero profits and Cobb-Douglas production, $\hat{X}^j = \hat{N}^j + \hat{w}^j$, $Y_X^j = \hat{X}^j - \hat{p}^j = \hat{N}^j + \hat{w}^j - \hat{p}^j$.

Market-clearing in the housing market simplifies as both households and firms have housing demand that obeys $\hat{Y} = \hat{N}^j + \hat{w}^j - \hat{p}^j$. Setting supply equal to demand, we have that $\hat{N}^j + \hat{w}^j = \hat{L}^j + \hat{p}^j/\phi_L = \hat{L}^j + \hat{r}^j$, the same as in the Glaser and Gottlieb model.

Summing up, the model imposes

$$\begin{aligned}\hat{A}_X &= \theta_N \hat{w} + (1-\theta_N)\,\hat{p}^j \\ \hat{Q} &= s_y \hat{p}^j - \hat{w}^j \\ \hat{L}^j &= \hat{N}^j + \hat{w}^j - \hat{p}^j/\phi\end{aligned}$$

The last equation may be seen as an imposition of the data, or a way of inferring true land supply. It implies that density should be equal to the ratio of the inferred rent to the wage:

$$\hat{N}^j - \hat{L}^j = \hat{p}^j/\phi - \hat{w}^j$$

Solving, the model inverts easily to

$$\begin{aligned}\hat{p}^j &= \frac{\phi_L \hat{A}_X^j + \theta_N \phi_L \hat{Q}^j}{\theta_L + s_y \theta_N \phi_L} \\ \hat{w}^j &= \frac{\phi_L \hat{A}_X^j + \theta_N \phi_L \hat{Q}^j}{\theta_L + s_y \theta_N \phi_L} \\ \hat{N}^j &= \frac{[s_y \phi_K + (1-s_y)]\hat{A}_X^j + (1-\theta_K)\hat{Q}^j}{\theta_L + s_y \theta_N \phi_L}\end{aligned}$$

The model is equivalent in most ways to the Glaeser Gottlieb model. Lee and Li (2013) concern themselves with fitting Zipf's Law rather than specific cities.

G.4 Saiz (2010) Model

This monocentric city around a CBD cannot be easily mapped to our framework. There is no mobile capital, and land is only in the housing sector. The initial wage, w_0, quality of life, Q_0, and arc of expansion Θ are exogenous. The key endogenous variables are $\bar{p}, N$. Housing demand per person is perfectly inelastic with respect to income and price: $y = 1, Y = N$. Households at a distance of z from the CBD pay tz for commuting. There is only labor income, so $x = w - p - tz$. Quality of life enters additively with wages, making it indistinguishable from what is effectively trade productivity: $U = (x + Q)\, I[y \geq 1]$, and $V = Q + w - p - tz$. Agglomeration diseconomies in production and consumption imply $w = w_0 - \alpha N^{1/2}, Q = Q_0 - \psi N^{1/2}$. With mobility, the downtown rent at $p(0)$ declines with population $p(0) = w_0 + Q_0 - (\alpha + \psi) N^{1/2}$, while rent at z is $p(z) = p(0) - tz$.

Housing supply is based on fixed coefficients with land and non-land costs $Y = \min\{v, L/\tilde{\gamma}\}$ where the price of $v = 1$ or i and $\tilde{\gamma}$ is a fixed population density. The land area of the city with radius $\underline{z}$ is $L = \Theta \underline{z}^2 = N/\tilde{\gamma} \Rightarrow \underline{z} = \sqrt{N/(\tilde{\gamma}\Theta)}$. $r(\underline{z}) = 0$ at the fringe, $r(0) = t\sqrt{N/(\tilde{\gamma}\Theta)}, \bar{r} = r(0) = t/3\sqrt{N/(\tilde{\gamma}\Theta)}$. Log-linearized we get $\hat{r} = \hat{t} + (1/2)\left(\hat{N} - \hat{\Theta} - \hat{\gamma}\right)$. This restricts the elasticity of land supply to 2. The price of a house is the capitalized value of the rent plus the construction costs. This has the inverse supply equation

$$\bar{p} = v + \bar{r} = v + \frac{t}{3}\sqrt{\frac{N}{\tilde{\gamma}\Theta}}, \text{and } \bar{\phi}_L = \frac{\bar{r}}{v + \bar{r}} \frac{t\sqrt{N}}{3v\sqrt{\tilde{\gamma}\Theta} + t\sqrt{N}}$$

is the typical cost share of land, which ranges from 0 to 1 as N goes from 0 to infinity.

G.5 Desmet Rossi-Hansberg (2013)

The authors impose a Cobb-Douglas production function for X, with $\theta_L = 0$, and $\sigma_X = 1$. Housing is produced directly from land, which is supplied through a monocentric city. This imposes $\phi_L = 1$ and $\Theta = 1$. Household demand for land is completely inelastic, thus, the elasticity of population as well as housing supply is always 2:

$$\hat{N} = \hat{Y} = 2.$$

At the household level, workers supply leisure in Cobb-Douglas utility function, unlike our model. They use data on non-housing consumption, C, capital, K, and hours worked, H. Their basic measures are

$$\begin{aligned} \hat{Q}^j &= \left(\hat{C}^j - \hat{N}^j\right) + \psi\left[\widehat{(1-H)}^j - \hat{N}^j\right] \\ \hat{A}_X^j &= \hat{X}^j - \left(1-\theta_N^j\right)\hat{K}_X^j - \theta_N \hat{H}_X^j \\ \widehat{(1-\tau)}^j &= \hat{C}^j - \hat{X}^j + \widehat{\left(\frac{H}{1-H}\right)}^j \end{aligned}$$

The model imposes other restrictions in the steady state, such as $\hat{A}_X^j = (1-\theta_N)\,\hat{w}^j$, which does not hold exactly in the data.

Table A.1: Parametrized Relationship between Amenities, Prices, and Quantities, with Feedback Effects

