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ABSTRACT

We develop a multicountry model in which default in one country triggers default in other countries. Countries are linked to one another by borrowing from and renegotiating with common lenders with concave payoffs. A foreign default increases incentives to default at home because it makes new borrowing more expensive and defaulting less costly. Foreign defaults tighten home bond prices because they lower lenders' payoffs. Foreign defaults make home default less costly by lowering future recoveries, because countries can extract more surplus if they renegotiate simultaneously. In our model, the home country may default only because the foreign country is defaulting. This dependency arises during fundamental foreign defaults, where the foreign country defaults because of high debt and low income, and also during self-fulfilling defaults, where both countries default only because the other is defaulting. The simultaneity in defaults induces a correlation in interest rate spreads across countries. The model can rationalize some of the recent economic events in Europe.

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1 Introduction

Sovereign debt crises tend to occur in tandem. During the 1980s, almost all Latin American countries defaulted and subsequently renegotiated their sovereign debt. Greece, Ireland, Italy, Portugal, and Spain have been struggling with their sovereign debt throughout the recent European debt crises, and Greece defaulted in 2012. During these crises, interest rates increased simultaneously for multiple countries. As Reinhart and Rogoff (2011) show, such clustering of default crises has been present throughout history for the last 200 years. Yet, despite sovereign debt crises occurring in tandem, theoretical work on sovereign default has mainly studied countries in isolation.

This paper develops a multicountry model in which default in one country triggers default in other countries. Countries are linked to one another by borrowing from and renegotiating with common lenders. A foreign default increases incentives to default at home because it makes new borrowing more expensive and defaulting less costly. Foreign defaults make it more difficult for the home country to service the debt because these defaults lower lenders' payoffs, which in turn tightens bond prices at home. Foreign defaults also make home default less costly by lowering future recoveries, because countries can extract more surplus if they renegotiate simultaneously. In our model, the home country may default only because the foreign country is defaulting. This dependency arises during *fundamental* foreign defaults, where the foreign country defaults because of high debt and low income, and also during *self-fulfilling* defaults, where both countries default only because the other is defaulting.

The model economy consists of two symmetric countries that borrow, default, and renegotiate their debt with competitive lenders that have concave payoffs. The price of debt reflects the risk-adjusted compensation for the loss lenders face in case of default. Default entails costs in terms of access to financial markets and direct output costs. After default, countries can renegotiate with a committee of lenders through Nash bargaining. Countries then pay the debt recovery and default costs are lifted. When multiple countries renegotiate, they do it simultaneously with lenders. We consider a dynamic recursive Markov equilibrium.

Countries are linked because the prices of debt and the recovery are determined jointly and depend on countries' choices of default, borrowing, and renegotiation, as well as on their states, which are their level of debt, credit standing, and income. Importantly, borrowing countries are strategically large players and understand that their choices have an impact on all debt prices and recoveries. They engage in Cournot competition between each other when optimizing.

The bond price schedule incorporates the lenders' cost of funds, the risk-adjusted default probability, and the risk-adjusted recovery rate. When the foreign country borrows a large loan or especially when it defaults, the bond price schedule worsens at home because lenders' marginal valuation rises, which increases the cost of funds, and because of higher future default probabilities and and lower future recovery rates at home. Such tightening of the price schedule increases incentives to default at home.

Recoveries also respond to other countries' choices and states. All parties renegotiating in a given period renegotiate simultaneously with Nash bargaining. If two countries renegotiate with lenders, their recoveries are lower than when only one renegotiates because the outside option for lenders is lower. During simultaneous renegotiations, the threat value for lenders in case of renegotiation failure is autarky, whereas during single renegotiations, this threat value consists of the value that arises from continuing to trade with the other country.[1] Hence, anytime the foreign country defaults, the home country's default incentive increases to take advantage of the lower recovery. Foreign defaults also delay renegotiations at home, because renegotiating simultaneously in the future is worth to wait.

We parameterize the model to Europe. To focus on our mechanisms, we study the case of uncorrelated income shocks across countries. The important parameters that determine the extent of debt market linkages are the curvature of lenders' payoff function and the parameters controlling the bargaining process. We calibrate these parameters to the observed volatility of the risk-free rate, the average recovery rate, and the lower recovery observed during multiple-country renegotiations taken from the historical data set of recoveries provided by Cruces and Trebesch (2013). Other parameters of the model are calibrated to match observed spreads in Greece.

We find that about 25% of the defaults at home occur only because the foreign country is defaulting. About 11% of these induced defaults happen because of fundamental foreign defaults, where the foreign country defaults due to high debt or low income. In 14% of these defaults, however, the dependency is self-fulfilling in that the foreign country defaults only because the home country is defaulting too. Repayment in the foreign country also induces repayment at home. This beneficial dependency arises 27% of the time the home country is repaying. Almost all of the renegotiations in our model are dependent on the foreign country renegotiating or repaying.

The model predicts that country interest rate spreads comove. The cross-country correlation of spreads across countries in the model is 0.43, which implies that about half of the correlation of spreads between Italy and Greece of 0.97 can be attributed to linkages in

[1] In practice, countries also renegotiate together when frequently in default. The Brady Plan of the early 1990s is an example in which many Latin American countries renegotiated together and received an unusually good deal. These countries were able to exchange their defaulted debt for new Brady bonds with principal collateralized by the U.S. government.

their debt markets. Our model also predicts that the correlation of countries' borrowing is positive, as shown in the data of Greece and Italy, and equal to 0.3 and 0.56 in the model and data, respectively.

The positive correlation in spreads arises largely because countries default together. The probability of default at home rises from an average of 4.5% to over 37% in states when the foreign country defaults. A second reason for the positive correlation in spreads is that large foreign borrowing also increases home default probabilities because large foreign loans tighten the home bond price schedule. When the foreign country borrows heavily and has spreads above its median, the default incidence at home is about 1.3%, higher than when foreign spreads are below the median. Hence, current foreign defaults not only induce home defaults today but also home defaults in the near future.

The model also predicts that foreign defaults hinder renegotiations at home because recoveries spike. Recoveries for the home country during foreign defaults increase from an average of 66% to 90%. These recoveries reduce the probability of renegotiation from 98% to 1%. The lack of foreign renegotiations also reduces home renegotiations to zero if the home country is in bad credit standing and increases default probabilities to 100% if the home country is in good credit standing.

Through comparative static exercises, we find that the majority of the correlation in spreads arises because of the strategic interactions between the two countries in default and renegotiation. If countries were linked only by common risk-free rates and were small and non-strategic, the correlation in spreads would drop from 0.43 to 0.17. Moreover, the fact that countries want to renegotiate together to take advantage of lower recoveries is quantitatively important. This effect alone would deliver a positive correlation across spreads of 0.28, as shown in an exercise in which lenders have linear payoffs. Nevertheless, concavity in the lenders' payoff function does increase the correlation across spreads as in the benchmark because the bond price functions at home respond not only to foreign defaults but also to the level of foreign borrowing.

The mechanism of the model rests on the idea that having a common lender generates financial linkages across countries. A large empirical literature on contagion has found support for the common lender theory. Kaminsky and Reinhart (2000) and Van Rijckeghem and Weder (2001) provide evidence that countries that borrow from the same lender as the country where the crisis started are more vulnerable to contagion than those countries that borrow from other lenders. Van Rijckeghem and Weder (2003) also show that spillovers through bank lending, as opposed to trade linkages and country characteristics, can help explain contagion in the Mexican, Thai, and Russian crises. Using a disaggregated database on mutual funds that hold emerging markets securities, Broner, Gelos, and Reinhart (2006) find that the extent of the mutual fund exposure to countries' shares helps explain the pattern of stock market comovement across countries as well as the pattern of contagion during crises.

The model in this paper builds on the benchmark model of equilibrium default with incomplete markets analyzed in Aguiar and Gopinath (2006) and Arellano (2008), and in a seminal paper on sovereign debt by Eaton and Gersovitz (1981). These papers analyze the case of risk-neutral lenders, abstract from recovery, and focus on the default experiences of single countries. Borri and Verdelhan (2009) and Lizarazo (2013) study the case of risk-averse lenders, and Pauzo and Presno (2011) study the case of lenders with uncertainty aversion. They show that deviations from risk neutrality allow the model to generate spreads larger than default probabilities, which is a feature of the data. Borri and Verdelhan also show empirically that a common factor drives a substantial portion of the variation observed. Lizarazo (2009) and Park (2013) study contagion in a model similar to ours in which multiple borrowers trade with risk-averse lenders. Their model can generate comovement in spreads across borrowing countries; however, they abstract from any debt renegotiation and strategic interactions because they both consider competitive borrowers. Yue (2010), D'Erasmo (2011), and Benjamin and Wright (2009) study debt renegotiation in a model with risk-neutral lenders. They find that debt renegotiation allows the model to better match the default frequencies and the debt-to-output ratios.

Our model also presents new types of self-fulfilling equilibria that lead to sovereign defaults. Coordination failures have been popular explanations for sovereign debt crises. The main channel analyzed in the literature, however, emphasizes coordination failures among lenders, whereas we focus on coordination issues among borrowers.[2] Cole and Kehoe (2000), for example, develop a model with multiple equilibria in which defaults are self fulling: lenders refuse to completely roll over the country's debt because they think that countries will default on the debt, which in turn leads to default. Relatedly, Lorenzoni and Werning (2013) develop a dynamic model with self-fulfilling defaults arising from high interest rates. Lenders charge higher interest rates because they predict high default rates. These high rates lead to faster debt accumulation and self-fulfilling high default rates. In contrast, the self-fulfilling equilibria of our model arise because of strategic interactions among large borrowers, which we view as also relevant for the case in which sovereign countries borrow from international lenders.

[2]In the context of private borrowing, Arellano and Kocherlakota (2012) present a model in which borrowers default when other borrowers are also defaulting in environments in which private debtors cannot be punished when many are in default.

2 Model

Consider an economy in which two symmetric countries, Home and Foreign, borrow from a continuum of foreign lenders. Countries are strategically large players who borrow, default, and renegotiate their debt. Lenders are competitive and have a concave payoff function. Countries that default receive a bad credit standing, are excluded from borrowing, and suffer a direct output cost. Countries in bad credit standing can renegotiate their debt with a committee of lenders and bargain over the debt recovery. After renegotiation is complete, countries regain their good credit standing.

The current period payoff to each borrowing country i is $u(c_{it})$, and the current payoff to lenders is $g(c_{Lt})$ where c_{it} is the consumption of the representative household in each country and c_{Lt} is the dividend to lenders. The functions $u(\bullet)$ and $g(\bullet)$ are increasing and concave. The lifetime payoff to each borrowing country i is $E\sum_{t=0}^{\infty}\beta^t u(c_{it})$, and the payoff to lenders is $E\sum_{t=0}^{\infty}\delta^t g(c_{Lt})$. Borrowing countries are more impatient than lenders: $0 < \beta < \delta < 1$.

Each borrowing country receives a stochastic endowment each period. Let $y = \{y_i\}_{\forall i}$ be the vector of endowments for each country in a period. These shocks follow a Markov process with transition matrix $\pi(y', y)$. We assume that lenders face no additional shocks. The endogenous aggregate states consist of the vector of countries' debt holdings $b = \{b_i\}_{\forall i}$ and their credit standing $h = \{h_i\}_{\forall i}$. The economy-wide state s incorporates the endogenous and exogenous states: $s = \{b, h, y\}$.

