NBER WORKING PAPER SERIES

OPTIMAL TARIFFS IN CONSISTENT CONJECTURAL VARIATIONS EQUILIBRIUM

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Working Paper No. 1872

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
March 1986

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ABSTRACT

This paper analyzes the determination of the optimal tariff under the assumption of Consistent Conjectural Variations (CCV). A general characterization of the CCV equilibrium is given. We show that (i) there are, in general, a multiplicity of such equilibria, and (ii) under certain restrictions, the Cournot equilibrium, which is based on the assumption of no retaliation can also be a CCV equilibrium. By contrast, free trade is never a CCV equilibrium. Finally the CCV equilibrium is solved explicitly in a simple example.

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1. INTRODUCTION

Until recently, the theory of optimal tariffs has been based on the assumption that neither country will retaliate to the imposition of the tariff by its trading partner.[1/] The existence of tariff wars, however, calls into serious question the realism of this assumption. This shortcoming is recognized in a recent paper by Thursby and Jensen (1983), which analyzes the question of optimal tariffs on the assumption of some constant conjectured form of retaliation by each country.[2/]

In this paper, we determine the optimal tariff under the assumption of consistent conjectural variations (CCV). That is, we assume that each country's conjecture about the retaliation by its trading partner is in fact consistent with that country's reaction curve. This form of equilibrium is appealing and has been introduced into oligopolistic models by a number of authors; see, e.g., Laitner (1980), Bresnahan (1981), Perry (1982), Kamien and Schwartz (1983), Eaton and Grossman (forthcoming). It has also been applied to the determination of tariff equilibrium by Jensen and Thursby (1984).

But despite its appeal and widespread adoption in the literature, the conjectural variations equilibrium is often criticized as not being a proper game-theoretic construct. The essence of this objection is that in a static game there is no possibility for action and policy reaction and hence there is no way an agent can conjecture, let alone correctly conjecture, his rival's response. One justification for the Consistent Conjectural Variations equilibrium in this circumstance is as a shorthand 'reduced form' for a fully dynamic repeated game. Also, Bresnahan (1981) justifies it in terms of comparative static responses in a situation where exogenous shocks occur with sufficient frequency for each decision maker to observe and to learn his rival's true response. However, the dynamic process by which these consistent conjectures are generated is an unsolved problem.[3/]

As we shall see in the course of the analysis below, the determination of the CCV equilibrium involves inherent nonlinearities and all we are able to establish is consistency to the first order of approximation. We give a general characterization of the CCV equilibrium for the optimal tariffs. We show that: (i) there are, in general, a multiplicity of such equilibria, and (ii) under certain restrictions, the Cournot equilibrium, which is based on the assumption of no retaliation, can also be a CCV equilibrium. By contrast, free trade is never a CCV equilibrium. Finally, we solve for the CCV equilibrium explicitly in a simple example.

2. DETERMINATION OF EQUILIBRIUM WITH CONSISTENT CONJECTURES

A. Framework

We consider the usual two country, two good trade model, where both countries are competitive and the only distortions are import tariffs.[4/] We index the countries by A, B, the goods by x, y, and assume without loss of generality that Country B imports good x in return for good y which it exports to Country A. The tariffs levied by Countries A, B, are t_y, t_x, respectively, and the two governments are assumed to redistribute the tariff revenues to their citizens.

The basic model is described by the following set of equations:

$$U^i = U^i(C_x^i, C_y^i) \qquad i = A,\ B \tag{1}$$

$$\frac{\partial U^i}{\partial C_x^i} = p\,\frac{\partial U^i}{\partial C_y^i} \qquad i = A,\ B \tag{2}$$

$$F^i(Q_x^i, Q_y^i) = 0 \qquad i = A,\ B \tag{3}$$

$$\frac{\partial F^i}{\partial Q_x^i} - p\frac{\partial F^i}{\partial Q_y^i} = 0 \qquad i = A,\ B \tag{4}$$

$$Q_j^i + E_j^i = C_j^i \qquad \begin{array}{l} i = A,\ B \\ j = x,\ y \end{array} \tag{5}$$

