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ABSTRACT

Many monetary and fiscal policy decision makers and economists hold the view that exchange rates are volatile even though nominal exchange rates vary less than many other financial market prices and yields. This paper seeks an explanation for this puzzle by contrasting exchange rate dynamics in a general equilibrium model to those presented in Dornbusch (1976) and Kouri (1978). Kouri introduced the "acceleration hypothesis", according to which the rate of currency depreciation is given by the ratio of the current account deficit to the sum of holdings of foreign assets by domestic agents and holdings of domestic assets by foreign agents. In this paper, we derive the "generalized acceleration hypothesis", assuming price flexibility but imperfect substitutability of assets. A Kouri type gradual adjustment of the current account induces stickiness in portfolio adjustments and exchange rate adjustment. Uncertainty in the model arises from monetary policy and supply side shocks. Due to general equilibrium constraints on wealth and investment behavior, the speed of adjustment is defined by the sum of speculative (expectations sensitive) demand for foreign (domestic) assets by domestic (foreign) agents, deducted by the stock of domestic assets traded out by domestic residents. The adjustment speed is then higher and the market correction mechanism through the current account stronger. The model developed in this paper includes the three key channels of external adjustment of an economy: the capital account or portfolio allocation channel as applied by Kouri (and also by Dornbusch, although under perfect substitutability of assets), the current account channel as applied by Kouri and the asset valuation channel as applied in Gourinchas & Rey (2007). In a linearized testing environment, we study three different cases of exchange rate dynamics. Sampling 10 000 continuous time paths of Monte Carlo simulations for 30 years, and using the 90% variation range as the metric, the Dornbusch formulation yields a 200% variation range about the mean, reduced to 100% in the Kouri case and to 20% in the general equilibrium case.

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1 Introduction

A common paradigm among many monetary and fiscal policy decision makers and economists has been that exchange rates tend to be rather volatile. This paradigm has its roots in open economy macroeconomics. In the mainstream literature the volatility of real exchange rates is often benchmarked against the volatility of inflation indices. The latter volatilities are typically found very low whereby real exchange rate variations appear relatively very high.

On the other hand if nominal exchange rate variations are benchmarked against variations of financial market prices and yields, the former appear to be relatively very moderate if not even low. When estimating standard deviations of yields on currency holdings of a number of currency pairs we find that both in several years long periods before and after the introduction of the euro currency average standard deviations were all within the range of 9-12 per cent in annual terms. Turbulent high volatility periods were clearly observable in the data but those appeared to be smoothed out by market correction mechanism rather fast. The introduction of the euro does not make any practical difference in the results. In financial market terms such standard deviations (or in market jargon volatilities) are rather low to the extent that only money market and up to medium term fixed income investments have significantly lower volatilities. In many countries e.g. household real estate and stock investments are considerably more risky than open currency exposures. Yet hedging possibilities of currency exposures are typically much better than those of real estate (and most other) investments.

High volatilities of exchange rates has also been one of the key arguments in favor of the European financial integration and in establishing the eurozone. However, if volatilities of exchange rates are moderate or low and the respective exposures can be hedged by private sector entities if desired, the weight of such an argument is correspondingly diminished¹.

In the literature the still dominating model of exchange rate behavior is the one presented by Rudiger Dornbusch in his seminal (1976) contribution. Currently the Dornbusch model is more broadly seen as an extension of the Mundell-Fleming model.

¹This has been noted since Frenkel and Mussa (1980) and Bergstrand (1983), both quoted in Obstfeld (1985), who also drew our attention to Sercu and Upall (2006).

The fundamental intuition in the Dornbusch partial equilibrium model is that only capital account adjustments matter in explaining exchange rate variations. This result relies on assumptions of the quantity equation and gradually adjusting price levels, and on perfect substitutability of domestic and foreign assets. With no correction mechanism possibly introduced through current account adjustments the Dornbusch model predicts a strong possibility of exchange rate overshooting and rather high volatilities of exchange rates at least in speculation driven market conditions.

As a Ph.D. student of Dornbusch's, Pentti Kouri in his July 1975 dissertation introduced the view of the exchange rate as a variable determined by the capital account of the balance of payments in the short run and by the current account in the long run². In 1978 he wrote "The balance of payments and the foreign exchange market: a dynamic partial equilibrium model", where he introduced the "acceleration hypothesis", arguing that, in the absence of central bank intervention, international capital flows must be financed by appropriate current account flows. This implies that the exchange rate must adjust so that in the equilibrium both capital flows and current account flows must be equal³. Furthermore, it implies that, without introducing any external price rigidity, but due to imperfect substitutability of assets and the flow nature of components of the current account, exchange rate adjustment is gradual.

In the Kouri model current account adjustments smooth exchange rate variations relative to the Dornbusch model. However, in some of Kouri's own demonstrating calculations and some empirical work the smoothing effect is so weak that it may well take 30 years before the new stationary state is achieved. This implies that the Kouri quantitative implications are not much different from those of Dornbusch.

²The first essay, presented at the Stockholm conference on Flexible Exchange Rates and Stabilization Policy in August 1975, became Kouri (1976a), one of his most influential papers see de Macedo and Lempinen (2011, p. 15, 32, 45). The fourth essay, which had been presented at the 1974 Wingspread conference became Kouri (1976c) and was published in a volume edited by Bob Aliber. It is relevant for our work because it uses stochastic calculus

³Kouri (1978) is a revision of Kouri(1976b), see de Macedo and Lempinen (2011, p. 29, 275). Obstfeld (2004) states that the Gourinchas and Rey (2007) "are also consistent with models of home currency portfolio bias, as embodied in the portfolio balance models of the 1970s developed by Branson, Henderson, Kouri and others-all inspired by Tobin's seminal 1969 general equilibrium model of monetary policy" and notes that "Economists devoted considerable effort to the microfoundations of the portfolio balance model, but they moved on to other more tractable issues after the mid-1980s, with the 1985 Handbook survey of Branson and Henderson (1985) a high-water mark (and to a large extent a terminus) of that earlier research effort".

In celebrating the 25th anniversary of the Dornbusch paper, Rogoff (2002, p.18) mentions that empirically “current account dynamics can have large medium term impacts on real exchange rates” and that this “is perhaps no less important than the connection between monetary expansion and real exchange rates highlighted by the Dornbusch model”. He underlines this with graphs showing the correlations in Thailand, Korea, Indonesia and Mexico in the 1990s as well as the United States, Japan and the United Kingdom. Except for this last case, he believes that “the wealth channel was quite important” and integrates the two models, with the Kouri (1975) mechanism being “central to the long run change in the real exchange rate” (p.19)⁴.

Blanchard, Giavazzi and Sa (2005) build on “an old (largely and unjustly forgotten) set of papers” who “relax the interest parity condition and assume instead imperfect substitutability of domestic and foreign assets”. They add in footnote 1 that there were two fundamental papers written in 1976, one by Dornbusch, who explored the implications of perfect substitutability, the other by Kouri, who explored the implications of imperfect substitutability. “The Dornbusch approach, and its powerful implications, has dominated research since then. But imperfect substitutability seems central to the issues we face today”⁵.

In his survey of current account imbalances, Obstfeld (2012) warns against a complete markets or “consenting adults” view of the world: “current account imbalances, while very possibly warranted by fundamentals and welcome, can also signal elevated macroeconomic and financial stresses, as was arguably the case in the mid-2000”. He adds that valuation changes in net international investment positions, “while possibly important in risk allocation, cannot be relied upon systematically to offset the changes in national wealth implied by the current account”⁶.

⁴Other parts are quoted in de Macedo and Lempinen (2011, p. 16-17), including the “unified model” in Obstfeld and Rogoff (1995).

⁵As stated in Rey (2005), the portfolio balance model in Blanchard et al. (2005) draws “on the work of Pentti Kouri, Stanley Black, Dale Henderson and Kenneth Rogoff, and William Branson in the 1980s to model jointly the dynamics of the current account and of the exchange rate, allowing for imperfect substitutability between assets and for (some) valuation effects”. This follows the line quoted in note 2 above and de Macedo and Lempinen (2011, p. 15-21).

⁶Obstfeld (2012) continues: “Thus, Germany itself experienced neither a current account deficit nor a housing boom in the 2000s, yet flows from German banks to economies that did display those symptoms led to problems later on. Unfortunately, the ways in which gross financial positions propagated the recent global crisis across borders became obvious only after the fact”.He also

At the other extreme of the quantitative implications of Dornbusch and Kouri is the famous Lucas (1982) complete markets model in which flexible prices, identical agents and instantaneously adjusting stocks are assumed and in which the exchange rate does not vary and can be normalized to unity ⁷.

In this paper we first develop a general equilibrium two-country model where we let representative agents, monetary policies, endowments and productivity processes differ. Following Dornbusch, domestic price levels and inflation rates are determined by quantity equations in the two countries. No price rigidities are introduced in the model in any other way than the ones potentially caused by assumptions of flow nature of the components of the current account and the imperfect substitutability of domestic and foreign assets, as in Kouri. The set-up allows us to analyze the impact of a variety of factors on exchange rate adjustment. These factors include consumption and investment behavior, saving and spending patterns, monetary policies, productivity processes and relative size of the two countries. Sections 2 through 6 build the general equilibrium model from the partial equilibrium one and compare key results with those of the Kouri partial equilibrium framework. In the latter the rate of change of the exchange rate is proportional to the ratio of current account deficit to the sum of holdings of foreign assets by domestic residents and holdings of domestic assets by foreign residents.

According to the “generalized acceleration hypothesis”, the acceleration coefficient is smaller and adjustment speed therefore higher than in the small open economy partial equilibrium framework. Specifically, speed of adjustment is not defined by the sum of total holdings of foreign assets by domestic residents and total holdings of domestic assets by foreign residents, but by the sum of the speculative (expectations sensitive) holdings of foreign (domestic) assets by domestic (foreign) residents deducted by the stock of domestic assets traded out by domestic residents in exchange for the corresponding stock of foreign assets. In the model developed in this paper, the external adjustment of an economy and the exchange rate is then determined through three channels: the capital account channel (portfolio allocation adjustments as in Kouri),

quotes in that connection Gourinchas and Rey (2007) and Devereux and Sutherland (2010). On the center/periphery schism in the Euro Zone, see de Macedo and Lempinen (2012).

⁷The influence of Kouri in Lucas (1978) is mentioned in Rossi and Sajari (2011, .p. 42): “It turned out to be one of my most popular papers. I wrote to Pentti and asked if he wanted to be the co-author of the paper, but he didn’t seem that interested. A generous guy, I guess.”

current account adjustments as in Kouri and Gourinchas & Rey (2007), and through valuation adjustments as in Gourinchas & Rey.

As it turns out, general equilibrium constraints on wealth and investment behavior reflect to a substantial degree the stylized facts of liquid foreign exchange markets reported above. These are demonstrated by Monte Carlo simulations reported and discussed in Section 7. A strong correction mechanism of the exchange rates through the current account implies that market disturbances originating in underlying shocks or in capital flows may be smoothed out faster than thought possible. This is contrary to the made up image of foreign exchange market volatility among practitioners ⁸.

2 The Kouri Partial Equilibrium Model and Key Results

Kouri (1978) specifies a two country model for demand and supply of foreign exchange under certainty and under conditions in which the central banks of the two countries do not intervene in the foreign exchange market. Consequently the supply of foreign exchange in the market must be financed by changes in holdings of foreign assets or by imbalances in the current account. The short term equilibrium in the Kouri model (eq. (14) in Kouri (1978); using the same notation as in the model to be introduced in the following section 3 and leaving out autonomous factors driving asset and goods demand for simplicity) is defined by the following condition:

$$(1) f(r_h, r_r + \mu)W/s - g(r_h - \mu, r_r)W^* = NFA^d = NFA^s = F_0 - G_0/s$$

where

r_h =nominal rate of return on domestic assets in domestic currency

r_r = nominal rate of return on foreign assets in foreign currency

μ = expected rate of change in the domestic currency price of foreign currency

W =domestic marketable wealth in domestic currency

W^* =foreign marketable wealth in foreign currency

s =domestic currency price of foreign exchange

⁸The discussion of Michael Mussa's paper by Jacob Frenkel, Dornbusch and Kouri at the 1983 Bellagio conference is summarized in de Macedo and Lempinen (2011, p. 17), together with a recent endorsement of "Rudi Dornbusch's classic overshooting analysis" model by Paul Krugman (ibid. note 1).

$f(g)$ =domestic (foreign) demand functions for foreign (domestic) assets

NFA =net foreign assets (supply and demand)

$F_0(G_0)$ are initial domestic (foreign) holdings of foreign exchange

The equilibrium condition simply states that in the absence of central bank intervention the difference between demand for foreign assets by domestic agents and demand for domestic assets by foreign agents must equal the difference of respective initial stocks of assets, all denominated in foreign currency.

Eq. (1) only defines equilibrium in a steady state. Over time foreign currency values of assets, due to capital gains and losses, will change when the exchange rate changes. Simultaneously the current account will adjust. Kouri postulates in eq. (16) the following dynamical balance of payments equilibrium condition (again leaving out autonomous determinants of the current account):

$$(2) CF \equiv (gW^* + fW/s) \frac{ds}{dt} \frac{1}{s} = -B(s)$$

where

CF = net outflow of capital

B = current account surplus

Kouri solves the equation for the rate of change of the exchange rate, obtaining the famous "acceleration equation" (eq. (17)):

(3) $\frac{ds}{dt} \frac{1}{s} = -B(s; x)/(f\frac{W}{s} + gW^*) \equiv -B(s; x)/A_K$; A_K is the Kouri acceleration coefficient

Eq. (3) states that the rate of depreciation (appreciation) of the exchange rate is equal to the ratio of the current account deficit (surplus) to the sum of domestic holdings of foreign assets and foreign holdings of domestic assets. Note that Kouri abstracts away from any autonomous factors potentially driving the exchange rate, such as differences in monetary policies and growth rates. He also assumes wealth levels W and W^* insensitive to potential changes in exchange rates, given portfolio allocations. The two factors together imply that current account imbalances simply drive portfolio re-allocations based on exchange rate expectations in the model.

