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AGGREGATE SAVINGS IN THE PRESENCE OF PRIVATE AND SOCIAL INSURANCE

Andrew B. Abel

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ABSTRACT

In the presence of uncertain lifetimes, social security has the characteristics of an annuity: a consumer pays a tax when young in exchange for receiving a social security benefit if he survives to be old. If consumers have identical ex ante mortality probabilities, then a fully funded social security system would offer a rate of return equal to the actuarially fair rate available on competitively supplied private annuities. In this case fully funded social security would be a redundant asset and would have no effect on consumption or national saving.

In this paper, consumers have different (publicly known) ex ante mortality probabilities and consequently can buy actuarially fair private annuities offering different rates of return. If the social security system does not discriminate on the basis of ex ante mortality probabilities, then the introduction of social security induces a redistribution of income from consumers with a high probability of dying young to consumers with a low probability of dying young. Under homothetic utility this redistribution reduces aggregate bequests and aggregate consumption of young consumers in the steady state; the steady state national capital stock can either increase or decrease. If consumers display at least as much risk aversion as the logarithmic utility function, then average steady state welfare is increased by the introduction of fully funded social security.

Andrew Abel
Department of Economics
Littauer Center 111
Harvard University
Cambridge, MA 02138

I. Introduction

Over the last decade there has been a lively debate over the effects of social security on consumption and capital accumulation. This debate has distinguished between a pay-as-you-go social security system and a fully funded social security system. A balanced-budget pay-as-you-go system levies taxes on young consumers and uses the tax revenue to pay the social security benefits of old consumers. In a standard overlapping generations model in which consumers have no bequest motive (e.g. Diamond (1965)), the saving of young consumers is reduced both by the tax they pay when they are young and by the benefit they receive when they are old. Thus, the private (and national) capital stock is reduced by the introduction of pay-as-you-go social security (See Feldstein (1974)). However, if consumers obtain utility from the utility of their heirs, as well as from their own consumption, then the introduction of pay-as-you-go social security will not affect consumption or private capital accumulation; as shown by Barro (1974), consumers will adjust their bequests in order to offset the lump-sum intergenerational transfers imposed by the social security system.

In contrast to pay-as-you-go social security, the introduction of fully funded social security has no effect on consumption or the national capital stock, regardless of whether or not consumers have bequest motives. The reason for the irrelevance of fully funded social security is that the implicit rate of return on social security is the same as the rate of return on private wealth. Consumers will offset an increase in social security taxes and benefits by reducing private saving while maintaining unchanged consumption.

In order for fully funded social security to affect consumption and the national capital stock, the rate of return on social security must differ from the rate of return on privately traded assets. If consumers have random dates of death, then a social security system which taxes young consumers and gives benefits only to consumers who survive to old age has the characteristics of an annuity. The gross rate of return on this publicly provided non-tradable annuity will exceed the rate of return on private assets which are not contingent on survival. If there is no private annuity market, and if consumers have no bequest motive, then the introduction of fully funded social security reduces the steady state national capital stock and narrows the distribution of wealth (See Abel (1985a)). However, if there were a competitive annuity market, then the rate of return on private annuities would equal the rate of return on social security and hence social security would have no effect.

The possibility for fully funded social security to have an effect on the consumption and portfolio decisions in the presence of a competitive annuity market arises when we introduce heterogeneous ex ante mortality probabilities. If, as in actual practice, the social security system does not discriminate across individuals in a cohort according to the probability of death, then social security has real effects under two alternative information structures. First, if annuity companies know the ex ante mortality probability of each individual, then a competitive annuity market will provide annuities with different rates of return to consumers with different mortality probabilities. Clearly, in such a case, some individuals must face different rates of return on private annuities and on social security. In general, the consumption and investment decisions of these people will be affected by changes in social security. Alternatively, if an individual's ex ante mortality probability is private information observable only by that individual, then heterogeneity introduces adverse selection into the private annuity market. However, the social security system is immune to the problem of adverse selection because of the compulsory nature of social security taxes and benefits. Again, for at least some consumers, the rate of return on social security differs from that on private annuities so that the consumption and portfolio decisions of these people are affected by changes in the level of social security. In this paper, I assume the first structure of information i.e., public information; in a companion paper (Abel (1985b)) I make the alternative assumption of private information.

In order to analyze the effects of social security on steady state welfare, it is not sufficient to determine the effect on the steady state capital stock, especially in an economy with heterogeneous consumers. The effects on the aggregate capital stock and aggregate welfare can be in opposite directions for two reasons: first, well-known Golden Rule considerations imply that aggregate consumption and the aggregate capital stock can move in opposite directions; second, and more importantly, it will be shown that social security narrows the steady state cross-sectional distribution of consumption which tends to increase average steady state welfare.

Much of the existing literature on uncertain lifetimes examines the consumption and portfolio behavior of an individual, taking as given any wealth received by the individual in the form of bequests.[1] In the next two sections of this paper, I also analyze the individual consumer's decision problem. In section II, I state and solve the consumption and portfolio decision problem of a consumer who lives for either one period or two periods and who can hold his wealth in the form of riskless bonds and actuarially fair annuities. As a step toward analyzing the aggregate behavior of heterogeneous consumers, I show in section III that consumption and bequests are increasing functions of expected lifetime wealth. In addition, for a given value of expected lifetime wealth, consumption and bequests are increasing functions of the probability of dying young.

After examining individual behavior, I then analyze steady state behavior allowing for the endogenous adjustment of bequests using an extension to uncertain lifetimes of the Modigliani-Brumberg (1954) - Samuelson (1958) - Diamond (1965) overlapping generations model. Previously, Abel (1985a,b), Eckstein, Eichenbaum and Peled (1985) and Sheshinski and Weiss (1981) have examined uncertain lifetimes in an overlapping generations framework. Sheshinski and Weiss (1981) assumed that all consumers in a given cohort have identical ex post mortality experiences whereas Abel (1985a,b) and Eckstein, Eichenbaum and Peled (1985) allow for consumers with the same ex ante mortality probabilities to have different mortality experiences ex post. By allowing ex post mortality experiences to differ, these models generate intra-cohort variation in bequests and have implications for the intergenerational transmission of inequality. In this paper, I allow for different ex post mortality experiences, but the presence of actuarially fair annuities eliminates the intra-cohort variation in bequests received and left by members of a given cohort with identical ex ante mortality probabilities.[2]

In section IV, I analyze steady state consumption and bequest behavior and present sufficient conditions for the existence of a unique steady state. Then I demonstrate that the steady state values of consumption and bequests are higher for families with a high probability of dying young. In addition, I show that fully funded social security narrows the steady-state intra-cohort distributions of consumption and bequests. In section V, I restrict the analysis to homothetic utility functions and show that fully funded social security reduces steady state aggregate bequests and steady state aggregate consumption of young consumers. The steady state aggregate private capital stock is crowded out by a degree greater than, equal to, or less than one-for-one depending on whether the steady state consumption of young consumers is less than, greater than, or equal to the inheritances received. Then in section VI, I show that if utility function is sufficiently concave (more concave than logarithmic utility), then average steady state welfare is increased by the introduction of fully funded social security.

