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Evaluating Durable Public Good Provision using Housing Prices

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**ABSTRACT**

Recent empirical work in public finance uses the housing price response to public investments to assess the efficiency of local durable public good provision. This paper investigates the theoretical foundations for this technique. In the context of a novel theoretical model developed to study the issue, it shows that there is little justification for the technique if citizens have rational expectations concerning future investment in their communities. An example in which investment is chosen by a budget-maximizing bureaucrat is developed to show why the technique can falsely predict under-provision. The technique is valid, however, when citizens have adaptive expectations, believing that whatever provision level that currently prevails will be maintained indefinitely.

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# 1 Introduction

A sizable fraction of government spending is devoted to investment in durable public goods. Such investment is undertaken by all levels of government - federal, state, and local. The goods in question include physical infrastructure (roads, bridges, airports, etc), basic research, defense equipment, environmental clean-ups, parks, and schools. A basic question of interest to economists and policy-makers is how the levels of durable public good provision emerging from the political process compare with socially optimal levels. This question arises in many different policy areas. For example, there seems broad agreement that government substantially underinvests in physical infrastructure and basic research. There is much less agreement concerning defense, environmental, and educational investments, with conservatives and liberals often coming down on opposing sides of the issue. Given the importance of the question, it would be helpful if economic analysis provided convincing ways of answering it.

There is a long tradition in public finance of using house prices to assess the social optimality of local public good provision (see, for example, Brueckner 1979, 1982, Lind, 1973, and Wildasin 1979). The underlying idea is that the demand of potential residents to live in a community will be influenced by the local public goods it provides and the taxes it levies to finance them (Oates 1969, Tiebout 1956). Accordingly, the net surplus generated by local public good provision will be reflected in housing prices.<sup>1</sup> In a well-known and elegant theoretical formulation of the idea, Brueckner develops a model in which if housing prices rise following a small, permanent increase in local public good provision, then it can be inferred that the good is under-provided. Conversely, if housing prices fall, the good is over-provided (Brueckner 1979, 1982). This model has been used as the basis for a number of empirical studies of the optimality of local public good provision (see, for example, Barrow and Rouse 2004, Brueckner 1982, and Lang and Jian 2004).

Can housing prices be used to assess the social optimality of local durable public good provision? In an ambitious and creative paper, Cellini, Ferreira, and Rothstein (2010) employ the approach to detect whether local school districts are over or under-providing public school facilities. Using a static version of Brueckner's model, they argue that if housing prices in a district rise following an investment in public school facilities, then such facilities are under-provided. Con-

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<sup>1</sup> A vast literature investigates the relationship between housing prices and local public good provision empirically, with particular focus on schooling. See Nguyen-Hoang and Yinger (2011) and Ross and Yinger (1999) for useful surveys.

versely, if housing prices fall, facilities are over-provided. To estimate what house prices would be in the counter-factual situation in which an observed investment is not undertaken, Cellini, Ferreira, and Rothstein exploit the fact that investments must be approved by residents in a referendum. Drawing on the regression discontinuity literature, they then compare housing prices in school districts in which referenda have just passed with those in which they have just failed. If prices are higher in the just passing districts, they argue that school facilities are under-provided. This is indeed what they find for California school districts.

The intuitive appeal of this approach notwithstanding, there are important conceptual differences between an investment in a durable public good and a permanent increase in a non-durable public good. First, because of depreciation, the benefits from investment in the durable public good will not be permanent. Rather, they will diminish over time. Second, again because of depreciation, whether or not the investment in question is undertaken, future investments will be made by the community. Moreover, the nature of these investments will depend on the stock of the public good and hence on the fate of the investment in question. This creates a linkage between the current investment and the future investment path in the community. These differences raise the question of whether the logic that underlies the standard test can be applied to justify using the housing price response to investments to evaluate durable public good provision. The purpose of this paper is to investigate this important question.

The paper begins by developing a novel theoretical model in which to study the issue. This model is designed to capture the recurring nature of investment in durable public goods and the linkages between decision-making periods that durability creates. The model is a partial equilibrium model of a single community whose government provides a durable public good. There is a pool of households who, for exogenous reasons, are potential residents of the community. Households move in and out of this pool, creating an active housing market in each period. Public good provision is managed by a bureaucrat who, in any period, can propose investment. Investment is financed by a tax on the residents and, to be implemented, the bureaucrat's proposal must be approved by the residents. The supply of houses in the community is perfectly inelastic, implying that the future surplus a resident is expected to receive from public good provision is fully capitalized into housing prices.

To set the stage for the analysis of durable public goods, the paper first uses the model to review why using the housing price response to a small, permanent increase in provision can be

used to evaluate non-durable public good provision. In addition, it extends the approach to show how the test can be used when the assumption of a small increase is not tenable. In particular, it is shown that a non-negative housing price response implies that the public good level without the increase is too low, while a non-positive response implies that the level with the increase is too high.

The paper then investigates whether a similar logic implies that the housing price response to an investment can be used to evaluate durable public good provision. To permit a general analysis, the paper starts out by modelling the behavior of the bureaucrat and residents in a reduced form way with an investment proposal function and a proposal approval probability function. It shows that the key implicit assumption of the test is that the socially optimal level of the public good maximizes the surplus residents are expected to receive from provision *in equilibrium*; that is, when future investment is governed by the investment proposal and proposal approval probability functions. The paper argues that there is no reason to expect this to be the case. The argument is basically an application of the *theory of the second best* (Lipsey and Lancaster 1956). The socially optimal level of the public good is derived under the assumption that all future investments will be socially optimal. This means that all future investments are approved with probability one and any increase in investment today is accommodated by a compensating reduction in the future. It cannot reasonably be presumed that this will be the case in equilibrium. For example, it may be the case that more investment today leads to a more than compensating reduction in the future or reduces the probability that future investments are approved. These second best impacts must be taken into account in the surplus maximization problem and this implies that the public good level that maximizes surplus in equilibrium could be much different from the socially optimal level.

To actually predict the second best impacts of an investment requires a specific model of bureaucrat and resident interaction. To illustrate more concretely how using the housing price response to an investment can provide misleading information, the paper next turns to such a model. Specifically, it assumes that the investment path is generated by the interaction between rational, forward-looking residents and a budget-maximizing bureaucrat who cares about the level of the public good but not its cost. Budget-maximization is a common assumption in the political economy literature and underlies Romer and Rosenthal's *agenda control model*, the leading alternative to the median voter model of local government spending (Romer and Rosenthal 1978,

1979).<sup>2</sup>

With a budget-maximizing bureaucrat and rational residents, the paper shows that the equilibrium price of housing if the bureaucrat's proposed investment is approved (which it will be in equilibrium) exactly equals that if it were not approved. This holds irrespective of the level of the public good prevailing at the time at which the investment is proposed. Intuitively, this reflects the fact that the bureaucrat proposes a level of investment which makes residents indifferent between undertaking it or not. This means that the future value of public good surplus is the same with or without the investment. Since surplus is fully capitalized, this implies that housing prices are the same with or without the investment.

Applying the housing price test, the fact that equilibrium housing prices are the same whether or not the bureaucrat's investment is approved, suggests that the socially optimal level of the public good should lie between the levels that would prevail with and without the investment. However, this is not always the case. Specifically, there exist public good levels at which the bureaucrat proposes an investment, the residents approve it, and the public good level that would prevail without the investment exceeds the socially optimal level. Intuitively, this reflects the fact that the level of public good that maximizes residents' surplus in equilibrium exceeds the socially optimal level. In equilibrium, the public good has a higher marginal value because more units reduce the bureaucrat's future ability to exploit his agenda-setting power. In this situation, therefore, the housing price test falsely predicts under-provision.

Finally, the paper points out that a justification for the housing price test is available if the assumption that residents have rational expectations concerning the future investment path in the community is relaxed. Specifically, the test is shown to work if citizens have adaptive expectations, believing that whatever level of public good they observe in the community at the beginning of a period will be maintained indefinitely. Thus, they observe the current quantity and quality of school facilities, say, and just assume they are at steady state levels. This is a form of myopia that is perhaps not too implausible, particularly for new residents moving into a community. This assumption means that residents perceive a successful investment as permanently increasing provision and this brings us back into the world studied by Brueckner.

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<sup>2</sup> The budget maximizing assumption was first proposed by Niskanen (1971). For analysis of the relative performance of the median voter and agenda control models see Romer and Rosenthal (1982), Romer, Rosenthal, and Munley (1992), and, in the specific context of school infrastructure investment, Balsdon, Brunner, and Rueben (2003).

Beyond providing a framework to analyze the theoretical question at hand, the model developed here makes a broader contribution. In particular, it provides a simple dynamic model of a housing market in which the market is active in each period and agents are rational and forward-looking. The model highlights the relationship between housing prices and fiscal variables, illustrating the phenomenon of capitalization. In contrast to standard treatments of capitalization in which values of policy variables are frozen through time, the model shows that it is both the current and future values of policy variables that are capitalized into housing prices. Moreover, by endogenizing policy choices via the agenda control framework, the model derives the dynamic implications of budget maximization for housing prices. More generally, the paper fits in with a growing literature that studies issues in housing markets using dynamic models with rational, forward-looking households (see, for example, Bayer, McMillan, Murphy, and Timmins 2011), particularly those papers that endogenize policy choices with political economy models (Coate 2011, Epple, Romano, and Sieg 2009, and Ortalo-Magne and Prat 2011).