Note also that stickiness in exchange rate adjustment in eq. (3) is due to the fact that the current account surplus is specified as a flow, which can most easily be seen in eq. (2). This is a very natural specification as it simply states that material transfers of goods and services between countries take time and that e.g. 5 % of GDP cannot be transferred across the border in a split second. In the Kouri portfolio

balance approach the core of gradual adjustments is in the flow specification of the current account surplus, while at the same time portfolio shares f and g might jump instantaneously to new optimal levels.

As mentioned, in a number of tables Kouri presents calculations about adjustment speeds and adjustment times, given different parameter values in equation (3) and its linearized version assuming that foreign agents do not hold domestic assets. Calculations suggest that for many parameter values rather long adjustment periods are to be expected, before the exchange rate converges within a set distance from the equilibrium value⁹.

Kouri also analyses the model under two alternative assumptions on exchange rate expectations - static expectations and rational expectations. He shows that given a shock in the market the initial jump in the exchange rate and its adjustment speed is smaller (larger) under static than under rational expectations. Furthermore, the equilibrium under static expectations is stable while it is of the saddle path type under rational expectations.

3 A Simple General Equilibrium Model

Kouri analysed consequences of shocks and decision making without giving a specific structure to the nature and source of uncertainty, to investment, consumption and saving decisions, monetary policies in the two countries, or to the relative sizes of the two economies. In his model analysis of autonomous factors in exchange rate determination, such as inflation and growth, was not natural or easy. In the following general equilibrium model framework, these drivers of analysis are specified in more detail, which is expected to lead to both new insights and more specific results than those obtained in the Kouri partial equilibrium framework.

While the model structure is on a general level not much different from Lucas (1982) complete markets model, we deliberately let representative agents, monetary policies, endowments, productivity processes differ, without introducing price rigidities in the model in any other way than through the gradual adjustment of portfolios

⁹In some empirical analyses, quoted in de Macedo and Lempinen (2011, p. 22 note 10) adjustment periods as long as 15-40 years appear possible.

and trade flows as specified by Kouri¹⁰. But unlike in the Kouri partial equilibrium model, the monetarist model background implies that domestic and foreign price levels and inflation rates are determined by the quantity equation.

The home country model and its specifications are stated and derived in detail, while the model of the foreign country may differ in parameter values but is qualitatively similar, and can hence be reviewed quickly. In the home country, the representative agent consumes home and foreign goods and invests in domestic and foreign assets. He makes decisions by maximising the expected utility of consumption and portfolio decisions. For convenience, just one good (or index of goods) is assumed to be produced in each country, and one (dominating) risky asset specifies investment opportunities in each country. A continuous time formulation is assumed throughout the analysis. By choice of preferences and stochastic process type, consumption and portfolio decisions separate and are stationary, unless there are changes in relevant parameter values.

The general decision making problem of the home country representative agent is the following:

$$(4) \text{Max} E_0 \int_{t=0}^{\infty} [e^{-v\tau} u(c_h(\tau), c_r(\tau)) dt] = E_0 J(W(t), t)$$

$$\{c_h, c_r, \beta\}$$

$$s.t. dW = \beta d(QK) + (1 - \beta) d(sQ^* K^*) - pc_h - sp^* c_r$$

where

$u(c_h, c_r)$ = instantaneous utility function

v = rate of time preference

c_h = consumption of domestic goods

¹⁰In connection with this model, Obstfeld (2012) writes: “Complete market models with investment can result in current account deficits or surpluses, but they are likely to be smaller than in the incomplete market case, sometimes much smaller. For example, Coeurdacier, Kollmann, and Martin (2010) analyze a model with equity claims and real consols, an asset structure sufficient to reproduce complete markets up to a first-order approximation in a model with two shocks, shocks to productivity and to investment efficiency. In that model (again up to a first-order approximation), optimal portfolio behavior implies a zero current account as bond flows always offset equity flows”. He continues: “For the rich industrial countries, much of the expansion of gross external asset and liability stocks has necessarily taken the form of debt instruments. There is considerable trade in equity too, as in the Lucas (1982) model, but the fact of home bias in equity ownership remains (although it is declining over time). Furthermore there is only so much real capital to underlie equity claims. The extreme ratios of external asset stocks to GDP that some countries display are not feasible except on the basis of extensive two way trade in debt or debt-like instruments, including derivatives”.

c_r =consumption of foreign goods

p =price of domestic goods, in home currency

p^* =price of foreign goods, in foreign currency

s =exchange rate (in €/\\$)

$W = \beta QK + (1 - \beta)sQ^*K^*$ =domestic wealth (€)

β = wealth share of domestic assets

$J(W(t), t)$ =Value function

Let us assume that the instantaneous utility function is of the following type:

$u(c_h, c_r) = \alpha \log c_h + (1 - \alpha) \log c_r$; α is the expenditure share of domestic goods.

The problem then yields the following optimal consumption and investment rules:

$$(5a) c_h = \frac{\alpha v W}{p}$$

$$(5b) c_r = \frac{(1-\alpha)vW}{sp^*}$$

$$(5c) \beta = \frac{\sigma_r^2 - \sigma_{hr}}{S^2} - \frac{J_W}{J_{ww}W} \frac{(r_h - r_r)}{S^2} \equiv b_m + b_s(r_h - r_r)$$

where

σ_r =standard deviation of return on the foreign asset

σ_h =standard deviation of return on the domestic asset

σ_{hr} =covariance of domestic and foreign assets

S^2 =portfolio variance term

$-\frac{J_w}{J_{ww}W}$ =the inverse of the coefficient of relative risk aversion

r_h, r_r =expected rate of return on domestic and foreign assets

b_m =minimum variance portfolio share

b_s =speculative portfolio share

In (5c) the separability of consumption and investment decisions under isoelastic preferences has been invoked in that the inverse of the coefficient of relative risk aversion has been used in b_s as the weight of the speculative portfolio share. In the limiting case of logarithmic preferences, of course this weight would be one.

Note that in this economy no stock of money is held as an asset by agents as money only has the roles of unit of account and means of exchange. This is based on the institutional structure such that the central bank is assumed to operate an electronic clearinghouse through which all payments in the economy are settled. The clearinghouse awards settlement limits to agents accepting claims on wealth as collateral for settlements. Monetary policy then consists of decisions of the central bank on how

large settlement limits it will award to agents for clearing of payments¹¹.

The shock structure in home country includes disturbances arising from production process and from monetary policy. The central bank controls money supply, and the quantity equation determines the domestic price level. Formally,

$$(6a) P(t) = \frac{M(t)}{y(t)}, \text{ and}$$

$$(6b) \frac{dP}{P} = \frac{dM}{M} - \frac{dy}{y} - \frac{dM}{M} \frac{dy}{y} + \frac{dy^2}{y^2}$$

where

$M(t)$ = (stochastic) money supply

$y(t)$ = (stochastic) production

$P(t)$ = (stochastic) price level

Production flow is assumed to be proportional to the resource endowment K , which is assumed to be constant. The rate of productivity of the endowment grows at a stochastic rate. Formally,

$$(7) y = K \exp(rdt + \sigma_h dz_h)$$

where

r = the drift of the productivity process

dz_h = normally distributed domestic productivity shocks with mean 0 and standard deviation 1

The firms operating the technology receive nominal revenues at price $p(t)$ for flow of production $y(t)$. For convenience the revenues are assumed to be net of any costs. The value of shares of the firms is the discounted present value of the revenue flows from the present time to infinity. Formally, and assuming the discount rate to be equal to the rate of time preference ν the price of the share $Q(t)$ is

$$Q(t) = \int_t^\infty [e^{-\nu\tau} P(\tau)y(\tau)] dt \equiv qP(t)y(t)$$

Differentiating the price $Q(t)$ the rate of return on shares follows the following stochastic process

$$(8) \frac{dQ}{Q} = \frac{dM}{M} - \frac{dy}{y} - \frac{1}{2} \frac{dy}{y} \frac{dM}{M} + \frac{1}{2} \left(\frac{dy}{y}\right)^2 + \frac{dy}{y} = \frac{dM}{M} - \frac{1}{2} \frac{dy}{y} \frac{dM}{M} + \frac{1}{2} \left(\frac{dy}{y}\right)^2$$

It is assumed that the central bank has an inflation target in executing its monetary policy. Specifically, it is assumed that the central bank calibrates the policy so that the expected inflation rate is π . However, it is also assumed that the central bank is only partially able to control the impact on inflation rate of productivity shocks.

¹¹ See the discussion in Woodford (2000).

Let the proportional control factor be γ . With these assumptions the inflation rate in (6b) and rate of return on domestic shares in (8) can be expressed in the following form:

$$(9a) \frac{dP}{P} = \pi dt - (1 - \gamma)\sigma_h dz_h$$

$$(9b) \frac{dQ}{Q} = (r + \pi + \rho)dt + \gamma\sigma_h dz_h \equiv r_h dt + \gamma\sigma_h dz_h$$

where $\rho = \frac{1}{2}\left(\frac{dy}{y} - \frac{dM}{M}\right)\frac{dy}{y}$

The stochastic processes of inflation and rate of return on domestic assets contain in this specification both monetary policy and productivity (supply) shocks.

In the foreign country the structure of the economy is modeled in a similar way, only allowing for differences in parameters determining demand and supply behavior and monetary policy. In most of the notation to follow, foreign country parameter values are referred to by a superscript *, while in some separately defined cases more specific notation is used. The general decision making problem of the foreign representative agent is (in \$):

$$(10) \text{Max} E_0 \int_{t=0}^{\infty} [e^{-v^* \tau} u^*(c_h^*(t), c_r^*(t)) dt] = E_0 J^*(W^*(t), t)$$

$$\{c_h^*, c_r^*, \beta^*\}$$

$$s.t. dW^* = \beta^* d(Q^* K^*) + (1 - \beta^*) d\left(\frac{QK}{s}\right) - \frac{p c_h^*}{s} - p^* c_r^*$$

where

$u^*(c_h^*, c_r^*)$ =instantaneous utility function

v^* =rate of time preference

c_h^* =consumption of domestic goods by foreign residents

c_r^* =consumption of foreign goods by foreign residents

p =price of domestic goods, in home currency

p^* =price of foreign goods, in foreign currency

s =exchange rate (in €/ \$)

$W^* = \beta^* Q^* K^* + (1 - \beta^*) \frac{QK}{s}$ =foreign wealth (in \$)

β^* = wealth share of foreign assets of foreign residents

$J^*(W^*(t), t)$ =Value function

Assuming again an instantaneous utility function of the type

$u(c_r^*, c_h^*) = \alpha^* \log c_r^* + (1 - \alpha^*) \log c_h^*$; α^* is the expenditure share of foreign goods by foreign residents.

Assuming again separable preferences and general isoelastic preferences in risk behavior, the problem then yields the following optimal consumption and investment

rules:

$$\begin{aligned}
(11a) c_r^* &= \frac{\alpha^* v^* W^*}{p^*} \\
(11b) c_h^* &= \frac{(1-\alpha^*) v^* W^* s}{p} \\
(11c) \beta^* &= \frac{\sigma_h^2 - \sigma_{hr}}{S^2} - \frac{J_W^*}{J_{ww}^* W^*} \frac{(r_r - r_h)}{S^2} \equiv b_m^* + b_s^* (r_r - r_h)
\end{aligned}$$

4 General Equilibrium Considerations

The general equilibrium framework imposes certain structure between some key variables of the model. Following the choice of Kouri, let us first consider the determination and distribution of wealth, when both domestic and foreign wealth are denoted in dollars. In capital market equilibrium, the following conditions must hold:

$$\begin{aligned}
(12a) \beta^* W^* + (1 - \beta) \frac{W}{s} &= Q^* K^* \\
(12b) \beta \frac{W}{s} + (1 - \beta^*) W^* &= \frac{QK}{s}
\end{aligned}$$

From these conditions solutions in terms of values of endowments for domestic and foreign wealth in \$ can be obtained as

$$\begin{aligned}
(13a) \frac{W}{s} &= \frac{1}{(\beta + \beta^* - 1)} [\beta^* \frac{QK}{s} - (1 - \beta^*) Q^* K^*] = \frac{1}{(\beta + \beta^* - 1)} (\beta^* W_G - Q^* K^*) = \\
&= \frac{1}{(\beta + \beta^* - 1)} [(\beta^* - (1 - \omega)) W_G] \\
(13b) W^* &= \frac{1}{(\beta + \beta^* - 1)} [\beta Q^* K^* - (1 - \beta) \frac{QK}{s}] = \frac{1}{(\beta + \beta^* - 1)} (\beta W_G - \frac{QK}{s}) = \\
&= \frac{1}{(\beta + \beta^* - 1)} [(\beta - \omega) W_G]
\end{aligned}$$

where $W_G = \frac{QK}{s} + Q^* K^* =$ global wealth in \$

and $\omega = \frac{QK}{W_G} =$ share of the value of domestic endowment of global wealth

Domestic (foreign) wealth is then the share of foreign (domestic) agents of global wealth, deducted by the value of foreign (domestic) endowment, scaled to units of wealth by a factor depending on portfolio shares. Intuitively this means that e.g. the domestic wealth is a multiple of the difference between the portfolio share of global wealth of foreign residents and the value of foreign endowment, after exchange of assets has taken place in the market. It is easy to verify that

$$\frac{W}{s} + W^* = \frac{QK}{s} + Q^* K^*$$

which reflects the assumption that there is no stock demand for money by agents.

In addition to requiring that the global wealth identity holds, it is natural to require in this type of model that national wealths must be positive. This requirement arises from the fact that e.g. consumption is positive in this model only, if the national

wealth of the respective country is positive.

