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ABSTRACT

We provide a theory of the determination of exchange rates based on capital flows in imperfect financial markets. Capital flows drive exchange rates by altering the balance sheets of financiers that bear the risks resulting from international imbalances in the demand for financial assets. Such alterations to their balance sheets cause financiers to change their required compensation for holding currency risk, thus impacting both the level and volatility of exchange rates. Our theory of exchange rate determination in imperfect financial markets not only helps rationalize the empirical disconnect between exchange rates and traditional macroeconomic fundamentals, but also has real consequences for output and risk sharing. Exchange rates are sensitive to imbalances in financial markets and seldom perform the shock absorption role that is central to traditional theoretical macroeconomic analysis. Our framework is flexible; it accommodates a number of important modeling features within an imperfect financial market model, such as non-tradables, production, money, sticky prices or wages, various forms of international pricing-to-market, and unemployment.

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I Introduction

We provide a theory of exchange rate determination based on capital flows in imperfect financial markets. In our model, exchange rates are governed by financial forces because global shifts in the demand and supply of assets result in large scale capital flows that are intermediated by the global financial system. The demand and supply of assets in different currencies and the willingness of the financial system to absorb the resulting imbalances are first order determinants of exchange rates. Despite extensive debates on these financial forces and their implications for exchange rates, there are very few tractable frameworks to provide a unified analysis of such phenomena.

Active risk taking in currency markets is highly concentrated in few large financial players.[1] These institutions range from the (former) proprietary desks and investment management arms of global investment banks such as Goldman Sachs and JP Morgan, to macro and currency hedge funds such as Soros Fund Management, to active investment managers and pension funds such as PIMCO and BlackRock. While there are certainly significant differences across these intermediaries, we stress their common characteristic of being active investors that profit from medium-term imbalances in international financial markets, often by bearing the risks (taking the other side) resulting from imbalances in currency demand due to both trade and financial flows. They also share the characteristic of being subject to financial constraints that limit their ability to take positions, based on their risk-bearing capacities and existing balance sheet risks.

Our model captures this element of reality by placing financiers at the core of exchange rate determination. In our model, financiers absorb a portion of the currency risk originated by imbalanced global capital flows. Alterations to the size and composition of financiers' balance sheets induce them to differentially price currency risk, thus affecting both the level and the volatility of exchange rates. Our theory of exchange rate determination in imperfect financial markets differs from conventional open macroeconomic model by introducing financial forces, such as portfolio flows, financiers' balance sheets and risk-bearing capacity, as first order determinants of exchange rates.

[1] Detailed data on risk taking in this international and opaque over-the-counter market are relatively scarce, particularly since a number of players, such as hedge funds, have low reporting requirements. It is precisely this nature of the market that favors specialization and concentration. Transaction volume data, however, also portray a highly concentrated market. The top 10 banks accounted for 80% of all flows in 2014, with the top two banks (Citigroup and Deutsche Bank) accounting for 32% of all flows (Euromoney (2014)). Not only are these institutions large players in currency markets, currency risk also accounts for a large fraction of their overall respective risk taking. Regulatory filings reveal that currency risk accounted for 26% to 35% of total (Stressed) Value at Risk at Deutsche Bank in 2013 and between 17% and 23% at Citigroup in the same period (Deutsche Bank (2013); Citigroup (2013)).

A number of stylized facts have emerged from the empirical analysis of international financial markets: the failure of the uncovered interest parity condition (UIP) and the associated profitable carry trade, the presence of large-scale global (gross) capital flows putting appreciation pressure on the currencies of inflow-recipient countries, the disconnect of exchange rates from macro fundamentals, the vulnerability of external-debtor countries' currencies to global financial shocks, and the impact of large scale currency interventions by governments. At the same time the global financial crisis has highlighted the importance of financial frictions not only for outcomes in financial markets but also for real outcomes such as output and risk sharing. The main purpose of this paper is to provide a tractable framework to both jointly analyze these issues (some classic, some new) and to provide a number of new insights.

Financiers actively trade currencies but have limited risk-bearing capacity: in equilibrium, a global imbalance that requires financiers to be long a currency generates an increase in the expected return of this currency. This has to occur to provide incentives to financiers to use part of their limited risk-bearing capacity to absorb the imbalance. All else equal, the currency has to depreciate today and be expected to appreciate in the future in order for financiers to earn compensation for their risk taking. This is the central exchange rate determination mechanism in the model.

Based on this framework, we analyze the importance of capital flows, i.e. demand for foreign-currency-denominated assets, in directly determining exchange rates. Whenever they are not matched globally, these global flows generate an imbalance and, via the constraints of the financiers, a direct effect on both the level and dynamics of the exchange rate. Consequently, countries that have recently received capital inflows tend to have risky currencies that depreciate if financiers' risk-bearing capacity is disrupted. Since these countries have borrowed from financiers, their currencies in equilibrium have high expected returns in order to incentivize financiers to lend. A financial disruption, by reducing financiers' risk-bearing capacity, generates an immediate currency depreciation and an expectation of further future currency appreciation in order to increase financiers' incentive to sustain the imbalance.

The model accounts for the failure of the UIP and provides a financial view of the carry trade whereby the trade performs poorly whenever adverse shocks to the financial system occur. UIP is violated because the financiers' limited risk-bearing capacity induces them to demand a currency risk premium. In this world the carry trade is profitable because, given an interest rate differential, financiers' limited risk-bearing capacity precludes them from taking enough risk to completely exploit the profitability of the trade. Similarly, financial disruptions generate a need to increase the expected returns of the carry trade: this is achieved with an immediate loss in the carry trade and an expectation of its recovery going forward.

The exchange rate is disconnected from traditional macroeconomic fundamentals such as imports, exports, consumption and output in as much as these same fundamentals correspond to different equilibrium exchange rates depending on financiers' balance sheets and risk-bearing capacity. Financiers both act as shock absorbers, by using their risk bearing capacity to accommodate flows that result from fundamental shocks, and are themselves the source of financial shocks that distort exchange rates.

The financial determination of exchange rates in imperfect financial markets has real consequences for output and risk sharing. To more fully analyze these consequences, we extend the basic model by introducing a simple model of production under both flexible and sticky prices. For example, in the presence of goods' prices that are sticky in the producers' currencies, a capital inflow or financial shock that produces an overly appreciated exchange rate causes a fall in demand for the inflow-receiving country's exports and a corresponding fall in output. This perverse effect of capital flows transmits frictions from financial markets to the real economy.

In our model, currency intervention by the government is effective because, as a capital flow, it alters the balance sheet of constrained financiers. The potency of the intervention relies entirely on the frictions; there would be no effect from the intervention absent financial imperfections. We show that a commonly-adopted policy combination of currency intervention and capital controls can be understood in our model as capital controls exacerbating financial imperfections, thus further segmenting the currency market and increasing the potency of currency intervention. We show that if a country has an overly appreciated currency and its output is demand-driven, i.e. output would increase via an increase in exports if the currency were to depreciate, then a currency intervention increases output at the cost of distorting consumption risk-sharing intertemporally.

Throughout the paper we stress tractability and make simplifying functional assumptions that make our model a convenient specification. We believe that the simple modeling we provide offers a number of insights with pencil-and-paper analysis. Of course, we also appreciate the need for optimization and general equilibrium, both of which are present in our set-up. However, we leave for the appendix and, in large part, to future research to provide deeper contracting foundations for the frictions that we study and to assess in a large-scale model their quantitative implications.

This paper is related to three broad streams of literature: early literature on portfolio balance, literature on portfolio demand in complete or incomplete markets, and literature on frictions and asset demand. Our paper is inspired by a number of ideas in the early literature on portfolio balance models by Kouri (1976) and Driskill and McCafferty (1980*a*).[2] A number of prominent economists have lamented that this earlier research effort "had its high watermark and to a large extent a terminus in Branson and Henderson (1985) handbook chapter" (see Obstfeld (2004)) and is "now largely and unjustly forgotten" (see Blanchard, Giavazzi and Sa (2005)). The literature that followed this earlier modeling effort has either focused on UIP-based analysis (Obstfeld and Rogoff (1995)) or mostly focused on currency risk premia in complete markets (Lucas, 1982; Backus, Kehoe and Kydland, 1992; Backus and Smith, 1993; Dumas, 1992; Verdelhan, 2010; Colacito and Croce, 2011; Hassan, 2013).[3] Pavlova and Rigobon (2007) analyze a real model with complete markets where countries' representative agents have logarithmic preferences affected by taste shocks similar to those considered in this paper.[4] A smaller literature has analyzed the importance of incomplete markets (for recent examples: Chari, Kehoe and McGrattan (2002); Corsetti, Dedola and Leduc (2008); Pavlova and Rigobon (2012)).

The most closely related stream of the literature is the small set of papers that focused on exchange rate modeling in the presence of frictions. One important set of papers by Jeanne and Rose (2002); Evans and Lyons (2002); Hau and Rey (2006); Bruno and Shin (2014) studies frictions in financial markets in the absence of a real side of the economy with production, imports, and exports. One other important set of papers has a very different focus: informational frictions, infrequent portfolio rebalancing, or frictions in access to domestic money/funding market. Evans and Lyons (2012) focuses on how disaggregate order flows from customers might convey information about the economy fundamentals to exchange rate market makers who observe the consolidated flow. Bacchetta and Van Wincoop (2010) studies the implications of agents that infrequently rebalance their portfolio in an OLG setting. Alvarez, Atkeson and Kehoe (2002, 2009) and Maggiori (2014) are models of exchange rates where the frictions, a form of market segmentation, are only present in the domestic

[2] An active early literature also includes: Allen and Kenen (1983); Henderson and Rogoff (1982); Dornbusch and Fischer (1980); Calvo and Rodriguez (1977); Branson, Halttunen and Masson (1979); Tobin and de Macedo (1979); Diebold and Pauly (1988); Driskill and McCafferty (1980*b*); de Macedo and Lempinen (2013). De Grauwe (1982) considers the role of the banking sector in generating portfolio demands.

[3] Among others see also: Farhi and Gabaix (2014); Martin (2011), and Stathopoulos (2012).

[4] Similar preferences are also used in Pavlova and Rigobon (2008, 2010).

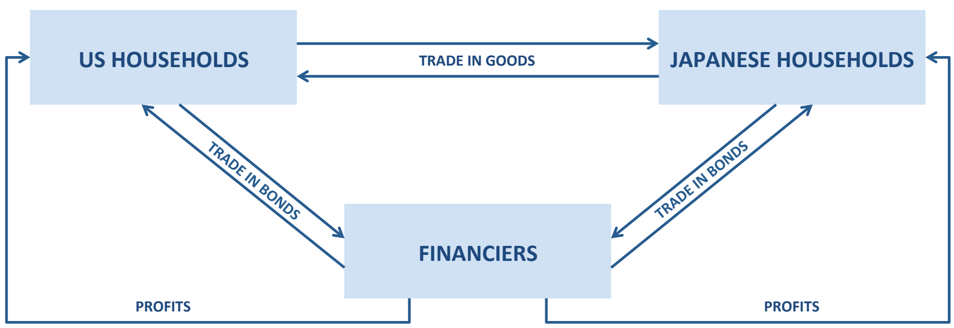
money market or funding market.

II Basic Gamma Model

Let us start with a minimalistic model of financial determination of exchange rates in imperfect financial markets. This simple real model carries most of the economic intuition and core modeling that we will then extend to more general set-ups.

Time is discrete and there are two periods: $t = 0, 1$. There are two countries, the USA and Japan, each populated by a continuum of households. Households produce, trade (internationally) in a market for goods, and invest with financiers in risk-free bonds in their domestic currency.[5] Financiers intermediate the capital flows resulting from households' investment decisions. The basic structure of the model is displayed in Figure I.

Figure I: Basic Structure of the Model



The players and structure of the flows in the goods and financial markets in the Basic Gamma Model.

Intermediation is not perfect because of the limited commitment of the financiers. The limited-commitment friction induces a downward sloping demand curve for risk taking by financiers. As a result, capital flows from households move financiers up and down their demand curve. Equilibrium is achieved by a relative price, in this case the exchange rate, adjusting so that international financial markets clear given the demand and supply of capital denominated in different currencies. In this sense, exchange rates are financially determined in an imperfect capital market.

We now describe each of the model's actors, their optimization problems, and analyze the resulting equilibrium.

[5] In the absence of a nominal side to the model, which we add in Section IV, we intentionally abuse the word "currency" to mean a claim to the numéraire of the economy, and "exchange rate" to mean the real exchange rate. Similarly we abuse the words "Dollar or Yen denominated" to mean values expressed in units of non-tradable goods in each economy.

II.A Households

Households in the US derive utility from the consumption of goods according to:

$$\theta_0 \ln C_0 + \beta \mathbb{E}\left[\theta_1 \ln C_1\right], \tag{1}$$

where C is a consumption basket defined as:

$$C_t \equiv \left[(C_{NT,t})^{\chi_t} (C_{H,t})^{a_t} (C_{F,t})^{\iota_t}\right]^{\frac{1}{\theta_t}}, \tag{2}$$

where $C_{NT,t}$ is the US consumption of its non-tradable goods, $C_{H,t}$ is the US consumption of its domestic tradable goods, and $C_{F,t}$ is the US consumption of Japanese tradable goods. We use the notation $\{\chi_t, a_t, \iota_t\}$ for non-negative, potentially stochastic, preference parameters and we define $\theta_t \equiv \chi_t + a_t + \iota_t$. The non-tradable good is the numéraire in each economy and, consequently, its price equals 1 in domestic currency ($p_{NT} = 1$).

Households can trade both tradable goods in a frictionless goods market across countries, but can only trade non-tradable goods within their domestic country. Financial markets are incomplete and each country trades a risk-free domestic currency bond. The assumption that each country only trades in its own currency bonds is made here for simplicity and to emphasize the currency mismatch that the financiers have to absorb; we relax the assumption in later sections. *Risk-free* here refers to paying one unit of non-tradable goods in all states of the world and is therefore akin to "nominally risk free".

US households' optimization problem is:

$$\max_{(C_{NT,t}, C_{H,t}, C_{F,t})_{t=0,1}} \theta_0 \ln C_0 + \beta \mathbb{E}\left[\theta_1 \ln C_1\right], \tag{3}$$

subject to (2),

$$\text{and} \quad \sum_{t=0}^{1} R^{-t}\left(Y_{NT,t} + p_{H,t} Y_{H,t}\right) = \sum_{t=0}^{1} R^{-t}\left(C_{NT,t} + p_{H,t} C_{H,t} + p_{F,t} C_{F,t}\right). \tag{4}$$

US households maximize the utility by choosing their consumption and savings in dollar bonds subject to the state-by-state dynamic budget constraint. The households' optimization problem can be divided into two separate problems. The first is a static problem, whereby households decide, given their total consumption expenditure for the period, how to allocate resources to the consumption of various goods. The second is a dynamic problem, whereby households decide intertemporally how much to save and consume.

The static utility maximization problem takes the form:

$$(5) \qquad \max_{C_{NT,t}, C_{H,t}, C_{F,t}} \quad \chi_t \ln C_{NT,t} + a_t \ln C_{H,t} + \iota_t \ln C_{F,t} + \lambda_t \left(CE_t - C_{NT,t} - p_{H,t} C_{H,t} - p_{F,t} C_{F,t} \right),$$

where CE_t is aggregate consumption expenditure, which is taken as exogenous in this static optimization problem and later endogenized in the dynamic optimization problem, λ_t is the associated Lagrange multiplier, $p_{H,t}$ is the Dollar price in the US of US tradables, and $p_{F,t}$ is the Dollar price in the US of Japanese tradables. First-order conditions imply: $\frac{\chi_t}{C_{NT,t}} = \lambda_t$, and $\frac{\iota_t}{C_{F,t}} = \lambda_t p_{F,t}$. We assume that non-tradable goods are produced by an endowment process that for simplicity follows $Y_{NT,t} = \chi_t$, unless otherwise stated. This simplifying assumption, combined with the market clearing condition for non-tradables $Y_{NT,t} = C_{NT,t}$, implies that in equilibrium $\lambda_t = 1$ in all states. The assumption, while stark, makes the analysis of the basic model most tractable by neutralizing variations in household marginal utility that are not at the core of our paper. The neutralization occurs because variation in household marginal utility is stabilized by a proportionate increase in the consumption of the non-tradable good.[6] With this assumption in hand, the Dollar value of US imports is:

$$p_{F,t} C_{F,t} = \iota_t,$$

Japanese households derive utility from consumption according to: $\theta_0^* \ln C_0^* + \beta^* \mathbb{E} \left[\theta_1^* \ln C_1^* \right]$, where starred variables denote Japanese quantities and prices. By analogy with the US case, the Japanese consumption basket is: $C_t^* \equiv \left[(C_{NT,t}^*)^{\chi_t^*} (C_{H,t}^*)^{\xi_t} (C_{F,t}^*)^{a_t^*} \right]^{\frac{1}{\theta_t^*}}$, where $\theta_t^* \equiv \chi_t^* + a_t^* + \xi_t$. The Japanese static utility maximization problem, reported for brevity in the online appendix, together with the assumption $Y_{NT,t}^* = \chi_t^*$, leads to a Yen value of US exports to Japan, $p_{H,t}^* C_{H,t}^* = \xi_t$, that is entirely analogous to the import expression derived above.

The exchange rate e_t is defined as the quantity of dollars bought by 1 yen, i.e. the strength of the Yen. Consequently, an increase in e represents a Dollar depreciation.[7] The Dollar value of US exports

[6] We stress that the assumption is one of convenience, and not necessary for the economics of the paper. Online appendix A.4 provides more general results that do not impose this assumption.

[7] In this real model, the exchange rate is related to the relative price of non-tradable goods. Online appendix A.1.D provides a detailed discussion of different exchange rate concepts in this economy including the nominal and CPI-based real exchange rate.

is: $e_t \xi_t$. US net exports, expressed in dollars, are given by: $NX_t = e_t p^*_{H,t} C^*_{H,t} - p_{F,t} C_{F,t} = \xi_t e_t - \iota_t$.[8] We collect these results in the Lemma below.

Lemma 1. (Net Exports) *Expressed in dollars, US exports to Japan are $\xi_t e_t$; US imports from Japan are ι_t; so that US net exports are $NX_t = \xi_t e_t - \iota_t$.*

Note that this result is independent of the pricing procedure (e.g. price stickiness under either producer or local currency pricing). Under producer currency pricing (PCP) and in the absence of trade costs, the US Dollar price of Japanese tradables is p_H/e, while under local currency pricing (LCP) the price is simply p^*_H.

It follows that under financial autarky, i.e. if trade has to be balanced period by period, the equilibrium exchange rate is: $e_t = \frac{\iota_t}{\xi_t}$. In financial autarky, the Dollar depreciates ($\uparrow e$) whenever US demand for Japanese goods increases ($\uparrow \iota$) or whenever Japanese demand for US goods falls ($\downarrow \xi$). This has to occur because there is no mechanism, in this case, to absorb the excess demand/supply of dollars versus yen that a non-zero trade balance would generate.

The optimization problem (3) for the intertemporal consumption-saving decision leads to a standard optimality condition (Euler equation):

$$1 = \mathbb{E}\left[\beta R \frac{U'_{1,C_{NT}}}{U'_{0,C_{NT}}}\right] = \mathbb{E}\left[\beta R \frac{\chi_1 / C_{NT,1}}{\chi_0 / C_{NT,0}}\right] = \beta R, \quad (6)$$

where $U'_{t,C_{NT}}$ is the marginal utility at time t over the consumption of non-tradables. Given our simplifying assumption that $C_{NT,t} = \chi_t$, the above Euler equation implies that $R = 1/\beta$. An entirely similar derivation yields: $R^* = 1/\beta^*$. It might appear surprising that in a model with risk averse agents the equilibrium interest rate equals the rate of time preference. Of course, this occurs here because the marginal utility of non-tradable consumption, in which the bonds are denominated, is constant in equilibrium given the assumption $C_{NT,t} = \chi_t$; so that the precautionary and intertemporal consumption smoothing desires simplify to be exactly zero.

We stress that the aim of our simplifying assumptions is to create a real structure of the basic economy that captures the main forces (demand and supply of goods), while making the real side of the economy as simple as possible. This will allow us to analytically flesh out the crucial forces of the paper in the financial markets in the next sections without carrying around a burdensome real

[8] Note that we chose the notation so that *i*mports are denoted by ι_t and e*x*ports by ξ_t.

structure. Should the reader be curious as to the robustness of our model to relaxing some of the assumptions made so far, the quick answer is that it is quite robust.[9]

II.B Financiers

Suppose that global financial markets are imbalanced, such that there is an excess supply of dollars versus yen resulting from, for example, trade or portfolio flows. Who will be willing to absorb such an imbalance by providing Japan those yen, and holding those dollars? We posit that the resulting imbalances are absorbed, at some premium, by global financiers.

We assume that there is a unit mass of global financial firms, each managed by a financier. Agents from the two countries are selected at random to run the financial firms for a single period.[10] Financiers start their jobs with no capital of their own and can trade bonds denominated in both currencies. Therefore, their balance sheet consists of q_0 dollars and $-\frac{q_0}{e_0}$ yen, where q_0 is the Dollar value of Dollar-denominated bonds the financier is long of and $-\frac{q_0}{e_0}$ the corresponding value in Yen of Yen-denominated bonds. At the end of (each) period, financiers pay their profits and losses out to the households.[11]

We assume that each financier maximizes the expected value of her firm:

$$V_0 = \mathbb{E}\left[\beta\left(R - R^*\frac{e_1}{e_0}\right)\right] q_0 = \Omega_0 q_0. \tag{7}$$

This valuation of currency trading corresponds to that of the household if they were allowed to trade optimally in foreign currency. Indeed, if US households were allowed to trade optimally Yen bonds as well as Dollar bonds we would recover the standard Euler equation:

$$0 = \mathbb{E}\left[\beta\frac{U'_{1,C_{NT}}}{U'_{0,C_{NT}}}\left(R - R^*\frac{e_1}{e_0}\right)\right] = \mathbb{E}\left[\beta\frac{\chi_1/C_{NT,1}}{\chi_0/C_{NT,0}}\left(R - R^*\frac{e_1}{e_0}\right)\right] = \mathbb{E}\left[\beta\left(R - R^*\frac{e_1}{e_0}\right)\right],$$

where the last equality follows from the assumption that $C_{NT,t} = \chi_t$ and the result that $\beta R = 1$ de-

[9] We make such robustness explicit in the online appendix.

[10] In this set-up, being a financier is an occupation for agents in the two countries rather than an entirely separate class of agents. The selection process is governed by a memoryless Poisson distribution. Of course, there are no selection issues in the one period basic economy considered here, but we proceed to describe a more general set-up that will also be used in the model extensions.

[11] An interesting literature also stresses the importance of global financial frictions for the international transmission of shocks, but does not study exchange rates: Kollmann, Enders and Müller (2011); Kollmann (2013); Dedola, Karadi and Lombardo (2013); Perri and Quadrini (2014).

rived in the previous subsection (see equation (6)). Households optimally value the currency trade according to its expected (discounted at R) excess returns. Notice that this mean-return criterion holds despite the households being risk averse. The simplification occurs because variations in marginal utility are exactly offset by variations in the relative price of non-tradable goods, so that marginal utility in terms of the numéraire (the NT good) is constant across states of the world. In the absence of frictions our financiers would simply be a veil and the optimality condition in maximization (7) would impose the household optimality criterion: $0 = \mathbb{E}\left[\beta\left(R - R^* \frac{e_1}{e_0}\right)\right]$.

