A TIME SERIES ANALYSIS OF
REPRESENTATIVE AGENT MODELS OF
CONSUMPTION AND LEISURE
CHOICE UNDER UNCERTAINTY


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Consumption and Leisure Choice Under Uncertainty

## ABSTRACT

This paper investigates empirically a model of aggregate consumption and
leisure decisions in which goods and leisure provide services over time. The
implied time non-separability of preferences introduces an endogenous source of
dynamics which affects both the co-movements in aggregate compensation and hours
worked and the cross-relations between prices and quantities. These cross-
relations are examined empirically using post-war monthly U.S. data on quan-
tities, real wages and the real return on the one-month Treasury bill. We find
substantial evidence against the overidentifying restrictions. The test results
suggest that the orthogonality conditions associated with the representative
consumer's intratemporal Euler equation underlie the failure of the model.
Additionally, the estimated values of key parameters differ significantly from
the values assumed in several studies of real business models. Several possible
reasons for these discrepancies are discussed.

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1. INTRODUCTION

The purpose of this paper is to investigate empirically a model which relates aggregate consumption, aggregate hours worked, aggregate compensation and interest rates. The model we consider has a representative consumer whose indirect preferences defined over current and past acquisitions of consumption goods and leisure choices are non-time-separable. This non-separability introduces an endogenous source of dynamics which is not present in the studies of aggregate labor supply by Altonji (1982), Ashenfelter and Card (1982) and Mankiw, Rotemberg and Summers (1985). Kydland and Prescott (1982) and Kydland (1983) argue that non-time-separable utility is an important ingredient in explaining the co-movements in aggregate compensation and hours worked. They do not, however, investigate empirically the cross-relations between prices and quantities that are implied by their model. It is these cross relations which are the focal point of the empirical analysis in this paper.

Kennan (1985) has studied an equilibrium model of the aggregate labor market in which preferences are not time separable and there is an intertemporal technology for producing consumption goods. He restricts preferences and technology so that the resulting model implies a linear time series representation for hours worked and wages. His model implies that the interest rate on risk free (in units of consumption) securities is constant. In contrast, our model is not a fully articulated equilibrium model but can accommodate equilibrium laws of motion for labor supply, consumption, and real wages that are not linear and allows for stochastic interest rates.

The empirical methodology that we use is an extended version of the nonlinear Euler equation methods suggested by Hansen and Singleton (1982). This approach to studying the implications of the model is quite different

from the approach used by Kydland and Prescott (1982) and Altug (1985). These authors study the implications of their real business cycle models by considering the implied equilibrium law of motion for quantity variables as calculated from an approximate social planning problem. In contrast, our analysis examines only the cross-relations between prices and quantities that are implied by our specifications of preferences of the representative consumer and not by the technology for producing new goods. Thus, our analysis is a limited information one because we abstain from studying any additional restrictions that might emerge from the specification of this technology.

A representative consumer framework is used in this study because it provides an analytically tractable way of deducing implications of consumption and leisure choice under uncertainty for the joint behavior of asset returns and other aggregates. Representative agent models of aggregate labor supply have been used by Lucas and Rapping (1969), Hall (1980), Kydland and Prescott (1982), and Mankiw, Rotemberg and Summers (1985), among others. We recognize that the assumptions commonly used to rationalize a representative agent model in the presence of heterogeneous consumers (e.g., see Rubinstein (1974), Brennan and Kraus (1978), and Eichenbaum, Hansen and Richard (1985)) are not very compelling in the case of aggregate labor supply. For instance, the common assumption of complete securities markets implies that the implicit price of leisure for all consumers be identical. For the particular specifications of preferences that we use, time invariant efficiency units scaling could be introduced and still preserve the rationalization for a representative consumer [see Muellbauer (1981) and Appendix A]. This, however, introduces only a very limited amount of diversity in skills among workers and still imposes restrictions which are not supported by the

microeconomic evidence (e.g., see Satinger [1978]). Further, the assumption that consumers choose optimally to be at interior points in their respective commodity spaces rules out consumers moving in and out of the labor force over time. Hence, the behavior of the fictitious representative agent confounds movements of some consumers into and out of the labor force with movements in hours worked by other consumers who are in the labor force. In fact there is substantial evidence that much of the variation in aggregate hours worked can be attributed to movements in and out of employment (e.g., see Coleman [1984]). In spite of these well known criticisms of the representative consumer paradigm, we still use it in this paper to help document its ability or inability to explain the aggregate time series.

The specifications of preferences considered are variations of the specification suggested by Kydland and Prescott (1982). In interpreting their specification of preferences, we introduce a hypothetical leisure service that depends on linear combinations of current and past values of leisure time. Kydland and Prescott assume that the representative agent has time separable preferences defined over leisure service and the consumption of a nondurable consumption good. In our analysis, we modify the preference specification used by Kydland and Prescott by introducing a consumption service that is a linear combination of current and past values of consumption acquisitions. Hence, our modification allows for the possibility that both current acquisitions of consumption goods and current period leisure time gives rise to consumption and leisure services in current and future time periods. Preferences of the representative agent are time separable over these services. Hence, nonseparabilities over time in the preference specification are most easily interpreted as emerging in the linear transformation of current and past values of leisure time and new consumption goods into current levels of leisure and consumption services.

We use an empirical methodology in this paper that was suggested by Hansen and Singleton (1982), Dunn and Singleton (1986), and Eichenbaum and Hansen (1985). Hansen and Singleton show how to exploit shock exclusion restrictions from preferences to estimate and test representative consumer models using generalized method of moments estimators. Although Hansen and Singleton only consider models in which a representative agent has time-separable preferences defined over a single consumption good, Eichenbaum and Hansen (1985) and Dunn and Singleton (1986) show how their methodology can be extended in a straightforward manner to apply to more general specifications of preferences. In addition to applying this methodology we illustrate how to test whether a subset of relations are contaminated by measurement errors (in this case, measurement errors in aggregate compensation).

The paper is organized as follows. In section 2, the preferences of the representative consumer are described and then, using this specification, relations among consumption, hours worked, compensation, and asset returns are deduced. In section 3 we describe the data used in our empirical analysis. In section 4 we show how to obtain estimates of preference parameters and test the relations derived in section 2. The empirical results are presented and discussed in section 5. Finally, concluding remarks are presented in section 6.

## 2. Preferences of the Representative Consumer

In this section we discuss the preferences of the representative consumer. Then, equilibrium relations among real wages, asset returns, consumption and leisure are deduced from the first-order conditions of the representative consumer's intertemporal optimum problem.

The representative consumer is assumed to have preferences defined over the services provided by the acquisitions of consumption goods and leisure time. Accordingly, we introduce two hypothetical services that are linear functions of current and past values of consumption and leisure respectively:

$$c_t^* = A(L)c_t, \qquad (2.1)$$

$$\ell_t^* = B(L)\ell_t, \qquad (2.2)$$

where $c_t$ is the amount of the consumption good purchased at date t and $\ell_t$ denotes hours of leisure at date t.[1] The polynomial in the lag operator $A(L)$ is given by

$$A(L) = 1 + \alpha L$$

and $B(L)$ is given by either

$$B_1(L) = 1 + \delta L/(1-\eta L) \qquad (2.4)$$

or

$$B_2(L) = 1 + bL. \qquad (2.5)$$

The time t leisure and consumption decisions are constrained to be in an exogenously specified information set $I_t$ of the representative agent.

Expression (2.1) and the assumed form of $A(L)$ imply that the service flow from consumption goods at date t, $c_t^*$, depends linearly on consumption

acquisitions at dates t and t-1. The coefficient $\alpha$ is assumed to be nonnegative so that consumption acquisitions at time t contribute consumption services (and not disservices) in the current and one future time period.

In (2.2), $\ell_t^*$ denotes a leisure service that depends linearly on current and lagged values of leisure time. The case in which $B(L) = B_1(L)$ corresponds to the leisure service specification suggested by Kydland and Prescott (1982). They assume that $\delta$ is greater than or equal to zero and that $\eta$ is between zero and one. In contrast, we do not restrict the sign of $\delta$ in our empirical analysis. Under this service technology, one unit of leisure time at date t contributes $\delta\eta^{\tau-1}$ units of leisure services at data $t + \tau$. Therefore, the sign of $\delta$ determines whether leisure time today provides leisure services or disservices in future time periods. Leisure time today augments leisure services in future time periods when $\delta$ is positive, diminishes leisure services in future time periods when $\delta$ is negative, and has no impact on leisure services in future time periods when $\delta$ is zero. The impact of current leisure time on future leisure services decays geometrically as dictated by the parameter $\eta$. Kydland (1983) provides an extensive motivation for this service technology.