$$E_y^A + p\tau_y E_x^A = 0 \tag{6a}$$

$$\tau_x E_y^B + qE_x^B = 0 \tag{6b}$$

$$E_y^A + E_y^B = 0 \tag{7a}$$

$$E_x^A + E_x^B = 0 \tag{7b}$$

$$q = p\tau_y\tau_x \tag{8}$$

$$\tau_y = 1 + \tau_y,\ \tau_x = 1 + \tau_x \tag{9}$$

where C_j^i denotes consumption of good in Country i $\quad i = A, B, j = x, y$

Q_j^i denotes production of good j in Country i

E_j^i denotes excess demand of good j in Country i

p = price of x relative to y in Country A

q = price of x relative to y in Country B

Equations (1) define the utility functions in Countries A, B to be functions of the respective consumptions of the two goods. The functions U are assumed to be continuous, twice differentiable, and quasi-concave. Equations (2) define the optimality conditions for consumers in each country. Equations (3) and (4) apply analogously to production. Equation (3) defines the production possibility curve, which is assumed to be concave, while (4) specifies the usual marginal product conditions for optimality. Equations (5) define excess demand as the difference between consumption and production. For the import good $E > 0$, while for the export good $E < 0$.[5/]

Equations (6) describe the balance of trade equilibrium for the two countries, while (7) specifies market clearance in the world markets for the two goods. With no transport costs and in the absence of frictions, (8) specifies the relationship between the relative prices of the two goods in the two economies and the tariffs being levied. Using this relationship, it is clear from (6), (7) that the goods markets are not both independent; equilibrium in one implies equilibrium in the other, so that one equation, say (7b), can be dropped.

B. Determination of Optimal Tariff with Conjectured Variations

Consider first Country A. Using (5), (6), (7b), its utility function may be expressed in the form

$$U^A = U^A[Q_x^A - E_x^B,\ Q_y^A + qE_x^B/\tau_x]$$

To determine the optimal tariff, differentiate U^A with respect to τ_y. Taking the differential of (3) and using conditions (2), (4), yields the optimality condition

$$\left[q\left(\frac{\tau_y - 1}{\tau_y}\right)\frac{\partial E_x^B}{\partial q} + E_x^B\right]\frac{\partial q}{\partial \tau_y}$$

$$+\left\{\left[q\left(\frac{\tau_y - 1}{\tau_y}\right)\frac{\partial E_x^B}{\partial q} + E_x^B\right]\frac{\partial q}{\partial \tau_x} + \left[q\left(\frac{\tau_y - 1}{\tau_y}\right)\frac{\partial E_x^B}{\partial \tau_x} - \frac{q}{\tau_x}E_x^B\right]\right\}\left(\frac{d\tau_x}{d\tau_y}\right)_B^* = 0 \qquad (10)$$

where $(d\tau_x/d\tau_y)_B^*$ denotes the conjectured variation of Country B in response to the tariff imposed by Country A. Performing the same calculations for Country B, we can show that the analogous optimality condition is

$$\left[p\left(\frac{\tau_x - 1}{\tau_x}\right)\frac{\partial E_y^A}{\partial p} - E_y^A\right]\frac{\partial p}{\partial \tau_x}$$

$$+\left\{\left[p\left(\frac{\tau_x - 1}{\tau_x}\right)\frac{\partial E_y^A}{\partial p} - E_y^A\right]\frac{\partial p}{\partial \tau_y} + \left[p\left(\frac{\tau_x - 1}{\tau_x}\right)\frac{\partial E_y^A}{\partial \tau_y} - \frac{p}{\tau_y}E_y^A\right]\right\}\left(\frac{d\tau_y}{d\tau_x}\right)_A^* = 0 \qquad (10')$$

where $(d\tau_y/d\tau_x)_A^*$ is the conjectured variation of Country A in response to the tariff imposed by Country B.

