NBER WORKING PAPER SERIES

THE IMPACT OF THE INTERNET ON ADVERTISING MARKETS FOR NEWS MEDIA

Susan Athey
Emilio Calvano
Joshua Gans

Working Paper 19419
http://www.nber.org/papers/w19419

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 2013

Financial support from the Toulouse Network on Information Technology is gratefully acknowledged. Part of this research was conducted at Microsoft Research, Cambridge MA. Both Athey and Gans have consulting relationships with Microsoft that has an interest in the efficiency of online advertising markets but does not have direct interests in the news media industry. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

The Impact of the Internet on Advertising Markets for News Media
Susan Athey, Emilio Calvano, and Joshua Gans
NBER Working Paper No. 19419
September 2013
JEL No. L11,L13,L82

ABSTRACT

In this paper, we explore the hypothesis that an important force behind the collapse in advertising revenue experienced by newspapers over the past decade is the greater consumer switching facilitated by online consumption of news. We introduce a model of the market for advertising on news media outlets whereby news outlets are modeled as competing two-sided platforms bringing together heterogeneous, partially multi-homing consumers with advertisers with heterogeneous valuations for reaching consumers. A key feature of our model is that the multi-homing behavior of the advertisers is determined endogenously. The presence of switching consumers means that, in the absence of perfect technologies for tracking the ads seen by consumers, advertisers purchase wasted impressions: they reach the same consumer too many times. This has subtle effects on the equilibrium outcomes in the advertising market. One consequence is that multi-homing on the part of advertisers is heterogeneous: high-value advertisers multi-home, while low- value advertisers single-home. We characterize the impact of greater consumer switching on outlet profits as well as the impact of technologies that track consumers both within and across outlets on those profits. Somewhat surprisingly, superior tracking technologies may not always increase outlet profits, even when they increase efficiency. In extensions to the baseline model, we show that when outlets that show few or ineffective ads (e.g. blogs) attract readers from traditional outlets, the losses are at least partially offset by an increase in ad prices. Introducing a paywall does not just diminish readership, but it furthermore reduce advertising prices (and leads to increases in advertising prices on competing outlets).

Susan Athey
Graduate School of Business
Stanford University
655 Knight Way
Stanford, CA 94305
and NBER
athey@stanford.edu

Emilio Calvano
Bocconi University
Via Rontgen, 1
20136, Milan
Italy
emilio.calvano@gmail.com

Joshua Gans
Rotman School of Management
University of Toronto
105 St. George Street
Toronto ON M5S 3E6
and NBER
joshua.gans@gmail.com

1 Introduction

A recent report of the Federal Communication Commission found that U.S. Newspaper advertising revenues dropped 47% from 2005 to 2009.[1] The ad revenue decline is pronounced even when controlling for obvious explanatory factors such as circulation, decline in revenues from classified ads and the business cycle.[2] From a public policy perspective, the likely reduction in investigative, enterprise and beat reporting represents a serious source of concern. The average newsroom shrunk by a quarter with more than 50% due to heavy cuts of editorial costs. The report concludes that "in very real ways, the dramatic newspaper industry cutbacks appear to have caused genuine harm to American citizens."

The decline in advertising revenue has been almost unanimously attributed to the rise of the Internet. However, the adverse impact of the web represents an economic puzzle because, in many respects, the forces influencing supply and demand appear to be as favorable for the industry, if not more so, than before. Online consumption of news media created new and improved advertising products and services that should be, in principle, *more* valuable to advertisers (e.g. enhanced ads, targeting capabilities, and improved measurement).[3] Moreover, the Internet dramatically increased the accessibility of many outlets for a wider audience.

A variety of theories have been proposed to explain the drop in advertising revenue. A common theme is that there is a glut in the supply of advertising space (Rice, 2010). However, this argument fails to account for the fact that while there may be space for every advertiser on the web, those ads must be still viewed by actual consumers:

---

[1] *The Information Needs of Communities* (2011), available at: http://www.fcc.gov/info-needs-communities.

[2] According to the Newspaper Association of America (www.naa.org), since 2000 total advertising revenue earned by its member US newspapers declined by 57% in real terms to be around $27 billion in 2009. Much of this decline was in revenue from classifieds but total display advertising revenue fell around 40%. In contrast, circulation over the same period declined by 18%. Ad revenue as a share of GDP also declined by 60%. According to ComScore, total US display advertising revenue online was around $10 billion in 2010 which includes all sites and not just newspapers.

[3] The Internet has also created new types of opportunities such as "search ads." However, many observers and regulators have noted that these new forms of advertising are complements rather than substitutes for the kind of advertising typically used by the news media; see Evans (2008, 2009). Chandra and Kaiser (2011) demonstrate that magazines who are better able to tailor content to specific consumer groups can continue to command a premium in ad rates and that this premium is associated with a consumer base with higher Internet use.

human attention is naturally scarce which limits the amount of advertising that can be supplied. Another theme is that online or digital ads are far less effective than ads that are on paper. However, the evidence is not consistent with that hypothesis (see Dreze and Hussherr, 2003; Lewis and Reiley, 2009; Goldfarb and Tucker, 2011b).

From an economics perspective, the industry-wide decline in advertising revenue remains a puzzle. A distinctive feature of the benchmark model in media economics pioneered by Anderson and Coate (2005) is that news outlets are the gatekeepers of their readers' attention; that is, consumers are assumed to *single home* with their attention concentrated on one outlet.[4] Thus, advertising revenues at the outlet and, hence, at the industry level reflect monopoly prices for access to those consumers. In particular, if the advertising space per outlet is constant, prices are independent of the number of outlets. Outlets compete for consumers by reducing advertising output. In this benchmark model, the comparative statics associated with changes in economic primitives are not consistent with the hypothesis that the rise of the internet reduced aggregate ad revenue. For instance, increased competition for consumers due to lower search costs or increased entry by new outlets would lead to higher advertising prices, as those outlets scale back levels of annoying advertising to attract consumers, and then charge monopoly prices to advertisers for the reduced advertising space. In contrast to these predictions, there is evidence that competition is associated with falling prices (Anderson, Foros and Kind, 2011).

This aim of this paper is to consider seriously the impact of increased consumer switching that many have observed is an essential distinguishing feature of on-line news consumption (Fahri, 2009; Gentzkow and Shapiro, 2011 and Varian, 2010). Switching refers to the proneness of online consumers to satisfy their content needs on *multiple* outlets as opposed to buy only one print newspaper. Web browsers, search engines, aggregators and social network make it easy for consumers to move between outlets and increase consumer switching among outlets (Athey and Mobius, 2012), while free access removes other constraints.

The paper revolves around a simple yet very powerful observation. Absent a technology that can track consumers as they move across outlets, switching makes

[4] For a general treatment for two-sided markets see Armstrong and Wright (2007).

advertising a relatively more daunting task. By placing ads on additional outlets, the advertisers take the risk of reaching same consumers multiple times. Switching thus degrades the market value of an outlet's advertising inventory. As we shall see, this has important welfare implications as, in equilibrium, it leads to inefficient depletion (duplication) and use (mismatches) of a scarce resource: the consumers' attention.

To address this we present a theory of advertising that has readers spreading their attention across multiple outlets. However, we do not assume that all consumers visit all outlets; instead, they switch outlets stochastically, that is, they *multi-home* but not fully (unlike the most of the existing literature on two-sided markets where agents either fully multi-home or single-home). We deploy an equilibrium model featuring a set of heterogeneous advertisers who profit from informing readers about their products, a mass of identical (from the advertisers' perspective) consumers with a fixed endowment of attention, and a finite number of outlets. Outlets use their consumers' attention as an input to produce advertising inventory; the fixed number of ads that can be shown to a consumer (capacity) and advertisers purchase ads to reach them. Given the stochastic nature of consumer switching, an additional ad has uncertain benefits from the perspective of an advertiser. The ad either reaches already informed consumers (and, hence, wastes some of their attention) or informs a new ones. The probability of success depends, among other things, on the outlets' "tracking technology." These technologies allow outlets to enhance the allocation of the ads and, hence, reduce wasteful duplication. We postulate that, as a baseline description of reality today, outlets have a superior ability to track the behavior of consumers within their outlets rather than between them (see, for example, Edelman, 2010). Finally ad prices are determined via a market clearing condition.

A key feature of our model is that the multi-homing behavior of advertisers is determined endogenously. With no consumer switching and a single market-clearing price for advertising, advertisers should place ads on all outlets. Consumer switching together with imperfect tracking of consumers across outlets creates inefficiencies in duplicated impressions. Switching by consumers is, thus, a source of diminishing returns to buying ad space on additional outlets (multi-homing). Consequently, in equilibrium, higher value advertisers choose to multi-home, as they have a higher opportunity cost of not informing readers, while lower-value advertisers single home, avoiding wasted impressions. As a result of the subtle effects of the mixed homing behavior on market prices, ad prices do not necessarily fall with switching. We show that the marginal return from an additional ad is a convex combination of marginal returns on switching consumers and loyal readers. Increased switching decreases marginal returns for multi-homing advertisers. We show that whether switching reduces profits depends on the total available ad capacity per unit of attention. With low or moderate ad capacity, fewer advertisers multi-home, and a greater range of advertiser values is served, leading to lower advertising prices. However, with high ad capacity, increased switching induces high value advertisers to purchase multiple impressions on each outlet, leading to a higher-value set of advertisers being served and higher advertising prices. Indeed, profits may exceed levels that can be achieved when either switching or imperfect tracking is not a problem. Interestingly, this implies that outlets can have suboptimal incentives to invest in technology.

Next, we consider several applications of the theory. We show that it offers a natural solution to a number of long-standing puzzles in media economics. First, there is evidence that larger outlets command a premium, and that advertisers are willing to pay for “reach” which refers to the number of users who can be impressed through an outlet.[5] However, the benchmark model with no switching predicts that prices per viewer equalize across outlets in equilibrium. We show that consumer switching makes larger outlets relatively more attractive to those advertisers who cannot afford the waste that comes with large (i.e. multi-homing) campaigns. Consequently, higher valued, single-homing advertisers sort onto the high readership outlet first, giving larger outlets a “positional advantage.” Second, rather than welcome regulation that requires public media to raise revenue from ads as opposed to be subsidized, existing outlets have typically lobbied against the lifting of advertising restrictions.[6] (Public subsidies, the argument goes, should make state-owned media *tougher* competitors on the market for readers). We demonstrate that, when some outlets cannot sell ads (as they might if they

---

[5] Recently, this has been referred to the “ITV Premium Puzzle.” (Competition Commission 2003). However the relationship has been noted previously by Fisher et al (1980) and Chwe (1988).

[6] For example see Filistrucchi, Luini and Mangani (2011) for an empirical analysis of the French advertising ban on prime-time state television.

are regulated public broadcasters or smaller blogs), ad prices will be higher. The more obvious effect behind that result is that when outlets capture consumer attention without selling ads, this reduces the capacity that can be sold to advertisers in the market, raising prices (but note, this effect is absent in traditional models). Further, because movements to and from such outlets do not create wasted impressions, efficiency and prices typically go up.

We then explore strategic implications arising from our model. The positional advantage arising from having a larger readership share can drive competition for consumers and, indeed, may cause outlets to invest more in quality than they would under benchmark cases or perfect tracking. This result is consistent with the stylized fact that media outlets that provide greater "reach" command higher ad prices, all else equal.[7] We also demonstrate that an outlet can gain a positional advantage by having limited content, but content that consumers visit reliably – something we term 'magnet content.' If outlets can ensure that a high share of consumers will at some point allocate attention to them, those outlets can command a premium in advertising markets. This suggests that outlets may focus their efforts on producing offerings that regularly attract the attention of many consumers rather than the focused attention of fewer consumers. Relatedly, we demonstrate that paywalls unilaterally imposed by an outlet can have the effect of reducing their positional advantage or giving their rivals a positional advantage in advertising markets. As a result, we identify additional competitive costs to outlets from introducing paywalls.

Finally, on the policy side, our model sheds light on a number of issues that we believe are important for antitrust policy. Specifically we discuss the impact of a merger (in terms of better technology and stronger discrimination power) on the allocative efficiency of consumer attention. Also, we discuss the impact of privacy regulation that reduces the extent of tracking.

We have focused thus far on comparing our model to the standard setup for analyses of media markets. While most models in the media economics literature assume

[7] A countervailing effect outside our model is that with more data about consumers, outlets can sell more targeted advertising. See Athey and Gans (2010) for an analysis of the impact of targeting technology on ad prices. See also Bergmann and Bonatti (2011) for an analysis of the interaction between online and offline media competition and targeted advertising.

that consumers single-home – that is, choose to allocate attention to only one outlet – there are a few recent papers that have considered what happens when consumers multi-home, including Ambrus, Calvano and Reisinger (2011) as well as independent contributions from Anderson, Foros and Kind (2011), Anderson, Foros, Kind and Peitz (2011), and George and Hogendorn (2011) among others. There are a few important distinctions between our model and the ones studied in the literature. First, our model explicitly models the consumer switching process in an environment with a fixed amount of consumer attention, allowing us to perform comparative statics with respect to the extent of switching. The alternative, where multi-homing consumers consume *more* media, are not as well suited for understanding trends in market prices for advertising, since they implicitly assume that total consumer attention and, thus, potential ad capacity increases with switching. Second, our model introduces a new force, which is the (potential) inefficiency created by consumer switching in the absence of perfect tracking technologies. We study the implications of the inefficiency for the advertising market equilibrium, and, in particular, for advertiser strategies and willingness to pay for ad space. In contrast, the existing literature focuses on the outlets' choice of advertising space and the tradeoff between the revenue gained from additional advertising space and consumer disutility for ads. Consumer switching increases competition between outlets and, thus, increases equilibrium ad space. Our paper treats the ad space as exogenous for much of the analysis, endogenizing it as an extension to the model, in order to highlight more clearly the novel forces introduced in our model. We thus view our model as complementary to prior literature. Moreover, by modeling explicitly the allocation process of scarce attention, we are able to identify and characterize additional outlet reactions as well as discuss the impact of different government policies towards the news media.[8]

We share the finding that larger outlets command a premium with the work of Crampes, Haritchabalet and Jullien (2009) and Anderson, Foros and Kind (2011). The former argue that by exploiting information from a large customer base, larger outlets

[8] This method of dealing with two-sided markets is itself novel. Rather than the outlet (or platform) choosing prices in a monopolistic or oligopolistic fashion (e.g.., see the general result of Weyl, 2010), on the advertising side, revenues to outlets are determined by market clearing prices. Thus, we can analyze how technology and other factors impact on the efficiency of advertising market outcomes and, in turn, how this impacts on outlet revenues.

have superior possibilities in targeting leading to increasing returns. In Anderson, Foros and Kind (2011), having relatively more exclusive viewers allows outlets to charge higher prices. In contrast, we show that a "positional advantage" can obtain regardless of the composition of one's viewership and absent returns to scale.[9] In accord with conventional wisdom among practitioners, large outlets command a premium because they reach relatively more viewers while minimizing duplication.[10]

Our paper also relates to a number of price-theoretic papers on multi-sided markets that explore the equilibrium implications of having one (or more) sides multi-homing.[11] A common theme in these works is the idea that increased multi-homing on one side of the market, in equilibrium, reduces (and, in the limit, annihilates) the incentives to multi-home of those on the opposite side. These latter become "competitive bottlenecks" leading to a number of important positive and normative implications. This paper shows a natural instance in which increased multi-homing on one side could lead to increased multi-homing on the opposite side. Contrary to the above papers, multi-homing readers are relatively harder to impress. This highlights a fundamental matching problem at the heart of advertising markets that the Internet potentially has disrupted but also, in the future, could resolve.

2 Model Set-up

2.1 Consumer Attention and advertising inventory

There is a continuum (unit mass) of consumers and two media outlets. Each consumer visits one outlet per period over a horizon of two periods. In each period spent visiting outlet i, a consumer is exposed to a_i ads. We refer to a_i as the (period) ad capacity of outlet i. Capacity is assumed exogenous (but, it is endogenized in an extension).