		A: Trade-productivity Feedback						B: Quality of Life Feedback					
		I: Current Regime			II: Neutral Taxes			I: Current Regime			II: Neutral Taxes		
		Quality of Life	Trade Productivity	Home Productivity	-	-	-	-	-	-	-	-	-
Price/quantity	Notation	$\hat{Q}$	$\hat{A}_{X0}$	$\hat{A}_Y$	$\hat{Q}$	$\hat{A}_{X0}$	$\hat{A}_Y$	$\hat{Q}_0$	$\hat{A}_X$	$\hat{A}_Y$	$\hat{Q}_0$	$\hat{A}_X$	$\hat{A}_Y$
Land value	$\hat{r}$	14.101	4.615	4.655	13.125	8.240	4.944	10.544	3.674	3.400	9.136	5.890	3.228
Wage	$\hat{w}$	0.256	1.253	0.100	0.194	1.311	0.105	-0.320	1.101	-0.103	-0.277	1.034	-0.098
Home price	$\hat{p}$	3.448	1.849	0.148	3.182	2.731	0.218	2.263	1.536	-0.270	1.961	2.012	-0.307
Trade consumption	$\hat{x}$	-0.249	0.401	0.032	0.764	0.656	0.052	-0.397	0.362	-0.020	0.471	0.483	-0.074
Home consumption	$\hat{y}$	-2.335	-0.713	-0.057	-1.358	-1.166	-0.093	-1.768	-0.563	0.143	-0.837	-0.859	0.131
Population density	$\hat{N}$	9.394	2.486	3.315	8.135	4.791	3.499	7.282	1.927	2.569	5.772	3.399	2.482
Capital	$\hat{K}$	9.546	3.293	3.349	8.322	5.645	3.537	7.064	2.637	2.473	5.647	4.070	2.386
Land	$\hat{L}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Trade production	$\hat{X}$	9.839	3.838	3.598	8.160	6.186	3.786	7.088	3.109	2.627	5.311	4.508	2.561
Home production	$\hat{Y}$	7.059	1.773	3.258	6.777	3.625	3.406	5.513	1.364	2.712	4.935	2.540	2.613
Trade labor	$\hat{N}_X$	9.481	2.619	3.465	7.868	4.883	3.646	7.301	2.042	2.696	5.496	3.486	2.626
Home labor	$\hat{N}_Y$	9.188	2.171	2.957	8.770	4.572	3.148	7.236	1.654	2.268	6.427	3.193	2.141
Trade capital	$\hat{K}_X$	9.651	3.455	3.532	7.997	5.757	3.716	7.088	2.776	2.627	5.311	4.175	2.561
Home capital	$\hat{K}_Y$	9.358	3.006	3.023	8.899	5.446	3.218	7.023	2.388	2.199	6.242	3.882	2.076
Trade land	$\hat{L}_X$	0.246	0.377	0.427	-0.757	0.261	0.418	0.055	0.326	0.360	-0.782	0.246	0.407
Home land	$\hat{L}_Y$	-0.047	-0.072	-0.081	0.144	-0.050	-0.080	-0.010	-0.062	-0.069	0.149	-0.047	-0.078

Each value in Table A.1 represents the partial effect that a one-point increase in each amenity has on each price or quantity. The values in Panel A include feedback effects on trade-productivity, where $A_X^j = A_{X0}^j (N^j)^\alpha$ and $\alpha = 0.06$. The values in Panel B include feedback effects on quality-of-life, where $Q^j = Q_0^j (N^j)^{-\gamma}$ and $\gamma = 0.015$. Each panel includes values for the current regime and geographically neutral taxes. All variables are measured in log differences from the national average.

Table A.2: List of Metropolitan and Non-Metropolitan Areas Ranked by Density

Name of Metropolitan Area	Population Density $\hat{N}^j$	Land Value $\hat{r}^j$	Quality of Life $\hat{Q}^j$	Inferred Costs Eq. (23)	Trade Productivity $\hat{A}_X^j$	Home Productivity $\hat{A}_Y^j$
New York, Northern New Jersey, Long Island, NY-NJ-CT-PA	2.294	3.405	0.031	0.218	0.272	0.504
Honolulu, HI	1.302	1.953	0.208	0.056	0.039	-0.166
Los Angeles-Riverside-Orange County, CA	1.258	1.946	0.080	0.154	0.163	0.088
San Francisco-Oakland-San Jose, CA	1.218	2.050	0.137	0.292	0.273	-0.171
Chicago-Gary-Kenosha, IL-IN-WI	1.200	1.789	0.007	0.130	0.160	0.276
Miami-Fort Lauderdale, FL	0.972	1.372	0.036	0.021	0.043	0.202
Philadelphia-Wilmington-Atlantic City, PA-NJ-DE-MD	0.967	1.409	-0.038	0.097	0.134	0.343
San Diego, CA	0.881	1.439	0.122	0.100	0.088	-0.108
Salinas (Monterey-Carmel), CA	0.847	1.443	0.141	0.145	0.123	-0.198
Boston-Worcester-Lawrence, MA-NH-ME-CT	0.806	1.278	0.050	0.132	0.136	0.035
Santa Barbara-Santa Maria-Lompoc, CA	0.722	1.299	0.181	0.117	0.082	-0.324
New Orleans, LA	0.697	0.875	0.005	-0.063	-0.036	0.255
Las Vegas, NV-AZ	0.693	0.998	-0.016	0.050	0.075	0.229
Washington-Baltimore, DC-MD-VA-WV	0.693	1.069	-0.009	0.120	0.137	0.162
Providence-Fall River-Warwick, RI-MA	0.593	0.850	0.012	0.019	0.035	0.146
Milwaukee-Racine, WI	0.582	0.804	-0.005	0.032	0.051	0.179
Stockton-Lodi, CA	0.538	0.813	-0.002	0.082	0.095	0.121
Laredo, TX	0.533	0.531	-0.009	-0.192	-0.157	0.329
Phoenix-Mesa, AZ	0.517	0.729	0.015	0.026	0.038	0.109
Denver-Boulder-Greeley, CO	0.476	0.734	0.049	0.065	0.063	-0.022
Buffalo-Niagara Falls, NY	0.457	0.625	-0.052	-0.046	-0.012	0.316
Provo-Orem, UT	0.456	0.577	0.014	-0.044	-0.029	0.139
Champaign-Urbana, IL	0.445	0.569	-0.011	-0.076	-0.052	0.225
Sacramento-Yolo, CA	0.442	0.716	0.032	0.075	0.075	0.005
Reading, PA	0.411	0.522	-0.050	-0.010	0.018	0.270
Salt Lake City-Ogden, UT	0.402	0.530	0.025	-0.017	-0.009	0.075
Modesto, CA	0.398	0.590	-0.008	0.048	0.060	0.115
El Paso, TX	0.395	0.345	-0.040	-0.166	-0.129	0.347
Detroit-Ann Arbor-Flint, MI	0.356	0.570	-0.046	0.107	0.124	0.161
Madison, WI	0.342	0.498	0.058	-0.027	-0.030	-0.025
Lincoln, NE	0.339	0.318	0.017	-0.118	-0.102	0.146
Cleveland-Akron, OH	0.338	0.453	-0.015	0.006	0.021	0.145

