NBER WORKING PAPER SERIES

SKI-LIFT PRICING, WITH AN APPLICATION TO THE LABOR MARKET

Robert J. Barro

Paul M. Romer

Working Paper No. 1985

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
July 1986

The research reported here is part of the NBER's research program in Economic Fluctuations. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

Working Paper #1985
July 1986

Ski-Lift Pricing, with an Application to the Labor Market

ABSTRACT

The market for ski runs or amusement rides often features lump-sum admission tickets with no explicit price per ride. Therefore, the equation of the demand for rides to the supply involves queues, which are systematically longer during peak periods, such as weekends. Moreover, the prices of admission tickets are much less responsive than the length of queues to variations in demand, even when these variations are predictable. We show that this method of pricing generates nearly efficient outcomes under plausible conditions. In particular, the existence of queues and the "stickiness" of prices do not necessarily mean that rides are allocated improperly or that firms choose inefficient levels of investment. We then draw an analogy between "ski-lift pricing" and the use of profit-sharing schemes in the labor market. Although firms face explicit marginal costs of labor that are sticky and less than workers' reservation wages, and although the pool of profits seems to create a common-property problem for workers, this method of pricing can approximate the competitive outcomes for employment and total labor compensation.

Robert J. Barro
Department of Economics
University of Rochester
Rochester, NY 14627

Paul M. Romer
Department of Economics
University of Rochester
Rochester, NY 14627

During Christmas or Spring vacation, most ski areas have long lines. The same is true for Disneyland and other amusement parks in peak season. This type of crowding does not depend on surprises in demand, but instead is systematic. Most economists look at chronic queuing and conjecture that the suppliers would do better by raising prices. Further, most economists would argue that the failure to price properly leads to inefficient allocations of rides, as well as improper investment decisions. But the regular occurrence of lines in some markets suggests that it is economists, rather than suppliers (who have survived), who are missing something.

We argue in this paper that competitive suppliers of ski-lift services (amusement rides, etc.) may rationally set prices so that queues occur regularly and are longer at peak times. Under plausible assumptions, this method of pricing can support efficient allocative decisions. In equilibrium, owners of ski areas set prices for all-day lift tickets (or equivalently, for admission tickets to amusement parks) by maximizing profits subject to a downward-sloping demand curve. Yet this appearance of monopoly power leads to no inefficiency. Moreover, the equilibrium price charged for a lift ticket may not rise with expansions of demand. Sticky prices may be consistent with optimization by suppliers and with efficient choices of quantities.

The explanation for this last result is that skiers calculate the effective price per lift ride by summing opportunity costs, transportation costs and the price of an all-day lift ticket, and then dividing this sum by the number of rides received. This effective price varies inversely with the number of skiers, and thereby changes in the correct direction with fluctuations in demand, even if the lift-ticket price is constant. Under plausible conditions, the magnitude of response of the effective price is also nearly "correct."

We go on to develop parallels between the ski-lift example and the labor market. Fully flexible wages correspond to fully flexible prices per lift ride. But, as in the case of skiing, it is possible to implement alternative methods of pricing that support the efficient outcomes. We show as an example that a profit-sharing scheme, such as that described in Weitzman (1985) parallels the use of admission tickets. Further, under some conditions—and without appealing to long-term contracts—there is little efficiency loss by having sticky wages and fixed parameters of the profit-sharing formula.

In the ski-lift and labor-market examples, the contracts and trading arrangements that agents use may depart significantly from the textbook description of a competitive market. Stated prices may differ from marginal values. Some agents may act explicitly as price setters facing finite elasticities, while others may face explicit or implicit quantity constraints. Without forcing a departure from these institutional conventions, competitive forces may nonetheless induce efficient outcomes. To decide whether a market is competitive or efficient, economists cannot accept at face value the description offered by agents of how a market operates. The prices quoted in actual contracts and trades may bear little relation to the theoretical contructs that economists call competitive equilibrium prices.

2. The Supply and Demand for Ski-Lift Services

We will show that the ski-area equilibrium with queues and sticky prices for all-day lift tickets is essentially a repackaged or disguised version of a conventional equilibrium with a price per lift ride that adjusts in the usual fashion. To do this, we analyze the conventional equilibrium first. Section 2.1 describes the market for ski-lift rides and works out the equilibrium. Section 2.2 then illustrates how the quantities and prices from this efficient solution can be replicated in an equilibrium with lift-ticket pricing and queues. Section 2.3 considers the factors that might influence the choice between these two institutional arrangements for supporting the equilibrium. Section 2.4 illustrates why lift-ticket prices may not respond to changes in demand. The results in these four subsections are derived under the assumption that all ski areas are identical and that individuals differ only in a limited sense. Section 3 shows how the results are modified with more interesting heterogeneity across consumers and producers. Then Section 4 shows how a parallel analysis applies to profit-sharing schemes in the labor market.

2.1 Equilibrium with Ride Tickets

Consider a group of identical, competitive ski-lift operators, each of whom sells ride tickets at a price P per ride. Each firm has a fixed capacity and therefore supplies inelastically the total quantity of rides x. Flexibility in this quantity at some positive marginal cost is more realistic, since suppliers can open more lift lines or perhaps operate the existing ones at greater speed. But these modifications would not change the nature of the problem. In the present case the industry's total supply of rides is Jx, where J is the fixed number of firms.