2.1 Borrowing Countries

The government of each country is benevolent, and its objective is to maximize household utility. The government trades one-period discount bonds with foreign lenders, decides whether to repay or default on its debt, and after a default, decides whether or not to renegotiate the debt. The government rebates back to households all the proceedings from its credit operations in a lump-sum fashion. We label country i as Home and country $-i$ as Foreign. Below we describe in detail the problem for the home country. The problem for the foreign country is symmetric.

We consider a Markov equilibrium where the governments take as given future decisions. The *current strategy* for the government at Home incorporates its repayment or renegotiation decision d_i and its borrowing decision b_i'. When the country is in good credit standing $h_i = 0$, it decides to repay the debt by setting $d_i = 0$. Only after deciding to repay can the country choose its new borrowing b_i'. If the government decides to default by setting $d_i = 1$, the government cannot borrow and its credit standing changes to bad the following period. When the home government is in bad credit standing $h_i = 1$, it decides to renegotiate by setting $d_i = 0$. Renegotiation changes the next period's credit standing of the government to good. After renegotiation the government starts with zero debt, $b'_i = 0$. The current strategy for both countries is summarized by $\{b', d\} = \{b'_i, d_i\}_{\forall i}$.

The home prices for loans $q_i(s, b', d)$ and recovery $\phi_i(s, b', d)$ are functions that depend on the current strategies for both countries as well as the aggregate state. In making decisions, the governments take as given the price and recovery functions. The bond price function compensates the lender for the risk-adjusted loss in case of default and depends on the strategies of both countries and the aggregate states because the lenders' kernel, and future defaults, renegotiations, and recoveries depend on all of these variables. The recovery function is the result of a bargaining process, the outcome of which depends on the countries' strategies and the aggregate state. Below we specify how the bond price and recovery functions are determined.

The current home consumption depends on the aggregate state and the current strategies of both countries $c_i(s, b', d)$. Consider a case where the home country is in good credit standing, $h_i = 0$, and an arbitrary strategy to repay $d_i = 0$ and to borrow b'_i. Consumption in this case is

$$c_i = y_i - b_i + q_i(s, b', d)b'_i \tag{1}$$

Note that consumption for country i also depends on the state and strategy of the other country by their effect on the price q_i. Now consider consumption with a strategy to default, such that $d_i = 1$. Default results in exclusion from trading international bonds and output costs $y_i - y_i^d$, with $y_i^d \leq y_i$. Consumption equals output during these periods:

$$c_i = y_i^d. \tag{2}$$

Following Arellano (2008) we assume that borrowers lose a fraction λ of output if output is above a threshold:

$$y_t^d = \begin{cases} y_t & \text{if } y_t \leq (1-\lambda)\bar{y} \\ (1-\lambda)\bar{y} & \text{if } y_t > (1-\lambda)\bar{y} \end{cases}$$

where $\bar{y}$ is the mean level of output.

Finally, consider the case when country i is in bad credit standing such that $h_i = 1$. When renegotiation is chosen, $d_i = 0$, the country pays the recovery $\phi_i(s, b', d)$, starts tomorrow with zero debt, $b'_i = 0$, and consumption is

$$c_i = y_i - \phi_i(s, b', d) \tag{3}$$

Here, the state and strategy of the other country also affect home consumption by their effect on the recovery. If the home country does not renegotiate, then consumption satisfies (2).

We represent the home borrowing country's payoffs as a dynamic programming problem. The government today takes as given all the decisions of future governments, which are summarized by the continuation value function from tomorrow on $v_{i,t+1}(s')$ when the state tomorrow is s'. The lifetime payoff of the home country today when the state today is s for arbitrary current strategies (b', d) is

$$w_{i,t}(s, b', d; v_{t+1}) = \{u(c_i(s, b', d)) + \beta \sum_{y'} \pi(y', y) v_{i,t+1}(s')\}. \tag{4}$$

Tomorrow's state $s' = \{b', h', y'\}$ depends on the current strategy of both countries. Specifically, the future credit standing and debt tomorrow depend on the default and renegotiation of each country, as follows

$$h'_i = \begin{cases} 1 & \text{if } d_i = 1 \\ 0 & \text{otherwise} \end{cases} \qquad for\ all\ i \tag{5}$$

$$b'_i = \begin{cases} b'_i & \text{if } h_i = 0 \text{ and } d_i = 0 \\ b_i & \text{if } d_i = 1 \\ 0 & \text{otherwise} \end{cases} \qquad for\ all\ i \tag{6}$$

In our model, each borrowing country internalizes the effects its strategies have on bond prices and recoveries. The intraperiod game between the two countries has two stages. In the first stage, countries make their default and renegotiation decisions. In the second stage, if countries chose to repay in the first stage, they make their borrowing decisions and engage in Cournot competition with one another.[3]

To develop the intraperiod game, we start with the second borrowing stage after default and renegotiation decisions d have been made. The nature of this subgame depends on the credit standing of countries and their repayment decisions. When all countries are in good credit standing and repay, $\{d_i = 0\}_{\forall i}$, equilibrium borrowing strategies $B(s, d) = \{B_i(s, d)\}_{\forall i}$ are Nash in that $\{B_i = x_i^b(B_{-i}, s, d)\}_{\forall i}$, where $x_i^b(b'_{-i}, s, d)$ is the borrowing best response of each country i for arbitrary borrowing strategies b'_{-i}, given states s and repayment choices d,

$$x_i^b(b'_{-i}, s, d) = \{b'_i : \max_{b'_i} w_i(s, b', d; v_i(s'))\} \text{ for all } i. \tag{7}$$

[3]We subdivide the intraperiod game between the two countries into a repayment and borrowing stage because it substantially simplifies our computational algorithm.

When each country starts with a bad credit standing or it defaults, it cannot borrow and hence does not enter the second borrowing stage of the game. Here, the remaining country i chooses its borrowing to satisfy (7), where b'_{-i} equals b_{-i} or 0 according to the default and renegotiation choices given by (6).

In the first stage of the game, each country i chooses its repayment strategy d_i taking as given the equilibrium borrowing strategies of the second stage. The equilibrium repayment strategies $D(s) = \{D_i(s)\}_{\forall i}$ are Nash in that $\{D_i = x_i^d(D_{-i}, s, B(s, D)\}_{\forall i}$, where $x_i^d(d_{-i}, s, B(s, d)\}$ is the repayment best response of each country i for arbitrary repayment strategies d_{-i}, given states s and taking into account the outcome of the second borrowing stage $B(s, d)$:

$$x_i^d(d_{-i}, s, B(s, d)) = \{d_i : \max_{d_i} w_i(s, B(s, d), d; v_i(s'))\} \qquad \text{for all } i. \tag{8}$$

The resulting outcome of the intraperiod game is summarized by the repayment and borrowing functions $\{D(s)\}$ and $\{B(s) = B(s, D(s))\}$, as well as the consumptions $c(s) = \{c_i(s)\}_{\forall i}$ and values $v(s) = \{v_i(s)\}_{\forall i}$.

Definition 1. *A Markov partial equilibrium takes as given price functions $\{q_i(s, b', d)\}_{\forall i}$ and recovery functions $\{\phi_i(s, b', d)\}_{\forall i}$ and consists of equilibrium strategies $\{B(s), D(s)\}$ and payoffs $c(s)$ and $v(s)$ such that*
(1) Given future value functions $v(s')$, period equilibrium strategies $\{B(s), D(s)\}$ are the solution of the intraperiod game such that they satisfy (7), (8), and (6).
(2) Equilibrium payoffs $v(s)$ implied by equilibrium strategies $\{B(s), D(s)\}$ are a fixed point

$$v_i(s) = w_i(s, B(s), D(s); v_i(s')) \qquad \textit{for all } i.$$

2.2 Lenders

Competitive lenders trade bonds with the two borrowing countries. Every period lenders receive a constant payoff from the net operations of other loans $r_L L$ and deposits $r_d D$, which we summarize by $y_L = r_L L - r_d D$. We assume that lenders honor all financial contracts.

Lenders take as given the evolution of the aggregate state,

$$s' = H(s) \tag{9}$$

and the corresponding decision rules for debt, default and renegotiation, $\{B(s), D(s)\}$. Lenders choose optimal dividends c_L and loans to the borrowing countries $\ell' = \{\ell'_i\}_{\forall i}$, taking as given the prices of bonds $Q = \{Q_i\}_{\forall i}$ and recoveries $\Phi = \{\Phi_i\}_{\forall i}$. The value function for the lender is given by

$$v^L(\ell, s) = \max_{\{c_L, \ell'_i \text{ if } h_i = h'_i = 0\}\forall i} \{g(c_L) + \delta \sum_{y'} \pi(y', y) v^L(\ell', s')\}. \tag{10}$$

Lenders maximize their value subject to their budget constraint that depends on the credit standing of each borrowing country and whether they repay,

$$c_L = y_L + \sum_i (1 - D_i(s)) \left((1 - h_i)(\ell_i - Q_i \ell'_i) + h_i \frac{\Phi_i \ell_i}{b_i} \right), \tag{11}$$

the evolution of the endogenous states when they do not trade with each country,

$$\ell'_i = \begin{cases} \ell_i \text{ if } h'_i = 1 \\ 0 \text{ if } (h_i = 1 \text{ and } h'_i = 0) \end{cases} \qquad \text{for all } i, \tag{12}$$

and the evolution of the aggregate state (9).

Using the first order conditions and envelope conditions for the lenders' problem, one can show that bond prices satisfy

$$Q_i = \sum_{s'} [m(s', s)(1 - D_i(s')(1 - \zeta_i(s'))] \qquad \text{for all } i, \tag{13}$$

where $\zeta_i(s')$ is the present value of recoveries and is defined recursively by

$$\zeta_i(s) = \sum_{s'} \left[m(s', s)(1 - D_i(s')) \frac{\Phi_i(s')}{b'_i} + D_i(s') \zeta_i(s') \right] \qquad \text{for all } i. \tag{14}$$

and $m(s', s)$ is the lenders' stochastic discount factor or pricing kernel,

$$m(s', s) = \frac{\delta \pi(y', y) g'(c_L(s'))}{g'(c_L(s))},$$

where $c_L(s)$ are the equilibrium dividends in state s.

The bond prices in (13) and the values of recoveries in (14) are easily interpretable. The bond price contains two elements: the payoff in nondefault states $D_i(s') = 0$ and the payoff in default states $D_i(s') = 1$. The lender discounts cash flows by the pricing kernel $m(s', s)$ and hence states are weighted by $m(s', s)$. For every unit of loan ℓ'_i, the lender gets one unit in the nondefault states and the value of recovery $\zeta_i(s')$ in default states. The recovery value is the expected payoff from defaulted debt the following period. It also contains two parts. If the country renegotiates next period, $D_i(s') = 0$, and the value of recovery for every unit of loan is $\frac{\Phi_i(s')}{b'_i}$. If the country does not renegotiate, $D_i(s') = 1$, and the present value of recovery is the discounted value of future recovery given by $\zeta_i(s')$. These future recovery values are weighted by the pricing kernel $m(s', s)$, which implies that recovery values are weighted more heavily for states s' that feature a higher pricing kernel.

The bond price compensates the lender for any covariation between its kernel and the bond payoffs. If default happens in states when $m(s', s)$ is low, the price contains a positive risk premia for low payoff in the default event. Moreover, if the value of recovery is low when $m(s', s)$ is low, the price also contains positive risk premia for the covariation of recovery.

