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ABSTRACT

This paper develops an analytical framework for the analysis of targeting rules for monetary policy. We derive the optimal money supply rule and analyze the implications of other monetary rules including rules that target nominal GNP, the price level, the monetary growth rate and the interest rate. An explicit welfare criterion is used in order to rank the alternative rules. In the model monetary policy is needed because labor market contracts set nominal wages in advance of the realization of the stochastic shocks. The principal result is that the welfare ranking of alternative targeting rules depends on whether the elasticity of labor demand exceeds or falls short of the elasticity of labor supply. Specifically, it is shown that if the demand for labor is more elastic than the supply, then targeting nominal GNP produces a smaller welfare loss than targeting the CPI which in turn produces a smaller welfare loss than interest-rate targeting.

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This paper develops an analytical framework for the analysis of targeting rules for monetary policy. We derive the optimal money-supply rule and analyze the implications of other rules governing the money supply process including the rules that target nominal GNP, the price level, the monetary growth rate and the interest rate. We use an explicit welfare criterion which permits ranking of the alternative rules. In our model, as in Fischer (1977), the need for monetary policy arises from the existence of labor market contracts which set nominal wages in advance of the realization of the stochastic shocks. We show that if the demand for labor is more elastic than the supply, then targeting nominal GNP produces a smaller welfare loss than targeting the CPI which in turn produces a smaller welfare loss than interest-rate targeting. We also explore the conditions under which a constant money-growth rule produces larger or smaller welfare losses than the nominal GNP targeting rule.

I. The Model

Output is assumed to be produced by a Cobb-Douglas production function using labor as the variable input. Hence, for period t,

$$(1) \qquad \log Y_t = \log B + \beta \log L_t + \mu_t , \qquad 0 \leq \beta < 1,$$

where Y_t denotes output, L_t denotes labor input, B denotes a fixed parameter and μ_t denotes a stochastic i.i.d. productivity shock assumed to be distributed normally with a fixed known variance σ_μ^2. In the subsequent analysis we suppress the time subscript and use lower-case letters to denote percentage discrepancies of a variable from the value obtained in the absence of shocks. Accordingly, equation (1) can be rewritten as

$$(1') \qquad y = \beta \ell + \mu$$

where y and ℓ denote the percentage deviations of output and employment from their corresponding non-stochastic levels. It is assumed that due to cost of negotiations nominal wages are set in advance at their expected market clearing level, and employment is determined by the demand for labor. Profit maximization along with the assumption that producers satisfy their demand for labor yields

$$(2) \qquad y = \eta[\beta(p-w) + \mu]$$

where $\eta = 1/(1-\beta)$ is the elasticity of labor demand (defined to be positive) and where p and w denote, respectively, the product price and the nominal wage rate (all measured as a percentage deviations from their non-stochastic levels).

This specification reflects the assumption that ℓ and thereby y are determined exclusively by the demand for labor rather than by the interaction between the demand and the supply. As a result actual employment and output may not be optimal. We evaluate the various rules for monetary policies in terms of their impact on the expected welfare loss arising from sub-optimal output. As a benchmark we first determine the <u>undistorted equilibrium</u>.

Let the supply of labor (measured in percentage deviations) be

$$(3) \qquad \ell^s = \varepsilon(w-p)$$

where ε denotes the elasticity of labor supply. Substituting (3) for ℓ in (1') yields the value of y consistent with the <u>supply</u> of labor; equating it with the value of y that is consistent with the <u>demand</u> for labor (eq. 2) yields the <u>equilibrium</u> level of output, $\tilde{y}$ (measured in terms of deviations):

$$(4) \qquad \tilde{y} = \frac{(1+\varepsilon)\eta}{\varepsilon+\eta}\,\mu\ .$$

Equilibrium output, $\tilde{y}$, may not coincide with realized output, y. The latter is determined by the contractual nominal wage and by the realized price. The difference between the two inflicts welfare loss which is proportional to the squared discrepancy [see Aizenman and Frenkel (1985)]. Thus, the expected welfare loss, H, is

$$(5) \qquad H = cE[\tilde{y}-y]^2$$

where c is a positive proportionality factor.

