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THE COST OF CAPITAL FOR ALTERNATIVE INVESTMENTS

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ABSTRACT

We document that the risks and pre-fee returns of broad hedge fund indices can be accurately matched with simple equity index put writing strategies, which provide monthly liquidity and complete transparency over their state-contingent payoff profiles. This nonlinear risk exposure combines with large allocations, typical among investors in alternatives, to produce required rates of return that are more than twice as large as those implied by popular linear factor models. Despite earning annualized excess returns over 6% between 1996 and 2010, many hedge fund investors have not covered their proper cost of capital.

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This paper develops a simple method for calculating the cost of capital for alternative investments. *Ex ante* cost of capital estimates are central to the efficient allocation of capital, and generally depend on the composition of the investor's total portfolio and the payoff profile of the new investment relative to the remainder of the portfolio. In the context of alternative investments, these allocations (1) constitute a large share of the investor's total portfolio and (2) have nonlinear payoffs relative to the remainder of the portfolio at horizons that permit rebalancing. We find that these two features interact to produce very large required rates of return relative to commonly used benchmarking techniques, including those developed recently to address the nonlinear payoffs of alternative investments.

Merton (1987) develops a model of capital market equilibrium with incomplete information in which investors must be informed about an investment before they allocate any capital to it. This creates specialization in investing and leads some investors to hold highly concentrated portfolios in equilibrium. As a result, the equilibrium required rate of the return on these investments exceeds what would be required in a frictionless model. An important class of real world decisions to which this applies are alternative investments. Conditional on investing in alternatives, allocations are typically large relative to the equilibrium supply of these risks, in order to amortize the fixed costs associated with expanding the traditional investment universe to include alternatives. For example, as of June 2010, the Ivy League endowments had 40% of their combined assets allocated to non-traditional assets (Lerner, et al. (2008)), whereas the share of alternatives in the global wealth portfolio was closer to 2%.[1] This paper argues that the wedge between the proper cost of capital and that implied by a frictionless model is likely to be particularly large here due to the non-linearity of the payoff profile relative to the remainder of the portfolio. Taking observed portfolio allocations as given, we study the role concentrated allocations play in the cost of capital for alternative investments.

To the extent that the real world equilibrium is affected by market frictions that lead some investors to hold highly concentrated portfolios, these consequences must be explicitly handled in cost of capital estimates, but typically are not. For example, traditional cost of capital computations are heavily reliant on linear factor models, which implicitly assume that: (a) investors trade in frictionless markets, and thus hold efficient portfolios; and (b) asset returns are well described by the considered set of traded factors. Merton (1987) highlights the theoretical and empirical challenges posed by the first assumption when the equilibrium is one where a small subset of investors hold relatively large shares of a particular risk, as appears to be the case for alternative investments. The linear factor model approach relies on the notion that all agents agree on the required rate of return for

[1] As of end of 2010, the total assets under management held by hedge funds stood at roughly $2 trillion (source: HFRI), in comparison to a combined global equity market capitalization of $57 trillion (source: World Federation of Exchanges) and a combined global bond market capitalization of $54 trillion, excluding the value of government bonds (source: TheCityUK, "Bond Markets 2011").

a marginal deviation from their efficient portfolios, but this will generally not be true when market frictions cause some investors to hold concentrated portfolios. Ignoring these issues altogether, the focus of the empirical literature has been on expanding the factor set (e.g. by adding non-linear factors) in an attempt to describe the downside risk exposure of alternatives. Nonlinear factors have been included in an *ad hoc* way, such that they often do not represent feasible investments, and are unlikely to capture the specific nonlinear risk profile of hedge funds.[2] Moreover, this does little to correct for the problems due to return smoothing or asset illiquidity (Asness, et al. (2001), Getmansky, et al. (2004)). We remedy these shortcomings, and produce cost of capital estimates for alternatives reflecting the economic reality of the concentrated portfolio to which they belong.

To derive estimates of the cost of capital in a setting where investors make large allocations to downside risks, we assemble a simple static portfolio selection framework that combines power utility (CRRA) preferences, with a state-contingent asset payoff representation, in the spirit of Arrow (1964) and Debreu (1959). We specify the joint structure of asset payoffs by describing each security's payoff as a function of the aggregate equity index (here, the S&P 500).[3] Finally, to capture the non-linear risk exposure of alternatives, we model hedge fund returns as a portfolio of cash and a short position in equity index put options. The contractual nature of index put options immediately provides a complete state-contingent description of an investable alternative to the aggregate hedge fund universe. In turn, the availability of a state-contingent risk profile allows us to determine the rate of return that an investor would require as a function of his risk aversion, portfolio allocation, and the underlying return distributions of other asset classes, all of which are necessary for any asset allocation decision.

Our first empirical contribution is to provide a new methodology for replicating the returns to a broad cross-section of hedge fund indices. While evidence of non-linear systematic risk exposures resembling those of index put writing has been provided by Mitchell and Pulvino (2001) for risk arbitrage, and Agarwal and Naik (2004) for a large number of equity-oriented strategies, the literature – aside from Lo (2001) – has been comparatively silent on exploring *non-linear* replicating strategies. We take seriously the problem of capital requirements (Santa-Clara and Saretto (2009)) and transaction costs to produce the returns of feasible put writing strategies, thus extending the linear hedge fund replication analysis of Lo and Hasanhodzic (2007). For a given hedge fund index, we identify suitable replicating strategies by matching the mean *pre-fee* returns of the index. Our procedure does not rely on linear regression, and therefore sidesteps problems due to return smoothing or asset illiquidity. When compared with our put writing strategies, linear strategies identified via in-sample regressions: (a) generate

[2]For example, Agarwal and Naik (2004) use options that are 1% out-of-the-money to explain hedge fund returns, which may not be the appropriate downside risk profile of these investments. Fung and Hsieh (2004) construct factors based on the theoretical returns to lookback straddle portfolios, which represent highly infeasible portfolio returns since they do not satisfy margin requirements.

[3]The same state-contingent payoff model is used in Coval, et al. (2009) to value tranches of collateralized debt obligations relative to equity index options, and in Jurek and Stafford (2013) to elucidate the time series and cross-section of repo market spreads and haircuts.

statistically significant shortfalls in replicating the returns of hedge funds; (b) produce residuals exhibiting greater skewness and excess kurtosis; (c) do a inferior job of matching the drawdown patterns of hedge fund indices; and, (d) deliver minimal improvements in explanatory R^2, despite being designed to essentially maximize this statistic. To further validate our replication methodology we examine the cross-section of HFRI subindices and compare the fit of our put writing strategies against linear factor models *out-of-sample*, by selecting the replicating strategies using the first half of the data (Jan. 1996 - Jun. 2003) and examining their performance using the second half (Jul. 2003 - Dec. 2010). We find that for the set of equity-related hedge fund subindices, where simple strategies based on equity index options can be expected to perform well, non-linear replicating portfolios dominate their linear counterparts both along the dimension of matching the risks, as well as, the returns.

While the preceding analysis indicates that *pre-fee* hedge fund returns can be replicated using put writing strategies, it has little to say about whether investors in either strategy have covered their proper cost of capital. For example, to the extent that put options are fairly priced, our results indicate that hedge funds may be earning compensation for jump and volatility risk premia. Another interpretation is that put options are themselves mispriced, reflecting an imperfectly competitive market for the provision of "pre-packaged" liquidity, such that a conclusion of zero after-fee alpha for hedge funds may still be rejected.[4] To confront this possibility directly, we use our state-contingent payoff framework combined with the composition of the put writing strategies and the realized path of market volatility, to examine the time series of the proper cost of capital for an investor in hedge funds. In the absence of any access to alternatives, the investor is assumed to have a baseline allocation of 40% to risk-free securities and a 60% allocation to risky securities, mimicking the standard benchmark invoked by pension plans and endowments. With access to alternatives, the investor has a 35% or 50% allocation to alternatives.

Our second empirical contribution demonstrates that cost of capital computations based on *ex post* factor regressions, or *ex ante* theoretical estimates based on the CAPM, meaningfully understate the investors' true cost of capital. For example, linear regressions (CAPM, Fama-French, Fung-Hsieh) suggest that hedge fund investors have required rates of return of 0% to 3% per annum, and have therefore earned alphas ranging from 3-6% net of fees. By contrast, our model indicates that relative to the proper cost of capital, which accounts for the payoff non-linearity of the aggregate hedge fund index and the concentrated investor allocations, the endowment investor earned statistically unreliable alphas of -2.5% to -0.6%. The proper cost of capital for the endowment with

[4]Option returns reflect the returns to bearing jump and volatility risk (e.g. Carr and Wu (2009), Todorov (2010)), as well as, compensation for systematic demand imbalances (e.g. Garleanu, et al. (2009), Constantinides, et al. (2012)). He and Krishnamurthy (2012) highlight the role of time-varying capital constraints of intermediaries on asset prices.

a large allocation to hedge funds stands at 6.9% to 8.7%, more than twice as large as the theoretical prediction based on the CAPM beta of the aggregate hedge fund universe (3%). We extend this analysis to the cross-section of equity-related hedge fund strategies, and find that net-of-fee investor alphas are statistically indistinguishable from zero for both investor types. In practice, many investors have likely underperformed their proper cost of capital estimates as the hedge fund indices we study are likely to suffer from survivorship and backfill bias (Malkiel and Saha (2005)), overstating the returns to actual hedge fund investors.

Finally, our state-contingent framework also allows us to provide a new perspective on the pricing of equity index options. We find that the pre-fee put writing returns are consistently about 3.5% to 4% higher than their associated hedge fund strategy return, which translates into consistently positive alphas. A highly specialized investor (allocating 50% to put writing) generally realized alphas that are statistically indistinguishable from zero, while less specialized investors realized statistically significant alphas from the put writing portfolios. While these findings qualitatively confirm that equity index put options are "expensive," our estimates of annualized alphas are (at least) an order of magnitude lower than reported in previous papers (e.g. Coval and Shumway (2001), Bakshi and Kapadia (2003), Bondarenko (2003), Frazzini and Pedersen (2011), Constantinides, et al. (2012)). From the perspective of our model, the marginal price setters in equity index options markets may simply hold concentrated portfolios, or believe the underlying equity index return distribution has a more severe left tail than in our calibration.

The remainder of the paper is organized as follows. Section 1 describes the risk profile of hedge funds. Section 2 presents a simple recipe for replicating the aggregate hedge fund risk exposure with index put options and empirically compares the returns of this replication strategy with those produced by linear factor models. Section 3 develops a generalized asset allocation framework for computing the cost of capital for investors with large allocation to nonlinear payoffs. Section 4 evaluates the empirical pricing of downside risks using *after-fee* hedge fund index returns, as well as, the *pre-fee* returns to put-writing strategies, and compares them to inference based on traditional linear factor models. Finally, Section 5 concludes the paper.

1 Describing the Risk Profile of Hedge Funds

We begin our investigation of hedge fund risk profiles with an assessment of the risk properties of the aggregate asset class. We proxy the performance of the hedge fund universe using two indices: the (value-weighted) Dow Jones/Credit Suisse Broad Hedge Fund Index, and the (equal-weighted) HFRI Fund Weighted Composite Index. Such indices are not investable, and typically provide an upward biased assessment of hedge fund performance due to the presence of backfill and survivorship bias. For example, Malkiel and Saha (2005) report that the difference between the mean annual fund return in the backfilled and non-backfilled TASS database was 7.34% per year in the 1994-2003 sample. Moreover, once defunct funds are added in the computation of the mean annual returns to correct for survivorship bias, the mean annual fund return declines by 4.42% (1996-2003). To the extent that the survivorship bias also affects the measured risks, it is unlikely that the true risks are lower than those estimated from the realized returns over this period.

Table 1 reports summary statistics for the HFRI and DJ/CS aggregates and their major sub-indices, computed using quarterly returns from 1996:Q1-2010:Q4 ($N = 60$ quarters), and compares them to the S&P 500 index and one-month T-bills. Although index returns are available at the monthly frequency, we focus on quarterly returns throughout the paper to ameliorate the effects of stale prices and return smoothing (Asness, et al. (2001), Getmansky, et al. (2004)). Finally, since our goal is to characterize compensation for bearing risk across various markets rather than investor returns *per se*, we report summary statistics for *pre-fee* index returns. To obtain pre-fee returns, we treat the observed *net-of-fee* time series as if it represented the return of a representative fund that was at its high watermark throughout the sample, and charged a 2% flat fee and a 10% incentive fee, both payable monthly.[5] The difference between the mean pre-fee and net-of-fee returns represents an approximation of the *all-in* investor fee. For comparison, using cross-sectional data from the TASS database for the period 1995-2009, Ibbotson, et al. (2010) find that the average fund collected an all-in annual fee of 3.43%. French (2008) reports an average total fee of 4.26% for U.S. equity-related hedge funds in the HFRI database using data from 1996 through 2007. We find that our crude computation of all in-fees coincides well with these estimates.

The attraction of hedge funds over this time period is clear: mean returns on alternatives exceeded that of the S&P 500 index, while incurring lower volatility. Moreover, the estimated linear systematic risk exposures (or CAPM β values) indicate that hedge fund performance was largely unrelated to the performance of the public equity index, and suggests that relative to this risk model they have outperformed. The realized pre-fee Sharpe ratios on alternatives were approximately three times higher than that of the S&P 500 index. Under all of the standard risk metrics inspired by the mean-variance portfolio selection criterion, hedge funds represented a highly attractive investment. Hedge funds also perform well when evaluated on the dimension of drawdowns, which measure the magnitude of the strategy loss relative to its highest historical value (or high watermark). Both hedge fund indices have a minimum drawdown of approximately -20%, which is less than half of the -50% drawdown sustained by investors in public equity markets. This is further illustrated in Figure 1, which plots the

[5]In practice most funds impose a “2-and-20” compensation scheme, comprised of a 2% flat fee and a 20% incentive allocation, subject to a high watermark provision. Our compensation scheme can therefore loosely be interpreted as describing the scenario where half of the funds in the universe are at their high watermark at each point in time. Our computation is also likely conservative in that the incentive component represents an option on the pre-fee return of a portfolio of funds, rather than a portfolio of options on the pre-fee returns of the underlying funds.

net-of-fee value of $1 invested in the various assets through time. By December 2010, the hedge fund investor had amassed a wealth roughly 50% larger than the wealth of the investor in public equity markets, and more than twice the wealth of an investor rolling over investments in short-term T-bills.

The data also reveal the presence of significant non-normalities in hedge fund returns, as demonstrated by the departures of the measured skewness and kurtosis from zero and three, respectively. The Jarque-Bera (JB) statistic evaluates whether a time series exhibits skewness and kurtosis, and is a popular test for normality. The 5% critical value for the JB test statistic in a sample of our size is 5.0, indicating that the null of normally-distributed returns is rejected for all, but four, of the eighteen hedge fund sub-indices. This raises the possibility that the high measured CAPM alphas may not only reflect manager skill, but also some compensation for exposure to (non-linear) downside risks.[6]

Figure 1 also confirms that the performance of hedge funds as an asset class is *not* market-neutral. For example, hedge funds experience severe declines during extreme market events, such as the credit crisis during the fall of 2008 and the LTCM crisis in August 1998. During the two-year decline following the bursting of the Internet bubble, hedge fund performance is flat. And, finally, in the "boom" years hedge funds perform well. Empirically, the downside risk exposure of hedge funds as an asset class is reminiscent of writing out-of-the-money put options on the aggregate index. Severe index declines cause the option to expire in-the-money, generating losses that exceed the put premium. Mild market declines are associated with losses comparable to the put premium, and therefore flat performance. Finally, in rising markets the put option expires out-of-the-money, delivering a profit to the option-writer.

There are structural reasons to view the aggregated hedge fund exposure as being similar to short index put option exposure. Many strategies explicitly bear risks that tend to realize when economic conditions are poor and when the stock market is performing poorly. For example, Mitchell and Pulvino (2001) document that the aggregate merger arbitrage strategy is like writing short-dated out-of-the money index put options because the underlying probability of deal failure increases as the stock market drops. Hedge fund strategies that are net long credit risk are effectively short long-dated put options on firm assets – in the spirit of Merton's (1974) structural credit risk model – such that their aggregate exposure is similar to writing long-dated index put options. Other strategies (e.g. distressed investing, leveraged buyouts) are essentially betting on business turnarounds at firms that have serious operating or financial problems. In the aggregate these assets are likely to perform well when purchased cheaply so long as market conditions do not get too bad. However, in a rapidly deteriorating economy these are likely to be the first firms to fail.

[6] Harvey and Siddique (2000) provide an asset pricing model where skewness is priced, and present empirical evidence of a systematic skewness risk premium in equity markets.

The downside exposure of hedge funds is induced not only by the nature of the economic risks they are bearing, but also by the features of the institutional environment in which they operate. In particular, almost all of the above strategies make use of outside investor capital and financial leverage. Following negative price shocks outside investors make additional capital more expensive, reducing the arbitrageur's financial slack, and increasing the fund's exposure to further adverse shocks (Shleifer and Vishny (1997)). Brunnermeier and Pedersen (2008) provide a complementary perspective highlighting the fact that, in extreme circumstances, the withdrawal of funding liquidity (i.e. leverage) from to arbitrageurs can interact with declines in market liquidity to produce severe asset price declines.

