THE DYNAMIC DEMAND
FOR CAPITAL AND LABOR

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The Dynamic Demand for Capital and Labor

ABSTRACT

A model of the dynamically interrelated demand for capital and labor
is specified and estimated. The estimates are of the first-order conditions
of the firm's problem rather than of the closed-form decision rules. This
use of the first-order conditions allows a random rate of return and a
flexible specification of the technology. The estimates do not imply the
very slow rates of adjustment displayed in other, related estimates of the
demand for capital. Because adjustment is estimated to be rapid, there is,
contrary to the standard view, scope for factor-prices to affect investment
at relatively high frequencies.

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I. Introduction

There is a significant gap between the theory and empirical work in the standard models of investment. This paper offers some plausible estimates of investment dynamics that are both consistent with a structural model and useful for policy analysis. Because the choice of capital stock is inherently connected with the choice of other factors, labor in particular, the model is of capacity choice in general instead of investment in particular. When a firm purchases or hires factors of production, it determines its productive capacity. Short-run variation in output comes principally from variation in factors that are adjustable at little or no cost. Investment responds relatively slowly to shocks because of adjustment costs.

In this paper I provide structural estimates of demand for factors--capital in particular. These estimates can be used for analyses of tax policy that are immune from the Lucas critique. The dynamic response of investment implied by the estimates is more plausible than that found in previous research. I use a model where the firm maximizes the present discounted value of profits subject to a technology with adjustment costs. Such a specification has a long history in the investment literature. Moreover, it is a generalization of Tobin's q-model, which is equivalent to an adjustment cost model where only capital is costly to adjust. The firm chooses its capital stock, its number of production and non-production workers, and hours of production workers.

I estimate the first-order conditions, or Euler equations, of the problem rather than the closed-form decision rules. The estimation technique is Hansen's [1982] generalized method of moments. To evaluate the performance of the model, I consider its dynamic properties. My estimates

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of the dynamic response of the capital stock do not have the implausibly long adjustment lags found in other research. Because adjustment is rapid, changes in interest rates and factor prices can affect the demand for capital at business cycle frequencies.

Factor prices determine investment in the neo-classical models of Jorgenson and Hall (see Jorgenson [1963] and Hall and Jorgenson [1967]), the cost-of-adjustment models (see Lucas [1967], Treadway [1969], and Lucas and Prescott [1971]), and the q-models (see Brainard and Tobin [1968] and Tobin [1969]). Abel [1979, 1982, 1983] and Hayashi [1982] show that the cost-of-adjustment models and the q-models are essentially equivalent. I use a generalization of the cost-of-adjustment model that takes into account the choice of number of employees and their hours of work as well as the choice of the stock of capital.

The q-approach has not yielded satisfactory estimates of the parameters of the investment function. Abel and Blanchard [1983b] resort to a sales-investment specification because of their difficulty in usefully explaining investment with q (see Abel and Blanchard [1983a] for the estimates with q). Their dissatisfaction with the q-approach does not arise from an a priori defect in the theory but in its empirical performance. Although the market value of publicly traded corporate capital is easy to measure, it is difficult to measure its replacement cost. Moreover, the appropriate concept in a q-equation is marginal rather than average q .[1] Marginal q cannot be directly measured, although it can in principal be constructed. Indeed, a q is implied in the calculations in this paper. Nonetheless, the implementation of the model in this paper does not depend on constructing a series for q .

Summers's [1981] paper highlights the problem with the estimates based

on q-theory. In particular, he finds extremely slow adjustment of the capital stock to changes in factor prices. The estimates offered in this paper do not have this shortcoming. By examining the adjustment problem directly, rather than through a summary statistic such as q , I obtain reasonable rates of adjustment.

II. The Firm's Decision Problem

The representative firm maximizes the present discounted value of cash flow. The choice variables of the firm are the purchases of capital, the net hires of employees, and the number of hours that the employees work. I distinguish between production and non-production workers. These choices determine output; thus, output is endogenous. The firm takes factor prices and the investors' ex ante required rate of return as exogenous.

Stocks of factors are costly to adjust, so lagged factor stocks enter the decision rule of the firm. That is, the firm in the short run does not hire the long-run, profit-maximizing level of inputs. Because the exogenous variables change over time, the firm takes into account their expected path in making its current decisions. Since the time path of factors determines the time path of output, no separate choice of capacity utilization is made.

I now consider the individual components of real cash-flow: real output, real labor cost, and real capital cost. I combine these below to obtain the expression for discounted cash-flow.

A. Production Function

The real output of the firm is given by the production function

(1) $y_t = f(K_t, L_t, N_t, H_t, K_t - dK_{t-1}, L_t - q_{t-1}L_{t-1},$

$N_t - q_{t-1}N_{t-1}, H_t - H_{t-1}, X_t) = f(Z_t, \Delta Z_t, X_t)$

where K_t is the stock of capital, L_t the number of production workers, N_t the number of non-production workers, and H_t the average hours of production workers. The parameter d is one minus the rate of depreciation of capital and q_{t-1} is one minus the quit rate, that is, the rate at which the stock of workers depreciates. For notational convenience, Z_t and ΔZ_t denote, respectively, vectors of the levels and gross changes of the factors. The vector X_t represents unobserved factors in the production function. Trend productivity and shocks to the production function are the obvious examples of such unobservables. Measured output, y_t, is value-added so intermediate inputs are neglected.[2]

Output depends on the level and gross rate of change of the factors of production. Output is allowed to depend on the rates of change of the inputs to allow for adjustment costs. Brechling and Mortenson [1969] and Brechling [1975] characterize in detail the properties of a production function such as (1). The adjustment costs are internal to the production process. That is, the cost of output lost when the factors of production are varied. This cost does not represent specific payment to a factor. External adjustment costs (such as the purchase cost of capital) are accounted for elsewhere.

In adjusting the stock of labor, output is lost through the inexperience of new workers and the time taken to readjust the schedule and pattern of production. Adjustment costs for the number of employees are likely to be much more important than that for the average hours worked. Output

is lost when capital is adjusted through the lost production time during installation, the difficulty of incorporating new machines into the production process, and the labor input diverted to install the new capital.

The production function must be parameterized in order to render the theory empirically useful. Some researchers estimate closed-form decision rules rather than Euler equations. Examples are Sargent [1978], Kennan [1979], Meese [1980], and Hansen and Sargent [1980, 1982]. They specify the production functions so the problems are linear quadratic. This specification yields closed-form decision rules under rational expectations.

Epstein and Denny [1983] offer a general functional form and then approximate it for estimation. They assume static expectations, which is a serious shortcoming given the explicitly dynamic nature of the problem. Indeed, they do not specify the source of the randomness in the firm's environment. As I discuss in Section IV, the structure of the error term in the factor demand equations determines what instruments are valid to identify the parameters. Therefore, it is difficult to evaluate the identification of their parameters in the absence of explicit assumptions about the stochastic environment facing the firm. Epstein and Denny do discuss the restrictions on the shape of the production function. In particular, they test whether it is concave. An exception to the practice of using quadratic approximations is Pindyck and Rotemberg [1983], who derive conditional factor demands on the basis of trans-log cost functions.

