NBER WORKING PAPER SERIES

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Working Paper 18777
http://www.nber.org/papers/w18777

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 2013

We thank the Kielts-Nielsen Data Center at the University of Chicago Booth School of Business for providing access to the Kielts-Nielsen Consumer Panel Dataset. We thank the editor, Harald Uhlig, and three anonymous referees for insightful and detailed suggestions that helped us revising the paper. We are also grateful to Mark Aguiar, Jim Albrecht, Michele Boldrin, Russ Cooper, Ben Eden, Chris Edmond, Roger Farmer, Veronica Guerrieri, Bob Hall, Erik Hurst, Martin Schneider, Venky Venkateswaran, Susan Vroman, Steve Williamson and Randy Wright for many useful comments. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 18777
February 2013, Revised October 2013
JEL No. D11,D21,D43,E32

ABSTRACT

We propose a novel theory of self-fulfilling unemployment fluctuations. According to this theory, a firm hiring an additional worker creates positive external effects on other firms, as a worker has more income to spend and less time to search for low prices when he is employed than when he is unemployed. In response to the increase in demand and prices, other firms enter or increase their presence in the product market by hiring additional workers. We quantify the external effects of employment on demand and prices and show that they are sufficiently strong to generate multiple rational expectations equilibria and, hence, self-fulfilling economic fluctuations.

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1 Introduction

We propose a novel theory of self-fulfilling unemployment fluctuations. According to our theory, a firm hiring an additional worker creates a positive external effect on other firms, as a worker has more income to spend and less time to search for low prices when he is employed than when he is unemployed. In response to the increase in demand and prices, other firms enter or increase their presence in the product market by hiring additional workers. The feedback between employment, demand and prices can lead to multiple rational expectations equilibria. If agents expect lower unemployment, labor demand is higher, more vacancies are created and, eventually, lower unemployment materializes. If agents expect higher unemployment, labor demand is weak, fewer vacancies are created and, eventually, higher unemployment materializes. Hence, economic fluctuations can be caused not only by changes in technology, preferences or other fundamentals, but also by self-fulfilling changes in the agents' expectations about future unemployment.

The theory is motivated by the observation that unemployed and employed people behave differently in the product market. First, unemployed people spend more time shopping. Using the American Time Use Survey (ATUS) and other time-use surveys, Kruger and Muller (2010) find that unemployed people spend between 15 and 30% more time shopping than employed people. Second, unemployed people pay lower prices. Using the Kielts-Nielsen Consumer Panel Data (KNCPD), Kaplan and Menzio (2013) find that households where at least one head is non-employed pay between 1 and 4% less for the same goods than households where all heads are employed. Third, unemployed people spend less. Using the Panel Study of Income Dynamics (PSID), Stephens (2001) finds that households reduce their food expenditures by approximately 15% after becoming unemployed.

We use search theory to build a model economy that captures the above differences in shopping behavior between employed and unemployed people. We model the labor market as in Mortensen and Pissarides (1994). In this market, unemployed workers and vacant jobs come together through a constant return to scale matching function. In equilibrium, there is unemployment as it takes time for a worker to find a viable job. Moreover, there is an income differential between employed and unemployed workers, because employed workers can extract a fraction of the surplus that they create when matched with a firm. We model the product market as in Burdett and Judd (1983). In this market, sellers (i.e. firm-worker pairs) post prices and buyers (i.e. workers) search for sellers with an intensity that depends on their employment status. In equilibrium, sellers post different prices for identical goods. Sellers who post relatively high prices enjoy a higher margin per unit of output sold but they sell fewer units, as they only attract buyers who did not find any other seller. Sellers who post relatively low prices enjoy a lower margin but they sell more units, as they attract both buyers who did not meet any other seller and buyers who met other, but more expensive sellers. Unemployed buyers search more, they are more likely to meet sellers posting relatively low prices and, on average, they pay lower prices than employed buyers.

We first carry out a theoretical analysis of the model. We find that the model admits multiple steady states if unemployed buyers have a sufficiently low income relatively to employed buyers or if unemployed buyers spend a sufficiently large amount of time searching for low prices relative to employed buyers. The result is intuitive. When a firm decides to increases its workforce, it creates external effects on other firms. On the one hand, it increases the tightness of the labor market and, hence, it makes it more costly for other firms to hire an additional worker. We refer to this effect as the *congestion externality*. On the other hand, when a firm increases its workforce, it makes it more profitable for other firms to hire additional workers as it tilts the composition of buyers towards types (i.e. employed buyers) who have higher income and, hence, spend more, and who have less time to devote to shopping and, hence, are more likely to be willing to purchase goods at the monopoly price. We refer to these effects as the *shopping externalities*. In particular, we refer to the effect of employment at one firm on other firms' demand as the *demand externality* and to the effect of employment at one firm on other firms' probability of making a sale at the monopoly price as the *market power externality*. If the difference in either income or search intensity between employed and unemployed buyers is sufficiently large, the shopping externalities dominate the congestion externality and the employment decisions of different firms become strategic complements. When this happens, multiple steady states obtain.

When the model admits multiple steady states, it also has multiple rational expectations equilibria for some initial conditions. Different equilibria are associated to different expectations that the agents have about future unemployment. Yet, in each equilibrium, the agents' expectations about future unemployment are correct, in the sense that the path of unemployment that materializes is exactly the one expected by the agent. The existence of multiple equilibria implies that the behavior of our model economy us not only determined by fundamentals (e.g., technology, preferences and policy), but also by the agents' expectations about future endogenous outcomes. Moreover, for some initial conditions, the model admits rational expectations equilibria that converge to different steady states. This implies that the agents' expectations about future unemployment can be so important as to affect the long-run outcomes of the economy.

We then carry out a quantitative analysis of the model. We calibrate the parameters of the model so as to match the difference in expenditures, shopping time and prices paid between employed and unemployed workers, as well as the frequency at which individual workers transition between the states of employment and unemployment. The calibrated model admits three steady states—two with some economic activity and one without trade—and, for any initial condition, it has rational expectations equilibria converging to each of the three steady states. Multiplicity obtains because the empirical differences in expenditure and shopping time between employed and unemployed buyers are so large that the shopping externalities dominate the congestion externality and, hence, the employment decisions of different firms are strategic complements. Interestingly, the key shopping externality is the market power externality. Specifically, the demand externality is proportional to 15% of the expenditures of unemployed buyers, which is the empirical difference in expenditures between employed and unemployed buyers. The market power externality is proportional to more than 30% of the expenditures of unemployed buyers, a magnitude implied by the empirical difference in shopping time between employed and unemployed buyers.

We use the calibrated model to measure the effect of a negative shock to the agents' expectations about the long-run. Formally, we model these expectation shocks as a 2-state Markov switching process. In what we call the optimistic state, agents expect the economy to converge to the steady state with the lowest unemployment rate. In the pessimistic state, agents expect the economy to converge to the steady state with the intermediate unemployment rate. We then examine the response of the economy to a switch from the optimistic to the pessimistic state. We find that the unemployment rate increases from 5 to 10% and then slowly declines towards 9%, which is the rate associated with the pessimistic steady state. Moreover, unemployment remains stuck at this higher rate for as long as the agents' expectations about the future remain pessimistic. We find that the equity value of a firm falls by approximately 30% and its decline precedes the increase in unemployment. Finally, we find that the increase in unemployment and the decline in the equity value of firms take place without any concurrent decline in real labor productivity. We show that the response of the economy to a negative shock to long-run expectations is qualitatively and quantitatively similar to the behavior of the US economy during the Great Recession and its aftermath. These findings suggest the possibility that the financial crises that caused the Great Recession might have been amplified and protracted by coordinating the agents' expectations towards a steady state with higher unemployment.

The theory of multiple equilibria in this paper hinges on two mechanisms. The first mechanism links unemployment, search and competition: when unemployment is lower, buyers spend less time searching for low prices and, in doing so, they make the product market less competitive and drive prices up. We find that this mechanism is consistent with the recent behavior of prices and unemployment. The second mechanism links revenues, entry and labor demand: when revenues are higher because of either higher demand or higher prices, new firms want to enter the product market, established firms want to scale-up their presence in the product market and, since both activities require some labor, labor demand increases. This mechanism is consistent with the findings in Bilbiie, Ghironi and Melitz (2012), who show that the introduction of new products and the entry of new firms is strongly procyclical and, hence, positively correlated with employment. Similarly, Hall (2013) shows that advertisement expenditures (which presumably measure an effort to reach new buyers) are procyclical.

Our contribution is to advance a novel and quantitatively relevant theory of multiple equilibria and non-fundamental fluctuations. In Benhabib and Farmer (1994), Farmer and Guo (1994), Christiano and Harrison (1999) and Mortensen (1999), multiplicity and non-fundamental shocks arise because of increasing returns to scale in production. Similarly, in Diamond (1982), Diamond and Fudenberg (1989) and Boldrin, Kyiotaki and Wright (1993), multiplicity obtains because of increasing returns to scale in matching. In contrast to these papers, we assume constant returns to scale in both production and matching and obtain multiplicity from the differences in the shopping behavior of employed and unemployed buyers. Moreover, while there is no clear empirical evidence of increasing returns to scale in either production or matching, the differences in the shopping behavior of employed and unemployed buyers are well documented. In Heller (1986), Roberts (1987) and Cooper and John (1988), multiplicity obtains because of demand externalities. The demand externality—which is due to the difference in the expenditures of employed and unemployed buyers—is an integral part of our theory as well. However, we find that the demand externality is quantitatively much less important than the market power externality—which is due to the difference in the search intensity of employed and unemployed buyers—and, alone, it is not sufficient to generate multiple equilibria.

More recently, Angeletos and La'O (2013) and Benhabib, Wang and Wen (2012) consider environments where agents have heterogeneous beliefs about the gains from producing and trading. They show that, even when the complementarity between the agents' production decisions is not strong enough to create multiplicity, non-fundamental fluctuations in economic activity can arise from correlated shocks to the agents' higher order beliefs. In contrast to these papers, we assume that all agents have common knowledge. Finally, Farmer (2012a,b) obtains non-fundamental fluctuations by letting sentiments, rather than bargaining, determine the equilibrium wage in a labor market with search frictions. As a result, non-fundamental shocks generate a positive correlation between wages of new hires and unemployment. In contrast to Farmer, we assume that wages are given as the unique Nash bargaining solution and, hence, non-fundamental shocks in our model generate a negative correlation between wages of new hires and unemployment, the same correlation found in the data (see, e.g. Pissarides 2009).

2 Environment and Equilibrium

We develop a model economy with search frictions in the labor and the product markets. The labor market is modeled as in Mortensen and Pissarides (1994). In this market, search frictions generate equilibrium unemployment and income differences between employed and unemployed workers, as employed workers can capture some of the quasi-rents associated with the creation of a firm-worker match. The output generated by a firm-worker pair is partly sold in a product market modeled as in Burdett and Judd (1983). In this market, search frictions induce identical sellers to post different prices for identical goods. Unemployed buyers, having more time available for shopping around, end up paying lower prices and, having less income, demanding lower quantities than employed buyers.

The equilibrium conditions for our model economy are identical to the equilibrium conditions in Mortensen and Pissarides (1994), except for one aspect. In Mortensen and Pissarides (1994), the output produced by a firm-worker pair generates revenues that are constant as it is sold in a perfectly competitive market where demand is infinitely elastic. In our model, the output produced by a firm-worker pair generates revenues that depend on the unemployment rate. This is because part of the output is sold in an imperfectly competitive market where the extent of competition and the level of demand depend on the unemployment rate. This small difference between our model and Mortensen and Pissarides (1994) may be of great consequence. In fact, as we shall see in Section 3, when revenues depend negatively on unemployment, multiple rational expectations equilibria may arise.

2.1 Environment

We consider a discrete-time, infinite-horizon economy populated by two types of agents—workers and firms—who exchange three goods—labor and two consumption goods. Labor is traded in a decentralized and frictional market modeled as in Mortensen and Pissarides (1994). The first consumption good is traded in a decentralized and frictional market modeled as in Burdett and Judd (1983). We shall refer to this good as the Burdett-Judd (BJ) good. The second consumption good is traded in a centralized and frictionless product market. We shall refer to this good as the Arrow-Debreu (AD) good.

The measure of workers in the economy is normalized to one. A worker has preferences described by the utility function $\sum_{t=0}^{\infty}(1+\rho)^{-t}u_w(x_t, y_t)$, where $1/(1+\rho) \in (0,1)$ is the discount factor and $u_w(x, y)$ is a periodical utility function defined over consumption of the

BJ good, x, and consumption of the AD good, y. We assume that $u_w(x, y)$ is of the Cobb-Douglas form $x^\alpha y^{1-\alpha}$, where $\alpha \in (0, 1)$. A worker has access to a technology that allows him to transform the AD good into the BJ good at the rate of r to 1, with $r > 0$. This technology guarantees that a worker can consume some of the BJ good even when he fails to meet a seller in the BJ market. In every period, a worker is endowed with one indivisible unit of labor, which he may sell to a firm (if he has found an employer) or use for home-production (if he is unemployed). When the worker is employed, his labor income is worth $w(u_t)$ units of the AD good, where $w(u_t)$ is a bargaining outcome which depends on the unemployment rate u_t. When the worker is unemployed, his labor income is worth y_u units of the AD good, $y_u \geq 0$. This income can be interpreted as either the value of home-production or as the value of an unemployment benefit that is financed by lump-sum taxes levied on the firms. For the sake of simplicity, we assume that workers can neither borrow nor save income or goods across periods.[1]

The measure of firms in the economy is also normalized to one. A firm has preferences described by the utility function $\sum_{t=0}^{\infty}(1+\rho)^{-t}u_f(x_t, y_t) - kv_t$, where $u_f(x_t, y_t)$ is a periodical utility function defined over consumption of the BJ and AD goods, v_t is the number of vacancies created by the firm and $k > 0$ is the disutility cost of creating a vacancy. A firm creates vacancies in order to find employees. Every employee of the firm can produce any combination of x units of the BJ good and y units of the AD good such that $cx + y = y_e$, with $c \in (0, r)$ and $y_e > 0$. The parameter y_e describes the productivity of labor, measured in units of the AD good. The parameter c describes the rate at which firm-worker matches can implicitly transform the AD good into the BJ good.[2] As we shall see momentarily, the allocation of production between the AD and the BJ good depends on whether the firm-worker pair meets a buyer in the BJ market and, if so, on the quantity demanded by that buyer. For the sake of simplicity, we assume $u_f(x, y) = y$. That is, we assume that firms only care about consuming the AD good.[3] Moreover, we assume that firms cannot store

[1] In the quantitative section of the paper, we will try to address this unrealistic feature of the model by making sure that the decline in expenditures experienced by a worker who becomes unemployed is in line with what we observe in the data.

[2] The production technology can be interpreted as follows. The firm has to allocate a unit of the worker's time between producing the AD good and the BJ good. Producing each unit of the AD good requires $1/y_e$ units of time and producing each unit of the BJ good requires c/y_e units of time. According to this interpretation, y_e is the highest quantity of the AD good that the worker can produce and c is the opportunity cost of allocating the worker's time to producing an extra unit of the BJ good rather than to producing the AD good. The assumption that every firm produces both the BJ and AD goods is not crucial. Indeed, we could have used a model where firms choose whether to specialize in the BJ good or in the AD good.

[3] We assume that the owners of the firms are different agents than the workers. In particular, we assume that the owners of the firms have linear preferences over the AD good. The assumption implies that firms simply want to maximize of the present value of profits measured in units of the AD good. If, in contrast, we were to assume that firms are owned by workers, firms would maximize the present value of profits with an endogenous discount factor that depends on the consumption path of workers. The model would be harder

goods from one period to the next.

Markets open sequentially in every period t. The first market to open is the Mortensen-Pissarides (MP) labor market. In this market, firms create vacancies at the disutility cost k. Then unemployed workers, u_{t-1}, and vacant jobs, v_t, come together through a constant return to scale matching function $M(u_{t-1}, v_t) < \min\{u_{t-1}, v_t\}$. The probability that an unemployed worker matches with a vacancy is $\lambda(\theta_t) \equiv M(1, \theta_t)$, where θ_t denotes the tightness of the labor market, v_t/u_{t-1}, and $\lambda : \mathbb{R}_+ \to [0,1]$ is a strictly increasing and concave function with boundary conditions $\lambda(0) = 0$ and $\lambda(\infty) = 1$. Similarly, the probability that a vacant job matches with an unemployed worker is $\eta(\theta_t) \equiv M(1/\theta_t, 1)$, where $\eta : \mathbb{R}_+ \to [0,1]$ is a strictly decreasing function with boundary conditions $\eta(0) = 1$ and $\eta(\infty) = 0$. When an unemployed worker and a vacant job match, they bargain over the current wage and enter the BJ and AD market to produce and sell output. While vacant jobs and unemployed workers search for each other in the MP market, existing firm-worker pairs are destroyed with probability $\delta \in (0,1)$.

The second market to open is the BJ product market. In this market, each firm-worker pair (henceforth, a seller) posts a price p, measured in units of the AD good, at which it is willing to sell the BJ good. Each worker (henceforth, a buyer) searches for sellers with an intensity that depends on his employment status.[4] In particular, if a buyer is unemployed, he makes one search with probability $1 - {}_u$, and two searches with probability ${}_u$, where ${}_u \in [0,1]$. If a buyer is employed, he makes one search with probability $1 - {}_e$ and two searches with probability ${}_e$, where ${}_e \in [0,1]$. We assume ${}_e \leq {}_u$ in order to capture the idea that a buyer has less time to search the product market when he is employed.[5]

Sellers and buyers come together through a constant return to scale matching function $N(b(u_t), s(u_t))$, where $b(u_t) \equiv 1 + {}_e + u({}_u - {}_e)$ is the measure of buyers' searches, $s(u_t) \equiv 1 - u_t$ is the measure of active sellers and u_t is the measure of unemployed workers at the opening of the BJ market. A seller meets a buyer with probability $\mu(\sigma(u_t)) \equiv N(1/\sigma(u_t), 1)$,

to solve but we believe that the main results would still go through.

[4] We do not interpret the search process in the BJ market as a process of discovery of prices. Rather, we interpret it as a constraint on the number and location of stores a buyer can visit in a given interval of time. On some day, the buyer may be busy tending to his kids and he is able to shop only at the local convenience store. On some other days, the buyer may be relatively free and he is able to shop both at the supermarket in the suburbs and at the local convenience store.