2 A New Method for Replicating Hedge Fund Risk Exposure

In order to replicate the aggregate risk exposure of hedge funds, we examine the returns to simple strategies that write naked (unhedged) put options on the S&P 500 index. Our initial focus on replicating the risk exposure of the aggregate hedge fund universe, rather than strategy sub-indices or individual funds, is motivated by the observation that sophisticated investors (e.g. endowment and pension plans) generally hold diversified portfolios of funds, either directly or via funds-of-funds. Consequently, a characterization of the asset class risk exposure provides a first-order characterization of their problem.

We consider a range of replicating strategies with different downside risk exposures, as measured by how far the put option is out-of-the-money and how much leverage is applied to the portfolio. Each replicating strategy writes a single, short-dated put option, and is rebalanced monthly. Our hedge fund replication methodology matches hedge fund indices to feasible put writing strategies on the basis of their realized *mean* returns, and evaluates the model fit based on a variety of distributional properties of the *feasible residuals*, defined as the difference between the quarterly returns of the hedge fund index and the feasible replicating portfolio. This approach takes seriously the notion that many hedge fund strategies primarily bear downside risks resembling put writing, and that risk premia across economically similar exposures should be equalized. To the extent that we incorrectly benchmark hedge fund performance against an explicitly nonlinear strategy, we will produce residuals that have large skewness and kurtosis relative to the residuals from linear factor models. On the other hand, to the extent that hedge fund returns share the risk properties of put writing strategies, the put-writing-portfolio residuals will have less skewness and kurtosis than those from linear factor model regressions.

Our proposed methodology, based on matching *mean returns*, contrasts starkly with existing approaches in the hedge fund replication literature, which fall into three broad categories: factor-based, rule-based, and distributional. The factor-based methods, inspired by the ICAPM and APT, rely on regression analysis to identify replicating portfolios of tradable indices, which in some cases include option-based strategies (Fung and Hsieh (2002, 2004), Agarwal and Naik (2003, 2004), Lo and Hasanhodzic (2007)). A major concern with the regression-based approach is that hedge fund return series are smoothed (Asness, et al. (2001), Getmansky, et al. (2004)), which results in downward biased estimates of factor loadings, and therefore upward biased estimates of hedge fund alphas. In principle, with additional assumptions about the smoothing technology one can try to improve the specification. From a practical perspective, after correcting for return smoothing by using lower frequency returns or adding lagged factors, the portfolios of traditional risks identified by these regressions typically fail to match the high excess returns delivered by hedge funds. The rule-based methods use mechanical algorithms to assemble portfolios mimicking basic hedge fund strategies (Mitchell and Pulvino (2001), Duarte, et al. (2007)). To the extent that hedge fund strategies bear risks distinct from those represented by commonly-used asset pricing factors, an attractive feature of this approach is that the replicating portfolio will earn the premia associated with those distinct risks by being directly exposed. For our purposes, a disadvantage of this method is that the issue of determining the appropriate cost of capital for these risks remains unresolved without a clear mapping into an asset pricing model. Finally, distributional methods focus on matching the unconditional distribution of hedge fund returns, with no emphasis on matching contemporaneous movements between hedge funds and other assets, such as the market portfolio. This approach is inspired by Dybvig's (1988) payoff distributional pricing theory, which examines the properties of the cheapest-to-deliver lottery matching a given distribution, and was first applied to hedge fund replication by Amin and Kat (2003). The general idea behind their approach is to identify a static payoff function that transforms the distribution of the index return into the distribution of hedge fund returns, and then replicate the static payoff through dynamic trading. While our approach shares the flavor of using a transformation of the index return, through the choice of option strike and leverage pairs, our model evaluation procedure explicitly relies on the contemporaneous replication residuals, rather than the properties of the unconditional return distributions.

2.1 Measuring Put Writing Portfolio Returns

To calculate returns and characterize risks associated with put writing portfolios, we begin by specifying feasible investment strategies. Implementing each strategy requires defining the (1) rebalancing frequency, (2) security selection rule, and (3) amount of financial leverage.

Each month from January 1996 through December 2010, we form a simple portfolio consisting of a short position in a single S&P 500 index put option, $\mathcal{P}(K(Z), T)$, and equity capital, $\kappa_E(L)$, where $K(Z)$ is the option strike price, T is the option expiration date, and L is the leverage of the portfolio. The portfolio buys (sells) put options at the ask (bid) prevailing at the market close of the month-end trade date.[7] If no market quotes are available for the option contract held by the agent at month-end, the portfolio rebalancing is delayed until such quotes become available. The proceeds from shorting the option, along with the portfolio's equity capital are invested at the risk-free rate for one month, earning $r_{f,t+1}$. This produces a terminal *accrued interest* payment of:

$$\mathrm{AI}_{t+1} = \Big(\kappa_E(L) + \mathcal{P}_t^{bid}(K(Z),T)\Big) \cdot \left(e^{r_{f,t+1}} - 1\right). \tag{1}$$

The monthly portfolio return, $r_{p,t+1}$, is comprised of the change in the value of the put option plus the accrued interest divided by the portfolio's equity capital:

$$r_{p,t+1} = \frac{\mathcal{P}_t^{bid}(K(Z),T) - \mathcal{P}_{t+1}^{ask}(K(Z),T) + \mathrm{AI}_{t+1}}{\kappa_E(L)}. \tag{2}$$

We construct strategies that write options at fixed strike Z-scores. Selecting strikes on the basis of their corresponding Z-scores ensures that the systematic risk exposure of the options at the rebalancing dates is roughly constant, when measured using their Black-Scholes deltas. This contrasts with previous studies, which have focused on strategies with fixed option moneyness (measured as the strike-to-spot ratio, K/S, or strike-to-forward ratio), such as Glosten and Jagannathan (1994), Coval and Shumway (2001), Bakshi and Kapadia (2003), Agarwal and Naik (2004). Options selected by fixing moneyness have higher systematic risk, as measured by delta or market beta, when implied volatility is high, and lower risk when implied volatility is low.

In particular, we define the option strike corresponding to a Z-score, Z, by:

$$K(Z) = S_t \cdot \exp\Big(\sigma_{t+1} \cdot Z\Big) \tag{3}$$

where S_t is the prevailing level of the S&P 500 index and σ_{t+1} is the one-month stock index implied volatility, observed at time t. We select the option whose strike is closest to, but below, the proposal value (3), and whose expiration date is closest, but after the end of the month. At trade initiation, the time to option expiration is roughly equal to seven weeks, since options expire on the third Friday of the following month. To measure volatility at the one-month horizon, σ_{t+1}, we use the CBOE VIX implied volatility index.

Option writing strategies require the posting of capital, or margin. The capital bears the risk of losses due to

[7] We aim to provide a conservative assessment of put writing returns by assuming the strategy demands immediacy by executing at the opposing side of the bid-ask spread. Returns measured on the basis of the option midprice are considerably higher given the wide bid-ask spread, especially in the early part of the sample.

changes in the mark-to-market value of the liability. The inclusion of margin requirements plays an important role in determining the profitability of option writing strategies (Santa-Clara and Saretto (2009)), and further distinguishes our approach from papers, where the option writer's capital contribution is assumed to be limited to the option price, as it would be for a long position. In the case of put writing strategies, the maximum loss per option contract is given by the option's strike value, K. Consequently, a put writing strategy is fully-funded or unlevered (i.e. can guarantee the terminal payoff) if and only if, the portfolio's equity capital is equal to (or exceeds) the maximum loss at expiration. For European options, this requires an initial investment of unlevered asset capital, κ_A, equal to the discounted value of the exercise price less the proceeds of the option sale:

$$\kappa_A \quad = \quad e^{-r_{f,t+\tau}} \cdot K(Z) - \mathcal{P}_t^{bid}(K(Z), T) \tag{4}$$

where $r_{f,t+\tau}$ is the risk-free rate of interest corresponding to the time to option expiration, and is set on the basis of the nearest available maturity in the OptionMetrics zero curves. The ratio of the unlevered asset capital to the portfolio's equity capital represents the portfolio leverage, $L = \frac{\kappa_A}{\kappa_E}$. Allowable leverage magnitudes are controlled by broker and exchange limits, with values up to approximately 10 being consistent with existing CBOE regulations.[8] We consider four put writing strategies, $[Z, L]$, all targeting an average realized return to match that of the hedge fund index being replicated. In particular, we consider options at four strike levels, $Z \in \{-0.5, -1.0, -1.5, -2.0\}$, which are progressively further out-of-the-money. The options we consider have strike prices that (at inception) are on average between 4% (Z = -0.5) and 13% (Z = -2.0) below the prevailing strike price. By contrast, Agarwal and Naik (2004), base their "out-of-the-money" put factor on options whose strike is 1% below the prevailing spot price. Consequently, their approach is essentially equivalent to a linear regression methodology which separately estimates the downside and upside betas, in the spirit of Glosten and Jagannathan (1994). For each strike level, Z, we choose the leverage, L, such that the average strategy-level return equals the mean realized return of the hedge fund strategy being considered over the estimation period.

2.1.1 An Example

To illustrate the portfolio construction mechanics consider the first portfolio rebalancing trade of the $[Z = -1,\ L = 2]$ strategy. The initial positions are established at the closing prices on January 31, 1996, and are held

[8] The CBOE requires that writers of uncovered (i.e. unhedged) puts "deposit/maintain 100% of the option proceeds plus 15% of the aggregate contract value (current index level) minus the amount by which the option is out-of-the-money, if any, subject to a minimum of [...] option proceeds plus 10% of the aggregate exercise amount:

$$\min\ \kappa_E^{CBOE} = \mathcal{P}^{bid}(K,\ S,\ T;\ t) + \max\left(0.10 \cdot K,\ 0.15 \cdot S - \max(0,\ S - K)\right).$$

until the last business day of the following month (February 29, 1996), when the portfolio is rebalanced. At the inception of the trade the closing level of the S&P 500 index was 636.02, and the implied volatility index (VIX) was at 12.53%. Together these values pin down a proposal strike price, $K(Z) = 613.95$, for the option to be written via (3). We then select an option maturing *after* the next rebalance date, whose strike is closest from below to the proposal value, $K(Z)$. In this case, the selected option is the index put with a strike of 610 maturing on March 16, 1996. The $[Z = -1, L = 2]$ strategy writes the put, bringing in a premium of \$2.3750, corresponding to the option's *bid* price at the market close. The required asset capital, κ_A, for that option is \$603.56, and since the investor deploys a leverage, $L = 2$, he posts capital of $\kappa_E = \$301.78$. The investor's capital is invested at the risk-free rate, with the positions held until February 29, 1996. The risk-free rates corresponding to the trade roll date (29 days) and maturity (45 days) are $r_{f,t+1} = 5.50\%$ and $r_{f,t+\tau} = 5.43\%$, respectively, and are obtained from the OptionMetrics zero-coupon yield curves. On the trade roll date, the option position is closed by repurchasing the index put at the close-of-business *ask* price of \$1.8750. This generates a profit of \$0.50 on the option and \$1.3150 of accrued interest, representing a 60 basis point return on investor capital. Finally, a new strike proposal value, which reflects the prevailing market parameters is computed, and the entire procedure repeats.

2.1.2 Comparison to Capital Decimation Partners

Lo (2001) and Lo and Hasanhodzic (2007) examine the returns to bearing "tail risk" using a related, naked put-writing strategy, employed by a fictitious fund called Capital Decimation Partners (CDP). The strategy involves "shorting out-of-the-money S&P 500 put options on each monthly expiration date for maturities less than or equal to three months, and with strikes approximately 7% out of the money (Table 2, Panel A)." This strike selection is comparable to that of a $Z = -1.0$ strategy, which between 1996-2010 wrote options that were *on average* about 7% out-of-the-money. By contrast, given the margin rule applied in the CDP return computations, the leverage, L, at inception is roughly three and a half times greater than our preferred hedge fund replication strategy. The CDP strategy is assumed "to post 66% of the CBOE margin requirement as collateral," where margin is set equal to $0.15 \cdot S - \max(0, S - K) - \mathcal{P}$. In what follows, we interpret this conservatively to mean that the strategy posts a collateral that is 66% *in excess* of the minimum exchange requirement. Abstracting from the value of the put premium, which is significantly smaller than the other numbers in the computation, and setting the risk-free interest rate to zero, the strategy leverage given our definition is:

$$L^{CDP} \quad = \quad \frac{\kappa_A}{\kappa_E} \approx \frac{0.93 \cdot S}{\left(1 + \frac{2}{3}\right) \cdot (0.15 \cdot S - \max(0, S - 0.93 \cdot S))} = 6.975 \tag{5}$$

This has led some to conclude that put-writing strategies do not represent a viable alternative to hedge fund replication, due to difficulties with surviving exchange margin requirements. As we demonstrate, this is not the case. The strategy that best matches the risk exposure of the aggregate hedge fund universe is comfortably within exchange margin requirements at inception, and also does not violate those requirements intra-month (unreported results).

2.2 Evaluating the Match for the Aggregate Index

Our hedge fund index replication procedure calls for applying varying degrees of leverage to strategies which write equity index puts at four, fixed strike Z-scores, in order to match the in-sample arithmetic mean pre-fee return of the target index. This procedure sidesteps the well-documented problems with regression analysis in the presence of return smoothing, and relies on a simple notion of capital equilibrium whereby downside risks embedded in hedge fund strategies and in derivative markets command similar risk premia. Even though hedge fund returns may be smoothed (Asness, et al. (2001), Getmansky, et al. (2004)), weakening the contemporaneous match, the strategies are anticipated to share a common trend due to the underlying risk premia being earned by both. When applied to the HFRI Fund Weighted Composite over the period from January 1996 to December 2010, this methodology suggests four candidate put writing strategies: $[Z = -0.5, L = 1.7]$, $[Z = -1.0, L = 2.0]$, $[Z = -1.5, L = 2.5]$, and $[Z = -2.0, L = 3.6]$.

To evaluate the quality of our replication procedure, we conduct a variety of statistical tests. The empirical literature studying the characteristics of hedge fund returns has focused on linear regressions of index (and individual fund) returns onto replicating portfolios of tradable indices (Fung and Hsieh (2002, 2004), Agarwal and Naik (2004), Lo and Hasanhodzic (2007)). Consequently, we are interested in how the derivative-based risk benchmarks introduced in this paper compare with commonly used factor models in characterizing the realized returns of the aggregate hedge fund universe. In particular, we consider several popular linear factor models, including the CAPM one-factor model; the Fama-French/Carhart four-factor model; and the Fung-Hsieh nine-factor model, which was specifically developed to describe the risks of well-diversified hedge fund portfolios (Fung and Hsieh (2001, 2004)). Five of the Fung-Hsieh factors are based on lookback straddle returns, to mimic trend-following strategies, whose return characteristics are similar to being *long* options, or volatility (Merton (1981)).[9]

[9] To facilitate comparisons with the other factor models, we represent each of the factors in the form of equivalent zero-investment factor mimicking portfolios. Specifically, we make the following adjustments: (a) returns on the S&P 500 and five trend following factors are computed in excess of the return on the 1-month T-bill (from Ken French's website); (b) the bond market factor is computed as the difference between the monthly return of the 10-year Treasury bond return (CRSP, *b10ret*) and the return on the 1-month T-bill; and (c) the credit factor is computed as the difference between the total return on the Barclays (Lehman) US Credit Bond Index and the return on 10-year Treasury bond return.

Table 2 reports results from regressions of hedge fund excess returns onto factor-mimicking portfolio returns, alongside the corresponding regressions for the four mean-matched put writing portfolios. The regressions use quarterly data spanning the period from January 1996 to December 2010. The linear factor models all achieve high explanatory R^2, ranging from 68% (CAPM) to 82% (Fama-French/Carhart), suggesting a good overall fit. Table 2 also indicates that relative to standard linear factor models hedge funds have delivered *pre-fee* alphas between 7-10% per year, accounting for 67-97% of the excess return earned by the aggregate hedge fund indices. Put differently, standard asset pricing factors account for no more than a third of the risk premium earned by alternatives in the context of the unconditional factor regression.

Table 2 also reports results from regressions of hedge fund excess returns onto the excess returns of the four mean-matched put writing portfolios and a composite put writing index that is formed as an equally-weighted portfolio of the four individual derivative-based strategies. Since the put writing strategies are selected to match the mean in-sample return of the hedge fund index, it is not surprising that the intercepts are statistically indistinguishable from zero, although this is not entirely mechanical since the slope coefficient estimates depart from one. The put writing portfolios have consistently lower explanatory R^2 when compared with the linear factor models that include an intercept, although R^2 values are reasonably large, averaging over 50%. For the put writing composite index, the intercept is essentially zero and the slope is statistically indistinguishable from one, producing a p-value of the joint test of 0.99. The last three specifications in the table include the excess return on the put writing composite index as an additional factor to each of the linear factor models. The coefficient on the put writing composite index is never statistically reliable in the presence of the other factors.

The results so far suggest that while the put writing portfolios replicate the risks of the hedge fund index reasonably well on their own, after controlling for other common factors they appear to be statistically unrelated, offering no improvement in explanatory power. However, this conclusion is highly sensitive to two key assumptions of the regression methodology. First, the regression evaluates the quality of the fit on the basis of a specific criterion – residual variance – which is interesting, but not the only criterion of interest to investors. We explore other distributional properties of residuals below. Second, the regression matches the mean excess return of the hedge fund index with a free parameter (intercept) that will not earned by an investor passively replicating the risks. The consequences of requiring a feasible replicating strategy are significant in both statistical and economic terms. For example, the overall fit suggested by the linear factor model regression is not feasibly achievable, when the measured intercept is positive. By contrast, the put writing portfolios impose feasibility before they are evaluated ensuring that the entirety of the fitted return is achievable. To the extent that feasibility is a desirable feature, we can compute a regression R^2 based on feasible residuals, obtained by constraining the intercept to zero before computing the fitted values from the linear factor model. For the put writing strategies, the feasible residuals are simply the difference in excess returns on the hedge fund index and those of the mean-matched put writing strategies. On this basis, the put writing replicating portfolios deliver R^2 values that are essentially in line with those of the linear factor models. The average feasible R^2 of the three linear models is 55%, while that of the non-linear models is 50%.