How do consumers allocate attention to different media outlets? We assume that whenever a consumer has an opportunity to choose, outlet i is chosen with probability x_i.

---

[9] See Goettler (2012) for recent empirical verification of such advantages.

[10] A related recent research strand that asks how the internet is changing consumers' choices through its impact on their choice set (e.g. Gentzkow (2009), Gentzkow and Shapiro (2011). Building on this, we ask we ask how this affects the advertisers' choices and the associated revenues.

[11] A (partial) list includes Caillaud and Jullien (2003), Anderson and Coate (2005), Armstrong (2006), Gabszewicz and Wauthy (2004), Anderson, Foros and Kind (2011) and Reisinger (2012)).

Thus, x_i is a measure of an outlet's relative quality.[12] Between attention periods, an opportunity for a consumer to switch outlets arrives (independently) with probability ρ.[13] Thus, the total expected amount of attention going to i is $x_i + x_i\left((1-\rho)+\rho x_i\right)+(1-x_i)\rho x_i = 2x_i$. We let $D_i^l = x_i - x_i(1-x_i)\rho$ denote the share of consumers who end up using the same outlet in each period (single-homers), that is, are (ex post) *loyal* to i and $D_{ij}^s = 2\rho x_i x_j$ denote the share of *switchers* or multi-homers. Note that when $\rho = 0$, $D_i^l = x_i$ and $D^s = 0$. A consumer loyal to an outlet i generates $2a_i$ in advertising inventory while a consumer switching between outlets generates $a_i + a_j$ in advertising inventory. This characterizes the *supply-side* of advertising markets. For future reference, note that the model can accommodate $n > 2$ outlets without difficulty. In that case, if outlets have asymmetric capacity, different consumer "switching types" generate different advertising inventories. Whenever outlets are assumed symmetric in readership (that is $x_i = x_j = \frac{1}{2}$) then the subscript is dropped: $D_i^l = D_j^l := D^l$.

2.2 Advertisers' preferences

There is a unit mass of advertisers. Advertisers want each consumer to see their ad sometime over the two periods but are indifferent about precisely when. They differ as to the value of putting an ad in front of a consumer. This value is denoted v and is distributed on $[0,V]$ according to a cumulative distribution F.[14] The value does not increase if the same consumer sees more than one ad from a given advertiser. Each time an ad is put in front of consumer it is referred to as one "impression." From the perspective of an advertiser, an "impressed" consumer is a consumer who has been exposed to *at least* one ad.

[12] Everything else held constant, a higher quality of outlet i increases x_i and decreases weakly x_j for all other outlets. In our baseline model these choice probabilities are exogenous, but later on we endogenize the quality.

[13] Here we treat this probability as independent of history (i.e., outlets a consumer may have visited earlier) or the future (i.e., outlets that they may visit later). We explore the implications of relaxing this assumption after characterizing the equilibrium of the baseline case.

[14] For the given advertiser, v is the same for all consumers and independent of the number of distinct consumers receiving an impression. An alternative specification might have advertisers aiming to reach a specific number of consumers (Athey and Gans, 2010) or a specific consumer type (Athey and Gans, 2010; Bergemann and Bonatti, 2010).

2.3 Tracking technology

Here, as attention is a primitive of the model, we need to specify how that attention is matched to ads – something that is usually not explicitly considered in the economics of advertising literature. If n_1 ads are allocated to a given advertiser on outlet 1 and n_2 on outlet 2 over both attention periods, then its payoff is equal to v times the expected number of unique consumers that (n_1, n_2) allows impressing. So, in order to close the model, one needs specify a function φ that maps (n_1, n_2) pairs to a real number in [0,1] that represents the fraction of the population that the advertiser expects to be impressed. That is the "reach" of the campaign. Note that the function implicitly captures the extent to which the outlets are able to control how single ads are matched to individual consumers; that is, their "tracking" capabilities. For example, if a public record of previous ad/consumer matches were available, it would in principle be possible to match each successive ad from a given advertiser to a different consumer so no consumer would be matched to the same ad twice. Then $\varphi = n_1 + n_2$ whenever $n_1 + n_2 < 1$. This is what we refer to as "perfect tracking." Under perfect tracking, individual ads are most valuable to advertisers since each ad is put in front of an unimpressed consumer with probability one. At the other end of the spectrum, suppose there is no control whatsoever over the matching process. Then it would be *as if* each successive ad from a given advertiser were put in front of a consumer chosen at random (*possibly* an already impressed one). This is what we refer to as "no tracking."[15]

In reality, outlets have some, but typically not full, scope to track consumers internally (the same consumer may arrive from different browsers and devices) but have little (or no) scope to track which ads their customers have been matched to on *other* outlets. Accordingly, in what follows we specialize to one of many formulations that displays two desiderata:

a) Tracking is internal: outlets cannot track consumers – and specifically, the ads they see – across outlets.
b) Internal tracking is not perfect.

[15] The (so called) "Butter's technology," firstly employed in Butter (1977) and then extensively in previous works approximates precisely the outcome under no tracking with φ taking an exponential form.

A simple, stylized way to capture a) and b) within our two-period model is to posit that the outlets can perfectly track consumers within each period but cannot track consumers across periods or across outlets. The easiest context to understand this is to imagine that each outlet has two units of content (e.g., web pages or articles) and consumers do not read the same content twice. Loyals get both units of content from the same outlet (one per period), while switchers get in each period a random piece of content from each.

From now on, as tracking is assumed perfect within periods, we adopt the convention that the arguments of φ, referred to as the number of "ads," denote the number of impressions per period-consumer. In every period, each outlet gets a number of unique consumers equal to x_i. So increasing n_i by one unit implies purchasing x_i additional impressions. If advertisers are restricted to an integer number of ads per period-consumer, (or per "unit of content" under the above interpretation) and the outlets are symmetric in readership, then the matching technology is fully described by the following five cases:

(i) (Single-home) $\varphi(1,0)=\varphi(0,1)=D^l+\frac{1}{2}D^s$;

(ii) (Intense single-home) $\varphi(2,0)=\varphi(0,2)=D^l+D^s$;

(iii) (Multi-home) $\varphi(1,1)=\varphi(1,1)=2D^l+\frac{3}{4}D^s$;

(iv) (Targeted multi-home) $\varphi(2,1)=\varphi(1,2)=2D^l+D^s$;

(v) (Intense multi-home) $\varphi(2,2)=\varphi(2,2)=2D^l+D^s$.

With just a single ad from a given advertiser (case (i)), the outlet matches the advertiser's ads to all its different period-consumers (or to all the readers of a given piece of content, under the above interpretation). Thus, the expected reach associated to "one ad on *i*" is all the loyals of *i* plus half of the switchers. By assumption there is no duplication as no consumer is matched to a given ad twice. Of course, if an advertiser allows a second ad in a different period (case (ii)) then all loyal consumers will be matched twice to the same ad. This is because there is no tracking across periods. The benefit of doing so, from the advertisers' perspective, is to exploit internal tracking in order to reach more switchers. On the contrary, as there is no across outlet tracking, buying a second impression on the other outlet (case (iii)) allows the advertiser to reach all of the second outlet's loyals at the cost of duplicating impressions on (some) of the switchers (those consumers who already saw the ad on the first outlet). As all consumers are assumed to consume one outlet in each period, strategy (iv) whereby the advertiser places two impressions on one outlet and one on the other (and a fortiori (v)) allows full reach.

Of course, there are many other micro-foundations that would lead to a φ function that satisfies the above desiderata; each drawing from some aspects of the constraints faced by outlets in reality.[16] We present and discuss notable alternatives in the online appendix. We shall remark here that our findings will hinge on a) and b) as opposed to the actual functional form chosen in this specific illustration. In addition the above formulation satisfies a third, simplifying, desideratum:

c) Duplication due to imperfect tracking occurs if only if there is consumer switching.[17]

Indeed if $D^s = 0$ then no duplication and full reach would occur when advertisers multi-home. So this as an additional assumption (which is an implication of our specified functional form, φ) analytically allows us to isolate the impact of consumer switching in what follows.

2.4 Market equilibrium and outlets' profits

To close the model, we assume that the outlets' inventory is allocated through a basic price mechanism that equates the individual outlets' demand and supply of ads. We adopt the convention that p_i is the price of a single impression. So, given a pair of prices, the expenditure required to implement a choice (n_1, n_2) is $p_1 n_1 x_1 + p_2 n_2 x_2$. Outlet i's profit denoted π_i is assumed equal to price times quantity of impressions: $p_i(2ax_i)$.

---

[16] One might wonder whether a pay-per-click model of advertising would alleviate the inefficiencies created by switching. The answer is no: whatever the payment model, displaying one advertisement necessarily displaces another. For this reason, most pay-per-click advertising networks charge advertisers a price per click that is inversely proportional to the click-through rate of the ad. Thus, the overall payment of the advertiser is "per impression"—an ad that is not clicked on often (perhaps because it is wasted, if the advertiser multi-homes) has to pay a proportionally higher price per click to justify displacing another advertiser.

[17] We can make an "if and only if" statement here because $D^s < 1$ even when $\rho = 1$. If, however, D^s can equal 1, there is also not duplication if advertisers single-home.

A *market equilibrium* is a tuple $\left\{(n_1^*(v,\hat{p}_1,\hat{p}_2),n_2^*(v,\hat{p}_1,\hat{p}_2))_{\forall v\in[0,V]},(\hat{p}_1,\hat{p}_2)\right\}$, where:

(i), $(n_1^*(v,\hat{p}_1,\hat{p}_2),n_2^*(v,\hat{p}_1,\hat{p}_2))=\arg\max_{n_1,n_2}\varphi(n_1,n_2)v-\hat{p}_1n_1x_1-\hat{p}_2n_2x_2$ for all v and

(ii) for each outlet, i, $\hat{p}_i$ is such that $x_i\int_0^V n_i^*(v,\hat{p}_1,\hat{p}_2)dF(v)=2x_ia_i$.

The first condition says that advertisers optimize their impression choices taking prices as given while the second condition says that the market for each outlet clears. To build intuition, we focus, initially, on the simplest case with symmetric outlets; i.e., $x_i=x_j=\frac{1}{2}$, and $a_i=a_j=:a$. In this case, it can be readily seen that $\hat{p}_1=\hat{p}_2=\hat{p}$.

2.5 Benchmark: perfect tracking

The first-best allocation of consumers' attention to advertisers is such that the highest value advertisers are allocated with priority to scarce advertising inventory and there is no duplication. Let v_i denote the marginal advertiser allocated to consumers loyal to outlet i and let v_s denote the marginal advertiser allocated to consumers who switch between outlets j and i. An efficient allocation of advertisers to consumers involves allocating all advertisers with $v\geq v_i$ to outlet i's loyal consumers and those with $v\geq v_s$ to those who switch between i and j. Thus, the marginal advertisers are defined as the unique solution to: $2a=1-F(v_i)$ and $a_i+a_j=1-F(v_s)$.

To see how this first best might be implemented in practice, consider a scenario where there exists a public record that keeps track of all consumer/ad matches. More realistically, suppose that both outlets outsource their advertising to a third party, labeled "ad-platform." The platform acquires the outlets' entire advertising inventory and can keep track, say by planting "cookies" on the consumers' web-browsers, of all previous consumer/ad matches. [18] Moreover suppose that the platform can *price discriminate* based on consumer-type (loyal or switcher).

[18] An alternative (but probably less realistic) assumption would be that the ad platform shares information with the outlet about the consumer type, so that the outlet can set different capacities for different types. This additional flexibility would lead to a scenario with essentially distinct markets, so that firms compete for switchers and but have a monopoly over access to loyal users. It is a bit more complicated to think how this would work in practice, since consumer types would only be fully determined in the second period,

In this scenario there would still be two markets: one market for impressions on loyals and another one for impressions on switchers. The prices that equate demand and supply are equal to $\hat{p}_i = v_i$ and $\hat{p}_s = v_s$ respectively. This as advertisers will choose to advertise so long as their value exceeds the impression price. Note that if $a_i = a_j$, then $\hat{p}_i = \hat{p}_j = \hat{p}_{ij}$ while if $a_i > a_j$, then $\hat{p}_i < \hat{p}_{ij} < \hat{p}_j$. In equilibrium the outlets' inventory is worth $\pi_i = \hat{p}_i 2a_i D_i^l + \hat{p}_s a_i D^s$. When $a_i = a_j = a$ and $x_i = x_j = \frac{1}{2}$ then $\pi = v_i 2a(D^l + \frac{1}{2}D^s)$. Note in this case that profits are independent of the mix of loyal and switching customers.

3 Equilibrium analysis

3.1 Switchers and the Demand for ads

Given consumers' allocation of attention and ad prices, we now proceed to characterize the advertisers' demand as a function of their idiosyncratic valuation. That is, we associate to each type v the pair (n_1, n_2) with $n_i \in \{0,1,2\}$ which solves:

$$\max_{n_1,n_2} \varphi(n_1,n_2)v - p_1 n_1 x_1 - p_2 n_2 x_2.$$

To better understand what drives the advertisers' choices it is useful to decompose the advertisers' program in two sub-programs. First how many ads shall each advertiser buy overall? Second, given $n_1 + n_2$, how do ads on different outlets substitute for one another? In other words what is and, most importantly, what drives the optimal *allocation* of these ads across outlets?

A key observation is that the combination of switching and imperfect tracking is a *source* of diminishing returns from purchasing additional impressions. That is, $\Delta\varphi$ (evaluated along the optimal allocation) decreases with $n_1 + n_2$ if and only if $0 < D^s < 1$. For example, with equal prices and symmetric outlets, the first ad is worth $\frac{1}{2}v$ in revenues at a cost of $\frac{1}{2}p$. A second ad, for instance allocated to a different outlet, costs

after the consumer had already experienced a first-period ad capacity. We omit the formal analysis of this case.

just the same. However, it is only worth $(D^l + \frac{1}{4}D^s)v < \frac{1}{2}v$ in additional revenues due to duplicated impressions. Diminishing returns immediately implies that advertisers sort in equilibrium with relatively higher types buying more ads overall. Let v_k denote the advertiser type indifferent between $n_1 + n_2 = k$ and $k-1$ ads. The above diagram illustrates the sorting that arises in this model. It distinguishes three cases depending on the level of D^s. The above thresholds as well as their analytical expression are marked below each line. Above the regions delimited by the threshold we report the optimal advertisers' strategy (or strategies in case of indifference).

Figure 1 highlights two important features of the solution to the advertisers' problem. First, and foremost, sorting takes place only if there are some switchers. Second, the threshold on D^s stresses the tension that the advertisers' face when deciding how to allocate a second, additional ad; i.e., the tension between diversifying (by spreading ads on different outlets) or concentrating all ads on a single outlet. When diversifying, the advertisers trade off increased reach on switchers, due to internal tracking, with increased reach on loyal consumers. In our formulation, diversifying pays off if and only if $D^s < \tilde{D}^s := \frac{2}{3}$. The threshold makes type v_2 simultaneously indifferent between purchasing an additional ad on the same outlet and on the other outlet (or doing without the additional ad altogether). It, therefore, equates two sources of "waste/duplication." That is, within outlet waste (due to imperfect internal tracking) whose manifestation here is duplication on loyals and across outlet waste (due to no tracking across outlets), which causes duplication on switchers.[19]

[19] In Athey, Calvano and Gans (2013) we show that this tension and in particular this threshold property holds for very general tracking technologies satisfying a) and b) above.