II. Consumption and Portfolio Behavior of an Individual

In this section we analyze the consumption and portfolio behavior of an individual consumer who does not know when he will die. We show that if the consumer can buy actuarially fair annuities, and if the utility from leaving a bequest is independent of the consumer's date of death, then the consumer will leave the same bequest whether he dies young or old. Furthermore, as shown by Sheshinski and Weiss (1981), the consumer will hold riskless bonds to provide for his bequests and will hold annuities to provide for consumption when old.

Consider consumers with the following life-cycle of events: At birth, each consumer receives an initial inheritance I from his parent. During the first period of his life the consumer earns a fixed labor income Y, pays a social security tax ($T<Y$) and consumes an amount c_1. At the end of the first period, the consumer selects a portfolio to carry his wealth, $I+Y-T-c_1$, to the next period. There are two assets: an actuarially fair annuity and a riskless bond. One unit of output invested in the annuity yields A units of output to the consumer if the consumer survives to the second period; if the consumer dies after one period, his estate receives nothing from the annuity. Let Q denote the number of units of output that the consumer invests in an annuity. The consumer invests the remainder of his wealth $I+Y-T-c_1-Q$ in a riskless bond which pays a gross rate of return R to the consumer if he survives, or to his estate if he dies after one period.

At the beginning of the second period, the consumer gives birth to $G\geq 1$ children. There is a probability p that the consumer dies at the beginning of the second period after giving birth to G heirs. If the consumer dies at the beginning of the second period, each of his heirs receives a bequest B^D/G, where B^D is equal to the consumer's riskless bonds with accrued interest,

$$B^D = (I+Y-T-c_1-Q)R \tag{1}$$

If the consumer survives in the second period, he receives a social security payment S ($S \geq 0$) in addition to the principal and interest on his portfolio of bonds and annuities. The consumer then consumes an amount c_2 and gives the remainder of wealth, B^S, to his heirs, where

$$B^S = (I+Y-T-c_1-Q)R + QA + S - c_2 \tag{2}$$

This total bequest, B^S, is divided equally among the consumer's G children.

At the end of the second period, the consumer dies. Because we have assumed that the consumer does not live beyond the second period, all uncertainty is resolved at the beginning of the second period. Therefore, the bequest, B^S, can be given to the consumer's heirs at the beginning of the second period, i.e., at the beginning of the first period of his heirs' lives. Thus, we can assume, as stated above, that all inheritances are received at birth.[3]

The consumer's utility function is assumed to be additively separable. In particular, the utility function is specified as

$$U(c_1) + (1-p)\delta U(c_2) + p\delta V(B^D) + (1-p)\delta V(B^S) \tag{3}$$

where $\delta>0$ is the one-period discount factor, U( ) is the utility index of the consumer's own consumption and V( ) is the index of utility derived from leaving a bequest. We assume that U( ) and V( ) are strictly concave and satisfy the Inada conditions $\lim_{c\to 0} U'(c) = \infty = \lim_{B\to 0} V'(B)$ and $\lim_{c\to\infty} U'(c) = 0 = \lim_{B\to\infty} V'(B)$.

The utility function in (3) can be viewed simply as the expected value of utility where the only uncertain element is the consumer's date of death. Since the utility function is a function of the bequest left to the consumer's heirs, it is an example of what Yaari (1965) has called a "Marshall utility function".[4,5]

The consumer's optimization problem is to maximize (3) subject to (1) and (2). Substituting (1) and (2) into (3) and then differentiating with respect to c_1, c_2, and Q, respectively, yields

$$U'(c_1) = \delta R[pV'(B^D) + (1-p)V'(B^S)] \tag{4a}$$

$$U'(c_2) = V'(B^S) \tag{4b}$$

$$pRV'(B^D) = (1-p)(A-R)V'(B^S) \tag{4c}$$

We now assume that annuities are actuarially fair which implies that

$$R = (1-p)A \tag{5}$$

That is, the expected return on an annuity is equal to the return on a riskless bond. Substituting (5) into (4c) yields

$$V'(B^D) = V'(B^S) \tag{6}$$

The strict concavity of V( ) then implies that $B^D = B^S$. Let $B = B^D = B^S$ denote the optimal level of bequests. Since $B^S = B^D$, it follows immediately from equations (1) and (2) that

$$c_2 = QA + S \tag{7}$$

Thus, in the presence of a market for actuarially fair annuities, annuities are used to provide for second-period consumption and riskless bonds are used to provide for bequests, and shown by Sheshinski and Weiss

(1981). The interpretation of (7) that second-period consumption is equal to the payoffs from annuities recognizes that the social security payment S is contingent on survival and thus is appropriately viewed as an annuity. It is clear from (7) that if the social security benefit S is less than second-period consumption c_2, the consumer will hold a positive amount of annuities. Alternatively, if S is greater than c_2, then the consumer would want a negative position in annuities. If actuarially fair life insurance (which pays a gross rate of return $\frac{R}{p}$ to the consumer's estate if he dies young and pays zero if he dies old) is available, then the consumer can, by holding life insurance and bonds,[6] achieve the same payoff structure as provided by a negative holding of annuities.

III. The Effects of Changes in the Probability of Death and Changes in Wealth

In this section we calculate the effects of variation in the probability of death and variation in expected lifetime wealth on consumption and portfolio decisions. We will show that consumption at each age and bequests are increasing functions of the expected present value of lifetime wealth. Also, for a given level of expected lifetime wealth, consumption at each age and the amount of bequests are increasing in p. These results will be useful in later sections when we aggregate over consumers with different probabilities of dying.

The income expansion path is easily derived from (4a,b) and (6)

$$U'(c_1) = \delta RV'(B) = \delta RU'(c_2) \tag{8}$$

The strict concavity of U( ) and V( ) implies that $\frac{dc_2}{dc_1} > 0$ and $\frac{dB}{dc_1} > 0$ along the income expansion path. Furthermore, because consumers can buy actuarially fair annuities, the income expansion path is independent of p.

The choice of c_1, c_2 and B is constrained by a lifetime budget constraint. Using (1), (2), (5) and the fact that $B = B^S = B^D$, the lifetime budget constraint can be written as[7]

$$c_1 + (1-p)R^{-1}c_2 + R^{-1}B = W \tag{9}$$

$$\text{where } W \equiv I + Y - T + (1-p)R^{-1}S.$$

According to (9), the expected present value of lifetime purchases (of consumption and of bequests) is equal to expected lifetime wealth. The optimal values of c_1, c_2, and B are determined by the intersection of the income expansion path in (8) and the lifetime budget constraint in (9).