The demand for skiing involves first, the decision to participate—that is, whether to go skiing—and second the quantity of rides to demand contingent on participation. To abstract from income effects, suppose that all individuals have common preferences that can be represented in the quasi-linear form $U(q) + z$, where q is the number of lift rides, U is a strictly concave utility function, and z is real expenditure on goods other than skiing. For someone who chooses to ski, the expenditure on other goods is $z = Y - Pq - c$, where Y is real income, Pq is the expenditure on lift rides, and c is an individual specific, lump-sum cost of going skiing. It is quasi-fixed in the sense that the cost for a non-skier is zero, but the cost for a skier is independent of the number of ski runs.

Since P is a price per ride, the quantity of rides demanded by those who ski follows from the condition, $U'(q_i) = P$. Inverting this equation, the number of rides demanded per person appears as a usual demand curve with a negative slope: $q_i = D(P)$, with $D'(P) < 0$. Since we assume in this section that U is the same for all individuals, each person who participates demands the same number of rides, q. Section 3 generalizes the results to allow for cross-sectional differences in the demand curves for ski-lift rides.

An individual participates if the net (money value) of the utility gained, $U[D(P)] - P{\cdot}D(P)$, exceeds the fixed cost c. In order to neglect integer constraints, we assume that the agents in the economy can be represented as a continuum $[0,M]$, arranged in terms of increasing costs of participating. We also want to allow the cost for all individuals to vary over time. Thus we describe the costs in terms of a family of cost functions $c_s(i)$, which are continuous and strictly increasing in agent type i and are indexed by a shift parameter s. For example, during vacation periods, the cost for each individual is shifted downward relative to that at other times. Given a price P for lift rides, all individuals with a cost $c_s(i)$ less than the benefits from skiing, U[D(P)] - P·D(P), choose to participate. Thus, the number of individuals N who choose to ski is given by

$$(1) \quad N = N(P) = c_s^{-1}\{U[D(P)] - P\cdot D(P)\}.$$

By the strict concavity of U, the benefits from skiing fall with an increase in the price per ride P, so the number of skiers falls as well. A downward shift in the distribution of fixed costs (as on weekends) raises the value of N for a given P.

By specifying that the ski areas are competitive, we mean that each is small enough that its actions have a negligible impact on aggregate quantities. In this model (though not in those that follow) competitive behavior implies that firms take prices as given. Equilibrium requires that the total capacity of rides, Jx, equal the total number demanded, qN—that is,

$$(2) \quad Jx = D(P)\cdot N(P) = D(P)\cdot c_s^{-1}\{U[D(P)] - P\cdot D(P)\}.$$

For a given value of Jx, this condition determines the equilibrium price per ride P. As one would expect, the price P falls with an increase in total capacity, Jx, and rises with an increase in the level of demand such as that generated from a downward shift in the level of fixed costs c_s.

Over the longer term the model also determines the size of the industry, Jx. This scale depends on the cost of building new capacity (either more firms J or more rides per firm x) and on the distribution of returns, as determined by equation (2) and the distribution function of the shift parameter s.

2.2 Equilibrium with Lift Tickets

We now show how the equilibrium described above can be implemented using an entry fee (i.e. an all-day lift ticket) and a price per ride set equal to zero. Let π_j denote the price of a lift ticket at area j, and let n_j be the number of skiers who ski there. Given the total capacity x, the maximum number of rides per skier will be $q_j = x/n_j$. In equilibrium each person will desire a greater number of rides than x/n_j at the zero marginal cost implied by lift-ticket pricing. Hence there is no problem in getting the customers to accept the quantity of rides available. In fact, people will queue up to receive the rides.

The assumption that the cost $c_s(i)$ is quasi-fixed implies that an individual cares only about the outlay on skiing, $c_s(i) + \pi_j$, and the number of rides available, $q_j = x/n_j$. It is important for our analysis that individuals do not care directly about the time spent waiting in lift lines, or about how the rides are distributed throughout the day. They would prefer shorter lift lines because they would prefer more rides; but given a fixed number of rides, they are indifferent between spending time outdoors in line or indoors in the lodge. We discuss these assumptions more fully in section 2.3.

Suppose that individual i considers the choice between areas j and k. Since the cost $c_s(i)$ of going skiing is assumed to be the same for each area, the individual will be indifferent between j and k if the total utility from skiing minus the cost of the lift ticket is the same. Since the number of lift rides per skier available at area j is determined by the number of people who attend, $q_j = x/n_j$, the equilibrium condition for individuals to be indifferent between areas can be written as

$$(3) \qquad U(x/n_j) - \pi_j = U(x/n_k) - \pi_k.$$

In this equilibrium areas do not take the price π_j for lift tickets as given. Suppose that each area is negligibly small relative to the size of the market. Then each area can choose its lift-ticket price π_j, but the number of individuals skiing at the area will adjust so that the net utility from skiing, $U(x/n_j) - \pi_j$, is the same as that offered at all other areas. Competitive behavior in this context implies that each area takes as given the net utility from skiing. By differentiating net utility with respect to n_j and π_j, we can evaluate the response of the number of skiers n_j to changes in the lift-ticket price π_j. Written in terms of elasticities, we have

$$(4) \qquad \frac{dn_j}{d\pi_j} \cdot \frac{n_j}{\pi_j} = \frac{-\pi_j}{(x/n_j)U'(x/n_j)} .$$

Since costs are assumed to depend on the fixed level of capacity x, and not on the number of skiers, each ski area seeks to maximize its revenues,

