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INTERNATIONAL LENDING AND BORROWING IN A STOCHASTIC SEQUENCE EQUILIBRIUM

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ABSTRACT

This paper is a theoretical investigation of international lending and borrowing in the context of a general equilibrium model in which national productivities are subject to random fluctuations and rates of time preference differ among countries. International capital flows arise from the efforts of risk-averse households situated in different countries to self-insure against random productivity fluctuations. We establish the existence of a rational expectations equilibrium in which the world interest rate is constant and strictly less than the rate of time preference of the least impatient countries. The rate of time preference, solvency restrictions on borrowing, and balanced-budget fiscal policies are rigorously analyzed.

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INTERNATIONAL LENDING AND BORROWING

IN A STOCHASTIC SEQUENCE EQUILIBRIUM

by

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1. Introduction

This paper is a theoretical investigation of international lending and borrowing in a stochastic sequence equilibrium. It shares with recent research on the determinants of the current account by Sachs (1982), Obstfeld (1982, 1983), and Dornbusch (1983) its emphasis on the intertemporal consumption and savings choices of infinitely lived, optimizing households.[1] However, in contrast to the deterministic, partial equilibrium approach taken by the above authors, we study international lending and borrowing in the context of a general equilibrium model in which national productivities are subject to random fluctuations. Rates of time preference and levels of lump sum taxation differ among countries. Countries have access to world bond market and face a borrowing limit which precludes bankruptcy with probability one.[2] International lending and borrowing arise in the stochastic sequence equilibrium we study from the efforts of risk-averse households situated in different countries to self-insure against random productivity fluctuations by accumulating in relatively productive periods financial claims against the rest of the world which can be used to finance a less variable profile of consumption relative to output when productivity is unexpectedly low.

The analysis is adopted from the work of Bewley (1984) and employs the device of a continuum of countries subject to independently and identically distributed productivity shocks. A number of interesting results are obtained. The equilibrium world interest rate is constant and strictly less than the rate of time preference of the least impatient countries. The equilibrium distribution of net foreign asset holdings among countries is non-degenerate and stationary. Individual countries in general run current account imbalances, and thus decumulate or accumulate foreign assets, as national productivities are continually shocked. However, in equilibrium, the fraction of countries with net foreign asset holdings less than or equal to any particular value is constant over time. In particular, we show that there is a constant fraction of countries which exhaust their debt limits in each period and exhibit a zero correlation between unanticipated, transitory productivity shocks and their current account balances.

The stochastic steady-state properties of equilibrium consumption, the current account, the trade balance, and net foreign asset holdings in each country are completely characterized by stationary probability distributions which depend on the aversion to risk and rate of time preference of the representative household, the probability distribution of productivity shocks, and the equilibrium rate of interest. While the expected asymptotic current account balance is zero in each country, the expected asymptotic trade balance, consumption, and net foreign asset holdings will in general depend upon preferences and the probability distribution of productivity shocks. We obtain the intuitive results that low time preference countries run expected asymptotic trade balance deficits, service account surpluses, and achieve higher expected asymptotic consumption relative to their high time preference counterparts. In the special case in which all countries share the identical rate of time preference, the expected asymptotic trade balance and net foreign asset holdings are zero and expected asymptotic consumption equals mean output.

We also investigate the influence of lump sum taxes, collected to finance government consumption which does not enter agents' utility functions, on the stochastic steady-state in each country. In the special case in which all governments levy the same lump sum tax, the results are unsurprising: expected asymptotic private consumption in each country falls by the amount of the tax and the expected asymptotic trade balance and foreign asset holdings are unaffected relative to the no tax (and thus, no government consumption) case. However, if tax rates differ among countries, we establish that expected asymptotic private consumption in <u>each</u> country falls by the <u>average</u> lump sum tax across all countries. This implies that 'high tax' economies run expected asymptotic trade deficits and service account surpluses relative to their 'low tax' counterparts.

Sach (1982) and Obstfeld (1983) investigate international lending and borrowing in the context of a small open economy populated by a representative, infinitely lived household who discounts utility at a constant rate of time preference and who can borrow completely against a deterministic output sequence at a constant, exgenous rate of interest. In this set up, a steady-state exists iff the exogenous world interest rate equals the rate of time preference. Household preferences for a flat consumption profile induce current account deficits (surpluses) in periods of below (above) average productivity. Furthermore, as emphasized by

Sachs (1982), consumption is constant and equal to permanent income in each period and the stock of net foreign assets converges asymptotically to its initial level so that, if output converges to its permanent income level, the trade balance converges to the additive inverse of the interest income on initial foreign asset holdings. Of course, if the world is comprised of countries with different rates of time preference, there cannot be a steady-state with international capital mobility in the deterministic case. Lucas (1982, part 2) studies a two-country Arrow-Debreu equilibrium model which features Markovian fluctuations in the national outputs of imperfectly substitutable goods. Although his primary concern is with asset pricing in the open economy, the strong implication of the risk pooling equilibrium studied by Lucas is that current account balances are zero in each period because countries hold identical portfolios in every period.

The plan of the paper is as follows. Section 2 establishes the existence of a stationary, rational expectations equilibrium. Section 3 investigates the correlation between unanticipated productivity shocks and the current account. Sections 4 and 5 establish, respectively, the links between time preference and lump sum taxation and the stochastic steady state behavior of consumption, the trade balance, and foreign asset holdings. Section 6 provides some concluding remarks.

2. The Model

2.1. Overview and Assumptions

We study a world economy with a continuum of countries indexed $i \in [0,1]$, each populated by a representative trader with an infinite planning horizon. Each trader has access to a stochastic, Ricardian technology which transforms labor into the single, tradable consumption good. We assume that labor is not mobile internationally and that no utility is attached to leisure.[3] Time is discrete, and at the beginning of each period $t = 0, 1, 2, \ldots$, the labor input requirement in each country is randomly shocked. These shocks are assumed to be independently and identically distributed across countries and across time so that, with a continuum of countries, there is no aggregate uncertainty.[4] Normalizing the labor endowment in each country to unity so that we may speak of output and productivity interchangeably, the output of consumption goods in country i at date t may be written as

$$(1) \qquad y_t^i = \underline{y} + \varepsilon_t^i .$$

$\underline{y}$ is a deterministic, constant lower bound on the productivity of labor in each country and ε_t^i is the stochastic productivity shock in i at t. Formally, ε_t^i is a drawing from a finite set of a non-negative random variable from the fixed c.d.f. G such that $G(0) > 0$ and $G(\bar{\varepsilon}) = 1$.