I estimate the Euler equations themselves, rather than an approximated decision rule, that is, the solution of the Euler equation. This procedure allows important complications to be introduced. These include non-quadratic specifications of the production function, a non-linear wage bill, and a variable rate of discount. Only efficiency of estima-

tion is lost by estimating the Euler equation instead of the decision rules. In particular, the technique of Hansen and Sargent [1980, 1982] of estimating the decision rules exploits the restrictions between the demand equations and the stochastic processes of the variables on the other side of the market. The decision rules also impose the transversality conditions. The Euler equation and decision rules estimate the same parameters. More information (cross equation restrictions and transversality conditions) are used to estimate the decision rules. This gain in efficiency seems a high price to pay for restricting the rate of return to be constant and for having to make strict assumptions about the technology. In this research I need not impose the extreme restrictions on the structure of the problem which are made by Hansen and Sargent to yield closed-form, rational expectations solutions.

In presenting the basic results, I choose the Cobb-Douglas production function for tractability and ease of interpretations. I also present alternative estimates with a constant elasticity of substitution production function. The Cobb-Douglass function has the advantage of simplicity so one can inspect the parameters for the plausibility of their magnitudes as well as for statistical significance. The results of other studies which report production function parameters in terms of the steady-state values of derivatives (that is, the quadratic specification commonly used in the rational expectations literature), are difficult to evaluate. In particular, assumptions of convexity and constant returns to scale are often neither imposed nor tested.[3]

Many authors assume that employees and hours enter multiplicatively in the production function. Both Sargent [1978] and Meese [1980] make this assumption. Indeed, Meese makes the much stronger assumption that

firms choose man-hours alone instead of separately choosing number of employees and hours of work. (Sargent handles this issue by distinguishing between straight-time and overtime hours.) Employees and hours have different marginal costs. Adding an hour may entail an overtime premium; adding an employee may involve fixed costs such as health insurance. Employees and hours also have different adjustment costs. Hence, assuming that the decision variable is man-hours rather than hours and employees separately distorts the firm's choice problem even if hours and employees enter the production function multiplicatively. Treating the decision variable as man-hours instead of employees and hours separately may also lead to an incorrect linearization of the production function even if those variables enter it multiplicatively. Additionally, straight-time and average overall wages vary insubstantially over the business cycle. Overtime hours do vary substantially causing the marginal cost of an hour worked to be higher than the basic data suggest. The dynamic factor demand models rely on variation in factor prices to explain the dynamics of demand; neglecting this substantial variation in cost is a mistake.

Some authors, such as Fair [1969], Nadiri and Rosen [1969], and Bernanke [1983], stress that hours and employment may not enter multiplicatively in the production function. In this paper, I also let them enter separately.

The production function I use for the basic results is

$$(2) \qquad \log y_t = \log[f(Z_t, \Delta Z_t, X_t)]$$

$$= \log Z_t a - \frac{1}{2} \Delta Z_t G \Delta Z_t' + X_t$$

where

$$a = (a_K, \ a_L, \ a_N, \ a_H)' \ ,$$

$$G = \begin{pmatrix} g_{KK} & g_{KL} & g_{KN} & g_{KH} \\ & g_{LL} & g_{LN} & g_{LH} \\ & & g_{NN} & g_{NH} \\ & & & g_{HH} \end{pmatrix} , \quad \text{symmetric,}$$

$$\log Z_t = \begin{pmatrix} \log K_t \\ \log L_t \\ \log N_t \\ \log H_t \end{pmatrix}' , \quad \text{and}$$

$$\Delta Z_t = \begin{pmatrix} K_t - dK_{t-1} \\ L_t - q_{t-1}L_{t-1} \\ N_t - q_{t-1}N_{t-1} \\ H_t - H_{t-1} \end{pmatrix}' .$$

That is, the log of output is a linear function of the log of the levels of inputs minus a quadratic function of the gross changes of the inputs. Hence, the production function is Cobb-Douglas augmented by adjustment cost terms. The vector X_t denotes the productivity shock. It, in general, will contain constant and trend terms as well as stochastic productivity shocks of arbitrary serial correlation. As long as X_t enters the production function additively in logs, it will drop out of the estimated Euler equation. Hence, the productivity shock term need not be parameterized.

I impose the constant-returns-to-scale restriction on output gross of adjustment costs so that $a_K + a_L + a_N = 1$. The form of the function

implies convexity. There is no natural CRS restriction on the cost of adjustment parameters. That part of the function is convex as long as the matrix of the g parameters is positive definite. I do impose the requirement that the matrix of the g parameters is symmetric. It is useful to note that I do not restrict the coefficient of hours, H_t, when I impose constant returns to scale. It is natural to leave a_H unrestricted: a replication of capital and labor would presumably leave hours per worker constant. The difference $a_H - a_L$ measures the departure of employees and hours from entering the production function multiplicatively.

I assume that capital and labor have adjustment costs related to the change in the stock. The adjustment cost for capital is a function of the square of gross investment, $K_t - dK_{t-1}$. The depreciation rate is taken to be parametric so the capital series can be derived from the gross investment series. An alternative specification would be to use net investment. The rationale for using net investment would be that replacement investment is somehow more routine and therefore less costly than net investment. Machines are rarely replaced one-for-one; consequently, there is not a clear distinction between replacement and new investment. Therefore, there is not a strong case of using net investment.

Likewise, the adjustment cost for employees is proportional to the gross change in the number of employees, $L_t - q_{t-1}L_{t-1}$ and $N_t - q_{t-1}N_{t-1}$, where q_{t-1} is one minus the quit rate. As with the depreciation rate, the firm takes the quit rate as given. Unlike the depreciation rate, the quit rate is published data and varies across time. The choice variable of the firm is the current stock of employees. In arriving at the figure it takes into account the probability that a fraction of the workers will

quit each period.

I also allow there to be interrelated adjustment costs in (2). These arise if the lost production time through adjusting one factor is greater or less when the firm adjusts other factors.

I expect there to be little or no cost to adjusting hours. Firms can adjust hours merely by extending the length of the shift. (Diminishing marginal product is, of course, captured in the production function.) Such an adjustment is likely to be much easier than adding a worker. In particular, extending the operation of the plant for an hour requires no realignment of workers and machines. An extra worker, on the other hand, must be incorporated into the pattern of production.

B. Cost of Labor

Now I consider the cost of the labor to the firm. Labor cost is a function of the number of employees, the hours they work, and rate of compensation of employees not sensitive to hours worked. Compensation not sensitive to hours worked includes all payments to salaried employees and certain non-wage payments to hourly workers.