To capture the role of limited financial risk-bearing capacity by the financiers we assume that in each period, after taking positions but before shocks are realized, the financier can divert a portion of the funds she intermediates. If the financier diverts the funds, her firm is unwound and the households that had lent to her recover a portion $1 - \Gamma \left|\frac{q_0}{e_0}\right|$ of their credit position $\left|\frac{q_0}{e_0}\right|$, where $\Gamma = \gamma\, var\left(e_1\right)^{\alpha}$, with $\gamma \geq 0, \alpha \geq 0$.[12] The appendix at the end of the paper provides further details and regularity conditions for this constraint. As will become clear below, our functional assumption regarding the diversion of funds is not only a convenient specification for tractability, but also stresses the idea that financiers' outside options increase in the size and volatility, or complexity, of their balance sheet. This constraint captures the relevant market practice in financial institutions whereby risk taking is limited not only by the overall size of the positions, position limits, but also by their expected riskiness, often measured by their variance. It is outside the scope of this paper to provide deeper foundations for this constraint. The reader can think of it as a convenient specification of a more complicated contracting problem.[13] Since creditors, when lending to the financier, correctly anticipate the incentives of the financier to divert funds, the financier is subject to a credit constraint of the form:

$$\underbrace{\frac{V_0}{e_0}}_{\substack{\text{Intermediary Value}\\ \text{in yen}}} \geq \underbrace{\left|\frac{q_0}{e_0}\right|}_{\substack{\text{Total}\\ \text{Claims}}} \underbrace{\Gamma\left|\frac{q_0}{e_0}\right|}_{\substack{\text{Diverted}\\ \text{Portion}}} = \underbrace{\Gamma\left(\frac{q_0}{e_0}\right)^2}_{\substack{\text{Total divertable}\\ \text{Funds}}}. \tag{8}$$

Limited commitment constraints in a similar spirit have been popular in the literature; for earlier use

[12]Given that the balance sheet consists of q_0 dollars and $-\frac{q_0}{e_0}$ yen, the Yen value of the financier's liabilities is always equal to $\left|\frac{q_0}{e_0}\right|$, irrespective of whether q_0 is positive or negative; hence the use of absolute value in the text above. More formally, the financier's creditors can recover a Yen value equal to: $\max\left(1 - \Gamma\left|\frac{q_0}{e_0}\right|, 0\right)\left|\frac{q_0}{e_0}\right|$.

[13]Such foundations could potentially be achieved in models of financial complexity where bigger and riskier balance sheets lead to more complex positions. In turn, these more complex positions are more difficult and costly for creditors to unwind when recovering their funds in case of a financier's default.

as well as foundations see among others: Caballero and Krishnamurthy (2001); Kiyotaki and Moore (1997); Hart and Moore (1994), and Hart (1995). Here we follow most closely the formulation in Gertler and Kiyotaki (2010) and Maggiori (2014).[14]

The constrained optimization problem of the financier is:

$$\max_{q_0} \; V_0 = \mathbb{E}\left[\beta\left(R - R^*\frac{e_1}{e_0}\right)\right] q_0, \qquad \text{subject to} \quad V_0 \geq \Gamma\frac{q_0^2}{e_0}. \tag{9}$$

Since the value of the financier's firm is linear in the position q_0, while the right hand side of the constraint is convex in q_0, the constraint always binds.[15] Substituting the firm's value into the constraint and re-arranging (using $R = 1/\beta$), we find: $q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right]$. Integrating the above demand function over the unit mass of financiers yields the aggregate financiers' demand for assets: $Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right]$. We collect this result in the Lemma below.

Lemma 2. (Financiers' downward sloping demand for dollars) *The financiers' constrained optimization problem implies that the aggregate financial sector's optimal demand for Dollar bonds versus Yen bonds follows:*

$$Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right]. \tag{10}$$

where

$$\Gamma = \gamma\left(var\left(e_1\right)\right)^{\alpha}. \tag{11}$$

The demand for dollars decreases in the strength of the dollar (i.e. increases in e_0), controlling for the future value of the Dollar (i.e. controlling for e_1). Notice that Γ governs the ability of financiers to bear risks; hence in the rest of the paper we refer to Γ as the financiers' risk bearing capacity. The higher Γ, the lower the financiers' risk bearing capacity, the steeper their demand curve, and the more segmented the asset market. To understand the behavior of this demand, let us consider two polar opposite cases. When $\Gamma = 0$, financiers are able to absorb any imbalances, i.e. they want to

[14] We generalize these constraints by studying cases where the outside option is directly increasing in the size of the balance sheet and its variance. Adrian and Shin (2013) provide foundations and empirical evidence for a value-at-risk constraint that shares some of the properties of our constraint above.

[15] Intuitively, given any non-zero expected excess return in the currency market, the financier will want to either borrow or lend as much as possible in Dollar and Yen bonds. The constraint limits the maximum position and therefore binds. We make the very mild assumption that the model parameters always imply: $\Omega_0 \geq -1$. That is, we assume that the expected excess returns from currency speculation never exceed 100% in absolute value. This bound is several order of magnitudes greater than the expected returns in the data (of the order of 0-6%) and has no economic bearing on our model.

take infinite positions whenever there is a non-zero expected excess return in currency markets. So uncovered interest rate parity (UIP) holds: $\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right] = 0$. When $\Gamma \uparrow \infty$, then $Q_0 = 0$; financiers are unwilling to absorb any imbalances, i.e. they do not want to take any positions, no matter what the expected returns from risk-taking. In the intermediate cases ($0 < \Gamma < \infty$) the model endogenously generates a deviation from UIP and relates it to financiers' risk taking. On the contrary, since the covered interest rate parity (CIP) condition is an arbitrage involving no risk it is always satisfied. Similarly the model smoothly converges to the frictionless benchmark ($\Gamma \downarrow 0$) as the economy becomes deterministic ($var(e_1) \downarrow 0$). Section III.A studies the carry trade and provides further analysis on UIP and CIP.

Since Γ, the financiers' risk bearing capacity, plays a crucial role in our theory, we refer hereafter to the setup described so far as the *basic Gamma model*. In many instances, like the one above, it is most intuitive to consider comparative statics on Γ rather than its subcomponents, and we do so for the remainder of the paper; in some instances it is interesting to consider the effect of each subcomponent γ and $var(e_1)$ separately.[16]

We stress that the above demand function captures the spirit of international financial intermediation by providing a simple and tractable specification for the constrained portfolio problem that generates the demand function that has been central to the limits of arbitrage theory pioneered by De Long et al. (1990*a*,*b*), Shleifer and Vishny (1997), and Gromb and Vayanos (2002). We follow the pragmatic tradition of macroeconomics and frictional finance, and we take as given the prevalence of frictions and short-term debt in different currencies, and proceed to analyze their equilibrium implications. This direct approach to modeling financial imperfections has a long standing tradition and has proved very fruitful with recent contributions by Kiyotaki and Moore (1997); Gromb and Vayanos (2002); Mendoza, Quadrini and Rıos-Rull (2009); Mendoza (2010); Gertler and Kiyotaki (2010); Garleanu and Pedersen (2011); Perri and Quadrini (2014).[17]

For simplicity, we assume (for now and for much of this paper) that financiers rebate their profits and losses to the Japanese households, not the US ones. This asymmetry gives much tractability to the model, at fairly little cost to the economics.[18]

[16]The reader is encouraged either to intuitively consider the case $\alpha = 0$, or to follow the formal proofs that show the sign of the comparative statics to be invariant in Γ and γ.

[17]Even in the most recent macro-finance literature in closed economy, intense foundations of the contracting environment have either been excluded or relegated to separate companion pieces (Brunnermeier and Sannikov (2014); He and Krishnamurthy (2013)). See Duffie (2010) for an overview.

[18]For completeness, note that this assumption had already been implicitly made in deriving the US households' inter-

Before moving to the equilibrium, note that we are modeling the ability of financiers to bear substantial risks over a horizon that ranges from a quarter to a few years. Our model is silent on the high frequency market-making activities of currency desks in investment banks. To make this distinction intuitive, let us consider that the typical daily volume of foreign exchange transactions is estimated to be $5.3 trillion.[19] This trading is highly concentrated among the market making desks of banks and is the subject of attention in the market microstructure literature pioneered by Evans and Lyons (2002). While these microstructure effects are interesting, we completely abstract away from these activities by assuming that there is instantaneous and perfect risk sharing across financiers, so that any trade that matches is executed frictionlessly and nets out. We are only concerned with the ultimate risk, most certainly a small fraction of the total trading volume, which financiers have to bear over quarters and years because households' demand is unbalanced.[20]

II.C Equilibrium Exchange Rate

Recall that for simplicity we are for now only considering imbalances resulting from trade flows (imbalances from portfolio flows will soon follow). The key equations of the model are the financiers' demand:

$$Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1 \frac{R^*}{R}\right], \tag{12}$$

and the equilibrium "flow" demand for dollars in the Dollar-Yen market at times $t = 0, 1$:

$$\xi_0 e_0 - \iota_0 + Q_0 = 0, \tag{13}$$

$$\xi_1 e_1 - \iota_1 - R Q_0 = 0. \tag{14}$$

temporal budget constraint in equation (4). This assumption is relaxed in the online appendix, where we solve for general and symmetric payoff functions numerically.

[19]Source: Bank of International Settlements (2013).

[20]This is consistent with evidence that market-making desks in large investment banks, for example Goldman Sachs, might intermediate very large volumes on a daily basis but are almost always carrying no residual risk at the end of the business day. In contrast, proprietary trading desks (before recent changes in legislation) or investment management divisions of the same investment banks carry substantial amounts of risk over horizons ranging from a quarter to a few years. These investment activities are the focus of this paper. Similarly, our financiers capture the risk-taking activities of hedge funds and investment managers that have no market making interests and are therefore not the center of attention in the microstructure literature.

Equation (13) is the market clearing equation for the Dollar against Yen market at time zero. It states that the net demand for Dollar against Yen has to be zero for the market to clear. The net demand has two components: $\xi_0 e_0 - \iota_0$, from US net exports, and Q_0, from financiers. Recall that we assume that US households do not hold any currency exposure: they convert their Japanese sales of ξ_0 yen into dollars, for a demand $\xi_0 e_0$ of dollars. Likewise, Japanese households have ι_0 dollars worth of exports to the US and sell them, as they only keep Yen balances.[21] At time one, equation (14) shows that the same net-export channel generates a demand for dollars of $\xi_1 e_1 - \iota_1$; while the financiers need to sell their dollar position RQ_0 that has accrued interest at rate R.[22] We now explore the equilibrium exchange rate in this simple setup.

Equilibrium exchange rate: a first pass To streamline the algebra and concentrate on the key economic content, we assume for now that $\beta = \beta^* = 1$, which implies $R = R^* = 1$, and that $\xi_t = 1$ for $t = 0, 1$. Adding equations (13) and (14) yields the US external intertemporal budget constraint:

$$e_1 + e_0 = \iota_0 + \iota_1. \tag{15}$$

Taking expectations on both sides: $\mathbb{E}[e_1] = \iota_0 + \mathbb{E}[\iota_1] - e_0$. From the financiers' demand equation we have:

$$\mathbb{E}[e_1] = e_0 - \Gamma Q_0 = e_0 - \Gamma(\iota_0 - e_0) = (1+\Gamma) e_0 - \Gamma \iota_0,$$

where the second equality follows from equation (13). Equating the two expressions for the time-one expected exchange rate, we have:

$$\mathbb{E}[e_1] = \iota_0 + \mathbb{E}[\iota_1] - e_0 = (1+\Gamma) e_0 - \Gamma \iota_0.$$

Solving this linear equation for the exchange rate at time zero, we conclude:

$$e_0 = \frac{(1+\Gamma)\iota_0 + \mathbb{E}[\iota_1]}{2+\Gamma}.$$

[21] These assumptions are later relaxed in Section II.D and in the online appendix where households are allowed to have (limited) foreign currency positions.

[22] At the end of period 0, the financiers own Q_0 dollars and $-Q_0/e_0$ yen. Therefore, at the beginning of period one, they hold RQ_0 dollars and $-R^* Q_0/e_0$ yen. At time one, they unwind their positions and give the net profits to their principals, which we assume for simplicity to be the Japanese households. Hence they sell RQ_0 dollars in the Dollar-Yen market at time one.

We define $\{X\} \equiv X - \mathbb{E}[X]$ to be the innovation to a random variable X. Then, the exchange rate at time $t = 1$ is:

$$\begin{aligned} e_1 &= \iota_0 + \iota_1 - e_0 = \iota_0 + \mathbb{E}[\iota_1] + \{\iota_1\} - e_0 \\ &= \{\iota_1\} + \iota_0 + \mathbb{E}[\iota_1] - \frac{(1+\Gamma)\,\iota_0 + \mathbb{E}[\iota_1]}{2+\Gamma} = \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\,\mathbb{E}[\iota_1]}{2+\Gamma}. \end{aligned}$$

This implies that $var\,(e_1) = var\,(\iota_1)$, so that, by (11), $\Gamma = \gamma\, var\,(\iota_1)^{\alpha}$.

We collect these results in the Proposition below.

Proposition 1. (Basic Gamma equilibrium exchange rate) *Assume that $\xi_t = 1$ for $t = 0, 1$, and that interest rates are zero in both countries. The exchange rate follows:*

$$(16) \qquad \begin{aligned} e_0 &= \frac{(1+\Gamma)\,\iota_0 + \mathbb{E}[\iota_1]}{2+\Gamma}, \\ e_1 &= \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\,\mathbb{E}[\iota_1]}{2+\Gamma}, \end{aligned}$$

where $\{\iota_1\}$ is the time-one import shock. The expected Dollar appreciation is: $\mathbb{E}\left[\frac{e_0 - e_1}{e_0}\right] = \frac{\Gamma(\iota_0 - \mathbb{E}[\iota_1])}{(1+\Gamma)\iota_0 + \mathbb{E}[\iota_1]}$. Furthermore, $\Gamma = \gamma\, var\,(\iota_1)^{\alpha}$.

Depending on Γ, the time-zero exchange rate varies between two polar opposites: the UIP-based and the financial-autarky exchange rates, respectively. Both extremes are important benchmarks of open economy analysis, and the choice of Γ allows us to modulate our model between these two useful benchmarks. $\Gamma \uparrow \infty$ results in $e_0 = \frac{\iota_0}{\xi_0}$, which we have shown in Section II.A to be the financial autarky value of the exchange rate. Intuitively, financiers have so little risk-bearing capacity that no financial flows can occur between countries and, therefore, trade has to be balanced period by period. When $\Gamma = 0$, UIP holds and we obtain $e_0 = \frac{\iota_0 + \mathbb{E}[\iota_1]}{2}$. Intuitively, financiers are so relaxed about risk taking that they are willing to take infinite positions in currencies whenever there is a positive expected excess return from doing so. UIP only imposes a constant exchange rate in expectation $\mathbb{E}[e_1] = e_0$; the level of the exchange rate is then obtained by additionally using the inter-temporal budget constraint in equation (15).

To further understand the effect of Γ, notice that at the end of period 0 (say, time 0^+), the US net foreign asset (NFA) position is $N_{0^+} = \xi_0 e_0 - \iota_0 = \frac{\mathbb{E}[\iota_1] - \iota_0}{2+\Gamma}$. Therefore, the US has positive NFA at $t = 0^+$ *iff* $\iota_0 < \mathbb{E}\,[\iota_1]$. If the US has a positive NFA position, then financiers are long the Yen and short the Dollar. For financiers to bear this risk, they require a compensation: the Yen needs to appreciate in expectation. The required appreciation is generated by making the Yen weaker at time zero. The magnitude of the effect depends on the extent of the financiers' risk bearing capacity (Γ), as formally shown here by taking partial derivatives: $\frac{\partial e_0}{\partial \Gamma} = \frac{\iota_0 - \mathbb{E}[\iota_1]}{(2+\Gamma)^2} = \frac{-N_{0+}}{2+\Gamma}$. We collect the result in the Proposition below.

Proposition 2. (Effect of financial disruptions on the exchange rate) *In the basic Gamma model, we have:* $\frac{\gamma}{\Gamma}\frac{de_0}{d\gamma} = \frac{\partial e_0}{\partial \Gamma} = \frac{-N_{0+}}{2+\Gamma}$, *where* $N_{0^+} = \frac{\mathbb{E}[\iota_1]-\iota_0}{2+\Gamma}$ *is the US net foreign asset (NFA) position. When there is a financial disruption* ($\uparrow \gamma, \uparrow \Gamma$)*, countries that are net external debtors* ($N_{0^+} < 0$) *experience a currency depreciation* ($\uparrow e$)*, while the opposite is true for net-creditor countries.*

Intuitively, net external-debtor countries have borrowed from the world financial system, thus generating a long exposure for financiers to their currencies. Should the financial system's risk bearing capacity be disrupted, these currencies would depreciate to compensate financiers for the increased (perceived) risk. This modeling formalizes a number of external crises where broadly defined global risk aversion shocks, embodied here in Γ, caused large depreciations of the currencies of countries that had recently experienced large capital inflows. Della Corte, Riddiough and Sarno (2014) confirm our theoretical prediction in the data. They show that net-debtor countries' currencies have higher returns than net-creditors' currencies, tend to be on the receiving end of carry trade related speculative flows, and depreciate when financial disruptions occur. In this basic model the entire external balance of a country is absorbed by the financier; we will relax this shortly by providing a distinct role between Q and the external balance. Here we clarify that the driving force behind the result in Proposition 2 is the position of the financiers, i.e. what matters for the effect of an increase in Γ on the exchange rate if whether Q is positive on negative. The proposition stresses the idea that financiers are more likely to be long the currency of debtor countries since these countries have borrowed from the world financial system.[23]

To illustrate how the results derived so far readily extend to more general cases, we report below expressions allowing for stochastic US export shocks ξ_t, as well as non-zero interest rates. Several more extensions can be found in Section IV and the online appendix.

[23]In mapping the proposition into the data one can think that the net foreign asset positions are correlated with Q, but the correlation can be less than perfect, with instances like the US where the two might be substantially different (see Shin (2012); Maggiori (2014)).

Proposition 3. *With general trade shocks and interest rates (ι_t, ξ_t, R, R^*), the values of exchange rate at times $t = 0, 1$ are:*

$$e_0 = \frac{\mathbb{E}\left[\frac{\iota_0 + \frac{\iota_1}{R}}{\xi_1}\right] + \frac{\Gamma \iota_0}{R^*}}{\mathbb{E}\left[\frac{\xi_0 + \frac{\xi_1}{R^*}}{\xi_1}\right] + \frac{\Gamma \xi_0}{R^*}}; \qquad e_1 = \mathbb{E}\left[e_1\right] + \{e_1\}, \tag{17}$$

where we again denote by $\{X\} \equiv X - \mathbb{E}\left[X\right]$ the innovation to a random variable X, and

$$\mathbb{E}\left[e_1\right] = \frac{R}{R^*} \frac{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\iota_0 + \frac{\iota_1}{R}\right)\right] + \Gamma \xi_0 \mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0 + \frac{\xi_1}{R^*}\right)\right] + \Gamma \xi_0}, \tag{18}$$

$$\{e_1\} = \left\{\frac{\iota_1}{\xi_1}\right\} + R \frac{\iota_0 - \mathbb{E}\left[\xi_0 \frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0 + \frac{\xi_1}{R^*}\right)\right] + \Gamma \xi_0} \left\{\frac{1}{\xi_1}\right\}. \tag{19}$$

When ξ_1 is deterministic, $\Gamma = \gamma var(\frac{\iota_1}{\xi_1})^{\alpha}$. The proof of this Proposition reports the corresponding solution for Γ when ξ_1 is stochastic.

II.D *The Impact of Portfolio Flows*

We now further illustrate how the *supply and demand of assets do matter for the financial determination of the exchange rate*. We stress the importance of portfolio flows in addition, and perhaps more importantly than, trade flows for our framework. The basic model so far has focused on current account, or net foreign asset, based flows; we introduce here pure portfolio flows that alter the countries' gross external positions. We focus here on the simplest form of portfolio flows from households, not so much for their complete realism, but because they allow for the sharpest analysis of the main forces of the model. The online appendix extends this minimalistic section to more general flows.

II.D.1 Asset Flows Matter in the Gamma Model

Consider the case where Japanese households have, at time zero, an inelastic demand (e.g. some noise trading) f^* of Dollar bonds funded by an offsetting position $-f^*/e_0$ in Yen bonds. Both transactions face the financiers as counterparties.

While we take these flows as exogenous, they can be motivated as a liquidity shock, or perhaps as a decision resulting from bounded rationality or portfolio delegation. Technically, the maximization problem for the Japanese household is the one written before, where the portfolio flow is not a decision variable coming from a maximization, but is simply an exogenous action.[24]

The flow equations are now given by:

$$\xi_0 e_0 - \iota_0 + Q_0 + f^* = 0, \qquad \xi_1 e_1 - \iota_1 - RQ_0 - Rf^* = 0. \tag{20}$$

The financiers' demand is still $Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - \frac{R^*}{R}e_1\right]$. The equilibrium exchange rate is derived in the Proposition below.

Proposition 4. (Gross capital flows and exchange rates) *Assume $\xi_t = R = R^* = 1$ for $t = 0, 1$. With an inelastic time-zero additional demand f^* for Dollar bonds by Japanese households who correspondingly sell $-f^*/e_0$ of Yen bonds, the exchange rates at times $t = 0, 1$ are:*

$$e_0 = \frac{(1+\Gamma)\,\iota_0 + \mathbb{E}[\iota_1] - \Gamma f^*}{2+\Gamma}; \qquad e_1 = \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\,\mathbb{E}[\iota_1] + \Gamma f^*}{2+\Gamma}.$$

Hence, additional demand f^* for dollars at time zero induces a Dollar appreciation at time zero, and subsequent depreciation at time one. However, the time-average value of the Dollar is unchanged: $e_0 + e_1 = \iota_0 + \iota_1$, independently of f^*. Furthermore, $\Gamma = \gamma\, var(\iota_1)^\alpha$.

Proof. Define: $\tilde{\iota}_0 \equiv \iota_0 - f^*$, and $\tilde{\iota}_1 \equiv \iota_1 + f^*$. Given equations (20), our "tilde" economy is isomorphic to the basic economy considered in equations (13) and (14). For instance, import demands are now $\tilde{\iota}_t$ rather than ι_t. Hence, Proposition 1 applies to this "tilde" economy, thus implying that:

$$\begin{aligned} e_0 &= \frac{(1+\Gamma)\,\tilde{\iota}_0 + \mathbb{E}[\tilde{\iota}_1]}{2+\Gamma} = \frac{(1+\Gamma)\,\iota_0 + \mathbb{E}[\iota_1] - \Gamma f^*}{2+\Gamma}, \\ e_1 &= \{\tilde{\iota}_1\} + \frac{\tilde{\iota}_0 + (1+\Gamma)\,\mathbb{E}[\tilde{\iota}_1]}{2+\Gamma} = \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\,\mathbb{E}[\iota_1] + \Gamma f^*}{2+\Gamma}. \quad \square \end{aligned}$$

An increase in Japanese demand for Dollar bonds needs to be absorbed by financiers, who correspondingly need to sell Dollar bonds and buy Yen bonds. To induce financiers to provide the desired bonds, the Dollar needs to appreciate on impact *as a result of the capital flow*, in order to then be expected to depreciate, thus generating an expected gain for the financiers' short Dollar positions. This example emphasizes that our model is an elementary one where a relative price, the exchange rate,

[24] The Japanese households' state-by-state budget constraint is $\sum_{t=0}^{1} \frac{Y^*_{NT,t} + p^*_{F,t} Y_{F,t} + \pi^*_t}{R^{*t}} = \sum_{t=0}^{1} \frac{C^*_{NT,t} + p^*_{H,t} C^*_{H,t} + p^*_{F,t} C^*_{F,t}}{R^{*t}}$, where π^*_t are FX trading profit to the Japanese, so $\pi^*_0 = 0$, $\pi^*_1 = (f^* + Q_0)(R - R^* \frac{e_1}{e_0})/e_1$ (recall that the financiers rebate their profits to the Japanese).

has to move in order to equate the supply and demand of two assets, Yen and Dollar bonds. The capital flows considered in this section are gross flows that do not alter the net foreign asset position, thus introducing a first example of the distinct role for the financiers' balance sheet from the country net foreign asset position. In the data gross flows are much larger than net flows and we provide a reason why they play an important role in determining the exchange rate.[25]

This framework can analyze concrete situations, such as the recent large scale capital flows from developed countries into emerging market local-currency bond markets, say by US investors into Brazilian Real bonds, that put upward pressure on the receiving countries' currencies. While such flows and their impact on currencies have been paramount in the logic of market participants and policy makers, they had thus far proven elusive in a formal theoretical analysis.

Hau, Massa and Peress (2010) provide direct evidence that plausibly exogenous capital flows impact the exchange rate in a manner consistent with the Gamma model. They show that, following a restating of the weights of the MSCI World Equity Index, countries that as a result experienced capital inflows (because their weight in the index increased) saw their currencies appreciate.

To stress the difference between our basic Gamma model of the financial determination of exchange rates in imperfect financial markets and the traditional macroeconomic framework, we next illustrate two polar cases that have been popular in the previous literature: the UIP-based exchange rate, and the complete market exchange rate.