Using Eq. (13a) and (13b) the positivity constraints can be written in the following form:

$$(14a) \frac{(\beta^* - (1 - \omega))}{(\beta + \beta^* - 1)} > 0$$

$$(14b) \frac{(\beta - \omega)}{(\beta + \beta^* - 1)} > 0$$

From Eq. (14a) and (14b) it can be seen that the numerators of the both conditions are positive as long as investment shares in domestic assets in the both countries are bigger than the shares of the values of endowments of global wealth. In order for the positivity requirement to hold, at the same time the common denominator should be positive, which requires that the sum $(\beta + \beta^*)$ of investment shares must be bigger than one. This would typically be quite natural, but not necessary, as investment shares are determined by the preferences of the representative investor. An interesting polar case often used in partial equilibrium open economy analyses is the one in which foreign agents do not hold domestic assets at all, i.e. $\beta^* = 1$. In such a case in Eq. (14b) the denominator is positive. In order for the positivity condition to be fulfilled in this case $\beta > \omega$ must hold, i.e. the willingness of domestic agents to hold domestic assets must exceed the share of the value of domestic endowment out of global wealth. As all the foreign assets are held by the foreign investors this is not possible.

Given the formulation of the model in terms of underlying stochastic processes it is to be expected that the solution for the exchange rate will also be a stochastic process, parameters of which will be determined in the general equilibrium. The solution will not be one with a fixed steady state towards which the exchange rate is gradually adjusting, but a stochastic process with a finite non-negative mean path. Under certain conditions certainty equivalence states for the process can be defined.

5 Derivation of the Stochastic Process of Exchange Rate in General Equilibrium

In the current general equilibrium model framework, the immediate counterpart of the Kouri acceleration equation can be written in a straight-forward way. The holdings in dollars of foreign (domestic) assets by domestic (foreign) agents are the following

$$(15a) (1 - \beta) \frac{W}{s} = \text{stock of foreign assets held by domestic agents} = F = f \frac{W}{s} \text{ in}$$

Kouri notation

$$(15b)(1 - \beta^*)W^* = \text{stock of domestic assets held by foreign agents} = \frac{G}{s} = gW^* \text{ in}$$

Kouri notation

Similarly, current account surplus, denoted by B , in the general equilibrium specification is the following

$$(16) B = c_h^* - \frac{c_r}{s} = \frac{(1-\alpha^*)v^*W^*}{p} - \frac{(1-\alpha)vW}{sp_r}$$

Applying (15 a and b) and (16) in the Kouri equation (3), we obtain the equation in the general equilibrium model corresponding directly with the Kouri equation

$$(17) \frac{ds}{s} = \frac{\frac{(1-\alpha)vW}{sp_r} - \frac{(1-\alpha^*)v^*W^*}{p}}{(1-\beta)\frac{W}{s} + (1-\beta^*)W^*}$$

According to (17) the euro rate of depreciation is determined precisely in the same way as in the Kouri partial equilibrium model. The acceleration coefficient is the sum of foreign assets held by domestic agents and domestic assets held by foreign agents. Two questions have to be raised regarding the solution. First, even assuming neutral foreign exchange policy of the central banks, whenever the exchange rate is such that B is not equal to zero, assets must move between countries. This implies that portfolio holdings of domestic and foreign agents must change, most likely both through deliberate allocation changes and through capital gains and losses. However, the latter is not allowed in the derivation of the Kouri acceleration equation. Secondly, it is clear from investment rules (5c) and (11c) that only a fraction of portfolio holdings are sensitive to changes in expected returns of assets, while - depending on investor risk tolerance - the rest of the asset demand is based on variance minimization.

In order to find answers to the above questions, let us move next to derivation of the stochastic process of the exchange rate in the general equilibrium model. Maintaining the denomination of all variables in dollars and assuming absence of central bank intervention, the net outflow of capital from the home country must be funded by the flow of current account deficits. In formal terms this condition can be written in the following form

$$(18) d[(1 - \beta)\frac{W}{s}] - d[(1 - \beta^*)W^*] = -Bdt = (\frac{c_r}{s} - c_h^*)dt$$

The left hand side of equation (18) is the difference of the (stochastic) differential of foreign assets held by domestic agents and differential of domestic assets held by foreign agents. The left hand side then is likely to include value changes due to autonomous factors (monetary policies and productivities), due to capital gains

and losses induced by exchange rate changes, and due to changes in portfolio allocations potentially induced by expectations of exchange rate changes. After performing appropriate differentiations in (18), we derive the general equilibrium stochastic differential equation for the exchange rate, which determines exchange rate adjustments such that the economy is in general equilibrium.

In order to take properly into account the impact of exchange rate expectations on portfolio decisions of domestic and foreign agents, we have to invoke the notions of minimum variance and speculative portfolio shares introduced in equations (5c) and (11c). From the general portfolio theory it is known that

$$\sum_{i=1}^n b_{mi} = 1; \text{ and } \sum_{i=1}^n b_{si}r_i = 0 \text{ for } i = 1, \dots, n \text{ assets.}$$

Intuitively this means that in a two-moment optimization model the investor always invests all of his capital on diversification (variance minimization) basis in the absence of risk-free assets. In the context of currency positions this reason for holding assets can be called the safe haven demand, and it is completely immune to changes in yields of alternatives. However, the investor takes also speculative positions in search for yield in various assets, but long speculative positions are always financed by short speculative positions. Hence the sum of speculative portfolio shares is 0. Let us call speculative portfolio holdings as yield enhancement demand.

In this simple two risky assets model the speculative portfolio shares of the domestic and foreign agents have opposite signs, as $-(r_h - r_r) = (r_r - r_h)$. The difference between these shares comes from differences of investor risk preferences and underlying risks of the two assets. The opposite signs represent the fact that in (18) expectations of depreciation of the home currency will lower the demand for domestic asset holdings by both domestic and foreign investors.

Investor behavior in deriving equations (5c) and (11c) assumed for convenience a flexible exchange rate regime, but with stable and constant expectations of exchange rate change. While solving for the general equilibrium stochastic process of the exchange rate this can no longer be the case. Allowing for a possibility of exchange rate movements and movements in rational expectations about them, the portfolio shares can be written as:

$$(19a)\beta = b_m + b_s[r_h - \mu(t, f) - r_r]$$

$$(19b)\beta^* = b_m^* + b_s^*[r_r - (r_h - \mu(t, f))]$$

where $\frac{ds}{s} = \mu(t, f)dt + \sigma_s(t, f)dz_s$ and $\mu(t, f)dt$ and $\sigma_s(t, f)dz_s$ are mean rate of

depreciation and standard deviation of that rate, both possibly depending on time and other factors f . The term dz_s is an i.i.d. disturbance with mean 0 and standard deviation 1.

Ignoring the time and other factor dependence of the exchange rate process in notation, equation (18) can be written in the following form:

$$(20) \quad -d\beta \frac{W}{s} + (1-\beta)d\left(\frac{W}{s}\right) + d\beta^* W^* - (1-\beta^*)dW^* = \left(\frac{c_r}{s} - c_h^*\right)dt,$$

and after several steps and regrouping of terms (see Appendix 1)

$$\left[b_s \frac{W}{s} + b_s^* W^* - \frac{(1-\beta)}{(\beta+\beta^*-1)} \frac{QK}{s}\right] \mu dt - \frac{(1-\beta)}{(\beta+\beta^*-1)} \frac{QK}{s} \sigma_s dz_s = \frac{(1-\beta^*)}{(\beta+\beta^*-1)} Q^* K^* (r_r dt + \sigma_r dz_r) - \frac{(1-\beta)}{(\beta+\beta^*-1)} \frac{QK}{s} \left[\left(r_h - \frac{1}{2}\sigma_{hs} + \sigma_s^2\right) dt + \sigma_h dz_h \right] + \left(\frac{c_r}{s} - c_h^*\right) dt$$

Using the fact that in order for a stochastic differential equation to have a solution the random terms must be equal on both sides of the equation, we have that

$$(21) \quad \sigma_s dz_s = \sigma_h dz_h - \frac{(1-\beta^*)}{(1-\beta)} \frac{Q^* K^* s}{QK} \sigma_r dz_r$$

Using (21) we are able to solve for the terms σ_{hs} and σ_s^2 , letting ρ_{hr} = correlation coefficient between disturbances dz_h and dz_r :

$$\begin{aligned} \sigma_{hs} &= \sigma_h^2 - \frac{(1-\beta^*)}{(1-\beta)} \frac{Q^* K^* s}{QK} \rho_{hr} \sigma_h \sigma_r \\ \sigma_s^2 &= \sigma_h^2 + \frac{(1-\beta^*)^2}{(1-\beta)^2} \frac{Q^{*2} K^{*2} s^2}{Q^2 K^2} \sigma_r^2 - 2 \frac{(1-\beta^*)}{(1-\beta)} \frac{Q^* K^* s}{QK} \rho_{hr} \sigma_h \sigma_r \end{aligned}$$

Having the disturbance term solved in (21) we move to solve for the mean term μdt in (20). The following solution is obtained:

$$(22) \quad \mu dt = \left[b_s \frac{W}{s} + b_s^* W^* - \frac{(1-\beta)}{(\beta+\beta^*-1)} \frac{QK}{s}\right]^{-1} \left[\frac{(1-\beta^*)}{(\beta+\beta^*-1)} Q^* K^* r_r - \frac{(1-\beta)}{(\beta+\beta^*-1)} \frac{QK}{s} (r_h - \sigma_{hs} + \sigma_s^2) + \left(\frac{c_r}{s} - c_h^*\right) \right] dt$$

It is illustrative at this point to change variables and use the definition of global wealth W_G as introduced above in (13a,b), using also the explicit form of the current account surplus as shown in (16).

Using these specifications the stochastic process of the exchange rate can be written in the following form:

$$(23) \quad \frac{ds}{s} = \left[b_s (\beta^* - (1-\omega)) + b_s^* (\beta - \omega) - (1-\beta) \right]^{-1} \left\{ (1-\beta^*) (1-\omega) r_r - (1-\beta) \omega (r_h - \sigma_{hs} + \sigma_s^2) + \left[\frac{(1-\alpha)v(\beta^* - (1-\omega))}{p_r} - \frac{(1-\alpha^*)v^*(\beta - \omega)}{p} \right] \right\} dt + \sigma_h dz_h - \frac{(1-\beta^*)(1-\omega)}{(1-\beta)\omega} \sigma_r dz_r$$

Equation (23) is the general equilibrium solution for the stochastic process of the exchange rate in the model. The process does not include explicitly notions of wealth or income, and is therefore seemingly stationary. However, as long as there

are differences in the nominal growth rates of the two economies (inflation targets, real growth rates or both) then the parameter $\omega = QK/W_g s$ (representing the share of the value of domestic endowment out of global wealth) becomes dependent on growth rates and time. Hence the process is not necessarily stationary. Considering long term empirical growth patterns, very persistent or eventually permanent material differences in growth and inflation rates between countries are not typical.

The exchange rate process comprizes both autonomous terms and the current account term. The general equilibrium acceleration coefficient is fundamentally different in structure from Kouri's partial equilibrium coefficient. First, while Eq. (22) contains terms $b_s \frac{W}{s} + b_s^* W^*$, which structurally correspond with the Kouri partial equilibrium terms $(1 - \beta) \frac{W}{s} + (1 - \beta^*) W^*$ in Eq. (17), the former coefficients are a subset of the latter and therefore smaller given absence of borrowing possibilities. Secondly, Eq. (22) also includes coefficient $-\frac{(1-\beta)}{(\beta+\beta^*-1)} \frac{QK}{s}$, which does not exist in the Kouri partial equilibrium model at all. This term has a negative sign, and therefore it will make the acceleration coefficient smaller and speed up exchange rate adjustment relative to the partial equilibrium model.

It is useful to observe that the Kouri terms in Eq. (22) imply simultaneous realisation of current account deficits and home currency depreciation, which many economists would find plausible. But they also imply simultaneous occurrence of home currency depreciation and inflationary monetary policy abroad. Furthermore, Kouri was aware that his analysis could not do justice to inflationary or real growth conditions as they would result in home currency appreciation. He indicates that, if the real interest earnings are spent on imports, his analysis applies to the real exchange rate and the real, or inflation adjusted, balance of payments. However, he continues to assume that "there is no inflation or real growth" as a satisfactory treatment thereof requires an analysis of its own ¹².

The additional general equilibrium wealth effect term in Eq. (22) has opposite qualitative implications. It appears to drive the currency weaker when the current account is in surplus, but associate domestic inflationary monetary policy with weakening currency and foreign inflationary policy with strengthening currency.

It is clear from Eq. (21) and (22) that with the Lucas (1982) assumptions of

¹²Section 4.2.5 titled "balance of payments equilibrium with inflation" (de Macedo and Lempinen, 2011, p. 348)

identical agents, endowments and technologies implemented in this model both μdt and $\sigma_s dz_s$ would equal zero, and the current account would always be in equilibrium. The exchange rate would be constant and could be normalized to 1.

According to equation (23) the exchange rate follows a stochastic process, which theoretically never stands still. A notion of a certainty equivalent stationary state can, however, be computed for the process. If assumptions are made that the shocks dz_h and dz_r are temporarily zero, that at the same time the autonomous terms net each other out, and that investment shares $\beta, \beta^*, (1 - \beta)$ and $(1 - \beta^*)$ remain unaffected, the following solution for the certainty equivalent stationary state exchange rate s_{G0} can be computed:

$$(24) s_{G0} = \frac{[(1-\beta)p_r p_r^h - (1-\alpha)v\beta^*p - (1-\alpha^*)v^*(1-\beta)p_r]QK}{[(1-\beta^*)p_r p_r^r - (1-\alpha^*)v^*\beta p_r - (1-\alpha)v(1-\beta^*)p]Q^*K^*}$$

The certainty equivalent stationary state exchange rate is still a simultaneous solution as e.g. portfolio shares, asset prices, wealth and consumption levels depend on productivity growth rates and central bank inflation targets. For a large vs. large economy case (the Eurozone vs. USA) assuming a candidate set of parameter values (altogether 21 germane parameters driving the model) some local comparative static properties of the steady state exchange rate can be determined. It can be illustrated e.g. that the home currency (euro) depreciates (increases) with increases in α , and K and appreciates (decreases) with increases in α^* and K^* . In intuitive terms the home currency is the stronger the more open the home country is in trade, and the smaller it is when measured in terms of the real endowment K . The home currency is also the stronger the more closed the foreign economy is in trade, and the larger it is when measured in terms of the real endowment K^* .