[5] We assume that the average number of searches of employed and unemployed buyers is exogenous. Thus, it is legitimate to wonder what would happen if we were to endogenize the search intensity of the buyer. In general, unemployment would have two countervailing effects on search intensity. On the one hand, an unemployed buyer has more time and, hence, faces a lower cost of searching. On the other hand, an unemployed buyer has lower consumption and, hence, faces a lower return to searching. Thus, in principle, an unemployed buyer could choose to search more or less than an employed one. Empirically, though, we find that unemployed buyers spend 20 to 30 percent more time shopping than employed buyers and, in the quantitative part of the paper, we use this information to discipline the choice of the exogenous parameters ${}_e$ and ${}_u$.

where $\sigma(u_t)$ denotes the tightness of the product market, $s(u_t)/b(u_t)$. Similarly, a buyer who makes one search meets a seller with probability $\nu(\sigma(u_t)) \equiv N(1, \sigma(u_t))$, while a buyer who makes two searches meets two sellers with probability $\nu(\sigma(u_t))^2$ and one seller with probability $2\nu(\sigma(u_t))(1 - \nu(\sigma(u_t)))$. When a buyer meets a seller, it observes the price and decides whether and how much of the BJ good to purchase. We assume $N(b(u_t), s(u_t)) = \min\{b(u_t), s(u_t)\}$ in order to focus on search frictions (i.e. buyers meeting a random subset of sellers and sellers meeting random buyers) and abstract from matching frictions.[6]

The last market to open is the AD product market. In this market, each firm-worker pair produces and sells a quantity of AD goods which depends on the quantity of BJ goods it produced and sold in the BJ market. Each worker purchases and consumes an amount of AD goods that depends on the income that he spent in the BJ market. The AD market is frictionless and perfectly competitive.[7]

The medium of exchange in our economy is a perfectly divisible and transferrable one-period IOU. In the MP market, firms pay wages to their workers by issuing IOUs promising a payment worth $w(u_t)$ units of AD goods in the current period. Firms also pay taxes to the government by issuing IOUs, which are then transferred by the government to unemployed workers. In the BJ market, sellers exchange the BJ good for the IOUs carried by employed and unemployed buyers. In the AD market, firms use the IOUs that they have collected in the BJ market to purchase AD goods and they repay the IOUs that they have issued to their workers by selling AD goods.

It may be useful to point out the two features of the environment that are critical to our theory of self-fulfilling unemployment fluctuations. The first feature is that some of the output produced by a firm-worker pair is sold in an imperfectly competitive market (the BJ market) where the fraction of employed and unemployed buyers affects demand

[6] The theoretical results in Section 3 carry over to more general matching functions $N(b, s)$ with the property that the elasticity of N with respect to b is not too large.

[7] The AD goods and the BJ goods are two groups of consumer goods which differ with respect to the structure of the market where they are traded. The AD goods are traded in a perfectly competitive market, where the Law of One Price holds. The BJ goods are traded in an imperfectly competitive market where price dispersion obtains. In Kaplan and Menzio (2013), we find that price dispersion is pervasive, but the extent of it varies across different types of goods. Thus, one can think of the AD goods as the subset of consumer goods that feature relatively little price dispersion and think of the BJ goods as the subset of consumer goods that feature a relatively large amount of price dispersion. We need both goods in our model. We need BJ goods for substantive reasons. Indeed, our theory of multiplicity is based on the idea that the extent of competition in the product market is endogenous and, hence, we need some goods to be traded in an imperfectly competitive market. We need the AD goods for technical reasons. In our model, the buyers of BJ goods compensate the sellers of BJ goods by giving them claims to contemporaneous AD goods. Hence, all trades are completed within each period. If the model did not have AD goods, buyers of BJ goods could only pay the sellers of BJ goods with claims to future BJ goods. Hence, trades would not be completed within each period and we would have to keep track of the credit/debit position of all the agents in the economy. The role of AD goods is to act as a medium of exchange; a sort of fiat money that is consumed and is perishable.

(because of differences in income) and the extent of competition (because of differences in search intensity).[8] As we shall see, this feature implies that the revenues generated by a firm-worker output pair tend to decrease with unemployment. The second feature is that a firm needs to hire labor in order to enter or scale up its presence in the product market.[9] As we shall see, this feature implies that when the product market features low demand and more competition, a firm demands less labor and creates fewer vacancies.

2.2 Individual Problems and Terms of Trade

Having described the environment, we now proceed to analyze the problem of individual agents and the determination of the terms of trade in the different markets.

2.2.1 Individual Problems

Problem of the buyer. Consider an unemployed buyer who enters the BJ market with an income worth y_u units of the AD good. With some probability, the buyer does not contact any seller in the BJ market. In this case, the buyer spends all of his income on the AD good and then transforms some of the AD goods into BJ goods at the rate of r to 1. With some probability, the buyer contacts one or two sellers. Let p denote the lowest price of the BJ good among the sellers contacted by the buyer. If $p > r$, the buyer spends all of his income on the AD good and then transforms some of the AD good into the BJ good at the rate of r to 1. If $p \leq r$, the buyer purchases both the AD good and the BJ good on the market. In particular, the buyer chooses how to allocate his income y_u between consumption of the

[8] Besides ours, there are many other models of the product market with the property that the fraction of employed and unemployed buyers affects the level of demand and the extent of competition. For instance, the property would obtain in a version of the monopolistic competition model of Dixit and Stiglitz (1974) where unemployed buyers have lower income and are more willing to substitute different varieties of goods. Similarly, the property would obtain in a version of the search-theoretic model of Albrecht, Gautier and Vroman (2006) where unemployed buyers have lower income and are more likely to make multiple searches, meet multiple sellers of identical goods and engage these sellers in Bertrand competition. In contrast, the property would not emerge in a version of the search-theoretic model of Bai, Rios-Rull and Storesletten (2012) where unemployed buyers search more than employed buyers. In their model, a buyer who searches more meets more sellers. However, as each seller has limited capacity, the buyer purchases from every seller that he meets rather than only from the one with the lowest price. Hence, in their model, higher search intensity does not increase the competitiveness of the product market.

[9] This feature follows from the assumption that each firm-worker pair represents a distinct seller in the product market. In the context of our search-theoretic model of imperfect competition, there is a natural interpretation for this assumption. New firms can enter the product market by opening a production-and-retail outlet and existing firms can expand their presence in the product market by opening additional outlets. Each outlet requires a fixed amount of labor for production and retail. Each outlet is a different seller because it is located in a different area and, hence, reaches a different set of buyers. In the context of a Dixit-Stiglitz model of monopolistic competition, the assumption can be interpreted as follows. New firms can enter the product market by setting up a plant producing a new variety of goods and existing firms can expand their presence in the product market by setting up new plants. Each plant requires a fixed amount of labor to be run. Each plant is a different seller because it produces a different variety of goods.

BJ good and consumption of the AD good so as to maximize its periodical utility function. Formally, the problem facing the buyer is

$$\max_{x,y} x^{\alpha} y^{1-\alpha}, \quad \text{s.t. } px + y = y_u. \tag{1}$$

The solution to the problem of an unemployed buyer is

$$px = \alpha y_u, \quad y = (1-\alpha) y_u. \tag{2}$$

That is, the buyer spends a fraction α of his income on the BJ good and a fraction $1-\alpha$ on the AD good. The solution to the problem of an employed buyer is the same as (2), except that the income is the wage $w(u_t)$ rather than the unemployment benefit y_u.

Problem of the seller. Consider a seller entering the BJ market. The seller chooses the relative price of the BJ good so as to maximize its total revenues from sales in the BJ and AD markets. The seller takes as given the composition of buyers between employed and unemployed, the demand of each type of buyer, the total number of sellers, and the distribution of posted prices, an endogenous object which we shall denote as $F(p, u_t)$. Formally, the problem facing the seller is to maximize $S(p, u_t) + y_e$ with respect to p, where $S(p, u_t)$ denotes the seller's expected revenues in the BJ market net of the opportunity cost of producing the BJ good.

The net revenue function $S(p, u_t)$ is given by

$$\begin{aligned} S(p, u_t) &= \mu(\sigma(u_t)) \frac{u_t(1+\ _u)}{b(u_t)} \left[1 - \frac{2\ _u \nu(\sigma(u_t)) F(p, u_t)}{1+\ _u} \right] \frac{\alpha y_u (p-c)}{p} \\ &+ \mu(\sigma(u_t)) \frac{(1-u_t)(1+\ _e)}{b(u_t)} \left[1 - \frac{2\ _e \nu(\sigma(u_t)) F(p, u_t)}{1+\ _e} \right] \frac{\alpha w(u_t)(p-c)}{p}. \end{aligned} \tag{3}$$

Consider the first line in the above expression. The probability that the seller meets a buyer is $\mu(\sigma(u_t))$. The probability that the buyer is unemployed is $u_t(1+\ _u)/b(u_t)$, the fraction of buyers' searches originating from unemployed workers divided by the total number of buyers' searches. Conditional on being unemployed, the probability that the buyer is not in contact with a seller charging a price less than p is given by the term in square brackets, which is equal to the complement to 1 of the product between the probability that the buyer is in contact with a second seller, $2\ _u \nu(\sigma(u_t))/(1+\ _u)$, and the probability that the second seller charges a price less than p, $F(p, u_t)$. Conditional on being unemployed and not being in contact with a seller charging less than p, the buyer purchases y_u/p units of the BJ good. For every unit of the BJ good sold, the seller's revenue net of the opportunity cost of production is $p-c$. Summarizing, the first line in (3) represents the seller's expected net revenue from meetings with unemployed buyers. Similarly, the second line in (3) represents the seller's expected net revenue from meetings with employed buyers. The two lines differ because unemployed and employed buyers are willing to purchase at the price p with different probabilities and because, when they do, they demand different quantities.

Problem of the firm. Consider a firm entering the MP market. The firm decides how many vacancies to create by comparing the cost and the benefit of opening a vacancy. The cost of opening a vacancy is given by the disutility cost k. The benefit of opening a vacancy is given by the product of the probability of filling a vacancy, $\eta(\theta_t)$, and the present discounted value of the profits generated by an additional employee, J_t. Since the firm operates a constant return to scale technology, the value of an additional employee to the firm is independent of the number of workers employed by the firms and, hence, the firm's problem is linear. The solution of the problem is such that the firm does not open any vacancies if $k > J_t$, it opens infinitely many vacancies if $k < J_t$, and it is indifferent between opening any number of vacancies if $k = J_t$.

The value of an employee to the firm is such that

$$J_t = \max_p \left[S(p, u_t) + y_e\right] \square\, w(u_t) + \frac{1 \square \delta}{1+\rho} J_{t+1}. \tag{4}$$

The above expression is easy to understand. In the current period, the firm collects $\max_p \left[S(p, u_t) + y_e\right]$ revenues in the BJ and AD markets and it pays the wage $w(u_t)$ to the worker. In the next period, the worker leaves the firm with probability δ and stays with the firm with probability $1 \square \delta$. In the first case, the continuation value of the employee is zero. In the second case, the continuation value of the employee is J_{t+1}.

2.2.2 Terms of Trade and Market Tightness

Terms of trade in the BJ market. The distribution of posted prices in the BJ market, F, is consistent with the seller's optimal pricing strategy if and only if any price on the support of the distribution maximizes the seller's total revenues $S(p, u_t) + y_e$ or, equivalently, the seller's net revenues in the BJ market, $S(p, u_t)$. Lemma 1 states that there exists a unique price distribution that is consistent with the seller's optimal pricing strategy. The proof of the lemma is similar to the one in Burdett and Judd (1983) or in Head, Liu, Menzio and Wright (2012) and is presented in Appendix A.

Lemma 1 (Equilibrium Price Distribution): *The unique price distribution consistent with* *the seller's optimal pricing strategy is*

$$
\begin{aligned}
F(p, u_t) = \quad & \left\{ u_t(1 + \quad_u) \left[1 - \left(1 - \frac{2 \quad_u \nu(\sigma(u_t))}{1 + \quad_u} \right) \frac{(r - c)p}{(p - c)r} \right] y_u \right. \\
& \left. + (1 - u_t)(1 + \quad_e) \left[1 - \left(1 - \frac{2 \quad_e \nu(\sigma(u_t))}{1 + \quad_e} \right) \frac{(r - c)p}{(p - c)r} \right] w(u_t) \right\} \Big/ \\
& 2\nu(\sigma(u_t)) \left\{ u_t \quad_u y_u + (1 - u_t) \quad_e w(u_t) \right\}
\end{aligned}
\tag{5}
$$

with support $[\underline{p}_t, \overline{p}_t]$, *where* $c < \underline{p}_t < \overline{p}_t = r$.

Proof: See Appendix A.

The price distribution F is continuous. In fact, if F had a mass point at some $p_0 > c$, a seller posting p_0 could increase its gains from trade by charging $p_0 - \epsilon$. This deviation would increase the probability of making a sale by a discrete amount, but it would leave the net revenues on each unit sold approximately constant.[10] The support of F is connected. In fact, if the support of F had a gap between p_0 and p_1, the seller's gains from trade would be strictly higher at p_1 than p_0, as the probability of making a sale is the same at p_0 and p_1 but the net revenues on each unit sold are strictly greater at p_1. For the same reason, the highest price on the support of F is the buyer's reservation price r.

An implication of the previous lemma is that the maximized net revenues of the seller in the BJ market, $S^*(u_t) = \max_p S(p, u_t)$, are equal to the net revenues for a seller who charges the buyers' reservation price, i.e. r, and only sells to captive buyers, i.e. buyers that contact the seller and nobody else. That is, $S^*(u_t)$ is given by

$$
\begin{aligned}
S^*(u_t) \quad & = \mu(\sigma(u_t)) \frac{u_t(1 + \quad_u)}{b(u_t)} \left[1 - \frac{2 \quad_u \nu(\sigma(u_t))}{1 + \quad_u} \right] \frac{\alpha y_u (r - c)}{r} \\
& + \mu(\sigma(u_t)) \frac{(1 - u_t)(1 + \quad_e)}{b(u_t)} \left[1 - \frac{2 \quad_e \nu(\sigma(u_t))}{1 + \quad_e} \right] \frac{\alpha w(u_t)(r - c)}{r}.
\end{aligned}
\tag{6}
$$

Terms of trade in the MP market. The wage in the MP market is determined as the outcome of a bargaining process between the firm and the worker. We assume that the wage outcome is such that the additional income that the worker and the firm can generate together is divided between the worker and the firm according to the constant and exogenous fractions $\quad$ and $1 - \quad$, with $\quad \in (0, 1)$. Formally, we assume that the wage outcome is given

[10] The price p_0 cannot be equal to c because the seller's net revenues in the BJ market are always strictly positive. To see this, note that the seller's net revenues are bounded below by the (strictly positive) revenues associated with posting the reservation price r and selling only to the buyers who are not in contact with any other seller.

by[11]

$$w(u_t) = y_u + \quad (S^*(u_t) + y_e - y_u)\,. \tag{7}$$

The wage outcome in (7) coincides with the Generalized Nash Bargaining Solution when the worker's and firm's outside options are as follows. The outside option of the worker is to enjoy an income worth y_u units of the AD good, to make one search in the BJ marker with probability $1 - \;{}_e$ and two searches with probability $\;{}_e$, and to enter the next MP market still matched with the firm. The outside option of the firm is to generate no revenues from the worker in the BJ and AD markets and to enter the next MP market still matched with the worker. That is, a failure to agree on the terms of trade results in the firm and the worker not producing together in the current period and trying to agree on the terms of trade again in the next period. Moreover, a failure to agree on the terms of trade costs the worker some time and results in him searching the BJ market with the same intensity as an employed buyer.[12]

Tightness of the MP market. The tightness of the MP market, θ_t, is consistent with the firm's optimal vacancy creation strategy if and only if it is equal to

$$\theta(J_t) = \begin{cases} 0 & \text{if } k > J_t, \\ \eta^{-1}(k/J_t) & \text{if } k \leq J_t. \end{cases} \tag{8}$$

If the cost of creating a vacancy, k, is strictly greater than the value of an additional worker to the firm, J_t, the tightness of the MP market is zero. In fact, if $k > J_t$, the cost of creating a vacancy certainly exceeds the benefit of creating a vacancy, which is given by the probability of filling the vacancy times the value of a worker to the firm. If, on the other hand, k is smaller than J_t, the tightness of the MP market is such that $\eta(\theta_t)J_t = k$. In fact, if $k \leq J_t$, firms continue to create new vacancies until the tightness of the MP market is high enough and, hence, the probability of filling a vacancy is low enough to equalize the cost and the benefit of opening an additional vacancy.

[11] Since employed and unemployed workers pay different prices in the BJ market, the wage bargaining outcome (7) does not guarantee that a worker is better off employed than unemployed. In the theoretical part of the paper, we proceed under the assumption that employed workers are always better off. In the quantitative part of the paper, we verify that employed workers are better off than unemployed workers in all rational expectations equilibria.

[12] The outside options here may be more or less realistic than the outside options in Pissarides (1985), Mortensen and Pissarides (1994) and many subsequent papers. They certainly simplify the analysis. The assumption that, in case of disagreement, the firm and the worker do not lose contact with each other simplifies the analysis by making the wage only a function of current variables. The assumption that, in case of disagreement, the worker searches with the same intensity as an employed buyer simplifies the analysis by making the wage independent of the price distribution F.

2.2.3 Unemployment Dynamics

The law of motion for unemployment is

$$u_t = u_{t\square 1}\,(1 \square \lambda(\theta_t)) + (1 \square u_{t\square 1})\delta. \tag{9}$$

The measure of unemployed workers at the opening of the MP market in period t is $u_{t\square 1}$. During the MP market, an unemployed worker becomes employed with probability $\lambda(\theta_t)$ and an employed worker becomes unemployed with the exogenous probability δ. Thus, the measure of unemployed workers at the opening of the BJ market in period t is given by the right-hand side of (9). Clearly, this is also the measure of unemployed workers at the opening of the MP market in period $t + 1$.

2.3 Rational Expectation Equilibrium

We are now in the position to define an equilibrium for our model economy.