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Seattle-Tacoma-Bremerton, WA	0.334	0.597	0.062	0.094	0.081	-0.122
Houston-Galveston-Brazoria, TX	0.332	0.453	-0.072	0.043	0.072	0.265
Dallas-Fort Worth, TX	0.327	0.472	-0.041	0.044	0.064	0.182
Allentown-Bethlehem-Easton, PA	0.317	0.422	-0.021	-0.006	0.011	0.160
State College, PA	0.301	0.346	0.037	-0.123	-0.114	0.085
Reno, NV	0.272	0.468	0.057	0.037	0.028	-0.088
Portland-Salem, OR-WA	0.250	0.387	0.050	0.040	0.032	-0.078
Lafayette, IN	0.245	0.274	-0.014	-0.059	-0.042	0.155
West Palm Beach-Boca Raton, FL	0.244	0.383	0.020	0.045	0.045	-0.004
Fresno, CA	0.241	0.332	-0.004	-0.023	-0.012	0.103
San Antonio, TX	0.231	0.193	-0.034	-0.100	-0.075	0.232
Norfolk-Virginia Beach-Newport News, VA-	0.218	0.239	0.031	-0.094	-0.088	0.053
Minneapolis-St. Paul, MN-WI	0.213	0.316	-0.023	0.068	0.077	0.082
Anchorage, AK	0.198	0.382	0.021	0.083	0.078	-0.048
Bakersfield, CA	0.198	0.233	-0.056	0.008	0.030	0.206
Omaha, NE-IA	0.174	0.090	-0.014	-0.084	-0.068	0.150
Columbus, OH	0.165	0.210	-0.027	0.011	0.024	0.115
Erie, PA	0.161	0.099	-0.037	-0.115	-0.091	0.228
Springfield, MA	0.152	0.244	0.005	-0.006	-0.002	0.041
Bloomington-Normal, IL	0.135	0.164	-0.061	0.003	0.025	0.201
Tucson, AZ	0.131	0.132	0.051	-0.086	-0.090	-0.031
Pittsburgh, PA	0.128	0.098	-0.043	-0.058	-0.037	0.194
Albuquerque, NM	0.122	0.088	0.051	-0.066	-0.072	-0.050
Toledo, OH	0.122	0.104	-0.043	-0.034	-0.015	0.175
Tampa-St. Petersburg-Clearwater, FL	0.118	0.090	0.002	-0.055	-0.047	0.069
Iowa City, IA	0.112	0.088	0.038	-0.075	-0.076	-0.011
Hartford, CT	0.108	0.294	-0.019	0.117	0.117	0.002
Lubbock, TX	0.084	-0.057	-0.008	-0.164	-0.147	0.163
Corpus Christi, TX	0.083	-0.021	-0.032	-0.112	-0.092	0.187
Austin-San Marcos, TX	0.078	0.147	0.020	0.014	0.010	-0.037
Non-metro, RI	0.074	0.222	0.062	0.068	0.048	-0.187
Bryan-College Station, TX	0.072	0.002	0.027	-0.121	-0.117	0.036

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Colorado Springs, CO	0.069	0.058	0.051	-0.063	-0.070	-0.068
St. Louis, MO-IL	0.061	0.022	-0.034	-0.008	0.004	0.115
Brownsville-Harlingen-San Benito, TX	0.057	-0.188	-0.063	-0.213	-0.177	0.330
Rochester, NY	0.033	0.061	-0.040	-0.032	-0.017	0.137
Fargo-Moorhead, ND-MN	0.024	-0.161	-0.022	-0.172	-0.152	0.186
Spokane, WA	0.019	-0.075	0.006	-0.091	-0.085	0.052
Pueblo, CO	0.009	-0.144	0.002	-0.150	-0.139	0.103
Lancaster, PA	0.006	-0.006	-0.011	-0.015	-0.011	0.042
Cincinnati-Hamilton, OH-KY-IN	0.005	-0.017	-0.035	0.019	0.028	0.081
Lawrence, KS	0.004	-0.110	0.028	-0.119	-0.118	0.010
Louisville, KY-IN	-0.003	-0.083	-0.021	-0.050	-0.041	0.088
Bloomington, IN	-0.006	-0.083	0.031	-0.114	-0.114	-0.005
Albany-Schenectady-Troy, NY	-0.019	0.003	-0.031	-0.022	-0.012	0.091
Amarillo, TX	-0.019	-0.179	-0.008	-0.146	-0.134	0.117
Memphis, TN-AR-MS	-0.026	-0.111	-0.055	-0.014	0.001	0.145
Fort Collins-Loveland, CO	-0.039	-0.044	0.067	-0.026	-0.044	-0.169
Scranton–Wilkes-Barre–Hazleton, PA	-0.040	-0.163	-0.024	-0.111	-0.097	0.128
Orlando, FL	-0.048	-0.119	0.008	-0.041	-0.042	-0.009
Syracuse, NY	-0.072	-0.126	-0.071	-0.058	-0.036	0.204
Altoona, PA	-0.073	-0.268	-0.044	-0.160	-0.138	0.203
Visalia-Tulare-Porterville, CA	-0.078	-0.132	-0.019	-0.033	-0.028	0.049
Green Bay, WI	-0.085	-0.157	-0.007	-0.029	-0.028	0.013
South Bend, IN	-0.085	-0.221	-0.048	-0.075	-0.059	0.149
Corvalis, OR	-0.102	-0.168	0.076	-0.074	-0.093	-0.181
Lexington, KY	-0.104	-0.266	-0.023	-0.094	-0.084	0.093
Yuma, AZ	-0.112	-0.263	0.008	-0.109	-0.107	0.019
Des Moines, IA	-0.118	-0.246	-0.009	-0.043	-0.041	0.015
Kansas City, MO-KS	-0.121	-0.259	-0.033	-0.020	-0.014	0.061
Oklahoma City, OK	-0.123	-0.365	-0.017	-0.137	-0.126	0.100
Waterloo-Cedar Falls, IA	-0.123	-0.346	-0.017	-0.142	-0.131	0.103
Sarasota-Bradenton, FL	-0.130	-0.196	0.073	-0.056	-0.077	-0.194
Dayton-Springfield, OH	-0.137	-0.238	-0.031	-0.029	-0.023	0.056

Table A.2: List of Metropolitan and Non-Metropolitan Areas Ranked by Density

Name of Metropolitan Area	Population Density $\hat{N}^j$	Land Value $\hat{r}^j$	Quality of Life $\hat{Q}^j$	Inferred Costs Eq. (23)	Trade Productivity $\hat{A}_X^j$	Home Productivity $\hat{A}_Y^j$
Odessa-Midland, TX	-0.145	-0.387	-0.064	-0.137	-0.114	0.215
Sioux City, IA-NE	-0.149	-0.404	-0.025	-0.156	-0.143	0.127
Eugene-Springfield, OR	-0.150	-0.230	0.087	-0.081	-0.105	-0.220
Dubuque, IA	-0.160	-0.412	-0.027	-0.149	-0.136	0.122
Indianapolis, IN	-0.175	-0.274	-0.036	-0.004	0.001	0.041
Appleton-Oshkosh-Neenah, WI	-0.176	-0.314	-0.019	-0.056	-0.053	0.033
Boise City, ID	-0.176	-0.366	0.010	-0.079	-0.082	-0.028
Wichita, KS	-0.178	-0.402	-0.047	-0.083	-0.070	0.124
Davenport-Moline-Rock Island, IA-IL	-0.205	-0.387	-0.034	-0.089	-0.080	0.084
Merced, CA	-0.206	-0.287	-0.015	-0.008	-0.011	-0.021
Lansing-East Lansing, MI	-0.209	-0.298	-0.054	-0.002	0.007	0.077
Harrisburg-Lebanon-Carlisle, PA	-0.213	-0.328	-0.028	-0.022	-0.020	0.020
Richmond-Petersburg, VA	-0.219	-0.348	-0.033	-0.009	-0.007	0.022
Elmira, NY	-0.221	-0.377	-0.057	-0.149	-0.129	0.182
Grand Rapids-Muskegon-Holland, MI	-0.225	-0.319	-0.046	-0.010	-0.004	0.055
Portland, ME	-0.235	-0.405	0.057	-0.054	-0.074	-0.188
Abilene, TX	-0.247	-0.568	0.004	-0.228	-0.220	0.068
Rockford, IL	-0.253	-0.401	-0.064	-0.031	-0.020	0.109
Jacksonville, FL	-0.256	-0.430	-0.009	-0.051	-0.054	-0.023
Cedar Rapids, IA	-0.257	-0.460	-0.005	-0.074	-0.076	-0.019
Muncie, IN	-0.264	-0.506	-0.043	-0.124	-0.112	0.115
Sioux Falls, SD	-0.264	-0.549	0.006	-0.147	-0.147	0.001
York, PA	-0.269	-0.422	-0.030	-0.041	-0.039	0.020
Yakima, WA	-0.278	-0.435	-0.005	-0.035	-0.041	-0.052
Atlanta, GA	-0.282	-0.371	-0.033	0.063	0.058	-0.046
Tulsa, OK	-0.285	-0.570	-0.026	-0.100	-0.095	0.046
Burlington, VT	-0.287	-0.489	0.054	-0.076	-0.096	-0.181
Gainesville, FL	-0.292	-0.543	0.022	-0.134	-0.140	-0.060
Binghamton, NY	-0.293	-0.460	-0.055	-0.125	-0.111	0.138
Sheboygan, WI	-0.293	-0.479	-0.016	-0.066	-0.067	-0.005
Lewiston-Auburn, ME	-0.298	-0.590	-0.013	-0.120	-0.117	0.021
Canton-Massillon, OH	-0.313	-0.515	-0.026	-0.082	-0.079	0.024