Money-market equilibrium requires that the supply of money, m, equals the demand:

$$(6) \qquad m = k + p + \xi y - \alpha(i - i_o).$$

In equation (6), i denotes the nominal rate of interest, i_o denotes the nominal interest rate in the absence of shocks, k denotes a stochastic com-ponent of money demand which is distributed normally and independently with zero mean and a fixed known variance σ_k^2, ξ denotes the income elasticity of the demand for money and α denotes the interest rate semi-elasticity of money demand. The real rate of interest also contains a stochastic component, ρ, which is distributed normally and independently with zero mean and a fixed known variance σ_ρ^2. It can be shown[1] that the specification of the stochastic shocks imply that $i-i_o = \rho-\dot{p}$ and, therefore, equation (6) can be written as:

$$(6') \qquad m = k + (1+\alpha)p + \xi y - \alpha\rho .$$

II. Active Money-Supply Rules

In this section we analyze the welfare implications of alternative active money-supply rules starting with a determination of the optimal rule.

II.1 The Optimal Money-Supply Rule

The optimal money-supply rule aims at eliminating the welfare loss. As seen from equation (5), the welfare loss is eliminated only if $\tilde{y} = y$. Since nominal wages are assumed given by wage contracts, the equilibrium price $\tilde{p}$ must generate an output level, y, equal to the optimal output, $\tilde{y}$. Equating y (from equation (2)) to $\tilde{y}$ (from equation (4) yields

$$(7) \qquad \tilde{p} = -\frac{\eta}{\varepsilon+\eta}\,\mu \,.$$

Using equations (4) and (7) in (6') yields $\tilde{m}$ as the optimal money-supply rule

$$(8) \qquad \tilde{m} = k - \alpha\rho + \frac{[\xi(1+\varepsilon) - (1+\alpha)]\eta}{\varepsilon+\eta}\,\mu \,.$$

Equation (8) also shows that the optimal monetary rule requires a complete accommodation to money-demand shocks. As is evident in the special case for which $\alpha = \varepsilon = 0$ and $\xi = 1$, monetary policy should not respond to real shocks. This case corresponds to the one examined by Fischer (1985).

The optimal monetary rule can also be expressed in terms of the observable variables p and y. Since at the optimum $\tilde{m} - k = (1+\alpha)\,\tilde{p} + \xi\tilde{y} - \alpha\rho$, it can be shown[2] from (7), (8) and (6') that

$$(9) \qquad \varepsilon\tilde{p} + (\tilde{p}+\tilde{y}) = 0 \,.$$

Thus, a monetary rule that targets the sum of $(p+y) + \varepsilon p$ to zero, eliminates the welfare loss. Equation (9) also reveals that in the special case for which $\varepsilon=0$, a nominal GNP targeting rule is optimal; likewise, in the special case for which $\tilde{y} = 0$ (i.e., in the absence of real shocks), a price level targeting rule is also optimal.

II.2 Nominal GNP Targeting

The preceding discussion specified the optimal money-supply rule. Suppose alternatively that the monetary authority targets nominal GNP.[3] With this rule $p+y=0$. Using equation (2) and recalling that $w=0$, the realized price is $-\mu$ and, correspondingly, $y = \mu$. Using the loss function (5) and the expression for $\tilde{y}$ from (4), the welfare loss associated with this targeting rule is

$$(10) \qquad H\Big|_{p+y=0} = a\left(\frac{\varepsilon}{\varepsilon+\eta}\right)^2 \sigma_\mu^2$$

where the notation indicates that this loss results from the stabilization of nominal income and where $a = (\eta\beta)^2 c$.

II.3 CPI Targeting

With CPI targeting $p=0$. In that case the value of the loss function (5) becomes

$$(11) \qquad H\Big|_{p=0} = a\left(\frac{\eta}{\varepsilon+\eta}\right)^2 \sigma_\mu^2 \; .$$

II.4 Interest-Rate Targeting

With interest-rate targeting monetary policy assures that $i=i_o$. Equivalently, since $i-i_o = \rho-p$,, this targeting rule sets $p=\rho$. In this case $y = \eta(\beta\rho+\mu)$ and the loss function (5) becomes

$$(12) \qquad H\Big|_{i=i_o} = a\,\sigma_\rho^2 + H\Big|_{p=0} \; .$$

III. Ranking the Targeting Rules

A comparison of (11) and (12) shows that price targeting produces a smaller welfare loss than interest-rate targeting (they are equivalent only in the case in which there are no interest-rate shocks). A comparison between the welfare cost associated with these rules and the cost induced by nominal GNP targeting (equation (10)) shows that the ranking depends on whether η exceeds or falls short of ε. If $\eta>\varepsilon$ then

$$(13)\qquad m = \tilde{m} \succsim p + y = 0 \succsim p = 0 \succsim i = i_0$$

where the symbol $x \succsim z$ indicates that x produces a smaller welfare loss than or is equivalent to z. On the other hand if $\eta < \varepsilon$ then price targeting produces a smaller welfare loss than both nominal GNP and interest-rate targeting. Nominal GNP targeting in turn produces a smaller welfare loss than interest-rate targeting if $\sigma_\rho^2/\sigma_\mu^2 > (\varepsilon-\eta)/(\varepsilon+\eta)$ and vice versa.