Definition 1: *A discrete-time Rational Expectation Equilibrium is a sequence* $\{J_t, u_t\}$ *such that:*

(i) *For* $t = 0, 1, 2, ...,$ J_t *satisfies the Bellman Equation*

$$J_t = (1 \square \quad)\,[S^*(u_t) + y_e \square y_u] + \frac{1 \square \delta}{1+\rho} J_{t+1}. \tag{10}$$

(ii) *For* $t = 0, 1, 2, ...,$ u_t *satisfies the law of motion*

$$u_t = u_{t\square 1}\,(1 \square \lambda(\theta(J_t))) + (1 \square u_{t\square 1})\delta. \tag{11}$$

(iii) $\lim_{t\to\infty} J_t$ *is finite and* $u_{\square 1}$ *is given.*

Condition (i) is the Bellman Equation (4) describing the value of an additional worker to a firm, where we have replaced the wage with its equilibrium value (7). Condition (ii) is the equation (9) describing the law of motion for the unemployment rate. Finally, condition (iii) describes the boundary conditions for the system of differential equations defined by conditions (i) and (ii). In particular, condition (iii) states that the initial value of unemployment is given and that the value of a worker to a firm satisfies a transversality condition. Given a sequence $\{J_t, u_t\}$ that satisfies the above equilibrium conditions, one can recover all the other equilibrium objects, such as the terms of trade in the MP and BJ markets (which are functions of the contemporaneous unemployment rate) and the tightness of the MP market (which is a function of the contemporaneous value of a worker).

In this section, which was mainly devoted to describing the environment and the equilibrium conditions, it was natural to make the assumption of discrete time. In the remainder of the paper, which is mainly devoted to characterizing the set of rational expectations equilibria, it is more convenient to work in continuous time. We formally derive a continuous-time version of our discrete-time model in Appendix B. There, we assume that, over a period of length Δ, the vacancy cost is $k\Delta$, the productivity of labor is $y_e\Delta$, the unemployment income is $y_u\Delta$, the job-loss probability is $1 - \exp(\delta\Delta)$, the MP matching function is $M(u, v)\Delta$, while the parameters $_e$, $_u$, N, r and c are independent of Δ. We then take the limit as Δ goes to zero and obtain the continuous-time equivalent to the equilibrium conditions (10) and (11). This leads to the following definition of equilibrium for the continuous-time version of the model.

Definition 2: *A continuous-time Rational Expectation Equilibrium is a path $\{u_t, J_t\}$ such that:*

(i) *For all $t \geq 0$, J_t satisfies the Bellman Equation*

$$(\rho + \delta) J_t = (1 - \quad) (S(u_t) + y_e - y_u) + \mathring{J}_t; \tag{12}$$

(ii) *For all $t \geq 0$, u_t satisfies the law of motion*

$$\mathring{u}_t = -u_t\lambda(\theta(J_t)) + (1 - u_t)\delta; \tag{13}$$

(iii) $\lim_{t\to\infty} J_t$ *is finite and u_0 is given.*

3 Equilibrium Characterization

In this section, we characterize the equilibrium set for our model economy. First, we analyze the set of steady states. We show that the revenues generated by a firm-worker pair are decreasing in unemployment if the income of employed buyers is sufficiently high relative to the income of unemployed buyers and/or if the search intensity of employed buyers in the BJ product market is sufficiently low relative to the search intensity of unemployed buyers. Moreover, we show that if the revenues generated by a firm-worker pair are decreasing in unemployment, so are the firms' incentives to employ workers and, given an appropriate choice of the vacancy cost and the labor market matching function, the model admits multiple steady states.

We then analyze the set of rational expectations equilibria. If the model admits a unique steady state, we find that there is also a unique rational expectations equilibrium for any initial condition of the economy. If the model admits multiple steady states, we find that there are always multiple rational expectations equilibria for some initial conditions of the economy. These equilibria differ with respect to the agents' expectations about future unemployment. Yet, in each equilibrium, the agents' expectations are correct, in the sense that the actual path of unemployment coincides with the one forecast by the agents. The existence of multiple rational expectations equilibria implies that the behavior of our model economy is determined not only by fundamentals (i.e. technology and preferences), but also by expectations.

3.1 Steady States

A steady state is a point (u, J) such that the unemployment rate and the value of a worker to a firm are stationary. In order to characterize the set of steady states, we use equation (13) to find the locus of points where the unemployment rate is stationary (henceforth, the *u-nullcline*) and equation (12) to find the locus of points where the value of a worker to a firm is stationary (henceforth, the *J-nullcline*). We then look for the set of intersections between the two loci.

The locus of points where unemployment is stationary is given by

$$u = \frac{\delta}{\delta + \lambda(\theta(J))}. \tag{14}$$

The stationary value of unemployment is given by the ratio of the worker's job loss rate, δ, and the sum of the worker's job loss and the job finding rates, $\delta + \lambda(\theta(J))$. When the value to the firm from employing an additional worker, J, is smaller than the cost of creating a vacancy, k, the tightness of the MP market, $\theta(J)$, and the job-finding rate, $\lambda(\theta(J))$, are both equal to zero and, hence, the stationary value of unemployment is equal to $\overline{u} \equiv 1$. When J is greater than k, $\theta(J)$ and $\lambda(\theta(J))$ are both strictly positive and strictly increasing in J. Hence, the stationary value of unemployment is strictly smaller than one and strictly decreasing in J. In the limit for $J \to \infty$, $\theta(J) \to \infty$ and $\lambda(\theta(J)) \to 1$. Hence, the stationary value of unemployment converges to $\underline{u} \equiv \delta/(1+\delta)$. Figure 1 plots the u-nullcline in the (u, J)-space.

The locus of points where the value of a worker to a firm is stationary is given by

$$J = \frac{(1 \,\square\quad) \left[S^*(u) + y_e \,\square\, y_u\right]}{\rho + \delta}. \tag{15}$$

The stationary value of a worker is given by the flow of revenues generated by a firm-worker pair and accruing to the firm, $(1 \,\square\quad) \left[S^*(u) + y_e \,\square\, y_u\right]$, capitalized by the factor $1/(\rho + \delta)$.

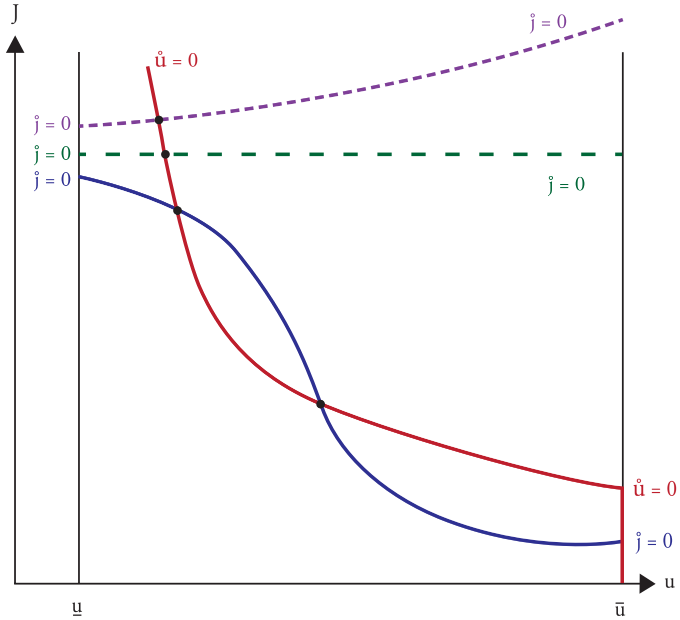


Figure 1: The u and J nullclines

The stationary value of a worker is bounded because the flow revenues, $S^*(u) + y_e$, are bounded. Moreover, the stationary value of a worker is increasing, constant or decreasing with respect to unemployment depending on whether the flow revenues are increasing, constant or decreasing with respect to unemployment. Figure 1 illustrates several possible shapes for the J-nullcline in the (u, J)-space.

From the properties of the u-nullcline and J-nullcline, we can already reach some general conclusions about the set of steady states. First, the model always admits one steady state, as the u-nullcline and the J-nullcline must cross at least once. Second, if the J-nullcline is constant or upward sloping, the model admits only one steady state. If, on the other hand, the J-nullcline is downward sloping, the model may admit multiple steady states. Whether this is the case depends on the exact shape of the u-nullcline, which depends on the vacancy cost, k, on the shape of the matching function in the MP market, M, and on the shape of the J-nullcline, which depends on the shape of the seller's revenue function $S^*(u) + y_e$.

In equilibrium, the seller's revenues are equal to the revenues for a seller that, in the BJ market, posts the reservation price r and sells only to buyers who are not in contact with any other sellers (i.e. captive buyers). The derivative of the seller's revenues with respect to unemployment, $S^{*\prime}(u)$, is a complicated expression that can be written as the sum of three terms—the *market power effect* $ME(u)$, the *demand effect* $DE(u)$, and the *tightness effect* $TE(u)$—scaled up by a multiplier $1/(1 - MLT(u))$. That is, the derivative of the seller's revenues can be written as

$$S^{*\prime}(u) = \frac{ME(u) + DE(u) + TE(u)}{1 \square MLT(u)}. \tag{16}$$

The market power effect of unemployment is given by

$$ME(u) = \square \frac{(1 + {}_u)(1 + {}_e)}{b(u)^2} \cdot \left[\frac{2 {}_u}{1 + {}_u} \frac{(1 \square u)}{b(u)} \square \frac{2 {}_e}{1 + {}_e} \frac{(1 \square u)}{b(u)} \right] \cdot \frac{\alpha y_u (r \square c)}{r}. \tag{17}$$

The first term in the expression above is the derivative with respect to unemployment of the probability that, in the BJ market, the seller meets a buyer who is employed rather than unemployed. The second term is the difference between the probability that a buyer is captive conditional on being employed rather than unemployed. The last term is the seller's revenue (net of the opportunity cost of production) from making a sale to an unemployed buyer. Thus, the market power effect measures the decline in seller's revenues caused by the fact that an increase in unemployment raises the seller's probability of meeting an unemployed buyer and, since unemployed buyers search more, lowers the probability that the buyer is captive.

The demand effect of unemployment is given by

$$DE(u) = \square \frac{(1 + {}_u)(1 + {}_e)}{b(u)^2} \cdot \left[1 \square \frac{2 {}_e}{1 + {}_e} \frac{(1 \square u)}{b(u)} \right] \cdot \frac{\alpha \left(w(u) \square y_u \right) (r \square c)}{r}. \tag{18}$$

The first term in the expression above is derivative with respect to unemployment of the probability that, in the BJ market, the seller meets a buyer who is employed rather than unemployed. The second term is the probability that an employed buyer is captive. The last term is the difference between the net revenues that the seller makes if it sells to a buyer who is employed rather than unemployed. Thus, the demand effect measures the decline in seller's revenues caused by the fact that an increase in unemployment raises the seller's probability of meeting an unemployed buyer and, since unemployed buyers have lower income, it lowers the quantity demanded by a captive buyer.

The tightness effect of unemployment is given by

$$\begin{aligned} TE(u) &= \frac{u(1 + {}_u)}{b(u)} \cdot \frac{2 {}_u}{b(u)^2} \cdot \frac{\alpha y_u (r \square c)}{r} \\ &\quad + \frac{(1 + {}_e)(1 \square u)}{b(u)} \cdot \frac{2 {}_e (1 + {}_u)}{b(u)^2 (1 + {}_e)} \cdot \frac{\alpha w(u)(r \square c)}{r} \end{aligned} \tag{19}$$

The tightness effect measures the increase in seller's net revenues due to the fact that an increase in unemployment lowers the seller-to-buyer ratio in the BJ market and, hence, increases the probability that a buyer in a particular employment state is captive. Formally, the first line is the product of the seller's probability of meeting a buyer who is unemployed, the derivative with respect to unemployment of the probability that the buyer is captive, and the seller's net revenues from trading with the buyer. Similarly, the second line is the product of the seller's probability of meeting a buyer who is employed, the derivative with respect to unemployment of the probability that the buyer is captive, and the seller's revenues from trading with the buyer.

An increase in unemployment affects the sellers' revenues through the market power, demand and tightness effects. But by affecting the sellers' revenues, an increase in unemployment also affects the wage of employed workers, the quantity sold by sellers upon meeting a captive employed buyer and, in the end, the sellers' revenues. This feedback effect between revenues and wages multiplies the market power, the demand and the tightness effects by the factor $1/(1 - MLT(u))$, where $MLT(u)$ is given by

$$MLT(u) = \frac{(1 + \ _e)(1 - u)}{b(u)} \left[1 - \frac{2 \ _e}{1 + \ _e} \frac{(1 - u)}{b(u)} \right] \frac{\alpha(r - c)}{r}, \tag{20}$$

which is the derivative of the seller's revenues with respect to the wage of employed workers times the derivative of the wage of an employed worker with respect to the revenues him and his employer generate.

Overall, the sign of the derivative of the seller's revenues with respect to unemployment—and, hence, the slope of the J-nullcline—is positive if the demand and the market power effects dominate and it is negative if the tightness effect dominates. The following theorem provides two necessary and sufficient conditions under which the derivative of the seller's revenues is negative and, hence, the J-nullcline downward sloping. Further, the theorem proves that, when $S^{*\prime}(u)$ is negative, one can find parameter values such that the u-nullcline crosses the J-nullcline multiple times.

Theorem 1 (Multiplicity of Steady States): (i) *For any $u \in [\underline{u}, \overline{u}]$ there is a $\overline{y}_e(u) \geq y_u$ such that $S^{*\prime}(u) < 0$ iff $y_e > \overline{y}_e(u)$.* (iii) *There exists a $\tilde{u} > 0$ such that, for any $u \in [\underline{u}, \tilde{u}]$ there is a $\overline{\ }_e(u) \leq \ _u$ such that $S^{*\prime}(u) < 0$ iff $0 \leq \ _e < \overline{\ }_e(u)$.* (i) *Iff $S^{*\prime}(u) < 0$ for some $u \in [\underline{u}, \overline{u}]$, there is a vacancy cost k and a labor market matching function M such that the model admits multiple steady states.*

Proof: See Appendix C.

The first part of Theorem 1 states that the seller's revenues are decreasing in unemployment if and only if the productivity of labor, y_e, is sufficiently high relative to the income of the unemployed, y_u. This result is intuitive. The higher is y_e relative to y_u, the larger is the difference in income between employed and unemployed workers and, hence, the larger is the demand effect of unemployment. The second part of Theorem 1 states that the seller's revenues are decreasing in unemployment if and only if the search intensity of employed buyers, $_e$, is sufficiently small relative to the search intensity of employed buyers, $_u$. This result is also intuitive. The lower is $_e$ relative to $_u$, the larger is the difference between the probabilities than an employed buyer and an unemployed buyer are captive and, hence, the larger is the market power effect of unemployment. The last part of Theorem 1 states that, if and only if the seller's revenues are decreasing in unemployment, one can find a value for the vacancy cost, k, and a legitimate matching function, M, such that the u-nullcline crosses the J-nullcline multiple times and, hence, the model admits multiple steady states. Intuitively, if $S^{*\prime}(u) < 0$, a higher unemployment rate lowers the firm's value of hiring a worker, it lowers the firm's incentive to create vacancies and, if the matching function M is chosen appropriately, the decline in vacancy creation sustains the higher unemployment rate.

It is useful to interpret the existence of multiple steady states as the outcome of the externalities that a firm imposes on other firms if it permanently increases its workforce. If a firm increases its workforce, it increases the other firms' cost of hiring an additional worker by increasing the tightness of the labor market. We refer to this external effect as the *congestion externality*. The congestion externality is negative and its strength is measured by the slope of the u-nullcline. Moreover, if a firm decides to increase its workforce, it affects the other firms' benefit from hiring an additional worker by lowering the unemployment rate and, thus, increasing the tightness of the product market and changing the composition of the buyers populating it. The increase in the tightness of the product market lowers the other firms' benefit from hiring an additional worker as it lowers the probability that a firm-worker pair meets an employed or unemployed buyer who is captive. This external effect is captured by the TE term in (19). The change in the composition of buyers increases the other firms' benefit from hiring an additional worker as it increases the probability that a firm-worker pair meets an employed buyer, which is the type that demands more and is more likely to be captive. This external effect is captured by the ME and DE terms in (17) and (18). We refer to the sum of TE, ME and DE as the *shopping externalities* of employment. The shopping externalities may be positive or negative and their strength is measured by the slope of the J-nullcline.

If, at some steady state, the shopping externalities are positive and dominate the congestion externality, the employment decisions of different firms are strategic complements, in the sense that if a firm increases its workforce, other firms want to increase their workforce as well. This strategic complementarity in employment leads to multiple steady states. Graphically, if, at some steady state, the shopping externalities are positive and dominate the congestion externality, the J-nullcline is downward sloping and crosses the u-nullcline from above, which, as one can see from Figure 1, guarantees the existence of multiple intersections between the two curves.

The existence of multiple steady states rests on the demand and the market power externalities which, in turn, originate from the fact that employed buyers demand more and search less than unemployed ones. Theoretically, either one of these externalities in isolation is sufficient to obtain multiplicity. Indeed, one can easily verify that the seller's revenues can be decreasing in unemployment (and, by virtue of Theorem 1, multiplicity may emerge) even when the search intensity of employed and unemployed buyers is the same and, hence, the market power externality is shut off. Similarly, one can easily verify that the seller's revenues can be decreasing in unemployment even when the income of employed and unemployed workers is the same and, hence, the demand externality is shut off. Empirically, though, we shall see that the market power externality—which is the genuinely novel feature of our theory—is much more important than the demand externality.

3.2 Rational Expectation Equilibria

When the model admits only one steady state, it is easy to prove that there exists a unique rational expectations equilibrium for any initial unemployment rate. When the model admits multiple steady states, the characterization of the set of rational expectations equilibria is more complicated. First, we need to characterize the solutions of the dynamical system given by the differential equations (12) and (13) around each steady state. Second, we need to characterize the solutions of the dynamical system (12)-(13) away from the steady states. Finally, we need to check the transversality condition, in order to understand which solutions are indeed rational expectations equilibria. For the sake of concreteness, we will carry out the analysis for the case in which there are three steady states.

