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ABSTRACT

We show that endogenous firm selection provides a new welfare margin for heterogeneous firm models of trade (relative to homogeneous firm models). Under some parameter restrictions, the trade elasticity is constant and is a sufficient statistic for welfare, along with the domestic trade share. However, even small deviations from these restrictions imply that trade elasticities are variable and differ across markets and levels of trade costs. In this more general setting, the domestic trade share and endogenous trade elasticity are no longer sufficient statistics for welfare. Additional empirically observable moments of the micro structure also matter for welfare.

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An online appendix is available at:
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1 Introduction

Over the last decade, new theories of trade with heterogeneous firms in differentiated product markets have been developed. These theories were designed to account for features of disaggregated trade data: only some firms export, exporters are more productive than non-exporters and trade liberalization induces intra-industry reallocations of resources between those different types of firms. These reallocations represent a new potential channel for the gains from trade. However, the implications of these models for aggregate welfare (combining together all welfare channels) were left unanswered.

In a recent paper, Arkolakis, Costinot and Rodriguez-Clare (2012, henceforth ACR) show that there exists a class of heterogeneous and homogeneous firm models in which a country's domestic trade share and the elasticity of trade with respect to variable trade costs are sufficient statistics for the aggregate welfare gains from trade. Therefore, if the different models within this class are calibrated to the same domestic trade share and the same trade elasticity, they imply the same welfare gains from trade. Based on this result, ACR summarizes the contribution of new theories of heterogeneous firms to the aggregate welfare implications of trade as "So far, not much."

In this paper, we compare a heterogeneous firm model to a homogeneous firm model that is a special case with a degenerate productivity distribution. We use a theoretical comparative static to show that the heterogeneous firm model has an extra adjustment margin that is absent in the homogeneous firm model: the endogenous decisions of heterogeneous firms to enter and exit the domestic and export markets. Furthermore, adjustment along this margin is efficient, in the sense that the market equilibrium corresponds to the constrained efficient allocation chosen by a social planner. As a result, if the degenerate productivity distribution in the homogeneous firm model is chosen so that the two models have the same welfare for an initial value of trade costs, this extra adjustment margin implies that the heterogeneous firm model has higher welfare for all other values of trade costs. It follows that the two models have different aggregate welfare implications: there are *larger welfare gains* from *reductions* in trade costs and *smaller welfare losses* from *increases* in trade costs in the heterogeneous firm model than in the homogeneous firm model. Quantitatively, we find that this extra adjustment margin is substantial for a calibration of our heterogeneous firm model to U.S. firm-level and aggregate data.

Under additional restrictions on the parameter space, our heterogeneous and homogeneous firm models belong to the class analyzed by ACR.[1] In this class of models, the elasticity of trade with respect to variable trade costs is constant (and then also serves as a sufficient statistic for welfare along with the domestic trade share). We show that this existence of a single constant trade elasticity and its sufficiency property for welfare are highly sensitive to small departures from those ACR parameter restrictions. In the heterogeneous firm model, the restrictions include an untruncated Pareto distribution for productivity. Even a slight generalization of this distribution to a truncated

[1] We focus on monopolistic competition models featuring imperfect competition, endogenous product variety, and increasing returns to scale. ACR also consider perfect competition models without those features, such as Armington (1969) and Eaton and Kortum (2002).

Pareto (with a finite upper bound for productivity) implies a variable trade elasticity that differs across markets and levels of trade costs. As a result, a trade elasticity estimated from one context need not apply for the evaluation of trade policy in another context. In this more general setting, evaluating a trade policy based on an estimated trade elasticity is subject to the Lucas Critique: This elasticity is not invariant with respect to trade policy.[2]

Furthermore, once we move beyond those ACR restrictions on the parameter space, the trade share and (endogenous) trade elasticity are no longer sufficient statistics for welfare. Even conditional on these variables, micro structure matters for the welfare gains from trade. In this more general setting, the impact on welfare of the extra adjustment margin in the heterogeneous firm model is not captured by the trade elasticity. We develop several examples of trade liberalization scenarios in which this additional impact of the micro structure on welfare can be substantial, even for small, empirically relevant, departures from the ACR parameter restrictions.

We extend the ACR approach of expressing the welfare gains from trade as a function of observable empirical moments (including the trade share and elasticity) to the more general cases of our homogeneous and heterogeneous firm models. We provide a framework for assessing whether the ACR formula provides a good approximation to the true welfare gains from trade liberalization. We quantitatively measure the discrepancies between the ACR formula and the true welfare gains using our more general model calibrated to U.S. aggregate and firm-level data. We find substantial discrepancies ranging up to a factor of four. Using an elasticity estimated *ex post* for the observed local changes in trade costs will reduce – but not eliminate – the discrepancy between the predicted and true welfare gains from trade. In addition to the two aggregate moments of the domestic trade share and trade elasticity, our more general welfare expression highlights differences in the hazard rate of the distribution of log firm size between the domestic and export markets and the response of firm entry to changes in trade costs, both of which can be examined empirically using firm-level data.

Our paper is related to other recent research on the welfare gains from trade when the ACR parameter restrictions are not satisfied. ACR and Costinot and Rodriguez-Clare (2014) explore multiple sectors, tradable intermediate inputs and multiple factors of production; Arkolakis, Costinot, Donaldson and Rodriguez-Clare (2012) and Edmond, Midrigan and Xu (2012) examine variable markups; Head, Mayer and Thoenig (2014) analyze a log normal productivity distribution; Feenstra (2014) introduces variable markups using Quadratic Mean of Order r preferences and considers a truncated Pareto productivity distribution; and Fajgelbaum and Khandelwal (2013) investigate non-homothetic preferences. In contrast to these studies, we show theoretically that endogenous firm selection provides an extra margin of adjustment in the heterogeneous firm model. We demonstrate the fragility of a constant trade elasticity to small departures from the ACR restrictions even in the benchmark case of a single sector with no intermediate inputs, constant elasticity of substitution (CES) preferences and monopolistic competition, as considered by Krugman (1980) and Melitz (2003).

[2]When there is a single constant trade elasticity – as in the class of models considered by ACR – this elasticity must be invariant so the Lucas Critique does not apply.

The remainder of the paper is organized as follows. In Section 2, we introduce the heterogeneous and homogeneous firm models. In Section 3, we use our theoretical comparative static to show that the heterogeneous firm model has an extra margin of adjustment that is absent in the homogeneous firm model. In Section 4, we extend the ACR approach of expressing welfare gains from trade as a function of observable empirical moments (including the trade share and elasticity) to the more general cases of the homogeneous and heterogeneous firm models. In Section 5, we provide several examples of trade liberalization scenarios where the additional impact of the model's micro structure on welfare can be substantial. In Section 6, we examine the quantitative relevance of our theoretical results. Section 7 concludes.

2 Heterogeneous and Homogeneous Firm Models

We compare the canonical heterogeneous firm model of Melitz (2003) to a homogeneous firm model that is a special case with a degenerate productivity distribution (as in Krugman 1980).[3] We hold all other parameters (including the trading technology) constant across the two models.

2.1 Closed Economy Heterogeneous Firm Model

The specification of preferences, production and entry is the same as in Melitz (2003).[4] There is a continuum of firms that are heterogeneous in terms of their productivity $\varphi \in (0, \varphi_{\max})$, which is drawn from a common cumulative distribution $G(\varphi)$ after incurring a sunk entry cost of f_e units of labor. We allow the upper bound of the support of the productivity distribution to be either finite ($\varphi_{\max} < \infty$) or infinite ($\varphi_{\max} = \infty$). Labor is the sole factor of production. Production involves a fixed production cost and a constant marginal cost that depends on firm productivity, so that $l(\varphi) = f_d + q(\varphi)/\varphi$ units of labor are required to supply $q(\varphi)$ units of output. Consumers have constant elasticity of substitution (CES) preferences with elasticity $\sigma > 1$ defined over the differentiated varieties supplied by firms. Profit maximization implies that variety prices are a constant mark-up over marginal cost that is determined by the elasticity σ. The revenue of a firm with productivity φ is then given by:

$$r_d(\varphi) = \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma-1} \varphi^{\sigma-1} R P^{\sigma-1} w^{1-\sigma}, \tag{1}$$

where R is aggregate revenue, P is the CES price index, and w is the wage. We use the subscript d to reference the domestic market.

We begin by considering the closed economy equilibrium, which can be summarized by the following three relationships, where we use the superscript A to denote the autarky equilibrium. First, fixed production costs imply a productivity cutoff (φ_d^A) below which firms exit. This cutoff is defined by a zero-profit condition equating operating profit to the fixed cost:

$$\frac{r_d(\varphi_d^A)}{\sigma} = \frac{R}{\sigma}\left(\frac{\sigma - 1}{\sigma}\frac{P\varphi_d^A}{w}\right)^{\sigma-1} = w f_d. \tag{2}$$

[3] A web-based technical appendix contains the derivations of all expressions in the paper.

[4] Following most of the subsequent international trade literature, including ACR, we consider a static version of Melitz (2003) in which there is zero probability of firm death.

Second, free entry requires that the probability of successful entry $(1 - G(\varphi_d^A))$ times average profits conditional on successful entry $(\bar{\pi})$ equals the sunk entry cost: $\left[1 - G\left(\varphi_d^A\right)\right]\bar{\pi} = wf_e$. Using the relationship linking relative firm revenue to relative firm productivity and the zero-profit cutoff condition above, this free entry condition can be expressed as:

$$f_d J\left(\varphi_d^A\right) = f_e, \tag{3}$$

where

$$J\left(\varphi_d^A\right) = \int_{\varphi_d^A}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_d^A}\right)^{\sigma-1} - 1\right] \mathrm{d}G(\varphi), \tag{4}$$

where $J\left(\varphi_d^A\right)$ is a monotonically decreasing function of the productivity cutoff. We can then write $J\left(\varphi_d^A\right)$ in terms of the ratio of average productivity to cutoff productivity:

$$J\left(\varphi_d^A\right) = \left[1 - G(\varphi_d^A)\right]\left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_d^A}\right)^{\sigma-1} - 1\right]. \tag{5}$$

Following Melitz (2003), we define $\tilde{\varphi}_d^A$ as a weighted average of firm productivity (corresponding to a harmonic mean weighted by output shares):

$$\tilde{\varphi}_d^A = \left[\int_{\varphi_d^A}^{\varphi_{\max}} \varphi^{\sigma-1} \frac{\mathrm{d}G\left(\varphi\right)}{1 - G\left(\varphi_d^A\right)}\right]^{\frac{1}{\sigma-1}}. \tag{6}$$

Third, the mass of producing firms (M) equals the mass of entrants (M_e) times the probability of successful entry $(1 - G(\varphi_d^A))$. This mass of producing firms also equals aggregate revenue (R) divided by average firm revenue $(\bar{r})$. Using the relationship linking relative firm revenue to relative firm productivity and the free entry condition, the mass of producing firms can be written in terms of the economy's labor supply (L) relative to average fixed costs per firm:

$$M = \left[1 - G(\varphi_d^T)\right] M_e = \frac{R}{\bar{r}} = \frac{L}{\sigma F^A}. \tag{7}$$

In this derivation, we choose labor as the numeraire so that aggregate revenue R equals labor payments L, and we define F^A to represent the average fixed cost paid per surviving firm:

$$F^A = \frac{f_e}{1 - G(\varphi_d^A)} + f_d. \tag{8}$$

Using the CES price index and the mass of firms (7), closed economy welfare can be then written in terms of the mass of firms $(L/\sigma F^A)$ and the weighted average productivity of these firms $(\tilde{\varphi}_d^A)$:

$$\mathbb{W}_{\text{Het}}^A = \frac{w}{P} = \frac{\sigma - 1}{\sigma}\left\{\frac{L}{\sigma F^A}\left(\tilde{\varphi}_d^A\right)^{\sigma-1}\right\}^{\frac{1}{\sigma-1}}. \tag{9}$$

2.2 Open Economy Heterogeneous Firm Model

We consider the case of trade between two symmetric countries. We use the subscript x to reference the export market and the superscript T to reference the open economy equilibrium. We assume that there is a fixed exporting cost of f_x units of labor in the source country and an iceberg variable trade cost, whereby $\tau > 1$ units of a variety must be shipped from one country in order for one unit to arrive in the other country. The open economy equilibrium is characterized by productivity cutoffs for serving the domestic market (φ_d^T) and export market (φ_x^T) that are defined by zero-profit conditions equating the operating profit in each market to the relevant fixed costs:

$$\frac{r_d(\varphi_d^T)}{\sigma} = \frac{R}{\sigma}\left(\frac{\sigma-1}{\sigma}\frac{P\varphi_d^T}{w}\right)^{\sigma-1} = wf_d, \tag{10}$$

$$\frac{r_x(\varphi_x^T)}{\sigma} = \frac{R}{\sigma}\left(\frac{\sigma-1}{\sigma}\frac{P\varphi_x^T}{\tau w}\right)^{\sigma-1} = wf_x. \tag{11}$$

The revenue functions $r_d(\varphi)$ and $r_x(\varphi)$ separate firm sales by destination market (domestic and export). Together these two zero-profit conditions imply that the export cutoff is a constant multiple of the domestic cutoff, where this multiple depends on the fixed and variable costs of trade:

$$\varphi_x^T = \tau\left(\frac{f_x}{f_d}\right)^{\frac{1}{\sigma-1}}\varphi_d^T. \tag{12}$$

For sufficiently high fixed and variable trade costs ($\tau\left(f_x/f_d\right)^{\frac{1}{\sigma-1}} > 1$), only the most productive firms export, consistent with an extensive empirical literature (see for example the review in Bernard, Jensen, Redding and Schott 2007).

The free entry condition again equates the expected value of entry to the sunk entry cost, $\left[1-G\left(\varphi_d^T\right)\right]\bar{\pi} = wf_e$, and can be written as:

$$f_d J\left(\varphi_d^T\right) + f_x J\left(\varphi_x^T\right) = f_e, \tag{13}$$

where $J(\cdot)$ is defined in (4). Using the relationship between the productivity cutoffs (12), and noting that $J(\cdot)$ is a decreasing function, the free entry condition (13) determines a unique equilibrium value of the domestic cutoff (φ_d^T), which in turn determines the export cutoff (φ_x^T). Furthermore, the domestic cutoff in the open economy is strictly greater than the domestic cutoff in the closed economy ($\varphi_d^T > \varphi_d^A$) for positive values of fixed exporting costs.

As in the closed economy, the mass of producing firms (M) equals the mass of entrants (M_e) times the probability of successful entry ($1-G(\varphi_d^T)$), and is determined by the economy's labor supply (L) relative to average fixed costs:

$$M = \left[1-G(\varphi_d^T)\right]M_e = \frac{R}{\bar{r}} = \frac{L}{\sigma F^T}, \tag{14}$$

where F^T summarizes average fixed costs per surviving firm in the open economy:

$$F^T = \frac{f_e}{1-G(\varphi_d^T)} + f_d + \chi f_x, \tag{15}$$

and $\chi = \left[1-G(\varphi_x^T)\right]/\left[1-G(\varphi_d^T)\right]$ is the proportion of exporting firms. In this derivation, we choose labor in one country as the numeraire and use country symmetry, which implies that aggregate revenue R still equals labor payments L in each country.

