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THE VALUATION OF SECURITY ANALYSIS

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ABSTRACT

Active portfolio management is commonly partitioned into two types of activities: market timing, which requires forecasts of broad-based market movements, and security analysis, which requires the selection of individual stocks that are perceived to be underpriced by the market. Merton (1981) has provided an inciteful and easily-implemented means to place a value on market timing skills. In contrast, while a normative theory of stock selection was outlined long ago in Treynor and Black's (1973) work, no convenient means of valuing potential selection ability has yet been devised.

We present a framework in which the value of a security analyst can be computed. We also treat market timing ability in this framework, and therefore can compare the relative values of each type of investment analysis. We find that stock selection is potentially extremely valuable, but that its value depends critically on the forecast interval, on the correlation structure of residual stock returns, and on the ability to engage in short sales. Finally, we show how to modify the value of selection for the important case in which analysts' forecasts of stocks' alphas are subject to error.

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THE VALUATION OF SECURITY ANALYSIS

I. Introduction

Active portfolio management is commonly partitioned into two types of activities: market timing, which requires forecasts of broad-based market movements, and security analysis, which requires the selection of individual stocks that are perceived to be underpriced by the market. Merton (1981) has provided an inciteful and easily-implemented means to place a value on market timing skills. In contrast, while a normative theory of stock selection was outlined long ago in Treynor and Black's (1973) work, no convenient means of valuing potential selection ability has yet been devised.

The goal of this paper is to provide a framework in which stock selection ability can be valued. One of the motivations for this research is to provide a framework for efficient portfolio management. A management firm that believes it has access to superior information or insight must allocate effort across the two broad tasks of market timing and stock selection. Optimal allocation of effort requires that the expected marginal value of each type of analysis be evaluated.

We demonstrate that a constant-relative-risk-aversion client would desire that a portfolio manager maximize the ex ante Sharpe measure of the managed portfolio. The annual fee that such a client would be willing to pay for management is proportional to the difference between the square of the Sharpe measures of the managed portfolio and of the market, and is inversely proportional to the client's risk aversion. The equilibrium fee for a given level of investment ability would then depend on the risk aversion of the marginal client.

One of the interesting implications of our model is that the value of security selection is sensitive to the correlation structure of residual returns from a market-model regression. Presumably, these residual correlations are determined by nonpriced factors that affect various blocks of stocks. Therefore, security analysts must be concerned with nonpriced as well as priced factors. This insight provides a useful distinction between the utility of the APT model of Ross (1976) and the intertemporal multifactor CAPM of Merton (1973). While each model yields quite similar predictions about the relationship between expected returns and exposure to systematic sources of risk, the factor-analysis methodology commonly used in empirical applications of the APT provides a natural means for determining the correlation structure of residual returns.

We do not address in this paper the issue of how the client would assess performance from ex post returns of the portfolio manager. Here, the client would face the problems addressed by Dybvig and Ross (1985) and Admati and Ross (1985). If the client can not overcome the statistical difficulties in obtaining appropriate estimates of the ex ante Sharpe measure, then portfolio managers will face the standard agency conflict.

The next section introduces a means for assigning a dollar value to selection ability. Section III provides estimates of the value of what we describe as an ex ante perfect security analyst. In Section IV, we place Merton's (1981) market timer in our framework and use market timing as a benchmark to which one may compare the potential value of stock selection. We find that stock selection is potentially extremely valuable; even for modest numbers of securities analyzed, selection can be far more valuable than timing. However, these results are extremely sensitive to the ability of the fund manager to engage in short sales, or to borrow funds to lever up the position in the active portfolio. Section V shows that if such activity is not feasible, the potential value of analysis is far below that of market timing. These results also are sensitive to the interval over which abnormal security performance can be forecasted and to the correlation structure of residual stock returns from a market-model regression. The expected value of security selection is shown to be inversely related to the forecast interval, and to decrease rapidly with the correlation among stock-return residuals. Finally, Section VI shows how our analysis can be modified for the case of unbiased but imperfect selection. Section VII concludes.

II. Valuation Framework

A. The Investor's Problem

We start with a common model of infinitely-lived individuals who maximize lifetime utility by finding the optimal rules for consumption, c, and portfolio weights (see Merton, 1969, 1971). The derived utility of wealth function, J(W), is

$$J(W_t) = \max E_t \int_t^{\infty} e^{-\pi s} U(c_s) ds \qquad (1)$$

where π is the rate of time preference and $U(c_t)$ is the utility of consumption at time t. Individuals choose a portfolio from a menu of N risky assets with covariance matrix C and one risk-free asset with rate of return r_f. If we let μ denote the vector of expected rates of return on the N risky assets and assign constant relative risk aversion to all investors with an index of risk aversion equal to $\delta > 1$, then upon optimizing, investors will find that (Merton, 1971):

$$J(W_t) = q\, W^{1-\delta}/(1-\delta) \qquad (2)$$

where

$$q = \left[\frac{\delta}{(\delta-1)\lambda}\right]^{\delta}$$

and where λ is a number which summarizes and is determined by the parameters of the investment opportunity set:

$$\lambda = r_f + \frac{(\mu - r_f i)' C^{-1} (\mu - r_f i)}{2\delta} . \qquad (3)$$

Finally, i is an N-dimensional unit vector.

Equations (2) and (3) define the derived utility of an optimizing investor freely choosing to hold some portfolio, which we will call P. Of course, the investor's welfare would be unchanged if we were to replace the entire investment opportunity set with the optimal portfolio P. Thus λ may be written

$$\lambda_P = r_f + (r_P - r_f)^2/2\delta\sigma_P^2 = r_f + S_P^2/2\delta \qquad (4)$$

where S_P is the Sharpe measure of the portfolio, the mean excess return divided by the standard deviation. Because J(W) is increasing in λ, equation (4) demonstrates that the determination of optimal portfolio weights is equivalent to the maximization of the Sharpe measure.

B. Treynor-Black Analysis

We assume that asset returns are generated according to the process

$$r_i = r_f + \beta_i(r_M - r_f) + \alpha_i + \varepsilon_i \quad (5)$$

where notation is standard except for α_i, which is the expected inferior or superior rate of return of asset i that is forecast by security analysis. Non-zero estimates of α_i can arise from either expenditure of resources on the discovery and processing of new information or from (perceived) superior insight. In the absence of security evaluation, any investor would assign a value of zero to α. In this sense, all assets are fairly priced, and offer expected rates of return commensurate with risk. Following Treynor and Black, we initially assume a diagonal model, so that the residuals are uncorrelated. We will explore the consequences of correlated residuals below.