Let $E_i = (u_i^*, J_i^*)$ denote the steady state with the i-th lowest unemployment rate. The linear approximation of the dynamical system (12)-(13) around E_i is

$$\begin{pmatrix} \mathring{u}_t \\ \mathring{J}_t \end{pmatrix} = \begin{pmatrix} \square\delta \,\square\, \lambda(\theta(J_i^*)) & \square\lambda'(\theta(J_i^*))\theta'(J_i^*) \\ \square(1 \,\square\quad)S^{*\prime}(u_i^*) & \rho + \delta \end{pmatrix} \begin{pmatrix} u_t \,\square\, u_i^* \\ J_t \,\square\, J_i^* \end{pmatrix}. \tag{21}$$

The behavior of the dynamical system (12)-(13) in a neighborhood of the steady state E_i depends on the sign of the two eigenvalues of the 2x2 matrix in (21). At the steady state with the lowest unemployment rate, E_1, the two eigenvalues are real and have opposite signs. This means that the steady state is a saddle and, hence, there exists one and only one trajectory—the stable manifold—that converges towards E_1. We find it useful to denote as $J_1^S(u)$ the set of J's such that the point (u, J) belongs to the stable manifold converging to E_1. At the steady state with the intermediate unemployment rate, E_2, the two eigenvalues are complex conjugates. The sign of the real part of these eigenvalues is negative if $u_i^* < \delta/(\rho+\delta)$ and positive if $u_i^* > \delta/(\rho+\delta)$. This means that, if $u_i^* < \delta/(\rho+\delta)$, the steady state is a sink and, hence, any trajectory starting in a neighborhood of E_2 converges towards E_2. If $u_i^* > \delta/(\rho+\delta)$, the steady state is a source and, hence, any trajectory starting in a neighborhood of E_2 diverges from E_2. Finally, at the steady state with the highest unemployment rate, E_3, the two eigenvalues are real and have opposite signs. Hence, the steady state E_3 is also a saddle. We denote as $J_3^S(u)$ the set of J's such that the point (u, J) belongs to the stable manifold converging to E_3.

The global behavior of the dynamical system (12)-(13) depends on the shape of the stable manifolds associated with the steady states E_1 and E_3 and on whether the steady state E_2 is a source or a sink. Figure 2 illustrate the solutions of the dynamical system (12)-(13) when: (i) the backward extension of J_1^S to the left of E_1 exits from $\underline{u}$ and the backward extension of J_1^S to the right of E_1 exits from $\overline{u}$; (ii) the backward extension of J_3^S to the left of E_3 exits from $\underline{u}$ and the backward extension of J_3^S to the right of E_3 exits from $\overline{u}$; (iii) E_2 is a sink. Fix any initial unemployment $u_0 \in [\underline{u}, \overline{u}]$. First, consider the point (u_0, J_0) where $J_0 = J_1^S(u_0)$. The solution to the dynamical system starting from (u_0, J_0) is the stable manifold that converges to E_1. This trajectory is an equilibrium because it satisfies the differential equations (12) and (13) in Definition 2, as well as the transversality condition. Second, consider the point (u_0, J_0) where $J_0 = J_3^S(u_0)$. The solution to the dynamical system starting from (u_0, J_0) is the stable manifold that converges to E_3. Clearly, this trajectory is also an equilibrium. Third, consider any point (u_0, J_0), where $J_0 \in (J_3^S(u_0), J_1^S(u_0))$. Given the initial condition (u_0, J_0), the solution to the dynamical system is a trajectory that remains in the shaded area in Figure 3 and converges to either E_2 or to a limit cycle surrounding E_2. Clearly, the trajectory constitutes an equilibrium in either case. Finally, consider any point (u_0, J_0), where $J_0 \notin [J_3^S(u_0), J_1^S(u_0)]$. Given the initial condition (u_0, J_0), the solution to the dynamical system is a trajectory such that the value of the worker to a firm, J_t, diverges to plus or minus infinity. Hence, the trajectory does not constitute an equilibrium because it violates the transversality condition. Overall, for any initial unemployment u_0, there are three types of rational expectations equilibria: the stable manifold converging to E_1, the stable manifold converging to E_3 and a continuum of equilibria starting between the two stable manifolds and converging to either E_2 or to a limit cycle around E_2.

In Appendix D, we characterize the solutions to the dynamical system (12)-(13) and the set of rational expectations equilibria when E_2 is a sink—which is the relevant case for most reasonable parametrizations of the model—and the stable manifolds associated with E_1 and E_3 take on shapes different than in Figure 2. All of these cases share two key features with the one illustrated in Figure 2. First, there are initial unemployment rates

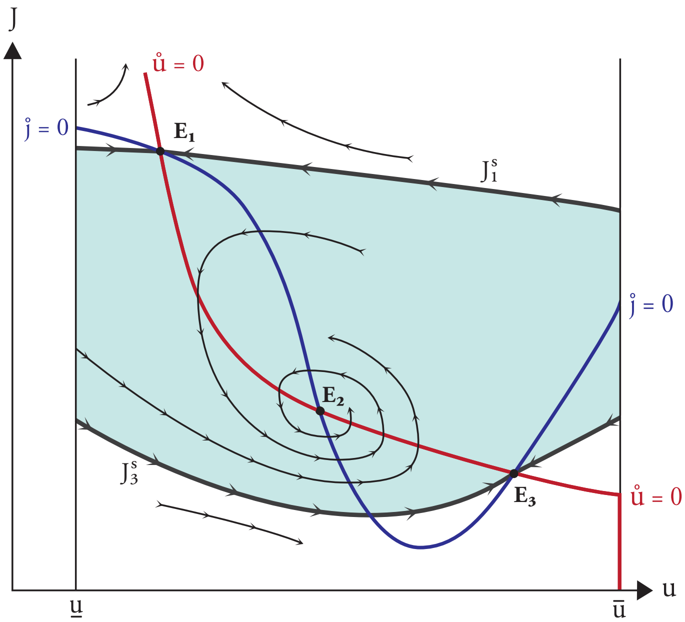


Figure 2: Rational Expectation Equilibria

for which the model admits multiple rational expectations equilibria. These equilibria differ with respect to the agents' expectations about future unemployment. However, in each one of these equilibria, the agents' expectations are correct, in the sense that the realized path of unemployment coincides with the one expected by the agents. The existence of multiple rational expectations equilibria is due to the fact that the expectation of higher unemployment in the future lowers the firm's value of a worker, it lowers the firm's incentives to create vacancies now and, in turn, lower vacancies can fulfill the expectation of higher future unemployment. The existence of multiple rational expectations equilibria implies that the behavior of our model economy is not only determined by fundamentals (e.g., preferences and technology), but also by the agents' expectations about future unemployment.

Second, there are initial unemployment rates for which the model admits rational expectations equilibria that converge to different steady states. The coexistence of equilibria converging to different steady states means that the agents' expectations can be so important as to determine the long-run outcomes of the economy, and not simply the path that the economy follows in the short run. This is a key feature of our model. In our model—like in any textbook search models of the labor market—the transition of the economy towards the steady state is rapid because, empirically, workers move rather quickly in and out of unemployment. Therefore, changes in the agents' expectations could not possibly have a significant impact on labor market outcomes if these expectations only affected the transition path of the economy towards the steady state, and not the steady state towards which the economy is headed.

4 Calibration and Validation

In this section, we calibrate the parameters of the model so as to match US data on the difference in shopping behavior between employed and unemployed workers, on the rate at which workers transit between employment and unemployment, and on the distribution of prices at which identical goods trade in the same market and in the same period of time. We find that the calibrated model admits multiple steady states. Moreover, we find that, for any initial condition, the calibrated model admits multiple rational expectations equilibria and that some of these equilibria converge to different steady states. Multiplicity obtains because unemployment has a very strong negative effect on the revenues generated by a firm-worker pair and, hence, on the firms' incentives to hire. In turn, unemployment has a strong negative effect on the revenues generated by a firm-worker pair not so much because unemployed buyers spend less than employed buyers, but mainly because unemployed buyers spend more time searching for low prices and, in doing so, they increase the competitiveness of the product market. Finally, we show empirical evidence of the effect that suggests that, indeed, when the unemployment rate is higher product markets are more competitive.

4.1 Calibration Strategy

Preferences are described by the discount factor, ρ, and by the periodical utility function of workers, $u_w(x,y) = x^{\alpha}y^{1-\alpha}$, and firms, $u_f(x,y) = y$. The technology operated by firm-worker pairs is described by the productivity of labor measured in units of the AD good, y_e, and by the implicit rate of transformation of the AD good in the BJ good, c. The technology operated by workers is described by the value of home production/unemployment benefits measured in units of the AD good, y_u, and by the rate of transformation of the AD good in the BJ good, r. Search and bargaining in the MP market are described by the vacancy cost, k, by the matching function M, which we assume to be of the CES form $M(u,v) = uv(u^{\phi}+v^{\phi})^{-1/\phi}$, by the worker's bargaining power, , and by the job-loss rate, δ. The search frictions in the BJ market are described by the probability that an unemployed worker searches twice, $_u$, by the probability that an employed worker searches twice, $_e$, and by the matching function N, which we assume to be of the form $N(b,s) = \min\{b,s\}$.

The model is described by a total of 12 parameters. The value of these parameters is chosen so that the model, evaluated at the steady-state with the lowest unemployment, matches features of the US economy over the period 1987-2007. We choose the vacancy cost, k, and the job-loss rate, δ, so that the model matches the empirical average of the worker's monthly transition rate between unemployment and employment (the UE rate) and between employment and unemployment (the EU rate). We choose the parameter ϕ in the MP matching function so that the model matches the empirical elasticity of the UE rate with respect to the vacancy to unemployment ratio. This part of our calibration strategy is standard (see, e.g., Shimer 2005).

The calibration strategy for the remaining 9 parameters is novel and we shall describe it in greater detail. We normalize labor productivity, y_e, to 1 and we choose y_u so that the model matches the empirical ratio between the expenditures of unemployed workers and the expenditures of employed workers. This is an appropriate calibration target because, in the model, the ratio of expenditures for unemployed and employed workers, y_u/w, is increasing in y_u. We assume that the rate of transformation between the AD and the BJ good in the market, c, equals one and we choose the rate of transformation at home, r, so that the model matches the average ratio between the highest and the lowest price at which identical goods trade in the same geographical market and time period. This is an appropriate target because, in the model, the ratio of the highest to the lowest price increases with r. The profit rate enjoyed by a firm, $(1 \square \;)(S^*(u) + y_e \square y_u)/(S^*(u) + y_e)$, is decreasing in the worker's bargaining power . Thus, it is natural to choose so as to match an empirical measure of profits.

We choose the search intensities $_u$ and $_e$ so that the model matches the empirical ratio between the amount of time spent shopping by employed and unemployed workers, as well as the empirical ratio between the prices paid by employed and unemployed workers. These are natural targets. Under the assumption that the average number of searches is proportional to the time spent shopping, one can recover $_u/ _e$ from the amount of time spent shopping by employed and unemployed workers. Then, one can recover $_e$ from the price paid by employed and unemployed workers because the return of $_u \square _e$ additional searches (measured by the decline in the average price paid) is decreasing in $_e$. Finally, we choose alpha so that the model matches the average standard deviation of prices for identical goods. Intuitively, alpha determines the size of the BJ market (where there is price dispersion) and the size of the AD market (where there is no price dispersion) and, hence, it determines the average dispersion of prices.

4.2 Data Sources and Target Values

Table 1 summarizes the value of the empirical targets used to calibrate the parameters of the model. We construct empirical measures of the workers' transition rates between employment

Table 1: Calibration targets

Product Market Targets		Labor Market Targets	
Expenditures of U relative to E	0.85	Monthly transition rate, UE	0.433
Shopping time of U relative to E	1.25	Monthly transition rate, EU	0.024
St dev log prices	0.15	Elasticity UE wrt tightness	0.650
Max-min ratio	1.70	Other Targets	
Price paid by U relative to E	0.98	Profit margin	0.05
		Real annual interest rate	0.035

and unemployment following the methodology developed by Shimer (2005). Over the period 1987-2007, we find that the average monthly UE rate was 43% per month and the average monthly EU rate was 2.4%. We also find that the elasticity of the UE rate with respect to the vacancy-to-unemployment rate was approximately 25%. However, as discussed in Petrongolo and Pissarides (2000) and Menzio and Shi (2011), this is a biased estimate of the elasticity of the UE rate with respect to labor market tightness because, in reality, both employed and unemployed workers search for jobs. Hence, we choose to target an elasticity of the UE rate with respect to theta of 65%, which is the value estimated by Menzio and Shi (2011) after accounting for search on the job. In Appendix E, we consider alternative calibrations where the targeted elasticity varies between 45 and 70%.

We choose the target for the difference in expenditures between employed and unemployed workers by looking at several of the existing estimates of the decline in expenditures experienced by households who transit from employment to unemployment. Bentolila and Ichino (2008) use the PSID to estimate the effect of moving into unemployment on a household's food expenditures. They find that a year of unemployment leads to a 19% decline in food expenditures. Restricting attention to households who move into unemployment because of either business closures or mass layoffs, Stephens (2001) finds that a year of unemployment leads to a 14% decline in food expenditures. Stephens (2004) obtains similar findings using data from the Health and Retirement Survey (HRS) in addition to the PSID.[13] Based on these estimates, we target a 15% difference in expenditures between employed and unemployed workers. In Appendix E, we vary this target between 10 and 40%.

We choose the target for the difference in shopping time between employed and unemployed workers using cross-sectional data from the American Time Use Survey (ATUS). Restricting attention to individuals aged 22-55, we find that employed people spend between 24 and 33% less time shopping than non-employed people, and between 13 and 20 % less

[13] The elasticity of food expenditures with respect to income is likely to be low compared to other expenditures categories, such as luxury goods or semi-durable goods. Therefore, the estimated effect of moving into unemployment on food expenditures is likely to be low compared with the effect on overall expenditures.

than unemployed people.[14] Krueger and Mueller (2010) also measure differences in shopping time between workers in different employment states. They find a difference in shopping time between employed and unemployed individuals of 29% in the US, 67% in Canada and Western Europe and 56% in Eastern Europe. On the basis of these findings, we target a 25% difference in shopping time between employed and unemployed workers. In Appendix E, we let this target vary between 10 and 40%.

We use the Kielts-Nielsen Consumer Panel Data (KNCDP) to measure the dispersion of prices for the same good, in the same market and at the same time.[15] We consider different definitions of a good. In our narrowest definition, we group together all products that have different barcodes, but are identical along all other dimensions (i.e. brand, size, color, shape, packaging, etc…). In our intermediate definition of a good, we group together all products that are identical except for their barcode and brand. In our broadest definition of a good, we group together all products that are identical except for their barcode, brand and size. We define a market as Scantrack Market Area, which is the notion of market used by Nielsen. We define a time period as a quarter. Then, for each triple of good, market and time, we compute the distribution of prices (expressed as ratios of the average price), the standard deviation of prices and various percentile ranges. We find that the average standard deviation of prices ranges between 19 and 36% and that the average 90-to-10 percentile ratio ranges between 1.7 and 2.6, depending on the definition of a good. Considering that some price dispersion is likely to be caused by factors that are not in our model (e.g., measurement error and seller's heterogeneity), we decided to target a standard deviation of prices of 15%, and a max-to-min price ratio of 1.7. In Appendix E, we consider alternative parametrizations.

We follow the methodology developed by Aguiar and Hurst (2007) to measure the difference in prices paid by employed and non-employed households. We restrict attention to households where the head of the family is aged 25 to 55. For each household in our sample, we construct a price index that is defined as the ratio of the household's actual expenditures to the counterfactual expenditures that the household would have incurred if it had purchased goods at their average market price. We then regress the household's price index on the household's employment status and on a number of other household's characteristics.

[14]The estimation results vary depending on the definition of shopping time. We consider a broad definition of shopping time which includes time spent purchasing all goods and services plus related travel time, and a narrow definition of shopping time which includes time spent purchasing consumer goods and groceries plus related travel time. All details are available upon request.

[15]The KNCPD is a panel dataset covering approximately 50,000 households over the period 2004-2009. Respondents use in-home UPC scanning devices to record information (price, quantity, outlet, etc…) about their purchases of grocery and non-grocery household items, which account for roughly 30 percent of total expenditures. This data is similar, although much broader in scope to that used by Aguiar and Hurst (2007) in their analysis of the shopping behavior of retired households. The reader can find all details about the data and the measurement of price dispersion in Kaplan and Menzio (2013).

Table 2: Calibrated parameters

Preference Parameters			Labor Market Parameters		
ρ	Discount factor	0.003	κ	Vacancy cost	8.02
α	BJ exponent in utility	1.00	δ	Exogenous destruction rate	0.024
Technology Parameters			ϕ	MP matching function parameter	1.24
y_e	Market production, AD goods	1.00		Workers' bargaining power	0.74
y_u	Home production, AD goods	4.91	Shopping Parameters		
c	Market transformation, AD to BJ	1.00	$_u$	Prob unemployed search twice	0.27
r	Home transformation, AD to BJ	15.7	$_e$	Prob employed search twice	0.02

We find that the presence of an additional non-employed household head leads to a decline in the price index ranging between 1 and 3.5%, depending on the definition of a good. Moreover, we find that at most one third of the effect can be accounted for by stores' fixed effects. Based on these findings, we choose to target a difference in prices paid by employed and unemployed workers of 2%.

4.3 Properties of the Calibrated Model

Table 2 reports the calibrated parameter values. Given these values, we find that the model admits three steady states. At the first steady state, the unemployment rate is 5.3% and the value of a worker to a firm is 11.3. At the intermediate steady state, the unemployment rate is 8.1% and the value of a worker to a firm is 9.5. At the last steady state, there is no economic activity as the unemployment rate is 100%. Moreover, we find that the set of rational expectations equilibria is as in Figure 2. For any initial unemployment rate, there is a rational expectations equilibrium converging to the low unemployment steady state, one converging to the high unemployment steady state and a continuum of equilibria converging to the intermediate steady state.

The calibrated model admits multiple steady states—and, consequently, multiple rational expectations equilibria—because unemployment has a sufficiently strong negative effect on the revenues generated by a firm-worker pair and, in turn, on the value of a worker to a firm. To understand why this is the case, it is useful to decompose the effect of unemployment on the revenues of a seller according to

$$S^{*\prime}(u) = \frac{ME(u) + DE(u) + TE(u)}{1 \square MLT(u)}. \tag{22}$$

At the low unemployment steady state, the demand effect DE, which measures the decline in seller's revenues due to the fact that unemployed buyers spend less than employed buyers, is □16% of $y_e + S^*(u)$. The market power effect ME, which measures the decline in seller's revenues due to the fact that unemployed buyers search more and, hence, are less likely to be captive, is -33% of $y_e + S^*(u)$. The tightness effect TE, which measures the increase in seller's revenues due to the fact that higher unemployment increases the tightness of the product market, is 6% of $y_e + S^*(u)$. The multiplier $1/(1 - MLT(u))$, which reflects the feedback between revenues and wages, is 2.7. Overall, the semi-elasticity of the seller's revenues with respect to the unemployment rate is -1.15% and the semi-elasticity of the value of a worker to a firm is -6%. Thus, if we increase unemployment from its lowest steady-state value of 5.3% to, say, 10%, the stationary value of a worker to a firm declines by approximately 28%. Since this is larger than the decline required to support a stationary unemployment of 10%, a second steady state obtains.