Define the following elasticities of the offer curves

$$\alpha_p \equiv \frac{\partial E_y^A}{\partial p}\frac{p}{E_y^A} > 0; \qquad \beta_q \equiv \frac{\partial E_x^B}{\partial p}\frac{q}{E_x^B} < 0$$

$$\alpha_y \equiv \frac{\partial E_y^A}{\partial \tau_y}\frac{\tau_y}{E_y^A} < 0; \qquad \beta_x \equiv \frac{\partial E_x^B}{\partial \tau_x}\frac{\tau_x}{E_x^B} < 0 \qquad (11)$$

The sign restrictions introduced are based on the assumption that an increase in the relative price of good x in either country reduces the excess demand for that good, while increasing it for the other good y. As usual, we invoke the Marshall-Lerner condition $\alpha_p - \beta_q > 1$. Also, an increase in the tariff rate reduces the excess demand for the imported good. Using these definitions the optimality conditions can be expressed in terms of the following elasticity expressions

Country A:

$$\left[(\frac{\tau_y - 1}{\tau_y})\beta_q + 1\right]\frac{\partial q}{\partial \tau_y} + \left\{\left[(\frac{\tau_y - 1}{\tau_y})\beta_q + 1\right]\frac{\partial q}{\partial \tau_x} + \frac{q}{\tau_x}\left[(\frac{\tau_y - 1}{\tau_y})\beta_x - 1\right]\right\}(\frac{d\tau_x}{d\tau_y})_B^* = 0 \quad (12)$$

Country B:

$$\left[(\frac{\tau_x - 1}{\tau_x})\alpha_p - 1\right]\frac{\partial p}{\partial \tau_x} + \left\{\left[(\frac{\tau_x - 1}{\tau_x})\alpha_p - 1\right]\frac{\partial p}{\partial \tau_y} + \frac{p}{\tau_y}\left[(\frac{\tau_x - 1}{\tau_x})\alpha_y - 1\right]\right\}(\frac{d\tau_y}{d\tau_x})_A^* = 0 \quad (12')$$

Differentiating the basic model with respect to τ_x, τ_y, one can express the partial price derivatives in terms of the elasticities as follows:

$$\frac{\partial p}{\partial \tau_y} = \frac{p}{\tau_y}\left[\frac{\beta_q - \alpha_y + 1}{\alpha_p - \beta_q - 1}\right]; \quad \frac{\partial p}{\partial \tau_x} = \frac{p}{\tau_x}\left[\frac{\beta_x + \beta_q}{\alpha_p - \beta_q - 1}\right]$$

$$\frac{\partial q}{\partial \tau_y} = \frac{q}{\tau_y}\left[\frac{\alpha_p - \alpha_y}{\alpha_p - \beta_q - 1}\right]; \quad \frac{\partial q}{\partial \tau_x} = \frac{q}{\tau_x}\left[\frac{\beta_x + \alpha_p - 1}{\alpha_p - \beta_q - 1}\right] \tag{13}$$

Note that in the case that no retaliation is anticipated (the Cournot case),

$$\left(\frac{d\tau_x}{d\tau_y}\right)^*_B = \left(\frac{d\tau_y}{d\tau_x}\right)^*_A = 0$$

and (12), (12') reduce to

$$t_y = \frac{-1}{1 + \beta_q}\;; \qquad t_x = \frac{1}{\alpha_p - 1}$$

which are just the usual formulae for the optimal tariff.[6/]

C. Determination of Consistent Conjectural Variations Equilibrium

Substituting for $\partial p/\partial \tau_i$, $\partial q/\partial \tau_i$, $i = x, y$, into (12), (12'), these conditions can be simplified to

$$(\alpha_p-\alpha_y)\tau_x[(\tau_y-1)\beta_q + \tau_y] + (\beta_q+\beta_x)\tau_y[\alpha_p(\tau_y-1) + 1]\left(\frac{d\tau_x}{d\tau_y}\right)^*_B = 0 \tag{14}$$

$$(\beta_x+\beta_q)\tau_y[(\tau_x-1)\alpha_p - \tau_x] + (\alpha_p-\alpha_y)\tau_x[\beta_q(\tau_x-1) - 1]\left(\frac{d\tau_y}{d\tau_x}\right)^*_A = 0 \tag{14'}$$