The paper also contributes to a growing literature on durable public goods. While the vast majority of the public good literature has focused on the provision of non-durable goods, such as firework displays and police protection, in practice many important public goods are durable. Durability not only complicates the conditions for efficient provision but also makes understanding political provision considerably more challenging. This is because today's political choices have implications for future choices, creating a dynamic linkage across policy-making periods. The practical importance of durable public goods and the theoretical challenges they pose is leading to increasing interest in their provision. A number of recent papers have studied the provision of such goods under varying political institutions (see, for example, Battaglini and Coate 2007, Battaglini, Nunnari, and Palfrey forthcoming, Coate 2012, and LeBlanc, Snyder, and Tripathi 2000). This paper shows that durability also has important implications for the evaluation of public provision.

The organization of the remainder of the paper is as follows. Section 2 describes the model and characterizes efficient public good provision. Section 3 reviews the logic underlying using the housing price response to an increase in provision to evaluate non-durable public good provision and then explains why a similar logic does not justify using the housing price response to an investment to evaluate durable public good provision. Section 4 assumes that public good investment is generated by the interaction between a budget-maximizing bureaucrat and rational,

forward-looking residents and shows that using the housing price response to an investment can erroneously predict under-provision. Section 5 points out the adaptive expectations justification for the housing price test and Section 6 concludes.

## 2 Preliminaries

### 2.1 The model

Consider a community such as a municipality or school district. This community can be thought of as one of a number in a particular geographic area. The time horizon is infinite and periods are discrete. There is a pool of potential residents of the community of size 1. These can be thought of as households who for exogenous reasons (employment opportunities, family ties, etc) need to live in the geographic area in which the community is situated. Potential residents are characterized by their desire to live in the community (as opposed to an alternative community in the area) which is measured by the preference parameter  $\theta$ . This desire, for example, may be determined by a household's idiosyncratic reaction to the community's natural amenities. The preference parameter takes on values between 0 and  $\bar{\theta}$ , and the fraction of potential residents with preference below  $\theta \in [0, \bar{\theta}]$  is  $\theta/\bar{\theta}$ . Reflecting the fact that households' circumstances change over time, in each period new households join the pool of potential residents and old ones leave. The probability that a household currently a potential resident will be one in the subsequent period is  $\mu$ . Thus, in each period, a fraction  $1 - \mu$  of households leave the pool and are replaced by an equal number of new ones.

The only way to live in the community is to own a house. There are a fixed number of houses sufficient to accommodate a population of size  $H$  where  $H$  is less than the size of the pool of potential residents (i.e.,  $H < 1$ ). These houses are infinitely durable.

The community provides a durable public good which depreciates at rate  $\delta \in (0, 1)$ . Provision is managed by a bureaucrat.<sup>3</sup> In any period, the bureaucrat can propose investment. Investment costs  $c$  per unit and is financed by a tax on those choosing to reside in the community. To be implemented, the bureaucrat's proposal must be approved by a majority of the residents. Once approved, the investment takes time to build and is not available for use until the next period. The community pays for the investment when it is complete and thus taxes to finance the investment

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<sup>3</sup> For now, we will not be specific about the bureaucrat's objectives.



are levied in the next period.<sup>4</sup>

When living in the community, households have preferences defined over the public good and consumption. A household with preference parameter  $\theta$  and consumption  $x$  obtains a period payoff of  $\theta + x + B(g)$  if they live in the community and the public good level is  $g$ . The benefit function  $B(g)$  is increasing, smooth, strictly concave, and satisfies  $B(0) = 0$ . When not living in the community, a household's per period payoff is  $\underline{u}$ .<sup>5</sup> Households discount future payoffs at rate  $\beta$  and can borrow and save at rate  $1/\beta - 1$ . This assumption means that households are indifferent to the intertemporal allocation of their consumption. Each household in the pool receives an exogenous income stream the present value of which is sufficient to pay taxes and purchase housing in the community.<sup>6</sup>

There is a competitive housing market which opens at the beginning of each period. Demand comes from new households moving into the community and supply comes from owners leaving the community. The price of houses is denoted  $P$ .

The timing of the model is as follows. Each period, the community starts with a public good level  $g$  and a tax obligation  $T$  (which may be zero). The public good level is the depreciated level from the prior period plus any investment approved in the prior period. The tax obligation is to finance any investment approved in the prior period. At the beginning of the period, those in the pool learn whether they will be remaining and new households join. Households in the pool then decide whether to live in the community. The housing market opens and the equilibrium housing price  $P(g, T)$  is determined. The government levies taxes on residents sufficient to meet its tax obligation and residents obtain their payoffs from living in the community. The bureaucrat decides how much investment  $I$  to propose and the residents vote. If the proposal is approved, the community's public good level and tax obligation in the next period  $(g', T')$  is  $((1 - \delta)g + I, cI)$ ; otherwise, it is  $((1 - \delta)g, 0)$ .

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<sup>4</sup> As will become clear below, the predictions of the model concerning the impact of an investment on the price of housing would not be changed if the cost of investment was financed by a bond issue rather than a tax.

<sup>5</sup> Note that  $\underline{u}$  is both the per period payoff of living in one of the other communities in the geographic area if a household is in the pool and the payoff from living outside the area when a household leaves the pool.

<sup>6</sup> The assumption that utility is linear in consumption means that there are no income effects, so it is not necessary to be specific about the income distribution.

## 2.2 Housing market equilibrium

We now explain how the housing market equilibrates for any given possible path of investment. We summarize the investment path in a reduced form manner with two functions  $\pi(g, I)$  and  $I(g)$ . The former describes the probability that the investment  $I$  will be approved by the residents in a period in which the public good level is  $g$ . The latter describes the level of investment the bureaucrat proposes when the public good level is  $g$ . We will sometimes use the notation  $\pi^*(g)$  to denote the probability  $\pi(g, I(g))$ .

**Decisions of households** At the beginning of any period, households fall into two groups: those who resided in the community in the previous period and those who did not, but could in the current period. Households in the first group own homes. The second group do not. Households in the first group who leave the pool sell their houses and obtain a continuation payoff of

$$P(g, T) + \frac{\underline{u}}{1 - \beta}. \quad (1)$$

The remaining households in the first group and all those in the second must decide whether to live in the community. Formally, they make a location decision  $l \in \{0, 1\}$ , where  $l = 1$  means that they live in the community. This decision will depend on their preference parameter  $\theta$ , current and future housing prices, and public goods and taxes. Since selling a house and moving is costless, there is no loss of generality in assuming that all households sell their property at the beginning of any period.<sup>7</sup> This makes each household's location decision independent of its property ownership state. It also means that the only future consequences of the current location choice is through the selling price of housing in the next period.

To make this more precise, let  $V_\theta(g, T)$  denote the expected payoff of a household with preference parameter  $\theta$  at the beginning of a period in which it belongs to the pool but does not own a house. Then, we have that

$$V_\theta(g, T) = \max_{l \in \{0, 1\}} \left\{ \begin{array}{l} l(\theta + B(g) - T/H - P(g, T) + \beta EP(g', T')) \\ + (1 - l)\underline{u} + \beta[\mu EV_\theta(g', T') + (1 - \mu)\frac{\underline{u}}{1 - \beta}] \end{array} \right\}, \quad (2)$$

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<sup>7</sup> It should be stressed that this is just a convenient way of understanding the household decision problem. The equilibrium we study is perfectly consistent with the assumption that the only households selling their homes are those who plan to leave the community.

where  $EP(g', T')$  denotes the expected price of housing next period; i.e.,

$$EP(g', T') = \pi^*(g)P((1 - \delta)g + I(g), cI(g)) + (1 - \pi^*(g))P((1 - \delta)g, 0), \quad (3)$$

and  $EV_\theta(g', T')$  denotes the expected payoff of a household in the pool next period; i.e.,

$$EV_\theta(g', T') = \pi^*(g)V_\theta((1 - \delta)g + I(g), cI(g)) + (1 - \pi^*(g))V_\theta((1 - \delta)g, 0). \quad (4)$$

Inspecting this problem, it is clear that a household of type  $\theta$  will choose to reside in the community if

$$\theta + B(g) - T/H - P(g, T) + \beta EP(g', T') \geq \underline{u}. \quad (5)$$

The left hand side of this inequality represents the per-period payoff from locating in the community, assuming that the household buys a house at the beginning of the period and sells it the next. This payoff depends on the preference parameter  $\theta$ , public good surplus, and the current and future price of housing. The right hand side represents the per period payoff from living elsewhere.