The main channel through which unemployment affects the seller's revenues and, hence, the value of a worker to a firm is the market power effect, not the demand effect. Indeed, the demand effect is proportional to the difference between the expenditures of employed and unemployed workers, which is calibrated to be 15% of y_u. The market power effect is proportional to the difference between the probability that an employed and an unemployed buyers are captive times the expenditures of an unemployed buyer. Since the search intensity of employed workers, $1 + \ _e$, is 1.02 and the search intensity of unemployed workers, $1 + \ _u$, is calibrated to be 25% larger, the difference in the probability that an employed and an unemployed buyers are captive is 36%. Thus, the market power effect is proportional to 36% of y_u, which is more than twice as large as the demand effect. If we were to abstract from the difference in search intensity between employed and unemployed buyers, the model would not feature multiple equilibria, as the empirical difference in expenditures between employed and unemployed buyers is only 15%. When we take into account the difference in search intensity, the model generates multiple equilibria because it behaves like a model where employed and unemployed buyers only differ in expenditures, but where the difference in expenditures is 51%!

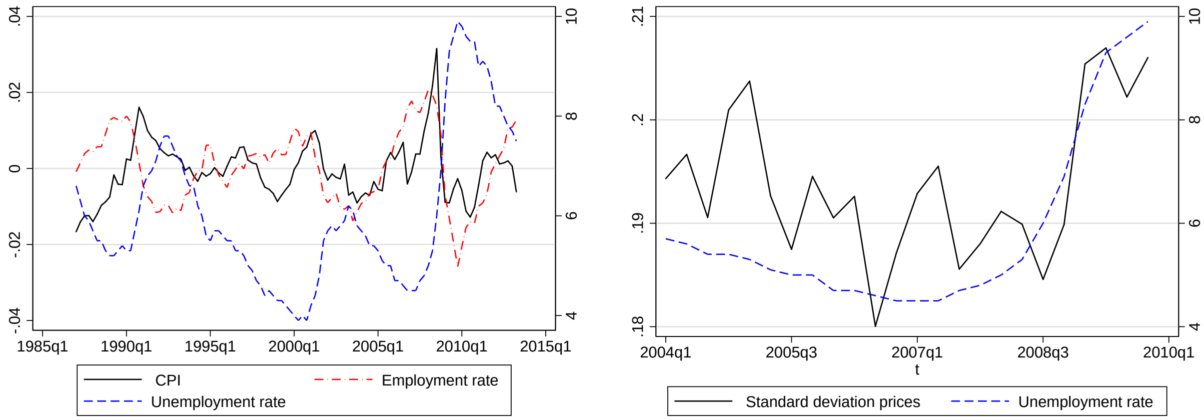
The market power effect captures the effect of unemployment on the competitiveness of the product market. For an individual seller posting the reservation price r, the market power effect manifests itself in a decline in the probability of meeting a captive buyer and, hence, in the probability of making a sale. For sellers as a whole, the market power effect manifests itself as a decline in the equilibrium distribution of prices. Therefore, one may wonder how a large market power effect can be consistent with the observation—matched by the calibrated model—that unemployed buyers pay only 2% less than employed buyers. In order to answer this question, it is helpful to take a detour in auction theory.

Consider a *first-price procurement auction with symmetric information*. In this auction, a seller submits a bid to a buyer knowing the number of sellers who are bidding for the buyer. The equilibrium in this auction is such that the seller bids the reservation price r if the buyer is in contact with no other seller (i.e. captive), and it bids the opportunity cost c if the buyer is in contact with other sellers (i.e. non-captive). Since unemployed buyers are more likely to be in contact with multiple sellers than employed buyers and since the difference between r and c is large (as implied by the empirical observations on price dispersion), an increase in unemployment causes a large decline in the seller's revenues. Note that in this auction, unemployed buyers end up paying much lower prices than employed buyers. The BJ market operates as a *first-price procurement auction with asymmetric information.* In this auction, a seller submits a bid to a buyer without knowing the number of sellers who are in contact with the buyer. The expected revenues for a seller in the asymmetric information auction are the same as in the symmetric information auction and, hence, they are very sensitive to changes in unemployment. But equilibrium prices in the asymmetric information auction are different than in the symmetric information one, as sellers cannot discriminate between captive and non-captive buyers. In particular, in the asymmetric information auction, part of the gains from trade that are extracted by non-captive buyers are "stolen" as informational rents by captive buyers. For this reason, although employed buyers are much less likely to be captive than unemployed buyers, they pay only marginally higher prices.

4.4 Discussion and Validation

In the previous subsection, we showed that the model features multiple rational expectations equilibria when calibrated to match the empirical differences in the shopping behavior of employed and unemployed workers and the empirical rate at which workers transit between employment and unemployment. In this subsection, we want to further convince the reader of the empirical importance of our theory of multiple rational expectation equilibria by providing some direct evidence in support of the two central mechanisms of our theory.

The first mechanism is the one linking aggregate unemployment to competition: when aggregate unemployment is higher, buyers spend more time for searching for low prices (or for cheaper substitutes) and, in doing so, they make the product market more competitive. We would expect this mechanism to be operative in imperfectly competitive markets where the fraction of unemployed buyers is equal to the aggregate unemployment rate. As long as employed workers search more when the value of their time is lower, we would also expect the mechanism to be operative in imperfectly competitive markets where the fraction of unemployed buyers is different from the aggregate unemployment rate. Indeed, when aggregate unemployment is higher, wages are lower, the value of employed workers' time is lower and search intensity should increase even in markets visited exclusively by employed buyers. In contrast, we would not expect the mechanism to be at work in pure monopolies or in perfectly competitive markets.



(a) Unemployment, cyclical employment and CPI

(b) Unemployment and price dispersion

Figure 3: Labor market variables, price level and price dispersion

Since it is difficult to indentify markets that are perfectly competitive, imperfectly competitive and pure monopolies in the data, we shall look for empirical evidence of the mechanism linking unemployment, search and competition at the aggregate level. Indeed, if the mechanism is at work in the aggregate economy, consumer prices should be lower when the unemployment rate is higher. Naturally, the predicted relationship between prices and unemployment may be obscured by other variables (e.g., productivity, money supply, etc...) that change at the same time as the unemployment rate. However, if changes in these other variables are not too large, we should still see a negative correlation between the price level and the unemployment rate. Figure 3(a) plots the cyclical component of the Consumer Price Index (CPI), the unemployment rate and the employment rate over the period 1987-2011. Clearly, the cyclical component of the CPI moves in the opposite direction of the unemployment rate, and in the same direction as the employment rate. These comovements are rather strong during the last 15 years and, in particular, during the Great Recession, a fact that is quite remarkable considering that the Federal Reserve Bank has followed a very expansive monetary policy since 2008.

There is a second testable implication of the mechanism linking unemployment, search and competition. Indeed, if the mechanism is at work in the aggregate economy, the ratio between the highest and the lowest price for identical goods should increase with unemployment, as unemployed buyers search more and induce some sellers to move their prices closer to the opportunity cost of production. Moreover, for our calibrated parameters, the standard deviation of prices for identical goods should increase with unemployment. Again, one should not expect the prediction to hold exactly in the data because the economy is likely to be subject to many other changes besides changes in unemployment. However, if these changes are relatively small, we should still see a positive correlation between price dispersion and the unemployment rate. Figure 3(b) plots the average standard deviation of prices for identical goods in KNCDP and the unemployment rate over the period 2004-2009, which is the period covered by KNCDP. In the period preceding the Great Recession, the unemployment rate slowly declined and so did price dispersion. Between 2008 and 2009, the unemployment rate rapidly rose from 4.5 to 10% and the standard deviation of prices rapidly increased by 15%.

The second key mechanism in our theory is the one linking entry and labor demand: when expected revenues are higher, new firms want to enter the product market, established firms want to scale-up their presence in the product market and, since both activities require some labor input, labor demand increases. The mechanism should be at work in any sector where the extent of a firm's access to buyers depends on labor input. For example, the mechanism should be at work in the retail sector, where firms can reach new buyers by opening new outlets, all requiring some labor input. Similarly, the mechanism should be at work in the manufacturing sector, where firms can reach new buyers by introducing new product lines, all requiring some labor to be managed and run. For the economy as a whole, the mechanism appears to be empirically important. For example, Bilbiie, Ghironi and Melitz (2012) show that the introduction of new products and the entry of new firms account for a sizeable fraction of yearly manufacturing output in the US and that this fraction is strongly procyclical and, hence, positively correlated with employment. Similarly, Hall (2013) shows that advertisement expenditures (which presumably measure an effort to reach new buyers) are procyclical.

5 Expectation Shocks

When calibrated to the US economy, the model features multiple rational expectation equilibria. This suggests that observed fluctuations in the US labor market may be caused not only by changes in preferences, technologies and other fundamentals, but also by non-fundamental, self-fulfilling changes to the agents' expectations about future unemployment. In this section, we explore the qualitative and quantitative effects of an expectation shock. In Section 5.1, we develop a version of the model in which the agents' expectations about long-run unemployment follow a simple 2-state Markov switching process. In Section 5.2, we use the augmented model to measure the effect of a negative shock to the agents' expectations about long-run unemployment. We find that, in response to the shock, the unemployment rate almost doubles, the tightness of the labor market falls by approximately 70%, the equity value of firms declines by almost 30% and that these changes take place without any concurrent decline in the real productivity of labor. We also find that the decline in the equity value of firms leads the rise in unemployment.

5.1 Modeling Expectation Shocks

We consider a version of the model in which the expectations of the agents about long-run unemployment follow a 2-state Markov process. In the optimistic state, G, agents expect the economy to converge to the steady state with the lowest unemployment rate (conditional on remaining in the optimistic state). In the pessimistic state, B, agents expect the economy to converge to the steady state with the intermediate unemployment rate (conditional on remaining in the pessimistic state). The agents' expectations switch from optimistic to pessimistic at the Poisson rate π_{GB}, in which case the value of a worker to a firm jumps by $D_{GB}(u, J)$. Similarly, the agents' expectations switch from pessimistic to optimistic at the Poisson rate π_{BG}, in which case the value of a worker to a firm jumps by $D_{BG}(u, J)$.

In the optimistic state, the evolution of the economy is described by the following pair of differential equations

$$\mathring{u}_t = -u_t \lambda(\theta(J_t)) + (1 - u_t)\delta, \tag{23}$$

$$(\rho + \delta) J_t = (1 - \quad) [S^*(u_t) + y_e - y_u] + \mathring{J}_t + \pi_{GB} D_{GB}(u_t, J_t). \tag{24}$$

The first differential equation describes the evolution of unemployment. The second differential equation describes the evolution of the value of a worker to a firm. The equation states that the annuitized value of a worker is equal to the sum of three terms. The first term is the flow of revenues generated by the firm-worker pair and accruing to the firm. The second term is the change in the value of a worker conditional on the economy remaining in the optimistic state. The last term is the rate at which the economy switches to the pessimistic state, π_{GB}, times the change in the value of a worker conditional on the economy switching states, $D_{GB}(u_t, J_t)$. We denote as $\{E_1^G, E_2^G, E_3^G\}$ the steady states associated with the optimistic state and with $J_{G,1}^S$ the stable manifold associated with the steady state with the lowest unemployment rate (see Figure 4).

In the pessimistic state, the evolution of the economy us described by the following pair of differential equations

$$\mathring{u}_t = -u_t \lambda(\theta(J_t)) + (1 - u_t)\delta, \tag{25}$$

$$(\rho + \delta) J_t = (1 - \quad) [S^*(u_t) + y_e - y_u] + \mathring{J}_t + \pi_{BG} D_{BG}(u_t, J_t). \tag{26}$$

The first differential equation describes the evolution of unemployment, which is the same as in the optimistic state. The second differential equation describes the evolution of the value of a worker to a firm. The first two terms in this equation are the same as in the optimistic

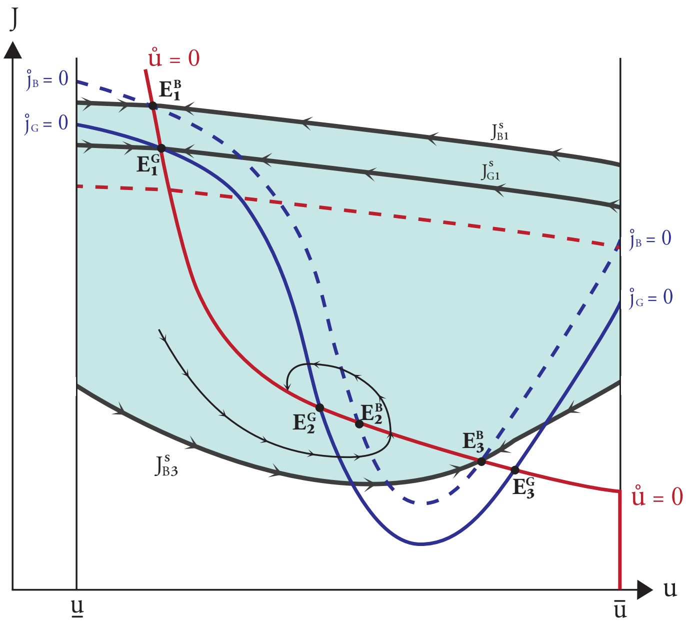


Figure 4: Expectation Shocks

state. The last term is the rate at which the economy switches to the optimistic state, π_{BG}, times the change in the value of a worker conditional on the economy switching states, $D_{BG}(u_t, J_t)$. We denote as $\{E_1^B, E_2^B, E_3^B\}$ the steady states associated with the pessimistic state and with $J^S_{B,2}$ the basin of attraction of the intermediate steady state (see Figure 4).

The expectations of the agents must be rational. First, when the economy switches from the optimistic to the pessimistic state, the value of a worker to a firm must fall in the basin of attraction $J^S_{B,2}$ of the intermediate steady state E_2^B. This condition guarantees that, if the economy remains in the pessimistic state forever after, it will converge to E_2^B. Second, when the economy switches from the pessimistic to the optimistic state, the value of a worker to a firm must land on the stable manifold $J^S_{G,1}$ associated with the low unemployment steady state E_1^G. This condition guarantees that, if the economy remains in the optimistic state forever after, it will converge to E_1^G. Finally, the initial value of a worker to a firm must be on the stable manifold associated with E_1^G if the initial state of the economy is optimistic, and it must belong to the basin of attraction of E_2^B if the initial state of the economy is pessimistic.

We are now in the position to formally define an equilibrium. Let h denote a history of realizations of the 2-state Markov process for the agents' expectations and let $t_n(h)$ the n-th time at which the state of the process switches in history h. Then:

Definition 3. *A Rational Expectation Equilibrium is a history-dependent path* $\{u_t(h), J_t(h)\}$ *such that, for every* h, *the following conditions are satisfied: (i) For all* $t \in [t_n, t_{n+1})$ *with* $h_{t_n} = G$, $\{u_t, J_t\}$ *satisfies (23)-(24); (ii) For all* $t \in [t_n, t_{n+1})$ *with* $h_{t_n} = B$, $\{u_t, J_t\}$ *satisfies (25)-(26); (iii) For any* $u \in [\underline{u}, \overline{u}]$ *and any* $J \in J^S_{G,1}(u)$, $J + D_{GB}(u, J) \in J^S_{B,2}(u)$. *For any* $u \in [\underline{u}, \overline{u}]$ *and any* $J \in J^S_{B,2}(u)$, $J + D_{BG}(u, J) \in J^S_{G,1}(u)$*; (iv)* $J_0 \in J^S_{G,1}(u_0)$ *if* $h_0 = G$, *and* $J_0 \in J^S_{B,2}(u_0)$ *if* $h_0 = B$.

Naturally, we could have specified any number of different stochastic processes for the agents' expectations about future unemployment. However, the one that we chose is the most natural and, arguably, the only one that has the potential to be empirically relevant. Indeed, shocks to the agents' expectations about the path that the economy will follow while converging to a given steady state are unlikely to be quantitatively important, as the transitional dynamics of the model are very fast. Hence, it is natural to focus on shocks to the agents' expectations about the steady state that the economy will reach. Second, notice that, according to our benchmark calibration, the high-unemployment steady state features no economic activity. For other reasonable calibration, the high-unemployment steady state features some economic activity, but still displays an unemployment rate much greater than anything observed in the history of the US economy. Hence, it is natural to focus on a stochastic process where the agent's expect to either reach the steady state with the lowest unemployment rate or the one with the intermediate unemployment rate. Finally, the assumption that the stochastic process is Markovian is made, as usual, for the sake of simplicity.