When $B(L) = B_2(L)$, leisure time today provides leisure services today and either leisure services or disservices one period in the future depending on whether b is positive or negative.

Following Kydland and Prescott (1982), the representative agent is assumed to rank alternative streams of consumption and leisure services using the time and state separable utility function

$$E \sum_{t=0}^{\infty} \beta^t \frac{(c_t^{*\gamma} \ell_t^{*(1-\gamma)})^\theta - 1}{\theta} , \qquad (2.6)$$

where $\beta$ and $\gamma$ are preference parameters between zero and one, $\theta$ is a preference parameter that is less than one, and E denotes the mathematical expectation. When $\theta$ is equal to zero, we interpret (2.6) to be the logarithmic specification

$$E \sum_{t=0}^{\infty} \beta^t \{\gamma \log c_t^* + (1-\gamma) \log \ell_t^* \}, \tag{2.7}$$

which is separable across consumption and leisure services. The marginal utilities of services implied by (2.6) are

$$MC_t^* = \beta^t \gamma c_t^{*\gamma\theta-1} \ell_t^{*(1-\gamma)\theta} \tag{2.8}$$

$$ML_t^* = \beta^t (1-\gamma) c_t^{*\gamma\theta} \ell_t^{*(1-\gamma)\theta-1} . \tag{2.9}$$

The joint specification of an intertemporal service technology and preferences defined over services can be viewed as inducing an indirect set of preferences defined over leisure time and consumption acquisitions. More precisely, letting $MC_t$ and $ML_t$ denote the indirect marginal utilities of consumption acquisitions and leisure at time t, it follows from (2.1) and (2.2) that [2]

$$MC_t = E[A(L^{-1})MC_t^* | I_t], \tag{2.10}$$

$$ML_t = E[B(L^{-1})ML_t^* | I_t]. \tag{2.11}$$

The indirect marginal utilities depend in general on the current and expected future direct marginal utilities because of the intertemporal service

technologies. For instance, if $B(L)$ is equal to $B_1(L)$ and $\delta \neq 0$, then $\ell_t$ continues to provide services in all future periods. Therefore, $ML_t$ depends on the current and expected values of all future direct marginal utilities of leisure services. Alternatively, if $\delta = 0$, then the leisure service is equal to leisure ($\ell_t^* = \ell_t$) and $ML_t = ML_t^*$. More generally, the indirect utility function is non-time-separable and $MC_t \neq MC_t^*$ or $ML_t \neq ML_t^*$, so long as $\alpha \neq 0$ or $\delta \neq 0$ ($b \neq 0$ when $B(L) = B_2(L)$).

The first-order conditions of the representative agent choosing optimally to allocate consumption and leisure over time imply that

$$w_t MC_t = ML_t,$$

where $w_t$ is the real wage. Substituting from (2.8) - (2.11) and rearranging terms gives

$$E[w_t \{A(\beta L^{-1})\{\gamma[A(L)c_t]^{\gamma\theta-1}[B(L)\ell_t]^{(1-\gamma)\theta}\}\}$$

$$- B(\beta L^{-1})\{(1-\gamma)[A(L)c_t]^{\gamma\theta}[B(L)\ell_t]^{(1-\gamma)\theta-1}\}|I_t] = 0. \qquad (2.12)$$

Note that when $A(L)$ and $B(L)$ are the identity operators relation (2.12) holds without taking conditional expectations. In this case, (2.12) implies an exact relation among current wages, consumption, and leisures: $c_t/w_t\ell_t = \gamma/(1-\gamma)$.

If the consumer can trade a one-period asset with a price of one unit of $c_t$ and with a random payoff of $r_{t+1}$ units of $c_{t+1}$ at date $t+1$, then a second necessary condition for utility maximization is that

$$E[r_{t+1}MC_{t+1}] = MC_t. \tag{2.13}$$

Substituting from (2.8) and (2.10) gives

$$E[r_{t+1}\beta\{A(\beta L^{-1})\{[A(L)c_{t+1}]^{\gamma\theta-1}[B(L)\ell_{t+1}]^{(1-\gamma)\theta}\}\}$$

$$- A(\beta L^{-1})\{[A(L)c_t]^{\gamma\theta-1}[B(L)\ell_t]^{(1-\gamma)\theta}\}|I_t] = 0. \tag{2.14}$$

Expressions (2.12) and (2.14) are used in Section 4 to deduce a set of estimation equations.

The analysis so far has assumed a single consumer. It turns out that the same implications can be obtained in an environment with many consumers who have identical preferences but possibly heterogeneous initial endowments of capital. These implications can also be derived in an environment in which consumers' marginal products of labor are distinct as long as there is a time invariant efficiency units transformation that makes consumers' labor perfectly substitutable. In this latter case efficiency units are priced and their relative price can be inferred from the aggregate compensation data after correction by a time-invariant translation factor [see Appendix A].

## 3. Description of the Data and Analysis of Trends

The formal justification of the econometric procedures described in Section 4 and implemented in Section 5 rely on the assumption that the variables entering the estimation equations are stationary (see Hansen (1982)). In fact, some of the time series considered exhibited pronounced trends during the sample period. Consequently, a stationary-inducing transformation of the data is required. The choice of detrending procedure is

restricted in our context by the requirement that the transformed series satisfy the stochastic Euler equations (2.12) and (2.14). Therefore, after briefly describing the data used in the empirical analysis, we discuss in detail a model of nonstationarity that rationalizes the particular transformation involved here. This transformation does not require a priori or simultaneous estimation of parameters governing the nonstationarities.

The monthly, seasonally adjusted observations on aggregate real consumption of nondurables and services were obtained from the Citibank Economic Database. The per capita consumption series was constructed by dividing each observation of the aforementioned measure of aggregate real consumption by the corresponding observation on the total adult (age sixteen and over) population, published by the Bureau of the Census. The asset return considered is the ex post real return on one-month Treasury bills.[3] Nominal returns reported in Ibbotson and Sinquefield (1979) were converted to ex post real returns using the implicit price deflator for nondurables and services. Nominal wages were measured by the seasonally adjusted averaged hourly compensation for all employees on nonagricultural payrolls, obtained from the Citibank Economic Database. Real wages were constructed by dividing each observation on nominal wages by the implicit price deflator associated with our measure of consumption.

We constructed a measure of hours worked, $h_t$, by forming the ratio of total hours worked by the civilian labor force and our measure of population. Like our compensation measure, this measure of hours averages across members of the population who were and were not employed, a point to which we shall return subsequently. The representative consumer was given a time endowment of 112 hours a week and 4.25 weeks per month, which gives a monthly time endowment ($h_0$) of 476 hours. The leisure series ($\ell_t$) was then

calculated by subtracting hours worked from the monthly time endowment. All data covered the period 1959:1 to 1978:12.

For the equilibrium relations (2.12) and (2.14) to be consistent with this data, certain relations among the respective growth rates of the series must be satisfied. The most desirable way to model nonstationarities in consumption and hours worked is to specify technologies for capital accumulation and the production of new consumption goods that include temporal shifts in the productivity of labor and/or capital. By combining such a specification of technology with a preference specification, one could in principle construct a stochastic growth model with the nonstationarities in consumption and hours worked modeled endogenously.