**Equilibrium** Given an initial state  $(g, T)$ , the price of housing  $P(g, T)$  adjusts to equate demand and supply. The demand for housing is the fraction of households for whom (5) holds. Given the uniform distribution of preferences, this fraction is

$$1 - \frac{\underline{u} - (B(g) - T/H - P + \beta EP(g', T'))}{\bar{\theta}}. \quad (6)$$

The supply of housing is, by assumption, perfectly inelastic at  $H$ . The equilibrium price of housing therefore satisfies

$$1 - \frac{\underline{u} - (B(g) - T/H - P(g, T) + \beta EP(g', T'))}{\bar{\theta}} = H. \quad (7)$$

To characterize the housing market equilibrium, define the present value of public good surplus  $S(g, T)$  recursively as follows:

$$S(g, T) = B(g) - T/H + \beta [\pi^*(g)S((1 - \delta)g + I(g), cI(g)) + (1 - \pi^*(g))S((1 - \delta)g, 0)]. \quad (8)$$

Intuitively,  $S(g, T)$  represents the discounted value of future public good surplus for a household who will be living in the community permanently starting in a period in which the community has public good level  $g$  and tax obligation  $T$ . Then, we have:

**Proposition 1** *In equilibrium, those households for whom  $\theta \in [(1 - H)\bar{\theta}, \bar{\theta}]$  choose to reside in the community and those for whom  $\theta \in [0, (1 - H)\bar{\theta}]$  do not. For households choosing to reside in the community there exists a constant  $\kappa(\theta)$  such that*

$$V_{\theta}(g, T) = \kappa(\theta) + S(g, T) - P(g, T), \quad (9)$$

*while for households choosing not to reside in the community*

$$V_{\theta}(g, T) = \frac{\underline{u}}{1 - \beta}. \quad (10)$$

*Furthermore, there exists a constant  $K$  such that the equilibrium housing price is given by*

$$P(g, T) = K + S(g, T). \quad (11)$$

**Proof:** See Appendix A.

The first part of the proposition tells us that the fraction  $H$  of households who choose to reside in the community are those in the pool with the highest preference parameters. This should make good sense intuitively since in all other respects potential residents are identical. The second part gives us expressions for the expected payoffs of the different types of households. These expressions will be useful later in the paper. The final part tells us that the equilibrium price can be expressed as the sum of a constant and the value of public good surplus. Equation (11) implies that the value of future public good levels and tax obligations is fully capitalized into the price of housing and follows from the assumption that the supply of houses is fixed. The constant  $K$  is tied down by the requirement that the marginal household with preference  $(1 - H)\bar{\theta}$  is just indifferent between living and not living in the community.<sup>8</sup>

It should be clear from Proposition 1 that households' equilibrium payoffs and the price of housing would be the same if the investment were financed via a bond issue rather than a tax increase. All that matters is the discounted present value of tax obligations and a policy change which held this constant but altered the future timing of taxes would have no impact on the current price of housing.<sup>9</sup> Ricardian Equivalence therefore holds in this model. Similar remarks

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<sup>8</sup> It is straight forward to show that  $K$  equals  $\kappa((1 - H)\bar{\theta}) - \underline{u}/(1 - \beta)$ . To guarantee that housing always has a positive value, it must be the case that the parameters and investment path are such that  $\kappa((1 - H)\bar{\theta}) + S(g, T)$  always exceeds  $\underline{u}/(1 - \beta)$ . We will assume this in what follows.

<sup>9</sup> The future housing price path would be impacted by the choice of debt versus taxes. Suppose, for example, that the cost of investment was financed by issuing one period bonds. Then, while the price of housing in the period

apply if, once approved, the investment comes on tap over a sequence of future periods rather than all in the next period as assumed here.

### 2.3 Optimal public good provision

We next characterize the path of investment that would be chosen by a bureaucrat that sought to maximize the residents' payoffs. Suppose that the current level of the public good is  $g$ . From Proposition 1, the residents of the community consist of those households for whom  $\theta$  lies between  $(1 - H)\bar{\theta}$  and  $\bar{\theta}$ . At the time the bureaucrat is choosing investment, these households all own houses. Thus, the expected continuation payoff of each of these households next period if the bureaucrat chooses  $I$  units of investment is given by

$$P((1-\delta)g+I, cI) + \mu V_{\theta}((1-\delta)g+I, cI) + (1-\mu) \frac{\underline{u}}{1-\beta} = \mu \kappa(\theta) + (1-\mu) \left( \frac{\underline{u}}{1-\beta} + K \right) + S((1-\delta)g+I, cI). \quad (12)$$

To understand the left hand side of (12), consider a home-owning household at the beginning of the next period. The public good level and tax obligation will be  $((1 - \delta)g + I, cI)$ . As noted above, we can assume wlog that the household will sell their house and obtain a payoff  $P((1 - \delta)g + I, cI)$ . With probability  $\mu$ , they will remain in the set of potential residents and obtain a payoff  $V_{\theta}((1 - \delta)g + I, cI)$  and with probability  $1 - \mu$  they will exit the pool and obtain payoff  $\underline{u}/(1 - \beta)$ . The right hand side of (12) follows immediately from equations (9) and (11) of Proposition 1 and tells us that the continuation payoff can be written as the sum of a type-specific constant and the value of public good surplus.

It follows from (12) that choosing investment to maximize resident payoffs is equivalent to maximizing public good surplus.<sup>10</sup> Letting  $S^o(g, T)$  denote maximized surplus, we know that

$$S^o(g, T) = \max_{I \geq 0} B(g) - T/H + \beta S^o((1 - \delta)g + I, cI). \quad (13)$$

Letting  $I^o(g)$  denote the surplus maximizing investment rule, we have:

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after the investment was approved would be the same as under tax finance, the price in the subsequent period when the bonds must be repaid would be lower. This is because taxes must be levied, whereas, with tax finance, the investment is already paid for. However, this is irrelevant for the purposes of this paper which is concerned solely with the immediate impact of an approved investment on housing prices.

<sup>10</sup> This conclusion arises despite the fact that residents may leave the community and thus not get to enjoy the fruits of their investment. The intuition is that, when they leave, residents will sell their homes and the price they get will reflect the future benefits.

**Proposition 2** *The optimal investment rule is that*

$$I^o(g) = \begin{cases} g^o - (1 - \delta)g & \text{if } g \leq g^o/(1 - \delta) \\ 0 & \text{if } g > g^o/(1 - \delta) \end{cases}, \quad (14)$$

where the public good level  $g^o$  satisfies the dynamic Samuelson Rule

$$HB'(g^o) = c[1 - \beta(1 - \delta)]. \quad (15)$$

**Proof:** See Appendix A.

Proposition 2 tells us that the optimal investment rule is to get the public good level to  $g^o$  as fast as possible and then keep it there. The optimal level  $g^o$  satisfies the condition that the sum of one period marginal benefits equals the “one period marginal cost”. The latter reflects the fact that investing one unit today saves  $c(1 - \delta)$  in investment costs tomorrow and these future cost savings have a present value of  $\beta c(1 - \delta)$ .

## 3 The housing price test

### 3.1 Non-durable public goods

We begin by reviewing the logic underlying the housing price test in the context of non-durable public goods. The claim to be evaluated is that the housing price response to a small, permanent increase in public good level reveals the efficiency of provision. Suppose therefore that the community currently provides  $g$  units of public good per period which generates a tax obligation of  $cg$  and consider a small, permanent increase in provision of  $\Delta g$  with associated tax obligation  $c\Delta g$ .

In our model, we capture this scenario by assuming 100% depreciation ( $\delta = 1$ ), which effectively makes the public good non-durable. In addition, we assume that the investment path is such that, whatever the current public good level  $\tilde{g}$ , the proposed investment  $I(\tilde{g})$  is always  $g + \Delta g$  and the probability that this is approved  $\pi^*(\tilde{g})$  equals 1. These assumptions imply that the community’s public good level and tax obligation each period will be  $(g + \Delta g, c(g + \Delta g))$ . From (8), public good surplus is

$$S(g + \Delta g, c(g + \Delta g)) = \frac{B(g + \Delta g) - c(g + \Delta g)/H}{1 - \beta}. \quad (16)$$

Now let  $\Delta P(g, \Delta g)$  denote the difference in housing prices with and without the public good increase.<sup>11</sup> Proposition 1 implies that the price difference is equal to the difference in surplus;

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<sup>11</sup> That is,  $\Delta P(g, \Delta g) = P(g + \Delta g, c(g + \Delta g)) - P(g, cg)$ .

that is,

$$\Delta P(g, \Delta g) = S(g + \Delta g, c(g + \Delta g)) - S(g, cg). \quad (17)$$

The price difference will therefore be positive if the increase has raised surplus, and negative if not. If the increase is small, then the difference in surplus is approximately equal to the derivative of surplus multiplied by the increase; that is,

$$S(g + \Delta g, c(g + \Delta g)) - S(g, cg) \approx \frac{dS(g, cg)}{d\Delta g} \Delta g. \quad (18)$$

From (16), the derivative of surplus is

$$\frac{dS(g, cg)}{d\Delta g} = \frac{B'(g + \Delta g) - c/H}{1 - \beta}. \quad (19)$$

The optimal level of the public good  $g^o$  satisfies the static Samuelson Rule that the sum of marginal benefits  $HB'(g)$  equals the marginal cost  $c$ . From (19) this implies that the derivative of surplus is zero at the optimal level; i.e.,  $dS(g^o, cg^o)/d\Delta g = 0$ . Thus, since surplus is concave in  $\Delta g$ , we have that

$$\frac{dS(g, cg)}{d\Delta g} \Delta g \begin{matrix} \geq \\ \leq \end{matrix} 0 \iff g \begin{matrix} \leq \\ \geq \end{matrix} g^o. \quad (20)$$

From (17) and (18), therefore, the housing price response to a small, permanent increase in the public good is, to a first approximation, positive if  $g$  is less than  $g^o$  and negative if  $g$  exceeds  $g^o$ .