2.3 Renegotiation Protocol

During renegotiation, countries renegotiate their debt with a committee of lenders. The renegotiation protocol we consider is one in which the committee of lenders bargain simultaneously with all the countries renegotiating using Nash bargaining.[4]

First consider the case in which only country i renegotiates its debt. Consider a candidate recovery value $\hat{\phi}_i$. The payoff for lenders from renegotiating and receiving recovery $\hat{\phi}_i$ equals the value of the representative lender evaluated at the aggregate debt values, $V^L(s; \hat{\phi}_i) \equiv v^L(b, s; \hat{\phi}_i)$. The payoff for the borrower from renegotiation is $v_i(s; \hat{\phi}_i)$ for this candidate value of recovery $\hat{\phi}_i$. If the two parties do not reach an agreement, the defaulter country is in permanent financial autarky with $y_i = y_i^d$ and gets a threat value equal to

$$v_{i,aut}(y) = \{u(y_i^d) + \beta \sum_{y'_i} \pi(y', y) v_{i,aut}(y')\}.$$

All lenders recover zero debt and are permanently precluded from trading with the defaulter country. Lenders, however, will still have access to financial trading with the other nondefaulting country. Let $V^L_{fail}(s_{-i})$ be the value to all lenders from trading only with the nondefaulting country. This value arises from the single-country Markov equilibrium described in detail in Appendix I.

The recovery ϕ_i maximizes the weighted surplus for borrowing country i and the lenders. The bargaining power for the borrower is θ and that for lenders is $(1-\theta)$. Recovery ϕ_i solves

$$\max_{\phi_i \epsilon [0,1]} \left[v_i(s; \phi_i) - v_{i,aut}(y)\right]^{\theta} \left[V^L(s; \phi_i) - V^L_{fail}(s_{-i})\right]^{1-\theta} \tag{15}$$

subject to both parties receiving a nonnegative surplus from the renegotiation: $v_i(s; \phi_i) -$

[4] Such bargaining protocol has often been used in industrial organization models of multifirms. See Dobson (1994) and Horn and Wolinsky (1988) for details.

$v_{i,aut}(y_i) \geq 0$, and $V^L(s;\phi_i) - V^L_{fail}(s_{-i}) \geq 0$, and law of motion (9).

Now consider states when both countries renegotiate simultanously with the committee of all lenders. If the parties do not reach an agreement, all parties remain in financial autarky thereafter. The recoveries $\{\phi_i\}$ for all i solve

$$\max_{\phi_i \epsilon [0,1]} \left[v_i(s;\phi_i) - v_{i,aut}(y)\right]^{\theta} \left[V^L(s;\phi_i,\phi_{-i}) - V^L_{aut}\right]^{1-\theta} \text{ for all } i \tag{16}$$

subject to all parties receiving a nonnegative surplus from the renegotiation and law of motion (9). The outside option for the lenders in this case is autarky $V^L_{aut} = \frac{g(y_L)}{1-\delta}$. The interpretation for lenders having autarky as their outside option is that countries have an agreement exante on a cooperative bargaining strategy to send offers to the committee of lenders, and the committee has to accept or reject both offers simultaneously.

An important aspect of the renegotiation protocol we consider is the simultaneity in bargaining between the committee of lenders and all countries renegotiating. Under such protocol, countries send offers to lenders, and they have to accept or reject all offers simultaneously.[5] Such simultaneity implies that the threat value for lenders depends on whether only one country renegotiates or two countries renegotiate. The differences between these two threat values have implications for the simultaneity of defaults and renegotiations across countries.

2.4 Functions for Bond Prices and Recoveries

The lenders' problem and the renegotiation protocol determine the functions for bond prices and recoveries. First consider the case when both countries are in good credit standing, $\{h_i = 0\}_{\forall i}$. Here, bond price functions $q(s,b',d) = \{q_i(s,b',d)\}_{\forall i}$ solve the demand system determined by lenders' first order conditions:

$$q_i = \sum_{s'} \left[m(s',s;q,b',d)(1 - D_i(s')(1-\zeta_i(s'))\right] \text{ for all } i, \tag{17}$$

where the state tomorrow $s' = \{b',h',y'\}$ depends on countries' current strategies (b',d) and the lenders' kernel $m(s',s;q,b',d)$ is itself a function of prices, countries' strategies, and current and future states.

Now consider the case when country i is in good credit standing and country $-i$ is in bad credit standing, $h_i = 0$ and $h_{-i} = 1$. The bond price function for country i and the recovery

[5] Dobson (1994) describes such protocol as *strict simultaneous bargaining*, where countries have an agreement beforehand to eliminate any other alternative bargaining strategy for lenders.

function derived from (15) for country $-i$, $\{q_i(s,b',d), \phi_{-i}(s,b',d)\}$ solve

$$q_i = \sum_{s'} [m(s',s;q,b',d)(1-D_i(s')(1-\zeta_i(s'))] \tag{18}$$

$$\frac{\theta u'(y_{-i}-\phi_{-i})}{[v_{-i}(s;\phi_{-i}) - v_{_{-i,aut}}(y_{-i})]} = \frac{(1-\theta)g'(c_L(s,q_i,\phi_{-i},b',d))}{[V^L(s,q_i,\phi_{-i},b',d) - V^L_{fail}(s_i)]}$$

where the lender's dividends and values are evaluated for every strategy and corresponding price and recovery.

Finally, when both countries are in bad credit standing, $\{h_i = 1\}_{\forall i}$ recovery functions $\phi(s,b',d) = \{\phi_i(s,b',d)\}_{\forall i}$ are derived from (16) and solve

$$\frac{\theta u'(y_i-\phi_i)}{[v_i(s;\phi_i) - v_{_{i,aut}}(y)]} = \frac{(1-\theta)g'(s,q_i,\phi_{-i},b',d)}{[V^L(s,\phi,d) - V^L_{aut}]} \text{ for all } i. \tag{19}$$

2.5 Equilibrium

We focus on recursive Markov equilibria in which all decision rules are functions only of the state variable s.

Definition 2. *A recursive Markov equilibrium for this economy consists of (i) countries' policy functions for repayment, borrowing, and consumption, $\{B(s), D(s), C(s)\}$, and values $v(s)$; (ii) lenders' policy functions for lending choices and dividends $\{\ell'(\ell,s), c_L(\ell,s)\}$ and value function $v^L(\ell,s)$; (iii) the functions for bond prices and recoveries $\{q(s,b',d), \phi(s,b',d)\}$; (iv) the equilibrium prices of debt $Q(s)$ and recovery rates $\Phi(s)$; (v) the evolution of the aggregate state $H(s)$; and (vi) the lenders' value in the case of renegotiation failure $\{v^L_{i,fail}(\ell_i,s_i)\}_{\forall i}$ such that given $b_0 = \ell_0$:*

1. *Taking as given the bond price and recovery functions, the policy and value functions for countries satisfy the Markov partial equilibrium in definition (1).*

2. *Taking as given the bond prices $Q(s)$, recoveries $\Phi(s)$, and the evolution of the aggregate states $H(s)$, the policy functions and value functions for the lenders $\{\ell'(\ell,s)$, $c_L(\ell,s)$, $v^L(\ell,s)\}$ satisfy their optimization problem.*

3. *Taking as given countries' policy and value functions, bond price and recovery functions $\{q(s,b',d), \phi(s,b',d)\}$ satisfy (17), (18), and (19).*

4. *The prices of debt $Q(s)$ clear the bond market for every country,*

$$\ell_i'(s) = B_i(s) \text{ for all } i.$$

5. *The recoveries $\Phi(s)$ exhaust all the recovered funds*

$$\phi_i(s, B(s), D(s)) = \Phi_i(s) \text{ for all } i.$$

6. *The goods market clears*

$$c_1 + c_2 + c_L = y_1 + y_2 + y_L.$$

7. *The law of motion for the evolution aggregate states (9) is consistent with countries' decision rules and shocks.*

8. *The lenders' value in the case of renegotiation failure $\{v^L_{i,fail}(\ell_i, s_i)\}_{\forall i}$ arises from the single-country Markov equilibrium.*

3 Joint Defaults

In this section, we develop a simple two-period example to illustrate why countries have incentives to default together.

Consider a two-period version of our model with no uncertainty, where countries have identical endowment paths y and y'. The lenders' payoff function is $g(c_L) = \frac{c_L^{1-\alpha}-1}{1-\alpha}$. In period 1 the two countries with debt b_i and b_{-i} are in good credit standing and are deciding whether to repay their current debt or default on it. If countries repay their debt, they choose to borrow. In period 2, countries either repay their debts if they borrowed in period 1 or pay the recovery ϕ' if they defaulted in period 1. In this example without uncertainty, in period 2 countries with good credit always repay and countries with bad credit always renegotiate, $\{d_i' = 0\}_{\forall i}$. Default does not happen in equilibrium in period 2 because default would be perfectly foreseen and the price of such a loan would be zero. Default incentives in period 2, however, limit the borrowing possibilities for period 1. In particular, in period 1 countries effectively face a borrowing limit $\bar{b}$, which is the maximum repayment that countries would be willing to make and equals the default penalty in period 2, $\bar{b} = y' - y^d$, where $y^d < y'$ is the income in case of default.

In this example, we assume that β is sufficiently less than δ such that it is optimal for countries to borrow to the limit in period 1. Hence, we abstract from the interdependence across countries in the borrowing decisions and focus on the interdependence in their repayment/default decisions. In this simplified environment, the relevant states for bond prices are the debt states b and the default decisions of both countries d, $\{q_i(b,d)\}_{\forall i}$. The relevant states for recovery tomorrow are the credit standing of both countries h', which is determined by d, $\{\phi_i'(h')\}_{\forall i}$. This example has these reduced states because we are assuming that endowments are constant for the countries. Here again, we label i as Home and $-i$ as Foreign.

In period 1, each country repays and sets $d_i = 0$ if the value of repayment is greater than the value of default:

$$u(y - b_i + q_i(b,d)\bar{b}) + \beta u(y' - \bar{b}) \geq u(y^d) + \beta u(y' - \phi_i'(h')) \text{ for all } i. \tag{20}$$

It is apparent that default is more likely for country i when debt b_i is high, the price q_i is low, and the recovery tomorrow ϕ_i' is low. The default decisions of the two countries are linked because bond prices today and recoveries tomorrow depend on the decisions of both countries through the lenders' problem.

It is useful to derive the home country's default best response conditional on the foreign country's default decision, $x_i^d(d_{-i}, b)$. The foreign default decision affects the home country's future recovery ϕ_i' and current debt price q_i. A foreign default today decreases the home recovery ϕ_i' tomorrow because the surplus from renegotiating is higher when both countries renegotiate together, $\phi_i'(h_{-i}' = 1) < \phi_i'(h_{-i}' = 0)$. A foreign repayment increases the recovery because here the country borrows $\bar{b}$ in period 1 and repays it in period 2. The $\bar{b}$ payment gives the lender a high outside option during renegotiation with the home country, which in turn increases the equilibrium $\phi_i'(h_{-i} = 0)$. This force implies that a foreign default $d_{-i} = 1$ increases the right-hand side of equation (20) and thus increases the incentive to default for the home country.