Using the CES price index and mass of firms (14), open economy welfare can be written in terms of the mass of varieties available for consumption ($L(1+\chi)/\sigma F^T$) and the weighted average productivity of these varieties ($\tilde{\varphi}_t^T$):

$$\mathbb{W}_{\text{Het}}^T = \frac{w}{P} = \frac{\sigma-1}{\sigma}\left\{\frac{L(1+\chi)}{\sigma F^T}\left(\tilde{\varphi}_t^T\right)^{\sigma-1}\right\}^{\frac{1}{\sigma-1}}. \tag{16}$$

This weighted average productivity ($\tilde{\varphi}_t^T$) in the open economy is constructed using the same weighting scheme (6) as we used for the closed economy. However the productivity of exporters is reduced by τ to account for the units "lost" in transit. Letting $\tilde{\varphi}_x^T$ denote the average productivity of exporters, defined as in (6), the overall productivity average for the open economy can be written:

$$\left(\tilde{\varphi}_t^T\right)^{\sigma-1} = \frac{1}{1+\chi}\left[\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi\left(\tau^{-1}\tilde{\varphi}_x^T\right)^{\sigma-1}\right]. \tag{17}$$

Aggregate trade between the two countries is inversely related to the domestic trade share (the proportion of domestic sales in total sales):

$$\lambda_{\text{Het}} = \frac{\int_{\varphi_d^T}^{\varphi_{\max}} r_d(\varphi)\mathrm{d}G(\varphi)}{R} = \frac{1}{1+\tau^{1-\sigma}\Lambda}, \tag{18}$$

where $\Lambda = \delta(\varphi_x^T)/\delta(\varphi_d^T) \leq 1$ is the market share of exporters in the domestic market and $\delta(\varphi_j) = \int_{\varphi_j}^{\varphi_{\max}} \varphi^{\sigma-1}\mathrm{d}G(\varphi)$ is a function that depends only on $G(\cdot)$ and σ. The sensitivity of aggregate trade to changes in variable trade costs is captured by the full trade elasticity:

$$\theta_{\text{Het}} = -\frac{\mathrm{d}\ln\left(\frac{1-\lambda_{\text{Het}}}{\lambda_{\text{Het}}}\right)}{\mathrm{d}\ln\tau} = \begin{cases} (\sigma-1) - \frac{\mathrm{d}\ln\Lambda}{\mathrm{d}\ln\tau} & > 0 \qquad \text{for } \tau\left(f_x/f_d\right)^{1/(\sigma-1)} > 1 \\ (\sigma-1) & > 0 \qquad \text{for } \tau\left(f_x/f_d\right)^{1/(\sigma-1)} < 1 \end{cases}, \tag{19}$$

where $\mathrm{d}\ln\Lambda/\mathrm{d}\ln\tau < 0$. When trade costs are sufficiently low, all firms export and there is no extensive margin of trade. Given CES preferences, the elasticity of the intensive margin of trade is constant at $\sigma-1$. When there is selection into the export market, the elasticity of trade θ_{Het} is the sum of the intensive margin elasticity $\sigma-1$ and the extensive margin elasticity $-\mathrm{d}\ln\Lambda/\mathrm{d}\ln\tau$.

2.3 Closed Economy Homogeneous Firm Model

We construct a homogeneous firm model that is a special case of the heterogeneous firm model with a degenerate productivity distribution. Firms pay the same sunk entry cost of f_e units of labor and draw a productivity of either zero or $\bar{\varphi}_d$ with exogenous probabilities $\bar{G}_d$ and $\left[1-\bar{G}_d\right]$ respectively. Fixed production costs imply that only firms drawing a productivity of $\bar{\varphi}_d$ find it profitable to produce. Therefore producing firms are homogeneous and there is a degenerate productivity distribution conditional on production at $\bar{\varphi}_d$.

The closed economy equilibrium of this homogeneous firm model is isomorphic to that in Krugman (1980), in which the representative firm's productivity is set equal to $\bar{\varphi}_d$ and the fixed production cost is scaled to incorporate the expected value of entry costs ($\bar{F}_d \equiv f_d + f_e/\left[1-\bar{G}_d\right]$). These values for the representative firm's productivity and the fixed production cost are exogenous and held constant.

To simplify the exposition, we adopt this Krugman (1980) interpretation. The representative firm's production technology is:

$$l = \frac{q}{\bar{\varphi}_d} + \bar{F}_d. \tag{20}$$

Consumers have the same CES preferences that we defined previously. Profit maximization implies that equilibrium prices are a constant markup over marginal cost. Profit maximization and free entry imply that equilibrium output and employment for the representative variety are proportional to the fixed production cost:

$$q = \bar{\varphi}_d \bar{F}_d (\sigma - 1), \qquad l = \sigma \bar{F}_d.$$

Using equilibrium employment for the representative variety, the mass of firms can be determined from the labor market clearing condition:

$$M = \frac{L}{\sigma \bar{F}_d}. \tag{21}$$

Using the CES price index and the mass of firms (21), closed economy welfare can be written in terms of the mass of firms ($L/\sigma\bar{F}_d$) and productivity ($\bar{\varphi}_d$):

$$\mathbb{W}^A_{\mathrm{Hom}} = \frac{w}{P} = \frac{\sigma - 1}{\sigma} \left\{ \frac{L}{\sigma \bar{F}_d} \left(\bar{\varphi}_d \right)^{\sigma - 1} \right\}^{\frac{1}{\sigma - 1}}, \tag{22}$$

where we again choose labor as the numeraire.

2.4 Open Economy Homogeneous Firm Model

We again consider trade between two symmetric countries and assume the same trading technology as in the heterogeneous firm model, so that there is a fixed exporting cost of f_x units of labor and an iceberg variable trade cost of $\tau > 1$ units of each variety.

In the homogeneous firm model, the probability of successful entry and productivity conditional on successful entry are exogenous and remain unchanged and equal to $\left[1 - \bar{G}_d\right]$ and $\bar{\varphi}_d$ respectively. For sufficiently high fixed and variable trade costs ($\tau^{\sigma-1} f_x / \bar{F}_d > 1$), the representative firm does not find it profitable to export. In contrast, for sufficiently low fixed and variable trade costs ($\tau^{\sigma-1} f_x / \bar{F}_d < 1$), the representative firm finds it profitable to export, and there is trade in both models. The open economy equilibrium of this homogeneous firm model is isomorphic to a version of Krugman (1980) with the same trading technology as in Melitz (2003).

Profit maximization again implies that equilibrium prices are a constant mark-up over marginal costs, with export prices a constant multiple of domestic prices due to the variable costs of trade. Profit maximization and free entry imply that equilibrium output and employment for the representative variety are proportional to fixed costs:

$$q = \bar{\varphi}_d \left(\bar{F}_d + f_x \right) (\sigma - 1),$$

$$l = \sigma \left(\bar{F}_d + f_x \right).$$

Therefore both output and employment rise for the representative firm following the opening of trade to cover the additional fixed costs of exporting.

Using equilibrium employment for the representative variety, the mass of firms can be determined from the labor market clearing condition:

$$M = \frac{L}{\sigma\left(\bar{F}_d + f_x\right)}. \tag{23}$$

Using the CES price index and the mass of firms (23), open economy welfare can be written in terms of the mass of varieties available for consumption ($2L/\sigma(\bar{F}_d + f_x)$) and average productivity ($\bar{\varphi}_t$):

$$\mathbb{W}^T_{\text{Hom}} = \frac{w}{P} = \frac{\sigma - 1}{\sigma}\left\{\frac{2L}{\sigma\left(\bar{F}_d + f_x\right)}\left(\bar{\varphi}_t\right)^{\sigma-1}\right\}^{\frac{1}{\sigma-1}}. \tag{24}$$

where average productivity ($\bar{\varphi}_t$) is constructed in the same way as in (17) for heterogeneous firms:[5]

$$\left(\bar{\varphi}_t\right)^{\sigma-1} = \frac{1}{2}\left[\left(\bar{\varphi}_d\right)^{\sigma-1} + \left(\tau^{-1}\bar{\varphi}_d\right)^{\sigma-1}\right]. \tag{25}$$

We again choose the wage in one of the symmetric countries as the numeraire.

In the case of homogeneous firms, the domestic trade share simplifies to:

$$\lambda_{\text{Hom}} = \frac{1}{1 + \tau^{1-\sigma}}. \tag{26}$$

There is no extensive margin of trade, so the trade elasticity is given by the constant elasticity for the intensive margin of trade (so long as there is some trade):

$$\theta_{\text{Hom}} = \begin{cases} \sigma - 1 & \text{for } \tau^{\sigma-1} f_x/\bar{F}_d < 1 \\ 0 & \text{otherwise} \end{cases}. \tag{27}$$

3 Theoretical Comparative Static

We now show that endogenous firm selection provides a new margin of adjustment through which the economy can respond to trade liberalization that leads to different aggregate welfare implications in the heterogeneous and homogeneous firm models. Holding all other structural parameters constant across the two models (same f_d, f_e, f_x, τ, L, σ), we first pick the parameters $\bar{G}_d$ and $\bar{\varphi}_d$ for the degenerate productivity distribution with homogeneous firms such that welfare in an initial equilibrium is the same in the two models. We next examine the effects of changes in trade costs from this initial equilibrium. We undertake this analysis both for the opening of the closed economy to trade and for changes in trade costs in the open economy equilibrium.

3.1 Opening the Closed Economy to Trade

We begin by picking the parameters $\bar{G}_d$ and $\bar{\varphi}_d$ of the degenerate productivity distribution with homogeneous firms such that the autarky equilibrium is isomorphic to that with heterogeneous firms, and examine the effect of opening the closed economy to trade.

[5] With a representative firm, average productivity across domestic firms and exporters is $\bar{\varphi}_t$, and the proportion of exporting firms is one.

Proposition 1 *Consider a homogeneous firm model that is a special case of the heterogeneous firm model with an exogenous probability of successful entry* $\left[1 - \bar{G}_d\right] = \left[1 - G(\varphi_d^A)\right]$ *and an exogenous degenerate distribution of productivity conditional on successful entry* $\bar{\varphi}_d = \tilde{\varphi}_d^A$*. Given the same value for all remaining parameters* $\{f_d,\ f_e,\ L,\ \sigma\}$*, all aggregate variables (welfare, wage, price index, mass of firms, and aggregate revenue) are the same in the closed economy equilibria of the two models.*

Proof. Comparing (9) and (22), equal welfare follows immediately from $\bar{\varphi}_d = \tilde{\varphi}_d^A$ and $\left[1 - \bar{G}_d\right] = \left[1 - G(\varphi_d^A)\right]$, which implies $\bar{F}_d = F^A$. This also implies equal price indices. Equal masses of firms follow immediately from equal price indices and $\bar{\varphi}_d = \tilde{\varphi}_d^A$. Equal aggregate revenue follows from $R = L$ in both models. ■

This first proposition reflects the aggregation properties of the heterogeneous firm model. All aggregate variables in this model take the same value as if there were a representative firm with productivity $\bar{\varphi}_d$ and fixed costs $\bar{F}_d$. The key difference between the heterogeneous firm model and such a representative firm model is that aggregate productivity in the heterogeneous firm model is endogenous and responds to changes in trade costs.

Proposition 2 *Choosing the degenerate productivity distribution in the homogeneous firm model so that the two models have the same closed economy welfare and the same structural parameters* ($f_d, f_e, f_x, \tau, L, \sigma$)*, the proportional welfare gains from opening the closed economy to trade are strictly larger in the heterogeneous firm model than in the homogeneous firm model* ($\mathbb{W}_{Het}^T/\mathbb{W}_{Het}^A > \mathbb{W}_{Hom}^T/\mathbb{W}_{Hom}^A$)*, except in the special case with no fixed exporting cost. In this special case, the proportional welfare gains from opening the closed economy to trade are the same in the two models.*

Proof. See the Appendix. ■

The intuition for this result involves revealed preference arguments of the kind commonly used in international trade.[6] Our starting point is to note that, with CES preferences and monopolistic competition, the open economy equilibrium in the heterogeneous firm model is efficient. As shown in the web appendix, a welfare-maximizing social planner faced with the same production and entry technology would choose the same allocation as the market equilibrium. When the economy is opened to trade, the planner *could* choose not to adjust the set of firms selected for production and exports. Average productivity would then remain constant, and this outcome would replicate the opening to trade in the homogeneous firm case. The latter is thus a feasible allocation for the planner with the heterogeneous firm production and entry technology. However, efficiency implies that this planner chooses to replicate the market equilibrium of the heterogeneous firm model, which involves adjustments in the set of firms selected for production and exports – and an associated increase in average productivity. This induced allocation therefore yields higher welfare than any other feasible allocation

[6] For the sake of parsimony, we focus on symmetric countries; however, this revealed preference argument applies more generally for asymmetric countries.

– including the homogeneous firm outcome. Additionally, the planner's objective function is strictly concave. Thus, welfare in the heterogeneous firm open economy must be strictly higher than in the homogeneous firm case whenever the cutoffs adjust to the opening to trade.[7]

The difference in aggregate welfare implications between the two models arises because of the additional efficient adjustment margin of firm entry and exit decisions in the heterogeneous firm model (for both the domestic and export markets). In the special case with no fixed exporting cost, the domestic productivity cutoff is unaffected by the opening of the closed economy to trade. As a result, the additional adjustment margin of firm entry and exit decisions is inoperative in the heterogeneous firm model, and the welfare effects of trade liberalization are the same in the two models. We consider this special case uninteresting, since firm productivity dispersion then plays no role in the heterogeneous firm model (the exit threshold and average productivity are the same in the closed and open economies). Furthermore, this special case stands at odds with an extensive body of empirical evidence that only some firms export, exporters are larger and more productive than non-exporters, and there are substantial fixed exporting costs.[8]

3.2 Changes in Trade Costs in the Open Economy Equilibrium

The role of the extra adjustment margin of firm entry and exit decisions for generating different aggregate welfare implications is not limited to the opening of the closed economy to trade and also holds for reductions in trade costs in the open economy equilibrium. To show this, we recast our heterogeneous and homogeneous firm models so that they have the same welfare in an initial open economy equilibrium. In order to ensure that the two models have the same initial welfare and only differ in their productivity distribution (keeping the same structural parameters f_d, f_e, f_x, τ, L, σ), we extend the homogeneous firm model to allow for two types of firms: exporters and non-exporters. In this extension, firms again pay a sunk entry cost of f_e units of labor before observing their productivity. With probability $\left[1 - \bar{G}_x\right]$ a firm draws a productivity of $\bar{\varphi}_x$ and can export; with probability $\bar{G}_{dx}$ the firm draws a productivity of $\bar{\varphi}_{dx}$ and cannot export; with probability $\left[\bar{G}_x - \bar{G}_{dx}\right]$ the firm draws a productivity of zero and does not find it profitable to produce. We pick the parameters of this "extended" homogeneous firm model ($\bar{\varphi}_x, \bar{\varphi}_{dx}, \bar{G}_x, \bar{G}_{dx}$) such that the open economy equilibrium features the same aggregate variables as the initial open economy equilibrium with heterogeneous firms (same welfare, price index, mass of firms, aggregate revenue, and domestic trade share).