Treynor-Black analysis recognizes that it is not feasible to analyze all securities, which would allow the analyst to obtain estimates of β_i, α_i, and $\sigma_{\varepsilon i}$ for all i, and then to construct a mean-variance efficient frontier. Instead, a two-tier strategy is suggested in which the portfolio manager maintains a core market-index portfolio, to which he adds incremental positions in a limited set of analyzed securities based on analysts' recommendations. The resultant portfolio is optimal given the constraints on the feasible extent of security analysis. The security analysts cover n securities and provide n estimation triplets (α, β, σ_ε). Based on these estimates an active portfolio, A, is constructed, and the active portfolio is mixed with the index fund.

A detailed analysis of the construction of the active fund is deferred to the Appendix. The major results, due originally to Treynor and Black (1973), can be sketched, however. The allocation of funds between the active fund, A, and the market index, M, is chosen to maximize the portfolio's Sharpe measure,

S_p, or equivalently, the square of that measure. Call w_A the fraction of funds allocated to the active fund. Then the goal is to

$$\max_{w_A} \left(\frac{\mu_P - r_f}{\sigma_P}\right)^2$$

where μ_P is the expected return on the entire managed portfolio (which includes the passive core plus the incremental active position).

The optimality condition (see Appendix) is that

$$w_A = \frac{w_A'}{1-\beta w_A'} \tag{6}$$

where β is the beta of the actively management portion of the portfolio, and

$$w_A' = \frac{\alpha_A/\sigma^2_{\varepsilon A}}{r_M/\sigma^2_M + \alpha_A/\sigma^2_{\varepsilon A}} \tag{7}$$

The interpretation of equations (6) and (7) is that w_A' is the weight that would be attached to the active portfolio if it turned out to have zero beta. The actual weight, given in (6), adjusts for the correlation of the active portfolio with the indexed core. The greater the beta, the greater is the weight given the active portfolio. When the active portfolio more closely follows the market, the investor will shift more funds out of the index fund into the active portfolio, since the diversification potential offered by the market is smaller and the costs of plunging into the active portfolio are correspondingly smaller. When the optimal strategy is followed, the managed portfolio will satisfy [see Appendix, equation (A.7)]

$$S_P^2 = S_M^2 + \left(\frac{\alpha_A}{\sigma_{\varepsilon A}}\right)^2 \qquad (8)$$

Equation (8) shows that the squared Sharpe measure of the managed portfolio is decomposable into the square of the market's Sharpe measure plus the square of the Jensen (1969) measure of the active portfolio, $\alpha/\sigma_\varepsilon$.[1] Thus, the active part of the managed portfolio should be chosen to maximize its squared Jensen measure. This equation links the Sharpe and Jensen measures of the managed portfolio. This relationship is valid, however, only for optimized portfolios; it will not hold for arbitrary portfolio weights.

Moreover, under the assumption of the diagonal model, when the squared Jensen measure of the active portfolio is maximized, it will satisfy the relationship [see Appendix, equation (A.10)]

$$JM_A^2 = \sum_{i=1}^{n} JM_i^2 \qquad (9)$$

Equation (9) establishes a second decomposition property of the managed portfolio, since it states that the active portfolio's squared Jensen measure equals the sum of those values for each component security.

C. The Valuation of Security Analysis

A passive investment strategy would simply mix bills with the market-index fund. In this case, the derived utility of wealth would be, from (2), $q_M W^{1-\delta}/1-\delta$ where q_M denotes the value that q would attain if the risky portfolio were placed in the market-index fund. For this passive strategy,

$$\lambda_M = r_f + S_M^2/2\delta \ , \qquad (10)$$

analogously to equation (4). S_M is the Sharpe measure of the market-index portfolio; equivalently, it is the slope of the capital market line.

If an analyst can indeed identify non-zero alphas, a fund with a larger Sharpe measure than S_M can be constructed and so will be more attractive to investors. Presumably, then, Treynor-Black managers can charge investors for their services. This charge may be a one-time up-front load to buy into the fund, or it may be an ongoing annual management fee.

To derive the one-time charge that investors would be willing to pay to buy into the fund, define L as that load charge per dollar invested, and equate the derived utility of wealth with and without access to the managed fund.

$$q_M W^{1-\delta}/(1-\delta) = q_p[(1-L)W]^{1-\delta}/(1-\delta)$$

which implies, using the definition of q from equation (2), that

$$L = 1 - (\lambda_M/\lambda_p)^{\delta/\delta-1}$$

$$= 1 - \left(\frac{2\delta r_f + S_M^2}{2\delta r_f + S_p^2}\right)^{\frac{\delta}{\delta-1}} \qquad (11)$$

The value of superior management, L, is a present value, reflecting the contribution to the derived utility of wealth.

The <u>annual</u> management fee that investors would be willing to pay for active management can be derived similarly. Call the annual fee as a percent of assets under management f. Then equation (4) immediately implies that the fee would reduce the value of λ for the managed portfolio to

$$\lambda_P = r_f - f + S_P^2/2\delta \ , \tag{12}$$

and equating λ_P with λ_M to obtain investor indifference we obtain

$$f = (S_P^2 - S_M^2)/2\delta \tag{13}$$

Therefore, the difference in squared Sharpe measures is directly proportional to the periodic fee investors are willing to offer for active management. In a full-fledged equilibrium model, with a specification of the distributions of risk aversion parameters and wealth across the population, equation (13) could be used to determine the equilibrium market price of management services.

Now consider a security firm that regularly analyzes n securities, each of which follows the process of equation (5). Our goal is to derive an expression for the difference in squared Sharpe measures for a fund managed by such a firm and the market-index fund. This expression can be evaluated to obtain an estimate of the value of analysis via equations (11) or (13).

We start our analysis with the case of what may be called "ex ante perfect foresight," deferring the issue of estimation risk to Section V. We define perfect foresight as the knowledge, at the beginning of each period, of the set of n triplets (α_i, β_i, $\sigma_{\varepsilon i}$). This definition comprises only limited foresight, since it does not imply advanced knowledge of ε_i. The analyst discovers only the ex ante value of alpha. This definition is somewhat analogous to Merton's (1981) definition of a perfect market timer as an individual who can forecast only whether the market will outperform bills. Perfect foresight as to the actual market return is not required. Our definition of perfect forecasting is more severe than Merton's since it requires quantitative estimates of mispricing.[2] However, we still require only ex ante perfection, in that the forecaster would not need to predict the ultimate realization of returns in order to be deemed perfect.

Note first from equation (8) that the difference in squared Sharpe measures of the managed and indexed portfolios equals the square of the Jensen measure of the actively managed position, $(\alpha_A/\sigma_{\varepsilon A})^2$. Because the analyst can not know in advance the values of the alphas and sigmas that he will uncover, the ex ante value of analysis will depend on the distribution of these parameters across all securities. To get a feel for the actual empirical distribution of these parameters, we observed 500 randomly selected stocks on the CRSP tape for the two five-year periods ending in 1978 and 1984. Using 60 monthly observations, we computed estimates of α_i and $\sigma^2_{\varepsilon i}$ for each stock by estimating equation (5). Because the value of analysis depends on the squared Jensen measure, we computed the frequency distribution of $(\alpha_i/\sigma_{\varepsilon i})^2$.