$\pi_j n_j$, taking as given the relation between the ticket price and the number of skiers implied by equation (4). As usual, maximization of revenue requires that the elasticity of customers n_j with respect to the price π_j be equal to -1, so that in equilibrium we have

$$(5) \qquad \frac{\pi_j}{q_j} = U'(q_j).$$

The left side of equation (5) is the lift-ticket price, π_j, divided by the number of rides per person at area j, $q_j = x/n_j$. For convenience, define the effective price per ride under lift-ticket pricing as

$$(6) \qquad \hat{P}_j = \pi_j/q_j.$$

The right side of equation (5) is the marginal valuation of rides, U', evaluated at the quantity q_j. Because the demand curve $D(\cdot)$ is simply the inverse of marginal utility, each person at area j ends up with the quantity from the demand curve q_j that corresponds to the effective price per ride $\hat{P}_j$. Although people wait in line and face an explicit marginal cost for rides of zero, the results are as if each skier gets the quantity of rides that he or she would demand at an explicit market price per ride $\hat{P}_j$. That is, equation (5) can be rewritten as

$$(7) \qquad q_j = D(\hat{P}_j).$$

Each area sets prices according to equation (5) and each must provide a given level of the net utility term, $U(x/n_j) - \pi_j$. Since the areas have the same capacity x and are otherwise identical, they end up with the same values for the lift-ticket price, $\pi_j = \pi$, the number of customers, $n_j = N/J$, and the effective price per ride, $\hat{P}_j = \hat{P}$.

To complete the description of the equilibrium, it remains to determine the value of the common lift-ticket price, π, or equivalently of the effective price per ride $\hat{P}$. We can analyze the decision to incur the fixed cost to go skiing just as in the first model, except that the explicit price per ride P is now replaced by the effective price $\hat{P}$. (Recall from equation (7) that people end up with the quantity of rides that they would demand at this price.) The analogue to equation (1)—which determined the total number of skiers N in the first model—is

$$(8) \qquad N = N(\hat{P}) = c_s^{-1}\{U[D(\hat{P})] - \hat{P}\cdot D(\hat{P})\}.$$

In equilibrium the effective price per ride $\hat{P}$ is such as to equate the total capacity for rides, Jx, to the total demand, qN, which implies

$$(9) \qquad Jx = D(\hat{P})\cdot N(\hat{P}) = D(\hat{P})\cdot c_s^{-1}\{U[D(\hat{P})] - \hat{P}\cdot D(\hat{P})\}.$$

Since this is the same equation that determined the price per ride P in the first equilibrium, the effective price $\hat{P}$ takes on the same value. Finally, equations (6) and (7) imply that the common lift-ticket price is determined by the effective price per ride,

$$(10) \quad \pi = \hat{P}q = \hat{P}\cdot D(\hat{P}).$$

Since the equilibrium with lift-ticket pricing yields an effective price per ride $\hat{P}$ equal to the explicit price per ride P in the first equilibrium, skiers receive the same number of rides at the same cost in each case. The same people end up participating, and each ski area receives the same revenue.

The equality of π_j and n_j across areas is special to the example here. Section 3 shows that π_j and n_j can vary across areas if there are differences in skiers' preferences, $U_i(\cdot)$, or in the characteristics of ski areas. What does generalize is the result that the lift-ticket equilibrium can replicate the quantities and effective prices from a ride-ticket equilibrium where prices are set equal to marginal valuations and no quantity constraints are present.

In the longer run context where the capacity Jx is variable, suppliers have the same incentives to invest under lift-ticket pricing as they did in the first model. Specifically, the effective price per ride correctly corresponds to the skiers' marginal valuation of rides, U'. Thus—given our assumption that people care about the number of rides but not directly about the time spent in line—there are no inefficiencies implied by the existence of queues, which reflect the explicit marginal cost of zero for rides. Allocative decisions are still based on the proper shadow price, $\hat{P} = U'(q)$.

Although the lift-ticket equilibrium is fundamentally only a repackaged form of the original competitive equilibrium, the superficial appearances are strikingly different. Specifically, the lift-ticket solution features quantity rationing by means of queues, as well as ticket prices that seem to be set by firms with market power. A regulator might note with concern that the "demand" for lift tickets at each area is the downward-sloping curve $n_j(\pi_j)$ determined by equation (4), and that each area maximizes revenue subject to this curve.

2.3 Ride Tickets versus Lift Tickets

Given the assumptions so far, there is no basis for predicting which of the two forms of pricing will be observed. They lead to identical allocations and effective prices. Ski areas charging on a per ride basis could coexist with others charging on a lift-ticket basis. One can readily verify that an area could also use a combination of a lift ticket (i.e. an entry fee) and a charge per ride.

The description of the world implicit in this model misses important features of reality. For some aspects, such as the determination of the price per ride P or $\hat{P}$, these features may be unimportant. However, in the choice between two otherwise equivalent pricing schemes, these features may be decisive. The most obvious elements neglected so far are:

a) the costs that must be incurred by an area to enforce contracts, for example, to avoid the theft of rides;

b) rides are not homogenous, but are heterogeneous goods indexed by the time of day and by contingencies such as breakdowns and arrivals of skiers;

c) time spent waiting in line is likely to have a positive opportunity cost.

We can conjecture what the inclusion of these features would imply. Given the allocation of rides common to the two kinds of equilibria (and to any mixture of these two), the form of pricing that minimizes the neglected costs will be selected. Ride-ticket pricing will generally have higher monitoring and set-up costs than lift-ticket pricing. Since it would be extremely expensive to set up a complete series of markets in time and contingency specific rides and to enforce contracts written in this form, some amount of queuing would be expected even under pure ride pricing. On the other hand, lift-ticket pricing imposes costs in terms of time. Relative to a system with an extensive system of reservations for lift rides at specified times, each individual must spend more time at a ski area to achieve the given allocation of rides. However, if the typical skier's fixed cost, c, for getting to the ski area is large, then this last element would be relatively unimportant.