The assumption that the increase in public good level is small is important to this logic. Nonetheless, the housing price test is still informative when this assumption is not satisfied. In this case, the public good level *with* the increase (i.e.,  $g + \Delta g$ ) must be distinguished from the level *without* (i.e.,  $g$ ). Since surplus is strictly concave in  $\Delta g$ , it follows from (17) that

$$\frac{dS(g + \Delta g, c(g + \Delta g))}{d\Delta g} \Delta g < \Delta P(g, \Delta g) < \frac{dS(g, cg)}{d\Delta g} \Delta g. \quad (21)$$

A non-negative price difference therefore signals that  $dS(g, cg)/d\Delta g$  is positive and hence the public good level without the increase is below optimal. By contrast, a non-positive difference signals that  $dS(g + \Delta g, c(g + \Delta g))/d\Delta g$  is negative and hence the level with the increase is too high.

### 3.2 Durable public goods

We now explore whether a similar logic implies that the housing price response to an investment sheds light on the efficiency of durable public good provision. Suppose the existing level of public

good is  $g$  and the bureaucrat proposes an investment level  $I(g)$ . Let the difference in the price of housing that would prevail next period with and without the investment be denoted  $\Delta P(g, I(g))$ .<sup>12</sup>

Proposition 1 implies that the price difference equals the difference in surplus; that is,

$$\Delta P(g, I(g)) = S((1 - \delta)g + I(g), cI(g)) - S((1 - \delta)g, 0). \quad (22)$$

A positive price difference implies that the investment has increased surplus, while a negative difference implies that surplus has decreased. Assuming the investment is small, we can approximate the change in surplus as follows:

$$S((1 - \delta)g + I(g), cI(g)) - S((1 - \delta)g, 0) \approx \frac{dS((1 - \delta)g, 0)}{dI} I(g). \quad (23)$$

If it were the case that the optimal level of the public good  $g^o$  satisfied the first order condition that  $dS(g^o, 0)/dI$  equals zero and if surplus were concave in  $I$ , we would have that

$$\frac{dS((1 - \delta)g, 0)}{dI} I(g) \begin{cases} \geq 0 \\ \leq 0 \end{cases} \iff (1 - \delta)g \begin{cases} \leq g^o \\ \geq g^o \end{cases}. \quad (24)$$

The logic from the non-durable case would then exactly carryover. The price impact of a small investment would be positive if  $(1 - \delta)g$  is less than  $g^o$  and negative if  $(1 - \delta)g$  exceeds  $g^o$ .

Again, the assumption that the investment  $I(g)$  is small is key to this logic. However, this is not a tenable assumption for a durable good subject to depreciation. After all, just to maintain public good levels, it will be necessary to have investment sufficient to offset depreciation. Thus, unless the depreciation rate is infinitesimal,  $I(g)$  cannot be small in steady state. But, as in the non-durable case, this not a major problem. We just need to distinguish the level of public good with the investment (i.e.,  $(1 - \delta)g + I(g)$ ) and the level without (i.e.,  $(1 - \delta)g$ ). Since surplus is linear in taxes, if surplus is strictly concave in the public good level, we have that<sup>13</sup>

$$\frac{dS((1 - \delta)g + I(g), 0)}{dI} I(g) < \Delta P(g, I(g)) < \frac{dS((1 - \delta)g, 0)}{dI} I(g). \quad (25)$$

Thus, provided the optimal level  $g^o$  satisfies the first order condition that  $dS(g^o, 0)/dI$  equals zero, a non-negative price difference signals that the public good level without the investment is below

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<sup>12</sup> This price difference corresponds to what Cellini, Ferreira, and Rothstein (2010) refer to in their empirical work as the “intent-to-treat” (ITT) effect of the investment on housing prices. It represents the reaction of housing prices to the investment *assuming that all future investment decisions will be made according to the community equilibrium*. They also discuss a “treatment on the treated” (TOT) effect which is the hypothetical reaction of housing prices to the investment *assuming there were no future investments*. As will be pointed out below in footnote #14, this paper’s critique also applies to this latter measure.

<sup>13</sup> Note that  $dS(g, 0)/dI = \partial S(g, 0)/\partial g + c\partial S(g, 0)/\partial T$ . However,  $\partial S(g, 0)/\partial T$  is equal to  $-1/H$ . Thus, if  $\partial^2 S(g, 0)/\partial g^2$  is negative,  $dS(g, 0)/dI$  is decreasing in  $g$ .



optimal, while a non-positive price difference signals that the level with investment is too high. This is the form of the housing price test that is most relevant in the durable good context.

### 3.3 The problem

The fundamental problem with the foregoing analysis lies in the assumption that the socially optimal public good level  $g^o$  satisfies the first order condition that  $dS(g^o, 0)/dI$  equals zero. This is true for the socially optimal surplus function  $S^o(g, T)$  but the *equilibrium* public good surplus function  $S(g, T)$  will not in general equal the *optimal* surplus function  $S^o(g, T)$ . The former assumes that future investment decisions are governed by the equilibrium rules  $\pi(g, I)$  and  $I(g)$ , while the latter assumes decisions are made optimally. As is well known from the theory of the second best, the fact that some decisions are not optimal typically means that the rules governing the decisions that can be optimized will change.

To see the difficulty formally, note from (8) that the derivative of surplus with respect to investment is

$$\frac{dS((1-\delta)g+I, 0)}{dI} = [B'((1-\delta)g+I) - c/H] + \beta \left[ \begin{aligned} & \frac{d\pi^*(\cdot)}{dg} \{S((1-\delta)[(1-\delta)g+I] + I(\cdot), cI(\cdot)) - S((1-\delta)[(1-\delta)g+I], 0)\} \\ & + \pi^*(\cdot) \left\{ \frac{\partial S((1-\delta)[(1-\delta)g+I] + I(\cdot), cI(\cdot))}{\partial g} [(1-\delta) + \frac{dI(\cdot)}{dg}] - c \frac{dI(\cdot)}{dg} / H \right\} \\ & + (1 - \pi^*(\cdot)) \frac{\partial S((1-\delta)[(1-\delta)g+I] + I(\cdot), cI(\cdot))}{\partial g} (1 - \delta) \end{aligned} \right] \quad (26)$$

where to compact notation  $\pi^*(\cdot)$  denotes  $\pi^*((1-\delta)g+I)$  and  $I(\cdot)$  denotes  $I((1-\delta)g+I)$ . The term in square brackets on the top line of (26) measures the immediate consequences of an increase in investment on surplus: public good benefits go up as do taxes. The second term measures the future consequences and these are evidently quite complicated. In particular, account must be taken of how an increase in investment will impact the level of tomorrow's investment (i.e.,  $dI(\cdot)/dg$ ) and also the probability that it passes (i.e.,  $d\pi^*(\cdot)/dg$ ).

A necessary condition for  $dS(g^o, 0)/dI$  to equal zero is that the second term in (26) equals  $\beta(1-\delta)c/H$  when evaluated at  $g^o$ . For only then will it be the case that

$$\frac{dS(g^o, 0)}{dI} = B'(g^o) - c[1 - \beta(1-\delta)]/H, \quad (27)$$

which, given the dynamic Samuelson Rule (15), is necessary for  $dS(g^o, 0)/dI$  to equal zero. Intuitively, when the second term in (26) equals  $\beta(1-\delta)c/H$ , the future consequence of a marginal

increase in investment today is just the discounted value of a compensating decrease in the amount of investment approved tomorrow. With depreciation, a unit of investment today creates  $1 - \delta$  of a unit tomorrow and so a compensating decrease would be  $1 - \delta$ . This would save each resident  $c(1 - \delta)/H$  in taxes and this has a present value of  $\beta(1 - \delta)c/H$ . Under an optimal investment plan, the second term in (26) will indeed equal  $\beta(1 - \delta)c/H$ . To see this, note from (14) that  $dI(\cdot)/dg = -(1 - \delta)$  and, since investments pass with probability one,  $\pi^*(\cdot) = 1$  and  $d\pi^*(\cdot)/dg = 0$ . In equilibrium, however, there is no reason to believe that the future consequences of an increase in investment will be so simple.

From equation (25), it is clear that if  $dS(g^o, 0)/dI$  exceeds zero, a non-negative price difference no longer implies that the public good level without investment is below optimal. Conversely, when  $dS(g^o, 0)/dI$  is less than zero, a non-positive price difference no longer implies that the public good level with investment is too high. To understand the potential bias in the housing price test, therefore, it is interesting to understand which of these cases is more likely to arise. The former (latter) case arises when the second term in (26) evaluated at  $g^o$  exceeds (falls short of)  $\beta(1 - \delta)c/H$ . Differencing the two expressions, we obtain

$$\beta \left[ \begin{aligned} & \frac{d\pi^*(g^o)}{dg} (S((1 - \delta)g^o + I(g^o), cI(g^o)) - S((1 - \delta)g^o, 0)) \\ & + \left( \frac{\partial S((1 - \delta)g^o + I(g^o), cI(g^o))}{\partial g} - c/H \right) \left( 1 - \delta + \frac{dI(g^o)}{dg} \right) \end{aligned} \right]. \quad (28)$$

When this expression is positive (negative), the future consequence of an increase in investment exceeds (falls short of)  $\beta(1 - \delta)c/H$ . The sign of the first term is unclear because the sign of  $d\pi^*(g^o)/dg$  is uncertain. While *ceteris paribus* having a higher public good level might be expected to reduce the probability of a proposed investment passing, it will also reduce the size of the proposed investment, so the net effect is uncertain. The sign of the second term is ambiguous because it is not clear how  $dI(g^o)/dg$  will compare with  $-(1 - \delta)$ : that is, will an increase in public good level lead to a more or less than compensating adjustment in investment? Moreover, even if that issue were resolved, the sign of the difference  $\partial S/\partial g - c/H$  is not obvious.<sup>14</sup>

All this suggests that to understand the bias in the housing price test, an explicit theory of

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<sup>14</sup> Suppose that we instead evaluated public good provision using the hypothetical housing price response to the investment assuming there were no future investments (the TOT effect discussed in footnote #12). With no future investments, public good surplus would be  $S(g, T) = \sum_{t=0}^{\infty} \beta^t B(g(1 - \delta)^t) - T/H$ . It is easily verified that with this surplus function,  $dS(g^o, 0)/dI$  exceeds zero. Intuitively, if no investment will take place in the future, the optimal public good level today will be much larger than  $g^o$ . As a consequence, a positive housing price response would not imply the public good level without investment was below optimal.

voter and bureaucrat behavior is necessary. The next section describes a simple but natural model in which  $dS(g^o, 0)/dI$  is positive and, as a consequence, the housing price test erroneously predicts under-provision.