Figure 2 summarizes the fit of various *feasible* replicating strategies. The two top panels display the value of an initial $1 investment in the hedge fund index and each of the feasible fitted linear factor model replicating portfolios (i.e. excluding the intercept) (left panel) and each of the fitted put writing replicating portfolios (right panel). The bottom panels plot the time series of drawdowns for each of the replicating models. The linear factor models produce return series that look highly dissimilar to the HFRI return series. On the other hand, all of the put writing strategies produce time series that look virtually identical to the aggregate hedge fund index. The top left panel of Figure 2 shows how the highly significant positive means in the feasible residuals from the linear factor models (Table 2), translate into large shortfalls in the terminal wealth levels, and that this feature is shared by all linear models under consideration. By contrast, the put writing strategies match the losses during the fall of 2008 and the LTCM crisis, the flat performance during the bursting of the Internet bubble, as well as the strong returns during boom periods. While the put writing strategy fails to explain some of the return variation in economically benign times like the bull market between 2002 and 2007, it captures the variation in economically important times remarkably well. Clearly, from the perspective of feasible replicating strategies, the put writing portfolios dominate those implied by the linear factor models because of the massive average return shortfall of the linear factor models.

The remaining statistical question is how different are the residuals across the various models based on criteria other than residual variance, given the residuals have been demeaned. Given the presence of both skewness and excess kurtosis in the raw hedge fund returns (Table 1), we evaluate the ability of the various replicating models to produce residuals free of skewness and excess kurtosis. We rely on the Jarque-Bera statistic (JB-statistic) to test whether the time series of demeaned residuals from the various models exhibit skewness and kurtosis, a popular test of normality. The JB-statistic has a χ-squared distribution with two degrees of freedom. Due to the well known deviations in the distribution of the JB-statistic from its asymptotic distribution in finite samples, we base inference on finite-sample distributions constructed by Monte Carlo. Table 3 reports the JB-statistic and p-values for the residuals from each of the linear factor models, the four put writing portfolios, and the put writing composite index. The JB-statistics are consistently higher for the linear factor models than for the put writing portfolios, rejecting normality at the 2% level for all but the CAPM, which is rejected at the 9% level; and failing to reject normality for all put writing specifications.

Table 3 also reports some additional summary statistics of the fitted replicating portfolio returns, including the estimated CAPM beta, annualized return volatility, the most severe drawdown, along with the root mean squared error (RMSE) of the deviations between the drawdown time series of the hedge fund index and those of the replicating strategy. The replicating portfolios implied by the linear factor models all have CAPM betas of approximately 0.45, and annualized volatilities between 8-9%, which match the HFRI Fund Weighted Composite well. The worst drawdowns range from -22% to -32%, and generally exceed the maximum drawdowns experienced by the aggregate index of -18.8%. The root mean squared errors of the deviations between the drawdown time series of the index and the feasible linear replicating strategies are economically large and range from 4.7% to 7.0%. This confirms the intuition conveyed by the bottom left panel of Figure 2, which illustrates that the drawdowns of the feasible linear replicating strategies are poorly matched with those of the hedge fund index. This owes in part to the failure of linear factor models to deliver enough drift to keep pace with the index, thus slowing down the recovery following adverse shocks (e.g. the bursting of the Internet bubble, the fall of 2008).

By contrast, the put-writing strategies exhibit noticeable cross-sectional variation in their estimated CAPM betas, which decline monotonically from 0.53 for the $[Z = -0.5, L = 1.7]$ strategy to 0.22 for the $[Z = -2.0, L = 3.6]$ strategy. Correspondingly, the volatilities and minimum drawdowns of the strategies also decline. Intuitively, strategies applying higher leverage to further out-of-the-money options reallocate losses to progressively worse states of nature, thus increasing their true economic risk. Linear CAPM betas fail to capture this feature, instead suggesting a declining required rate of return. We return to this point in Section 3, where we evaluate investors' proper cost of capital for allocations to non-linear risk exposures. When compared on the ability to match the time series of the hedge fund index drawdowns, the $Z = \{-1, -1.5, -2\}$ strategies are preferred to the $Z = -0.5$ strategy; all of the non-linear replicating strategies strongly dominate their linear counterparts.

The final analysis reported in Table 3 is a joint test of the mean shortfall being zero (regression intercept for linear factor models equals zero) and the normality of the demeaned residuals. We augment the Jarque-Bera tests statistic by combining it with the square of the *t*-statistic for the mean of the of the feasible residuals (or intercept from the linear factor model regressions). The new test statistic, which we refer to as the JS-statistic, penalizes the residuals for deviations from normality, as well as, large mean shortfalls in replicating a desired returns series, both of which are of interest to an investor seeking a feasible replicating strategy. The JS-statistic ($JS = JB +$ (mean t-stat)^2) is asymptotically χ-square distributed with 3 degrees of freedom, since the JB-statistic has a χ-squared distribution with two degrees of freedom, and the t-statistic of the mean is asymptotically Gaussian and independent of the other two moment estimators. Again, we base inference on finite-sample distributions constructed by Monte Carlo.

The annualized shortfalls of the linear factor models range from 7-10% per annum with t-statistics ranging from 5.8 to 9.5. The feasible residuals exhibit some skewness and kurtosis, though the Jarque-Bera statistic fails to reject normality in the case of the CAPM. The JS-statistic, which evaluates the model fit on the basis of normality and the ability to match mean returns, strongly rejects all linear factor model replicating strategies. Since the non-linear put writing strategies were selected to match the in-sample mean of the hedge fund index, the (annualized) mean of the feasible residuals is close to zero by construction. The JB-statistics are uniformly smaller for the put writing models than for the linear models, indicating that all of the considered put writing replicating portfolios do a good job of removing the skewness and excess kurtosis from the returns of the aggregate hedge fund index. Finally, the JS-statistic of the joint test of mean zero shortfall and normally distributed residuals does not reject any of the put writing models.

The evidence suggests that the overall distributional properties of residuals from the two classes of models are quite similar, with the linear regression models having a slight edge in terms of residual variance, and with the mean-matched put writing portfolios having a slight edge in terms of being free of skewness and excess kurtosis. The dominant difference between the two classes of models concerns the shortfall in mean returns coming from the contribution of passive exposure to capital market risks. One interpretation of these results is that the put writing strategies capture a dimension of hedge fund risk that the linear factor models do not capture and that this risk is associated with an economically large risk premium. For example, it is well understood that option returns reflect the returns to bearing jump and volatility risk (e.g. Carr and Wu (2009), Todorov (2010)), as well as, compensation for systematic demand imbalances (e.g. Garleanu, et al. (2009), Constantinides, et al. (2012)). This is consistent with the notion that hedge funds specialize in the bearing of a particular class of non-traditional, positive *net* supply risks, that may be highly unappealing to a majority of investors. If the non-traditional risks of hedge funds are more heavily smoothed than the traditional risks, then regression analysis will be unable to detect this exposure while controlling for traditional risk factors. Matching on the mean realized return over a sufficiently long sample period may provide a better method for identifying a feasible risk matched alternative.

We explore the robustness of our results by conducting out-of-sample tests in the next section. Additionally, we repeat our analysis using the Dow Jones/Credit Suisse Broad Index, which is a value-weighted index designed to capture the performance of the aggregate hedge fund universe. Given the similarities between the DJ/CS index and the HFRI index evident in Figure 1, it is perhaps unsurprising that our results are qualitatively unchanged. Finally, we verify that our results do not depend on our choice of working with *pre-fee* returns, rather than the *after-fee* returns provided by HFRI and DJ/CS. To maintain consistency with our main analysis, we apply a 2% flat fee and a 10% incentive fee to the put writing portfolios. Our results continue to hold for both indices.

2.3 Replicating Hedge Fund Strategy Returns Out-of-Sample

The close fit of the put writing replicating strategy indicates that in spite of variation in the popularity of individual hedge fund strategies and institutional changes in the industry, the underlying economic risk exposure of hedge funds in the aggregate has remained essentially unchanged over the 15-year sample. At the individual strategy level, there are many reasons to anticipate that risks change through time. The risk properties of the investable universe for a specific strategy changes through time (e.g. the mix of cash-financed and stock-financed deals affects the overall risk properties of merger arbitrage), views about appropriate leverage levels change over time, and the actual classification of strategies is subject to modification through time.

Despite these potential challenges, we evaluate the performance of our replication methodology out-of-sample using a cross-section of hedge fund sub-indices. Specifically, we use the first half of the sample (Jan. 1996 - June 2003) to identify candidate replicating strategies for each hedge fund sub-index; linear factor model replicating portfolios via in-sample regression and put-writing replicating portfolios by matching mean in-sample returns. We then evaluate the quality of the match using the second half of the data (July 2003 - December 2010). We split the full cross-section of twenty HFRI and Dow Jones/Credit Suisse hedge fund indices reported in Table I into two groups: equity-related and non-equity-related. The first group includes the aggregate indices studied earlier and strategies that are likely to share the short-term downside exposure of the put writing portfolios (Event Driven, Distressed, Merger Arbitrage, Equity Long/Short, Equity Market Neutral, and Equity Directional). The second group includes strategies that trade primarily in intermediate or long-term credit, currencies, and commodities (Relative Value, Convertible Arbitrage, Corporate, Macro and Managed Futures). We report the results for the non-equity related strategies for completeness, though economic intuition suggests that these are unlikely to be well described by the short-dated put writing strategies we focus on in this paper. Many strategies in the non-equity grouping are exposed to interest rate risk, which we do not model, unlike the Fung-Hsieh specification which includes two interest rate factors (term and credit spread).[10]

The evaluation procedure involves producing out-of-sample returns and feasible residuals (differences between the out-of-sample returns of each sub-index and the returns of the replicating strategies) for each of the three linear factor models, and the four put-writing strategies. Since there are twenty hedge fund indices and

[10]Coval, et al. (2009) and Jurek and Stafford (2013) show that long-dated credit exposures of structured and traditional corporate can be accurately described using portfolios of U.S. Treasuries and 5-year equity index options. Since the dynamics of long-dated volatility are generally distinct from those of short-dated volatility, the short-dated put-writing strategies we explore are *a priori* not expected to capture interest rate or credit risk.

seven replicating portfolios we have a total of 140 out-of-sample time series of returns and feasible residuals. To parsimoniously characterize our out-of-sample results for each index, we collapse the time series of the replicating returns produced by the three linear models into a single time series by forming an equally-weighted portfolio. We apply the same weighting scheme to the put writing replicating strategies. This produces two time series of replicating returns per hedge fund index to be evaluated out-of-sample.[11]

Tables IV and V report the results of this out-of-sample analysis. Panel A of each table reports the fit of the linear replicating strategies, and Panel B summarizes the fit of the put writing strategies. Using the out-of-sample returns, we report the feasible R^2 to characterize the strategies' ability to explain monthly variation in returns, and the root mean squared error of the out-of-sample drawdown time series to evaluate the downside risk exposure match. Finally, we examine the distributional properties of the feasible residuals as in Table III.

Panel A shows that the linear replicating strategies fail to match the out-of-sample mean returns of all twelve indices, as indicated by the presence of statistically significant mean residuals. The mean shortfalls range from 2.1% (HFRI Equity Hedge: Market Neutral) to 8.8% (HFRI Event Driven: Distressed) and t-statistics between 1.2 and 4.0. Moreover, the linear replicating strategies produce highly non-normal residuals, indicating they have failed to match the downside risk properties of the hedge fund indices. The Jarque-Bera test rejects the normality of the the feasible residuals at the 5% level for all but one investment style. Taken together these facts combine to produce JS-statistics that strongly reject the ability of linear models to replicate the returns *and* risks of all hedge fund styles at the 5% level.

Panel B reports the corresponding values for the out-of-sample fit of the put-writing strategies. On average, the non-linear replicating strategies produce higher out-of-sample feasible R^2 and match the drawdown patterns of the hedge fund indices more closely. The put-writing strategies continue to match the mean returns of the hedge fund strategies out-of-sample, producing mean feasible residuals that are statistically significant in only one case (HFRI Equity Hedge - Market Neutral), where they are negative, indicating that the put writing portfolio *outperformed* the corresponding hedge fund index. The JB statistics are uniformly smaller for the put writing replicating portfolios than those of the linear factor model replicating portfolios for each index individually. Normality of the feasible residuals is not rejected at the 5% level for *any* hedge fund sub-index within this grouping. Correspondingly, the JS-statistic never rejects the joint test that the put-writing strategy has matched the means returns and risks of the hedge fund index at the 5% level.

Figure 3 summarizes these results and provides intuition for the JS-statistic by plotting the pairs of (t-statistic

[11] Because inferences are similar within each class of model (linear factor models or put writing portfolios), our results are qualitatively unchanged if we simply select the strategy that provides the best in-sample fit based on various criteria within each model class, and then use that model to construct out-of-sample replicating returns.

of the mean feasible residual, JB-statistic) for each strategy/model class, along with the 5% confidence level for each test statistic. The left panel corresponds to the in-sample estimation period using the first half of the sample, while the right panel corresponds to the out-of-sample evaluation period based on the second half of the sample. The out-of-sample plot shows that the put writing model is never rejected on both dimensions across the twelve strategies considered, and only rejected once for the mean shortfall. On the other hand, the linear factor model is rejected on both dimensions for ten of the strategies considered. This highlights the robustness of the mean-matched put writing view of hedge fund risks over the linear factor model regression view. The estimation period, January 1996 to June 2003, covers a mostly benign economic environment where regression analysis identifies risk-matched portfolios that result in large unexpected outcomes in the second half of the sample in terms of skewness and kurtosis when the systematic risks of 2008 are realized. The investor using the mean-matched put writing portfolios as a benchmark is not surprised.

Finally, Table V reports the out-of-sample fitting results for the non-equity-related hedge fund subindices. The quality of the fit here is noticeably worse for both linear and non-linear replicating portfolios as evidenced by the lower feasible R^2 and higher root mean squared errors between the drawdown time series of the actual index and the replicating strategies. The returns of the Macro and Managed Futures categories are particularly poorly characterized. This is consistent with the summary statistics reported in Table I, which indicate that the returns of these categories are largely unrelated to the equity market index (low CAPM beta), and are in some instances, positively skewed. Across the various sub-indices, the linear strategies continue to generate positive shortfalls, but their significance is now diminished. Overall, the results from Table V suggest that the JS-test has power to reject the put writing replicating strategies when the match is dissimilar and that even when there is little reason to expect that the short-dated put writing portfolio represents a reasonable replicating strategy, it does no worse than the linear factor model replicating strategies.

3 Required Rates of Return for Downside Risks

The evidence presented so far suggests that it is reasonable to view a portfolio of cash and short positions in short-dated index put options as a feasibly investable alternative to the hedge fund index. In this sample period, hedge fund investors as a group would have been better off bearing this risk in the put writing replicating portfolio, given the large fees that they paid for this exposure. The question that remains is whether either of these strategies have actually covered their proper cost of capital, given they are typically held in such large allocations relative to their equilibrium wealth share.

3.1 The Investor's Cost of Capital

To study investor required rates of return in the context of concentrated portfolios, we rely on the insights of Merton's (1987) model of capital market equilibrium with incomplete information. In this framework investors must become informed about an investment before they allocate capital to it, creating specialization in investing. This leads some investors to hold highly concentrated portfolios, and causes the investor required rate of return to depart from that applicable under the fully diversified equilibrium. In this sprit, we assume that the universe of investors consists of two types: (1) traditional investors who have access to cash and the equity index, but do not invest in alternatives, and (2) endowment investors who have access to both traditional and alternative investments. Consistent with the notion that alternative risks account for a small fraction of the aggregate wealth and are held by a small set of investors in large allocations, we assume that the equity index is priced solely by traditional investors, while alternatives are priced by endowment investors.

We assemble a static framework that combines power utility (CRRA) preferences with a state-contingent asset payoff representation originating in Arrow (1964) and Debreu (1959). To specify the joint structure of asset payoffs, we describe each security's payoff as a function of the log return, $\tilde{r}_m$ on the aggregate equity index (here, the S&P 500).[12] For every $1 invested, the state-contingent payoffs of the three assets are as follows: the risk-free asset pays $\exp(r_f \cdot \tau)$ in all states, the equity index payoff is, by definition, $\exp(r_m)$, and the payoff to the hedge fund investment is $f(r_m)$. The analysis in Section 2 indicates that the state-contingent payoff to alternatives can be accurately characterized using simple levered portfolios of index put options justifying the existence of a suitable payoff representation. To emphasize that the payoff of the replicating portfolio depends on the initial put premium, as in (2), we write, $f(r_m, \mathcal{P})$. Given a realization of the market return, $\tilde{r}_m$, the agent's utility is given by:

$$U(\tilde{r}_m) = \frac{1}{1-\gamma} \cdot \Big((1-\omega_m-\omega_a) \cdot \exp(r_f \cdot \tau) + \omega_m \cdot \exp(\tilde{r}_m) + \omega_a \cdot f(\tilde{r}_m, \mathcal{P}) \Big)^{1-\gamma} \tag{6}$$

where, ω_m and ω_a, are his allocations to the equity market and alternatives, respectively. Finally, to operationalize the framework we need to specify the investor's risk aversion, γ, and the distribution of the log market index return, $\phi(r_m)$.

