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TARIFFS, SAVING AND THE CURRENT ACCOUNT

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ABSTRACT

We investigate the effects of higher tariffs on the current account. Tariffs may increase or decrease investment depending on the capital intensity of the sector protected. We find that the response of saving to tariffs is sensitive to the modelling of saving behavior. In a model in which consumers' discount rate varies endogenously (in the Uzawa preference form), saving falls with higher tariffs. This result may, however, be reversed in the Blanchard-Yaari type model in which consumers have uncertain lifetimes. We find that in both models the response of saving depends on a production distortion effect which changes steady-state income and an effect on steady-state expenditures.

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Tariffs are frequently proposed as a policy to reduce or eliminate current account deficits. This paper explores the effectiveness of those policies for a small country in the context of a two-sector neoclassical growth model. We find it less insightful to examine the current account when written in its net exports form. A naive analysis leads one to conclude that since tariffs reduce imports, there must be a tendency to improve the current account. This is a very partial equilibrium viewpoint that ignores adjustments throughout the economy. In static neoclassical trade models (in which the current account is zero by assumption) shifts in production and consumption patterns ensure that any reduction in imports is matched by an equal drop in exports. In a large class of macroeconomic models with flexible exchange rates the tariff also has no impact on the current account, because an exchange rate appreciation will immediately offset all changes from higher tariffs. To understand the long run consequences of tariff policies, we want to consider the components of the current account in its saving less investment form. This allows us to see clearly that if a tariff is to reduce a current account deficit it must have the effect of decreasing the country's international borrowing.

We concentrate mainly on how tariffs change the level of saving. The response of saving is found to be sensitive to the specific modelling of saving behavior. In particular, we compare saving behavior in two popular intertemporal optimizing models of small open economies - the endogenous discount rate approach of Uzawa (1968) (which has been taken in the international context by Obstfeld (1981, 1982) among others) and the uncertain lifetime model of Blanchard (1985) and Yaari (1965). In some respects, the saving functions derived from these models are quite similar. Both assume that savers maximize utility over an infinite horizon, and both generate

Metzlerian saving equations in which the level of saving is proportional to the difference between some target level of wealth and current wealth. Yet, the response of national saving to tariffs can be quite different in the two formulations. The target wealth level will change according to two effects that we identify - a steady-state expenditure effect and a production distortion effect. The distortion effect always works to increase saving when tariff levels rise. In the Uzawa-type model tariffs also increase saving through the expenditure effect, in contrast to the uncertain lifetime model in which the expenditure effect may lead to lower saving as tariffs go up.

Our small open economy consists of two sectors - one that produces a good that can be used either for current consumption or for investment, and the other being a pure consumption good. The composite good is manufactured with labor and capital, while the pure consumption good uses labor and land in its production. This particular structure was chosen because it represents the simplest possible arrangement that allows a capital good to be produced and traded, and that allows international borrowing.[1] It is well known that the more familiar two sector models in which labor and capital are used to produce both goods yield, in general, an indeterminate capital stock when foreign borrowing is permitted.[2]

The model is dynamic, since international borrowing is inherently not a static phenomenon. Furthermore, we examine the dynamics of the current account over an infinite span of time. Another approach would have been to look at a two-period model of the economy, but there are drawbacks to such a tact. It is impossible in the two-period world to distinguish between the short-run and long-run effects of policy changes. Also, the two-period view can be limiting when trying to study the dynamics of borrowing. A dollar borrowed today must be paid back with interest in that set-up. With an infinite horizon, the principal on the loan never needs to be paid back - the present-value of the stream of interest payments equals the value of the principal.

Since we wish to focus on the effect of tariffs on saving, we have assumed that there are no costs to adjusting the capital stock. Thus, the entire investment response occurs on impact when the tariff rates change. If the tariff is placed on the pure consumption good then production of that good expands, drawing labor out of the composite good sector. This lowers the marginal productivity of capital in that sector, thus making capital a less attractive asset than foreign bonds. Capital is immediately traded internationally for bonds until the marginal productivity of capital increases into equality with the world interest rate. If the tariff were levied on the composite good the opposite reaction would occur - there would be an immediate export of foreign bonds for capital. Thus, the impact effect from investment changes on the current account depends on which good the tariff is levied.

Countries that are small in international capital markets, and in which individuals are infinitely lived and have constant discount rates, cannot reach a steady-state with non-zero wealth unless the knife-edge condition is met that the discount rate at home equals the world interest rate. The Yaari-Blanchard model endogenizes the interest rate, in a sense which will be made clear later. The Uzawa-Obstfeld approach assumes the discount rate changes in response to changes in expenditure levels. Both models yield a saving equation near the steady-state that can be written as

$$s = \theta(\bar{a} - a),$$

where s is saving, a is tradeable assets (capital plus bonds) and $\bar{a}$ is the steady-state level of a. Since a cannot change immediately when tariffs are imposed (capital can only be acquired through borrowing in the very short run), the effect on saving of a tariff is directly related to the effect on steady-state holdings of tradeable assets. The response of $\bar{a}$ to the tariff, however, may be very different in the two models.