## 4 The housing price test and the budget-maximizing bureaucrat

To illustrate the problem with the housing price test more concretely, we now analyze what it tells us when the investment path is generated by the interaction between a budget-maximizing bureaucrat and rational, forward-looking residents. More specifically, we assume that the probability  $\pi(g, I)$  and investment rule  $I(g)$  are generated by the equilibrium of the dynamic version of Romer and Rosenthal's agenda control model analyzed by Coate (2012). We derive the equilibrium value of public good surplus in this model and, using Proposition 1, show that whether or not an investment is approved makes no difference in the prices that prevail next period. The housing price test therefore suggests that the socially optimal public good level lies between the levels with and without investment. However, we will see that the socially optimal level can be smaller than the level without investment. This reflects the fact that  $dS(g^o, 0)/dI$  is positive.

Romer and Rosenthal's agenda control model studies the interaction between a budget-maximizing bureaucrat who manages the provision of a public good or service for a community and the residents of that community. The model assumes that the level of the public good is chosen by the bureaucrat but is subject to resident approval via a ratification vote. If the bureaucrat's proposed spending level is not approved by a majority, then spending reverts to an exogenously specified reversion level. A tension exists between the bureaucrat and residents, because the former cares just about the size of his budget, while the latter also care about costs. The equilibrium proposal depends on the reversion level and exceeds the median voter's preferred level whenever this is larger than the reversion level. Essentially, the bureaucrat exploits his agenda setting power to extract additional spending from the residents. In the dynamic version of this model studied by Coate, the bureaucrat manages the provision of a durable public good and chooses investment proposals that must be approved by the voters.<sup>15</sup> The main difference created by the durable

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<sup>15</sup> The model studied here differs from that studied by Coate (2012) in that it incorporates a housing market. This permits the implications of the equilibrium for housing price dynamics to be derived. Furthermore, the analysis here is limited to deriving the equilibrium public good surplus function, which is all that is necessary to evaluate the performance of the housing price test. By contrast, the point of Coate (2012) is to provide a comprehensive

public good is that the reversion level is simply the depreciated current level of the good rather than an exogenously set level.

In the dynamic agenda control model, residents approve the bureaucrat's investment proposal if it raises their expected continuation payoffs. As we saw in Section 2.3, each resident's continuation payoff is the sum of a constant and public good surplus (equation (12)). Thus, residents approve the bureaucrat's proposal if it raises public good surplus, implying that

$$\pi(g, I) = \begin{cases} 1 & \text{if } S((1 - \delta)g + I, cI) \geq S((1 - \delta)g, 0) \\ 0 & \text{if } S((1 - \delta)g + I, cI) < S((1 - \delta)g, 0) \end{cases}. \quad (29)$$

Letting  $U(g, T)$  denote the bureaucrat's value function, he chooses an investment proposal  $I(g)$  where

$$I(g) = \arg \max \left\{ \begin{array}{l} g + \beta U((1 - \delta)g + I, cI) \\ \text{s.t. } S((1 - \delta)g + I, cI) \geq S((1 - \delta)g, 0) \ \& \ I \geq 0 \end{array} \right\}. \quad (30)$$

The bureaucrat's value function is defined recursively by the equation

$$U(g, T) = g + \beta U((1 - \delta)g + I(g), cI(g)), \quad (31)$$

while from (8), the residents' public good surplus function is defined by

$$S(g, T) = B(g) - T/H + \beta S((1 - \delta)g + I(g), cI(g)). \quad (32)$$

An *equilibrium* of the dynamic agenda control model consists of an investment proposal function  $I(g)$  and value functions  $U(g, T)$  and  $S(g, T)$  satisfying equations (30), (31), and (32).

Coate (2012) focuses on a particular type of equilibrium, he terms a Romer-Rosenthal equilibrium. In Romer and Rosenthal's static model, equilibrium involves the bureaucrat proposing the largest level of public spending which leaves the median voter at least as well off as with the reversion level. This is just the reversion level if it exceeds the median voter's optimal level, and otherwise exceeds the reversion level. The Romer-Rosenthal equilibrium is the analogue to this in the dynamic setting. The defining feature is that in each period the bureaucrat proposes the maximum level of investment the residents will approve.<sup>16</sup>

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analysis of equilibrium in the dynamic agenda control model.

<sup>16</sup> This is as opposed to holding back in some period to boost the amount of investment that the residents approve in the next period.

To define the Romer-Rosenthal equilibrium concept formally, let  $g^*$  denote the residents' preferred level of the public good given the equilibrium value function  $S(g, T)$ ; i.e.,  $g^*$  maximizes  $S(g, cg)$ . When  $g$  exceeds  $g^*/(1 - \delta)$  the residents will prefer the reversion level  $g(1 - \delta)$  to any higher level. Accordingly, the bureaucrat can propose no investment. When  $g$  is less than  $g^*/(1 - \delta)$ , there exist investment levels that will be supported by the residents. In a Romer-Rosenthal equilibrium, the bureaucrat will choose the largest of these. Thus, he will choose an investment level  $I(g) > 0$  such that

$$S((1 - \delta)g + I, cI) = S((1 - \delta)g, 0). \quad (33)$$

Intuitively, at this investment level, the future benefits to the residents are just offset by the tax cost. Accordingly, an equilibrium  $(I(g), U(g, T), S(g, T))$  is a *Romer-Rosenthal equilibrium* if  $I(g)$  is zero when  $g$  exceeds  $g^*/(1 - \delta)$  and satisfies (33) otherwise.

In a Romer-Rosenthal equilibrium, the residents' public good surplus function takes a very simple form.<sup>17</sup> Notice that when  $g$  is less than  $g^*/(1 - \delta)$ , equation (33) implies that the strategy  $I(g)$  is such that surplus with the investment  $S((1 - \delta)g + I(g), cI(g))$  must equal surplus without  $S((1 - \delta)g, 0)$ . Substituting this equality into (32), we see that the surplus function satisfies

$$S(g, T) = B(g) - T/H + \beta S((1 - \delta)g, 0). \quad (34)$$

Moreover, equation (34) also holds when  $g$  exceeds  $g^*/(1 - \delta)$  since  $I(g)$  is zero. Applying equation (34) repeatedly, we conclude that in a Romer-Rosenthal equilibrium, the public good surplus function is

$$S(g, T) = \sum_{t=0}^{\infty} \beta^t B(g(1 - \delta)^t) - T/H. \quad (35)$$

Intuitively, the residents get the same level of surplus in equilibrium as they would do if there were never any more investment. This reflects the fact that the bureaucrat extracts all the surplus from any new investment.

With this information, we can now evaluate the performance of the housing price test. Note first that, if investment takes place, the difference in housing prices with and without the investment must be zero. This follows immediately from Proposition 1 and equation (33). The housing price test therefore implies that if  $g$  is such that investment takes place (i.e.,  $g < g^*/(1 - \delta)$ ) it must be

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<sup>17</sup> The residents' surplus function also takes this form in any equilibrium in which whenever the bureaucrat does invest, he proposes the maximum possible level.

the case that the socially optimal level  $g^o$  lies between  $(1 - \delta)g$  and  $(1 - \delta)g + I(g)$ .<sup>18</sup> However, this is incorrect since it is perfectly possible that  $g^o$  is less than  $(1 - \delta)g$ . To see why, note first that given the surplus function (35), it is easy to show that

$$\frac{dS(g^o, 0)}{dI} > 0 = \frac{dS(g^*, 0)}{dI}. \quad (36)$$

This implies that  $g^o$  is less than the residents' preferred level in equilibrium  $g^*$ . Intuitively, there is an additional benefit of investing in equilibrium: namely, higher public good levels reduce future exploitation by the bureaucrat. It follows that for all  $g$  between  $g^o/(1 - \delta)$  and  $g^*/(1 - \delta)$ , investment takes place but  $(1 - \delta)g$  exceeds  $g^o$ . This leads to the following proposition:

**Proposition 3** *Let  $(I(g), U(g, T), S(g, T))$  be a Romer-Rosenthal equilibrium. Then, if  $g$  is such that  $I(g) > 0$ , the housing price difference with and without the investment is zero. The housing price test therefore predicts that the socially optimal public good level  $g^o$  lies between  $(1 - \delta)g$  and  $(1 - \delta)g + I(g)$ . However, this is not the case. In particular, while  $(1 - \delta)g + I(g)$  always exceeds  $g^o$ , there exists an open interval of public good levels  $g$  with the property that  $I(g) > 0$  and  $(1 - \delta)g$  exceeds  $g^o$ .*

**Proof:** See Appendix A.

Proposition 3 tells us that, for a range of public good levels, the housing price test will provide misleading information. Specifically, it erroneously predicts that the public good level that would prevail without investment is below optimal. It is important to note that the quantitative magnitude of the discrepancy can be substantial. The difference between  $(1 - \delta)g$  and the optimal level  $g^o$  is increasing in  $g$  and converges to  $g^* - g^o$  as  $g$  approaches  $g^*/(1 - \delta)$ . Using (15) and (35), it is straightforward to compute the difference  $g^* - g^o$  for specific benefit functions. In the case of a quadratic benefit function (i.e.,  $B(g) = b_0g - b_1g^2$ ), for example, we have that

$$g^* = g^o \left( \frac{1 - \beta(1 - \delta)^2}{1 - \beta(1 - \delta)} \right). \quad (37)$$

Over the possible domain of parameters, the multiplicative coefficient on the right hand side of (37) ranges from one to two. Thus, the housing price test could tell us that  $(1 - \delta)g$  was less than  $g^o$  despite it being almost twice as big!