Countries have access to a world capital market in one-period bonds and are allowed to borrow in amounts which can be repaid with probability one. However, in contrast to the recent work of Lucas (1982), we assume that Arrow-Debreu contingent claims markets do not exist. Although we do not rigorously derive this restricted market structure, we argue, as have Lucas (1980), Bewley (1980c), and Scheinkman and Weiss (1984) in related closed-economy constructs, that it captures apparently important precautionary and self-insurance considerations which underlie the lending, borrowing, and consumption decisions of 'liquidity constrained' households. This being said, let a_t^i denote the net holdings of foreign claims maturing in period t purchased last period by country i, and let $\rho_t = 1 + r_t$ be the gross rate of return on these claims.

To establish the existence of stationary, rational expectations sequence equilibrium, we begin by examining the partial equilibrium consumption and accumulation behavior of a representative country confronting a constant rate of interest, i.i.d. productivity fluctuations, and a deterministic limit on borrowing which precludes bankruptcy with probability one. Schechtman and Escudero (1977) prove the existence of a stationary distribution for wealth in the special case in which borrowing is not allowed and the rate of interest is zero, and show that the wealth accumulation process is bounded above if the rate of interest is strictly less than the rate of time preference and the elasticity of marginal utility is bounded above. Clarida (1985) establishes that these latter conditions are sufficient to insure the existence of a stationary distribution for asset holdings in the more general case in which lending and borrowing (in amounts which can be repaid with probability one) are allowed at a constant rate of interest. Bewley (1984) proves that the mean of this distribution is a continuous function of the interest rate and that there exists at least one interest rate, strictly less than the rate of time preference, at which the mean of this distribution is zero.

In a world with homogeneous preferences, a continuum of countries, and i.i.d. shocks, the per-country supply of loans is given by the mean of the stationary distribution for lending if loans are initially distributed among countries according to this distribution. As Bewley shows, these results together imply that there exists at least one stationary rational expectations sequence equilibrium in which the rate of interest is constant and the per-capita excess demand for goods and net supply of loans is zero in every period. We extend in an entirely straightforward manner Bewley's existence argument to incorporate heterogeneous rates of time preference and, in Section 5, rates of lump sum taxation among countries.

2.2. Representative Country's Optimization Problem

The representative country confronting a constant rate of interest r acts so as to

$$\max_{\{c_t\}_{t=0}^{\infty}} E\sum_{t=0}^{\infty}(1+\delta)^{-t}u(c_t) \tag{2}$$

$$\text{s.t.}\quad c_t + a_{t+1} = (1+r)a_t + y_t \tag{3}$$

$$a_t \geq -\underline{y}/r \tag{4}$$

$$c_t \geq 0\ . \tag{5}$$

Here $\beta = (1+\delta)^{-1}$ is the discount factor and $\delta \in [\underline{\delta},\bar{\delta}]$ is the positive rate of time preference, $u : R^+ \to R$ is a bounded, strictly concave, and twice differentiable utility function shared by all agents, and c_t is consumption at t . We assume that all countries confront (4), an institutional restriction on borrowing which precludes bankruptcy with probability one.

In this simple set up, the current account is equal to domestic savings, s_t, since investment spending is, for simplicity, ignored:[5]

$$(6) \qquad s_t = a_{t+1} - a_t = ra_t + y_t - c_t .$$

The trade balance, τ_t, is just the difference between output and consumption:

$$(7) \qquad \tau_t = y_t - c_t .$$

Let $v(w; r; \delta)$ be the value of (2) for a country facing interest rate r beginning the period with wealth w, where

$$(8) \qquad w = \rho a + y + \underline{y}/r .$$

The trader in this country makes a consumption/lending decision which solves

$$(9) \quad v(w; r; \delta) = \max\{u(c) + \beta \int v(\rho(w-c) + \varepsilon')dG(\varepsilon')\} , \quad 0 \leq c \leq w .$$

Denote the optimal consumption decision by $c(w; r; \delta)$ and note from (3) and (8) that the implied optimal lending/borrowing decision is given by

$$(10) \qquad a' = w - c(w; r; \delta) - \underline{y}/r = \ell(w; r; \delta; \underline{y}) .$$

The existence and several key properties of v and c are established in Schechtman and Escudero (1977) and Clarida (1985) and are now stated without proof.

Proposition 2.1 [Schechtman and Escudero (1977) Property 1.3, p. 153, Theorem 1.2, p. 154, Theorem 3.4, p. 59; Clarida (1985), Proposition 2.1, 6]: There exists a unique, bounded, continuous, strictly increasing, strictly increasing function c which solves (9). For all $0 \leq w \leq \hat{w}_\delta$, where $\hat{w}_\delta$ is the unique solution to[6]

$$(11) \qquad u'(\hat{w}_\delta) = \beta\rho\int u'(\varepsilon')dG(\varepsilon') ;$$

optimal consumption is given by

$$(12) \qquad c(w; r; \delta) = w .$$

For all $w > \hat{w}_\delta$ optimal consumption satisfies

$$(13) \qquad u'(c(w)) = \beta\rho\int v'(\rho(w - c(w)) + \varepsilon')dG(\varepsilon')$$

From the strict concavity of v and u, it follows that optimal lending $\ell(w; r; \delta; \underline{y})$ is a unique, continuous, and strictly increasing function for $w \geq \hat{w}_\delta$. Furthermore, for $0 \leq w \leq \hat{w}_\delta$

$$(14) \qquad \ell(w; r; \delta; \underline{y}) = -\underline{y}/r .$$

That is, there exists a threshold level of wealth, $\hat{w}$, at which it is optimal to exhaust the borrowing limit so long as $r < \delta$.

For a given rate of interest, the decision rule $c(w; r; \delta)$, the transition equation $w_{t+1} = \rho(w_t - c_t) + \varepsilon_{t+1}$, and the c.d.f. G together define a Markov process for wealth

$$(15) \qquad w_{t+1} = \rho(w_t - c(w_t; r; \delta)) + \varepsilon_{t+1} .$$

The following proposition, which is proved in the appendix, establishes the existence of a unique limiting distribution for wealth which is independent of initial wealth (and thus initial asset holdings).

