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# WINNERS AND LOSERS:
# CREATIVE DESTRUCTION AND THE STOCK MARKET

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**ABSTRACT**

We develop a general equilibrium model of asset prices in which the benefits of technological innovation are distributed asymmetrically. Financial market participants do not capture all the economic rents resulting from innovative activity, even when they own shares in innovating firms. Economic gains from innovation accrue partly to the innovators, who cannot sell claims on the rents that their future ideas will generate. We show how the unequal distribution of gains from innovation can give rise to a high risk premium on the aggregate stock market, return comovement and average return differences among firms, and the failure of traditional representative-agent asset pricing models to account for cross-sectional differences in risk premia.

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Technological innovation is arguably the main driver of economic growth in the long run. However, the economic value generated by new ideas is usually not shared equally. The popular press is rife with rags-to-riches stories of new entrepreneurs, whose net worth rose substantially as a result of their innovative ideas during technological booms such as those experienced in the 1990s and the 2000s. In addition to the large wealth gains for successful innovators, technological progress can also create losses through creative destruction, as new technologies render old capital and processes obsolete.

We show that the asymmetric sharing of gains and losses from technological innovation can give rise to well-known, prominent empirical patterns in asset price behavior, including a high risk premium on the aggregate stock market, return comovement and average return differences among growth and value firms. We build a tractable general equilibrium model in which the benefits of technological progress are distributed unevenly across investors and firms. Our model allows for two forms of technological progress. Some advances take the form of improvements in labor productivity, and are complementary to existing investments, while others are embodied in new vintages of capital. Throughout the paper we refer to the first type of technological progress as disembodied, and the second type as embodied.[1] The latter type of technological progress leads to more creative destruction, since old and new capital vintages are substitutes.

A prominent feature of our model is that the market for new ideas is incomplete. Specifically, shareholders cannot appropriate all the economic rents generated by new technologies, even when they own equity in the firms that develop those technologies. Our motivation for this market incompleteness is that ideas are a scarce resource, and the generation of ideas relies heavily on human capital. As a result, innovators are able to capture a fraction of the economic rents that their ideas generate. The key friction is that potential innovators cannot sell claims to these future rents. This market incompleteness implies that technological progress has an asymmetric impact on household wealth. Most of the financial benefits from innovation accrue to a small fraction of the population, while the rest bear the cost of creative destruction. This reallocative effect of technological progress is particularly strong when innovations are embodied in new capital goods. By exposing households to idiosyncratic randomness in innovation outcomes, improvements in technology can thus reduce households' indirect utility. This displacive effect on indirect utility is amplified when households also care about their consumption relative to the economy-wide average, since households dislike being 'left behind'.

Displacement risk contributes to the equity risk premium and also leads to cross-sectional differences in asset returns. Owning shares in growth firms helps offset potential utility losses

[1] Berndt (1990) gives the following definitions for these two types of technology shocks: "Embodied technical progress refers to engineering design and performance advances that can only be embodied in new plant or equipment; older equipment cannot be made to function as economically as the new, unless a costly remodelling or retrofitting of equipment occurs," and "by contrast, disembodied technical progress refers to advances in knowledge that make more effective use of all inputs, including capital of each surviving vintage (not just the most recent vintage). In its pure form, disembodied technical progress proceeds independently of the vintage structure of the capital stock. The most common example of disembodied technical progress is perhaps the notion of learning curves, in which it has been found that for a wide variety of production processes and products, as cumulative experience and production increase, learning occurs which results in ever decreasing unit costs."

brought on by technological improvements. In our model, firms differ in their ability to acquire projects that implement new technologies. This difference in future growth opportunities implies that technological progress has a heterogeneous impact on the cross-section of asset returns. Firms with few existing projects, but many potential new investment opportunities, benefit from technological advances. By contrast, profits of firms that are heavily invested in old technologies and have few growth opportunities decline due to increased competitive pressure. In equilibrium, investors hold growth firms, despite their lower expected returns, as a hedge against the potential wealth reallocation that may result from future technological innovation. Aggregate consumption does not accurately reflect all the risks that households face as a result of technological progress, implying the failure of traditional representative-agent asset pricing models to account for cross-sectional differences in risk premia.

We estimate the parameters of the model using indirect inference. The baseline model performs well at replicating the joint properties of aggregate consumption, investment, and asset returns that we target. In particular, the model generates an aggregate consumption process with moderate low-frequency fluctuations; volatile equity returns; a high equity premium; a low and stable risk-free rate; and the observed differences in average returns between value and growth stocks (the value premium). Jointly replicating these patterns has proven challenging in existing representative-agent general equilibrium models. These results depend on three key features of our model: technology advances that are embodied in new capital goods, incomplete market for ideas, and preferences over relative consumption. Restricted versions of the model that eliminate any one of these three features have difficulty replicating the empirical properties of asset returns, and especially the cross-sectional differences in stock returns between value and growth firms. In sum, these three features are responsible for generating sufficient displacement risk in the model, and ensuring that households care sufficiently about displacement to endure that the embodied shock carries a negative risk price.

In addition to the features of the data that we target, we evaluate the model's other implications for asset returns. The model generates realistic predictions about income and wealth inequality, especially at the top of the distribution. Importantly, the model replicates the observed patterns of comovement between value and growth stocks (the value factor) and the failure of the Capital Asset Pricing Model (CAPM) and the Consumption CAPM to explain the value premium.[2] In addition,

[2]The value 'puzzle' consists of two robust empirical patterns. First, firms with higher than average valuations–growth firms–experience lower than average future returns. These differences in average returns are economically large and comparable in magnitude to the equity premium. This finding has proven to be puzzling because growth firms are typically considered to be riskier and therefore should command higher average returns. Existing asset pricing models, such as the CAPM and the CCAPM, largely fail to price the cross-section of value and growth firms. Second, stock returns of firms with similar valuation ratios exhibit comovement, even across industries. These common movements are typically unrelated to the firms' exposures to fluctuations in the overall market value. See Fama and French (1992, 1993) for more details. Similar patterns to the returns of high market to book firms have been documented for firms with high past investment (Titman, Wei, and Xie, 2004), price-earnings ratios (Rosenberg, Reid, and Lanstein, 1985), labor hiring (Belo, Lin, and Bazdresch, 2014), new share issuance (Loughran and Ritter, 1995). The strong patterns of return comovement among firms with similar characteristics have motivated the use of empirical factor models (Fama and French, 1993). However, the economic origins of these empirical return factors are yet to be fully understood. Our work can thus be viewed as providing a micro-foundation for including the value factor in reduced-form asset pricing models.

the model generates an upward-sloping real yield curve and a downward-sloping term structure of risk premia on 'corporate payout strips'.[3]

In the last part of the paper, we provide additional evidence that directly relates to the main mechanism in the model. Most of the model's predictions rely on the fluctuations in the share of economic value that is generated by new innovative ideas, or blueprints – a quantity that is challenging to measure empirically. We construct an empirical estimate of this share using data on patents and stock returns collected by Kogan, Papanikolaou, Seru, and Stoffman (2016). Following Kogan et al. (2016), we infer the economic value of patents from the firms' stock market reaction following a successful patent application.[4] The correlations between the estimated value of new blueprints and aggregate quantities and prices are largely consistent with the model. Increases in the value of new blueprints are associated with higher aggregate investment, lower market returns, and higher returns for growth firms relative to value firms. Further, increases in the relative value of new blueprints are associated with increases in consumption, income, and wealth inequality. Consistent with the model, high-$Q$ firms are more likely to innovate in the future than low-$Q$ firms. Last, we find that rapid technological progress within an industry is associated with lower future profitability for low Tobin's $Q$ (value) firms relative to high-$Q$ (growth) firms. We replicate these results in simulated data from the model, the empirical estimates are in most cases close to those implied by the model. We interpret these findings as providing support for the model's main mechanism.

In sum, the main contribution of our work is to develop a general equilibrium model that introduces a new mechanism: it relates increases in inequality following technological innovations to the pricing of shocks to technology in financial markets.[5] Our work thus adds to the growing literature studying asset prices in general equilibrium models.[6] Conceptually closest to our paper is

[3] Binsbergen, Brandt, and Koijen (2012) argue that, in the data, the term structure of risk premia for dividend strips is downward sloping, and argue that this presents a challenge for existing general equilibrium models.

[4] Relative to other measures of innovation, such as patent citations, the stock market reaction to patent issues has the unique advantage of allowing us to infer the economic – as opposed to the scientific – value of the underlying innovations. Focusing on the days around the patent is issued allows us to infer the economic value over a narrow time window. However, the stock market reaction may underestimate the value of the patent when some of the information about the innovation is already priced in by the market. Further, our analysis misses the patents issued to private firms, as well as those inventions that are not patented. Our analysis should thus be viewed as a joint test of the model and the assumption that movements in the economic value of patented innovations are representative of the fluctuations in the overall value of new inventions in the economy.

[5] Kogan and Papanikolaou (2013, 2014) feature a similar structural model of the firm in a partial equilibrium setup. Partial equilibrium models are useful in connecting factor risk exposures to firm characteristics. However, they take the stochastic discount factor (SDF) as given. Reduced-form specifications of the SDF can arise in economies in which all cross-sectional variation in expected returns is due to sentiment (see, e.g. Nagel, Kozak, and Santosh, 2014). A general equilibrium model is necessary to connect the SDF to the real side of the economy.

[6] Similar to this paper, Papanikolaou (2011) and Garleanu, Panageas, and Yu (2012) are representative agent models which also feature two types of technological progress: disembodied shocks that affect the productivity of all capital, and embodied (also termed 'investment-specific' shocks) that affect the productivity only of *new* capital. Papanikolaou (2011) focuses on the pricing of embodied shocks in an environment with a representative firm and complete markets. In his model, capital-embodied shocks carry a negative risk premium due to agents' aversion to short-run consumption fluctuations. The mechanism in Papanikolaou (2011) requires that positive IST shocks are associated with reductions in aggregate consumption on impact. The empirical evidence for this is mixed. By contrast, we propose a different mechanism that leads to a negative risk price for embodied shocks. Garleanu et al. (2012) focus on understanding the joint time-series properties of consumption and excess asset returns; our main focus is on the model's cross-sectional implications. Closely related work also features heterogenous firms. Gomes, Kogan, and Zhang (2003) study the cross-section of risk premia in a model where technology shocks are complementary to all capital.

Garleanu, Kogan, and Panageas (2012), who study the value premium puzzle in an overlapping-generations economy where technological improvements lead to inter-generational displacement risk. Our paper shares some of the elements in their model, namely incomplete markets. However, whereas the analysis in Garleanu et al. (2012) is largely qualitative, we aim for a quantitative exploration of the mechanism. Specifically, in Garleanu et al. (2012), consumption inequality is an inter-generational phenomenon; there is no inequality in consumption within household cohorts. This stylized assumption makes the model analytically tractable but makes it difficult to reconcile the model with the patterns of inequality in the data.[7] Allowing for intra-cohort heterogeneity is a nontrivial analytical challenge, because models of individual heterogeneity with incomplete markets and aggregate shocks typically do not aggregate to a finite-dimensional state space. Another contribution of our paper is thus to provide a tractable model with individual heterogeneity and aggregate shocks that has realistic implications for inequality. Unlike the Garleanu et al. (2012) model, our model also incorporates firm investment and generates a realistic (and stationary) cross-sectional distribution of firms. In sum, the fact that the model has realistic predictions about inequality and firm heterogeneity, allows for a more rigorous evaluation of the quantitative importance of the mechanism.