<sup>18</sup> We use the form of the housing price test that applies to large investments since equilibrium investments will only be small when the current level of the public good is close to  $g^*/(1 - \delta)$ .

One possible reaction to Proposition 3 is that it will not be very damning if the range of public good levels for which the housing price test provides false information do not arise on the equilibrium path. It would then be the case that the test would only give misleading results for initial public good levels that were for some reason out of equilibrium. While this is a reasonable point, it does not save the test. Coate (2012) shows that when benefits are quadratic, the equilibrium public good level converges to a unique steady state  $g_s$ . What this means is that, in the long run, in every period the community approves an investment of  $\delta g_s$ . Furthermore, he shows that for sufficiently low depreciation rates,  $(1 - \delta)g_s$  exceeds the optimal level  $g^o$ . Thus, the difficulty arises for the investments that the community makes repeatedly in long run equilibrium.

Another possible reaction is that the result is not compelling because the example has the property that investments are approved with probability one. On the one hand, this would seem to disqualify the example as an adequate description of reality. On the other, even if it were, the property implies that it would not be possible to empirically implement the housing price test. This reaction misses the point of the example. The point is to illustrate why the socially optimal public level need not maximize the equilibrium level of surplus and the difficulties this creates for the housing price test. The intuition revealed by the example is very general: namely, with public good levels chosen by a bureaucrat with stronger preferences than the residents, the bureaucrat will exploit a lower reversion level to force through higher investment levels. This effect increases the marginal value of the public good to residents which means that their preferred level in equilibrium exceeds the socially optimal level. It is also likely to be the case in reality that investment impacts the probability that future investments are passed, but this does not negate the first point. Rather it just reinforces the idea that there is no good reason to believe the housing price test.

To make this point more formally, it is useful to refer back to equation (28) which determines the direction of bias in the housing price test. In the example, the term on the top line is zero because the bureaucrat's proposals are passed with probability one. The second term is positive. On the one hand, from (33) and (35), it is clear that  $\partial S/\partial g$  is less than  $c/H$ . On the other, (33) and (35) also imply that  $dI/dg$  is less than  $-(1 - \delta)$  meaning that a higher public good level creates a more than compensating decrease in investment. It follows that the expression in equation (28) is positive, which is why the housing price test can falsely predict under-provision. If the example were extended so that the public good level influenced the probability of a proposal passing, the

term on the top line would come in to play. While the impact of this is unclear, there is no reason to believe that the whole expression would magically become zero and the bias in the housing price test would be eliminated.<sup>19</sup>

## 5 An adaptive expectations justification

The analysis so far has been conducted under the assumption that citizens have rational expectations concerning the future investment path in their community. Thus, they understand the dynamic environment they are in and correctly predict how investment in the public good will evolve over time. This is clear in the game-theoretic budget-maximizing bureaucrat example of Section 4. In the general model of Section 3, the assumption is reflected in the idea that the functions  $\pi(g, I)$  and  $I(g)$  represent accurate predictions about what is going to happen in the future. This section relaxes this assumption and interprets the functions  $\pi(g, I)$  and  $I(g)$  as simply representing citizens' beliefs about what will happen. A simple adaptive expectations model is adopted and is shown to provide one way of rationalizing the housing price test.

Specifically, suppose that citizens expect that whatever level of public good they observe in the community at the beginning of a period will be maintained indefinitely. Thus, they observe the current quantity and quality of school facilities, say, and just assume they are at steady state levels. This is a form of myopia that is perhaps not too implausible, particularly for new citizens moving into a community. Formally, this assumption means that when the initial level of public good is  $g$ , citizens expect that investment  $I(g)$  will equal  $\delta g$  and the probability of passing this investment  $\pi(g, \delta g)$  is equal to 1. These rules imply that citizens believe that the present value of public good surplus is given by

$$S(g, T) = B(g) - T/H + \frac{\beta}{1 - \beta} [B(g) - c\delta g/H]. \quad (38)$$

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<sup>19</sup> It would of course be interesting to incorporate random election outcomes into the budget-maximizing bureaucrat example. Such an extension would shed light on exactly how the impact of investment on the future probability of proposals passing (i.e., the top line of equation (28)) influences the bias of the housing price test. Unfortunately, introducing randomness significantly complicates the example and it is no longer possible to solve for the equilibrium surplus function in closed form. Indeed, it is not even clear that it is possible to solve for it numerically. The interaction between the bureaucrat and the residents defines a dynamic game and the surplus function  $S(g, T)$  is the equilibrium value function for the residents. As is well known, such value functions are very difficult to compute in dynamic games and typically cannot be solved for in closed form. What allows us to derive the residents' equilibrium value function so simply (i.e., in equation (35)), is the fact that the bureaucrat is pinning the residents to their reservation utility. If the bureaucrat were unsure of the election outcome, then he would no longer want to do this and this is what makes things much more complicated. These complications are explained in Appendix B which introduces randomness in election outcomes via probabilistic voting.



It then follows that the derivative of surplus with respect to investment is

$$\frac{dS((1-\delta)g+I, 0)}{dI} = \frac{B'((1-\delta)g+I) - c[1-\beta(1-\delta)]/H}{1-\beta}. \quad (39)$$

In particular, therefore, it is the case that  $dS(g^o, 0)/dI$  is equal to zero. Accordingly, the housing price test will work as advertised. The intuition is straightforward: a successful investment is interpreted as creating a permanent increase in public good levels and hence we are back in a world where the standard logic applies.

In our judgement, this represents the best way to justify the housing price test. But still there is an important caveat. Observe that

$$\frac{dS((1-\delta)g+I, cI)}{dI} = \left(\frac{1}{1-\beta}\right) \left(\frac{dS^o((1-\delta)g+I, cI)}{dI}\right). \quad (40)$$

The multiplicative factor  $1/(1-\beta)$  means that the change in housing prices over-estimates the willingness to pay for investments. Intuitively, this reflects the fact that, under adaptive expectations, citizens interpret a small increase in investment as signalling a permanent increase in the level of the public good. If the good is under-provided, this permanent increase will have a considerably higher value to citizens than a temporary one.

## 6 Conclusion

This paper has explored the theoretical foundation for using the housing price response to investments to evaluate local durable public good provision. This is a potentially very useful technique, as the recent analysis of Cellini, Ferreira, and Rothstein demonstrates. Unfortunately, the exploration reveals that there is scant justification for the idea under the assumption that residents and potential residents have rational expectations about the future investment path in their communities. The most compelling way of justifying the test appears to be to assume that residents have adaptive expectations. However, even with such a non-standard assumption, caution must be exercised in interpreting the test.

These findings are unquestionably disappointing. Nonetheless, the idea of using the housing price response to investments to infer something about the efficiency of durable public good provision still appears promising. It is just that a more structural approach may be necessary to exploit this connection. Ultimately, inefficient provision must be driven either by bureaucratic objectives or by resident heterogeneity. Different bureaucratic objectives, for example, may have different

implications for the behavior of housing prices over the investment cycle. If so, information about objectives and thus efficiency may be recovered from housing price dynamics.