Financial Flows in a UIP Model. Much of the now classic international macroeconomic analysis spurred by Dornbusch (1976) and Obstfeld and Rogoff (1995) either directly assumes that UIP holds or effectively imposes it by solving a first order linearization of the model.[26] The closest analog to this literature in the basic Gamma model is the case where $\Gamma = 0$, such that UIP holds by assumption. In this world, financiers are so relaxed, i.e. their risk bearing capacity is so ample, about supplying liquidity to satisfy shifts in the world demand for assets that such shifts have no impact on expected returns. Consider the example of US investors suddenly wanting to buy Brazilian Real bonds; in this case financiers would simply take the other side of the investors' portfolio demand with no effect on the exchange rate between the Dollar and the Real. In fact, equation (21) confirms that if $\Gamma = 0$, then

[25] One could extend the distinction between country level positions and financiers' balance sheet further by modeling situations where not all gross flows are stuck, either temporarily or permanently, on the balance sheet of the financiers.

[26] Intuitively, a first order linearization imposes certainty equivalence on the model and therefore kills any risk premia such as those that could generate a deviation from UIP.

portfolio flow f^* has no impact on the equilibrium exchange rate.[27]

Financial Flows in a Complete Market Model. Another strand of the literature has analyzed risk premia predominantly under complete markets. We now show that the exchange rate in a setup with complete markets (and no frictions) but otherwise identical to ours is *constant*, and therefore trivially not affected by the flows.

Lemma 3. (Complete Markets) *In an economy identical to the set-up of the basic Gamma model, other than the fact that financial markets are complete and frictionless, the equilibrium exchange rate is constant: $e_t = \nu$, where ν is the relative Negishi weight of Japan.*

Here, we only sketch the logic and the main equations; a full treatment is relegated to the online appendix. Under complete markets, the marginal utility of US and Japanese agents must be equal when expressed in a common currency. Intuitively, the full risk sharing that occurs under complete markets calls for Japan and the US to have the same marginal benefit from consuming an extra unit of non-tradables. In our set-up, this risk sharing condition takes a simple form: $\frac{\chi_t / C_{NT,t}}{\chi_t^* / C_{NT,t}^*} e_t = \nu$, where ν is a constant.[28] Simple substitution of the conditions $C_{NT,t} = \chi_t$ and $C_{NT,t}^* = \chi_t^*$ shows that $e_t = \nu$, i.e. the exchange rate is constant.[29]

II.D.2 Flows, not just Stocks, Matter in the Gamma model

In frictionless models only stocks matter, not flows per se. In the Gamma model, instead, flows per se matter. This is a distinctive feature of our model. To illustrate this, consider the case where the US has an exogenous Dollar-denominated debt toward Japan, equal to D_0 due at time zero, and D_1 due at time one.[30] For simplicity, assume $\beta = \beta^* = R = R^* = \xi_t = 1$ for $t = 0, 1$. Hence, total debt is $D_0 + D_1$. The flow equations now are:

$$e_0 - \iota_0 - D_0 + Q_0 = 0; \qquad e_1 - \iota_1 - D_1 - Q_0 = 0.$$

[27]These gross flows do not play a role in determining the exchange rate even in models, for example Schmitt-Grohé and Uribe (2003), that assume reduced-form deviations from UIP to be convex functions of the net foreign asset position.

[28]Formally, the constant is the relative Pareto weight assigned to Japan in the planner's problem that solves for complete-market allocations.

[29]The irrelevance of the f gross flows generalizes also to complete, and incomplete, market models where the exchange rate is not constant and the presence of a risk premium makes the two currencies imperfect substitutes. Intuitively in these models the state variables are ratios of stocks of assets, such as wealth, and since these gross flows do not alter the value of such stocks, they have no equilibrium effects because the agents can frictionlessly unwind them. In our model they have effects because these flows alter the balance sheet of constrained financiers.

[30]Hence, the new budget constraint is $\sum_{t=0}^{1} R^{-t} \left(Y_{NT,t} + p_{H,t} Y_{H,t} - D_t \right) = \sum_{t=0}^{1} R^{-t} \left(C_{NT,t} + p_{H,t} C_{H,t} + p_{F,t} C_{F,t} \right).$

The exchange rate at time zero is:

$$e_0 = \frac{(1+\Gamma)\,\iota_0 + \mathbb{E}\left[\iota_1\right]}{2+\Gamma} + \frac{(1+\Gamma)\,D_0 + D_1}{2+\Gamma}.$$

Hence, when finance is imperfect ($\Gamma > 0$), both the timing of debt flows, as indicated by the term $(1+\Gamma)\,D_0 + D_1$, and the total stock of debt $(D_0 + D_1)$ matter in determining exchange rates. The early flow, D_0, receives a higher weight $\left(\frac{1+\Gamma}{2+\Gamma}\right)$ than the late flow, D_1, $\left(\frac{1}{2+\Gamma}\right)$. In sum, flows, not just stocks, matter for exchange rate determination.

To highlight the contrast, let us parametrize the debt repayments as: $D_0 = F$ and $D_1 = -F + S$. The parameter F alters the flow of debt repayment at time zero, but leaves the total stock of debt ($D_0 + D_1 = S$) unchanged. The parameter S, instead, alters the total stock of debt, but does not affect the flow of repayment at time zero. We note that: $\frac{de_0}{dF} = \frac{\Gamma}{2+\Gamma}$, and $\frac{de_0}{dS} = \frac{1}{2+\Gamma}$. When $\Gamma \uparrow \infty$, only flows affect the exchange rate at time zero; this is so even when flows leave the total stocks unchanged $\left(\frac{de_0}{dF} > 0 = \frac{de_0}{dS}\right)$. In contrast, when finance is frictionless ($\Gamma = 0$), flows have no impact on the exchange rate, and only stocks matter $\left(\frac{de_0}{dF} = 0 < \frac{de_0}{dS}\right)$. We collect the result in the Proposition below.

Proposition 5. (Stock Vs flow matters in the Gamma model) *Flows matter for the exchange rate when* $\Gamma > 0$*. In the limit when financiers have no risk bearing capacity* ($\Gamma \uparrow \infty$)*, only flows matter. When risk bearing capacity is very ample* ($\Gamma = 0$)*, only stocks matter.*

II.D.3 The Exchange Rate Disconnect

The Meese and Rogoff (1983) result on the inability of economic fundamentals such as output, inflation, exports and imports to predict, or even contemporaneously co-move with, exchange rates has had a chilling and long-lasting effect on theoretical research in the field (see Obstfeld and Rogoff (2001)).[31] The Gamma model helps to reconcile the disconnect by introducing financial forces, both the risk bearing capacity Γ and the balance sheet Q, as determinants of exchange rates. Intuitively a disconnect occurs because economies with identical fundamentals feature different equilibrium exchange rates depending on the incentives of the financiers to hold the resulting (gross) global imbalances.

[31] Some forecastability of exchange rates using traditional fundamentals appears to occur at very-long horizons (e.g. 10 years) in Mark (1995) or for specific currencies, such as the US Dollar, using transformations of the balance of payments data (Gourinchas and Rey (2007*b*); Gourinchas, Govillot and Rey (2010)).

Recently new evidence has been building in favor of these new financial channels. In addition to the instrumental variable approach in Hau, Massa and Peress (2010) discussed earlier, Froot and Ramadorai (2005); Adrian, Etula and Groen (2011); Hong and Yogo (2012); Kim, Liao and Tornell (2014), and Adrian, Etula and Shin (2014) find that flows, financial conditions, and financiers' positions provide information about expected currency returns. Froot and Ramadorai (2005) show that medium-term variation in expected currency returns is mostly associated with capital flows, while long-term variation is more strongly associated with macroeconomic fundamentals. Hong and Yogo (2012) show that speculators' positions in the futures currency market contain information that is useful, beyond the interest rate differential, to forecast future currency returns. Adrian, Etula and Groen (2011); Adrian, Etula and Shin (2014) show that empirical proxies for financial conditions and the tightness of financiers' constraints help forecast both currency returns and exchange rates. Kim, Liao and Tornell (2014) show that information extracted from the speculators' positions in the futures currency market helps to predict exchange rate changes at horizons between 6 and 12 months.

The model can also help to rationalize the co-movement across bilateral exchange rates and between exchange rates and other asset classes. Intuitively, this occurs because all these assets are traded by financiers and are therefore affected to some degree by the same financial forces. Verdelhan (2013) shows that there is substantial co-movement between bilateral exchange rates both in developed and emerging economies, while Dumas and Solnik (1995); Hau and Rey (2006); Farhi and Gabaix (2014); Verdelhan (2013); Lettau, Maggiori and Weber (2014) link movements in exchange rates to movements in equity markets.

II.E Closing the Economy: Endowments, Production, and Unemployment

Very little has been said so far about output; we now close the model by describing the output market. To build up the intuition for our framework, we consider first a full endowment economy, and then consider production economies under both flexible and sticky prices.

Endowment Economy Let all output stochastic processes $\{Y_{NT,t}, Y_{H,t}, Y^*_{NT,t}, Y_{F,t}\}_{t=0}^{1}$ be exogenous strictly-positive endowments. Assuming that all prices are flexible and that the law of one price (LOP) holds, one has: $p_{H,t} = p^*_{H,t}e_t$, and $p_{F,t} = p^*_{F,t}e_t$.

Summing US and Japanese demand for US tradable goods ($C_{H,t} = \frac{a_t}{p_{H,t}}$ and $C^*_{H,t} = \frac{\xi_t e_t}{p_{H,t}}$, respectively, which are derived as in Section II.A), we obtain the world demand for US tradables: $D_{H,t} \equiv C_{H,t} + C^*_{H,t} = \frac{a_t+\xi_t e_t}{p_{H,t}}$. Clearing the goods market, $Y_{H,t} = D_{H,t}$, yields the equilibrium price in dollars of US tradables: $p_{H,t} = \frac{a_t+\xi_t e_t}{Y_{H,t}}$. An entirely similar argument yields: $p^*_{F,t} = \frac{a^*_t+\frac{\iota_t}{e_t}}{Y_{F,t}}$.

Production Without Price Rigidities. Let us introduce a minimal model of production that will allow us to formalize the effects of the exchange rate on output and employment. While we maintain the assumption that non-tradable goods in each country are given by endowment processes, we now assume that tradable goods in each country are produced with a technology linear in labor with unit productivity. In each country, labor L is supplied inelastically and is internationally immobile.

Simple profit maximization at the firm level yields a Dollar wage in the US of $w_{H,t} = p_{H,t}$. Under flexible prices, goods market clearing then implies full employment $Y_{H,t} = L$ and a US tradable price in dollars of: $p^{\circ}_{H,t} = \frac{a_t+\xi_t e_t}{L}$, where the circle in p° denotes a frictionless quantity. Likewise, for Japanese tradables the equilibrium features both full employment $Y_{F,t} = L$ and a Yen price of: $p^{*\circ}_{F,t} = \frac{a^*_t+\iota_t/e_t}{L}$.

Production With Price Rigidities. Let us now assume that wages are "downward rigid" in domestic currency at a preset level of $\{\bar{p}_H, \bar{p}^*_F\}$, where these prices are exogenous. Let us further assume that firms do not engage in pricing to market, so that prices are sticky in producer currency (PCP). Firm profit maximization then implies that: $p_{H,t} = \max\left(\overline{p}_H, p^{\circ}_{H,t}\right)$; or more explicitly: $p_{H,t} = \max\left(\overline{p}_H, \frac{a_t+e_t\xi_t}{L}\right)$. Hence:

$$(21) \qquad Y_{H,t} = \min\left(\frac{a_t + e_t\xi_t}{\bar{p}_H}, L\right).$$

If demand is sufficiently low ($a_t + \xi_t e_t < \overline{p}_H L$), then output is demand-determined (i.e., it depends directly on: e_t, ξ_t,and a_t) and there is unemployment: $L - Y_{H,t} > 0$. Notice that in this case the exchange rate has an *expenditure-switching* effect: if the Dollar depreciates ($e_t \uparrow$), unemployment falls and output expands in the US. Intuitively, since US tradables' prices are sticky in dollars, these goods become cheap for Japanese consumers to buy when the Dollar depreciates. In a world that is demand constrained, this expansion in demand for US tradable is met by expanding production, thus raising US output and employment.

Clearly, a similar expression and mechanism apply to Japanese tradables:

$$Y_{F,t} = \min\left(\frac{a_t^* + \iota_t/e_t}{\bar{p}_F^*}, L\right). \tag{22}$$

The expenditure switching role of exchange rates has been central to the Keynesian analysis of open macroeconomics of Dornbusch (1976); Obstfeld and Rogoff (1995). In the Gamma model, it is enriched by being the central channel for the transmission of financial forces affecting the exchange rate, such as the risk-bearing capacity and balance sheet of the financiers, into output and employment.

The financial determination of exchange rates has real consequences. Let us reconsider our earlier example of a sudden inflow of capital from US investors into Brazilian Real bonds. The exchange rate in this economy with production and sticky prices is still characterized by equation (21). As previously discussed, the capital inflow in Brazil causes the Real to appreciate and,[32] if the flow is sufficiently strong (f sufficiently high) or the financiers' risk bearing capacity sufficiently low (Γ sufficiently high), the appreciation (the increase in e_0) can be so strong as to make Brazilian goods uncompetitive on international markets; the corresponding fall in world demand for Brazilian output ($\downarrow C_H^* = \frac{\iota_0}{e_0 \bar{p}_F^*}$) causes an economic slump in Brazil with both falling output and increasing unemployment.[33]

The main focus of our model is to disconnect the exchange rate from fundamentals by altering the structure of financial markets. Of course, part of the disconnect in practice also comes from frictions in the goods makers. These frictions can be analyzed in our model; we illustrate this by considering prices that are sticky in the export destination currency (LCP). To make the point sharp, assume that prices for US tradable goods are exogenously set at $\{\bar{p}_H, \bar{p}_H^*\}$ in dollars in the US and in yen in Japan, respectively.

Lemma 4. (LCP vs PCP) *Under Local Currency Pricing the value of the exchange rate is the same as under Producer Currency Pricing, but US tradable output does not depend on the exchange rate:* $Y_{H,t} = \min\left(\frac{a_t}{\bar{p}_H} + \frac{\xi_t}{\bar{p}_H^*}, L\right)$.

Proof Because of the log specification, the dollar value of US imports and exports is unchanged:

[32] When $\alpha = 0$, $\frac{\partial e_0}{\partial f} = -\frac{\Gamma}{2+\Gamma} < 0$. More generally, a sufficient condition for this effect is that α is small.

[33] The Brazilian Finance Minister Guido Mantega complained, as reported in Forbes Magazine (2011), that: "We have to face the currency war without allowing our productive sector to suffer. If we allow [foreign] liquidity to [freely] enter [the economy], it will bring the Dutch Disease to the economy."

they are still ι_t and $e_t \xi_t$. Consequently, the value of net exports is unchanged, and the exchange rate is unchanged from the previous formulae. Total demand is derived as above. $\square$.

LCP helps to further the disconnect between the exchange rate and fundamentals by preventing output in the tradable sector from responding to the exchange rate.[34]

III Revisiting Canonical Issues with the Gamma Model

We consider in this section a number of canonical issues of international macroeconomics via the lenses of the Gamma model. While these classic issues have also been the subject of previous literature, our analysis not only provides new insights, but also allows us to illustrate how the framework built in the previous section provides a unified and tractable rationalization of empirical regularities that are at the center of open-economy analysis.

III.A The Carry Trade in the Presence of Financial Shocks

In the Gamma model there is a profitable carry trade. Let us give the intuition in terms of the most basic model first and then extend it to a set-up with shocks to the financiers' risk bearing capacity (Γ shocks).

First, imagine a world in which countries are in financial autarky because the financiers have zero risk bearing capacity ($\Gamma = \infty$), suppose that Japan has a 1% interest rate while the US has a 5% interest rate, and that all periods $(t = 0, ..., T)$ are ex-ante identical with $\xi_t = 1$ and ι_t a martingale. Thus, we have $e_t = \iota_t$, and the exchange rate is a random walk $e_0 = \mathbb{E}[e_1] = ... = \mathbb{E}[e_T]$. A small financier with some available risk bearing capacity, e.g. a small hedge fund, could take advantage of this trading opportunity and pocket the 4% interest rate differential. In this case, there is a very profitable carry trade. As the financial sector risk bearing capacity expands (Γ becomes smaller, but still positive), this carry trade becomes less profitable, but does not disappear entirely unless $\Gamma = 0$, in which case the UIP condition holds. Intuitively, the carry trade in the basic Gamma model reflects the risk compensation necessary to induce the financiers to intermediate global financial flows.

[34] Devereux and Engel (2003) stressed the absence of exchange rate effects on output under LCP. The empirical evidence shows that, in practice, a combination of PCP, LCP and limited pass-through are present in the data (see Gopinath and Itskhoki (2010); Gopinath, Itskhoki and Rigobon (2010); Amiti, Itskhoki and Konings (2014); Burstein and Gopinath (2015)). For much of this paper, we focus on flexible prices or PCP as the basic cases. As shown in Lemma 4 above, our qualitative analysis can easily accommodate a somewhat more limited pass-through of exchange rate changes to local prices of internationally traded goods.

In the most basic model, the different interest rates arise from different rates of time preferences, such that $R = \beta^{-1}$ and $R^* = \beta^{*-1}$. Without loss of generality, assume $R < R^*$ so that the Dollar is the "funding" currency, and the Yen the "investment" currency. The return of the carry trade is: $R^c \equiv \frac{R^*}{R}\frac{e_1}{e_0} - 1$. For notational convenience we define the carry trade expected return as $\overline{R}^c \equiv \mathbb{E}[R^c]$. The calculations in Proposition 3 allow us to immediately derive the equilibrium carry trade.

Proposition 6. *Assume $\xi_t = 1$. The expected return to the carry trade in the basic Gamma model is:*

$$\overline{R}^c = \Gamma \frac{\frac{R^*}{R}\mathbb{E}\left[\iota_1\right] - \iota_0}{\left(R^* + \Gamma\right)\iota_0 + \frac{R^*}{R}\mathbb{E}\left[\iota_1\right]}, \qquad \textit{where} \quad \Gamma = \gamma\, var(\iota_1)^{\alpha}. \tag{23}$$

Hence the carry trade return is bigger (i) when the return differential R^*/R is larger (ii) when the funding country is a net foreign creditor (iii) when finance is more imperfect (higher Γ).

To gain further intuition on the above result, consider first the case where $\iota_0 = \mathbb{E}\left[\iota_1\right]$. The first order approximation to $\overline{R}^c$ in the case of a small interest rate differential $R^* - R$ is: $\overline{R}^c = \frac{\Gamma}{2+\Gamma}\left(R^* - R\right)$. Notice that we have both $\frac{\partial \overline{R}^c}{\partial \Gamma} > 0$ and $\frac{\partial \overline{R}^c}{\partial (R^* - R)} > 0$, so that the profitability of the carry trade increases the more limited the risk-bearing capacity of the financiers and the larger the interest rate differential.[35]

The effects of broadly defined "global risk aversion", here proxied by Γ, on the profitability of the carry trade have been central to the empirical analysis of for example Brunnermeier, Nagel and Pedersen (2009); Lustig, Roussanov and Verdelhan (2011), and Lettau, Maggiori and Weber (2014). Here we have shown that the carry trade is more profitable the lower the risk bearing capacity of the financiers; we next formally account for shocks to such capacity in the form of a stochastic Γ.

In addition to a pure carry force due to the interest rate differential, our model features global imbalances as a separate risk factor in currency risk premia. The reader should recall Proposition 2 that showed how net-external-debtor countries' currencies have a positive excess return and depreciate whenever risk bearing capacity decreases ($\uparrow \Gamma$). This effect occurs even if both countries have the same interest rate, thus being theoretically separate from the pure carry trade. Della Corte, Riddiough and Sarno (2014) test these theoretical predictions and find evidence of a global imbalance risk factor in currency excess returns.[36]

[35] The first effect occurs because, given an interest rate differential, expected returns to the carry trade have to increase whenever the risk bearing capacity of the financiers goes down to induce them to intermediate financial flows. The second effect occurs because, given a level of risk bearing capacity for the financiers, an increase in the interest rate differential will not be offset one to one by the expected exchange rate change due to the risk premium.

[36] Notice that we built the model so that financial forces have no effect on the interest rates and the exchange rate makes

The exposure of the carry trade to financial disruptions We now expand on the risks of the carry trade by studying a three period ($t = 0, 1, 2$) model with stochastic shocks to the financiers' risk bearing capacity in the middle period. To keep the analysis streamlined, we take period 2 to be the "long run". Intuitively, period 2 will be a long-run steady state where countries have zero net foreign assets and run a zero trade balance. This allows us to quickly focus on the short-to-medium-run exchange rate dominated by financial forces and the long-run exchange rate completely anchored by fundamentals. We jump into the analysis, and provide many of the background details of this model in the appendix.[37]

We assume that time-1 financial conditions, Γ_1, are stochastic. In the 3-period economy with a long-run last period, the equilibrium exchange rates are:

$$e_0 = \frac{\Gamma_0 \iota_0 + \frac{R^*}{R} \mathbb{E}_0 \left[\frac{\Gamma_1 \iota_1 + \iota_2 R^* / R}{\Gamma_1 + 1} \right]}{\Gamma_0 + 1}; \qquad e_1 = \frac{\Gamma_1 \iota_1 + \frac{R^*}{R} \mathbb{E}_1 [\iota_2]}{\Gamma_1 + 1}; \qquad e_2 = \iota_2. \tag{24}$$

Recall that the carry-trade return between period 0 and 1 is: $R^c \equiv \frac{R^*}{R} \frac{e_1}{e_0} - 1$. Interestingly, in this case the carry trade also has "exposure to financial conditions". Notice that $\frac{\partial e_1}{\partial \Gamma_1} < 0$ in the equations above, so that the Dollar (the funding currency) appreciates whenever there is a negative shock to the financiers' risk bearing capacity ($\uparrow \Gamma_1$, $\downarrow e_1$). Since in our chosen parametrization the carry trade is short Dollar and long Yen, we correspondingly have: $\frac{\partial R^c}{\partial \Gamma_1} < 0$, the carry trade does badly whenever there is a negative shock to the financiers' risk bearing capacity ($\uparrow \Gamma_1$). This is consistent with the intuition and the empirical findings in Brunnermeier, Nagel and Pedersen (2009); we obtain this effect here in the context of an equilibrium model. We formalize and prove the results obtained so far in the proposition below.

Proposition 7. (Determinants of expected carry trade returns) *Assume that $R^* > R$, $1 = \iota_0 = \mathbb{E}_0 [\iota_1]$ and $\iota_1 = \mathbb{E}_1 [\iota_2]$. Define the "certainty equivalent" $\overline{\Gamma}_1$ by $\frac{\overline{\Gamma}_1 + R^*/R}{\overline{\Gamma}_1 + 1} \equiv \mathbb{E}_0 \left[\frac{\Gamma_1 + R^*/R}{\Gamma_1 + 1} \right]$. Consider the returns to the carry trade, R^c. The corresponding expected return $\overline{R}^c \equiv \mathbb{E}_0 [R^c]$ is*

$$\overline{R}^c = (\mathcal{R}^* - 1) \, \Gamma_0 \frac{\overline{\Gamma}_1 + 1 + \mathcal{R}^*}{\overline{\Gamma}_1 (\Gamma_0 + \mathcal{R}^*) + \Gamma_0 + (\mathcal{R}^*)^2}.$$

all the adjustment; while this sharpens the model, we could extend the framework to allow for effects of imbalances on both the exchange rate and interest rates.

[37] The flow demand equations in the Yen / Dollar market are: $e_t - \iota_t + Q_t = 0$ for $t = 0, 1$, and in the long-run period $e_2 - \iota_2 = 0$, with the financiers' demand for dollars: $Q_t = \frac{e_t - \mathbb{E}[e_{t+1}] \frac{R^*}{R}}{\Gamma_t}$. with $\Gamma_t = \gamma \, var_t(e_{t+1})$.

with $\mathcal{R}^* \equiv \frac{R^*}{R}$*. We have:*

1. *An adverse shock to financiers affects the returns to carry trade negatively :* $\frac{\partial R^c}{\partial \overline{\Gamma}_1} < 0$*.*
2. *The carry trade has positive expected returns:* $\overline{R}^c > 0$*.*
3. *The expected return to the carry trade is higher the worse the financial conditions are at time 0* $\left(\frac{\partial \overline{R}^c}{\partial \Gamma_0} > 0\right)$*, the better the financial conditions are expected to be at time 1* $\left(\frac{\partial \overline{R}^c}{\partial \overline{\Gamma}_1} < 0\right)$*, and the higher the interest rate differential (*$\frac{\partial \overline{R}^c}{\partial R^*} > 0$*,* $\frac{\partial \overline{R}^c}{\partial R} < 0$*).*

The Fama Regression The classic UIP regression of Fama (1984) is in levels:[38]

$$\frac{e_1 - e_0}{e_0} = \alpha + \beta_{\text{UIP}} \left(R - R^*\right) + \varepsilon_1.$$

Under UIP, we would find $\beta_{\text{UIP}} = 1$. However, a long empirical literature finds $\beta_{\text{UIP}} < 1$, and sometimes even $\beta_{\text{UIP}} < 0$. The proposition below rationalizes these findings in the context of our model.