We are interested in studying the asset pricing implications of a segmented market equilibrium in which the investor's allocation to alternatives, ω_a, is pre-specified exogenously to satisfy market clearing. We estimate this

[12] By specifying the joint distribution of returns using state-contingent payoff functions, we can allow security-level exposures to depend on the market state non-linearly, generalizing the linear correlation structure implicit in mean-variance analysis. Patton (2004), Harvey, et al. (2010), and Martellini and Ziemann (2010) emphasize the importance of higher-order moments and the asset return dependence structure for portfolio selection.

value based on observed allocations by Ivy League endowments to be between $\omega_a = 35\%$ to 50%. Taking the alternative allocation, ω_a, as given, we solve for the equilibrium equity market allocation and a valuation for the put option, $(\omega_m^*, \mathcal{P}^*)$, which jointly satisfy the investor's two first order conditions with respect to the portfolio weights. At the constrained equilibrium, the endowment investor's subjective valuations of the equity index and the alternative payoff, $f(\tilde{r}_m, \mathcal{P})$, match their market prices, which have both been normalized to one:

$$E_t\left[\Lambda\left(\omega_m^*, \omega_a, \mathcal{P}^*\right) \cdot \exp\left(\tilde{r}_m\right)\right] = 1 \quad (7)$$

$$E_t\left[\Lambda\left(\omega_m^*, \omega_a, \mathcal{P}^*\right) \cdot f\left(\tilde{r}_m, \mathcal{P}^*\right)\right] = 1 \quad (8)$$

where: $\Lambda = \exp(-r_f \cdot \tau) \cdot \frac{U'(\cdot)}{E_t[U'(\cdot)]}$, is the investor's subjective pricing kernel, and ω_a is the allocation to alternatives, which is determined by the equilibrium supply of this type of risk relative to the aggregate wealth of endowment investors. The first equation ensures the investor is at his optimal allocation to equities, and therefore that subsequent required rate of return computations are based on (constrained) optimal portfolios. The second equation pins down his subjective valuation for the put option embedded in the alternative investment, $\mathcal{P}^*$, and is used to determine the required rate of return on alternatives via (9). Both of these equations are computed taking the distribution of equity index returns as exogenous, reflecting the assumption that the endowment investor is assumed to be a price taker in this market. Once we have solved for the equilibrium value of $\mathcal{P}^*$, the endowment investor's required excess rate of return on the alternative investment is:

$$\begin{aligned} r_a^*\left(\omega_a\right) &= \frac{1}{\tau} \cdot \ln E_t\left[\frac{f\left(\tilde{r}_m, \mathcal{P}^*\right)}{E_t\left[\Lambda\left(\omega_m^*, \omega_a, \mathcal{P}^*\right) \cdot f\left(\tilde{r}_m, \mathcal{P}^*\right)\right]}\right] - r_f \\ &= \frac{1}{\tau} \cdot \ln E_t\left[f\left(\tilde{r}_m, \mathcal{P}^*\right)\right] - r_f \end{aligned} \quad (9)$$

Given our focus on a single-factor payoff representation, we contrast the proper required rate of return, (9), with the corresponding rate of return based on the linear CAPM rule, $\beta \cdot \lambda$, where $\beta = \frac{Cov[r_a, r_m]}{Var[r_m]}$ is the CAPM β of the alternative on the equity index and λ is the market risk premium. The CAPM equilibrium logic identifies the market risk premium, λ, as the rate of return, under which the representative investor is fully invested in the portfolio of risky assets. Given a risk aversion, γ, for the representative agent and a Gaussian distribution for the equity index, the equilibrium market risk premium is given by, $\lambda = \gamma \cdot \sigma_m^2$.

3.2 Baseline Model Parameters

The investor's cost of capital is a function of model parameters describing the distribution of the (log) market return, investor's risk tolerance, investor's allocation to other assets, and the structure of the alternative investment (e.g. option strike price and leverage). Before turning to a discussion of the comparative statics of the investor's cost of capital, we describe the baseline model parameters.

3.2.1 Investor types and risk aversion, γ

We consider two investor types in our analysis and model calibration. The first investor type – the *traditional* investor – is assumed to have access to cash (risk-free bonds) and equities, whereas the second investor – the *endowment* investor – additionally has access to alternative investments. The investors have a risk aversion, $\gamma = 3.3$, and in the absence of alternatives, are each assumed to hold a portfolio of 40% cash and 60% equities, corresponding to an allocation commonly used as a benchmark by endowments and pension plans.

3.2.2 Equity index return distribution, $\phi(r_m)$

Given our focus on pricing payoffs with non-linear downside risk exposures, we choose a parametrization for the equity index distribution, which can accommodate the empirical evidence of skewness and kurtosis in index returns. Specifically, we rely on the normal inverse Gaussian (NIG) distribution, which allows us to flexibly specify the first four moments (Appendix A). Since required rates of return will be increasing in the severity of tail outcomes, we conservatively calibrate the distribution to match the properties of historical returns. To the extent that our sample understates the severity of the possible outcomes, our calibration will produce downward biased estimates of required rates of return. A similar effect would occur if the tails of the true return distribution are heavier than implied by the NIG parametrization, e.g. exhibit power law decay (Gabaix (2009)).

We set the annualized volatility, σ, of the distribution to 17.8%, or 0.8 times the average value of the CBOE VIX index our sample (1996-2010: 22.2%). This scaling is designed to remove the effect of jump and volatility risk premia embedded in index option prices used to compute the index (e.g. Carr and Wu (2009), Todorov (2010)), as well as, the effect of demand imbalances (e.g. Garleanu, et al. (2009), Constantinides, et al. (2012)).[13] The remaining moments are chosen to roughly match historical features of monthly S&P 500 Z-scores, obtained by demeaning the time-series of monthly log returns and scaling them by 0.8 of the VIX as of the preceding month end. Specifically, we target a monthly Z-score skewness, $\mathcal{S}$, of -1, and kurtosis, $\mathcal{K}$, of 7. These parameters

[13]The scaling parameter was chosen on the basis of a historical regression of monthly realized S&P 500 volatility, computed using daily returns, onto the value of the VIX index as of the close of the preceding month (1986 to 2010). The slope of this regression is 0.82, with a standard error of 0.05.

combine to produce a left-tail "event"once every 5 years that results in a mean monthly Z-score of -3.6. For comparison, the mean value of the Z-score under the standard normal (Gaussian) distribution, conditional on being in the left 1/60 percent of the distribution, is -2.5.[14]

We fix the equilibrium equity risk premium by imposing that a $\tilde{\gamma} = 2$ investor is fully invested in the equity market. In a Gaussian setting, this is equivalent to an optimal equity allocation of 60% for the traditional investor ($\gamma = 3.3$), since the risky asset allocation is inversely related to the coefficient of relative risk aversion. This pins down the conditional distribution from which we simulate τ-period log index returns:

$$r_m = (r_f + \lambda - k_Z(1)) \cdot \tau + \tilde{Z}_\tau, \quad \tilde{Z}_\tau \sim \text{NIG}\,(0,\, \mathcal{V},\, \mathcal{S},\, \mathcal{K}) \tag{10}$$

where $\mathcal{V} = \sigma^2 \cdot \tau$ is the τ-period variance. The market risk premium, $\lambda = k_Z(-\tilde{\gamma}) + k_Z(1) - k_Z(1 - \tilde{\gamma})$, and the convexity adjustment (Jensen) term $k_Z(1)$, depend on the cumulant generating function, $k_Z(u)$, of the shock, $\tilde{Z}_\tau$, and are given in Appendix A. Under the baseline model parameters, the Gaussian component of the equity risk premium equals 6.31%, with the higher order cumulants contributing an additional 0.25%. Finally, we set the risk-free rate, r_f, and equity market dividend yield, δ, to their sample averages, which are equal to 3.1% and 1.7%, respectively.

3.2.3 Alternative investment, $[Z,\, L]$

The payoff of the alternative investment is represented using a levered, naked put writing portfolio, as in the empirical analysis in Section 2. Specifically, we assume that the investor places his capital, ω_a, in a limited liability company (LLC) to eliminate the possibility of losing more than his initial contribution. Limited liability structures are standard in essentially all alternative investments, private equity and hedge funds alike, effectively converting their payoffs into put spreads. In practice, the cost of establishing this structure is minimal relative to the assets under management, hence we approximate its cost as zero. Given a leverage of L, the quantity of puts that can be supported per \$1 of investor capital is given by:

$$q = \frac{L}{\exp\left(-r_f \cdot \tau\right) \cdot K(Z) - \mathcal{P}(K(Z),\, \tau)} \tag{11}$$

where $K(Z)$ is the strike corresponding to a Z-score, Z. The put premium and the agent's capital grow at the risk free rate over the life of the trade, and are offset at maturity by any losses on the index puts to produce a terminal

[14] Based on a preceding month-end VIX value of 22.4%, and our parameterization of the NIG distribution, the -21.6% return of the S&P 500 index in October 1987 corresponds to a Z-score of -4.7. The probability of observing a monthly return at least as bad as this is 0.2% under the NIG distribution, and 0.0001% under the Gaussian distribution.

state-contingent payoff:

$$f(\tilde{r}_m, \mathcal{P}) \quad = \quad \max\Big(0, \exp(r_f \cdot \tau) \cdot (1 + q \cdot \mathcal{P}(K(Z), \tau)) - q \cdot \max(K(Z) - \exp(\tilde{r}_m), 0)\Big) \quad (12)$$

The terminal payoff of the alternative depends on the initial put premium, $\mathcal{P}$, and the terminal realization of the equity index. We substitute this payoff function into the endowment investor's first order conditions to determine his shadow valuation of the put option, and therefore, his required rate of return on the alternative investment.

3.3 Comparative Statics

The novel components of the framework as applied to alternative investments are the nonlinear downside exposure as proxied by the put writing strategy and the large allocation to alternatives among the few who invest at all. The large allocation relative to the aggregate share of total risks is presumably due to market frictions that result in market segmentation. Figure 4 explores the consequences of this friction for the endowment investor as a comparative static in ω_a for the optimal portfolio weights and required returns. In the absence of market frictions these risks can be diffusely held and ω_a will be small, whereas if market frictions force high degrees of specialization then ω_a can be quite large. To illustrate the comparative statics we proxy the alternative investment with the $[Z = -1, L = 2]$ put writing strategy. The top left panel plots the portfolio weights. By construction, the allocation to alternatives is a 45 degree line. The optimal allocation to the equity index declines monotonically as market frictions force ω_a to be large. The top right panel illustrates the consequences of changing ω_a for required rates of return. By assumption, the endowment investor takes the price of the equity index as given, so the required return on the equity index remains constant as ω_a changes. So too, does the required return on the alternative investment based on the CAPM implementation ($\beta \cdot \tilde{\gamma}\sigma^2$, with $\beta = 0.4$), which ignores both the nonlinearity and the large allocation. The proper required return for the endowment investor is increasing and convex in ω_a. Moreover, even at a tiny allocation to alternatives the proper required return meaningfully exceeds the CAPM calculation due to the nonlinear risk profile. Another notable feature of this analysis is that the endowment investors optimal allocation to risky assets (alternatives + equity) is increasing in ω_a over the considered range. The allocation to alternatives is initially removing risk from the endowment investors portfolio allowing the total allocation to increase from the 60% risky and 40% safe benchmark, which rationalizes observed endowment portfolio risk-on tilts.

Another interesting comparative static is the sensitivity of the endowment investor's portfolio and required rates of return as volatility changes. The bottom panels of Figure 4 explore the same model properties as a function of volatility, around a fixed equilibrium allocation to alternatives of $\omega_a = 35\%$. The bottom left panel shows that there is a subtle adjustment to the equity index allocation, whereby the endowment investor reduces equity exposure as volatility increases. The bottom right panel shows that required returns for both the equity index (determined by the traditional investor) and the alternative investment increase sharply as volatility increases. Moreover, the proper required return for alternatives increases much more rapidly than implied by the CAPM.

4 Evaluating Downside Risks against a Proper Cost of Capital

Traditional cost of capital computations, based on linear factor models, assume that: (1) an investor's allocation to the risk being evaluated represents an infinitesimal deviation from their efficient portfolio, and (2) the payoff structure of the risk can be well-described via its covariances with the factors. In practice, neither of these assumptions fits the problem of a typical investor in alternatives. First, given the specialized investment expertise required to evaluate and monitor these investments, it is common for allocations to be large relative to the supply of these risks, in order to amortize the fixed costs associated with expanding the investment universe to include alternatives (Merton (1987)). Second, the analysis of feasible replicating residuals in Section 2 demonstrates that the returns of many hedge fund strategies are matched more closely by the returns of (non-linear) put writing strategies, than by the returns of replicating portfolios suggested by commonly used linear factor models (CAPM, Fama-French, Fung-Hsieh). Taken together, these deviations suggest that inferences regarding the investor's cost of capital based on standard factor regressions are likely to be biased. Recall, such regressions suggest a cost of capital ranging between 0-3% per annum (Table II). This section revisits the excess returns to hedge fund strategies from the perspective of the state-contingent framework assembled in Section 3, which was explicitly designed to handle large allocations to non-linear risks.

4.1 The Time Series of the Proper Cost of Capital

To evaluate the realized performance of the aggregate hedge fund universe and the equity-related sub-indices that were well-described by the put writing portfolios, we use the state-contingent payoff model developed in Section 3 to produce a time series of required rates of return for the endowment investor introduced in the previous section. The endowment investor without access to alternatives would normally allocate 60% to stocks and 40% to risk-free securities, but we assume they allocate either 35% or 50% to alternatives to match the holdings of various Ivy League endowments.

To produce the time series of proper required rates of return for each considered downside risk profile, at each rebalancing date we supply the model the specific composition of the fitted put writing replicating portfolio for various hedge fund indices considered in Section 2, along with parameters characterizing the terminal distribution of the (log) equity index return. At each point in time, the composition of the put writing replicating portfolio is pinned down by the option strike, $K(Z)$, and the option price, $\mathcal{P}(Z)$, which jointly with L, determine the quantity of options sold, and the investor's capital. For parsimony, we hold the skewness and kurtosis of the market return distribution fixed at their baseline values, and only let the market return volatility, σ_t, vary through time, by setting it equal to 0.8 times the prevailing value of the VIX on each rebalancing date. The time series of market volatility also pins down the time series of the equilibrium market risk premium, λ_t (Appendix A). For comparison, we also produce a time series of required rates of return for hedge funds based on the linear CAPM model by multiplying the β_t of the option replicating portfolio (at inception) by the CAPM market risk premium, $\tilde{\gamma} \cdot \sigma_t^2$, where $\tilde{\gamma} = 2$ is the risk aversion of the "all equity" investor.

Before comparing the model required rates of return to the realized returns of hedge funds and put writing strategies, we convert the continuously compounded required rates of return, $r_a^*(\omega_a)$, given by (9) and plotted in Figure 4, into discretely-compounded net returns, and compute the required rate of return given the investor's allocation to alternatives, ω_a. To obtain the discretely compounded monthly return, we scale the annualized continuously compounded rate by $\frac{1}{12}$, exponentiate it, and subtract one. We repeat this procedure at each rebalancing date to produce a monthly time series of average required rates of return for use in performance evaluation.

4.2 Estimates of Hedge Fund Alphas

Panel A of Table 6 reports the annual time series from 1996 through 2010 of various measures of volatility, as well as the excess realized and required returns for the S&P 500 index, the after-fee HFRI Fund Weighted Composite, and the pre-fee put-writing replicating portfolio. The returns to put portfolio are computed as an equal-weighted average of the returns to the four put writing strategies identified in-sample. The table shows that the simple estimate of volatility ($\sigma_t = 0.8 \cdot VIX_t$) corresponds closely to realized volatility year-by-year and on average. *Mean* reports the full-sample average with *t*-statistics reported in square brackets. Over this period, the stock market index realized, on average, an annualized excess return of 5.1%, while the traditional investor with no allocation to alternatives required 7.6% per year, given the realized path of volatility over the sample. As a point of comparison, we also report the CAPM required return for the equity index, $r_{m,t}^* = \tilde{\gamma}\sigma_t^2$, which averages 7.2%. The small 40 basis point wedge in required returns for the equity index is created by the proper accounting for skewness and kurtosis of the assumed equity index distribution. These estimates reflect the severe consequences of 2008 and 2009, when realized returns were low and realized volatility was high. Over the more economically benign period of 1996 through 2007, the stock market index average annual excess return is 6.5%, the required excess return for the traditional investor is 6.2%. These computations suggest that our model calibration produces sensible required rates of return for traditional investments and that the sample period is not particularly unusual.

We now turn to an evaluation of the after-fee HFRI Fund Weighted Composite, which has a mean realized excess return is 6.3%. An endowment investor, proxying this risk with the put writing composite index identified in Section 2, who allocates 35% to alternatives requires 6.9% for this exposure; and 8.7% with a 50% allocation to alternatives. Both of these requirements are considerably higher than the CAPM required excess rate of return, which stands at 3%. The wedge between the model and linear CAPM risk premia reflects the non-linearity of the downside risk exposure, as well as the concentrated portfolio allocation. This can be seen by setting the alternative allocation to a tiny amount, which produces an average required excess return of 4.1% over the sample period. In other words, this calculation isolates the effect of the nonlinearity (1.1% over the CAPM required excess return). The model risk premium is very volatile, averaging nearly 20% for the endowment investor with a large allocation in 2008 and 2009, when both the VIX and realized volatility are high.