To this point, the conjectured variations $\left(\frac{d\tau_x}{d\tau_y}\right)^*_B$, $\left(\frac{d\tau_y}{d\tau_x}\right)^*_A$, are arbitrarily specified. We now impose the condition of consistency of these conjectures by equating them to the slopes of the actual reaction functions. For notational convenience, let

$$\Gamma^A(\tau_y,\tau_x) \equiv (\alpha_p-\alpha_y)\tau_x[(\tau_y-1)\beta_q + \tau_y] \tag{15a}$$

$$\Omega^A(\tau_y,\tau_x) \equiv (\beta_q+\beta_x)\tau_y[(\tau_y-1)\alpha_p + 1] \tag{15b}$$

$$\Gamma^B(\tau_y,\tau_x) \equiv (\beta_q+\beta_x)\tau_y[(\tau_x-1)\alpha_p - \tau_x] \tag{15c}$$

$$\Omega^B(\tau_y,\tau_x) \equiv (\alpha_p-\alpha_y)\tau_x[(\tau_x-1)\beta_q - 1] \tag{15d}$$

and denote the slopes of the reaction functions by

$$\phi_A \equiv \left(\frac{d\tau_y}{d\tau_x}\right)_A ; \quad \phi_B \equiv \left(\frac{d\tau_x}{d\tau_y}\right)_B$$

Following the usual procedures (see, e.g., Bresnahan), the CCV solution is determined by[7/]

$$\Gamma^A(\tau_y,\tau_x) + \Omega^A(\tau_y,\tau_x)\phi_B = 0 \tag{16a}$$

$$\Gamma^B(\tau_y,\tau_x) + \Omega^B(\tau_y,\tau_x)\phi_A = 0 \tag{16b}$$

$$\frac{\partial\Omega^B}{\partial\tau_x}\phi_A\phi_B + \frac{\partial\Omega^B}{\partial\tau_y}\phi_A + \frac{\partial\Gamma^B}{\partial\tau_x}\phi_B + \frac{\partial\Gamma^B}{\partial\tau_y} = 0 \tag{16c}$$

$$\frac{\partial\Omega^A}{\partial\tau_y}\phi_A\phi_B + \frac{\partial\Omega^A}{\partial\tau_x}\phi_B + \frac{\partial\Gamma^A}{\partial\tau_y}\phi_A + \frac{\partial\Gamma^B}{\partial\tau_x} = 0 \tag{16d}$$

These equations jointly determine the optimal tariffs τ_x, τ_y, as well as the slopes of the reaction curves ϕ_A, ϕ_B. Because of the nonlinearity, these solutions need not be unique. There may be a multiplicity of solutions; alternatively there may be no real solution to these equations, in which case there is no CCV solution.[8/]

3. CHARACTERIZATION OF CCV EQUILIBRIUM

Equations (16) give general characterizations of the CCV equilibrium. The following propositions can be established and are proved in the Appendix.

<u>Proposition 1</u>: A Cournot equilibrium is CCV if and only if the elasticity of each country's offer curve is independent of the other country's tariff.[9]

In their analysis of conjectural variation, Thursby and Jensen (1983) have shown that under certain conditions, free trade may emerge as a special case of their conjectured tariff equilibrium. By contrast, we can show:[10]

<u>Proposition 2</u>: Free trade is never a CCV equilibrium.

4. CONSTANT ELASTICITY OFFER CURVES

The expressions for the optimal tariffs under CCV, embodied in (15), (16) are extremely complicated to determine. In this section we consider the case where all elasticities β_q, β_x, α_p, α_y, are constant. Eliminating ϕ_A, ϕ_B, under these conditions, the solutions in (16) simplify to the following pair of equations in τ_y, τ_x

$$\frac{(\alpha_p-\alpha_y)[(\tau_y-1)\beta_q+\tau_y]}{(\tau_y-1)\alpha_p+1}=\frac{(\beta_q+\beta_x)[(\tau_x-1)\alpha_p-\tau_x][(\tau_x-1)\beta_q-1]}{\alpha_p[(\tau_x-1)\beta_q-1]-\beta_q\tau_x[(\tau_x-1)\alpha_p-\tau_x]} \tag{17a}$$

$$\frac{(\beta_q+\beta_x)[(\tau_x-1)\alpha_p-\tau_x]}{(\tau_x-1)\beta_q-1}=\frac{(\alpha_p-\alpha_y)[(\tau_y-1)\alpha_p+1][(\tau_y-1)\beta_q+\tau_y]}{\beta_q[(\tau_y-1)\alpha_p+1]-\alpha_p\tau_y[(\tau_y-1)\beta_q+\tau_y]} \tag{17b}$$