Our model contains features that connect it to several other strands of the literature. A key part of our model mechanism is that technological progress endogenously increases households' uninsurable consumption risk. The fact that time-varying cross-sectional dispersion of consumption can increase the volatility of the stochastic discount factor is well known (Constantinides and Duffie, 1996; Storesletten, Telmer, and Yaron, 2007; Constantinides and Ghosh, 2014). However, whereas the existing literature uses reduced-form specifications of idiosyncratic labor income risk, in our setting the time variation in households' uninsurable risk arises as an equilibrium outcome. The resulting effect of idiosyncratic risk on asset prices is further amplified by households' preferences over relative consumption. Our work thus builds upon the extensive literature that emphasizes the role of consumption externalities and relative wealth concerns for asset prices, investment, and consumption dynamics (Duesenberry, 1949; Abel, 1990; Gali, 1994; Roussanov, 2010). Closest to our work is Roussanov (2010), who argues that households may invest in risky, zero-mean gambles whose payoffs are uncorrelated with the aggregated state when they have preferences over their rank in the consumption distribution. In our setting, preferences over relative consumption induce agents to accept low risk premia (or equivalently high valuations) to hold assets that increase in value when technology prospects improve—growth firms. Last, the idea that a significant fraction of the rents from innovation accrue to human capital is related to Atkeson and Kehoe (2005), Lustig, Syverson, and Van Nieuwerburgh (2011) and Eisfeldt and Papanikolaou (2013). In contrast to these

Pastor and Veronesi (2009) study the pricing of technology risk in a model with a life-cycle of endogenous technology adoption. Ai, Croce, and Li (2013) analyze the value premium in a model where some technology shocks only affect the productivity of old capital. However, all of these studies consider representative-agent models.

[7]For instance, income inequality within cohorts is typically much larger than inequality between cohorts (O'Rand and Henretta, 2000). Also, Song, Price, Guvenen, Bloom, and von Wachter (2015) document that most of the rise in income inequality over the 1978 to 2012 period is within-cohorts. In Table A.1 in the Online Appendix, we perform a variance decomposition exercise and show that most of the inequality in consumption, income, or wealth exists within cohorts.

papers, we explore how fluctuations in the value of these rents affect the equilibrium pricing of technology shocks.

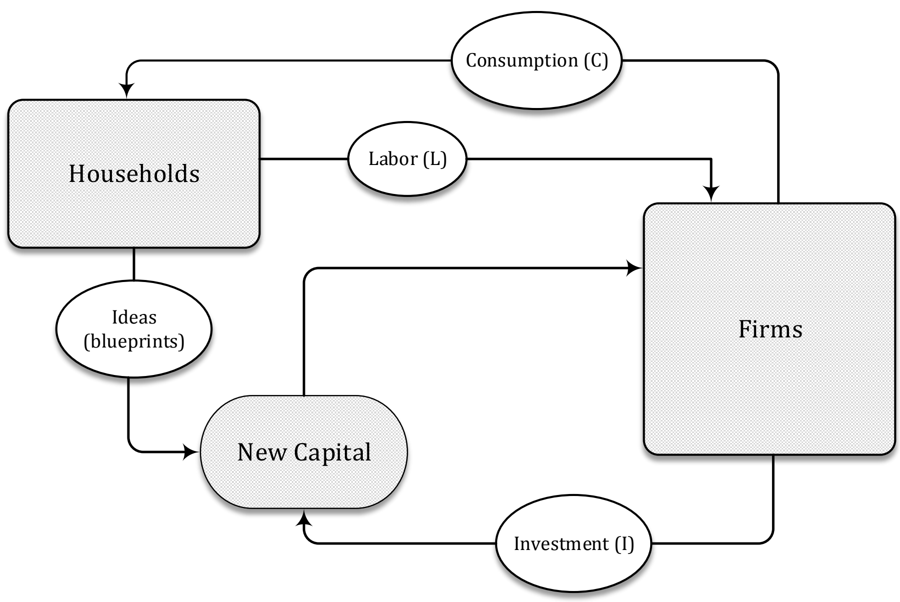
# 1 The Model

We consider a dynamic continuous-time economy, with time indexed by $t$. We first introduce the productive sector of the economy – firms and the projects they own. We next introduce households, and describe the nature of market incompleteness in our setup.

## 1.1 Firms and Technology

The basic production unit in our economy is called a project. Projects are owned and managed by firms. Each firm hires labor services to operate the projects it owns. The total output of all projects can be used to produce either consumption or investment. New production units are created using investment goods and project blueprints (ideas). Households supply labor services and blueprints to firms, and derive utility from consumption. Figure 1 summarizes the structure of our model.

**Figure 1:** Production




*Active projects*

Each firm $f$ owns a constantly evolving portfolio of projects, which we denote by $\mathcal{J}_{ft}$. We assume that there is a continuum of infinitely lived firms in the economy, which we index by $f \in [0,1]$.

Projects are differentiated from each other by three characteristics: a) their operating scale, determined by the amount of capital goods associated with the project, $k$; b) the systematic component of project productivity, $\xi$; and c) the idiosyncratic, or project-specific, component of productivity, $u$. Project $j$, created at time $\tau(j)$, produces a flow of output equal to

$$Y_{j,t} = \left(u_{j,t}\, e^{\xi_{\tau(j)}}\, K_{j,t}\right)^{\phi} \left(e^{x_t}\, L_{j,t}\right)^{1-\phi}, \tag{1}$$

where $L_{j,t}$ is labor allocated to project $j$. In contrast to the scale decision, the choice of labor allocated to the project $L_{j,t}$ can be freely adjusted every period. Firms purchase labor services at the equilibrium wage $\mathrm{w}_t$. We denote by

$$\Pi_{j,t} = \sup_{L_{j,t}} \left[ \left(u_{j,t}\, e^{\xi_{\tau(j)}}\, K_{j,t}\right)^{\phi} \left(e^{x_t}\, L_{j,t}\right)^{1-\phi} - \mathrm{w}_t\, L_{j,t} \right] \tag{2}$$

the profit flow of project $j$ under the optimal hiring policy.

We emphasize one important dimension of heterogeneity among technological innovations by modeling technological progress using two independent processes, $\xi_t$ and $x_t$. First, the shock $\xi$ reflects technological progress *embodied* in new projects. It follows an arithmetic random walk

$$d\xi_t = \mu_\xi\, dt + \sigma_\xi\, dB_{\xi,t}, \tag{3}$$

where $B_\xi$ is a standard Brownian motion. $\xi_s$ denotes the level of frontier technology at time $s$. Growth in $\xi$ affects only the output of new projects created using the latest frontier of technology. In this respect our model follows the standard vintage-capital model (Solow, 1960).

Second, the labor-augmenting productivity process $x_t$ follows an arithmetic random walk

$$dx_t = \mu_x\, dt + \sigma_x\, dB_{x,t}. \tag{4}$$

Here, $B_x$ is a standard Brownian motion independent of all other productivity shocks. In particular, the productivity process $x$ is independent from the embodied productivity process $\xi$. Labor in our model is complementary to capital. Thus, in contrast to the embodied shock $\xi$, the technology shock $x$ affects the output of *all* vintages of existing capital.

The level of project-specific productivity $u_j$ is a stationary mean-reverting process that evolves according to

$$du_{j,t} = \kappa_u(1 - u_{j,t})\, dt + \sigma_u\, u_{j,t}\, dB^u_{j,t}, \tag{5}$$

where $B^u_j$ are standard Brownian motions independent of $B_\xi$. We assume that $dB^u_{j,t} \cdot dB^u_{j',t} = dt$ if projects $j$ and $j'$ belong in the same firm $f$, and zero otherwise. As long as $2\,\kappa_u \geq \sigma_u^2$, the ergodic distribution of $u$ has finite first two moments (see Lemma 1 in Appendix A for details). All new projects implemented at time $s$ start at the long-run average level of idiosyncratic productivity, i.e., $u_{j,\tau(j)} = 1$. Thus, all projects created at a point in time are ex-ante identical in terms of productivity, but differ ex-post due to the project-specific shocks.

The firm chooses the initial operating scale $k$ of a new project irreversibly at the time of its creation. Firms cannot liquidate existing projects and recover their investment costs. Over time, the scale of the project diminishes according to

$$dK_{j,t} = -\delta\, K_{j,t}\, dt, \tag{6}$$

where $\delta$ is the economy-wide depreciation rate. At this stage, it is also helpful to define the aggregate stock of installed capital, adjusted for quality,

$$K_t = \int_0^1 \left( \sum_{j \in \mathcal{J}_{f,t}} e^{\xi_{\tau(j)}}\, K_{j,t} \right) df \tag{7}$$

The aggregate capital stock $K$ also depreciates at rate $\delta$.

*Creation of new projects*

Creating a new project requires a blueprint and new investment goods. Firms are heterogeneous in their ability to acquire new blueprints. Inventors initially own the blueprints for creation of new projects. We assume that inventors lack the ability to implement these ideas on their own, and instead sell the blueprints for new projects to firms (we outline the details of the process for blueprint sales below).

Firms acquire projects by randomly meeting inventors who supply blueprints. The likelihood of acquiring a new project is exogenous to each firm, driven by a doubly stochastic Poisson process $N_{f,t}$ with increments independent across firms. The arrival rate of new projects equals $\lambda_{f,t}$. This arrival rate is time-varying and follows a two-state continuous-time Markov chain with high and low growth states $\{\lambda_H, \lambda_L\}$, $\lambda_H > \lambda_L$. The transition rate matrix is given by

$$\begin{pmatrix} -\mu_L & \mu_L \\ \mu_H & -\mu_H \end{pmatrix}. \tag{8}$$

We denote the unconditional average of $\lambda_{f,t}$ by $\lambda$. The stochastic nature of $\lambda_{ft}$ has no effect on the aggregate quantities in the model. Parameters of this process control the cross-sectional differences in investment, valuation ratios and systematic risk among firms.

To implement a new blueprint as a project $j$ at time $t$, a firm purchases new capital goods in quantity $I_{j,t}$. Investment in new projects is subject to decreasing returns to scale,

$$K_{j,t} = I_{j,t}^{\alpha}. \tag{9}$$

The parameter $\alpha \in (0, 1)$ parameterizes the investment cost function and implies that costs are convex at the project level.

The value of new ideas (blueprints) plays a key role in our analysis. We denote by

$$\nu_t \equiv \sup_{K_{j,t}} \left\{ \mathrm{E}_t \left[ \int_t^{\infty} \frac{\Lambda_s}{\Lambda_t}\, \Pi_{j,s}\, ds \right] - K_{j,t}^{1/\alpha} \right\} \tag{10}$$

the net value of a new project implemented at time $t$ under the optimal investment policy, where $\Lambda_t$

is the equilibrium stochastic discount factor defined in Section 1.4. Since all projects created at time $t$ are identical ex-ante, $\nu$ is independent of $j$. Equation (10) is also equal to the value of a new blueprint associated at time $t$.

*Aggregate output*
The total output in the economy is equal to the aggregate of output of all active projects,

$$Y_t = \int_0^1 \left( \sum_{j \in \mathcal{J}_{f,t}} Y_{j,t} \right) df. \tag{11}$$

The aggregate output of the economy can be allocated to either investment $I_t$ or consumption $C_t$,

$$Y_t = I_t + C_t. \tag{12}$$

The amount of new investment goods $I_t$ produced is used as an input in the implementation of new projects, as given by the investment cost function defined in (9).

## 1.2 Households

There is a continuum of households, with the total measure of households normalized to one. Households die independently of each other according to the first arrival of a Poisson process with arrival rate $\delta^h$. New households are born at the same rate, so the total measure of households remains constant. All households are endowed with the unit flow rate of labor services, which they supply inelastically to the firms producing the final good.

Households have access to financial markets, and optimize their life-time utility of consumption. Households are not subject to liquidity constraints; hence, they sell their future labor income streams and invest the proceeds in financial claims. We denote consumption of an individual household $i$ by $C_{i,t}$.