Before presenting these results, we note that the squared Jensen measure of each stock depends on the time horizon of the investment and the average measured JM across the population of stocks depends on the length of the time series used in the regression. If securities are priced fairly with respect to the uninformed investor's information set, then the alpha uncovered by the analyst for any small time period is equally likely to be positive or negative, and the alpha over any investment period will be the "average" of the alphas for each subperiod. With an expected alpha of zero, the expected squared alpha equals the variance. Therefore, because the full-period alpha is the average of the independent subperiod alphas, the expected value of the full-period squared alpha will be inversely proportional to the length of the investment horizon, according to the usual relationship between the variance of an average of i.i.d. random variables and the variance of each of those variables.

Essentially, by measuring alpha over a long period, we fail to see possible episodes of major under and overpricing of the stock during subperiods. Over the long haul, these episodes partially offset each other, and result in average alphas per month that appear small. The analyst, however, could exploit these episodes if he were to rebalance the active portfolio more frequently than the measurement interval. Of course, this issue is present for any analysis of subperiod performance. It always is better to be able to predict abnormal performance on a more frequent basis. For example, Merton's (1981) market timer is far more valuable as a (perfect) monthly timer than as an annual timer.

Our estimates of equation (5) generate the average alphas (per month) over 5-year sample periods. Therefore, the expected squared alpha for a randomly selected stock for a posited investment horizon other than five years should equal the average of the estimated alphas across the population of stocks, multiplied by the ratio of five years to the length of that investment horizon.

In contrast, the regression estimates of $\sigma^2_{\varepsilon i}$ are unaffected by the length of the time series used to estimate equation (5). We observe a residual each month, and use that monthly residual to calculate $\sigma^2_{\varepsilon i}$.

Longer time series simply generate more observations. Thus, under the quite reasonable assumption that the distributions of α_i^2 and $\sigma_{\varepsilon i}^2$ across the population of stocks are independent, the expected value of their ratio, i.e., of the squared Jensen measure, should be inversely proportional to the length of the time period considered.

These theoretical arguments are borne out by the data. For each of the five-year periods we examined, we computed regression equation (5) using only the first 40 months of data from the period. Our considerations suggest that the average of the squared Jensen measures should have increased by a factor of 1.5. In fact, it rose by a factor of 1.44 in the earlier period and 1.77 in the latter period.

We have presented the histogram of squared Jensen measures for the 500-stock sample in Figure 1. We normalized our regression estimates to a posited investment interval of one-quarter year. This horizon is motivated by the quarterly evaluations of performance to which analysts are subjected. Dividend and earnings announcement also are made quarterly, and so provide a natural interval over which to predict stock performance. For this interval, the mean of the distribution using the 1979-83 regressions was .244 and was taken as an estimate of $E(JM_i^2)$ for the analysis to follow below. The mean of the distribution for the 1974-78 period was .324.

These values are typical squared Jensen measures for individual stocks. When n securities are combined into an active portfolio, the squared Jensen measure can be derived from equation (9), which implies that

$$E(JM_A^2) = nE(JM_i^2) = .244n \tag{14}$$

The economic value of this potential-performance measure is presented in the next section.

This analysis assumes that securities with negative α estimates may be sold short. If short sales are prohibited, securities with negative alphas simply will be dropped from the active portfolio. Even if the firm runs an in-house passive portfolio, the position in each security will be so small that dropping the negative alpha security from the passive portfolio would not be a sufficient substitute for a short sale. Therefore, short sale restrictions mean that the fund will have to drop, on average, one half of covered securities from the active portfolio. Consequently, if short sales were to be disallowed, the number of analyzed securities to achieve any particular value from analysis would need to be doubled. We will assume that short sales of individual securities are not feasible. Therefore, the number of analyzed securities, n, henceforth must be interpreted as net of negative alpha stocks on which no action is taken. On average, 2n stocks must be analyzed to obtain positions in n of them.

D. Correlated Residuals and Nonpriced Factors

When residuals are correlated, active management will be less valuable. While the general case does not yield any easily-implemented solutions, we may assess the impact of such correlation by examining a simple correlation structure. Imagine for simplicity that there are two nonpriced factors in the economy that affect distinguishable sectors of stocks (for example, industry effects). To keep the algebra tractable, suppose that one block of stocks is affected by the first factor only, while another equal-sized block is affected by the second factor. The two blocks are uncorrelated with each other, so that the covariance matrix of residual stock returns is block diagonal.

Consider a symmetric example in which for $\rho > 0$,

$$\text{cov}(\varepsilon_i, \varepsilon_j) = \begin{cases} \sigma_\varepsilon^2 & ; \quad i = j \\ \rho\sigma_\varepsilon^2 & ; \quad i \neq j \text{ ; i and j in same block} \\ 0 & ; \quad i \neq j \text{ and i, j in different blocks} \end{cases} \tag{15}$$

and in which the magnitude of α_i for all securities is some common value, α. Given the symmetry of this framework, it is clear that the active portfolio must be equally weighted with one half of the active portfolio invested in each block. It then follows that

$$\sigma_{\varepsilon A}^2 = \frac{\sigma_\varepsilon^2}{n}\left[1 + \frac{\rho}{2}(n-2)\right]$$

More generally, for k blocks, we would find that[3]

$$\sigma_{\varepsilon A}^2 = \frac{\sigma_\varepsilon^2}{n}\left[1 + \frac{\rho}{k}(n-k)\right]$$

and

$$E(JM_A^2) = \frac{nE(JM_i^2)}{1+(\rho/k)(n-k)} = \frac{.244n}{1 + (\rho/k)(n-k)} \tag{16}$$

For $\rho = 0$, equation (16) reduces to (14). For $\rho = 1$, n drops out of (16): analysis of many securities is no more valuable than analysis of only one, since the residuals of all securities in any block are perfectly correlated. Correlation among residuals hurts the investor because it prevents the variance of the active portfolio from falling rapidly as stocks are combined into the active position. Recall that the Jensen measure for each stock is the ratio of the stock's alpha to the standard deviation of the residual return. This residual standard deviation measures the diversifiable risk the investor willingly assumes in order to obtain superior risk-adjusted expected returns. When the residuals are correlated, the active position is worth less because it imposes greater diversifiable risk on the investor.

This analysis demonstrates the importance of nonpriced factors to the value of selection. To the extent that such factors result in correlated security residuals, security analysis is devalued. The portfolio manager faces a tradeoff between economies of information-gathering for related firms and the loss of value suffered by specializing in related securities. Therefore, an analysis of the factor structure of stock residuals emerges as a potentially critical component of the portfolio selection process.