Nevertheless these two models respond differently to changes in trade costs from this common initial equilibrium along a key dimension. In the heterogeneous firm model, the endogenous selection responses to trade costs lead to changes in the average productivity of exporting and non-exporting

[7] In contrast, if the elasticity of substitution between varieties is variable, the market equilibrium is not in general efficient (see Dixit and Stiglitz 1977 for the case of homogeneous firm models and Dhingra and Morrow 2012 for the case of heterogeneous firm models). Endogenous firm selection still provides an extra margin of adjustment in the heterogeneous firm model relative to the homogeneous firm model. This again generates different aggregate welfare implications in the two models, as considered in the web appendix.

[8] For reviews of the extensive empirical literatures on firm export market participation, see Bernard, Jensen, Redding and Schott (2007) and Melitz and Redding (2014). For evidence of substantial fixed exporting costs, see Roberts and Tybout (1997) and Das, Roberts and Tybout (2007).

firms and in the proportion of exporting firms. In contrast, in the extended homogeneous firm model, the average productivity levels of exporters and non-exporters and the proportion of exporting firms remain constant.[9] The presence of this extra adjustment margin in the heterogeneous firm model implies that welfare following the change in trade costs must be strictly higher than in the extended homogeneous from model. This argument holds irrespective of whether trade costs decrease or increase. Therefore, welfare *gains* are *larger* in the heterogeneous firm model whenever trade costs fall, and welfare *losses* are *smaller* in the heterogeneous firm model whenever trade costs *increase.*

Proposition 3 *Starting from an initial open economy equilibrium with the same welfare and the same structural parameters in the two models ($f_d, f_e, f_x, \tau, L, \sigma$), a common decrease (increase) in variable or fixed trade costs generates larger welfare gains (smaller welfare losses) in the heterogeneous firm model than in the extended homogeneous firm model.*

Proof. See the Appendix. ■

Note that the extended homogeneous firm model is equivalent to a version of the heterogeneous firm model in which the domestic and export productivity cutoffs are held constant at their values in an initial open economy equilibrium. Put another way, consider a planner who is constrained to keep the same set of firms operating in both the domestic and export markets – i.e. the endogenous selection margin is inoperative. Under this constraint, the welfare-maximizing allocation coincides with the market equilibrium of the extended homogeneous firm model. In contrast, in the absence of this constraint, the welfare-maximizing allocation coincides with the market equilibrium of the heterogeneous firm model. Therefore, the welfare differential between the two models provides a direct measure of the impact of selection on aggregate welfare. In other words, it isolates the *additional* contribution to aggregate welfare of the new endogenous selection/productivity channel highlighted by the heterogeneous firm model of trade – this represents the new welfare implications that we refer to in the title of this paper. Later in Section 6, we show that this additional welfare channel is quantitatively substantial for a model calibrated to U.S. aggregate and firm statistics.

Atkeson and Burstein (2010) considers this welfare differential from endogenous firm selection in a model with product and process innovation. They find that this welfare differential is of second-order. Proposition 3 is consistent with this result. As discussed above and shown formally in the web appendix, the initial equilibrium of the heterogeneous firm model is efficient. Therefore the envelope theorem implies that the changes in the productivity cutoffs in the heterogeneous firm model have only second-order effects on welfare. But, as we show later, these second-order welfare effects can be quite substantial for larger changes in trade costs.

[9] Unless trade costs become sufficiently high that firms with productivity $\bar{\varphi}_x$ no longer find it profitable to export or firms with productivity $\bar{\varphi}_{dx}$ no longer find it profitable to produce. In both cases, the average productivity of the two types of firms remains constant at $\bar{\varphi}_x$ and $\bar{\varphi}_{dx}$

3.3 Untruncated Pareto Distribution

Since the homogeneous firm model is a special case of the heterogeneous firm model, our comparison of the two models above is equivalent to a discrete comparative static of moving from a non-degenerate to a degenerate productivity distribution within the heterogeneous firm model. In the special case of an untruncated Pareto productivity distribution, the degree of firm heterogeneity is summarized by a single parameter: the shape parameter k. Lower values of k correspond to greater firm heterogeneity and the homogeneous firm model corresponds to the limiting case in which $k \to \infty$. Therefore, we can complement the above discrete comparative static with a continuous comparative static in the degree of firm heterogeneity (k), holding all other structural parameters constant.

Proposition 4 *Assuming that productivity in the heterogeneous firm model has an untruncated Pareto distribution ($g(\varphi) = k\varphi_{\min}^{k}\varphi^{-(k+1)}$, where $\varphi \geq \varphi_{\min} > 0$ and $k > \sigma - 1$) and fixed exporting costs are positive, greater dispersion of firm productivity (smaller k) implies:* ***(a)*** *larger welfare gains from opening the closed economy to trade (larger $\mathbb{W}_{Het}^{T}/\mathbb{W}_{Het}^{A}$),* ***(b)*** *larger (smaller) welfare gains (losses) from a decrease (increase) in variable trade costs in the open economy equilibrium.*

Proof. See the appendix. ■

Intuitively, a larger dispersion of firm productivity (smaller k) implies greater scope for adjustment along the margin of endogenous firm entry and exit decisions, which implies different aggregate welfare effects from a change in trade costs.

4 Welfare and Trade Policy Evaluation

To isolate the extra adjustment margin from endogenous firm selection, our theoretical comparative static changes the distribution of productivity holding all other exogenous variables fixed across models. This exercise does not restrict the equilibrium values of the endogenous variables (in particular the domestic trade share λ and the trade elasticity θ) to be the same in the two models. Instead the equilibrium values for these endogenous variables differ systematically across the two models. In the appendix, we show that the heterogeneous firm model generates a higher trade elasticity than either the homogeneous firm model or its extension given the same value of the exogenous variables. On the one hand, moving from the closed economy to the open economy, there is *less* trade (higher λ) in the heterogeneous firm model than in the homogeneous firm model. On the other hand, starting from an open economy equilibrium, trade liberalization generates *more* trade (lower λ) in the heterogeneous firm model than in the extended homogeneous case.[10]

[10] For sufficiently high trade costs, the domestic trade share is *higher* in the homogeneous firm model than in the heterogeneous firm model, because the representative firm does not find it profitable to export. As trade costs fall below the threshold at which the representative firm exports, the domestic trade share in the homogeneous firm model falls from one to a value *below* that in the heterogeneous firm case, and the trade elasticity jumps from zero to $\sigma - 1$ (less than the trade elasticity in the heterogeneous firm case).

ACR show that there exists a restricted subset of our heterogeneous and homogeneous firm models (in terms of parameter space restrictions) in which the trade elasticity is constant. Under these parameter restrictions, this constant trade elasticity and the domestic trade share become sufficient statistics for welfare. Even then, the micro structure of the underlying model still matters for the welfare gains from trade, but only through its effect on the trade share and trade elasticity. Therefore, if aggregate data can be used to measure the trade elasticity independently of a model (the trade share, by definition, is directly observed from aggregate data) then these aggregate data can be used to accurately measure the welfare gains from trade; and this welfare computation will be independent of the micro structure of the underlying model. Furthermore, since the trade elasticity is constant under the ACR parameter restrictions, it has a structural interpretation, and hence its use in trade policy evaluations is not subject to the Lucas Critique.

The key feature of those parameter restrictions is to induce a single constant trade elasticity that can be applied across models. However, when using the ACR sufficient statistics for an *ex ante* trade policy evaluation, one needs to assume more than a data generation process conforming to one of those models within the ACR class. One also needs to assume that these models are universal and eternal, in the sense that their structural parameters are always the same, independent of the time or country to which they are applied. If this assumption is not satisfied, and one wants to estimate the welfare gains from trade in a new context where the trade elasticity is unknown and cannot plausibly be taken from an existing context, one needs to start with a specific structural model and assumptions about its behavioral parameters. As shown in our theoretical comparative static, this structural model will generate different *ex ante* predictions for the aggregate welfare implications of changes in trade costs, depending on whether or not it features firm heterogeneity.

In particular, we show how the existence of a single constant elasticity breaks down under very small departures from the ACR parameter restrictions. In such a setting, a trade elasticity estimated from one context need not apply for the evaluation of trade policy in another context, even when the model parameters remain unchanged. Therefore trade policy evaluations using an estimated trade elasticity become subject to the Lucas Critique, because this elasticity is not invariant to trade policy. More fundamentally, we show that even the endogenous trade elasticity is no longer a sufficient statistic for welfare (along with the domestic trade share): even conditioning on those two aggregate moments, the micro structure influences the welfare gains from trade. The reason is that welfare depends on the entire distribution of firms producing and selling in a market – which is summarized by the domestic productivity cutoff. Therefore, changes in welfare depend on the change in the domestic productivity cutoff (which can be measured using a domestic trade elasticity). Only in the case of an untruncated Pareto distribution is the domestic trade elasticity equal to the export trade elasticity. Departing from this parametrization, these two elasticities diverge and depend on the micro structure and the level of the trade costs.

In the remainder of this section, we extend the ACR approach of expressing the welfare gains from trade as a function of observable empirical moments to the more general cases of the homogeneous and heterogeneous firm models from Section 2 (without imposing the ACR parameter restrictions). These empirical moments include the trade share and trade elasticity, but also additional ones that capture micro structure (and differ between the two models). In Section 5, we provide several examples of trade liberalization scenarios in which the additional impact of the micro structure on welfare can be substantial, even for small empirically relevant departures from the ACR restrictions. In Section 6, we quantitatively assess these differences in welfare predictions.

4.1 ACR Welfare Derivation

ACR show how the domestic trade share (λ) and trade elasticity (θ) are sufficient statistics for the welfare gains from trade in a large class of trade models (including special cases of our homogeneous and heterogeneous firm models), so long as three macro-level restrictions are satisfied: (R1) balanced trade; (R2) aggregate profits are a constant share of aggregate revenues; and (R3) a CES import demand system with a constant elasticity of trade with respect to variable trade costs. Under these restrictions, the welfare gains from trade regime T_0 to T_1 can be written:

$$\frac{\mathbb{W}^{T_1}}{\mathbb{W}^{T_0}} = \left(\frac{\lambda^{T_0}}{\lambda^{T_1}}\right)^{\frac{1}{\theta}}. \tag{28}$$

Thus, (28) will characterize the welfare gains from trade for both our homogeneous and heterogeneous firm models so long as (R1)-(R3) are satisfied. Trade is balanced in both of these models, so (R1) is always satisfied. However, the general versions of both models imply departures from a constant aggregate share of profits embodied in (R2). Given CES preferences, the constant aggregate trade elasticity restriction (R3) will be satisfied in all versions of our homogeneous firm model, whereas it will be violated along with (R2) in our general heterogeneous firm model.

4.2 Gains from Trade in the Homogeneous Firm Model

We consider a lowering of trade costs from τ_0 and f_{x0} (trade regime T_0) to τ_1 and f_{x1} (trade regime T_1). To simplify notation, we assume that τ_0 and f_{x0} may be high enough such that no trade is generated in T_0. Let χ^{T_0} denote an indicator variable for positive trade. Then, using the expressions for welfare in the closed economy (22) and open economy (24) and the domestic trade share (26), we can write the welfare gains from trade in the heterogeneous firm model as:

$$\frac{\mathbb{W}^{T_1}}{\mathbb{W}^{T_0}} = \left[\frac{\lambda^{T_0}\left(\bar{F}_d + \chi^{T_0} f_{x0}\right)}{\lambda^{T_1}\left(\bar{F}_d + f_{x1}\right)}\right]^{\frac{1}{\sigma-1}}, \tag{29}$$

where $\theta = \sigma - 1$ is the elasticity of trade with respect to variable trade costs.

In this more general setting, the welfare gains depend on the same two aggregate moments (the domestic trade share and trade elasticity) as in ACR (28), but also on the change in firm size (captured in (29) by the total fixed costs paid by the representative firm). This change in firm size is an observable empirical moment, but one that characterizes a change in micro structure. Even after controlling for the two aggregate moments, these changes in micro structure will affect the welfare gains from trade.

Such changes in micro structure will occur whenever the fixed exporting cost changes in an open economy with trade ($\chi^{T_0} = 1$) or even in the presence of any positive fixed exporting costs in an economy that opens up to trade (from $\chi^{T_0} = 0$). These changes represent a violation of (R2) as the share of profits in revenue changes with firm size in the homogeneous firm model.

4.3 Gains from Trade in the Heterogeneous Firm Model

We now seek to express the welfare gains from trade liberalization in terms of observable empirical moments for the general case of our heterogeneous firm model. Since trade continuously drops to zero when trade costs increase, we can start from an open economy trade regime T without loss of generality. To simplify notation, we drop the T superscript. For now, we also assume that there is export market selection in this trade regime so that $\varphi_x > \varphi_d$.

From (19), the full trade elasticity with export market selection is $\theta_{\text{Het}} = (\sigma - 1) - \mathrm{d}\ln\Lambda/\mathrm{d}\ln\tau$, where $\Lambda = \delta(\varphi_x)/\delta(\varphi_d)$ represents the domestic market share of exporters (and hence changes in Λ capture changes in the extensive margin of trade). This full trade elasticity θ_{Het} incorporates the direct effect of τ on the extensive margin of trade via its impact on the export cutoff $\varphi_x = \tau(f_x/f_d)^{1/(\sigma-1)}\varphi_d$ (see (11)), as well as indirect effects through the price index via its impact on the domestic cutoff φ_d.

As argued by ACR, only the *partial* trade elasticity capturing the direct effect of τ is observed empirically, since it is estimated from a gravity equation with exporter and importer fixed effects. In the context of our symmetric country model, this partial elasticity can be derived from (18), which relates the domestic trade share to variable trade costs and the two productivity cutoffs ($\lambda = \lambda(\tau, \varphi_d, \varphi_x)$), and from (12), which relates the two productivity cutoffs to one another ($\varphi_x = \varphi_x(\tau, \varphi_d)$).[11] Taking the partial derivative of the domestic trade share with respect to τ holding φ_d constant, we have:

$$\vartheta = -\left.\frac{\partial \ln\left(\frac{1-\lambda}{\lambda}\right)}{\partial \ln\tau}\right|_{\varphi_d} = (\sigma - 1) - \left.\frac{\partial \ln\Lambda}{\partial \ln\varphi_x}\,\frac{\partial \ln\varphi_x}{\partial \ln\tau}\right|_{\varphi_d},$$

where the relationship between the productivity cutoffs (12) implies $\partial\ln\varphi_x/\partial\ln\tau|_{\varphi_d} = 1$. Therefore the partial elasticity can be further written as:

$$\begin{aligned}\vartheta &= (\sigma - 1) - \left.\frac{\partial\ln\Lambda}{\partial\ln\varphi_x}\right|_{\varphi_d}, \\ &= (\sigma - 1) + \gamma(\varphi_x),\end{aligned} \tag{30}$$

where $\gamma(\varphi_j) = -\mathrm{d}\ln\delta(\varphi_j)/\mathrm{d}\ln\varphi_j$ is the elasticity of $\delta(\varphi_j)$ for market $j \in \{d, x\}$.