The stochastic process for the agents' expectations is fully characterized by the four variables $\{\pi_{GB}, \pi_{BG}, D_{GB}, D_{BG}\}$. The arrival rates π_{GB} and π_{BG} determine the average duration of the optimistic state and of the pessimistic state. The jump D_{GB} determines the path that the economy will follow in reaching the pessimistic steady state. The jump D_{BG} is actually not a free variable because there is only one path (i.e. the stable manifold $J^S_{G,1}$) that leads to the optimistic steady state and, hence, $D_{BG}(u, J)$ must be equal to $J^S_{G,1}(u) \square J$. Armed with a version of the model that allows for shocks to technology, preferences and expectations, one could estimate the values for π_{GB}, π_{BG} and D_{GB} using time-series data on unemployment, vacancies, productivity and the stock market. This estimation would be well beyond the scope of this paper. Here we simply take the view that negative expectation shocks are rather rare and persistent events associated with a large decline in the value of firms. In particular, we choose π_{GB} so that a negative expectation shock occurs on average once every 50 years, we choose π_{BG} so that the shock lasts on average 15 years and we choose D_{GB} so that, when the shock hits the economy, the value of the firms falls by 20%.[16]

[16] The response of the economy to a negative expectation shock is not very sensitive to the choice of these

5.2 Response to a Negative Expectation Shock

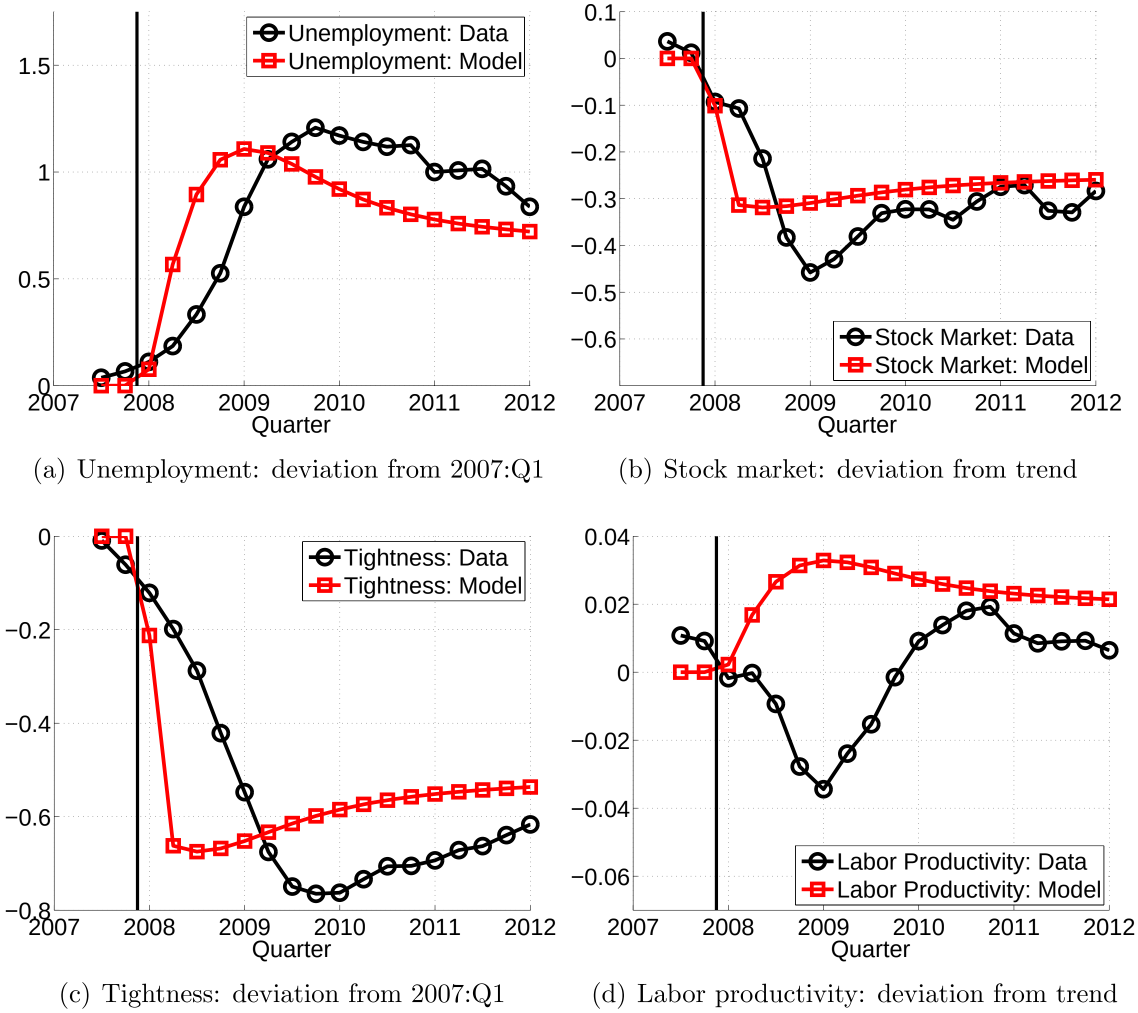
Figure 5 reports the behavior of unemployment, labor market tightness, labor productivity and the value of the firm when the economy is at the optimistic steady state and the agents' expectation about future unemployment become pessimistic. Panel (a) plots the response of the unemployment rate, measured as a percentage deviation from its value at the optimistic steady state. The unemployment rate increases by 120% during the first 6 quarters after the economy is hit by the negative shock. After reaching its peak, the unemployment rate begins a slow decline towards the pessimistic steady-state, where its value is approximately 80% higher than at the optimistic steady state. This illustrates the first key feature of a negative expectation shock: it causes an increase in unemployment that is very large relative to, say, the one caused by a productivity shock (see, e.g., Shimer 2005).

Panel (b) plots the response of the stock market value of an active firm, measured as a percentage deviation from its value at the optimistic steady state. The value of an active firm in the stock market, V, is equal to the value of the firm, J, net of the repayment of firm's debt, D. Under the assumption that D is a constant fraction $d = 1/3$ of the value of the firm at the optimistic steady state, $J^*_{G,1}$, this implies that V is equal to $J \square dJ^*_{G,1}$.[17] As one can see from panel (b), when the economy is hit by the negative shock, the stock market value of a firm falls instantly by 30%. Then, the stock market value of a firm begins a slow and modest rise towards the pessimistic steady-state, where its value is approximately 25% lower than at the optimistic steady state. Figure 6 is a scatter plot of unemployment and the stock market value of a firm. In this picture, one can clearly see the second key feature of a negative expectation shock: the decline in the stock market takes place before the increase in unemployment, as the value responds instantaneously to the change in expectations about future unemployment, revenues and profits, while the expected increase in unemployment takes some time to materialize.

Panel (c) plots the response of labor market tightness (i.e. the vacancy-to-unemployment ratio), measured as a percentage deviation from its value at the optimistic steady state. When the economy is hit by a negative shock, the tightness of the labor market falls by approximately 70%. Then, it begins a slow increase towards the pessimistic steady state, where its value is 60% lower than at the optimistic steady state. The tightness of the labor market behaves exactly like the value of an active firm and, hence, its decline precedes the

particular calibration targets. Indeed, the behavior of the model is driven by changes in the value of an additional worker to a firm. Since worker-firm relationships are on average relatively short (i.e. they last approximately 4 years), it does not matter much whether a switch in expectations occurs on average once every 15, 20 or 50 years.

[17] Over the period 1987-2007, the average ratio of credit market debt to net worth in the non-financial corporate business sector was approximately 50 percent, i.e. $d/(1 \square d) = 1/2$ which implies $d = 1/3$.



(a) Unemployment: deviation from 2007:Q1 (b) Stock market: deviation from trend

(c) Tightness: deviation from 2007:Q1 (d) Labor productivity: deviation from trend

Figure 5: Response to a negative expectation shock

Figure 6: Unemployment rate and stock market

increase in unemployment. This finding is not surprising as the tightness of the labor market is determined by the value of an active firm through the free entry condition.

Panel (d) plots the response of real labor productivity, measured as a percentage deviation from its value at the optimistic steady state. We define real labor productivity as the ratio between nominal labor productivity and a consumer price index. Nominal labor productivity is given by the revenues generated by each employed worker $S^*(u) + y_e$. The consumer price index is given by $Q^*_{BJ}P_{BJ}(u) + Q^*_{AD}P_{AD}(u)$, where Q^*_{BJ} and Q^*_{AD} are the quantities of the BJ and AD goods sold at the optimistic steady state and $P_{BJ}(u)$ and $P_{AD}(u)$ are the average prices at which the BJ and AD goods are sold when the unemployment rate is u. As one can see from panel (d), the response of real labor productivity to a negative expectation shock is very small compared to the response of the other variables. This finding is surprising but easy to understand. Nominal labor productivity (the seller's revenues) declines substantially in response to the negative expectation shock. However, the decline in nominal labor productivity takes place mostly because of a decline in prices and not because of a decline in quantities. Therefore, when we deflate nominal labor productivity, we find that real labor productivity does not change much and, indeed, the small change in real labor productivity is positive. This is the third key feature of a negative expectation shock: the large increase in unemployment and the large decline in the value of a firm take place while real labor productivity remains approximately constant.

During the Great Recession and its aftermath, the US economy displayed the same features that characterize the response to a negative expectation shock. First, the unemployment rate more than doubled between the last quarter of 2007 and the second quarter of 2009 and, since then, it has remained at a level substantially higher than before the beginning of the recession. Second, the stock market fell 40% below trend during 2008 and its decline preceded the bulk of the increase in the unemployment rate. Third, real labor productivity only fell below trend by a few percentage points during the recession and was already back on trend by the second quarter of 2009, a time when the unemployment rate was still twice as high as before the recession. Fourth, as discussed in Section 4, prices fell and price dispersion increased during the Great Recession. Figure 6 overlays the behavior of the US economy from 2007 to 2012 and the response of the model economy to a negative expectation shock. From this figure, it is clear that the behavior of the US economy during this period is not only qualitatively but also quantitatively rather similar to the response to a negative expectation shock.

The above observations suggest the possibility that the fundamental shocks to the financial and housing sectors that have ushered in and, presumably, started the Great Recession may have affected the economy not only directly, but also indirectly by triggering a non-fundamental change in the agents' expectations about long-run unemployment. This hypothesis would explain why labor market variables responded so strongly to fundamental shocks that had a rather small impact on labor productivity. Moreover, this hypothesis would explain why the labor market is still sluggish several years after any effect of the fundamental shocks on labor productivity has disappeared.

6 Conclusions

The paper advances a novel theory of self-fulfilling unemployment fluctuations. According to our theory, when unemployment is higher, firms' revenues are lower because unemployed buyers spend less income and devote more time looking for low prices than employed buyers. Conversely, when revenues are lower, new firms are less inclined to enter the product market and existing firms are less inclined to expand their presence in the product market and, hence, labor demand is lower and fewer vacancies are created. In the theoretical part of the paper, we prove that the feedback loop between unemployment, prices and demand can generate multiple rational expectations equilibria if the differences between employed and unemployed buyers in either expenditures or shopping time are sufficiently large. In the empirical part of the paper, we show that the observed differences in the shopping behavior of employed and unemployed buyers are indeed strong enough to generate multiplicity. Surprisingly, we find that the firms' revenues are very sensitive to the unemployment rate mainly because of the difference in the amount of time spent shopping by employed and unemployed buyers and not so much because of the difference in their expenditures. In the last part of the paper, we show that, due to the presence of multiple equilibria, the economy may fluctuate not only because of shocks to technology, preferences or other fundamentals, but also because of self-fulfilling shocks to the agents' expectations about future unemployment. We find that the response of the economy to a negative expectation shock about long-run unemployment has three distinctive features: (i) a large and persistent increase in unemployment, (ii) a large decline in the equity value of firms, which precedes the increase in unemployment, and (iii) relatively small fluctuations in the real productivity of labor.

Appendix

A Proof of Lemma 1

Let F denote an arbitrary distribution of prices for the BJ good. Let $\mathcal{F}$ denote the support of the distribution F and let $\xi(p)$ denote the measure of prices equal to p in the distribution F. Given F, the expected revenues for a seller are given by $S(p) + y_e$, where p denotes the price of the BJ good posted by the seller, $S(p)$ denotes the revenues from sales in the BJ market net of the opportunity cost of producing the BJ good, and y_e are the revenues from sales in the AD market plus the opprtunity cost of producing the BJ good. Formally, the seller's net revenues in the BJ market are

$$\begin{aligned} S(p) \quad &= \mu(\sigma)\frac{u(1+ \ {}_u)}{b}\left[1 \square \frac{2 \ {}_u\nu(\sigma)\left[F(p) \square \xi(p)/2\right]}{1+ \ {}_u}\right]\frac{\alpha y_u(p \square c)}{p} \\ &+\mu(\sigma)\frac{(1 \square u)(1+ \ {}_e)}{b}\left[1 \square \frac{2 \ {}_e\nu(\sigma)\left[F(p) \square \xi(p)/2\right]}{1+ \ {}_e}\right]\frac{\alpha w(p \square c)}{p}, \end{aligned} \tag{A1}$$

Note that (A1) assumes that a buyer meeting two sellers charging the price p purchases from either seller with probability $1/2$. However, Lemma 1 holds for any other tie-breaking rule.

A seller chooses the price p for the BJ good so as to maximize its expected revenues, $S(p) + y_e$, or, equivalently, to maximize its net revenues in the BJ market, $S(p)$. Therefore, the price distribution F is consistent with the seller's optimal pricing strategy if and only if

$$\begin{aligned} &S(p_0) = S^* \text{ for all } p_0 \in \mathcal{F} \\ &S^* \equiv \max_p S(p). \end{aligned} \tag{A2}$$

Claim 1. For any F, $S^* > 0$.

Proof. For a seller posting the reservation price r, net revenues are

$$\begin{aligned} S(r) \quad &\geq \mu(\sigma)\frac{u(1+ \ {}_u)}{b}\frac{1 \square \ {}_u}{1+ \ {}_u}\frac{\alpha y_u(r \square c)}{r}+ \\ &+\mu(\sigma)\frac{(1 \square u)(1+ \ {}_e)}{b}\frac{1 \square \ {}_e}{1+ \ {}_e}\frac{\alpha w(r \square c)}{r} > 0. \end{aligned}$$

As $S^* \geq S(r)$ and $S(r) > 0$, it follows that $S^* > 0$. ■

Claim 2. If F satisfies (A2), then F is continuous.

Proof. Suppose that there is a price p_0 such that $\xi(p_0) > 0$. For a seller posting the price p_0, net revenues are

$$
\begin{aligned}
S(p_0) \quad &= \mu(\sigma)\frac{u(1+\ {}_u)}{b}\left[1 \square \frac{2\ {}_u\nu(\sigma)\left[F(p_0) \square \xi(p_0)/2\right]}{1+\ {}_u}\right]\frac{\alpha y_u(p_0 \square c)}{p_0} \\
&+\mu(\sigma)\frac{(1 \square u)(1+\ {}_e)}{b}\left[1 \square \frac{2\ {}_e\nu(\sigma)\left[F(p_0) \square \xi(p_0)/2\right]}{1+\ {}_e}\right]\frac{\alpha w(p_0 \square c)}{p_0}.
\end{aligned}
$$

For a seller posting the price $p_0 \square \epsilon$, with $\epsilon > 0$, net revenues are

$$
\begin{aligned}
&S(p_0 \square \epsilon) \\
= \quad &\mu(\sigma)\frac{u(1+\ {}_u)}{b}\left[1 \square \frac{2\ {}_u\nu(\sigma)\left[F(p_0 \square \epsilon) \square \xi(p_0 \square \epsilon)/2\right]}{1+\ {}_u}\right]\frac{\alpha y_u(p_0 \square \epsilon \square c)}{p_0 \square \epsilon} \\
&+\mu(\sigma)\frac{(1 \square u)(1+\ {}_e)}{b}\left[1 \square \frac{2\ {}_e\nu(\sigma)\left[F(p_0 \square \epsilon) \square \xi(p_0 \square \epsilon)/2\right]}{1+\ {}_e}\right]\frac{\alpha w(p_0 \square \epsilon \square c)}{p_0 \square \epsilon} \\
> \quad &\mu(\sigma)\frac{u(1+\ {}_u)}{b}\left[1 \square \frac{2\ {}_u\nu(\sigma)\left[F(p_0) \square \xi(p_0)\right]}{1+\ {}_u}\right]\frac{\alpha y_u(p_0 \square \epsilon \square c)}{p_0 \square \epsilon} \\
&+\mu(\sigma)\frac{(1 \square u)(1+\ {}_e)}{b}\left[1 \square \frac{2\ {}_e\nu(\sigma)\left[F(p_0) \square \xi(p_0)\right]}{1+\ {}_e}\right]\frac{\alpha w(p_0 \square \epsilon \square c)}{p_0 \square \epsilon},
\end{aligned}
$$

where the above inequality follows from $F(p_0 \square \epsilon) \square \xi(p_0 \square \epsilon)/2 \leq F(p_0 \square \epsilon)$ and $F(p_0 \square \epsilon) < F(p_0) \square \xi(p_0)/2$. Since $p_0 \in \mathcal{F}$, (A2) implies $S(p_0) = S^* > 0$ and, hence, $p_0 > c$. In turn, if $p_0 > c$, there exists an ϵ small enough that $S(p_0) < S(p_0 \square \epsilon)$. As $S(p_0 \square \epsilon) \leq S^*$ and $S(p_0) < S(p_0 \square \epsilon)$, $S(p_0) < S^*$, which contradicts the hypothesis that F satisfies (A2). ■

Claim 3. If F satisfies (4), then $\overline{p} = r$.

Proof. Suppose that $\overline{p} < r$. For a seller posting the price $\overline{p}$, net revenues are

$$
\begin{aligned}
S(\overline{p}) \quad &= \mu(\sigma)\frac{u(1+\ {}_u)}{b}\left[1 \square \frac{2\ {}_u\nu(\sigma)}{1+\ {}_u}\right]\frac{\alpha y_u(\overline{p} \square c)}{\overline{p}} \\
&+\mu(\sigma)\frac{(1 \square u)(1+\ {}_e)}{b}\left[1 \square \frac{2\ {}_e\nu(\sigma)}{1+\ {}_e}\right]\frac{\alpha w(\overline{p} \square c)}{\overline{p}}.
\end{aligned}
$$

For a seller posting the price r, net revenues are

$$
\begin{aligned}
S(r) \quad &= \mu(\sigma)\frac{u(1+\ {}_u)}{b}\left[1 \square \frac{2\ {}_u\nu(\sigma)}{1+\ {}_u}\right]\frac{\alpha y_u(r \square c)}{r} \\
&+\mu(\sigma)\frac{(1 \square u)(1+\ {}_e)}{b}\left[1 \square \frac{2\ {}_e\nu(\sigma)}{1+\ {}_e}\right]\frac{\alpha w(r \square c)}{r}.
\end{aligned}
$$

Clearly $S(\overline{p}) < S(r) \leq S^*$, which contradicts the hypothesis that F satisfies (A2). Next, suppose that $\overline{p} > r$. In this case, the net revenues for a seller posting the price $\overline{p}$ are $S(\overline{p}) = 0$. The net revenues for a seller posting the price r are $S(r) > 0$. Hence $S(\overline{p}) < S(r) \leq S^*$, which contradicts the hypothesis that F satisfies (A2). Having ruled out $\overline{p} < r$ and $\overline{p} > r$, it follows that $\overline{p} = r$. ■

Claim 4. If F satisfies (A2), then $\mathcal{F}$ is connected.