Panel B of Table 6 reports estimated alphas based on the CAPM and the generalized model required rate of return for the HFRI Fund Weighted Composite under various assumptions. The annualized CAPM alpha is 3.3% (t-statistic = 1.7), which is nearly 1% lower than the one reported in Table 1 due to the average of the time varying CAPM risk premium being somewhat higher than the market risk premium realized in-sample. The endowment investor with a 35% allocation to alternatives realizes an annualized alpha of -0.6% (t-statistic = -0.3), while the endowment investor with a 50% alternatives allocation realizes an annualized alpha of -2.5% (t-statistic = -1.2), neither of which are statistically distinguishable from zero. These results indicate that sophisticated endowment investors, who had access to performance comparable to that of the survivorship-biased index, have barely covered their properly computed cost of capital.

Table 6 also reports the results from these same analyses conducted on the pre-fee put writing replicating portfolio, which was found to match well the risk properties of the HFRI index, and therefore commands the same required return. The mean annualized excess return of the put writing portfolio exceeds that of the after-fee HFRI index by 4% per year, indicating that a passive low cost exposure to downside risk through index derivatives may be preferable to direct investment in hedge funds. For example, the endowment investor with a 35% allocation to the put writing composite index earns an alpha of 3.3% (t-statistic: 1.8) per year; and with a 50% allocation, earns an alpha of 1.5% (t-statistic: 0.8) per year. We investigate this result in more detail below.

Table 7 reports alpha estimates for the equity-related HFRI sub-indices whose risks were generally well-explained by the put writing replicating portfolios. As before, we compute alpha estimates relative to the linear CAPM model, and the generalized model for the endowment investor at two relatively large allocations to alternative investments. Panel A displays results for the after-fee hedge fund indices and Panel B displays those for the associated pre-fee put writing portfolios. The endowment investor at either a 35% or 50% allocation to alternatives realizes consistently small negative, but statistically unreliable alphas across all of the considered hedge fund strategies over the period 1996-2010, while the endowment investor realized consistently negative, and again, statistically unreliable alphas, ranging from -3.9% to 1.5%. This suggests that the investors in equity-related hedge funds have barely earned their cost of capital, despite a finding of generally significantly positive CAPM alpha estimates, ranging from 1.0% to 4.5%.

Overall, the evidence presented in Tables 6 and 7 is inconsistent with claims that sophisticated endowment investors earned an "illiquidity premium" for investing in alternatives. To the extent that such a risk premium is in fact responsible for explaining the realized returns on the HFRI Fund Weighted Composite and the put writing strategy, this channel has been left unmodelled in our cost of capital computations. Consequently, our cost of capital estimates are biased *downward*. Even relative to this impoverished benchmark, we find that investors with concentrated hedge fund allocations did not earn statistically reliable alphas after fees between 1996 and 2010. This casts doubt on the so-called "endowment model" which is based on the premise that illiquid investments earn an additional risk premium that long-term institutional investors will have a comparative advantage in bearing (Swensen (2000)). Finally, while it is reasonable to expect that the first endowments to allocate to alternatives have actually earned the returns associated with the published indices, thus covering their cost of capital, more recent investors in alternatives are likely to have earned something closer to the average fund-of-fund return. These returns are on average over 300 basis points lower *per year*, than the reported average aggregate hedge fund return, suggesting these investors have not covered their cost of capital.

4.3 A New Perspective on the Expensiveness of Index Put Options

The pre-fee put writing returns are consistently about 3.5% to 4% higher than their associated hedge fund strategy return, which translates into consistently positive alphas for both investor types. The endowment investor with a 35% allocation to the put writing composite (equal-weighted average of the four strategies identified in sample) generally realizes statistically significant alphas from the put writing portfolios, while the endowment investor with a 50% allocation generally realized alphas that are statistically indistinguishable from zero. These findings contrast with much of the existing literature, which documents high negative (positive) risk-adjusted returns to buying (selling) index options (e.g. Coval and Shumway (2001), Bakshi and Kapadia (2003), Bondarenko (2003), Frazzini and Pedersen (2011), Constantinides, et al. (2012)).[15] The conclusion of index put

[15]Coval and Shumway (2001) report that zero-beta, at-the-money straddle positions produce average losses of approximately 3% per week. Bakshi and Kapadia (2003) conclude in favor of a negative volatility risk premium by examining delta-hedged options returns.

options being highly expensive implicitly assumes that an investor who is short these portfolios would earn the negative of the long portfolio returns. This is far from the reality, as an investor with a short position would be required to post sufficient margin to initiate the position and maintain sufficient margin to survive the sample paths realized *ex post* in the data (Santa-Clara and Saretto (2009)).

The annualized alphas we report are an order of magnitude lower than reported in previous papers. This difference is due to: (1) incorporating margin requirements, as emphasized by Santa-Clara and Saretto (2009); and (2) a cost of capital computation that explicitly accounts for the non-linearity of the payoff profiles and investor portfolio concentration. Importantly, the large margin requirements for short positions in index put options effectively make these positive net supply assets; the supplier of these payoffs has to allocate considerably more capital to this activity than that implied in the frictionless models of Black-Scholes/Merton (1973). Moreover, this is a risk that is not well distributed throughout the economy, as the suppliers of these securities are typically highly specialized in bearing this risk. The same channel highlighted in our paper – concentrated portfolios require additional risk premium above the frictionless model, especially when a nonlinear downside exposure is present – manifests itself here. From the perspective of the frictionless model, both alternative investments and index put options seem expensive, but much less so from the perspective of specialized investors (see also, Garleanu, Pedersen, and Poteshman (2009)). These two frictionless model anomalies are fairly consistent with one another after accounting for these two notable features.

Finally, it is important to recall that these calculations rely upon a specific distributional assumption about the underlying stock market index, which is roughly consistent with the historical experience. A slightly worse left tail will have a meaningful effect on the required returns for these portfolios, given their nonlinear risk profiles and the large allocation sizes.

5 Conclusion

This paper argues that the risks borne by hedge fund investors are likely to be positive net supply risks that are unappealing to average investors, such that they may earn a premium relative to traditional assets. A distinguishing feature of many of these risks is that their payoff profiles have a distinct possibility of being non-linear with respect to a broad portfolio of traditional assets. These non-linearities can arise either directly from the underlying economic risk exposure (e.g. credit risk, merger arbitrage), or through the institutional structure through which they are borne (e.g. funding liquidity). Our analysis focuses attention on investors with potentially

Frazzini and Pedersen (2011) report mean monthly delta-hedged excess returns between -9.5% (at-the-money) and -30% (deep out-of-the-money) for one-month index put options.

large allocations to such non-linear risk exposures, who may only be allowed to rebalance infrequently. This setup is common in practice, but infrequently examined in the literature, which has placed emphasis on continuous trading and/or assets whose risks can be well described by their covariance with each other over the rebalancing horizon.

We begin by documenting that simple put writing strategies can be used to match both the risks and *pre-fee* returns of the aggregate hedge fund universe, as well as, many equity-related hedge fund sub-indices. This contrasts starkly with replicating strategies suggested by linear factor models (CAPM, Fama-French/Carhart, Fung-Hsieh), which deliver high R^2, but consistently fail to match the mean rate of return of a hedge fund indices. Our non-linear replicating strategies dominate linear replicating portfolio both in-sample and out-of-sample. Along the way, we introduce a novel test statistic, which can be used to evaluate the ability of a feasible replicating strategy to match the returns and higher-order moments of the target return series.

We then exploit the transparency of the state-contingent payoffs of the risk-matched put writing portfolios to develop estimates of the cost of capital for allocations to alternatives with downside risks. The model required rates of return vary as a function of investor preferences and allocations, the non-linearity of the portfolio (option strike price and leverage), and the properties of the underlying equity market return distribution (volatility and tail risks). One of the attractive features of this simple generalized framework is that it conceptually requires no information beyond the traditional analysis, although in practice it will require more sophisticated judgment over the state-contingent risk profile of alternative investments.

An accurate assessment of the cost of capital is fundamental to the efficient allocation of capital throughout the economy. Investment managers should select risks that are expected to deliver returns at least as large as those required by their capital providers. The investors in alternatives should require returns for each investment that compensate them for the marginal contribution of risk to their overall portfolio. In the case of investments with downside exposure, the magnitude of these required returns is large relative to those implied by linear risk models. As the allocation to downside risks gets large, the marginal contribution of risk to the overall portfolio expands quickly, requiring further compensation. In practice, investors frequently seem surprised by increases in return correlations between alternatives and traditional assets (or between alternatives themselves) as economic conditions deteriorate, suggesting they may not fully appreciate their portfolio-level downside risk exposure. This *ex post* surprise likely coincides with meaningful *ex ante* errors in estimates of required rates of return, and therefore inappropriate capital allocations. The calibrations in this paper suggest that despite the seemingly appealing return history of alternative investments, many investors have not covered their cost of capital.

A Asset Pricing with NIG Distributions

The normal inverse Gaussian (NIG) distribution is characterized by four parameters, (a, b, c, d). The first two parameters control the tail heavyness and asymmetry, and the second two – the location and scale of the distribution. The density of the NIG distribution is given by:

$$f(x; a,\, b,\, c,\, d) \quad = \quad \frac{a \cdot d \cdot K_1\left(a \cdot \sqrt{d^2 + (x-c)^2}\right)}{\pi \cdot \sqrt{d^2 + (x-c)^2}} \cdot \exp\Big(d \cdot \eta + b \cdot (x-c)\Big) \tag{A.1}$$

where K_1 is the modified Bessel function of the third kind with index 1 (Abramowitz and Stegun (1965)) and $\eta = \sqrt{a^2 - b^2}$ with $0 \leq |b| < a$). Given the desired set of moments for the NIG distribution – mean ($\mathcal{M}$), variance ($\mathcal{V}$), skewness ($\mathcal{S}$), and kurtosis ($\mathcal{K}$) – the parameters of the distribution can be obtained from:

$$a \quad = \quad \sqrt{\frac{3 \cdot \mathcal{K} - 4 \cdot \mathcal{S}^2 - 9}{\mathcal{V} \cdot \left(\mathcal{K} - \frac{5}{3} \cdot \mathcal{S}^2 - 3\right)^2}} \tag{A.2}$$

$$b \quad = \quad \frac{\mathcal{S}}{\sqrt{\mathcal{V}} \cdot \left(\mathcal{K} - \frac{5}{3} \cdot \mathcal{S}^2 - 3\right)} \tag{A.3}$$

$$c \quad = \quad \mathcal{M} - \frac{3 \cdot \mathcal{S} \cdot \sqrt{\mathcal{V}}}{3 \cdot \mathcal{K} - 4 \cdot \mathcal{S}^2 - 9} \tag{A.4}$$

$$d \quad = \quad \frac{3^{\frac{3}{2}} \cdot \sqrt{\mathcal{V} \cdot \left(\mathcal{K} - \frac{5}{3} \cdot \mathcal{S}^2 - 3\right)}}{3 \cdot \mathcal{K} - 4 \cdot \mathcal{S}^2 - 9} \tag{A.5}$$

In order for the distribution to be well-defined we need, $\mathcal{K} > 3 + \frac{5}{3} \cdot \mathcal{S}^2$. The NIG-distribution has closed-form expressions for its moment-generating and characteristic functions, which are convenient for deriving equilibrium risk premia and option prices. Specifically, the moment generating function is:

$$E\left[\exp(u \cdot x)\right] \quad = \quad \exp\left(c \cdot u + d \cdot \left(\eta - \sqrt{a^2 - (b+u)^2}\right)\right) \tag{A.6}$$

A.1 Pricing Kernel and Risk Premia

Suppose the value of the aggregate wealth portfolio evolves according to:

$$W_{t+\tau} \quad = \quad W_t \cdot \exp\Big(\left(\mu - k_Z\left(1\right)\right) \cdot \tau + Z_{t+\tau}\Big) \tag{A.7}$$

where $k_Z(u)$ the cumulant generating function of random variable $Z_{t+\tau}$:

$$k_Z(u) \quad = \quad \frac{1}{\tau} \cdot \ln E_t\left[\exp\left(u \cdot Z_{t+\tau}\right)\right] = c \cdot u + d \cdot \left(\eta - \sqrt{a^2 - (b+u)^2}\right) \tag{A.8}$$

If markets are complete, there will exist a unique pricing kernel, $\Lambda_{t+\tau}$, which prices the wealth portfolio, as well as, the risk-free asset. Assuming the representative agent has CRRA utility with coefficient of relative risk aversion, γ, the pricing kernel in the economy is an exponential martingale given by:

$$\frac{\Lambda_{t+\tau}}{\Lambda_t} \quad = \quad \exp\Big(-r_f \cdot \tau - \gamma \cdot Z_{t+\tau} - k_Z\left(-\gamma\right) \cdot \tau\Big) \tag{A.9}$$

Now consider assets whose terminal payoff has a linear loading, β, on the aggregate shock , $Z_{t+\tau}$, and an independent idiosyncratic shock, $Z_{i,t+\tau}$:

$$P_{t+\tau} \quad = \quad P_t \cdot \exp\Big(\left(\mu(\beta) - k_Z(\beta) - k_{Z_i}(1)\right) \cdot \tau + \beta \cdot Z_{t+\tau} + Z_{i,t+\tau} \Big) \tag{A.10}$$

where $\mu(\beta)$ is the equilibrium rate of return on the asset, and the two $k(\cdot)$ terms compensate for the convexity of the systematic and idiosyncratic innovations. For example, when $\beta = 1$ and the variance of the idiosyncratic shocks goes to zero, the asset converges to a claim on the aggregate wealth portfolio. Assets with $\beta < 1$ ($\beta > 1$) are concave (convex) with respect to the aggregate wealth portfolio.

To derive the equilibrium risk premium for such assets, we make use of the equilibrium pricing condition:

$$\Lambda_t \cdot P_t = E_t\left[\Lambda_{t+\tau} \cdot P_{t+\tau}\right] \quad \Leftrightarrow \quad 0 = \frac{1}{\tau} \cdot \ln E_t\left[\frac{\Lambda_{t+\tau}}{\Lambda_t} \cdot \frac{P_{t+\tau}}{P_t}\right] \tag{A.11}$$

Substituting the payoff function into the above condition and taking advantage of the independence of the aggregate and idiosyncratic shocks, yields the following expression for the equilibrium risk premium on an asset with loading β on the aggregate wealth shock:

$$\mu(\beta) - r_f \quad = \quad k_Z\left(-\gamma\right) + k_Z\left(\beta\right) - k_Z\left(\beta - \gamma\right) \tag{A.12}$$

This expression generalizes the standard CAPM risk-premium expression from mean-variance analysis to allow for the existence of higher moments in the shocks to the aggregate market portfolio. In particular, the risk premium of the equity index, λ, is given by:

$$\lambda \quad = \quad k_Z\left(-\gamma\right) + k_Z\left(1\right) - k_Z\left(1 - \gamma\right) \tag{A.13}$$

and is a function of the (instantaneous) moments of the shocks $Z_{t+\tau}$. For a Gaussian-distributed shock, $Z_{t+\tau}$, the cumulant generating function is given by $k_Z(u) = \frac{1}{\tau} \cdot \frac{\left(\sigma \cdot \sqrt{\tau} \cdot u\right)^2}{2}$, such that, $(A.12)$, specializes to:

$$\begin{aligned} \mu(\beta) - r_f \quad &= \quad \frac{\sigma^2}{2} \cdot \left((-\gamma)^2 + \beta^2 - (\beta - \gamma)^2\right) \\ &= \quad \beta \cdot \gamma \cdot \sigma^2 = \beta \cdot \left(\mu(1) - r_f\right) \end{aligned} \tag{A.14}$$

In our generalized setting, the risk premium on an asset with loading β on the innovations to the market portfolio does not equal β times the market risk premium, unlike in the standard CAPM. The discrepancy is specifically related to the existence of higher moments in the shocks to the aggregate market portfolio.

Equilibrium risk premia can also be linked to the moments of the underlying distribution of the shocks to the aggregate portfolio, by taking advantage of an infinite series expansion of the cumulant generating function and the underlying cumulants of the distribution of $Z_{t+\tau}$:

$$\mu(\beta) - r_f \quad = \quad \frac{1}{\tau} \cdot \sum_{n=2}^{\infty} \frac{\kappa_n \cdot \left((-\gamma)^n + \beta^n - (\beta - \gamma)^n\right)}{n!} \tag{A.15}$$

Note, that the leading term in the above expression is $\beta \cdot \gamma\sigma^2$, consistent with the standard linear CAPM. The consecutive cumulants, κ_n, are obtained by evaluating the n^{th} derivative of the cumulant generating function at $u = 0$. The cumulants can then be mapped to central moments: $\kappa_2 = \mathcal{V}$, $\kappa_3 = \mathcal{S} \cdot \mathcal{V}^{\frac{3}{2}}$, and $\kappa_4 = \mathcal{K} \cdot \mathcal{V}^2$. Using the value for the first four terms, the equilibrium risk premium is approximately equal to:

$$\mu(\beta) - r_f \quad \approx \quad \frac{1}{\tau} \cdot \left\{ \beta \cdot \gamma \cdot \mathcal{V} + \frac{\beta^2 \cdot \gamma - \beta \cdot \gamma^2}{2} \cdot \mathcal{S} \cdot \mathcal{V}^{\frac{3}{2}} + \frac{2 \cdot \beta^3 \cdot \gamma - 3 \cdot \beta^2 \cdot \gamma^2 + 2 \cdot \beta \cdot \gamma^3}{12} \cdot \mathcal{K} \cdot \mathcal{V}^2 \right\} \tag{A.16}$$

This expression demonstrates the degree to which the agent demands compensation for exposure to higher moments, and illustrates the degree to which the standard linear CAPM over- or understates the required rate of return for asset with a given market beta, β.