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Rochester, MN	-0.316	-0.491	-0.066	0.001	0.008	0.070
Charlottesville, VA	-0.320	-0.479	0.049	-0.084	-0.103	-0.173
Billings, MT	-0.321	-0.664	0.008	-0.164	-0.165	-0.009
Savannah, GA	-0.327	-0.506	0.004	-0.078	-0.085	-0.061
St. Joseph, MO	-0.338	-0.656	-0.028	-0.169	-0.160	0.081
Topeka, KS	-0.340	-0.642	-0.029	-0.133	-0.126	0.058
Melbourne-Titusville-Palm Bay, FL	-0.351	-0.606	-0.001	-0.100	-0.104	-0.039
Pocatello, ID	-0.362	-0.743	-0.060	-0.150	-0.134	0.145
Casper, WY	-0.362	-0.757	-0.011	-0.208	-0.202	0.058
Utica-Rome, NY	-0.366	-0.571	-0.068	-0.123	-0.107	0.147
Fort Walton Beach, FL	-0.374	-0.669	0.062	-0.177	-0.194	-0.161
Decatur, IL	-0.377	-0.635	-0.087	-0.086	-0.068	0.167
Bismarck, ND	-0.381	-0.848	-0.041	-0.257	-0.240	0.164
La Crosse, WI-MN	-0.385	-0.634	-0.010	-0.126	-0.127	-0.010
Yuba City, CA	-0.386	-0.546	0.004	-0.059	-0.069	-0.092
Janesville-Beloit, WI	-0.386	-0.610	-0.049	-0.024	-0.021	0.021
Peoria-Pekin, IL	-0.390	-0.595	-0.064	-0.038	-0.030	0.070
Columbia, MO	-0.392	-0.688	0.013	-0.155	-0.160	-0.051
Evansville-Henderson, IN-KY	-0.396	-0.667	-0.032	-0.106	-0.102	0.031
Victoria, TX	-0.403	-0.720	-0.073	-0.110	-0.095	0.138
Naples, FL	-0.415	-0.483	0.106	0.020	-0.026	-0.425
Great Falls, MT	-0.416	-0.881	0.022	-0.264	-0.265	-0.008
Medford-Ashland, OR	-0.416	-0.606	0.092	-0.098	-0.131	-0.306
Springfield, IL	-0.422	-0.654	-0.038	-0.085	-0.082	0.023
Lawton, OK	-0.425	-0.858	-0.021	-0.251	-0.241	0.093
Waco, TX	-0.430	-0.755	-0.044	-0.129	-0.122	0.068
Richland-Kennewick-Pasco, WA	-0.431	-0.632	-0.052	0.014	0.013	-0.011
Chico-Paradise, CA	-0.436	-0.564	0.053	-0.066	-0.091	-0.232
Columbus, GA-AL	-0.436	-0.740	-0.026	-0.150	-0.147	0.032
San Angelo, TX	-0.441	-0.812	-0.024	-0.184	-0.178	0.049
Tallahassee, FL	-0.442	-0.714	0.025	-0.104	-0.118	-0.135
Fort Myers-Cape Coral, FL	-0.446	-0.678	0.049	-0.083	-0.106	-0.215

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Name of Metropolitan Area	Population Density $\hat{N}^j$	Land Value $\hat{r}^j$	Quality of Life $\hat{Q}^j$	Inferred Costs Eq. (23)	Trade Productivity $\hat{A}_X^j$	Home Productivity $\hat{A}_Y^j$
San Luis Obispo-Atascadero-Paso Robles, CA	-0.449	-0.404	0.131	0.064	0.007	-0.532
Baton Rouge, LA	-0.456	-0.730	-0.026	-0.061	-0.065	-0.036
Grand Forks, ND-MN	-0.459	-0.868	-0.042	-0.208	-0.196	0.108
Roanoke, VA	-0.461	-0.744	-0.017	-0.109	-0.112	-0.028
Williamsport, PA	-0.462	-0.770	-0.032	-0.133	-0.130	0.027
Pittsfield, MA	-0.470	-0.635	0.016	-0.054	-0.071	-0.156
Saginaw-Bay City-Midland, MI	-0.480	-0.714	-0.080	-0.029	-0.021	0.077
Raleigh-Durham-Chapel Hill, NC	-0.494	-0.712	0.011	0.017	-0.004	-0.199
Charleston-North Charleston, SC	-0.495	-0.755	0.035	-0.088	-0.109	-0.190
Youngstown-Warren, OH	-0.503	-0.806	-0.051	-0.095	-0.091	0.038
Grand Junction, CO	-0.512	-0.799	0.076	-0.148	-0.176	-0.261
Beaumont-Port Arthur, TX	-0.517	-0.866	-0.104	-0.077	-0.060	0.160
Fort Wayne, IN	-0.521	-0.829	-0.063	-0.071	-0.066	0.049
Nashville, TN	-0.530	-0.759	-0.001	-0.018	-0.035	-0.155
McAllen-Edinburg-Mission, TX	-0.532	-1.027	-0.081	-0.226	-0.205	0.199
Daytona Beach, FL	-0.550	-0.910	0.027	-0.150	-0.165	-0.143
Springfield, MO	-0.551	-0.935	0.006	-0.186	-0.193	-0.062
Birmingham, AL	-0.553	-0.847	-0.042	-0.034	-0.039	-0.043
Missoula, MT	-0.554	-0.962	0.094	-0.203	-0.234	-0.283
Columbia, SC	-0.557	-0.874	-0.006	-0.075	-0.087	-0.110
Fayetteville, NC	-0.557	-0.906	0.028	-0.179	-0.193	-0.129
Cheyenne, WY	-0.560	-0.990	0.049	-0.209	-0.226	-0.163
Duluth-Superior, MN-WI	-0.560	-0.904	-0.069	-0.122	-0.113	0.085
Montgomery, AL	-0.570	-0.921	-0.005	-0.125	-0.134	-0.082
Shreveport-Bossier City, LA	-0.574	-0.964	-0.038	-0.133	-0.132	0.008
Owensboro, KY	-0.593	-1.000	-0.041	-0.148	-0.146	0.020
Kalamazoo-Battle Creek, MI	-0.593	-0.860	-0.056	-0.044	-0.045	-0.012
Wichita Falls, TX	-0.600	-1.051	0.003	-0.224	-0.229	-0.045
Sharon, PA	-0.601	-0.982	-0.035	-0.154	-0.153	0.007
Eau Claire, WI	-0.604	-0.959	-0.031	-0.119	-0.122	-0.029
Kokomo, IN	-0.609	-0.906	-0.101	0.015	0.021	0.058
Fort Pierce-Port St. Lucie, FL	-0.611	-0.934	0.022	-0.089	-0.110	-0.191