In our analysis, we assume that the following vector of ratios

$$x_t = (c_t/c_{t-1}, \ \ell_t, \ w_t\ell_t/c_t, \ r_t-1) \tag{3.1}$$

forms a strictly stationary stochastic process. Notice that the assumption that $\ell_t$ and $w_t\ell_t/c_t$ are stationary implies that $\ell n w_t$ and $\ell n c_t$ have a common trend. This assumption is consistent with Altug's modification of the Kydland-Prescott model in which there is a geometric trend in the technology. It is also consistent with Christiano's (1986) growth model in which the technology shock can have a random walk component with drift.[4]

It is possible to derive relations from (2.12) and (2.14), respectively, that involve only current, past and future values of $x_t$. We illustrate this point for the case in which $B(L) = B_2(L) = 1 + bL$. Let $\sigma_0 = (\beta,\theta,\gamma,\alpha,\beta)$,

$$H_c[c_t, c_{t-1}, \ell_t, \ell_{t-1}, \sigma_0] = \{\gamma[c_t + \alpha c_{t-1}]^{\gamma\theta-1}[\ell_t + b\ell_{t-1}]^{(1-\gamma)\theta}\}, \tag{3.2}$$

$$H_\ell[c_t, c_{t-1}, \ell_t, \ell_{t-1} \cdot \sigma_0] = \{(1-\gamma)[c_t + ac_{t-1}]^{\gamma\theta}[\ell_t + b\ell_{t-1}]^{(1-\gamma)\theta-1}\}. \quad (3.3)$$

The expressions given in (3.2) and (3.3) are in the information set at time t. Therefore, (2.12) implies that

$$E[H_w(x_t, x_{t+1}, x_{t-1}, \sigma_0)|I_t] = 0, \quad (3.4)$$

where

$$H_w(x_t, x_{t+1}, x_{t-1}, \sigma_0) =$$

$$\frac{w_t(1+\alpha\beta L^{-1})H_c[c_t, c_{t-1}, \ell_t, \ell_{t-1}, \sigma_0] - (1+b\beta L^{-1})H_\ell(c_t, c_{t-1}\ell_t, \ell_{t-1}, \sigma_0)}{H_\ell(c_t, c_{t-1}, \ell_t, \ell_{t-1}, \sigma_0)}. \quad (3.5)$$

Even though $H_c(\bullet)$ and $H_\ell(\bullet)$ depend on $c_t$, $c_{t-1}, \ell_t$, and $\ell_{t-1}$ separately, $H_w(\bullet)$ depends only on $x_t, x_{t-1}$, and $x_{t+1}$, where $x_t$ is defined in (3.1). A similar strategy can be employed in transforming equation (2.14) to obtain

$$E[H_r(x_t, x_{t+1}, x_{t+2}, x_{t-1}, \sigma_0)|I_t] = 0, \quad (3.6)$$

where

$$H_r(x_t, x_{t+1}, x_{t+2}, x_{t-1}, \sigma_0) =$$

$$\frac{\beta r_{t+1}\{(1+\alpha\beta L^{-1})H_c(c_{t+1},c_t,\ell_{t+1},\ell_t,\sigma_0)\} - (1+\alpha\beta L^{-1})H_c(c_t,c_{t-1},\ell_t,\ell_{t-1},\sigma_0)}{H_c(c_t,c_{t-1},\ell_t,\ell_{t-1},\sigma_0)}.$$

(3.7)

Relations (3.4) and (3.6) are used in Section 4 to derive the estimation equations.

## 4. Estimation and Inference

Our approach to estimation and inference follows closely that of Eichenbaum and Hansen (1985) and Dunn and Singleton (1986). These authors show how to modify the analysis of Hansen and Singleton (1982) to allow for multiple consumption goods and preferences which are not separable over time.

First we consider the case in which $B(L) = B_2(L) = 1 + bL$. Using the notation from Section 3, consider the following two estimation equations:

$$d_{t+2} = H_w(x_t, x_{t+1}, x_{t-1}, \sigma_0) \qquad (4.1)$$

$$H_r(x_t, x_{t+1}, x_{t+2}, x_{t-1}, \sigma_0)$$

Relations (3.4) and (3.6) imply that the $E[d_{t+2}|I_t] = 0$. Consequently, the disturbance vector $d_{t+2}$ is orthogonal to any random variables in $I_t$. Such random variables can be used as instruments in estimating the true parameter vector. Let $z_t$ be an R-dimensional vector of elements in $I_t$, where 2R is greater than or equal to five. Using the components of $z_t$ as instruments, the 2R-dimensional function $g_T$

$$g_T(\sigma) = (1/T) \sum_{j=1}^{T} z_t \otimes d_{t+2}(\sigma) \qquad (4.2)$$

can be formed from the sample information. Since the vector $g_T(\sigma)$ is a consistent estimator of $Ez_t \otimes d_{t+2}(\sigma)$ and the expectation $Ez_t \otimes d_{t+2}(\sigma)$ is in general nonzero except at the point $\sigma = \sigma_0$, we estimate $\sigma_0$ by the choice of $\sigma$, say $\sigma_T$, in an admissible parameter space that makes $g_T(\sigma)$ close to zero in the sense of minimizing the quadratic form

$$g_T(\sigma)'W_T g_T(\sigma). \tag{4.3}$$

Here $W_T$ is a symmetric positive definite distance matrix that can depend on sample information.

Hansen (1982) shows that the choice of $W_T$ that minimizes the asymptotic covariance matrix of $\sigma_T$, depends on the autocovariance structure of the disturbance vector $d_{t+2}$. Although this vector is serially correlated, it is in the information set at time t+2. Hence the theory implies the restrictions

$$E\{(z_{t+k} \otimes d_{t+k+2})(z_t \otimes d_{t+2})'\} = 0, \text{ for } |k| \geq 2 \tag{4.4}$$

It follows that the optimal estimator is obtained by choosing $W_T^{-1}$ to be a consistent estimator of

$$S_0 = \sum_{k=-1}^{1} E(z_{t+k} \otimes d_{t+k+2})(z_t \otimes d_{t+2})'. \tag{4.5}$$

Hansen (1982) discusses a candidate estimator of $S_0$. In Appendix B, we describe an alternative estimator that, unlike the estimator suggested by Hansen, is constrained to be positive definite in finite samples.

Recall from the discussion of (2.12) that if the induced preferences defined over consumption acquisitions and leisure are time separable, then

there is an exact relationship between hours worked, consumption acquisitions, and wages. In this case, the first component of $d_{t+2}$ is actually in $I_t$ and hence is zero. An analogous observation applies to any specification of time separable preferences that like ours exclude unobservable shocks to preferences. Hence, temporal nonseparabilities in preferences are necessary in our analysis in order for one of the disturbances terms to be different from zero.

The estimation approach we use relies in an essential way on the exclusion of unobservable shocks to preferences and the absence of measurement errors. The introduction of such unobservables does not lead to additive error terms for the specification of preferences given in Section 2. Accommodation of these unobservables seems to require explicit or numerical solutions to the stochastic general equilibrium model while the approach adopted here avoids the need for such solutions.

The parameters of the model with $B(L) = B_1(L)$ can be estimated in a similar, but not entirely, analogous fashion. Two additional problems emerge. The first problem is that for hypothetical values of the parameters, the leisure service at any point in time depends on the entire infinite past of the consumption of leisure time. For instance, in the first time period we have that the leisure service is given by

$$\ell_1^* = \ell_1 + \delta \sum_{j=0}^{\infty} \eta^j \ell_{-j}. \tag{4.6}$$

Since we do not have observations on values of leisure time prior to time period one, we approximate the infinite sum

$$\sum_{j=0}^{\infty} \eta^j \ell_{-j} \tag{4.7}$$

by the average of the consumption of leisure time in our sample divided by $(1-\eta)$ for each hypothetical value of $\eta$.[5] Then, given an initial value of leisure services, the remaining values of leisure services for our sample can be calculated using the sample observations on leisure time consumption and hypothetical values of $\eta$ and $\delta$. In this manner we are able to calculate values of $MC_t^*$ and $ML_t^*$ for hypothetical values of the preference parameters.[6]

The second problem that occurs is that $ML_t$ as given by (2.11) now depends on the current and expected infinite future of $ML_t^*$. However, following Hotz, Kydland, and Sedlacek (1985), the relation

$$w_t MC_t = ML_t \tag{4.8}$$

also implies that

$$E\{(1-\eta L^{-1})\{w_t(1-\alpha L^{-1})MC_t^*\}|I_t\} = E\{[1+(\delta-\eta)L^{-1}]ML_t^*|I_t\} \tag{4.9}$$

for $B(L) = B_1(L)$. A virtue of the expression in (4.9) is that it only depends on terms involving $MC_t^*$, $MC_{t+1}^*$, $MC_{t+2}^*$, $ML_t^*$ and $ML_{t+1}^*$.