At this point it is worth emphasizing that we are interested only in the positive question of how tariffs affect the current account in a small economy, and do not examine welfare questions.

Section 2 sets up the model and explores the effects of tariffs on investment. In section 3, the response of saving to a tariff on the pure consumption good is explored when consumers have Uzawa preferences, while the same issue is dealt with in section 4 under the assumption that consumers have uncertain lifetimes. Section 5 takes up briefly the case of a tariff on the composite good. Conclusions are drawn in the final section. Much of the formal mathematics is included in an appendix.

2. The Model

There are two goods produced in our model - a pure consumption good and a composite good that can be consumed or used as an investment good. The composite good, which is labelled good 1, uses capital and labor in its production. The production function is assumed to be constant returns to scale, and output is given by

$$y_1 = kf(x/k)$$

where k is the stock of capital and x is the amount of labor employed in industry 1. Output in the second industry uses land and labor in its production, and the technology is again constant returns to scale. Labor is mobile between industries and it is assumed that the total labor supply as well as the total land stock are fixed at 1. We can write

$$y_2 = g(1 - x) .$$

Capital depreciates at a rate n, so

$$i = \dot{k} + nk$$

where i equals the rate of investment.

The current account is equal to the trade surplus added to interest earned on holdings of foreign bonds. We have

$$(1) \qquad \dot{b} = rb - \tau ,$$

where b is domestic holdings of foreign assets, τ is the trade deficit and r is the given world interest rate. This equation says that the current account surplus equals the rate of accumulation of foreign assets.

For the economy as a whole

$$(2) \qquad y_1 + py_2 = z + i - \tau \ ,$$

where z is the value of total current consumption expenditure on the two goods (valued at world prices) by domestic residents, and p is the world price of good 2. This simply states that the value of output equals consumption plus investment less the trade deficit. (There is no government sector per se. Tariff revenue is redistributed back to consumers with lump-sum transfers.)

It is convenient at this point to introduce the notation

$$\ell \equiv x/k \ .$$

Competitive asset markets and the free mobility of capital internationally ensure that bonds and physical capital offer the same rate of return:

$$(3) \qquad r = f(\ell) - \ell f'(\ell) - n \ .$$

The right side of the equation is the net marginal productivity of capital. For a given world rate of interest and depreciation rate this equation implies ℓ is fixed over time.

Since labor is mobile between sectors, the marginal productivity of labor will be equalized in the two industries:

$$(4) \qquad \tilde{p}g'(1 - \ell k) = f'(\ell) \ .$$

Here, $\tilde{p}$ is the domestic price of good 2, which will differ from the world price if tariffs are in place. Except at the instant of a change in the tariff rate, $\tilde{p}$ does not change over time. Given that ℓ is fixed from eq. (3), the capital stock k will only change at the moment the tariff is altered. So,

$$\dot{k} = 0 ,$$

and

$$i = nk .$$

This country may be net importers of both goods, only one good or neither good at any point in time. From eq. (4)

$$dk/d\tilde{p} = g'/\ell\tilde{p}g'' < 0 .$$

If the tariff on the pure consumption good is increased, the capital stock falls. This occurs because production of good 2 increases, drawing labor out of sector 1 which is the capital-using industry. So, there is an incipient drop in the marginal productivity of capital. Disinvestment occurs as capital is traded for foreign bonds. If the tariff on the pure consumption good is lowered--or, equivalently, the tariff on the composite good is raised--the capital stock increases.

We can see now that the direction of the investment effect on the current account of a change in the level of protection depends upon which good the tariff is levied. If the capital-using good is protected, investment increases and the current account falls. The current account balance will go up, on the other hand, if tariffs on the good that uses land and labor in its production are raised.

In this set-up the capital stock can change discretely. If investment is to occur over time there must be some rigidity that prevents the immediate adjustment of the stock of capital. A popular, though somewhat ad hoc, way of modelling this is to impose adjustment costs for both increasing and decreasing the capital stock. A more natural way of allowing gradual investment and disinvestment is to assume capital, once in place, cannot be moved. Disinvestment could take place only at the rate of depreciation. New investment could only occur as new capital goods are produced. A small country might reasonably be able to meet its capital needs in a very short period of time with capital imports, since its desired investment might be a small fraction of current production of capital goods. However, a large country could not increase its capital stock quickly since its desired investment might exceed current investment goods production.

Another direction in which the model of investment could be altered is to allow a more general production structure. For example, we might allow all three factors to move between industries. In this case, if the elasticities of substitution between factors are equal in the two industries, then protection of a good will unambiguously lead to an increase (decrease) in the capital stock if that industry uses a larger (smaller) share of the country's capital stock than its share of the supply of labor or land. If there are more goods and factors, some weaker general results are available in Ruffin (1984) and the references cited therein.

3. Saving in the Uzawa-Obstfeld Model

In this sector we will consider a model with a representative consumer who has Uzawa preferences. We will assume the pure consumption good is protected and look at the effects on saving of increasing the level of protection.