All shareholders have the same preferences, given by

$$J_t = \lim_{\tau \to \infty} \mathrm{E}_t \left[ \int_t^\tau \Upsilon(C_s, J_s; \bar{C}_s)\, ds \right], \tag{13}$$

where $\Upsilon(C, J; \bar{C})$ is the utility aggregator:

$$\Upsilon(C, J; \bar{C}) = \frac{\rho}{1-\theta^{-1}} \left( \frac{\left( C^{1-h} \left( C/\bar{C} \right)^h \right)^{1-\theta^{-1}}}{((1-\gamma)J)^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} - (1-\gamma)\, J \right). \tag{14}$$

Households' preferences fall into the class of stochastic differential utility proposed by Duffie and Epstein (1992), which is a continuous-time analog of the preferences proposed by Epstein and Zin (1989). Relative to Duffie and Epstein (1992), our preference specification also incorporates a relative-consumption concern (otherwise termed as "keeping up with the Joneses", see, e.g., Abel, 1990). That is, households also derive utility from their consumption relative to aggregate consumption,

$$\bar{C} = \int_0^1 C_{i,t}\, di. \tag{15}$$

The parameter $h$ captures the strength of the relative consumption effect; $\gamma$ is the coefficient of relative risk aversion; $\theta$ is the elasticity of intertemporal substitution (EIS); and $\rho$ is the effective time-preference parameter, which includes the adjustment for the likelihood of death (see Garleanu and Panageas, 2014, for a model with random life spans and non-separable preferences).

### 1.3 Household Innovation

The key feature of our model is imperfect risk sharing among investors. Households are endowed with ideas, or blueprints, for new projects. Inventors do not implement these project on their own. Instead, they sell the ideas to firms. Inventors and firms bargain over the surplus created by new projects; the inventor captures a share $\eta$ of the net present value of a new project.

Each household receives blueprints for new projects according to an idiosyncratic Poisson process with arrival rate $\mu_I$. In the aggregate, households generate blueprints at the rate equal to the total measure of projects acquired by firms, $\lambda$. Not all innovating households receive the same measure of new blueprints. Each household $n$ receives a measure of projects in proportion to her wealth $W_{n,t}$ – that is, equal to $\lambda W_{n,t} \left(\mu_I \int_0^1 W_{i,t}\, di\right)^{-1}$. This is a technical assumption that is important for tractability of the model, as we discuss below. Thus, conditional on innovating, wealthier households receive a larger measure of blueprints.

Importantly, households cannot trade in securities contingent on future successful individual innovation. That is, they cannot sell claims against their proceeds from future innovations. This restriction on risk sharing plays a key role in our setting. In equilibrium, wealth creation from innovation leads to changes in the cross-sectional distribution of wealth and consumption, and therefore affects households' financial decisions.

### 1.4 Financial Markets

We assume that agents can trade a complete set of state-contingent claims contingent on the paths of the aggregate and idiosyncratic productivity processes, as well as paths of project arrival rates and project arrival events at the firm level. We denote the equilibrium stochastic discount factor by $\Lambda_t$, so the time-$t$ market value of a time-$T$ cash flow $X_T$ is given by

$$\mathrm{E}_t\left[\frac{\Lambda_T}{\Lambda_t} X_T\right]. \tag{16}$$

In addition, we follow Blanchard (1985) and assume that investors have access to competitive annuity markets that allow them to hedge their mortality risk. This assumption implies that, conditional on surviving during the interval $[t,\, t+dt]$, investor $n$ collects additional income proportional to her wealth, $\delta^h\, W_{n,t}\, dt$.

## 1.5 Discussion of the Model's Assumptions

Most existing production-economy general equilibrium models of asset returns build on the neoclassical growth framework. We depart from this literature in three significant ways.

*a. Technological progress is embodied in new capital vintages.* Most existing general equilibrium models that study asset prices assume that technological progress is complementary to the entire existing stock of capital – as is the case for the $x$ shock in our model. However, many technological advances are embodied in new capital goods and thus only benefit firms which invest in the new capital vintages. Several empirical studies show substantial vintage effects in plant productivity. For instance, Jensen, McGuckin, and Stiroh (2001) finds that the 1992 cohort of new plants was 50% more productive than the 1967 cohort in their respective entry years, controlling for industry-wide factors and input differences. Further, an extensive literature documents a significant impact of embodied technological progress on economic growth and fluctuations (see, e.g. Solow, 1960; Cooley, Greenwood, and Yorukoglu, 1997; Greenwood, Hercowitz, and Krusell, 1997; Fisher, 2006). Since technological progress can take many forms, we distinguish between embodied and disembodied technological progress to obtain a broader understanding of how technological change affects asset returns.

In our model, capital-embodied technical change introduces a wedge between the value of installed capital and the value of future growth opportunities. This allows our model to deliver a realistic degree of heterogeneity in investment, valuations, and stock returns among firms. Furthermore, because of their heterogenous impact on the different sources of wealth, embodied shocks combined with incomplete markets help generate realistic levels of consumption inequality within the model.

*b. Incomplete markets for innovation.* Incomplete markets play a key role in our analysis. In the model, inventors generate new ideas and sell them to firms. Importantly, the economic value that is generated by new ideas cannot be fully pledged to outside investors. The goal of this assumption is to allow for a heterogeneous impact of technology shocks on households, which implies that the technology risk exposures of individual investors are not fully captured by aggregate consumption dynamics.

We should emphasize that our interpretation of 'inventors' is quite broad. For example, the term 'inventors' can include: highly skilled personnel, who generate new inventions or business ideas; entrepreneurs and startup employees, who can extract a large share of the surplus created by new ideas; angel investors and venture capitalist, who help bring these ideas to market; and corporate executives who decide how to optimally finance and implement these new investment opportunities. In sum, 'inventors' in our model encapsulate all parties that share the rents from new investment opportunities besides the owners of the firm's publicly-traded securities. Further, the exact process by which inventors and shareholders share the rents from new technologies can take many forms. One possibility is that inventors work for existing firms, generate ideas, and receive compensation commensurate with the economic value of their ideas. Since their talent is in scarce supply, these skilled workers may be able to capture a significant fraction of the economic value of their ideas. Another possibility is that inventors implement the ideas themselves, creating startups that are partly funded by outside investors. Innovators can then sell equity in these startups to investors and thus capture a substantial share of the economic value of their innovations.[8]

The assumption that households cannot share rents from innovation ex-ante can be motivated on theoretical grounds. New ideas are the product of human capital, which is inalienable. Hart and Moore (1994) show that the inalienability of human capital limits the amount of external finance that can be raised by new ventures. Bolton, Wang, and Yang (2015) characterize a dynamic optimal contract between a risk averse entrepreneur with risky inalienable human capital, and firm investors. The optimal contract involves a trade-off between risk sharing and incentives, and leaves the entrepreneur with a significant fraction of the upside.

*c. Preferences over relative consumption.* Quantitative asset pricing models often assume relative consumption preferences, which help increase household's aversion to consumption growth volatility and thus raising the equilibrium compensation for bearing consumption risk (Abel, 1990; Constantinides, 1990; Campbell and Cochrane, 1999). In our model relative consumption preferences make households averse to consumption inequality, thus magnifying the effect of limited sharing of innovation risk.

The assumption of relative consumption preferences is theoretically appealing and has direct empirical support. Rayo and Becker (2007) propose a theory in which peer comparisons are an integral part of the "happiness function" as a result of an evolutionary process. DeMarzo, Kaniel, and Kremer (2008) show that competition over scarce resources can make agents' utilities dependent on the wealth of their cohort. In particular, if households have preferences over a consumption basket $\hat{C} = C^{1-\kappa} H^{\kappa}$, where $C$ is non-durable consumption and $H$ is a consumption good that is in inelastic supply – for instance, land or local services such as education – then the household makes intertemporal choices as if it has preferences over the composite good $\hat{C}' = C^{1-\kappa} w^{\kappa}$, where $w$ denotes the household's wealth share.

In a series of seminal papers, Easterlin notes that income and self-reported happiness are positively correlated across individuals within a country but that average happiness within countries does not seem to rise over time as countries become richer (see, e.g. Easterlin, 1974, 1995, 2001). Easterlin interprets these findings as evidence that relative—rather than absolute—income matters for well-being. Consistent with this view, several empirical studies document that, controlling for household income, income of a peer group is negatively related to self-reported measures of happiness and satisfaction (Clark and Oswald, 1996; Solnick and Hemenway, 1998; Ferrer-i Carbonell, 2005; Luttmer, 2005; Card, Mas, Moretti, and Saez, 2012). These relative income concerns are substantial. For example, the point estimates in Luttmer (2005) imply that the income of households in the same metropolitan area is more important for happiness than the households' own level of income. Frydman (2015) finds strong evidence for utility preferences over relative wealth in an experimental setting using neural data collected through fMRI.

In Section 3.6 we explore the sensitivity of our quantitative results to assumptions (a) to (c). In addition to these three main assumptions, our model deviates in some other respects from the

[8]Possibly, the threat of implementing the idea themselves may be what ensures that existing employees can appropriate some fraction of the benefits. According to Bhide (1999), 71% of the founders of firms in the Inc 500 list of fast growing technology firms report that they replicated or modified ideas encountered through previous employment.

neoclassical framework. These deviations make the model tractable but do not drive our main results.

First, we assume that projects arrive independently of the firms' own past decisions, and firms incur convex investment costs at the project level. Together, these assumptions ensure that the optimal investment decision can be formulated as a static problem, thus implying that the cross-sectional distribution of firm size does not affect equilibrium aggregate quantities and prices. Second, the assumption that innovating households receive a measure of projects that is proportional to their existing wealth, together with homotheticity of preferences, implies that all households are solving the same consumption-portfolio choice problem. In equilibrium, households do not trade with each other – similar to Constantinides and Duffie (1996) – and their optimal consumption and portfolio choices scale in proportion to their wealth. The cross-sectional distribution of household wealth then does not affect equilibrium prices.[9] Third, households in our model have finite lives. This assumption ensures the existence of a stationary distribution of wealth among households. Fourth, our assumption that project productivity shocks are perfectly correlated at the firm level ensures that the firm state vector is low-dimensional. Last, there is no cross-sectional heterogeneity among the quality of different blueprints. We could easily allow for an idiosyncratic part to $\xi$, perhaps allowing for substantial skewness in this component, to capture the notion that the distribution of profitability of new ideas can be highly asymmetric. Our conjecture is that such an extension would strengthen our main results by raising the level of idiosyncratic risk of individual households' consumption processes.

### 1.6 Competitive Equilibrium

Here, we describe the competitive equilibrium of our model. Our equilibrium definition is standard, and is summarized below.

**Definition 1 (Competitive Equilibrium)** *The competitive equilibrium is a sequence of quantities* $\{C_t, I_t, Y_t, K_t\}$*; prices* $\{\Lambda_t, \mathrm{w}_t\}$*; household consumption decisions* $\{C_{i,t}\}$*; and firm investment and hiring decisions* $\{I_{j,t}, L_{j,t}\}$ *such that given the sequence of stochastic shocks* $\{x_t, \xi_t, u_{j,t}, N_{f,t}\}$, $j \in \bigcup_{f\in[0,1]} \mathcal{J}_{f,t}$, $f \in [0,1]$*: i) households choose consumption and savings plans to maximize their utility (13); ii) household budget constraints are satisfied; iii) firms maximize profits; iv) the labor market clears,* $\int_0^1 \left(\sum_{j\in\mathcal{J}_{f,t}} L_{j,t}\right) df = 1$*; v) the demand for new investment equals supply,* $\int_0^1 I_{n,t} dn = I_t$*; vi) the market for consumption clears* $\int_0^1 C_{n,t}\, dn = C_t$*, and vii) the aggregate resource constraint* (11) *is satisfied.*

We solve for equilibrium prices and quantities numerically. Because of market incompleteness, standard aggregation results do not apply. Specifically, there are two dimensions of heterogeneity in the model: on the supply side, among firms; and on the demand side, among households.

[9] The assumption that the magnitude of innovation is proportional to households' wealth levels likely weakens our main results compared to the case where all households received the same measure of blueprints upon innovating. In the latter case, wealthier households would benefit less from innovation, raising their exposure to innovation shocks relative to our current specification.

Both of these sources of heterogeneity can potentially make the state space of the model infinite-dimensional. However, the first two assumptions discussed above in Section 1.5 enable a relatively simple characterization of equilibrium; we can solve for aggregate-level quantities and prices in Definition 1 as functions of a low-dimensional Markov aggregate state vector.