Relation (4.9) can be used in deriving an expression analogous to (2.12) by substituting in for $MC_t^*$ and $ML_t^*$ from (2.8) and (2.9). This expression together with (2.14) then can be used to define two estimation equations with disturbance terms arising from expectational errors. The stationary-inducing transformation described in Section 3 can be modified appropriately to convert these relations to relations among variables that are assumed to be components of a strictly stationary stochastic process. Estimation then proceeds in the same fashion as in the case in which $B(L) = B_2(L)$.

## 5. Empirical Results

Estimates for the Kydland and Prescott specification of B(L) were obtained using the following orthogonality conditions:

$$E(d_{1t+2}) \begin{vmatrix} 1 \\ V_t \end{vmatrix} = 0 \quad \text{and} \quad E(d_{2t+2}) \begin{vmatrix} 1 \\ V_t \\ V_{t-1} \end{vmatrix} = 0, \tag{5.1}$$

where

$$V'_t = [(c_t-c_{t-1})/c_{t-1}, (\ell_t-\ell_{t-1})/\ell_{t-1}, (w_t-w_{t-1})/w_{t-1}, r_t-1].$$

Thus, fourteen orthogonality conditions were imposed. The results are displayed in Table 1.

The estimates displayed under the heading "Wage 1" were obtained using the data described in Section 3. All of the parameter estimates are economically meaningful except for $\hat{\beta}$, which is slightly larger than unity. The latter finding is common to several recent empirical studies of intertemporal Euler equations using treasury bill returns (see Singleton (1986)). The estimates of $\theta$ and $\gamma$ imply that the representative consumer's utility function is concave. The estimate of $\theta$ is about four times its standard error suggesting that logarithmic separability ($\theta=0$) is empirically implausible. We defer discussion of $\gamma$ until later in this section.

Next consider the parameters which govern the intertemporal aspects of the service technologies. In all cases the estimate of $\alpha$ is both positive and large relative to its estimated standard error.[7] This implies that consumption good acquisitions today give rise to consumption services both

today and one period in the future. The estimates of $\eta$ and $\delta$ raise some interesting quandries. The estimate of $\delta$ is negative implying that current leisure acquisitions give rise to future leisure disservices. The estimate of $\delta$, however, is small relative to its estimated standard error. When $\delta$ is zero, $\eta$ ceases to be identified if the model is specified correctly. The results in Table 1 indicate that $\eta$ is estimated quite accurately even though $\delta$ is estimated quite imprecisely. The econometric equation obtained from (4.9) is filtered forward by $(1-\eta L^{-1})$. When $\delta$ is zero this forward filter should leave the population orthogonality conditions intact for any value of $\eta$. Our finding that $\eta$ is estimated accurately, while $\delta$ is not, may just reflect the fact that the model is fundamentally misspecified. The forward filtering is exploited in allowing the orthogonality conditions to be approximately satisfied when in fact this filtering should have little impact.

We also studied a specification of the mapping from leisure to leisure services that does not require forward filtering. We estimated the model using the parsimonious representation of B(L) given by (2.5) and fourteen orthogonality conditions. The results are reported in the first column of Table 2. Notice that the estimated values of $\theta$ are closer to zero than those reported in Table 1. Also, there is little evidence against the hypothesis that preferences are logarithmically separable. Perhaps more importantly, the point estimates again imply that current leisure decisions impact negatively upon future leisure services. Unlike the estimates of $\delta$, the estimates of b are large in absolute value relative to their standard errors.

The representative consumer always chooses positive values of $\ell_t^*$. Therefore, when b is negative, he always must choose enough leisure to offset the negative impact of past leisure choices on the level of current leisure

services.  For example, if $B(L) = B_2(L)$ and $b < 0$, then it must be the case
that

$$\ell_t > |b\ell_{t-1}| \quad \text{for all } t.$$

Thus, based on the estimates of b reported in Table 2, the representative
consumer will always choose a value of $\ell_t$ that is greater than approximately
2/3 of $\ell_{t-1}$.  It follows that increases in hours worked will be accomplished
in a relatively gradual way, while decreases in hours worked are
unrestricted.[8]

The finding that current leisure decisions provide leisure disservices in
the future is inconsistent with the assumptions in Kydland and Prescott.
However, it is consistent with some of the empirical findings in Hotz, Kydland
and Sedlacek (1985) in a panel data analysis with $\ell_t^*$ given by $B_1(L)\ell_t$.
It is also consistent with Kennan's (1985) time series analysis of a model in
which $\ell_t^*$ is given by $B_2(L)\ell_t$.  Thus qualitatively similar properties of
the leisure technology have been obtained in studies using other data and
different identifying assumptions.

For comparison, estimates were also obtained using the ratios of
aggregate total employee compensation from the National Income and Product
Accounts to our measure of aggregate hours as the nominal wage rate.  These
results are displayed in Tables 1 and 2 under the heading "Wage 2".  The
estimated parameters are similar to those obtained using "Wage 1".

We now return to the discussion of $\gamma$.  Kydland and Prescott (1982)
argue that $\gamma$ should be approximately 1/3.  Their rationale for this choice
is "motivated by the fact that households' allocation of time to nonmarket
activities is about twice as large as the allocation to market activities"

[page 1352]. Since our estimates of $\gamma$ are considerably smaller than 1/3, it is of interest to understand why. One rough set of calculations involves abstracting from uncertainty as well as dynamics and conducting a steady state analysis. The steady state that we consider treats the growth rate of consumption, leisure, and the valuation of leisure relative to consumption as constants, but accommodates geometric growth in consumption and wages. Letting [c/wℓ] be the steady state ratio of consumption to the valuation of leisure, it follows from (2.12) that

$$\gamma = [c/(w\ell)]/(1 + [c/(w\ell)]). \tag{5.3}$$

Relation (5.3) is the standard relation between $\gamma$ and expenditure shares for Cobb-Douglas preferences.

Recall that relation (2.12) was also used to construct relation (3.4) which is utilized in our econometric analysis. In fact one of the orthogonality conditions which we imposed in our estimation procedure amounts to scaling (2.12) in order to induce stationarity and then taking unconditional expectations ($Ed_{1t+2} = 0$). This orthogonality condition imposes the stochastic counterpart of the steady-state relation (5.3). Substituting time averages of consumption relative to the valuation of leisure for c/[wℓ] in (5.3) gives values of $\gamma = .13$ and $\gamma = .16$ for the "Wage 1" and "Wage 2" measures of compensation, respectively. These values are quite similar to the point estimates reported in columns 1 and 2 of Tables 1 and 2, respectively. For our choice of total time endowment and measure of hours worked, the ratio of average hours worked to leisure is about .20 which is considerably less than one-half, the number assumed by Kydland and Prescott (1982). We have chosen to include all individuals age 16 and over in our sample when

calculating leisure time. Hence our sample includes unemployed adults. Of course other choices of time endowments will alter this conclusion. One reason for the ambiguity in defining total time endowments is that the representative consumer model confounds the behavior of employed and non-employed individuals, and the total time endowment is obviously sensitive to whether non-employed adults are included in the sample.

Formula (5.3) also suggests that the value of $\gamma$ will be sensitive to the measure of compensation. One possible problem is that wages should be measured in efficiency units. Interpreting the model as applying to efficiency units of labor in an environment where consumers have distinct marginal products of labor complicates the relation between observed total compensation and efficiency unit wages (see Appendix A). A second possible problem is that the measure of compensation used in obtaining the results reported in columns 1 and 2 of Tables 1 and 2 are not corrected for taxes. For the sake of comparison we also estimated the model using after-tax wages and returns. Our results are displayed in the last columns of Tables 1 and 2. The time series on annual marginal tax rates was taken from Seater (1985). The annual rates were interpolated linearly to obtain monthly rates. The adjustment for taxes lowers the average real wage. Equation (5.4) implies that this should result in a larger value of $\gamma$. Furthermore, the estimated values of $\gamma$ in Tables 1 and 2 are larger for the tax-adjusted data than the corresponding estimates from the unadjusted data. In fact, for the specification $B_1(L)$, the estimates of $\gamma$ are within one standard error of the value of one-third which was imposed by Kydland and Prescott (1982). The estimates of $\gamma$ are less precise when tax adjustments are made, however.