<u>Proposition 2.2</u> [Schechtman and Escudero (1977), Theorem 3.8, p. 161 and Theorem 3.4, p. 59; Clarida (1985), Theorem 2.1, p. 9-13]: If $r < \delta$ and $-cu''/u' < \infty$, wealth is a delayed renewal process with state $w_t = 0$ the renewal point. There exists a unique stationary distribution for wealth, denoted $F(w; r; \delta)$, which is independent of initial wealth. Furthermore

(16) $\quad F(0; r; \delta) > 0$.

<u>Corollary 2.3</u>: Under the conditions of Proposition 2.2, end-of-period asset holdings evolve according to a delayed renewal process with state $a_t = -\underline{y}/r$ the renewal point. There exists a unique stationary distribution for assets, denoted $X(a; r; \delta; \underline{y})$ which is independent of initial assets. Furthermore (see Figure 1)

(17) $\quad X(-\underline{y}/r; r; \delta; \underline{y}) > 0$.

<u>Proof</u>: End-of-period foreign asset holdings are a continuous, monotonic function of beginning-of-period wealth $a_{t+1} = \ell(w_t; r; \delta; \underline{y})$. Thus $X(-\underline{y}/r; r; \delta; \underline{y}) = F(\hat{w}; r; \delta)$, and for $a' > -\underline{y}/r$, $X(a'; r; \delta; \underline{y}) = F(\ell^{-1}(a'; r; \delta; \underline{y}))$ since ℓ is strictly increasing for $w > \hat{w}$. Q.E.D.

Key properties of the mean of the stationary distribution of asset holdings are established in Bewley (1984) and are now stated without proof.

<u>Proposition 2.4</u> [Bewley (1984), Lemma 17, p. 8 (Section 2), Lemma 24, p. 19 (Section 2), Proposition 6, p. 1 (Section 4)]:

(18) (i) $E(a|X(a;\, r;\, \delta;\, \underline{y}))$ is a continuous function of r if $0 < r < \delta$;

(19) (ii) $\lim_{r \uparrow \delta} E(a|X(a;\, r;\, \delta;\, \underline{y}) = \infty$;

(20) (iii) $\lim_{r \downarrow 0} E(a|X(a;\, r;\, \delta;\, \underline{y}) = -\infty$.

2.3. <u>Existence of a Stationary, Rational Expectations Equilibrium</u>

We shall assume, to simplify the exposition, that there are two rates of time preference $\underline{\delta}$ and $\bar{\delta}$, with $0 < \underline{\delta} < \bar{\delta}$, exhibited in the world economy. Let α be the strictly positive fraction of countries sharing $\underline{\delta}$. With a continuum of countries in each fraction, the per capita net supply of bonds in α in each period is given by

(21) $E(a|X(a;\, r;\, \underline{\delta};\, \underline{y}))$;

where $X(a;\, r;\, \underline{\delta};\, \underline{y})$ is the stationary distribution for net foreign asset holdings for each country in α at the interest rate r . For there to be a stationary rational expectations sequence equilibrium, we must find at least one interest rate such that

(22) $\alpha E(a|X(a;\, r;\, \underline{\delta};\, \underline{y}) + (1-\alpha)E(a|X(a;\, r;\, \bar{\delta};\, \underline{y})) = 0$.

If (20) is satisfied, then from (6), per-country consumption equals per-country output.[7]

(23) $r[\alpha Ea(r;\underline{\delta}) + (1-\alpha)Ea(r;\bar{\delta})] + Ey = \alpha Ec(r;\underline{\delta}) + (1-\alpha)Ec(r;\bar{\delta})$;

(24) $Ey = \alpha Ec(r;\underline{\delta}) + (1-\alpha)Ec(r;\bar{\delta})$.

The interpretation of such an equilibrium, if it exists, is as follows. If foreign asset holdings are initially distributed among low (high) time preference countries according to $X(a; r; \bar{\delta}; \underline{y})$ $(X(a; r; \bar{\delta}; \underline{y})$ such that r satisfies (22), then the goods and asset markets clear in every period at this interest rate and the distribution of assets among countries remains constant over time. Of course, consumption and asset holdings in any particular country will fluctuate over time as countries are continually shocked by new drawings from G. In a stationary, rational expectations equilibrium, the stochastic steady-state behavior of the current account, foreign asset holdings, consumption, and the trade balance are completely characterized by non-degenerate stationary probability distribution. We investigate some key properties of these distributions in Sections 3, 4 and 5. However, we must first establish the existence of a stationary rational expectations sequence equilibrium. Bewley (1984) provides the existence argument for the special case in which all countries share the same rate of time preference. The following proposition is a straightforward extension of Bewley.

<u>Proposition 2.5</u>: There exists at least one stationary, rational expectations sequence equilibrium. In equilibrium, the rate of interest is constant and satisfies:

(25) $\quad 0 < r < \underline{\delta} < \bar{\delta}$.

<u>Proof</u>: From (22) and (18), the world per-country demand for assets is a continuous function of r so long as $0 < r < \underline{\delta}$. Since we assume that α is strictly positive, (19) implies that the world per-country demand for assets becomes positive as $r \uparrow \underline{\delta}$, since the per-country demand of

the $\bar{\delta}$ countries is bounded below by $(1-\alpha)(-\underline{y}/r)$. Conversely, as $r \downarrow 0$, the per-country demand for assets must become negative. It follows that there exists at least one r such that the per-capita supply of loans is identically zero if assets are initially distributed in $\underline{\delta}$ countries according to $X(a;\ r;\ \underline{\delta};\ \underline{y})$ and in $\bar{\delta}$ countries according to $X(a;\ r;\ \bar{\delta};\ \underline{y})$. Furthermore, since these are stationary distributions, the per-country supply of loans will be equal to zero in every period at this interest rate and per-country consumption will equal per-country output in every period. Thus, we have proven existence of a rational expectations, sequence equilibrium with interest rate r . Q.E.D.

Figure 2 illustrates the above existence argument. Note that nothing guarantees that the world per-country demand for assets is a monotonic function of r ; we cannot rule out multiple equilibria.