6 Exchange Rate Adjustment and the Current Account: Acceleration in General Equilibrium and Partial Equilibrium

The key reasons for the different implications of the Kouri terms and the additional general equilibrium term is that the Kouri terms represent capital flows effected by actual portfolio restructuring decisions induced by changes in expectations, while the

general equilibrium term represents impact of direct capital gains and losses on the existing portfolio holdings. This fact can be easily seen by observing that it is only the additional general equilibrium term that drives the impact of both anticipated and unanticipated changes in market factors.

In order to compare the general equilibrium and the Kouri partial equilibrium models in a consistent manner and purely in the context of current account adjustments, let us assume that by feasible policy coordination in (23) the autonomous drift terms net each other. Then the process of the exchange rate becomes the following:

$$(25) \frac{ds}{s} = [b_s(\beta^* - (1-\omega)) + b_s^*(\beta - \omega) - (1-\beta)]^{-1} \left[\frac{(1-\alpha)v(\beta^* - (1-\omega))}{p_r} - \frac{(1-\alpha^*)v^*(\beta - \omega)}{p} \right] dt + \sigma_h dz_h - \frac{(1-\beta^*)(1-\omega)}{(1-\beta)\omega} \sigma_r dz_r$$

and exchange rate changes are essentially driven by random shocks. These induce changes in the current account which are then translated into exchange rate adjustments by the acceleration coefficient. The general equilibrium acceleration coefficient A_G , written in terms of domestic, foreign and global wealth, is

$$(26) A_G = [b_s \frac{W}{s} + b_s^* W^* - \frac{(1-\beta)\omega}{(\beta + \beta^* - 1)} W_G]$$

Note first that the general equilibrium acceleration coefficient A_G makes it possible that a J-curve type of adjustment of the exchange rate can result in the model, which is not possible in the partial equilibrium framework. Specifically, if

$$\frac{(1-\beta)\omega}{(\beta + \beta^* - 1)} W_G > b_s \frac{W}{s} + b_s^* W^*$$

then the acceleration coefficient becomes negative and an exchange rate depreciation is expected to coincide with a current account surplus and an appreciation to coincide with a current account deficit, at least in the short term. The negative acceleration coefficient raises some technical issues about stability of the solution. Typically, a negative adjustment coefficient in a feedback mechanism of the type of Eq. (22) implies unstable adjustment paths, which cannot be feasible at least in the long term in an economics application. These issues are discussed in more detail in Appendix 2.

Comparison of the general equilibrium acceleration coefficient A_G with the Kouri partial equilibrium coefficient A_K requires interpretation of the latter in terms of the current model. As indicated in (2) above, Kouri defines cash flow as the capital gain due to exchange rate changes in percentage terms multiplied by the sum of holdings of domestic assets by foreign agents and holdings of foreign assets by domestic agents. This is a very specific way of defining cash outflow, as was discussed in the previous

section.

A natural way to understand the Kouri definition of cash flow is to conjecture that Kouri in fact only considered yield enhancement demand in his cash flow identity. As was discussed above, in a two-asset model speculative portfolio shares of different agents are of opposite sign relative to yield and only differ in magnitude from each other in terms of risk preferences. This interpretation would make Kouri's postulation of Eq. (2) understandable. In empirical applications, in which total asset holdings have been typically used, this re-interpretation would likely lead to a quite substantial reduction in the value of A_K . But Kouri himself appears to have used total asset holdings in his numerical calculations.

Kouri's omission of the wealth effect may be due to the facts that he used a partial equilibrium framework, and that his analysis was performed in a nonstochastic framework, in which unanticipated capital gains and losses on existing portfolio holdings are not a very natural issue to address. But it is clear that in particular, when long term projections of exchange rates are simulated, asset valuation adjustments cannot be ignored.

Following the interpretation discussed above and denoting the Kouri acceleration coefficient in the current model by A_{KG} we have:

$$A_{KG} = b_s \frac{W}{s} + b_s^* W^*, \text{ and that}$$

$$(27) A_{KG} - A_G = \frac{(1-\beta)\omega}{(\beta+\beta^*-1)} W_G > 0$$

Clearly, the Kouri coefficient as interpreted in the general equilibrium framework - applying the separation between safe haven demand and yield enhancement demand - in the general equilibrium form, is (considerably) larger than A_G , the general equilibrium acceleration coefficient. This implies that exchange rate adjustment will be faster and adjustment periods shorter in the general equilibrium model than in the partial equilibrium model. The difference between the coefficients has a natural interpretation. It represents the component of wealth the holding of which is subject to wealth effects due to exchange rate adjustment. This component of wealth is the share of domestic endowment which the domestic agent does not want to hold but trades in exchange for foreign assets to benefit from diversification gains.

The possibility of the J-curve type effect in the model depends crucially on parameter values. Gourinchas & Rey (2007) estimated that for the USA 27% of the

external adjustment is explained by asset valuation effects. The sum of the speculative portfolio holdings $b_s \frac{W}{s} + b_s^* W^*$ in A_{KG} can be rather small in many economies under normal conditions. This may be the case especially due to holdings of US assets by foreign investors, which are likely motivated among many foreign investors by the safe haven demand. In fact Gourinchas & Rey also maintain that altogether capital account transactions are sufficiently small in case of the US market that they can be omitted from the analysis. This view would suggest that for some markets it would be quite natural for the J-curve type conditions to prevail.

It is useful to keep in mind, however, that speculative portfolio holdings are sensitive to the state of the market. Whenever there are larger shifts in the market, expectations of market movements are typically generated. These would instantaneously increase the size of speculative portfolio holdings, which would begin to offset and possibly dominate asset valuation effects. On these grounds it is possible that in some markets under normal market conditions asset valuation effects dominate while during more turbulent periods speculative portfolio holdings effects prevail.

7 Simulation experiments

In this section the relevance and importance of the findings of analysis are investigated by means of a set of Monte Carlo simulation experiments with parameter values and diagrams presented in Appendices 3 and 4 respectively. Two broad issues are covered in the simulation experiments. First, the impact of the partial equilibrium and general equilibrium acceleration models in smoothing out exchange rate variations through the current account mechanism are compared with a Dornbusch style specification, in which no direct current account mechanism affects exchange rate adjustments. Second, the quantitative significance of the general equilibrium specification relative to the partial equilibrium specification in terms of the adjustment speed of the exchange rate is investigated. As an important specific topic the nature of the J curve style adjustment process in the general equilibrium specification is analysed.

To provide background for the analysis it is useful to review briefly some stylized facts of liquid foreign exchange markets. We estimated volatilities of numerous exchange rates and found that they were quite moderate, even low, considering their

made up image among central bank and other practitioners. In broad terms the rolling 3 month volatilities (one of the most used measures of risk in e.g. currency option markets) estimated from daily market data through the Euro period and the pre-Euro floating rates period were all between 9 and 12 percent in annual terms, averaging at about 10. These are lower than most assets volatilities. Only money market and short-to-medium term fixed income markets and other comparable assets are significantly less risky than currencies. Real estate markets, stock markets and commodities markets are typically much more risky than currency markets. A number of eurozone and other currencies, including smaller and larger economies, were analysed relative to the US dollar. Relatively short financial crisis periods generated higher volatilities, but these were smoothed out quite rapidly within the two estimation periods (pre-euro 9/1992-12/1998, euro 1/1999-12/2011), summarized in diagrams 1-2 (rolling daily estimates of standard deviations in the two periods) and in diagrams 3-4 (sample averages of standard deviations in the two periods). Given the fact that foreign exchange markets have experienced numerous very turbulent sub-periods during the years covered, including several speculative attacks involving large reshuffling of portfolios, the findings would seem to suggest that a rather strong smoothing out mechanism exists, which prevents crises from escalating.

For the simulation experiments a very simple testing environment is set up. First, autonomous factors are assumed to net each other so that only current account adjustments translate impact of changes in fixed parameters and random shocks to changes in the exchange rate. Secondly, the testing platform includes two rather similar large, relatively open economies, which could be considered remotely relevant for the analysis of the euro/USD rate. Other specifications could naturally be rather easily considered and analysed. Thirdly, impact of two shifts in a single fixed parameter α^* are studied, letting α^* to jump permanently up and down. These shifts imply jumps up and down in the temporary steady state exchange rate s_{G0} as defined in Eq. (24). However, the actual exchange rate adjusts gradually towards the new steady state due to the acceleration mechanism. Fourth, random events with 10% annual standard deviation feed unexpected shocks in the adjustment process, causing perturbations in the adjustment process towards the steady state.

Note that we have deliberately postponed any linearization until as late as the specification of the testing environment because we prefer direct simulation of the

nonlinear general equilibrium solution, which would be the only way to establish reliable comparative dynamics properties of the heavily simultaneous stochastic model. By linearizing the model as in the original Kouri contribution, we could elaborate the discussion of comparative properties of the stationary state. Meanwhile, the testing environment is specified by the following general equation, consistent with Eq. (25):

$$(28) ds = \Delta_i[s_{G0} - s]dt + \sigma dz, dz \text{ is } (0, 1) \text{ distributed noise and } \sigma = 0.1$$

Specification (28) allows us to study three different cases. The general equilibrium framework includes the Kouri style imperfect substitutibility of assets, whereby a Dornbusch specification cannot be directly implemented in the testing environment. However, a rather close formulation is obtained by assuming that in the Dornbusch case the coefficient Δ_D equals zero and only random shocks drive the exchange rate. The adjustment coefficients in the three cases to be analysed are then the following:

$$\begin{aligned} \Delta_D &= 0; && \text{The Dornbusch case} \\ \Delta_K &= \frac{1}{A_K}; && \text{The partial equilibrium Kouri case} \\ \Delta_G &= \frac{1}{A_G}; && \text{The general equilibrium case} \end{aligned}$$

In the three cases 10 000 continuous time paths of Monte Carlo simulations are sampled typically for a period of 30 years. In some cases, when we analyse the model with the negative coefficient Δ_G resulting in the J curve type of effect with stability issues, shorter time horizons are applied. In each case three statistics, the mean path and the 5% lower tail and upper tail fractile paths are reported. The mean path is used for assessing adjustment speeds of the different cases towards the long term steady state. The 90% variation range between low and high fractiles is used as the metric to evaluate exchange rate variability implied by the three different specifications.

The starting value of the exchange rate is normalized at 1 and shifts in the foreign propensity to consume α^* are calibrated so that in the two types of experiments the value of $s_{G0} = 1$ jumps 10 % up and down to values $s_{G0+} = 1.1$ and $s_{G0-} = 0.91$. In both the cases the actual value of the exchange rate then starts to adjust gradually from 1 towards either 1.1 or 0.91. All other parameter values are presented in Appendix 3.

Simulations of the Dornbusch formulation in the current framework are reported in diagram 5. Only one diagram is needed in this case as the changes in the steady state values of the exchange rate are irrelevant when $\Delta_D = 0$. The non-stationary nature of exchange rate behaviour is seen in the pattern where the 90% range about

the median path expands all the time. At 30 years time point the 90% variation range is approximately 200% of the mean level.

Simulations of the Kouri partial equilibrium case are shown in diagrams 6 and 7. The adjustment process both upwards and downwards is very slow and convergence of the mean exchange rate towards the new steady states is still continuing at a noticeable rate at the 30 years time point. While the partial equilibrium adjustment process is rather slow, the impact of that in smoothing out exchange rate variations is substantial compared with the Dornbusch case. In the Kouri case the 90% variation range at 30 years time point is about 100% of the level of the initial steady state value, i.e. just about one half of the corresponding range in the Dornbusch case.

Simulations of the general equilibrium case are shown in diagrams 8 and 9. The adjustment process is rather fast in this case. Already 2 years after the start of simulations the actual exchange rate is very close to the new steady state value. The significance of the smoothing effect through current account adjustment in the general equilibrium case can be seen very clearly in looking at the variation range defined by the 90% confidence level boundaries. The variation range of the exchange rate at the 30 years time point is now just about 20% of the initial steady state value, which is about 10% of the corresponding range of the Dornbusch case and 20% of the range in the Kouri partial equilibrium case.

In the J curve case the coefficient Δ_G is negative, which would imply unstable adjustment paths of the exchange rate converging eventually to either plus or minus infinity. However, it can be concluded that if there is a feasible solution in the negative Δ_G case, such a solution must involve overshooting of the exchange rate. In Eq. (28) consider a transform in which the sign of Δ_G is reversed. The equation remains valid only if the sign of the term $[s_{G0} - s]$ is also reversed. The value of s_{G0} is determined by exogenous parameter values so that it cannot be changed. However, if the value of s changes in an appropriate direction and by a sufficient amount the sign of $[s_{G0} - s]$ will be reversed. Thus, when the news about the increase or decrease of foreign demand of domestic goods enters the market s_{G0} appreciates or depreciates by 10%, respectively, and the term $[s_{G0} - s]$ becomes negative or positive. The desired sign reversal is achieved if at the same time the nominal exchange rate s depreciates or appreciates more than 10%. The exchange rate will then overshoot the steady state value, and during the adjustment process in the case of overshooting up (immediate

jump up) the exchange rate will gradually appreciate towards the new steady state value (diagram 10). In the case of overshooting down the exchange rate will gradually depreciate towards the new steady state value (diagram 11). Further analysis of the J curve case is in Appendix 2.

8 Conclusions

In this paper we first developed a general equilibrium two-country complete markets model to analyze a variety of factors on exchange rate adjustment, focusing in particular on the role and nature of the Kouri Acceleration Hypothesis in general equilibrium. In the Kouri partial equilibrium framework the rate of change of the exchange rate is proportional to the ratio of current account imbalance to the sum of holdings of foreign assets by domestic residents and holdings of domestic assets by foreign residents.