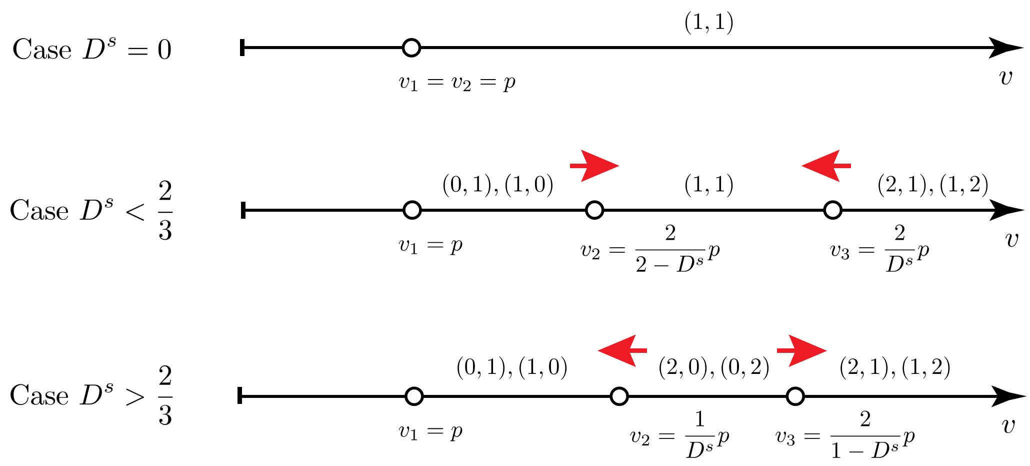


Figure 1

The aggregate demand integrates the individual demands across advertisers' types. In case of indifference between alternate strategies we assume that the advertisers split equally.

$$x_i \int_0^V n_i^*(v, p, p) dF(v) = \begin{cases} (D^l + \frac{1}{2} D^s)\left(\frac{1}{2}(F(v_2) - F(v_1)) + 1 - F(v_2) + \frac{1}{2}(1 - F(v_3))\right) & v_3 \le V \\ (D^l + \frac{1}{2} D^s)\left(\frac{1}{2}(F(v_2) - F(v_1)) + 1 - F(v_2)\right) & \text{if } v_2 \le V \le v_3 \\ (D^l + \frac{1}{2} D^s)\frac{1}{2}(1 - F(v_1)) & v_1 \le V \le v_2 \end{cases}$$

In what follows, we will refer to "high," "medium" and "low" types, all types that belong to $[v_3, \infty), [v_2, v_3)$ and $[v_1, v_2)$ respectively.

3.2 The impact of switching on aggregate demand

The arrows in Figure 1 illustrate the effect of a marginal increase in D^s on the aggregate demand of each outlet. Once more we shall distinguish between two cases depending whether switching exceeds the threshold or not. The intuition is as follows. Suppose $D^s < \tilde{D}^s$. As switching increases, the marginal medium types scale back on advertising and become low types due to the increased duplication on switchers. On the contrary the set of high types (who have a higher opportunity cost from missing out consumers) expands as incremental returns go up for them. The overall effect is ambiguous. As we will discuss after characterizing the equilibrium this is somewhat counterintuitive, as switching always *degrades* the value of the inventory in the sense that it lowers the expected value of an additional impression for all types. Note that the exact opposite intuition holds for the case $D^s > \tilde{D}^s$. In any case, as the two arrows always point in opposite directions, the overall effect is once more ambiguous.

The following proposition goes one step further by observing that when switching is sufficiently low, it must be that the aggregate demand decreases as switching increases. As D^s approaches zero, eventually nobody would purchase multiple impressions on one outlet ($v_3 \to \infty$). However $v_2 < v_1$ for all positive values of D^s. Then, increased switching in this range has a first order negative impact on the demand of the medium types while it has no impact on the demand of the high types. Thus, outlet demand necessarily falls; that is, for any given price, *p*, fewer impressions are purchased.

Proposition 1. *Outlet (and aggregate) demand is decreasing with D^s around $D^s = 0$.*

3.3 Equilibrium

To solve for the market equilibrium, the outlets' respective aggregate demands have to equal supply. For an outlet, the total supply of advertising inventory is $x_i 2a$. With fixed supply, the equilibrium properties are basically inherited from the properties of the aggregate demand for ads. In particular, a lower aggregate demand implies lower prices and profits. The following result directly follows from Proposition 1.

Proposition 2. *Equilibrium prices and profits are decreasing in D^s around $D^s = 0$.*

As discussed, it is also the case that outside of this range a greater number of switchers could reverse the result. Recall that in general an increase in $D^s < \tilde{D}^s$ increases v_2 while reduces v_3. Depending upon the rate of change of these thresholds and, specifically on the rate of change of the probability *measure* of the sets defined by them profits may go up or down.[20]

What can be said about the equilibrium allocation? As stressed, duplication can cause demand and, therefore, prices to drop below the prices that would arise without switching. This means that some low value advertisers who would not have access to

[20] By construction, if $D^s = 0$ then the competitive equilibrium yields the first best / perfect tracking outcome and no duplication occurs. Furthermore consumer attention is allocated in the way that maximizes total advertising surplus.

consumer attention in a hypothetical first best, now would. So two sorts of "mismatches" can occur in equilibrium. While consumer surplus is not explicitly modeled here, the combination of inefficient depletion (due to duplication) and inefficient use of attention (due to ad/consumer mismatches) would be necessarily suboptimal, had we spelled out a richer model on the consumer side.

The above observation raises the question of whether more switching is always detrimental. (Say, from a total surplus maximizing perspective). A key insight is that so long as $D^s < \tilde{D}^s$, *all* advertisers' are worse off *regardless* of the direction of change of prices and profits. This is well understood for marginal medium types, who scale back on (less valuable) impressions as D^s increases. The intuition is subtler for marginal high types who, on the contrary, *increase* their advertising effort with switching. For this group, while the gross advertising surplus falls with switching[21] (much as it does for all other types), the *incremental* value of buying a third impression *increases*. In other words, while these high value advertisers are less enthusiastic about advertising, at the same time they feel more compelled to advertise *more* as the option of doing without looses relatively more appeal. This is due to the property a) of the tracking technology.

On the contrary beyond $\tilde{D}^s$, the intuition is, somewhat surprisingly, reversed. Switching is *beneficial* as it allows increasing reach by exploiting internal tracking. So the advertising surplus generated in the economy goes *up*. That is due to property b) of the tracking technology. To see this via an extreme example, note that if $D^s = 1$ then one can easily verify that the market equilibrium once more yields the perfect tracking outcome. That is, all advertisers purchase 2 ads on one outlet only (it does not matter which one), no duplication occurs and consumer attention in allocated in a way that maximizes the total advertising surplus.

More generally (that is abstracting away from our tracking technology) the combination of $D^s = 1$ and *perfect* internal tracking always induces a first-best equilibrium allocation *despite* the absence of across outlet tracking. For this reason, in the rest of the paper we focus attention on the, arguably most relevant, case in which the gross advertising surplus falls switching for all strategies. That is $D^s < \tilde{D}^s$.

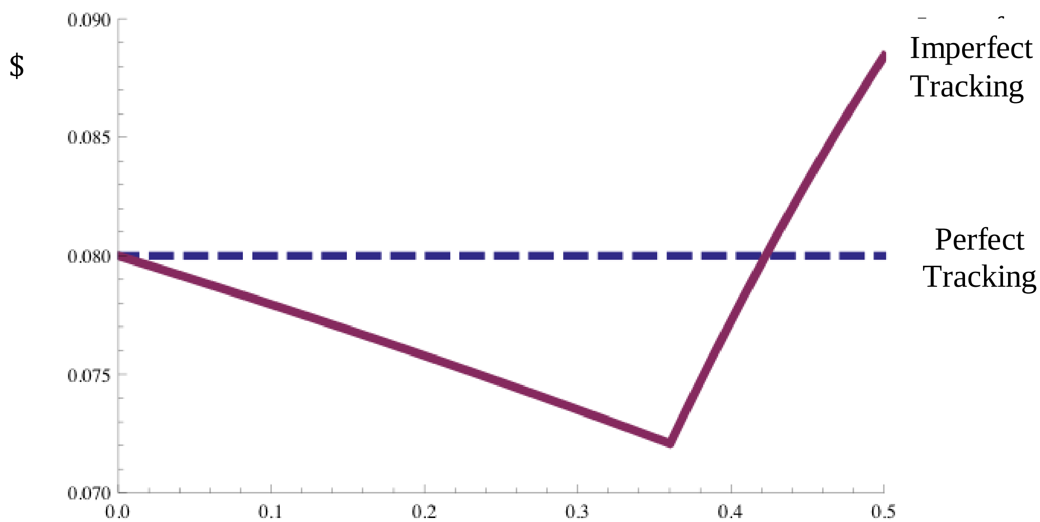
[21] That is the payoff associated with any of the strategies available to the advertisers weakly decreases.

3.4 An illustration with uniformly distributed valuations

In this sub-section, we further develop the model by choosing a distribution that allows for explicit solutions. As we shall see, this allows answering a broader set of questions while developing intuition at the same time.

A first issue to consider is the possibility that switching *increases* profits and, most importantly, whether it could do so beyond the level that would be obtained absent switching. The conjecture is that the higher demand from the high types may eventually more than compensate the decreased demand from the medium types. We now proceed proving this conjecture though an example. If advertisers' valuations are uniformly distributed then it is possible to derive a closed form expression for the equilibrium prices and profits (algebra in appendix). Figure 2 (a) depicts individual outlets' profits when $a_1 = a_2 = \frac{2}{5}$, $D^s = \frac{1}{2} < \hat{D}^s$ and v is uniformly distributed on the unit line.

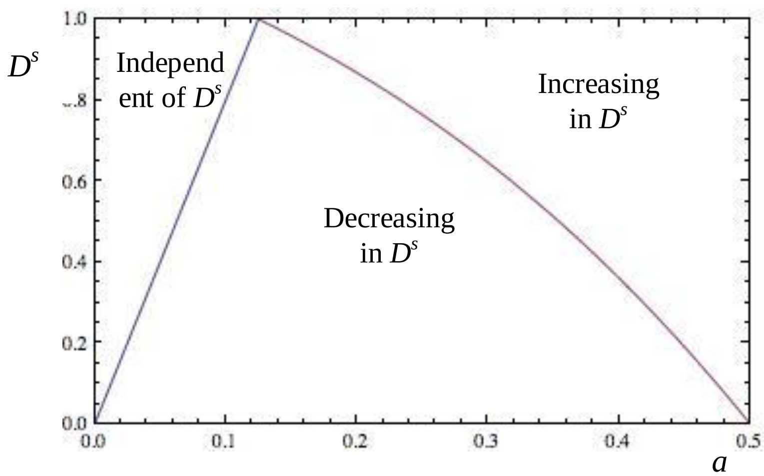
Figure 2 (a): Outlet Profits as a function of D^s ($a = 0.4$)



The decreasing portion of the curve corresponds to the case in which all of the aggregate demand variation comes from medium types ($v_2 < V < v_3$). The increasing portion corresponds to the case in which high types become active $v_2 < v_3 < V$. It easy to verify that that for all distributions high types are more sensitive than lower types to marginal changes in D^s. The intuition is that these latter target precisely switchers. This fact together with the uniform distribution assumption is exploited here to show that profits can, in general, increase. In particular, this parameterization shows that they can increase *beyond* what the outlets would get in the perfect tracking benchmark. This result, as we shall see, has important implications for the outlets' incentives to invest and/or coalize to allow for "better" tracking of their common consumers.

Finally, this exercise sheds light on the subtle interaction between switching and ad capacity. Figure 2 (b) summarizes how each outlet's profit changes with switching for all pairs (a, D^s).

Figure 2 (b): Direction of Change in Outlet Profits with D^s



Recall that the equilibrium price and, therefore, aggregate demand is monotone decreasing in capacity. In the region on the left hand side of the diagram, supply is so scarce that, at the equilibrium price, no advertiser multi-homes. So switching has no impact. In contrast, the central region corresponds to the case in which the equilibrium price is such that $v_1 < v_2 < V < v_3$. So all variation in aggregate demand due to switching comes from medium types who scale back (becoming low types) due to duplication. Therefore, prices and profits decrease with D^s. Finally, the top right region corresponds to the case in which $v_1 < v_2 < v_3 < V$. Here, as discussed, the variation in the aggregate demand due to switching comes from two sources. The fact that the increased demand due to a decrease in v_3 more than compensates the decreased demand due to medium types scaling back is a particular feature of the uniform distribution assumption. As discussed, in general, the effect is ambiguous. In addition, as shown in the Online Appendix, if ad capacities are endogenized in a particular way and under the assumption of uniform distribution, capacities will never be chosen in the region where profits increase with switching. However, a full analysis of endogenous ad capacities would incorporate other forces affecting optimal capacities (such as competition for users) and is thus left for future work.

4 Asymmetric outlets and positional advantage

As mentioned in the introduction, one puzzle associated with the economic theory on media advertising is differing advertising rates across outlets. Our baseline model with symmetric outlets involved an equilibrium outcome whereby advertising rates were the same across outlets. Here we explore asymmetric outlets and, in particular, what types of asymmetries may account for differing advertising rates: that is, when might outlets have a positional advantage in advertising markets?

4.1 Asymmetric readership shares

Here we consider what happens when a relatively higher quality allows one outlet to generate a higher readership share than the other. That is, we assume that $x_1 > x_2$ (which implies $D_1^l > D_2^l$) but the outlets are otherwise symmetric.

To build intuition, we work here through an indirect argument. Specifically, we suppose that the market clearing unit prices are equal across outlets and conclude that these prices cannot be part of a market equilibrium. For this purpose, consider the advertisers' demand. Recall that, in the baseline case, relatively low-valued advertisers were indifferent between single-homing on either outlet. Figure 3 illustrates the sorting

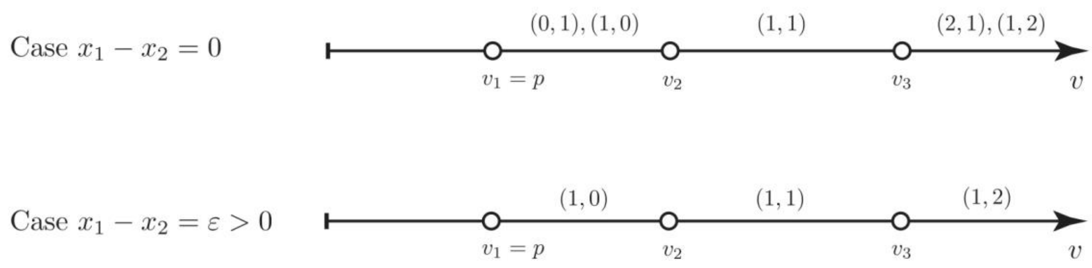


Figure 3

that would arise instead with equal prices, assuming that one outlet is marginally more attractive than the other (with epsilon denoting an arbitrarily small number) and $D^s < \tilde{D}^s$. Observe that when outlet 1 is larger, advertisers will sort on to that outlet first. The reason is that revenues from single-homing on outlet one exceed that from single-homing on outlet 2 for all active advertisers: $(D_1^l + \frac{1}{2}D^s)(v-p) > (D_2^l + \frac{1}{2}D^s)(v-p)$. Everything else held constant, this creates upward pressure on the relative price of outlet 1's ads to rebalance supply and demand on both markets. On the other hand, relatively high valued advertisers purchase a second ad on outlet 2 first since this is the most cost-efficient way to increase reach among switchers. This creates downward pressure on relative prices. Again, the overall effect of a marginal increase of x_1 beyond $\frac{1}{2}$ is ambiguous and depends on the local properties of the distribution function.

Once more, if most of the variation in demand comes from medium types, then a (small) competitive edge in quality gives a (big) positional advantage to one outlet in the sense that its impressions command a price premium. In the special context of the uniform distribution, that requires a and D^s to be "sufficiently" small (proof in the appendix).

Proposition 3. *Let $F(v) \sim U[0,1]$, $a_1 = a_2 = a$ and $x_1 > x_2$. Then, $\pi_1 > \pi_2$ if and only if $\frac{1}{2} - \frac{D^s}{16}\left(\frac{1}{1-x_1} + \frac{4}{4(1-x_1)-D^s}\right) > a$; that is, D^s and a are sufficiently small.*

The fact that 'larger' outlets, in terms of readership share, command a premium for their ad space is a known puzzle in traditional media economics. The reason being that in a canonical model consumers are equally valuable regardless of the outlet they are on, yet in practice advertising rates are typically higher on larger outlets. Here, because ads are tracked more effectively internally, placing ads on the larger outlet only involves less expected waste than when you place ads on the smaller outlet or spread them across outlets. So, the larger outlet can command a (tracking-related) premium and this fact can, arguably, contribute to account for the observed wedge in impression prices.