Consequently, the exclusive focus of the empirical arbitrage pricing debate [Roll and Ross (1980, 1984), Dhrymes, Friend, and Gultekin (1984)] on priced factors is overly limited since it seems to imply that only priced factors are of economic interest. While priced factors determine fair (equilibrium) rates of return, any attempt to exploit security mispricing must account for the covariance structure of residual returns. This structure is determined by nonpriced factors.

III. Numerical Estimates of the Value of Selection

The monthly fee that a managed fund could charge its clients is given by equation (14) while the one-time load that could be charged is given by (11).

Using our estimate of $E(JM_i^2)$, these fees can be calculated for any value of risk aversion and number of analyzed securities. Table 1 presents the results of such calculations. This table is computed under the assumption of the diagonal model: the disturbance terms in (5) are taken to be mutually uncorrelated.

Column (1) of Panel A is the individual's risk aversion, δ; column (2) is the monthly fee that an investor would pay to a perfect market timer (see Section IV); the remaining columns are the monthly fees than an ex ante perfect analyst could charge clients as a function of the numbers of securities analyzed. The value of selection rises rapidly with securities included in the active position, and falls with risk aversion. As n increases, firm-specific risk is diversified away, and the analyst's abnormal expected return becomes increasingly less risky. By the time n = 20, the fee that can be charged is huge. For example, with $\delta = 4$, the monthly fee (expressed at a continuously compounded rate[4]) is 61 percent. This value is much larger than the typical (absolute) magnitude of stock alphas. The immense value arises from the ability to safely borrow and lever up the portfolio alpha when firm-specific risk is eliminated through diversification.

The values in Table 1 therefore correspond to portfolios with weights of greater than 1.0 in the active portfolio. These positions would be accompanied by short positions in the passive index fund or by short positions in index futures plus borrowing positions, either of which simultaneously would hedge out market risk and lever up the stake in the active portfolio. The ability to acquire short positions in the market or to borrow is crucial to the potential value of security analysis. We will see later that a limit on short-sales of the index fund or on the ability to borrow to finance positions in the active fund drastically reduces the value of analysis.

Note also that these values are calculated for ex ante perfect analysts. When analysts' predictions contain noise, the value of their forecasts will be diminished and estimated values of ability will appear more reasonable. Section VI pursues this issue in greater detail.

For greater risk aversion, it is more difficult to coax the investor out of Treasury bills. With a smaller position in the risky portfolio, the value of analysis is diminished. Table 1 bears this out: the value of analysis falls with δ. One also should remember that the values in Table 1 depend on the horizon over which alpha can be predicted. For longer horizons, corresponding perhaps to long-run fundamental analysis, the value of analysis would be diminished.

Panel B of Table 1 presents the one-time load that the analyst could charge for his services. The figures in Panel B are percentages of gross assets invested that could be taken as a fee by the portfolio manager. Corresponding to the large monthly fees, the one-time loads also are fantastic. For example, an individual with $\delta = 4$ would be willing to pay an analyst of only five securities a load of 99.35 cents per dollar invested, leaving net assets of only 0.65 cents per dollar to be invested in his name.

In Table 2 we examine the impact of correlation across residuals using the symmetric-security case laid out in equation (15), and the modified formula for the portfolio's squared Jensen measure, equation (16). We take k=2 in this table, so that the value of rho may be loosely interpreted as twice the average correlation coefficient among the residual stock returns in the market. Because there are two blocks of stocks, we restrict the number of securities in the active portfolio to be even, with half the securities allocated to each block. Table 2, like Table 1, is calculated for a one-quarter year investment horizon; δ is fixed at 4. Adjustments for other horizons are similar to those in the uncorrelated-residual case: the expected value of security selection is inversely proportional to the horizon over which alphas can be forecasted.

Table 2 documents the large impact of residual correlation on the value of security analysis. The monthly fee investors would pay for analysis falls from 91.5 percent for 30 uncorrelated securities to only 17.6 percent when the residual correlation is .30. Likewise, the one-time load falls from 99.94 cents per dollar to 99.46 cents, increasing the client's initial net claim per dollar invested from .06 cents to .54 cents, a nine-fold increase.

The rows of particular interest in Table 2 are those for ρ = .12. We calculated the average correlation coefficient among 40 randomly selected stocks (which gives a sample of 780 correlation coefficients) and found that the average was .06, and reasonably symmetrically distributed around this value. With two uncorrelated blocks, a value of ρ=.12 would lead to an average correlation between randomly selected securities of .06. Figure 2 is a histogram of those correlation coefficients.

IV. Comparison to the Value of Market Timing

Merton (1981) valued the ability of a market timer who could predict whether the market index would outperform Treasury bills in any period. Merton showed that this ability has a payoff equivalent to that of a particular call option, and thus must have the same value as that option. For illustration, Merton reported the following results based on the 52 years ending 1978. While $1,000 invested in the NYSE index would have grown to $67,527, the month-by-month perfect foresight market timer would have amassed $5.36 billion.

To place Merton's results in our framework, note that the average monthly excess returns of the timed portfolio were $r_p - r_f = 2.37$ percent, with a standard deviation of 3.82 percent. In contrast, during this period monthly r_M averaged .85 percent, with standard deviation 5.89 percent, and r_f averaged .21 percent. Therefore, the squared Sharpe measures of the active and market portfolios were, respectively, .385 and .012. Thus, from equation (11) we obtain the value of the perfect foresight fund as

$$L = 1 - \left(\frac{.0042\delta + .012}{.0042\delta + .385}\right)^{\frac{\delta}{\delta-1}}$$

which for $\delta = 4$ yields a value of $L = .9705$, a 97 percent one-time load fee. An investor would be willing to pay \$1 for a \$.03 share in the perfect-foresight (monthly) market timing fund. Using the alternative periodic-management-fee formula, equation (13), we have for $\delta = 4$,

$$f = \frac{.385 - .012}{2 \times 4} = .0466$$

which means that an investor with $\delta = 4$ would be willing to pay a management fee of 4.66 percent <u>per month</u> to obtain perfect market timing. Notice that the perfect timer earns a mean rate in excess of the market index of "only" 1.73 percent per month, but in so doing, also obtains a lower standard deviation (3.82 percent versus 5.89 percent per month).

In a similar manner, one may obtain the value of market timing for other relative risk aversion coefficients; these values, expressed as monthly fees or one-time loads, are given in column 2, Panels A and B, of Table 1 and may be compared to the values for security selection. An interesting means to compare the values of market timing versus stock selection abilities is to consider the number of securities that must be analyzed in order to obtain equal performance as a perfect market timer. For the case of uncorrelated residuals, we can set n $E(JM_i^2)$ equal to the value of $(S_P^2 - S_M^2)$ for the market timer. Given the values earlier calculated for these parameters, we find that the critical value is $n^* = 1.53$ when $\delta = 4$. Given the volatility of individual stocks relative to that of the (diversified) market index, it is not too surprising that an ex ante perfect analyst of only one or two securities could capture the same fee as a perfect market timer. We will see below that this result is quite sensitive to any extraneous noise in the analyst's forecast.