At any moment in time, current felicity depends on consumption of both goods - $u(c_1, c_2)$. It is convenient, however, to express the level of felicity by the indirect utility function

$$v(I, p) = \max \{u(c_1, c_2) | c_1 + \tilde{p}c_2 \leq I\}$$

where I represents the level of expenditure at any given time, expressed in terms of domestic prices.

A consumer maximizes the integral

$$V = \int_0^{\infty} v_t e^{-\Delta_t} dt$$

where

$$\Delta_t = \int_0^t \delta_s ds$$

and δ_s is the instantaneous subjective discount rate at time s. Following Uzawa, we take δ_s to be a function of utility at time s:

$$\delta_s = \delta(v_s) \; .$$

As in Uzawa, we assume

$$\delta > 0 \ , \ \ \delta' > 0 \ , \ \ \delta - \delta' v > 0 \ , \ \ \delta'' > 0 \ .$$

Consumers choose their level of expenditure subject to the constraint

$$\dot{w} = rw + f'(\ell) + R - I \ , \tag{5}$$

where w is the value of non-human wealth owned by the individual. By definition

$$w = b + k + \tilde{p}(g - (1-x)g'(1-x))/r \ .$$

The last term in this equation represents the value of land. In eq. (5) R is tariff revenue distributed to the individual. Consumers take R as given and do not perceive that their choices alter its level. Since the capital stock and the value of land do not change over time $\dot{b} = \dot{w}$. Therefore, eq. (5) could be rewritten as

$$\dot{b} = rb + y_1 + \tilde{p}y_2 + R - i - I \ .$$

Then we can see from eqs. (1) and (2)

$$R = (\tilde{p} - p)(c_2 - y_2) \ .$$

The aggregate model is shown in the Appendix to be characterized by saddle stability. Therefore, near the steady-state we have

$$\dot{b} = \theta_1(\bar{b} - b) \ , \quad \theta_1 > 0$$

where $-\theta_1$ is the negative eigenvalue of the dynamic system. Define tradeable assets a to be

$$a = b + k \ .$$

Since equation (4) tells us capital is fixed over time, $k = \bar{k}$. Thus, we can write

$$(6) \qquad \dot{b} = \theta_1(\bar{a} - a) \ .$$

This is a particularly useful equation to analyze. It represents the accumulation of foreign bonds over time - i.e., the current account. The current account just equals saving, because all investment changes occur discretely at a point in time.

The level of a is given to the economy at any given time. Capital can be traded for bonds, and vice-versa, but their sum can only change over time. Thus, if tariffs are to affect saving it can only be through effects on $\bar{a}$. According to equation (6), as $\bar{a}$ rises so does saving and the current account.

Given our assumptions on the mobility of international capital, any model of saving that has a stable steady-state will yield a saving equation such as (6) near the steady-state. However, different models of saving behavior may imply that the target level of traded assets $\bar{a}$ responds differently as tariffs are increased.

From eqs. (1) and (2), we have that in steady-state when $\dot{b} = 0$,

$$\dot{\bar{b}} = (i - y_1 - py_2)/r + \bar{z}/r .$$

Therefore,

(7) $$\bar{a} = [k + (i - y_1 - py_2)/r] + [\bar{z}/r] .$$

It is useful to look at the change in the two bracketed terms on the right side of eq. (7) separately. We would like to know how each term changes with an increase in tariffs. First let us note

$$d(y_1 + py_2)/dk = d(kf(\ell) + pg(1 - k\ell))/dk$$

$$= f(\ell) - p\ell g(1 - x)$$

$$= r + n + (\tilde{p} - p)\ell g'(1 - x) .$$

The last step uses eqs. (3) and (4). Remembering that $i = nk$, we then have

$$d[k + (i - y_1 - py_2)/r]/d\tilde{p} = [1 + (1/r)d(i - y_1 - py_2)/dk]dk/d\tilde{p}$$

$$= -[(\tilde{p} - p)\ell g'(1 - x)/r]dk/d\tilde{p} .$$

This derivative is positive because $\tilde{p} > p$ and $dk/d\tilde{p} < 0$.

We see that from the first term of eq. (7), steady-state holdings of traded assets must rise with an increase in the tariff on the pure consumption good. Intuitively, in order to maintain the same level of income in steady-state after the tariff is imposed, the capital that is shipped abroad must be replaced by bonds. But, in fact, the economy needs to replace the capital with more than an equal amount of bonds to generate the same level of income. With a tariff already in place there was a distortion that caused the economy to have a lower capital stock than it would under free trade. An increase in the tariff worsens the distortion as it moves more resources to the protected sector. So, to maintain the same level of income, bonds must be imported not only to offset the lost capital but also to counteract the aggravation of the distortion. We call this effect on steady-state holdings of b + k the distortion effect, and it causes $\bar{a}$ to rise with an increase in the tariff rate irrespective of the model of saving behavior.