Proposition 8. (Fama regression and market conditions) *The coefficient of the Fama regression is* $\beta_{\text{UIP}} = \frac{1+\overline{\Gamma}_1-\Gamma_0}{(1+\Gamma_0)(1+\overline{\Gamma}_1)}$*. Therefore one has* $\beta_{\text{UIP}} < 1$ *whenever* $\Gamma_0 > 0$*. In addition, one has* $\beta_{\text{UIP}} < 0$ *if and only if* $\overline{\Gamma}_1 + 1 < \Gamma_0$*, i.e. if risk bearing capacity is very low in period 0 compared to period 1.*

Intuitively financial market imperfections always lead to $\beta_{\text{UIP}} < 1$ and very bad current market imperfections compared to future ones lead to $\beta_{\text{UIP}} < 0$. This occurs because any positive Γ leads to a positive risk premium on currencies that the financiers are long of and hence to a deviation from UIP ($\beta_{\text{UIP}} < 1$). If, in addition, financial conditions are particularly worse today compared to tomorrow the risk premium is so big as to induce currencies that have temporarily high interest rates to appreciate on average ($\beta_{\text{UIP}} < 0$).

The intuition for $\beta_{\text{UIP}} < 1$ is as follows. In the language of Fama (1984), when Japan has high interest rates, the risk premium on the Yen is high. The reason is that the risk premium is not entirely eliminated by financiers, who have limited risk-bearing capacity. In the limit where finance is eliminated ($\Gamma = \infty$), an interest rate of 1% translate one-for-one into a risk premium of 1% ($\beta_{\text{UIP}} = 0$). If riskiness (assuming $\alpha > 0$) or financial frictions go to 0, then β_{UIP} goes to 1.[39] In all cases, Covered

[38] The regression is most commonly performed in its logarithmic approximation version, but the levels prove more convenient for our theoretical treatment without loss of economic content.

[39] As riskiness ($var\left(e_1\right), var\left(e_2\right)$) goes to 0, Γ_0 and $\overline{\Gamma}_1$ go to 0, so β_{UIP} goes to 1.

Interest Rate Parity (CIP) holds in the model. This is because we allow financiers to eliminate all riskless arbitrages. Online Appendix Section A.3.C) provides full details on arbitrage trading in our model. There, we formulate a version of our basic demand (equation (10)), that applies to an arbitrary number of assets, and is arbitrage-free. One corollary of that extension is that CIP is respected.

Exchange Rate Excess Volatility In the data exchange rates are more volatile than fundamentals, a fact often referred to as exchange rate excess volatility. The Gamma model helps to rationalize this volatility not only by directly introducing new sources of variation, for example shocks to the risk bearing capacity of the financiers (γ_t) and gross flows (f_t), but also indirectly by endogenously amplifying fundamental volatility via the financial constraints. The intuition is that higher fundamental volatility tightens financial constraints, tighter constraints lead to higher volatility, thus generating a self-reinforcing feedback loop. We formalize this more subtle effect in the Lemma below and sharpen it by not only maintaining the assumption that $\xi_t = 1$ at all dates, but also by considering the case of deterministic (γ_t), so that the only source of volatility is fundamental, and no information revelation about future shocks $E_1[\iota_2] = E_0[\iota_2]$ and $Var_1[\iota_2] = Var_0[\iota_2]$.

Lemma 5. (Endogenous Amplification of Volatility) *The volatility of the exchange rate at time one is:* $var(e_1) = \left(\frac{\Gamma_1}{1+\Gamma_1}\right)^2 var(\iota_1)$*, where* $\Gamma_1 = \gamma_1 var(\iota_2)^{\alpha}$*. If* $\alpha > 0$ *and* $\gamma_1 > 0$*, then fundamental volatility is endogenously amplified by the financial constraint:* $\frac{\partial var(e_1)}{\partial var(\iota_2)} > 0$*. Notice that if* $\gamma_1 = 0$*, then* $\frac{\partial var(e_1)}{\partial var(\iota_2)} = 0$*.*

III.B Foreign Exchange Rate Intervention

The Gamma model of exchange rates considered so far has emphasized the central role of financial forces and in particular capital flows in the determination of exchange rates. We study here one very prominent type of flow: currency intervention by the official sector (the central bank or the treasury department).

Large-scale currency interventions have recently been undertaken by the governments of Switzerland and Israel.[40] Both governments aimed to relieve their currency appreciation in the face of turmoil in financial markets. By most accounts, the interventions successfully weakened the exchange rate and boosted the real economy.[41] Empirical studies, however, have yet to confront the thorny

[40]The Czech Republic also intervened in the currency market in November 2013 with the aim of depreciating the Koruna to boost the domestic economy

[41]Israel central bank governor Stanley Fisher remarked: "I have no doubt that the massive purchases [of foreign ex-

issue of endogeneity of the policy and future empirical work is necessary to provide a full empirical assessment.[42]

Here we focus on proving a framework to understand under which conditions foreign exchange rate intervention can be a powerful tool to combat exchange rate movements generated by financial turmoil. The limited risk bearing capacity of the financiers in our model is at the core of the effects of FX intervention on exchange rates. Indeed, Backus and Kehoe (1989) show that in a general class of models in which currencies are imperfect substitutes due to risk premia, but in which importantly there are no financial frictions, FX interventions have no effect on the exchange rate.

For notational simplicity, we set most parameters at 1: e.g. $\iota_0 = \xi_t = a_t = a_0^* = \beta = \beta^* = 1$. We allow ι_1 to be stochastic (keeping $\mathbb{E}[\iota_1] = 1$, and setting $a_1^* = \iota_1$ for symmetry) so that currency trading is risky.

At time 0, the Japanese government intervenes in the currency market vis-à-vis the financiers: it buys q^* dollars and sells q^*/e_0 yen. By Proposition 4 we immediately obtain the result below (as the government creates a flow $f^* = q^*$ in the currency market):

Lemma 6. *If the Japanese government buys q^* dollars and sells q^*/e_0 yen at time 0, the exchange rates satisfy : $e_0 = 1 - \frac{\Gamma}{2+\Gamma} q^*$, and $e_1 = 1 + \frac{\Gamma}{2+\Gamma} q^* + \{\iota_1\}$, with $\Gamma = \gamma\, var(\iota_1)^\alpha$.*

The intervention has no impact on the *average* exchange rate: $e_0 + \mathbb{E}[e_1] = 1$ irrespective of q^*. The intervention induces a depreciation at time 0, and an appreciation at time 1. We call this effect the "*boomerang effect*". A currency intervention can change the level of the exchange rate in a given period, but not the average level of the exchange rate over multiple periods. Lemma 6 highlights the importance of the frictions: if $\Gamma = 0$, a frictionless set-up analogous to that in Backus and Kehoe (1989), there is no effect of the intervention on the exchange rate. Correspondingly, the potency of the intervention is strictly increasing in the severity of the frictions: the higher the Γ the more the exchange rate moves for a given size of the intervention.

A classic criticism of portfolio balance models is that only extremely big interventions are effective because for an intervention to be effective it needs to alter very large stocks of assets: either

change] we made between July 2008 and into 2010 [...] had a serious effect on the exchange rate which I think is part of the reason that we succeeded in having a relatively short recession." Levinson (2010)

[42] Blanchard, de Carvalho Filho and Adler (2014) find empirical support for the efficacy of this policy. An earlier skeptical empirical literature, that mostly focused on interventions of considerable smaller size, is summarized by Sarno and Taylor (2001). Dominguez and Frankel (1993*a*,*b*) find empirical support for the effect of foreign exchange rate intervention via a portfolio balance channel.

the entire stock of assets outstanding or the country level gross external assets and liabilities. In our framework interventions are more effective because they need only alter Q, the balance sheet of financiers, which is potentially substantially smaller than the entire stock of assets.

Our framework also sheds light on the real consequence of FX intervention on output and risk-sharing. We assume that in the short run, i.e. period $t = 0$, Japanese tradables' prices are sticky in domestic currency (PCP) as in Section II.E; prices are flexible in the long run, i.e. period $t = 1$. We postulate that at time zero the price is downward rigid at a level $\overline{p}_F^*$ that is sufficiently high as to cause unemployment in the Japanese tradable sector. US tradable prices are assumed to be flexible. This captures a situation in which one country is in a recession, with high slack capacity and unemployment, so much so that its output is demand driven.

Proposition 9. (FX intervention) *Assume that $\Gamma > 0$ and that at time zero Japanese tradable goods' prices are downward rigid at a price $\overline{p}_F^*$ that is sufficiently high to cause unemployment in the Japanese tradable sector. A Japanese government currency intervention, whereby the government buys $q^* \in [0, \overline{q}^*]$ worth of Dollar bonds and sells q^*/e_0 yen bonds at time zero, depreciates the Yen and increases Japanese output. $\overline{q}^*$ is the smallest intervention that restores full employment in Japan. The intervention distorts consumption with the consumption shares determined by: $\frac{C_{H,t}^*}{L} = s_t^*$ and $\frac{C_{F,t}}{Y_{F,t}} = 1 - s_t^*$ with $s_t^* = \frac{e_t}{1+e_t}$ for $t = 0, 1$.*

Note that there are two preconditions for this intervention analysis. The first one is that prices are sticky (fixed) in the short run at a level that generates a fall in aggregate demand and induces an equilibrium output below the economy's potential. This condition, i.e. being in a *demand driven* state of the world, is central to the Keynesian analysis where a depreciation of the exchange rate leads to an increase in output via an increase in export demand. If this condition is satisfied a first order output loss would occur even in a world of perfect finance. The second precondition is that financial markets are imperfect, i.e. $\Gamma > 0$. Recall from Lemma 6 that the ability of the government to affect the time-zero exchange rate is inversely proportional to Γ. When markets are frictionless ($\Gamma = 0$) the government FX policy has no effect on the time-zero exchange rate, even if prices are sticky, because financiers would simply absorb the intervention without requiring a compensation for the resulting risk.

The intervention has two distinct effects on consumption. The first effect is an increase in world consumption because, as described above, the intervention expands Japanese output without decreasing US output. The second effect is a distortion in the share of world output consumed by each country. Both effects are clearly illustrated by the Japanese consumption of Japanese tradable goods:[43]

$$C^*_{F,0} = \frac{1}{\overline{p}^*_F} = \frac{e_0}{1+e_0}\frac{Y_{F,0}}{L}L$$

The term $\frac{e_0}{1+e_0}$ is the equilibrium share of Japanese tradables consumed by Japanese households. The intervention reduces this share via a Dollar appreciation ($e_0 \downarrow$). At the same time, the intervention increases total Japanese output by reducing slack: the term $\frac{Y_{F,0}}{L} \in (0,1]$ is decreasing in e_0 since output is demand driven. The functional specifications of the model (logarithmic utility and linear production) make the two effect cancel out and keep $C^*_{F,0}$ unchanged; the boomerang effect, however, will induce an expected increase in the consumption next period ($\mathbb{E}[C^*_{F,1}] \uparrow$). US consumption of both tradable goods increases at time zero, due both to an increase in US share of world consumption and an increase in output, but then falls at time 1.

Overall the intervention boosts world output with the cost of intertemporal distortions in consumption. Interestingly, the suggested policy is not of the *beggar thy neighbor* type: the Japense currency intervention, even with its aim to weaken the Yen, actually increases consumption, at least in the short run, in the US. The US benefits from an increase in Japanese output with no loss of US output. We highlight that *currency wars* can only occur when both countries are in a slump and the post-intervention weaker Yen causes a first order output loss in the US.[44]

The intervention has real effects even in this calibration that has been chosen so that before the intervention households have no incentives to trade in financial markets even if they were allowed to do so freely and optimally. Similarly, the financiers have no incentives to trade at equilibrium prices both before and after the intervention.[45] To further isolate the sole effect of financial frictions on the intervention outcome, we have assumed that the intervention's proceeds and losses are rebated lump sum (i.e. non-distortionary) by the Japanese government to its citizens.

[43]The first equality follows from the demand function of Japanese households for Japanese tradables ($C_{F,t} = \frac{a^*_t}{p^*_{F,t}}$), the second equality follows from the equilibrium output function of Japanese tradables in Section II.E.

[44]Cavallino (2014) builds on this analysis and analyzes the joint use of FX intervention and monetary policy.

[45]Indeed, in this economy (before the government intervention) the exchange rate is at 1, and is expected to remain at 1 on average, therefore financiers optimally choose to not trade at all. Similarly US household are on their "shadow" Euler equation and would not want to trade Yen bonds even if allowed to do so. Japanese household would have a small incentive to trade since their shadow Euler equation has a Jensen inequality term (an additional term compared to the US household Euler equation). After the intervention, financiers still have no further incentives to trade having already optimized their positions in response to the intervention. Of course, household would now like to trade but these unlimited optimal trades are not possible (here as in the rest of the model) due to the frictions in the intermediation process. Therefore the policy success relies on the presence of financial frictions rather than a direct failure of Ricardian equivalence.

The Potency of Intervention: Combining FX Intervention and Capital Controls It is often argued by policy markers that currency intervention should be undertaken together with capital controls. The Gamma model provides a unified view of this policy combination because capital controls increase the financial market segmentation thus enhancing the potency of currency intervention.

We introduce a second policy instrument, taxation of the financiers, which is a form of capital controls.[46] We consider a proportional (Japanese) government tax on each financier's profits; the tax proceeds are rebated lump sum to financiers as a whole. Recall the imperfect intermediation problem in Section II.B, we now assume that the after-tax value of the intermediary is $V_t(1-\tau)$, where τ is the tax rate. The financiers' optimality condition, derived in a manner entirely analogous to the optimization problem in equation (9), is now: $Q_0 = \frac{\mathbb{E}\left[e_0 - e_1 \frac{R^*}{R}\right](1-\tau)}{\Gamma}$. Notice that this is equivalent to changing Γ to an effective $\Gamma^{\text{eff}} \equiv \frac{\Gamma}{1-\tau}$, so that the financiers' demand can be rewritten as $Q_0 = \frac{\mathbb{E}\left[e_0 - e_1 \frac{R^*}{R}\right]}{\Gamma^{\text{eff}}}$. We consider the leading case of ξ_t deterministic and collect the result in the Proposition below.

Proposition 10. *Assume ξ_t is deterministic, a tax τ on finance is equivalent to lowering the financiers' risk bearing capacity by increasing Γ to $\Gamma^{\text{eff}} \equiv \frac{\Gamma}{1-\tau} = \frac{\gamma}{1-\tau} \, var\left(\frac{\iota_1}{\xi_1}\right)^{\alpha}$. A higher tax increases the effective Γ^{eff}, thus reducing the financiers' risk bearing capacity. The sign of the effect of the tax on exchange rates depends on the position that the financiers would have taken absent the tax (Q^-): if the financiers are long (short) dollars $Q^- > 0$ ($Q^- < 0$), then a tax depreciates (appreciates) the Dollar. The potency of FX intervention increases in the tax.*

First we note that if the equilibrium before the government intervention features zero risk taking by the financiers ($Q_0 = 0$), as was the case in the economy studied in the previous analysis of FX intervention, then the tax τ is entirely ineffective. Intuitively, this occurs because there are zero expected profits to tax, and therefore the tax has no effect on ex-ante incentives.

More generally we recall from Proposition 2 that an increase in Γ, in this case an increase in Γ^{eff} due to an increase in τ, has the opposite effect on the exchange rate depending on whether the financiers are long or short the Dollar to start with, i.e. depending on the sign of Q_0 before the tax is imposed. For example, the tax would make the Dollar depreciate on impact if the financiers were long dollars to start with ($Q_0 > 0$), but the same tax would make the Dollar appreciate if the

[46] There is a recent and interesting literature on the use of capital controls: Rey (2013); Costinot, Lorenzoni and Werning (2014); Farhi and Werning (2014, 2013); Farhi, Gopinath and Itskhoki (2014); Farhi and Werning (2012*a*,*b*); Magud, Reinhart and Rogoff (2011); Schmitt-Grohé and Uribe (2012); Mendoza (2010); Bianchi (2010); Korinek (2011).

financiers had the opposite position to start with. In practice this means that policy makers who are considering imposing capital controls, or otherwise taxing international finance, should pay close attention to the balance sheets of financial institutions that have exposures to their currency. Basing the policy on reduced form approaches or purely on traditional macroeconomics fundamentals can not only be misguiding, but might actually generate the opposite outcome for the exchange rate from the desired one. Finally, recall from Lemma 6 that the effect of currency intervention on the exchange rate is bigger the lower the financiers' risk bearing capacity (the higher the Γ). It follows that a tax on finance or a capital control, by implicitly reducing risk bearing capacity, increases the potency of FX intervention.

IV Analytical Generalization of the Model

The basic version of the Gamma model presented so far was real, and we now show that it can readily be extended to a nominal version where the nominal exchange rate is determined, similarly to our baseline model, in an imperfect financial market.[47]

We assume that money is only used domestically by the households and that its demand is captured, in reduced form, in the utility function of households in each country.[48] Financiers do not use money, but they trade in nominal bonds denominated in the two currencies. The US consumption basket is now extended to include a real money balances component such that the consumption aggregator is: $C_t \equiv \left[\left(\frac{M_t}{P_t}\right)^{\omega_t}(C_{NT,t})^{\chi_t}(C_{H,t})^{a_t}(C_{F,t})^{\iota_t}\right]^{\frac{1}{\theta_t}}$, where M is the amount of money held by the households and P is the nominal price level so that $\frac{M}{P}$ is real money balances. We maintain the normalization of preference shocks by setting $\theta_t \equiv \omega_t + \chi_t + a_t + \iota_t$. Correspondingly, the Japanese consumption basket is now: $C_t^* \equiv \left[\left(\frac{M_t^*}{P_t^*}\right)^{\omega_t^*}(C_{NT,t}^*)^{\chi_t^*}(C_{H,t}^*)^{\xi_t}(C_{F,t}^*)^{a_t^*}\right]^{\frac{1}{\theta_t^*}}$.

Money is the numéraire in each economy, with local currency price equal to 1. The static utility maximization problem is entirely similar to the one in the basic Gamma model in Section II.A, and standard optimization arguments lead to demand functions: $M_t = \frac{\omega_t}{\lambda_t}$; $p_{NT,t}C_{NT,t} = \frac{\chi_t}{\lambda_t}$; $p_{F,t}C_{F,t} =$

[47] Notice that we have indeed set up the "real" model in the main text in such a way that non-tradables in each country play a role very similar to money and where, therefore, the exchange rate is rather similar to a nominal exchange rate (see Obstfeld and Rogoff (1996)[Ch. 8.3]). In this section we make such analogy more explicit. Online Appendix A.1.D provides a full discussion of the CPI-based real exchange rate and the nominal exchange rate in our model. Alvarez, Atkeson and Kehoe (2009) provide a model of nominal exchange rates with frictions in the domestic money markets, while our model has frictions in the international capacity to bear exchange-rate risk.

[48] A vast literature has focused on foundations of the demand for money; such foundations are beyond the scope of this paper and consequently we focus on the simplest approach that delivers a plausible demand for money and much tractability.

$\frac{\iota_t}{\lambda_t}$, where, we recall from earlier sections, λ_t is the Lagrange multiplier on the households' static budget constraint.[49] Substituting for the value of the Lagrange multiplier, money demand is given by $M_t = \omega_t P_t C_t$ and is proportional to total nominal consumption expenditures; the coefficient of proportionality, ω_t, is potentially stochastic.[50]

Let us define $m_t \equiv \frac{M_t^s}{\omega_t}$ and $m_t^* \equiv \frac{M_t^{s*}}{\omega_t^*}$, where M_t^s and M_t^{s*} are the money supplies.[51] Notice that since money (as in actual physical bank notes) is non-tradable across countries or with the financiers (but bonds that pay in units of money are tradable with the financiers as in the previous sections), the money market clearing implies that the central bank can pin down the level of nominal consumption expenditure ($m_t = \lambda_t^{-1}, m_t^* = \lambda_t^{*-1}$).[52] The nominal exchange rate e_t is the relative price of the two currencies. It is defined as the strength of the Yen, so that an increase in e_t is a Dollar depreciation.[53]

US nominal imports in dollars are: $p_{F,t}C_{F,t} = \frac{\iota_t}{\lambda_t} = \iota_t m_t$. Similarly, Japanese demand for US tradables is: $p_{H,t}^* C_{H,t}^* = \xi_t m_t^*$. Hence, US nominal exports in dollars are: $p_{H,t}^* C_{H,t}^* \, e_t = \xi_t e_t m_t^*$. We conclude that US nominal net exports in dollars are: $NX_t = \xi_t e_t m_t^* - \iota_t m_t$.

We assume that the financiers solve:[54]

$$\max_{q_0} \; V_0 = \Omega_0 q_0, \quad \text{subject to } \; V_0 \geq \min\left(1, \Gamma \frac{|q_0|}{m_0^* e_0}\right) |q_0|, \quad \text{where } \Omega_0 \equiv \mathbb{E}_0\left[1 - \frac{R^*}{R}\frac{e_1}{e_0}\right].$$

Notice that m_0^* is now scaling the portion of nominal assets that the financiers' can divert to ensure that such fraction is scale invariant to the level of the Japanese money supply and hence the nominal

[49] The budget constraint of the households is now:

$$\sum_{t=0}^{1} R^{-t}\left(p_{NT,t}Y_{NT,t} + p_{H,t}Y_{H,t} + M_t^s\right) = \sum_{t=0}^{1} R^{-t}\left(p_{NT,t}C_{NT,t} + p_{H,t}C_{H,t} + p_{F,t}C_{F,t} + M_t\right).$$

where M_t^s is the seignorage rebated lump-sum by the government, which is equal to M_t in equilibrium.

[50] The money demand equation is similar to that of a cash in advance constraint where money is only held by the consumers within the period, i.e. they need to have enough cash at the beginning of the period to carry out the planned period consumption. For constraints of this type see Helpman (1981); Helpman and Razin (1982).

[51] It is often convenient to consider the *cashless* limit of our economies by taking the limit case when $\{M_t^s, M_t^{s*}, \omega_t, \omega_t^*\} \downarrow 0$ such that $\{m_t, m_t^*\}$ are finite; however, this is not needed for positive analysis.

[52] The central bank in each period choses money supply after the preference shocks are realized so that m and m^* are policy variables. We abstract here from issues connected with the zero lower bound on nominal interest rates. Notice the duality between money in the current setup and non-tradable goods in the basic Gamma model of Section II. If $M_t = \omega_t$ and $C_{NT,t} = \chi_t$, one recovers the equations in Section II, because the demand for money implies $\lambda_t = 1$, in which case the demand for non-tradables implies that $p_{NT,t} = 1$.

[53] To keep simpler notations, we denote the nominal exchange rate by e_t, the same symbol used for the exchange rate in the basic Gamma model.

[54] When we consider setups that are more general than the basic Gamma model of Section II, we maintain the simpler formulation of the financiers' demand function. We do not directly derive the households' valuation of currency trades in these more general setups. Our demand functions are very tractable and carry most of the economic content of more general treatments; we leave it for the extension Section A.4 to characterize numerically financier value functions more complex than those analyzed in closed form here.

value in Yen of the assets.[55]

Finally, the nominal interest rates are given by the households' intertemporal optimality conditions (Euler Equations):

$$1 = \mathbb{E}\left[\beta R \frac{U'_{1,C_{NT}}}{U'_{0,C_{NT}}} \frac{p_{NT,0}}{p_{NT,1}}\right] = \mathbb{E}\left[\beta R \frac{\chi_1 / C_{NT,1}}{\chi_0 / C_{NT,0}} \frac{p_{NT,0}}{p_{NT,1}}\right] = \beta R \mathbb{E}\left[\frac{m_0}{m_1}\right],$$

so that $R^{-1} = \beta \mathbb{E}\left[\frac{m_0}{m_1}\right]$. Similarly, $R^{*-1} = \beta^* \mathbb{E}\left[\frac{m_0^*}{m_1^*}\right]$. These interest rate determination formulas extend those in equation (6) to the nominal setup.