Our discussion of the point estimates must be qualified by the fact that the $J_T$ statistics reported in Tables 1 and 2 are large relative to the degrees

of freedom. One possible reason for these large test statistics is that all of the measures of compensation that we used are contaminated by measurement error and do not reflect the correct measure of consumers' marginal value of time. In order to explore this possibility we tested the null hypothesis that the five orthogonality conditions associated with the intratemporal Euler equation (2.12) relating $ML_t$, $MC_t$ and $w_t$ hold, conditional on the orthogonality conditions associated with the intertemporal Euler equation (2.14) being satisfied. We examined this null hypothesis using a statistical test that is analogous to a likelihood ratio test. A formula for the test statistic is presented formally in Appendix C and its asymptotic properties are discussed. In Tables 1 and 2 the value of this test statistic is denoted by $C_T$. The values of $C_T$ do suggest that the large $J_T$ statistics are indicative of the failure of the orthogonality conditions associated with the Euler equation relating $ML_t$, $MC_t$ and $w_t$ to hold in the sample.

To explore this possibility further, we re-estimated the parameters using only the orthogonality conditions associated with the intertemporal relation (2.14). In conducting this exercise, it was necessary to fix the value of $\gamma$ and $\eta$ in the model with $B(L) = B_1(L)$ and the value of $\gamma$ in the specification of the model with $B(L) = B_2(L)$ in order to obtain convergence of the minimization algorithm. (Recall that $\hat{\gamma}$ seems to be determined largely by the intratemporal Euler equation). The results are displayed in Table 3 for the second measure of wages (Wage 2). Notice first that the probability values of the $J_T$ statistics are substantially smaller than the probability values for the corresponding statistics in Tables 1 and 2. Second, with $B(L) = B_1(L)$ the point estimates are qualitatively similar to the corresponding estimates reported in Table 1. The primary difference is the loss of precision when only the intertemporal Euler equation is used in the empirical

analysis. On the other hand, for the model with $B(L) = B_2(L)$, the sign of b changes form negative to positive when the intratemporal Euler equation is omitted from the analysis. For both models, the estimates of $\alpha$ remain positive and are estimated precisely. Taken together, the results obtained when only the intertemporal Euler equation is used in the empirical analysis provide less evidence against the hypothesis of intertemporal substitution of leisure.

## 6. Conclusion

In this paper we estimate and test a representative consumer model which relates per capita consumption, per capita hours worked, per capita consumption and interest rates. The analysis focuses upon the cross-relations between prices and quantities that are implied by the representative consumer's non-time-separable preferences. When both the inter and intratemporal Euler equations of the representative consumer are utilized in the estimation procedure, we find substantial evidence against the over-identifying restrictions implied by the model. The results from the specification tests developed in the paper suggest that this can be attributed to the failure of the orthogonality conditions associated with the representative consumer's intratemporal Euler equation to hold in the sample.

TABLE 1[a]

$$A(L) = 1 + \alpha L \qquad B(L) = 1 + \delta L/(1-\eta L)$$

| Parameters | Wage 1[b] | Wage 2[b] | Tax-Adjusted [b] |
|---|---|---|---|
| $\beta$ | 1.0012 (.0002) | 1.0009 (.0002) | 1.0013 (.0003) |
| $\theta$ | .85585 (.0827) | .8014 (.1880) | -.1690 (.4337) |
| $\gamma$ | .14299 (.0237) | .1676 (.0213) | .14390 (.1062) |
| $\alpha$ | .30744 (.0741) | .3520 (.0655) | .3654 (.0592) |
| $\eta$ | .98302 (.0146) | .9816 (.0177) | .9884 (.0123) |
| $\delta$ | -.02394 (.0212) | -.0269 (.0272) | -.0169 (.0203) |
| $J_T$ | 25.102 (.9985) | 20.30 (.9907) | 16.55 (.9648) |
| $C_T$ Test | 23.529 (.9997) | 12.48 (.8689) | 65.12 (1.000) |

a Standard errors of the estimates and probability values of the test statistics are given in parentheses.

b The estimates under the heading Wage 1 were obtained using the data described in Section 3. The estimates under Wage 2 were obtained with nominal wages measured as the ratio of aggregate employee compensation (from the National Income and Product Accounts) divided by our constructed measure of aggregate hours worked. The Tax-Adjusted run is identical to the Wage 2 run, except that wages and asset returns are calculated on an after-tax basis.

TABLE 2[a]

| Parameters | $A(L) = 1 + \alpha L$ Wage 1[b] | $B(L) = 1 + bL$ Wage 2[b] | Tax-Adjusted[b] |
|---|---|---|---|
| $\beta$ | 1.0013 (.0002) | 1.0009 (.0002) | 1.0020 (.0002) |
| $\theta$ | .0061 (.0680) | -.0761 (.0681) | -.0009 (.0352) |
| $\gamma$ | .1158 (.0002) | .1459 (.0002) | .1832 (.0006) |
| $\alpha$ | .7304 (.1471) | .4032 (.0820) | .4405 (.0778) |
| b | -.6824 (.0386 | -.7562 (.0429) | -.8321 (.0216) |
| $J_T$ | 56.067 (1.000) | 25.46 (.9975) | 35.15 (.9999) |
| $C_T$ Test | 48.119 (1.000) | 17.52 (.9749) | 23.61 (1.000) |

a   Standard errors of the estimates and probability values of the test statistics are given in parentheses.

b   The estimates under the heading Wage 1 were obtained using the data described in Section 3.  The estimates under Wage 2 were obtained with nominal wages measured as the ratio of aggregate employee compensation (from the National Income and Product Accounts) divided by our constructed measure of aggregate hours worked.  The Tax-Adjusted run is identical to the Wage 2 run, except that wages and asset returns are calculated on an after-tax basis.

TABLE 3

ESTIMATES BASED ON INTERTEMPORAL EULER EQUATION[a]

$$B(L) = (1 + \delta L/(1-\eta L)) \qquad \gamma = .14 \qquad \eta = .98$$

| $\beta$ | $\theta$ | $\alpha$ | $\delta$ |
|---|---|---|---|
| 1.00164 | −.02867 | .33049 | −.01564 |
| (.0006) | (1.9831) | (.0626) | (.3048) |

$$J_T{}^{**} = 8.663 \qquad (.8767)$$

$$B(L) = (1 + bL) \qquad \gamma = .14 \qquad \eta = .98$$

| $\beta$ | $\theta$ | $\alpha$ | $b$ |
|---|---|---|---|
| 1.00143 | .69126 | .31175 | .70621 |
| (.0003) | (.7944) | (.0754) | (.6234) |

$$J_T{}^{**} = 8.1206 \qquad (.8503)$$

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a  Standard errors of the estimates and probability values of the test statistics are given in parentheses.

## APPENDIX A

In this appendix we consider the implications for our econometric analysis of consumers having distinct marginal products of labor. We consider only the special case in which individual labor supply can be converted into efficiency units that are comparable across consumers. Consumers are presumed to be compensated for the quantities of efficiency units of labor they supply. Muellbauer (1981) studies this problem in a single period context and obtains necessary and sufficient conditions for aggregation. Here we allow for multiple time periods but restrict our attention to the class of preferences used in our empirical analysis.

First, we introduce some notation. Let $c_t^j$ denote the consumption of person $j$ at time $t$ and $\ell_t^j$ denote the leisure of person $j$ at time $t$. We assume that hours worked at time $t$ by person $j$ can be converted to efficiency units by multiplying the hours worked by $e^j$, where $e^j$ is a positive number not indexed by time. Hence the efficiency units of leisure of person $j$ at time $t$ are $e^j \ell_t^j$. Similarly, the efficiency units of leisure services are given by $e^j \ell_t^{*j}$, where $\ell_t^{*j} = B(L)\ell_t^j$.