The second term in eq. (7) involves the steady-state level of expenditure, $\bar{z}$. If consumers have the endogenous discount rate of Uzawa preferences, then over time expenditure adjusts so that in the steady state the discount rate equals the world rate of interest:

$$\delta(v(I, \tilde{p})) = r \; .$$

The steady-state level of felicity is determined by this relationship, and will not change if a tariff is imposed. (Thus, all welfare loss from the imposition of a tariff comes along the transition to the steady-state, but not in the steady-state itself.)

An increase in the tariff rate will raise the long-run level of expenditure at world prices. Figure 1 demonstrates this increase in expenditure for a finite tariff starting from free trade. Before the tariff, steady-state consumption is at point a, and the expenditure is z. With the

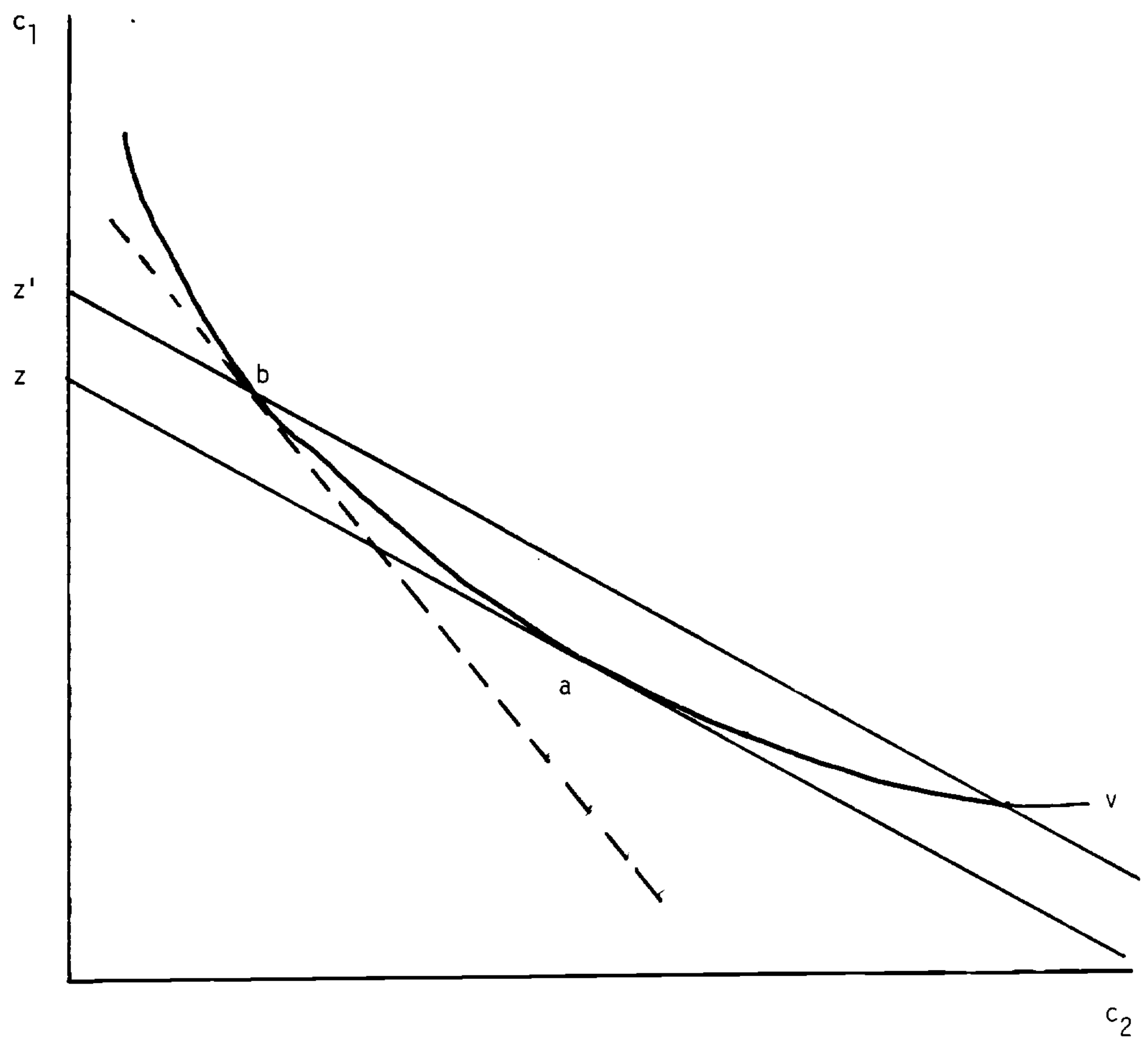

Figure 1

tariff, consumers set their marginal rate of substitution equal to the domestic price given by the slope of the dotted line. So, in order to maintain the same long-run felicity, expenditure rises to z'. This same point can be demonstrated mathematically by using properties of expediture functions. We can define

$$I(1, \tilde{p}, v) = \min \{c_1 + \tilde{p}c_2 | u(c_1, c_2) \geq v\} .$$

Then

$$z = I_1(1, \tilde{p}, \bar{v}) + pI_2(1, \tilde{p}, \bar{v}) .$$

Holding felicity constant

$$dz/d\tilde{p} = I_{12} + pI_{22}$$

$$= (I_{12} + \tilde{p}I_{22}) + (p - \tilde{p})I_{22}$$

$$= (p - \tilde{p})I_{22} > 0 .$$

We see that steady-state expenditure rises when the tariff increases. Thus, in this model both the distortion effect and the expenditure effect contribute to higher saving as tariffs go up. In the next section, we will see that in the uncertain lifetime model steady-state expenditure falls with a raise in the tariff rate.