Specifically, the first moment of the cross-sectional distribution of installed capital $K$ summarizes all the information about the cross-section of firms relevant for the aggregate dynamics in the model. As a result, the real side of the model aggregates to a model with a representative firm, where aggregate output is equal to

$$Y_t = K_t^{\phi} \, (e^{x_t} \, L_t)^{1-\phi}, \tag{17}$$

where the effective stock of capital $K$ defined in equation (7), evolves according to

$$\frac{dK_t}{K_t} = -\delta \, dt + \lambda \, \frac{e^{\xi_t}}{K_t} \left( \frac{I_t}{\lambda} \right)^{\alpha} \, dt, \tag{18}$$

where $I_t$ is aggregate investment expenditures, which satisfy the aggregate resource constraint (12). The capital accumulation equation (18) illustrates that the embodied shock $\xi$ acts as an investment-specific shock.

The net present value of new projects $\nu_t$ summarizes the marginal value of new investments. Specifically, the first-order condition for investment in (10), combined with market clearing, imply that in equilibrium,

$$I_t = \frac{\lambda \, \alpha}{1-\alpha} \nu_t. \tag{19}$$

Examining (19), we see that, conceptually, $\nu_t$ plays a similar role in our model as the Tobin's Q in a neoclassical model. There are two key differences however. First, in our setting, the market value of a new project is not directly linked to the average Q of the firm. Second, equation (19) holds in levels, not ratios, as in the neoclassical model.

Aggregate quantities in the model (denominated in units of consumption) have both a permanent and a temporary component. To see this, note that we can express most quantities of interest as functions of two aggregate state variables: a random walk component,

$$\chi_t = \frac{1-\phi}{1-\alpha \, \phi} \, x_t + \frac{\phi}{1-\alpha \, \phi} \xi_t, \tag{20}$$

and a stationary component,

$$\omega_t = \xi_t + \alpha \, \chi_t - \log K_t. \tag{21}$$

The trend variable $\chi_t$ is a function of the two technology shocks, and thus determines long-run growth in the model. The variable $\omega$ determines the conditional growth rate in the effective capital stock $K$ in equilibrium, and therefore expected consumption growth. In a non-stochastic model, $\omega$ would be constant; in our stochastic model, $\omega$ is stationary. As a result, we can write aggregate output (17) as

$$\log Y_t = \chi_t - \phi \, \omega_t, \tag{22}$$

where we have used the fact that aggregate labor supply is constant, $L_t = 1$.

Aggregate consumption $C$, investment $I$, labor income $\mathrm{W}$, and asset prices are cointegrated with aggregate output $Y$—they share the same stochastic trend, $\chi_t$. Their transitory deviations from output (or $\chi_t$) are functions of $\omega_t$ – which can be interpreted as the deviation of the current capital stock from its target level. Put differently, the state variable $\omega$ summarizes the transitory fluctuations of the model variables around the stochastic trend $\chi_t$.

# 2 Examining the Model's Mechanism

To obtain some intuition about the asset pricing predictions of the model, it is helpful to analyze the relation between technological progress, the stochastic discount factor, and asset returns. As is the case in any general equilibrium model, the behavior of asset prices are determined by the covariance of asset returns with the investors' stochastic discount factor (SDF). We split the discussion in two parts. In Section 2.1 we examine how technology shocks enter into the investors' stochastic discount factor (SDF), and in n Section 2.2 we examine how these two shocks impact asset returns in the cross-section. To aid the exposition, we present plots of the key variables of interest.[10]

## 2.1 The pricing of technology risk

The two technology shocks $x$ and $\xi$ affect the equilibrium SDF through their impact on the consumption of individual agents. Because of imperfect risk sharing, there is a distinction in how these shocks affect aggregate quantities and how they affect individual households. To emphasize this distinction, we first analyze the impact of technology on aggregate economic output and consumption, and then examine its impact on the distribution of consumption for individual households.

### 2.1.1 Aggregate quantities and asset prices

We compute impulse responses for aggregate output $Y_t$, consumption $C_t$, investment $I_t$, labor income $\mathrm{W}_t$ and aggregate payout to shareholders $D_t$ to the two technology shocks $x$ and $\xi$. The aggregate payout to shareholders is equal to total firm profits minus investment expenditures and payout to new inventors,

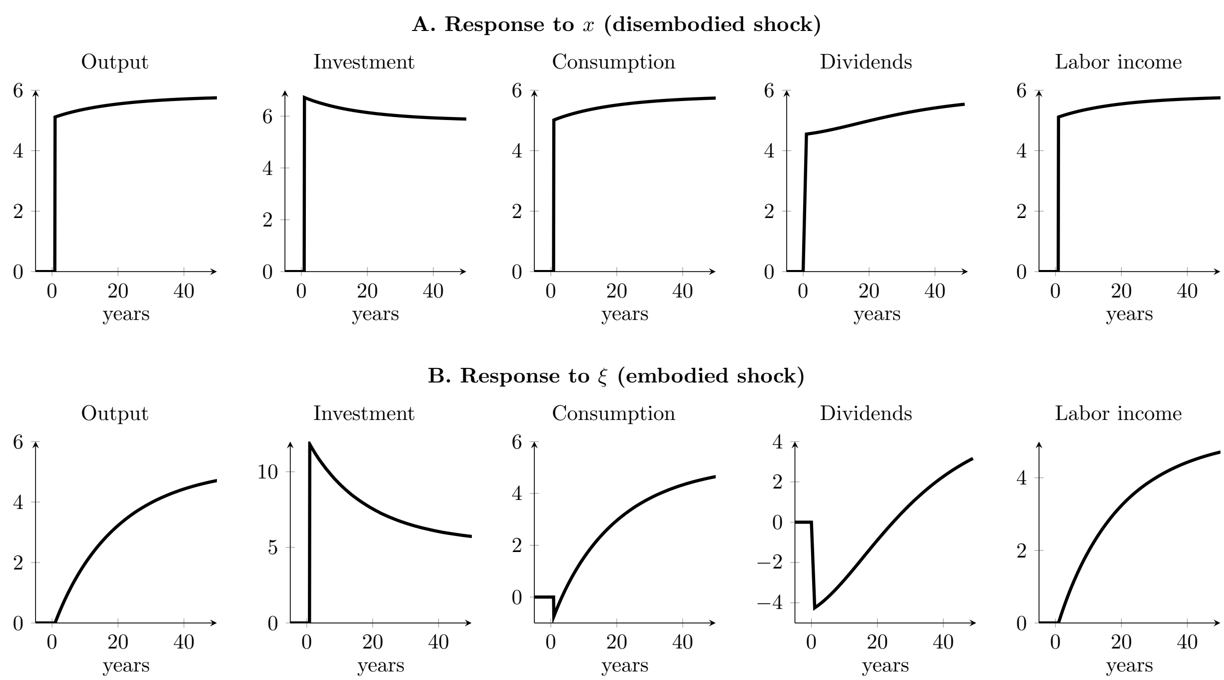
$$D_t = \phi\, Y_t - I_t - \eta\, \lambda\, \nu_t. \tag{23}$$

In the model, the aggregate payout $D$ is not restricted to be positive. However, using the baseline parameter estimates, $D$ becomes negative only in the extreme ranges of the state space that are rarely reached in model simulations. We compute impulse responses taking into account the nonlinear dynamics of the economy. The shape of these impulse responses depends on the current state vector $X$. In our model, the scalar state variable $\omega$ summarizes all relevant information for the model's non-linear dynamics. We compute impulse responses at the mean of the stationary distribution of $\omega$.

[10] To create plots, we use the parameter values in column three of Table 3. We postpone the discussion of how these parameters are estimated until Section 3. Since the goal of this section is to describe the qualitative features of the model, the exact values of the parameters used are of secondary important; the model's qualitative implications are similar across several parametrizations.

We plot the impulse responses to the technology shocks in Figure 2. Panel A shows that a positive disembodied technology shock $x$ leads to an increase in output, consumption, investment, payout, and labor income. The increase in investment leads to higher capital accumulation, so the increase in output is persistent. However, since $x$ is complementary to existing capital, most of its benefits are immediately realized. In panel B, we plot the response of these equilibrium quantities to a technology shock $\xi$ that is embodied in new capital. In contrast to the disembodied shock $x$, the technology shock $\xi$ affects output only through the formation of new capital stock. Consequently, it has no immediate effect on output, and only leads to a reallocation of resources from consumption to investment on impact. Further, shareholder payout declines immediately after the shock, as firms cut dividends to fund new investments. In the medium run, the increase in investment leads to a gradual increase in output, consumption, payout, and the equilibrium wage (or labor income).

**Figure 2: Technology and Aggregate Quantities**
This figure plots the impulse response of aggregate output, investment and consumption expenditures to the two technology shocks in the model. We construct the impulse responses taking into account the nonlinear nature of equilibrium dynamics: we introduce an additional one-standard deviation shock at time $t = 0$ without altering the realizations of all future shocks. The impulse responses are computed at the mean of the stationary distribution of $\omega$. We report the log difference between the mean response of the perturbed and unperturbed series (multiplied by 100).




Next, we examine the impact of technology on the prices of financial assets and human capital (i.e., the households tradeable wealth). The total wealth of all existing households,

$$W_t \equiv \int_0^1 W_{n,t}\, dn = V_t + G_t + H_t, \tag{24}$$

equals the sum of three components. The first part is the value of a claim on the profits of all *existing* projects $\mathcal{J}_t$,

$$V_t \equiv \int_0^1 \mathrm{E}_t \left[ \sum_{j \in \mathcal{J}_{f,t}} \int_t^\infty \frac{\Lambda_s}{\Lambda_t} \, \Pi_{j,s} \right] df. \tag{25}$$

The second component is the value of new growth opportunities that accrues to shareholders,

$$G_t \equiv (1-\eta) \int_0^1 \mathrm{E}_t \left[ \int_t^\infty \frac{\Lambda_s}{\Lambda_t} \, \lambda_{f,s} \nu_s \, ds \right] df, \tag{26}$$

where $\nu_t$ is the net present value of a new project implemented at time $t$ – defined in equation (10). Inventors capture a share $\eta$ of the surplus created by new projects, while the remaining share $1-\eta$ accrues to shareholders of these firms.