3. Productivity Shocks and the Current Account

Consumption smoothing plays a key role in recent theoretical research on the determinants of the current account [see in particular Sachs (1981, 1982) and Obstfeld (1983)]. This work implies that households' preferences for a smooth consumption profile relative to income lead, *ceteris paribus*, to current account surpluses in relatively productive periods and deficits in relatively unproductive periods. If such productivity fluctuations are perfectly foreseen, countries select a consumption profile consistent with permanent income and use their access to the world bond market to finance trade deficits when output is temporarily low. In the present framework, productivity fluctuations are stochastic and, as we have seen, the equilibrium interest rate is strictly less than the rate of time preference of the least impatient countries. These observations suggest that countries may run up against their debt limits when faced with a sequence of adverse productivity shocks. Without access to additional borrowing, such countries are forced to cut back consumption one for one with output when the latter is unexpectedly low, and to use whatever output is produced, $\underline{y}$, to service the debt already incurred. We now establish that in equilibrium, a constant fraction of countries do in fact exhaust their debt limited in every period and exhibit a zero correlation between adverse productivity shocks and the current account. This fraction of 'liquidity constrained' economies is strictly less than one; the remaining fraction of countries exhibits a strictly positive correlation between unanticipated productivity shocks and the current account.

<u>Proposition 3.1</u>: The equilibrium current account in each country is non-negatively correlated with unanticipated, transitory productivity shocks. A constant fraction of countries $\alpha F(\hat{w}_\delta; r; \underline{\delta}) + (1-\alpha)F(\hat{w}_\delta; r; \delta) < \bar{1}$ individually exhaust their debt limits and exhibit a zero correlation between the current account and productivity disturbances. The remaining fraction of countries individually exhibit a strictly positive correlation between the current account and productivity shocks.

<u>Proof</u>: For all $(a;\varepsilon)$ such that $\rho a + \underline{y} + \varepsilon + \underline{y}/r > \hat{w}_\delta$ optimal consumption satisfies

$$(26) \qquad u'(c) = \beta\rho\int v'(\rho a + \underline{y} + \varepsilon + \underline{y}/r - c;\ \varepsilon')dG(\varepsilon')) ;$$

so that $c(a;\varepsilon) = c(\rho a + \underline{y} + \varepsilon + \underline{y}/r)$ is a strictly increasing function of ε given a. For all $(a;\varepsilon) \subseteq \omega = \{(a;\varepsilon) : \rho a + y + \varepsilon + \underline{y}/r \lesssim \hat{w}_\delta\}$

$$(27) \qquad c(a;\varepsilon) = \rho a + \varepsilon \quad \underline{y} + \underline{y}/r .$$

The current account in each country is given by

(28) $s(a;\varepsilon) = ra + \underline{y} + \varepsilon - c(a;\varepsilon)$,

so that, for all $(a;\varepsilon) \subseteq \omega$

(29) $s(a;\varepsilon) = -(a + \underline{y}/r) \leq 0$.

The fraction of countries which begin each period with assets a and receive a productivity shock ε such that $(a;\varepsilon) \subseteq \omega$ is given by

(30) $0 < \alpha F(\hat{w}_{\underline{\delta}};\ r;\ \underline{\delta}) + (1-\alpha)F(\hat{w}_{\bar{\delta}};\ r;\ \bar{\delta}) < 1$.

This fraction is strictly positive by Propositions 2.2 and 2.5. It is strictly less than unity by the equilibrium condition that

(31) $\alpha E(a;\ r;\ \underline{\delta};\ \underline{y}) + (1-\alpha)E(a;\ r;\ \bar{\delta};\ \underline{y}) = 0$;

and the fact that $F(\hat{w}_{\underline{\delta}}) = F(\hat{w}_{\bar{\delta}}) = 1$ implies $E(a;\underline{\delta}) = E(a;\bar{\delta}) = -\underline{y}/r$ which cannot be an equilibrium. In the remaining fraction of countries, the current account is strictly positively correlated with unanticipated productivity shocks. This follows from the fact that, for all $(a;\delta) \not\subseteq \omega$, $c(a;\ \varepsilon^{(2)}) - c(a;\ \varepsilon^{(1)}) < \varepsilon^{(1)} - \varepsilon^{(2)}$ where $\varepsilon^{(2)} > \varepsilon^{(1)}$ are drawings from G , since v is strictly concave. Q.E.D.

4. Asymptotic Properties

In this section we invstigate the behavior of the expected asymptotic equilibrium current account, trade balance, foreign asset holdings, and consumption in each country. We begin by deriving a central implication of the equilibrium established in Section 2.

Proposition 4.1: The expected asymptotic current account balance in each country is zero.

Proof: From Proposition 2.2, there exists a unique limiting distribution for assets. In particular, if

$$(32) \qquad \text{Prob}(a_t \leq a' | a_0 = a;\ \delta;\ r;\ \underline{y}) = X(a';\ r;\ \delta;\ \underline{y}) \ ;$$

then, from (6)

$$(33) \qquad Es_t = Ea_{t+1} - Ea_t = rEa_t + Ey - Ec_t = 0 \ .$$

Since

$$(34) \qquad \lim_{t \to \infty} \text{Prob}(a_t \leq' | a_0 = a) = X(a;\ r;\ \delta;\ \underline{y})$$

it follows from Feller (1971), Theorem 1, p. 249

$$\lim_{t \to \infty} Es_t = 0 \ . \qquad \text{Q.E.D.}$$

In general, a country's expected asymptotic trade balance, foreign asset holdings, and consumption will depend upon its rate of time preference, aversion to risk, and the probability distribution of productivity shocks. The following proposition establishes the equilibrium relationships between a country's rate of time preference and its expected asymptotic consumption, trade balance, and net foreign asset holdings.

Proposition 4.2: Each low time preference country enjoys strictly higher expected asymptotic consumption than does its high time preference counterpart, and it runs as expected asymptotic trade balance deficit which is financed by an expected asymptotic service account surplus.