As far as we know, ski areas use only the lift-ticket form of pricing. Oi (1971) describes how Disneyland once followed a combination form of pricing with an entry fee and a per ride fee. (In contrast to the explanation offered here, Oi interprets this scheme as evidence of market power.) Disneyland has since shifted to a pure entry fee. We take these observations as evidence that the costs of allocating rides using ride tickets are higher than those using entry fees. Presumably the cost of implementing reservations and collecting ride tickets outweigh the value of the savings in the time required to acquire a given number of rides. This outcome is especially likely if the lump-sum costs of participating are large, and if time spent at a ski area or amusement park is valued for its own sake.

2.4 Shifts in Demand

The foregoing arguments demonstrate that there may be little or no deadweight loss associated with the use of lift-ticket pricing, rather than ride-ticket pricing. But the results do not yet explain why ticket prices would be "sticky." Over the course of a season, variations in the shift parameter s—such as those reflecting weekends and vacation periods—cause predictable changes in demand. Lift lines vary markedly, as do prices for accommodations, but lift-ticket prices apparently change relatively little.

As one would expect, equation (9) implies that the effective price per ride $\hat{P}$ varies in the same direction as the level of demand, with the sensitivity depending inversely on the magnitude of the price elasticity of the overall demand for rides (that is, of $D(\hat{P})\cdot N(\hat{P})$). Thus, the effective price per ride is high when the level of demand is high, and vice versa. However, the price π for a lift ticket does not necessarily vary in the same direction as the level of demand. From equation (10), the effective price per ride is $\hat{P} = \pi/q = \pi n/x$. Even with π (and x) fixed, the extra crowding associated with the increase in n (which equals N/J) itself generates a higher effective price per ride. The lift-ticket price π increases when $\hat{P}$ increases only if the associated fall in rides per person, $x/n = D(\hat{P})$, is less than equiproportional. Using equation (10), the effect of a change in $\hat{P}$ on π is

$$(11) \qquad d\pi/d\hat{P} = D(\hat{P})(1 + \eta_{D,\hat{P}}) \gtreqless 0,$$

where $\eta_{D,\hat{P}} < 0$ is the elasticity of rides demanded per person with respect to the price per ride. If this elasticity is greater than -1 (i.e. less than 1 in absolute value), π rises along with $\hat{P}$ and, hence, with the level of demand. But if the elasticity is less than -1 (i.e. greater than 1 in absolute value), π falls when $\hat{P}$ increases. Finally, if the elasticity is close to -1, π shows little sensitivity to fluctuations in demand. In this case competitive forces are consistent with nearly constant lift-ticket prices, even though the times of peak demand exhibit lines that are much longer than those during non-peak times.

This result suggests an additional advantage to lift-ticket pricing. If the elasticity of demand for rides per person is close to -1, it is unnecessary to incur the "menu costs" of changing the stated price at a ski area in response to changes in demand. The effective price per ride changes in nearly the right way if the price of lift tickets is held constant.

The same mechanism may explain why the explicit prices for goods such as airline tickets and restaurants often do not vary betweeen peak and off-peak periods. At busy times the effective amount of service diminishes because planes and restaurants are more crowded. Thus, the price per effective unit of service rises automatically if the explicit price is held fixed. Under such circumstances, the results with fixed explicit prices may roughly replicate the equilibrium with a flexible price per effective unit of service. (This flexible price would rise at peak times.)

Constant lift-ticket prices work exactly only if the elasticity of the demand for rides per person equals -1. But if the menu costs are large enough to play a decisive role in the choice of the pricing format, a two-part pricing scheme can be implemented to avoid price changes even when the elasticity differs from -1. This consideration does not appear to be relevant for ski areas, where per ride charges do not seem to be used, but may be a factor in the choice of such a scheme by some amusement parks.

To see how this would work, consider an amusement park with capacity x, which charges an entry fee π and a price per ride r. Exactly as was the case for a ski area, if the park is small relative to the industry as a whole, it takes the net utility level for attending the park, $U[D(\hat{P})] - \hat{P} \cdot D(\hat{P})$, as given. Hence, the park cannot change the effective price per ride $\hat{P}$. However, it does not take as given the number of people attending or the price of the admission ticket. When n people visit the park, the supply of rides per person is x/n and the effective price per ride is $r + \pi n/x$. Since the effective price must equal $\hat{P}$ and since the number of rides demanded must equal the demand, $D(\hat{P})$,[1] the set of possible combinations of r and π is given by the relation,

$$(12) \quad \hat{P} = r + \pi/D(\hat{P}).$$

For fixed r, this equation determines how π must vary with the changes in $\hat{P}$ that are induced by changes in demand. The analogue to equation (10) above is

$$(13) \quad \pi = (\hat{P}-r)D(\hat{P}),$$

---

[1]Note that, since $\hat{P}>r$, the demand at the explicit price r, D(r), exceeds the quantity available, $q=D(\hat{P})$. Therefore, although the explicit price is now positive, the demanders still queue up for the available rides. These queues typically applied at Disneyland, even when ride tickets were used.

and the analogue to equation (11) is

$$(14)\quad \frac{d\pi}{d\hat{P}} = D(\hat{P})[1 + \frac{\hat{P}-r}{\hat{P}}\eta_{D,\hat{P}}].$$