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## 7 Appendix A: Proofs

### 7.1 Proof of Proposition 1

Consider a period in which the community's level of public good is  $g$  and its tax obligation is  $T$ . Recall that the supply of housing is fixed at  $H$ . Potential residents just differ in their preference for living in the community  $\theta$ . Clearly, those with higher  $\theta$  will have a greater willingness to pay to live in the community. Thus, in equilibrium, the fraction  $H$  of potential residents with the highest preference parameters will live in the community. Since

$$\frac{\bar{\theta} - \bar{\theta}(1 - H)}{\bar{\theta}} = H,$$

the marginal resident will have preference parameter  $\bar{\theta}(1 - H)$ . This will be the case in each and every period irrespective of the community's level of public good and tax obligation. It follows that, in equilibrium, potential residents with types  $\theta \in [0, \bar{\theta}(1 - H))$  never reside in the community. For these types, therefore,

$$V_{\theta}(g, T) = \frac{u}{1 - \beta},$$

which yields equation (10) of Proposition 1. Types  $\theta \in [\bar{\theta}(1 - H), \bar{\theta}]$ , on the other hand, will reside in the community as long as they remain in the pool of potential residents. For these types, therefore, irrespective of  $g$  and  $T$

$$V_{\theta}(g, T) = \theta + B(g) - T/H - P(g, T) + \beta EP(g', T') + \beta[\mu EV_{\theta}(g', T') + (1 - \mu)\frac{u}{1 - \beta}].$$

We now show that the value functions of the resident households can be written as equation (9) of Proposition 1. Let future periods be indexed by  $t = 1, \dots, \infty$  and let  $(g_t, T_t)$  denote the public good level and tax obligation in period  $t = 1, \dots, \infty$ . If  $\theta \in [\bar{\theta}(1 - H), \bar{\theta}]$ , we know that

$$V_{\theta}(g, T) = \theta + \beta(1 - \mu)\frac{u}{1 - \beta} + B(g) - T/H - P(g, T) + \beta E[P(g_1, T_1) + \mu V_{\theta}(g_1, T_1)], \quad (41)$$

where expectations are taken over the possible values of  $(g_1, T_1)$ ; that is,  $((1 - \delta)g + I(g), cI(g))$  and  $((1 - \delta)g, 0)$ . But, since the household will reside in the community in period 1 if it remains in the pool, we also know that

$$\begin{aligned} \beta[P(g_1, T_1) + \mu V_{\theta}(g_1, T_1)] &= \beta(1 - \mu)P(g_1, T_1) \\ &+ \beta\mu \left[ \theta + \beta(1 - \mu)\frac{u}{1 - \beta} + B(g_1) - T_1/H + \beta E\{P(g_2, T_2) + \mu V_{\theta}(g_2, T_2)\} \right]. \end{aligned}$$

Moreover, period 1's housing price  $P(g_1, T_1)$  satisfies the equilibrium condition

$$1 - \frac{\underline{u} - (B(g_1) - T_1/H - P(g_1, T_1) + \beta EP(g_2, T_2))}{\bar{\theta}} = H,$$

which implies that

$$P(g_1, T_1) = \bar{\theta}(1 - H) - \underline{u} + B(g_1) - T_1/H + \beta EP(g_2, T_2).$$

Substituting this into the above expression, we can write

$$\begin{aligned} \beta [P(g_1, T_1) + \mu V_\theta(g_1, T_1)] &= \beta(1 - \mu) [\bar{\theta}(1 - H) - \underline{u} + B(g_1) - T_1/H + \beta EP(g_2, T_2)] \\ &\quad + \beta\mu \left[ \theta + \beta(1 - \mu) \frac{\underline{u}}{1 - \beta} + B(g_1) - T_1/H + \beta E \{P(g_2, T_2) + \mu V_\theta(g_2, T_2)\} \right] \\ &= \kappa_1(\theta) + \beta [B(g_1) - T_1/H] + \beta^2 E [P(g_2, T_2) + \mu^2 V_\theta(g_2, T_2)], \end{aligned}$$

where

$$\kappa_1(\theta) = \beta \left\{ (1 - \mu) [\bar{\theta}(1 - H) - \underline{u}] + \mu\theta + \beta\mu(1 - \mu) \frac{\bar{u}}{1 - \beta} \right\}.$$

Again, since the household will reside in the community in period 1 if it remains in the pool, we also know that

$$\begin{aligned} \beta^2 [P(g_2, T_2) + \mu^2 V_\theta(g_2, T_2)] &= \beta^2(1 - \mu^2) P(g_2, T_2) \\ &\quad + \beta^2 \mu^2 \left[ \theta + \beta(1 - \mu) \frac{\underline{u}}{1 - \beta} + B(g_2) - T_2/H + \beta E \{P(g_3, T_3) + \mu V_\theta(g_3, T_3)\} \right]. \end{aligned}$$

Equilibrium in the housing market implies that

$$P(g_2, T_2) = \bar{\theta}(1 - H) - \underline{u} + B(g_2) - T_2/H + \beta EP(g_3, T_3).$$

Substituting this in, we can write

$$\begin{aligned} \beta^2 [P(g_2, T_2) + \mu^2 V_\theta(g_2, T_2)] &= \beta^2(1 - \mu^2) (\bar{\theta}(1 - H) - \underline{u} + B(g_2) - T_2/H + \beta EP(g_3, T_3)) \\ &\quad + \beta^2 \mu^2 \left[ \theta + \beta(1 - \mu) \frac{\underline{u}}{1 - \beta} + B(g_2) - T_2/H + \beta E \{P(g_3, T_3) + \mu V_\theta(g_3, T_3)\} \right] \\ &= \kappa_2(\theta) + \beta^2 [B(g_2) - T_2/H] + \beta^3 E [P(g_3, T_3) + \mu^3 V_\theta(g_3, T_3)] \end{aligned}$$

where

$$\kappa_2(\theta) = \beta^2 \left\{ (1 - \mu^2) [\bar{\theta}(1 - H) - \underline{u}] + \mu^2 \left[ \theta + \beta(1 - \mu) \frac{\underline{u}}{1 - \beta} \right] \right\}.$$

By similar logic, for all periods  $t \geq 3$ , we have that

$$\beta^t [P(g_t, T_t) + \mu^t V_\theta(g_t, T_t)] = \kappa_t(\theta) + \beta^t [B(g_t) - T_t/H] + \beta^{t+1} E [P(g_{t+1}, T_{t+1}) + \mu^{t+1} V_\theta(g_{t+1}, T_{t+1})]$$

where

$$\kappa_t(\theta) = \beta^t \left\{ (1 - \mu^t) [\bar{\theta}(1 - H) - \underline{u}] + \mu^t \left[ \theta + \beta(1 - \mu) \frac{\underline{u}}{1 - \beta} \right] \right\}.$$

Successively substituting these expressions into (41), reveals that

$$\begin{aligned} V_{\bar{\theta}}(g, T) &= \theta + \beta(1 - \mu) \frac{\underline{u}}{1 - \beta} + B(g) - T/H - P(g, T) + \sum_{t=1}^{\infty} E \left\{ \kappa_t(\theta) + \beta^t (B(g_t) - T_t/H) \right\} \\ &= \sum_{t=0}^{\infty} \kappa_t(\theta) + S(g, T) - P(g, T). \end{aligned}$$

Letting  $\kappa(\theta) = \sum_{t=0}^{\infty} \kappa_t(\theta)$  yields equation (9).

It remains to show that equation (11) is satisfied. In equilibrium, it must be the case that the marginal household, which is the household with preference  $\bar{\theta}(1 - H)$ , is just indifferent between residing in the community or not. Thus, it must be the case that

$$\kappa(\bar{\theta}(1 - H)) + S(g, T) - P(g, T) = \frac{\bar{u}}{1 - \beta}.$$

This implies that

$$P(g, T) = \kappa(\bar{\theta}(1 - H)) - \frac{\bar{u}}{1 - \beta} + S(g, T).$$

Letting

$$K = \kappa(\bar{\theta}(1 - H)) - \frac{\bar{u}}{1 - \beta},$$

yields equation (11). ■

## 7.2 Proof of Proposition 2

Letting  $g'$  denote next period's public good level (i.e.,  $g' = (1 - \delta)g + I$ ), we have that

$$S^o(g, T) = \max_{g' \geq (1 - \delta)g} B(g) - T/H + \beta S^o(g', c(g' - (1 - \delta)g)). \quad (42)$$

Moreover, letting  $g'(g)$  denote the optimal policy function for (42), we have that  $I^o(g)$  is equal to  $g'(g) - (1 - \delta)g$ . Now note from (42) that

$$\frac{\partial S^o(g, T)}{\partial g} = B'(g) - \beta(1 - \delta)c \frac{\partial S^o(g', c(g' - (1 - \delta)g))}{\partial T}, \quad (43)$$

and that

$$\frac{\partial S^o(g, T)}{\partial T} = -1/H. \quad (44)$$

The first order condition for the optimal policy  $g'$  is that

$$\frac{\partial S^o(g', c(g' - (1 - \delta)g))}{\partial g} + c \frac{\partial S^o(g', c(g' - (1 - \delta)g))}{\partial T} \leq 0 \quad (= \text{ if } g' > (1 - \delta)g).$$

Using (43) and (44), this can be rewritten as

$$B'(g') - \beta(1 - \delta)c/H \leq c/H \quad (= \text{ if } g' > (1 - \delta)g).$$

Thus, if  $g' > (1 - \delta)g$

$$HB'(g') = c(1 - \beta(1 - \delta)).$$

Letting  $g^o$  satisfy the dynamic Samuelson Rule  $HB'(g^o) = c[1 - \beta(1 - \delta)]$ , we conclude that the optimal policy function is

$$g'(g) = \begin{cases} g^o & \text{if } g \leq g^o/(1 - \delta) \\ (1 - \delta)g & \text{if } g > g^o/(1 - \delta) \end{cases}.$$

Since  $I^o(g)$  is equal to  $g'(g) - (1 - \delta)g$ , Proposition 2 follows immediately.  $\blacksquare$

### 7.3 Proof of Proposition 3

If  $g$  is such that investment takes place, then we know that  $g \in (0, g^*/(1 - \delta))$ . That the housing price difference with and without the investment is zero follows from Proposition 1 and the fact that surplus with the investment must equal that without the investment (by equation (33)). To see that  $(1 - \delta)g + I(g)$  must exceed  $g^o$ , note from equation (35) that

$$\sum_{t=0}^{\infty} \beta^t B([(1 - \delta)g + I(g)](1 - \delta)^t) - cI(g)/H = \sum_{t=0}^{\infty} \beta^t B([(1 - \delta)g](1 - \delta)^t).$$