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Jackson, MS	-0.618	-1.009	-0.030	-0.103	-0.109	-0.049
Jamestown, NY	-0.624	-0.960	-0.082	-0.161	-0.147	0.126
Las Cruces, NM	-0.629	-1.066	0.019	-0.188	-0.202	-0.121
Santa Fe, NM	-0.632	-0.832	0.125	-0.013	-0.069	-0.523
Killeen-Temple, TX	-0.637	-1.066	0.035	-0.215	-0.230	-0.146
Charlotte-Gastonia-Rock Hill, NC-SC	-0.651	-0.955	-0.010	0.002	-0.018	-0.183
Mobile, AL	-0.667	-1.070	-0.012	-0.135	-0.144	-0.090
Pensacola, FL	-0.667	-1.092	0.009	-0.156	-0.170	-0.129
Terre Haute, IN	-0.668	-1.088	-0.065	-0.135	-0.130	0.050
Little Rock-North Little Rock, AR	-0.690	-1.107	-0.007	-0.107	-0.121	-0.129
Bellingham, WA	-0.692	-0.938	0.069	-0.029	-0.070	-0.383
Elkhart-Goshen, IN	-0.699	-1.052	-0.038	-0.069	-0.078	-0.076
Tuscaloosa, AL	-0.705	-1.093	-0.011	-0.104	-0.117	-0.125
Lake Charles, LA	-0.712	-1.126	-0.067	-0.079	-0.078	0.003
Jackson, MI	-0.713	-1.028	-0.067	-0.035	-0.038	-0.028
Panama City, FL	-0.714	-1.130	0.033	-0.148	-0.171	-0.213
New London-Norwich, CT-RI	-0.719	-0.875	-0.000	0.061	0.031	-0.273
Athens, GA	-0.720	-1.074	0.019	-0.132	-0.152	-0.189
Lakeland-Winter Haven, FL	-0.750	-1.193	-0.019	-0.122	-0.134	-0.105
Mansfield, OH	-0.751	-1.154	-0.043	-0.114	-0.119	-0.048
Greenville, NC	-0.757	-1.162	-0.014	-0.093	-0.108	-0.141
Enid, OK	-0.768	-1.319	-0.032	-0.222	-0.223	-0.008
Lima, OH	-0.778	-1.202	-0.059	-0.110	-0.112	-0.018
Charleston, WV	-0.781	-1.277	-0.047	-0.128	-0.132	-0.038
Biloxi-Gulfport-Pascagoula, MS	-0.787	-1.253	-0.016	-0.138	-0.150	-0.115
Huntington-Ashland, WV-KY-OH	-0.789	-1.328	-0.072	-0.183	-0.177	0.063
Parkersburg-Marietta, WV-OH	-0.801	-1.335	-0.074	-0.180	-0.173	0.063
Bangor, ME	-0.803	-1.323	-0.025	-0.169	-0.177	-0.074
Rapid City, SD	-0.806	-1.348	0.034	-0.213	-0.234	-0.199
Pine Bluff, AR	-0.806	-1.367	-0.046	-0.181	-0.183	-0.013
Macon, GA	-0.835	-1.262	-0.070	-0.078	-0.081	-0.030
Greensboro–Winston Salem–High Point, NC	-0.840	-1.251	-0.012	-0.056	-0.078	-0.199

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Punta Gorda, FL	-0.850	-1.302	0.054	-0.151	-0.184	-0.310
St. Cloud, MN	-0.851	-1.274	-0.055	-0.107	-0.112	-0.055
Albany, GA	-0.851	-1.297	-0.060	-0.105	-0.110	-0.042
Monroe, LA	-0.859	-1.356	-0.031	-0.140	-0.150	-0.097
Steubenville-Weirton, OH-WV	-0.870	-1.416	-0.057	-0.197	-0.196	0.007
Tyler, TX	-0.875	-1.334	-0.021	-0.115	-0.131	-0.146
Lafayette, LA	-0.882	-1.395	-0.045	-0.137	-0.144	-0.070
Augusta-Aiken, GA-SC	-0.897	-1.363	-0.048	-0.095	-0.105	-0.096
Huntsville, AL	-0.899	-1.360	-0.057	-0.061	-0.071	-0.097
Hattiesburg, MS	-0.901	-1.462	-0.019	-0.202	-0.213	-0.100
Johnstown, PA	-0.902	-1.459	-0.068	-0.194	-0.191	0.024
Wilmington, NC	-0.906	-1.301	0.067	-0.095	-0.138	-0.401
Non-metro, HI	-0.915	-1.218	0.128	0.009	-0.059	-0.635
Non-metro, CA	-0.917	-1.204	0.046	-0.023	-0.066	-0.400
Knoxville, TN	-0.923	-1.416	-0.008	-0.125	-0.145	-0.188
Benton Harbor, MI	-0.929	-1.329	-0.031	-0.082	-0.099	-0.160
Auburn-Opelika, AL	-0.942	-1.446	-0.013	-0.132	-0.151	-0.178
Chattanooga, TN-GA	-0.962	-1.455	-0.023	-0.111	-0.130	-0.172
Redding, CA	-1.003	-1.363	0.043	-0.078	-0.118	-0.379
Fort Smith, AR-OK	-1.047	-1.684	-0.024	-0.193	-0.208	-0.139
Non-metro, PA	-1.049	-1.609	-0.059	-0.144	-0.153	-0.083
Fayetteville-Springdale-Rogers, AR	-1.057	-1.622	0.007	-0.139	-0.167	-0.260
Wausau, WI	-1.057	-1.584	-0.054	-0.090	-0.104	-0.136
Jackson, TN	-1.064	-1.631	-0.060	-0.106	-0.118	-0.112
Danville, VA	-1.079	-1.671	-0.054	-0.173	-0.182	-0.084
Wheeling, WV-OH	-1.083	-1.714	-0.055	-0.197	-0.204	-0.066
Jacksonville, NC	-1.085	-1.659	0.053	-0.253	-0.287	-0.311
Flagstaff, AZ-UT	-1.104	-1.558	0.077	-0.105	-0.157	-0.482
Houma, LA	-1.110	-1.704	-0.049	-0.129	-0.143	-0.139
Alexandria, LA	-1.119	-1.744	-0.033	-0.174	-0.190	-0.153
Barnstable-Yarmouth (Cape Cod), MA	-1.125	-1.397	0.107	0.034	-0.037	-0.665
Texarkana, TX-Texarkana, AR	-1.125	-1.818	-0.070	-0.199	-0.203	-0.038