A.2 The Risk-Neutral Distribution

Suppose the historical ($\mathbb{P}$-measure) distribution of the shocks, $Z_{t+\tau}$, is NIG(a, b, c, d). The risk-neutral distribution, $\pi^{\mathbb{Q}} = \pi^{\mathbb{P}} \cdot \frac{\Lambda_{t+\tau}}{\Lambda_t}$, can also be shown to be the NIG class, but with perturbed parameters NIG(a, $b - \gamma$, c, d). To see this, substitute the expression for the $\mathbb{P}$-density into the definition of the $\mathbb{Q}$-density to obtain:

$$\pi^{\mathbb{Q}} = \frac{a \cdot d \cdot K_1\left(a \cdot \sqrt{d^2 + (Z_{t+\tau} - c)^2}\right)}{\pi \cdot \sqrt{d^2 + (Z_{t+\tau} - c)^2}} \cdot \exp\left(d \cdot \eta + (b - \gamma) \cdot (Z_{t+\tau} - c) - \gamma \cdot c - k_Z\left(-\gamma\right) \cdot \tau\right) \quad \text{(A.17)}$$

where $\eta = \sqrt{a^2 - b^2}$. Making use of the expression for the cumulant generating function of the NIG distribution the above formula can be rearranged to yield:

$$\pi^{\mathbb{Q}} = \frac{a \cdot d \cdot K_1\left(a \cdot \sqrt{d^2 + (Z_{t+\tau} - c)^2}\right)}{\pi \cdot \sqrt{d^2 + (Z_{t+\tau} - c)^2}} \cdot \exp\left(d \cdot \tilde{\eta} + \tilde{b} \cdot (Z_{t+\tau} - c)\right) \quad \text{(A.18)}$$

where we have introduced the perturbed parameters, $\tilde{b} = b - \gamma$, and $\tilde{\eta} = \sqrt{a^2 - \tilde{b}}$. This verifies that the risk-neutral ($\mathbb{Q}$-measure) distribution is also an NIG distribution, but with shifted parameters, (a, $\tilde{b}$, c, d). The ratio of the historical volatility, $\sigma^{\mathbb{P}}$, to the risk-neutral volatility, $\sigma^{\mathbb{Q}}$, is related to the NIG distribution parameters through:

$$\frac{\sigma^{\mathbb{P}}}{\sigma^{\mathbb{Q}}} = \left(\frac{a^2 - (b - \gamma)^2}{a^2 - b^2}\right)^{\frac{3}{4}}$$

References

[1] Amin, G. S., and H. Kat, 2003, "Hedge Fund Performance 1990-2000: Do the "Money Machines" Really Add Value?," *Journal of Financial and Quantitative Analysis*, 38, p. 251-274.

[2] Agarwal, V., and N. Naik, 2004, "Risks and Portfolio Decisions Involving Hedge Funds," *Review of Financial Studies*, 17, p. 63-98.

[3] Arrow, K. J., 1964, The Role of Securities in the Optimal Allocation of Risk Bearing, *Review of Economic Studies*, 31, p. 91-96.

[4] Asness, C., R. Krail, and J. Liew, 2001, "Do Hedge Funds Hedge?," *Journal of Portfolio Management*, 28, p. 1-19.

[5] Bakshi, G., and N. Kapadia, 2003, "Delta-Hedged Gains and the Negative Market Volatility Risk Premium," *Review of Financial Studies*, 16, p. 527-566.

[6] Black, F., and M. Scholes, 1973, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81, p. 617-654.

[7] Bondarenko, O., 2003, "Why Are Put Options So Expensive?," University of Illinois - Chicago, working paper.

[8] Brunnermeier, M., and L. Pedersen, 2009, "Market Liquidity and Funding Liquidity," *Review of Financial Studies*, 2009, 22, p. 2201-2238.

[9] Carr, P., and L. Wu, 2009, "Variance Risk Premiums," *Review of Financial Studies*, 11, p. 1311-1241.

[10] Constantinides, G., J. C. Jackwerth, and A. Z. Savov, 2012, "The Puzzle of Index Option Returns," University of Chicago, working paper.

[11] Coval, J. D., and T. Shumway, 2001, "Expected Option Returns," *Journal of Finance*, 56, p. 983–1009.

[12] Coval, J., J. W. Jurek, and E. Stafford, 2009, "Economic Catastrophe Bonds," *American Economic Review*, 99, p. 628-666.

[13] Debreu, G., 1959, *Theory of Value*, Wiley.

[14] Dybvig, P. H., 1988, "Distributional Analysis of Portfolio Choice," *Journal of Business*, 61(3), p. 369-393.

[15] Frazzini, A. and L. Pedersen, 2011, "Embedded Leverage," NYU Stern School of Business, working paper.

[16] French, K., 2008, "Presidential Address: The Cost of Active Investing," *Journal of Finance*, 58, p. 1537–1573.

[17] Fung, W., and D. Hsieh, 2001, "The Risk in Hedge Fund Strategies: Theory and Evidence from Trend Followers," *Review of Financial Studies*, 14, p. 313-341.

[18] Fung, D. and W. Hsieh, 2002, "Asset-Based Hedge-Fund Styles Factors for Hedge Funds," *Financial Analysts Journal*, 58, p. 16–27.

[19] Fung, D. and W. Hsieh, 2004, "Hedge Fund Benchmarks: A Risk Based Approach," *Financial Analysts Journal*, 60, p. 65–80.

[20] Gabaix, X., (2009), "Power Laws in Economics and Finance," *Annual Review of Economics*, 1, p. 255–93.

[21] Garleanu, N., L. H. Pedersen, and A. Poteshman, 2009, "Demand-Based Option Pricing," *Review of Financial Studies*, vol. 22, pp. 4259-4299.

[22] Getmansky, M., A. Lo, I. Makarov, 2004, "An Econometric Model of Serial Correlation and Illiquidity in Hedge Fund Returns," *Journal of Financial Economics*, 74, p. 529-609.

[23] Glosten, L., and R. Jagannathan, 1994, "A Contingent Claim Approach to Performance Evaluation," *Journal of Empirical Finance*, 1, p. 133-160.

[24] Harvey, C., and A. Siddique, 2000, "Conditional Skewness in Asset Pricing Tests," *Journal of Finance*, 55, p. 1263-1295.

[25] Harvey, C., J. Liechty, M. Liechty, and P. Muller, 2010, "Portfolio Selection with Higher Moments," *Quantitative Finance*, 10(5), p. 469-485.

[26] Hasanhodzic, J., and A. Lo, 2007, "Can Hedge-Fund Returns Be Replicated?: The Linear Case," *Journal of Investment Management*, 5, p. 5-45.

[27] He, Z., and A. Krishnamurthy, 2012, "Intermediary Asset Pricing," Northwestern University, working paper.

[28] Ibbotson, R. G., M. Spiegel, W. Goetzmann, and I. Welch, 2007, "Portfolio Performance Manipulation and Manipulation-proof Performance Measures," *Review of Financial Studies*, 20(5), p. 1503-1546.

[29] Ibbotson, R. G., P. Chen, and K. Zhu, 2010, "The ABCs of Hedge Funds: Alphas, Betas, and Costs," Yale School of Management, working paper.

[30] Ingersoll, J. E., 1987, *Theory of Financial Decision Making*, Rowman and Littlefield Publishers.

[31] Jarque, C. M, and Anil Bera, 1980, "Efficient Tests for Normality, Homoscedastcity, and Serial Independent of Regression Residuals," *Economic Letters*, 6, p. 255-259.

[32] Jurek, J. W., and E. Stafford, 2013, "The Risk of Safe Assets," Princeton University, working paper.

[33] Lerner, J., A. Shoar, and J. Wang, 2008, "Secrets of the Academy: The Drivers of University Endowment Success," *Journal of Economic Perspectives*, 22, p. 207-222.

[34] Lo, Andrew, 2001, "Risk Management for Hedge Funds: Introduction and Overview," *Financial Analysts Journal*, 6, p. 16-33.

[35] Malkiel, B., and A. Saha, 2005, "Hedge Funds: Risk and Return," *Financial Analysts Journal*, Vol. 61(6), p. 80-88.

[36] Martellini, L., and V. Ziemann, 2010, "Improved Estimates of Higher-Order Comoments and Implications for Portfolio Selection," *Review of Financial Studies*, p. 1467-1502.

[37] Merton, R. C., 1973, "Theory of rational option pricing," *Bell Journal of Economics and Management Science*, 141-183.

[38] Merton, R. C., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 29, p. 449-470.

[39] Merton, R. C., 1981, "On Market Timing and Investment Performance: An Equilibrium Theory of Value for Market Forecasts," *The Journal of Business*, 54, p. 363-406.

[40] Merton, R. C., 1987, "A Simple Model of Capital Market Equilibrium with Incomplete Information," *Journal of Finance*, 42, p. 483-510.

[41] Mitchell, M., and T. Pulvino, 2001, "Characteristics of Risk in Risk Arbitrage," *Journal of Finance*, 56, p. 2135-2175.

[42] Patton, A., 2004, "On the Out-of-Sample Importance of Skewness and Asymmetric Dependence for Asset Allocation," *Journal of Financial Econometrics*, 2(1), p. 130-168.

[43] Patton, A., 2009, "Are "Market Neutral" Hedge Funds Really Market Neutral?," *Review of Financial Studies*, 22(7), p. 2495-2530.

[44] Santa-Clara, P. and A. Saretto, 2009, "Option Strategies: Good Deals and Margin Calls," *Journal of Financial Markets*, 12, p. 391-417.

[45] Shleifer, A. and R. Vishny, 1997, "The Limits of Arbitrage," *Journal of Finance*, 52, p. 35-55.

[46] Swensen, D., 2000, *Pioneering Portfolio Management: An Unconventional Approach to Institutional Investment*, Free Press.

[47] Todorov, V., 2010, "Variance Risk Premium Dynamics: The Role of Jumps," *Review of Financial Studies*, 23, p. 345-383.

Table I
Pre-fee Hedge Fund Performance

This table reports the performance of investments in risk-free bills, public equities and hedge funds between January 1996 and December 2010. *T-bill* is the return on the one-month U.S. Treasury T-bill obtained from Ken French's website. *S&P 500* is the total return on the S&P 500 index obtained from the CRSP database. The *HFRI* and *DJ/CS* series are *pre-fee* hedge fund index return series based on data from Hedge Fund Research Inc. and Dow Jones/Credit Suisse, respectively. To compute pre-fee returns, we treat the observed net-of-fee time series as if it represents the return of a representative fund that is at its high watermark throughout the sample, and charges a 2% flat fee and a 10% incentive fee, both payable monthly. Before computing summary statistics, monthly return time series are compounded to the quarterly frequency. Means, volatilities, CAPM alphas ($\hat{\alpha}$), and Sharpe Ratios (SR) are reported in annualized terms. Skewness and kurtosis estimates are based on quarterly returns. *JB* and p_{JB} report the value of the Jarque-Bera test statistic for normality, and its associated p-value based on a finite sample distribution obtained by Monte Carlo. CAPM $\hat{\alpha}$ and $\hat{\beta}$ report the intercept (annualized) and slope coefficient from a regression of the quarterly excess return of each asset onto the quarterly excess return of the market (S&P 500). *Minimum Drawdown* measures the magnitude of the largest strategy loss relative to its highest historical value, and is computed using the monthly return time series. *All-in Fee* is an estimate of the total annual management fee (flat + incentive) paid by investors in a given hedge fund strategy, and is equal to the difference between the (annualized) mean pre- and net-of-fee strategy return.

								CAPM		Minimum	All-in
Asset	Mean	Vol.	Skew	Kurt.	JB	p_{JB}	SR	$\hat{\alpha}$	$\hat{\beta}$	Drawdown	Fee
T-bill	3.1%	1.0%	-0.24	1.51	19.41	0.00	NaN	0.0%	0.00	0.0%	-
S&P 500	8.5%	18.1%	-0.38	2.96	1.70	0.30	0.30	0.0%	1.00	-50.2%	-
HFRI Fund-Weighted Composite	13.6%	9.8%	-0.27	3.74	2.86	0.13	1.07	8.0%	0.45	-18.8%	4.0%
DJ/CS Broad Index	13.6%	9.2%	-0.19	5.88	3.98	0.07	1.15	8.6%	0.35	-18.8%	3.9%
HFRI Event Driven	14.7%	9.8%	-0.93	4.36	10.08	0.01	1.18	9.2%	0.45	-22.4%	4.0%
DJ/CS Event Driven	14.4%	8.6%	-1.79	7.15	20.18	0.00	1.30	9.3%	0.35	-16.5%	3.9%
HFRI ED - Distressed	13.9%	9.9%	-1.36	6.65	12.91	0.01	1.08	8.7%	0.39	-25.0%	3.9%
DJ/CS ED - Distressed	14.9%	9.3%	-1.60	6.27	18.20	0.00	1.26	9.7%	0.37	-20.1%	3.9%
HFRI ED - Merger Arbitrage	11.3%	4.9%	-0.37	3.06	12.03	0.01	1.80	7.3%	0.17	-6.0%	3.4%
DJ/CS ED - Merger Arbitrge	10.2%	5.5%	-0.83	4.66	11.06	0.01	1.33	6.1%	0.17	-7.3%	3.3%
HFRI Equity Hedge	15.5%	12.7%	0.15	4.26	1.64	0.31	0.98	9.3%	0.58	-28.1%	4.4%
HFRI EH - Market-neutral	8.8%	4.1%	-0.70	4.06	12.85	0.01	1.57	5.3%	0.07	-7.6%	3.1%
HFRI EH - Directional	15.8%	17.1%	0.13	2.96	0.04	0.98	0.74	8.4%	0.81	-28.5%	5.0%
DJ/CS Long/Short Equity	15.4%	12.9%	1.09	7.85	39.15	0.00	0.96	9.5%	0.53	-20.5%	4.4%
HFRI Relative Value	12.2%	6.5%	-1.67	8.14	18.09	0.00	1.40	7.8%	0.22	-16.4%	3.5%
HFRI RV - Convertible Arbitrage	12.7%	11.0%	-0.27	8.66	19.56	0.00	0.87	7.9%	0.31	-33.7%	3.7%
DJ/CS Convertible Arbitrage	12.5%	10.5%	-0.88	6.42	7.69	0.02	0.89	7.9%	0.26	-31.0%	3.8%
HFRI RV - Corporate	9.9%	9.0%	-1.32	8.19	15.93	0.01	0.74	5.0%	0.34	-25.2%	3.5%
DJ/CS Fixed Income	8.7%	8.2%	-2.56	14.14	66.41	0.00	0.67	4.4%	0.21	-27.4%	3.3%
HFRI Macro	12.6%	6.3%	0.05	3.43	5.15	0.05	1.50	8.7%	0.15	-6.8%	3.7%
DJ/CS Global Macro	17.0%	10.7%	-0.33	4.57	4.21	0.07	1.32	13.2%	0.13	-24.4%	4.4%
DJ/CS Managed Futures	12.2%	12.9%	0.56	2.71	1.16	0.45	0.70	10.2%	-0.19	-13.0%	4.5%

Table II
Comparison of Derivative-Based and Linear Factor Hedge Fund Replicating Models

This table reports coefficients from quarterly excess return regressions under several risk models over the period January 1996 through December 2010 ($N = 60$). The dependent variable is the quarterly excess return on the HFRI Fund Weighted Composite, computed as the difference between the quarterly pre-fee HFRI return and the quarterly return from rolling investments in 1-month T-bills, r_f. All independent variables represent zero-investment portfolios, and are obtained by compounding the corresponding monthly return series. Specification 1 corresponds to a CAPM-style model with a single factor calculated as the total return on the S&P 500 minus r_f. Specification 2 corresponds to the Fama-French (1993) model (RMRF, SMB, HML) with the addition of a momentum factor (MOM). Specification 3 corresponds to the 9-factor model proposed by Fung-Hsieh (2004). Specifications 4 through 7 correspond to derivative-based models with a single factor calculated as the quarterly return of the in-sample mean-matched put-writing strategy $[Z, L]$ less the compounded return from rolling investments in 1-month T-bills. Specification 8 uses a single factor computed as the equal-weighted excess return on the four put writing strategies. Specifications 9 thru 11 add the equal-weighted put writing composite to the CAPM, Fama-French/Carhart, and Fung-Hsieh factor sets. OLS standard errors are reported in parentheses; coefficients significant at the 5% level are reported in bold. *Adj.* R^2 is the adjusted R^2 measure of the goodness-of-fit of the linear regression. *Adj.* R^2 *[feasible]* is the goodness-of-fit based on *feasible residuals*, which are defined as the difference in the returns between the hedge fund index and a feasible replicating portfolio. For models (1)-(3) and (9)-(11), we obtain feasible residuals by differencing the returns of the index with the fitted value obtained from the regression after setting the intercept to zero. For models (4)-(8), we obtain feasible residuals by differencing the returns of the index and the put writing strategy. Finally, we report the p-value of the joint test that the intercept and slope of a regression of the hedge fund index returns onto the returns of the feasible replicating portfolio are zero and one, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Intercept (x100)	**2.01**	**1.74**	**2.52**	0.72	0.14	-0.59	-0.74	-0.06	**1.67**	**1.52**	**2.24**
	(0.36)	(0.29)	(0.42)	(0.46)	(0.51)	(0.59)	(0.64)	(0.52)	(0.57)	(0.46)	(0.53)
RMRF	**0.45**	**0.42**	**0.31**						**0.39**	**0.39**	**0.25**
	(0.04)	(0.03)	(0.05)						(0.08)	(0.06)	(0.08)
SMB		**0.22**								**0.23**	
		(0.06)								(0.06)	
HML		-0.04								-0.04	
		(0.04)								(0.04)	
MOM		**0.07**								**0.08**	
		(0.03)								(0.03)	
SIZE			**0.24**								**0.25**
			(0.07)								(0.07)
TSY			-0.11								-0.13
			(0.12)								(0.12)
CREDIT			0.19								0.14
			(0.16)								(0.17)
TF-BD			-0.00								-0.00
			(0.01)								(0.01)
TF-FX			0.01								0.01
			(0.01)								(0.01)
TF-COM			-0.00								-0.00
			(0.02)								(0.02)
TF-IR			**-0.02**								**-0.02**
			(0.01)								(0.01)
TF-STK			0.02								0.03
			(0.02)								(0.02)
Put Writing				**0.71**							
$[Z = -0.5, L = 1.7]$				(0.08)							
Put Writing					**0.95**						
$[Z = -1.0, L = 2.0]$					(0.11)						
Put Writing						**1.27**					
$[Z = -1.5, L = 2.5]$						(0.16)					
Put Writing							**1.29**				
$[Z = -2.0, L = 3.6]$							(0.17)				
Put Writing Composite								**1.03**	0.16	0.10	0.19
(equal-weighted)								(0.12)	(0.21)	(0.16)	(0.21)
Adj. R^2	68.4%	82.4%	78.5%	57.5%	55.1%	52.3%	48.0%	56.0%	68.2%	82.2%	78.4%
Adj. R^2 [feasible]	50.1%	68.0%	45.3%	47.9%	55.0%	49.8%	45.5%	55.2%	55.1%	71.0%	51.7%
p-value ($H_0 : \alpha = 0, \beta = 1$)	0.0001	0.0001	0.0001	0.0049	0.8592	0.0151	0.0107	0.9894	0.0001	0.0001	0.0001