I assume only hours of production workers are variable and that all non-production workers are salaried. The average wage is an increasing function of hours worked due to the overtime premium. Suppose that the wage bill for production workers is given by

$$(3) \qquad w_t^* L_t H_t = w_t L_t [w_0 + H_t + w_1 (H_t - 40) + w_2 (H_t - 40)^2] + v_t^W$$

where w_t^* is the average wage, w_t the straight-time wage, and v_t^W is a measurement error. (Abel [1979] gives a theoretical discussion of such a wage bill in a similar context.) The coefficient w_1 is the overtime

premium. I include parameters w_0 and w_2 to allow a more general specification.[4]

Labor cost is the sum of the wage bill for the production workers, their non-wage compensation, and the compensation of non-production workers. Summing these gives

(4) total labor cost

$$= w_t L_t [w_0 + H_t + w_1 (H_t - 40) + w_2 (H_t - 40)^2] + s_t^L L_t + s_t^N N_t + v_t^W,$$

where s_t^L is the non-wage compensation of production workers per worker and s_t^N is the total compensation of non-production workers per worker. Non-wage compensation includes pension contributions and health insurance. It should include only payments not a function of hours worked. Since I assume a firm cannot choose the hours of non-production workers their compensation is a fixed rate per employee.

C. Cost of Capital

The purchase price of capital is

(5) $$p_t^K = p_t (1 - t_t^K PVCCA_t - ITC_t)$$

where p is the price of new capital relative to the price of output (the ratio of the deflators from the National Accounts), PVCCA is the present discounted value of depreciation allowances, and ITC is the effective investment tax credit rate weighted for the composition of investment. The present discounted value of depreciation figures are weighted statutory depreciation rates discounted by the term structure of interest rates. Jorgenson and Sullivan [1981] outline the method for producing such estimates.[5]

Meese [1980] uses the rental cost of capital rather than the purchase cost for the price of capital in a model similar to (1). The rental cost (excluding tax terms for ease of exposition) is

$$p(\rho + \delta)$$

where ρ is the required return of investors and δ is the rate of depreciation. In the problem of maximizing discounted cash flow the investors' required rate of return enters through R_t, the discount rate. Meese's use of the rental rate rather than purchase cost allows him to vary the required rate of return while assuming R_t is constant. Such an assumption allows for closed-form solution, but it makes it difficult to interpret the discount rate; the same required rate of return should enter in both the rental rate and the discount rate.

D. The Firm's Objective

The problem of the representative firm is to maximize expected present discounted value of cash flow. The expected value of real, discounted, after-tax cash flow is

$$(6) \quad E_t \sum_{i=0}^{\infty} R_{t+i} \{ f(Z_{t+i}, \Delta Z_{t+i}, X_{t+i})(1 - t_{t+i}^K) - p_{t+i}^K(K_{t+i} - dK_{t+i-1})$$

$$- [w_{t+i} L_{t+i}(w_0 + H_{t+i} + w_1(H_{t+i} - 40) + w_2(H_{t+i} - 40)^2)$$

$$+ s_{t+i}^L L_{t+i} + s_{t+i}^N N_{t+i}](1 - t_{t+i}^K) \} ,$$

where E_t denotes expectation conditional on information available at time t where Z_t, ΔZ_t, and X_t are as defined above and where

K_t = capital stock

L_t = employees, production workers

N_t = employees, non-production workers

H_t = hours per production worker

f = production function

$d = 1 - \delta$, where δ = depreciation rate

q_t = one minus the quit rate

p_t^K = <u>after-tax</u> purchase price of capital (equation (5))

w_t = straight-time wage

s_t^L = fringe benefits per production worker

s_t^N = compensation per non-production worker

t_t^K = corporate tax rate

$R_{t+i} = \Pi_{j=t+1}^{t+i} r_{j-1}$, where $r_t = 1/(1+\rho_t)$ and ρ_t = required rate of return from periods t to t+1.

The previous sections discuss the individual components of (6) in detail. I estimate the Euler equations implied by (6) together with the equation for the wage bill (4).

The firm's decision variables at time t are its capital stock, number of production workers and non-production workers, and average hours of production workers. Substituting the production function (2) into (6) and differentiating yields the following four first-order conditions:

(7a) $E_t\{[a_K/K_t - g_{KK}(K_t - dK_{t-1}) - g_{KL}(L_t - q_{t-1}L_{t-1}) - g_{KN}(N_t - q_{t-1}N_{t-1})$

$- g_{KH}(H_t - H_{t-1})]y_t(1 - t_t^K) + [g_{KK}(K_{t+1} - dK_t) + g_{KL}(L_{t+1} - q_tL_t)$

$+ g_{KN}(N_{t+1} - q_tN_t) + g_{KH}(H_{t+1} - H_t)]y_{t+1}(1 - t_{t+1}^K)dr_t$

$- p_t^K + dr_t p_{t+1}^K\} = 0$

(7b) $E_t\{[a_L/L_t - g_{KL}(K_t - dK_{t-1}) - g_{LL}(L_t - q_{t-1}L_{t-1}) - g_{LN}(N_t - q_{t-1}N_{t-1})$

$- g_{LH}(H_t - H_{t-1})]y_t(1 - t_t^K) + [g_{KL}(K_{t+1} - dK_t) + g_{LL}(L_{t+1} - q_tL_t)$

$+ g_{LN}(N_{t+1} - q_tN_t) + g_{LH}(H_{t+1} - H_t)]y_{t+1}(1 - t_{t+1}^K)q_t r_t$

$- [w_t(w_0 + H_t + w_1(H_t - 40) + w_2(H_t - 40)^2) + s_t^L](1 - t_t^K)\} = 0$

(7c) $E_t\{[a_N/N_t - g_{KN}(K_t - dK_{t-1}) - g_{LN}(L_t - q_{t-1}L_{t-1}) - g_{NN}(N_t - q_{t-1}N_{t-1})$

$- g_{NH}(H_t - H_{t-1})]y_t(1 - t_t^K) + [g_{KN}(K_{t+1} - dK_t) + g_{LN}(L_{t+1} - q_tL_t)$

$+ g_{NN}(N_{t+1} - q_tN_t) + g_{NH}(H_{t+1} - H_t)]y_{t+1}(1 - t_{t+1}^K)q_t r_t - s_t^N(1 - t_t^K)\} = 0$

(7d) $E_t\{[a_H/H_t - g_{KH}(K_t - dK_{t-1}) - g_{LH}(L_t - q_{t-1}L_{t-1}) - g_{NH}(N_t - q_{t-1}N_{t-1})$

$- g_{HH}(H_t - H_{t-1})]y_t(1 - t_t^K) + [g_{KH}(K_{t+1} - dK_t) + g_{LH}(L_{t+1} - q_tL_t)$

$+ g_{NH}(N_{t+1} - q_tN_t) + g_{HH}(H_{t+1} - H_t)]y_{t+1}(1 - t_{t+1}^K)r_t$

$- w_t L_t[1 + w_1 + 2w_2(H_t - 40)](1 - t_t^K)\} = 0.$

These equations state that, in expectation, marginal product equals marginal cost. I estimate these equations together with the equation for the wage bill (4) with quarterly data for U.S. manufacturing. Thus, I am assuming that the manufacturing sector can be modeled as a representative firm.