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Greenville-Spartanburg-Anderson, SC	-1.134	-1.691	-0.022	-0.084	-0.111	-0.247
Clarksville-Hopkinsville, TN-KY	-1.169	-1.821	-0.002	-0.207	-0.231	-0.226
Jonesboro, AR	-1.174	-1.900	-0.028	-0.240	-0.254	-0.137
Non-metro, WA	-1.184	-1.686	0.025	-0.064	-0.107	-0.402
Cumberland, MD-WV	-1.188	-1.785	-0.050	-0.172	-0.186	-0.132
Glens Falls, NY	-1.192	-1.662	-0.028	-0.105	-0.130	-0.236
Joplin, MO	-1.207	-1.900	-0.012	-0.252	-0.271	-0.181
Non-metro, NY	-1.277	-1.811	-0.056	-0.120	-0.139	-0.179
Non-metro, UT	-1.299	-1.894	0.002	-0.116	-0.153	-0.342
Non-metro, CT	-1.313	-1.683	-0.018	0.084	0.038	-0.432
Dover, DE	-1.318	-1.891	-0.009	-0.087	-0.123	-0.339
Lynchburg, VA	-1.322	-1.967	-0.029	-0.147	-0.173	-0.246
Sherman-Denison, TX	-1.323	-1.982	-0.030	-0.139	-0.165	-0.249
Asheville, NC	-1.334	-1.923	0.066	-0.146	-0.199	-0.499
Longview-Marshall, TX	-1.347	-2.044	-0.046	-0.155	-0.177	-0.204
Decatur, AL	-1.347	-2.016	-0.070	-0.092	-0.111	-0.184
Non-metro, ID	-1.368	-2.062	0.009	-0.166	-0.203	-0.347
Non-metro, NV	-1.380	-1.886	-0.003	0.005	-0.042	-0.439
Sumter, SC	-1.382	-2.117	-0.024	-0.201	-0.227	-0.242
Florence, AL	-1.389	-2.101	-0.048	-0.143	-0.166	-0.221
Myrtle Beach, SC	-1.393	-2.026	0.050	-0.163	-0.212	-0.466
Florence, SC	-1.397	-2.115	-0.039	-0.147	-0.173	-0.244
Non-metro, OH	-1.402	-2.057	-0.054	-0.112	-0.137	-0.230
Johnson City-Kingsport-Bristol, TN-VA	-1.409	-2.149	-0.020	-0.188	-0.217	-0.269
Gadsden, AL	-1.437	-2.194	-0.068	-0.153	-0.172	-0.175
Non-metro, OR	-1.465	-2.090	0.053	-0.109	-0.167	-0.533
Non-metro, NM	-1.473	-2.254	-0.001	-0.196	-0.232	-0.334
Non-metro, IN	-1.493	-2.196	-0.055	-0.114	-0.141	-0.256
Goldsboro, NC	-1.500	-2.229	0.002	-0.187	-0.225	-0.356
Non-metro, WY	-1.508	-2.264	0.001	-0.152	-0.193	-0.381
Dothan, AL	-1.524	-2.323	-0.037	-0.193	-0.220	-0.257
Non-metro, IL	-1.524	-2.251	-0.064	-0.158	-0.181	-0.213

Table A.2: List of Metropolitan and Non-Metropolitan Areas Ranked by Density

Name of Metropolitan Area	Population Density $\hat{N}^j$	Land Value $\hat{r}^j$	Quality of Life $\hat{Q}^j$	Inferred Costs Eq. (23)	Trade Productivity $\hat{A}_X^j$	Home Productivity $\hat{A}_Y^j$
Non-metro, KS	-1.533	-2.362	-0.042	-0.239	-0.262	-0.216
Non-metro, MD	-1.552	-2.129	-0.021	-0.037	-0.082	-0.417
Non-metro, NE	-1.570	-2.438	-0.034	-0.247	-0.273	-0.243
Anniston, AL	-1.570	-2.399	-0.038	-0.206	-0.234	-0.262
Ocala, FL	-1.573	-2.364	-0.010	-0.167	-0.205	-0.363
Non-metro, MA	-1.576	-2.093	0.063	-0.020	-0.091	-0.656
Non-metro, ND	-1.590	-2.529	-0.056	-0.263	-0.282	-0.181
Hickory-Morganton-Lenoir, NC	-1.615	-2.358	-0.005	-0.130	-0.175	-0.414
Rocky Mount, NC	-1.631	-2.386	-0.018	-0.123	-0.165	-0.392
Non-metro, IA	-1.688	-2.560	-0.038	-0.191	-0.224	-0.309
Non-metro, MT	-1.768	-2.687	0.046	-0.226	-0.283	-0.530
Non-metro, MN	-1.774	-2.573	-0.056	-0.157	-0.191	-0.312
Non-metro, FL	-1.777	-2.636	0.007	-0.171	-0.222	-0.469
Non-metro, WI	-1.823	-2.636	-0.034	-0.117	-0.162	-0.413
Non-metro, WV	-1.878	-2.852	-0.058	-0.219	-0.251	-0.297
Non-metro, LA	-1.879	-2.825	-0.064	-0.189	-0.222	-0.303
Non-metro, TX	-1.885	-2.844	-0.055	-0.204	-0.238	-0.318
Non-metro, MI	-1.887	-2.691	-0.058	-0.108	-0.149	-0.379
Non-metro, AZ	-1.898	-2.737	0.030	-0.154	-0.216	-0.580
Non-metro, VA	-1.910	-2.790	-0.033	-0.162	-0.207	-0.414
Non-metro, AK	-1.922	-2.579	-0.003	0.054	-0.015	-0.647
Non-metro, MS	-1.961	-2.989	-0.068	-0.221	-0.253	-0.299
Non-metro, OK	-2.009	-3.061	-0.044	-0.259	-0.297	-0.349
Non-metro, SD	-2.014	-3.120	-0.022	-0.281	-0.323	-0.393
Non-metro, MO	-2.039	-3.059	-0.030	-0.253	-0.296	-0.401
Non-metro, VT	-2.048	-2.985	0.044	-0.159	-0.230	-0.661
Non-metro, NC	-2.149	-3.110	-0.011	-0.152	-0.212	-0.554
Non-metro, NH	-2.156	-3.054	0.021	-0.080	-0.154	-0.692
Non-metro, ME	-2.176	-3.179	0.015	-0.176	-0.242	-0.615
Non-metro, GA	-2.186	-3.156	-0.048	-0.151	-0.202	-0.470
Non-metro, KY	-2.298	-3.424	-0.076	-0.199	-0.241	-0.400
Non-metro, DE	-2.313	-3.232	0.011	-0.072	-0.149	-0.721

Table A.2: List of Metropolitan and Non-Metropolitan Areas Ranked by Density

Name of Metropolitan Area	Population Density $\hat{N}^j$	Land Value $\hat{r}^j$	Quality of Life $\hat{Q}^j$	Inferred Costs Eq. (23)	Trade Productivity $\hat{A}_X^j$	Home Productivity $\hat{A}_Y^j$
Non-metro, SC	-2.318	-3.373	-0.030	-0.148	-0.209	-0.563
Non-metro, CO	-2.324	-3.231	0.086	-0.091	-0.188	-0.908
Non-metro, TN	-2.507	-3.692	-0.042	-0.195	-0.254	-0.558
Non-metro, AR	-2.577	-3.836	-0.034	-0.239	-0.300	-0.572
Non-metro, AL	-2.865	-4.198	-0.072	-0.194	-0.258	-0.597

Population density is estimated from Census data, while the last five columns come from the parametrized model. See text for estimation procedure. Inferred costs equal $(\theta_L/\phi_L)\hat{p} + (\theta_N - \phi_N\theta_L/\phi_L)\hat{w}$, as given by equation (23). Quality-of-life and inferred costs are identical to those reported in Albouy (2016).