4.2 Asymmetric ad capacities

What if instead capacities differed? Suppose $\frac{a_1}{a_2} < 1$ and that the outlets are otherwise symmetric. A first observation is that whenever a market equilibrium exists then $\frac{\hat{p}_1}{\hat{p}_2} > 1$ [22]. Figure 4 illustrates this point showing the sorting that arises when advertisers face unequal prices. While symmetry (on the consumers' share) preserves the incremental reach associated to buying an additional ad, the lowest priced outlet always features an excess demand relative to its cheaper counterpart. But if this is the case then it must be that, in equilibrium, relative prices reflect relative scarcity in the usual fashion. That is $\hat{p}_1 > \hat{p}_2$ if and only if $a_1 < a_2$. The reason, of course, is that lower priced outlets are more attractive to low types, as they offer a better bargain. This is also the case for high types, whose objective is to increase penetration among the switchers. Intermediate types instead, as in the baseline case, demand one ad each.

More interestingly, a robust intuition that emerges from the diagram is that in any asymmetric equilibrium of this sort, the two outlets serve two very *different* products

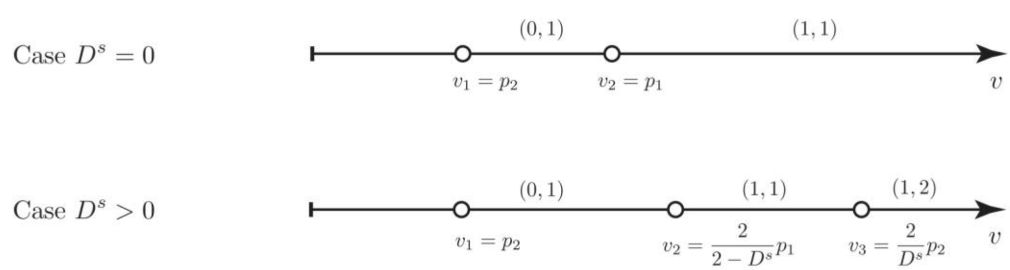


Figure 4: $a_1 < a_2$

[22] The issue of existence is sidestepped here to focus on the properties of the market allocation. In the online appendix we provide a full characterization for the case of uniformly distributed valuations.

whenever $D^s > 0$. The "smaller" outlet, due to the endogenous higher price, serves *additional* ads in equilibrium; i.e., the second impression for high valued multi-homing adverisers. This segmentation, in part, insulates the outlets from the externalities that their capacity choices, if endogenized, would exert on their respective market price. To see this point, consider how the unilateral incentives to exert market power of outlet 1, given a_2 are affected by D^s. Clearly, by reducing the efficiency of the matching process and therefore the value of the inventory being sold, switching acts as a vertical demand shifter. Everything else held constant this leads the outlet to expand capacity. The market clearing price of a given capacity is lower. A subtler effect is that it also makes the demand schedule *flatter*. That is, the extent to which a decrease in quantities increases prices, for given demand shift is smaller. In other words more capacity has to be sacrificed to achieve a given per impression price (or vice-versa).

Also note that locally, in the asymmetric equilibrium, there are no strategic externalities in the sense that a marginal change in outlet's 1 capacity has no impact on outlet's two revenues. This is the sense in which advertisers' sorting insulates the two firms. This is perhaps contrary to expectations induced by related work (e.g., Anderson, Foros and Kind, 2011), that switching would have been a source of externalities, the higher switching the higher the extent of competitive pressure when it comes to simultaneously choose the quantity of ads. This would have arisen here had the markets for loyals and switching consumers been identified. But imperfect tracking prevents that and so prevents impressions on switchers giving rise to competitive pressure. In an online appendix, we discuss endogenous capacity in more detail.

5 Strategic Implications

5.1 Incentives to compete for readers

In this section, we present an extension that endogenizes the quality of the outlets. Specifically, we develop a sequential move game where initially the outlets may exert effort to produce higher quality content. In turn, consumers choose which outlet to patronize. Effort is rewarded through a higher expected probability of being patronized whenever consumers face a choice. That is, a higher expected market share.

That consumer switching can increase the equilibrium level of effort is quite intuitive when the primitives are such that impression prices are *higher* relative to the case where $D^s = 0$, as discussed. In what follows we unveil an additional, subtler, mechanism that allows the equilibrium effort to rise with switching even when switching is *detrimental* to profits. That is, when switching depresses the outlets' market clearing prices. The key to understand the result is that switching engenders a *rent*. The rent is associated to the positional advantage created by asymmetries in market shares as demonstrated in the previous section. As we will see, this rent comes hand in hand with competition for it, which in turn engenders effort.

To fix ideas, we present here a very simple extension that yields the outcome described above. Suppose that prior to consumers and advertisers making any choices, outlets can invest an amount e_i at cost $c(e_i) = \frac{1}{2}e_i^2$ which generates a probability $e_i \in (0,1)$ of being a high rather than a low quality outlet. The probabilities are independent across outlets. Therefore, if outlets choose (e_1, e_2) then with probability $e_1(1-e_2)$ only outlet 1 has high quality while with probability $e_2(1-e_1)$ the reverse is true. Finally with probability $e_1e_2 + (1-e_1)(1-e_2)$ both outlets have the same quality. To isolate the impact of the positional advantage on the incentives to invest, we assume that in those states of nature where one outlet has relatively higher quality, it earns a positional advantage by being marginally more attractive than the other. That is $x_i - x_j = \varepsilon$ where epsilon denotes, once more, an arbitrarily positive small number. The outlets choose their effort levels simultaneously. If a quality gap realizes then the higher quality outlet earns $\pi^H(D^s)$ while the low quality outlet earns $\pi^L(D^s)$ otherwise they both earn $\pi(D^s)$ with $\pi^L < \pi < \pi^H$ (arguments omitted from now on) for all $D^s > 0$. If $D^s = 0$ then there cannot be any positional advantage and $\pi^H = \pi^L = \pi(0)$.

The unique Nash equilibrium level of effort is (algebra in the appendix):

$$e_1 = e_2 = \frac{\pi^H - \pi}{1 + \pi^H + \pi^L - 2\pi}. \tag{1}$$

Note that, by construction, effort is positive only if $D^s = 0$. The reason being that the only role of effort in this model is that of increasing the chances of appropriating the positional advantage rent. Crucially, equilibrium effort is positive for all levels of switching. Therefore, the incentives to invest in quality are enhanced regardless on how switching impacts the market clearing prices. Finally, as $\pi(0)$ is also the level of profits that would obtain under perfect tracking, it follows that according to this particular model equilibrium effort (here interpreted as quality) is, somewhat surprisingly, higher with imperfect tracking. Perfect tracking will induce lower investments.

Note that, in a more general formulation, for instance, one that allows effort to affect the market shares, then the opposite result could obtain. Nonetheless the mechanism identified here would interact with the effect of switching on equilibrium prices to amplify it or mitigate it. The following proposition takes stock (proof omitted).

Proposition 4. *The equilibrium level of effort in enhancing quality is higher under imperfect tracking than under perfect tracking.*

5.2 Limited content for reach

The analysis thus far has assumed that outlets have sufficient content to attract attention of loyal consumers throughout the relevant attention period. Of course, on the Internet, much content is provided on a smaller scale. For providers of that content, there is no possibility of attracting loyal consumers. However, here we demonstrate how such providers may still achieve a positional advantage in advertising markets; that is, what they lose in their inability to attract frequent visits from consumers, they can make up in terms of their *reach* across all consumers – acting as a "magnet" for attention in the relevant advertising period.

To see this, we amend the model as follows. Assume outlet 2 only has enough content to satisfy consumers for a single period. To assist in identifying it expositionally, we rename it outlet *f*. Outlet 1 is unchanged. To focus on the impact of limited content, we will confine ourselves here to the case where $\rho = 1$. In this situation, the total expected traffic (over both periods) to outlet 1 is $x_1(1+x_1)+x_f$ and to outlet *f* is $x_f(1+x_1)$. Thus, $D_1^l = x_1(1-x_f)$ while $D_f^l = 0$ and outlet *f* only has consumers who are switchers, $D^s = x_f(1+x_1)$. Thus, while outlet 1 supplies ad capacity of $D_1^l 2a + D^s a$ into the market, outlet *f* only supplies $D^s a$.

The significant change that arises here is that, in addition to targeted multi-homing, advertisers now have an additional option to reach the entire market by intensively single-homing on outlet 1 with 2 impressions (i.e., $n_1 = 2, n_f = 0$). This yields surplus of $v - p_1$ which always exceeds targeted multi-homing ($n_1 = 2, n_f = 1$), which has expected surplus to advertisers of $\frac{1}{2}(1+D^s)v - p_1 - \frac{1}{2}D^s p_f$. Thus, when x_f is low, intense single-homers on that outlet set the price for marginal advertisers in the market. The following proposition characterizes outcomes when one outlet has limited content.

Proposition 5. *Suppose $F(v) \sim U[0,1]$, $a_1 = a_2 = a$ and outlet f has limited content. The only non-dominated advertiser choices (n_1, n_f) are $\{(1,2),(1,0),(2,0),(0,2)\}$. In equilibrium, (i) for x_f low, the marginal advertiser in the market chooses (0,2) and $\hat{p}_1 > \hat{p}_f$ while (ii) for x_f high, the marginal advertiser in the market chooses (1,0), $\hat{p}_1 < \hat{p}_f$ and there are no multi-homing advertisers. As x_f approaches 1, π_f approaches π_1.*

The structure of the equilibrium is interesting. When *f*'s share is low ($\frac{1}{2}D^s < D_1^l$) and begins to rise, outlet 1, who was exclusively selling to single-homing advertisers (1 impression) continues to do so, but high valued advertisers also purchase 2 impressions on outlet *f*. The same is true of low valued advertisers who now become the marginal advertisers in the market at a price of p_f. Consequently, $p_f < p_1$ but as x_f rises, outlet 1's profit falls as does total profits from advertising in the industry. This changes when x_f reaches a critical level (i.e., 0.42265 so that $\frac{1}{2}D^s > D_1^l$). At that point, marginal advertisers prefer to bid for 2 impressions on outlet *f* and so single-homing advertisers with a single impression on outlet 1 become the marginal advertisers at a price of p_1. This implies that $p_f > p_1$. In addition, the high valued advertisers no longer choose to multi-home and become exclusive to outlet 1 with 2 impressions. Nonetheless, as x_f rises outlet 1's profits continue to fall. In this case, however, industry profits rise again and indeed, when $x_f \to 1$ they approach the same level as when $x_f = 0$. In this case, the profits are split evenly between the two outlets rather than held entirely by outlet 1. Intuitively, at this point, all consumers are switchers and so there is no longer any inefficiency resulting from wasted impressions.

An interesting observation is that at this limit, there may be negative incentives to provide additional content. The small content outlet can earn exactly the same profits as the other outlet. Indeed, when x_f is such that $\frac{1}{2}D^s > D_1^l$, outlet *f* earns more than half of outlet 1's profits. Thus, the rate of return for providing that additional content is lower for outlet 1 than for outlet *f*.

We can get a sense as to whether limited but magnet content is becoming relatively more important by looking at the type of outlets that now attract display ad impressions. ComScore reports that in the first quarter of 2011, Facebook (arguably a limited content provider) attracted over 30 percent of all display ad impressions in the US; around 350 billion impressions. In contrast, traditional, in-depth, news outlets such as Turner International, Fox Interactive and CBS Digital Attracted between 11 and 18 billion impressions (less than 2% of impressions).

5.3 Paywalls

In response to declining ad revenues, efforts to charge consumers for content are taking many different forms. While most of the debate insists on the adverse impact of paywalls on readership we contribute to the debate by assessing the impact of three of the most common pay models on the advertisers' incentives. Needless to say, a thorough assessment of paywalls would require a full-fledged model of consumer behavior. Nonetheless the model presented here allows focusing on the impact of paywalls on switching behavior.

Specifically, we assume that outlets are asymmetric in the probabilities that a consumer might have an opportunity to switch *away* from them. We define ρ_{ij} as the probability that a consumer who has visited outlet *i*, has an opportunity to switch from it. Consequently, the three consumer classes are now determined by:

$$D_1^l = x_1 - x_1(1-x_1)\rho_{12} \quad (2)$$

$$D_2^l = x_2 - x_2(1-x_2)\rho_{21} \quad (3)$$

$$D_{12}^s = (\rho_{21} + \rho_{12})x_1x_2 \quad (4)$$

A higher ρ_{ij} may result from the consumer having a higher cost associated with remaining with outlet *i*. Of course, a paywall may impact upon x_i. However, for the most part, we will hold that effect fixed and comment on the impact of such movements below.

We begin by considering *micropayments* whereby outlet 1 charges consumers for each period they visit its website. Holding the impact on x_1 fixed, a micropayment makes it less likely that visitors to outlet 1 will stay on that outlet another period (increasing ρ_{12}) while making it less likely visitors to outlet 2 will switch to outlet 1 (decreasing ρ_{21}). This has two impacts on advertising markets. First, D_{12}^s could rise or fall depending upon what happens to $\rho_{21}+\rho_{12}$. If it falls, then this will put upward pressure on advertising prices if ad capacity is relatively low. Second, recall that when readership shares were asymmetric, an outlet commanded a positional advantage if its expected share of loyal consumers was relatively high. However, holding x_1 fixed and starting from a symmetric position prior to the paywall, micropayments on outlet 1 will lead to more loyal users on outlet 2 than on 1 ($D_2^l > D_1^l$). Consequently, outlet 2 will be given a positional advantage in the advertising market so that $p_2 > p_1$. Add to that the reduction in x_1 due to the paywall, and this effect is only reinforced. Outlet 1 would have to not only make up for lost advertising revenues as a loss in visitors but also from the loss in positional advantage, while outlet 2 clearly benefits in both of these dimensions from the paywall.

In contrast to a micropayment system, a *subscription* system will have a more directed impact. In such a system, a visitor to outlet 1 only pays on their first visit and not thereafter. This means that a subscriber to outlet 1 may be just as likely – should the opportunity and desire arise – to switch to outlet 2 (i.e., ρ_{12} will not change). However, a non-subscriber who had visited outlet 2 previously would be less likely to then subscribe to outlet 1 for what remained of the attention period (i.e., ρ_{21} would fall). Once again, starting from a position of symmetry, this implies that $D_2^l > D_1^l$ and so the paywall would not only lead to relatively more visitors to outlet 2 but a positional advantage for it in advertising markets. This is an interesting result since one of the claims associated with subscription paywalls is that they will increase consumer loyalty to an outlet. While it is true that such loyalty, if generated, would increase an outlet's advertising revenues per consumer, here a subscription generates increased loyalty for the rival outlet rather than the outlet imposing the paywall. Of course, this effect could be mitigated if, say because they are subscribers, consumers are more inclined to be loyal to outlet 1 thereby increasing ρ_{12}. The point here is that that outcome is not straightforward.

Finally, some outlets have proposed a *limited paywall* (as recently implemented by the *Financial Times* and the *New York Times*). In this case, outlets allow access to some content for free and then charge should a consumer wish to consume more. In the context of the model here, such a paywall would only be imposed, say, if a consumer chose to stay on outlet 1 for both attention periods. This type of paywall would be unlikely to have any impact on those who had previously visited outlet 2 as they could still freely switch to outlet 1 (i.e., ρ_{21} would be unchanged). However, this paywall would impose a penalty for staying on outlet 1 making consumers there more inclined to switch (i.e., ρ_{12} would rise). It is clear again, that other things being equal, the paywall would result in $D_2^l > D_1^l$.

The analysis here demonstrates that putting in a paywall may give an outlet a positional disadvantage in advertising markets. Of course if an outlet already has a positional advantage, the likelihood that this occurs is lower. Nonetheless, the impact of a paywall does confer benefits on rivals in advertising markets as well as increasing their readership. These consequences may explain the low use of paywalls for online news media.