Given the fixed value of R, the optimal values of c_1, c_2, and B can each be expressed as functions of W and p. Clearly, c_1, c_2 and B are each increasing functions of W. As for the effect of an increase in p, note that if some bundle (c_1, c_2, B) satisfies (9), then an increase in p holding W constant will make the relevant expected present value of purchases on the left hand side of (9) smaller than W. Hence, c_1, c_2, and B will all be increased along the expansion path until the budget line (9) is satisfied. Therefore, we have

$$c_i = c_i(W,p); \qquad \frac{\partial c_i}{\partial W} > 0, \; \frac{\partial c_i}{\partial p} > 0; \; i = 1, 2 \tag{10a}$$

$$B = B(W, p); \qquad \frac{\partial B}{\partial W} > 0, \; \frac{\partial B}{\partial p} > 0. \tag{10b}$$

We have shown that the partial effect of an increase in p, holding W constant, is to increase c_1, c_2, and B. However, since $W = I + Y - T + (1-p)R^{-1}S$, an increase in p will, if $S > 0$, decrease W and tend to offset the increases in c_1, c_2, and W. Of course, if $S = 0$, then this offsetting effect is absent. In general, the total effect of an increase in p is to increase (decrease) c_1, c_2, and B if S is less (greater) than c_2. If $S = c_2$, then the consumer holds no private annuities or life insurance and the optimal values of c_1, c_2, and B are invariant to p.

In addition to determining the qualitative effect of W on B as above, it will be useful to calculate the magnitude of this effect. Totally differentiating the lifetime budget constraint (9) with respect to c_1, c_2, B and W yields

$$dc_1 + (1-p)R^{-1}dc_2 + R^{-1}dB = dW \tag{11}$$

Logarithmically differentiating the income expansion path (8) yields

$$\frac{\sigma_U(c_1)}{c_1}\,dc_1 = \frac{\sigma_V(B)}{B}\,dB \tag{12a}$$

$$\frac{\sigma_U(c_2)}{c_2}\,dc_2 = \frac{\sigma_V(B)}{B}\,dB \tag{12b}$$

where $\sigma_U(c) \equiv -cU''(c)/U'(c)>0$ is the coefficient of relative risk aversion for the utility index U( ) and $\sigma_V(B) \equiv -BV''(B)/V'(B)>0$ is the coefficient of relative risk aversion of V( ). Substituting (12a,b) into (11) yields

$$\frac{\partial B}{\partial W} = \frac{1}{\Phi(p, c_1, c_2, B)} > 0 \tag{13a}$$

where $$\Phi(p, c_1, c_2, B) = \frac{\sigma_V(B)}{\sigma_U(c_1)}\;\frac{c_1}{B} + (1-p)R^{-1}\,\frac{\sigma_V(B)}{\sigma_U(c_2)}\;\frac{c_2}{B} + R^{-1} \tag{13b}$$

IV. The Steady State Cross-Sectional Distributions of Consumption and Bequests

In this section we demonstrate that if $R \leq G$, then there exists a unique positive steady state level of bequests. It is well-known from the Golden Rule literature that $R<G$ characterizes a dynamically inefficient steady state, and hence we would also like to analyze steady state behavior under the alternative assumption that $R>G$. However, if $R>G$, then the existence of a positive steady state level of bequests depends on the parameters of the utility function as well as on R and G; we defer discussion of the existence of a positive steady state level of bequests with $R>G$ until section V where we restrict attention to homothetic utility. However, before restricting the utility function to be homothetic, we are able to show in this section that the introduction of fully funded social security narrows the steady state distributions of consumption and bequests.

In previous sections we derived the optimal consumption and portfolio behavior of an individual with probability p of dying after one period. Henceforth, we assume that all descendents of an individual face the same probability p as the individual. However, we allow for heterogeneity of p across members of the same cohort and we index consumers by their probability of dying after one period. We will say that a consumer is a type p consumer if his probability of an early death is equal to p. In order to rule out a known date of death, we assume that $0 < p < 1$. Let $H(p)$ be the fraction of consumers in each cohort who have a probability of early death less than or equal to p. Let $\bar{p}$ be the population average probability of early death so that $\bar{p} = \int_0^1 p\,dH(p)$. In order to rule out aggregate uncertainty, we assume that a fraction p of type p consumers does indeed die early. Thus $\bar{p}$ is the fraction of consumers of each cohort who die early.

Since each consumer has G children, the assumption that a consumer's bequest is divided equally among his heirs implies that $I_t = B_{t-1}/G$. (The consumer born in period t receives an inheritance I_t and leaves a total bequest B_t.) Using (10b) it follows that the sequence of bequests in a family with a given probability of dying, p, evolves according to the first-order nonlinear difference equation

$$B_t = B((B_{t-1}/G) + Y - T + A^{-1}S,\ p) \tag{14}$$

A steady state level of bequests, B*, must satisfy the difference equation (14) with $B_{t-1} = B_t = B^*$. Below we specify a simple sufficient condition for the existence of a unique positive steady state level of bequests.

<u>Proposition 1</u>. If $R \leq G$, then there exists a unique steady state level of bequests $B^*>0$.

<u>Proof</u>. Existence: Since $\lim_{B\to 0} V'(B) = \infty$, the optimal bequest is positive if $(B_{t-1}/G) + Y - T + A^{-1}S>0$. Therefore, since $Y - T + A^{-1}S>0$, if $B_{t-1}=0$, then $B_t>B_{t-1}$. Observe from (9) that, setting $I_t = B_{t-1}/G$, we have

$$c_{1,t} + (1-p)R^{-1}c_{2,t} + R^{-1}B_t = (B_{t-1}/G) + Y-T+A^{-1}S \tag{15}$$

where $c_{i,t}$ is the consumption of a consumer of age i born in period t. Using (15) we obtain

$$B_{t-1} - B_t = (1-R/G)B_{t-1} + Rc_{1,t} + (1-p)c_{2,t} - R(Y-T+A^{-1}S) \tag{16}$$

Since $\lim_{B\to\infty} V'(B) = 0$ and $U'(c)>0$ for finite c, it follows that $Rc_{1,t} + (1-p)c_{2,t}$ exceeds $R(Y-T+A^{-1}S)$ for sufficiently large B_{t-1}. Therefore, for large enough B_{t-1}, the right hand side of (16) is positive and hence $B_t<B_{t-1}$. Since $B_t = B((B_{t-1}/G) + Y-T+A^{-1}S,p)$ is a continuous function, there exists some $B^*>0$ such that $B^* = B(B^*/G+Y-T+A^{-1}S,p)$.

Uniqueness: It suffices to show that $dB_t/dB_{t-1}<1$ at any positive steady state level of bequests. It follows from (13a) and (14) that

$$\frac{dB_t}{dB_{t-1}} = \frac{1}{G\Phi\ (p,\ c_1,\ c_2\ B)} \tag{17}$$

It follows (13b) that $G\Phi(p,\ c_1,\ c_2,\ B)>GR^{-1}$ so that if $G\geq R$, then $G\Phi(p,\ c_1,\ c_2,\ B)>1$. q.e.d.