Suppose all $J$ consumers have identical preferences given by (2.6). These preferences could equivalently be expressed in terms of efficiency units of leisure services. The conversion to efficiency units simply scales the utility function. Since preferences are homothetic, in a competitive equilibrium with complete markets in consumption and leisure services,

$$c_t^{*j} = \omega^j [c_t^{*1} + c_t^{*2} + \ldots + c_t^{*J}]/J$$

(A.1)

$$e^j \ell_t^{*j} = \omega^j [e^1 \ell_t^{*1} + e^2 \ell_t^{*2} + \ldots + e^J \ell_t^{*J}]/J$$

where $\omega^j$ is strictly positive and $[\omega^1 + \omega^2 + \ldots = \omega^J]/J = 1$. The proportionality relations in (A.1) do not imply corresponding proportional relations for acquisitions of consumption goods or efficiency units of leisure. It turns out, however, that an asymptotic result can be obtained when $A(L)$ and $B(L)$ satisfy certain invertibility conditions. That is, proportionality will be obtained for appropriately defined stochastic steady states. Therefore, we strengthen (A.1) to be

$$c_t^j = \omega^j[c_t^1 + c_t^2 + \ldots + c_t^J]/J$$

(A.2)

$$e^j \ell_t^j = \omega^j[e^1 \ell_t^1 + e^2 \ell_t^2 + \ldots + e^J \ell_t^J]/J,$$

although we will not address formally the approximation involved.

We define the efficiency units so that

(A.3)     $(\omega^1/e^1 + \omega^2/e^2 + \ldots + \omega^J/e^J)/J = 1.$

Then

(A.4)     $(1/J)(\ell_t^1 + \ell_t^2 + \ldots + \ell_t^J) = (1/J)(e^1 \ell_t^1 + e^2 \ell_t^2 + \ldots + e^J \ell_t^J)$

so that the average amount of leisure is equal to the average amount of efficiency units of leisure.

Since consumers are compensated in terms of efficiency units, person $j$ receives $w_t^* e^j [h - \ell_t^j]$ units of the consumption good at time $t$ where $w_t^*$ is the wage rate in terms of efficiency units and $h$ is the total time endowment. Average compensation $w_t^a$ is then equal to

(A.5) $\qquad w_t^a = w_t^* (h^* - \ell_t^a),$

where

(A.6) $\qquad h^* = (e^1 + e^2 + \ldots e^J) h / J.$

Solving for $w_t^*$ gives

(A.7) $\qquad w_t^* = w_t^a / (h^* - \ell_t^a).$

The efficiency wage $w_t^*$ is equal to average compensation divided by the number of efficiency units worked. The parameter $h^*$ depends on both $h$ and the efficiency units correction. In the special case in which the $e^j$ are one for all $j$, $h^* = h$ as is assumed in our empirical analysis. Otherwise, it could be treated as a free parameter to be estimated. This describes one possible source of measurement error in our wage series that could in principle be accommodated by augmenting the parameter vector to include $h^*$.

APPENDIX B

APPENDIX B: <u>Estimating the Asymptotic Covariance Matrices</u>

In this appendix we describe the procedure used to estimate the distance matrix in our IV criterion function and the asymptotic covariance matrix of $\sigma_T$, the minimizer of (4.3).

Suppose the KxI vector of disturbances in the estimation equations is observed by agents at date t+q and satisfies $E_t d_{t+q}(\sigma_0) = 0$, for some finite integer $q \geq 1$. Also, let

$$g_T(\sigma) = \frac{1}{T} \sum_{t=1}^{T} z_t \otimes d_{t+q}(\sigma),$$

where $z_t$ is an R x 1 vector of elements of $I_t$, and suppose that the estimator of $\sigma_0$ is chosen from the admissible parameter space to minimize $g_T(\sigma)'W_T g_T(\sigma)$, where $W_T$ is a consistent estimator of the inverse of the matrix

(B.1) $$S_0 = \sum_{i=-q}^{q} E(z_t \otimes d_{t+q})(z_{t-i} \otimes d_{t+q-i})'.$$

Finally, let

(B.2) $$D_0 = E \; z_t \otimes \frac{\partial d_{t+q}}{\partial \sigma_0}.$$

Then Hansen (1982) shows under certain regularity conditions that the limiting distribution of $\{\sqrt{T}\sigma_T: T \geq 1\}$ is normal with mean vector zero and covariance matrix $(D_0'S_0^{-1}D_0)^{-1}$. To implement this estimator and conduct inference about $\sigma_0$ requires consistent estimators of $S_0$ and $D_0$. Here we describe such estimators for the case of arbitrary q. The results can be applied to study (4.2), for example, by setting q = 2.

Hansen supplies sufficient conditions to guarantee that if $\{\sigma_T : T \geq 1\}$ converges in probability to $\sigma_0$, then $\{\frac{\partial g_T}{\partial \sigma}(\sigma_T) : T \geq 1\}$ converges in probability to $D_0$. Therefore, in our empirical analysis we use $D_T = \frac{\partial g_T}{\partial \sigma}(\sigma_T)$ as our estimator of $D_0$. Estimation of $S_0$ is somewhat more involved. The matrix $S_0$ is a covariance matrix and is therefore positive

semidefinite. In this paper we impose the stronger requirement that it be positive definite. Hansen (1982) suggests estimating $S_0$ by replacing the population moments in (B.1) by their sample counterparts evaluated at $\sigma_T$. Although the resulting estimator converges almost surely to $S_0$, it is not constrained algorithmically to be positive definite in finite samples. There have been several empirical applications in which this estimator has turned out to be positive definite, but we encountered cases in which it was not positive definite.[B.1] For this reason we consider an alternative estimator of $S_0$ that is constrained to be positive definite in finite samples.[B.2]

Specifically, we estimate the coefficients of a Wold decomposition of the process $\{u_{t+q} = z_t \otimes d_{t+q} : -\infty < t < +\infty\}$, and then use these coefficient estimates in estimating the covariance matrix of the one-step-ahead linear least squares forecast errors and $S_0$. The zero restrictions on the autocovariances imply that the Wold decomposition can be represented as

$$(B.3) \quad u_t = e_t + B_1 e_{t-1} + \ldots + B_q e_{t-q},$$

where $e_t$ is the one-step ahead forecast error in forecasting $z_{t-q} \otimes d_t$ from linear combinations of past values of $z_{t-q} \otimes d_t$ and $B_1, \ldots, B_q$ are RK x RK dimensional matrices. The matrix $S_0$ is related to the $B_j$'s via the formula

$$(B.4) \quad S_0 = (I + B_1 + \ldots + B_q)\Omega_0(I + B_1' + \ldots + B_q'),$$

where $\Omega_0 = Ee_t e_t'$. Once we obtain consistent estimators of $B_1, \ldots, B_q$ and a consistent estimator of $\Omega_0$ that is constrained to be positive semidefinite in finite samples, we can use formula (B.4) to obtain a consistent estimator of $S_0$ that will be positive semidefinite.[B.3]

To estimate the moving average coefficients $B_1, B_2, \ldots, B_q$, we use a procedure suggested by Durbin (1960) with some minor modifications. A virtue of Durbin's procedure is that it provides estimators of the moving average coefficients without resorting to numerical search procedures. Numerical search procedures become intractable in our application because of the large number of elements in the $B_j$ matrices that have to be estimated simultaneously.

The first step of our modified Durbin procedure is to use the Yule-Walker equations to obtain estimates of $A_1, A_2, \ldots, A_{NLAG}$ in the finite order autoregression

$$(B.5) \qquad u_t = A_1 u_{t-1} + \ldots + A_{NLAG} u_{t-NLAG} + \tilde{e}_t.$$

These estimates are then used to construct estimates $\tilde{e}_t^T$ of the one-step-ahead forecast errors of the finite order vector autoregression. The sample forecast errors $\{\tilde{e}_t^T : t=NLAG+1, \ldots, T\}$ are used subsequently as estimates of the forecast errors $\{e_t : t=NLAG+1, \ldots, T\}$ in (B.4). Since the autoregressive representation of the process $\{u_t : -\infty < t < +\infty\}$ has infinite order when q is greater than zero, the choice of NLAG should be an increasing function of sample size in order that sample forecast errors will converge to the true forecast errors.[B.4] Recall that in our applications there is <u>a</u> <u>priori</u> information that all but a finite number of the autocovariances are zero. Therefore the number of nonzero sample autocovariances used in estimation of (B.5) does not need to increase with sample size even though NLAG does.[B.5]