<u>Proof</u>: Clarida (1984), Theorem 3, pp. 17, shows that $v'(w;\, r;\, \underline{\delta}) \geq v'(w;\, r;\, \overline{\delta})$. Using (13), we obtain for $w > \hat{w}_{\delta}$

(36) $$u'(c(w;\overline{\delta})) < \underline{\beta}\rho\int v'(\rho(w - c(w;\overline{\delta})) + \varepsilon';\, \underline{\delta})dG(\varepsilon') .$$

Thus, from the strict concavity of u and v

(37) $$c(w;\underline{\delta}) < c(w;\overline{\delta}) \quad \forall w > \hat{w}_{\underline{\delta}} .$$

Using this result, we now show that expected asymptotic asset holdings are inversely related to a country's rate of time preference. We compare the accumulation of wealth and assets of two countries, $\underline{\delta}$ and $\overline{\delta}$, which reach period t with w_t and behave optimally.[8] Let $\theta = (\omega_t, \omega_{t+1}, \ldots)$ be any potential sequence of random events with associated random realizations $\varepsilon(\theta) = (\varepsilon_t, \varepsilon_{t+1}, \ldots)$. Define $\gamma(\theta)$ and $\eta(\theta)$ as

(38) $$w^{\underline{\delta}}(\theta) = w^{\overline{\delta}}(\theta) + \eta(\theta) ,$$

(39) $$a^{\underline{\delta}}(\theta) = a^{\overline{\delta}}(\theta) + \gamma(\theta) .$$

Since $c(w;\underline{\delta}) \leq c(w;\overline{\delta})$, with strict inequality if $w > \hat{w}_{\underline{\delta}}$, it follows that

(40) $$a^{\underline{\delta}}_{t+1}(\omega_t) \geq a^{\overline{\delta}}_{t+1}(\omega_t) ;$$

(41) $$w^{\underline{\delta}}_{t+1}(\omega_t, \omega_{t+1}) \geq w^{\overline{\delta}}_{t+1}(\omega_t, \omega_{t+1}) .$$

From Proposition 3.1, we know that $\ell(w; r; \delta; \underline{y})$ is a strictly increasing function of w $\forall w > \hat{w}_{\underline{\delta}}$. Thus

$$(42) \qquad a^{\underline{\delta}}_{t+2}(\omega_t, \omega_{t+1}) \geq a^{\overline{\delta}}_{t+2}(\omega_t, \omega_{t+1}) .$$

Hence, the sequence $\{\gamma(\omega_t), \gamma(\omega_{t+1}), \dots\}$ is a converging sequence of non-negative numbers. Thus

$$(43) \qquad Ea^{\underline{\delta}}(\theta) = Ea^{\overline{\delta}}(\theta) + E\gamma(\theta) \geq Ea^{\overline{\delta}}(\theta) .$$

Using the fact that the expected asymptotic current account is zero, we obtain

$$(44) \qquad Ec^{\underline{\delta}} = rEa^{\underline{\delta}} + Ey \geq rEa^{\overline{\delta}} + Ey = Ec^{\overline{\delta}} .$$

Finally, using the definition of the trade balance, we have

$$(45) \qquad E\tau^{\underline{\delta}} = Ey - Ec^{\underline{\delta}} \leq Ey - Ec^{\overline{\delta}} = E\tau^{\overline{\delta}} .$$

In equilibrium

$$(46) \qquad \alpha Ea^{\underline{\delta}}(r) + (1-\alpha)Ea^{\overline{\delta}}(r) = 0 .$$

If $Ea^{\underline{\delta}}(r)$ and $Ea^{\overline{\delta}}(r)$ are equal, they must, from Propositions 3.1 and 4.2, equal $-\underline{y}/r$. But this contradicts (46). Thus

$$(47) \qquad \begin{aligned} & Ec^{\underline{\delta}} > Ec^{\overline{\delta}} ; \\ & Ea^{\underline{\delta}} > 0 > Ea^{\overline{\delta}} ; \\ & E\tau^{\underline{\delta}} < 0 < E\tau^{\overline{\delta}} . \end{aligned}$$

Q.E.D.

We conclude this section by investigating expected asymptotic consumption, foreign asset holdings, and the trade balance in a world in which all countries share a common rate of time preference.

Proposition 4.3: In the special case $\alpha = 1$, expected asymptotic consumption in each country is equal to mean productivity and the expected asymptotic trade balance is zero. Furthermore, initial debtors (creditors) run an expected asymptotic cumulative current account surplus (deficit). The fraction of countries which run an expected asymptotic cumulative current account deficit is given by $1 - X(0)$. The fraction of countries which run an expected asymptotic cumulative surplus is given by $X(0) - \text{Prob}(a = 0 | X(a;\ r;\ \delta;\ \underline{y}))$.

Proof: In equilibrium

$$(48) \qquad Ea^{\delta}(r) = 0 ;$$

so that

$$(49) \qquad Ec^{\underline{\delta}} = Ey ;$$

$$(50) \qquad E\tau^{\underline{\delta}} = 0 .$$

The expected cumulative current account surplus at t is given by

$$(51) \qquad Ea_t^{\delta} - a_0^{\delta} = E\sum_{n=0}^{t-1} s_n .$$

The final result follows from the existence of a limiting distribution for assets which, at the equilibrium interest rate, has mean zero. Q.E.D.

As discussed in the Introduction, in the special case in which output evolves according to a deterministic sequence a steady-state exists iff the rate of interest equals the rate of time preference. In this case, the limiting value of the trade balance is uniquely determined by initial asset holdings, the cumulative current account surplus converges to zero, and consumption is constant and equal to permanent income. Thus, our findings enrich those obtained in existing partial equilibrium, deterministic frameworks. The results contained in Proposition 4.3 are intuitive and related to the findings of Buiter (1981) who analyzes international investment flows in a two country version of Diamond's (1965) overlapping generations model. Blanchard (1985) studies a small open economy in which agents face a constant probability of death in each period. In this framework, a steady state exists if the exogenous world interest rate is less than the sum of the rate of time preference and the probability of death. In Blanchard's model, if the rate of interest is less than the rate of time preference, the small open economy is a net debtor in steady state and thus runs a steady-state trade balance surplus.