The last part of tradeable wealth consists of the present value of labor services to households,

$$H_t \equiv \mathrm{E}_t \left[ \int_t^\infty e^{-\delta^h (s-t)} \, \frac{\Lambda_s}{\Lambda_t} \, \mathrm{w}_s \, ds \right]. \tag{27}$$

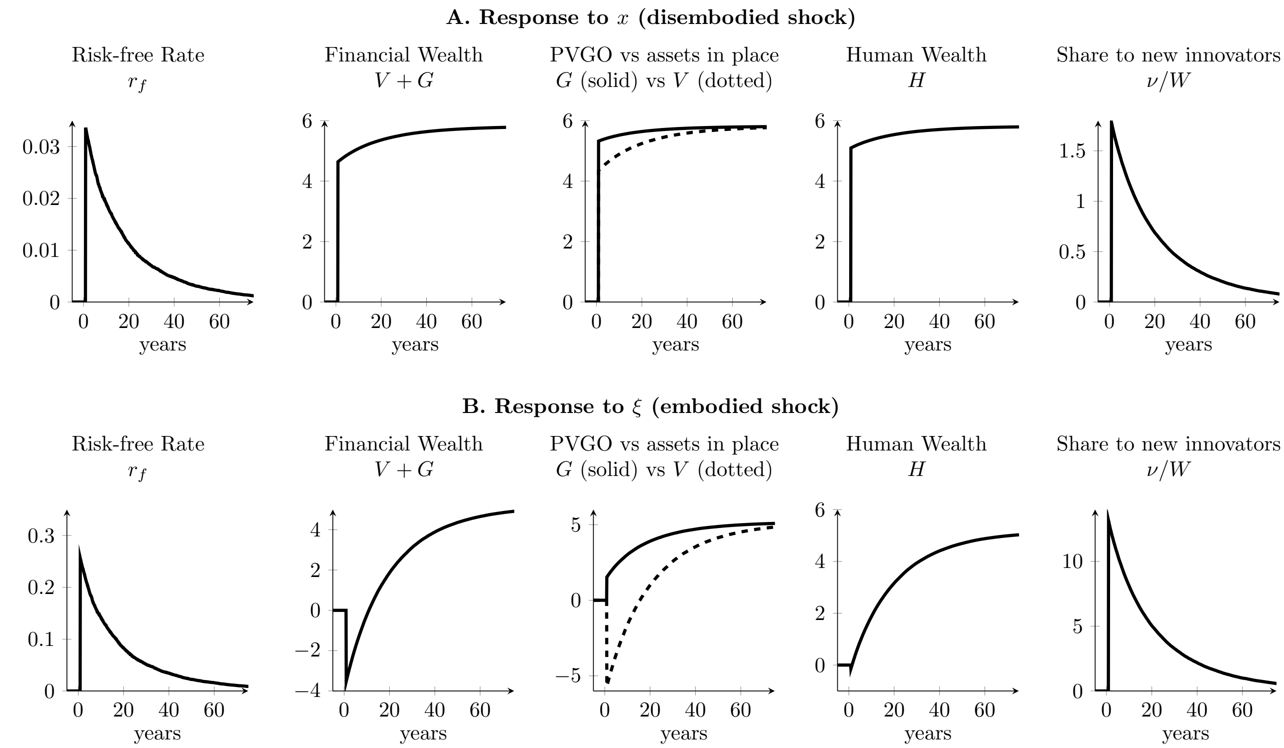
In Figure 3 we see how technology shocks affect the risk-free rate, the value of installed assets $V_t$, growth opportunities $G_t$, and human capital. In our subsequent analysis, the ratio of the value of new blueprints $\nu_t$ to total wealth $W_t$ plays a key role. Thus, we also plot the response of $\nu_t/W_t$ to a positive technology shock. A positive technology shock increases expected consumption growth; hence, as we see in the first column, the risk-free rate rises on impact. The next three columns plot the response of aggregate assets in place ($V_t$), the present value of growth opportunities ($G_t$), and human capital ($H_t$) to a technology shock. A positive disembodied shock $x$ is complementary to installed capital, hence the value of assets in place and growth opportunities rises on impact. By contrast, the technology shock $\xi$ that is embodied in new capital lowers the value of existing assets $V$ but it increases the value of growth opportunities $G$. In terms of net effects, a positive embodied shock lowers total financial wealth $V+G$; by contrast, a positive disembodied shock leads to an increase in total financial wealth.

The value of new blueprints $\nu$ relative to total wealth $W$ rises in response to either technology shock, as we see in the last column of Figure 3. Importantly, the effect is quantitatively much larger for the technological shock that is embodied in *new* projects compared to advances in technology that affect both existing and new projects. This difference is important because the responses of aggregate consumption in Figure 2 to technology shocks mask substantial heterogeneity in the consumption paths of individual households. As we show next, larger changes in $\nu/W$ lead to greater reallocation of wealth among households.

### 2.1.2 Technology and individual consumption

The consumption of an individual household differs from aggregate consumption $C_t$ due to to imperfect risk sharing. The current state of a household can be summarized by its current share of total wealth, $w_{i,t} \equiv W_{i,t}/W_t$. The functional form of preferences (13-14) together with our assumption that the scale of the household-level innovation process is proportional to individual

**Figure 3: Technology and Asset Prices**
This figure plots the impulse response of the risk-free rate ($r_f$), and the value of assets in place ($V$), growth opportunities ($G$) and human capital ($H$), as well as the relative value of payments to new innovators $\nu/W$ to the two technology shocks in the model. We construct the impulse responses taking into account the nonlinear nature of equilibrium dynamics: we introduce an additional one-standard deviation shock at time $t = 0$ without altering the realizations of all future shocks. The impulse responses are computed at the mean of the stationary distribution of $\omega$. In the first row, we report the difference between the mean response of the perturbed and unperturbed risk-free rate (multiplied by 100). For all other series we report the log of the ratio of the perturbed to the unperturbed series (multiplied by 100).




wealth imply that optimal individual consumption and portfolio plansholdings are proportional to individual wealth. Then, a household's consumption share is the same as its wealth share, $c_{i,t} = w_{i,t}$. Therefore, individual consumption satisfies

$$C_{i,t} = C_t \, w_{i,t}. \tag{28}$$

The dynamic evolution of households' share of aggregate wealth is

$$\frac{dw_{i,t}}{w_{i,t}} = \delta^h \, dt + \frac{\lambda}{\mu_I} \frac{\eta \, \nu_t}{W_t} \left( dN^I_{i,t} - \mu_I \, dt \right), \tag{29}$$

where $N^I_{i,t}$ is a Poisson process that counts the number of times that household $i$ has acquired a new blueprint.

The evolution of a household's relative wealth in (29) is conditional on the household survival; thus, the first term captures the flow payoff of the annuity, as it is standard in perpetual youth OLG models (Blanchard, 1985). The second term captures changes in the households' wealth resulting from innovation. Both the drift and the return to successful innovation depend on the fraction of shareholder wealth that accrues to all successful inventors, $\eta \, \nu_t / W_t$. This ratio is a monotonically increasing function of $\omega$, defined in (21). Each period, a household yields a fraction $\lambda \, \eta \, \nu_t / W_t$ of its wealth share to successful innovators. This wealth reallocation occurs because households own shares in all firms, and these firms make payments to new inventors in return for their blueprints. During each infinitesimal time period, with probability $\mu_I \, dt$, the household is itself one of the innovators, in which case it receives a payoff proportional to $\eta \, \nu_t \, W_{i,t}$. The magnitude of wealth reallocation depends on the contribution of new investments to total wealth $\nu_t / W_t$. As we saw in Section 2.1.1, an increase in either $x$ or $\xi$ implies that the equilibrium value of new blueprints to total wealth $\nu_t / W_t$ increases. This increase in $\nu_t / W_t$ leads to an increase in the households' idiosyncratic risk – similar to the model of Constantinides and Duffie (1996).

The process of wealth reallocation following positive technology shocks is highly skewed. Equation (29) shows that the rise in gains from successful innovation, i.e., the rise in $\nu_t / W_t$, implies that most households experience higher rates of relative wealth decline, as captured by a reduction in the drift of $dw_t$. By contrast, the few households that innovate increase their wealth shares greatly, as captured by the jump term $\nu_t / W_t \, dN_t^I$. From the perspective of a household at time $t$, the distribution of future consumption becomes more variable and more skewed following a positive technological shock, even though on average the effect is zero – a positive technological shocks magnifies both the extremely high realizations of $w$ and the paths along which $w$ declines persistently.

Figure 4 illustrates the impact of technology shocks on the consumption path of an individual household. Our objects of interest are the households' relative wealth share $w_i$, the households' consumption $C_i$, and household consumption adjusted for relative preferences $C_i^{1-h} w_i^h$. In the first three columns, we plot the impulse response of these variables to the two technology shocks. In addition to the response of the mean, we also plot how the median of the future distribution of these variables changes in response to the two technology shocks. Unlike the mean, the median of $w$ is not influenced by the rare but extremely positive outcomes, and instead reflects the higher likelihood of large gradual relative wealth declines in response to technology shocks.

The first column of Figure 4 summarizes the role of incomplete risk sharing in our model. A technology shock – either $x$ or $\xi$ – has no impact on the expected future wealth share $w$ at any horizon, because in our model technology shocks have ex-ante a symmetric effect on all households. However, the lack of an effect on the average wealth share masks substantial heterogeneity in individual outcomes. Specifically, the response of the median of the distribution is significantly negative at all horizons. The very different responses of the mean and the median wealth share suggest a highly skewed effect of technological shocks on individual households, which is key to understanding the effects of technology shocks on the stochastic discount factor in our model.

The next two columns of Figure 4 examine the response of household consumption. In the second column, we see that the responses of the mean and median future consumption to a disembodied shock $x$ are not substantially different. The difference in responses between the mean and the median consumption is most stark when technology is embodied in new vintages. A positive embodied shock

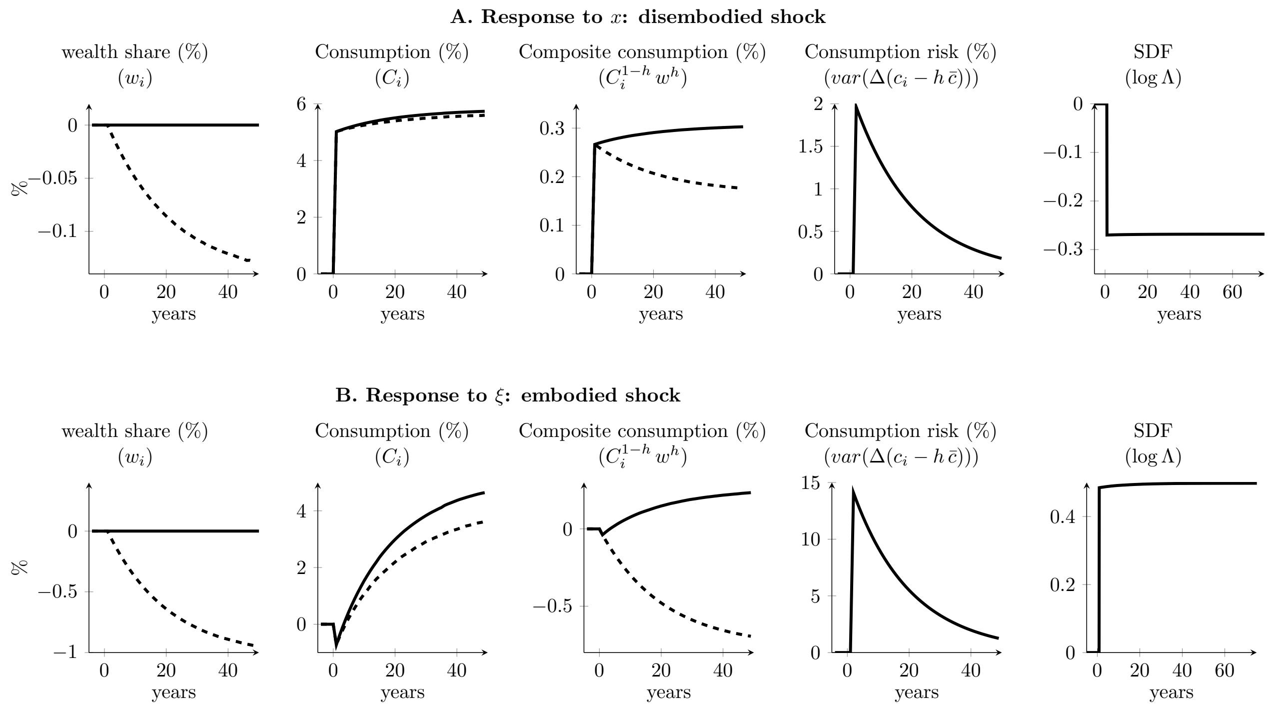
$\xi$ leads to an increase in the share of value due to new blueprints $\nu_t/W_t$, and thus to greater wealth reallocation among households. The next column shows the role of relative consumption preferences. If households care about their relative consumption $w_i$ in addition to their own consumption $C_i$, then the impact of technology shocks on their adjusted consumption flow is a weighted average of the first two columns.