6 Policy Implications

6.1 The Impact of Mergers

The evaluation of mergers between media outlets has always posed some difficult issues for policy-makers. On the one hand, if it is accepted that outlets have monopoly power over access to their consumers, mergers are unlikely to reduce the extent of competitive pressure in advertising markets. On the contrary, it is argued that a merger may indeed increase market power enhancing ad revenues and stimulating outlet’s incentives to attract consumers. We add to this debate by considering various cases depending on what the merger does with regard to inter-outlet tracking. The number of outlets affects the equilibrium outcome only through its impact on tracking and thus the efficiency of advertising on multiple outlets. A full analysis of mergers would in addition need to consider the extension of our model to endogenous capacity.

Mergers improve inter-outlet tracking. Suppose a merger reduces the number of duplicated and missed impressions in the advertising market. For example, as noted earlier, a move to perfect tracking generates, for a fixed ad capacity, the first best allocation of scarce advertising inventory. While these gains could potentially provide efficiency grounds for approval, the analysis showed that it is not clear that the outlets have indeed incentives to facilitate tracking ex-post due as profits need not be monotone.[23]

Mergers permit coordinated sales to advertisers. Suppose that creating a single entity can allow discrimination between single-homers and multi-homers; something that cannot be done without joint ownership. To explore this, suppose that parameters are such that no advertiser wants to purchase multiple impressions on any one outlet, outlet readership quality is symmetric and that $F(v) \sim U[0,1]$. Further, suppose that, on each outlet, the monopoly owner can commit to an ad capacity allocated to multi-homers, a_m, and an ad capacity allocated to single homers, a_s. Price discrimination is achieved by charging all advertisers the same price for their first impression on one of the outlets and a different price for their second impression. The price the outlet can charge multi-homers, p_m for their second impression and single-homers, p_s, for their single impression are determined by:

$$a_m = 1 - v_2 \text{ and } a_s = \tfrac{1}{2}(v_2 - v_1) \qquad (5)$$

Solving for prices and substituting into the profit function, $(p_s + p_m)a_m + p_s a_s$, gives:

$$\tfrac{1}{4}\Big((1 - 2a_s - a_m)(2a_s + a_m) + a_m \tfrac{12 - D^s}{2}(1 - a_m)\Big) \qquad (6)$$

Maximizing with respect to (a_m, a_s) and subject to $a_s + a_m = 2a$ yields:

$$a_m = \tfrac{16a - D^s}{2(4 - D^s)} \text{ and } a_s = \tfrac{D^s}{2(4 - D^s)}(1 - 4a) \qquad (7)$$

[23] In addition, as demonstrated above, moving to perfect tracking may not result in higher quality to readers leading to potentially ambiguous overall welfare effects.

so long as $16a > D^s$.[24] Profits are: $\frac{64a(2-D^s)(1-2a)+D^{s2}}{32(4-D^s)}$ which are greater than profits in the absence of price discrimination.

Price discrimination allows the outlet to separate advertisers' types exploiting the usual sorting condition: higher types value attention relatively more. With differential prices comes a different allocation of attention. Specifically, note that, for a given D^s, with no discrimination allocative efficiency obtains; i.e., there is no way to re-allocate attention to different advertisers to increase total surplus. What the price discrimination analysis shows is that a monopoly will introduce a *further* allocative distortion. Although characterizing this "rent-extraction / allocative efficiency of user attention" trade-off is beyond the scope of this paper, we believe this issue is important and should be addressed at the level of merger control.

6.2 The Impact of Blogs and Public Broadcasting

One of the factors that traditional newspapers have argued are contributing to their decline is the rise of user generated content (blogs) and also competition from government-subsidized media. Both types of outlets have in common that they either do not accept advertising or accept very little of it. Somewhat in contradiction to this position, newspapers and television broadcasters have objected to plans to allow public broadcasters to sell advertisements rather than rely on subsidies. This latter objection remains a puzzle from the perspective of traditional media economics, because these public broadcasters should in principle be tougher competitors precisely because they do not need to carry those annoying ads. Here we use our model to explore the impact of competition from non-advertising media outlets.

The baseline model is modified as follows. There are three outlets: one blog and two mainstream media. In line with the previous notation let x_b be the probability that consumers choose a blog if given a choice. Let $x_1 = x_2 = \frac{1}{2}(1-x_b)$ be the corresponding (symmetric) probability for the "sponsored" outlets. This implies that:

$$D^l = \tfrac{1}{2}(1-x_b)\left(1-\tfrac{1}{2}(1-x_b)\rho\right) \tag{8}$$

---

[24] If this condition does not hold, the outlet would not choose to price discriminate.

$$D_{12}^{s} = \rho \tfrac{1}{2}(1 - x_b)^2 \qquad (9)$$

$$D_{ib}^{s} = \rho x_b (1 - x_b) \qquad (10)$$

where D_{ij}^{s} denotes the fraction of consumers who switch between outlets i and j.

Proposition 6. *For $\rho > 0$ equilibrium impression prices are increasing in the popularity of the ad free outlet, x_b .*

Intuitively, an increase in x_b has two effects. First, as blogs are a sink of (scarce) attention, it decreases the effective supply of advertising capacity in the market. Second, unlike switchers between mainstream outlets, switchers between blogs and mainstream outlets do not contribute to the wasted impressions problem. Consequently, a greater share of blog readers increases the share of blog-mainstream switchers as well and so reduces duplication. This increases the demand for advertisements.[25] These two effects – a decrease in supply and an increase in demand – combine to raise equilibrium impression prices. It is instructive to note that, even under perfect tracking, the supply-side effect remains and so impression prices would be expected to rise with blog readership share in that case too.

Nonetheless, in terms of the impact on overall outlet profits, the price effect of an increased blog share may not outweigh the quantity effect (in terms of lost readers). If it is the case that we are comparing a situation where one output sells advertising to one where it does not (absent any quantity changes in readership), then it is clear that advertising-selling outlets prefer the situation where its rival is prohibited from selling ads.[26]

6.3 The Impact of Prohibiting Tracking

In 2010, the Federal Trade Commission explored a policy that would give consumers the right to 'opt out' of tracking of any kind by websites. If widely adopted, this would eliminate tracking options for media outlets. The impact of prohibitions on tracking depends on the incentives to adopt such tracking in the first place. The analysis

[26] A recent paper by de Corniere and Taylor (2013) develops this point. They show that if the mainstream outlet (e.g. Google) can also "divert" attention (i.e. set x_b) then they would trade-off higher ad-prices (due to scarcer supply) with higher quantity (due to lower attractiveness). So they show that the supply-side effect described here can be a basis for "search engine bias."

provides some insight into this by examining what happens to outlet profits as we move from imperfect to perfect tracking. As shown, outlet profits need not be monotone in the extent of tracking. It follows that perfect tracking technology might not be adopted despite their ability to generate efficient outcomes in advertising markets.[27]

The possibility that advertisers will purchase multiple impressions at a rate that likely leads to waste is borne out by the ComScore data. For instance, they estimate that in the first quarter of 2011, almost 1.1 trillion display ads were delivered in the US.[28] Of these, 19.5 billion were purchased by AT&T, 16.6 billion by Experian Interactive and 11.2 billion by Scottrade. If the entire US population surfed the net daily during that time, they would see one AT&T ad per day.

Note that outlets do not have a unilateral incentive to adopt perfect tracking as it has no value unless the other outlet also cooperates to share consumer information. This fact also makes it challenging for a provider of perfect tracking services to appropriate the rents from that activity as we would expect each outlet to have some hold-out power.

7 Conclusions and Directions for Future Research

This paper presents a theory that may resolve long-standing puzzles in media economics regarding the impact of competition by constructing a model where consumers can switch between media outlets and those outlets can only imperfectly track those consumers across outlets. This model generates a number of predictions, including: (i) as consumer switching increases total advertising revenue falls; (ii) outlets with a larger readership share command premiums for advertisements; (iii) greater switching may lead advertisers to increase the frequency of impressions purchased on outlets; (iv) an increase in attention from non-advertising sources will increase advertising prices; (v) mergers may allow outlets to increase advertising prices by practicing price discrimination in advertising markets; (vi) ad platforms that provide tracking services may not increase advertising prices; (vii) investments in content quality will be associated

[27] This also highlights the importance of how ad capacities are chosen; something we analyze in the online appendix. That analysis demonstrates that it is, in fact, an inability to commit to not selling advertisements when ad capacity is relatively high that permits the outcome that perfect tracking may lead to lower profits than imperfect tracking.

[28] http://www.comscore.com/Press_Events/Press_Releases/2011/5/U.S._Online_Display_Advertising_Market_Delivers_1.1_Trillion_Impressions_in_Q1_2011

with the nature of tracking technology; (viii) outlets that supply magnet content may be more profitable per unit of attention than outlets offering a deeper set of content. These predictions await thoughtful empirical testing but are thusfar consistent with stylized facts associated with the impact of the Internet on the newspaper industry. In addition, we note that while we have qualitative results, it is an open question as to whether the effects here are quantitatively significant; especially given the magnitude of the observed changes. Of course, the general characteristics of the model would also apply to other media industries including television following the introduction of cable television channels and remote controls and also the newly emerging mobile application industry that has so far struggled to be advertising supported.

While the model here has a wide set of predictions, extensions could deepen our understanding further. Firstly, the model involves two outlets usually modeled as symmetric with a distribution of advertisers with specific qualities. Generalizing these could assist in developing more nuanced predictions for empirical analysis; specifically, understanding the impact of outlet heterogeneity on advertising prices, incentives to invest in quality and incentives to invest in tracking technology.

Related, in this paper, we focused on frequency-based tracking noting that other forms of tracking have been part of the news industry. An open question is what the incentives are for firms to unilaterally improve their internal tracking of consumers. As noted throughout this paper, the adoption of more efficient matching may increase marginal demand but reduce inframarginal demand from advertisers. When ad capacity is scarce, it is not clear that such moves will prove profitable for outlets.

Finally, throughout this paper we have assumed that advertisements were equally effective on both outlets. However, in some situations, it may be that the expected value from impressing a consumer on one outlet is higher than that from impressing consumers on another. For instance, consider (as in Athey and Gans, 2010), a situation where all advertisers are in a given local area. One outlet publishes in that local area only while the other is general and publishes across local areas.[29] Absent the ability to identify consumers based on their location, a consumer impressed on the local outlet will still

[29] Location is only one aspect upon which consumers and advertisers might sort according to common interests. Any specialized media content can perform this function and give an outlet a matching advantage over more general outlets.

generate an expected value of v to advertiser v whereas one impressed on the general outlet will only generate an expected value of θv with $\theta < 1$. In this situation, even if there are no switching consumers, advertisers on the general outlet will be paying for wasted impressions.

While this situation may be expected to generate outcomes similar to when readership shares are asymmetric, the effects can be subtle. A general outlet may have fewer consumers who are of value to advertisers but also may have a larger readership.[30] Also, when consumers switch between outlets, the switching behavior is information on those hidden characteristics. Thus, switching behavior may actually increase match efficiency. Consequently, the effects of tailored content, self-selection and incentives to adopt targeting technologies that overcome these are not clear and likely to be an area where future developments can be fruitful.

[30] Levin and Milgrom (2010) argue that targeting may be limited because it conflicts with goals of achieving market thickness (see also Athey and Gans, 2010).

8 Appendix

8.1 Market equilibrium with uniformly distributed valuations, symmetric outlets and $D^s < \frac{2}{3}$

Suppose that v is uniformly distributed on [0,1]. Recall that in this case we have:

$$v_1 = p, \qquad v_2 = \frac{2}{2-D^s}p, \qquad v_3 = \frac{2}{D^s}p.$$

Let $\hat{D}^s > 0$ denote the unique solution to the equation $v_3 = 1$ (provided $p > 0$). Equating aggregate supply with aggregate demand for each outlet leads to (symmetric) market clearing impression prices:

$$\hat{p} = \begin{cases} \frac{D^s(2-D^s)}{4+D^s(2-D^s)}(3-4a) & \text{if } D^s > \hat{D}^s \\ \frac{2(2-D^s)}{4-D^s}(1-2a) & D^s \le \hat{D}^s \end{cases}$$

where $\hat{D}^s > 0$ denotes the smallest value of D^s such that v_3 evaluated at $p = \hat{p}$ equals one. These market clearing prices give rise to the kinked profit function plotted in figure 1(a). To rule out multiplicity note that market clearing prices in both cases above are equal if:

$$\frac{D^S(2-D^S)}{4+D^S(2-D^S)}(3-4a) = \frac{2(2-D^S)}{4-D^S}(1-2a) \Rightarrow D^S = 2\left(2(1-a) - \sqrt{2(1-2a)+4a^2}\right)$$

At this level of Ds, $p = 2(1-a) - \sqrt{2(1-2a)+4a^2}$; i.e., $D^s/2$. So, for given ad capacities, there is no issue of multiple equilibria.

8.2 Proof of Proposition 3

The fact that impression prices will differ in equilibrium together with asymmetric readership shares raises the possibility that the type indifferent between buying one ad on 1 or nothing does not coincide with the type indifferent between buying one ad on 2 or nothing. In fact when $p_1 > p_2$, the marginal advertiser of outlet 1 must be indifferent between the two outlets. We therefore adopt a different notation here. v_i denotes the marginal single-homer of outlet *i*. That is the type just indifferent between purchasing one ad on outlet *i* and either nothing or one ad on outlet *j*. v_{11} denotes the marginal multi-homer with one ad on each outlet. v_{12} is the type just indifferent between (1,1) and (1,2) (where the second ad is purchased on the smaller outlet. If $p_1 > p_2$ then $v_1 = \frac{2(D_2^l p_2 - D_1^l p_1) + D^s(p_2 - p_1)}{2(D_2^l - D_1^l)}$, while the marginal single-homer on outlet 2 is indifferent between outlet 2 and not advertising at all, so that $v_2 = p_2$. Note that given these expressions, $v_1 > v_2 \Rightarrow (2D_1^l + D^s)(p_2 - p_1) < 0$; this confirms the price premium of outlet 1.[31] The marginal multi-homing advertiser (v_{11}) will be determined by:

$$(D_1^l + D_2^l + \tfrac{3}{4}D^s)v_{11} - (D_1^l + \tfrac{1}{2}D^s)p_1 - (D_2^l + \tfrac{1}{2}D^s)p_2 \\ = \max\left[(D_1^l + \tfrac{1}{2}D^s)(v_{11} - p_1), (D_2^l + \tfrac{1}{2}D^s)(v_{11} - p_2)\right] \quad (11)$$

Note that if $p_1 \le p_2$ or there are single-homers on outlet 1, then $(D_1^l + \frac{1}{2}D^s)(v_{11} - p_1) \ge (D_2^l + \frac{1}{2}D^s)(v_{11} - p_2)$ implying that $v_{11} = \frac{D_2^l + \frac{1}{2}D^s}{D_2^l + \frac{1}{4}D^s} p_2$. Lastly the threshold, $v_{12} = \frac{2(2D_2^l + D^s)}{4(1 - D_2^l - D_1^l) - 3D^s} p_2$ leaves the advertiser indifferent between (1,1) and purchasing an additional ad on the smaller outlet. Given this, market clearing implies that the following equations (for each outlet) be simultaneously satisfied:

$$\underbrace{1 - F(v_1)}_{\text{Demand for 1}} = 2a \quad (12)$$

$$\underbrace{2(1 - F(\min\{v_{12}, V\})) + F(\min\{v_{12}, V\}) - F(v_{11}) + F(v_1) - F(v_2)}_{\text{Demand for 2}} = 2a \quad (13)$$

Thus, high-valued advertisers sort on to outlet 1, while outlet 2 attracts low-valued advertisers who single-home, as well as a set of high valued advertisers who multi-home. There is an intermediate interval of advertisers who do not advertise on outlet 2.