4. Saving in the Yaari-Blanchard Model

In this section we take up a model in which there is a continuum of agents, each of whom faces a constant probability of death π. At each instant of time a new cohort of size π is born. The population is constant and has a size equal to 1.

Each agent can own physical wealth in the form of bonds, capital or claims on land. Since these assets are perfect substitutes, they all earn the world rate of interest r. In addition, each agent is assumed to make a deal with an insurance company that he receive an additional rate of return π from the company if he lives, but that the company receive his physical wealth if he dies. Conversely, if the individual has net holdings of physical wealth less than zero, he agrees to pay a premium rate of π per unit of debt on the condition the insurance company assumes his debt if he dies. The expected profit for the insurance company is zero.

There are two types of wealth that are assumed not transferable to the insurance company for an annuity. The individual's human wealth (the discounted value of labor income) has no value upon death, so the insurance company is unwilling to pay anything to have the privilege of owning this asset after the person's death. Similarly, the individual has no claim on tariff revenue after death. Tariff revenue is distributed only to living persons, and not to anybody's estate.

Individuals are assumed to maximize expected utility, which, given the constant probability of death implies they maximize

$$\int_0^{\infty} v(I, \tilde{p})e^{-(\delta+\pi)t}dt \ .$$

They face the constraint

(8) $$\dot{w}_i = (r + \pi)w_i + f'(\ell) + R - I_i$$

where w_i is the value of non-human wealth owned by individual i. That is,

(9) $$w_i = b_i + k_i + \tilde{p}(g - (1 - x)g'(1 - x))/r \ .$$

The last term in eq. (9) represents the value of land. This can be seen by noting that the return to owning a unit of land is $\tilde{p}(g - (1 - x)g'(1 - x))$ plus π times the value of a unit of land.

We make an additional restriction on preferences in this section in order to be able to aggregate individual consumption into an aggregate consumption function. In particular, we assume that preferences are homothetic and can be written in constant relative risk aversion form:

$$v(I, \tilde{p}) = [I^{1-\sigma}/(1 - \sigma)]\nu(\tilde{p}) \ .$$

The Appendix shows that aggregate expenditure is proportional to wealth of all forms:

(10) $$z = \Delta[w + (f'(\ell) - (\tilde{p} - p)y_2)/(r + \pi))]$$

where

$$\Delta = r + \pi - (r - \delta)/\sigma \ .$$

Human wealth is given by $f'(\ell)/(r + \pi)$. The term $-(\tilde{p} - p)y_2$ is the sum of tariff revenue and z - I. It represents the cost to the individual of a tariff - he receives revenue R, but the price of good 2 is higher.

For the types of wealth with which the individual cannot purchase an annuity, there is no difference between the rate of return for society and the individual. They might both be discounted at a rate $r + \pi$. So, we can say the value of these assets is $[f'(\ell) - (\tilde{p} - p)y_2]/(r + \pi)$. The return on these assets is simply $f'(\ell) - (\tilde{p} - p)y_2$. On the other hand, the rate of return on physical wealth for society is only r. The annuity payment π is merely a transfer from the insurance company to individuals. Thus, for society

$$\dot{w} = rw + f'(\ell) - (\tilde{p} - p)y_2 - z \; . \tag{11}$$

This economy can reach a steady-state even though it faces a given world interest rate and has a constant discount rate. In a sense, the total rate of return on assets varies endogenously. The rate of return on physical wealth is r and on non-tangible assets $r + \pi$. The total return on the economy's portfolio is $r + \pi\lambda$, where λ is the share of wealth in non-tangible assets. As λ changes over time, the economy-wide rate of return adjusts.

If $r < \Delta$ then the system is saddle-stable, and

$$\dot{b} = \theta_2(\bar{b} - b) \; , \quad \theta_2 > 0 \; .$$

Unlike the previous section, this equation holds globally (not just near the steady-state) because of our constant relative risk aversion assumption. Once again, we can write

$$\dot{b} = \theta_2(\bar{a} - a)$$

because the capital stock jumps immediately to its long run value. So, the tariff will raise the current account if it causes $\bar{a}$ to jump up.

Eqs. (2), (3), (4), (5) and (9) can be used to show that the asset accumulation equation (11) above is equivalent to eq. (1). Thus, there is no difference from the previous section in the expression for the long-run level of traded assets

$$(7) \qquad \bar{a} = [k + (i - y_1 - py_2)/r] + [\bar{z}/r] .$$

As before, the distortion effect will imply that an increase in the tariff will cause $\bar{a}$ to go up.