5. Effect of Taxes on the Stochastic Steady State

We now investigate the effect of lump sum taxes, used to finance government consumption which does not enter agents' utility functions, on the stochastic steady state behavior of the trade balance, foreign asset holdings, and private consumption in each country. We consider a world comprised of a fraction $\alpha_{\underline{\theta}}$ of 'low tax' economies and a fraction $(1-\alpha_{\underline{\theta}})$ of 'high tax' economies; rates of time preference are assumed to be the same across all countries. In the special case in which all governments levy the same per capita tax, the results are as expected:

expected asymptotic private consumption in each country falls by the amount of the lump sum tax, and the expected asymptotic trade balance and foreign asset holdings are unaffected relative to the no-tax (no government consumption) case. However, if tax rates differ among countries, we show that expected asymptotic equilibrium consumption in <u>each</u> country falls by the <u>average</u> lump sum tax across all countries. High tax economies are shown to run expected asymptotic trade deficits and service account surpluses relative to their low tax counterparts.

We begin by demonstrating that, at any given interest rate, the consumption decision rule is identical in low and high tax economies. Let $0 \leq \underline{\theta}\underline{y} < \underline{y}$ be the lump sum tax collected to finance government consumption in 'low tax' countries, and let $0 < \overline{\theta}\underline{y} \leq \underline{y}$ be the lump sum tax collected in 'high tax' economies with $\overline{\theta} > \underline{\theta}$. Using the fact that

$$(52) \qquad a_{t+1} = (1+r)a_t + \underline{y}(1-\theta^j) + \varepsilon - c \;, \quad \theta^j = \underline{\theta}, \overline{\theta} \;,$$

and the definition of wealth

$$(53) \qquad w_t = (1+r)a_t + \underline{y}(1 - \theta^j) + \varepsilon_t + \underline{y}(1 - \theta^j)/r \;,$$

we obtain

$$(54) \qquad (1+r)a_{t+1} = (1+r)(w_t - c_t) - \underline{y}(1 - \theta^j)(1+r)/r$$

Re-arranging and adding ε_{t+1} to both sides we obtain

$$(55) \qquad w_{t+1} = (1+r)(w_t - c_t) + \varepsilon_{t+1} \;;$$

that is, the transition equation for wealth is independent of the lump sum tax so long as borrowing is allowed up to $\underline{y}(1 - \theta^j)/r$. Of course, households in countries with higher taxes begin, ceteris paribus, with lower initial wealth. But the optimal consumption decision rule depends only upon preferences, the probability distribution of productivity shocks, and the transition equation (technology for transferring wealth between periods), and not the initial level of wealth. Formally, we have

Proposition 5.1: At any given interest rate, the consumption decision rule is independent of lump sum taxes.

Proof: Clarida (1985), Theorem 4.2, p. 19.

It follows immediately from Proposition 5.1 that the Markov processes for wealth in low and high tax countries differ only in initial conditions. This implies from Proposition 2.2 that the limiting probability distributions for wealth in low and high tax economies are identical for any given interest rate less than δ , the common rate of time preference. Thus

$$(56) \qquad \rho Ea^{\underline{\theta}} + \underline{y}(1-\theta) + E\varepsilon + \underline{y}(1-\underline{\theta})/r = \rho Ea^{\bar{\theta}} + \underline{y}(1-\bar{\theta}) + E\varepsilon + \underline{y}(1-\bar{\theta})/r \ .$$

Re-arranging we obtain

$$(57) \qquad Ea^{\bar{\theta}} - Ea^{\underline{\theta}} = \underline{y}(\theta-\underline{\theta})/r \ .$$

The interpretation of (57) is that, because low and high lump sum tax countries accumulate so as to attain the identical expected asymptotic level of wealth, the savings behavior of high tax economies must be such that their expected asymptotic foreign asset holdings are large enough to offset their lower after tax income and borrowing capacity.

Consider now the equilibrium in a world comprised of a fraction $\alpha_{\underline{\theta}}$ low tax countries and a fraction $(1 - \alpha_{\underline{\theta}})$ high tax countries. The existence of a stationary rational expectations equilibrium can be established using exactly the arguments of Section 2.3; the proof will be omitted. In equilibrium, it must be the case that

(58) $$\alpha_{\underline{\theta}}E(a|X(a; r; \delta; \underline{y}(1-\underline{\theta})) + (1 - \alpha_{\underline{\theta}})E(a|X(a, r, \delta, \underline{y}(1-\bar{\theta})) = 0 .$$

Furthermore, at any interest rate less than δ ;

(59) $$Ea^{\bar{\theta}} - Ea^{\underline{\theta}} = \underline{y}(\bar{\theta}-\underline{\theta})/r .$$

Substituting (58) into (59) we obtain, after routine calculation,

(60) $$Ea^{\bar{\theta}} = \alpha_{\underline{\theta}}\underline{y}(\bar{\theta}-\underline{\theta})/r ,$$

(61) $$Ea^{\underline{\theta}} = (1 - \alpha_{\underline{\theta}})\underline{y}(\underline{\theta}-\bar{\theta})/r ,$$

where r is the equilibrium interest rate. We are now in a position to prove the following proposition.

<u>Proposition 5.2</u>: Each high tax economy runs an expected asymptotic trade balance deficit and service account surplus relative to its low tax counterpart. Expected asymptotic private consumption is identical across countries and equals

(62) $$Ec = \underline{y}(1 - \alpha_{\underline{\theta}}\underline{\theta} - (1 - \alpha_{\underline{\theta}})\bar{\theta}) + E\varepsilon ,$$

that is, relative to the zero tax (and government consumption) case, expected asymptotic consumption in each country falls by the average lump sum tax across countries.