Solving (12) and (13) for outlet prices and substituting them into outlet profits while checking to see what allocations of advertising choices these imply allows to derive the following lemma:

Lemma. *Assume that* $F(v) \sim U[0,1]$, $a_1 = a_2 = a$ *and* $x_1 > x_2$. *Then each outlet's equilibrium profits are as follows:*

(i) For $\frac{8 - x_1\rho(8 - x_1\rho)}{8(2 - x_1\rho)} < a$, $\pi_1 = x_1 \frac{4-\rho}{4 - x_1\rho}(1 - 2a)2a$ *and* $\pi_2 = x_2 \frac{x_1\rho(2 - x_1\rho)}{4 + x_1\rho(2 - x_1\rho)}(3 - 4a)2a$;

(ii) For $\frac{x_1\rho}{8} < a < \frac{8 - x_1\rho(8 - x_1\rho)}{8(2 - x_1\rho)}$, $\pi_1 = x_1 \frac{4-\rho}{4 - x_1\rho}(1 - 2a)2a$ *and* $\pi_2 = x_2 \frac{2(2 - x_1\rho)}{4 - x_1\rho}(1 - 2a)2a$

(iii) For $\frac{x_1\rho}{8} \ge a$, $\pi_1 = x_1(1 - \frac{2a}{x_1})2a$ *and* $\pi_2 = x_2(1 - 4a)2a$.

Note that, for case (i), $\left(2(1-a) - x_1 - \frac{4(3-4a)(1-x_1)}{4 + x_1\rho(2 - x_1\rho)}\right) > x_2 \frac{x_1\rho(2 - x_1\rho)}{4 + x_1\rho(2 - x_1\rho)}(3 - 4a) \Rightarrow a > \frac{1}{2}$, which can't hold. Thus, $\pi_2 > \pi_1$ always for this case. Note that this arises when $\frac{8 - x_1\rho(8 - x_1\rho)}{8(2 - x_1\rho)} < a$; which, substituting D^s for ρ gives the converse of the condition in the proposition. The LHS is decreasing in ρ so that when both ρ and a are high, this condition holds as stated in the proposition.

For case (ii), $x_1 \frac{4-\rho}{4 - x_1\rho}(1 - 2a)2a > x_2 \frac{2(2 - x_1\rho)}{4 - x_1\rho}(1 - 2a)2a \Rightarrow x_1(2 - \frac{1}{2}\rho) > x_2(2 - x_1\rho)$ which always holds for $x_1 > ½$. Thus, $\pi_1 > \pi_2$ for this case.

For case (iii), it is readily apparent that $x_1 > ½$, $\pi_1 > \pi_2$ for this case.

[31] Of course, in equilibrium, there may be no single-homers on outlet 2 which will alter this intuition as we discuss below.

8.3 Proof of Proposition 5

In this case, while outlet 1 supplies ad capacity of $D_1^l 2a + D^s a$ into the market, outlet *f* only supplies $D^s a$. The following table identifies the surplus to an advertiser with value *v* from pursuing different choices.

(n_1, n_f)	Expected Advertiser Surplus
(1,0)	$(D_1^l + \frac{1}{2}D^s)(v - p_1) = \frac{1}{2}(v - p_1)$
(2,0)	$(D_1^l + D^s)v - (2D_1^l + D^s)p_1 = \frac{1}{2}(1 + D^s)v - p_1$
(0,1)	$\frac{1}{2}D^s(v - p_f)$
(0,2)	$D^s(v - p_f)$
(1,1)	$(D_1^l + \frac{3}{4}D^s)v - (D_1^l + \frac{1}{2}D^s)p_1 - \frac{1}{2}D^s p_f = (\frac{1}{2} + \frac{1}{4}D^s)v - \frac{1}{2}p_1 - \frac{1}{2}D^s p_f$
(1,2)	$(D_1^l + D^s)v - (D_1^l + \frac{1}{2}D^s)p_1 - D^s p_f = \frac{1}{2}(1 + D^s)v - \frac{1}{2}p_1 - D^s p_f$
(2,1)	$(D_1^l + D^s)v - (2D_1^l + D^s)p_1 - \frac{1}{2}D^s p_f = \frac{1}{2}(1 + D^s)v - p_1 - \frac{1}{2}D^s p_f$

Notice that there are now three options for an advertiser to cover the entire consumer market – single homing on 1 with 2 impressions, and multi-homing with two impressions on at least one outlet. It is clear that multi-homing with 2 impressions on outlet 1 is dominated by single-homing on outlet 1 (as the former involves paying for impressions on *f* without any benefit). In addition, note that any advertiser who wants to single home on outlet *f* will prefer to do so with two impressions as there is no waste from the additional impression. We can always rule out multi-homing with one impression on each outlet. For this to be preferred to single-homing on outlet 1 (with one impression) it must be the case that $\frac{1}{4}D^s v > \frac{1}{2}D^s p_f$. However, this condition also means that by moving from multi-homing with single impressions to multi-homing on outlet *f* with 2 impressions is preferable. Consequently, if an advertiser wants to capture an additional $\frac{1}{4}D^s$ by purchasing an impression on outlet *f*, it will also want to do this by purchasing two additional impressions on outlet *f*.

This still leaves four choices that might be undertaken by advertisers. As a means of covering the entire market, single-homing on outlet 1 with 2 impressions and multi-homing with 2 impressions on *f* are substitutes. Indeed, multi-homing will only be chosen if $\frac{1}{2}p_1 > D^s p_f$; a condition that must hold if D^s is very small. At any point in time, we will only observe one of these strategies being chosen. In each case, it will be the highest value advertisers who pursue them.

For the remaining choices, advertisers single homing on *f* (with 2 impressions) or on 1 (with 1 impression) are candidates to be the marginal advertiser in the market. If $\frac{1}{2}D^s > D_1^l$, higher value advertisers prefer (holding prices constant) purchasing impressions on f rather than 1. Under this condition, the marginal advertiser, with value p_1, would earn $D^s(p_1 - p_f)$ by switching to outlet f which is negative if $p_1 < p_f$. Similarly, if the marginal advertiser has value, p_f, it will earn $(D_1^l + \frac{1}{2}D^s)(p_f - p_1)$ by switching to outlet 1. This reduces its surplus if $p_f < p_1$. Hence, the marginal advertiser will be on the lowest priced outlet.

We now turn to derive the equilibrium prices and profits. Case 1: $\frac{1}{2}D^s > D_1^l$. Suppose that $(D_1^l + \frac{1}{2}D^s)p_1 < D^s p_f$. Then consider a candidate equilibrium where high value advertisers sort as single-homers (2 impressions) on 1, then single-homers (2 impressions) on f and finally as single-homers (1 impression) on 1. In this case, equilibrium prices will be the solution to:

$$D_1^l 2a + D^s a = (D_1^l + \tfrac{1}{2}D^s)\big(2(1 - v_{1f}) + (v_f - p_1)\big) \tag{14}$$

$$\tfrac{1}{2}D^s 2a = \tfrac{1}{2}D^s 2(1 - v_{1f}) \tag{15}$$

where $v_{1f} = \frac{(2D_1^l + D^s)p_1 - D^s p_f}{D_1^l}$ and $v_f = \frac{2D^s p_f - (2D_1^l + D^s)p_1}{D^s - 2D_1^l}$. Solving this gives:

$$p_1 = \frac{aD_1^l + D^s(1-2a)}{D_1^l + D^s} \tag{16}$$

$$p_f = 1 - 2a - 2(1-3a)\tfrac{D_1^{l2}}{D^s} \tag{17}$$

(recalling that we assume that $a \le \frac{1}{4}$). It is easy to demonstrate that $p_f > p_1$ and that $(D_1^l + \frac{1}{2}D^s)p_1 < D^s p_f$. This confirms the equilibrium.

Is it possible that $(D_1^l + \frac{1}{2}D^s)p_1 > D^s p_f$? In this case, a candidate equilibrium would have high value advertisers sort as multi-homers (2 impressions) on f and then single-homers (2 impressions) on f. In this case, no advertiser will choose single-homing on 1. Thus, equilibrium prices will be the solution to:

$$D_1^l 2a + D^s a = (D_1^l + \tfrac{1}{2}D^s)(1 - v_{1f}) \tag{18}$$

$$\tfrac{1}{2}D^s 2a = \tfrac{1}{2}D^s 2(1 - p_f) \tag{19}$$

where $v_{1f} = \frac{(D_1^l + \frac{1}{2}D^s)p_1}{D_1^l}$. Solving this gives:

$$p_1 = \frac{D_1^l(1-2a)}{2D_1^l + D^s} \tag{20}$$

$$p_f = 1 - a \tag{21}$$

It is easy to demonstrate that $p_f > p_1$ but that $(D_1^l + \frac{1}{2}D^s)p_1 - D^s p_f = (\frac{1}{2} - a)D_1^l - D^s(1-a) > 0 \Rightarrow \frac{D^s}{D_1^l} < \frac{\frac{1}{2}-a}{1-a}$ which cannot hold as the LHS is greater than 2 while the RHS is less than 2. Thus, this cannot be an equilibrium.

Case 2: $\frac{1}{2}D^s < D_1^l$. Suppose that $(D_1^l + \frac{1}{2}D^s)p_1 > D^s p_f$. Then consider a candidate equilibrium where high value advertisers sort as multi-homers (2 impressions) on *f*, then single-homers (1 impression) on 1 and finally single-homers (2 impressions) on *f*. In this case, equilibrium prices will be the solution to:

$$D_1^l 2a + D^s a = (D_1^l + \tfrac{1}{2}D^s)(1 - v_1) \tag{22}$$

$$\tfrac{1}{2}D^s 2a = \tfrac{1}{2}D^s 2(1 - v_{1f} + v_1 - p_f) \tag{23}$$

where $v_{1f} = 2p_f$ and $v_1 = \frac{(2D_1^l + D^s)p_1 - 2D^s p_f}{2D_1^l - D^s}$. Solving this gives:

$$p_1 = \frac{6D_1^l(1-2a) + D^s}{3(2D_1^l + D^s)} \tag{24}$$

$$p_f = \tfrac{2}{3} - a \tag{25}$$

(recalling that we assume that $a \le \frac{1}{4}$). It is easy to demonstrate that $p_f < p_1$ and that $\frac{D^s(2-D^s)}{4+D^s(2-D^s)}(3-4a) = \frac{2(2-D^s)}{4-D^s}(1-2a) \Rightarrow D^s = 2\left(2(1-a) - \sqrt{2(1-2a) + 4a^2}\right)$. This confirms the equilibrium.

8.4 Proof of Proposition 6

The main difference between this case and the previous two outlet model is that some advertisers may choose to multi-home with two impressions on each outlet so as to impress a greater share of those switching between blogs and mainstream outlets. Since $x_1 = x_2$ then we have that the share of loyals of either outlets and of switchers must be equal. We can drop the outlet index: $D_1^l = D_2^l := D^l$ and $D_{1b}^s = D_{1b}^s := D_b^s$. Note that by construction: $2D^l + D_b^l + D_{12}^s + 2D_b^s = 1$. The advertiser expected surplus from the different advertising strategies are:

(n_1, n_2)	Expected Advertiser Surplus
(1,0)	$(D^l + \frac{1}{2}D_{12}^s + \frac{1}{2}D_b^s)(v - p)$
(2,0)	$(D^l + D_{12}^s + D_b^s)v - (2D^l + D_{12}^s + D_b^s)p$
(1,1)	$(2D^l + \frac{3}{4}D_{12}^s + D_b^s)v$ $-p(2D^l + D_{12}^s + D_b^s)$
(2,1)	$(2D^l + D_{12}^s + \frac{3}{2}D_b^s)v$ $-(3D^l + \frac{3}{2}(D_{12}^s + D_b^s))p$

(2,2)	$(1-D_b^l)v$ $-(4D^l+2D_{12}^s+2D_b^s)p$

Note that strategy (1,1) dominates (2,0) for all values of ρ. Therefore type v_2 is defined as the type indifferent between (1,1) and (1,0) or (0,1). The threshold advertiser rates become (under symmetric ad capacities and equal prices):

$$
\begin{aligned}
v_i &= p \\
v_2 &= \frac{D^l+\frac{1}{2}D_{12}^s+\frac{1}{2}D_b^s}{D^l+\frac{1}{4}D_{12}^s+\frac{1}{2}D_b^s}\,p = \frac{4}{4+\rho(x_b-1)}\,p \\
v_3 &= \frac{D^l+\frac{1}{2}D_{12}^s+\frac{1}{2}D_b^s}{\frac{1}{4}D_{12}^s+\frac{1}{2}D_b^s}\,p = \frac{4}{\rho(1+3x_b)}\,p \\
v_4 &= \frac{D^l+\frac{1}{2}D_{12}^s+\frac{1}{2}D_b^s}{\frac{1}{2}D_b^s}\,p = \frac{2}{\rho x_b}\,p
\end{aligned}
\tag{26}
$$

All thresholds are ordered ($v_{22} > v_{21} > v_{11} > v_i$) and decreasing in x_b. Note that when $x_b = 0$ we revert to the baseline case. It follows that for all prices, advertisers' demand is monotone increasing in x_b. Clearly, as the blog is a sink of attention, the aggregate supply decreases with x_b. Therefore the (symmetric) market equilibrium prices are necessarily strictly increasing in the blog's market share x_b.

9 References

Ambrus, A., E. Calvano and M. Reisinger. 2012. "Either or both competition: A two-sided theory of advertising with overlapping viewerships," *mimeo.*, Duke.

Anderson, S.P. and S. Coate. 2005. "Market Provision of Broadcasting: A Welfare Analysis." *Review of Economic Studies*, 72(4): 947-972.

Anderson, S.P., O. Foros and H.J. Kind. 2011. "Competition for advertisers in media markets," *mimeo*., Norwegian School of Economics and Business Administration.

Anderson, S.P., O. Foros, H.J. Kind and M. Peitz. 2012. "Market Media Concentration, Advertising Levels and Ad Prices," *International Journal of Industrial Organization*, 30: 321-325.

Armstrong, M. (2006), "Competition in Two-sided Markets", *Rand Journal of Economics* 37: 668-691.

Armstrong, M. and J. Wright (2007), "Two-Sided Markets, Competitive Bottlenecks and Exclusive Contracts," *Economic Theory*, 32 (2), pp.353-380.

Ashlagi, I., B. Edelman and H.S. Lee. 2010. "Competing Ad Auctions: Multihoming and Participation Costs," *Working Paper* No.10-055, Harvard Business School.

Athey, S., E. Calvano and J.S. Gans. 2013. "Consumer Tracking and Efficient Matching in Online Advertising Markets," *mimeo*., Toronto.

Athey, S., and J.S. Gans. 2010. "The Impact of Targeting Technology on Advertising Markets and Media Competition" *American Economic Review*, 100 (2), pp.608-613.

Athey, S. and M. Mobius, 2012. "The Impact of News Aggregators on Internet News Consumption: The Case of Localization," *mimeo*., Harvard University.

Bergemann, D. and A. Bonatti. 2011. "Targeting in Advertising Markets: Implications for Offline versus Online Media" *RAND Journal of Economics*, 42 (3): pp.417-443.

Butters, G.R. 1977. "Equilibrium Distributions of Sales and Advertising Prices," *Review of Economic Studies*, 44 (3), pp.465-491.

Chandra, A. and U. Kaiser. 2011. "Targeted Advertising in Magazine Markets and the Advent of the Internet," Available at SSRN: http://ssrn.com/abstract=1653435.

Chiou, L. and C.E. Tucker. 2010. "News, Copyright and Online Aggregators," *mimeo*., MIT.

Chwe, M.S-Y. 1988. "Believe the Hype: Solving Coordination Problems with Television Advertising," *Working Paper*, University of Chicago.

Competition Commission 2003. "Carlton Communications Plc and Granada Plc; A Report on the Proposed Merger."

Crampes, C., C. Haritchabalet and B. Jullien (2009), "Advertising, competition and entry in media industries," *Journal of Industrial Economics*, 57 (1), 7-31.

de Corniere, A. and G. Taylor (2013), "Integration and Search Engine Bias," mimeo., Oxford.

Dreze, X. and F-X. Hussherr, 2003. "Internet advertising: Is anybody watching?" *Journal of Interactive Marketing*, 17 (3), pp.8-23.

Edelman, B. 2010. "The Design of Online Advertising Markets," *Handbook of Market Design* (Neeman et.al., eds.), Oxford, forthcoming.