Suppose that we consider small fluctuations in demand that induce fluctuations in the effective price $\hat{P}$ around some level $\hat{P}_0$. Let $\eta_{D,\hat{P}}$ be the elasticity of the demand for rides with respect to the effective price. Then, for a given value of $\eta_{D,\hat{P}}$, the price per ride r can be chosen so that $\frac{\hat{P}-r}{\hat{P}}\eta_{D,\hat{P}}$ is equal to -1 when evaluated at $\hat{P} = \hat{P}_0$.[2] Then $\frac{d\pi}{d\hat{P}}$ will be zero when evaluated at $\hat{P}_0$ and it will be small in a neighborhood of $\hat{P}_0$. For small fluctuations in demand, an equilibrium with constant prices r and π will be approximately equivalent to the conventional equilibrium with no entry fee and a fluctuating price P per ride.

3. Elaborations of the Ski Area Model

In Section 2, ski areas were identical and agents differed only in terms of the fixed cost $c_s(i)$; conditional on participation, they too were identical. In this section we illustrate the extent to which the previous results hold when there are differences in characteristics of ski areas and in individuals' preferences for ski runs. Differences among ski areas lead to results that complement those above about sticky prices. The conditions that

---

[2] The value of r would be negative if $\eta_{D,\hat{P}}$ were less than 1 in absolute value.

cause lift-ticket prices to be invariant with demand also cause these prices to be the same at areas with different characteristics. However, differences in preferences can lead to differences in ticket prices among areas, even if the areas are identical.

3.1. Differences Among Ski Areas.

Consider a pool of N identical skiers who have decided to go skiing. These skiers choose among J areas, indexed by j. Let b_j denote the cost for any skier to travel to area j; for example, b_j could be determined by the distance of the area from a major urban center. Ski areas may also differ in terms of the length or "quality" of their ski runs. We represent these differences by assuming that area j has the capacity x_j, which is measured in terms of numbers of runs of a specified length (or quality). When modeled in this way, the differences across areas in x_j turn out to add little to the analysis. For a given value of the transportation cost, b_j, an increase in x_j leads solely to a one-for-one change in the number of skiers who come to area j. That is, variations in x_j do not lead to variations across areas in the number of standardized ski runs per skier or in the price per standardized ski run.

As before, π_j is the lift-ticket price and $q_j = x_j/n_j$ is the number of rides (of standardized length) per person at area j. For an individual to be indifferent between areas j and k, it must be that

$$(15) \quad U(q_j) - \pi_j - b_j = U(q_k) - \pi_k - b_k.$$

For each area, this condition implies that a change in the lift-ticket price causes the number of skiers to adjust so that the net utility from going skiing, $U(x_j/n_j) - \pi_j$, remains constant. In contrast to the case considered in Section 2.2, this net utility will not be the same for all areas; it varies one for one with the cost b_j. But for a given area, it is invariant to the choice of π_j. Area j maximizes revenue subject to the constraint that net utility remain constant. Then, by the argument leading to equation (5), the first-order condition for the revenue maximization problem of the firm is

$$(16) \quad \pi_j = U'(q_j)\cdot q_j .$$

Inserting this expression into equation (15) gives

$$(17) \quad U(q_j) - U'(q_j)\cdot q_j - b_j = U(q_k) - U'(q_k)\cdot q_k - b_k .$$

Suppose that area j is closer than area k, so that $b_j < b_k$. Since U is strictly concave, $U(q) - U'(q)\cdot q$ is increasing in q, and $q_j < q_k$. Thus, closer areas will be more crowded and offer fewer rides per person. Since U' is the inverse demand curve for lift rides, the effective price per ski run, $\hat{P}_j = U'(q_j)$, will be higher at the closer area. But whether the lift-ticket price π_j is higher or lower depends again on the elasticity of the demand curve for rides per person. If the elasticity equals -1 in a neighborhood of the equilibrium number of ski runs, lift-ticket prices will be identical at all areas.

In spite of the explicit terms of trade used by participants, the equilibrium here is one that equates the supply and demand for ski runs of standardized length, not the supply and demand for all-day lift tickets. The last result illustrates the importance of this distinction. If one thinks in terms of a demand for lift tickets, one is naturally led to the incorrect conclusion that the ticket price should vary one for one with the cost b_j, so that π_j+b_j would be the same for all areas. If this total cost of going skiing were the same at all areas, and the number of ski runs per person q_j were also the same, individuals would be indifferent between areas; that is, equation (15) would be satisfied.

To see why this cannot be an equilibrium, it is useful to consider the analysis from the viewpoint of a social planner. By the first welfare theorem, a competitive equilibrium with ride tickets is Pareto optimal. Because we assume that total utility is linear in income, we can calculate the unique, Pareto optimal allocation of ski rides by maximizing an unweighted sum over individuals of the utility from skiing minus the cost of going skiing.[3] With N identical skiers and J ski areas indexed on the interval [0,J], the planner must choose the number of skiers n_j at each area. The problem is then to maximize the integral,

---

[3]There are two equivalent ways to see why this is so. The first is to note that linearity of utility in other goods effectively converts the Pareto problem into a problem with transferable utility. Therefore, the optimum must maximize total utility. Redistribution takes place by means of transfers of income. Alternatively, note that the function U(q) measures the area under the demand curve for lift rides. The sum of these functions is the usual measure of aggregate consumer surplus, and quasi-linear utility was chosen precisely because it permits a simple Marshallian analysis of our problem.

$$\int_0^J [n_j U(x_j/n_j) - n_j b_j]\, dj,$$

subject to the constraint

$$\int_0^J n_j dj = N.$$

At the suggested allocation where $q_j = x_j/n_j$ is the same at all areas, individuals get the same utility from skiing at each area. At the margin, the planner could shift skiers from distant areas to closer ones with no loss in satisfaction from skiing, but in so doing would reduce total transportation costs. Thus this cannot be the social optimum. Using the substitution $q_j = x_j/n_j$, a quick calculation demonstrates that the first-order condition for this problem is precisely equation (17). To support this optimum, it makes no difference whether areas charge the price per standardized ski run of $P_j = U'(q_j)$, or offer q_j runs at a price $\pi_j = P_j \cdot q_j$.