Proof: Using the fact that the expected asymptotic current account is zero, we have, from (60) and (61)

$$(63) \qquad Ec^{\bar{\theta}} = \alpha_{\underline{\theta}}\underline{y}(\bar{\theta}-\underline{\theta}) + \underline{y}(1-\bar{\theta}) + E\varepsilon \ ,$$

$$(64) \qquad Ec^{\underline{\theta}} = (1 - \alpha_{\underline{\theta}})\underline{y}(\underline{\theta}-\bar{\theta}) + \underline{y}(1-\underline{\theta}) + E\varepsilon \ .$$

Re-arranging terms, we obtain

$$(65) \qquad Ec^{\bar{\theta}} = Ec^{\underline{\theta}} = \underline{y}(1 - \alpha_{\underline{\theta}}\underline{\theta} - (1 - \alpha_{\theta})\bar{\theta}) + E\varepsilon \ .$$

The expected asymptotic trade balance deficit in high tax economies is just the additive inverse of the service account surplus

$$(66) \qquad E\tau^{\bar{\theta}} = \alpha_{\underline{\theta}}\underline{y}(\underline{\theta}-\bar{\theta}) \ ,$$

and similarly for the expected asymptotic trade surplus in low tax countries

$$(67) \qquad E\tau^{\underline{\theta}} = (1 - \alpha_{\underline{\theta}})\underline{y}(\underline{\theta}-\bar{\theta}) \ . \qquad \text{Q.E.D.}$$

6. Concluding Remarks

The objective of this paper has been to study international lending and borrowing in general equilibrium framework in which countries are subject to stochastic productivity fluctuations. The role of time preference, borrowing limits, and lump sum taxation were rigorously analyzed, yielding results which enrich those obtained in the existing literature. Two useful extensions of the present analysis would be the incorporation of capital accumulation and the derivation of equilibrium loan agreements from an underlying model of asymmetric information between borrowers and lenders.

APPENDIX

This appendix demonstrates the existence of a limiting distribution for wealth. Schechtman and Escudero (S-E) (1977) prove existence for the case in which borrowing is not allowed, the rate of interest is zero, and the rate of time preference is positive. These authors also demonstrate that, so long as $r < \delta$, the wealth accumulation process is bounded above so long as $-cu''/u' < \infty$. Clarida (1985) extends the S-E existence proof to the case in borrowing is allowed at a positive interest rate in amounts which can be repaid with probability one. The case in which y_t is countable is straightforward.

Consider

$$\text{Max } E\sum_{t=0}^{\infty}\beta^t u(c_t)$$

(1) $$\text{s.t. } 0 \leq c_t \leq w_t$$

$$w_{t+1} = (1+r)(w_t - c_t) + y_t$$

$$w_t = (1+r)a_t + y_t$$

where y_t is a countable r.v. with lower support $\underline{y}$ and $r < \delta$, $\beta = 1/1+\delta$. This is S-E (1977) case (b) and, by their Theorem 3.8, $\exists$a $\bar{w}$ s.t.

(2) $$(1+r)(w_t - c(w_t)) + \bar{y} \leq \bar{w} \quad \forall w_t \geq \bar{w}$$

where $c(w)$ is the solution to the d.p. problem (9). Now, if $w > \hat{w}$

$$(3) \qquad u'(c(w)) = \beta\rho\int v'(\rho(w - c(w)) + \underline{y} + \varepsilon')dG(\varepsilon')$$
$$\leq \beta\rho v'(\rho(w - c(w)) + \underline{y})$$

since v' is non-increasing. By the envelope theorem

$$(4) \qquad u'(c(w)) \leq \beta\rho u'(c(\rho(w - c(w)) + \underline{y})$$

so that

$$(5) \qquad w_t > \rho(w_t - c(w_t)) + \underline{y} = w_{t+1} .$$

Thus, if the agent receives the worst shock $\underline{y}$ in T consecutive periods, $w_t > w_{t+1} > \ldots > w_{t+T}$. By Schechtman and Escudero (1977), Theorem 2.3, p. 156, in a finite number of drawings of $\underline{y}$, w_t hits y . By S-E (1977), Theorem 3.3, wealth is a delayed renewal and by Feller (1968), Theorem 2, p. 315, there exists a limiting distribution of wealth invariant to initial wealth. From the arguments in Clarida (1985), it can be shown that if $w_t > \bar{w}$, wealth will converge almost surely to the interval $[\underline{y},\bar{w}]$.

If borrowing is allowed

$$(6) \qquad w_t = \rho a_t + \underline{y} + \varepsilon_t + \underline{y}/r ,$$

and the wealth accumulation process evolves according to

$$w_{t+1} = \rho(w_t - c_t) + \varepsilon_{t+1} .$$

The proof goes through exactly as above so long as $u'(0) < \infty$, a condition which insures that $\hat{w}_\delta > 0$. Q.E.D.

NOTES

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1. Each of these authors also examines relationships between a relative price and current account — in Sachs (1982) it is the relative price of semi-tradeable goods; in Obstfeld (1983), the terms of trade; in Dornbusch (1983), the real exchange rate. To simplify the general equilibrium model studied here, we abstract from relative prices so as to focus on the determination of the equilibrium rate of interest and the asymptotic properties of consumption, the trade balance, and asset holdings.

2. It would certainly be preferable to derive optimal lending and borrowing contracts from an underlying model of asymmetric information. Although progress has been made on this front by Townsend (1983), the difficulties in characterizing incentive-compatible arrangements in a form susceptible to the analysis of this paper preclude this more satisfactory approach at this time.

3. Indeed, if labor were mobile internationally and preferences were identical, each country would consume $\underline{y} + \varepsilon$ — the maximum average product of labor — in each period and there would be no lending or borrowing in equilibrium.

4. See Bewley (1980b) for a rigorous treatment of the measurability issues involved with the device of a continuum of traders.

5. Empirically, investment fluctuations appear to be an important determinant of current account fluctuations (Sach (1981)). For analytical tractability we must unfortunately abstract from investment as do Sachs (1982), Obstfeld (1983), Dornbusch (1983), and Blanchard (1985).

6. $\hat{w}_\delta$ is unique if $r < \delta$, a condition which will be satisfied in equilibrium.

7. The existence of a stationary distribution for consumption follows immediately from Proposition 2.2 and the result that consumption is a strictly increasing function of wealth.