Equilibrium Exchange Rate in the Extended Setup. When we include all the extensions to the basic Gamma model considered so far, the key equations to solve for the equilibrium nominal exchange rate are the flow equations in the international bond market:

$$m_0^* \xi_0 e_0 - m_0 \iota_0 + Q_0 + f^* - f e_0 - D^{US} + D^J e_0 = 0, \tag{25}$$

$$m_1^* \xi_1 e_1 - m_1 \iota_1 - R Q_0 - R f^* + R^* f e_1 = 0, \tag{26}$$

and the financiers' demand curve:

$$Q_0 = \frac{m_0^*}{\Gamma} \mathbb{E}\left[e_0 - e_1 \frac{R^*}{R}\right]. \tag{27}$$

Equations 25-26 allow for household trading of foreign currency. They extend Section II.D.1, that only considered liquidity/noise trading, by allowing these demand functions for foreign bonds to depend on all fundamentals, but not directly depend on the exchange rate.[56] Equations 25-26 also allow for each country to start with a stock of foreign assets and liabilities. The US net foreign liabilities in dollars are D^{US} and Japan net foreign liabilities in yen are D^J.[57]

[55]The constraint $\Gamma = \gamma var(e_1)^\alpha$ can become $\Gamma = \gamma var(e_1 \frac{m_1^*}{m_1})^\alpha$, to make the model invariant to predictable changes to money supply.

[56]We allow the demand functions for foreign bonds from US and Japanese households, denoted by f and f^* respectively, to depend on all present and expected future fundamentals. We use the shorthand notation f and f^* to denote the generic functions: $f(R, R^*, \iota, \xi, ...)$ and $f^*(R, R^*, \iota, \xi, ...)$. For example, demand functions that load on a popular trading strategy, the carry trade, that invests in high interest rate currencies while funding the trade in low interest rate currencies can be expressed as $f = b + c(R - R^*)$ and $f^* = d + g(R - R^*)$, for some constants b, c, d, g. Interestingly, both gross capital flows and trade flows could be ultimately generated by financial frictions (see Antràs and Caballero (2009)). Dekle, Hyeok and Kiyotaki (2014) employ the reduced form approach and put holdings of foreign bonds directly in the utility function of domestic agents to generate flows. The online appendix extends the present results to demand functions that depend on the exchange rate directly by solving the model numerically.

[57]We could have alternatively assumed that only a fraction η of the debt had to be intermediated in which case we would

We show in the proposition below that the solution method, even in this more general case, follows the simple derivation of the basic model by representing the current economy as a "pseudo" basic economy. We also note that these results do not impose that $Y_{NT,t} = \chi_t$ and $Y^*_{NT,t} = \chi^*_t$, thus generalizing the analysis in Section II.

Proposition 11. *In the richer model above (with money, portfolio flows, external debt, and shocks to imports and exports) the values for the exchange rates e_0 and e_1 are those in Proposition 3, replacing imports (ι_t), exports (ξ_t), and the risk bearing capacity (Γ) by their "pseudo" counterparts $\{\widetilde{\iota}_t, \widetilde{\xi}_t, \widetilde{\Gamma}\}$, defined as: $\widetilde{\iota}_0 \equiv m_0\iota_0 + D^{US} - f^*$; $\widetilde{\xi}_0 \equiv m^*_0\xi_0 + D^J - f$; $\widetilde{\iota}_1 \equiv m_1\iota_1 + Rf^*$; $\widetilde{\xi}_1 \equiv m^*_1\xi_1 + R^*f$; $\widetilde{\gamma} \equiv \gamma/m^*_0$, $\widetilde{\Gamma} \equiv \Gamma/m^*_0$.*

Proof: Equations (25)-(26) reduce to the basic flow equations, equations (13)-(14), provided we replace ι_t and ξ_t by $\widetilde{\iota}_t$ and $\widetilde{\xi}_t$. Similarly, equation (27) reduces to equation (10), provided we replace Γ by $\widetilde{\Gamma}$. Then the result follows from the proof of Proposition 3.□

Intuitively, the pseudo imports $(\widetilde{\iota})$ are composed of factors that lead consumers and firms to sell dollars and hence "force" financiers to be long the Dollar. An entirely symmetric intuition applies to the pseudo exports $\left(\widetilde{\xi}\right)$.

We collect here a number of qualitative results for the generalized economy. While some properties do not strictly depend on $\Gamma > 0$ and therefore can be derived even in UIP models, it is nonetheless convenient to provide a unified treatment in the present model. We assume that $\widetilde{\iota}_t$ and $\widetilde{\xi}_t$ are positive at dates 0 and 1. Otherwise, various pathologies can happen, including the non-existence of an equilibrium (e.g. formally, a negative exchange rate).

Proposition 12. *The Dollar is weaker: 1)* (Imports-Exports) *when US import demand for Japanese goods (ι_t) is higher; when Japanese import demand for US goods (ξ_t) is lower; 2)* ("Myopia" from an imperfect financial system) *higher Γ increases the effects in point 1) by making current imports matter more than future imports;*[58] *3)* (Debts and their currency denomination) *when US net external liabilities in dollars (D^{US}) are higher; when Japanese net external liabilities in Yen (D^J) are lower; 4)* (Financiers' risk-bearing capacity) *when financial conditions are worse (Γ is higher), conditional on Japan being a net creditor at time 0^+ ($N_{0^+} < 0$); 5)* (Demand pressure) *when the noise demand for the Dollar (f^*) is lower, as long as $\Gamma > 0$; 6)* (Interest rates) *when the US real interest rate is lower; when the Japanese real interest rate is higher; 7)* (Money supply) *when the US current money supply (m_0) is higher; when the Japanese current money supply*

get a flow of ηD at time zero and a flow $(1-\eta)RD$ at time 1.

[58] That is, $\partial e_0/\partial \iota_0$ and $\partial e_0/\partial \iota_1$ are positive and respectively increasing and decreasing in Γ.

(m_0^*) is lower.

Point 3 above highlights a valuation channel to the external adjustments of countries. The exchange rate moves in a way that facilitates the re-equilibration of external imbalances. Interestingly, it is not just the net-external position of a country, its net foreign assets, that matters for external adjustment, but actually the (currency) composition of its gross external assets and liabilities (D^{US} and D^J). This basic result is consistent with the valuation channel to external adjustment highlighted in Gourinchas and Rey (2007*a*,*b*); Lane and Shambaugh (2010).

V Conclusion

We presented a theory of exchange rate determination in imperfect capital markets where financiers bear the risks resulting from global imbalances in the demand and supply of international assets. Exchange rates are determined by the balance sheet risks and risk bearing capacity of these financiers. Exchange rates in our model are disconnected from traditional macroeconomic fundamentals, such as output, inflation and the trade balance and are instead more connected to financial forces such as the demand for assets denominated in different currencies. Our model is tractable, with simple to derive closed form solutions, and can be generalized to address a number of both classic and new issues in international macroeconomic analysis.

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Appendix

The financiers' optimization problem. We clarify here the role of the mild assumption, made in footnote 15, that $1 \geq \Omega_0 \geq -1$. Formally, the financiers' optimization problem is:

$$\max_{q_0} V_0 = \Omega_0 q_0, \quad \text{subject to } V_0 \geq \min\left(1, \Gamma \frac{|q_0|}{e_0}\right)|q_0|, \quad \text{where } \Omega_0 \equiv \mathbb{E}\left[1 - \frac{R^*}{R}\frac{e_1}{e_0}\right].$$

Notice that Ω_0 is unaffected by the individual financier's decisions and can be thought of as exogenous in this constrained maximization problem.

Consider the case in which $\Omega_0 > 0$, then the optimal choice of investment has $q_0 \in (0, \infty)$. Notice that $\Omega_0 \leq 1$ trivially. Then one has $V_0 \leq q_0$. In this case, the constraint can be rewritten as: $V_0 \geq \Gamma \frac{q_0^2}{e_0}$, because the constraint will always bind before the portion of assets that the financiers can divert $\Gamma \frac{|q_0|}{e_0}$ reaches 1. This yields the simpler formulation of the constraint adopted in the main text.

Now consider the case in which $\Omega_0 < 0$, then the optimal choice of investment has $q_0 \in (-\infty, 0)$. It is a property of currency excess returns that Ω_0 has no lower bound. In this paper, we assume that the parameters of the model are as such that $\Omega_0 > -1$, i.e. we assume that the worst possible (discounted) expected returns from being long a Dollar bond and being short a Yen bond is -100%. Economically this is an entirely innocuous assumption given that the range of expected excess returns in the data is approximately [-6%,+6%]. With this assumption in hand we have $V_0 \leq |q_0|$, and hence we can once again adopt the simpler formulation of the constraint because the constraint will always bind before the portion of assets that the financiers can divert $\Gamma\frac{|q_0|}{e_0}$ reaches 1.

As pointed out in the numerical generalization section of the Online Appendix (Section A.4), more general (and non-linear) value functions would apply once the simplifying assumptions made in the text are removed and depending on who the financiers' repatriate their profit and losses to. In the main body of the paper, we maintain the assumption that the financiers' use the US household valuation criterion; this makes the model most tractable while very little economic content is lost. The numerical generalizations in the online appendix provide robustness checks by solving the non-linear cases.

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Online Appendix to "International Liquidity and Exchange Rate Dynamics"

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A.1 Further Details for the Main Body of the Paper

A.1.A A More Abstract Version of the Market Structure

It may be useful to have a more abstract presentation of the basic model. We focus on the US side, as the Japanese side is entirely symmetric.

For generality, we present the monetary model, and then show how the real model can be viewed as a special case of it. We call

$$c = (C_H, C_F, C_{NT}, M, 0, 0),$$

the vector of consumptions of C_H US tradables, C_F Japanese tradables, C_{NT} US non-tradables, and a quantity M of US money, respectively. The last 2 slots in vector c (set at 0) are the consumption of Japanese non-tradables, and Japanese money: they are zero for the US consumer. Likewise, the Japanese consumer has consumption:

$$c^* = (C_H^*, C_F^*, 0, 0, C_{NT}^*, M^*).$$

The Japanese household consumes C_H^* US tradables, C_F^* Japanese tradables, 0 US non-tradables, 0 US money, C_{NT}^* Japanese non-tradables, and M^* Japanese money.

The US production vector is

$$y = (y_H, 0, y_{NT}, M^s, 0, 0).$$

This shows that the US produces y_H US tradables, 0 Japanese tradables, y_{NT} US non-tradables, and 0 Japanese non-tradables and money. Here M^s is the money supply given by the government to the household. Japanese production is similarly

$$y^* = (0, y_F^*, 0, 0, y_{NT}^*, M^{s*}).$$

The vector of prices in the US is

$$p = (p_H, p_F, p_{NT}, 1, 0, 0).$$

Utility is $u(c_t, \phi_t)$, where ϕ_t is a taste shock. In the paper, $\phi_t = (a_t, \iota_t, \chi_t, \omega_t, 0, 0)$, so that in the utility function is

$$u(c_t, \phi_t) = \sum_{i=1}^{6} \phi_{it} \ln c_{it}, \text{ for } t = 0, \ldots, T.$$

Consumptions are non-negative, $c_{it} \geq 0$ for all i, t.

The fourth and sixth components of the above vectors correspond to money. In the real model they are set to 0. Then, in this real model, the numéraire is the non-tradable good, so that $p_{NT} = 1$.

We call $\Theta_t = \left(\Theta_t^{US}, \Theta_t^{J}\right)$ the holding by the US of US bonds and Japanese bonds, $P_t = (1, e_t)$ the price of bonds in dollars.

The US consumers' problem is:

$$\max_{\left(c_t, \Theta_t^{US}\right)_{t \leq T}} \mathbb{E} \sum_{t=0}^{T} \beta^t u(c_t, \phi_t), \tag{A.1}$$

s.t.

(A.2)
$$p_t \cdot (y_t - c_t) + P_t \cdot D_t \Theta_{t-1} + \pi_t^F = P_t \cdot \Theta_t, \text{ for } t = 0, ..., T,$$

and

(A.3)
$$\Theta_T = 0.$$

Here $D_t = diag\,(R, R^*)$ is the diagonal matrix expressing the gross rate of return of bonds in each currency and π_t^F is a profit rebated by financiers. The left-hand side of (A.2) is the households' financial wealth (in dollars) after period t. US firms are fully owned by US households. Because the economy is fully competitive, they make no profit. The entire production comes as labor income, whose value is $p_t \cdot y_t$. The budget constraint is the terminal asset holdings should be 0, which is expressed by (A.3). Finally, as is usual, c_t and Θ_t^{US} are adapted process, i.e. they depend only on information available at date t.

In the above maximization problems, US consumers choose optimally their consumption vector c_t and their dollar bond holdings Θ_t^{US}. However, they do not choose their holding of Japanese bonds Θ_t^J optimally. In the basic model we preclude such holdings and set $\Theta_t^J = 0$. In the extended model, we allow for such holdings and study simple and intuitive cases: for instance, at time 0 the holdings of Japanese bonds can be an endowment $\Theta_{-1}^J = D^J$ (or Japanese debt denoted in Yen). Alternatively, they could be a liquidity (noise trader) shock $\Theta_0^J = -f$, or we could have f be a function of observables, but not the exchange rate directly, e.g. $f = b + c\,(R - R^*)$ for a carry-trader. We do not focus on the foundations for each type of demand, but actually take the demands as exogenously specified. Possible microfoundations for these demands range from rational models of portfolio delegation where the interest rate is an observable variable that is known, in equilibrium, to load on the sources of risk of the model (see Section III.A), to models of "reaching for yield" (Hanson and Stein, 2014), or to the "boundedly rational" households who focus on the interest rate when investing without considering future exchange rate changes or covariance with marginal utility (as in Gabaix (2014)).

To summarize, while all goods are frictionlessly traded within a period (with the non-tradable goods being traded only within a country), asset markets are restricted: only US and Japanese bonds are traded (rather than a full set of Arrow-Debreu securities).

The *goods market clearing condition* is:

(A.4)
$$y_t + y_t^* = c_t + c_t^* \text{ at all dates } t \leq T.$$

Firms produce and repatriate their sales at every period. They have net asset flows,

$$\Theta_t^{firms} = p_{Ht}^* c_{Ht}^* \,(e_t, -1)\,,$$
$$\Theta_t^{firms,*} = p_{Ft} c_{Ft} \left(-1, \frac{1}{e_t}\right).$$

The first equation expresses the asset flows of US exporters: in Japan, they have sales of $p_{Ht}^* c_{Ht}^*$ Yen in Japan market; they repatriate those yens (hence a flow of $-p_{Ht}^* c_{Ht}^*$ in Yen), to buy dollars (hence a flow of $p_{Ht}^* c_{Ht}^* e_t$ dollars).

For instance, in the model with the log specification,

$$\Theta_t^{firms} = p_{Ht}^* c_{Ht}^* \,(e_t, -1) = m_t^* \xi_t \,(e_t, -1)\,,$$
$$\Theta_t^{firms,*} = p_{Ft} c_{Ft} \left(-1, \frac{1}{e_t}\right) = m_t \iota_t \left(-1, \frac{1}{e_t}\right),$$

so that $\Theta_t^{firms} + \Theta_t^{firms,*} = (m_t^* \xi_t e_t - m_t \iota_t)\left(1, -\frac{1}{e_t}\right)$. The real model is similar, replacing m_t and m_t^* by 1.

The gross demand by financiers is $Q_t\,(1, -1/e_t)$. Each period the financiers sell the previous period position, so that their net demand is:

$$Q_t\left(1,-1/e_t\right)-D_tQ_{t-1}\left(1,-1/e_{t-1}\right)=\left(1-D_t\mathcal{L}\right)Q_t\left(1,-1/e_t\right), \tag{A.5}$$

where $\mathcal{L}$ is the lag operator, $\mathcal{L}X_t = X_{t-1}$.

Financiers choose Q_t optimally, given the frictions, as in the main body of the paper and we do not restate their problem here for brevity. In the last period, holdings are 0, i.e. $Q_T = 0$.

The *asset market clearing condition* is that the net demand for bonds is 0

$$\Theta_t^{firms}+\Theta_t^{firms,*}+\left(1-D_t\mathcal{L}\right)\left(\Theta_t+\Theta_t^*+Q_t\left(1,-1/e_t\right)\right)=0. \tag{A.6}$$

For instance, for consumers, $(1-D_t\mathcal{L})\,\Theta_t$ is the increased asset demand by the agent. To gain some intuition, the first coordinate of equation (A.6), evaluated at time $t=0$, in the case where $\Theta_t = \Theta_t^* = 0$, gives equation (25) of the paper:

$$\xi_0 e_0 m_0^* - \iota_0 m_0 + Q_0 = 0;$$

and in the real case (corresponding to $m_0 = m_0^* = 1$), we obtain the basic equation (13) of the paper:

$$\xi_0 e_0 - \iota_0 + Q_0 = 0.$$

We now state formally the definition of equilibrium in the case of flexible prices. Recall that we assume the law of one price in goods market to hold such that:

$$p_{Ht}^* = p_{Ht}/e_t,\, p_{Ft}^* = p_{Ft}/e_t. \tag{A.7}$$

Definition A competitive equilibrium consists of allocations $\left(c_t, c_t^*, \Theta_t, \Theta_t^*, \Theta_t^{firms}, \Theta_t^{firms,*}, Q_t\right)$, prices p_t, p_t^*, exchange rate e_t, for $t=0,\ldots,T$ such that the US consumers optimize their utility function (A.1) under the above constraints (A.2-A.3), Japanese consumers optimize similarly, goods markets clear (A.4), and asset markets clear (A.6), and the law of one price (A.7) holds.

As explained in the paper (Lemma 4), if we use local currency pricing (i.e. change (A.7), and replace the value of p_{Ht}^* and p_{Ft} by other, potentially arbitrary, values), the equilibrium value of the exchange rate does not change (though consumptions do change).

The timing was already stated in the paper, but for completeness we restate it here. At time 0, producers produce, consumers demand and consume, exporters repatriate their sales, financiers take their FX positions, and asset and goods market clear (simultaneously, like in Arrow-Debreu). The potential diversion by the financiers happens at time 0^+, right after time 0 (of course, no diversion happens on the equilibrium path). Then, at time 1 and potentially future periods, the same structure is repeated (with no financiers' position in the last period).

A.1.B Maximization Problem of the Japanese Household

We include there many details excluded from Section II for brevity. The dynamic budget constraint of Japanese households (which holds state by state) is:

$$\sum_{t=0}^{1}\frac{Y_{NT,t}^*+p_{F,t}^*Y_{F,t}+\pi_t^*}{R^{*t}}=\sum_{t=0}^{1}\frac{C_{NT,t}^*+p_{H,t}^*C_{H,t}^*+p_{F,t}^*C_{F,t}^*}{R^{*t}},$$

where π_t^* are the financiers' profits remittances to the Japanese, $\pi_0^*=0$, $\pi_1^*=Q_0(R-R^*e_1/e_0)/e_1$.

The static utility maximization problem of the Japanese household:

$$\max_{C_{NT,t}^*,C_{H,t}^*,C_{F,t}^*}\quad \chi_t^*\ln C_{NT,t}^*+\xi_t\ln C_{H,t}^*+a_t^*\ln C_{F,t}^*+\lambda_t^*\left(CE_t^*-C_{NT,t}^*-p_{H,t}^*C_{H,t}^*-p_{F,t}^*C_{F,t}^*\right),$$

where CE_t^* is aggregate consumption expenditure of the Japanese household, λ_t^* is the associated Lagrange multiplier, p_H^* is the Yen price in Japan of US tradables, and p_F^* is the Yen price in Japan of Japanese tradables. Standard optimality conditions imply:

$$C^*_{NT,t} = \frac{\chi^*_t}{\lambda^*_t}; \qquad p^*_{H,t}C^*_{H,t} = \frac{\xi_t}{\lambda^*_t}; \qquad p^*_{F,t}C^*_{F,t} = \frac{a^*_t}{\lambda^*_t}.$$

Our assumption that $Y^*_{NT,t} = \chi^*_t$, combined with the market clearing condition for Japanese non-tradables $Y^*_{NT,t} = C^*_{NT,t}$, implies that in equilibrium $\lambda^*_t = 1$. We obtain:

$$p^*_{H,t}C^*_{H,t} = \xi_t; \qquad p^*_{F,t}C^*_{F,t} = a^*_t.$$

A.1.C The Euler Equation when there are Several Goods

We state the general Euler equation when there are several goods.

With utility $u^t(\mathsf{C}_t) + \beta u^{t+1}(\mathsf{C}_{t+1})$, where C_t is the vector of goods consumed (for instance, $\mathsf{C}_t = (C_{NT,t}, C_{H,t}, C_{F,t})$ in our setup), if the consumer is at his optimum, we have:

Lemma A.1. *When there are several goods, the Euler equation is:*

$$1 = \mathbb{E}_t\left[\beta R \frac{u^{t+1}_{c_{j,t+1}}/p_{j,t+1}}{u^t_{c_{i,t}}/p_{i,t}}\right] \text{ for all } i, j. \tag{A.8}$$

This should be understood in "nominal" terms, i.e. the return R is in units of the (potentially arbitrary) numéraire.

Proof. It is a variant on the usual one: the consumer can consume $d\varepsilon$ fewer dollars' worth (assuming that the "dollar" is the local unit of account) of good i at time t (hence, consume $dc_{i,t} = -\frac{d\varepsilon}{p_{i,t}}$), invest them at rate R, and consume the proceeds, i.e. $Rd\varepsilon$ more dollars of good j at time $t+1$ (hence, consume $dc_{j,t+1} = \frac{Rd\varepsilon}{p_{j,t+1}}$). The total utility change is:

$$dU = u^t_{c_{i,t}} dc_{i,t} + \beta \mathbb{E}_t u^{t+1}_{c_{j,t+1}} dc_{j,t+1} = \mathbb{E}_t\left(-u^t_{c_{i,t}}/p_{i,t} + \beta R u^{t+1}_{c_{j,t+1}}/p_{j,t+1}\right) d\varepsilon.$$

At the margin, the consumer should be indifferent, so $dU = 0$, hence (A.8). □

Applying this to our setup, with $i = j = NT$, with $p_{NT,t} = 1$ and $u^t_{c_{NT,t}} = \frac{\chi_t}{C_{NT,t}} = 1$ for $t = 0, 1$, we obtain: $1 = \mathbb{E}\left[\beta R \frac{1/1}{1/1}\right]$, hence $R = 1/\beta$.

A.1.D Price Indices, Nominal and Real Exchange Rates

We explore here the relationship between the nominal and the real CPI-based exchange rate in our framework. The real exchange rate can be defined as the ratio of two broad price levels, one in each country, expressed in the same numéraire. It is most common to use consumer price indices (CPI) adjusted by the nominal exchange rate, in which case one has: $\mathcal{E} \equiv \frac{P^* e}{P}$. Notice that a fall in $\mathcal{E}$ is a US Dollar real appreciation.

Consider the nominal version of the basic Gamma model in Section IV. Standard calculations reported below imply that the real CPI-based exchange rate is:

$$\mathcal{E} = \tilde{\theta} \frac{(p^*_H)^{\xi'} (p^*_F)^{a^{*\prime}} (p^*_{NT})^{\chi^{*\prime}}}{(p_H)^{a'} (p_F)^{\iota'} (p_{NT})^{\chi'}} e_t, \tag{A.9}$$

where $\tilde{\theta}$ is a function of exogenous shocks and primed variables are normalized by θ. The above equation is the most general formulation of the relationship between the CPI-RER and the nominal exchange rate in the Gamma model.