The difference between the response of the mean and the median of the distribution of future consumption highlights the asymmetric benefits of technology shocks. Since households are risk averse, the mean response is insufficient to characterize the impact of technology on their indirect utility. When evaluating their future utility, households place little weight on the extremely high paths of $w$. Hence, the median response is also informative. In other words, in addition to their effect on the mean consumption growth, technology shocks also affect the variability of consumption because they affect the magnitude of the jump term in (29). Even though the conditional risk of individual wealth shares is idiosyncratic, this risk depends on the aggregate state of the economy. Therefore, innovation risk affects the stochastic discount factor, similarly to the model of Constantinides and Duffie (1996). To illustrate this connection, the fourth column of Figure 4 plots the increase in the variance of instantaneous consumption growth – adjusted for relative preferences. We see that both technology shocks lead to higher consumption volatility. The effect is substantially higher for $\xi$ than for $x$, again due to its higher impact on the returns to innovation $\nu_t/W_t$.

In sum, Figure 4 shows that the economic growth that results from technological improvements is not shared equally across households. Specifically, innovation reallocates wealth shares from most households to a select few. Although an increase in $\nu_t/W_t$ does not affect the *expected* wealth share of any household, it raises the magnitude of unexpected changes in households' wealth shares. Since households are risk averse, they dislike the resulting variability of changes in their wealth. Importantly, preferences over relative consumption ($h > 0$) magnify the negative effect of relative wealth shocks on indirect utility. Households are averse to displacement risk not only because it exposes them to additional consumption risk, but also because they fear being 'left behind' following subsequent increases in aggregate economic growth. Indeed, as we can see in the second row of Panel B, a positive embodied shock increases the median consumption growth; however, the third row shows that, once relative preferences are taken into account, the impact on the 'consumption bundle' that incorporates relative consumption preferences is negative. The resulting effect on households' indirect utility has important implications about the pricing of these shocks, as we discuss next.

Last, we examine the stochastic discount factor. Financial markets in our model are incomplete, since some of the shocks (specifically, the acquisition of blueprints by individual households) are not spanned by the set of traded financial assets. As a result, there does not exist a unique stochastic discount factor (SDF) in our model. Similar to Constantinides and Duffie (1996), the utility gradients of various agents are not identical and each can serve as a valid SDF. To facilitate the discussion of the aggregate prices, we construct an SDF that is adapted to the market filtration $\mathcal{F}$ generated by the aggregate productivity shocks $(B_x, B_\xi)$. This SDF is a projection of agent-specific SDFs (utility gradients) on $\mathcal{F}$. The following proposition illustrates how to construct a valid SDF in our economy.

**Figure 4: Technology, Household Consumption and the Stochastic Discount Factor**
The first three columns of this figure plot the impulse response of the household wealth share ($w_i$), household consumption ($C_i$), and consumption adjusted for relative consumption preferences $C_i^{1-h}\, w_i^h$. We construct the impulse responses taking into account the nonlinear nature of equilibrium dynamics: we introduce an additional one-standard deviation shock at time $t = 0$ without altering the realizations of all future shocks. The impulse responses are computed at the mean of the stationary distribution of $\omega$. We report the log difference between the mean (median) response of the perturbed and unperturbed series in the solid (dashed) line. The fourth column plots the impulse response of the conditional variance of instantaneous log consumption growth, adjusted for relative preferences. The last column shows the impulse response of the equilibrium stochastic discount factor.




**Proposition 1 (Stochastic Discount Factor)** *The process $\Lambda_t$, given by the following equation, is adapted to the market filtration $\mathcal{F}$ and is a valid SDF:*

$$\log \Lambda_t = \int_0^t b(\omega_s)\, ds - \gamma_1 \chi_t - \frac{1}{\theta_1}\left(\log C_t - \chi_t\right) - \frac{1-\kappa}{\kappa} \log f(\omega_t). \tag{30}$$

*In the above equation, $\kappa \equiv \frac{1-\gamma}{1-\theta^{-1}}$, $\gamma_1 \equiv 1-(1-\gamma)(1-h)$, and $\theta_1 \equiv \big(1-(1-\theta^{-1})(1-h)\big)^{-1}$. In the first term, the function $b(\omega)$ is defined in the proof of the proposition in the Appendix. In the last term, the function $f(\omega)$ is related to the value function $J$ of an investor with relative wealth $w_{it}$,*

$$f(\omega_t) = (1-\gamma)\, J(w_{i,t}, \chi_t, \omega_t)\left(w_{i,t}^{1-\gamma} e^{(1-\gamma_1)\chi_t}\right)^{-1}. \tag{31}$$

**Proof:** See Appendix A.

The risk prices of the two technology shocks can be recovered by the impulse response of the log SDF on impact. We plot these responses in the last column of Figure 4. Comparing panels A and B, we see that the two technology shocks carry opposite prices of risk in our model. A positive disembodied shock $x$ negatively affects the SDF on impact, implying a positive risk premium. By contrast, a positive embodied shock $\xi$ leads to a rise in the SDF on impact, implying a negative risk premium. As a result, households value securities that provide a hedge against states of the world when $\xi$ is high and $x$ is low.

The difference in how the SDF responds to the two technology shocks stems primarily from the response of the indirect utility term $f(\omega_t)$ in the SDF. Both technology shocks $x$ and $\xi$ lead to an increase in the permanent component $\chi_t$ of consumption, which by itself causes the SDF to fall. However, in the case of the embodied shock, the fall in indirect utility due to the unequal sharing of benefits from technological progress is sufficiently large to offset the benefits of higher aggregate consumption. The resulting demand for insurance against high realizations of $\xi$ is driven by the endogenous increase in the consumption uncertainty of individual investors.

## 2.2 Technology Shocks and the Cross-section of Firms

Cross-sectional differences in risk premia are partly driven by differences in how firm returns respond to technology shocks. We next examine the impact of technology shocks on individual firms.

The firm's current state is fully characterized by the aggregate state $X_t$; the probability of acquiring new projects $\lambda_{f,t}$; the firm's relative size,

$$k_{f,t} \equiv \frac{K_{f,t}}{K_t}, \quad \text{where} \qquad K_{f,t} \equiv \sum_{j \in \mathcal{J}_{f,t}} e^{\xi_{\tau(j)}} K_{j,t}; \tag{32}$$

and the firm's average profitability across projects

$$\bar{u}_{f,t} \equiv \frac{1}{K_{f,t}} \left( \sum_{j \in \mathcal{J}_{f,t}} e^{\xi_{\tau(j)}} \, u_{j,t} \, K_{j,t} \right). \tag{33}$$

The assumption that shocks to projects' productivity $u_j$ are perfectly correlated if they are owned by the same firm, implies that there is considerable dispersion in average profitability (33) across firms.

Our focus is on understanding differences in asset returns between 'value' and 'growth' firms. Typically, these categories are defined based on firms' valuation ratios, for instance the firm's average Tobin's $Q$, or market-to-book ratio. In our model, a firm's Tobin's $Q$ can be written as

$$\log Q_{f,t} - \log Q_t = \log \left[ \frac{V_t}{V_t + G_t} \, (1 + \widetilde{p}(\omega_t) \, (\bar{u}_{ft} - 1)) + \frac{G_t}{P_t + G_t} \frac{1}{k_{f,t}} \left( 1 + \widetilde{g}(\omega_t) \, \left( \frac{\lambda_{f,t}}{\lambda} - 1 \right) \right) \right], \tag{34}$$

where $Q_t$ is the market-to-book ratio of the market portfolio; $V_t$ and $G_t$ are defined in equations (25) and (26) respectively; and $\widetilde{p}(\omega)$ and $\widetilde{g}(\omega)$ are defined in the Appendix.

Cross-sectional differences in valuation ratios (34) are mainly driven by the firm's current likelihood of future growth $\lambda_{f,t}$ relative to its current size $k_{f,t}$. Quantitatively, differences in $\bar{u}_f$ play only a minor role, as we demonstrate in the Online Appendix. Firms with relatively high levels of $\lambda_f / k_f$ are 'growth firms', since they derive more of their value from their future growth opportunities rather than their existing operations. Conversely, firms with low levels of $\lambda_f/k_f$ derive most of their market value from their existing operations, and we therefore term them as 'value' firms.

The impact of technology shocks on firm outcomes is related to the differences in their valuation ratios. Specifically, how firms respond to technology shocks depends on whether they are 'value' or 'growth' firms. Technological progress lowers the cost of new investments, hence it benefits firms with new investment opportunities (high $\lambda_{f,t}$). However, it also leads to displacement of installed capital due to general equilibrium effects and therefore harms firms with a lot of installed assets ($k_{f,t}$). To see these heterogenous effects, we can express the profit flow of a firm $f$ as

$$\Pi_{f,t} \equiv \sum_{j \in \mathcal{J}_{f,t}} \Pi_{j,t} = \phi \, Y_t \, \bar{u}_{f,t} \, k_{f,t}. \tag{35}$$

Equation (35) implies that cross-sectional differences in the sensitivity of firm profits to aggregate technology shocks can be summarized by how the firm's relative size $k_f$ responds to shocks. The dynamics of $k_f$ are given by

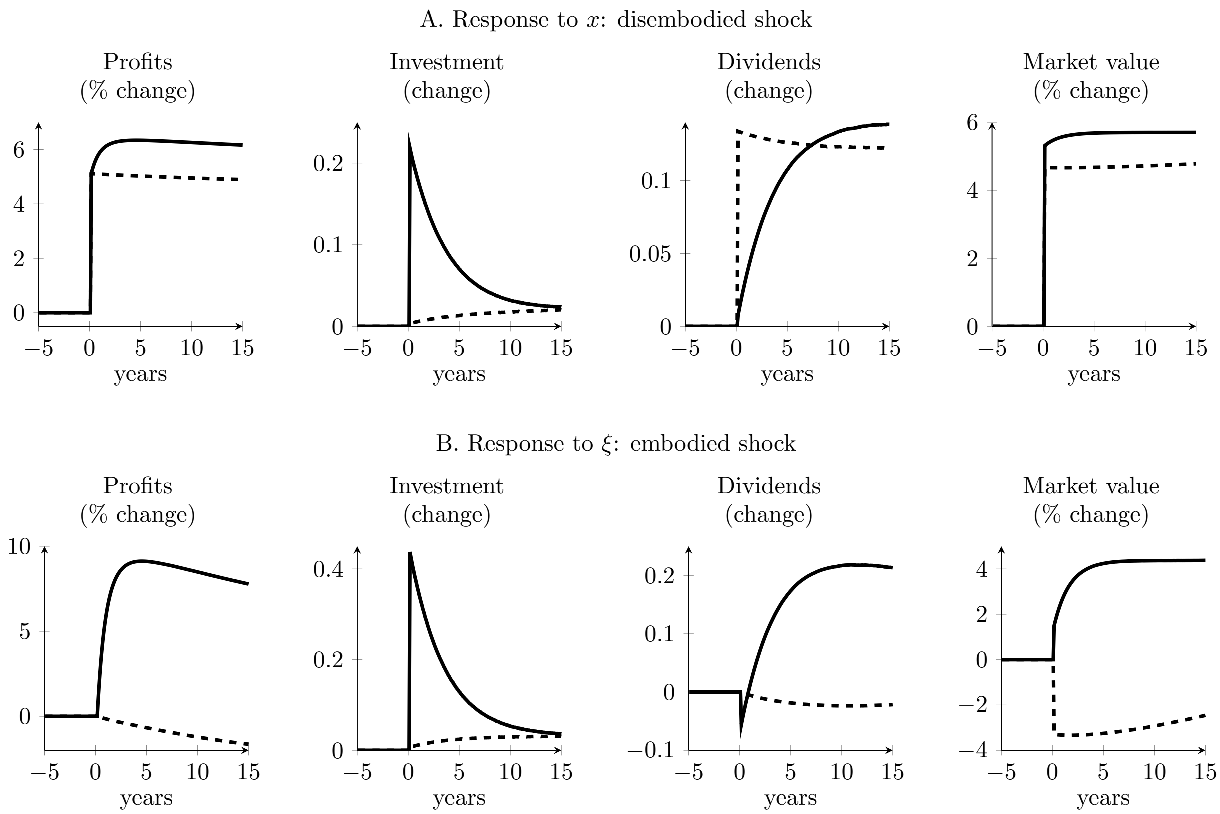
$$\frac{dk_{f,t}}{k_{f,t}} = a_0 \, \frac{\nu_t}{V_t} \, \left( \frac{\lambda_{f,t}}{\lambda \, k_{f,t}} - 1 \right) \, dt + a_0 \, \frac{\nu_t}{V_t} \, \frac{1}{\lambda \, k_{f,t}} \Big( dN_{f,t} - \lambda_{f,t} \, dt \Big), \tag{36}$$

where $a_0$ is a constant and $N_{f,t}$ is a Poisson process that counts the number of times that firm $f$ has acquired a new project.