Table III
Distributional Properties of Replicating Portfolio Returns and Residuals

This table compares the risk characteristics of feasible replicating portfolios and the distributional properties of the corresponding feasible residuals obtained on the basis of commonly-used factor models (CAPM, Fama-French/Carhart, Fung-Hsieh) and the non-linear put writing strategies ($[Z, L]$). For the factor models, the feasible replicating return is defined as the in-sample fitted regression return after setting the intercept to zero. For the put writing strategies, the feasible replicating return is the return of a put-writing strategy chosen to match the in-sample mean return of the hedge fund index. Feasible residuals are computed as the difference in the quarterly *pre-fee* return of the HFRI Fund-Weighted Composite and that of feasible replicating strategy. The Put Writing Composite is defined as the equal-weighted average of the four put writing strategies. The returns of the index and the replicating strategies span the period from January 1996 to December 2010. We report the slope from the regression of the quarterly excess return of the index and the feasible replicating strategies onto the excess return of the S&P 500 index (CAPM $\hat{\beta}$), the annualized volatility of the replicating strategy (*Vol.*), and the minimum drawdown sustained by each strategy. To characterize the goodness-of-fit of the drawdown time series, we report the root mean squared error between the monthly drawdown time series of the HFRI Composite and each of the replicating strategies. Finally, we report the distributional properties of the feasible residuals. *Mean* is the annualized mean of the quarterly replicating residuals, and *t-stat* is the t-statistic of the test that the mean residual is statistically distinguishable from zero. *JB* is the value of the Jarque-Bera test statistic for normality (zero skewness, kurtosis equal to three), and p_{JB} is its associated p-value based on a finite-sample distribution obtained by Monte Carlo. *JS* reports the value of a statistic designed to jointly test for mean-zero feasible residuals and their normality, computed by summing the squared t-statistic of the mean residual and the Jarque-Bera test statistic. p_{JS} is the p-value of the test statistic based on its finite sample distribution.

	Returns		Drawdowns		Feasible Residuals					
	CAPM $\hat{\beta}$	Vol.	Min.	RMSE	Mean	t-stat	JB	p_{JB}	JS	p_{JS}
HFRI Fund-Weighted Composite	0.45	9.8%	-18.8%	0.0%	-	-	-	-	-	-
CAPM	0.45	8.0%	-25.1%	5.5%	8.0%	5.76	3.5	0.09	36.7	0.00
Fama-French/Carhart 4-factor	0.46	8.9%	-21.7%	4.7%	7.0%	6.88	16.3	0.01	63.6	0.00
Fung-Hsieh 9-factor	0.45	8.8%	-31.7%	7.0%	10.1%	9.46	9.6	0.02	99.1	0.00
Put Writing Composite	0.34	7.4%	-21.4%	1.4%	0.1%	0.06	1.9	0.27	1.9	0.52
$[Z = -0.5, L = 1.7]$	0.53	10.4%	-29.1%	2.9%	-0.1%	-0.07	0.5	0.76	0.5	0.91
$[Z = -1.0, L = 2.0]$	0.37	7.6%	-21.8%	1.5%	0.1%	0.06	1.6	0.34	1.6	0.61
$[Z = -1.5, L = 2.5]$	0.25	5.6%	-18.8%	1.7%	0.4%	0.20	1.1	0.49	1.1	0.73
$[Z = -2.0, L = 3.6]$	0.22	5.3%	-18.5%	1.9%	0.1%	0.03	1.2	0.45	1.2	0.71

Table IV
Out-of-sample Evaluation: Equity strategies

This table compares the out-of-sample goodness-of-fit of feasible replicating strategies based on linear factor models (Panel A) and put-writing portfolios (Panel B) for equity-related hedge fund subindices. We use the first half of the sample (January 1996-June 2003) to identify three linear replicating strategies via regression, and four non-linear, put writing strategies by matching the mean in-sample return of each hedge fund subindex. We use the second half of the sample (July 2003-December 2010) to compute out-of-sample returns and feasible residuals (differences between the out-of-sample returns of each subindex and the returns of the feasible replicating strategies). We evaluate the out-of-sample performance of each subindex relative to an equal-weighted average of the three linear replicating strategies (Panel A), and an equal-weighted average of the four put writing strategies (Panel B). R^2 reports the adjusted R-squared goodness-of-fit measure computed using the out-of-sample feasible residuals. $RMSE_{DD}$ reports the root mean squared error between the monthly drawdown time series of each hedge fund subindex and the replicating strategy. Finally, we report the distributional properties of the feasible residuals. *Mean* is the annualized mean of the quarterly replicating residuals, and *t-stat* is the t-statistic of the test that the mean residual is statistically distinguishable from zero. *JB* is the value of the Jarque-Bera test statistic for normality (zero skewness, kurtosis equal to three), and p_{JB} is its associated p-value based on a finite-sample distribution obtained by Monte Carlo. *JS* reports the value of a statistic designed to jointly test for mean-zero feasible residuals and their normality, computed by summing the squared t-statistic of the mean residual and the Jarque-Bera test statistic. p_{JS} is the p-value of the test statistic based on its finite sample distribution.

Panel A: Linear Replication

	R^2	$RMSE_{DD}$	Mean	t-stat	JB	p_{JB}	JS	p_{JS}
HFRI Fund-Weighted Composite	53.1%	5.2%	7.7%	4.64	98.0	0.00	119.6	0.00
DJ/CS Broad Index	42.8%	4.0%	7.7%	4.10	151.2	0.00	168.0	0.00
HFRI Event Driven	57.0%	2.4%	7.7%	3.96	62.6	0.00	78.3	0.00
DJ/CS Event Driven	28.3%	2.1%	8.4%	4.03	166.3	0.00	182.6	0.00
HFRI ED - Distressed	48.7%	3.9%	8.8%	3.70	34.4	0.00	48.1	0.00
DJ/CS ED - Distressed	33.3%	2.9%	8.0%	3.53	47.6	0.00	60.1	0.00
HFRI ED - Merger Arbitrage	-46.4%	2.0%	4.8%	3.13	12.5	0.01	22.3	0.00
DJ/CS ED - Risk Arbitrage	-73.5%	1.8%	2.9%	1.48	110.7	0.00	112.9	0.00
HFRI Equity Hedge	57.8%	6.2%	7.3%	3.08	51.6	0.00	61.1	0.00
DJ/CS Long Short Equity	45.4%	8.1%	8.4%	3.73	37.3	0.00	51.2	0.00
HFRI EH - Market Neutral	-91.7%	1.8%	2.1%	1.23	191.3	0.00	192.9	0.00
HFRI EH - Directional	55.2%	9.6%	7.0%	2.63	0.2	0.88	7.2	0.05

Panel B: Non-Linear Replication

	R^2	$RMSE_{DD}$	Mean	t-stat	JB	p_{JB}	JS	p_{JS}
HFRI Fund-Weighted Composite	71.1%	1.2%	-0.1%	-0.06	1.1	0.41	1.1	0.70
DJ/CS Broad Index	70.1%	1.2%	-0.7%	-0.42	0.6	0.70	0.7	0.84
HFRI Event Driven	69.0%	1.5%	-0.2%	-0.09	0.6	0.67	0.6	0.87
DJ/CS Event Driven	70.2%	1.5%	1.6%	0.97	0.7	0.61	1.7	0.53
HFRI ED - Distressed	63.1%	2.8%	1.8%	0.73	4.5	0.05	5.0	0.11
DJ/CS ED - Distressed	71.4%	1.8%	-0.3%	-0.17	2.1	0.15	2.2	0.40
HFRI ED - Merger Arbitrage	-35.1%	3.5%	-1.1%	-0.64	1.8	0.20	2.2	0.39
DJ/CS ED - Risk Arbitrage	36.7%	2.1%	0.3%	0.20	0.0	0.98	0.1	1.00
HFRI Equity Hedge	71.5%	2.1%	-3.3%	-1.56	0.7	0.65	3.1	0.26
DJ/CS Long Short Equity	62.2%	1.8%	-1.3%	-0.59	0.4	0.82	0.7	0.85
HFRI EH - Market Neutral	-30.8%	2.0%	-3.1%	-2.34	0.3	0.87	5.7	0.09
HFRI EH - Directional	56.2%	4.0%	-0.4%	-0.15	1.8	0.21	1.8	0.49

Table V
Out-of-sample Evaluation: Non-equity strategies

This table compares the out-of-sample goodness-of-fit of feasible replicating strategies based on linear factor models (Panel A) and put-writing portfolios (Panel B) for non-equity-related hedge fund subindices. We use the first half of the sample (January 1996-June 2003) to identify three linear replicating strategies via regression, and four non-linear, put-writing strategies by matching the mean in-sample return of each hedge fund subindex. We use the second half of the sample (July 2003-December 2010) to compute out-of-sample returns and feasible residuals (differences between the out-of-sample returns of each subindex and the returns of the feasible replicating strategies). We evaluate the out-of-sample performance of each subindex relative to an equal-weighted average of the three linear replicating strategies (Panel A), and an equal-weighted average of the four put writing strategies (Panel B). R^2 reports the adjusted R-squared goodness-of-fit measure computed using the out-of-sample feasible residuals. $RMSE_{DD}$ reports the root mean squared error between the monthly drawdown time series of each hedge fund subindex and the replicating strategy. Finally, we report the distributional properties of the feasible residuals. *Mean* is the annualized mean of the quarterly replicating residuals, and *t-stat* is the t-statistic of the test that the mean residual is statistically distinguishable from zero. *JB* is the value of the Jarque-Bera test statistic for normality (zero skewness, kurtosis equal to three), and p_{JB} is its associated p-value based on a finite-sample distribution obtained by Monte Carlo. *JS* reports the value of a statistic designed to jointly test for mean-zero feasible residuals and their normality, computed by summing the squared t-statistic of the mean residual and the Jarque-Bera test statistic. p_{JS} is the p-value of the test statistic based on its finite sample distribution.

Panel A: Linear Replication

	R^2	$RMSE_{DD}$	Mean	t-stat	JB	p_{JB}	JS	p_{JS}
HFRI Relative Value	2.8%	3.2%	6.5%	2.57	13.6	0.01	20.2	0.00
HFRI RV - Convert Arb	9.3%	8.1%	6.9%	1.38	5.0	0.04	6.9	0.06
DJ/CS Convert Arb	6.9%	8.0%	5.9%	1.31	6.5	0.03	8.2	0.04
HFRI RV - Corporate	43.7%	4.9%	5.9%	2.14	20.7	0.00	25.3	0.00
DJ/CS Fixed Income	0.4%	7.4%	4.9%	1.30	43.3	0.00	45.0	0.00
HFRI Macro	-57.3%	4.1%	8.6%	4.37	4.0	0.06	23.1	0.00
DJ/CS Global Macro	-84.4%	2.5%	8.8%	3.13	35.2	0.00	45.0	0.00
DJ/CS Managed Futures	-42.7%	8.1%	12.3%	2.43	1.5	0.27	7.4	0.05

Panel B: Non-Linear Replication

	R^2	$RMSE_{DD}$	Mean	t-stat	JB	p_{JB}	JS	p_{JS}
HFRI Relative Value	63.9%	1.3%	-0.1%	-0.06	0.2	0.91	0.2	0.98
HFRI RV - Convert Arb	48.5%	3.6%	-2.2%	-0.57	7.7	0.02	8.0	0.04
DJ/CS Convert Arb	51.4%	3.3%	-3.4%	-1.03	0.5	0.76	1.5	0.57
HFRI RV - Corporate	33.0%	5.1%	3.0%	0.95	16.3	0.00	17.2	0.01
DJ/CS Fixed Income	36.0%	5.6%	-0.5%	-0.15	37.5	0.00	37.5	0.00
HFRI Macro	-39.5%	4.6%	0.5%	0.22	7.7	0.02	7.8	0.04
DJ/CS Global Macro	-77.4%	5.2%	-2.4%	-0.76	1.7	0.23	2.3	0.39
DJ/CS Managed Futures	-19.4%	5.6%	0.8%	0.16	1.1	0.43	1.1	0.71

Table VI
Risk-Adjusted Returns of the HFRI Composite (1996-2010)

Panel A of this table compares the realized excess rates of return for S&P 500 index, the HFRI Fund-Weighted Composite index and put writing, with *ex ante* required risk premia. The returns of the HFRI Fund-Weighted Composite are reported *after* fees. The returns of the put writing strategy are computed as the equal-weighted average of four put writing strategies matching the mean HFRI index return in the first half of the sample. Investor required rates of return are computed at the beginning of each month in the sample (January 1996 - December 2010) using investor portfolios and an estimate of equity market volatility ($0.8 \cdot VIX_t$) based on the CBOE VIX index. Realized volatility is computed using the standard deviation of daily returns within each month, annualized, and reported as a year-by-year average. The required risk premia are computed based on the linear CAPM benchmark ($\beta_t \cdot \tilde{\gamma}\sigma_t^2$) and the non-linear model introduced in Section 3. The CAPM benchmark is computed using the risk aversion of an all-equity investor ($\tilde{\gamma} = 2$), and the market beta of the put writing portfolio at inception (β_t). The model required rate of return is computed for two investor types: a traditional investor with no allocation to alternatives ($\omega_a = 0$), and an endowment investor with a large allocation to alternatives ($\omega_a = 0.35$ or $\omega_a = 0.50$). Each investor is assumed to have a CRRA risk aversion, γ, equal to 3.3, such that – in the absence of alternatives – their optimal portfolio is roughly comprised of 60% equities and 40% cash. The table reports the sum of monthly excess returns within each year, as well as, the mean annualized excess return for the full sample (*Mean*). The t-statistic for the mean excess return is reported in square brackets. Panel B reports the annualized values of the arithmetic mean monthly (excess) returns, and computes investor alphas as the difference in the realized and required excess returns with respect to the linear CAPM benchmark and the model implied excess return for the endowment investor (t-statistics in brackets).