Let us first derive the price indices $\{P, P^*\}$. The US price index P is defined as the minimum cost, in units of the numéraire (money), of obtaining one unit of the consumption basket:

$$C_t \equiv \left[\left(\frac{M_t}{P_t} \right)^{\omega_t} (C_{NT,t})^{\chi_t} (C_{H,t})^{a_t} (C_{F,t})^{\iota_t} \right]^{\frac{1}{\theta_t}}.$$

Let us define a "primed" variable as being normalized by the sum of the preference coefficients θ_t; so that, for example, $\chi_t^{'} \equiv \frac{\chi_t}{\theta_t}$. Substituting the optimal demand for goods (see the first order conditions at the beginning of Section IV) in the consumption basket formula we have:

$$1 = \left(\omega^{'} P \right)^{\omega^{'}} \left(a^{'} \frac{P}{p_H} \right)^{a^{'}} \left(\iota^{'} \frac{P}{p_F} \right)^{\iota^{'}} \left(\chi^{'} \frac{P}{p_{NT}} \right)^{\chi^{'}}.$$

Hence:

$$P = (p_H)^{a^{'}} (p_F)^{\iota^{'}} (p_{NT})^{\chi^{'}} \left[(\omega_t^{'})^{-\omega_t^{'}} (\iota_t^{'})^{-\iota_t^{'}} (a_t^{'})^{-a_t^{'}} (\chi_t^{'})^{-\chi_t^{'}} \right].$$

The part in square brackets is a residual and not so interesting. Similarly for Japan, we have:

$$P^* = (p_H^*)^{\xi^{'}} (p_F^*)^{a^{*'}} (p_{NT}^*)^{\chi^{*'}} \left[(\omega_t^{*'})^{-\omega_t^{*'}} (\xi_t^{'})^{-\xi_t^{'}} (a_t^{*'})^{-a_t^{*'}} (\chi_t^{*'})^{-\chi_t^{*'}} \right].$$

The CPI-RER in equation (A.9) is then obtained by substituting the price indices above in the definition of the real exchange rate $\mathcal{E} \equiv \frac{P^* e}{P}$. For completeness, we report below the full expression for the function $\tilde{\theta}$ that enters in equation (A.9):

$$\tilde{\theta}_t = \frac{(\omega_t^{*'})^{-\omega_t^{*'}} (\xi_t^{'})^{-\xi_t^{'}} (a_t^{*'})^{-a_t^{*'}} (\chi_t^{*'})^{-\chi_t^{*'}}}{(\omega_t^{'})^{-\omega_t^{'}} (\iota_t^{'})^{-\iota_t^{'}} (a_t^{'})^{-a_t^{'}} (\chi_t^{'})^{-\chi_t^{'}}}.$$

If we impose further assumptions on Equation (A.9), we can derive some useful special cases.

The Basic Gamma Model Assume that $\omega = \omega^* = 0$ and $p_{NT} = p_{NT}^* = 1$ so that there is no money and the numéraire in each economy is the non-tradable good. Recall that in the basic Gamma model of Section II the law of one price holds for tradables, so we have $p_H = p_H^* e$ and $p_F = p_F^* e$. Equation (A.9) then reduces to: $\mathcal{E} = \tilde{\theta} \, (p_H)^{\xi^{'} - a^{'}} (p_F)^{a^{*'} - \iota^{'}} e^{\chi^{*'}}$. This equation describes the relationship between the RER as defined in the basic Gamma model and the CPI-based RER. Notice that the two are close proxies of each other whenever the baskets' shares of tradables are symmetric across countries (i.e. $\xi^{'} \approx a^{'}$ and $a^{*'} \approx \iota^{'}$) and the non-tradable goods are a large fraction of the Japanese overall basket (i.e. $\chi^{*'} \approx 1$).

The Basic Complete Market Model We maintain all the assumptions from the paragraph above on the Basic Gamma model, except that we now assume markets to be complete and frictionless. Recall from Lemma 3 that we then obtain $e_t = \nu$. Hence, the CPI-RER now follows: $\mathcal{E} = \tilde{\theta} \, (p_H)^{\xi^{'} - a^{'}} (p_F)^{a^{*'} - \iota^{'}} \nu^{\chi^{*'}}$. Notice that while the real exchange rate (e) is constant in complete markets in the basic Gamma model, the CPI-RER will in general not be constant as long as the CPI baskets are not symmetric and relative prices of goods move.

A.1.E The Backus and Smith Condition

In the spirit of re-deriving some classic results of international macroeconomics with the Gamma model, let us analyze the Backus and Smith condition (Backus and Smith (1993)). Let us first consider the basic Gamma set-up but with the additional assumption of complete markets as in Lemma 3. Then by equating marginal utility growth in the two countries and converting, via the exchange rate, in the same units, we have: $\frac{P_0 C_0/\theta_0}{P_1 C_1/\theta_1} = \frac{P_0^* C_0^*/\theta_0^*}{P_1^* C_1^*/\theta_1^*}\frac{e_0}{e_1}$. Re-arranging we conclude:

$$\frac{C_0/\theta_0}{C_1/\theta_1} = \frac{C_0^*/\theta_0^*}{C_1^*/\theta_1^*}\frac{\mathcal{E}_0}{\mathcal{E}_1}, \tag{A.10}$$

where the reader should recall the definition $\mathcal{E} = \frac{P^* e}{P}$. This is the Backus and Smith condition in our set-up under complete markets: the perfect risk sharing benchmark equation.

Of course, this condition fails in the basic Gamma model because agents not only cannot trade all Arrow-Debreu claims, but also have to trade with financiers in the presence of limited commitment problems. In our framework (Section II), however, an extended version of this condition holds:

$$\frac{C_0/\theta_0}{C_1/\theta_1} = \frac{C_0^*/\theta_0^*}{C_1^*/\theta_1^*}\frac{\mathcal{E}_0}{\mathcal{E}_1}\frac{e_1}{e_0}. \tag{A.11}$$

The simple derivation of this result is reported below. The above equation is the extended Backus-Smith condition that holds in our Gamma model. Notice that our condition in equation (A.11) differs from the standard Backus-Smith condition in equation (A.10) by the growth rate of the "nominal" exchange rate $\frac{e_1}{e_0}$. Since exchange rates are much more volatile in the data than consumption, this omitted term creates an ample wedge between the complete market and the Gamma version of the Backus-Smith condition.

The condition in equation (A.11) can be verified as follows:

$$\frac{C_0/\theta_0}{C_1/\theta_1} = \frac{C_0^*/\theta_0^*}{C_1^*/\theta_1^*}\frac{\mathcal{E}_0}{\mathcal{E}_1}\frac{e_1}{e_0} \iff \frac{P_0 C_0/\theta_0}{P_1 C_1/\theta_1} = \frac{P_0^* C_0^*/\theta_0^*}{P_1^* C_1^*/\theta_1^*} \iff \frac{1}{1} = \frac{1}{1},$$

where the first equivalence simply makes use of the definition $\mathcal{E} \equiv \frac{P^* e}{P}$, and the second equivalence follows from $P_t C_t = \theta_t$ and $P_t^* C_t^* = \theta_t^*$ for $t = 0, 1$. These latter equalities (we focus here on the US case) can be recovered by substituting the households' demand functions for goods in the static household budget constraint: $P_t C_t = C_{NT,t} + p_{H,t} C_{H,t} + p_{F,t} C_{F,t} = \chi_t + a_t + \iota_t = \theta_t$.

A.2 Extensions of the Model

A.2.A Japanese Households and the Carry Trade

In most of the main body of the paper, consumers do not do the carry trade themselves. In this subsection, we extend Proposition 6 by analyzing the case in which Japanese consumers buy a quantity f^* of dollar bonds, financing the purchase by shorting an equivalent amount of Yen bonds. We let this demand take the form:

$$f^* = b\left(R - R^*\right).$$

Recall that Proposition 6 assumes $R < R^*$, so that if $b \geq 0$ the Japanese household demand is a form of carry trade. The flow equations now are:

$$NX_0 + Q + f^* = 0; \qquad NX_1 - R\left(Q + f^*\right) = 0.$$

We summarize the implications for the equilibrium carry trade in the Proposition below.

Proposition A.1. *Assume $\xi_t = 1$ for $t = 0, 1$, $R < R^*$ and that Japanese consumers do the carry trade in amount f^*, the expected return to the carry trade in the Gamma model is:*

$$\overline{R}^c = \Gamma \frac{\frac{R^*}{R}\mathbb{E}\left[\iota_1\right] - \iota_0 + f^*(1 + R^*)}{(R^* + \Gamma)\,\iota_0 + \frac{R^*}{R}\mathbb{E}\left[\iota_1\right] - \Gamma f^*}.$$

Hence the carry trade return is bigger: (i) when R^*/R is higher, (ii) when the funding country is a net foreign creditor, and (iii) when consumers do the carry trade less (f^* increases).

If consumers do the carry trade on too large a scale (f^* too negative), then the carry trade becomes unprofitable, $\overline{R}^c < 0$.

A.2.B Endogenizing the Number of Financiers

In the basic model, there is a fixed quantity of financiers. We now show a possible way to endogenize entry of financiers. This will confirm that the first order results of the paper are unchanged, except that γ is now endogenous.

We call $\Omega = \mathbb{E}_0\left[1 - \frac{e_1}{e_0}\frac{R^*}{R}\right]$ the expected discounted return of currency trading. Suppose that each potential trader has an incentive constraint of the form:

$$V_0 \equiv \Omega q_0 = \mathbb{E}_0\left[\beta\left(R - R^*\frac{e_1}{e_0}\right)\right]q_0 \geq G\frac{q_0^2}{e_0},$$

and we have $G = g\left(var_0\left(e_1\right)\right)^{\alpha}$ for a parameter g. Hence g and G are the agent's γ and Γ. Using $R\beta = 1$, this entails an individual demand:

$$q_0 = \frac{\Omega e_0}{G},$$

and a benefit

$$V_0 = \Omega q_0 = \frac{\Omega^2 e_0}{G} = \frac{\Omega^2 e_0}{g\ var_0\left(e_1\right)^{\alpha}}.$$

In the spirit of Jeanne and Rose (2002), we posit that financiers decide to enter at date -1 (before the values of $\iota_0, \mathbb{E}_0\left[\iota_1\right]$ are realized, hence before the actual expected currency trading return is known). Potential financier i enters if and only if $\mathbb{E}_{-1}\left[V_0\right] \geq \kappa_i$, where κ_i is a (perhaps psychological) cost drawn from a distribution with CDF $F\left(x\right) = P\left(\kappa_i \leq x\right)$. This implies that the mass n of financiers is

$$n = F\left(\mathbb{E}_{-1}\left[\Omega^2 e_0 var_0\left(e_1\right)^{-\alpha}\right]/g\right).$$

The aggregate demand at time 0 is then:

$$Q_0 = nq_0 = n\frac{\Omega e_0}{g\ var_0\left(e_1\right)^{\alpha}}.$$

Hence, we have $Q_0 = \frac{\Omega e_0}{\gamma\ var_0(e_1)^{\alpha}}$ with

$$\gamma = \frac{g}{n}, \tag{A.12}$$

so that

$$\gamma = \frac{g}{F\left(\mathbb{E}_{-1}\left[\Omega^2 e_0 var_0\left(e_1\right)^{-\alpha}\right]/g\right)}. \tag{A.13}$$

Hence, we have a fixed point determining γ, since e_0 and e_1 depend on γ.

Starting after date 0, the analysis is exactly like in the paper, except that the value of γ is pinned down by considerations at time -1.

For instance, take our baseline case, where $R = R^* = 1$. From Proposition 1, $e_0 = \frac{(1+\Gamma)\iota_0 + \mathbb{E}_0[\iota_1]}{2+\Gamma}$, $\Omega = \frac{\Gamma(\iota_0 - \mathbb{E}_0[\iota_1])}{(1+\Gamma)\iota_0 + \mathbb{E}_0[\iota_1]}$, and γ solves:

$$g = \gamma F\left(\mathbb{E}_{-1}\left[\frac{\Gamma\left(\gamma\right)^2\left(\iota_0 - \mathbb{E}_0\left[\iota_1\right]\right)^2 var_0\left(e_1\right)^{-\alpha}}{\left[\left(1+\Gamma\left(\gamma\right)\right)\iota_0 + \mathbb{E}_0\left[\iota_1\right]\right]\left(2+\Gamma\left(\gamma\right)\right)g}\right]\right) \text{ with } \Gamma\left(\gamma\right) = \gamma\ var_0\left(\iota_1\right)^{\alpha}. \tag{A.14}$$

We note that the government might wish to subsidize entry in the financial sector so to effectively remove the financial constraint. This is a property common to many models of financial imperfections: for example if the financiers had limited capital as in Kiyotaki and Moore (1997); Gertler and Kiyotaki (2010); Brunnermeier and Sannikov (2014); He and Krishnamurthy (2013), the government would want to recapitalize them in many states of the world.[59] Like those papers, we do not consider the optimal subsidy to financiers. One reason for this is that in practice, it is difficult as the government might be facing frictions with the financiers such as moral hazard or adverse selection. For example, the government might want to screen for "smart" FX traders that stabilize FX markets, and not subsidize noise traders, who might actually worsen the situation (they would be creating f, f^* shocks in our model).[60]

A.2.C A "Short-Run" Vs "Long-Run" Analysis

As in undergraduate textbooks, it is handy to have a notion of the "long run". We develop here a way to introduce it in our model. We have periods of unequal length: we say that period 0 is short, but period "1" lasts for a length T. The equilibrium flow equations in the dollar-yen market become:

$$\begin{aligned} \xi_0 e_0 - \iota_0 + Q_0 &= 0, \\ T(\xi_1 e_1 - \iota_1) - RQ_0 &= 0. \end{aligned} \tag{A.15}$$

The reason for the "T" is that the imports and exports will occur over T periods. We assume a zero interest rate "within period 1". This already gives a good notion of the "long run".[61]

Some extra simplicity is obtained by taking the limit $T \to \infty$. The interpretation is that period 1 is "very long" and period 0 is "very short". The flow equation (A.15) can be written: $\xi_1 e_1 - \iota_1 - \frac{RQ_0}{T} = 0$. So in the large T limit we obtain: $\xi_1 e_1 - \iota_1 = 0$. Economically, it means the trades absorbed by the financiers are very small compared to the trades in the goods markets in the long run. We summarize the environment and its solution in the following proposition.[62]

Proposition A.2. *Consider a model with a "long-run" last period. Then, the flow equations become* $\xi_0 e_0 - \iota_0 + Q_0 = 0$ *and* $\xi_1 e_1 - \iota_1 = 0$*, while we still have* $Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1 \frac{R^*}{R}\right]$*. The exchange rates become:*

$$e_0 = \frac{\frac{R^*}{R}\mathbb{E}\left[\frac{\iota_1}{\xi_1}\right] + \Gamma \iota_0}{1 + \Gamma \xi_0}; \qquad e_1 = \frac{\iota_1}{\xi_1}.$$

In this view, the "long run" is determined by fundamentals $e_1 = \frac{\iota_1}{\xi_1}$, while the "short run" is determined both by fundamentals and financial imperfections (Γ) with short-run considerations (ι_0, ξ_0). In the simple case $R = R^* = \xi_t = 1$, we obtain: $e_0 = \frac{\Gamma\iota_0 + \mathbb{E}[\iota_1]}{\Gamma + 1}$ and $e_1 = \iota_1$.

Application to the carry trade with three periods. In the 3-period carry trade model of Section III.A, we take period 2 to be the "long run". We assume that in period $t = 1$ financiers only intermediate the new flows; stocks arising from previous flows are held passively by the households (long term investors) until t=2. That allows us to analyze more clearly the dynamic environment. Without the "long-run" period 2, the expressions are less intelligible, but the economics is the same.

[59] We thank a referee for remarks along these lines.

[60] With endogenous entry, the FX intervention considered in Section III.B will also ex ante affect entry, similarly to the analysis in Jeanne and Rose (2002). We leave this interesting analysis to future research.

[61] The solution is simply obtained by Proposition 3, setting $\tilde{\iota}_1 = T\iota_1, \tilde{\xi}_1 = T\xi_1$.

[62] One derivation is as follows. Take Proposition 3, set $\tilde{\iota}_1 = T\iota_1, \tilde{\xi}_1 = T\xi_1$, and take the limit $T \to \infty$.

A.2.D The Fama Regression over Longer Horizons

We take the context of the Fama regresssion in the paper, and now consider the Fama regression over a 2-period horizon:

$$\frac{1}{2}\frac{e_2 - e_0}{e_0} = \alpha + \beta_{UIP,2}\left(R - R^*\right) + \varepsilon_1$$

i.e. regressing (normalized) the 2-period return on the interest rate differential. We assume that Γ_1 is deterministic.

Lemma A.2. *The coefficient* $\beta_{UIP,2} = \frac{1+\Gamma_1/2}{(1+\Gamma_0)(1+\Gamma_1)}$*, while the UIP coefficient in a 1-period regression is* $\beta_{UIP,1} \equiv \beta_{UIP} = \frac{1+\Gamma_1-\Gamma_0}{(1+\Gamma_0)(1+\Gamma_1)}$*, as in the main text.*

Proof We evaluate the derivative at $R = R^* = 1$, and for simplicity take the case Γ_1 deterministic.

$$\begin{aligned}\beta_{UIP,2} &= \frac{-1}{2}\frac{\partial \mathbb{E}\left[\frac{e_2-e_0}{e_0}\right]}{\partial R^*} = \frac{-1}{2}\frac{\partial}{\partial R^*}\frac{\Gamma_0+1}{\Gamma_0\iota_0 + \mathcal{R}^*\mathbb{E}\left[\frac{\Gamma_1\iota_1+\mathcal{R}^*\iota_2}{\Gamma_1+1}\right]} \\ &= \frac{1}{2}\frac{\Gamma_0+1}{(\Gamma_0+1)^2}\left(\mathbb{E}\left[\frac{\Gamma_1\iota_1+\iota_2}{\Gamma_1+1}\right] + \mathbb{E}\left[\frac{\iota_2}{\Gamma_1+1}\right]\right)] = \frac{1}{2}\frac{1}{\Gamma_0+1}\left(1+\mathbb{E}\left[\frac{1}{\Gamma_1+1}\right]\right) \\ &= \frac{1}{2}\frac{2+\Gamma_1}{(\Gamma_0+1)\left(1+\Gamma_1\right)} = \frac{1+\Gamma_1/2}{(1+\Gamma_0)\left(1+\Gamma_1\right)}.\end{aligned}$$

□

Hence, we have $1 \geq \beta_{UIP,2}$ as often found empirically. Furthermore, we have $\beta_{UIP,2} \geq \beta_{UIP,1}$ if and only if $\Gamma_1 \leq 2\Gamma_0$. For instance, suppose that Γ_1 and Γ_0 are drawn from the same distribution. Then, $\mathbb{E}\left[\beta_{UIP,2}\right] \geq \mathbb{E}\left[\beta_{UIP,1}\right]$: this means that as the horizon expands, the coefficient of the Fama regression is closer to 1. This is consistent with the empirical evidence that the Fama regression coefficient is higher, and closer to 1, at long horizons (Chinn and Meredith (2005)).

A.3 Model Extensions: Multi-Country, Multi-Asset Model, and Additional Material on the Variance in the Constraint

We provide below generalizations of the model. In particular, we develop a multi-asset, multi-country model.

A.3.A Verification of the tractability of the model when the variance is in the constraint

In the paper, we propose a formulation of $\Gamma = \gamma var(e_1)^\alpha$. We verify that it leads to a tractable model in the core parts of the paper. In this subsection of the appendix, we check that we also keep a tractable model in a more general model with T periods.

When ξ_t is deterministic, the formulation remains tractable. We obtain each Γ_t in closed form.[63] Let us

[63]However, when ξ_t is stochastic, the formulation is more complex. We obtain a fixed point problem not just in Γ_0 (like in the 2-period model), but in $(\Gamma_{0,...,}\Gamma_{T-1})$.

work out explicitly a 3-period example. We take $\xi_t = R = R^* = 1$ for simplicity. The equations are:

$$\begin{aligned}
e_0 - \iota_0 + Q_0 &= 0, \\
e_1 - \iota_1 - Q_0 + Q_1 &= 0, \\
e_2 - \iota_2 - Q_1 &= 0, \\
Q_t &= \frac{\mathbb{E}_t\left[e_t - e_{t+1}\right]}{\Gamma_t} \text{for } t = 0, 1, \\
\Gamma_t &= \gamma\, var_t\left(e_{t+1}\right)^{\alpha}.
\end{aligned}$$

Notice that the model at $t = 1, 2$ is like the basic model with 2 periods, except for the pseudo-import term $\widetilde{\iota}_1 = \iota_1 - Q_1$. Hence, we have $\{e_2\} = \{\iota_2\}$, and

$$\Gamma_1 = \gamma \sigma_{\iota_2}^{2\alpha}. \tag{A.16}$$

This also implies that (by Proposition 3 applied to (e_1, e_2) rather than (e_0, e_1)): $\{e_1\} = \frac{1+\Gamma_1}{2+\Gamma_1}\{\iota_1\}$, which gives:

$$\Gamma_0 = \gamma\left(\frac{1+\Gamma_1}{2+\Gamma_1}\sigma_{\iota_1}\right)^{2\alpha}, \tag{A.17}$$

so we endogenize Γ_0. Note that the σ_{ι_1} is, in general, the variance of pseudo-imports, hence it would include the volatility due to financial flows. Notice also that fundamental variance is endogenously amplified by the imperfect financial market: $var(e_1)$ depends positively on Γ_1, that itself depends positively on fundamental variance.

The same idea and procedure applies to an arbitrary number of periods, and indeed to the infinite period model. We could also have correlated innovations in ι_t.

A.3.B A tractable multi-country model

We call e_i^t the exchange rate of country i at date t, with a high e_i^t being an appreciation of country i's currency versus the USD. There is a central country 0, for which we normalize $e_0^t = 1$ at all dates t. As a short hand, we call this country "the US". For $i \neq j$, call $\xi_{ij} < 0$ exports of country i to country j (*minus* the Cobb-Douglas weight), and $x_i = -\xi_{i0} > 0$ exports of country i to country 0. Define the import weight as:

$$\xi_{ii} \equiv -\sum_{j=0,\dots,n, j\neq i} \xi_{ji} > 0,$$

so that ξ_{ii} equals total imports of country i. Call θ_i the holdings of country i's bonds by financiers, expressed in number of bonds: so, the dollar value of those bond holdings is $q_i \equiv \theta_i e_i^0$.

Hence, the net demand for currency i in the currency i / USD spot market, expressed in dollars, is:

$$-\sum_{j\neq 0} \xi_{ij}^0 e_j^0 + x_i^0 + \theta_i e_i^0, = 0, \tag{A.18}$$

and has to be 0 in market equilibrium. Indeed, at time 0 the country imports a dollar value $\xi_{ii}^0 e_i^0$, creating a negative demand $-\xi_{ii}^0 e_i^0$ for the currency. It also exports a dollar value $-\sum_{j\neq 0} \xi_{ij}^0 e_j^0 + x_i^0$ (recall that $\xi_{ij} < 0$ for $i \neq j$); as those exports are repatriated, they lead to a demand for the currency. Finally, financiers demand a dollar value $\theta_i e_i^0$ of the country's bonds. Using $q_i \equiv \theta_i e_i^0$, equation (A.18) can be rewritten in vector form:

$$-\xi^0 e^0 + x^0 + q = 0. \tag{A.19}$$

The flow equation at time $t = 1$ is (again, net demand for currency i in the dollar-currency i market, expressed in dollars):

$$- \sum_{j \neq 0} \xi^1_{ij} e^1_j + x^1_i - \theta_i e^1_i + \Pi_i = 0 \tag{A.20}$$

where Π_i is the time-1 rebate of financiers profits to country i. In the first equation, imports enter as $-\xi^0_{ii} e^0_i < 0$, creating a net negative demand for currency i, and exports to other countries enter as $- \sum_{j \neq 0, i} \xi^0_{ij} e^0_j > 0$.

Total financiers' profit is: $\Pi \equiv \sum_i \Pi_i = \sum_i \theta_i \left(e^1_i - e^0_i \right)$. We posit the following rule for the rebate Π_i to country i: $\Pi_i = \theta_i \left(e^1_i - e^0_i \right)$. Then, (A.20) becomes: $- \sum_{j \neq 0} \xi^1_{ij} e^1_j + x^1_i - \theta_i e^0_i = 0$, i.e., in vector form:

$$- \xi^1 e^1 + x^1 - q = 0. \tag{A.21}$$

Finally, we will have the generalized demand for assets:

$$q = \Gamma^{-1} \mathbb{E} \left[e^1 - e^0 \right], \tag{A.22}$$

where q, e^t are vectors, and Γ is a matrix. We provide a derivation of this demand in section A.3.C. The financiers buy a dollar value q_i of country i's bonds at time 0, and $- \sum_{i=1}^n q_i$ dollar bonds, so that the net time-0 value of their initial position is 0. The correspondence with the basic Gamma model (with only 2 countries) is $q = -Q$, $x_{it} = \iota_{it}$.