Steady-state expenditures can be derived from eqs. (10) and (11) when $\dot{w} = 0$. These expenditures are simply proportional to the value of what we have called non-tangible assets:

$$\bar{z} = [\Delta\pi/(\Delta - r)][(f'(\ell) - (\tilde{p} - p)y_2)/(r + \pi)] .$$

In this model, long run expenditures fall as the tariff is increased:

$$d\bar{z}/d\tilde{p} = [\Delta\pi(\Delta - r)(r + \pi)][\ell(\tilde{p} - p)g' \frac{dk}{d\tilde{p}} - g] .$$

Here, at any point in time consumption is proportional to wealth - much as in a permanent income model. The steady-state requires that w be proportional to non-tangible assets, which in turn implies steady-state consumption expenditures are proportional to the value of non-tangible assets. Since the value of these assets falls with a tariff, so does long-run expenditure.

So, in the Yaari-Blanchard world of uncertain lifetimes and a constant discount rate, the distortion and expenditure effects of a change in tariffs work in opposite directions on the current account. The expenditure effect may in fact dominate the distortion effect, so higher tariffs could cause a decrease in the current account balance.

5. Tariffs on the Composite Good

In this section we will briefly trace through the effects of a tariff on the composite good. The results are not too different from the previous section. The only change is that in the Yaari-Blanchard model the sign of the expenditure effect is indeterminate.

It is useful in this section to change numeraires so that expenditure levels are expressed in terms of the pure consumption good. So, we now have

$$z = py_1 + y_2 + p\tau - pi \tag{12}$$

and

$$I = \tilde{p}y_1 + y_2 + R + \tilde{p}\tau - \tilde{p}i \ . \tag{13}$$

We will also express the value of bonds and the current account in terms of good 2:

(14) $$\dot{b} = rb + py_1 + y_2 - z - pi \ .$$

This implies that in steady-state, when $\dot{b} = 0$, long-run bond holdings are given by

$$\bar{b} = (pi - py_1 - y_2)/r + \bar{z}/r \ .$$

We now write the labor market equilibrium as: $\tilde{p}f'(\ell) = g'(1 - k\ell)$.

Both models are saddle-stable again, under the same set of assumptions. So, foreign bond accumulation can once again be expressed as

$$\dot{b} = \theta(\bar{b} - b) \ , \quad \theta > 0 \ .$$

The capital stock will equal its long-run value at all times, so

(15) $$\dot{b} = \theta(\bar{a} - a) \ ,$$

where

$$a = b + pk \ .$$

It follows that the tariff will affect saving only to the extent it influences $\bar{a}$. The steady-state level of $\bar{a}$ is given by

(16) $$\bar{a} = [pk + (pi - py_1 - y_2)/r] + [\bar{z}/r] \ .$$

Taking the change in the term in the first bracket in eq. (16) will yield the distortion effect. It is again positive, so that higher tariffs tend to lead to higher levels of $\bar{a}$, and a higher current balance, through this channel.

$$d[pk + (pi - py_1 - y_2)/r]/d\tilde{p} = [p + (1/r)d(pi - py_1 - y_2)/dk]dk/d\tilde{p}$$

$$= [(\tilde{p} - p)\ell f'(\ell)/r]dk/d\tilde{p} \ .$$

This derivative is greater than zero because $\tilde{p} > p$ and $dk/d\tilde{p} > 0$.

In the model with Uzawa preferences, the increase in tariffs again leads to a higher steady-state expenditure level. The current account rises from both the distortion effect and the expenditure effect. Formally, the long-run felicity level is determined by the condition

$$\delta(v(I, \tilde{p})) = r \ .$$

Now, let us define

$$I(\tilde{p}, 1, v) = \min \{\tilde{p}c_1 + c_2 | u(c_1, c_2) \geq v\} \ .$$

Then

$$z = pI_1(\tilde{p}, 1, \bar{v}) + I_2(1, \tilde{p}, \bar{v}) \ .$$

Holding felicity constant

$$d\dot{z}/d\tilde{p} = pI_{11} + I_{21}$$

$$= (p - \tilde{p})I_{11} + (I_{21} + \tilde{p}I_{11})$$

$$= (p - \tilde{p})I_{11} > 0 .$$

In the uncertain lifetime formulation, aggregate expenditure is given by

(17) $$z = \Delta[w + (\tilde{p}f'(\ell) + (\tilde{p} - p)(i - y_1))/(r + \pi)] ,$$

where

(18) $$w = b + \tilde{p}k + (g - (1 - \ell k))g')/r .$$

Human wealth is given by $\tilde{p}f'(\ell)/(r + \pi)$. The term $(\tilde{p} - p)(i - y_1)$ again represents the change in the individual consumer's income from the tariff, equalling the sum of tariff revenue and $z - I$.