Just as technology shocks lead to wealth reallocation among households, they also lead to resource reallocation among firms. Equation (36) shows that the value of new blueprints $\nu$ relative to the value of installed capital $V$ affects both the conditional growth rate as well as the dispersion in growth rates across firms. The ratio $\nu/V$ is a monotonically increasing function of the state variable $\omega$. Focusing on the first term in (36), we see that high values of $\nu/V$ magnify the difference in conditional firm growth rates between 'growth' firms (high levels of $\lambda_f/k_f$) and 'value' firms (low levels of $\lambda_f/k_f$). The second term in (36) shows that high values of $\nu/V$ also increase the cross-sectional dispersion in firm growth rates. Recall that the ratio $\nu/V$ responds sharply to a positive embodied shock ($\xi$) but only modestly to a positive disembodied shock ($x$). Thus, technological innovation, especially when it is embodied in new capital, increases the rate of reallocation across firms – that is, the process of 'creative destruction'.

To illustrate the heterogeneous impact of technology shocks on firm profitability and investment, we examine separately two firms with high and low levels of $\lambda_f/k_f$ in Figure 5. As we see in panel A, improvements in technology that are complementary to all capital lead to an immediate increase in profitability for both types of firms. Growth firms are more likely to have higher investment opportunities than the value firms; hence, on average, they increase investment. While growth firms pay lower dividends in the short run, their payouts rise over the long run. As a result, the market value of a growth firm appreciates more than the market value of a value firm. In panel B, we see that value and growth firms have very different responses to technology improvements embodied in new vintages. The technology shock $\xi$ leaves the output of existing projects unaffected, since it

**Figure 5: Technology Shocks and Firms**
This figure plots the dynamic response of firm profits, investment, dividends and stock prices to the two technology shocks $x$ and $\xi$ in the model. We construct the impulse responses taking into account the nonlinear nature of equilibrium dynamics: we introduce an additional one-standard deviation shock at time $t = 0$ without altering the realizations of all future shocks. We report separate results for two types of firms. The solid line represents the responses for a Growth firm, defined as a firm with $\lambda_{f,t} = \lambda_H$ and $K_{f,t} = 0.5$. The dotted line indicates the responses for a value firm, defined as a firm with low investment opportunities $\lambda_{f,t} = \lambda_L$ and large size $K_{f,t} = 2$. For both firms, the level of average profitability is equal to its long-run mean, $u_{f,t} = 1$. The initial value of the state variable $\omega$ is set to its unconditional mean, $\omega_0 = \mathrm{E}[\omega_t]$. Columns one and four plot percentage changes, columns two and three plot changes in the level (since both dividends and investment need not be positive) normalized by the aggregate dividend and investment at time $t = 0$.




only increases the productivity of new investments. Due to the equilibrium response of the price of labor services, the profit flow from existing operations falls. Growth firms increase investment, and experience an increase in profits and market valuations. In contrast, value firms have few new projects to invest in, and therefore experience a decline in their profits and valuations.

In sum, growth firms have higher cashflow and stock return exposure to either technology shock than value firms, and the difference is quantitatively larger for technological improvements embodied in new capital vintages $\xi$ versus shocks to labor productivity $x$. These differential responses of growth and value firms to technology shocks translate into cross-sectional differences in risk and risk premia.

# 3 Estimation

Next, we describe how we calibrate the model to the data. Given that the majority of households do not participate in financial markets, we first introduce an extension of the model that features limited participation in Section 3.1. In Section 3.2 we integrate the Kogan et al. (2016) measure of the value of innovation into our analysis. Next, in Section 3.3, we describe our estimation strategy. In Section 3.4, we examine the model's performance in matching the features of the data that we target, and the resulting parameter estimates. In Section 3.5 we discuss parameter identification. In Section 3.6 we examine which of the model's non-standard features are important for the model's quantitative performance.

## 3.1 Limited participation

A stylized feature of the data is that only a relatively small subset of households participate in financial markets. For instance, Poterba and Samwick (1995) report that the households in the top 20% in terms of asset ownership consistently own more than 98% of all stocks. Given the important role that inequality plays in the model, allowing for this type of limited participation is likely quantitatively important.

We model limited participation by assuming that newly born households are randomly assigned to one of two types, *shareholders* (with probability $q_S$) and *workers* (with probability $1 - q_S$). Shareholders have access to financial markets, and optimize their life-time utility of consumption, just like the households in our baseline model. Workers are instead hand-to-mouth consumers. They do not participate in financial markets, supply labor inelastically, and consume their labor income as it arrives. Workers can also successfully innovate (just like shareholders); those that do so become shareholders. Hence,

$$\psi \equiv \frac{\mu_I + q_S\,\delta^h}{\mu_I + \delta^h} \tag{37}$$

represents the fraction of households that participate in financial markets.

## 3.2 Measuring the value of new blueprints

The market value of new blueprints $\nu_t$ plays a key role in the model's predictions, both for the dynamics of firm cashflows (36) and for the evolution of investors' wealth (29). To take the model to the data, we use data on patents and stock returns to construct an empirical proxy for $\nu_t$. Specifically, we view patents as empirical equivalents to the blueprints in our model. The Kogan et al. (2016) methodology allows us to assign a dollar value to each patent, that is based on the change in the dollar value of the firm around a three-day window after the market learns that the firm's patent application has been successful. To replicate this construction in simulated data, we employ an approximation that does not require the estimation of new parameters (see Appendix for details).

We construct an estimate of the aggregate value of new blueprints $\nu$ at time $t$ as

$$\hat{\nu}_t = \sum_{j \in P_t} \hat{\nu}_j, \tag{38}$$

where $P_t$ denotes the set of patents granted to firms in our sample in year $t$. Similarly, we measure the total dollar value of innovation produced by a given firm $f$ in year $t$ by summing the estimated values for all patents $\nu_j$ that were granted to the firm during that year $t$,

$$\hat{\nu}_{f,t} = \sum_{j \in P_{f,t}} \nu_j, \tag{39}$$

where now $P_{f,t}$ denotes the set of patents issued to firm $f$ in year $t$. To obtain stationary ratios, we scale both (38) and (39) by the market capitalization $M$ of the market portfolio and firm $f$, respectively. We label the corresponding ratios $\nu_t/M_t$ and $\nu_{f,t}/M_{f,t}$ as estimates of the (relative) value of innovation at the aggregate and firm level, respectively.

In the model, the ratio of the value of new blueprints $\nu$ to the value of the stock market $M$ is a monotone function of the state variable $\omega$, as we can see in Panel A of Figure 6. Hence, we view the ratio $\nu/M$ as a useful proxy for the state variable $\omega$ in the model and define

$$\hat{\omega}_t \equiv \log\left(\frac{\hat{\nu}_t}{M_t}\right). \tag{40}$$

In Panel B, we plot $\hat{\omega}_t$ in the data. Similar to Kogan et al. (2016), this time-series series lines up well with the three major waves of technological innovation in the U.S. – the 1930s, 1960s and early 1970s, and 1990s and 2000s.

### 3.3 Estimation strategy

The model has a total of 21 parameters. We choose the probability of household death as $\delta^h = 1/40$ to guarantee an average working life of 40 years. We estimate the remaining parameters of the model using a simulated minimum distance method (Ingram and Lee, 1991). Specifically, given a vector $X$ of target statistics in the data, we obtain parameter estimates by

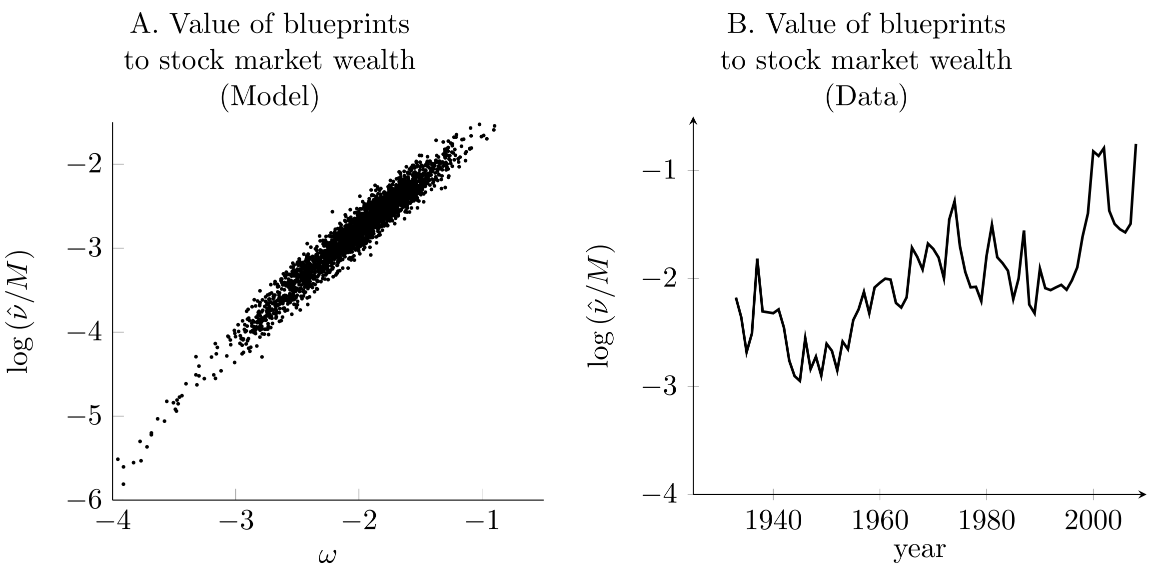
$$\hat{p} = \arg\min_{p \in \mathcal{P}} \left(X - \frac{1}{S}\sum_{i=1}^{S} \hat{X}_i(p)\right)' W \left(X - \frac{1}{S}\sum_{i=1}^{S} \hat{X}_i(p)\right), \tag{41}$$

where $\hat{X}_i(p)$ is the vector of statistics computed in one simulation of the model. Our choice of weighting matrix $W = diag(XX')^{-1}$ penalizes proportional deviations of the model statistics from their empirical counterparts.

Our estimation targets are reported in the first column of Table 1. They include a combination of first and second moments of aggregate quantities, asset returns, and firm-specific moments. We discuss the construction of these variables in detail in Appendix B. Due to data limitations, each of these statistics is available for different parts of the sample. We use the longest available sample

**Figure 6: Value of blueprints**

Panel A plots the value of blueprints (38) to total stock market wealth $M$ relative to the value of the state variable $\omega$ in the model, using a long panel of 3,000 years with 1,000 firms. Panel B plots the time series of the ratio of the estimated value of new blueprints $\hat{\nu}_t$ to the value of the stock market $M_t$ in the data. See the appendix and Kogan et al. (2016) for more details. The estimated value of blueprints is constructed in the same way in both simulated as well as actual data. The total value of innovation in year $t$ is scaled by end of year $t$ market capitalization. Data period is 1933 to 2008.




to compute them. Many of the moments that we target are relatively standard in the literature. Others are less common, but they are revealing of the main mechanisms of our paper. We discuss them next.

First, the dynamic behavior of our model is summarized by the dynamics of the stationary variable $\omega_t$. In the model, both the investment-to-output ratio ($I_t/Y_t$) as well as the relative value of new innovations ($\nu_t/M_t$) are increasing functions of $\omega$; therefore, the unconditional volatility of these ratios is informative about the model parameters. Similarly, fluctuations in $\omega$ lead to predictable movements in consumption growth. We therefore also include as a target an estimate of the long-run volatility of consumption growth using the methodology of Dew-Becker (2014) in addition to its short-term (annual) volatility. In contrast to short-term volatility, the long-run estimate takes into account the serial correlation in consumption growth, and is therefore revealing of the magnitudes of the fluctuations in $\omega$.