The second step is to estimate the regression equation

$$z_{t-q} \boxtimes d_t(\sigma_T) = \tilde{B}_1 \tilde{e}_{t-1}^T + \ldots + \tilde{B}_q \tilde{e}_{t-q}^T + \nu_t,$$

where $\nu_t$ is the vector disturbance term. Let $\tilde{B}_1^T, \ldots, \tilde{B}_q^T$ denote the resulting estimators of $\tilde{B}_1, \ldots, \tilde{B}_q$, respectively, and let

$$\Omega_T = \frac{1}{T-NLAG-q} \sum_{t=NLAG+q+1}^{T} \nu_t^T \nu_t^{T\prime},$$

where

$$\nu_t^T = z_{t-q} \boxtimes d_t(\sigma_T) - \tilde{B}_1^T \tilde{e}_{t-1}^T - \ldots - \tilde{B}_q^T \tilde{e}_{t-q}^T.$$

As an estimator of $S_0$ in our empirical work we use

$$S_T = (I + \tilde{B}_1^T + \ldots + \tilde{B}_q^T)\tilde{\Omega}_T(I + \tilde{B}_1^{T\prime} + \ldots + \tilde{B}_1^{T\prime}).^{B.6}$$

APPENDIX C

APPENDIX C:   <u>Testing Subsets of Orthogonality Conditions</u>

In this appendix we consider the problem of testing whether a subset of the orthogonality conditions hold (see Appendix B for notation).   More precisely, partition the vector $u_{t+q} = z_t \otimes d_{t+q}(\sigma_0)$ into two subvectors $u_{t+q}^1$ and $u_{t+q}^2$, where $u_{t+q}^1$ is $J_1$ dimensional with $J_1$ greater than or equal to the number of parameters, Q, and $u_{t+q}^2$ is a $J_2$ dimensional vector, $J_2 = RK-J_1$.   Let the assumptions that $E[u_{t+q}^1] = 0$ and $E[\frac{\partial u_{t+1}^1(\sigma_0)}{\partial \sigma}] = D_0^1$ has rank Q be maintained as true.   Suppose a researcher wishes to test the null hypothesis that $E[u_{t+q}^2] = 0$. The elements of the vector $u_{t+q}^1$ may be chosen, for example, to be the orthogonality conditions associated with a particular disturbance.

Throughout this discussion we shall assume that the matrices $S_0$ and $D_0$ can be consistently estimated by $\{S_T : T\geq 1\}$ and $\{D_T : T\geq 1\}$, and that $S_0$ is nonsingular.   Partitioning $W_0$, $S_0$, $S_0^{-1}$ and $D_0$ in accord with the two sets of orthogonality conditions, gives

$$D_0 = \begin{vmatrix} D_0^1 \\ D_0^2 \end{vmatrix} \qquad\qquad W_0 = \begin{vmatrix} W_0^{11} & W_0^{12} \\ W_0^{21} & W_0^{22} \end{vmatrix}$$

$$S_0 = \begin{vmatrix} S_0^{11} & S_0^{12} \\ S_0^{21} & S_0^{22} \end{vmatrix} \qquad\qquad S_0^{-1} = \begin{vmatrix} \tilde{S}_0^{11} & \tilde{S}_0^{12} \\ \tilde{S}_0^{21} & \tilde{S}_0^{22} \end{vmatrix}$$

Similarly, $g_T(\sigma)'$ is partitioned as $[g_{1T}(\sigma)' \; g_{2T}(\sigma)']$, where

$$g_{1T}(\sigma) = \frac{1}{T} \sum_{t=1}^{T} u_{t+q}^1(\sigma) \quad \text{and} \quad g_{2T}(\sigma) = \frac{1}{T} \sum_{t=1}^{T} u_{t+q}^2(\sigma).$$

The test which we consider exploits the fact that the sample orthogonality conditions $\{g_T(\sigma_T) : T>1\}$ converge in distribution to a normally distributed random vector with mean zero and covariance matrix $V_0$, where $V_0 = S_0 - D_0(D_0'S_0D_0)^{-1}D_0'$ (see Hansen 1982).

Gallant and Jorgenson (1979) have proposed a procedure for testing nonlinear restrictions on the parameter vector using instrumental variable estimators that is analogous to the likelihood ratio test. While they assumed that disturbance terms were serially independent and conditionally homoskedastic, their procedure is easily modified to apply to the inference problem considered here for subsets of orthogonality conditions. To implement this test, first one obtains an estimator $\{\sigma_{2T} : T\geq1\}$ of $\sigma_0$ by minimizing the objective function $g_T(\sigma)'S_T^{-1}g_T(\sigma)$ by choice of $\sigma$. This estimator exploits all of the orthogonality conditions appropriate under the null hypothesis. Next the estimator $\{\sigma_{1T} : T\geq1\}$ of $\sigma_0$ is formed using only the first $J_1$ orthogonality conditions that are presumed to hold under the alternative hypotheses, and the weighting matrix $(S_T^{11})^{-1}$. Using both estimators

$$(C.1) \qquad C_T = Tg_T(\sigma_{2T})'S_T^{-1}g_T(\sigma_{2T}) - Tg_{1T}(\sigma_{1T})'(S_T^{11})^{-1}g_{1T}(\sigma_{1T})$$

is then calculated. Under the null hypothesis the asymptotic distribution of $\{C_T: T\geq1\}$ is chi square with $J_2$ degrees of freedom. To see this, factor $S_0^{-1}$ and $(S_0^{11})^{-1}$ as $P_0'P_0$ and $P_1'P_1$, respectively. In proving Theorem 3.1, Hansen (1982) shows that $\{\sqrt{T}P_0g_T(\sigma_0) :T\geq1\}$ and $\{\sqrt{T}P_1g_{1T}(\sigma_0) : T\geq1\}$ have limiting distributions under the null hypothesis that are normals with zero means and covariance matrices $I_R$ and $I_Q$, respectively. These results, together with Lemma 4.1 in Hansen, imply that $\{\sqrt{T}P_{\sigma T}g_T(\sigma_{2T}) : T\geq1\}$ has the same limiting distribution as

$$\{\sqrt{T}(I_R-P_0D_0(D_0'S_0^{-1}D_0)^{-1}D_0'P_0')P_0g_T(\sigma_0) : T\geq1\} \equiv \{\sqrt{T}\ NP_0g_T(\sigma_0) : T\geq1\}$$

and $\{\sqrt{T}P_{1T}g_{1T}(\sigma_{1t}) : T\geq1\}$ has the same limiting distribution as

$$\{\sqrt{T}(I_Q - P_1D_0^1(D_0^1{}'(S_0^{11})^{-1}D_0^1)^{-1}D_0^1{}'P_1') P_1g_{1T}(\sigma_0) : T\geq1\} \equiv \{\sqrt{T}\ MP_1g_{1T}(\sigma_0) : T \geq 1\}.$$

Thus, under the null hypothesis, $C_T$ has the same asymptotic distribution as the statistic

$$(C.2) \qquad T g_T(\sigma_0)' P_0' \{N - (P_0')^{-1} \begin{vmatrix} I_Q \\ 0 \end{vmatrix} P_1' M P_1 [I_Q \ 0] P_0^{-1} \} P_0 g_T(\sigma_0).$$

Now the matrix in brackets in (C.2) is idempotent with rank equal to $J_2$ and, therefore, $C_T$ is distributed asymptotically as chi-square with $J_2$ degrees of freedom.

To conclude the discussion, note that the test procedure is easily modified to handle restrictions on parameters of the form,

$$(C.3) \qquad f_2(\sigma_2) = 0,$$

where $f_2$ has $J_2$ coordinates and where $J_2$ is less than Q. We simply view (C.3) as being a set of orthogonality conditions that we wish to test just as above. Now, however, there is no randomness in the orthogonality conditions that we wish to test so the $S_0$ matrix has the partitioned form

$$S_0 = \begin{vmatrix} S_0^{11} & 0 \\ 0 & 0 \end{vmatrix}$$

and is therefore singular. Subject to this modification, the analysis above carries over immediately to testing restrictions on the unknown parameters.

## FOOTNOTES

1.  A more general specification of this technology would allow $c_t^*(\ell_t^*)$ to also depend upon current and lagged values of $\ell_t(c_t)$. However for reasons of empirical tractability, we consider the specifications given by (2.1) and (2.2).

2.  Relations (2.10) and (2.11) ignore any nonnegativity constraints on $c_t$ and $\ell_t$.

3.  We also considered the value-weighted average of returns on the New York Stock Exchange. The results of the empirical analysis were qualitatively the same as those reported in this paper.