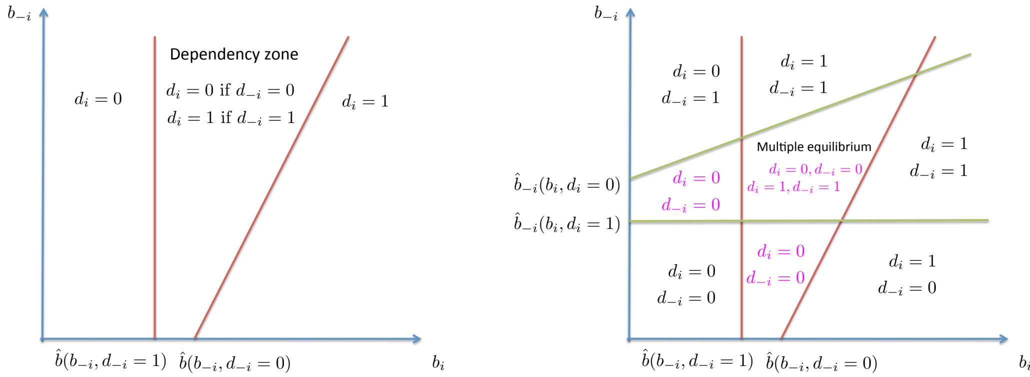
Proposition 3. *Suppose $\alpha \geq 1$. When two countries renegotiate simultaneously, recovery is smaller than when one country renegotiates alone: $\phi_i'(h_{-i}' = 1) < \phi_i'(h_{-i}' = 0)$.*

Proof. See Appendix II.

The second effect to consider is how a foreign default affects price q_i. This effect depends on the net capital flows that lenders forgo with the foreign default, $b_{-i} - q_{-i}\bar{b}$. The larger the foreign forgone capital flows, the more unfavorable the home bond price becomes with a foreign default. The following proposition shows that capital flows are increasing with b_{-i}, and the effect of a foreign default is increasingly detrimental for q_i the higher b_{-i}.

Proposition 4. *Home bond prices increase with the foreign country's debt when the foreign country repays:* $q_i(b, d)$ *is increasing in* b_{-i} *when* $d_{-i} = 0$.

Proof. See Appendix II.



(a) Home Best Response (b) Equilibrium

Figure 1: Debt Linkages

As in single-country default models, the home country will default when its current debt b_i is sufficiently high. It is useful to consider two home debt cutoffs $\hat{b}(b_{-i}, d_{-i} = 0)$ and $\hat{b}(b_{-i}, d_{-i} = 1)$, which depend on the foreign state and default decision. Home defaults when its debt level is above these two cutoffs.

The effects of a foreign default on the price q_i and the future recovery ϕ_i' imply that $\hat{b}(b_{-i}, d_{-i} = 0)$ is increasing in b_{-i} and that $\hat{b}(b_{-i}, d_{-i} = 1)$ is independent of b_{-i}. The ranking of $\hat{b}(b_{-i}, d_{-i} = 0)$ and $\hat{b}(b_{-i}, d_{-i} = 1)$ at $b_{-i} = 0$ depends on the details of the utility of lenders. We assume that the effect of default on recovery is strong enough such that $\hat{b}(b_{-i} = 0, d_{-i} = 0) > \hat{b}(b_{-i} = 0, d_{-i} = 1)$.

To summarize this analysis, Figure 1(a) plots the home best responses for default as a function of its own debt level b_i and the foreign country's debt level b_{-i} conditional on the foreign default decision d_{-i}. For sufficiently low (or high) levels b_i, the home country always repays (or defaults) independently of the foreign decision. For intermediate levels of b_i, however, the home country repays only if the foreign country repays. We label this region the *dependency zone.* By symmetry, the best response of the foreign country is identical to that of the home country, such that for intermediate levels of debt, the foreign country repays only if the home country repays.

Figure 1(b) illustrates the equilibrium in this example by considering both best response functions. The figure shows that in the dependency zones, both countries have joint repayments and joint defaults. Consider the dependency zone for country 1. When the foreign debt is low enough, the foreign repayment guarantees a home repayment. For high foreign debt, a foreign default guarantees a home default. When the foreign debt is in the intermediate region, our model features multiple equilibrium: either both countries default or both countries repay. Nevertheless, even in this region the equilibrium features either joint defaults or joint repayments.

This example has highlighted the forces that in our model lead to joint defaults due to a common lender. The main idea is that foreign defaults lead to home defaults because foreign defaults lead to lower future recoveries and tighter current bond prices for the home country. Joint defaults and joint repayments occur for fundamental and self-fulfilling reasons. In this example, however, we have abstracted from debt dynamics and have considered an arbitrary level of initial debt. In practice, the level of debt is endogenous to countries' decisions and their choices interact with defaults and renegotiations. In the following section, we analyze the general dynamic model with endogenous borrowing and default.

4 Quantitative Analysis

We solve the model numerically and analyze the linkages across the two borrowing countries in terms of spreads, defaults, recoveries, and renegotiations. Debt market linkages are quantitatively important and can generate strong positive comovements among spreads and debt exposures.

4.1 Calibration

The utility function for the borrowing countries is $u(c) = \dfrac{c^{1-\sigma}}{1-\sigma}$. We set the intertemporal elasticity of substitution (IES) $1/\sigma$ to $1/2$, which is a common value used in real business cycle studies. The utility for lenders is $g(c_L) = \frac{c_L^{1-\alpha}}{1-\alpha}$. The IES for lenders $1/\alpha$ is calibrated below.

The length of a period is one year. We assume the stochastic process for output for the borrowing countries is independent of one another and follows a lognormal AR(1) process: $\log(y_{t+1}) = \rho \log(y_t) + \varepsilon_{t+1}$ with $E[\varepsilon^2] = \eta^2$. We discretize the shocks into a nine-state Markov chain using a quadrature-based procedure (Tauchen and Hussey, 1991). To calibrate the volatility and persistence of output, we use an annual series of linearly detrended GDP for Greece for the period of 1960–2011, taken from the World Development Indicators .

We calibrate six parameters: the lenders' and borrowers' discount rates δ and β , the lenders' IES $1/\alpha$, the lenders' endowment y_L, the default cost λ, and the borrower's bargaining parameter θ, to match seven moments: the average yield and volatility of German one-year bonds of 4% and 1.4%, the average spread and volatility of Greek euro bonds of 1.5% and 2.6%, the volatility of German exposure to Greek debt of 15%, the average recovery of 60% and the difference between recoveries when many countries renegotiate their debt, and recoveries in single-country renegotiations of 16%.

The German exposure to Greek debt is measured as the total level of Greek debt held by the German financial sector. The series is taken from the Bank of International Settlements data set on cross-border claims. The volatility is computed from a log and linearly detrended series.

The average recovery of 60% is the one reported in Cruces and Trebesch (2013) across 182 sovereign restructures for the period 1970-2010. With this data set we compute the difference in recoveries for single renegotiations and joint renegotiations. We find that recoveries are 16 percentage points lower when the recovery occurs in a year during which four or more countries finish their renegotiations. To construct this moment, we regress the recovery rate for each renegotiation episode between 1970 and 2010, $recovery_{i,t}$, on a dummy variable that takes the value of 1 if in the renegotiation year there were four or more final renegotiations, $multi_i$. The estimated regression is

$$recovery_{it} = 0.76 - 0.16 * multi_t + \epsilon_{it}.$$

The coefficient on the dummy variable is statistically significant at the 5% level. Introducing country fixed effects in the regression changes only slightly the coefficient on $multi_t$ to -0.17, and it continues to be significant. [6]

Table 1 summarizes the parameter values.

We solve the model as the limit of a finite horizon model in which each period both countries engage in Cournot competition with one another, taking as given the future decisions that are encoded in the future values. As in the simple example, for a certain region of the parameter space, our model features multiple equilibria. We select the equilibrium that maximizes the joint values for the two borrowing countries, $v_1 + v_2$. The numerical algorithm is explained in detail in Appendix III.

4.2 Main Calibration Results

We simulate the model and report statistics summarizing debt markets for the home country. Because of symmetry, statistics for the foreign country are equal.

[6] We found similar results using an alternative data set of renegotiations provided by Benjamin and Wright (2009). In this data set, recovery rates are 13% lower in multiple-country renegotiations.

Table 1: Parameter Values

	Value	Target
Borrowers' IES	$1/\sigma = 1/2$	Standard value
Stochastic structure for shocks	$\rho = 0.88$, $\eta = 0.03$	Greek output
Calibrated parameters		
Output cost after default	$\lambda = 0.016$	German yield: mean and volatility
Borrowers' discount factor	$\beta = 0.82$	Greek spread: mean and volatility
Lenders' discount factor	$\delta = 0.96$	Recovery rate: mean and conditional
Lenders' endowment	$y_L = 1.4$	Volatility of exposure
Lenders' IES	$1/\alpha = 1/0.65$	
Bargaining power	$\theta = 0.38$	

Table 2 reports the calibration results as well as the correlation of spreads and exposures across countries predicted in the model and their empirical counterparts. The risk-free rate is defined as the inverse of the lender's kernel $r^f = 1/Em - 1$. Spreads are defined as the difference between the country interest rate and the risk-free rate $spr = 1/q - r_f - 1$. Recovery rates are defined as the recovery relative to the debt in default $100 \times \phi/b$. Exposure equals the market value of debt every period, qb'.

The calibration generates a fairly tight fit between the model predictions and the targets. In the model, the mean and volatility of the risk-free rate are 4.2% and 1.6%, which are close to the data statistics of 4.0% and 1.4%. In the model, the mean and volatility of the spread are 1.6% and 1.8%. The mean spread is close to its empirical counterpart of 1.4%, whereas volatility in the model is lower than the 2.6% found in the data. The volatility of detrended exposure in the model is 16%, close to 15% in the data. In the model, the average recovery and the difference in recoveries between single and multiple renegotiations are 66% and -13%, which are in line with the empirical estimates of 60% and -16%.

Although the calibrated moments are jointly controlled by all parameters, certain parameters affect certain moments more. The mean risk-free rate is mostly determined by the lenders' discount factor. The mean spread is mainly controlled by the borrowers' discount factor and the output cost of default. The volatility of the risk-free rate is controlled by the lenders' average output and their IES. The volatility of exposure is controlled by the lenders' IES, the borrowers' discount factor, and the output cost of default. The mean recovery and the recovery difference are controlled by the bargaining power and by the output cost of default.

Table 2: Main Statistics

	Data	Model
Calibrated moments:		
Mean risk-free rate	4.0	4.2
Mean spread	1.4	1.6
Volatility risk-free rate	1.4	1.6
Volatility spread	2.6	1.8
Volatility of exposure	15	16
Mean recovery	60	66
Change in recovery with multiple renegotiations	-16	-13
Other moments:		
Correlation of spreads	0.97	0.43
Correlation of exposure	0.56	0.30

Table 2 also shows that the model generates a substantial cross-country correlation of spreads and exposure of 0.43 and 0.30. The correlations of Greek spreads and those for Italy, Portugal, and Spain are 0.96, 0.97, and 0.97, respectively. The correlations of German exposure to Greek debt and German exposure of debt from Italy, Portugal, and Spain are 0.78, 0.31, and 0.58, respectively. Recall that the process for output is assumed to be uncorrelated and that the model generates positive correlations only because of the debt market linkages across countries. Hence, through the lens of our model, about half of the correlations in spreads and exposures across countries are attributed to the linkages in lending, default, and renegotiation.

4.3 Prices and Recoveries

In our model, the shapes of bond price and recovery functions determine the linkages across countries in their default, renegotiation, and borrowing. We now explain these functions in detail.

Figure 2 plots the bond price schedules for the home country $q_i(s, b', d)$ as a function of their borrowing level, b'_i. The schedules are for a level of income that is two standard deviations lower than the mean and debt at the mean $b_i = b_{-i} = 0.06$ for both countries. We plot the schedules as a function of the two foreign credit states $h_{-i} = \{0, 1\}$ and for various foreign choices for loans b'_{-i} and repay/renegotiate d_{-i}. For any foreign state or choice, bond prices are always decreasing in borrowing levels because both default probabilities and risk-free rates increase with larger loans. Risk-free rates increase with loans because the lenders' marginal utility increases with larger transfers to the home country. Nevertheless, foreign states and choices change the bond price schedule at home.