III. Data

The previous dynamic factor demand studies use annual or higher frequency data depending on whether or not they use output to explain demand. Consistent output, employment, and investment figures for manufacturing are available in the national income and product accounts only on an annual basis. Examples of studies using such data are Berndt and Morrison [1979], Pindyck and Rotemberg [1983], and Epstein and Denny [1983]. Indeed, the latter two pair of authors use data supplied by Berndt. On the other hand, authors using only factor prices as forcing variables choose data of higher frequency. Examples of such studies are Sargent [1978], Kennan [1979], and Meese [1980].

The data here are quarterly data for manufacturing from 1955 through 1980. I use manufacturing as a compromise between wide coverage and homogeneity of the underlying firms. Output data appear in the estimating stage to reduce the nonlinearity of the estimation problem. The dynamic demands are functions of factor prices and rates of return. Output is determined endogenously in the model. The output data are the quarterly index of manufacturing output produced by the Federal Reserve Board scaled to equal actual output in 1967.

The quarterly data for investment are from the Department of Commerce's Survey of Plant and Equipment Expenditure. Structures and equipment are aggregated. There are conceptual and data-related reasons for considering total plant and equipment rather than each component separately. In building a new plant, it is often difficult to distinguish between the structure and the machines. Is a bolted-down assembly line plant or equipment? The difficulty in answering such questions leads some firms to fail to separate capital expenditures into the two components when they respond to the Survey.

Moreover, such a breakdown has only been requested since 1972 in the quarterly Survey. Consequently, there are no separate quarterly plant and equipment series. Furthermore, in the annual series "the two components are less reliable than the total" [Survey of Current Business, October 1980, p. 30]. The Survey data for investment, which are available by industry, should not be confused with the National Income and Product Accounts data, which are available by function. The NIPA investment data are based on the output of the capital goods industry; the survey data are based on reported purchases by the investing firms.

I construct the capital stock data using a fixed depreciation rate of 0.0175 per quarter and a benchmark net capital stock of 311.8 billion 1972 dollars at the end of 1981 [see the Survey of Current Business, October 1982, p. 33].

In the model, the discount rate varies across time. This feature strongly distinguishes the results from those based on closed-form solution, which require a fixed discount rate. The discount rate, R_t , is defined above. In these estimates, I take the required rate of return to be the after-tax, real return on three-month Treasury bills plus a constant risk premium of two percent per quarter.

I calculate the premium by taking a weighted average of the return in excess of the return on Treasury bills of the stock market and of corporate bonds. The weight for equity is 0.8. The excess return of the stock market is 6.7 percent; the excess return of corporate bonds is 0.6. (See Ibbotson and Sinquefeld [1982, p. 15].) Therefore, the premium is about eight percent at annual rate or about two percent at quarterly rate.

The data for employment are the BLS establishment survey figures for the number of production workers and non-production workers. The wage series is the average straight-time rate per hour for the production workers.

The hours series are total average weekly hours. Hours are multiplied by the number of weeks in the quarter in the marginal cost expressions in the Euler equations to express cash flow at quarterly rate. To construct the fixed cost of employing a worker (S^L and S^N), I divide the compensation minus wages, salaries, and contribution to social insurance into the number of workers using annual national income and product accounts data.[6]

The expression for the price of capital is given in equation (5). The purchase price is the implicit deflator from the BEA. The quarterly series for the present value of depreciation allowances and the investment tax credit are those computed by Data Resources, Inc.

IV. Estimation and Results

A. Basic Results

To estimate the first-order conditions (7), I replace the conditional expectations with actual values and use instrumental variables. Moreover, I substitute output data for the production function f( ) . The equations I estimate are then the same as (7) except that the zeros on the right-hand sides are replaced with a vector of error terms u_t . If the equations are specified correctly, the error term u_t equals only a forecast error e_t . I consider a more general error term $u_t = e_t + v_t$. The added component v_t is either measurement error or specification error or both. What instruments are valid depends on whether u_t is serially uncorrelated.

To estimate the system of Euler equations together with the equation for the wage bill (5), I use Hansen's [1982] generalized method of moments (GMM).[7] The procedure is essentially three-stage least squares with a covariance matrix that allows for general conditionally heteroskedastic

and moving average errors. Under the maintained hypothesis that the model is exactly correct, the errors are serially uncorrelated. Any instrument known at time t is valid. In the case of misspecification, the errors may be serially correlated. Details of how the moving average error term may arise are given in Hansen and Sargent [1980]. The intuition for it is that v_t, the measurement or specification error is part of the information set, and therefore contributes to the forecastability of e_t. If the error term is a first-order moving average, then only instruments dated at time $t-1$ are valid. I present estimates using both time t and time $t-1$ instruments.

To carry out the estimation, I substitute y_t for $f( )$. This procedure has several advantages. First, it makes what would otherwise be a highly nonlinear system linear in parameters. Second, it brings output data to bear on the problem without estimating demand functions that are conditional on output; that is, it imposes equation (2). Third, it eliminates the need to explicitly parameterize the shock to technology and the rate of technological progress, X_t. The substitution reduces the amount of noise in the estimated system by eliminating X_t, the error term in equation (2).

Garber and King [1983] criticize methodology of the type used in this paper. In particular, they note that if unobserved shocks move the production function (factor demands) the estimated curves will be factor supplies. This paper makes a substantial advance in addressing the problem raised by Garber and King. The specification explicitly allows for a productivity shock without losing identification. The productivity shock, X_t in equation (2), can have arbitrary serial correlation, but must enter additively in logs. Given the parameterization of the shock, output data can be used

to make X_t observable. Hence, the Garber and King critique does not apply. Indeed, using output data accommodates a wide range of productivity shocks without compromising identification.

A criticism of this approach to identification is that the system will not be identified if the shock is not additive in logs, or more generally, separable from the function. The appropriate rejoinder is that the separability is typical of identifying restrictions needed in any econometric application. It is much weaker than assuming no shock at all. Moreover, most of the minimum distance estimators generally used in econometrics have separable errors (see Amemiya [1983]).

Hence, the approach used in this paper may open the way to more plausible parameterizations of stochastic Euler equations when both inputs and outputs are observed. Unfortunately, the approach can not be readily used in the consumption or labor supply literature (e.g., Mankiw, Rotemberg, and Summers [1985]) because utility is not observable.

The instruments used in the estimates for Table I are the factor prices (p_t^K , w_t , s_t^L , and s_t^N), the factor stocks (K_t , L_t , N_t , and H_t), their logs, the tax rate, t_t^K , the required rate of return r_t , the quit rate, a constant, and a trend. (The required rate of return is not known at time t because of uncertainty about the inflation rate. Therefore, even in estimates with current instruments, r_t is lagged.)

Table I presents estimates of system (7) and equation (4). The cross-equation restrictions are imposed and the production function gross of adjustment costs is constrained to be constant returns to scale. I report both the standard three-stage least squares (3SLS) and the Hansen generalized method of moments (GMM) standard errors. The GMM errors are substantially larger than the 3SLS ones. The 3SLS errors are inconsistent unless

the errors are homoskedastic and serially uncorrelated. If the errors are serially correlated, only lagged instruments are valid.