To establish the existence of a unique steady state when R>G we need some additional restrictions on the utility function. We postpone the analysis of this case until section V when we introduce homothetic utility. The following useful proposition allows us to characterize the steady state cross-sectional distributions of consumption and bequests. Note that it does not require $R\leq G$.

Proposition 2. If there exists a unique positive steady state level of bequests, then $\partial B/\partial W<G$ when evaluated in the steady state.

Proof. Since $\lim_{B\to 0} V'(B)=\infty$, it follows that if $B_{t-1}=0$, then $B_t>B_{t-1}=0$. In addition, the existence of a unique positive steady state, $B_t=B_{t-1}=B^*$, implies that

$$B((B_{t-1}/G) + Y-T+A^{-1}S,p) - B_{t-1} \gtrless 0 \qquad \text{as } B_{t-1} \lessgtr B^* \tag{18}$$

The proposition follows immediately from (18). q.e.d.

We can now compare the steady state behavior of families with different probabilities of dying young. We begin by comparing bequests. Let $B^*(p)$ denote the steady state level of bequests for a type p family.

Proposition 3. If there exists a unique $B^*(p)>0$ for every p, and if the social security payment $S\geq 0$ is sufficiently small, then $dB^*(p)/dp>0$.

Proof. Observe from (14) that B^* satisfies

$$B^* = B((B^*/G) + Y-T + (1-p)R^{-1}S,p) \tag{19}$$

Totally differentiating (19) with respect to B^* and p yields

$$(1 - G^{-1} \frac{\partial B}{\partial W}) \, dB^* = (\frac{\partial B}{\partial p} - R^{-1}S \frac{\partial B}{\partial W}) \, dp \tag{20}$$

The existence of a unique positive steady state level of bequests implies that $G^{-1} \frac{\partial B}{\partial W} = \frac{dB_t}{dB_{t-1}} < 1$ when evaluated at B^* (Proposition 2). Therefore, the coefficient of dB^* in (20) is positive. Recall from (10b) that $\frac{\partial B}{\partial p} > 0$ so that for small enough S, the coefficient of dp is also positive. Therefore $dB^*/dp>0$. q.e.d.

One may be tempted to explain Proposition 3 by arguing that an increase in p is an increase in the frequency with which people die young leaving large bequests. However, we have shown that with a market for actuarially fair annuities, consumers leave the same bequest whether they die after one period or after two periods. The explanation for Proposition 3 is that, provided S is small, an increase in p reduces the expected present value of expenditures on the left side of the budget constraint (9) for given values of c_1, c_2, and B. Provided that $S \geq 0$ is small, (in particular, if $S<c_2$), the reduction in expected expenditure exceeds the reduction in expected lifetime wealth on the right hand side of (9), thereby permitting the consumer to increase c_1, c_2 and B. This reasoning suggests the following Corollary.

Corollary 3.1. If there exists a unique positive steady state level of bequests and if the social security payment $S \geq 0$ is sufficiently small, then $\frac{dc_i}{dp} > 0$, i=1,2.

Proof. Observe that for i = 1,2, $c_i^* = c_i((B^*/G) + Y-T + (1-p)R^{-1}S, p)$,

$\frac{\partial c_i}{\partial w} > 0$, $\frac{\partial c_i}{\partial p} > 0$ and $\frac{dB^*}{dp} > 0$. q.e.d.

We now consider a fully funded social security system. In such a system, the total benefits to a cohort are equal to the return on the system's investment of that cohort's contribution. We will limit our attention to a social security system which does not discriminate on the basis of an individual's probability of dying early.[8] Therefore, in an actuarially fair system, the taxes and benefits satisfy

$$RT = (1-\bar{p})S. \tag{21}$$

It follows from (5) and (21) that the expected net present value of social security benefits for a type p consumer is

$$-T + A^{-1}S = -T \frac{p-\bar{p}}{1-\bar{p}} \tag{22}$$

According to (22), the introduction of fully funded social security increases the expected lifetime wealth of consumers with a low p (less than $\bar{p}$) and decreases the expected lifetime wealth of consumers with a high p (greater than $\bar{p}$). The effects on the steady state distributions of consumption and bequests are given by the following proposition.

Proposition 4. Suppose that there exists a unique positive steady state level of bequests for families of every type p. Then, provided that $S \geq 0$ is small, an increase in fully funded social security, $RdT=(1-\bar{p})dS>0$, will narrow the steady state cross-sectional distributions of c_1, c_2 and B^*.

Proof. Substituting (22) into (19) yields

$$B^*(p) = B((B^*(p)/G) + Y - \frac{p-\bar{p}}{1-\bar{p}} T, p) \tag{23}$$

Applying the implicit function theorem to (23) yields

$$\frac{dB^*(p)}{dT} = \frac{-\frac{\partial B}{\partial W}}{1-G^{-1}\frac{\partial B}{\partial W}} \cdot \frac{p-\bar{p}}{1-\bar{p}} \tag{24}$$

Since $0 < G^{-1}\frac{\partial B}{\partial W} < 1$, the coefficient of $(p-\bar{p})$ in (24) is negative so that $\frac{dB^*(p)}{dT} \lesseqgtr 0$ as $p \gtreqless \bar{p}$. Thus $B^*(\bar{p})$ is invariant to T. Since $dB^*(p)/dp > 0$ (Proposition 3), it follows that an increase in T causes $B^*(p)$ to move toward $B^*(\bar{p})$.

Let $c_i^*(p)$ be the steady state level of c_i for type p consumers. It follows from (9), (10a) and (22) that

$$c_i^*(p) = c_i((B^*(p)/G)+Y - \frac{p-\bar{p}}{1-\bar{p}}\ T,\ p) \tag{25}$$

Differentiating (25) with respect to T yields

$$\frac{dc_i^*(p)}{dT} = (G^{-1}\frac{dB^*(p)}{dT} - \frac{p-\bar{p}}{1-\bar{p}})\ \frac{\partial c_i}{\partial W} \tag{26}$$

Now substitute (24) into (26) to obtain

$$\frac{dc_i^*(p)}{dT} = \frac{-\frac{\partial c_i}{\partial W}}{1 - G^{-1}\frac{\partial B}{\partial W}}\ \frac{p-\bar{p}}{1-\bar{p}} \tag{27}$$

Since $\frac{\partial c_i}{\partial W} > 0$ and $G^{-1}\frac{\partial B}{\partial W} < 1$, the coefficient of $(p-\bar{p})$ in (27) is negative. Therefore $\frac{dc_i^*}{dT} \lesseqgtr 0$ as $p \gtreqless \bar{p}$. Hence $c_i^*(\bar{p})$ is invariant to T. Since $dc_i^*(p)/dp > 0$ (Corollary 3.1), it follows that an increase in T causes $c_i^*(p)$ to move toward $c_i^*(\bar{p})$. q.e.d.