First consider the case in which the foreign country is in good credit standing. We plot the schedule for three foreign choices: the optimal borrowing choice $b'_{-i} = B(s, d)$, a large borrowing choice 30% larger than optimal, and default $d_{-i} = 1$. When the foreign country repays and borrows an optimal amount, which is modest here, the schedule for the home country is the most favorable. When foreign borowing is large or when the foreign country defaults, the schedule is tighter because of the increase in the risk-free rate and because of higher default probabilities in the future. The change in the risk-free rate can be read from the vertical distance in the bond price schedule for small home borrowing levels. Risk-free rates rise with large foreign borrowing but especially with foreign defaults because these choices tilt the lenders' consumption path upward. With default, lenders' consumption is lower today because they lose the transfer of the defaulted debt and is higher tomorrow because of the recovery lenders obtain during renegotiation.

Large foreign borrowing and foreign default also increase the probabilities that the home country defaults the next period, as illustrated by the steeper slope of the bond price function at small levels of debt. These foreign actions increase future default incentives at home because debt renegotiation after default is more beneficial when renegotiating simultaneously with the foreign country. When the foreign country borrows a large amount, it naturally has a higher default probability the next period.

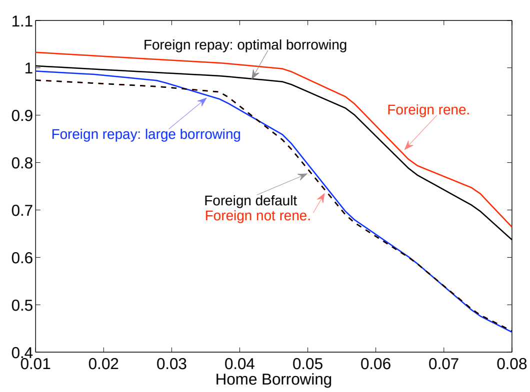


Figure 2: Bond Price Functions

Figure 2 also plots the price function when the foreign country has bad credit $h_{-i} = 1$. It considers two foreign choices, renegotiate $d_{-i} = 1$ and not renegotiate $d_{-i} = 0$. The bond price schedule at home is most lenient when the foreign country renegotiates. Here risk-free rates are very low because there is no possibility of an immediate default for the foreign country that just renegotiated. The bond price schedule for not renegotiating is tight and coincides with that for default. Hence, the same forces as for the case of foreign default are at play in this case.

We now turn the focus to recoveries. Figure 3 plots the recovery rate for the home country $\phi_i(s, b', d)/b_i$ as a function of the home country's debt state b_i. The levels of income and foreign debt are as in the bond price figure. We plot the schedules as a function of the two foreign credit states $h_{-i} = \{0, 1\}$ and for foreign choices for optimal loans $b'_{-i} = B(s, d)$ and repay/renegotiate d_{-i}.

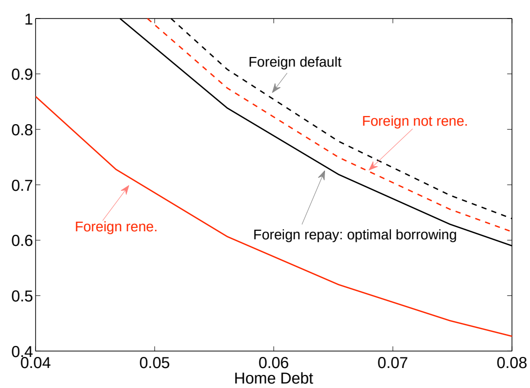


Figure 3: Recovery Functions

Recovery rates are decreasing in the level of defaulted debt because the recovery level $\phi_i(s, b', d)$ is independent of b_i. The home country faces the most lenient recovery function when the foreign country is also renegotiating because with joint renegotiations, the outside option of lenders, which is autarky, is lower. With single renegotiations, the outside value for lenders is the value of trading with the foreign country. As long as lenders attach any positive value from holding the foreign country debt (which are lenders' assets), the outside option of lenders is higher with single renegotiations than it is with joint renegotiations. Nevertheless, the extent of this effect is controlled by the bargaining parameters. For example, if lenders have all the bargaining power, then their outside options are irrelevant for the equilibrium.

The recovery functions are the tightest if the foreign country would default or not renegotiate. In these cases, the lenders' outside option relative to the value of renegotiation is the highest because default or not renegotiating lowers the lenders' value of renegotiation while the outside option is fixed across these potential choices for a given state.

Foreign policies to default and borrow affect home countries' bond prices and recoveries, and these prices shape home default, borrowing, and renegotiation decisions. One implication of this analysis is that, as in the example in Section 3, our model features *joint defaults* for two reasons. First, a foreign default makes the home debt price schedule tighter, which makes it harder to roll over the debt and hence can induce a default. Second, countries want to renegotiate together because recoveries are lower with joint renegotiations. A foreign default lowers the future recoveries for the home country, which can also induce a default. When countries default together, their default probabilities and hence their spreads are correlated.

4.4 Debt Linkages

To further our understanding of debt market linkages, Table 3 reports observed debt market statistics for the home country conditional on the debt market conditions of the foreign country. We partition the limiting distribution into states when the foreign country is in good credit standing and states when it is in bad credit standing. When the foreign country is in good credit standing, we further partition the sample into states when its spreads are above and below the median of 2.41 and states when it defaults. When the foreign country is in bad credit standing, we partition the states into those when it renegotiates and states when it does not renegotiate.

Table 3: Debt Linkages

	Overall	Foreign Good Credit			Foreign Bad Credit	
Home	Average	*spr* >p50	*spr* ≤p50	Default	Renego	Not Renego
Spread	1.6	2.7	1.3	1.9	1.1	–
Default prob.	4.5	3.9	2.6	37.3	0.03	100
Recovery	66	67	72	90	58	–
Renegotiation prob.	98	100	100	1	100	–
Risk-free rate	4.2	4.3	4.3	6.3	-0.0	7.4

Note: *spr* denotes spreads and *Renego* denotes renegotiation.

Table 3 shows that foreign conditions affect home spreads, default probabilities, recoveries, and renegotiation probabilities. First consider the states when the foreign country is in good credit standing. Spreads are correlated: the average spread for the home country equals 2.7%

when the spread for the foreign country is above its median, whereas it is 1.3% when the foreign country spread is below the median. As explained in the previous section, a major reason for this positive correlation of spreads is the prevalence of joint defaults. The average default probability for the home country jumps to about 37% when the foreign country defaults. This strong incidence of joint defaults implies that high foreign spreads forecast a foreign default and a home default, which in turn is priced in home spreads. A second force that induces a positive correlation of spreads is that the home country is more likely to default when the foreign country borrows heavily which coincides with high and persistent spreads. As illustrated in the bond price schedules above, a large foreign loan tightens the home bond price schedule, which leads to more defaults. The home country's default rate is 3.9% when foreign spreads are high compared with 2.6% when spreads are low. Given that spreads are persistent, a high spread in the foreign country also predicts a default at home in states when the foreign country repays and continues to have high spreads. The correlation of spreads across countries is hence positive because of joint defaults and because both countries induce each other to default when they borrow heavily.

Recoveries for home also vary with the conditions in the foreign country. When foreign spreads are low, home recovery equals 72%; when foreign spreads are high, recoveries are lower and equal to 67%; and when the foreign country defaults, recoveries are highest and equal to 90%. Nevertheless, these high recoveries during foreign defaults are rarely observed in equilibrium because the home country renegotiates during these states with less than 1% probability. To obtain a lower recovery, the home country would rather delay renegotiation and renegotiate the following period along with the foreign country. Lower recoveries at home coincide with high foreign spreads because a low recovery tightens the price schedule for the foreign country and leads to a higher spread.

Now consider the case when the foreign country is in bad credit standing because of a previous default. When the foreign country renegotiates, the home country spreads are low, and recoveries are the lowest across the board. Hence, with foreign renegotiations, the home country almost never defaults if it is in good credit standing and always renegotiates if it is in bad credit standing. When the foreign country does not renegotiate, the home country always defaults if it is in good credit standing. The limiting distribution does not have any mass in states in which both countries are in bad credit standing and only one renegotiates. When countries are in bad credit standing, they renegotiate jointly.

Note that the risk-free rate does not vary much when countries are borrowing and repaying their debt. When countries default, however, the risk-free rate rises about 2%, and during renegotiations, it drops to about zero. The risk-free rate is high during defaults because the lenders stop collecting their payments and have a lower marginal value. During renegotiations, the opposite occurs: lenders receive the recovery, which lowers their marginal value.

In analyzing these results, it is important to keep in mind that in the limiting distribution of the model, the probability of different credit standing states varies. The probability that both countries are borrowing and repaying their debt is the highest and equal to 86%. Each country defaults alone about 2.6% of the time, and countries default together about 1.5% of the time. Countries also renegotiate and repay together often. About 5% of the time, one country is repaying its debt while the other country is renegotiating. When both countries are in bad credit standing, they always renegotiate together, which happens about 1.8% of the time.

Our model also provides a laboratory in which to analyze precisely whether the observed defaults and renegotiations for the home country are induced by the defaults and renegotiations of the foreign country. We find that many defaults in one country could be avoided if other countries were to not default, and most renegotiations can be facilitated if other countries renegotiate. To conduct this experiment, we consider the home best responses for default or renegotiation, $x_i^d(d_{-i}, s, B(s,d))$, as a function of the foreign country strategy for default or renegotiation d_{-i}. We define home events as independent if the event continues to occur even if the foreign country changes its strategy from default to repay, from renegotiate to not renegotiate, or vice versa. If the home event changes when the foreign country changes its default/renegotiation strategy, we label such events as dependent.[7] Self-fulfilling events are those dependent events that have two equilibria. Table 4 reports the fraction of the defaults, repayments, renegotiations, and nonrenegotiations for the home country that are independent and dependent. As the table shows, a substantial portion of the home events are induced by the foreign country decisions; 25% of the defaults, 27% of the repayments, 93% of the renegotiations, and 100% of the nonrenegotiations are dependent. Self-fulfilling equilibria are a substantial portion of the equilibria during renegotiations and nonrenegotiations but are also sizable for defaults. Nevertheless, the majority of the default and repayment events are independent with a portion equal to 75% and 73%, respectively.

[7] More precisely, default and renegotiation events are independent for country i if $D_i(s) = x_i^d(1 - D_{-i}(s), s, B(s,d))$, where $D_i(s)$, and $D_{-i}(s)$ are the equilibrium policy functions, x_i^d is the home best response function, and $B(s,d)$ is the outcome of the second stage intraperiod game when default/renegotiation strategies are $d_{-i} = 1 - D_{-i}(s)$ and $d_i = x_i^d(1 - D_{-i}(s), s, B(s,d))$. If $D_i(s) \neq x_i^d(1 - D_{-i}(s), s, B(s,d))$, the event is dependent.

Table 4: Types of Defaults and Renegotiations (%)

	Default	Repay	Renegotiation	Nonrenegotiation
Independent	75	73	7	0
Dependent	25	27	93	100
Self-fulfilling	14	0	36	87

The dependent defaults at home happen mostly because the foreign country is defaulting, although 2% of the defaults happen because the foreign country is not renegotiating. All of the dependent repayments happen because the foreign country is repaying. Of the dependent renegotiations at home, 55% happen because the foreign country is renegotiating and 39% because the foreign country is repaying. Of the nonrenegotiations, 100% happen because the foreign country is defaulting.