The next section shows that differences in individual preferences cause lift-ticket prices to vary among areas. But to the extent that these differences are small, the present observation permits a kind of cross-sectional check on the explanation proposed above for the stickiness of lift-ticket prices. Many explanations can be offered for price stickiness over time, but it is harder to explain cross-sectional stickiness. If the demand curve for ski runs per person is close to being unit elastic, then there should be less variation in lift-ticket prices than in the number of skiers or the length of lift lines, both in comparisons over time and among areas at a point in time. In both dimensions, it will appear that quantities respond more than prices.

3.2. Differences in Preferences

Suppose now that utility as a function of lift rides, $U_i(q)$, differs among individuals. At an effective price per ride $\hat{P}$, the demand for rides per person, $D_i(\hat{P})$, also differs. All of the previous equilibria deliver the same number of rides to all skiers at a given area by means of a queuing mechanism. Since this mechanism does not discriminate among people with different preferences, it cannot allocate different numbers of rides per day, $D_i(\hat{P})$, to them. To achieve an allocation that does discriminate, different areas (or different classes of tickets at a single area) will have to cater to different types of individuals.

Consider again the case where ski areas are identical. For simplicity we deal with only two types of consumers: there are N_1 agents with preferences $U_1(q) = u(q)$, and N_2 agents with preferences $U_2(q) = \sigma u(q)$, where $\sigma > 1$. Thus, type 2 individuals are more avid skiers. Think again of a social planner, who is free to allocate skiers among areas. Assume that type one skiers are allocated to areas in the interval $[0,J_1]$, and type 2 skiers to the interval $[J_1,J]$. Simple arguments show that the number of skiers should be the same for all areas in a given interval. Since the total capacity of areas serving type 1 skiers is J_1x, and that serving type 2 skiers is $[J-J_1]x$, the planning problem reduces to choosing J_1 to maximize the sum

$$(18)\qquad N_1U_1(J_1x/N_1) + N_2U_2[(J-J_1)x/N_2]$$
$$= N_1u(J_1x/N_1) + N_2\sigma u[(J-J_1)x/N_2].$$

As one would expect from an analysis of the demand for rides, the first-order condition for this problem implies that the marginal utility of a ride is the same at each area, $U_1'(q_1) = U_2'(q_2)$.

As in the analysis of Section 2, competition among areas using lift-ticket pricing will lead to lift-ticket prices $\pi_1 = U_1'(q_1)\cdot q_1$, and $\pi_2 = U_2'(q_2)\cdot q_2$. In general, these prices need not be the same. In the case where $u(q) = \ln(q)$, so that the demand curve for each type of individual is unit elastic, we have $q_2 = \sigma q_1$ and $\pi_2 = \sigma\pi_1$. In this case, the areas catering to type two skiers charge a higher lift-ticket price and offer more rides per person. In general, the areas with high ticket prices are less crowded and attract more avid skiers (that is, people who are willing to pay more per ski run).

Each ski area still meets the effective price per ride, $\hat{P} = U_1'(q_1) = U_2'(q_2)$, just as if it charged this price directly. Moreover, each area faces a given capacity constraint x. Since x/n_j, the number of rides per person, times $\hat{P}$, the effective price per ride, must equal the ticket price π, the number of customers who show up as a function of the ticket price is

$$(19) \qquad n_j = x\hat{P}/\pi_j.$$

Therefore, over the range of values for π_j that are observed in equilibrium, each area faces a demand in terms of numbers of skiers, n_j, that has an elasticity of precisely -1 with respect to the lift-ticket price π_j. Correspondingly, the area's revenue, $\pi_j n_j = \hat{P}x$, is invariant with respect to the choice of the ticket price π_j from the set of ticket prices that are observed in equilbrium. The areas are indifferent between charging a high price and catering with short lines to the skiers who demand lots of rides per person, or charging a low price and servicing with long lines those who demand ew rides.

This equiproportional change in the number of skiers in response to a change in the lift-ticket price does not depend on the elasticity of the aggregate demand curve for lift rides. It obtains whenever a range of lift-ticket prices π_j is observed in equilibrium. As an area changes its lift-ticket price, it also changes the entire class of skiers that choose to patronize it.

Except for the restriction to a finite number of individual types, and hence a finite number of observed prices π_j, the lift-ticket equilibrium in the presence of different tastes resembles the equilibrium with differentiated products and hedonic prices as described in Rosen (1974). Each type of ski area offers a different type of skiing experience, indexed by q_j, the number of ski runs available per skier. With identical competitive producers, profit is invariant to the type of good offered, and the price function $\pi(q)$ traces out the structure of the demand side of the market. In Rosen's model, producers with no market power choose the type of good offered and the price charged from the locus described by $\pi(q)$. Here, the departure from the appearance of price-taking behavior is even sharper. Firms simply choose a price π; quality adjusts endogenously. It is interesting to note that, until recently, the Metro in Paris used a similar scheme to sort people by taste. Purchasers of first-class tickets rode in separate cars, which were not physically different from second-class cars, except that the first-class cars were less crowded (in equilibrium).[4] Roughly speaking, individuals with a stronger preference for ski rides or with a greater distaste for congestion are willing to pay more for the opportunity to pay more.