Proof. Suppose that $\mathcal{F}$ is not connected, i.e. there exist p_0, $p_1 \in \mathcal{F}$ such that $p_0 < p_1$ and $F(p_0) = F(p_1)$. For a seller posting the price p_0, net revenues are

$$\begin{aligned} S(p_0) \quad &= \mu(\sigma)\frac{u(1+\ {}_u)}{b}\left[1 \square \frac{2\ {}_u\nu(\sigma)F(p_0)}{1+\ {}_u}\right]\frac{\alpha y_u(p_0 \square c)}{p_0} \\ &+\mu(\sigma)\frac{(1 \square u)(1+\ {}_e)}{b}\left[1 \square \frac{2\ {}_e\nu(\sigma)F(p_0)}{1+\ {}_e}\right]\frac{\alpha w(p_0 \square c)}{p_0}. \end{aligned}$$

For a seller posting the price p_1, net revenues are

$$\begin{aligned} S(p_1) \quad &= \mu(\sigma)\frac{u(1+\ {}_u)}{b}\left[1 \square \frac{2\ {}_u\nu(\sigma)F(p_0)}{1+\ {}_u}\right]\frac{\alpha y_u(p_1 \square c)}{p_1} \\ &+\mu(\sigma)\frac{(1 \square u)(1+\ {}_e)}{b}\left[1 \square \frac{2\ {}_e\nu(\sigma)F(p_0)}{1+\ {}_e}\right]\frac{\alpha w(p_1 \square c)}{p_1}. \end{aligned}$$

Clearly $S(p_0) < S(p_1) \leq S^*$, which contradicts the hypothesis that F satisfies (A2). ■

Claim 5. F satisfies (A2) if and only if

$$\begin{aligned} F(p) = \quad & \left\{u(1+\ {}_u)\left[1 \square \left(1 \square \frac{2\ {}_u\nu(\sigma)}{1+\ {}_u}\right)\frac{(r \square c)p}{(p \square c)r}\right]y_u\right. \\ & \left.+(1 \square u)(1+\ {}_e)\left[1 \square \left(1 \square \frac{2\ {}_e\nu(\sigma)}{1+\ {}_e}\right)\frac{(r \square c)p}{(p \square c)r}\right]w\right\}\Big/ \\ & 2\nu(\sigma)\left\{u\ {}_u y_u + (1 \square u)\ {}_e w\right\} \end{aligned} \tag{A3}$$

Proof. First, suppose F satisfies (A2). Claim 3 implies that $S(r) = S^*$ and claim 4 implies that $S(p) = S^*$ for all $p \in [\underline{p}, r]$, with $\underline{p} \in (c, r)$. Thus, $S(p) = S(r)$ for all $p \in [\underline{p}, r]$. The solution to the equation $S(p) = S(r)$ with respect to $F(p)$ is (A3). Conversely, suppose that F is given by (A3) for all $p \in [\underline{p}, \overline{p}]$, with $\overline{p} = r$ and $\underline{p}$ such that $F(\underline{p}) = 0$. Given F, it is easy to verify that $S(p) = S^* > 0$ for all $p \in [\underline{p}, \overline{p}]$, $S(p) < S^*$ for all $p \notin [\underline{p}, \overline{p}]$, where $S^* \equiv S(r)$ Hence, F satisfies (A2). ■

B Continuous Time Limit

Let $\Delta \in (0, 1]$ denote the lenght of a period. A worker has preferences described by the utility function $\sum_{i=0}^{\infty} e^{\square\rho\Delta i}x^{\alpha}(\Delta i)^{\alpha}y(\Delta i)^{1\square\alpha}$, where $x(\Delta i)$ denotes consumption of the BJ good and $y(\Delta i)$ denotes consumption of the AD good in period i. In period i, a worker has an income of $y_u\Delta$ units of the AD good if he is unemployed and an income of $w_\Delta(u(\Delta i))$ units of the AD good if he is employed, where $u(\Delta i)$ denotes the unemployment rate. A firm has preferences described by the utility function $\sum_{i=0}^{\infty} e^{\square\rho\Delta i}y(\Delta i) \square k\Delta v(\Delta i)$, where $y(\Delta i)$ denotes consumption of the AD good and $v(\Delta i)$ denotes the number of vacancies created in the MP market. In period i, every worker employed by the firm is able to produce any combination of AD and BJ goods such that $cx + y = y_e\Delta$.

Consider the economy at some arbitrary date $t = i\Delta$. In the MP market, unemployed workers, $u(t - \Delta)$, and vacant jobs, $v(t)$, create $M(u(t - \Delta), v(t))\Delta$ matches. Hence, an unemployed worker meets a vacant job with probability $\lambda(\theta(t))\Delta$ and a vacant job meets an unemployed worker with probability $\eta(\theta(t))\Delta$, where $\theta(t) = v(t)/u(t - \Delta)$. In the MP market, existing firm-worker matches are destroyed with an exogenous probability $1 - e^{-\delta\Delta}$. In the BJ market, an unemployed buyer searches for a seller once with probability $1 - \;{}_u$ and twice with probability ${}_u$. Similarly, an employed buyer searches once with probability $1 - \;{}_e$ and twice with probability ${}_e$. Buyer's searches, $b(u(t)) = u(t)(1 + \;{}_u) + (1 - u(t))(1 + \;{}_e)$, and sellers, $s(u(t)) = 1 - u(t)$, create $N(b(u(t)), s(u(t)))$ matches. Hence, a buyer's search matches with a seller with probability $\nu(\sigma(t))$ and a seller matches with a buyer with probability $\mu(\sigma(t))$, where $\sigma(t) = s(t)/b(t)$.

It is straightforward to verify that the seller's net revenues in the BJ market are

$$
\begin{aligned}
S^*_\Delta(u(t)) \quad &= \mu(\sigma(u(t)))\frac{u(1 + \;{}_u)}{b(u(t))}\left[1 - \frac{2\;{}_u\nu(\sigma(u(t)))}{1 + \;{}_u}\right]\frac{(r - c)}{r}\alpha y_u\Delta \\
&+\mu(\sigma(u(t)))\frac{(1 - u(t))(1 + \;{}_e)}{b(u(t))}\left[1 - \frac{2\;{}_e\nu(\sigma(u(t)))}{1 + \;{}_e}\right]\frac{(r - c)}{r}\alpha w_\Delta(u(t)),
\end{aligned} \tag{B1}
$$

where

$$
w_\Delta(u(t)) = y_u\Delta + \quad \left[S^*_\Delta(u(t)) + y_e\Delta - y_u\Delta\right]. \tag{B2}
$$

Similarly, it is straightforward to verify that the tightness $\theta(t)$ of the MP market is

$$
\theta_\Delta(J) = \eta^{-1}\left(\min\left\{\frac{k\Delta}{J\Delta}, 1\right\}\right). \tag{B3}
$$

The Bellman Equation for the value of an additional worker to a firm is

$$
\begin{aligned}
J(t) \quad &= (1 - \quad)\left(S^*(u(t), \Delta) + y_e\Delta - y_u\Delta\right) + e^{-(\rho+\delta)\Delta}J(t + \Delta) \\
&= (1 - \quad)\left(S^*(u(t)) + y_e - y_u\right)\Delta + e^{-(\rho+\delta)\Delta}J(t + \Delta),
\end{aligned} \tag{B4}
$$

where the second line makes use of the fact that $S^*_\Delta(u(t)) = S^*(u(t))$ and $w_\Delta(u(t)) = \Delta w(u(t))$, with $S^*(u)$ and $w^*(u)$ defined as in (6) and (7). The law of motion for unemployment is

$$
\begin{aligned}
u(t) \quad &= u(t - \Delta)(1 - \lambda(\theta(J(t), \Delta))\Delta) + (1 - u(t))(1 - e^{-\delta\Delta}) \\
&= u(t - \Delta)(1 - \lambda(\theta(J_t))\Delta) + (1 - u(t))(1 - e^{-\delta\Delta})
\end{aligned} \tag{B5}
$$

where the second line makes use of the fact that $\theta_\Delta(J) = \theta(J)$, with $\theta(J)$ defined as in (8). The limit for $\Delta \to 0$ of the difference equation (B4) is the differential equation (12). Similarly, the limit for $\Delta \to 0$ of the difference equation (B5) is the differential equation

(13). ■

C Proof of Theorem 1

Proof of parts (i)-(ii): Given $u \in [\underline{u}, \overline{u}]$ and $y_e \geq y_u$, the seller's net revenues in the BJ market are

$$S^*(u, y_e) = \frac{\alpha(r \square c)}{r} \left\{ \frac{u(1 + {}_u)}{b(u)} \left[1 \square \frac{2 {}_u}{1 + {}_u} \frac{(1 \square u)}{b(u)} \right] y_u + \frac{(1 \square u)(1 + {}_e)}{b(u)} \left[1 \square \frac{2 {}_e}{1 + {}_e} \frac{(1 \square u)}{b(u)} \right] w^*(u, y_e) \right\},$$

where

$$w^*(u, y_e) = y_u + \quad [S^*(u, y_e) + y_e \square y_u].$$

The derivative of $S^*(u, y_e)$ with respect to u has the same sign as

$$\Sigma_u(u, y_e) = \frac{(1 + {}_u)(1 + {}_e)}{b(u)^2} \left[\left(1 \square \frac{2 {}_u}{1 + {}_u} \frac{(1 \square u)}{b(u)} \right) y_u \square \left(1 \square \frac{2 {}_e}{1 + {}_e} \frac{(1 \square u)}{b(u)} \right) w(u) \right] + 2 \frac{1 + {}_u}{b(u)^2} \left[\frac{(1 + {}_u) u}{b(u)} \frac{{}_u}{1 + {}_u} y_u + \frac{(1 + {}_e)(1 \square u)}{b(u)} \frac{{}_e}{1 + {}_e} w(u) \right].$$

The derivative of Σ_u with respect to y_e is

$$\frac{\partial \Sigma_u(u, y_e)}{\partial y_e} = \square \frac{1 + {}_u}{b(u)^2} \left[1 + {}_e \square \frac{4(1 \square u) {}_e}{b(u)} \right] \left[1 + \frac{\partial S^*(u, y_e)}{\partial y_e} \right], \tag{C1}$$

where

$$\frac{\partial S^*(u, y_e)}{\partial y_e} = \frac{\alpha(r \square c)}{r} \left\{ \frac{(1 \square u)(1 + {}_e)}{b(u)} \left[1 \square \frac{2 {}_e}{1 + {}_e} \frac{(1 \square u)}{b(u)} \right] \left(\frac{\partial S^*(u, y_e)}{\partial y_e} + 1 \right) \right\}. \tag{C2}$$

After substituting (C2) into (C1), we obtain

$$\frac{\partial \Sigma_u(u, y_e)}{\partial y_e} = \frac{\square \frac{1 + {}_u}{b(u)^2} \left[1 + {}_e \square \frac{4(1 \square u) {}_e}{b(u)} \right]}{1 \square \frac{1 \square u}{b(u)} \left[1 + {}_e \square \frac{2(1 \square u) {}_e}{b(u)} \right] \frac{\alpha(r \square c)}{r}}. \tag{C3}$$

Letting $\phi(u)$ denote the right-hand side of (C3), we can write $\Sigma_u(u, y_e)$ as

$$\Sigma_u(u, y_e) = \Sigma_u(u, y_u) + \phi(u) (y_e \square y_u). \tag{C4}$$

For any $u \in [\underline{u}, \overline{u}]$, $\Sigma_u(u, y_u)$ is finite and $\varphi(u)$ is strictly negative. Therefore, there exists a $\overline{y}_e(u) \geq y_u$ such that $\Sigma_u(u, y_e) < 0$ for all $y_e \in [y_u, \overline{y}_e(u))$ and $\Sigma_u(u, y_e) < 0$ for all $y_e > \overline{y}_e(u)$.

Since $\partial S^*(u, y_e)/\partial u$ has the same sign as $\Sigma_u(u, y_e)$, this completes the proof of part (i). The proof of part (ii) is similar and it is omitted for the sake of brevity. ■

Proof of part (iii): Suppose $S^{*\prime}(u) < 0$ for some $u \in [\underline{u}, \overline{u}]$. In this case, there exist u_0 and u_1 such that $\underline{u} < u_0 < u_1 < \overline{u}$ and $J_0 > J_1$, where $J_0 \equiv (1 - \;)(S^*(u_0) + y_e - y_u)/(\rho + \delta)$ and $J_1 \equiv (1 - \;)(S^*(u_1) + y_e - y_u)/(\rho + \delta)$. In what follows, we will find a vacancy cost k and a matching function M such that (u_0, J_0) and (u_1, J_1) are steady states.

Define x_0 as $(1 - u_0)\delta/u_0$ and x_1 as $(1 - u_1)\delta/u_1$. From the inequalities $\underline{u} < u_0 < u_1 < \overline{u}$, it follows that $0 < x_1 < x_0 < 1$. Choose the vacancy cost k to be equal to $J_1 - \epsilon$, where $\epsilon > 0$ and $\epsilon < \min\{x(J_0 - J_1)/(x_0 - x_1), J_1\}$. Such a choice for ϵ is always possible because $J_0 > J_1$, $x_0 > x_1$ and $J_1 > 0$. Choose the inverse of the job-finding probability, $\varphi(x) \equiv \lambda^{-1}(x)$, to be such that $\varphi(0) = 0$ and

$$\varphi'(x) = \begin{cases} 1 + 2\;{}_0 x, & \text{if } x \in [0, x_1] \\ 1 + 2\;{}_0 x_1 + 2\;{}_1 (x - x_1), & \text{if } x \in [x_1, x_0] \\ 1 + 2\;{}_0 x_1 + 2\;{}_1 (x_0 - x_1) + \dfrac{(x - x_0)}{(1 - x_0)(1 - x)}, & \text{if } x \in [x_0, 1], \end{cases} \tag{C5}$$

where the parameters ${}_0$ and ${}_1$ are

$${}_0 = \frac{J_1 - k}{k x_1},$$

$${}_1 = \frac{x_0 J_0 - x_1 J_1}{k(x_0 - x_1)^2} - \frac{1 + 2\;{}_0 x_1}{x_0 - x_1}.$$

First, notice that $\varphi(x)$ is strictly increasing and strictly convex for all $x \in [0, 1]$. In fact, $k < J_1$ implies ${}_0 > 0$ and $k > J_1 - x_0(J_0 - J_1)/(x_0 - x_1)$ implies ${}_1 > 0$. In turn, ${}_0 > 0$ and ${}_1 > 0$ imply that $\varphi'(x)$ is strictly positive and strictly increasing for all $x \in [0, 1]$. Second, notice that $\varphi(x)$ is such that $\varphi(0) = 0$ and $\varphi(1) = \infty$. Third, $\varphi(x)$ is such that

$$\begin{aligned} \varphi(x_1) &= \varphi(0) + \textstyle\int_0^{x_1} (1 + 2\;{}_0 x) dx = \dfrac{J_1 x_1}{k}, \\ \varphi(x_2) &= \varphi(x_1) + \textstyle\int_{x_1}^{x_0} (1 + 2\;{}_0 x_1 + 2\;{}_1 (x - x_1)) dx = \dfrac{J_0 x_0}{k}. \end{aligned} \tag{C6}$$

From the properties of $\varphi(x)$, it follows that the job-finding probability function $\lambda(\theta)$ is strictly increasing, strictly concave and such that $\lambda(0) = 0$, $\lambda(\infty) = 1$ and $\lambda'(0) = 1$. In turn, from the properties of $\lambda(\theta)$, it follows that the job-filling probability function $\eta(\theta) \equiv \lambda(\theta)/\theta$ is strictly decreasing and such that $\eta(0) = 1$ and $\eta(\infty) = 0$. Therefore, the function $\varphi(x)$ defined in (C5) implies a matching process $\lambda(\theta)$, $\eta(\theta)$, $M(u, v) = u\lambda(u/v)$ that satisfies all of the regularity assumptions made in Section 2. Moreover, since $\epsilon < J_1$, $k = J_1 - \epsilon > 0$. Therefore, the vacancy cost k the assumptions made in Section 2.

Now, notice that both (u_0, J_0) and (u_1, J_1) satisfy the stationarity conditions (14)-(15) and, hence, both (u_0, J_0) and (u_1, J_1) are steady states. In fact, the definition of J_0 implies that

$$J_0 = \frac{(1 - \square)\,(S^*(u_0) + y_e - y_u)}{\rho + \delta},$$

and the first line in (C6) implies that

$$\begin{aligned} \frac{x_0}{\varphi(x_0)} = \frac{k}{J_0} \quad &\iff \quad \frac{\lambda(\varphi(x_0))}{\varphi(x_0)} = \frac{k}{J_0} \\ &\iff \quad x_0 = \lambda\left(\eta^{-1}\left(\frac{k}{J_0}\right)\right) \\ &\iff \quad u_0 = \frac{\delta}{\delta + \lambda(\theta(J_0))}. \end{aligned}$$

Similarly, the definition of J_1 and the second line in (C6) imply that

$$\begin{aligned} J_1 &= \frac{(1 - \square)\,(S^*(u_1) + y_e - y_u)}{\rho + \delta}, \\ u_1 &= \frac{\delta}{\delta + \lambda(\theta(J_1))}. \end{aligned}$$

Now, suppose $S^{*\prime}(u) \geq 0$ for all $u \in [\underline{u}, \overline{u}]$. On the way to a contradiction, suppose that there exist two steady states (u_0, J_0) and (u_1, J_1). From the stationarity condition (15) and the fact that $S^{*\prime}(u) \geq 0$ for all $u \in [\underline{u}, \overline{u}]$, it follows that

$$J_0 = \frac{(1 - \square)\,(S^*(u_0) + y_e - y_u)}{\rho + \delta} \leq \frac{(1 - \square)\,(S^*(u_0) + y_e - y_u)}{\rho + \delta} = J_1. \tag{C7}$$

From the stationarity condition (14), it follows that

$$u_0 = \frac{\delta}{\delta + \lambda(\theta(J_0))} < \frac{\delta}{\delta + \lambda(\theta(J_1))} = u_1. \tag{C8}$$

Since $J_0 \leq J_1$, $\lambda(\theta)$ is increasing in θ and $\theta(J)$ is increasing in J, $\lambda(\theta(J_0)) \leq \lambda(\theta(J_1))$, which contradicts the inequality in (C8). ■

D Global Dynamics

In this appendix, we provide a complete characterization of the set of rational expectations equilibria of the model. We carry out the analysis under the assumption that the dynamical system (12)-(13) admits exactly three steady states, E_1, E_2 and E_3. It is straightforward to verify that the steady state with the lowest unemployment rate, E_1, is a saddle. Similarly, the steady state with the highest unemployment rate, E_3, is a saddle. In contrast, the intermediate steady state, E_2, is a sink or a source depending on whether the unemployment rate is greater or smaller than $\delta/(\delta+\rho)$. Here, we assume that E_2 is a sink, as this is the relevant case for most reasonable parametrizations of the model. Moreover, for the sake of exposition, we assume that the J-nullcline is quasi-convex. When the J-nullcline is not quasi-convex, the analysis is slightly different, but the qualitative features of the set of rational expectations equilibria are unchanged.