Table I presents estimates with and without interrelated adjustment costs. The estimated coefficients are plausible and significant. Consider the elasticities in the Cobb-Douglas part of the production function. The elasticity of production workers (L_t) is about 0.46 and the elasticity of non-production workers (N_t) is about 0.27 implying a total labor elasticity of about three-quarters, which is broadly consistent with its share in national income. The implied value of a_K is 0.27. The coefficients are estimated very precisely and change little when the instrument list or specification is varied.

The estimate of the a_H , the hours elasticity, is substantially greater than a_L . Thus, labor input should be treated as hours and workers separately and not just as man-hours. The coefficients are consistent with the theory developed above where short-run variation in the utilization of capital comes from variation in the number of man-hours worked at a fixed plant. That is, increasing average hours increases the work week of capital as well as adding labor input.

The estimates of the cost of adjustment parameters are also plausible. They all have the correct sign except two of four of the estimates of g_{LL} and one of the four estimates of g_{HH} , all of which differ unimportantly and insignificantly from zero. Capital has important adjustment costs. They are significant with the 3SLS standard errors but not so with the GMM standard errors. In any case, the estimated coefficient is substantial. Varying non-production workers induces important and significant adjustment costs. Varying production workers or their hours induces small and insignificant adjustment costs. Such a finding is not surprising. It is likely

that hours are not costly to adjust because of the ease of lengthening
shifts. The result that production workers are not costly to adjust is
more difficult to rationalize. Perhaps given the institutionalization of
the temporary layoff, such a result should not be too surprising. The
interrelated adjustment costs are never significant with the GMM standard
errors; g_{KL} is significant with 3SLS standard errors. Even though there
are no adjustment costs for production workers alone, when capital is varied,
there is an added cost to varying production workers.

It is difficult to evaluate the magnitude of the adjustment cost based
on the coefficients alone. Therefore, I calculate the marginal cost of
adjustment for typical values of the variables when the gross change in
the other variables is zero. The marginal reduction in output from adjust-
ment cost due to investment is

$$-g_{KK} y_t (K_t - dK_{t-1})$$

and the reduction from changing the number of non-production workers is

$$-g_{NN} y_t (N_t - q_{t-1} N_{t-1}) \ .$$

The average level of output for the period is 64 billion 1972 dollars at
a quarterly rate. The average gross investment is 4.8 billion 1972 dollars.
The estimate of g_{KK} from column (b) in Table I is 0.0014. Therefore, the
marginal cost of adjustment from a representative amount of investment
is 0.7 percent of the output for the quarter or about nine percent of
the cost of the investment. Previous estimates of the marginal cost of
investment are implausibly high. For example, Summers [1981] finds
very high marginal costs of adjustment from estimates based on the q
approach. This problem with his result is seen as a major barrier to their

use for practical, policy-oriented discussion (see Tobin and White [1981]).
In particular, the high adjustment costs imply extremely long lags in ad-
justing to permanent changes in factor prices or the required rate of
return.

Alternative estimates of the response of investment to q yield re-
markably similar results. Summers [1981] constructs an estimate of q
based on tax-adjusted financial market data. Abel and Blanchard [1983b]
estimate marginal q from the present discounted value of marginal profits.
Abel [1979] calculates a q implied from the firm's profit maximizing labor-
capital choice. Summers and Abel and Blanchard report regressions of the
investment/capital ratio on their respective measures of q . Summers
finds the sum of the coefficients of current and lagged q to be 0.031
[1981, equation 4-6]. Abel and Blanchard report estimates of about half
that amount [1983b, Tables 5a and 5b]. Abel reports an elasticity of the
investment/capital ratio of between one-half and three-quarters [1979,
pp. 99-100]. Given an average investment/capital ratio of ten percent at
annual rate and Abel's average q of 1.73 [1979, p. 78], this range of
elasticities implies a range of regression coefficients of 0.029 to 0.044.
Hence, estimates of q-theory investment equations based on very different
data have similar results.

Consideration of the data on which the q-model is estimated demonstrates
why it produces such high estimates of adjustment costs. The stock market
is much more variable than investment. The q-theory as Summers implements
it would have investment respond to these changes except for adjustment
costs. Therefore, estimated adjustment costs must be very high to ration-
alize the relatively small response of investment to changes in the stock
market. The model used in this paper takes the price of and required return

to capital <u>per se</u> as its data. Therefore, it has the potential to produce more plausible estimates such as the ones presented here.

The argument about stock market data does not apply to the results of Abel and Blanchard. Their estimates, like those in this paper are based on direct study of the prices and quantities of factor inputs. Nonetheless, an attempt to summarize their affect through a reduced-form q or similar variable probably fails to wholly capture their effect.

The adjustment costs for non-production workers are also plausible. The average stock of non-production workers is 4.5 million. Consider a five percent change in the number of employees. An estimate of g_{NN} of 0.081 implies an adjustment cost of 1.8 percent of output for the quarter. Similar arguments establish that the costs of adjusting production workers, L_t, and their hours, H_t, are insubstantial.

The estimates of the wage function (4) are highly plausible. The estimated overtime premium is 0.43, a value close to 0.5, the typical premium in contracts. It differs significantly from 0.5 with the 3SLS but not with the GMM standard errors.

The overtime premium may not be paid symmetrically. I estimate separate w_1 coefficients depending on whether $H_t - 40$ is greater or less than zero. The t-statistic for the hypothesis that the coefficients are equal is 0.3, so one cannot reject the hypothesis that the premium is symmetric.

The last line of Table I gives J, the value of the minimized objective function in the GMM estimation. It gives a test of the overidentifying restrictions of the model which Hansen [1982] discusses. J is

distributed as chi-squared with N-k degrees of freedom where N is the number of instruments and k the number of parameters estimated. The number of instruments here is 85 (17 times 5 equations). The number of parameters is either 10 or 16 depending on the specification. The over-identifying restrictions are rejected in the estimates without the interrelated adjustment costs (columns a and b) at the five percent level but not at the one percent level. They are rejected at the one percent level in the estimates with interrelated adjustment costs (c and d). The value of the statistic is about the same for all the equations, but more parameters are estimated in (c) and (d). Given that there is no well-defined alternative model, it is difficult to see what direction such a rejection implies for further research. Moreover, the rejection notwith-standing, the estimated parameters and the dynamic response they imply are plausible.

The coefficients change very little when lagged instruments are used to allow for a moving-average error (a versus b and c versus d). Conse-quently, one cannot reject the hypothesis that the current instruments are valid.[8]

The GMM standard errors exceed the 3SLS ones by a factor of two or three. None of the adjustment cost parameters are significant at the cus-tomary levels with the GMM standard errors but some are strongly signifi-cant with the 3SLS standard errors. One could draw several conclusions from these findings. The first is that these estimates--and many past estimates using least squares--need to be reevaluated in light of Hansen's covariance estimator. If the magnitude of the change in the standard errors from 3SLS to GMM in this paper is typical, many estimates previously believed to be significant may indeed be insignificant. A second conclusion is

that the small sample properties of GMM are not well understood and therefore some weight should be given to the 3SLS estimates. Moreover, the parameter values are the best point estimates. Finally, these signs and magnitudes accord with economic theory and with priors about their size.