We summarize the set-up below.

Lemma A.3. *In the extended n-country model, the basic equations describing the vectors of exchange rates e^t are:*

$$\xi^0 e^0 - x^0 - q = 0, \tag{A.23}$$

$$\xi^1 e^1 - x^1 + q = 0, \tag{A.24}$$

$$\mathbb{E} \left[e^1 - e^0 \right] = \Gamma q. \tag{A.25}$$

Those are exactly the equations of the 2−country model (with $Q^{\text{Gamma}} = -q^{\text{here}}$), and $\iota^{\text{Gamma}} = x^{\text{here}}$, but with n countries (so $e^t \in \mathbb{R}^{n-1}$). Hence the solution is the same (using matrices). We assume that ξ^1 is deterministic.

Proposition A.3. *The exchange rates in the $n-$country model are given by the following vectors:*

$$e^0 = \left(\xi^0 + \xi^1 + \xi^1 \Gamma \xi^0 \right)^{-1} \left(\left(1 + \xi^1 \Gamma \right) x^0 + \mathbb{E} \left[x^1 \right] \right), \tag{A.26}$$

$$e^1 = \left(\xi^0 + \xi^1 + \xi^0 \Gamma \xi^1 \right)^{-1} \left(x^0 + \left(1 + \xi^0 \Gamma \right) \mathbb{E} \left[x^1 \right] \right) + \left(\xi^1 \right)^{-1} \left\{ x^1 \right\}. \tag{A.27}$$

Hence, the above model has networks of trade in goods, and multi-country asset demand.

A.3.C Derivation of the multi-asset, multi-country demand

We derive the financiers' demand function in a multi-asset case. We start with a general asset case, and then specialize our results to exchange rates.

A.3.C.1 General asset pricing case

Basic case We use notations that are valid in general asset pricing, as this makes the exposition clearer and more general. We suppose that there are assets $a = 1, \ldots, A$, with initial price p^0, and period 1 payoffs p^1 (all in $\mathbb{R}^A$). Suppose that the financiers hold a quantity position $\theta \in \mathbb{R}^A$ of those assets, so that the terminal value is $\theta \cdot p^1$. We want to compute the equilibrium price at time 0.

Let

$$\pi = \mathbb{E} \left[p^1 \right] - p^0,$$

denote the expected gain (a vector), and

$$V = var\left(p^1\right),$$

denote the variance-covariance matrix of period 1 payoffs.

Given a matrix G, our demand will generate the relation

$$\pi = G\theta^*, \tag{A.28}$$

This is a generalization to an arbitrary number of assets of the basic demand of Lemma 2, $Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right]$. The traditional mean-variance case is $G = \gamma V$. The present machinery yields more general terms: for example, we could have $G = VH'$, for a "twist" matrix H. The mean-variance case is $H = \gamma I_n$, for a risk-aversion scalar γ. The H can, however, represent deviations from that benchmark, e.g. source-dependent risk aversion (if $H = diag(\gamma_1, ..., \gamma_A)$, we have a "risk aversion" scalar γ_a for source a), or tractability-inducing twists (our main application here). Hence, the machinery we develop here will allow to go beyond the traditional mean-variance setup.

The financiers' profits (in dollars) are: $\theta \cdot \left(p^1 - p^0\right)$, and their expected value is $\theta'\pi$, where $\pi := \mathbb{E}\left[p^1\right] - p^0$. We posit that financiers solve:

$$\max_{\theta \in R^A} \theta'\pi \text{ s.t. } \theta'\pi \geq \theta' S\theta,$$

where S is a symmetric, positive semi-definite matrix. This is a limited commitment constraint: the financiers' outside option is $\theta' S\theta$. Hence, the incentive-compatibility condition is $\theta'\pi \geq \theta' S\theta$. Again, this is a generalization (to an arbitrary number of assets) of the constraint in the paper in Equation (8).

The problem implies:

$$\pi = S\theta^*,$$

where θ^* is the equilibrium θ.[64]

Hence, we would deliver (A.28) if we could posit $S = G$. However, this is not exactly possible, because S must be symmetric, and G is not necessarily symmetric.

We posit that the outside option $\theta' S\theta$ equals:[65]

$$\theta' S\theta \equiv \sum_{i,j} \theta_i^2 \frac{1_{\theta_i^* \neq 0}}{\theta_i^*} G_{ij}\theta_j^*, \tag{A.29}$$

where θ is chosen by the financier under consideration, and θ^* is the equilibrium demand of *other* financiers (in equilibrium, $\theta = \theta^*$). This functional form captures the fact that as the portfolio or balance sheet expands (θ_i high), it is "more complex" and the outside option of the financiers increases. In addition (if say $G = \gamma V$), it captures that high variance assets tighten the constraint more (perhaps again because they are more "complex" to monitor). The non-diagonal terms indicate that "similar" assets (as measured by covariance) matter. Finally, the positions of other financiers matter. Mostly, this assumption is made for convenience. However, it captures the idea (related to Basak and Pavlova (2013)) that the positions of other traders influence the portfolio choice of a given trader. The influence here is mild: when G is diagonal, there is no influence at all.

We will make the assumption that

$$\forall i,\ sign\left(\pi_i^*\right) = sign\left(\theta_i^*\right), \text{ where } \pi^* \equiv G\theta^*. \tag{A.30}$$

This implies that S is a positive semi-definite matrix: for instance, when $\theta_i^* \neq 0$, $\sum_j \frac{1}{\theta_i^*} G_{ij}\theta_j^* \geq 0$. Equation (A.30) means that the sign of the position θ_i^* is equal to the sign of the expected return π_i. This is a mild

[64] The proof is as follows. Set up the Lagrangian $\mathcal{L} = \theta'\pi + \lambda\left(\theta'\pi - \theta' S\theta\right)$. The first-order condition reads $0 = \mathcal{L}_{\theta'} = (1+\lambda)\pi - 2\lambda S\theta$. So, $\pi = \frac{2\lambda}{1+\lambda} S\theta$. Left-multiplying by θ' yields $\theta'\pi = \frac{2\lambda}{1+\lambda}\theta' S\theta$. Since $\theta'\pi \geq \theta' S\theta$, we need $\lambda \geq 1$. Hence, $\pi = S\theta$.

[65] This is, $S_{ij} = 1_{i=j}\frac{1}{\theta_i^*} G_{ij}\theta_j^*$ if $\theta_i^* \neq 0$, $S_{ij} = 0$ if $\theta_i^* = 0$.

assumption that rules out situations where hedging terms are very large.

We summarize the previous results. Recall that we assume (A.30).

Proposition A.4. *(General asset pricing case: foundation for the financiers' demand) With the above set-up, the financiers' equilibrium holdings θ^* satisfy:*

$$\mathbb{E}\left[p^1 - p^0\right] = G\theta^*, \tag{A.31}$$

with G a matrix. When G is invertible, we obtain the demand $\theta^* = G^{-1}\mathbb{E}\left[p^1 - p^0\right]$.

Proof: First, take the case $\theta_i^* \neq 0$. Deriving (A.29) w.r.t. θ_i: $2\,(S\theta)_i = \sum_j \frac{2\theta_i}{\theta_i^*} G_{ij}\theta_j^*$, so that $(S\theta^*)_i = \sum_j G_{ij}\theta_j^* = (G\theta^*)_i$. When $\theta_i^* = 0$, assumption (A.30) implies again $(S\theta^*)_i = \sum_j S_{ij}\theta_j^* = 0 = \pi_i^* = (G\theta^*)_i$.

Thus, $S\theta^* = G\theta^*$. Hence, the set-up induces $\pi = S\theta^* = G\theta^*$. $\square$

Proposition A.5. *Suppose that we can write $G = VH'$, for some matrix H. Then, a riskless portfolio simply offers the riskless US return, and in that sense the model is arbitrage-free.*

Proof: Suppose that you have a riskless, 0-investment portfolio κ: $\kappa' V = 0$. Given $\pi = VH'\theta^*$, we have $\kappa'\pi = \kappa' G\theta^* = \kappa' VH'\theta^* = 0$, i.e. the portfolio has 0 expected return, hence, as it is riskless, the portfolio has 0 return. $\square$

Proposition A.7 offers a stronger statement that the model is arbitrage-free.

A.3.C.2 Extension with derivatives and other redundant assets

The reader may wish to initially skip the following extension. When there are redundant assets (like derivatives), some care needs to be taken when handling indeterminacies (as many portfolios are functionally equivalent). Call Θ the full portfolio, including redundant assets, and P^t the full price vector. We say that assets $a \leq B$ are a basis, and we reduce the portfolio Θ into its "basis-equivalent" portfolio in the basis, $\theta \in \mathbb{R}^B$, with price p^t, defined by:

$$\Theta \cdot P^1 = \theta \cdot p^1, \quad \text{for all states of the world.}$$

For instance, if asset c is redundant and equal to asset a minus asset b ($p_c^1 = p_a^1 - p_b^1$), then $(\theta_a, \theta_b) = (\Theta_a + \Theta_c, \Theta_b - \Theta_c)$.

More generally, partition the full portfolio into basis assets Θ_B and derivative assets Θ_D, $\Theta = (\Theta_B, \Theta_D)$, and similarly partition prices in $P = (p, p_D)$. We sometimes write p_B rather than p when this clarifies matters. As those assets are redundant, there is a matrix Z such that

$$p_D^1 = Zp^1.$$

Then, the basis-equivalent portfolio is $\theta = \Theta_B + Z'\Theta_D$.[66]

Then, we proceed as above, with the "basis-equivalent portfolio". This gives the equilibrium pricing of the basis assets, p_B^0. Then, derivatives are priced by arbitrage:

$$p_D^0 = Zp^0,$$

A.3.C.3 Formulation with a Stochastic Discount Factor

The following section is more advanced, and may be skipped by the reader.

It is often useful to represent pricing via a Stochastic Discount Factor (SDF). Let us see how to do that here. Call $w = P_1'\Theta = p_1'\theta$ the time-1 wealth of the financiers. Recall that we have $\pi = G\theta^*$, with $G = VH'$.

If we had traditional mean-variance preferences, with $\pi = \gamma V\theta^*$, we could use a SDF: $M = 1 - \gamma\{w\}$, for a scalar γ. We want to generalize that idea.

[66] Proof: the payoffs are $\Theta' P^1 = \Theta_B' p^1 + \Theta_D' p_D^1 = \Theta_B' p^1 + \Theta_D' Z p_B^1 = \theta' p_B^1$ with $\theta' = \Theta_B' + \Theta_D' Z$.

As before, we define

$$\{X\} \equiv X - \mathbb{E}[X]$$

to be the innovation to a random variable X.

Recall that we are given B basis assets $a = 1, \ldots, B$ (i.e., $\left(\{p_a^1\}\right)_{a=1,\ldots,B}$ are linearly independent), while assets $a = B+1, ..., A$ are derivatives (e.g. forward contracts), and so their payoffs are spanned by the vector $\left(p_a^1\right)_{a \leq B}$.

Next, we choose a linear operator Ψ for the basis assets that maps random variables into random variables.[67] It is characterized by:

$$\Psi\left\{p_a^1\right\} = \sum_b H_{ab}\left\{p_b^1\right\} \text{ for } a = 1, \ldots, B, \text{ and for } b = 1, \ldots, B,$$

or, more compactly:

$$\Psi\left\{p^1\right\} = H\left\{p^1\right\}.$$

This is possible because $\{p_1^a\}$ are linearly independent. The operator extends to the whole space S of traded assets (including redundant assets).

Proposition A.6. *The pricing is given by the SDF:*

$$M = 1 - \Psi\{w\}, \tag{A.32}$$

where $w = P_1'\Theta = p_1'\theta$ is the time-1 wealth of the financiers.

Proposition A.7. *If the shocks $\{p^1\}$ are bounded and the norm of matrix H, $\|H\|$, is small enough, then $M > 0$ and the model is arbitrage-free.*

In addition, it shows that the SDF depends linearly on the agents' total terminal wealth w, including their proceeds from positions in derivatives.

Proof. We need to check that this SDF generates: $p^0 = \mathbb{E}\left[Mp^1\right]$. Letting $M = 1 - m$ with $m = \Psi\{w\}$, we need to check that $p^0 = \mathbb{E}\left[p^1 - mp^1\right] = \mathbb{E}\left[p^1\right] - \mathbb{E}\left[mp^1\right]$, i.e. $\pi := \mathbb{E}\left[p^1 - p^0\right] = \mathbb{E}\left[mp^1\right]$. Recall that we have $\pi = G\theta = VH'\theta$.

Hence, we compute:

$$\begin{aligned}
\mathbb{E}\left[mp_a^1\right] &= \mathbb{E}\left[p_a^1\left(\Psi\{w\}\right)\right] \\
&= \mathbb{E}\left[p_a^1 \sum_{b,c} \theta_c H_{cb}\left\{p_b^1\right\}\right] = \sum_{b,c} \mathbb{E}\left[p_a^1\left\{p_b^1\right\}\right] H_{cb}\theta_c \\
&= \sum_{b,c} V_{ab}\left(H'\right)_{bc}\theta_c = \left(VH'\theta\right)_a
\end{aligned}$$

i.e., indeed, $\mathbb{E}\left[mp^1\right] = VH'\theta = \pi$. □

A.3.C.4 Application to the FX case in a multi-country set-up

We now specialize the previous machinery to the FX case. In equilibrium, we will indeed have (with $q = (q_i)_{i=1,\ldots,n}$):

$$q = \Gamma^{-1}\mathbb{E}\left[e^1 - e^0\right], \tag{A.33}$$

[67] Mathematically, call S the space of random payoffs spanned by (linear combinations of) the traded assets, $\left(p_a^1\right)_{a=1,\ldots,B}$. S is a subset of $L^2(\Omega)$, where Ω is the underlying probability space. Ψ: $S \to S$ is an operator from S to S, while H is a $B \times B$ matrix.

and $q_0 = -\sum_{i=1}^{n} q_i$ ensures $\sum_{i=0}^{n} q_i = 0$. We endogenize this demand, with

$$\Gamma = \gamma V^{\alpha}, \tag{A.34}$$

where $V = var\left(e^1\right)$, and $var\left(x\right) = \mathbb{E}\left[xx'\right] - \mathbb{E}\left[x\right]\mathbb{E}\left[x'\right]$ is the variance-covariance matrix of a random vector x. Note that $var\left(e^1\right) = var\left(x^1\right)$ is independent of e^0. Hence, with this endogenous demand, we have a model that depends on variance, is arbitrage free, and (we believe) sensible.

Let us see how the general asset pricing case applies to the FX case. The basis assets are the currencies, with $p^t = e^t$, θ_a is the position in currency a, and $q_a = \theta_a e_a^0$ is the initial dollar value of the position. The position held in dollars is q_0 (and we still have $e_0^t = 1$ as a normalization). We define

$$D = diag\left(e^0\right), \tag{A.35}$$

so that $q = D\theta$. We take the G matrix to be

$$G = \gamma V^{\alpha} D, \tag{A.36}$$

for scalars $\gamma > 0$ and $\alpha \geq 0$. Recall that $V = var\left(e^1\right)$ is a matrix. The reader is encouraged to consider the leading case where $\alpha = 1$. In general, V^{α} is the variance-covariance matrix to the power α: if we write $V = U^{'}\Lambda U$ for U an orthogonal matrix and $\Lambda = diag\left(\lambda_i\right)$ a diagonal matrix, $V^{\alpha} = U^{'} diag\left(\lambda_i^{\alpha}\right) U$.

Proposition A.8. *(FX case: Foundation for the financiers' demand (A.22)) With the above set-up, the financiers' equilibrium holding q satisfies:*

$$\mathbb{E}\left[e^1 - e^0\right] = \Gamma q, \tag{A.37}$$

with

$$\Gamma = \gamma V^{\alpha},$$

where $\gamma > 0$ and $\alpha \geq 0$ are real numbers, and $V = var\left(e^1\right)$ is the variance-covariance matrix of exchange rates. In other terms, when Γ is invertible, we obtain the Gamma demand (A.22), $q = \Gamma^{-1}\mathbb{E}\left[e^1 - e^0\right]$.

Proof. This is a simple correlate of Proposition A.4. This Proposition yields

$$\mathbb{E}\left[p^1 - p^0\right] = G\theta^*,$$

Using $p^t = e^t$, $\Gamma \equiv \gamma V^{\alpha}$, $G \equiv \Gamma D$, $q = D\theta^*$, we obtain

$$\mathbb{E}\left[e^1 - e^0\right] = G\theta^* = \Gamma D\theta^* = \Gamma q.$$

□

It may be useful to check the logic by inspecting what this yields in the Basic Gamma model. There, the outside option of the financiers is given by (A.29) (using $\theta = -q/e_0$, since in the basic Gamma model the dollar value of the yen position is $-q$)

$$\theta' S\theta = \gamma\theta^2 var\left(e_1\right)^{\alpha} e_0 = \gamma var\left(e_1\right)^{\alpha}\frac{q^2}{e_0}.$$

The financiers' maximization problem is thus:

$$\max_q V_0 \text{ where } V_0 := \mathbb{E}\left[1 - \frac{e_1}{e_0}\right] q,$$
$$\text{s.t. } V_0 \geq \gamma\, var\left(e_1\right)^{\alpha}\frac{q^2}{e_0},$$

i.e., the divertable fraction is $\gamma var\left(e_1\right)^{\alpha}\frac{q}{e_0}$. It is increasing in q and the variance of the trade (a "complexity" effect).

The constraint binds, and we obtain:

$$\mathbb{E}\left[1-\frac{e_1}{e_0}\right]q=\gamma var\left(e_1\right)^{\alpha}\frac{q^2}{e_0},$$

or,

$$\mathbb{E}\left[e_0-e_1\right]=\gamma var\left(e_1\right)^{\alpha}q, \tag{A.38}$$

which confirms the intuitive properties of this derivation.

A.3.C.5 Application to the CIP and UIP trades

Suppose that the assets are: dollar bonds paying at time 1, yen bonds paying at time 1 (so that their payoff is e_1), and yen futures that pay e_1-F at time 1, where F is the futures' price. The payoffs (expressed in dollars) are:

$$P_1=\left(1,e_1,e_1-F\right)',$$

and the equilibrium time-0 price is:

$$P_0=\left(1,e_0,0\right)',$$

as a futures position requires 0 initial investment.

Suppose that financiers undertake the CIP trade, i.e. they hold a position:

$$\Theta^{CIP}=\left(e_0,-1,1\right)',$$

where they are long the dollar, short the yen, and long the future. To review elementary notions in this language, the initial price is $\Theta^{CIP}\cdot P^0=0$. The terminal payoff is $\Theta^{CIP}\cdot P^1=e_0-F$, hence, by no arbitrage, we should have $F=e_0$.

The financiers can also engage in the UIP trade; in the elementary UIP trade they are long 1 dollar, and short the corresponding yen amount:

$$\Theta^{UIP}=\left(1,\frac{-1}{e_0},0\right)'.$$

Assume that financiers' portfolio is composed of C CIP trades, and q UIP trades:

$$\Theta=C\Theta^{CIP}+q\Theta^{UIP}.$$

We expect the risk premia in this economy to come just from the risk currency part (q), not the CIP position (C). Let us verify this.

In terms of the reduced basis, we have

$$\theta^{CIP}=\left(e_0-F,0\right)',$$
$$\theta^{UIP}=\left(1,\frac{-1}{e_0}\right)',$$

so that

$$\theta=C\theta^{CIP}+q\theta^{UIP}.$$

Hence, the model confirms that the financiers have 0 exposure to the yen coming from the CIP trade. We then have

$$\mathbb{E}\left[e_0-e_1\right]=\Gamma q,$$

with $\Gamma=\gamma var\left(e_1\right)^{\alpha}$. The CIP trade, causing no risk, causes no risk premia. We summarize the results in the following lemmas.

Lemma A.4. *If the financiers undertake both CIP and UIP trades, only the net positions coming from the UIP trades induce risk premia.*

Lemma A.5. *Assume that $\alpha \geq 1$, or that $V = var\left(x^1\right)$ is invertible (and $\alpha \geq 0$). Then, in the FX model risk-less portfolios earn zero excess returns. In particular, CIP holds in the model, while UIP does not.*

Proof: Define $W = \gamma V^{\alpha-1}D$, which is well-defined under the lemma's assumptions. Then, we can write $G = \gamma V^{\alpha}D$ as $G = VW$, and apply Proposition A.5. □

A.4 Numerical Generalization of the Model

We include here a generalization of the basic Gamma model of Section II that relaxes some of the assumptions imposed in the main body of the paper for tractability. The generalization of the model in Section II has to be solved numerically. Our main aim is to verify, at least numerically, that all the core forces of the basic model carry through to this more general environment. We provide a brief numerical simulation and stress that this is only a numerical example without any pretense of being a full quantitative assessment. A full quantitative assessment, with its need for further channels and numerical complications, while interesting, is the domain of future research.

Model Equations Since the model is a generalization of the basic one, we do not restate, in the interest of space, the entire structure of the economy. We only note here that the model has infinite horizon, symmetric initial conditions (both countries start with zero bond positions), and report below the system of equations needed to compute the solution.

$$\text{(A.39)} \quad R_{t+1} = \frac{\chi_t / Y_{NT,t}}{\beta_t \mathbb{E}_t[\chi_{t+1}/Y_{NT,t+1}]},$$

$$\text{(A.40)} \quad R^*_{t+1} = \frac{\chi^*_t / Y^*_{NT,t}}{\beta^*_t \mathbb{E}_t[\chi^*_{t+1}/Y^*_{NT,t+1}]},$$

$$\text{(A.41)} \quad Q_t = \frac{1}{\Gamma}\mathbb{E}_t\left[\left(\eta\beta_t \frac{Y_{NT,t}/\chi_t}{Y_{NT,t+1}/\chi_{t+1}} + (1-\eta)\beta^*_t \frac{e_t}{e_{t+1}} \frac{Y^*_{NT,t}/\chi^*_t}{Y^*_{NT,t+1}/\chi^*_{t+1}}\right)\left(e_t R_{t+1} - R^*_{t+1}e_{t+1}\right)\right]$$

$$\text{(A.42)} \quad Q_t = f_t e_t - f^*_t - D_t,$$

$$\text{(A.43)} \quad D_t = D_{t-1}R_t + (\eta Q_{t-1} - e_{t-1}f_{t-1})\left(R_t - R^*_t \frac{e_t}{e_{t-1}}\right) + e_t \frac{\xi_t}{\chi^*_t} Y^*_{NT,t} - \frac{\iota_t}{\chi_t} Y_{NT,t},$$

where η is the share of financiers' profits repatriated to the US, and D are the US net foreign assets. This is a system of five nonlinear stochastic equations in five endogenous unknowns $\{R, R^*, e, Q, D\}$. We solve the system numerically by second order approximation. The exogenous variables evolve according to:

$$\text{(A.44)} \quad \ln \iota_t = (1-\phi_\iota)\ln \iota_{t-1} + \sigma_\iota \varepsilon_{\iota,t}; \qquad \ln \xi_t = (1-\phi_\xi)\ln \xi_{t-1} + \sigma_\xi \varepsilon_{\xi,t},$$

$$\text{(A.45)} \quad f_t = (1-\phi_f) f_{t-1} + \sigma_f \varepsilon_{f,t}; \qquad f^*_t = (1-\phi_f) f^*_{t-1} + \sigma_f \varepsilon_{f^*,t},$$

$$\text{(A.46)} \quad \beta_t = \bar{\beta}\exp(x_t); \qquad \beta^*_t = \bar{\beta}\exp(x^*_t),$$

$$\text{(A.47)} \quad x_t = (1-\phi_x)x_{t-1} + \sigma_x \varepsilon_{x,t}; \qquad x^*_t = (1-\phi_x)x^*_{t-1} + \sigma_x \varepsilon_{x^*,t},$$

where $[\varepsilon_\iota, \varepsilon_\xi, \varepsilon_f, \varepsilon_{f^*}, \varepsilon_x, \varepsilon_{x*}] \sim N(0, I)$. We assume that all other processes, including the endowments, are constant.