This implies that

$$\sum_{t=0}^{\infty} \beta^t [B([(1 - \delta)g + I(g)](1 - \delta)^t) - B([(1 - \delta)g](1 - \delta)^t)] = cI(g)/H.$$

Since  $B(\cdot)$  is strictly concave, it is clear that

$$\sum_{t=0}^{\infty} \beta^t [B'([(1 - \delta)g + I(g)](1 - \delta)^t)](1 - \delta)^t I(g) < cI(g)/H.$$

But we have that

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t [B'([(1 - \delta)g + I(g)](1 - \delta)^t)](1 - \delta)^t I(g) &> \sum_{t=0}^{\infty} [\beta(1 - \delta)]^t B'((1 - \delta)g + I(g))I(g) \\ &= \frac{B'((1 - \delta)g + I(g))I(g)}{1 - \beta(1 - \delta)}. \end{aligned}$$

Thus, we have that

$$\frac{B'((1 - \delta)g + I(g))}{1 - \beta(1 - \delta)} < c/H,$$



or, equivalently,

$$HB'((1 - \delta)g + I(g)) < c[1 - \beta(1 - \delta)].$$

Given the definition of  $g^o$ , this implies that  $(1 - \delta)g + I(g)$  must exceed  $g^o$ .

It remains to show that there exists a public good level  $\tilde{g}$  strictly less than  $g^*/(1 - \delta)$  such that for any  $g$  in the interval  $(\tilde{g}, g^*/(1 - \delta))$ ,  $(1 - \delta)g$  exceeds  $g^o$ . By definition,  $g^*$  maximizes  $S(g, cg)$ . Given (35), this implies that

$$g^* = \arg \max \sum_{t=0}^{\infty} \beta^t B(g(1 - \delta)^t) - cg/H.$$

This in turn implies that  $g^*$  satisfies the first order condition

$$\sum_{t=0}^{\infty} [\beta(1 - \delta)]^t B'(g^*(1 - \delta)^t) = c/H.$$

It follows from this that

$$\sum_{t=0}^{\infty} [\beta(1 - \delta)]^t B'(g^*) < c/H,$$

which implies that

$$HB'(g^*) < c[1 - \beta(1 - \delta)] = HB'(g^o).$$

It follows that  $g^* > g^o$ , which means that for all  $g \in (g^o/(1 - \delta), g^*/(1 - \delta))$ ,  $(1 - \delta)g$  exceeds  $g^o$ .

The proof is now complete. ■

## 8 Appendix B: Probabilistic voting<sup>20</sup>

The simplest way to extend the budget-maximizing bureaucrat example to make election outcomes uncertain is to follow the probabilistic voting approach to voting behavior. Under this approach, a household votes for a proposal if the difference in his continuation payoff with and without the proposed investment exceeds the value of a voting preference shock. This shock consists of an aggregate component  $\eta$  and an idiosyncratic component  $\epsilon$ . For each household, the idiosyncratic component is the realization of a random variable uniformly distributed on  $[-\bar{\epsilon}, \bar{\epsilon}]$  and the aggregate component is the realization of a random variable uniformly distributed on  $[-\bar{\eta}, \bar{\eta}]$ . The parameters  $\bar{\epsilon}$  and  $\bar{\eta}$  are assumed to be such as to ensure that the probabilities that each household votes for or against the proposal are both positive.

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<sup>20</sup> This Appendix is not intended for publication.

Formally, with probabilistic voting, when the state is  $(g, T)$ , a resident household of type  $\theta$  votes in favor of investment proposal  $I$  if

$$P((1 - \delta)g + I, cI) + \mu V_\theta((1 - \delta)g + I, cI) - [P((1 - \delta)g, 0) + \mu V_\theta((1 - \delta)g, 0)] \geq \epsilon + \eta. \quad (45)$$

Given Proposition 1, this simplifies to

$$S((1 - \delta)g + I, cI) - S((1 - \delta)g, 0) \geq \epsilon + \eta. \quad (46)$$

Under the assumption that  $\epsilon$  is distributed uniformly on  $[-\bar{\epsilon}, \bar{\epsilon}]$ , the fraction of voters voting in favor of the proposal given the aggregate shock  $\eta$  is

$$\frac{S((1 - \delta)g + I, cI) - S((1 - \delta)g, 0) - \eta}{2\bar{\epsilon}} + \frac{1}{2}. \quad (47)$$

Under majority rule, the proposed investment is approved when this fraction exceeds 1/2. The probability the proposed investment passes is therefore the probability that

$$S((1 - \delta)g + I, cI) - S((1 - \delta)g, 0) \geq \eta. \quad (48)$$

Given the assumption that  $\eta$  is distributed uniformly on  $[-\bar{\eta}, \bar{\eta}]$ , this probability is

$$\pi(g, I) = \frac{S((1 - \delta)g + I, cI) - S((1 - \delta)g, 0)}{2\bar{\eta}} + \frac{1}{2}. \quad (49)$$

This expression implies a simple relationship between the probability of a proposal passing and the surplus it generates for residents. Moreover, it is a straightforward generalization of the expression in (29).

Letting the bureaucrat's value function be denoted  $U(g, T)$  as before, the bureaucrat will choose a proposed investment to solve the problem

$$\max_{I \geq 0} g + \pi(g, I)U((1 - \delta)g + I, cI) + (1 - \pi(g, I))U((1 - \delta)g, 0). \quad (50)$$

The solution to this problem yields the investment proposal function  $I(g)$ . The bureaucrat's value function then satisfies the functional equation

$$U(g, T) = g + \pi^*(g)U((1 - \delta)g + I(g), cI(g)) + (1 - \pi^*(g))U((1 - \delta)g, 0). \quad (51)$$

An equilibrium with probabilistic voting consists of an investment proposal function  $I(g)$ , a proposal approval probability function  $\pi(g, I)$ , and value functions  $U(g, T)$  and  $S(g, T)$  such that:

i)  $I(g)$  solves problem (50); ii)  $\pi(g, I)$  satisfies equation (49); iii)  $S(g, T)$  satisfies equation (8); and iv)  $U(g, T)$  satisfies equation (51).

This definition can be simplified somewhat by using (49) and the first order condition for problem (50). From the latter, we know that the investment proposal function  $I(g)$  satisfies

$$\begin{aligned} & \frac{\partial \pi(g, I(g))}{\partial I} (U((1-\delta)g + I(g), cI(g)) - U((1-\delta)g, 0)) + \\ & \pi(g, I(g)) \frac{dU((1-\delta)g + I(g), cI(g))}{dI} \leq 0 \quad (= \text{ if } I(g) > 0). \end{aligned} \quad (52)$$

Using (49), we can write this as

$$\begin{aligned} & \frac{dS((1-\delta)g + I(g), cI(g))}{dI} \left( \frac{U((1-\delta)g + I(g), cI(g)) - U((1-\delta)g, 0)}{2\bar{\eta}} \right) \\ & + \left( \frac{S((1-\delta)g + I(g), cI(g)) - S((1-\delta)g, 0)}{2\bar{\eta}} + \frac{1}{2} \right) \frac{dU((1-\delta)g + I(g), cI(g))}{dI} \leq 0 \quad (= \text{ if } I(g) > 0). \end{aligned} \quad (53)$$

We can also use (49) to write equations (8) and (51) as

$$\begin{aligned} S(g, T) = & B(g) - T/H + \frac{S((1-\delta)g + I(g), cI(g)) + S((1-\delta)g, 0)}{2} \\ & + \frac{(S((1-\delta)g + I(g), cI(g)) - S((1-\delta)g, 0))^2}{2\bar{\eta}}, \end{aligned} \quad (54)$$

and

$$\begin{aligned} U(g, T) = & g + \left( \frac{S((1-\delta)g + I(g), cI(g)) - S((1-\delta)g, 0)}{2\bar{\eta}} + \frac{1}{2} \right) U((1-\delta)g + I(g), cI(g)) \\ & + \left( \frac{1}{2} - \frac{S((1-\delta)g + I(g), cI(g)) - S((1-\delta)g, 0)}{2\bar{\eta}} \right) U((1-\delta)g, 0). \end{aligned} \quad (55)$$

We can now define an equilibrium to be an investment proposal function  $I(g)$  and value functions  $U(g, T)$  and  $S(g, T)$  satisfying (53), (54), and (55).

It should be clear that solving these three equations for the equilibrium functions  $I(g)$ ,  $U(g, T)$ , and  $S(g, T)$  is going to be challenging. However, it does seem possible that an iterative procedure might permit the solution to be obtained numerically. The  $t$ th round of this iteration would begin with value functions  $U_{t-1}(g, T)$  and  $S_{t-1}(g, T)$ . These would then be used with (53) to obtain the associated investment proposal function  $I_t(g)$ . This would be substituted into (54) to solve for a new surplus function  $S_t(g, T)$ . This, together with  $I_t(g)$ , would be substituted in to (55) to solve for the new bureaucrat's value function  $U_t(g, T)$ . If this process converged, we would have a solution. Unfortunately, however, it is not clear that the functions involved are well behaved. As shown in Coate (2012), in the dynamic agenda control model, even without probabilistic voting, the bureaucrat's value function is not concave.