Evans, D.S. 2008. "The Economics of the Online Advertising Industry," *Review of Network Economics*, 7 (3).

Evans, D.S. 2009. "The Online Advertising Industry: Economics, Evolution and Privacy," *Journal of Economic Perspectives*, 23 (3), pp.37-60.

Farhi, P. 2010. "Build that Pay Wall High," *American Journalism Review*, June/July, http://ajr.org/Article.asp?id=4800 (accessed 27th October 2009).

Filistrucchi, L., L. Luini and A. Mangani 2011. "Regulating one side of a two-sided market: the french advertising ban on prime-time state television," *mimeo*., Siena.

Fisher, F.M., J.J. Mc Gowan, and D.S. Evans. 1980. "The Audience-Revenue Relationship for Local Television Stations," *Bell Journal of Economics*, 11, pp.694-708.

Gabszewicz, J.J. and X.Y. Wauthy (2004), "Two-Sided Markets and Competition with Multi-homing," *mimeo*., CORE.

Gentzkow, M.A. (2007), "Valuing New Goods in a Model with Complementarity: Online Newspapers," *American Economic Review*, 97(3), 713-744.

Gentzkow, M.A., and J.M. Shapiro (2011), "Ideological segregation online and offline. Quarterly Journal of Economics, 126(4), 1799-1839.

Ghosh, A., P. McAfee, K. Papineni and S. Vassilvitskii. 2009. "Bidding for Representative Allocations for Display Advertising," *Internet and Network Economics, Lecture Notes in Computer Science*, 5959, pp.208-219.

Goldfarb, A. and C.E. Tucker. 2011a. "Privacy Regulation and Online Advertising. Management Science," 57(1), 57-71.

Goldfarb, A. and C.E. Tucker. 2011b. "Online Display Advertising: Targeting and Obtrusiveness," *Marketing Science*, 30(3), 389-404.

Goettler, R. (2012), "Advertising Rates, Audience Composition and Network Competition in the Television Industry," *mimeo*., Chicago.

Levin, J. and P. Milgrom. 2010. "Online Advertising: Heterogeneity and Conflation in Market Design," *American Economic Review*, 100 (2), pp.603-607.

Lewis, R., and D. Reiley, 2009. "Retail Advertising Works! Measuring the Effects of Advertising on Sales via a Controlled Experiment on Yahoo!" *Yahoo Research Technical Report*.

Reisinger, M. 2011. "Platform competition for advertisers and users in media markets," *International Journal of Industrial Organization*, doi: 10.1016/j.ijindorg.2011.10.002

Rice, A. 2010. "Putting a Price on Words," *New York Times Magazine*, 16th May, (http://www.nytimes.com/2010/05/16/magazine/16Journalism-t.html?hpw).

Varian, H. 2010. "Newspaper economics: Online and offline," presentation at FTC Workshop on "How Will Journalism Survive the Internet Age" Washington, March 9.

Weyl, G. 2010. "A Price Theory of Multi-Sided Platforms," *American Economic Review*, 100 (4): pp.1642-72.

10 Online Appendix:

10.1 Note on tracking technologies

Here we discuss alternative tracking technologies to the one used in the paper. That one is best described as "content-based tracking," which leads to tracking within an outlet and period, but not across periods or across outlets. The following table illustrates potential alternative tracking technologies and the type of offers each permits outlets to make to advertisers.

Table A: Alternative Tracking Technologies and Offer Types

Technology	Example of Offer Type
Perfect tracking	Over the two attention periods, we will impress a given set of consumers just once regardless of where their attention is allocated at a price of *p* per consumer/impression.
No tracking	Over the two attention periods, we will place a given number of impressions on our outlet for a price of *p* per impression.
Perfect internal tracking	Over the two attention periods, we will impress each unique consumer on the outlet once at a price of *p* per impression/consumer.
Content-based tracking	We will associate an ad with a given piece of content and you will pay a price of *p* each time that ad is viewed
Frequency-capping	Over the two attention periods, we will place at most *x* impressions per consumer at a price of *p* per impression

We considered perfect tracking above as a possible ideal. Such offers were made by ad platforms and were outlet independent. At the opposite extreme from perfect tracking is no tracking that arises when neither outlets nor a common platform are unable to internally (or externally) track impressions and to control matching between advertisers and consumers. In the early days of the Internet (circa 2000), websites had no ability to track consumers even within outlets, and even today with privacy settings such tracking may not be possible. The models of Butters (1977) and, more recently, Bergemann and Bonatti (2011) assume that advertisers choose the intensity of their advertising on an outlet, but that advertising messages (impressions) are distributed independently (across messages) and uniformly across consumers. This means that a given consumer might see the same advertisement multiple times, which involves waste.[32]

The next three technologies are intermediate ones where outlets can track impressions internally but not externally. Thus, outlets cannot offer inter-outlet arrangements (such as different prices for different switching categories of consumer) that would be possible under perfect tracking because they cannot track consumers across outlets. Under perfect internal tracking, no consumer receives more than one impression from an advertiser on a given outlet. Thus, an advertiser could purchase impressions on

[32] In such models, the expected number of unique impressions received by an advertiser with advertising intensity n in a market of size x is given by $x\left(1-(1-\frac{1}{x})^n\right) \approx x(1-e^{-n/x})$, where $1/x$ is the probability that a given consumer is selected for a given ad impression.

$D_i^l + D^s$ consumers on outlet *i*. However, this creates a capacity management issue if the outlet cannot distinguish loyal from switching consumers. If an outlet does not impress all consumers in the 'first period' it will have to impress them in the second period. However, unless it can distinguish between loyals and switchers in the first period, some consumers may move to the other outlet and it will be unable to fulfill its contract. Alternatively, it could impress all consumers in the first period and perhaps identify the new switchers as unique consumers to impress in the second period. But, even in that case, loyals, who remain with the same outlet through the second period, will have additional capacity that can be sold. In principle, that capacity could be sold under a "impress all unique consumers" contract but this would mean that the outlet would have to offer a range of distinct products to advertisers. This is an interesting and potentially realistic scenario, but we leave it for future work.

10.2 Market Equilibrium with asymmetric capacities and uniformly distributed valuations.

Proposition A1. *Suppose that outlets are symmetric in readership,* $F(v) \sim U[0,1]$ *but that* $a_1 < a_2$ *. Suppose* $a_2 < \frac{2-D^s}{4}$ *. Then an equilibrium exists if and only if* $1-2a_2 < \dfrac{2-D^s}{2}(1-2a_1)$ *. If* $a_2 > \frac{2-D^s}{4}$ *then an equilibrium exists if and only if* $\dfrac{2D^s}{2+D^s}(1-a_2) < \dfrac{2-D^s}{2}(1-2a_1)$ *.*

Proof: Suppose that $a_1 < a_2$. As the aggregate demand of outlet two exceeds that of outlet one only if $\hat{p}_1 > \hat{p}_2$ then a pair of prices is part of a market equilibrium only if $\hat{p}_1 > \hat{p}_2$. We shall divide the proof in two cases depending on whether the set of high types is empty or not. Suppose this is the case ($v_3 > 1$). In this case, the market clearing condition amounts to:

$$2a_1 = 1 - v_2 \tag{27}$$

$$2a_2 = 1 - v_2 + v_2 - v_1. \tag{28}$$

as outlet 1 only sells to multi-homers while outlet 2 sells to all of the single-homers. If equilibrium exists then the market clearing prices must be

$$\left(\hat{p}_2 = 1 - 2a_2,\ \hat{p}_1 = \tfrac{D^l + \frac{1}{4}D^s}{D^l + \frac{1}{2}D^s}(1-2a_1)\right) \tag{29}$$

Such equilibrium exists if and only if

$$\hat{p}_2 = 1 - 2a_2 < \tfrac{D^l + \frac{1}{4}D^s}{D^l + \frac{1}{2}D^s}(1-2a_1) = \frac{2-D^s}{2}(1-2a_1) = \hat{p}_1. \tag{30}$$

Using the identity $2D^l + D^s = 1$ delivers:

$$1 - 2a_2 < \frac{2-D^s}{2}(1-2a_1)$$

That is if $a_2 - a_1$ is large enough. Note that an equilibrium always exists if $D^s = 0$ as the condition reduces to $a_1 < a_2$. By continuity an equilibrium also exists for $D^s > 0$.

This derivation assumes that $v_{21} > 1$. If this was not the case and if $\hat{p}_1 > \hat{p}_2$ then the market clearing conditions for the asymmetric equilibrium would become:

$$2a_1 = 1 - v_2 \tag{31}$$

$$2a_2 = 2(1 - v_3) + v_3 - v_1 \tag{32}$$

as only outlet 2 sells additional impressions to some multi-homers. Thus, outlet 1's price would remain as in (29) while outlet 2's pricing condition would satisfy (substituting v_{21} into (32)):

$$\hat{p}_2 = \tfrac{2D^s}{2+D^s}(1 - a_2) \tag{33}$$

This would be an equilibrium so long as $v_{21}(p_2) < 1$ or $a_2 > \frac{2-D^s}{4}$ in addition to the ad capacity asymmetries as identified earlier. The condition provided in the statement is equivalent to requiring $\hat{p}_1 > \hat{p}_2$.

10.3 Endogenous Ad Capacities

The results in the paper focused on comparative statics for exogenous ad capacities. We now endogenize capacity choice, so that outlets can commit to smaller capacity levels than could be potentially supplied, focusing on how it relates to both readership share and the share of multi-homing consumers. Observe that the choice here for outlets is capacity *per consumer per unit of attention*. We do *not* allow outlets to sell different quantities of advertising to different types of consumers.

First, however, it is useful to provide a general discussion of how the extension to our model affects our results. Consumer switching creates competition among outlets, and so standard Cournot-type forces operate. This analysis allows us to isolate the novel forces introduced by consumer switching. First, consider the case of perfect tracking, whereby switching affects outcomes by increasing competition among the outlets. Then, with endogenous capacity, outlet profits always fall when consumer switching rise so long as outlets are sufficiently symmetric in terms of readership share. In contrast, for sufficiently asymmetric outlets, increased switching increases profits for the less popular outlet and decreases profits for the more profitable outlet.

Next, consider the more interesting case of imperfect tracking, where we focus on symmetric readership across outlets. The analysis is complicated by the fact that for positive but low levels of switching, there is no pure strategy equilibrium in capacity choices. For moderate to high levels of switching, we find that, relative to no switching, equilibrium capacity levels are higher and profits are lower. In addition, as switching increases, outlet profits fall. Thus, the case (ii) above, whereby outlet profits increase as switching increases, cannot occur.

Finally, as discussed in the introduction, our model contrasts with the prior literature where outlets compete for users by reducing ad capacities, where ads are disliked by consumers. In Anderson and Coate (2005), the advertising prices are always monopoly prices, so the level of ad capacity is determined by trading off deviations from monopoly levels against increasing the user base. The ad capacity of other firms does not affect advertising prices. Although we leave this extension for future work, we can say a few things about what would happen in an extension to our model where ad capacities are endogenous and consumers dislike ads. In our model the problem is more complex, since opponent ad capacities do affect advertising prices directly. Nonetheless, the comparative statics about outlet profits and asymmetric ad capacities suggest that there can be novel incentives created by the impact of capacity on advertising prices. For example, the advantage gained by having the larger consumer base may intensify competition for consumers, creating a force in favor of lower equilibrium ad capacities.

10.3.1 Perfect Tracking

We first consider the perfect tracking case. We assume that there are only two outlets to focus on the impact of outlet asymmetry.[33] This means that an outlet will face demands for two sets of consumers – one set that it has monopoly control over and the other for which it competes with its rival a la Cournot. We now consider an analysis of the comparative statics of competition in this set-up.

We can write profits as a function of capacity, readership share and ρ:

$$\pi_i(a_i, a_j; x_i, \rho) = P(a_i + a_j) a_i D_{ij}^s(x_i, 1 - x_i, \rho) + P(2a_i) 2a_i D_i^l(x_i, \rho)$$

Let $MR_i^D(a_i, a_T) = \left(a_i P'(a_T) + P(a_T)\right)$ and $MR_i^M(a_i) = 2\left(2a_i P'(2a_i) + P(2a_i)\right)$. The first-order conditions for outlet i imply:

$$MR_i^D(a_i, a_i + a_j) D_{ij}^s(x_i, x_j, \rho) + MR_i^M(a_i) D_i^l(x_i, \rho) = 0.$$

This shows that the outlet considers the relative proportion of switchers and loyals when choosing output, and it will select capacity so that one of the marginal revenue terms is positive while the other is negative. Note that if $a_i > a_j$, then if P is decreasing and concave, $MR_i^D(a_i, a_i + a_j) < 0$ implies that $MR_i^M(a_i) < 0$. Thus, for the outlet with the larger equilibrium capacity, we must have $MR_i^D(a_i, a_i + a_j) \geq 0$ in equilibrium: capacity is chosen lower than the Cournot best response, but higher than the monopoly level for that outlet. The converse is not necessarily true, however; the outlet with small equilibrium capacity may also have $MR_i^D(a_i, a_i + a_j) \geq 0$ (and indeed, this holds in the case of uniformly distributed advertiser valuation).

The impact of an increased readership share on the incentive to expand capacity is:

$$\begin{aligned} \tfrac{\partial^2}{\partial a_i \partial x_i} \pi_i &= MR_i^D(a_i, a_i + a_j) \tfrac{\partial}{\partial x_i} D_{ij}^s(x_i, 1 - x_i, \rho) + MR_i^M(a_i) \tfrac{\partial}{\partial x_i} D_i^l(x_i, \rho) \\ &= MR_i^D(a_i, a_i + a_j) 2\rho(1 - x_i) + MR_i^M(a_i)(1 - \rho + 2x_i\rho) \end{aligned}$$

---

[33] All of the qualitative predictions in this subsection apply for a general $F(.)$ assumed to be log-concave. (Proofs available from the authors).

At an equilibrium choice of capacity, the ratio of the marginal revenue terms is equal to the ratio of switchers to loyal users, so that we will have (where $\hat{a}_i$ is the equilibrium capacity for *i*):

$$\tfrac{\partial^2}{\partial a_i \partial x_i}\pi_i\Big|_{(a_i,a_j)=(\hat{a}_i,\hat{a}_j)} = MR_i^D(\hat{a}_i,\hat{a}_i+\hat{a}_j)\left(\tfrac{\partial}{\partial x_i}D_{ij}^s(x_i,1-x_i,\rho)-\frac{D_{ij}^s(x_i,x_j,\rho)}{D_i^l(x_i,\rho)}\tfrac{\partial}{\partial x_i}D_i^l(x_i,\rho)\right)$$
$$= -MR_i^D(\hat{a}_i,\hat{a}_i+\hat{a}_j)2\rho(1-x_i)\left(\frac{x_i\rho}{1-(1-x_i)\rho}\right)$$

Since higher readership share increases the proportion of loyal users, its direct effect on capacity is negative if and only if $MR_i^D(\hat{a}_i,\hat{a}_i+\hat{a}_j)\geq 0$. Intuitively, becoming larger causes a firm to put more weight on loyal users, giving it the incentive to reduce output. However, clear equilibrium comparative statics are complicated by the fact that Cournot outputs are strategic substitutes.

We can also consider the impact of switching on capacity choice:

$$\tfrac{\partial^2}{\partial a_i \partial \rho}\pi_i = MR_i^D(a_i,a_i+a_j)\tfrac{\partial}{\partial \rho}D_{ij}^s(x_i,1-x_i,\rho)+MR_i^M(a_i)\tfrac{\partial}{\partial \rho}D_i^l(x_i,\rho)$$
$$= MR_i^D(a_i,a_i+a_j)2x_i(1-x_i)-MR_i^M(a_i)x_i(1-x_i)$$

At an equilibrium capacity choice, we will have

$$\tfrac{\partial^2}{\partial a_i \partial \rho}\pi_i\Big|_{(a_i,a_j)=(\hat{a}_i,\hat{a}_j)} = MR_i^D(\hat{a}_i,\hat{a}_i+\hat{a}_j)2x_i^2(1-x_i)\left(\frac{1-2\rho(1-x_i)}{1-\rho(1-x_i)}\right)$$

So long as switching is not too prevalent and outlets are not too asymmetric, switching decreases the share of loyal users, so that the direct effect of switching on capacity is positive if and only if $MR_i^D(\hat{a}_i,\hat{a}_i+\hat{a}_j)(1-2\rho(1-x_i))\geq 0$. Thus, the direct effect is unambiguously positive for the outlet with the larger share.