Note that $\delta(\varphi_j)$ is proportional to the cumulative market share (in any given market) of firms above any cutoff φ_j. Therefore $\gamma(\varphi_j)$ represents the hazard function for the distribution of log firm size within a market. If the distribution of productivity φ is an untruncated Pareto(k), then the distribution of firm size (in any given market) also will be an untruncated Pareto($k - \sigma + 1$) and the hazard function $\gamma(\cdot)$ will be constant at $k - (\sigma - 1)$. In this case, the partial and full trade elasticity

[11] In the web appendix, we show how a multi-country version of our model yields an expression for log bilateral trade that is linear in exporter and importer fixed effects and $\vartheta\ln\tau$.

are equal to one another and constant at k. Even a slight departure from an untruncated Pareto to a truncated Pareto implies that the partial and full trade elasticity are distinct from one another and variable. In this case, the hazard function $\gamma(\varphi_j)$ becomes:

$$\gamma(\varphi_j) = (k - (\sigma - 1)) \frac{\left(\frac{\varphi_{\min}}{\varphi_j}\right)^{k-(\sigma-1)}}{\left(\frac{\varphi_{\min}}{\varphi_j}\right)^{k-(\sigma-1)} - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^{k-(\sigma-1)}}, \tag{31}$$

where $\varphi_{\max}$ is the upper bound to the support of the productivity distribution. As $\varphi_{\max} \to \infty$, the hazard function $\gamma(\varphi_j)$ converges to its constant value for an untruncated Pareto distribution: $\lim_{\varphi_{\max}\to\infty} \gamma(\varphi_j) = k - (\sigma - 1)$. More generally, for $\varphi_{\max} < \infty$, $\gamma(\varphi_j)$ takes a strictly higher value than for an untruncated Pareto productivity distribution and differs between the domestic and export market. The hazard function for each market is increasing in the productivity cutoff, attaining its minimum value as $\varphi_j \to \varphi_{\min}$, and converging towards infinity as $\varphi_j \to \varphi_{\max}$. Since higher variable trade costs reduce the domestic productivity cutoff and increase the export productivity cutoff, they imply a *lower* $\gamma(\varphi_d)$ and a *higher* $\gamma(\varphi_x)$.

Using welfare (16) and the trade share (18), welfare in the heterogeneous firm model can be written:

$$\mathbb{W}_{\text{Het}} = \frac{\sigma - 1}{\sigma} M_e^{\frac{1}{\sigma-1}} \left(\frac{\delta(\varphi_d)}{\lambda}\right)^{\frac{1}{\sigma-1}}, \tag{32}$$

Since welfare (16) also implies that changes in welfare are proportional to changes in the domestic productivity cutoff ($\mathrm{d}\ln\mathbb{W} = \mathrm{d}\ln\varphi_d$ and $\mathrm{d}\ln\delta(\varphi_d) = -\gamma(\varphi_d)\mathrm{d}\ln\varphi_d$), we can then write the welfare change using the (observable) partial trade elasticity from (30):

$$\mathrm{d}\ln\mathbb{W} = \frac{1}{\vartheta + [\gamma(\varphi_d) - \gamma(\varphi_x)]} \left(\mathrm{d}\ln M_e - \mathrm{d}\ln\lambda\right). \tag{33}$$

As highlighted by ACR, restricting the distribution of productivity draws $G(\varphi)$ to be untruncated Pareto and assuming that there is export market selection ($\varphi_x^T > \varphi_d^T$), ensures that the macro restrictions (R1)-(R3) are satisfied. In this case, the hazard function is constant so that the difference $\gamma(\varphi_d) - \gamma(\varphi_x)$ is zero, and entry does not respond to changes in trade costs ($\mathrm{d}\ln M_e = 0$). In this case, we recover the welfare gain derivation (28) from ACR. Since the partial trade elasticity ϑ is constant in this case, the welfare differential can be integrated to capture proportional welfare changes between any two trade regimes, so long as there is export market selection in both.

However, the welfare differential (33) highlights how, in the general case, the welfare gains from trade liberalization will change with the micro structure. Even after controlling for the trade share and trade elasticity, this micro structure matters for welfare through the hazard differential $\gamma(\varphi_d) - \gamma(\varphi_x)$. In Section 6, we show quantitatively how small changes in the shape of the distribution of firm productivity $G(\varphi)$ away from an untruncated Pareto distribution can lead to large changes in the hazard differential $\gamma(\varphi_d) - \gamma(\varphi_x)$. This issue is distinct from the challenge of measuring the "appropriate" trade elasticity ϑ in a world where this elasticity is variable (both across countries and within each country for different values of trade costs). The predicted welfare effects of trade liberalization based on the ACR formula will also diverge from the true welfare effects because of the variable nature of the partial trade elasticity ϑ.

Finally, the welfare differential (33) also shows that, in cases where trade liberalization leads to responses in firm entry ($\mathrm{d}\ln M_e \neq 0$), then this change in micro structure will also affect the welfare gains from trade, even conditional on the domestic trade share and trade elasticity.

Our analysis also highlights the direction of the bias in the ACR formula. With a truncated Pareto productivity distribution, the hazard function $\gamma(\varphi_j)$ is monotonically increasing in the productivity cutoff φ_j. Furthermore, in an equilibrium with export market selection, the domestic productivity cutoff (φ_d) is less than the export productivity cutoff (φ_x), which implies a negative hazard differential $\gamma(\varphi_d) - \gamma(\varphi_x)$. Therefore, even with a correct estimate of the variable partial trade elasticity ϑ, an evaluation of welfare changes (33) that does not control for the hazard differential will tend to understate the absolute magnitude of changes in welfare in response to changes in trade costs, since $\vartheta > \vartheta + \gamma(\varphi_d) - \gamma(\varphi_x)$.

To make our argument as clearly as possible, we have developed these results for two symmetric countries. But the expression for welfare in the heterogeneous firm model with a general productivity distribution (32) holds more generally in a setting with many asymmetric countries, as shown in the web appendix. In such a setting, there is a separate partial trade elasticity for each exporter-importer pair. Empirical estimates of the coefficient on variable trade costs from a gravity equation including exporter and importer fixed effects capture the average value of this elasticity across all exporter-importer pairs in the regression sample. This average elasticity need not provide a good approximation to the partial trade elasticity for any one individual exporter-importer pair either inside or outside the regression sample. The appropriate elasticity for welfare in (33) is the partial trade elasticity for any one individual exporter-importer pair *adjusted* for the hazard differential between that exporter-importer pair and the domestic market.

A somewhat separate implication of an untruncated Pareto distribution is that the increase in product variety from imports (following trade liberalization) is exactly offset by a decrease in domestic product variety (associated with tougher selection). Hsieh and Ossa (2011) establish this result for a multi-sector setting with asymmetric countries and CES preferences (see also Feenstra 2010). Feenstra (2014) shows that this implication of the untruncated Pareto productivity distribution extends to a general class of non-CES preferences, but that it is similarly broken by small departures away from an untruncated Pareto distribution (to a truncated Pareto distribution). In our setting with a general productivity distribution, the response of firm entry to trade liberalization implies changes in product variety available for consumption.

Lastly, we briefly characterize the gains from trade (in terms of observable moments) when trade costs are sufficiently low that all surviving firms export. In other words, there is no export market selection and $\varphi_d^T = \varphi_x^T$. As we previously discussed, the equilibrium in this case will have identical aggregate properties to an equilibrium with homogeneous firms, in which all firms have a common productivity level $\tilde{\varphi}_d^T$ and face a fixed cost $F^T = \left(f_e/\left[1 - G(\varphi_d^T)\right]\right) + f_d + f_x$. Thus, the welfare gains associated with a transition from trade regime T_0 to T_1 can be measured using:

$$\frac{\mathbb{W}^{T_1}}{\mathbb{W}^{T_0}} = \left(\frac{\lambda^{T_0} F^{T_0}}{\lambda^{T_1} F^{T_1}}\right)^{\frac{1}{\sigma-1}}, \tag{34}$$

where in this case the full and partial trade elasticities are equal to one another: $\theta = \vartheta = \sigma - 1$.

As in the homogeneous firm case, we see that the welfare gains from trade depend on changes in average firm size (captured by F^T) as well as the domestic trade share and trade elasticity (which is now constant at $\sigma - 1$). Average firm size is now endogenous and varies with the domestic productivity cutoff (φ_d^T affects F^T). Any change in the fixed exporting costs between T_0 and T_1 will induce changes in average firm size - even when productivity has an untruncated Pareto distribution (as was the case in the homogeneous firm model, this situation represents a violation of ACR's macro restriction R2).

Taking the results of this subsection together, our generalization of the ACR welfare representation provides a way of quantitatively assessing whether predictions for the welfare gains from trade liberalization based on the domestic trade share and the assumption of a constant trade elasticity provide a good approximation to the true welfare gains. The success of this approximation depends on the extent to which the partial trade elasticity is constant, the size of the hazard rate differential between the domestic and export markets and the degree to which firm entry responds to changes in trade costs. If firm-level data are available, these differences in hazard rates and the response of firm entry can be examined empirically. Admittedly, measuring the response of firm entry to changes in trade costs raises challenges. But these challenges are similar to those faced in estimating a partial trade elasticity and recovering the change in trade induced by a change in variable trade costs alone. Head, Mayer and Thoenig (2014) propose a goodness of fit test of firm size distribution to the Pareto distribution that is similar to checking for changes in the hazard rate (which is constant under Pareto).

Even in cases where only aggregate trade data is available, one can in principle estimate differences in trade elasticities across country-partner pairs. Helpman, Melitz and Rubinstein (2008) and Novy (2013) both implement gravity estimation procedures that allow for variation in the elasticity of trade with respect to observable trade frictions (such as distance). Both papers find substantial variation in these elasticities. Unless this variation is exactly offset by an equal and opposite variation in the elasticity of trade costs with respect to the observable trade frictions, these results imply a variable elasticity of trade with respect to trade costs.

In the setting with many asymmetric countries discussed above, our generalized welfare derivations highlight that the discrepancy between the predicted and true welfare effects of trade liberalization will be minimized by choosing a trade elasticity for country-partners that most closely approximates the elasticity for a country's trade with itself. If the hazard rate function $\gamma(\varphi_j)$ is monotonic in the cutoff φ_j, then the hazard differential $\gamma(\varphi_{ii}) - \gamma(\varphi_{ik})$ will be minimized when φ_{ik} is closest to φ_{ii}, which occurs for the trading partner with the highest share of exporting firms.

5 Trade Policy Evaluation

In the previous section, we introduced small deviations from the ACR parameter restrictions and showed how the micro structure then affects the measurement of the welfare gains from trade – even when conditioning on a given trade elasticity and a given domestic trade share. This led to discrepancies between the true welfare effects of trade liberalization and those predicted by the ACR formula. We now illustrate more concretely how such discrepancies may arise when evaluating the welfare gains generated from a few specific trade liberalization scenarios. Our starting point is the heterogeneous firm model with two symmetric countries developed in Section 2. We consider the welfare gains from liberalizing trade first from trade regime $T_0(\tau_0, f_{x0})$ to $T_1(\tau_1, f_{x1})$, and then to $T_2(\tau_2, f_{x2})$. We contrast the true welfare gains from (16) with those measured by a policy analyst who applies the ACR formula (28). We also contrast the cases of *ex post* and *ex ante* policy evaluation using a similar approach to ACR and Costinot and Rodriguez-Clare (2014).

Specifically, we assume that trade liberalization from $T_0 \to T_1$ is evaluated *ex post* so that the domestic trade shares λ^{T_0} and λ^{T_1} are observed, and the (arc) trade elasticity therefore can be directly measured as:[12]

$$\hat{\theta}_{01} = -\frac{\ln\left(\frac{1-\lambda^{T_1}}{\lambda^{T_1}}\right) - \ln\left(\frac{1-\lambda^{T_0}}{\lambda^{T_0}}\right)}{\ln \tau_1 - \ln \tau_0}. \tag{35}$$

The ACR predicted welfare gains from trade are then $\hat{\mathbb{W}}_{01} = \left(\lambda^{T_0}/\lambda^{T_1}\right)^{1/\hat{\theta}_{01}}$. On the other hand, we assume that trade liberalization from $T_1 \to T_2$ is evaluated *ex ante*, so the domestic trade share λ^{T_2} is unobserved and is recovered from the model using the elasticity $\hat{\theta}_{01}$. That is, $\hat{\lambda}_{T_2}$ solves:

$$\hat{\theta}_{01} = -\frac{\ln\left(\frac{1-\hat{\lambda}^{T_2}}{\hat{\lambda}^{T_2}}\right) - \ln\left(\frac{1-\lambda^{T_1}}{\lambda^{T_1}}\right)}{\ln \tau_2 - \ln \tau_1}. \tag{36}$$

Ex ante, the ACR welfare derivation yields predicted welfare gains from trade given by $\hat{\mathbb{W}}_{12} = \left(\lambda^{T_1}/\hat{\lambda}^{T_2}\right)^{1/\hat{\theta}_{01}}$. We assume that the trade costs in the trade regimes T_0 and T_1 are high enough to generate export market selection. However, we do not impose this restriction on the hypothetical trade regime T_2: A policy analyst may be interested in evaluating the welfare gains from trade for scenarios that go most (or all) of the way to free trade.

5.1 Scenario 1: Untruncated Pareto Productivity Distribution

We assume that $G(\varphi)$ is distributed untruncated Pareto(k) and initially assume no change in the fixed export costs: $f_{x0} = f_{x1} = f_{x2}$. Then, the measured elasticity $\hat{\theta}_{01}$ will recover the constant elasticity k, and $\hat{\mathbb{W}}_{01}$ will exactly measure the "true" welfare gains from the *ex post* liberalization $T_0 \to T_1$. Also, if the hypothetical trade regime T_2 features export market selection (the trade costs τ_2 and f_{x2} are high enough), then $\hat{\theta}_{01} = k$ will also capture the trade elasticity between T_1 and T_2, and the analyst would

[12] When the distribution of productivity draws $G(\varphi)$ is an untruncated Pareto – a necessary condition for the ACR macro restrictions to hold – there is no difference between the full and partial trade elasticities θ and ϑ.

also correctly predict the attained domestic trade share in regime T_2. Thus, the predicted welfare gain $\hat{\mathbb{W}}_{12}$ will again recover the "true" welfare gain from (16).

However, if the trade costs in regime T_2 are low enough – such that all firms export in T_2 – then the *ex ante* welfare evaluation will be incorrect. The true trade elasticity drops from k to $\sigma - 1$ once there is no export market selection, and this change will not be reflected in the elasticity $\hat{\theta}_{01}$. Consequently, the analyst will also incorrectly predict the attained domestic trade share in regime T_2. This transition between the case of export market selection and no selection represents a violation of ACR's macro restriction (R3). Yet, this transition occurs endogenously in our model; the only structural change is a reduction in trade costs.