4.5 Comparative Statics

Standard quantitative default models as in Arellano (2008) abstract from debt linkages across countries because each country is considered in isolation. Our model generates strong linkages in debt markets across countries by deviating from a standard default model along three dimensions. First, the standard model considers one large borrowing country, whereas we consider two large borrowing countries, which leads to the analysis of the strategic interactions among them. Second, the standard model considers risk-neutral lending, whereas we consider lenders that have concave payoffs. Third, the standard model does not consider renegotiation, whereas we add renegotiation in an environment with two borrowing countries interacting with lenders.[8]

In this section we show that these three forces are important for our results and that they interact with each other. To that end, we compute three versions of our model. We compute a *linear* model, where we set $\alpha = 0$. This version highlights the roles of two large borrowing countries interacting with one another through renegotiation. We also compute a *low IES* model by lowering the IES to $1/\alpha = 1/5$. This version explores the role of the elasticity of substitution.[9] Finally we compute a *small country* model, where we add a competitive small

[8] As noted in the introduction, several papers have analyzed some of these extensions in isolation: Yue (2010) and D'Erasmo (2011) study renegotiation in the case of one borrowing country, and Lizarazo (2013, 2009) studies the impact of risk-averse lenders.

[9] With power utility, risk aversion and elasticity of substitution are controlled by the same parameter. We focus on describing the effects of the elasticity of substitution because when we extended our analysis to an Epstein-Zin utility function, we found that the key parameter controlling this result is the IES and not risk aversion. We found that risk aversion playeds only a minor role.

country that is otherwise identical to the home country to the benchmark model.[10] This version highlights the role of strategic interactions.

Table 5: Sensitivity

	Benchmark	Linear	Low IES	Small Country
Mean (%)				
Default probability	4.5	4.2	1.3	5.7
Spread	1.6	1.7	0.6	2.8
Recovery	66	66	62	77
Recovery *multiple* − *single*	-13	-10	-18	-2.5
Debt service / GDP	6.3	6.3	5.9	7.4
Volatility (%)				
Risk-free rate	1.6	0.0	4.0	1.6
Spread	1.8	1.7	1.2	5.4
Exposure	15	15	17	8.5
Correlations across countries				
Spreads	0.42	0.28	0.52	0.17
Exposure	0.30	0.34	0.51	0.07
Default	0.34	0.45	0.32	0.11
Fraction dependent events (%)				
Default	25	35	31	–
Repay	27	27	22	–
Renegotiation	93	94	95	–
Nonrenegotiation	100	100	100	–

Table 5 reports the sensitivity results for the three versions of our model as well as for the benchmark model. First consider the results for the linear model. In terms of means (default probabilities, spread, recovery, and debt), the linear model behaves very similarly to the benchmark. Having linear lenders in our model, of course, implies a zero risk-free rate volatility, which is far from that observed in the data. The volatilities of spreads and exposures are comparable to the benchmark. The correlations of spreads are greatly reduced in the linear model from 0.44 to 0.28 even though the correlation of defaults increases from 0.34 to 0.45. The reason for the increase in the default correlation is the higher incidence of more dependent states. The fraction of defaults that are dependent increases from 25% to 35%, and such events are mainly due to foreign defaults.

In the linear model, the main force that operates for linkages is that countries want to renegotiate together because their recoveries will be lower. The results from the linear model show that this effect is powerful and important for the results in our benchmark model.

[10] In Appendix I, we lay out the small country problem in detail .

Nevertheless, the correlation of spreads is lower in the linear model because foreign borrowing does not affect the risk-free rates. Recall that in the benchmark model, a home default is induced not only by a foreign default but also by large foreign loans, which increase the risk-free rate. In the linear model, this effect is absent thereby lowering the correlation of spreads.

Now consider the results from the low IES model. When lenders have low IES, the bond price functions are much tighter, which limits borrowing and leads to a lower default probability and spread in equilibrium. The volatility of the risk-free rate is the highest in this model because risk-free prices are more sensitive to lenders' consumption paths when they have a low IES. This model generates a higher correlation of spreads than the benchmark, 0.52 relative to 0.42, despite generating a comparable correlation of defaults. More curvature in lenders' utility amplifies the effects from large foreign borrowings on home bond prices.

Finally, consider the results from the small country model. In this model, the small country takes as given the evolution of the aggregate states and decisions of the two large borrowing countries arising from the benchmark model. This assumption matters for the small country because it determines the evolution of the risk-free rate. Moreover, the income shock of the small country is identical to that of the home country. Table 5 reports the statistics for the small country across its own limiting distribution of debt. The small country borrows more, defaults more, and faces higher spreads because the small country does not internalize that large borrowing increases the risk-free rate. The correlations across the spreads and defaults of the small country and the foreign country are small and equal 0.17 and 0.11. The positive correlations reflect the fact that countries face common risk-free rates. Nevertheless, correlations are small and less than half of that observed across the home country and the foreign country because the small country does not engage in any strategic interactions with the foreign country. This experiment shows that the large cross-country correlations in the benchmark are mainly driven by the strategic interactions across countries and that the modest variation in the lenders' condition plays only a minor role when countries are not strategic.

5 Conclusion

We developed a multicountry model of sovereign default and renegotiation in which default in one country triggers default in other countries. Debt market conditions for borrowing countries are linked to one another because they borrow from a common lender with concave payoffs. In our model, country interest rates are correlated because countries tend to default together. Joint defaults occur because a default abroad makes the price of debt more stringent and recoveries lower at home. In this way, our model provides a framework in which to study some of the recent economic events in Europe.

References

[1] Aguiar, M., and G. Gopinath (2006). Defaultable Debt, Interest Rates and the Current Account. *Journal of International Economics,* 69(1): 64–83.

[2] Arellano, C. (2008). Default Risk and Income Fluctuations in Emerging Economies. *American Economic Review,* 98(3): 690–712.

[3] Arellano, C. and N. Kocherlakota (2012). Internal Debt Crises and Sovereign Defaults. Manuscript, Research Department, Federal Reserve Bank of Minneapolis.

[4] Bank for International Settlements, Consolidated Banking Statistics, http://www.bis.org/statistics/consstats.htm.

[5] Benjamin, D., and M. Wright (2009). Recovery Before Redemption: A Theory of Delays in Sovereign Debt Renegotiations. Manuscript, State University of New York at Buffalo.

[6] Borri, N., and A. Verdelhan (2009). Sovereign Risk Premia. Manuscript, LUISS Guido Carli University.

[7] Broner, F. A., R. G. Gelos, and C. M. Reinhart (2006). When in Peril, Retrench: Testing the Portfolio Channel of Contagion. *Journal of International Economics*, 69(1): 203-230.

[8] Cole, H. L., and T. J. Kehoe (2000). Self-Fulfilling Debt Crises. *Review of Economic Studies*, 67(1), 91–116.

[9] Cruces, J. J., and C. Trebesch (2013). Sovereign Defaults: The Price of Haircuts. *American Economic Journal: Macroeconomics*, 5(3): 85-117.

[10] D'Erasmo, P. (2011). Government Reputation and Debt Repayment. Manuscript, University of Maryland.

[11] Dobson, P. W.(1994). Multifirm Unions and the Incentive to Adopt Pattern Bargaining in Oligopoly. *European Economic Review*, 38(1): 87-100.

[12] Eaton, J., and M. Gersovitz (1981). Debt with Potential Repudiation: Theoretical and Empirical Analysis. *Review of Economic Studies*, 48(2): 289–309.

[13] Horn, H., and A. Wolinsky (1988). Bilateral Monopolies and Incentives for Merger. *RAND Journal of Economics*, 19(3): 408-419.

[14] Kaminsky, G. L., and C. M. Reinhart 2000. On Crises, Contagion, and Confusion. *Journal of International Economics*, 51(1): 145-168.

[15] Lizarazo, S. V. (2013). Default Risk and Risk Averse International Investors. *Journal of International Economics*, 89(2): 317-330.

[16] Lizarazo, S. V. (2009). Contagion of Financial Crises in Sovereign Debt Markets. Working Paper no. 0906, Centro de Investigacion Economica, ITAM.

[17] Lorenzoni, G., and I. Werning (2013). Slow Moviing Debt Crises. Working Paper no. 13-18, MIT Department of Economics.

[18] Park, J. (2013). Contagion of Sovereign Default Risk: The Role of Two Financial Frictions. Manuscript, University of Wisconsin-Madison.

[19] Puozo. D., and I. Presno (2011). Sovereign Default Risk and Uncertainty Premia. Manuscript, University of California at Berkeley.

[20] Reinhart, C. M., and K. S. Rogoff (2011). From Financial Crash to Debt Crisis. *American Economic Review*, 101(5): 1676-1706.

[21] Tauchen, G., and R. Hussey (1991). Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models. *Econometrica*, 59(2): 371-396.

[22] Van Rijckeghem, C., and B. Weder (2001). Sources of Contagion: Is It Finance or Trade? *Journal of International Economics*, 54 (2): 293-308.

[23] Van Rijckeghem, C., and B. Weder (2003). Spillovers Through Banking Centers: A Panel Data Analysis of Bank Flows. *Journal of International Money and Finance*, 22 (4): 483-509.

[24] World Bank, World Development Indicators, http://data.worldbank.org/data-catalog/world-development-indicators.

[25] Yue, V. Z. (2010). Sovereign Default and Debt Renegotiation. *Journal of International Economics*, 80(2): 176-187.

Appendix I. Auxiliary Models

One Large Country Model

Let $v^L_{i,fail}(\ell_i, s_i)$ be the value to the lender when trading only with country i:

$$v^L_{i,fail}(\ell, s_i) = \max_{\{d_L, \ell'_i \text{ if } h_i = h'_i = 0\}} \{g(c_L) + \delta \sum_{y'_i} \pi(y'_i, y_i) v^L_{i,fail}(\ell', s'_i)\}, \tag{21}$$

subject to its budget constraint,

$$c_L = y_L + [1 - D_i(s)] \left((1 - h_i)(\ell - Q_i \ell') + h_i \frac{\Phi_i \ell}{b_i} \right),$$

the evolution of the endogenous states akin to equation (12), and a law of motion of aggregate states for the case that country i is dealing alone with lenders $s'_i = H_{fail}(s_i)$. The optimal solution of the lender is given by $c_{L,fail}(\ell, s_i)$ and $\ell'_{fail}(\ell, s_i)$

The problem for country i in the case when it trades alone with the lenders is similar to one described in Section 2.1 with three main differences. First, its aggregate states are only $s_i = \{b_i, h_i, y_i\}$. Second, the price function $q_{i,fail}(s_i, b'_i, d_i)$ and recovery $\phi_{i,fail}(s_i, b'_i, d_i)$ depend only on its own states and its own strategies. Third, the intraperiod Nash game between countries is absent. The decision rules for this problem are labeled $B_{i,fail}(s_i)$ for borrowing and $D_{i,fail}(s_i)$ for repayment. These decisions in turn determine the evolution of the aggregate state $s'_i = H_{fail}(s_i)$.

When $h_i = 0$, the price function $q_{i,fail}(s_i, b'_i, d_i)$ solves

$$q_{i,fail} = \sum_{s'} m_{fail}(s'_i, s_i; q_{i,fail}, b'_i, d_i) \left[1 - D_{i,fail}(s'_i)(1 - \zeta_{i,fail}(s'_i))\right]. \tag{22}$$

Here, the decision rules of the country and the lender's kernel are those corresponding to the problem when country i trades alone with the lender.