4. Application to the Labor Market

In this section we apply the paradigm of ski-lift pricing to the labor market. In this context a fully flexible wage rate corresponds to a flexible price per lift ride. The case of a lift ticket relates to alternative methods of labor compensation, such as profit-sharing schemes. From the standpoint of an individual worker, the firm's total profit looks like a ski operator's total capacity. In particular, the amount that each person gets (share of profits or number of lift rides) varies inversely with the number of other people who show up. (Profit per worker falls with more workers as long as the average product of labor exceeds the marginal product.) But, as in the ski example, competition among firms causes the parameters of the profit-sharing rule to adjust so as to reproduce the outcomes that would emerge under flexible wages. Further, at least as an approximation, it is satisfactory to have fixed wages and fixed parameters for the sharing formula. Thus—even in the absence of any long-term contracts—these kinds of rigidities need not imply any inefficiencies.

---

[4] We are told that the abolition of this vestige of the class system was one of the promises made in the presidential campaign of Francois Mitterand. After his election, a compromise was reached whereby this system was not allowed to operate during the morning and evening periods of peak demand (where it presumably would be most useful), but remained in effect during the middle of the day.

Suppose that each of J identical, competitive firms has the production function,

$$(20) \quad Y = A \cdot F(n),$$

where Y is output, A is a technological shift parameter, and n is the number of workers. We assume that each worker works a fixed number of hours per day. Given the real wage rate w, profit maximization for each firm entails

$$(21) \quad F'(n) = w/A.$$

Equation (21) determines each firm's labor demand. Aggregate labor demand is the multiple J of the demand per firm.

The economy has a population of M potential workers. Those who work consume the quantity $q = w+R$, where R is non-wage (profit) income. Those who do not work consume the amount $q = R$. Utility for person i is given by $U[q(i)] - c(i)$, where $c(i)$ is the reservation wage. As before, we think of the index i as running over the continuum, [0,M], with c(i) increasing in i. Assuming that w and R are the same for all persons, individual i works if $U(w+r) - U(R)$ exceeds c(i). Hence, aggregate labor supply can be written as

$$(22) \quad N = c^{-1}[U(w+r) - U(R)].$$

For a given distribution of reservations wages and a given value of R, the slope of the labor-supply function is $(c^{-1})' > 0$.

The equation of aggregate labor suply to aggregate labor demand determines the market-clearing values of the wage rate, w*, and employment, N* (which we assume to be less than the potential population M). Then each firm's employment is $n^* = N^*/J$. We assume that variations in wages rates and employment reflect shifts in the technological parameter A, and we make assumptions on preferences which guarantee that an increase in A causes an increase in employment. Equations (21) and (22) imply that an increase in A leads to an increase in w*. If the production function F is strictly concave, profits will also increase, which we assume raises the value of R for each potential worker. In equation (22), an increase in w, with R held constant, causes N to increase by the usual substitution effect. An increase in R causes N to fall by the positive income effect on the demand for leisure. We assume that the substitution effect from a higher w is dominant, which is especially likely if the technological disturbance is temporary. If we define $\eta_{x,y}$ to be the elasticity of x with respect to y, it follows that

$$\eta_{n^*,A} > 0 \text{ and } 0 < \eta_{w^*,A} < 1.$$ [5]

For concreteness we focus on technological shocks, but we could equally well have assumed other types of exogenous shocks. We have also assumed implicitly that the market for goods is competitive, but this assumption is also not crucial. We could have used instead the kind of monopolistically competitive market structure assumed by Weitzman (1985). All that matters for our analysis is that workers and firms lack market power in the labor market, and that employment and the marginal product of workers move together.

As in the ski-lift example, the competitive equilibrium in the labor market can be supported by pricing mechanisms other than the obvious one of freely flexible wage rates per worker. For example, assume that the wage rate is fixed at some value w' < w*. The wage w' parallels the explicit price per ride, r, in the ski-lift case. Therefore, w' = 0 corresponds to pure lift-ticket pricing, where r = 0.

---

[5]From equation (21), $\eta_{n^*,A} > 0$ implies $\eta_{w^*/A,A} < 0$, which implies $\eta_{w^*,A} < 1$.

Assume now that a worker's total compensation consists of the wage w' plus a bonus B. We want to consider specifications of B that allow the labor market to reproduce the competitive outcomes, despite the rigidity of explicit wages. Let A_0 denote the value of A at an initial position for the economy. For total compensation under the two schemes to be the same initially, we require first that

$$(23)\quad w' + B(A_0) = w^*(A_0).$$

In order for the compensation under the two schemes to vary in the same way with small departures of A from A_0, it is also necessary that

$$(24)\quad \eta_{w'+B,A} = \eta_{w^*,A},$$

where these elasticities are evaluated at $A = A_0$. If equation (24) holds at the initial position, then the total compensation, w'+B, reacts in the same way as w* to changes in A. Using equation (23) and the fixity of w', equation (24) implies that

$$(25)\quad \eta_{B,A} = \frac{[w'+B(A_0)]}{B(A_0)}\,(\eta_{w^*,A}).$$

Hence, if $0 < w' < w^*$, the bonus B must be proportionately more sensitive than w* to changes in A. Therefore, the results suggest tying the bonus to something that is correlated with but more volatile than w*.