We now consider the case where trade liberalization from $T_0 - T_1$ involves a change in both the variable and fixed trade cost. In this case, the measured trade elasticity $\hat{\theta}_{01}$ will be biased, because it captures the effects of the change in both the variable and fixed trade costs. In turn, this will generate discrepancies between the true and predicted welfare gains from trade liberalization for both the *ex post* and *ex ante* policy evaluations. This case does not represent any violation of ACR's macro restrictions; it represents a measurement issue for the trade elasticity.[13]

5.2 Scenario 2: Truncated Pareto Productivity Distribution

We now assume that $G(\varphi)$ is distributed Pareto, but that there is a finite upper bound to the support of the productivity distribution ($\varphi_{\max} < \infty$). We return to our initial assumption of no change in the fixed exporting cost (so $f_{x0} = f_{x1} = f_{x2}$). As we highlighted in the previous section, this small departure from an untruncated Pareto distribution changes the derivation of the welfare gains from trade from the ACR formula (28) to (33). We abstract from measurement issues for the trade elasticity induced by differences between arc versus point elasticities, and between the full versus the partial trade elasticity. Thus, we assume that the measured elasticity $\hat{\theta}_{01}$ yields an accurate estimate for any point partial trade elasticity ϑ between τ_0 and τ_1.[14]

Nevertheless, the analyst will obtain an incorrect measure of the *ex post* welfare gains from trade because both the difference in the hazard rates $\gamma(\varphi_d) - \gamma(\varphi_x)$ and the response of entry $\triangle \ln M_e$ will be non-zero. In our quantitative analysis in the next section, we find the former effect to be substantial while the latter effect is relatively small for our parametrization.

Lastly, we consider the evaluation of an *ex ante* hypothetical trade liberalization from T_1 to T_2. The same discrepancies between the true and predicted welfare gains from trade liberalization as mentioned above for the *ex post* evaluation will also apply to the *ex ante* case. In addition, the measured elasticity $\hat{\theta}_{01}$ will no longer apply to changes in τ between τ_1 and τ_2 – even abstracting from

[13] ACR note that, in a multi-country world, the trade elasticity can be recovered from the estimation of a gravity equation when variations in bilateral tariffs are observed. This estimation also requires that the variation in *any* fixed trade cost that is correlated with the tariffs is also observed. In the absence of controls for this variation in fixed costs, the gravity equation estimation will be subject to omitted variables bias. While the seriousness of this concern depends on the source of variation used, to our knowledge such data on fixed trade costs are not available.

[14] In our calibration based on a truncated Pareto distribution in the next section, we find that differences between the arc partial elasticity and the arc full elasticity have a relatively small effect on the measured welfare gains from trade.

the differences between arc versus point elasticities and between the full and partial elasticities: All of these elasticities are different for $\tau \in (\tau_1, \tau_2)$ relative to $\tau \in (\tau_0, \tau_1)$.[15] This also means that the analyst will incorrectly predict the domestic trade share in regime T_2.

In closing, we highlight that all the different trade liberalization scenarios that we have described in this section satisfy the assumptions for our open economy comparative static exercise described in Subsection 3.2. Thus, in all these cases, the extra margin of adjustment in the heterogeneous firm model is operative. Under the ACR parameter restrictions, the trade elasticity is constant and the heterogeneous and homogeneous firm models can be calibrated to both generate the same welfare gains from trade across the two models. Even for small departures from these parameter restrictions, the trade elasticity is not constant, and is not a sufficient statistic for the welfare gains from trade (along with the domestic trade share). Instead, micro structure also matters for the welfare gains from trade and differs between the heterogeneous and homogeneous firm models, because of the extra adjustment margin in the heterogeneous firm model.

6 Quantitative Relevance

In this section, we examine the quantitative relevance of our results. In subsection 6.1, we show that the extra margin of adjustment in the heterogeneous firm model is associated with substantial differences in the aggregate welfare implications of trade between the heterogeneous and homogeneous firm models. In subsection 6.2, we show that the assumption of a constant trade elasticity when the true elasticity is variable can lead to quantitatively relevant discrepancies between the true and predicted welfare effects of trade liberalization.

6.1 Theoretical Comparative Static

In this subsection, we compare the welfare properties of the heterogeneous and homogeneous firm models holding all structural parameters other than the productivity distribution constant between the two models. We assume an untruncated Pareto distribution for productivity in the heterogeneous firm model; this satisfies the ACR macro restrictions so long as trade costs are high enough to generate export market selection. The homogeneous firm model satisfies the ACR restrictions so long as the fixed exporting cost remains constant. We choose standard values for the heterogeneous firm model's parameters based on central estimates from the existing empirical literature and moments of the U.S. data. Those same structural parameters then carry over to the homogeneous firm case, except that we set a degenerate productivity distribution as described below.

We set the elasticity of substitution between varieties $\sigma = 4$, which is consistent with the estimates using plant-level U.S. manufacturing data in Bernard, Eaton, Jensen and Kortum (2003). The Pareto shape parameter for the productivity distribution (k) determines the elasticity of trade flows with

[15] A growing body of empirical research reports results that are consistent with a variable trade elasticity, including Helpman, Melitz and Rubinstein (2008), Novy (2013), and Head, Mayer and Thoenig (2014). Simonovska and Waugh (2014b) provides evidence that the estimated trade elasticity is model dependent. Imbs and Méjean (2009) and Ossa (2012) argue that aggregate trade elasticities are influenced by sectoral composition in multi-sector models.

respect to variable trade costs in the heterogeneous firm model under export market selection. We set $k = 4.25$ as a central value for estimates of the trade elasticity.[16] A choice for the Pareto scale parameter is equivalent to a choice of units in which to measure productivity, and hence we normalize $\varphi_{\min} = 1$.

We consider trade between two symmetric countries, and choose labor in one country as the numeraire ($w = 1$), which implies that the wage in both countries is equal to one. With an untruncated Pareto productivity distribution, scaling L and $\{f_e, f_d, f_x\}$ up or down by the same proportion leaves the productivity cutoffs $\{\varphi_d^T, \varphi_x^T\}$ and the mass of entrants unchanged (M_e), and merely scales average firm size ($\bar{r}$) up or down by the same proportion. Therefore we set L equal to the U.S. labor force and normalize f_d to one. With an untruncated Pareto productivity distribution, the sunk entry cost f_e affects the absolute levels of the productivity cutoffs and welfare but not their relative levels for different values of trade costs. As a result, the relative comparisons below are invariant to the choice of f_e, and hence we normalize f_e to one.

We calibrate τ to match the average fraction of exports in firm sales in U.S. manufacturing ($\frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}} = 0.14$, as reported in Bernard, Jensen, Redding and Schott 2007), which implies $\tau = 1.83$ (which is in line with the estimate of 1.7 in Anderson and van Wincoop 2004). Given our choice for the parameters $\{\sigma, k, \varphi_{\min}, f_d, f_e, \tau\}$, we choose f_x to ensure that the model is consistent with the average fraction of U.S. manufacturing firms that export (0.18, as reported in Bernard, Jensen, Redding and Schott 2007).

We choose the degenerate productivity distribution in the homogeneous firm model so that the two models generate the same aggregate variables in an initial equilibrium. In our baseline specification here, we do so for an initial open economy equilibrium using our calibrated values of trade costs of $\tau = 1.83$ and $f_x = 0.545$. Thus we compare the heterogeneous firm model to the extended homogeneous firm model introduced in subsection 3.2. In the web appendix, we undertake a similar analysis for an initial closed economy equilibrium, as analyzed in subsection 3.1.

In our baseline specification here, we solve for the initial open economy equilibrium of the heterogeneous firm model, including the probability of successful firm entry $\left[1 - G\left(\varphi_d^{T_0}\right)\right]$, the proportion of exporting firms $\left[1 - G\left(\varphi_x^{T_0}\right)\right] / \left[1 - G\left(\varphi_d^{T_0}\right)\right]$, weighted average productivity in the export market ($\tilde{\varphi}_x^{T_0}$), weighted average productivity in the domestic market ($\tilde{\varphi}_d^{T_0}$), and the weighted average productivity of domestic firms ($\tilde{\varphi}_{dx}^{T_0}$). In the extended homogeneous firm model, we choose the probabilities of entry and exporting and the weighted average productivities for domestic and exporting firms to equal to their values in the initial open economy equilibrium of the heterogeneous firm model: $\left[1 - \bar{G}_x\right] = \left[1 - G\left(\varphi_x^{T_0}\right)\right]$, $\bar{G}_{dx} = \left[G\left(\varphi_x^{T_0}\right) - G\left(\varphi_d^{T_0}\right)\right]$, $\bar{\varphi}_x = \tilde{\varphi}_x^{T_0}$, and $\bar{\varphi}_{dx} = \tilde{\varphi}_{dx}^{T_0}$. All parameters apart from the productivity distribution are held constant across the two models (same f_d, f_e, f_x, τ, L, σ), which implies $\bar{F}_d = f_d + f_e / \left[1 - G\left(\varphi_d^{T_0}\right)\right]$. The key difference between the two models is that the market entry probabilities and weighted average productivities in each market respond to changes

[16] Simonovska and Waugh (2014a) estimate a trade elasticity of 4.10 or 4.27 depending on the data used. Costinot and Rodriguez-Clare (2013)'s benchmark value for the trade elasticity is 5. Any of these values would lead to quantitatively similar results.

in trade costs in the heterogeneous firm model. In contrast, in the extended homogeneous firm model, these probabilities and weighted average productivities are parameters.

In Figure 1, we show the effects of adjusting variable trade costs from their calibrated value of $\tau^{T_0} = 1.83$ to values of $\tau^{T_1} \in [1, 3]$ (for which there is trade in both models).[17] Panel A displays relative welfare, measured as welfare for each value of variable trade costs relative to welfare in the initial open economy equilibrium: $\mathbb{W}^{T_1}/\mathbb{W}^{T_0}$; Panel B shows the probability of exporting: $\chi^{T_1} = [1 - G(\varphi_x^{T_1})]/[1 - G(\varphi_d^{T_1})]$; Panel C displays weighted average productivity in the domestic market: $\tilde{\varphi}_d^{T_1} = \left[\frac{G(\varphi_x^{T_1}) - G(\varphi_d^{T_1})}{1 - G(\varphi_d^{T_1})}\right] \tilde{\varphi}_{dx}^{T_1} + \left[\frac{1 - G(\varphi_x^{T_1})}{1 - G(\varphi_d^{T_1})}\right] \tilde{\varphi}_x^{T_1}$; Panel D shows the domestic trade share: λ^{T_1}. The solid blue line corresponds to values in the heterogeneous firm model, while the red dashed line corresponds to values in the extended homogeneous firm model.

As shown in Panel A, welfare in the two models is the same for the calibrated value of variable trade costs, but is strictly higher in the heterogeneous firm model than in the extended homogeneous firm model for all other values of variable trade costs. Therefore the welfare gains (losses) from reductions (increases) in variable trade costs are greater (smaller) in the heterogeneous firm model than in the extended homogeneous firm model. The differences in welfare between the two models are quantitatively relevant for empirically plausible changes in variable trade costs. A reduction in variable trade costs from $\tau = 1.83$ to $\tau = 1$ generates welfare that is five percentage points of real GDP higher in the heterogeneous firm model than in the extended homogeneous firm (this represents around a third of the overall welfare gains of 17 percentage points in the heterogeneous firm model).

As shown in Panels B and C, the source of these welfare differences is endogenous selection into the domestic and export markets in the heterogeneous firm model. For the calibrated value of variable trade costs of $\tau = 1.83$, the probability of exporting and weighted average productivity in the domestic market are the same in the two models. As variable trade costs fall from their calibrated value to $\tau = 1$, the probability of exporting rises from 0.18 to 1 in the heterogeneous firm model, but remains constant in the extended homogeneous firm model (Panel B). Additionally, weighted average productivity in the domestic market rises by around 8 percent in the heterogeneous firm model, while remaining constant in the extended homogeneous firm model (Panel C).

As shown in Panel D, the domestic trade share is the same in the two models for the calibrated value of variable trade costs. Given the same structural parameters, the elasticity of trade with respect to variable trade costs is higher in the heterogeneous firm model with an untruncated Pareto distribution (k) than in the extended homogeneous firm model ($\sigma - 1$). Therefore, the domestic trade share is lower in the heterogeneous firm model for variable trade costs below the calibrated value, and is higher in the heterogeneous firm model for variable trade costs above the calibrated value.

Taken together, these results show that the extra margin of adjustment in the heterogeneous firm model relative to the homogeneous firm model is of quantitative relevance for aggregate welfare. In the web appendix, we show that we find similar results when we calibrate the heterogeneous and

[17] For brevity, we concentrate on changes in variable trade costs, but we find a similar pattern of results for changes in fixed exporting costs, as shown in the calibration in the working paper version of this paper.

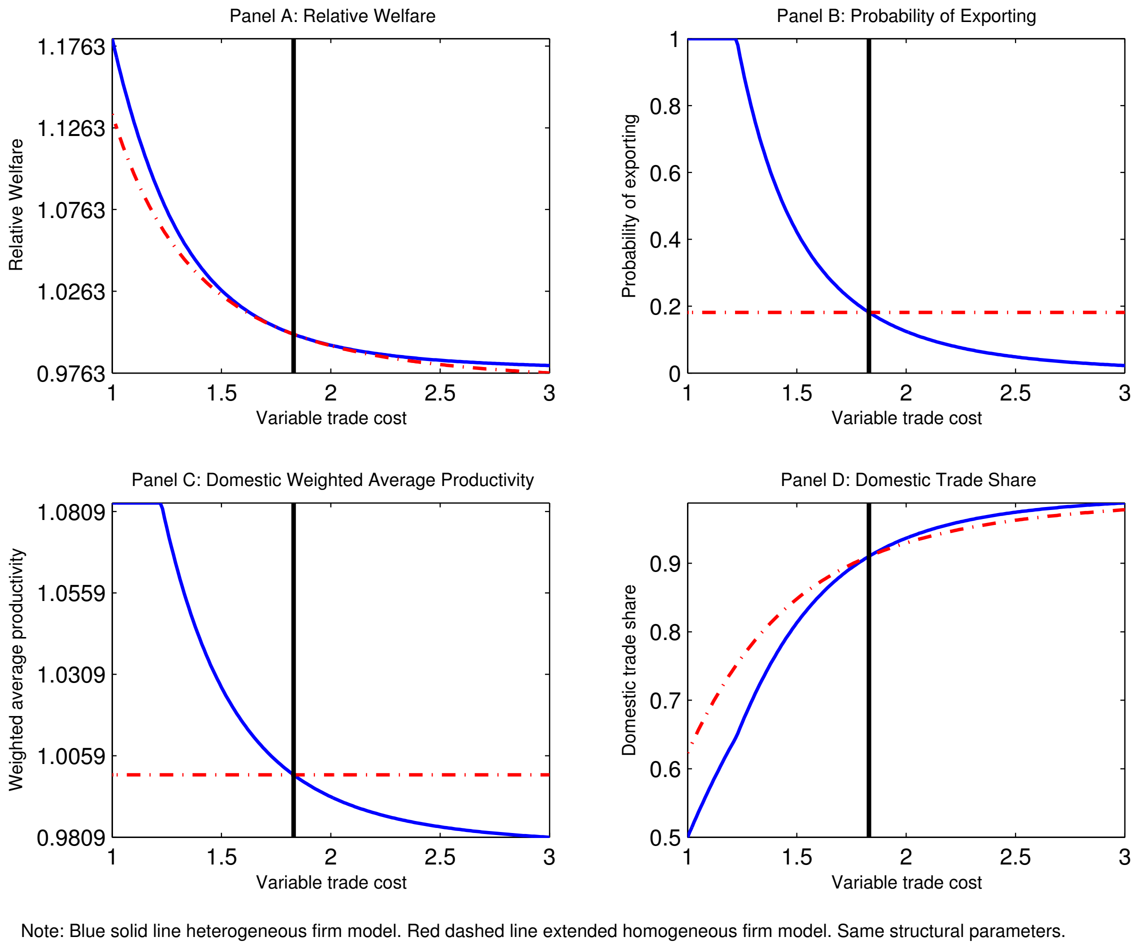

Figure 1: Reductions in variable trade costs in the heterogeneous firm model with an untruncated Pareto distribution and in the extended homogeneous firm model

homogeneous firm models to an initial closed economy equilibrium.