Table A.3: Relationship between Model-Implied Variables and Data

Model-Implied Variable	Notation	Data: Wage $\hat{w}$	Data: Home Price $\hat{p}$	Data: Population Density $\hat{N}$
Quality-of-life	$\hat{Q}$	-0.480	0.325	0.000
Trade-productivity	$\hat{A}_X$	0.837	0.008	0.034
Home-productivity	$\hat{A}_Y$	0.731	-0.926	0.321
Land value	$\hat{r}$	0.491	0.316	1.374
Trade consumption	$\hat{x}$	0.478	-0.107	0.000
Home consumption	$\hat{y}$	0.483	-0.713	0.000
Land	$\hat{L}$	0.000	0.000	0.000
Capital	$\hat{K}$	0.619	0.031	0.989
Trade production	$\hat{X}$	1.117	-0.100	1.055
Home production	$\hat{Y}$	0.474	-0.705	0.999
Trade labor	$\hat{N}_X$	0.171	-0.103	1.044
Home labor	$\hat{N}_Y$	-0.436	0.270	0.892
Trade land	$\hat{L}_X$	0.510	-0.314	0.128
Home land	$\hat{L}_Y$	-0.097	0.060	-0.024
Trade capital	$\hat{K}_X$	0.838	-0.103	1.044
Home capital	$\hat{K}_Y$	0.231	0.270	0.892

Each row presents the relationship between a model-implied amenity, price, or quantity and data on wages, home prices, and population density. For example, the parametrized model implies $\hat{Q}^j = -0.480\hat{w}^j + 0.325\hat{p}^j$. All variables are measured in log differences from the national average.

Table A.4: Summary statistics, land supply

Variable	Mean	Std. Dev	N
Log urban area	6.642	1.313	276
Inferred land rent	0.249	0.956	276
Wharton Land-Use Regulatory Index (s.d.)	0	1	276
Average slope of land (s.d.)	0	1	274
Log land share (s.d.)	0	1	227
Interaction between inferred land rent and			
Wharton Land-Use Regulatory Index (s.d.)	0.563	0.911	276
Average slope of land (s.d.)	0.396	1.191	274
Log land share (s.d.)	-0.470	0.807	227

Table A.5: The Determinants of Land Supply, Inferred Land Rent Measured using Density

Dependent variable: Log urban area, square miles	(1)	(2)	(3)
Inferred land rent	0.783***	0.788***	0.845***
	(0.0620)	(0.0762)	(0.0801)
Wharton Land-Use Regulatory Index (s.d.)		0.184**	0.125
		(0.0721)	(0.0777)
Average slope of land (s.d.)		-0.253***	-0.252***
		(0.0672)	(0.0601)
Log land share (s.d.)		0.184***	0.117
		(0.0685)	(0.0710)
Interaction between inferred land rent and			
Wharton Land-Use Regulatory Index (s.d.)			0.110
			(0.0828)
Average slope of land (s.d.)			-0.0368
			(0.0515)
Log land share (s.d.)			0.128*
			(0.0744)
Constant	6.123***	6.177***	6.163***
	(0.0844)	(0.0793)	(0.0942)
Observations	276	227	227
R-squared	0.621	0.672	0.682

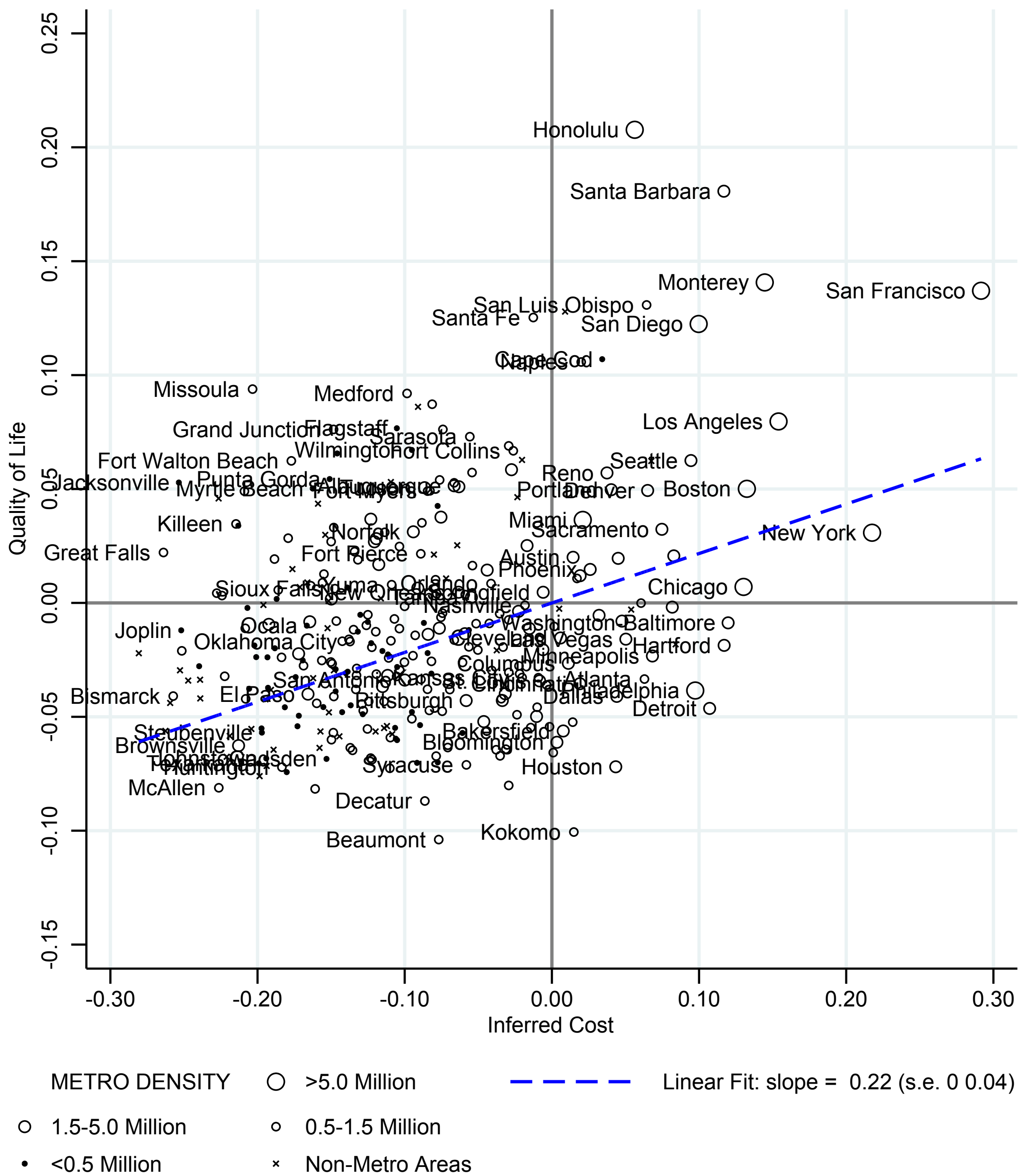
Inferred land rent is constructed using price and density data. All explanatory variables are normalized to have mean zero and standard deviation one. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table A.6: Fraction of Population Density Explained by Quality of Life, Trade-productivity, and Home Productivity, with Neutral Taxes and Feedback Effects