The intuition underlying Proposition 4 is quite straightforward. For consumers with a low probability of dying early, the annuity offered by social security system has a rate of return, $\frac{R}{1-\bar{p}}$, which exceeds the rate of return available from private annuity companies $\frac{R}{1-p}$. Thus an increase in social security effectively raises the wealth of the consumers with $p < \bar{p}$. Hence these consumers increase consumption and bequests. As for consumers with $p > \bar{p}$, an increase in social security forces them to hold annuities with a lower rate of return than on annuities in the private market; for these consumers, an increase in T effectively lowers wealth and leads to a reduction in bequests and consumption. Finally observe that for consumers with $p=\bar{p}$, an increase in social security has no effect since these consumers can undo the effects of an increase in social security by reducing their holdings of private annuities.

In Abel (1985a) it was also shown that an increase in the level of actuarially fair social security will narrow the steady state distributions of bequests and consumption. It is worth noting how Proposition 4 differs from the result in Abel (1985a). In the previous paper, there are no annuity markets, no bequest motive and no heterogeneity of ex ante mortality probabilities. In the model presented there, all bequests are "accidental"; bequests are equal to the wealth of consumers who die after one period. The introduction of social security reduces the need to save for retirement consumption and thus reduces the size of accidental bequests. Since all intra-cohort variation is due to intra-cohort variation in bequests, the reduction in all positive bequests reduces intra-cohort variation. However, in the current paper with actuarially fair annuities, consumers with the same wealth and the same ex ante mortality probabilities leave the same bequest whether they die young or old. Thus, for consumers with a given p, there is no intra-cohort variation in bequests or consumption. The intra-cohort variation is across consumers with different ex ante mortality probabilities. To the extent that the social security system forces everyone to hold a particular asset in their portfolios, it reduces the intra-cohort variation in portfolios and hence in bequests and consumption.

V. The Effects of Social Security on Steady State Aggregate Capital and Consumption

Much of the literature on the effects of social security has focussed on its effects on the long-run aggregate capital stock. Presumably the reason for examining the effects on the capital stock is that if the long-run capital stock is below the Golden Rule capital stock, then if social security reduces the long-run aggregate capital stock, it will also reduce long-run aggregate consumption. The implication of the reduction in aggregate consumption is evidently that aggregate welfare is reduced. We argue in this section and the next section that, for two reasons, the emphasis on the long-run capital stock is misplaced if one is actually interested in social welfare. First, as is well-known from the Golden Rule literature, the long-run aggregate capital stock and long-run aggregate consumption can move in opposite directions in response to social security. Second, and more importantly, with heterogeneous consumers, it can happen that aggregate consumption is reduced but aggregate welfare is increased by social security. This apparent contradiction can be explained by observing that social security narrows the distribution of consumption.

We restrict our attention henceforth to the case of homothetic utility as in Hakansson (1969), Fischer (1973), and Richard (1975). There are two reasons for restricting the utility function to be homothetic. First, we can present necessary and sufficient conditions for the existence of a unique steady state which do not require $R \leq G$. Second, homothetic utility implies linear decision rules which are easily aggregated.

Suppose that U(c) and V(B) are characterized by constant and equal coefficients of relative risk aversion σ

$$U(c) = \frac{c^{1-\sigma}-1}{1-\sigma} \tag{28a}$$

$$V(B) = \lambda \frac{B^{1-\sigma}-1}{1-\sigma} \tag{28b}$$

Note that U( ) and V( ) are each strictly concave and satisfy the Inada conditions. Therefore, Propositions 1-4 apply to this specification of the utility function.

Homothetic utility is particularly convenient because it implies that the income expansion path is a ray through the origin. Using (28a,b), the income expansion path in (8) can be expressed as

$$c_1 = \theta_1 B \quad \text{where} \quad \theta_1 = (\delta\lambda R)^{-\frac{1}{\sigma}} \tag{29a}$$

$$c_2 = \theta_2 B \quad \text{where} \quad \theta_2 = \lambda^{-\frac{1}{\sigma}} \tag{29b}$$

Substituting (29a,b) into the lifetime budget constraint (9) yields

$$B(p) = \frac{1}{\phi(p)} (I + Y - T + A^{-1}S) \tag{30a}$$

where

$$\phi(p) \equiv \theta_1 + (1-p)R^{-1}\theta_2 + R^{-1} \tag{30b}$$

It follows from (13b) and (29a,b) that $\phi(p)$ in (30b) is simply $\Phi(p, c_1, c_2, B)$ evaluated under the assumption of homothetic utility. In this case, $\Phi(p, c_1, c_2, B)$ is independent of c_1, c_2, and B and is simply a decreasing function of p. Recall from the proof of Proposition 1 that the steady state will be unique if $G\Phi>1$. The analogous result for homothetic utility is given below.

<u>Proposition 5</u>. Suppose that U( ) and V( ) have equal constant degrees of relative risk aversion as specified in (28a,b). There will be a unique positive steady state level of bequests, $B^*(p)$, if and only if $G\phi(p)>1$, where $\phi(p)$ is defined in (30b).

<u>Proof</u>. Setting I equal to B/G in (30a) yields

$$B^*(p) = \frac{G}{G\phi(p)-1} (Y-T+A^{-1}S) \tag{31}$$

which immediately proves the proposition.

Corollary 5.1. If $\delta\lambda R\leq 1$, then there exists a unique steady state level of bequests.

Proof. If $\delta R\lambda \leq 1$, then $\theta_1 \equiv (\delta R\lambda)^{-\frac{1}{\sigma}} \geq 1$. Therefore $\phi > 1$ and $G\phi > 1$. q.e.d.

In the remainder of this section we analyze the effects of changes in fully funded social security on various aggregate magnitudes in the steady state. We adopt the notational convention of using two asterisks to denote the average value of a variable in the steady state. For example, $B^{**} \equiv \int_0^1 B^*(p)\,dH(p)$. We will demonstrate below that an increase in fully funded social security decreases aggregate bequests and aggregate consumption of young consumers in the steady state. The steady state national capital stock will be increased or decreased depending on whether consumption of the young is greater or less than the inheritance they receive. Whether aggregate consumption in the steady state increases or decreases depends on whether the interest rate exceeds the growth rate as well as whether the consumption of the young is greater or less than the inheritance they receive.

A. Steady State Bequests

To examine the effects of fully funded social security on steady state bequests, substitute (22) into (31) and integrate over all types p to obtain

$$B^{**} = \int_0^1 \frac{G}{G\phi(p) - 1} \left[Y - \frac{p-\bar{p}}{1-\bar{p}} T\right] dH(p) \tag{32}$$

It is evident from (32) that an increase in T redistributes resources away from consumers with $p>\bar{p}$ toward consumers with $p<\bar{p}$. Since $\frac{G}{G\phi(p)-1}$, the ratio of steady state bequests to non-inheritance income, is increasing in p, the redistribution of resources is from consumers with a high value of $\frac{G}{G\,\phi(p)-1}$ to consumers with a low value of this factor; hence B** declines.