6.2 Practical Evaluation of Trade Policies

We now examine the quantitative implications of a variable trade elasticity for the practical evaluation of trade policies. We assume a truncated Pareto productivity distribution in the heterogeneous firm model (Scenario 2 from Section 5). We keep all of the other parameters in the heterogeneous firm model the same as for the untruncated Pareto distribution in the previous subsection. To calibrate the upper bound to the support of the productivity distribution ($\varphi_{\max}$), we use data on average size differences between exporters and non-exporters. For the assumed value of $f_x = 0.545$, the heterogeneous firm model with an untruncated Pareto distribution from the previous subsection matches the average fraction of U.S. manufacturing firms that export (0.18), but implies larger differences in average revenue between exporters and non-exporters than observed for U.S. manufacturing firms (2.09 log points compared to 1.48 log points in Bernard, Jensen, Redding and Schott 2007). Therefore, we choose f_x and $\varphi_{\max}$ in the heterogeneous firm model with a truncated Pareto distribution so that it matches both of these moments ($f_x = 0.535$ and $\varphi_{\max} = 2.85$).

In Figure 2, we examine each of the components of the proportional change in welfare (33) for this truncated Pareto productivity distribution. Panel A shows the partial trade elasticity (ϑ); Panel B displays the hazard differential between the domestic and export markets ($\gamma(\varphi_d) - \gamma(\varphi_x)$); Panel C shows the domestic trade share (λ); Panel D displays the mass of entrants (M_e). We change variable trade costs from their calibrated value of $\tau^{T_0} = 1.83$ to values of $\tau^{T_1} \in [1, 3]$ for which trade occurs.

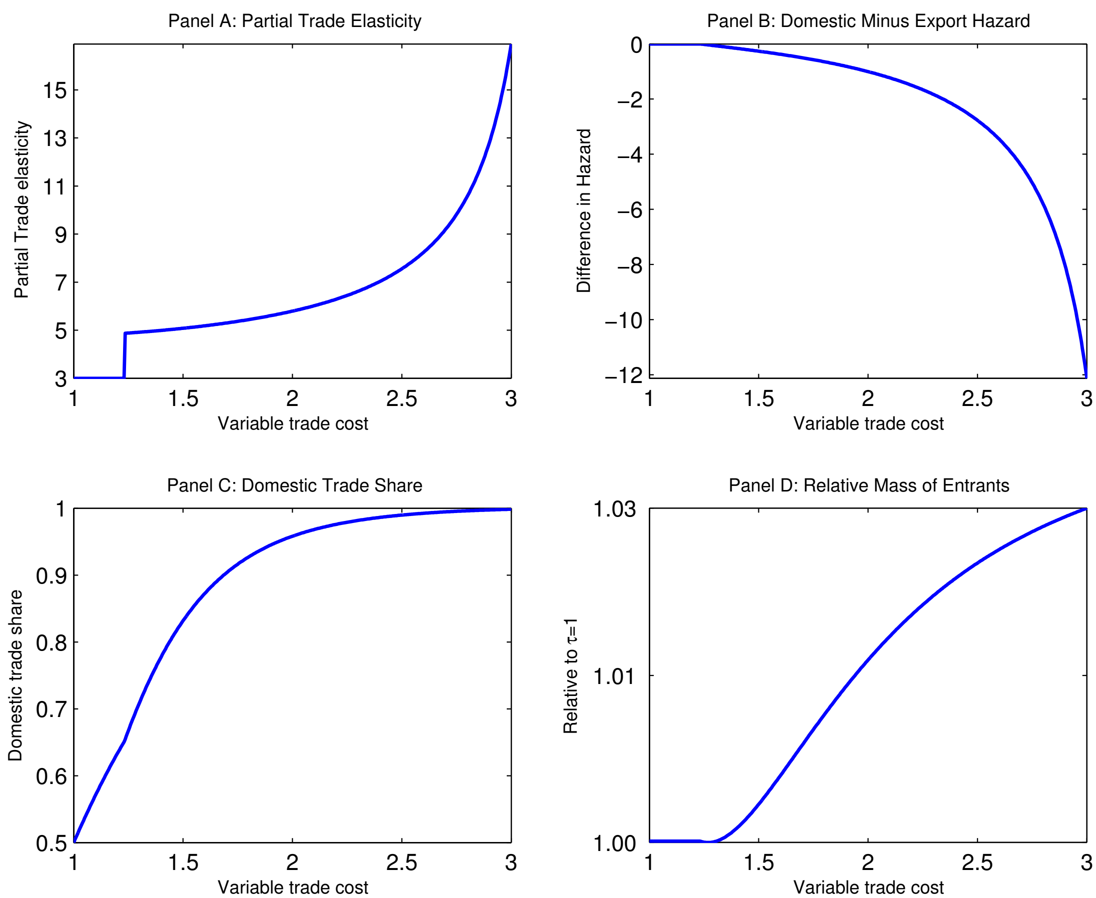

Figure 2: Trade elasticity, domestic trade share and mass of entrants in the heterogeneous firm model with a truncated Pareto productivity distribution

In the special case in which the upper bound to the support of the productivity distribution converges to infinity ($\varphi_{\max} \to \infty$), the truncated Pareto distribution converges to an untruncated Pareto distribution. In this special case, the partial trade elasticity (ϑ) is constant and equal to the full trade elasticity (θ), which is equal to the Pareto shape parameter ($k = 4.25$). Furthermore, in this special case, the mass of entrants depends only on parameters and hence is constant. In contrast, for a truncated Pareto distribution with a finite upper bound to the support of the productivity distribution

($\varphi_{\max} < \infty$), the partial trade elasticity (ϑ) is variable and differs from both the full trade elasticity (θ) and the Pareto shape parameter ($k = 4.25$). As we vary variable trade costs from one to three, the partial trade elasticity in Panel A ranges from around three to more than fifteen.[18]

The relationship between the partial trade elasticity and trade costs can be seen from our closed form expression for the hazard function with a truncated Pareto distribution (31). As variable trade costs increase, the export productivity cutoff (φ_x^T) rises, which increases the export hazard ($\gamma(\varphi_x^T)$) and hence in turn increases the partial trade elasticity (ϑ). As variable trade costs become sufficiently large that the export productivity cutoff approaches the upper bound to the support of the productivity distribution ($\varphi_x \to \varphi_{\max}$), the partial trade elasticity converges towards infinity ($\vartheta \to \infty$). From the hazard function (31), this result is robust to the choice of any finite value for the upper bound to the support of the productivity distribution ($\varphi_{\max}$), because there exists a sufficiently high trade cost such that φ_x^T converges to any finite value of $\varphi_{\max}$.[19] As variable trade costs become sufficiently small that all firms export, the export and domestic productivity cutoffs become equal to one another ($\varphi_x^T = \varphi_d^T$) and independent of variable trade costs. At the threshold value for variable trade costs below which all firms export, the partial trade elasticity (ϑ) falls discretely to $\sigma - 1$ and remains equal to this constant value for all lower variable trade costs. Taken together, these results suggest that the partial trade elasticity can vary quite substantially from one context to another.

As shown in Panel B, these changes in variable trade costs have implications for the difference in hazard functions between the domestic and export markets ($\gamma(\varphi_d^T) - \gamma(\varphi_x^T)$). As variable trade costs increase, the resulting rise in the export productivity cutoff (φ_x^T) *increases* the hazard function in the export market ($\gamma(\varphi_x^T)$), but the associated reduction in the domestic productivity cutoff (φ_d^T) *reduces* the hazard function in the domestic market ($\gamma(\varphi_d^T)$). As a result, as we vary variable trade costs from one to three, the hazard rate differential between the two markets ranges from zero to minus twelve. When variable trade costs are low enough, all firms export, there is a single domestic/export productivity cutoff ($\varphi_x = \varphi_d$), and there is no difference between the two hazard functions, $\gamma(\varphi_d) - \gamma(\varphi_x) = 0$. However, as variable trade costs increase, generating export market selection, the hazard differential monotonically decreases from zero (as the proportion of exporting firms decreases). When variable trade costs become sufficiently large, the proportion of exporting firm goes to zero ($\varphi_x \to \varphi_{\max}$) and the hazard differential converges to minus infinity ($\gamma(\varphi_d) - \gamma(\varphi_x) \to -\infty$). This differential is directly related to the bias associated with using the foreign trade elasticity instead of the (unobserved) domestic trade elasticity when evaluating welfare gains from trade using the ACR formula (see equation (33)). Thus, when this evaluation is performed for country pairs with a low proportion of exporting firms, this bias can be arbitrarily large.

As shown in Panel C, increases in variable trade costs raise the domestic trade share, which converges towards one as variable trade costs rise towards three, and converges towards a value of one

[18] Assuming a constant elasticity of trade costs with respect to distance, Novy (2013) estimates elasticities of trade with respect to trade costs that range from less than five to more than twenty.

[19] To illustrate this robustness of our results to the choice for the upper bound to the support of the truncated Pareto productivity distribution, the web appendix reports a robustness check using $\varphi_{\max} = 4$.

half as variable trade costs fall to one (reflecting country symmetry). As shown in Panel D, increases in variable trade costs raise the mass of entrants, which is shown for each value of variable trade costs in the figure relative to its value for $\tau = 1$. With a truncated Pareto distribution, higher variable trade costs reduce average firm size conditional on successful entry. With a fixed labor endowment, this in turn leads to a larger mass of entrants. For the parameterization considered here, these changes in the mass of entrants are relatively small, with the mass of entrants increasing by around 3 percent as variable trade costs increase from one to three. As variable trade costs become sufficiently small that all firms export, the export and domestic productivity cutoffs become equal to one another ($\varphi_x^T = \varphi_d^T$) and independent of variable trade costs. Therefore, for this range of variable trade costs, both average firm size and the mass of entrants are constant.

We now examine the quantitative implications of the above changes in micro structure for the evaluation of trade policies. Table 1 compares the true welfare gains from trade liberalization with a truncated Pareto distribution to the welfare gains that would be predicted by a policy analyst who falsely assumed a constant trade elasticity and applied the ACR formula. We examine trade liberalization from high variable trade costs for which the economy is relatively closed ($\tau = 3$ and $\lambda = 0.998$), through intermediate values of variable trade costs ($\tau = 1.5$ and $\lambda = 0.832$), and to low values of variable trade costs for which the economy is relatively open but still only some firms export ($\tau = 1.25$ and $\lambda = 0.668$).

In Column (1), we report the true relative change in welfare ($\mathbb{W}^{T_1}/\mathbb{W}^{T_0}$) in the heterogeneous firm model with a truncated Pareto distribution (as computed using (32)). Reducing variable trade costs from $\tau = 3$ to $\tau = 1.25$ increases welfare by 8.07 percent, which is broadly in line with estimates of the welfare gains from trade in recent quantitative trade models. Around half of these welfare gains are achieved from the reduction in variable trade costs from $\tau = 3$ to $\tau = 1.5$ (3.36 percent), with the remaining half realized from a further reduction in variable trade costs to $\tau = 1.25$ (4.56 percent). Since the variable partial trade elasticity is increasing in variable trade costs, larger welfare gains are generated from a given percentage reduction in variable trade costs when the economy is relatively open than when it is relatively closed. This property of a variable trade elasticity has important implications for the evaluation of future efforts at multilateral trade liberalization. Even if variable trade costs already have been reduced to relatively low levels, this does not necessarily mean that

	(1)	(2)	(3)	(4)
Trade Liberalization	Actual (Truncated Pareto)	Predicted (ACR formula) ϑ^{start}	Predicted (ACR formula) $\vartheta^{\text{average}}$	Predicted (ACR formula) $\theta^{\text{start}}_{\text{end}}$
$\tau = 3$ to $\tau = 1.25$	108.07%	102.41%	105.80%	106.30%
$\tau = 3$ to $\tau = 1.5$	103.36%	101.09%	102.47%	102.64%
$\tau = 1.5$ to $\tau = 1.25$	104.56%	104.43%	104.52%	104.55%

Table 1: Actual and Predicted Welfare Gains from Trade Liberalization

most of the welfare gains from reductions in variable trade costs already have been achieved.

In Column (2), we report the results of an *ex ante* policy evaluation under the (false) assumption of a constant trade elasticity. We consider a policy analyst who has access to estimates of the partial trade elasticity for an initial value of trade costs (ϑ^{start}). The policy analyst considers each of the reductions in variable trade costs (e.g. from $\tau = 3$ to $\tau = 1.5$) and uses the ACR formula to predict the welfare effects of these trade liberalizations based on the observed change in the domestic trade share and the assumption of a constant trade elasticity.

For trade liberalizations starting from high variable trade costs (the first and second rows), we find substantial discrepancies between the true and predicted welfare gains from trade liberalization. Reducing variable trade costs from $\tau = 3$ to $\tau = 1.25$ is predicted in Column (2) to increase welfare by 2.41 percent (a discrepancy of around six percentage points or 70 percent). These discrepancies arise because the true trade elasticity is variable rather than constant and because the hazard function differs between the domestic and export markets. For high values of variable trade costs, the partial trade elasticity changes substantially across different values of trade costs (Panel A of Figure 2) and the difference in the hazard function between the domestic and export market is large (Panel B of Figure 2). In contrast, for reductions in variable trade costs from intermediate to low values (the third row), we find that the predicted and true welfare effects of trade liberalization are relatively close to one another (a discrepancy of less than one percentage point). At these lower values of variable trade costs, the partial trade elasticity is relatively stable (Panel A of Figure 2), and the difference in the hazard function between the domestic and export markets is small (Panel B of Figure 2), because the export and domestic productivity cutoffs are close together.

In Columns (3) and (4), we report the results of an *ex post* policy evaluation under the (false) assumption of a constant trade elasticity. We consider a policy analyst who has access to an estimate of the average trade elasticity in between the start and end values of variable trade costs. The policy analyst considers each of the reductions in variable trade costs (e.g. from $\tau = 3$ to $\tau = 1.5$) and uses the ACR formula to predict the welfare effects of these trade liberalizations based on the observed change in the domestic trade share and the estimated average trade elasticity. We consider two different estimates for the average trade elasticity. In Column (3), we compute an average partial trade elasticity by considering variable trade costs at intervals of 0.005, evaluating the partial trade elasticity at each of these points, and taking the arithmetic mean of the partial trade elasticities across these points ($\vartheta^{\text{average}}$). In Column (4), we compute an average full trade elasticity by evaluating the logarithmic percentage growth in trade between the start and end values of trade costs and dividing this by the logarithmic percentage reduction in variable trade costs ($\theta^{\text{start}}_{\text{end}}$). Although the estimated average trade elasticity in Column (4) is a full elasticity rather than a partial elasticity, in practice we find similar results in both Columns (3) and (4).