Panel A: Excess returns

	Volatility		S&P 500 Index			Alternatives					
			Realized	Required		Realized		Required			
				CAPM	Traditional	HFRI Composite	Put Writing	CAPM	Traditional	Endowment	
Year	$0.8 \cdot VIX$	Realized	S&P 500	$(\tilde{\gamma}\sigma_t^2)$	$(\omega_a = 0)$	(after-fee)	(pre-fee)	$(\beta_t \cdot \tilde{\gamma}\sigma_t^2)$	$(\omega_a = 0)$	$(\omega_a = 0.35)$	$(\omega_a = 0.50)$
1996	13.1%	11.4%	16.5%	3.5%	3.6%	14.4%	10.1%	1.4%	1.8%	3.0%	3.7%
1997	18.4%	17.2%	25.4%	7.0%	7.3%	10.8%	12.2%	2.7%	3.7%	6.3%	8.0%
1998	20.8%	18.6%	23.4%	9.4%	10.0%	-1.5%	14.7%	4.0%	5.3%	9.0%	11.4%
1999	19.6%	18.0%	15.7%	7.8%	8.2%	23.3%	19.3%	3.3%	4.3%	7.2%	9.2%
2000	18.5%	21.9%	-13.1%	7.1%	7.4%	-0.4%	7.3%	2.5%	3.7%	6.3%	8.0%
2001	20.6%	21.0%	-14.6%	8.8%	9.2%	1.0%	4.3%	3.4%	4.9%	8.3%	10.6%
2002	20.9%	24.6%	-24.0%	9.3%	9.9%	-3.0%	2.6%	3.7%	5.1%	8.7%	11.2%
2003	18.1%	16.5%	25.1%	7.0%	7.3%	17.0%	19.6%	2.9%	3.8%	6.5%	8.2%
2004	12.5%	11.0%	9.5%	3.1%	3.2%	7.6%	13.0%	1.3%	1.7%	2.7%	3.4%
2005	10.4%	10.1%	2.4%	2.2%	2.3%	6.1%	8.5%	0.9%	1.1%	1.8%	2.2%
2006	10.1%	9.8%	10.1%	2.1%	2.1%	7.6%	9.7%	0.8%	1.1%	1.7%	2.0%
2007	13.5%	15.1%	1.5%	3.9%	3.9%	5.1%	10.1%	1.7%	2.1%	3.5%	4.4%
2008	24.1%	35.2%	-44.0%	14.1%	15.5%	-22.1%	-10.7%	6.3%	8.9%	14.6%	18.4%
2009	26.7%	25.0%	25.9%	15.2%	16.0%	18.5%	20.3%	6.9%	9.4%	16.1%	20.3%
2010	19.3%	16.8%	16.0%	7.8%	7.9%	9.9%	12.4%	3.7%	4.7%	8.0%	10.0%
Mean	17.8%	18.1%	5.1%	**7.2%**	**7.6%**	**6.3%**	**10.2%**	**3.0%**	**4.1%**	**6.9%**	**8.7%**
			[1.2]	[15.8]	[14.9]	[3.2]	[5.4]	[14.5]	[13.8]	[13.9]	[13.9]

Panel B: Investor alphas

	HFRI Composite	Put Writing
Realized excess return, R*	6.3%	10.2%
CAPM R*	3.0%	3.0%
alpha	3.3%	**7.2%**
	[1.7]	[3.9]
Model R* (endowment, $\omega_a = 0.35$)	6.9%	6.9%
alpha	-0.6%	3.3%
	[-0.3]	[1.8]
Model R* (endowment, $\omega_a = 0.50$)	8.7%	8.7%
alpha	-2.5%	1.5%
	[-1.2]	[0.8]

Table VII
Risk-Adjusted Returns of Hedge Fund Sub-Indices (1996-2010)

This table reports the investor alphas for equity-related hedge fund strategies (Panel A) and their associated put writing replicating portfolios (Panel B) relative to the linear CAPM benchmark ($\beta_t \cdot \tilde{\gamma}\sigma_t^2$) and the model implied required rates of return for the endowment investor, at two different allocations to alternatives ($\omega_a = 0.35$ and $\omega_a = 0.50$). For each subindex, the four mean-matched put writing strategies are identified using the first half of the sample (January 1996-June 2003). We evaluate performance of each index (Panel A) and the corresponding equal-weighted composite of the four put-writing strategies (Panel B) using the full sample (January 1996-December 2010). The required rate of return for both strategies is computed on the basis of the equal-weighted put writing composite. To capture feasible investor returns we use net-of-fee returns for the hedge fund indices, as reported by HFRI and Dow Jones/Credit Suisse, and pre-fee returns for the put writing strategies. The cost of capital estimates are based on an estimate of market volatility, σ_t, given by $0.8 \cdot VIX_t$). The CAPM benchmark is computed using the risk aversion of an all-equity investor ($\tilde{\gamma} = 2$), and the market beta of the put writing portfolio at inception (β_t). Alphas are computed monthly as the difference in the realized and required excess returns, and are reported in annualized terms along with their t-statistic.

Panel A: After-Fee Alphas (Hedge Funds)

	CAPM	t-stat	Endowment ($\omega_a = 0.35$)	t-stat	Endowment ($\omega_a = 0.50$)	t-stat
HFRI	3.3%	1.7	-0.6%	-0.3	-2.5%	-1.2
DJCS	3.3%	1.6	-0.6%	-0.3	-2.5%	-1.1
HFR Event Driven	4.0%	2.1	-0.4%	-0.2	-2.6%	-1.3
DJCS Event Driven	3.9%	2.3	-0.3%	-0.2	-2.3%	-1.2
HFR ED - Distressed	3.3%	1.8	1.2%	0.7	-1.0%	-0.5
DJCS ED - Distressed	4.5%	2.5	1.5%	0.8	-1.1%	-0.6
HFR ED - Merger Arbitrage	2.4%	2.5	-0.3%	-0.3	-1.4%	-1.3
DJCS ED - Risk Arbitrage	1.7%	1.5	-0.3%	-0.3	-1.2%	-1.0
HFR Equity Hedge	4.1%	1.6	-0.9%	-0.3	-3.3%	-1.3
DJCS Long Short Equity	4.1%	1.5	-0.9%	-0.3	-3.4%	-1.2
HFR EH - Market Neutral	1.0%	1.2	-0.3%	-0.3	-0.8%	-0.9
HFR EH - Directional	3.7%	1.0	-1.4%	-0.4	-3.9%	-1.1

Panel B: Pre-Fee Alphas (Put Writing)

	CAPM	t-stat	Endowment ($\omega_a = 0.35$)	t-stat	Endowment ($\omega_a = 0.50$)	t-stat
HFRI	7.2%	3.9	3.3%	1.8	1.5%	0.8
DJCS	7.3%	3.9	3.3%	1.8	1.5%	0.8
HFR Event Driven	7.8%	3.8	3.4%	1.7	1.2%	0.6
DJCS Event Driven	7.6%	3.8	3.4%	1.7	1.3%	0.7
HFR ED - Distressed	7.3%	3.9	3.3%	1.7	1.4%	0.7
DJCS ED - Distressed	8.0%	3.8	3.4%	1.6	1.2%	0.5
HFR ED - Merger Arbitrage	5.8%	4.0	3.1%	2.1	2.0%	1.3
DJCS ED - Risk Arbitrage	5.0%	4.0	3.0%	2.4	2.1%	1.7
HFR Equity Hedge	8.4%	3.8	3.4%	1.5	1.0%	0.4
DJCS Long Short Equity	8.4%	3.8	3.4%	1.5	1.0%	0.4
HFR EH - Market Neutral	4.0%	4.2	2.7%	2.8	2.2%	2.2
HFR EH - Directional	8.5%	3.8	3.4%	1.5	0.9%	0.4

Figure 1. Asset Class Performance Comparison. This figure plots the total return indices for two hedge fund indices – the Hedge Fund Research Inc. (HFRI) Fund Weighted Composite Index, and the Dow Jones Credit Suisse Broad Hedge Fund Index – the HFRI Fund-of-Funds Composite Index, the S&P 500 Index, and a strategy that rolls over one-month U.S. Treasury bills over the period from January 1996 to December 2010 ($N = 180$ months). Hedge fund index returns are reported net of fees.

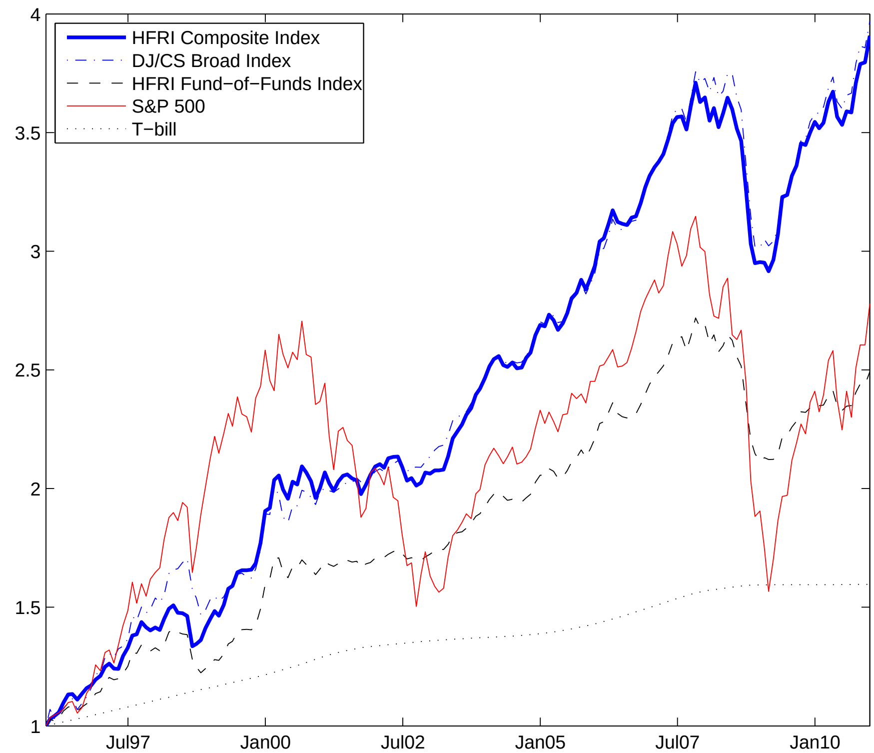


Figure 2. Replicating the Risks and Returns of the HFRI Fund-Weighted Composite In-Sample. The top panels compare the compounded realized *pre-fee* returns of a direct investment in the HFRI Fund-Weighted Composite with the returns from feasible replicating strategies identified in-sample (January 1996-December 2010). The left panel compares the hedge fund index to portfolios based on linear factor model regressions (CAPM, Fama-French/Carhart, and Fung-Hsieh). The returns to the feasible linear factor replicating portfolio are obtained by compounding the fitted regression values after setting positive intercepts to zero. The right panel compares the hedge fund index to the compounded return of feasible put-writing strategies selected by matching the mean arithmetic return in-sample. Each put writing strategy applies a progressively higher amount of leverage to options that are written further out-of-the-money. The bottom panels plot the corresponding monthly drawdown series for the hedge fund index and the feasible replicating strategies.

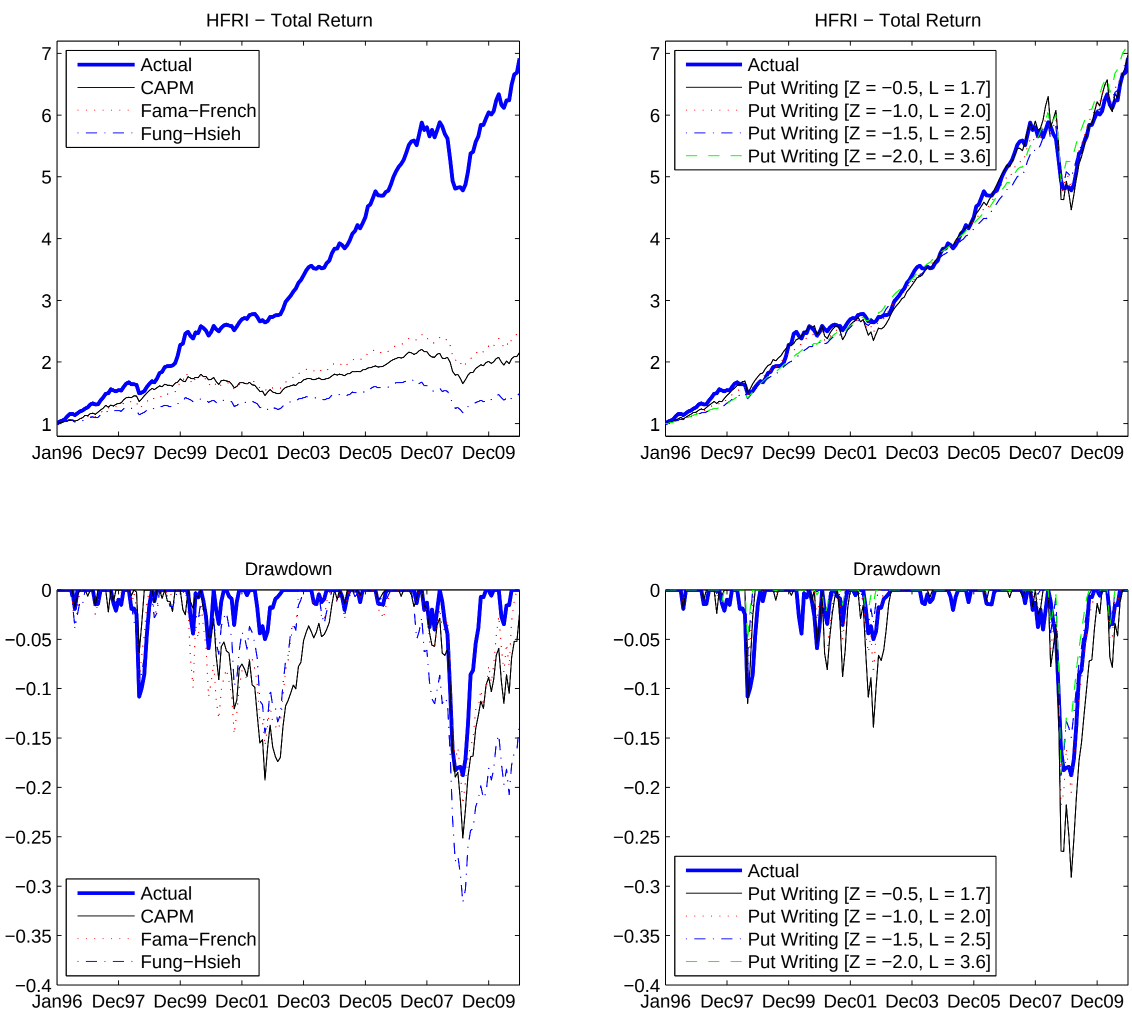
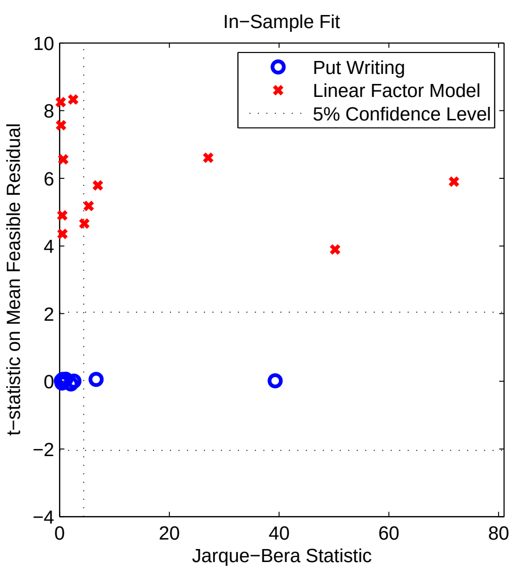


Figure 3. Replicating the Risks and Returns of the Hedge Fund Indices Out-of-Sample. This figure summarizes the goodness-of-fit analysis based on the properties of the feasible residuals for the linear factor models and put writing strategies replicating the equity-related hedge fund strategies. The left panel summarizes the fit in-sample (January 1996-June 2003), and the right panel summarizes the fit out-of-sample (July 2003-December 2010). Feasible residuals are computed as the difference between the *pre-fee* returns of the hedge fund sub-index and an equal-weighted average of three feasible linear replicating strategies, and an equal-weighted average of four feasible put writing strategies. The feasible linear factor model replicating strategies are identified via in-sample regression; the corresponding put writing strategies are identified by matching the mean in-sample hedge fund index return. The x-axis reports the value of the Jarque-Bera test statistic for the normality of the feasible residuals; the y-axis reports the value of the t-statistic for the mean of the feasible residuals. A large value for the Jarque-Bera test statistic indicates the feasible residuals are skewed or heavy-tailed. JS values larger than 80 are set to 80. A positive and statistically significant value of the t-test indicates the returns of the hedge fund index exceed those of the replicating strategy. To highlight the quality of the fit we plot the critical value of each test statistic at the 5%-significance level computed on the basis of their finite-sample distributions.



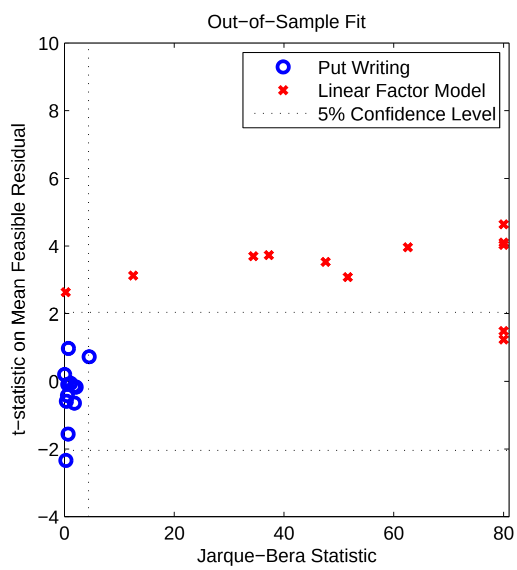


Figure 4. Required Rates of Return for Large Allocations to Non-Linear Risk Exposures. This figure illustrates the comparative statics of the endowment investor's optimal portfolio composition (left panels) and cost of capital (right panels) as a function of his allocation to alternatives (top panels) and the level of market volatility (bottom panels). The payoff profile of the alternative investment is assumed to be described by the [Z = -1, L = 2], put writing strategy. The top left panel illustrates the investor's optimal allocation to equities as a function of the share of his portfolio held in alternatives (ω_a), and the total allocation to risky assets (equities and alternatives). The volatility of the equity index is assumed to be fixed at 0.8 times the sample average of the VIX index ($\sigma = 17.8\%$). The top right panel plots the model required rate of return on the equity index and the alternative investment as a function of the alternative allocation, and the risk premium for the alternative investment based on the linear CAPM benchmark ($\beta \cdot \tilde{\gamma}\sigma^2$) with $\beta = 0.4$. The bottom left panel plots the optimal portfolio allocation as a function of the prevailing level of equity market volatility (σ), while holding the allocation to alternatives fixed at 35%. The bottom right panel plots the corresponding model required rates of return, along the CAPM risk premium for the alternative.