When residuals are correlated, the value of security analysis is determined by (16) so that equating the value of selection to that of perfect timing, we obtain (allowing short sales)

$$n^* = \frac{(1-\rho)(S_P^2 - S_M^2)}{E(JM_i^2) - \frac{\rho}{k}(S_P^2 - S_M^2)} \qquad (17)$$

which depends on the correlation coefficient, ρ, and the number of blocks, k. As ρ/k approaches $E(JM_i^2)/(S_P^2 - S_M^2)$, n^* approaches infinity. The intuition here derives from equation (16) which shows that even for large n, the squared Jensen measure of the active portfolio is bounded by $kE(JM_i^2)/\rho$.

In our sample, the average value of JM_i^2 was .244 and the average correlation, ρ, was .06. For these values, n^* is finite. In fact, for the parameter values we have estimated, n^* always will equal two for the two-block case. In this case, the two stocks are uncorrelated by virtue of being selected from the different blocks, and the problem reduces trivially to the earlier-analyzed diagonal model for which n* is only 1.53. Thus, with more than one block, our parameters indicate that analysis of one stock in each block will be sufficient to dominate market timing, at least for the ex ante perfect analyst. However, when the value of security analysis is limited, either by short-sale constraints or by noise in the analyst's forecasts, the critical value, n*, will exceed the number of blocks; in that event, the correlation among the stocks in each block will affect the value of analysis and the value of n*.

V. Short-Sale Restrictions

With even modest perceived values of Jensen measures, the optimal weight in the active component of the managed portfolio quickly exceeds 1.0. As noted, this occurs because firm-specific risk may be easily eliminated through diversification when many securities are analyzed, and market risk may be eliminated by shorting either the index fund or an index futures contract. If short positions and borrowing are disallowed, the ability to lever up superior returns is eliminated and the value of security analysis is correspondingly reduced.

Table 3 examines the fees that $\delta = 4$ investors would pay for security analysis when the maximum allowed weight in the active position is 1.0. The fees are dramatically lower than in Table 2, never exceeding 0.5 percent per month even for 30 securities analyzed. In this case, n* is infinite: the upper bound on the value of the ex ante perfect analyst is exceeded by the value of the perfect monthly market timer.

Nevertheless, on an absolute scale, the value of analysis is still large. A fee of one half percent per month is far greater than anything observed in the capital market. Correspondingly, the one-time loads in the neighborhood of 50 percent are unheard of.

VI. Estimation Risk

In practice, we do not expect managers to exhibit perfect foresight, even in an ex ante sense. However, imperfect forecasting ability also can be valued in the framework established above.

Suppose that an analyst's estimate of α is imperfect, but unbiased. Calling η the prediction error in α, the process generating asset returns would be modified to

$$r_i = r_f + \beta_i(r_M - r_f) + \alpha_i^* + \eta_i + \varepsilon_i \qquad (18)$$

where α_i^* is the analyst's forecast of alpha. However, comparing the asset-return generating equations (18) and (5), it is clear that the two are identical except for the error term. Under the quite reasonable assumption that η_i is independent of ε_i, the entire analysis would go through with the simple modification that $\text{var}(\eta_i + \varepsilon_i) = \sigma^2_{\eta i} + \sigma^2_{\varepsilon i}$ would replace $\sigma^2_{\varepsilon i}$ in all equations.

The value of an unbiased analyst is easily compared to a perfect foresight analyst. Equations (8) and (9) show that the analyst's contribution to the squared Sharpe measure of the passive strategy is the sum of the squared Jensen measures of the analyzed securities. When forecast error is present, each Jensen measure decreases from $(\alpha_i/\sigma_{\varepsilon i})^2$ to $\alpha_i^2/(\sigma_{\varepsilon i}^2 + \sigma_{\eta i}^2)$, a proportional decrease of

$$D = \sigma_{\varepsilon i}^2/(\sigma_{\varepsilon i}^2 + \sigma_{\eta i}^2) \ . \tag{19}$$

Therefore, this ratio is a natural metric of the value of an unbiased forecaster relative to an ex ante perfect forecaster. Notice that the ratio ranges from one, when $\sigma_{\eta i} = 0$, to zero when $\sigma_{\eta i}$ becomes large. The relative value of unbiased forecasts therefore depends in a straightforward manner on the noise embodied in the forecast.

In a similar manner the number of securities needed to be analyzed to achieve any level of superior performance would vary directly with the metric of relative value. For example, in the symmetric-security case, the noisy analyst would need to analyze 1/D securities for every security analyzed by a perfect-foresight analyst in order to realize the same expected performance level.

Table 4 investigates the value of security analysis for D = .05, for various correlations among residuals, and for $\delta = 4$. The values here are far lower than in Table 2, although even here, the value of analysis is potentially extremely large when many securities can be evaluated. Column 2, Panel A, of Table 4 presents the critical values n* for correlation coefficients 0 through .30. As ρ increases, the number of analyzed stocks necessary to attain any given level of performance increases. One implication of these results is that there is a tradeoff between economies of specialization realized by studying several firms in the same or related industries, and the potential reduction of the value of the active portfolio due to correlation among residuals. Note also that the effect of correlation among residuals is greatly attenuated in this case, because the noise in analysts' forecasts is assumed to be independent across securities. This has the effect of decreasing ρ by a factor of D, since the relevant correlation coefficient for portfolio performance is between $(\varepsilon_i+\eta_i)$ and $(\varepsilon_j+\eta_j)$. For larger values of D, n* would be more sensitive to ρ.

The foregoing analysis suggests that the estimation of σ_ε and σ_η might be necessary for the evaluation and compensation of analysts. This estimation is not difficult, however. Letting R_i denote excess returns, and adding time subscripts to (18) gives

$$R_{it} = \beta_i R_{Mt} + \alpha_i^* + \eta_i + \varepsilon_{it} \tag{20}$$

A time series regression of R_{it} on R_{Mt} and a constant would yield estimates of α_i (the intercept) and $\sigma_{\varepsilon i}^2$ (the standard error of the regression). A time series regression of $R_{it} - \alpha_i^*$ on R_{Mt} and a constant would yield estimates of η_i (the intercept) and $\sigma_{\varepsilon i}^2$ (the standard error of the regression). To obtain the variance of η_i, one would need many observations of η_i. These could be obtained in principle by observing the analyst's predictions over many non-overlapping time periods. A simpler procedure could be followed if the analyst covers many securities, so that the recent history of each would yield its own estimate of η_i. Then, on the assumption that the variance of the forecast errors is equal across securities, the observations of η_i could be used to compute σ_η, and the value of the unbiased analyst could be calculated via (19).