For trade liberalizations starting from high variable trade costs (the first and second rows), we continue to find quantitatively relevant discrepancies between the true and predicted welfare gains from trade liberalization. Reducing variable trade costs from $\tau = 3$ to $\tau = 1.25$ is predicted in

Column (3) to increase welfare by 5.80 percent (a discrepancy of around 2.27 percentage points or around 28 percent of the true welfare gain from trade liberalization). In contrast, for reductions in variable trade costs from intermediate to low values (the third row), we find that the predicted and true welfare effects of trade liberalization are relatively close to one another (a discrepancy of less than one percentage point). Again this reflects the relative stability of the partial trade elasticity (Panel A of Figure 2) and the small difference between the domestic and export hazards (Panel B of Figure 2) at low values of variable trade costs. Unsurprisingly, the difference between the true and predicted welfare effects of trade liberalization is smaller using an average estimated trade elasticity in an *ex post* evaluation than using an initial estimated trade elasticity in an *ex ante* evaluation.

As discussed in section 4.3, with a truncated Pareto productivity distribution, the hazard function $\gamma(\varphi_j)$ is monotonically increasing in the productivity cutoff φ_j, and hence the hazard differential $\gamma(\varphi_d) - \gamma(\varphi_x)$ is negative under selection into export markets. Therefore, even with a correct estimate of the variable partial trade elasticity ϑ, an evaluation of welfare changes (33) without controlling for the hazard differential will tend to understate the absolute magnitude of changes in welfare in response to changes in trade costs, since $\vartheta > \vartheta + \gamma(\varphi_d) - \gamma(\varphi_x)$. Consistent with this direction of bias, the predicted welfare changes using the ACR formula in Table 1 are all smaller in absolute magnitude that the true changes in welfare with a truncated Pareto productivity distribution.

Key takeaways from this section are that both the partial trade elasticity and the hazard differential between the domestic and export markets can vary substantially across different values for variable trade costs (and hence in a multi-country world across relatively open and relatively closed economies). Taking a trade elasticity estimated from a relatively closed economy and applying this elasticity to a relatively open economy without controlling for the difference in hazard functions between the two markets can lead to quantitatively relevant discrepancies between the predicted and true welfare effects of trade liberalization in both *ex ante* and *ex post* evaluations. In contrast, taking a trade elasticity estimated from a relatively open economy and applying it to another relatively open economy provides a much closer approximation to the true welfare effects of trade liberalization.

We focus our quantitative analysis in this subsection on the truncated Pareto distribution to highlight that simply changing the upper bound to the support of the productivity distribution can induce substantial variation in partial trade elasticities and substantial differences in the hazard function between the domestic and export markets. But the point that the partial trade elasticity is variable and the hazard function differs across markets is much more general, and also applies for example with a log normal distribution, as examined in Head, Mayer and Thoenig (2014).

7 Conclusions

We examine whether firm heterogeneity matters for the aggregate welfare implications of trade. We use a theoretical comparative static to show that endogenous firm selection provides an extra welfare margin for trade liberalization in the heterogeneous firm model relative to the homogeneous firm model. Under additional restrictions on the parameter space, ACR show that two aggregate statistics, the domestic trade share and a constant trade elasticity, are sufficient statistics for the welfare gains from trade. But the existence of a single constant trade elasticity is highly sensitive to small departures from those parameter restrictions, such as generalizing the productivity distribution in the heterogeneous firm model from an untruncated to a truncated Pareto. In this more general setting, the endogenous trade elasticity and domestic trade share are no longer sufficient statistics for welfare. Even conditioning on these two aggregate statistics, the extra margin of adjustment highlighted by our theoretical comparative static implies that micro structure matters for the measurement of the welfare gains from trade.

We develop several examples of trade liberalization scenarios in which this additional impact of micro structure on welfare can be substantial, even for small, empirically relevant departures from the ACR parameter restrictions. We show that assuming a constant trade elasticity when the true elasticity is variable can lead to substantial quantitative discrepancies between the predicted and true welfare effects of trade liberalization. We extend the ACR approach of expressing the welfare gains from trade as a function of observable empirical moments to the more general cases of the homogeneous and heterogeneous firm models. We show that using a trade elasticity estimated for a local change in trade costs in a similar context will reduce – but not eliminate – the discrepancy between the predicted and true welfare gains from trade. In addition to the two aggregate moments of the domestic trade share and trade elasticity, our more general welfare expression highlights differences in the hazard rate of the distribution of log firm size between the domestic and export markets and the response of firm entry to changes in trade costs, both of which can be examined empirically using firm-level data.

A Appendix

A.1 Proof of Proposition 2

Proof. We establish the proposition for the various possible types of open economy equilibria depending on parameter values. **(I)** First, we consider parameter values for which the representative firm does not find it profitable to export in the homogeneous firm model ($\tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} > 1$). For these parameter values, the proposition follows immediately from the fact that the two models have the same closed economy welfare, there are welfare gains from trade, and trade only occurs in the heterogeneous firm model. **(II)** Second, we consider parameter values for which the representative firm exports in the homogeneous firm model and there is selection into export markets in the heterogeneous firm model ($0 < \tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} < 1 < \tau\left(f_x/f_d\right)^{1/(\sigma-1)}$). From (16) and (24), open economy welfare is higher in the heterogeneous firm model than in the homogeneous firm model if the following inequality is satisfied:

$$\frac{\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi\tau^{1-\sigma}\left(\tilde{\varphi}_x^T\right)^{\sigma-1}}{\frac{f_e}{1-G\left(\varphi_d^T\right)} + f_d + \chi f_x} > \frac{\left(1+\tau^{1-\sigma}\right)\left(\tilde{\varphi}_d^A\right)^{\sigma-1}}{\bar{F}_d + f_x}. \tag{37}$$

To show that this inequality must be satisfied, we use the open economy free entry condition in the heterogeneous firm model, which implies:

$$f_d \int_{\varphi_d^T}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] \mathrm{d}G(\varphi) + f_x \int_{\varphi_x^T}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_x^T} \right)^{\sigma-1} - 1 \right] \mathrm{d}G(\varphi) = f_e,$$

$$f_d \left[1 - G\left(\varphi_d^T\right)\right] \left[\left(\frac{\tilde{\varphi}_d^T}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] + f_x \left[1 - G\left(\varphi_x^T\right)\right] \left[\left(\frac{\tilde{\varphi}_x^T}{\varphi_x^T} \right)^{\sigma-1} - 1 \right] = f_e,$$

$$f_d \left(\frac{\tilde{\varphi}_d^T}{\varphi_d^T} \right)^{\sigma-1} + f_x \frac{1 - G\left(\varphi_x^T\right)}{1 - G\left(\varphi_d^T\right)} \left(\frac{\tilde{\varphi}_x^T}{\varphi_x^T} \right)^{\sigma-1} = \frac{f_e}{1 - G\left(\varphi_d^T\right)} + f_d + \chi f_x.$$

Using $\left(\varphi_x^T\right)^{\sigma-1} = \left(\varphi_d^T\right)^{\sigma-1} \tau^{\sigma-1} f_x / f_d$, we obtain:

$$\frac{f_d}{\left(\varphi_d^T\right)^{\sigma-1}} \left[\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi \tau^{1-\sigma} \left(\tilde{\varphi}_x^T\right)^{\sigma-1} \right] = \frac{f_e}{1 - G\left(\varphi_d^T\right)} + f_d + \chi f_x. \tag{38}$$

Note that the open economy free entry condition in the heterogeneous firm model also implies:

$$f_d \int_{\varphi_d^A}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] \mathrm{d}G(\varphi) + f_x \int_{\varphi_d^A}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_x^T} \right)^{\sigma-1} - 1 \right] \mathrm{d}G(\varphi) < f_e, \tag{39}$$

since $\varphi_d^A < \varphi_d^T < \varphi_x^T$ and

$$\left[\left(\frac{\varphi}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] < 0, \qquad \text{for} \qquad \varphi < \varphi_d^T,$$

$$\left[\left(\frac{\varphi}{\varphi_x^T} \right)^{\sigma-1} - 1 \right] < 0 \qquad \text{for} \qquad \varphi < \varphi_x^T.$$

Rewriting (39), we have:

$$f_d \left[1 - G\left(\varphi_d^A\right)\right] \left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] + f_x \left[1 - G\left(\varphi_d^A\right)\right] \left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_x^T} \right)^{\sigma-1} - 1 \right] < f_e,$$

$$f_d \left(\frac{\tilde{\varphi}_d^A}{\varphi_d^T} \right)^{\sigma-1} + f_x \left(\frac{\tilde{\varphi}_d^A}{\varphi_x^T} \right)^{\sigma-1} < \frac{f_e}{1 - G\left(\varphi_d^A\right)} + f_d + f_x.$$

Using $\left(\varphi_x^T\right)^{\sigma-1} = \left(\varphi_d^T\right)^{\sigma-1} \tau^{\sigma-1} f_x / f_d$, we obtain:

$$\frac{f_d}{\left(\varphi_d^T\right)^{\sigma-1}} \left(1 + \tau^{1-\sigma}\right) \left(\tilde{\varphi}_d^A\right)^{\sigma-1} < \frac{f_e}{1 - G\left(\varphi_d^A\right)} + f_d + f_x. \tag{40}$$

From (38) and (40), we have:

$$\frac{\frac{f_d}{\left(\varphi_d^T\right)^{\sigma-1}} \left[\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi \tau^{1-\sigma} \left(\tilde{\varphi}_x^T\right)^{\sigma-1} \right]}{\frac{f_e}{1 - G\left(\varphi_d^T\right)} + f_d + \chi f_x} = 1, \tag{41}$$

$$\frac{\frac{f_d}{\left(\varphi_d^T\right)^{\sigma-1}} \left(1 + \tau^{1-\sigma}\right) \left(\tilde{\varphi}_d^A\right)^{\sigma-1}}{\frac{f_e}{1 - G\left(\varphi_d^A\right)} + f_d + f_x} = \frac{\frac{f_d}{\left(\varphi_d^T\right)^{\sigma-1}} \left(1 + \tau^{1-\sigma}\right) \left(\tilde{\varphi}_d^A\right)^{\sigma-1}}{\bar{F}_d + f_x} < 1,$$

which establishes that inequality (37) is satisfied. In an open economy equilibrium of the heterogeneous firm model with export market selection, welfare also can be expressed as:

$$\mathbb{W}_{\text{Het}}^{T} = \frac{w}{P} = \left(\frac{L}{\sigma f_d}\right)^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\sigma} \varphi_d^T. \tag{42}$$

From (42) and (24), the condition for open economy welfare to be higher in the heterogeneous firm model with export market selection than in the homogeneous firm model can be also written as:

$$\left(\frac{1}{f_d}\right)^{\frac{1}{\sigma-1}} \varphi_d^T > \left(\frac{1+\tau^{1-\sigma}}{\bar{F}_d + f_x}\right)^{\frac{1}{\sigma-1}} \tilde{\varphi}_d^A.$$

Using (37) and (41), this (equivalent) inequality is necessarily satisfied. Since closed economy welfare is the same in the two models, and open economy welfare is higher in the heterogeneous firm model than in the homogeneous firm model, it follows that the proportional welfare gains from trade are larger in the heterogeneous firm model ($\mathbb{W}_{\text{Het}}^{T}/\mathbb{W}_{\text{Het}}^{A} > \mathbb{W}_{\text{Hom}}^{T}/\mathbb{W}_{\text{Hom}}^{A}$). **(III)** Third, we consider parameter values for which the representative firm exports in the homogeneous firm model and all firms export in the heterogeneous firm model, but fixed exporting costs are still positive ($0 < \tau \left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} < \tau \left(f_x/f_d\right)^{1/(\sigma-1)} \leq 1$). This is simply a special case of **(II)** in which $\varphi_x^T = \varphi_d^T$, $\tilde{\varphi}_x^T = \tilde{\varphi}_d^T$ and $\frac{1-G\left(\varphi_x^T\right)}{1-G\left(\varphi_d^T\right)} = 1$. Therefore the same line of reasoning as in **(II)** can be used to show that the inequality (37) is satisfied and hence open economy welfare is higher in the heterogeneous firm model than in the homogeneous firm model. In this special case in which all firms export, the free entry condition in the open economy equilibrium of the heterogeneous firm model implies:

$$\begin{aligned} (f_d + f_x) \int_{\varphi_d^T}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1} - 1\right] \mathrm{d}G(\varphi) &= f_e, \\ (f_d + f_x) \int_{\varphi_d^A}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1} - 1\right] \mathrm{d}G(\varphi) &< f_e, \end{aligned} \tag{43}$$

since $\varphi_d^A < \varphi_d^T$ and

$$\left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1} - 1\right] < 0, \qquad \text{for} \qquad \varphi < \varphi_d^T.$$

Rewriting (43), we obtain:

$$(f_d + f_x) \left(\frac{\tilde{\varphi}_d^A}{\varphi_d^T}\right)^{\sigma-1} < \frac{f_e}{1 - G\left(\varphi_d^A\right)} + f_d + f_x = \bar{F}_d + f_x. \tag{44}$$

In an open economy equilibrium of the heterogeneous firm model in which all firms export, welfare can be also expressed as:

$$\mathbb{W}_{\text{Het}} = \frac{\sigma-1}{\sigma} \left(\frac{\left(1+\tau^{1-\sigma}\right) L}{\sigma\left(f_d + f_x\right)}\right)^{\frac{1}{\sigma-1}} \varphi_d. \tag{45}$$

Therefore the condition for open economy welfare in the heterogeneous firm model without export market selection (45) to be higher than open economy welfare in the homogeneous firm model (24) can be also written as:

$$\left(\frac{1}{f_d + f_x}\right)^{\frac{1}{\sigma-1}} \varphi_d^T > \left(\frac{1}{F_d + f_x}\right)^{\frac{1}{\sigma-1}} \tilde{\varphi}_d^A.$$

From (44), this inequality is necessarily satisfied. Since closed economy welfare is the same in the two models, and open economy welfare is higher in the heterogeneous firm model than in the homogeneous firm model, it follows that the proportional welfare gains from trade are larger in the heterogeneous firm model ($\mathbb{W}^T_{\text{Het}}/\mathbb{W}^A_{\text{Het}} > \mathbb{W}^T_{\text{Hom}}/\mathbb{W}^A_{\text{Hom}}$). **(IV)** Finally, when fixed exporting costs are zero, we have $0 = \tau \left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} = \tau \left(f_x/f_d\right)^{1/(\sigma-1)}$. This is a special case of **(III)** in which $\varphi^T_x = \varphi^T_d = \varphi^A_d$, $\tilde{\varphi}^T_x = \tilde{\varphi}^T_d = \tilde{\varphi}^A_d$ and $\frac{1-G\left(\varphi^T_x\right)}{1-G\left(\varphi^T_d\right)} = 1$. In this special case of zero fixed exporting costs, the free entry condition in the open economy equilibrium of the heterogeneous firm model implies:

$$(f_d + f_x) \int_{\varphi^A_d}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi^T_d} \right)^{\sigma-1} - 1 \right] \mathrm{d}G(\varphi) = f_e,$$

$$(f_d + f_x) \left(\frac{\tilde{\varphi}^A_d}{\varphi^T_d} \right)^{\sigma-1} = \frac{f_e}{1 - G\left(\varphi^A_d\right)} + f_d + f_x = \bar{F}_d + f_x,$$

where we have used $\varphi^A_d = \varphi^T_d$. From (45) and (24), it follows immediately that open economy welfare is the same in the two models when fixed exporting costs are equal to zero. ■