When the country is in bad credit standing and chooses to renegotiate, the recovery function $\phi_{i,fail}(s_i, d_i)$ solves

$$\frac{\theta u'(y_i - \phi_{i,fail})}{[v_i(s_i; \phi_{i,fail}) - v_{i,aut}(y_i)]} = \frac{(1-\theta) g'(s_i, \phi_{i,fail}, d_i)}{[V^L(s_i, \phi_{i,fail}, d_i) - V^L_{aut}]} \tag{23}$$

Definition 5. *A single-country recursive Markov equilibrium consists of (i) the country i's policy functions for repayment, borrowing, and consumption,* $\{B_{i,fail}(s_i), D_{i,fail}(s_i), C_{i,fail}(s_i)\}$, *and values* $v_{i,fail}(s_i)$*; (ii) lenders' policy functions for lending choices and dividends* $\{\ell'_{fail}(\ell, s_i),$

$c_{L,fail}(\ell, s_i)\}$ *and value function* $v^L_{i,fail}(\ell, s_i)$*; (iii) the functions for bond prices and recoveries* $\{q_{i,fail}(s_i, b'_i, d_i), \phi_{i,fail}(s_i, d_i)\}$*; (iv) the equilibrium prices of debt* $Q_{i,fail}(s_i)$ *and recovery rates* $\Phi_{fail}(s_i)$*; (v) the evolution of the aggregate state* $H_{fail}(s_i)$ *such that given* $b_0 = \ell_0$*:*

1. *Taking as given the bond price and recovery functions, the country i's policy functions for repayment, borrowing, and consumption,* $\{B_{i,fail}(s_i), D_{i,fail}(s_i), C_{i,fail}(s_i)\}$*, and values* $v_{i,fail}(s_i)$ *solves country i's problem when it trades alone with the lenders.*
2. *Taking as given the bond prices* $Q_{fail}(s_i)$*, recoveries* $\Phi_{fail}(s_i)$*, and the evolution of the aggregate states* $H_{fail}(s_i)$*, the policy functions and value functions for the lenders* $\{\ell'_{fail}(\ell, s_i),\ c_{L,fail}(\ell, s_i)\},\ v^L_{i,fail}(\ell, s_i)\}$ *satisfy lenders' optimization problem in (21).*
3. *Taking as given countries' policy and value functions, bond price and recovery functions* $\{q_{i,fail}(s_i, b'_i, d_i), \phi_{i,fail}(s_i, d_i)\}$ *satisfy (22) and (23).*
4. *The prices of debt* $Q_{fail}(s_i)$ *clear the bond market,* $\ell'_{i,fail}(b_i, s_i) = B_{i,fail}(s_i)$*.*
5. *The recoveries* $\Phi_{i,fail}(s_i)$ *exhaust all the recovered funds:* $\phi_{i,fail}(s_i, D_{i,fail}(s_i)) = \Phi_{fail}(s_i)$*.*
6. *The law of motion for the evolution aggregate states* $H_{fail}(s_i)$ *is consistent with country i's decision rules and shocks.*

Small Country Model

The model for the small country is a one-country competitive version of the benchmark model. This model is studied in Yue (2010), but here the risk-free rate is time varying and depends on the evolution of the aggregate states. The recursive problem for the small country takes as given the law of motion of aggregate states (9). Given the individual state $(b_s, y_s, h_s$ and aggregate state s, the small country's problem is given by

$$v_s(b_s, y_s, h_s = 0, s) = \max_{d_s=\{0,1\}} \{(1 - d_s)v^0_s(b_s, y_s, h_s = 0, s) + d_s v^1_s(b_s, y_s, h_s = 0, , s)\}.$$

If it repays, the small country chooses optimal consumption and savings:

$$v^0_s(b_s, y_s, h_s = 0, s) = \max_{c_s, b'_s} \{u(y_s - b_s + q_s(b'_s, y_s, s)b'_s) + \beta E v_s(b'_s, y'_s, h'_s = 0, s')\}.$$

If it defaults, the small country's value is given by

$$v^1_s(b_s, y_s, h_s = 0, s) = \{u(y^d_s) + \beta E v_s(b_s, y'_s, h'_s = 1, s')\}. \tag{24}$$

If the country is in bad credit standing, it chooses whether to renegotiate according to

$$v_s(b_s, y_s, h_s = 1, s) = \max_{d_s=\{0,1\}} \{(1 - d_s)v_s^0(b_s, y_s, h_s = 1, s) + d_s v_s^1(b_s, y_s, h_s = 1, s)\}.$$

Its renegotiation value depends on the recovery $\phi_s(b_s, y, s)$ and is given by

$$v_s^0(b_s, y_s, h_s = 1, s) = u(y_s - \phi_s(b_s, y, s)) + \beta E v_s(0, y'_s, h'_s = 0, s').$$

Without renegotiation, its value is the same as the default value given by equation (24).

In equilibrium, bond price and recovery functions for the small country satisfy the following equations

$$\begin{aligned} q_s &= E\left[1 - d'_s(b'_s, y'_s, h'_s, s')(1 - \zeta_s(b'_s, y'_s, h'_s, s'))\right] Em(s', s), \\ \zeta_s(b_s, y_s, h_s, s) &= E[(1 - d'_s(b_s, y'_s, h'_s, s'))\frac{\phi_s(b_s, y'_s, h'_s, s')}{b_s} + d'_s(b_s, y'_s, h'_s, s')\zeta_s(b_s, y'_s, h'_s, s')]E\left[m(s', s)\right], \\ 1 - \theta &= \frac{\theta u'(y_s - \phi_s)}{[v_s^0(b_s, y_s, h_s = 1, s; \phi_s) - v_{aut}(y_s)]}. \end{aligned}$$

where $m(s', s)$ is the equilibrium pricing kernel from the two-big-country problem.

Appendix II. Proofs

Proof for Proposition 3. Let us call ϕ_2^i and ϕ_2^{-i} the recovery values for country i and $-i$ respectively when the two countries renegotiate jointly with lenders, and ϕ_1^i be the recovery value when country i renegotiates alone with lenders. Nash bargaining implies that ϕ_2^i satisfies

$$\frac{\theta u_c(y_2'^i)}{u(y_2'^i) - u(y^d)} = \frac{(1-\theta)g'(y_L + \phi_2^i + \phi_2^{-i})}{g(y_L + \phi_2^i + \phi_2^{-i}) - g(y_L)} \leq \frac{(1-\theta)g'(y_L + \phi_2^i + \phi_2^{-i})}{g(y_L + \phi_2^i + \phi_2^{-i}) - g(y_L + \phi_2^{-i})}.$$

The inequality holds because g is an increasing function. Suppose the following condition holds:

$$\frac{(1-\theta)g'(y_L + \phi_2^i + \phi_2^{-i})}{g(y_L + \phi_2^i + \phi_2^{-i}) - g(y_L + \phi_2^{-i})} \leq \frac{(1-\theta)g'(y_L + \phi_2^i + \bar{b})}{g(y_L + \phi_2^i + \bar{b}) - g(y_L + \bar{b})}. \tag{25}$$

Then, the recovery under two borrowing countries ϕ_2^i satisfies

$$\frac{\theta u_c(y_2'^i)}{u(y_2'^i) - u(y^d)} \leq \frac{(1-\theta)g'(y_L + \phi_2^i + \bar{b})}{g(y_L + \phi_2^i + \bar{b}) - g(y_L + \bar{b})}$$

where recovery alone ϕ_1^i satisfies

$$\frac{\theta u_c(y_1'^i)}{u(y_1'^i)-u(y^d)} = \frac{(1-\theta)g'(y_L+\phi_1^i+\bar{b})}{g(y_L+\phi_1^i+\bar{b})-g(y_L+\bar{b})}.$$

It is easy to show by contradiction that $\phi_2^i \leq \phi_1^i$ due to concavity of u and g.

We still need to show that inequality (25) holds. Given $\bar{b} \geq \phi_2^{-i}$, we need to show that the function $\frac{g'(y_H+x)}{g(y_H+x)-g(y_L+x)}$ with $y_H = y_L+\phi_2^i \geq y_L$ increases with x. Given $g(c) = c^{1-\alpha}/(1-\alpha)$, we need to show

$$\left(\frac{y_H+x}{y_L+x}\right)^{\alpha-1} - 1 - \frac{\alpha-1}{\alpha}\left[\left(\frac{y_H+x}{y_L+x}\right)^{\alpha} - 1\right] \leq 0$$

When $\alpha \geq 1$ and $y_H \geq y_L$, the above inequality holds. Q.E.D.

Proof for Proposition 4. Conditional on repaying, country i's net capital flow to lenders increases with its initial debt holding b_i. To see this, let $\omega_L(b_{-i}, d_{-i})$ and $\omega_L'(d_{-i})$ be the lenders' wealth from trading with the other country $-i$ in period 1 and period 2, respectively. In particular,

$$\omega_L(b_{-i}, d_{-i}) \equiv y_L + (1-d_{-i})TB(b_{-i})$$

We can define the net capital flow from country i as $TB_i = b_i - q_i\bar{b}$, where q_i solves

$$q_i = \frac{\delta g'[\omega_L'(d_{-i})+\bar{b}]}{g'[\omega_L(b_{-i}, d_{-i})+b_i-q_i\bar{b}_i]}.$$

It is easy to show that

$$\partial TB_i/\partial b_i = \frac{g'[\omega_L(d_{-i})+b_i-q_i\bar{b}]}{g'(\omega_L(b_{-i}, d_{-i})+b_i-q_i\bar{b}) - q_i g''(\omega_L(b_{-i}, d_{-i})+b_i-q_i\bar{b})\bar{b}} \geq 0.$$

Higher b_{-i} therefore leads to higher net capital flow TB_{-i} and so higher lenders' wealth from country $-i$ since $\omega_L(b_{-i}, d_{-i})$. The bond price of country i thus increases with b_{-i} conditional on country $-i$ repaying.

Appendix III. Computational Algorithm

We compute the model as the limit of a finite horizon model with T periods. In each period, we compute two models: a single-country model facing a continuum of lenders and a two-country model facing a continuum of lenders. We need to compute the first model, since its equilibrium values are used in solving for the Nash bargaining allocations of the second model.

We start with the last period T. Given that this is the last period, there will be no borrowing and lending. First, we compute the equilibrium when lenders face only country i. When the country is in good credit standing, it defaults $D^1_{i,T}(s) = 1$ if and only if the debt is large enough $b_i > y_i - y_i^d$. The value of lenders is therefore given by $v^L_{i,T}(s) = g(y_L + (1 - D^1_{i,T}(s))b_i)$, and the stochastic discount factor is given by $m^1_{i,T}(s) = g'(y_L + (1 - D^1_{i,T}(s))b_i)$. If the country is in bad credit standing, its recovery value of debt $\phi^1_{i,T}(s)$ from Nash bargaining satisfies

$$\frac{(1-\theta)g'(y_L + \phi^1_{i,T}(s))}{g(y_L + \phi^1_{i,T}(s)) - g(y_L)} = \frac{\theta u_c(y - \phi^1_{i,T}(s))}{u(y - \phi^1_{i,T}(s)) - u(y_i^d)}.$$

The country chooses to renegotiate with lenders $D^1_{i,T}(s) = 0$ if and only if $u(y - \phi^1_{i,T}(s)) \geq u(y_i^d)$. Correspondingly, the lenders' value is given by $V^L_{i,T} = g(y_L + (1 - D^1_{i,T}(s))\phi^L_{i,T}(s))$.