Equation (17a) defines the reaction curve for Country A, <u>correctly taking account</u> of the reaction function of Country B. Likewise, (17b) does the same for Country B. The pair of equations may therefore be termed as being "generalized" reaction functions. Because of the interactions they embody, both of these equations are highly nonlinear, raising the possibility of a multiplicity of solutions. Note that

$$(\tau_y-1)\beta_q+\tau_y=0, \quad (\tau_x-1)\alpha_p-\tau_x=0$$

satisfy the two equations. This verifies Proposition 2, that with offer curves having constant elasticities, the Cournot solution is also a CCV solution. But there are other solutions as well.

In order to study (17a), (17b) further, and to obtain an explicit solution, we shall consider the case where not only all elasticities are constant, but also both countries are symmetric with

$$\alpha_p = -\beta_q \equiv \alpha, \quad \alpha_y = \beta_x$$

Given the symmetry it is clear that the solutions to τ_x, τ_y, obtained at the intersection of the reaction functions (17a), (17b) must imply $\tau_x = \tau_y$, a fact which can be verified directly. Thus setting $\tau_x = \tau_y = \tau$ in (17a) say and simplifying the common CCV tariffs are obtained as the solutions to the equation

$$[(1-\alpha)\tau + \alpha][\alpha\tau^2 - 2\alpha\tau - (1-\alpha)] = 0 \tag{18}$$

Letting $\tau = 1+\tau$, the solutions to this cubic equation are

(i) $t = t_1^* = \dfrac{1}{\alpha - 1}$

(ii) $t = t_2^* = \dfrac{1}{\sqrt{\alpha}}$

(iii) $t = t_3^* = \dfrac{-1}{\sqrt{\alpha}}$

Thus, in general, there are <u>three</u> CCV equilibria, one of which is $t = t_1^*$, the usual Cournot equilibrium.

These solutions may be ranked over varying values of the elasticity α. First, in order for the Marshall-Lerner condition to hold $\alpha > 1/2$. Secondly, for the range $1/2 < \alpha < 1$, both t_1^* (Cournot) and t_3^* imply corresponding values $\tau_1^*, \tau_3^* < 0$. That is, the price of the imported good in each country is negative, which is obviously infeasible. Hence, for $1/2 < \alpha < 1$ there is a unique feasible CCV solution, namely

$$t = t_2^* = \frac{1}{\sqrt{\alpha}}$$

For $\alpha > 1$, all three solutions are feasible, although t_3^* represents a subsidy. If $1 < \alpha < \alpha^*$, where $\alpha^* = 2.618$ is the positive solution to the equation

$$\alpha^2 - 3\alpha + 1 = 0$$

the rankings of the optimal tariffs are

$$t_1^* > t_2^* > t_3^*$$

On the other hand, if $\alpha > \alpha^* > 1$, the rankings are altered to

$$t_2^* > t_1^* > t_3^*.$$

5. CONCLUSIONS

This paper has determined the optimal tariff under the assumption that each country correctly anticipates the response by its rival. We have given both a general characterization of this CCV equilibrium and considered a specific example. In general, there are a multiplicity of CCV equilibrium tariffs. Under certain restrictions one of these may be the Cournot equilibrium, which (in this case correctly) assumes no retaliation. But free trade is never such an equilibrium outcome.

FOOTNOTES

*I am grateful to two referees for their comments and to Tamer Basar for helpful discussions on the game-theoretic underpinnings of the Consistent Conjectural Variations equilibrium.

1/ See, e.g., Johnson (1954), Gorman (1958), Horwell (1966), and more recently Otani (1980). Some of the literature is summarized in a survey article by McMillan (1985).

2/ Alternative tariff strategies are considered by Mayer (1981), Riezman (1982).