A.2 Proof of Proposition 3

Proof. In the initial open economy equilibrium before the change in trade costs, (16) implies that welfare in both the heterogeneous firm model and in the extended homogeneous firm model can be written as:

$$\left(\mathbb{W}^{T_1}_{\text{Het}}\right)^{\sigma-1} = \frac{L\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \left[\left(\tilde{\varphi}^{T_1}_d\right)^{\sigma-1} + \chi_1 \tau_1^{1-\sigma} \left(\tilde{\varphi}^{T_1}_x\right)^{\sigma-1} \right]}{\sigma \left[\frac{f_e}{1-G\left(\varphi^{T_1}_d\right)} + f_d + \chi_1 f_{x1} \right]}.$$

In the new open economy equilibrium after the change in trade costs, (16) implies that welfare in the heterogeneous firm model is:

$$\left(\mathbb{W}^{T_2}_{\text{Het}}\right)^{\sigma-1} = \frac{L\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \left[\left(\tilde{\varphi}^{T_2}_d\right)^{\sigma-1} + \chi_2 \tau_2^{1-\sigma} \left(\tilde{\varphi}^{T_2}_x\right)^{\sigma-1} \right]}{\sigma \left[\frac{f_e}{1-G\left(\varphi^{T_2}_d\right)} + f_d + \chi_2 f_{x2} \right]}.$$

In contrast, in the new open economy equilibrium after the change in trade costs, welfare in the extended homogeneous firm model is:

$$\left(\mathbb{W}^{T_2}_{\text{Hom}}\right)^{\sigma-1} = \frac{L\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \left[\left(\tilde{\varphi}^{T_1}_d\right)^{\sigma-1} + \chi_1 \tau_2^{1-\sigma} \left(\tilde{\varphi}^{T_1}_x\right)^{\sigma-1} \right]}{\sigma \left[\frac{f_e}{1-G\left(\varphi^{T_1}_d\right)} + f_d + \chi_1 f_{x2} \right]}$$

To show that welfare in the new open economy equilibrium is higher in the heterogeneous firm model than in the homogeneous firm model, we need to show that:

$$\frac{\left(\tilde{\varphi}^{T_2}_d\right)^{\sigma-1} + \chi_2 \tau_2^{1-\sigma} \left(\tilde{\varphi}^{T_2}_x\right)^{\sigma-1}}{\frac{f_e}{1-G\left(\varphi^{T_2}_d\right)} + f_d + \chi_2 f_{x2}} > \frac{\left(\tilde{\varphi}^{T_1}_d\right)^{\sigma-1} + \chi_1 \tau_2^{1-\sigma} \left(\tilde{\varphi}^{T_1}_x\right)^{\sigma-1}}{\frac{f_e}{1-G\left(\varphi^{T_1}_d\right)} + f_d + \chi_1 f_{x2}}. \tag{46}$$

To establish this inequality, we use the free entry condition in the new open economy equilibrium of the heterogeneous firm model, which implies:

$$f_d \int_{\varphi_d^{T_2}}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_d^{T_2}} \right)^{\sigma-1} - 1 \right] \mathrm{d}G(\varphi) + f_{x2} \int_{\varphi_x^{T_2}}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_x^{T_2}} \right)^{\sigma-1} - 1 \right] \mathrm{d}G(\varphi) = f_e,$$

$$f_d \left[1 - G\left(\varphi_d^{T_2}\right) \right] \left[\left(\frac{\tilde{\varphi}_d^{T_2}}{\varphi_d^{T_2}} \right)^{\sigma-1} - 1 \right] + f_{x2} \left[1 - G\left(\varphi_x^{T_2}\right) \right] \left[\left(\frac{\tilde{\varphi}_x^{T_2}}{\varphi_x^{T_2}} \right)^{\sigma-1} - 1 \right] = f_e,$$

$$f_d \left(\frac{\tilde{\varphi}_d^{T_2}}{\varphi_d^{T_2}} \right)^{\sigma-1} + f_{x2} \frac{1 - G\left(\varphi_x^{T_2}\right)}{1 - G\left(\varphi_d^{T_2}\right)} \left(\frac{\tilde{\varphi}_x^{T_2}}{\varphi_x^{T_2}} \right)^{\sigma-1} = \frac{f_e}{1 - G\left(\varphi_d^{T_2}\right)} + f_d + \frac{1 - G\left(\varphi_x^{T_2}\right)}{1 - G\left(\varphi_d^{T_2}\right)} f_{x2}.$$

Using $\left(\varphi_x^{T_2}\right)^{\sigma-1} = \left(\varphi_d^{T_2}\right)^{\sigma-1} \tau_2^{\sigma-1} f_{x2}/f_d$, we obtain:

$$\frac{f_d}{\left(\varphi_d^{T_2}\right)^{\sigma-1}} \left[\left(\tilde{\varphi}_d^{T_2}\right)^{\sigma-1} + \chi_2 \tau_2^{1-\sigma} \left(\tilde{\varphi}_x^{T_2}\right)^{\sigma-1} \right] = \frac{f_e}{1 - G\left(\varphi_d^{T_2}\right)} + f_d + \chi_2 f_{x2}. \tag{47}$$

Note that the free entry condition in the new open economy equilibrium of the heterogeneous firm model also implies:

$$f_d \int_{\varphi_d^{T_1}}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_d^{T_2}} \right)^{\sigma-1} - 1 \right] \mathrm{d}G(\varphi) + f_{x2} \int_{\varphi_x^{T_1}}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_x^{T_2}} \right)^{\sigma-1} - 1 \right] \mathrm{d}G(\varphi) < f_e, \tag{48}$$

since $\varphi_d^{T_1} < \varphi_d^{T_2}$ and $\varphi_x^{T_1} > \varphi_x^{T_2}$ and

$$\left[\left(\frac{\varphi}{\varphi_d^{T_2}} \right)^{\sigma-1} - 1 \right] < 0, \qquad \text{for} \qquad \varphi < \varphi_d^{T_2},$$

$$\left[\left(\frac{\varphi}{\varphi_x^{T_2}} \right)^{\sigma-1} - 1 \right] > 0 \qquad \text{for} \qquad \varphi_x^{T_2} < \varphi < \varphi_x^{T_1}.$$

Rewriting (48), we have:

$$f_d \left[1 - G\left(\varphi_d^{T_1}\right) \right] \left[\left(\frac{\tilde{\varphi}_d^{T_1}}{\varphi_d^{T_2}} \right)^{\sigma-1} - 1 \right] + f_{x_2} \left[1 - G\left(\varphi_x^{T_1}\right) \right] \left[\left(\frac{\tilde{\varphi}_x^{T_1}}{\varphi_x^{T_2}} \right)^{\sigma-1} - 1 \right] < f_e,$$

$$f_d \left(\frac{\tilde{\varphi}_d^{T_1}}{\varphi_d^{T_2}} \right)^{\sigma-1} + f_{x2} \frac{1 - G\left(\varphi_x^{T_1}\right)}{1 - G\left(\varphi_d^{T_1}\right)} \left(\frac{\tilde{\varphi}_x^{T_1}}{\varphi_x^{T_2}} \right)^{\sigma-1} < \frac{f_e}{1 - G\left(\varphi_d^{T_1}\right)} + f_d + \frac{1 - G\left(\varphi_x^{T_1}\right)}{1 - G\left(\varphi_d^{T_1}\right)} f_{x2}.$$

Using $\left(\varphi_x^{T_2}\right)^{\sigma-1} = \left(\varphi_d^{T_2}\right)^{\sigma-1} \tau_2^{\sigma-1} f_{x2}/f_d$, we obtain:

$$\frac{f_d}{\left(\varphi_d^{T_2}\right)^{\sigma-1}} \left[\left(\tilde{\varphi}_d^{T_1}\right)^{\sigma-1} + \chi_1 \tau_2^{1-\sigma} \left(\tilde{\varphi}_x^{T_1}\right)^{\sigma-1} \right] < \frac{f_e}{1 - G\left(\varphi_d^{T_1}\right)} + f_d + \chi_1 f_{x2}. \tag{49}$$

From (47) and (49), we have:

$$\frac{\frac{f_d}{\left(\varphi_d^{T_2}\right)^{\sigma-1}}\left[\left(\tilde{\varphi}_d^{T_2}\right)^{\sigma-1}+\chi_2\tau_2^{1-\sigma}\left(\tilde{\varphi}_x^{T_2}\right)^{\sigma-1}\right]}{\frac{f_e}{1-G\left(\varphi_d^{T_2}\right)}+f_d+\chi_2 f_{x2}}=1,$$

$$\frac{\frac{f_d}{\left(\varphi_d^{T_2}\right)^{\sigma-1}}\left[\left(\tilde{\varphi}_d^{T_1}\right)^{\sigma-1}+\chi_1\tau_2^{1-\sigma}\left(\tilde{\varphi}_x^{T_1}\right)^{\sigma-1}\right]}{\frac{f_e}{1-G\left(\varphi_d^{T_1}\right)}+f_d+\chi_1 f_{x2}}<1,$$

which establishes the inequality (46). ■

A.3 Proof of Proposition 4

Proof. **(a)** First, consider parameter values for which the representative firm exports in the homogeneous firm model and there is selection into export markets in the heterogeneous firm model ($0<\tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)}<1<\tau\left(f_x/f_d\right)^{1/(\sigma-1)}$). From (42), we have:

$$\frac{\mathbb{W}_{\text{Het}}^T}{\mathbb{W}_{\text{Het}}^A}=\frac{\varphi_d^T}{\varphi_d^A}. \tag{50}$$

In the special case of an untruncated Pareto productivity distribution and for these parameter values for which there is selection into export markets in the heterogeneous firm model, we have:

$$\frac{\varphi_d^T}{\varphi_d^A}=\left[1+\left(\frac{1}{\tau\left(f_x/f_d\right)^{1/(\sigma-1)}}\right)^k\frac{f_x}{f_d}\right]^{1/k},$$

which can be written as:

$$\ln\left(\frac{\varphi_d^T}{\varphi_d^A}\right)=k^{-1}\ln\left[1+\left(\frac{1}{\tau\left(f_x/f_d\right)^{1/(\sigma-1)}}\right)^k\frac{f_x}{f_d}\right].$$

Note that

$$\frac{\mathrm{d}\ln\left(\varphi_d^T/\varphi_d^A\right)}{\mathrm{d}k}=-k^{-2}\ln\left[1+\left(\frac{1}{\tau\left(f_x/f_d\right)^{1/(\sigma-1)}}\right)^k\frac{f_x}{f_d}\right]-\frac{k^{-1}\ln\left(\tau\left(f_x/f_d\right)^{1/(\sigma-1)}\right)\left(\frac{1}{\tau\left(f_x/f_d\right)^{1/(\sigma-1)}}\right)^k\frac{f_x}{f_d}}{\left[1+\left(\frac{1}{\tau\left(f_x/f_d\right)^{1/(\sigma-1)}}\right)^k\frac{f_x}{f_d}\right]}<0, \tag{51}$$

where we have used $d\left(a^x\right)/dx=\left(\ln a\right)a^x$. Since a smaller value of k corresponds to greater productivity dispersion, it follows that greater productivity dispersion implies larger φ_d^T/φ_d^A. Second, consider parameter values for which the representative firm exports in the homogeneous firm model and all firms export in the heterogeneous firm model, but fixed exporting costs are still positive ($0<\tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)}<\tau\left(f_x/f_d\right)^{1/(\sigma-1)}\leq 1$). From (42) and (45), we have:

$$\frac{\mathbb{W}_{\text{Het}}^T}{\mathbb{W}_{\text{Het}}^A}=\left(\frac{\left(1+\tau^{1-\sigma}\right)f_d}{f_d+f_x}\right)^{\frac{1}{\sigma-1}}\frac{\varphi_d^T}{\varphi_d^A}.$$

In the special case of an untruncated Pareto productivity distribution and for these parameter values for which all firms export in the heterogeneous firm model, we have:

$$\frac{\varphi_d^T}{\varphi_d^A} = \left[1 + \frac{f_x}{f_d}\right]^{1/k},$$

which can be written as:

$$\ln\left(\frac{\varphi_d^T}{\varphi_d^A}\right) = k^{-1} \ln\left[1 + \frac{f_x}{f_d}\right].$$

Note that

$$\frac{\mathrm{d}\ln\left(\varphi_d^T/\varphi_d^A\right)}{\mathrm{d}k} = -k^{-2} \ln\left[1 + \frac{f_x}{f_d}\right] < 0. \tag{52}$$

Since a smaller value of k corresponds to greater productivity dispersion, it follows that greater productivity dispersion again implies larger φ_d^T/φ_d^A. Taking (51) and (52) together and using (50), it follows that greater dispersion of firm productivity (smaller k) implies larger proportional welfare gains from opening the closed economy to trade. **(b)** Consider parameter values for which there is selection into export markets in the open economy equilibrium of the heterogeneous firm model ($\tau\left(f_x/f_d\right)^{1/(\sigma-1)} > 1$). In the special case of an untruncated Pareto productivity distribution, we have:

$$\varphi_d^T = \left(\frac{\sigma-1}{k-(\sigma-1)}\right)^{1/k} \left[\frac{f_d + \left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k f_x}{f_e}\right]^{1/k} \varphi_{\min}.$$

Therefore:

$$\begin{aligned} \frac{\mathrm{d}\varphi_d^T}{\mathrm{d}\tau}\frac{\tau}{\varphi_d^T}\mathrm{d}\tau &= -\frac{\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k f_x}{f_d + \left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k f_x}\mathrm{d}\tau \\ &= -\xi \mathrm{d}\tau. \end{aligned}$$

Hence:

$$\frac{\mathrm{d}\left(\frac{\mathrm{d}\varphi_d^T}{\mathrm{d}\tau}\frac{\tau}{\varphi_d^T}\mathrm{d}\tau\right)}{\mathrm{d}k} = \frac{\ln\left(\tau\left(f_x/f_d\right)^{1/(\sigma-1)}\right)\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k f_x}{f_d + \left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k f_x}\left(1-\xi\right)\mathrm{d}\tau,$$

which implies:

$$\begin{aligned} &\frac{\mathrm{d}\left(\frac{\mathrm{d}\varphi_d^T}{\mathrm{d}\tau}\frac{\tau}{\varphi_d^T}\mathrm{d}\tau\right)}{\mathrm{d}k} < 0 \qquad \text{for} \qquad \mathrm{d}\tau < 0, \\ &\frac{\mathrm{d}\left(\frac{\mathrm{d}\varphi_d^T}{\mathrm{d}\tau}\frac{\tau}{\varphi_d^T}\mathrm{d}\tau\right)}{\mathrm{d}k} > 0 \qquad \text{for} \qquad \mathrm{d}\tau > 0. \end{aligned}$$

Therefore greater dispersion of firm productivity (smaller k) implies a larger elasticity of the domestic productivity cutoff with respect to reductions in variable trade costs, which from (42) implies greater proportional welfare gains from reductions in variable trade costs. By the same reasoning, greater dispersion of firm productivity (smaller k) implies a smaller elasticity of the domestic productivity cutoff with respect to increases in variable trade costs, which from (42) implies smaller proportional welfare costs from increases in variable trade costs. ■

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