VII. Conclusion

We have presented a framework in which the value of a security analyst can be computed. We also are able to treat market timing ability in this framework, and therefore can compare the relative values of each type of investment analysis. We find that stock selection is potentially extremely valuable. However, this potential is sensitive to several factors. First, it depends on the ability of the portfolio manager to lever up the position in the actively analyzed stocks and to hedge out market risk by shorting the market index. If the leverage is unavailable, the value of analysis is drastically reduced, although by conventional standards, the potential value of analysis is still quite large. Second, the value of analysis depends critically on the forecast interval over which returns are projected. The periodic fee that can be charged for analysis is inversely proportional to the length of the forecast interval. In addition, the correlation structure of residual returns has a potentially great impact on the value of analysis. Finally, when analyst's forecasts are subject to error, the signal-to-noise ratio will be a crucial determinant of the analyst's value.

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APPENDIX

In this appendix we derive equations (6)-(9) which appear originally in Treynor and Black (1983). The investor's goal is to maximize the portfolio's Sharpe measure. This is to be done by optimally mixing the active portfolio, A, with the index fund, M, and by optimally choosing the weights of the stocks evaluated in the active portfolio. We start with the problem of the active/index-fund mix, and progress to the composition of the active fund.

Denote excess expected returns by R, so that $R_i = E(r_i) - r_f$. Note that

$$R_P = w_A R_A + (1-w_A)R_M = w_A(R_A - R_M) + R_M \tag{A.1}$$

$$\sigma_A^2 = \beta_A^2 \sigma_M^2 + \sigma_{\varepsilon A}^2 \tag{A.2}$$

$$\text{cov}(R_A, R_M) = \beta_A \sigma_M^2 \tag{A.3}$$

$$\sigma_P^2 = w_A^2[\sigma_{\varepsilon A}^2 + \sigma_M^2(1-\beta_A)^2] - 2w_A\sigma_M^2(1-\beta_A) + \sigma_M^2 \tag{A.4}$$

The investor's goal is to

$$\max S_P^2 = \frac{R_P^2}{\sigma_P^2} \tag{A.5}$$

which has first order condition

$$2\sigma_P^2 \frac{\partial R_P}{\partial w_A} = R_P \frac{\partial \sigma_P^2}{\partial \sigma_{\varepsilon A}^2} \tag{A.6}$$

Solving (A.5) using the above relationship and the relationships in (A.1) through (A.4) leads directly to equations (6) and (7). To obtain equation (8), note that the squared Sharpe measure of the portfolio is R_p^2/σ_p^2. Use the right-hand side of (A.1) for the numerator, and (A.4) for the denominator. Then substitute from equations (6) and (7) for w_A. Multiplying numerator and denominator through by $[R_M/\sigma_M^2 + \alpha_A/\sigma_{\varepsilon A}^2]^2$ yields

$$S_p^2 = \frac{R_M^2}{\sigma_M^2} + \frac{\alpha_A^2}{\sigma_{\varepsilon A}^2} \tag{A.7}$$

which is equation (8). Because the optimal value for w was used in obtaining (A.7), the relationship (A.7) will be satisfied only by portfolios utilizing the optimal weights for the indexed and managed component portfolios. From (A.7), it follows that maximization of S_p^2 is accomplished by the maximization of $\alpha_A^2/\sigma_{\varepsilon A}^2$. Hence, we need to choose weights for the active portfolio, x_i, in order to

$$\max \frac{\alpha_A^2}{\sigma_{\varepsilon A}^2} = \frac{\left(\sum_{i=1}^{n} x_i \alpha_i\right)^2}{\sum_{i=1}^{n} x_i^2 \sigma_{\varepsilon i}^2} \tag{A.8}$$

which has first order condition

$$x_j = \frac{\alpha_j/\sigma_j^2}{\sum x_i \alpha_i / \sum x_i^2 \sigma_i^2}$$

Since x_j is proportional to α_j/σ_j^2, and the sum of all x_j must equal 1.0, we obtain

$$x_j = \frac{\alpha_j/\sigma_j^2}{\sum \alpha_i/\sigma_i^2} \tag{A.9}$$

Substituting (A.9) into (A.8) yields

$$JM_A^2 = \sum_j \left(\frac{\alpha_j}{\sigma_j}\right)^2 = \sum_j JM_j^2 \tag{A.10}$$

which is equation (9).

Footnotes

1. This might be a bit of a misnomer. Jensen actually refers to alpha alone as the performance measure. The ratio $\alpha/\sigma_\varepsilon$ is sometimes called the appraisal ratio.
2. We note, however, that Merton's model envisions plunging into the market in response to a forecast that the market will outperform bills. If Merton's market timer were not perfect, then an optimal use of his services would result in interior solutions, and in this more general application, the market timing portfolio manager also would need to provide quantitative forecasts.
3. These equations also embody the fact that only long positions are allowed in stocks. The presence of a short position would reverse the signs of the correlations coefficient between that security position and others in the portfolio.
4. The interpretation of the monthly fee as a continuously compounded rate is important since otherwise, fees in excess of 100 percent would appear to be logically nonsensical.