Saving is given by the relation

(19) $$\dot{w} = rw + \tilde{p}f'(\ell) + (\tilde{p} - p)(i - y_1) - z .$$

Eqs. (12), (13) and (18) can be used to show that equation (19) is identical to eq. (14). Thus, eq. (16) gives the steady-state holdings of $\bar{a}$ in this model.

We can use eqs. (18) and (19) to solve for long run consumption expenditures:

$$\dot{\bar{z}} = [\Delta\pi/(\Delta - r)][(\tilde{p}f'(\ell) + (\tilde{p} - p)(i - y_1))/(r + \pi)] .$$

The change in $\bar{z}$ from an increase in the tariff on the composite good is given by:

$$d\bar{z}/d\tilde{p} = [\Delta\pi/(\Delta - r)(r + \pi)][(\tilde{p} - p)(n - f(\ell)) \frac{dk}{d\tilde{p}} - (kf(\ell) - f'(\ell) - nk)] .$$

The first term in the second bracket is negative because $f(\ell) > n$ on the assumption that the world interest rate is positive. This means the entire expression for $d\bar{z}/d\tilde{p}$ is less than zero if $(kf(\ell) - f'(\ell) - nk)$ is positive. This condition will be met if capital's income exceeds the income of labor employed in the consumption goods sector.

The expenditure effect on steady-state traded asset holdings can be negative (though it need not be). If it is negative, it may outweigh the distortion effect. So, once again in the uncertain lifetime model raising tariffs may lower the current account balance.

6. Conclusions

We have examined in this paper how the current account responds to increases in the level of protection, according to two popular models for small economies that can borrow internationally. The two models--the Uzawa-Obstfeld endogenous time preference set-up and the Yaari-Blanchard uncertain lifetime formulation--have much in common. Both assume consumers have foresight and optimize; and both, under the proper set of assumptions, describe economies that converge to a steady-state. Yet, the reaction of the current account to tariff changes can be quite dissimilar in these models.

This conclusion is of practical importance. There is increasing use in the profession of general equilibrium models for small less developed countries to try to give answers to policy questions. These models are empirically based, and some (such as Kharas and Shishido (1985), and Ghanem (1985)) have allowed intertemporal optimization. A lesson of this paper is that some policy conclusions drawn about the effects of tariffs might be quite sensitive to the structure of the model. It is necessary to know whether savers have a target level of long-run welfare or whether their expenditure levels are just proportional to permanent income.

Although in some ways the models studied in this paper are limited, the structure is rich enough to highlight the usefulness of the dynamic intertemporal approach to current account analysis. The effect on saving of higher tariffs is seen to depend on matters that are not at all relevant in static or two-period models. The impact of the tariff on the steady-state levels of income and consumption through the distortion effect and the expenditure effect are of primary importance.

Mathematical Appendix

A. Uzawa-Obstfeld Model

In setting up the formal optimization problem for the model described in section 3, it is useful to note

$$d\Delta = \delta dt .$$

We can write the level of utility as

$$V = \int_0^\infty (v/\delta)d\Delta .$$

We also have

$$db/dt = (1/\delta)db/d\Delta .$$

With these facts in mind, we can write the Hamiltonian for the individual's optimization problem as

$$H = (1/\delta)(v + q(rb + y_1 + \tilde{p}y_2 + R - i - I)) .$$

We also impose the constraint that

$$\lim_{t\to\infty} b_t e^{-rt} \geq 0 .$$

Without this constraint, given the infinite planning horizon, an individual could achieve an arbitrarily high level of utility by borrowing a large sum now and meeting interest payments through future borrowing. When we solve the individual's optimum problem, if the transversality condition is satisfied then the constraint is also satisfied.

The first-order conditions are given by

$$-(\delta' v'/\delta)q(rb + y_1 + \tilde{p}y_2 + R - i - I) + (v'/\delta)(\delta - \delta' v) = q$$

or

$$q = v'(\delta - \delta' v)/[\delta + \delta' v(rb + y_1 + \tilde{p}y_2 + R - i - I)]$$

and

$$\dot{q} = (\delta - r)q \ .$$

Taking the time derivative of the log of the first condition and equating it to the second we have

$$\delta - r = [v''(\delta - \delta' v) - \delta'' v'^2 v]\dot{I}/v'(\delta - \delta' v)$$

$$- \delta' v'[r\dot{b} + \dot{I} - \dot{z}]/[\delta + \delta' v'\dot{b}]$$

$$- [\delta'' v'^2 + \delta' v'']\dot{b}\dot{I}/[\delta + \delta' v'\dot{b}] \ .$$

We will use the fact that $\dot{z} = (1 + (p - \tilde{p})c_2')\dot{I}$ where c_2' is the derivative of c_2 with respect to income, which is assumed to be positive.