Geographically Neutral Taxes		No	Yes	No	Yes
Feedback Effects		No	No	Yes	Yes
		(1)	(2)	(3)	(4)
Variance/Covariance Component	Notation				
Quality-of-life	Var$(\varepsilon_{N,Q}\hat{Q})$	0.238	0.110	0.314	0.182
Trade-productivity	Var$(\varepsilon_{N,A_X}\hat{A}_X)$	0.103	0.236	0.045	0.130
Home-productivity	Var$(\varepsilon_{N,A_Y}\hat{A}_Y)$	0.439	0.302	0.446	0.383
Quality-of-life and trade-productivity	Cov$(\varepsilon_{N,Q}\hat{Q}, \varepsilon_{N,A_X}\hat{A}_X)$	0.137	0.141	0.118	0.152
Quality-of-life and home-productivity	Cov$(\varepsilon_{N,Q}\hat{Q}, \varepsilon_{N,A_Y}\hat{A}_Y)$	-0.153	-0.087	-0.036	-0.025
Trade and home-productivity	Cov$(\varepsilon_{N,A_X}\hat{A}_X, \varepsilon_{N,A_Y}\hat{A}_Y)$	0.236	0.297	0.113	0.178
Total variance of prediction		0.757	0.976	0.757	0.780

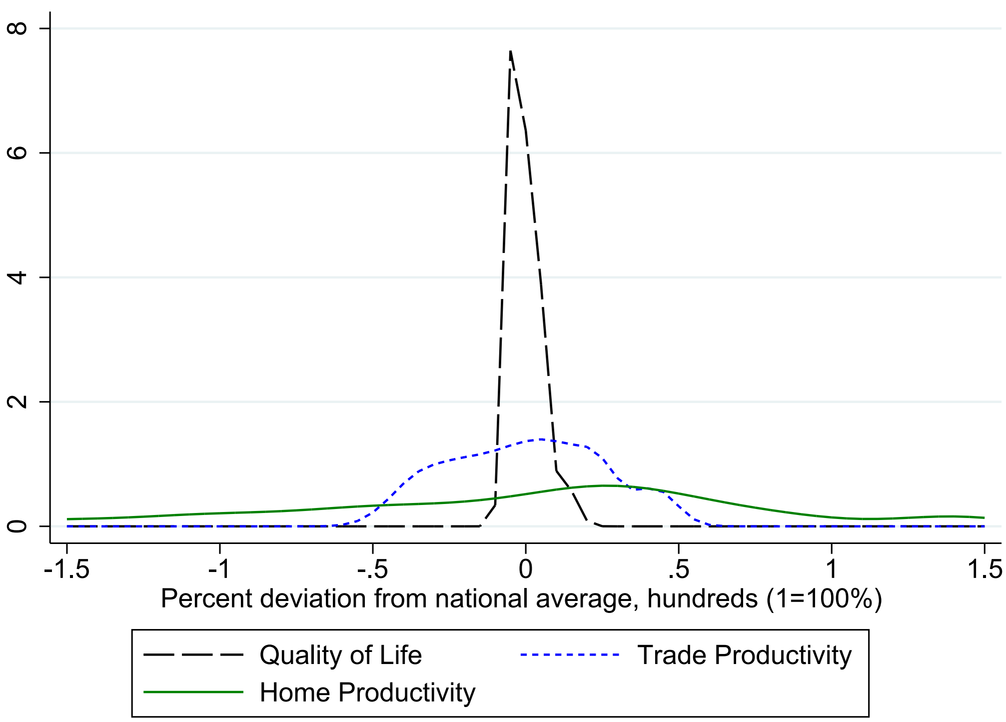
Columns 3 and 4 include both quality-of-life and trade-productivity feedback effects.

Figure A.1: Quality of Life and Inferred Costs, 2000



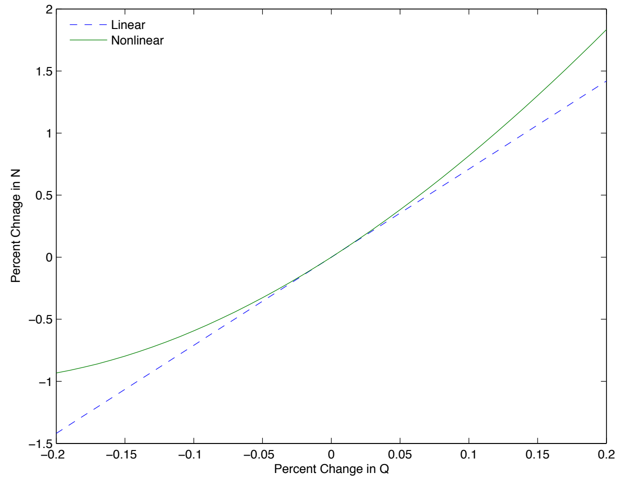
See note to figure 4 for metro density definitions.

Figure A.2: Estimated Amenity Distributions, 2000

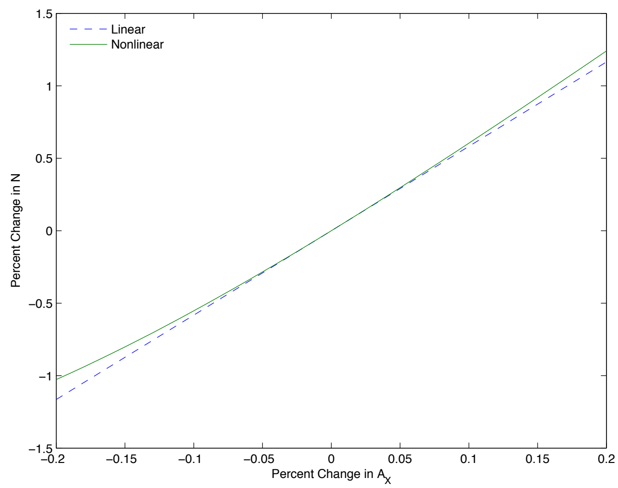


Amenities are normalized to have equal value: trade-productivity corresponds to $\hat{A}_X/s_x$ and home-productivity to $\hat{A}_Y/s_y$.

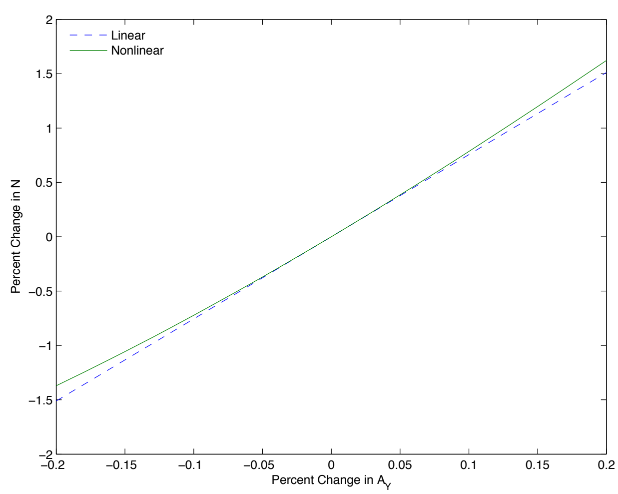
Figure A.3: Comparison of Nonlinear and Linear Model



(a) Quality of Life



(b) Trade-productivity



(c) Home Productivity