I report both standard errors. I discuss the economic interpretation of the point estimates with the reservation that they may be subject to substantial error. It is difficult to evaluate Euler equation estimates using traditional diagnostics. In particular, R^2 and SEE are irrelevant because there is no dependent variable per se. Instead one can appeal to the plausibility of the parameter estimates and of the dynamics that the estimates imply.

To quantify the rates of adjustment of the capital stock implied by the estimated Euler equations, I study their dynamic properties. Specifically, the lower the root of the capital equation, the more rapidly the capital stock will adjust to steady state following a change in the cost of capital. The root of the Euler equation for capital is 0.75.[9] The root of 0.75 implies a rapid rate of adjustment. The speed of adjustment greatly exceeds that estimated by others. Summers [1981] estimates that after twenty years just over half the adjustment to a shock in the required rate of return would have occurred. In these estimates, over half the adjustment occurs in the first year.[10] After four years, almost all the adjustment has occurred. The Euler equation approach generates what are possibly more plausible results because it permits more explicit consideration of the decision problem of the firm.

B. Alternative Estimates

In all applied research, special assumptions must be made to yield estimable relationships. The Euler equations estimated in this paper place tight restrictions on the data. The tightness of these restrictions has the advantage of giving a precise theoretical interpretation to each of the estimated coefficients. The tightness of the restrictions has the disadvantage, however, of perhaps making the results overly dependent on assumptions about functional form. Consequently, I present estimates of the Euler equations under different assumptions about the production function and the data to see whether the basic results discussed above depend critically on the auxiliary assumptions.

The basic results are based on the assumption of a Cobb-Douglas technology. I now consider a technology which allows the possibility of much less substitutability among the inputs--the constant elasticity of substitution (CES) production function (see Arrow, Chenery, Minhas, and Solow [1961]). Use of the CES functional form breaks the tight link between the empirically constant factor shares and the estimated coefficients.

I also consider two additional modifications of the production function relating to the cost of adjustment of the inputs. First, the marginal cost of adjustment is made a function of the percentage change of the input rather than the change in the level. This formulation makes the marginal cost of adjustment independent of the scale of the economy. The second modification of the cost of adjustment is the inclusion of a target in the expression for the cost of adjustment. This formulation posits that there is cost of adjustment only when the gross change in a factor input deviates from a normal or target level rather than from zero.

The alternative form of the production function is

$$(2') \qquad \log y = \frac{1}{\gamma}\log[a_K K^\gamma + (a_L + \tilde{a}_H)L^\gamma H^\gamma - \tilde{a}_H L^\gamma + a_N N^\gamma]$$

$$- \frac{1}{2}\left[\tilde{g}_{KK}\frac{(K_t - dK_{t-1} - b_K)^2}{dK_{t-1}} \right.$$

$$+ \tilde{g}_{LL}\frac{(L_t - q_{t-1}L_{t-1} - b_L)^2}{q_{t-1}L_{t-1}}$$

$$+ \tilde{g}_{NN}\frac{(N_t - q_{t-1}N_{t-1} - b_N)^2}{q_{t-1}N_{t-1}}$$

$$\left. + \tilde{g}_{HH}\frac{(H_t - H_{t-1} - b_H)^2}{H_{t-1}} \right] + X_t \;.$$

The coefficient $\tilde{a}_H$ measures the departure of the hours and production workers measures multiplicatively. The restriction that $a_K = 1 - a_L - a_N$ is again imposed. Hence, as γ approaches zero, the CES function approaches the Cobb-Douglas function defined in (2).[11]

Because the interrelated adjustment costs prove unimportant in the basic estimates, they are constrained to be zero here. The quadratic terms are scaled by the level of the input so that the marginal cost of adjustment will be a function of the rate of change rather than the absolute change of the factor. The b coefficients denote the target amounts of adjustment.

The alternative production function is substituted into the expression for the present discounted value of profits (6). The alternative Euler equation for capital is

$$(7a') \quad E_t \left\{ \left[\frac{a_K}{(a_K K^\gamma + (a_L + \tilde{a}_H) L^\gamma H^\gamma - \tilde{a}_H L^\gamma + a_N N^\gamma) K^{1-\gamma}} \right. \right.$$

$$\left. - \frac{\tilde{g}_{KK} (K_t - dK_{t-1} - b_K)}{dK_{t-1}} \right] y_t (1 - t_t^K)$$

$$+ \tilde{g}_{KK} \frac{(K_{t+1} - dK_t - b_K)}{dK_t} y_{t+1} (1 - t_{t+1}^K) r_t$$

$$\left. - p_t^K + dr_t p_{t+1}^K \right\} = 0 \; .$$

The Euler equations for the labor inputs are defined analogously. The marginal product is a complicated non-linear function of γ. The marginal adjustment cost is a function of the rate of change of the input rather than the absolute change.[12]

Estimates of the parameters of the Euler equations and equation for the wage bill are given in Table II. Estimates are given in columns a, b, and c for various values of the elasticity of substitution. The elasticity of substitution is constrained rather than estimated because the value of econometric objective is essentially the same for a wide range of plausible values for γ. As in the basic results discussed above, the target adjustments b_K, b_L, b_N, and b_H are set to zero. The instrumental variables are the levels of the factors, their gross rates of change, the price and tax variables, and the lagged required rate of return.[13] Again, the level of output is not used as an instrument.

Consider first the results for the Cobb-Douglas case $(\gamma = 0)$. The coefficients of the levels of the factors are about the same as in Table I. The marginal adjustment cost implied by the alternative functional form is very similar to the results reported in Table I. The adjustment cost at the margin implied by the estimate of g of 0.25 is 0.6 percent of output

for the typical gross investment.[14] The cost is almost identical to the estimate of 0.7 percent implicit in Table I. Again, the adjustment cost of hours and number of production workers is nil while that of non-production workers is substantial. Hence, the estimates in column a of Table II imply the same response of the capital stock to a change in input prices as do the basic results.

The estimates for the CES functional form are given in columns b and c of Table II. In column b, the value of γ is -0.4. In column c, it is -1.0, which implies very low substitutability among the factors. The results demonstrate that the marginal cost of adjustment is low, and hence the rate of adjustment is high, does not depend on the Cobb-Douglas functional form. The coefficient $\tilde{g}_{KK}$, which determines the rate of adjustment of the capital stock, remains unchanged across the wide range of elasticities of substitution. The coefficient $\tilde{g}_{NN}$, which governs the adjustment of non-production workers, only increases modestly as the assumed elasticity of substitution decreases. The coefficients of the levels of the inputs-- a_K , a_L , a_N , and $\tilde{a}_H$ --do change importantly when the assumed elasticity of substitution is varied. In particular, as the assumed elasticity of substitution decreases the estimated marginal productivity of labor increases and that of capital falls. Moreover, the estimated productivity of non-production workers relative to production workers falls as the assumed elasticity of substitution decreases. Hence, the estimates with the low elasticity of substitution predict relatively low steady-state capital and non-production worker intensities. The steady-state level of the capital stock is ultimately determined, given the technology, by the willingness of agents to work and supply capital. Hence, with factor demand alone, one cannot fully characterize the level of steady-state factor

inputs. In any case, the estimates imply rapid adjustment of adjustment of the capital stock to the steady state. This rapid adjustment implies that factor price or tax policy induced changes in factor demand can be important at business cycle frequencies.