3/ The notion of consistent conjectural variations equilibrium has been refined recently by Basar (1986) who defines CCV equilibria of different orders. He shows that when appropriately defined, the CCV can indeed be a legitimate equilibrium concept. According to this classification, the Cournot equilibrium is a zero'th order CCV equilibrium. The usual definition, and the one we shall consider here, is a first order CCV equilibrium, in which the players correctly conjecture on the slopes (first derivatives) of the reaction functions. More generally in a CCV equilibrium of the n'th order, the rivals correctly conjecture on the first n derivatives of the rival's reaction function.

4/ The model we employ is a standard trade model such as in Bhagwati and Srinivasan (1983) or Takayama (1974).

5/ Under our assumptions, $E_y^A > 0$, $E_x^B > 0$.

6/ See, e.g., Bhagwati and Srinivasan (1983) or Takayama (1974).

7/ Details of this derivation are available from the author on request.

8/ Problems of nonexistence can arise when the two countries are too dissimilar.

9/ A similar proposition has been obtained by Jensen and Thursby (1984).

10/ That is, in the Thursby-Jensen model free trade may emerge as a noncooperative equilibrium. This is not so under our CCV assumptions. By contrast, in the Eaton and Grossman (1983) duopoly model, with no home consumption, free trade emerges as the CCV equilibrium. Mayer (1981) and Riezman (1982) show how free trade may result from a cooperative equilibrium.

APPENDIX

Proof of Propositions 1 and 2

Proof of Proposition 1

Assuming a Cournot equilibrium and setting $\phi_A = \phi_B = 0$ in (16a), (16b) of the text, these equations reduce to

$$\Gamma^A(\tau_y,\tau_x) = \Gamma^B(\tau_y,\tau_x) = 0.$$

Noting the definitions in (15a), (15b) of the text, these conditions can be expressed in terms of elasticities, as

$$(\tau_y-1)\beta_q + \tau_y = 0$$

$$(\tau_x-1)\alpha_p - \tau_x = 0$$

The equilibrium will be consistent with CCV (with zero conjectured variation) if and only if

$$\frac{d\beta_q}{d\tau_x} = 0; \qquad \frac{d\alpha_p}{d\tau_y} = 0$$

Since $\beta_q = \beta_q(q,\tau_x)$, we have

$$\frac{d\beta_q}{d\tau_x} = \frac{\partial\beta_q}{\partial q}\frac{\partial q}{\partial\tau_x} + \frac{\partial\beta_q}{\partial\tau_x}$$

and substituting for $\partial q/\partial\tau_x$ from (13), see that $d\beta_q/d\tau_x = 0$ if and only if

$$\frac{\partial\beta_q}{\partial q}\frac{q}{\beta_q}[\beta_x + \alpha_p - 1] + \frac{\partial\beta_q}{\partial\tau_x}\frac{\tau_x}{\beta_q}[\alpha_p - \beta_q - 1] = 0$$

An analogous condition holds for $d\alpha_p/d\tau_y = 0$.

Proof of Proposition 2

To establish this proposition suppose that free trade were consistent with CCV. In this case $\tau_x = \tau_y = 1$ (i.e., $t_x = t_y = 0$) are solutions to (16a)-(16d). Setting $\tau_x = \tau_y = 1$ in the definitions in (15) we have

$$\Gamma^A = \alpha_p - \alpha_y; \quad \Omega^A = \beta_q + \beta_x$$

$$\Gamma^B = -(\beta_q + \beta_x); \quad \Omega^B = -(\alpha_p - \alpha_y)$$

so that (16a), (16b) become

$$(\alpha_p - \alpha_y) + (\beta_q + \beta_x)\phi_B = 0 \tag{A.1a}$$

$$(\beta_q + \beta_x) + (\alpha_p - \alpha_y)\phi_A = 0 \tag{A.1b}$$

Multiplying these two equations together yields

$$\phi_A \phi_B = 1 \tag{A.2}$$

Differentiating the expressions in (15) with respect to τ_y, τ_x, and evaluating at the free trade equilibrium $\tau_y = \tau_x = 1$, yields