It was shown that in general equilibrium the acceleration coefficient is for two reasons considerably smaller than the Kouri coefficient. First, in the Kouri partial equilibrium model the coefficient is the sum of foreign assets held by domestic agents and the domestic assets held by foreign agents. In the general equilibrium model only the sum of expectations sensitive (speculative) asset holdings is relevant, likely to be a rather small subset of the total asset holdings at least under normal market conditions and assuming risk averse preferences. Secondly, this sum is further reduced by an additional term omitted in the Kouri partial equilibrium framework. This term is the stock of domestic assets not held by domestic investors but traded in exchange for foreign assets to benefit from diversification gains and yield enhancement. The result implies that current account and exchange rate adjustments are likely to be substantially faster than predicted by Kouri's analysis and empirical implementations of it.

One interesting connection of the result is that it reflects some stylized facts of liquid foreign exchange markets. Estimated exchange rate volatilities observed in market data appear to be lower than thought among market practitioners. This is consistent with a correction mechanism of the exchange rates through the current account being stronger than the one implied by Kouri's partial equilibrium analysis.

Consequently market disturbances may be smoothed out faster than thought possible, both when they originate in underlying shocks, or in capital flows induced by shocks and by changes in expectations and preferences.

Furthermore the finding has an important implication regarding the significance of current account relative to capital market factors and disturbances. Long current account adjustment periods would undermine the relevance of current account in analysis of exchange rate movements and capital flows. Conversely short current account adjustment periods enhance the relevance of the current account. In short, the general equilibrium analysis brings additional relevance to the current account dynamics. In addition to this, it can be seen in Eq. (24) that the general equilibrium certainty equivalent stationary state exchange rate is determined jointly by factors typically associated with the current account or the capital account, and that these contributions are effectively inseparable.

The significance of the general equilibrium specification can also be seen through the J-curve effect: in driving expected exchange rate movements the partial equilibrium terms and the general equilibrium capital gains and losses term have opposite signs.

While associating current account deficits with a weakening home currency the Kouri terms associate e.g. domestic inflationary policy with an appreciating currency. The general equilibrium term works in the opposite way. The Kouri adjustment process is intuitively appealing in terms of the current account channel but much less so in terms of the autonomous factors of the exchange rate. The general equilibrium term has intuitively plausible implications relative to the autonomous factors. The key reasons for the differences are that the Kouri terms represent capital flows through forward-looking reshuffling portfolio decisions, while the general equilibrium term represents impact of direct capital gains and losses on the existing portfolios.

The view held by many monetary and fiscal policy decision makers and economists that exchange rates tend to be rather volatile may reflect exchange rate dynamics implied by the partial equilibrium models of especially Dornbusch (1976) and to a lesser degree Kouri (1978). A complete opposite of this view is obtained in the general equilibrium complete markets model by Lucas (1982) in which the exchange rate does not vary.

After contrasting Dornbusch's fundamental intuition that only capital account

shocks matter in exchange rate adjustments to Kouri's "acceleration hypothesis", we derived the "generalized acceleration hypothesis" in a model with two countries with potentially different endowments, production technologies, consumption and saving/investment preferences and monetary policies, where uncertainty arises from monetary policy and supply side shocks. We extended the analyses of Kouri and Lucas by introducing in a general equilibrium Lucas type model a Kouri style portfolio balance adjustment mechanism with imperfect substitutibility of assets. A key element of the Kouri specification towards introducing a partial exchange rate rigidity in the model is that portfolio allocations can be adjusted towards new optima only over time, through gradually accumulating flows of the current account. Due to general equilibrium constraints on wealth and investment behavior, the general equilibrium acceleration coefficient is defined by the stock of domestic assets not held by domestic residents, adjusted for speculative (expectations sensitive) portfolio allocation terms.

In a testing environment defined by a general linearized equation for exchange rate dynamics, we studied three different cases. We assumed that in the Dornbusch case only random shocks drive the exchange rate. Sampling 10 000 continuous time paths of Monte Carlo simulations for 30 years and using the 90% confidence level variation range about the mean at the 30 years time point relative to the mean as the metric of exchange rate variability, the Dornbusch formulation implied a variation range of 200%, reduced to 100% in the Kouri case and to 20% in the general equilibrium case.

In addition to the variation range results, the simulations also showed very clearly that the speed of adjustment in the general equilibrium case is much faster than in the partial equilibrium, let alone in the Dornbusch case where it is not defined. Already at 2 year time point the extent of exchange rate adjustment in the general equilibrium model exceeded the corresponding adjustment at 30 year timepoint in the Kouri framework.

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Appendix 1: Derivation of Equation (20)

Recall Equation (19) in the text

$$(19)d[(1 - \beta)\frac{W}{s}] - d[(1 - \beta^*)W^*] = -Bdt = (\frac{c_r}{s} - c_h^*)dt$$

Substitute in (18) the explicit forms of β and β^* given by

$$(19a)\beta = b_m + b_s[r_h - \mu(t, f) - r_r]$$

$$(19b)\beta^* = b_m^* + b_s^*[r_r - (r_h - \mu(t, f))]$$

Use also the general notation for the stochastic rate of depreciation

$$\frac{ds}{s} = \mu(t, f)dt + \sigma_s(t, f)dz_s$$

Then (19) can be written in the following form

$$d\{[1 - b_m - b_s[r_h - \mu(t, f) - r_r]] W/s\} - d\{[1 - b_m^* - b_s^*[r_r - (r_h - \mu(t, f))]] W^*\} = (\frac{c_r}{s} - c_h^*)dt$$

and, differentiating with respect to s:

$$b_s W/s E ds/s + (1 - \beta)d(W/s) + b_s^* W^* E ds/s - (1 - \beta^*)dW^* = (\frac{c_r}{s} - c_h^*)dt$$

Calculate next $d(W/s)$ and dW^*

$$\begin{aligned} d(W/s) &= d\left[\left(\frac{1}{\beta + \beta^* - 1}\right)\left(\beta^*\frac{QK}{s} - (1 - \beta^*)Q^*K^*\right)\right] \\ &= \left(\frac{1}{\beta + \beta^* - 1}\right)\left[\beta^*\frac{dQ}{Q} + \frac{dK}{K} - \frac{ds}{s} - \frac{dQ}{Q}\frac{ds}{s} - \frac{dK}{K}\frac{ds}{s} + \frac{dQ}{Q}\frac{dK}{K} + \frac{1}{2}\frac{ds^2}{s^2}\right]\frac{QK}{s} - \\ &\quad (1 - \beta^*)\left(\frac{dQ^*}{Q^*} + \frac{dK^*}{K^*} + \frac{dQ^*}{Q^*}\frac{dK^*}{K^*}\right)Q^*K^* \end{aligned}$$

Let Q, K, Q^*, K^* also vary to include all factors driving s

$$\begin{aligned}
dW^* &= d \left[\left(\frac{1}{\beta + \beta^* - 1} \right) (\beta Q^* K^* - (1 - \beta) \frac{QK}{s}) \right] = \\
&= \left(\frac{1}{\beta + \beta^* - 1} \right) \left[\beta \left(\frac{dQ^*}{Q^*} + \frac{dK^*}{K^*} + \frac{dQ^*}{Q^*} \frac{dK^*}{K^*} \right) Q^* K^* - \right. \\
&\quad \left. (1 - \beta^*) \left(\frac{dQ}{Q} + \frac{dK}{K} - \frac{ds}{s} - \frac{dQ}{Q} \frac{ds}{s} - \frac{dK}{K} \frac{ds}{s} + \frac{dQ}{Q} \frac{dK}{K} + \frac{1}{2} \frac{ds^2}{s^2} \right) \frac{QK}{s} \right]
\end{aligned}$$

Substitute $d(W/s)$ and dW^* back into cash flow equation to get:

$$\begin{aligned}
&(b_s W/s + b_s^* W^*) E ds/s + \left(\frac{1}{\beta + \beta^* - 1} \right) \\
&\left\{ [(1 - \beta)\beta^* + (1 - \beta^*)(1 - \beta)] \left(\frac{dQ}{Q} + \frac{dK}{K} - \frac{ds}{s} - \frac{dQ}{Q} \frac{ds}{s} - \frac{dK}{K} \frac{ds}{s} + \frac{dQ}{Q} \frac{dK}{K} + \frac{1}{2} \frac{ds^2}{s^2} \right) \frac{QK}{s} - \right. \\
&\quad \left. [(1 - \beta^*)\beta + (1 - \beta^*)(1 - \beta)] \left(\frac{dQ^*}{Q^*} + \frac{dK^*}{K^*} + \frac{dQ^*}{Q^*} \frac{dK^*}{K^*} \right) Q^* K^* \right\} \\
&= \left(\frac{c_r}{s} - c_h^* \right) dt
\end{aligned}$$

Compile terms. In the form following equation (19) let

$$\begin{aligned}
r_h &= E \left(\frac{dQ}{Q} + \frac{dK}{K} + \frac{dQ}{Q} \frac{dK}{K} \right) \\
\sigma_{hs} &= E \left[-\frac{1}{2} \left(\frac{dQ}{Q} + \frac{dK}{K} \right) \frac{ds}{s} \right] \\
\sigma_h dz_h &= \frac{dQ}{Q} - E \left(\frac{dQ}{Q} \right) + \frac{dK}{K} - E \left(\frac{dK}{K} \right) \\
\sigma_s^2 &= E \left(\frac{ds^2}{s^2} \right)
\end{aligned}$$

In $r_r dt + \sigma_r dz_r$ let:

$$\begin{aligned}
r_r &= E \left(\frac{dQ^*}{Q^*} + \frac{dK^*}{K^*} + \frac{dQ^*}{Q^*} \frac{dK^*}{K^*} \right) \\
\sigma_r dz_r &= \frac{dQ^*}{Q^*} - E \frac{dQ^*}{Q^*} + \frac{dK^*}{K^*} - E \frac{dK^*}{K^*}
\end{aligned}$$

Let also

$$\frac{ds}{s} = \mu dt + \sigma_s dz_s$$

as there clearly is no state dependency of μ and σ_s

With these specifications the cash flow equation becomes

$$(b_s W/s + b_s^* W^*) \mu dt + \left(\frac{1}{\beta + \beta^* - 1} \right) (1 - \beta) \left[r_h dt + \sigma_h dz_h - \frac{1}{2} \sigma_{hs} dt - (\mu - \sigma_s^2) dt + \sigma_s dz_s \right] \frac{QK}{s} - (1 - \beta^*) (r_r dt + \sigma_r dz_r) Q^* K^* = \left(\frac{c_r}{s} - c_h^* \right) dt$$

Compiling terms involving $\frac{ds}{s}$ and moving them to the left hand side of the equation we obtain the form following equation (20) in the text:

$$\left[b_s \frac{W}{s} + b_s^* W^* - \frac{(1 - \beta)}{(\beta + \beta^* - 1)} \frac{QK}{s} \right] \mu dt - \frac{(1 - \beta)}{(\beta + \beta^* - 1)} \frac{QK}{s} \sigma_s dz_s = \frac{(1 - \beta^*)}{(\beta + \beta^* - 1)} Q^* K^* (r_r dt + \sigma_r dz_r) - \frac{(1 - \beta)}{(\beta + \beta^* - 1)} \frac{QK}{s} \left[(r_h - \frac{1}{2} \sigma_{hs} + \sigma_s^2) dt + \sigma_h dz_h \right] + \left(\frac{c_r}{s} - c_h^* \right) dt$$

Equations (21)-(27) in the text follow from these steps.

Appendix 2: Negative acceleration coefficient

The negative acceleration coefficient in exchange rate adjustment is a real possibility, as can be seen from e.g. Eq. (26). If the negative coefficient is inserted as such in the testing environment (28) and the same simulation experiments are run as in the other cases, the following family of exchange rate adjustment paths result, reported for convenience for only 5 years time horizon.

Diagram 2.1 Negative acceleration the unfeasible case downwards

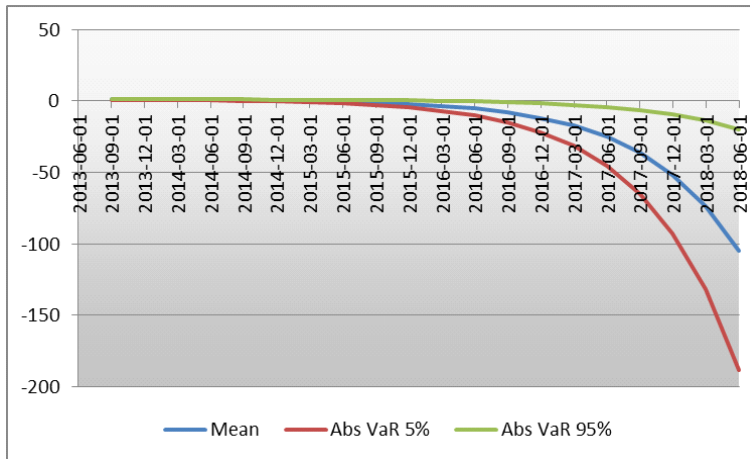
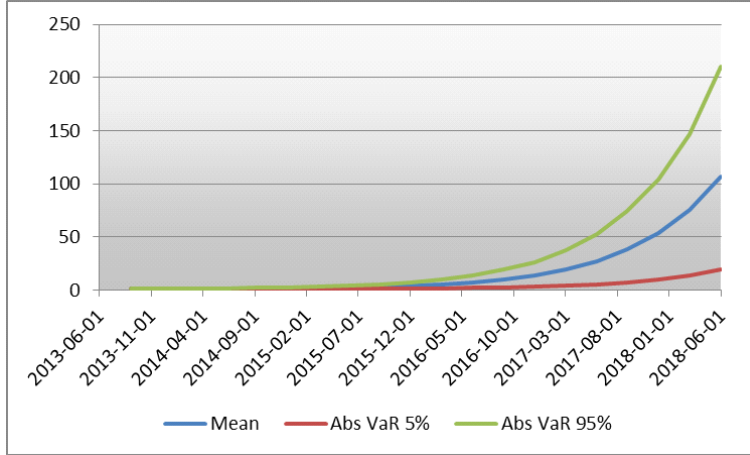


Diagram 2.2 Negative acceleration the unfeasible case upward



In the first case in which the new steady state value shifts upwards (home currency depreciates) and exceeds the starting value of the market rate the simulated graphs begin to dive rapidly downwards in a way, which clearly has no end. In the case in which the new steady state value shifts downwards (home currency appreciates) and undercuts the starting value of the market rate the simulated graphs begin to rise upwards clearly without an end. Both these adjustment processes might be for some time possible during temporary chaotic market panic periods, but they cannot be credible and valid general equilibrium solutions for a market in which rational agents are making decisions.