4.  To obtain this result Christiano assumes preferences are logarithmically separable in consumption and leisure and time separable in consumption.

5.  Under our assumption that the $\ell_t$ process is stationary, $E \sum_{j=0}^{\infty} \eta^j \ell_{-j} = E\ell_t/(1-\eta)$. Thus our procedure amounts to replacing (4.7) with the sample estimate of its unconditional mean.

6.  It can be shown that neither the consistency of our estimators nor the relevant asymptotic distribution theory is affected by the fact that our measure of the initial condition is undoubtedly incorrect.

7.  Interestingly, Eichenbaum and Hansen (1985) and Dunn and Singleton (1986) in their analyses of purchases of nondurable and durable consumption goods also present evidence of intertemporal nonseparabilities in the mapping from nondurable consumption goods to nondurable consumption good services.

8.  There is a literature which models temporally nonseparable preferences defined over consumption goods as reflecting the presence of "habit-formation." Negative estimated values of b and $\delta$ are consistent with this interpretation. See Pollak (1970) for an overview of habit-formation models.

B.1 Brown and Maital (1981) and Hansen and Hodrick (1980, 1983) have used the estimator proposed by Hansen without encountering any problem.

B.2 A third alternative is to estimate $S_0$ using procedures developed for estimating spectral density matrices. While this method gives rise to a positive definite estimate of $S_0$, it ignores the implication of the theory that all but a finite number of the autocovariances of $\{z_t d_t : -\infty < t < +\infty\}$ are zero. Under the alternative hypotheses considered in Section 4, the zero restrictions in the autocovariance function may not hold. In conducting tests with respect to these alternatives it is not clear for power considerations whether one should or should not impose these zero restrictions. Under the null hypothesis the asymptotic distribution of the test statistics are likely to approximate more accurately their finite sample distributions if the zero restrictions are imposed.

B.3 As long as $\Omega_0$ is nonsingular, this approach will, in general, give rise to a nonsingular estimate of $S_0$ in finite samples.

B.4 When $\det B(z)$ has zeroes on the unit circle even an infinite order autoregressive representation will not exist. In our discussion we are implicitly ruling our zeroes with unit moduli.

B.5 Durbin's (1960) procedure is designed to handle mixed autoregressive-moving average models which do not, in general, have only a finite number of nonzero autovariances.

B.6 Cumby, Huizinga and Obstfeld (1982) propose a related method for estimating $S_0$. They use a Yule-Walker equation to obtain estimates of the autoregressive parameters, inverts the autoregressive polynomial, and then uses the resulting first q moving average coefficient matrices to estimate $B_1, \ldots, B_q$. Durbin (1960) suggests a third step in the procedure described here that increases the asymptotic efficiency of $\tilde{B}_1, \ldots, \tilde{B}_q^T$ when the underlying time series process in linear.

## REFERENCES

Altonji, J. G., "The Intertemporal Substitution Model of Labor Market Fluctuations: An Empirical Analysis," Review of Economic Studies, 49, 783-824, 1982.

Altug, S. "Gestation Lags and the Business Cycle: An Empirical Analysis," manuscript, University of Minnesota, 1985.

Ashenfelter, O. and Card, D., "Time Series Representations of Economic Variables and Alternative Models of the Labor Market," Review of Economic Studies, 49, 761-782, 1982.

Brennan, M. J. and Kraus, A., "Necessary Conditions for Aggregation in Securities Markets," Journal of Financial and Quantitative Analysis, September, 407-418, 1978.

Brown, B. W. and J. Maital, "What Do Economists Know? An Empirical Study of Experts' Expectations," Econometrica 49, 491-504, 1981.

Christiano, L., "Inventory Fluctuations and the Business Cycle," Manuscript, Federal Reserve Bank of Minneapolis, 1986.

Coleman, T., "Essays on Aggregate Labor Market Business Cycle Fluctuations," Unpublished Ph.D. thesis, University of Chicago, 1984.

Cumby, R. E., Huizinga, J., and Obstfeld, M., "Two-Step Two-Stage Least Squares Estimation in Models with Rational Expectations," Journal of Econometrics, April, 1983.

Dunn, K., and K. J. Singleton, "Modeling the Term Structure of Interest Rates Under Nonseparable Utility and Durability of Goods," Journal of Financial Economics, Sept. 1986.

Durbin, J., "The Fitting of Time Series Models," Review of the International Statistical Institute 28, 233-244, 1960.

Eichenbaum, M. S., Hansen, L. P., and S. F. Richard, "The Dynamic Equilibrium Pricing of Durable Consumption Goods," manuscript, Carnegie-Mellon University, 1985.

Eichenbaum, M. S., and L. P. Hansen, "Estimating Models with Intertemporal Substitution Using Aggregate Time Series Data," manuscript, Carnegie-Mellon University, 1985.

Gallant, A. R., and D. W. Jorgenson, "Statistical Inference for a System of Simultaneous, Nonlinear, Implicit Equations in the Context of Instrumental Variable Estimation," Journal of Econometrics 11, 275-302, 1979.

Hall, R. E., "Labor Supply and Aggregate Fluctuations," in K. Brunner and A. Meltzer eds., On the State of Macroeconomics, Carnegie-Rochester Conference on Public Policy, Vol. 12, North Holland, Amsterdam, 1980.

Hansen, L. P., "Large Sample Properties of Generalized Method of Moment Estimators," Econometrica 50, 1029-1054, 1982.

Hansen, L. P. and R. J. Hodrick, "Forward Exchange Rates as Optimal Predictors of Future Spot Rates: An Econometric Analysis," Journal of Political Economy, 88, 829-53, 1980.

Hansen, L. P. and R. H. Hodrick, "Risk Averse Speculation in the Forward Foreign Exchange Market: An Econometric Analysis of Linear Models," in Exchange Rates and International Macroeconomics, edited by J. A. Frenkel, Chicago: University of Chicago Press for the National Bureau of Economic Research, 1983.

Hansen, L. P. and K. J. Singleton, "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models," Econometrica 50, 1269-1286, 1982.

Hotz, V. J., G. Sedlacek and F. E. Kydland, "Intertemporal Preferences and Labor Supply," NORC Working Paper No. 85-18, 1985.

Ibbotson, R. G. and R. A. Sinquefield, <u>Stocks, Bonds, Bills and Inflation:</u> <u>Historical Returns, 1926-1978</u>, 2d. ed. Charlottesville, VA: Financial Analysts Res. Found. 1979.

Kennan, J., "An Econometric Analysis of Equilibrium Labor Market Fluctuations," manuscript, University of Iowa, 1985.

Kydland, F. E., "Nonseparable Utility and Labor Supply," Working Paper No. E-83-16, The Hoover Institution, Stanford University, August, 1983.

Kydland, F. E. and E. C. Prescott, "Time to Build and Aggregate Fluctuations," <u>Econometrica</u>, 50, 1345-1370, 1982.

Lucas, R. E. Jr. and L. A. Rapping, "Real Wages, Employment, and Inflation," <u>Journal of Political Economy</u> 77, 721-754, 1969.

Mankiw, N. G., J. Rotemberg and L. H. Summers, "Intertemporal Substitution in Macroeconomics," <u>Quarterly Journal of Economics</u>, Vol. C, 225-252, February, 1985.

Muellbauer, J., "Linear Aggregation and Neoclassical Labor Supply," <u>Review of Economic Studies</u>, Volume XLVIII: 21-36, January, 1981.

Pollak, R. A., "Habit Formation and Dynamic Demand Functions," <u>Journal of Political Economy</u> 78, 60-78, 1970.

Rubinstein, M., "An Aggregation Theorem for Securities Markets," <u>Journal of Financial Economies</u> 1, 225-244, 1974.

Satinger, M., "Comparative Advantage in Individuals," <u>Review of Economics and Statistics</u>, 60: 259-267, 1978.

Seater, J. J., "On the Construction of Marginal Federal Personal and Social Security Tax Rates in the U.S.," Journal of Monetary Economics, Vol. 15, 121-136, 1985.

Singleton, K. J., "Specification and Estimation of Intertemporal Asset Pricing Models," forthcoming, Handbook of Monetary Economics, eds. B. Friedman and F. Hahn, North Holland, 1986.