Proposition 6.[9,10] Suppose that U( ) and V( ) have equal constant relative risk aversion as specified in (28a,b) and that $G\phi(p)>1$ for all p in the support of H(p). Then an increase in fully funded social security reduces aggregate bequests in the steady state, B**.

B. Steady State Consumption

Now we examine the effects on aggregate consumption of consumers of each age. The steady state aggregate (per capita) consumption of the young cohort is $c_1^{**} \equiv \int_0^1 c_1^*(p)dH(p)$. Since the ratio of c_1 to B is θ_1 for all consumers, it follows immediately that

$$c_1^{**} = \theta_1 B^{**} \tag{33}$$

Equation (33) and Proposition 6 lead to

Proposition 7. Under the assumptions of Proposition 6, an increase in fully funded social security reduces the aggregate consumption of the young.

Although social security unambiguously reduces the steady state consumption of the young, it can either reduce, raise, or leave unchanged the steady state aggregate consumption of the old cohort. To understand why the effect on the consumption of the old is ambiguous, recall that, in the long run, social security raises both the first-period and second-period consumption of consumers with $p<\bar{p}$ and reduces the first-period and second-period consumption of consumers with $p>\bar{p}$. Consumers with $p<\bar{p}$ represent a larger share of the old generation than of the young generation because of their higher survival rates. Thus, even though average first-period consumption is unambiguously reduced in the long run, it is possible for average second-period consumption to be increased in the long run.

Let $c_2{}^{**}$ denote the steady state consumption (per capita) of the old generation and observe that

$$c_2{}^{**} = \int_0^1 (1-p)G^{-1}c_2{}^{*}(p)dH(p) \tag{34}$$

The factor $(1-p)G^{-1}$ reflects the facts that (a) only a fraction (1-p) of type p consumers survives, and (b) each cohort is only G^{-1} times as large as the succeeding cohort. Substituting (25) into (31), and using (29b) and (31), equation (34) may be rewritten as

$$c_2{}^{**} = \int_0^1 \frac{(1-p)\theta_2}{G\phi(p)-1}\left[Y - \frac{p-\bar{p}}{1-\bar{p}}T\right]dH(p) \tag{35}$$

The factor $\frac{(1-p)\theta_2}{G\phi(p)-1}$, which is the steady type p consumers to their bequests, can be rewritten using (30b) as

$$\frac{(1-p)\theta_2}{G\phi(p)-1} = \left[1 + \frac{1-(\theta_1 + R^{-1})G}{G\phi(p)-1}\right]\frac{R}{G} \tag{36}$$

It is clear from (36) that the factor $\frac{(1-p)\theta_2}{G\phi(p)-1}$ is decreasing in p if and only if $(\theta_1 + R^{-1})G>1$. Therefore, since social security transfers resources from high p consumers to low p consumers, the redistribution is from consumers with a low value of the factor $\frac{(1-p)\theta_2}{G\phi(p)-1}$ to consumers with a high value of this factor if and only if $(\theta_1 + R^{-1})G>1$. Therefore we have

Proposition 8.[11] Suppose that U( ) and V( ) have equal constant coefficients of relative risk aversion as specified in (28a,b) and that $G\phi(p)>1$ for all p in the support of H(p). Then for an increase in fully funded social security,

$$\frac{dc_2^{**}}{dT} \gtrless 0 \quad \text{as} \quad G(\theta_1 + R^{-1}) \gtrless 1 \tag{37}$$

C. The Steady State Capital Stock

Let $K^*(p)$ be the steady state capital stock of type p consumers and $K^{**} \equiv \int_0^1 K^*(p)dH(p)$ be the aggregate steady state private capital stock. We will measure K^{**} at the end-of-period, that is, before interest is accrued. At the end of a period, all privately owned capital is held by young consumers; the surviving old consumers have already consumed c_2 and have already given the remaining wealth to their heirs. Thus, the private capital stock is equal to inheritances received by the young, I^{**}, plus net labor income, Y-T, minus first-period consumption, so that

$$K^{**} = I^{**} + Y-T - c_1^{**} \tag{38}$$

Since $I^{**}=B^{**}/G$ and $c_1^{**} = \theta_1 B^{**}$, equation (38) can be rewritten as

$$K^{**} = Y - T + (G^{-1} - \theta_1)B^{**} \tag{39}$$

Equation (39) and Proposition (6) imply

Proposition 9. Suppose that U( ) and V( ) have equal constant relative risk aversion as specified in (28a,b) and that $G\phi(p) > 1$ for all p in the support of H(p). Then the effect on the long-run aggregate private capital stock of an increase in fully funded social security is

$$\frac{dK^{**}}{dT} \gtrless -1 \quad \text{as} \quad \theta_1 \gtrless G^{-1} \tag{40}$$

In a fully funded social security system, the long-run aggregate national capital stock K_N^{**}, is equal to $K^{**} + T$, the sum of aggregate private capital K** and the government capital stock. Therefore, the following corollary to Proposition 9 is obvious.

Corollary 9.1. If the assumptions of Proposition 9 hold, then under fully funded social security

$$\frac{dK^{**}_N}{dT} \gtreqless 0 \quad \text{as} \quad \theta_1 \gtreqless G^{-1} \tag{41}$$

To interpret the condition in (40) and (41), observe that since $c_1^{**} = \theta_1 B^{**}$ and $B^{**} = GI^{**}$, we have $c_1^{**} = G\theta_1 I^{**}$. That is, in comparing steady states, $G\theta_1$ is the response of consumption of young consumers to changes in the inheritances they receive. The introduction of fully funded social security reduces B** and hence reduces I** and c_1^{**}. If $G\theta_1$ is less than 1, the reduction in first period consumption is smaller than the reduction in inheritances and the national capital stock falls. Alternativey, if $G\theta_1$ is greater than 1, the reduction in c_1^{**} exceeds the reduction in inheritances and the national capital stock rises.

It is useful at this point to present the aggregate resource constraint of the economy. In the steady state, the aggregate disposable resources of the private sector are given by $Y + (R/G)K^{**} + (1-\bar{p})S/G - T$ where $Y - T$ is the net labor income of the young, $(R/G)K^{**}$ is the per capita gross income accruing to privately-held capital carried over from the previous period, and $(1-\bar{p})S/G$ is the per capita social security income of the old. The private sector uses these resources for consumption $c^{**} \equiv c_1^{**} + c_2^{**}$ and (gross) capital accumulation K**. Equating $c^{**}+K^{**}$ with total disposable resources yields

$$c^{**} = Y-T + (1-\bar{p})S/G + (R/G - 1)K^{**} \tag{42}$$

Using condition (21) for fully funded social security we obtain

$$c^{**} = Y + (R/G - 1)(K^{**} + T) \tag{43}$$

Equation (43) displays the well-known result from the Golden Rule literature that an increase in the steady state national capital stock leads to an increase, decrease, or no change in aggregate consumption depending on whether the net rate of return to capital is greater than, less than, or equal to the population growth rate.