We then compute the equilibrium with two countries. A good-credit country i defaults $D_{i,T}(s) = 1$ if and only if $b_i > y_i - y_i^d$. The value of country i is given by $v_{i,T}(s) = u(y_i - (1 - D_{i,T}(s))b_i)$. For a bad-credit country i, its renegotiation decision depends on the recovery value from the Nash bargaining. If country $-i$ is in good credit standing, country i's recovery value $\phi_{i,T}(s)$ solves

$$\frac{(1-\theta)g'(y_L + \phi_{i,T}(s) + (1 - D_{-i,T}(s))b_{-i})}{g(y_L + \phi_{i,T}(s) + (1 - D_{-i,T}(s))b_{-i}) - g(y_L + (1 - D^1_{-i,T}(s))b_{-i})} = \frac{\theta u_c(y_i - \phi_{i,T}(s))}{u(y_i - \phi_{i,T}(s)) - u(y_i^d)}.$$

Country i chooses to renegotiate $D_{i,T}(s) = 0$ if and only if $u(y_i - \phi_{i,T}(s)) \geq u(y_i^d)$. If both countries are in bad credit standing, we need to find the Nash equilibrium $\{D_{i,T}(s), D_{-i,T}(s)\}$ which jointly satisfies

$$\begin{aligned} D_{i,T}(s) = &\quad argmax_{\{d_i\}}\ (1 - d_i)u(y_i - \phi_{i,T}(s, d_i, D_{-i,T}(s))) + d_i u(y_i^d) \\ D_{-i,T}(s) = &\quad argmax_{\{d_{-i}\}}\ (1 - d_{-i})u(y_{-i} - \phi_{-i,T}(s, D_{i,T}(s), d_{-i})) + d_{-i} u(y_{-i}^d), \end{aligned}$$

where $\{\phi_{i,T}(s, d_i, d_{-i}), \phi_{-i,T}(s, d_i, d_{-i})$ solve

$$\begin{aligned} \frac{(1-\theta)g'(y_L + \phi_{i,T} + (1 - d_{-i})\phi_{-i,T})}{g(y_L + \phi_{i,T} + (1 - d_{-i})\phi_{-i,T}) - g(y_L)} &= \frac{\theta u_c(y_i - \phi_{i,T})}{u(y_i - \phi_{i,T}) - u(y_i^d)} \\ \frac{(1-\theta)g'(y_L + \phi_{-i,T} + (1 - d_i)\phi_{i,T})}{g(y_L + \phi_{-i,T} + (1 - d_i)\phi_{i,T}) - g(y_L)} &= \frac{\theta u_c(y_{-i} - \phi_{-i,T})}{u(y_{-i} - \phi_{-i,T}) - u(y_{-i}^d)}. \end{aligned}$$

In an abuse of notation, let us call the equilibrium recovery $\phi_{i,T}(s) = \phi_{i,T}(s, D_{i,T}(s), D_{-i,T}(s))$. The bad-credit country i's value is then given by

$$v_{i,T}(s) = (1 - D_{i,T}(s))u(y_i - \phi_{i,T}(s)) + D_{i,T} u(y_i^d).$$

Finally, we update the expected discounted recovery value $\zeta_{i,T}$ for period T, $\zeta_{i,T}(s) = g'(c_{L,t}(s))(1 - D_{i,T}(s))\phi_{i,T}(s)$.

We now describe the algorithm for a generic period $t < T$. Again, let us first solve for the case with only one country. If this country i is in good credit standing, it makes a default decision and a borrowing decision to maximize its value:

$$v_{i,t}^{1}(s) = \max_{d,b'} \ (1-d)v_{i,t}^{1,r}(s,b') + dv_{i,t}^{1,d}(s,b),$$

where

$$v_{i,t}^{1,r}(s,b') = u(y_i - b_i + q_{i,t}^{1}(s,b')b') + \beta E v_{i,t+1}^{1}(s')$$

$$v_{i,t}^{1,d}(s,b) = u(y_i^{d}) + \beta E v_{i,t+1}^{1}(s')$$

$$q_t^{1}(s,b')g'[y_L + b - q_t^{1}(s,b')b'] = \int_{y^*(b')} \left\{\delta\pi(y'|y)m_{i,t+1}^{1}(y',b')\right\} dy'.$$

If the country is in bad credit standing, it chooses whether to renegotiate:

$$v_{i,t}^{1}(s) = (1 - D_{i,t}^{1}(s))[u(y - \phi_{i,t}^{1}(s)) + \beta E v_{i,t+1}^{1}(s')] + D_{i,t}^{1}(s)[u(y_i^{d}) + \beta E v_{i,t+1}^{1}(\hat{s}')],$$

where the recovery $\phi_{i,t}^{1}(s)$ solves

$$\frac{(1-\theta)g'(y_L + \phi_{i,t}^{1}(s))}{g[y_L + \phi_{i,t}^{1}(s)] + \delta E v_{i,t+1}^{L}(s') - g(y_L)(1+\delta)} = \frac{\theta u_c(y - \phi_{i,t}^{1}(s))}{u(y - \phi_{i,t}^{1}(s)) + \beta E v_{i,t+1}^{1}(0,s') - \left[u(y_i^{d}) + \beta E u_{i,t+1}(y_i^{d})\right]}.$$

The lender's value is evaluated at optimal choices:

$$v_{i,t}^{L}(s) = g[y_L + (1-h_i)(1-D_{i,t}^{1})\left(b_i - q_{i,t}^{1}(B_{i,t}^{1}(s);s)B_{i,t}^{1}(s)\right) + h_i(1-D_{i,t}^{1}(s))\phi_{i,t}^{1}(s)] + \delta E v_{i,t+1}^{L}(s',s).$$

We then solve the equilibrium in period t with two countries in two steps. In the first step, taking as given the default/renegotiation decisions of the two countries, we compute the optimal borrowing decisions. When both countries are in good credit standing and choose not to default, the Cournot equilibrium $\{B_{i,t}(s,d), B_{-i,t}(s,d)\}$ satisfies

$$B_{i,t}(s,d) = argmax_{\{b_i'\}} \ w_{i,t}(s,b_i',B_{-i,t}(s,d),d),$$

where $w_{i,t}(s,b',d) = u(y_i - b_i + q_{i,t}(s,b',d)b_i') + \beta\sum_{y'}\pi(y'|y)v_{i,t+1}(s'(b',d))$ and the bond prices

$\{q_{i,t}(s,b',d), q_{-i,t}(s,b',d)\}$ satisfy two equations, one for i

$$q_{i,t}g'[y_L + (b_i - q_{i,t}b_i') + (1-d_{-i})(b_{-i} - q_{-i,t}b_{-i}')] =$$
$$\sum_{s'} \delta\pi(s'|s)\left\{g'(c_{L,t+1}(s'(b',d))[1-d_{i,t+1}(s')] + d_{i,t+1}(s')\delta E_t\zeta_{i,t+2}(s'')\right\}$$

and one for $-i$ in a similar fashion. When both countries are in good credit standing but country $-i$ chooses to default and country i chooses not to, only country i can renew its debt $B_{i,t}(s,d) = argmax_{\{b_i'\}}\, w_{i,t}(s,b_i',d)$. The bond price function $\{q_{i,t}(s,b',d)\}$ solves

$$q_{i,t}g'[y_L + (b_i - q_{i,t}b_i')] = \sum_{s'} \delta\pi(s'|s)\left\{g'\left(c_{L,t+1}(s'(b',d))\right)[1-d_{i,t+1}(s')] + d_{i,t+1}(s')\delta E_t\zeta_{i,t+2}(s'')\right\}.$$

The value of country i is given by $w_{i,t}(s, B_{i,t}(s,d), d)$, and the value of country $-i$ is given by $w_{-i,t}(s, B_{i,t}(s,d), d) = u(y_{-i}^d) + \beta\sum_{s'}\pi(s'|s)v_{-i,t+1}(s'(B_{i,t}(s,d),d))$.

When country i is in good credit standing and country $-i$ is in bad credit standing, if both choose to repay $d_{i,t} = d_{-i,t} = 0$, the price functions $\{q_{i,t}(s,b',d), \phi_{-i,t}(s,b',d)\}$ satisfy the following two equations jointly:

$$q_{i,t}g'[y_L + (b_i - q_{i,t}b_i') + \phi_{-i,t}] = \sum_{s'} \delta\pi(s'|s)\left\{g'\left(c_{L,t+1}(s')\right)[1-d_{i,t+1}(s')] + d_{i,t+1}(s')\delta E_t\zeta_{i,t+2}(s'')\right\}$$
$$\frac{\theta u'(y_{-i} - \phi_{-i,t})}{v_{-i}(s;\phi_{-i,t}) - v_{-i,aut}(y_{-i})} = \frac{(1-\theta)g'(s, q_i, \phi_{-i}, b', d)}{V_t^L(s, q_{i,t}, \phi_{-i,t}, b_{fail}'^L(s_i)}.$$

Under these price functions, the good-credit country chooses $B_{i,t}(s,d)$ to maximize its value $w_{i,t}(s,b',d)$. If the bad-credit country chooses not to renegotiate already, the bond price function of the good-credit country $q_{i,t}$ solves the following one equation alone:

$$q_{i,t}g'[y_L + (b_i - q_{i,t}b_i')] = \sum_{s'} \delta\pi(s'|s)\left\{g'\left(c_{L,t+1}(s')\right)[1-d_{i,t+1}(s')] + d_{i,t+1}(s')\delta E_t\zeta_{i,t+2}(s'')\right\}.$$

If the good-credit country chooses to default, the recovery function of the bad-credit country solves the following equation alone:

$$\frac{\theta u'(y_{-i} - \phi_{-i,t})}{v_{-i}(s;\phi_{-i,t}) - v_{-i,aut}(y_{-i})} = \frac{(1-\theta)g'(s, \phi_{-i}, b', d)}{V_t^L(s, \phi_{-i,t}, b_{fail}'^L(s_i)}.$$

Similarly, when both countries are in bad credit standing, the two recovery functions solve the Nash bargaining problem jointly. Otherwise, the two recovery functions are independent of each other.

In the second step, we find the equilibrium default/renegotiation decisions $\{D_{i,t}(s), D_{-i,t}(s)\}$ that solve jointly

$$D_{i,t}(s) \in argmax_{\{d_{i,t}\}} w_{i,t}(s; d_{i,t}, D_{-i,t}(s), B(d_{i,t}, D_{-i,t}(s)))$$

$$D_{-i,t}(s) \in argmax_{\{d_{-i,t}\}} w_{-i,t}(s; D_{i,t}(s), d_{-i,t}, B(D_{i,t}(s), d_{-i,t})).$$

If there are multiple pairs of $(D_{i,t}, D_{-i,t})$ as equilibrium for a state s, we take the pair that maximizes $w_{i,t}(s, D_{i,t}(s), B_{i,t}(s, D_{i,t}(s))) + w_{-i,t}(s, D_{-i,t}(s), B_{-i,t}(s, D_{-i,t}(s)))$. We finally update the period t value for each country i:

$$v_{i,t}(s) = w_{i,t}(s, D(s), B(s, D(s)))$$

and the expected discounted recovery ζ according to

$$\zeta_{i,t}(s) = g'(c_{L,t}(s))(1 - D_{i,t}(s))\phi_{i,t}(s) + D_{i,t}(s)\delta E\zeta_{i,t+1}(s').$$

We continue the process until we reach the first period or until the value functions and price functions converge.