Figure 1

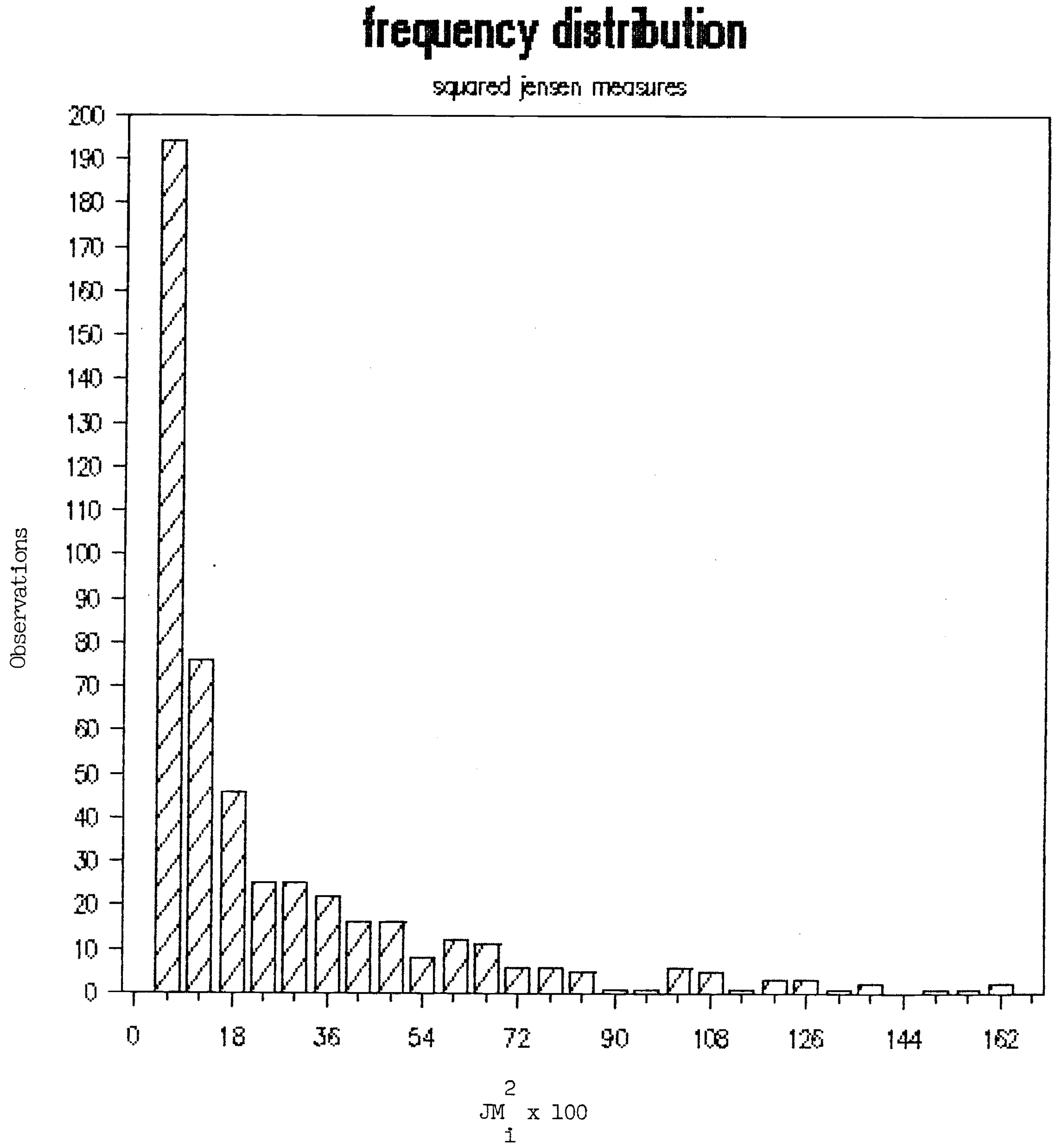
frequency distribution
squared jensen measures
Observations
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
0
18
36
54
72
90
108
126
144
162
2
JM x 100
i

Figure 2

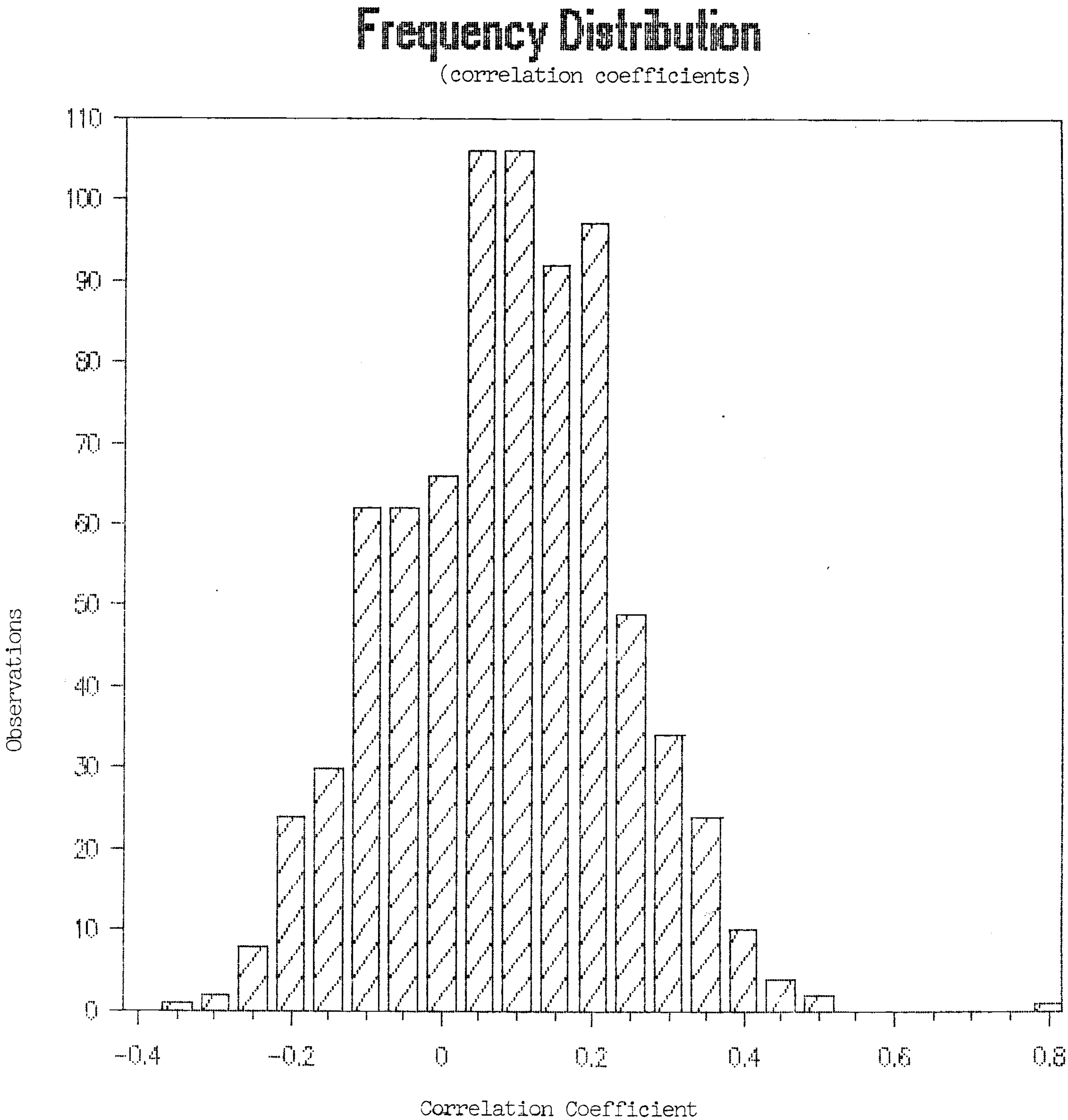
Frequency Distribution
(correlation coefficients)
110
100
90
80
70
60
50
40
30
20
10
0
Observations
-0.4
-0.2
0
0.2
0.4
0.6
0.8
Correlation Coefficient

TABLE 1: FEES CHARGED FOR PORTFOLIO MANAGEMENT
(EX ANTE PERFECT ANALYSIS)

PANEL A: MONTHLY FEES

DELTA	VAL OF TIMING	VALUE OF SELECTION FOR GIVEN N N=1	N=2	N=5	N=10	N=20	N=30
2.	9.3	6.1	12.2	30.5	61.0	122.0	183.0
3.	6.2	4.1	8.1	20.3	40.7	81.3	122.0
4.	4.7	3.0	6.1	15.2	30.5	61.0	91.5
5.	3.7	2.4	4.9	12.2	24.4	48.8	73.2
6.	3.1	2.0	4.1	10.2	20.3	40.7	61.0