Equation (43) and Corollary (9.1) imply the following:

Corollary 9.2. Suppose that the assumptions of Proposition 9 hold. Then, under fully funded social security

$$\frac{dc^{**}}{dT} \gtrless 0 \qquad \text{as} \qquad [\theta_1 - G^{-1}][R-G] \gtrless 0 \tag{44}$$

The condition in (44) has a simple interpretation[12]. The direction of the effect of social security on the national capital stock is given by the sign of $\theta_1 - G^{-1}$ (Corollary 9.1). The direction of the effect of a change in the capital stock on aggregate consumption is given by the sign of R-G, as is well known from the Golden Rule literature.

Propositions 6 through 9 and their corollaries describe the conditions under which various aggregate magnitudes either increase or decrease in response to an increase in fully funded social security. Only bequests and consumption of the young have unambiguous responses to social security. The effects on consumption of the old, aggregate consumption, and the national capital stock are summarized in Figure 1. If a steady state exists, then the directions of the effects of social security on c_2^{**}, c^{**} and K_N^{**} depend only on $\theta_1 \equiv (\delta\lambda R)^{-\frac{1}{\sigma}}$ and on R; the

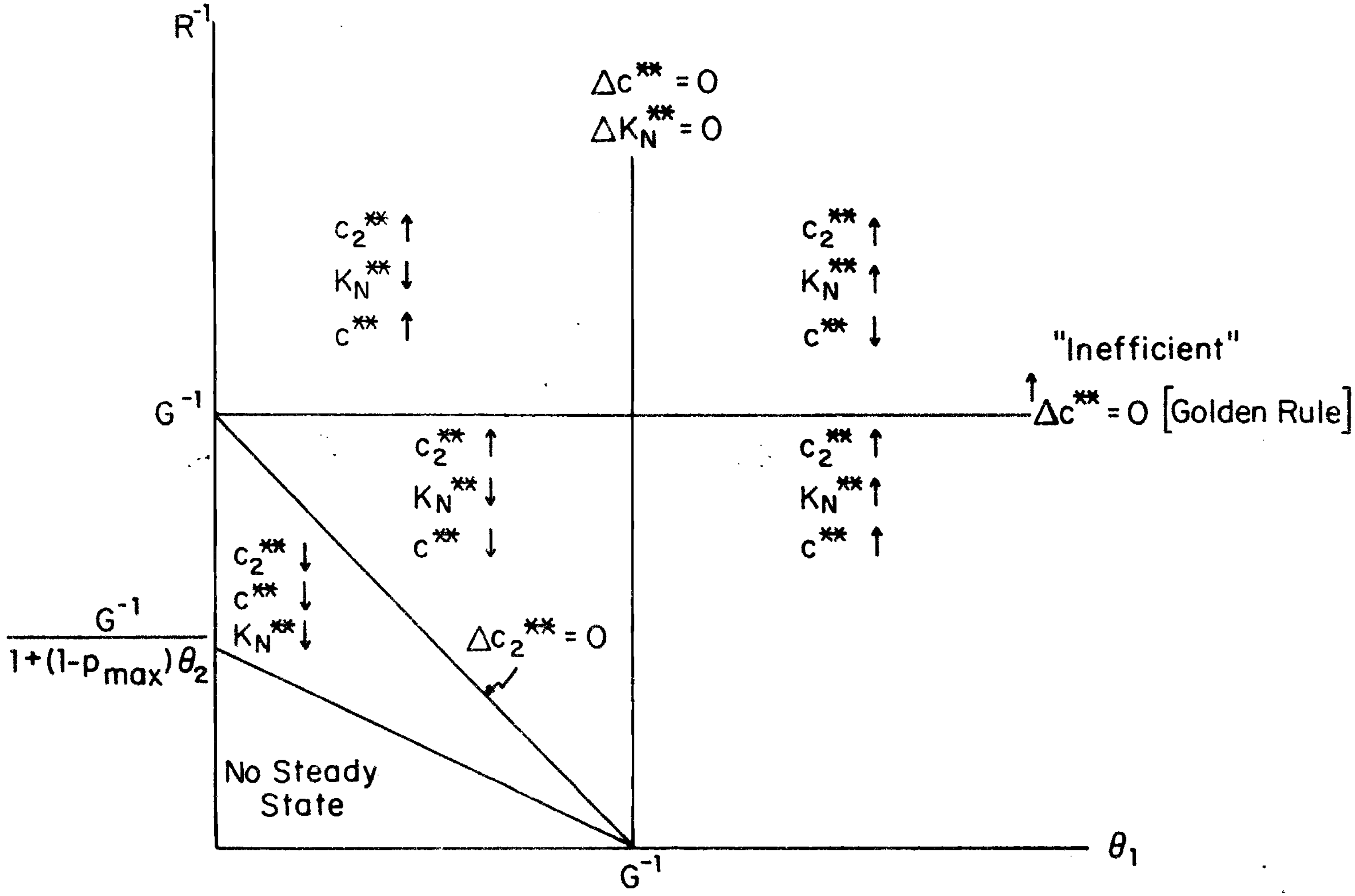

Figure 1

Effects of $R\,dT = (1-\bar{p})\,dS > 0$:

$\Omega^{**} \uparrow$, if $\sigma \geq 1$; $c_1^{**} \downarrow$

existence of a steady state depends on $\theta_2 \equiv \lambda^{-\frac{1}{\sigma}}$ and p_{max} (the largest value of p in the population) as well.

It is not obvious how one might best choose an estimate for the crucial parameter θ_1 in the two-period lifetime model presented above. However, I will offer a casual guess without pretending it is anything more than a guess. As is clear from Proposition 9 and its Corollaries, the critical value of θ_1 is G^{-1}. Recalling that $\theta_1 G$ is the steady state ratio of aggregate consumption of the young to aggregate inheritances received at birth, a reasonable guess is that the bequest motive is sufficiently weak so that this ratio is greater than one, which implies that $\theta_1 > G^{-1}$.[13] In addition, it appears that the marginal product of capital is greater than the population growth rate so that $R^{-1} < G^{-1}$. As is clear from Figure 1, these two guesses imply that the national capital stock, consumption of the old, and aggregate consumption are all increased by fully funded social security, whereas aggregate bequests and aggregate consumption of the young are decreased.

VI. Steady State Welfare

In this section, we examine the effect of an increase in social security on the steady state level of aggregate welfare. Our measure of aggregate welfare is simply the sum of the individual utilities of all consumers in a given cohort. We demonstrate that if consumers are sufficiently risk-averse, then the introduction of social security will increase steady state aggregate welfare.