8. These arguments draw on Danthine and Donaldson (1981).

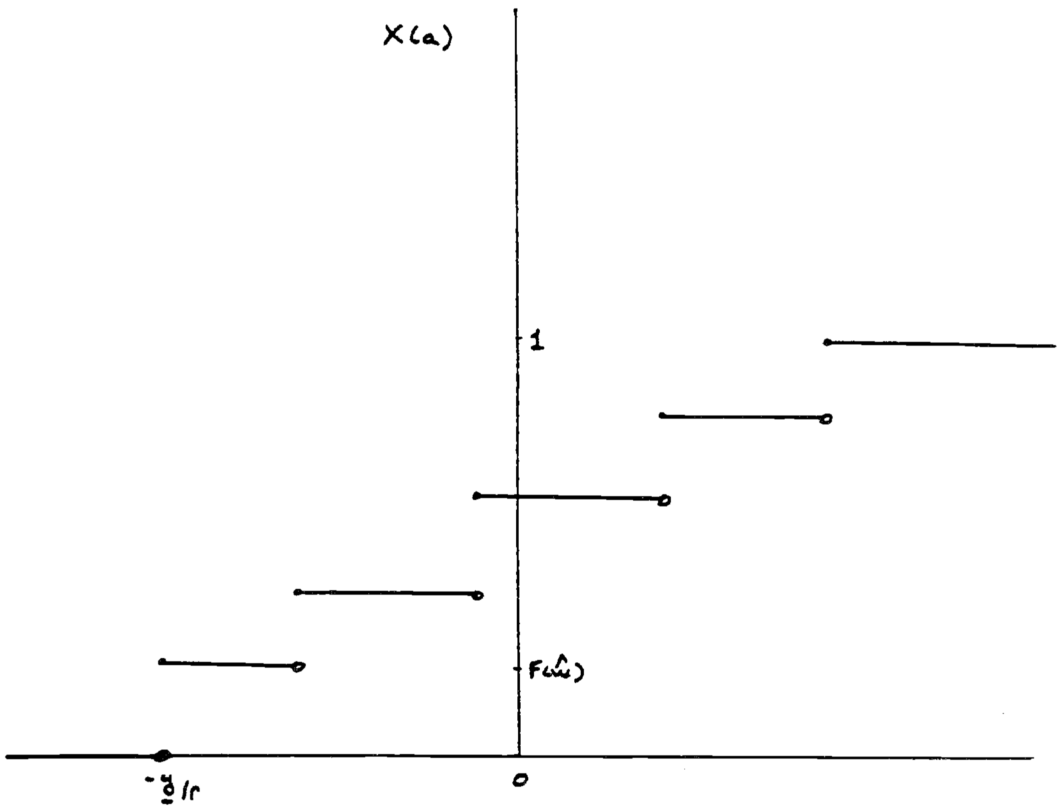

Figure 1

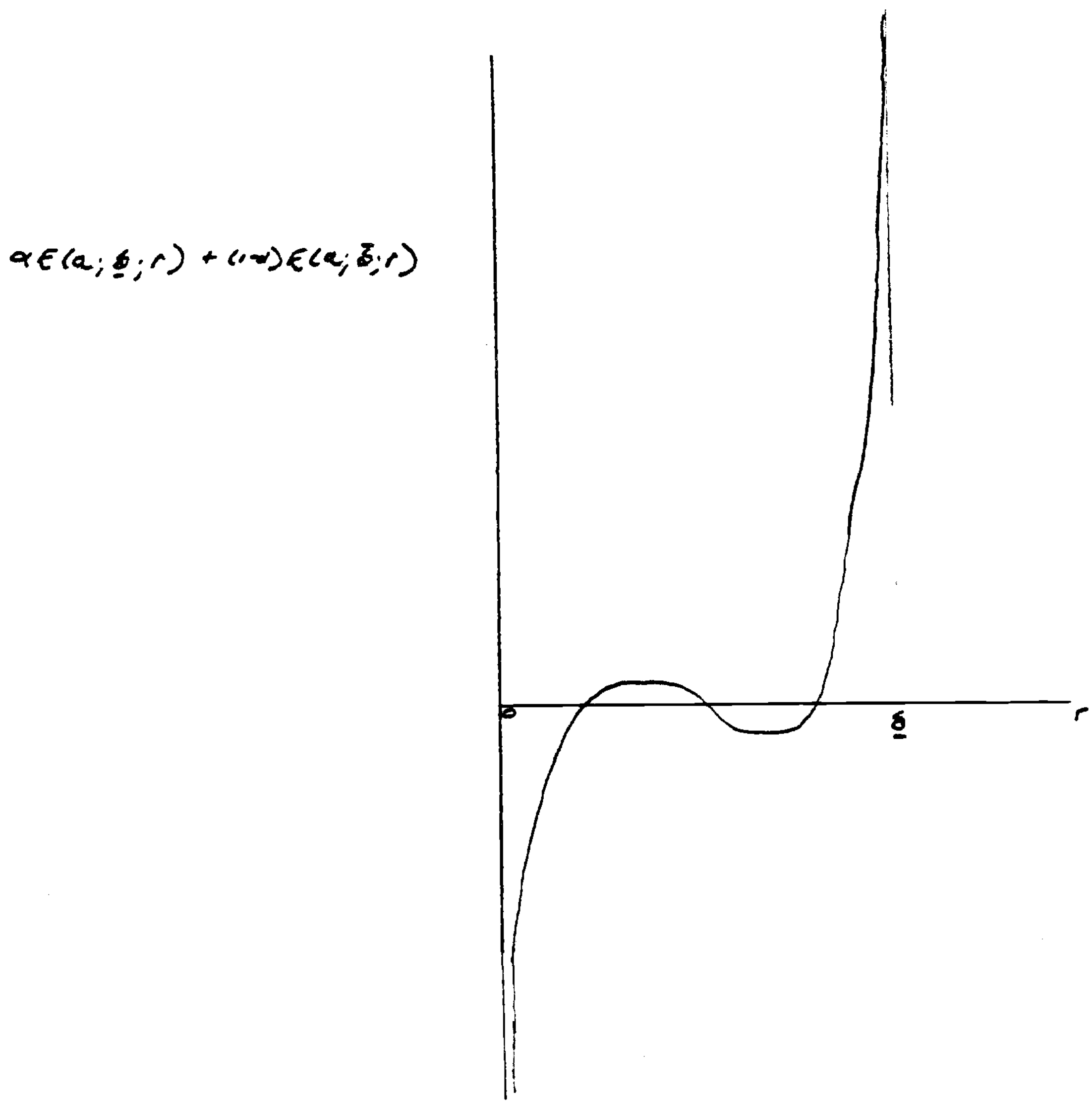
$\alpha\epsilon(a;\underline{\delta};r) + (1-\alpha)\epsilon(a;\bar{\delta};r)$
0
$\underline{\delta}$
r

Figure 2

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