Solving out, we get

$$\dot{z} = \frac{[(1+(p-\tilde{p})c_2')v'(\delta-\delta'v)(\delta-r+\delta'v'(rb-z+y_1+py_2-i)]}{[v''(\delta-\delta'v)-\delta''v'^2(v+v'\dot{b})]-(\delta'v'^2/\delta)(\delta-\delta'v)c_2'(\tilde{p}-p)}$$

This is an expression for $\dot{z}$ in terms of z and b. If we linearize this expression near the steady-state we get

$$\dot{z} = [(1+(p-\tilde{p})c_2')v'(\delta-\delta'v)\delta'v'r](b-\bar{b})/D$$

$$+ [(\tilde{p}-p)c_2'v'(\delta-\delta'v)\delta'v'](z-\bar{z})/D$$

where $D = v''(\delta-\delta'v)-\delta''v'^2v - (\delta'v'^2/\delta)(\delta-\delta'v)c_2'(\tilde{p}-p) < 0$. Both coefficients in the above equation are negative. The first is less than zero because $1 + (p-\tilde{p})c_2' > 0$. To see this, note that $1 - \tilde{p}c_2' = c_1'$ by Engel aggregation (no relation). Assuming good 1 is normal, $c_1' > 0$.

This dynamic equation combined with the bond accumulation equation

$$\dot{b} = r(b - \bar{b}) - (z - \bar{z})$$

yields a two-equation, two-variable linear dynamic system near steady state.

The system has a positive and negative root, so it exhibits saddle stability. As in the text, we have

$$\dot{b} = \theta_1(\bar{b} - b)$$

where $-\theta_1$ is the negative root of the dynamic system.

The transversality condition gives a sufficient condition for a path of b that satisfies the first-order conditions to be optimal. In this case, it is:[3]

$$\lim_{\Delta \to \infty} e^{-\Delta} q b_{\Delta} = 0 \ .$$

The path that leads to the steady state is an optimal path, so we will concentrate on the dynamics along the saddle path. Notice that the intertemporal budget constraint is satisfied along the saddle path. If initial conditions are given which are not on the path, then there will be a jump in the state variables b and k, such that wealth remains constant.[4]

In the case of a tariff on the composite good, the derivation of the equations of the dynamic system follows in a straightforward fashion the methods for the tariff on the pure consumption good. The dynamic system once again is saddle-stable.

B. Yaari-Blanchard Model

The optimization problem described in the text is for an individual at the beginning of his lifetime who is born at time 0. This particular problem was set up for notational simplicity.

The Hamiltonian for the problem described is

$$H = [I_i^{(1-\sigma)}/(1-\sigma)]\nu(\tilde{p}) + q[(r+\pi)w_i + f'(\ell) + R - I_i] \ .$$

The first order conditions are given by

$$I_i^{-\sigma} v(\tilde{p}) = q$$

and

$$\dot{q}/q = r - \delta \ .$$

Taking the time derivative of the log of the first condition and equating it to the second yields

$$\sigma \dot{I}_i / I_i = r - \delta \ .$$

This implies

$$I_{it} = I_{i0} e^{((r-\delta)/\sigma)t} \ .$$

We would like to get an expression for the evolution of z. Because we assume homothetic preferences, c_2 is a constant proportion of I. Therefore

$$(\tilde{p} - p)c_2 = \alpha I_i$$

where α is a function only of $\tilde{p}$ and not of I. Therefore,

$$z_i = (1 - \alpha) I_i \ ,$$

and we have

$$\dot{z}_{it} = z_{i0}e^{((r-\delta)/\sigma)t} .$$

We impose the transversality condition

$$\lim_{t\to\infty} w_{it}e^{-(r+\pi)t} = 0 .$$

Rewriting the dynamic budget constraint as

$$\dot{w}_i = (r + \pi)w_i + f'(\ell) - (\tilde{p} - p)y_2 - z_i ,$$

we can integrate the transversality condition to get

$$\int_0^\infty z_{it}e^{-(r+\pi)t}\, dt = w_{it} + [f'(\ell) - (\tilde{p} - p)y_2]/(r + \pi) .$$

Plugging in our expression for z_{it} and solving, we get

$$z_i = \Delta[w_i + [f'(\ell) - (\tilde{p} - p)y_2]/(r + \pi)] .$$

Aggregating as in Blanchard (1985) gives us the aggregate expenditure function in the text.

We have a two-equation, two-variable linear dynamic system, with the equations of motion given by

$$\dot{w} = r(w - \bar{w}) - (z - \bar{z})$$

and

$$\dot{z} = [(r - \delta)/\sigma](z - \bar{z}) - \Delta\pi(w - \bar{w}) .$$

This system is saddle-stable if $r - \Delta < 0$. We then have

$$\dot{b} = \theta_2(\bar{b} - b)$$

where

$$\theta_2 = \Delta - r .$$

The analysis of the case of the tariff on the composite good is a simple copy of the work in this section.

Footnotes

1. This model of the production side was used, for example, by Eaton (1984a, 1984b) to study various dynamic trade issues.

2. See, for example, Mundell (1957).

3. See Arrow and Kurz (1970) and Obstfeld (1982) for a discussion of this transversality condition.

4. Arrow and Kurz (1970) show that if a jump in the state variable is ever optimal, it will only occur initially.

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