PANEL B: ONE-TIME LOADS

DELTA	VAL OF TIMING	VALUE OF SELECTION FOR GIVEN N N=1	N=2	N=5	N=10	N=20	N=30
2.	99.74	99.42	99.84	99.97	99.99	100.00	100.00
3.	98.48	97.26	98.96	99.73	99.90	99.96	99.98
4.	97.05	95.05	97.89	99.35	99.74	99.90	99.94
5.	95.69	93.05	96.84	98.95	99.55	99.81	99.88
6.	94.42	91.22	95.85	98.55	99.36	99.72	99.83

TABLE 2: FEES CHARGED FOR PORTFOLIO MANAGEMENT
(EX ANTE PERFECT ANALYSIS)

PANEL A: MONTHLY FEES

RHO	FEE CHARGED FOR SECURITY SELECTION FOR GIVEN N N=2	N=4	N=6	N=10	N=20	N=30
0.0	6.10	12.20	18.30	30.50	61.00	91.50
0.03	6.10	11.84	17.26	27.23	48.03	64.44
0.06	6.10	11.51	16.34	24.60	39.61	49.73
0.09	6.10	11.19	15.51	22.43	33.70	40.49
0.12	6.10	10.89	14.76	20.61	29.33	34.14
0.15	6.10	10.61	14.08	19.06	25.96	29.52
0.18	6.10	10.34	13.46	17.73	23.28	25.99
0.21	6.10	10.08	12.89	16.58	21.11	23.22
0.24	6.10	9.84	12.36	15.56	19.30	20.99
0.27	6.10	9.61	11.88	14.66	17.78	19.14
0.30	6.10	9.38	11.44	13.86	16.49	17.60

PANEL B: ONE-TIME LOADS

RHO	FEE CHARGED FOR SECURITY SELECTION FOR GIVEN N N=2	N=4	N=6	N=10	N=20	N=30
0.0	97.89	99.13	99.49	99.74	99.90	99.94
0.03	97.89	99.10	99.45	99.70	99.86	99.90
0.06	97.89	99.06	99.41	99.65	99.81	99.86
0.09	97.89	99.03	99.36	99.61	99.77	99.82
0.12	97.89	98.99	99.32	99.56	99.72	99.77
0.15	97.89	98.96	99.28	99.51	99.68	99.73
0.18	97.89	98.92	99.23	99.47	99.63	99.68
0.21	97.89	98.89	99.19	99.42	99.57	99.62
0.24	97.89	98.85	99.15	99.37	99.52	99.57
0.27	97.89	98.82	99.10	99.32	99.47	99.52
0.30	97.89	98.78	99.05	99.26	99.41	99.46

TABLE 3: FEES CHARGED FOR PORTFOLIO MANAGEMENT
NO LEVERAGE ALLOWED

PANEL A: MONTHLY FEES

RHO	FEE CHARGED FOR SECURITY SELECTION FOR GIVEN N N=2	N=4	N=6	N=10	N=20	N=30
0.0	0.18	0.29	0.35	0.41	0.46	0.48
0.03	0.18	0.29	0.34	0.39	0.44	0.46
0.06	0.18	0:28	0.33	0.38	0.43	0.45
0.09	0.18	0.28	0.33	0.37	0.42	0.43
0.12	0.18	0.27	0.32	0.36	0.40	0.42
0.15	0.18	0.27	0.31	0.35	0.39	0.40
0.18	0.18	0.26	0.30	0.34	0.38	0.39
0.21	0.18	0.26	0.30	0.33	0.37	0.38
0.24	0.18	0.26	0.29	0.33	0.35	0.36
0.27	0.18	0.25	0.29	0.32	0.34	0.35
0.30	0.18	0.25	0.28	0.31	0.33	0.34

PANEL B: ONE-TIME LOAOS

RHO	FEE FOR TIMING	FEE CHARGED FOR SECURITY SELECTION FOR GIVEN N N=2	N=4	N=6	N=10	N=20	N=30
0.0	97.05	41.51	54.70	59.56	63.61	66.75	67.81
0.03	97.05	41.51	54.28	58.96	62.87	65.89	66.92
0.06	97.05	41.51	53.86	58.37	62.13	65.04	66.03
0.09	97.05	41.51	53.43	57.78	61.40	64.20	65.15
0.12	97.05	41.51	53.02	57.20	60.68	63.36	64.27
0.15	97.05	41.51	52.60	56.62	59.96	62.53	63.40
0.18	97.05	41.51	52.18	56.04	59.24	61.71	62.54
0.21	97.05	41.51	51.77	55.47	58.53	60.89	61.69
0:24	97.05	41.51	51.36	54.89	57.82	60.08	60.84
0.27	97.05	41.51	50.94	54.33	57.12	59.27	60.00
0.30	97.05	41.51	50.54	53.76	56.42	58.47	59.16

TABLE 4: FEES CHARGED FOR PORTFOLIO MANAGEMENT
(NOISY FORECASTS)

PANEL A: MONTHLY FEES

RHO	N*	FEE CHARGED FOR SECURITY SELECTION FOR GIVEN N N=2	N=4	N=6	N=10	N=20	N=30
0.0	30.58	0.30	0.61	0.91	1.52	3.05	4.57
0.03	31.25	0.30	0.61	0.91	1.52	3.01	4.48
0.06	31.96	0.30	0.61	0.91	1.51	2.97	4.39
0.09	32.70	0.30	0.61	0.91	1.50	2.93	4.30
0.12	33.47	0.30	0.61	0.90	1.49	2.89	4.22
0.15	34.29	0.30	0.61	0.90	1.48	2.86	4.14
0.18	35.14	0.30	0.60	0.90	1.47	2.82	4.06
0.21	36.05	0.30	0.60	0.90	1.46	2.79	3.99
0.24	37.01	0.30	0.60	0.89	1.46	2.75	3.92
0.27	38.02	0.30	0.60	0.89	1.45	2.72	3.85
0.30	39.09	0.30	0.60	0.89	1.44	2.69	3.78

PANEL B: ONE-TIME LOADS

RHO	FEE CHARGED FOR SECURITY SELECTION FOR GIVEN N N=2	N=4	N=6	N=10	N=20	N=30
0.0	56.06	73.48	81.60	89.08	95.05	96.98
0.03	56.06	73.45	81.54	89.01	94.97	96.90
0.06	56.06	73.41	81.49	88.94	94.89	96.82
0.09	56.06	73.38	81.44	88.87	94.81	96.74
0.12	56.06	73.35	81.38	88.80	94.73	96.66
0.18	56.06	73.28	81.28	88.66	94.57	96.50
0.21	56.06	73.25	81.23	88.59	94.49	96.42
0.24	56.06	73.21	81.17	88.52	94.41	96.34
0.27	56.06	73.18	81.12	88.45	94.33	96.26
0.30	56.06	73.15	81.07	88.38	94.25	96.18