Column d of Table II gives estimates where cost of adjustment is presumed to be incurred only when the adjustment deviates from a target level. The estimates are derived with the value of the targets-- b_K , b_L , b_N , and b_H --set at the sample average or the respective gross changes in the inputs. When the targets are set to zero, adjustment cost is presumed to occur even at very low levels of change of the factor. It is possible, on the other hand, that only an atypical adjustment causes output to be lost. Hence, because of the linearity of the marginal adjustment cost, the formulation with zero targets could understate the truly marginal adjustment cost by averaging it with the inframarginal adjustment. If marginal adjustment cost is non-linear in this manner, then including the target should increase the cost of adjustment coefficients. By comparing columns a and d of Table II, one sees that the coefficients relating to the adjustment cost change little.[15] Hence, the linearity of marginal adjustment cost is an adequate approximation.

Finally, I carry out another type of experiment again to determine whether the auxiliary assumption needed to tightly restrict the data are responsible for the conclusions. On the theoretical level, the Euler equation for capital, (7a) or (7a'), relates the marginal product of capital net of adjustment cost to the implied rental rate. On an empirical level, they relate, among other things, current and lagged investment to output. The marginal product is conveniently expressed as a function of output for the Cobb-Douglas and CES production functions. Use of the output data allows

the Euler equation to be estimated even in the presence of certain unobserved productivity shocks. Nonetheless, the use of output data invites the suggestion that the estimates are only picking up the well-known accelerator. To consider this possibility, I present the estimates in column e of Table II. For these estimates, the output data are set to equal the mean value for the sample. Hence, the parameter estimates cannot be attributed to a disguised accelerator. The parameterization is the same as that in column a, that is, γ and the targets equal zero.[16] The estimates with and without the variation in the output data are very similar. The estimated $\tilde{g}_{KK}$ is somewhat reduced without the output data, but not by enough to alter the major results of the paper. Hence, the strength of the results cannot be attributed to a disguised accelerator.

A similar experiment can be carried out by likewise shutting down the variation in the factor price and tax variables. The point of such an exercise is to verify that identification of the parameters hinges, as the theory suggests, on variation in factor prices. When the equations are estimated with the factor prices at constant values, the coefficient estimates change substantially.[17] Specifically, when the price of capital terms are excluded from the equations, the crucial cost of adjustment of capital parameter, $\tilde{g}_{KK}$, triples. Hence, factor prices, but not output, have an important role in identifying the coefficients of the model.

VI. <u>Conclusion</u>

This paper offers estimates of the dynamic demand for capital and labor based on explicit consideration of the firm's decision problem. The estimates are based on the Euler equations. The q-theory and the linear-quadratic approach examine closed-form decision rules of the firm. The advantage of the Euler equation approach over the linear-quadratic approach is that it allows more flexible functional forms and a varying and uncertain rate of discount. The advantage of the approach over the q-theory approach is that the adjustment cost in the Euler equation is not summarized with a single, reduced-form variable.

The estimated structural parameters have reasonable values and the capital stock responds at a credible rate to innovations in the factor prices. These results are robust to a variety of changes in specification of the functional form and the data. The best defense of the Euler equation approach is the plausibility of its empirical results. Specifically, the estimates presented in this paper do not imply the excessively large lags in the adjustment of the capital stock found in estimates based on q . Therefore, the investment equation of this paper may be useful to analyze the effects of changes in tax policy on the demand for capital.

The standard view is that rates of adjustment are so slow that the cost of capital will not effect investment in the short run.[18] To fully study the effects of changing the cost of capital on investment would require a complete model with product demand and capital and labor supply. Yet, the rapid rate of adjustment implied by the estimates in this paper provides a clear challenge to the view that factor prices do not matter for short run fluctuations of investment.

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FOOTNOTES

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1. Hayashi [1982] shows that average q equals marginal q under certain circumstances such as constant returns to scale.

2. The cost of intermediate inputs do not appear in the expression for profits so real value added minus real factor costs equals real profits.

3. Sargent [1978] and Meese [1980] use a quadratic production function and do not impose or test CRS or convexity. Pindyck and Rotemberg [1983] impose and test CRS and convexity when estimating a trans-log cost function. Morrison and Berndt [1981] show how to impose CRS in a quadratic cost function. I tried a trans-log specification but failed to identify the production function parameters. In some cases the parameter of the first-order term in L_t exceeded one. It is difficult to untangle the adjustment cost parameters from the other production function parameters in the trans-log case because they both multiply second-order terms in the factors.

4. I consider specifications where the overtime premium is asymmetric so it is paid only when hours exceed 40. The data strongly do not reject symmetry. In aggregate data some overtime is always paid; a symmetric premium appears to be a good approximation.

5. I use the quarterly PVCCA and ITC constructed by Data Resources, Inc.

6. The compensation data are annual. I interpolate to obtain quarterly data. Because separate data are not available, the same quit rate is used for production and non-production workers.

7. I carried out the estimation in a FORTRAN program which I wrote to perform GMM. The program will handle general, non-linear problems.

8. Rotemberg [1984] argues that if the overidentifying restrictions fail, different instrument lists could lead to vastly different estimates. That the estimates remain essentially unchanged when the timing of the instruments is changed is informal evidence that his problem does not arise with these estimates even though the J statistic is large.

9. To calculate the root, I must make more explicit assumptions about the environment of the representative firm than in the Euler equations. With the Euler equations, I assume that the representative firm is a price taker. It would be incorrect, however, to assume that there is no feedback from factor and product markets to the representative firm. Even if the market is competitive, the representative firm will move down the market demand curve and up the labor supply curve. The competitive assumption means only that the firm does not take its effect on prices into account in its decision rule. I assume that labor is supplied inelastically, so L_t, N_t, and H_t are held constant. Wages relative to output prices are also held constant. Summers [1981] makes a similar assumption. Using parameter values from Table 1, column b, the linearized rule for representative form is

$$K_t = 0.75 \, K_{t-1} - 7.1 \sum_{i=0}^{\infty} 0.93^i \, (E_t p^K_{t+i} - dr E_t p^K_{t+i+1})$$

when the required rate of return is constant.

10. Meese [1980, p. 151], using the technique advocated by Hansen and Sargent [1980, 1982] estimates that the root in the capital equation is 0.9563. Again, the adjustment lags are much larger than in my estimates, but not as large as in Summers. The root of 0.9563 implies that half the adjustment to the steady state takes place after four years. Moreover, it takes the economy over 25 years to get within one percent of the steady state under Meese's estimate compared to four years under the estimates presented here.