The unfeasible solution paths can be ruled out by specifying appropriate terminal conditions for the differential equation (28). If the equation is constrained with following terminal conditions:

$$(28) ds = \Delta_i [s_{G0} - s] dt + \sigma dz;$$

$$s(T^+) < \infty, T \rightarrow \infty$$

$$s(T^-) > 0, T \rightarrow \infty$$

no such paths can result from the adjustment process, as the paths are monotonous. Feasible solutions for the case of negative A_G can clearly only exist in cases in

which the coefficient is positive. This results if we instead of using the value A_G use its inverse value $-A_G$. But then Eq. (28) does not hold any longer. The sign of the mean term on the right hand side $[s_{G0} - s]$ must also be reversed for the differential equation to hold. The value of the temporary steady state exchange rate s_{G0} is determined by the structural parameters of the balance of payments. It is, therefore, always given to the market, and is the rate towards which the actual exchange rate will adjust. This being the case the sign of the right hand side of equation (28) can only be reversed by adjusting the starting level of the actual exchange rate accordingly.

Let us denote by reference time t_0 the moment when the market realizes that the steady state value shifts to s_{G1} either upwards or downwards. At that moment the spot exchange rate has reached the level s_{G0} and starts to adjust from there. In order for the sign reversal of $[s_{G0} - s]$ to occur the following immediate adjustments must happen in the spot exchange rate:

$$(29a)[s_{G1} - s_{G0}] > 0 \approx s(t_0) > s_{G1}$$

$$(29b)[s_{G1} - s_{G0}] < 0 \approx s(t_0) < s_{G1}$$

With an immediate jump in the spot exchange rate such that an overshooting occurs in the respective direction in which the steady state value adjusts stable exchange rate adjustment processes will result. Such adjustment processes always involve a gradual appreciation of the exchange rate towards the new (home currency depreciated) steady state value, and a gradual depreciation of the exchange rate towards the new (home currency appreciated) steady state value.

The terminal conditions are not sufficient to determine the size of the overshooting. This could be determined by appropriate initial conditions of Eq. (28), but no obvious guidelines for specifying such conditions can be stated. For the feasibility of exchange rate adjustments it is sufficient that conditions (29a,b) are satisfied.

Appendix 3: Parameter values

The key parameter values used in determining the steady state value s_{G0} are the following:

$$\alpha = \alpha^* = 0,7$$

$$\beta = \beta^* = 0.7$$

$$v = v^* = 0.03$$

$$p_0 = p_0^* = 1$$

$$Q_0 = Q_0^* = 3$$

$$K_0 = K_0^* = 1000$$

$$b_s = 0.532$$

$$b_s^* = 0.34$$

Appendix 4: Diagrams

Diagram 1: 3 Month Rolling Volatilities for 1992-09-08 – 1998-12-31

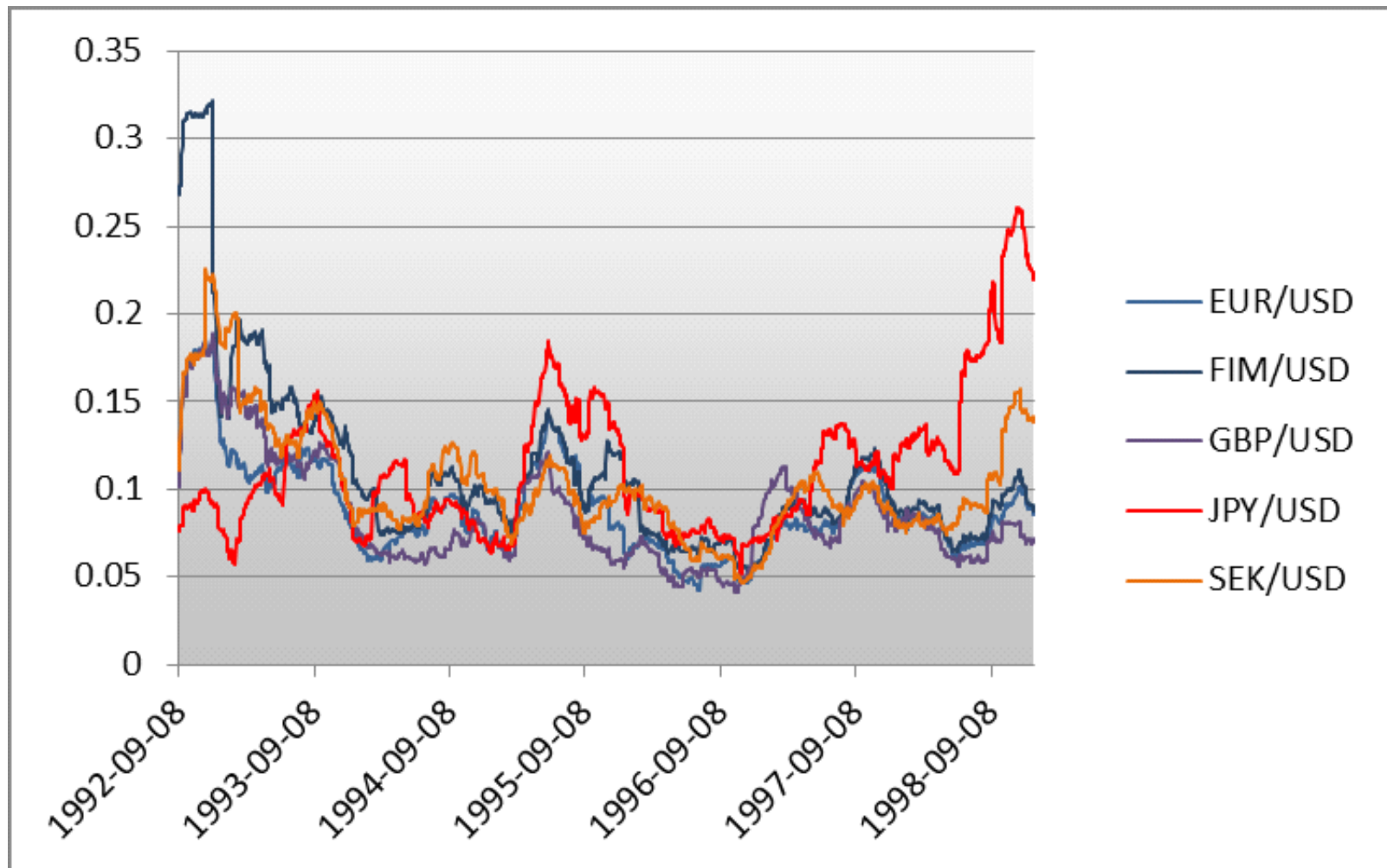


Diagram 2: 3 Month Rolling Volatilities for 1999-01-04 – 2013-09-30

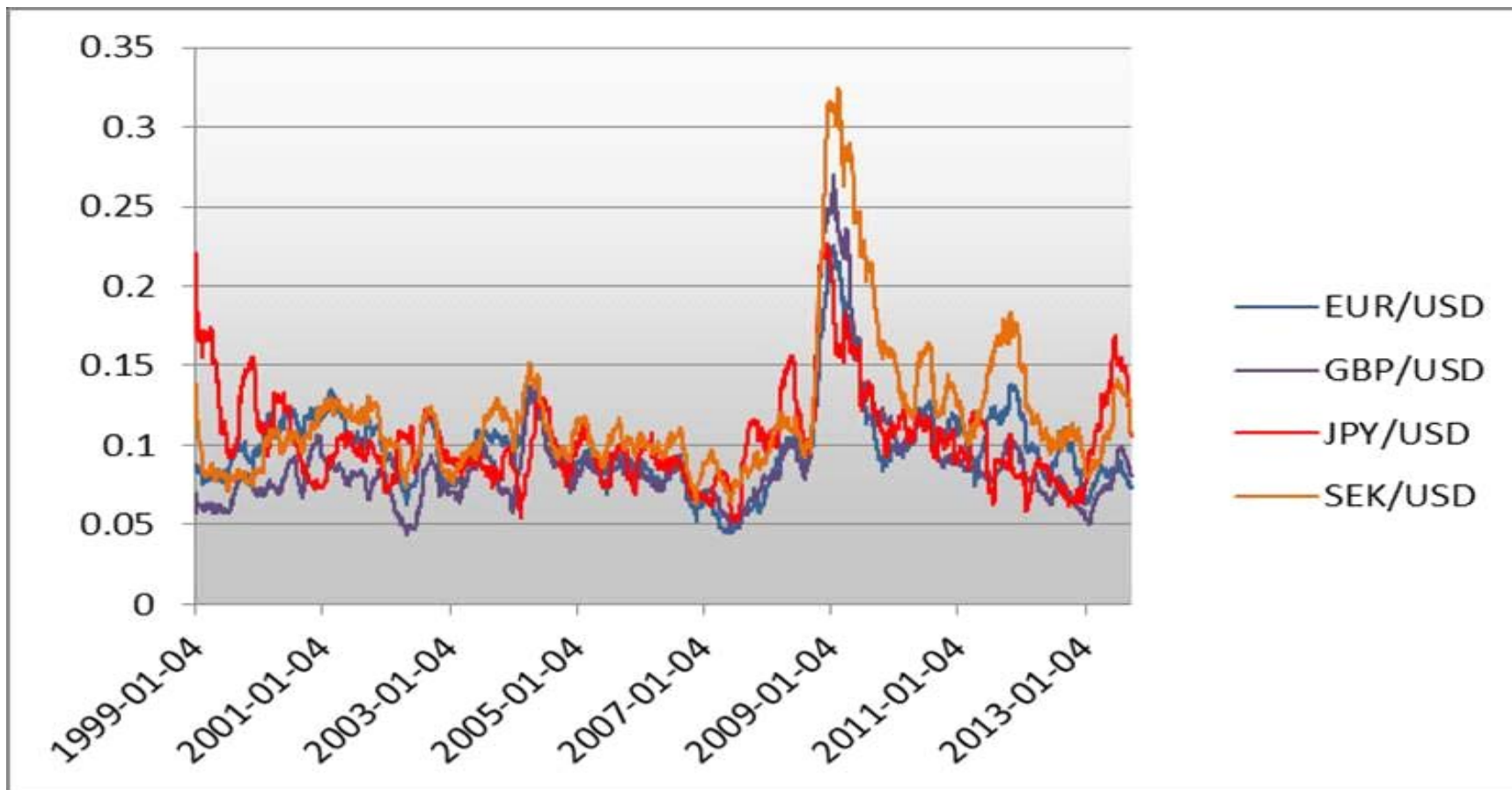


Diagram 3: Averages of 3 Month Rolling Volatilities for 1992-09-08 – 1998-12-3 1

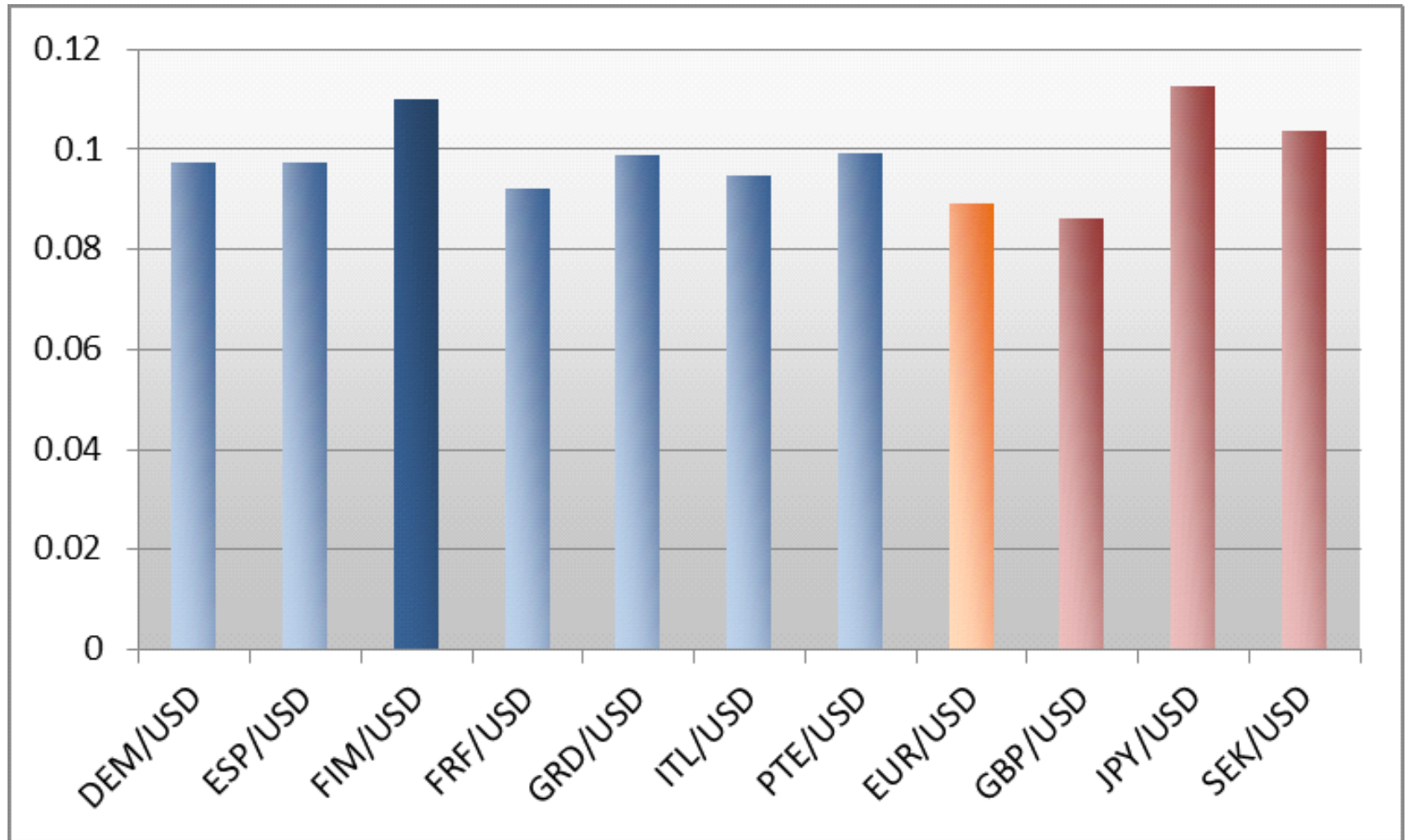


Diagram 4: Averages of 3 Month Rolling Volatilities for 1999-01-04 – 2013-09-30

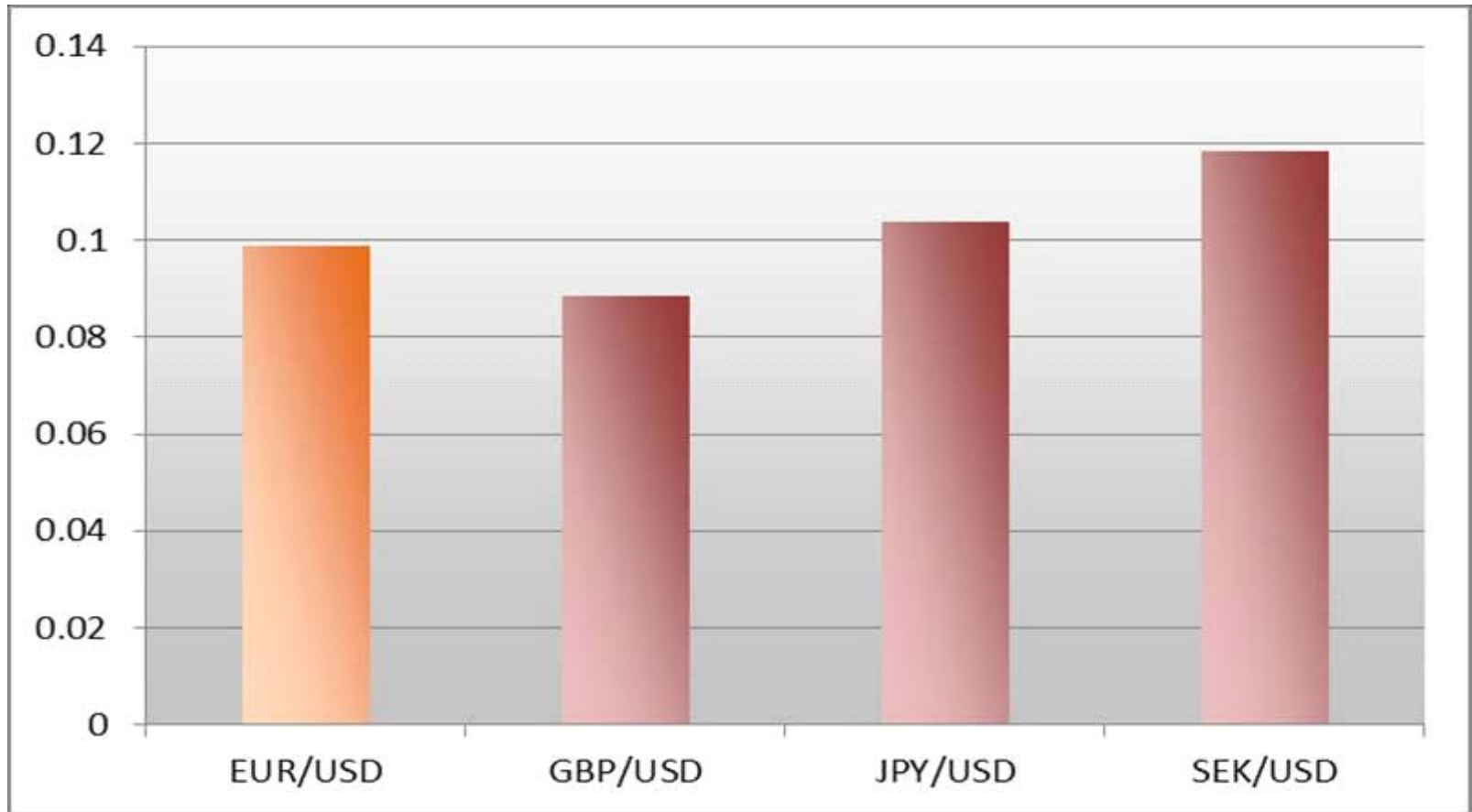


Diagram 5: Exchange rate adjustment in Dornbusch case

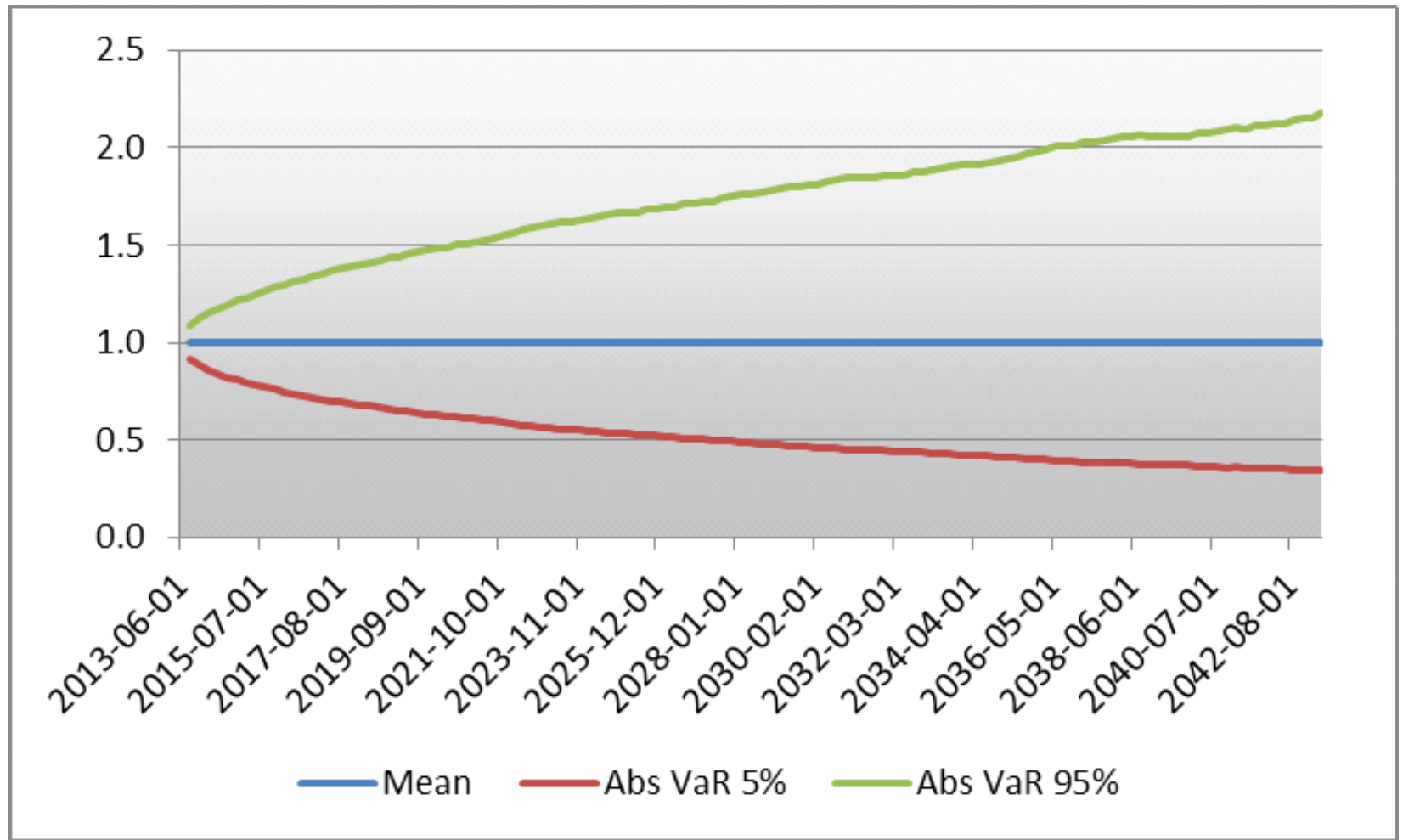


Diagram 6: Exchange rate adjustment with Kouri acceleration, case of steady state depreciation

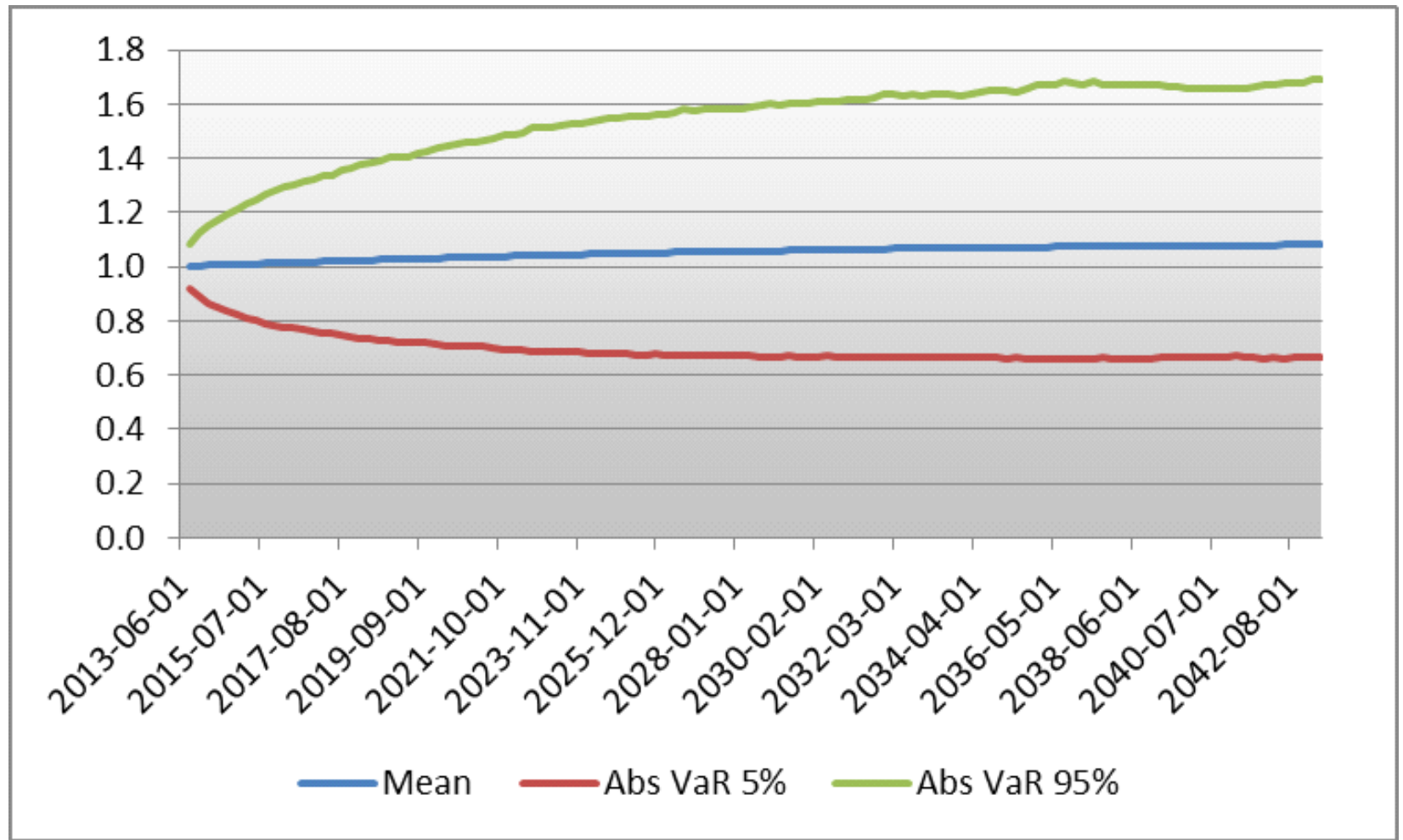


Diagram 7: Exchange rate adjustment with Kouri acceleration, case of steady state appreciation