Using the envelope theorem, we can write the impact of ρ on profits as follows:

$$\tfrac{d}{d\rho}\pi_i(a_i^*(x_i,\rho),a_j^*(x_i,\rho);x_i,\rho) = P'(a_i^*+a_j^*)a_i^*D_{ij}^s(x_i,1-x_i,\rho)\tfrac{\partial}{\partial\rho}a_j^*(x_i,\rho)$$
$$+P(a_i^*+a_j^*)a_i^*\tfrac{\partial}{\partial\rho}D_{ij}^s(x_i,1-x_i,\rho)+P(2a_i^*)2a_i^*\tfrac{\partial}{\partial\rho}D_i^l(x_i,\rho)$$
$$= P'(a_i^*+a_j^*)a_i^*D_{ij}^s(x_i,1-x_i,\rho)\tfrac{\partial}{\partial\rho}a_j^*(x_i,\rho)$$
$$+2P(a_i^*+a_j^*)a_i^*x_i(1-x_i)-2P(2a_i^*)a_i^*x_i(1-x_i)$$

Switching has an indirect effect through increasing the opponent's output, which (if it increases opponent capacity) lowers price and thus profits. It also has a direct effect of increasing the proportion of switchers and decreasing the proportion of loyals. The sum of the last two terms is negative if and only if $a_i^*\leq a_j^*$: for the lower-capacity outlet, switchers are less profitable. The analysis for the outlet with the higher equilibrium output appears ambiguous if its competitor's output is increasing in ρ, as the price effect and the switcher/loyal effect move in opposite directions.

Summarizing the discussion so far, we can gain some intuition about the direct effects of parameter changes on outlet capacity choices and profits, but some additional structure on demand is required to obtain unambiguous comparative statics results. To do so, we focus on the case of linear demand (uniformly distributed advertiser valuations). The following proposition demonstrates that the larger outlet will provide the lowest advertising capacity.

Proposition A2. *Suppose that there are two outlets and that* $F(v) = v$*. Equilibrium advertising for each outlet,* $\hat{a}_i$ *are non-increasing in readership share,* x_i*. Equilibrium advertising* $\hat{a}_i$ *is non-decreasing in* ρ *if* $x_i \le (21-\sqrt{249})/6 \approx .87$ *or* $\rho \le (2/3)(3-\sqrt{3}) \approx .84$ *. Total ad capacity,* $\hat{a}_i + \hat{a}_j$ *, is non-decreasing in* ρ*. For sufficiently symmetric firms (*$.33 \le x_i \le .67$*), profits of both firms are decreasing in* ρ*, while for sufficiently asymmetric firms, profits are decreasing (increasing) in* ρ *for* $x_i > (<) x_j$*.* $\pi_i^{PT}/x_i < \pi_j^{PT}/x_j$ *when* $x_i > x_j$*.* $\pi_i^{PT} - \pi_j^{PT}$ *is decreasing in* ρ *for* $x_i > x_j$*.*

PROOF: Solving for the unique Nash equilibrium with the uniform distribution we have:

$$\hat{a}_i = \frac{16D_i^l D_j^l + 6D_j^l D^s + 4D_i^l D^s + D^{s2}}{64D_j^l D_i^l + 16(D_j^l + D_i^l)D^s + 3D^{s2}} \tag{34}$$

$$\pi_i^{PT} = \frac{(4D_i^l + D^s)(16D_i^l D_j^l + 6D_j^l D^s + 4D_i^l D^s + D^{s2})^2}{(64D_j^l D_i^l + 16(D_j^l + D_i^l)D^s + 3D^{s2})^2} \tag{35}$$

The rest of the proposition follows from manipulating these expressions.

We have already developed some intuition for these results, but the uniform distribution gives us more definitive conclusions. Consider the comparative statics of switching on profits. The increase in capacity of an opponent's outlet has a negative impact on each outlet. However, the increase in the share of switchers has a positive (resp. negative) effect on the smaller (larger) outlet, as the share of consumers coming from the switchers goes up. Switchers are more (less) profitable than loyals for the smaller (larger) outlet, because the larger outlet serves less capacity than the smaller outlet. With the uniform distribution, for the small outlet the latter effect dominates the negative effect of increase in capacity and small outlet profits go up.

Note that switching also affects the impact of an increase in readership share on profits. Under the benchmark single-homing consumer case, more readers simply improved profits in a linear fashion; that is π_i^{PT}/x_i was independent of x_i. With perfect tracking, an additional reader attracted from a rival outlet not only causes an outlet to restrict advertising capacity but for that capacity to increase elsewhere (since capacities are strategic substitutes in our Cournot setup), decreasing impression prices for switchers. Thus, outlets with a lower readership share have a higher incentive to attract marginal readers.

It is also useful to note that if the two outlets were commonly owned, their owner would maximize joint outlet profits by setting $a_1 = a_2 = \frac{1}{4}$. In this case, realized profits in this case will be the same as those generated when there are no switchers. Thus, under perfect tracking with $D^s > 0$ there will be an incentive for outlets to merge.

In the absence of common ownership, multi-homing consumers cause outlets to compete for advertisers and a greater proportion of them increases available advertising space and decreases overall profits. However, the question of interest is what this does to the marginal incentive to attract an additional reader at the expense of rivals. What we can demonstrate is that as $x_i \to 0$ or $x_i \to 1$, then $\frac{\partial \pi_i^{PT}}{\partial x_i} > \frac{\partial \pi_i^{NS}}{\partial x_i} = \frac{1}{4}$. It is useful to note that if both outlets are commonly owned (i.e., in a monopoly), then profits under perfect tracking are the same as profits earned for each outlet in the no switching case. Thus, competition is the source of any reduction in profits as a result of switching but this competition can, in turn, promote higher incentives to attract readership when there are asymmetric readership shares.

10.3.2 Imperfect tracking

We now turn to consider endogenous capacity for the case of imperfect tracking. Our goal here is to explore the robustness of the comparative static results on D^s, with ad capacity was exogenous; recalling our main finding that as D^s rose, impression prices and outlet prices fell except for high a when D^s was large. As in the perfect tracking case, we suppose that competition comprises two. In stage 1, both outlets simultaneously choose their ad capacities. In stage 2, the market clears based on those capacities and prices and profits are realized. It turns out that, in this situation, a pure strategy equilibrium in the Stage 1 (Cournot) game does not exist for a non-trivial rage of D^s. Given this, we then consider a Stackelberg Stage 1. Significantly, we demonstrate that advertising capacity, while asymmetric in this equilibrium between the two outlets, does not reach a level whereby an increase in D^s leads to an increase in the profits of either outlet.

The Cournot game equilibrium outcomes are summarized by the following

Proposition A3. *Suppose that outlets are symmetric in readership, $F(v) = v$ and V = 1. With endogenous capacity, $F(v)$ = v and symmetric readership shares, the pure strategy equilibrium outcomes are:*

(i) For $D^s = 0$, $a_i = \frac{1}{4}$ with per consumer profits of $\pi_i = \frac{1}{4}$ for all i.

(ii) For $D^s \geq \frac{4}{9}$, $a_i = \frac{1}{3}$ with per consumer profits of $\pi_i = \frac{2(2-D^s)}{4-D^s}\frac{2}{9}$ for all i.

Otherwise no pure strategy equilibrium exists.

PROOF: Note that for $D^s = 0$, $v_{12} = p$ and the asymmetric equilibrium holds for any $(a_1, a_2) \notin (\frac{1}{4}, \frac{1}{4})$. In any asymmetric equilibrium, per consumer profits equal $(1-2a_i)2a_i$ for each outlet; which is maximized at a capacity of ¼. Hence, by deviating, each would receive no greater profits than they do under the equilibrium as specified in (i).

To check that outcome (ii) is an equilibrium, observe that if each outlet plays a local best response, they each choose capacity equal to $\frac{1}{3}$. Now consider a choice $a_1 >> \frac{1}{3}$ so that $p(a_1, a_2) \leq \frac{1}{2} D^s$. In this case, the highest profits outlet 1 could earn are: $\max_{a_1} \frac{D^s(2-D^s)}{4+D^s(2-D^s)}(3-2(a_1+\frac{1}{3}))2a_1$ which is maximized at $a_1 = \frac{7}{12}$; which would create the asymmetric equilibrium. Thus, the maximum capacity 1 would chose would be $\frac{1}{12}(4+D^s)$ resulting in profits of $\frac{1}{36}(2-D^s)(4+D^s) < \frac{2(2-D^s)}{4-D^s}\frac{2}{9}$. Now consider a choice $a_1 << \frac{1}{3}$ so that $\sigma_1 = 0$; specifically, $a_1 \leq \frac{4-\frac{1}{3}D^s}{6(2-D^s)}$. In this case, outlet 1 maximizes profits with a choice of $a_1 = \frac{1}{4}$ earning profits of $p_1 2a_1 = \frac{D_1^l + \frac{1}{4}D^s}{D_1^l + \frac{1}{2}D^s}\left(1-2a_1\right)2a_1 = \frac{1}{8}(2-D^s)$ which is greater than $\frac{2(2-D^s)}{4-D^s}\frac{2}{9}$ for $D^s \leq \frac{4}{9}$. When $D^s > \frac{4}{9}$, this deviation is not profitable. Finally, we need to check that, in fact, $p(a_1, a_2) \geq \frac{1}{2} D^s$. This implies that $\frac{1}{2}D^s < \frac{2(2-D^s)}{4-D^s}\frac{1}{3} \Rightarrow D^s < \frac{2}{3}(4-\sqrt{10})$ which always holds for $D^s \leq \frac{1}{2}$.

We now turn to establish that there are no other pure strategy equilibria. First, note when $p < \frac{1}{2}D^s$, it is easy to see that $a_1 = \frac{1}{2}$ is a local best response to $a_2 = \frac{1}{2}$. At this point, each outlet earns profits of $\frac{D^s(2-D^s)}{4+D^s(2-D^s)}$. Note, however, that any deviation from these capacities generates the asymmetric equilibrium. Thus, setting $a_1 >> \frac{1}{2}$ would earn that outlet profits of $\frac{2D^s}{2+D^s}(1-a_1)2a_1$ which are maximized at ½ and exceed $\frac{D^s(2-D^s)}{4+D^s(2-D^s)}$ at this point. A reduction in capacity would involve maximum profits at $a_1 = \frac{1}{4}$. In this case, it is easy to establish that $\frac{D^s(2-D^s)}{4+D^s(2-D^s)} < \frac{1}{8}(2-D^s)$ and so a large reduction in ad capacity is a profitable deviation for outlet 1. Thus, no equilibria of this type exists.

What about an asymmetric equilibrium? Any equilibrium would involve the outlet with the smaller capacity, say 1, choosing $a_1 = \frac{1}{4}$ while the other outlet chooses $a_2 = \frac{1}{8}(2+D^s)$. Note that this is consistent with $v_{12'} > 1$ and it is straightforward to establish that outlet 2 would not want to choose a higher ad capacity to change this. In this case, outlet 2 earns per consumer profits of $(1-2a_2)2a_2$ and it is easy to determine that these are decreasing in a_2 at $a_2 = \frac{1}{8}(2+D^s) > \frac{1}{4}$. Therefore, given 1's choice, 2 would not find it profitable to expand output. Contracting it would generate profits of $\frac{2(2-D^s)}{4-D^s}(1-\frac{1}{4}-a_2)2a_2$; maximized at 3/8 which would involve too much asymmetry to generate that outcome. Thus, any contraction involves profits less than $\frac{1}{16}(4-D^{s2})$. For 1, $a_1 = \frac{1}{4}$ is a local best response, but by choosing a higher ad capacity, it may earn different profits depending upon the resulting impression price. For $p \geq \frac{1}{2}D^s$, outlet 1 would earn per consumer profits of $\frac{2(2-D^s)}{4-D^s}(1-\frac{1}{8}(2+D^s)-a_1)2a_1$ which is maximized at $a_1 = \frac{1}{16}(6-D^s)$. However at this capacity, ad capacities would be sufficiently asymmetric that this would not be feasible. Instead, outlet 1 is constrained to a capacity no more than $\frac{1}{16}(4+4D^s-D^{s2})$. Note that this results in a price $p = \frac{2(2-D^s)}{4-D^s}(1-\frac{1}{8}(2+D^s)-\frac{1}{16}(4+4D^s-D^{s2})) \geq \frac{1}{2}D^s$. It is straightforward to demonstrate that this deviation is profitable for 1. A similar reasoning holds for the case where $p < \frac{1}{2}D^s$. Thus, there is no pure strategy equilibrium involving asymmetric capacity choices.

Intuitively, for smaller levels of D^s, each outlet would prefer to be the outlet with the larger capacity so long as the required asymmetry is not too large. When that occurs, their preferences switch. Consequently, there is a (downwards) discontinuity in the best response functions of each outlet for $D^s \in (0, \frac{4}{9})$ and no pure strategy equilibrium exists.

Given the lack of a pure strategy equilibrium for a non-trivial set of parameters, we might consider a mixed strategy equilibrium. However, given this application, it is unclear whether mixing in its strict form is something that we would expect to see; specifically, because ad capacity may be a design decision for web pages.[34] As an alternative, the following proposition characterizes the Stackelberg outcome where one outlet chooses its ad capacity prior to the other.

Proposition A4. *Suppose that outlets are symmetric in readership, $F(v) = v$ and V = 1. In a sequential move game where outlet 1 chooses a_1 before outlet 2 chooses a_2, the unique equilibrium outcome involves $a_1 = \frac{2+\sqrt{2D^s}-2D^s}{4(2-D^s)}$ and $a_2 = \frac{1}{4}$ with per consumer profits of $\pi_1 = \frac{2-3D^s+\sqrt{2D^{s3}}}{2(2-D^s)^2}$ and $\pi_2 = \frac{1}{8}(2-D^s)$.*

PROOF: If $a_1 = \frac{2+\sqrt{2D^s}-2D^s}{4(2-D^s)}$, then outlet 2 is indifferent between $a_2 = \frac{1}{4}$ or setting its capacity high enough to ensure that outlet 1 only has multi-homers; that is, $a_2 \geq \frac{1}{4}(2a_1(2-D^s)+D^s = \frac{1}{8}(2+\sqrt{2D^s})$. So 2 has no incentive to deviate. Outlet 1 has no incentive to increase capacity as this lowers its asymmetric equilibrium profits. It could, however, decrease capacity. This would result in 2 no longer being indifferent between a high and low capacity and choosing a high capacity, $\frac{1}{4}(2a_1(2-D^s)+D^s$. This would result in profits for 1 as the low capacity outlet in the asymmetric equilibrium which are maximized at ¼ yielding $\frac{1}{8}(2-D^s)$. These

[34] Frankly, we have also been unable to identify the mixed strategy equilibrium although we know the set that contains its support and that that set converges to (¼, ¼) as D^s goes to 0.

are less than the equilibrium profits and hence, there is no profitable deviation for 1.

The result here is related to Proposition A1 where the low capacity outlet always had profits of the form $(1-\frac{1}{2}D^s)(1-2a)2a$ and did not have any single-homing advertisers. These profits are maximized with a capacity of ¼. Thus, if outlet 1 chooses a1 high enough, outlet 2 will be the low capacity outlet and choose a capacity of ¼. The proof then demonstrates that outlet 1 will prefer to be the high capacity rather than the low capacity outlet.

Importantly, an examination of the equilibrium profits of both outlets shows that in each case these are decreasing in D^s. Thus, outlet 1's ad capacity never reaches the level whereby for high enough D^s, impression prices and its profits would fall as D^s increases.