11. The limiting argument is a straight-forward application of L'Hôpital's rule.

12. A term with the square of the level of capital in the denominator arises from differentiating the lead expression of the production function by the current capital stock. This term is two orders of magnitude smaller than the other terms in the Euler equation and hence has negligible numerical effect on any of the calculations. Consequently, it is omitted from (7a') and the other Euler equations.

13. Because use of the GMM estimates did not alter any conclusions for the results in Table I, only the three-stage least squares estimates are given here. The values of the three-stage least squares objective M for the parameter estimates given in Table I are similar to those given in Table II for the alternative estimates.

14. Over the sample period, the average quarterly rate of gross change of K is 2.4 percent, of L is 6.2 percent, of N is 6.6 percent, and of H is -0.01 percent.

15. The estimate in column d presumes γ equal to zero. Similar results hold for other values of that parameter.

16. The conclusions of this paragraph are unaltered for estimates for other elasticities of substitution and for the non-zero values of the targets.

17. The exceptions are a_L and a_N, which are still tied down by the factor intensities. On the other hand, $\tilde{a}_H$, which depends on the wage function, and coefficients relating to the wage function and cost of adjustment change substantially and nonscenically.

18. The standard view of the effect of the cost of capital on investment is well-represented by the following:

> The effect of interest rates and tax changes...are
> likely to be felt only gradually, over long periods
> of time. For short-term forecasting (two years or
> less), the effect of moderate variations in taxes
> and interest rates is likely to be negligible.
> (Clark [1979, p. 104])

TABLE I

Estimates of the First-Order Conditions (7)

and Equation for the Wage Bill (4)

1955 QIII to 1980 QIII

Instruments	(a) Current	(b) Lagged	(c) Current	(d) Lagged
w_0	1.49 (0.02) (0.06)	1.50 (0.02) (0.08)	1.50 (0.02) (0.06)	1.50 (0.02) (0.08)
w_1	0.43 (0.02) (0.05)	0.43 (0.02) (0.06)	0.43 (0.02) (0.05)	0.43 (0.02) (0.09)
w_2	-0.05 (0.006) (0.011)	-0.05 (0.006) (0.020)	-0.05 (0.007) (0.018)	-0.05 (0.008) (0.023)
a_L	0.45 (0.005) (0.009)	0.46 (0.005) (0.009)	0.46 (0.006) (0.017)	0.46 (0.006) (0.018)
a_N	0.27 (0.004) (0.007)	0.27 (0.003) (0.009)	0.27 (0.004) (0.017)	0.27 (0.004) (0.014)
a_H	0.53 (0.01) (0.02)	0.53 (0.01) (0.02)	0.52 (0.011) (0.022)	0.52 (0.011) (0.028)
g_{KK}	0.0013 (0.0004) (0.0011)	0.0014 (0.0006) (0.0014)	0.0011 (0.0005) (0.0014)	0.0014 (0.0005) (0.0020)
g_{LL}	-0.0002 (0.0008) (0.0018)	-0.0003 (0.0010) (0.0030)	0.0007 (0.0016) (0.0043)	0.0006 (0.0023) (0.0088)
g_{NN}	0.088 (0.027) (0.097)	0.088 (0.023) (0.062)	0.088 (0.039) (0.193)	0.088 (0.047) (0.23)
g_{HH}	0.00047 (0.00027) (0.00068)	-0.0001 (0.0005) (0.0019)	0.0007 (0.0005) (0.0012)	0.0003 (0.0006) (0.0025)
g_{KL}			0.0010 (0.0004) (0.0012)	0.0010 (0.0005) (0.0019)

TABLE I (continued)

Instruments	(a) Current	(b) Lagged	(c) Current	(d) Lagged
g_{KN}			-0.0005 (0.0021) (0.0083)	-0.0006 (0.0025) (0.0011)
g_{KH}			-0.0003 (0.0003) (0.0009)	-0.0007 (0.0004) (0.0014)
g_{LN}			0.0005 (0.0044) (0.017)	0.0008 (0.0074) (0.038)
g_{LH}			0.0004 (0.0007) (0.0017)	0.0011 (0.0011) (0.0035)
g_{NH}			0.0008 (0.0020) (0.0080)	0.0006 (0.0040) (0.014)
J*	103.3	104.6	104.1	103.9
Significance	0.017	0.014	0.004	0.004

Standard errors in parentheses. The first standard errors are from 3SLS, the second from GMM.

*J test of overidentifying restrictions (see Hansen [1982]). J is distributed as chi-squared (85-k) where k is the number of parameters estimated. It is here calculated using the GMM rather than the least squares objective.

TABLE II

Parameter Estimates under Alternative Assumptions

	(a) targets = 0	(b) targets = 0	(c) targets = 0	(d) targets = means	(e) targets = 0; output data = mean
γ	0.0	-0.4	-1.0	0.0	0.0
w_0	1.50 (0.02)	1.50 (0.02)	1.50 (0.02)	1.50 (0.02)	1.51 (0.02)
w_1	0.33 (0.02)	0.32 (0.02)	0.31 (0.02)	0.33 (0.02)	0.33 (0.02)
w_2	-0.02 (0.005)	-0.02 (0.003)	-0.01 (0.009)	-0.02 (0.005)	-0.02 (0.005)
a_K	0.26 (0.01)	0.25 (0.01)	0.16 (0.01)	0.26 (0.01)	0.26 (0.01)
a_L	0.47 (0.01)	0.69 (0.01)	0.83 (0.01)	0.47 (0.01)	0.46 (0.01)
a_N	0.27 (0.003)	0.06 (0.001)	<0.005 (<0.0005)	0.27 (0.003)	0.28 (0.004)
$\tilde{a}_H$	0.05 (0.01)	0.02 (0.002)	<0.005 (<0.0005)	0.05 (0.01)	0.05 (0.01)
$\tilde{g}_{KK}$	0.25 (0.08)	0.25 (0.08)	0.25 (0.08)	0.25 (0.05)	0.21 (0.04)
$\tilde{g}_{LL}$	-0.01 (0.01)	-0.01 (0.01)	-0.01 (0.02)	-0.01 (0.01)	-0.01 (0.01)
$\tilde{g}_{NN}$	0.23 (0.07)	0.27 (0.08)	0.34 (0.09)	0.23 (0.07)	0.23 (0.06)
$\tilde{g}_{HH}$	0.002 (0.007)	<0.0005 (<0.00005)	<0.0005 (<0.00005)	0.002 (0.007)	-0.01 (0.006)
M	305	305	309	304	297

Three stage least-squares standard errors in parentheses. M is the value of the three-stage least squares objective.

In columns a, b, c, and e, the target coefficients-- b_K , b_L , b_N , and b_H --are zero. In column d, they are equal to the sample means of the gross changes in the factors. In column e, the output data are constrained to equal the mean value.