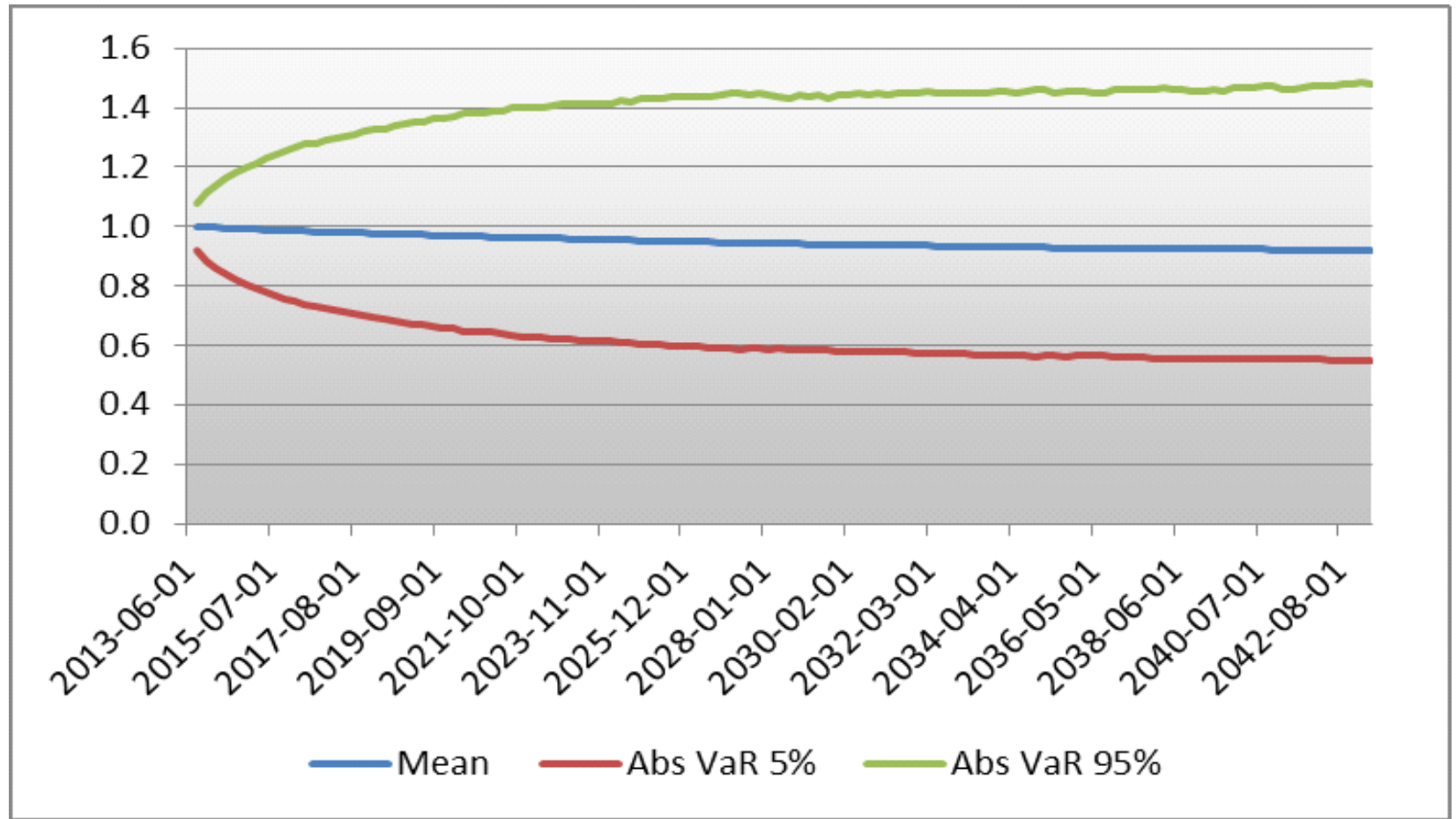


Diagram 8: Exchange rate adjustment with General Equilibrium acceleration, case of steady state depreciation

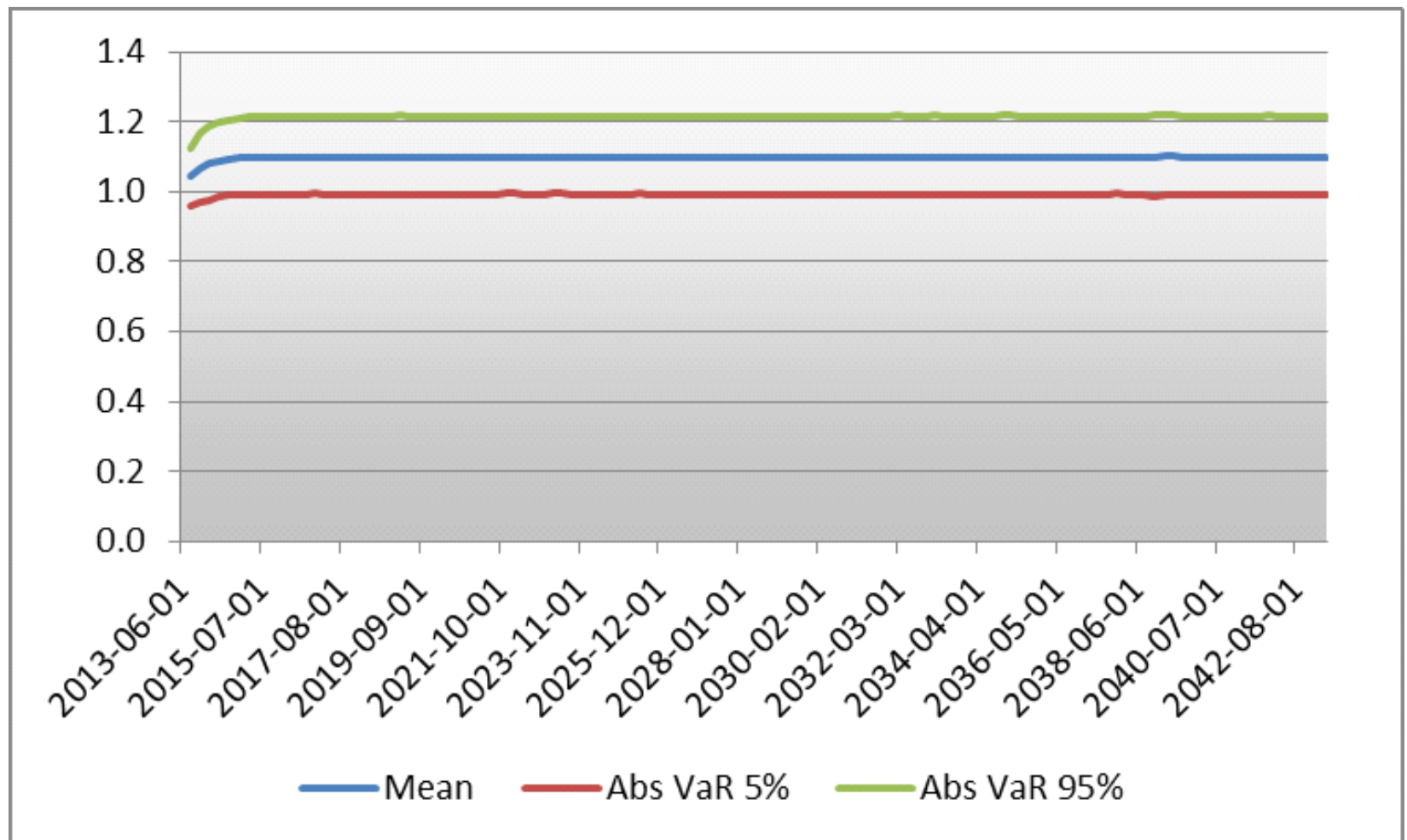


Diagram 9: Exchange rate adjustment with General Equilibrium acceleration, case of steady state appreciation

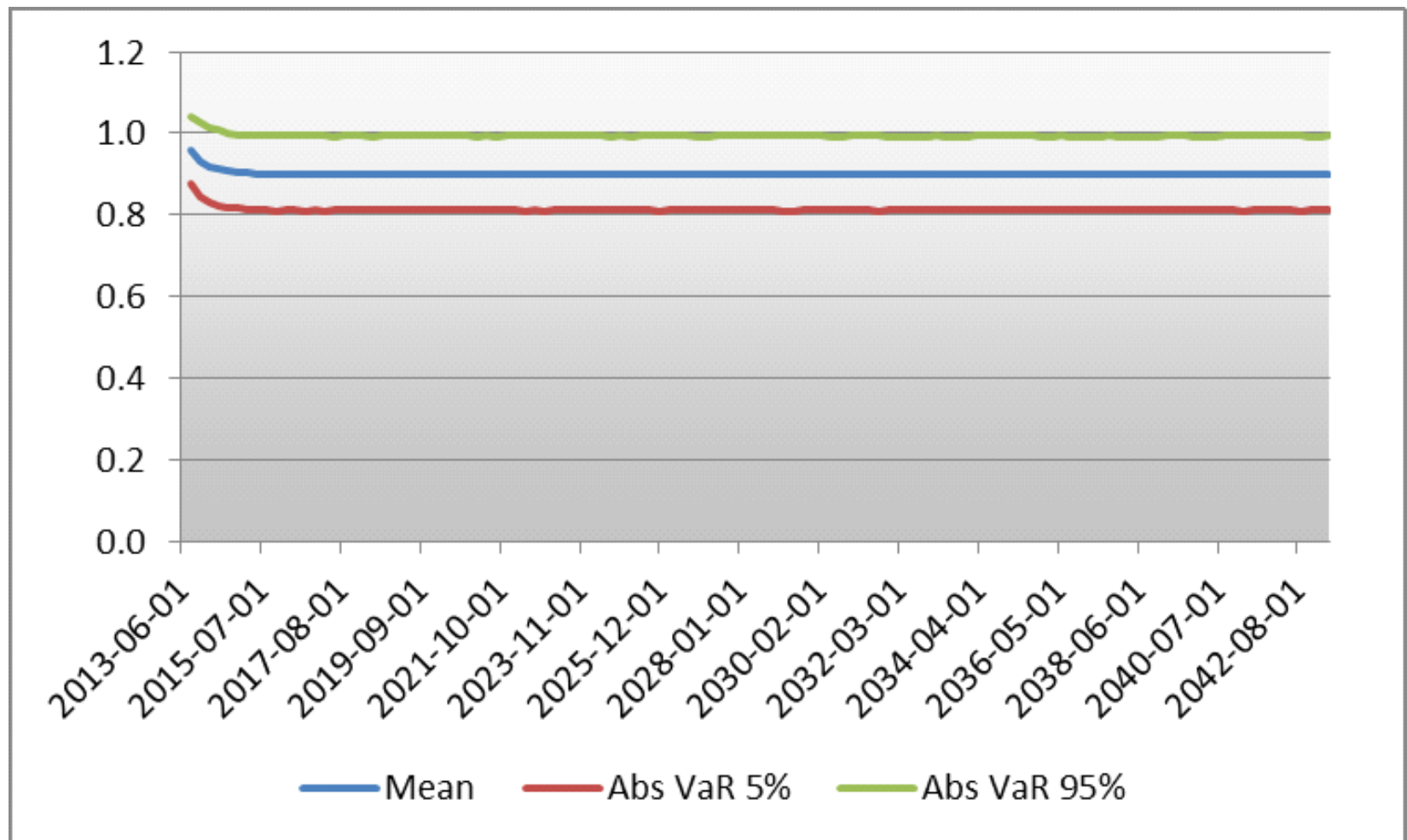


Diagram 10: Exchange rate adjustment with upward shift and overshooting

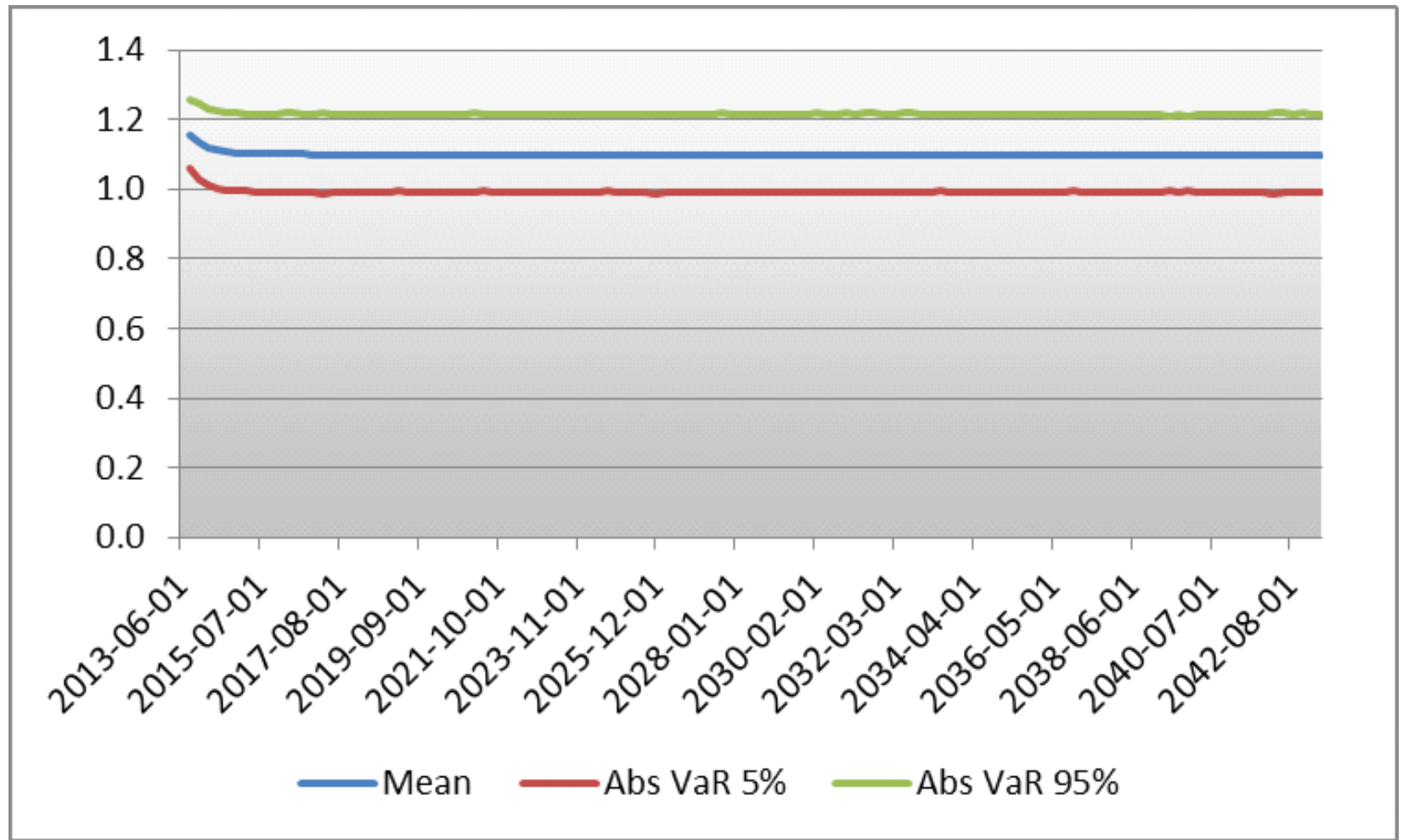


Diagram 11: Exchange rate adjustment with downward shift and overshooting