IV. A Constant Money-Growth Rule

In sections II and III we analyzed the implications of alternative targeting rules requiring active policy actions. In this section we analyze a targeting rule which does not allow for a policy response to the stochastic shocks. Specifically, we consider a constant money-growth rule by which $m=0$. Using equations (2) and (6') and recalling that $m=w=0$, yields the value of p which, together with equation (2) implies that the value of y associated with the constant money-growth rule is $\eta\,[\beta(\alpha\rho-k) + (1+\alpha)\mu]/(1+\alpha + \xi\eta\beta)$. Using equation (5), the welfare loss associated with the constant money-growth rule is

$$(14)\qquad H\big|_{m=0} = \frac{a}{(1+\alpha+\xi\eta\beta)^2}\left[(\sigma_k^2 + \alpha\,\sigma_\rho^2) + \left(\frac{[(1+\alpha)-\xi(1+\varepsilon)]\eta}{\varepsilon+\eta}\right)^2\right]\sigma_\mu^2 .$$

As is evident from equation (14) one of the components of the welfare loss arises from shocks to the demand for money. This may be contrasted with the welfare cost of the CPI and the nominal GNP targeting rules in which money demand shocks did not play a role. This assymetry stems from the fact that these targeting rules respond in one way or another to the realized price and, thereby, neutralize the welfare effects of the monetary shocks.

Using equation (11) we can also write equation (14) as

$$(14') \qquad H\Big|_{m=0} = \frac{1}{(1+\alpha+\xi\eta\beta)^2}\left\{a\left(\sigma_k^2 + \alpha\ \sigma_\rho^2\right) + \left[(1+\alpha) - \xi(1+\varepsilon)\right]^2 H\Big|_{p=0}\right\} .$$

In general, the relation between this welfare cost and the cost associated with the other targeting rules depends on the value of the parameters and on the variance of the shocks. It may be seen, however, that for a relatively high value of σ_k^2 the constant money-growth rule produces a larger welfare loss than the other targeting rules. Similarly, for a relatively high value of σ_ρ^2, the constant money-growth rule produces a smaller welfare loss than the interest-rate targeting rule. In these two cases both rules produces a larger welfare loss than the CPI and to the nominal-GNP targeting rules.

Footnotes

[1]The equilibrium nominal rate of interest i_t satisfies the Fisher relation $i_t = i_o + \rho_t + E_t[\log P_{t+1} - \log P_t]$ where the last term on the right hand side denotes expected inflation. The absence of trends and the zero-mean i.i.d. shocks imply that the expected price level $E_t \log P_{t=1}$ equals the non-stochastic stationary level $\log P_o$. Hence, expected inflation is $-p_t$. In the presence of trends our formulation applies to the transitory deviations from the trend.

[2]Substituting (7) for μ into (8) yields $\tilde{m}$ as a function of $\tilde{p}$. Substituting the resulting expression for $\tilde{m}$ into equation (6') (evaluated at the optimum, i.e., $\tilde{m}-k = (1+\alpha)p + \xi\tilde{y} - \alpha\rho$) yields equation (9).

[3]For recent discussions of this targeting rule, see Bean (1983), Hall (1983), McCallum (1984) and Aizenman and Frenkel (1986).

REFERENCES

Aizenman, Joshua and Frenkel, Jacob A., 1985, Optimal wage indexation, foreign exchange intervention and monetary policy, American Economic Review 75, No. 3, 402-23.

__________, 1986, Supply shocks, wage indexation and monetary accommodation, Journal of Money Credit and Banking XVIII, No. 3, forthcoming.

Bean, Charles R., 1983, Targeting nominal income: an appraisal, Economic Journal 93, No. 4, 806-19.

Fischer, Stanley, 1977, Long term contracts, rational expectations and the optimal money supply rule, Journal of Political Economy 85, No. 1, 191-205.

Hall, Robert E., 1983, Macroeconomic policy under structural change, in Industrial Change and Public Policy, Federal Reserve Bank of Kansas City, 85-111.

McCallum, Bennett T., 1984, Monetarist rules in the light of recent experiences, American Economic Review 74, No. 2, 388-91.

Taylor, John B., 1985, What would nominal GNP targeting do to the business cycle? in: Karl Brunner and Allan H. Meltzer, eds., Understanding Monetary Regimes, Carnegie-Rochester Conference Series on Public Policy, Vol. 22, North-Holland, Amsterdam 61-84.