One candidate for this tie, proposed by Weitzman (1985), is the firm's profit. In this case the bonus per worker could be determined from the linear profit-sharing formula,

$$(26)\quad B = (\alpha/n)[A\cdot F(n) - nw' + \beta],$$

where α and β are parameters. If α and/or β were flexible, then—as in the ski-lift example—competition would ensure the adjustments necessary to support the employment level n^* at each firm. If α and β are restricted to be constants (that is, invariant with respect to A), then the necessary values of these parameters follow from equations (23) and (25). In particular, for a given value of w', we can show that $0 < \alpha < 1$ and $\beta \gtreqless 0$.[6] Further, given the other parameters of the problem, it is possible to choose a value of $w' < w^*$ such that $\beta = 0$, in which case the profit-sharing rule takes the simple form described by Weitzman.[7] In any event, if α and β are such that equations (23) and (25) are satisfied, then—at least locally—the fluctuations in total compensation correspond to the fluctuations in w^*, and thereby support the variations in n^* that would have resulted with flexible wages.

In this solution workers face the explicit wage $w' < w^*$ ($w' = 0$ is possible), but also get a share of the profits. In deciding whether to work, each person looks only at his own total compensation and neglects the fact

---

[6]The result for α is

$$\alpha = \frac{[w^*\eta_{w^*,A} + (w^*-w')\eta_{n^*,A}]}{[A\cdot F(n)/n + (w^*-w')\eta_{n^*,A}]},$$

where $\alpha < 1$ follows because $A\cdot F(n)/n > w^*$ and $\eta_{w^*,A} < 1$. The result for β is

$$\beta/n = \frac{(w^*-w')AF(n)/n - (w^*-w')[AF(n)/n - w^*]\eta_{w^*,A} - w^*[AF(n)/n - w']\eta_{w^*,A}}{w^*\eta_{w^*,A} + (w^*-w')\eta_{n^*,A}} \gtreqless 0$$

($\beta < 0$ applies if $w^* = w'$.) All expressions above are evaluated at some initial position, denoted before by the superscript 0.

[7]However, the necessary value of w' could be negative. See the formula for β/n in n.6 above.

that his participation reduces the profit distributed to the other workers (which occurs if the marginal product of labor is below the average product). This interaction parallels the negative effect of an additional skier's participation on the rides available for others. Nevertheless, as in our previous example for skiing, the appropriate profit-sharing scheme reproduces the results for employment and total compensation per worker that would arise under flexible wages. (This result is exact if α and/or β are flexible and determined by the competitive interaction of firms. If α and β are fixed, but at the appropriate values, then the result holds locally.)

It also follows that firms would eagerly hire more workers than are available at the going wage w' ($< w^*$) and the prescribed terms for sharing profits. (This result parallels the eagerness of skiers to queue up for lift rides.) But more workers than n^* do not present themselves because the total compensation, $w'+B$, would then fall below the value w^*, which is the reservation wage of the marginal worker (when total employment is $N^* = Jn^*$). As with flexible wages, employment is determined so that labor's marginal product equals the competitive wage w^*. In other words, profits are maximized subject to the constraint that firms pay each worker a total compensation that equals the marginal worker's reservation wage. Thus, the outcomes are Pareto optimal despite sticky wages and the apparent common-property problem associated with the sharing of profits. Even though the marginal cost to the firm of an additional worker is less under the profit-sharing arrangement, the equilibrium level of employment is the same as that with flexible wages. Correspondingly, the firms in each regime face the appropriate shadow price of labor (w^*), and thereby make correct decisions with respect to investment in capital, entry and exit, etc.

A profit-sharing scheme will not work if disturbances sometimes cause employment and wages to move in opposite directions. For example, this pattern would emerge from shifts to workers' reservation wages. Even barring such disturbances, the results do not imply that profit sharing is superior to other schemes that allow the bonus (and thereby total labor compensation) to move along with w*. Also, the analysis does not suggest that a framework with fixed wages and a flexible bonus (related, say, to profits) would be superior to a setup with flexible wages. As was the case for ski areas, the choice of compensation scheme must be based on elements of reality that are excluded from this model.

One element that is not a candidate for explaining a preference for profit sharing is a pure menu cost of changing the amount of compensation received by each worker. Total compensation per worker is the same at every point in time under any equivalent scheme, and there is no reason to believe that it is easier to vary bonuses than to vary wages. A more promising approach may be to consider asymmetric information. If profits are observable by workers, but a technological shock like A is not, then problems of enforcement may lead to the selection of a contract that is contingent on profits rather than on A. But this line of argument suggests that an observable like total sales, which is less susceptible to manipulation by firms, might be an even better choice. In any case, a call for the widespread use of profit-sharing contracts and for subsidies to encourage their adoption, as in Weitzman (1985), requires further analysis of why this system is more attractive than alternatives, and why firms and workers do not settle voluntarily on the preferred format for pricing.

REFERENCES

Oi, W. Y., "A Disneyland Dilemma: Two-Part Tariffs for a Mickey Mouse Monopoly," Quarterly Journal of Economics, 85, February 1971, 77-96.

Rosen, S., "Hedonic Prices and Implicit Markets," Journal of Political Economy, 82, January/February 1974, 34-55.

Weitzman, M.L., "The Simple Macroeconomics of Profit Sharing," American Economic Review, 75, December 1985, 937-53.