The deterministic steady state is characterized by: $\{\bar{e} = 1, \bar{R} = \bar{R}^* = \bar{\beta}^{-1}, \bar{Q} = \bar{D} = \bar{D}^* = 0\}$.[68] In order to provide a numerical example of the solution, we briefly report here the chosen parameter values. We stress that this is not an estimation, but simply a numerical example of the solutions. We set $\bar{\beta} = 0.985$

[68]Note that the deterministic steady state is stationary whenever $\Gamma > 0$, which we always assume here (i.e $\alpha = 0$ from the main text). Similarly the portfolio of the intermediary is determinate via the assumption that households only actively save in domestic currency and via the limited commitment problem of the intermediary.

to imply a steady state annualized interest rate of 6%. We set the share of financiers' payout to households at $\eta = 0.5$, so that it is symmetric across countries. We set all constant output parameters at 1 ($Y_H = Y_F = a = a^* = 1$), except for the value of non-tradables set at 18 ($Y_{NT} = Y^*_{NT} = \chi = \chi^* = 18$), so that they account for 90% of the consumption basket. We set $\Gamma = 0.1$.[69] Finally, we set the shock parameters to: $\phi_\iota = \phi_\xi = 0.018, \sigma_\iota = \sigma_\xi = 0.037, \phi_f = 0.0001, \sigma_f = 0.05, \phi_x = 0.0491, \sigma_x = 0.0073$.

We report in Table A.1 below a short list of simulated moments.[70] For a rough comparison, we also provide data moments focusing on the GBP/USD exchange rate and US net exports.

Table A.1: Numerical Example of Simulated Moments

Moment	*Data*	*Model*
$SD\left(\frac{e_{t+1}}{e_t} - 1\right)$	0.1011	0.1269
$\phi(e_{t+1}, e_t)$	0.2442	0.0831
$\bar{R}^c$	0.0300	0.0408
$SD\left(R^c_t\right)$	0.1011	0.1269
$SD(nx_t)$	0.0335	0.0143
$\phi(nx_t, nx_{t-1})$	0.0705	0.1438
$SD(R_t)$	0.0479	0.0479
$\phi(R_{t+1}, R_t)$	0.1821	0.1824

Data and model-simulated moments. The first column reports the standard deviation ($SD(\frac{e_{t+1}}{e_t} - 1)$) and (one minus) autocorrelation ($\phi(e_{t+1}, e_t)$) of exchange rates, the average carry trade return ($\bar{R}^c$) and its standard deviation ($SD\,(R^c_t)$), the standard deviation ($SD(nx_t)$) and (one minus) the autocorrelation coefficient ($\phi(nx_t, nx_{t-1})$) of net exports over GDP for the US, and the standard deviation ($SD(R_t)$) and (one minus) autocorrelation of interest rates ($\phi(R_{t+1}, R_t)$). Data sources: exchange rate moments are for the GBP/USD, the carry trade moments are based on Lettau, Maggiori and Weber (2014) assuming the interest rate differential is 5%, the interest rate moments are based on the yield on the 6-month treasury bill minus a 6-year moving average of the 6-month rate of change of the CPI. All data are quarterly 1975Q1-2012Q2 (150 observations). The reported moments are annualized. Model implied moments are computed by simulating 500,000 periods (and dropping the first 100,000). The carry trade moments are computed selecting periods in the simulation when the interest rate differential is between 4% and 6%.

Finally, we provide a numerical example of classic UIP regressions. The regression specification follows:

$$\Delta \ln(e_{t+1}) = \alpha + \beta_{UIP}[\ln(R_t) - \ln(R^*_t)] + \varepsilon_t.$$

The above regression is the empirical analog to the theoretical results in Section III.A.[71] We find a regression coefficient well below one ($\hat{\beta} = 0.33$), the level implied by UIP. Indeed, on average we strongly reject UIP with an average standard error of 0.19. The regression adjusted R^2 is also low at 0.018. The results are broadly in line with the classic empirical literature on UIP.

[69] We set this conservative value of Γ based on a thought experiment on the aggregate elasticity of the exchange rate to capital flows. We suppose that an inelastic short-term flow to buy the Dollar, where the scale of the flow is comparable to 1 year worth of US exports (*i.e.* $f^* = 1$), would induce the Dollar to appreciate 10%. The numbers are simply illustrative, but are in broad congruence with the experience of Israel and Switzerland during the recent financial crisis. Let us revert to the basic Gamma model. Suppose that period 1 is a "long run" during which inflows have already mean-reverted (so that the model equations are: $e_0 - 1 + f^* + Q = 0$, $e_1 = 1$, $Q = \frac{1}{\Gamma}(e_0 - e_1)$). Then, we have $e_0 = 1 - \frac{\Gamma}{1+\Gamma} f^*$. Hence, the price impact is $e_0 - 1 = -\frac{\Gamma}{1+\Gamma} \simeq -0.1$. This leads to $\Gamma \simeq 0.1$.

[70] The moments are computed by simulating 500,000 periods with pruning. We drop the first 100,000 observations (burn-in period).

[71] To estimate the regression based on model-produced data, we simulate the model for 500,000 periods, dropping the first 100,000, and then sample at random 10,000 data intervals of length 150. The length is chosen to reflect the data span usually available for the modern period of floating currencies (150 quarters). On each data interval, we estimate the above regression. Finally, we average across the regression output from the 10,000 samples.

A.5 Proofs for the Main Body of the Paper

Proof of Proposition 3 The flow equilibrium conditions in the dollar-yen markets are:

$$\xi_0 e_0 - \iota_0 + Q_0 = 0, \tag{A.48}$$
$$\xi_1 e_1 - \iota_1 - RQ_0 = 0. \tag{A.49}$$

Summing (A.48) and (A.49) gives the intertemporal budget constraint: $R(\xi_0 e_0 - \iota_0) + \xi_1 e_1 - \iota_1 = 0$. From this, we obtain:

$$e_1 = \xi_1^{-1}(R\iota_0 + \iota_1 - R\xi_0 e_0). \tag{A.50}$$

The market clearing in the Dollar / Yen market, $\xi_0 e_0 - \iota_0 + \frac{1}{\Gamma}\mathbb{E}\left[e_0 - \frac{R^*}{R}e_1\right] = 0$, gives:

$$\frac{R^*}{R}\mathbb{E}[e_1] = e_0 + \Gamma(\xi_0 e_0 - \iota_0) = (1+\Gamma\xi_0)e_0 - \Gamma\iota_0. \tag{A.51}$$

Combining (A.50) and (A.51),

$$\mathbb{E}[e_1] = \mathbb{E}\left[\xi_1^{-1}(R\iota_0 + \iota_1)\right] - \mathbb{E}\left[\xi_1^{-1}\right]\xi_0 R e_0 = \frac{R}{R^*}(1+\Gamma\xi_0)e_0 - \frac{R}{R^*}\Gamma\iota_0,$$

i.e.

$$\begin{aligned}
e_0 &= \frac{\frac{R}{R^*}\Gamma\iota_0 + \mathbb{E}\left[\xi_1^{-1}(R\iota_0+\iota_1)\right]}{\frac{R}{R^*}(1+\Gamma\xi_0) + \mathbb{E}\left[\xi_1^{-1}\right]\xi_0 R} = \frac{\left(\mathbb{E}\left[R^*\xi_1^{-1}\right]+\Gamma\right)\iota_0 + \mathbb{E}\left[\frac{R^*}{R}\xi_1^{-1}\iota_1\right]}{\left(\mathbb{E}\left[R^*\xi_1^{-1}\right]+\Gamma\right)\xi_0 + 1} \\
&= \frac{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\iota_0 + \frac{\iota_1}{R}\right)\right] + \Gamma\iota_0}{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0 + \frac{\xi_1}{R^*}\right)\right] + \Gamma\xi_0}.
\end{aligned}$$

We can now calculate e_1. We start from its expected value:

$$\begin{aligned}
\frac{R^*}{R}\mathbb{E}[e_1] &= (1+\Gamma\xi_0)e_0 - \Gamma\iota_0 = (1+\Gamma\xi_0)\frac{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\iota_0 + \mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1} - \Gamma\iota_0 \\
&= \frac{\left\{(1+\Gamma\xi_0)\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right) - \Gamma\left[\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1\right]\right\}\iota_0 + (1+\Gamma\xi_0)\mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1} \\
&= \frac{\mathbb{E}\left[\frac{R^*}{\xi_1}\right]\iota_0 + (1+\Gamma\xi_0)\mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1} = \frac{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\iota_0+\frac{\iota_1}{R}\right)\right] + \Gamma\xi_0\mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0+\frac{\xi_1}{R^*}\right)\right]+\Gamma\xi_0}.
\end{aligned}$$

To obtain the time-1 innovation, we observe that $e_1 = \frac{1}{\xi_1}(R\iota_0 + \iota_1 - R\xi_0 e_0)$ implies:

$$\{e_1\} = \left\{\frac{\iota_1}{\xi_1}\right\} + R(\iota_0 - \xi_0 e_0)\left\{\frac{1}{\xi_1}\right\}.$$

As:

$$\iota_0 - \xi_0 e_0 = \iota_0 - \xi_0\frac{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\iota_0 + \mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1} = \frac{\iota_0 - \mathbb{E}\left[\xi_0\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1},$$

we obtain:

$$\{e_1\} = \left\{ \frac{\iota_1}{\xi_1} \right\} + R \frac{\iota_0 - \mathbb{E}\left[\xi_0 \frac{R^*}{\xi_1} \frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right] + \Gamma\right) \xi_0 + 1} \left\{ \frac{1}{\xi_1} \right\}.$$

We next derive the value of Γ. Notice that we can write the above equation as:

$$\begin{aligned}
\{e_1\} &= \varepsilon + \frac{1}{a+\Gamma}\eta, \\
\varepsilon &\equiv \left\{ \frac{\iota_1}{\xi_1} \right\}, \\
\eta &\equiv \left(\iota_0 - \mathbb{E}\left[\xi_0 \frac{R^*}{\xi_1} \frac{\iota_1}{R}\right] \right) \frac{1}{\xi_0} \left\{ \frac{1}{\xi_1} \right\}, \\
a &\equiv \mathbb{E}\left[\frac{R^*}{\xi_1} \left(\xi_0 + \frac{\xi_1}{R^*} \right) \right] \frac{1}{\xi_0}.
\end{aligned}$$

Then,

$$var\left(e_1\right) = \sigma_\varepsilon^2 + \frac{2\sigma_{\varepsilon\eta}}{a+\Gamma} + \frac{\sigma_\eta^2}{\left(a+\Gamma\right)^2}.$$

Letting $G(\Gamma)$ be

$$G\left(\Gamma\right) \equiv \Gamma - \gamma \left(\sigma_\varepsilon^2 + \frac{2\sigma_{\varepsilon\eta}}{a+\Gamma} + \frac{\sigma_\eta^2}{\left(a+\Gamma\right)^2} \right)^\alpha, \tag{A.52}$$

then Γ is defined as

$$G\left(\Gamma\right) = 0. \tag{A.53}$$

When $\alpha = 0$, we get the basic Gamma model. When $\alpha = 1$, we have a polynomial of degree 3 in Γ. When there is no noise and $\alpha > 0$, $\Gamma = 0$. In general, it is still amenable to computation: there is a unique positive solution of $G\left(\Gamma\right)$ (as $G\left(\Gamma\right)$ is increasing in Γ, and $G\left(0\right) < 0$, $\lim_{\Gamma\to\infty} G\left(\Gamma\right) = \infty$).

Proof of Lemma 3 In the decentralized allocation, the consumer's intra-period consumption, Equation (5), gives the first order conditions:

$$\begin{aligned}
p_{NT}C_{NT} &= \frac{\chi}{\lambda}; & p^*_{NT}C^*_{NT} &= \frac{\chi^*}{\lambda^*}; \\
p_H C_H &= \frac{a}{\lambda}; & \frac{p_H}{e}C^*_H &= \frac{\xi}{\lambda^*}; \\
ep^*_F C_F &= \frac{\iota}{\lambda}; & p^*_F C^*_F &= \frac{a^*}{\lambda^*}.
\end{aligned} \tag{A.54}$$

so that

$$e = \frac{\frac{C^*_H \lambda^*}{\xi}}{\frac{C_H \lambda}{a}}.$$

Suppose that the Negishi weight is ν. The planner maximizes $U + \nu U^*$ subject to the resource constraint; hence, in particular $\max_{C_H + C^*_H \le Y_H} a \ln C_H + \nu\xi \ln C^*_H$, which gives the planner's first order condition $\frac{a}{C_H} = \frac{\nu\xi}{C^*_H}$. Hence, in the first best exchange rate satisfies:

$$e^{FB}_t = \nu \frac{\lambda^*_t}{\lambda_t} = \nu \frac{p_{NT}C_{NT,t}/\chi_t}{p^*_{NT}C^*_{NT,t}/\chi^*_t}.$$

In the basic case of Lemma 3, we have $\lambda_t = \lambda_t^* = 1$, so $e_t^{FB} = \nu$. Note that this is derived under the assumption of identical discount factor $\beta = \beta^*$. □

Proof of Proposition 6

$$\overline{R}^c = \frac{\mathbb{E}\left[\frac{R^*}{R}e_1 - e_0\right]}{e_0} = \frac{-\Gamma Q_0}{e_0}$$
$$= -\Gamma\left(\frac{\iota_0 - e_0}{e_0}\right) = \Gamma\left(1 - \frac{\iota_0}{e_0}\right).$$

Recall that:

$$e_0 = \frac{(R^* + \Gamma)\,\iota_0 + \frac{R^*}{R}\mathbb{E}\left[\iota_1\right]}{R^* + \Gamma + 1},$$

so that we conclude:

$$\overline{R}^c = \Gamma\left(1 - \iota_0 \frac{R^* + \Gamma + 1}{(R^* + \Gamma)\,\iota_0 + \frac{R^*}{R}\mathbb{E}\left[\iota_1\right]}\right).$$

which, rearranged, gives the announced expression.

Derivation of 3-period economy exchange rates We will use the notation:

$$\mathcal{R}^* \equiv \frac{R^*}{R}.$$

Recall that we assume that in period $t = 1$ financiers only intermediate the new flows; stocks arising from previous flows are held passively by the households (long term investors) until t=2. Therefore, from the flow demand equation for $t = 1$, $e_1 - \iota_1 + Q_1 = 0$, and the financiers' demand, $Q_1 = \frac{e_1 - \mathcal{R}^*\mathbb{E}[e_2]}{\Gamma_1}$, we get an expression for e_1:

$$e_1 = \frac{\Gamma_1 \iota_1 + \mathcal{R}^*\mathbb{E}_1\left[e_2\right]}{\Gamma_1 + 1}.$$

The flow demand equation for $t = 2$ gives $e_2 = \iota_2$, so we can rewrite e_1 as:

$$e_1 = \frac{\Gamma_1 \iota_1 + \mathcal{R}^*\mathbb{E}_1\left[\iota_2\right]}{\Gamma_1 + 1}.$$

Similarly for e_0, we have

$$e_0 = \frac{\Gamma_0 \iota_0 + \mathcal{R}^*\mathbb{E}_0\left[e_1\right]}{\Gamma_0 + 1},$$

and we can use our expression for e_1 above to express e_0 as:

$$e_0 = \frac{\Gamma_0 \iota_0 + \mathcal{R}^*\mathbb{E}_0\left[\frac{\Gamma_1 \iota_1 + \mathcal{R}^* \iota_2}{\Gamma_1 + 1}\right]}{\Gamma_0 + 1}. \square$$

Proof of Proposition 7 We have already derived Claim 1. For Claim 2, we can calculate, from the definition of carry trade returns ($R^c \equiv \frac{R^*}{R}\frac{e_1}{e_0} - 1$) and equation (24):

$$\overline{R}^c = (\mathcal{R}^* - 1)\,\Gamma_0 \frac{\overline{\Gamma}_1 + 1 + \mathcal{R}^*}{\overline{\Gamma}_1(\Gamma_0 + \mathcal{R}^*) + \Gamma_0 + (\mathcal{R}^*)^2} > 0.$$

Hence, the expected carry trade return is positive.

For Claim 3, recall that a function $\frac{ax+b}{cx+d}$ is increasing in x iff $\Delta^x \equiv ad - bc > 0$. For Γ_0,

$$\Delta^{\Gamma_0} = \left(1 + \overline{\Gamma}_1 + \mathcal{R}^*\right)\left(\overline{\Gamma}_1 \mathcal{R}^* + (\mathcal{R}^*)^2\right) > 0,$$

which proves $\frac{\partial \overline{R}^c}{\partial \Gamma_0} > 0$.

For $\overline{\Gamma}_1$, the discriminant is

$$\frac{\Delta^{\overline{\Gamma}_1}}{(\mathcal{R}^* - 1)\,\Gamma_0} = \Gamma_0 + (\mathcal{R}^*)^2 - (1 + \mathcal{R}^*)\,(\Gamma_0 + \mathcal{R}^*) = -\mathcal{R}^*\,(1 + \Gamma_0) < 0,$$

so that $\frac{\partial \overline{R}^c}{\partial \overline{\Gamma}_1} < 0$.

Finally, for $\mathcal{R}^*$, we simply compute:

$$\frac{\partial \overline{R}^c}{\partial \mathcal{R}^*} = \frac{\Gamma_0\,(1 + \Gamma_0)\,(1 + \overline{\Gamma}_1)\,(2\mathcal{R}^* + \overline{\Gamma}_1)}{\left(\Gamma_0\,(1 + \overline{\Gamma}_1) + \overline{\Gamma}_1 \mathcal{R}^* + (\mathcal{R}^*)^2\right)^2} > 0.\square$$

Proof of Proposition 8 The regression corresponds to: $\beta_{\text{UIP}} = \frac{-\partial}{\partial R^*}\mathbb{E}\left[\frac{e_1}{e_0} - 1\right]$. For simplicity we calculate this derivative at $R = R^* = \mathbb{E}\iota_t = 1$, and with deterministic $\Gamma_1 = \overline{\Gamma}_1$. Equation (24) yields, for those values but keeping R^* potentially different from 1:

$$e_0 = \frac{\Gamma_0 + R^* \frac{\overline{\Gamma}_1 + R^*}{\overline{\Gamma}_1 + 1}}{\Gamma_0 + 1}; \qquad \mathbb{E}e_1 = \frac{\overline{\Gamma}_1 + R^*}{\overline{\Gamma}_1 + 1}.$$

Calculating $\beta_{\text{UIP}} = \frac{-\partial}{\partial R^*}\mathbb{E}\left[\frac{e_1}{e_0} - 1\right] = \frac{-\partial}{\partial R^*}\frac{\mathbb{E}e_1}{e_0}$ gives:

$$\beta_{\text{UIP}} = \frac{1 + \overline{\Gamma}_1 - \Gamma_0}{(1 + \Gamma_0)\,(1 + \overline{\Gamma}_1)}.$$

Hence, $\beta_{\text{UIP}} \leq \frac{1 + \overline{\Gamma}_1}{(1+\Gamma_0)(1+\overline{\Gamma}_1)} = \frac{1}{1+\Gamma_0} < 1$. $\square$

Proof of Proposition 9 Lemma 6 shows that the Yen (strictly) monotonically depreciates as a function of the intervention q^*. Let $e_0(q^*)$ be the exchange rate as a function of the intervention. From Section II.E and the assumption in this proposition that output is demand determined under PCP, we know that:

$$Y_{F,0} = \frac{1 + \frac{1}{e_0(q^*)}}{\overline{p}_F^*} \quad \forall q^* \in [0, \overline{q}^*), \tag{A.55}$$

so that Japanese tradable output increases monotonically as a function of the intervention. We define $\overline{q}^* \equiv \min\{\text{argmax}_{q^*}\ Y_{F,0}(q^*)\}$ as the smallest intervention that achieves full employment. Strict monotonicity of $Y_{F,0}(q^*)$ for all q^* such that $Y_{F,0} < L$ and the fact that $Y_{F,0}$ is bounded above by L guarantee that $\overline{q}^*$ exists and is unique.

The consumption shares are obtained from the household demand functions plus market clearing, so that:

$$C_{H,t} = (1 - s_t^*)L; \qquad C_{F,t} = (1 - s_t^*)Y_{F,t};$$
$$C_{H,t}^* = s_t^* L; \qquad C_{F,t}^* = s_t^* Y_{F,t};$$

where $s_t^* \equiv \frac{e_t}{1+e_t}$. To derive the solution for $C_{F,t}$, recall that the US household demand function is given by $C_{F,t} = \frac{\iota_t}{p_{F,t}}$. At time $t = 0$ we have $\iota_0 = 1$ and $p_{F,0} = \overline{p}^*_F e_0$, and substituting in the output expression in (A.55), we obtain $C_{F,0} = \frac{1}{1+e_0} Y_{F,0}$. At time $t = 1$ we have $p_{F,1} = e_1 p^*_{F,1} = e_1 \frac{a^*_1 + \iota_1/e_1}{L} = \frac{\iota_1(1+e_1)}{L}$, so that $C_{F,1} = \frac{1}{1+e_1} L$. The rest of the expressions can be derived by analogy. □

Proof of Proposition 12 We first prove a Lemma.

Lemma A.6. *In the setup of Proposition 3, e_0 is increasing in ι_t and R^* and decreasing in ξ_t and R; $\frac{\partial e_0}{\partial \iota_0}$ increases in Γ. In addition, e_0 increases in Γ if and only if the US is a natural net debtor at time 0^+, i.e. $N_{0^+} \equiv \xi_0 e_0 - \iota_0 < 0$.*

Proof: The comparative statics with respect to ι_t, ξ_0, and R are simply by inspection. We report here the less obvious ones:

$$\frac{\partial e_0}{\partial \xi_1} = \frac{\mathbb{E}\left[\frac{e_0\xi_0 - \iota_0 - \frac{\iota_1}{R}}{\xi_1^2}\right]}{\mathbb{E}\left[\frac{\xi_0 + \frac{\xi_1}{R^*}}{\xi_1}\right] + \frac{\Gamma\xi_0}{R^*}} = -\frac{\mathbb{E}\left[\frac{e_1}{R\xi_1}\right]}{\mathbb{E}\left[\frac{\xi_0 + \frac{\xi_1}{R^*}}{\xi_1}\right] + \frac{\Gamma\xi_0}{R^*}} < 0,$$

where we made use of the state-by-state budget constraint $e_0\xi_0 - \iota_0 + \frac{e_1\xi_1 - \iota_1}{R} = 0$. To be very precise, a notation like $\frac{\partial e_0}{\partial \xi_1}$ is the sensitivity of e_0 to a small, deterministic increment to random variable ξ_1.

$$\frac{\partial e_0}{\partial R^*} = \frac{1}{R^{*2}} \frac{e_0 - \Gamma Q}{\mathbb{E}\left[\frac{\xi_0 + \frac{\xi_1}{R^*}}{\xi_1}\right] + \frac{\Gamma\xi_0}{R^*}} = \frac{1}{RR^*} \frac{\mathbb{E}\left[e_1\right]}{\mathbb{E}\left[\frac{\xi_0 + \frac{\xi_1}{R^*}}{\xi_1}\right] + \frac{\Gamma\xi_0}{R^*}} > 0,$$

where we made use of the financiers' demand equation, $\Gamma Q_0 = \mathbb{E}\left[e_0 - \frac{R^*}{R} e_1\right]$, and the flow equation, $\xi_0 e_0 - \iota_0 + Q_0 = 0$.

We also have,

$$\frac{\partial e_0}{\partial \Gamma} = -N_{0^+} \frac{1}{1 + R^*\mathbb{E}\left[\frac{\xi_0}{\xi_1}\right] + \xi_0\Gamma} < 0,$$

where we made use of the definition $N_{0^+} = e_0\xi_0 - \iota_0$. This implies:

$$\frac{\partial^2 e_0}{\partial \Gamma \partial \iota_0} = \frac{1}{\left(R^*\mathbb{E}\left[\frac{\xi_0}{\xi_1}\right] + 1 + \Gamma\xi_0\right)^2} > 0. \square$$

This implies all the points of Proposition 12 with two exceptions. The effects with respect to interest rate changes, both domestic and foreign, hold for f, f^* sufficiently small. Finally, we focus on the impact of f^*. Simple calculations yield:

$$\frac{\partial e_0}{\partial f^*} = -\frac{\Gamma}{R^*\mathbb{E}\left[\frac{\xi_0}{\xi_1}\right] + 1 + \Gamma\xi_0} < 0.$$

We notice that the comparative statics with respect to f are less clear-cut, because f affects the value of $\Gamma\widetilde{\xi}_1$, and hence affects risk-taking. However, we have $\frac{\partial e_0}{\partial f} > 0$ for typical values (e.g. $R = R^* = 1, \widetilde{\xi}_0 = \widetilde{\xi}_1$). □

Appendix References

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