$$\frac{\partial \Gamma^A}{\partial \tau_y} = \left(\frac{\partial \alpha_p}{\partial \tau_y} - \frac{\partial \alpha_y}{\partial \tau_y}\right) + (\alpha_p - \alpha_y)(\beta_q + 1); \quad \frac{\partial \Gamma^A}{\partial \tau_x} = \frac{\partial \alpha_p}{\partial \tau_x} - \frac{\partial \alpha_y}{\partial \tau_x} + \alpha_p - \alpha_y$$

$$\frac{\partial \Omega^A}{\partial \tau_y} = \frac{\partial \beta_q}{\partial \tau_y} + \frac{\partial \beta_x}{\partial \tau_y} + (\beta_q + \beta_x)(1 + \alpha_p); \quad \frac{\partial \Omega^A}{\partial \tau_x} = \frac{\partial \beta_q}{\partial \tau_x} + \frac{\partial \beta_x}{\partial \tau_x}$$

$$\frac{\partial \Gamma^B}{\partial \tau_y} = -\left(\frac{\partial \beta_q}{\partial \tau_y} + \frac{\partial \beta_x}{\partial \tau_y}\right) - (\beta_q + \beta_x); \quad \frac{\partial \Gamma^B}{\partial \tau_x} = -\left(\frac{\partial \beta_q}{\partial \tau_x} + \frac{\partial \beta_x}{\partial \tau_x}\right) + (\beta_q + \beta_x)(\alpha_p - 1)$$

$$\frac{\partial \Omega^B}{\partial \tau_y} = -\left(\frac{\partial \alpha_p}{\partial \tau_y} - \frac{\partial \alpha_y}{\partial \tau_y}\right); \quad \frac{\partial \Omega^B}{\partial \tau_x} = -\left(\frac{\partial \alpha_p}{\partial \tau_x} - \frac{\partial \alpha_y}{\partial \tau_x}\right) + (\alpha_p - \alpha_y)(\beta_q - 1)$$

Next, substituting these quantities into (16c) and (16d) and using (A.2), we obtain

$$(\frac{\partial\beta_q}{\partial\tau_y} + \frac{\partial\beta_x}{\partial\tau_y}) + (\beta_q+\beta_x)(1+\alpha_p) + (\frac{\partial\alpha_p}{\partial\tau_x} - \frac{\partial\alpha_y}{\partial\tau_x}) + (\alpha_p-\alpha_y)$$

$$+ \left[\frac{\partial\alpha_p}{\partial\tau_y} - \frac{\partial\alpha_y}{\partial\tau_y} + (\alpha_p-\alpha_y)(1+\beta_q)\right]\phi_A + (\frac{\partial\beta_q}{\partial\tau_x} + \frac{\partial\beta_x}{\partial\tau_x})\phi_B = 0 \tag{A.1c}$$

$$- (\frac{\partial\alpha_p}{\partial\tau_x} - \frac{\partial\alpha_y}{\partial\tau_y}) - (\alpha_p-\alpha_y)(1-\beta_q) - (\frac{\partial\beta_x}{\partial\tau_y} + \frac{\partial\beta_q}{\partial\tau_y}) - (\beta_q+\beta_x)$$

$$+ \left[-(\frac{\partial\beta_q}{\partial\tau_x} + \frac{\partial\beta_x}{\partial\tau_x}) + (\beta_x+\beta_q)(\alpha_p-1)\right]\phi_B - (\frac{\partial\alpha_p}{\partial\tau_y} - \frac{\partial\alpha_y}{\partial\tau_y})\phi_A = 0 \tag{A.1d}$$

Finally, summing these equations and substituting for (A.1a), (A.1b), these equations reduce to

$$(\beta_q + \beta_x - \alpha_p + \alpha_y)(\beta_q - \alpha_p + 1) = 0 \tag{A.3}$$

But this is a contradiction. By the Marshall-Lerner condition $\alpha_p - \beta_q - 1 > 0$, while the condition

$$\beta_q + \beta_x - \alpha_p + \alpha_y = 0$$

contradicts the sign restrictions imposed in (11) of the text. Thus we conclude that free trade is never a CCV equilibrium.

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