Let $\Omega(p)$ be the maximized value of the individual utility function (3) subject to the constraints in (1) and (2). Restricting U( ) and V( ) to have equal constant relative risk aversion as in (28a,b) , we can use the income expansion path in (29a,b) to obtain

$$\Omega(p) = \frac{\theta_1^{-\sigma}}{1-\sigma} \phi(p)[B(p)]^{1-\sigma} - \frac{\gamma(p)}{1-\sigma} \tag{45}$$

where $\gamma(p) \equiv 1 + \delta(1-p+\lambda)$.

Let $\Omega^*(p)$ be the steady state value of $\Omega(p)$.

It is clear from (45) that for a given p steady state welfare $\Omega^*(p)$ is an increasing function of the steady state bequest $B^*(p)$. Therefore, in view of Proposition 4, an increase in fully funded social security increases steady state utility for consumers with $p<\bar{p}$ and reduces steady state utility for consumers with $p>\bar{p}$.

To examine the effects of social security on social welfare, we of course, need to specify a social welfare function. We use a utilitarian social welfare function which is the sum of the utility of all consumers in a given cohort. The steady state level of social welfare Ω^{**} is $\Omega^{**} \equiv \int_0^1 \Omega^*(p)dH(p)$ so that evaluating (45) in the steady state we have

$$\Omega^{**} = \frac{\theta_1^{-\sigma}}{1-\sigma} \int_0^1 \phi(p)[B^*(p)]^{1-\sigma}dH(p) - \frac{\gamma(\bar{p})}{1-\sigma} \tag{46}$$

We will limit our attention to the introduction of a small amount of social security into an economy without social security. Substituting (31) into (46), differentiating with respect to T, and evaluating the derivative of T=0 yields

$$\left.\frac{d\Omega^{**}}{dT}\right|_{T=0} = \frac{(\theta_1 Y)^{-\sigma}}{1-\bar{p}} \int_0^1 J(p)(p-\bar{p})dH(p) \tag{47a}$$

where

$$J(p) = -\phi(p) \left[\frac{G}{G\phi(p)-1} \right]^{1-\sigma} \tag{47b}$$

We now state and prove

<u>Proposition 10</u>. Suppose that U( ) and V( ) have equal and constant relative risk aversion as specified in (28a,b) and that $G\phi(p) > 1$ for all p in the support of H(p). Then if $G\phi(p) > \frac{1}{\sigma}$, the introduction of actuarially fair social security increases steady state welfare.

<u>Proof</u>. From (47a) and the Lemma, it is clear that $\left.\frac{d\Omega^{**}}{dT}\right|_{T=0} > 0$ if $J'(p) > 0$ for all p in the support of H(p). Differentiating (47b) with respect to p and simplifying yields

$$J'(p) = -\phi'(p) \left[\frac{G}{G\phi(p) - 1} \right]^{1-\sigma} \left[\frac{\sigma G\phi(p) - 1}{G\phi(p) - 1} \right] \tag{48}$$

Since $\phi'(p) < 0$ and $G\phi(p) -1 > 0$, it is clear that if $\sigma G\phi(p) - 1 > 0$, then $J'(p) > 0$. q.e.d.

<u>Corollary 10.1</u>. Suppose that $G\phi(p) > 1$ for all p in the support of H(p) and that $\sigma \geq 1$. Then the introduction of actuarially fair social security increases steady state social welfare.

We have shown that the introduction of actuarially fair social security can reduce steady state aggregate consumption but, if the CRRA utility functions U( ) and V( ) display at least as much risk aversion as the logarithmic utility function, it increases steady state social welfare. Although a reduction in aggregate consumption may seem, at first, to be inconsistent with an increase in social welfare, these results are easily reconciled by the observation that social security reduces the variation in bequests and consumption (Proposition 4). If the individual utility functions are sufficiently risk averse, the welfare-improving effects of reduced variance outweigh the welfare-reducing effects of reduced aggregate consumption.

Corollary 10.1 indicates that if the coefficient of relative risk aversion is greater than or equal to one, then the welfare-improving effects of reduced variance are strong enough to raise social welfare. Alternatively, if the coefficient of relative risk aversion is sufficiently small, then the introduction of social security will reduce steady state welfare. A sufficient condition is given in the following corollary.

Corollary 10.2. Suppose that $1 < G\phi(p) < \frac{1}{\sigma}$ for all p in the support of H(p). Then the introduction of fully funded social security reduces steady state welfare.

VII. Conclusion

The social security system essentially forces all workers to hold an annuity in their portfolios. If the rate of return on this annuity is equal to the rate of return on the private annuities which a consumer holds, then the consumer can offset the effects of social security simply by reducing his holding of private annuities. However, if the rate of return on social security differs from the rate available to the consumer in the private annuity market, then changes in the level of social security will, in general, force the consumer to change his consumption and/or portfolio behavior. Thus, in a world in which consumers all face the same rate of return on social security but face different rates of return on private annuities, changes in social security will affect the behavior of at least some individuals.

In this paper, we have assumed that each consumer can buy private annuities at an actuarially fair rate of return. After presenting sufficient conditions for the existence of a unique steady state equilibrium, we then established that increased social security narrows the steady state distributions of bequests and consumption. We showed that fully funded social security will crowd out steady state private wealth by more than, less than, or exactly one-for-one depending on whether steady state consumption of the young is less than, greater than, or equal to steady state inheritance received by the young. We also established simple conditions which determine whether steady state aggregate consumption rises, falls, or remains unchanged. Finally, we showed that if individual utility functions are sufficiently risk-averse, then fully funded social security increases steady state social welfare because it reduces inequality; however, if individual utility functions display very little risk-aversion, then social welfare is reduced by social security.

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Footnotes

1. See, for example, Yaari (1965), Hakansson (1969), Fischer (1973), Richard (1975), Barro and Friedman (1977), Levhari and Mirman (1977) and Kotlikoff and Spivak (1981). Kotlikoff and Spivak (1981) analyze the role of the family in providing annuities but stop short of a general equilibrium model in which the distribution of bequests is determined endogenously.

2. If there are annuities and if consumers derive some utility from leaving bequests, then in general there will be intra-cohort variation in bequests received and left by members of the same cohort with identical ex ante mortality probabilities. (See Abel (1985b).) Only if the rate of return on annuities is actuarially fair will there be no intra-cohort variation in bequests by consumers who can live either one period or two periods.

3. We are using the term "inheritance" to refer to a transfer received from one's parent, regardless of whether the parent is alive.

4. We follow Yaari (1965), Hakansson (1969), Fischer (1973) and Richard (1975) in specifying utility as a function of the size of the bequest left to one's heirs. An alternative formulation which gives rise to a bequest motive is to specify utility as a function of the utility of one's heirs' as in Barro (1974).

5. More generally we might specify the utility of bequests as a function $\tilde{V}(B,G)$ where B is the size of the total bequests and each child receives B/G. However, since G is assumed to be fixed exogenously, we can write the utility of bequests simply as a function of B.