In Figure D1, we plot the u-nullcline, the J-nullcline and the direction of motion of the solutions to the dynamical system (12)-(13) in the six regions defined by the intersections of the two nullclines. Using the direction of motion of the solutions to (12)-(13) and using the fact that any solution to (12)-(13) must cross the u-nullcline vertically and the J-nullcline horizontally, we can characterize the shape of the stable manifold J_1^S associated with the steady state E_1 and of the stable manifold J_3^S associated with the steady state E_3. In particular, it is straightforward to verify that the left backward extension of the stable manifold J_1^S to the left of E_1 must go through region III and exit the domain at $\underline{u}$. The backward extension of J_1^S to the right of E_1 must go through region II and then it may either: (i) exit the domain at $\overline{u}$; (ii) go through regions V and III and exit the domain at $\underline{u}$; (iii) circle around the regions V, III, IV and II without ever exiting the domain.[18] Similarly, the backward extension of the stable manifold J_3^S to the right of E_3 must go through region VI and exit the domain at $\overline{u}$. The backward extension of J_3^S to the left of E_3 goes through region V and then it may either: (i) exit the domain at $\underline{u}$; (ii) go through regions III, IV and II and exit the domain at $\overline{u}$; (iii) circle around the regions III, IV, II and V without ever exiting the domain. After eliminating incompatible cases, the above classification leaves us with five possible scenarios.

In the main text, we have already analyzed the case in which the right branch of J_1^S exits at $\overline{u}$ and the left branch of J_3^S exits at $\underline{u}$. Figure D2(a) plots the solution to the dynamical system (12)-(13) when both the right branch of J_1^S and the left branch of J_3^S exit at $\underline{u}$. Let $\overline{u}_1$ denote the easternmost point on the stable manifold associated with E_1. For any initial unemployment $u_0 \in [\underline{u}, \overline{u}_1]$, there are three types of rational expectations equilibria. In particular, there are two equilibria along the stable manifold converging to E_1 (one starting on the upper branch of J_1^S and one starting on the lower branch of J_1^S), there is one equilibrium along the stable manifold converging to E_3, and there is a continuum of equilibria starting between the upper and the lower branches of J_1^S and converging to either E_2 or to a limit cycle around E_2. For any initial unemployment $u_0 \in (\overline{u}_1, \overline{u}]$, the only rational

[18] For the sake of brevity, the analysis abstracts from the knife-edge cases in which the stable manifolds are either homoclinic—i.e. the backward extension of the stable manifold associated with one saddle steady state converges to the same steady state—or heteroclinic—i.e. the backward extension of the stable manifold associated with one saddle steady state converges to the other saddle steady state.

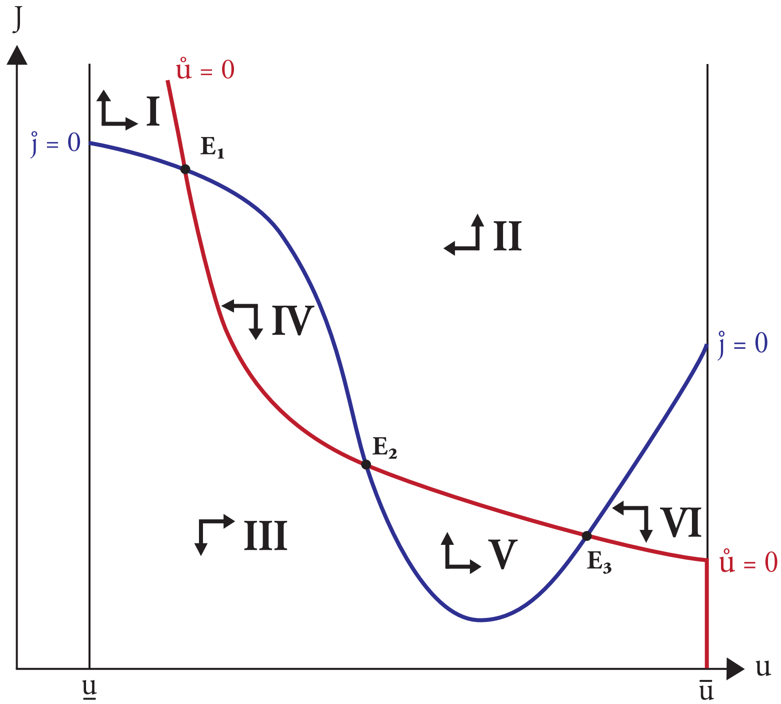


Figure D.1: Direction of Motion

expectations equilibrium is the stable manifold converging to E_3. Figure D2(b) illustrates a case that is symmetric to the one in Figure D2(a).

Figure D2(c) plots the solution to the dynamical system (12)-(13) when the right branch of J_1^S coils around E_2, while the left branch of J_3^S exits at $\underline{u}$. In this case, one can prove that there exists a repellent limit cycle around E_2 (see Boldrin, Kiyotaki and Wright 1993, Proposition 5). Let J_2^C denote the limit cycle. Moreover, let $\underline{u}_1$ and $\overline{u}_1$ the westernmost and easternmost points on the stable manifold J_1^S and let $\underline{u}_2$ and $\overline{u}_2$ the westernmost and easternmost points on the limit cycle J_2^C. For any initial unemployment $u_0 \in [\underline{u}, \underline{u}_2) \cup (\overline{u}_2, \overline{u}_1]$, there are two types of rational expectations equilibria: the stable manifold converging to E_1 and the stable manifold converging to E_3. For any initial unemployment $u_0 \in [\underline{u}_2, \overline{u}_2]$, there are two additional types of rational expectations equilibria: the limit cycle and a continuum of equilibria starting inside the limit cycle and converging to E_2. For any initial unemployment $u_0 \in (\overline{u}_1, \overline{u}]$, the only rational expectations equilibrium is the stable manifold converging to E_3. Figure D2(d) illustrates a case that is symmetric to the one in Figure D2(c).

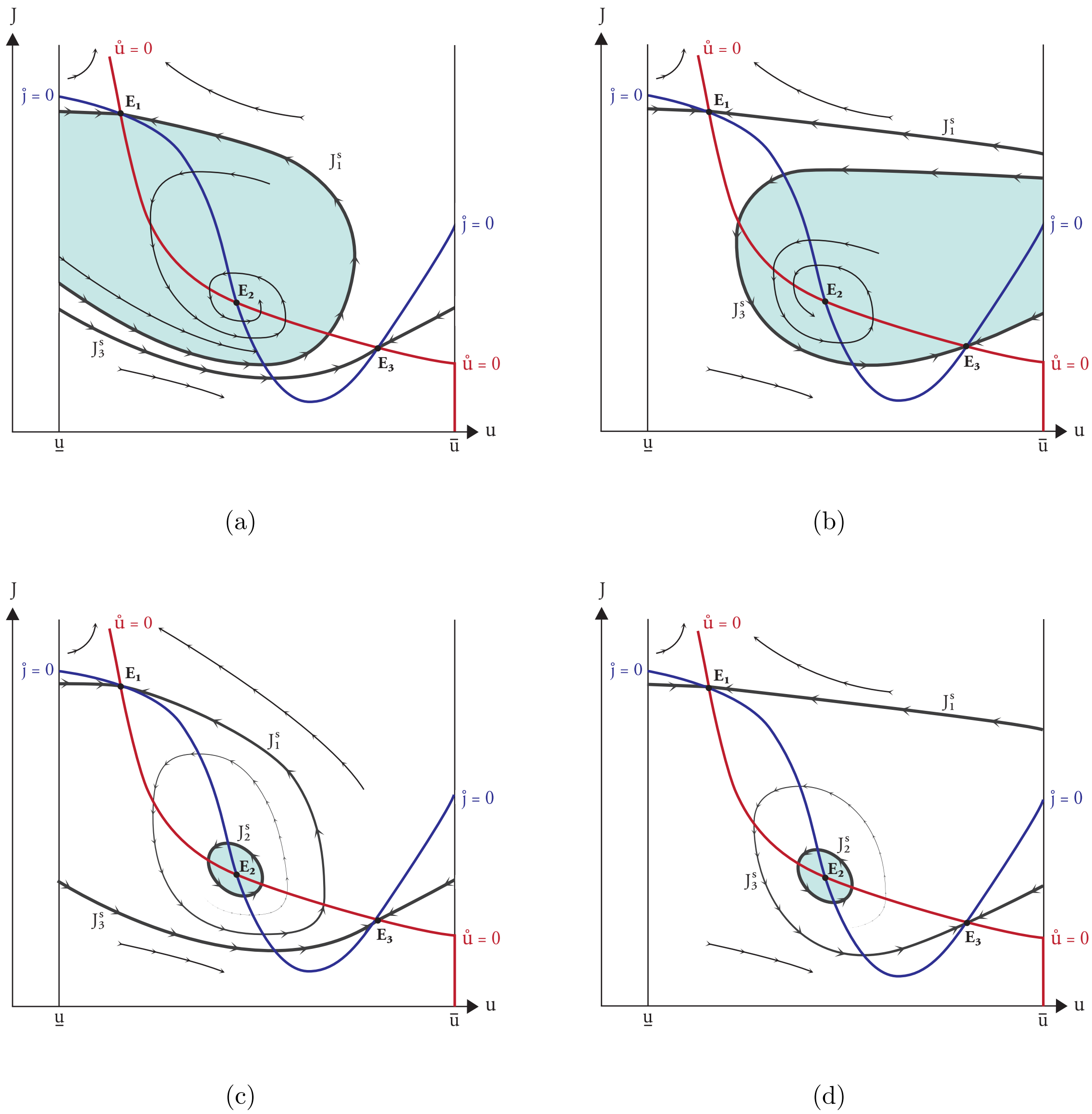


Figure D.2: Rational Expectation Equilibria

E Robustness

In Section 4, we showed that a reasonably calibrated version of the model admits multiple steady states and multiple rational expectations equilibria. In this appendix, we want to understand whether the existence of multiple steady states and multiple rational expectations equilibria is robust to alternative choices of calibration targets—especially those describing the shopping behavior of employed and unemployed buyers—and parameter values—especially those describing the product market.

Table E1(a) reports the steady-state unemployment rates as we let the target for the relative expenditures of unemployed buyers vary between 70 and 90% of the expenditures of employed buyers (horizontal axis), and the target for the relative time spent shopping by unemployed buyers vary between 110 and 140% of the time spent shopping by employed buyers (vertical axis), while keeping the product market parameters α, $_e$ and R constant and recalibrating all the other parameters so as to match their respective empirical targets. We choose not to recalibrate the product market parameters because they are not precisely pinned down by the associated empirical targets, in the sense that the calibrated value of the product market parameters changes quite dramatically in response to relatively small changes in the value of the other parameters of the model.

Table E1(a) shows that, for all alternative calibrations, the model admits three steady states. The unemployment rate at the first steady-state is pinned down by the calibration targets for the workers' transition rates between employment and unemployment and, hence, does not vary. The unemployment rate at the intermediate steady-state is decreasing in the relative shopping time of unemployed buyers and increasing in the relative expenditures of unemployed buyers. These results are intuitive because, whenever we increase the difference in shopping behavior of unemployed and employed buyers (on either the expenditure or the shopping time dimension), the seller's revenues become more sensitive to the unemployment rate, the J-nullcline becomes steeper and the unemployment rate at the intermediate steady state falls. Note that, for all alternative calibrations, the unemployment rate at the intermediate steady state is between 5.5 and 11%, values that are lower than $\delta/(\delta+\rho) \sim 0.9$. Therefore, the intermediate steady state is always a sink and, in light of Appendix D, the model admits multiple rational expectations equilibria.

Since we do not recalibrate the product market parameters α, $_e$ and R, it is useful to report the value of the associated calibration targets: the standard deviation of prices, the ratio of the highest to the lowest price and the relative price paid by unemployed workers (see Table E1(b)). For most calibration, the extent of price dispersion generated by the model is broadly consistent with the empirical evidence in Kaplan and Menzio (2013)—once we consider the fact that some of the empirical price dispersion may be due to factors that are not accounted for in our model (e.g., measurement error, store quality, etc. . .).

Table E2(a) reports the steady-state unemployment rates as we let α vary between 0.8 and 1 (vertical axis) and R vary between 10 and 20 (horizontal axis), while keeping $_e$ constant and recalibrating all the other parameters so as to match their respective empirical targets. For all alternative calibrations, we find that the model admits three steady states. Again, the unemployment rate at the first steady state is pinned down by the calibration targets for the workers' transition rates and, hence, it does not vary. The unemployment rate at the second steady state is decreasing in α and R. This finding is intuitive, since increasing either α or R makes the seller's revenues more sensitive to unemployment, the J-nullcline steeper and, hence, it lowers the unemployment rate at the second steady state. Over the entire parameter range, the unemployment rate at the second steady state goes from a minimum of 8% to a maximum of 14%, values which are all lower than $\delta/(\delta + \rho)$. Hence, over the entire parameter range, the second steady state is a sink and the model admits multiple rational expectations equilibria. Table E2(b) shows that both measures of price dispersion increase with R and α, while the relative price paid by unemployed buyers hardly changes.

Finally, Table E3 reports the steady-state unemployment rates as we vary the target for the elasticity of the UE rate with respect to labor market tightness from 0.45 to 0.75 (vertical axis) and the firm's profit margin from 5 to 20% (horizontal axis), while keeping the parameters α, $_e$ and R constant and recalibrating all the remaining parameters. We find that, in the region where the elasticity of the UE rate is relatively high or the profit margin is relatively low, the model admits three steady states. In this region, the unemployment rate associated with the intermediate steady state is decreasing in the elasticity of UE and increasing in the firm's profit margin. These findings are intuitive because the u-nullcline becomes more curved as we increase the elasticity of the UE rate, and the J-nullcline becomes steeper as we lower the profit margin. It is only in the region where the elasticity of the UE rate is relatively low and the profit margin of the firm is relatively high that the model has a unique steady state. Moreover, we find that this region shrinks when we increase the targeted difference in either expenditures or shopping time between employed and unemployed workers.

Table E.1: Robustness: steady-state unemployment rates and price statistics for combinations of targets for expenditure ratio and shopping time ratio of unemployed to employed

	Unemployed-employed expenditure ratio			
	0.75	0.8	0.85	0.9
Panel A: Steady-state unemployment rates				
Shopping time ratio				
1.1	5.3, 13.4, 100	5.3, 13.9, 100	5.3, 14.5, 100	5.3, 15.5, 100
1.2	5.3, 9.8, 100	5.3, 9.7, 100	5.3, 9.5, 100	5.3, 9.3, 96.9
1.3	5.3, 8.0, 100	5.3, 7.7, 100	5.3, 7.4, 100	5.3, 7.1, 94.5
1.4	5.3, 6.9, 100	5.3, 6.6, 100	5.3, 6.3, 100	5.3, 6.0, 92.6
Panel B: Price statistics (see notes)				
Shopping time ratio				
1.1	0.18, 1.56, 0.99	0.19, 1.57, 0.99	0.19, 1.57, 0.99	0.21, 1.58, 0.99
1.2	0.27, 1.65, 0.99	0.28, 1.66, 0.99	0.3, 1.67, 0.99	0.33, 1.68, 0.99
1.3	0.4, 1.74, 0.98	0.42, 1.75, 0.98	0.46, 1.77, 0.98	0.51, 1.79, 0.98
1.4	0.58, 1.82, 0.97	0.62, 1.84, 0.97	0.67, 1.87, 0.97	0.75, 1.89, 0.97

Notes: In Panel B, the numbers in each cell refer to the coefficient of variation of prices, the ratio of the maximum price to minimum price, and the price paid by unemployed relative to employed.

Table E.2: Robustness: steady-state unemployment rates and price statistics for combinations of α and R

	Home production technology for BJ goods (R)			
	10.00	13.33	16.67	20.00
Panel A: Steady-state unemployment rates				
Weight on BJ goods (α)				
0.80	5.3, 13.9, 100	5.3, 13.2, 100	5.3, 12.8, 100	5.3, 12.5, 100
0.87	5.3, 11.9, 100	5.3, 11.3, 100	5.3, 10.9, 100	5.3, 10.7, 100
0.93	5.3, 10.2, 100	5.3, 9.7, 100	5.3, 9.3, 100	5.3, 9.1, 100
1.00	5.3, 8.8, 100	5.3, 8.3, 100	5.3, 8.0, 100	5.3, 7.8, 100
Panel B: Price statistics (see notes)				
Weight on BJ goods (α)				
0.80	0.23, 1.2, 0.99	0.31, 1.35, 0.99	0.4, 1.5, 0.98	0.49, 1.65, 0.98
0.87	0.24, 1.3, 0.99	0.33, 1.47, 0.98	0.42, 1.63, 0.98	0.51, 1.79, 0.98
0.93	0.25, 1.4, 0.99	0.34, 1.58, 0.98	0.43, 1.75, 0.98	0.53, 1.93, 0.98
1.00	0.26, 1.5, 0.99	0.36, 1.69, 0.98	0.45, 1.88, 0.98	0.54, 2.07, 0.98

Notes: In Panel B, the numbers in each cell refer to the coefficient of variation of prices, the ratio of the maximum price to minimum price, and the price paid by unemployed relative to employed.

Table E.3: Robustness: steady-state unemployment rates and price statistics for combinations of targets for elasticity of UE rate with respect to θ and firms' profit margin

	Firms' profit margin			
	.05	.10	.15	.20
Panel A: Steady-state unemployment rates				
UE elasticity				
0.45	5.3, 20.7, 70.3	5.3	5.3	5.3
0.55	5.3, 12.6, 92.4	5.3, 23.1, 78.1	5.3, 48.8, 100	5.3
0.65	5.3, 8.3, 100	5.3, 13.2, 100	5.3, 18.8, 94.9	5.3, 25.9, 85.6
0.75	5.3, 5.7, 100	5.3, 8.2, 100	5.3, 10.7, 100	5.3, 13.4, 100
Panel B: Price statistics (see notes)				
UE elasticity				
0.45	0.37, 1.72, 0.98	0.41, 1.72, 0.98	0.44, 1.72, 0.98	0.48, 1.72, 0.98
0.55	0.37, 1.72, 0.98	0.41, 1.72, 0.98	0.44, 1.72, 0.98	0.48, 1.72, 0.98
0.65	0.37, 1.72, 0.98	0.41, 1.72, 0.98	0.44, 1.72, 0.98	0.48, 1.72, 0.98
0.75	0.37, 1.72, 0.98	0.41, 1.72, 0.98	0.44, 1.72, 0.98	0.48, 1.72, 0.98

Notes: In Panel B, the numbers in each cell refer to the coefficient of variation of prices, the ratio of the maximum price to minimum price, and the price paid by unemployed relative to employed.

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