Second, a large fraction of the parameters of the model affect the behavior of individual firms, specifically, the parameters governing the evolution of $\lambda_{f,t}$ and the persistence and dispersion of firm-specific shocks in (5). To estimate these parameters, we therefore include as targets the cross-sectional dispersion and persistence in firm investment, innovation, Tobin's Q, and profitability.

Third, our model connects embodied technology shocks to the return differential between value and growth firms. We thus include as estimation targets not only the first two moments of the market portfolio, but also the average value premium, defined as the difference in risk premia between firms in the bottom versus top decile in terms of their market-to-book ratios ($Q$), following Fama and French (1992). Given that the model has no debt, we create returns to equity by levering the value of corresponding dividend (payout) claims by a factor of 2.5.[11]

The statistics reported in Table 1 are largely insensitive to the likelihood of repeat innovation $\mu_I$. The reason why $\mu_I$ has a negligible impact on asset returns is risk aversion. As long as $\mu_I$ is small, and given moderate amounts of utility curvature, households make portfolio decisions by effectively ignoring the likelihood that they will themselves innovate—that is, they behave approximately as if $\mu_i = 0$. To see this, note that the certainty equivalent of a bet that pays with probability $\mu_I$ an amount that is proportional to $1/\mu_I$ is negligible for small $\mu_I$. We therefore calibrate the probability of repeat innovation to equal $\mu_I = 0.13\%$ so that, conditional on the other parameters, the model average top 1% income shares that are in line with the data.[12] We postpone this discussion until Section 4.1.

### 3.4 Model Fit

Examining columns two through five of Table 1 we see that the baseline model fits the data reasonably well. The model not only generates realistic patterns for aggregate consumption and investment, but can also fit both the mean as well as the dispersion in risk premia in the cross-section of firms. In addition, the model generates realistic levels of dispersion and persistence in firm-level profitability, investment, innovation output and valuation ratios.

On the asset pricing side, the main success of our model relative to existing work is that it delivers realistic predictions not only about the equity premium, but also the cross-section of asset returns. Specifically, the model generates realistic differences in risk premia between high-$Q$ (growth) and low-$Q$ (value) firms. These patterns arise primarily from the fact that the two technology shocks in the model carry opposite risk prices (as we saw in the last column of Figure 4), and the fact that value and growth firms have differential exposures to the two technology shocks (as we saw in Figure 5). Further, even though these are not our main objects of interest, the model also generates a low and stable risk free rate. In this respect our model represents an improvement compared to many equilibrium asset pricing models with production (see, e.g. Jermann, 1998; Kaltenbrunner and Lochstoer, 2010).[13] The model does have some difficulty in generating sufficiently volatile stock returns—stock returns in the model are smoother than their empirical counterparts by approximately

[11] This value lies between the estimates of the financial leverage of the corporate sector in Rauh and Sufi (2011) (which is equal to 2) and the values used in Abel (1999) and Bansal and Yaron (2004a) (2.74-3).

[12] Including $\mu_I$ in the full estimation along with the inequality moment is computationally costly because of the number of simulations required to estimate inequality in the model with sufficient precision. Our algorithm therefore alternates between fixing $\mu_I$ and optimizing over all other parameters, while targeting all moments other than the income shares, and optimizing over $\mu_I$ with other parameters fixed, while targeting the top income shares.

[13] Recent contributions include Boldrin, Christiano, and Fisher (2001); Guvenen (2009); Campanale, Castro, and Clementi (2010); Kaltenbrunner and Lochstoer (2010); Garleanu et al. (2012); Croce (2014). Some of these models succeed in generating smooth consumption paths with low-frequency fluctuations and volatile asset returns, sometimes at the cost of a volatile risk-free rate. The models are often hampered by the fact that consumption rises while dividends fall after a positive technology shock, leading to a negative correlation between aggregate payouts of the corporate sector and consumption (see e.g., Rouwenhorst, 1995). In our setup, consumption and dividends are positively correlated, which helps the model deliver a sizeable equity premium.

**Table 1:** Benchmark model: goodness of fit

| Statistic | Data | Model | | | |
|---|---|---|---|---|---|
| | | Mean | (5%) | (95%) | SQRD |
| *Aggregate quantities* | | | | | |
| Consumption growth, mean | 0.015 | 0.014 | 0.003 | 0.024 | 0.003 |
| Consumption growth, volatility (short-run) | 0.036 | 0.038 | 0.035 | 0.043 | 0.005 |
| Consumption growth, volatility (long-run) | 0.041 | 0.053 | 0.039 | 0.068 | 0.083 |
| Shareholder consumption share, mean | 0.429 | 0.464 | 0.437 | 0.491 | 0.006 |
| Shareholder consumption growth, volatility | 0.037 | 0.039 | 0.025 | 0.053 | 0.002 |
| Investment-to-output ratio, mean | 0.089 | 0.083 | 0.043 | 0.123 | 0.005 |
| Investment-to-output ratio (log), volatility | 0.305 | 0.288 | 0.149 | 0.506 | 0.003 |
| Investment growth, volatility | 0.130 | 0.105 | 0.083 | 0.126 | 0.037 |
| Investment and consumption growth, correlation | 0.472 | 0.373 | 0.170 | 0.537 | 0.044 |
| Aggregate Innovation, volatility | 0.370 | 0.369 | 0.219 | 0.619 | 0.000 |
| Capital Share | 0.356 | 0.354 | 0.312 | 0.385 | 0.001 |
| *Asset Prices* | | | | | |
| Market portfolio, excess returns, mean | 0.063 | 0.067 | 0.052 | 0.080 | 0.003 |
| Market portfolio, excess returns, volatility | 0.185 | 0.131 | 0.119 | 0.143 | 0.087 |
| Risk-free rate, mean | 0.020 | 0.020 | 0.015 | 0.029 | 0.003 |
| Risk-free rate, volatility | 0.007 | 0.007 | 0.003 | 0.013 | 0.000 |
| Value factor, mean | 0.065 | 0.063 | 0.031 | 0.093 | 0.001 |
| *Cross-sectional moments* | | | | | |
| Investment rate, IQR | 0.175 | 0.163 | 0.124 | 0.204 | 0.005 |
| Investment rate, persistence | 0.223 | 0.228 | 0.053 | 0.457 | 0.000 |
| Tobin Q, IQR | 1.139 | 0.882 | 0.611 | 1.147 | 0.051 |
| Tobin Q, persistence | 0.889 | 0.948 | 0.928 | 0.961 | 0.004 |
| Firm innovation, 90-50 range | 0.581 | 0.542 | 0.441 | 0.609 | 0.005 |
| Firm innovation, persistence | 0.551 | 0.567 | 0.519 | 0.623 | 0.001 |
| Profitability, IQR | 0.902 | 0.936 | 0.856 | 1.073 | 0.001 |
| Profitability, persistence | 0.818 | 0.815 | 0.796 | 0.830 | 0.000 |
| Distance (mean squared relative deviation) | | | | | 0.014 |

This table reports the fit of the model to the statistics of the data that we target. Growth rates and rates of return are reported at annual frequencies. See main text for details on the estimation method and Appendix B for details on the data construction. We report the mean statistic, along with the 5% and 95% percentiles across simulations. We also report the squared relative deviation of the mean statistic to their empirical counterparts, $SQRD_i = (X_i - \mathcal{X}_i(p))^2/X_i^2$.

30%. As is the case with almost all general equilibrium models, the need to match the relatively smooth dynamics of aggregate quantities imposes tight constraints on the shock volatilities $\sigma_x$ and $\sigma_\xi$.[14]

The model's success in fitting the asset pricing moments does not come at the cost of counterfactual implications for quantities. Examining the first ten rows of Table 1, we see that the model

[14]Mechanisms that lead to time-variation in risk premia – for example, time-variation in the volatility of the shocks $\sigma_x$ and $\sigma_\xi$ – may help increase the realized variation in asset returns. Due to the associated increased computational complexity, we leave such extensions to future research.

generates realistic implications for aggregate quantities. Even in the few cases in which the point estimates between the model and the data differ, the empirical moments can still be plausibly observed in simulated data—that is, they are covered by the standard 90% confidence intervals based on model simulations. Similarly, the bottom nine rows of Table 1 show that the model also generates realistic firm-level moments. This is notable in itself, since existing models that fit the cross-section of asset returns fail to match the cross-section of firm investment, and vice versa (see, e.g. Clementi and Palazzo, 2015). Further, a notable feature of the data is that firm-level investment and innovation exhibit relatively low persistence, while valuation ratios ($Q$) are highly persistent. The model can largely accommodate this behavior because realized investment at the firm level exhibits spikes—that is, firms only invest conditional on having a project. Valuation ratios depend partly on expectations of future investment, and therefore are much more persistent.

**Table 2:** Benchmark model: parameter estimates

| Parameter | Symbol | Estimate | SE |
|---|---|---|---|
| *Preferences* | | | |
| Risk aversion | $\gamma$ | 56.734 | 41.163 |
| Elasticity of intertemporal substitution | $\theta$ | 2.341 | 2.867 |
| Effective discount rate | $\rho$ | 0.044 | 0.015 |
| Preference weight on relative consumption | $h$ | 0.836 | 0.067 |
| *Technology* | | | |
| Disembodied technology growth, mean | $\mu_x$ | 0.016 | 0.011 |
| Disembodied technology growth, volatility | $\sigma_x$ | 0.082 | 0.010 |
| Embodied technology growth, mean | $\mu_\xi$ | 0.004 | 0.018 |
| Embodied technology growth, volatility | $\sigma_\xi$ | 0.110 | 0.025 |
| Project-specific productivity, volatility | $\sigma_u$ | 0.533 | 0.065 |
| Project-specific productivity, mean reversion | $\kappa_u$ | 0.210 | 0.023 |
| *Production and Investment* | | | |
| Cobb-Douglas capital share | $\phi$ | 0.427 | 0.023 |
| Decreasing returns to investment | $\alpha$ | 0.446 | 0.096 |
| Depreciation rate | $\delta$ | 0.029 | 0.011 |
| Transition rate to low-growth state | $\mu_L$ | 0.364 | 0.098 |
| Transition rate to high-growth state | $\mu_H$ | 0.021 | 0.006 |
| Project mean arrival rate, mean | $\lambda$ | 0.812 | 0.273 |
| Project mean arrival rate, rel. difference high-low growth states | $\lambda_D$ | 15.674 | 3.303 |
| *Incomplete Markets* | | | |
| Fraction of project NPV that goes to inventors | $\eta$ | 0.767 | 0.391 |
| Fraction of households that is a shareholder | $\psi$ | 0.148 | 0.060 |

This table reports the estimated parameters of the model. When constructing standard errors, we approximate the gradient of $\frac{1}{S}\sum_{i=1}^{S}\hat{X}_i(p)$ using a five-point stencil centered at the parameter vector $\hat{p}$. In addition, we approximate the sampling distribution of $X$ (the empirical moment covariance matrix) with the sampling distribution of $X_i(p)$ in the model (the covariance matrix of model moments across simulations. See the Appendix for more details.

### 3.5 Parameter Estimates and Identification

Table 2 reports the parameter estimates in the model, along with standard errors. These standard errors depend on how sensitive the model's implications (the model-implied statistics) are on the underlying parameters, and how precisely these statistics can be estimated in the data. To obtain some intuition for which features of the data identify individual parameters, we compute the Gentzkow and Shapiro (2014) measure of (local) sensitivity of parameters to moments (GS). To conserve space, we only summarize the results here, and relegate a more comprehensive discussion of identification, along with plots of the GS sensitivity measures, to the Online Appendix.

The parameters governing the volatility of the two technology shocks are of primary importance for the model's implications about the dynamics of aggregate quantities and asset prices. Accordingly, their estimated values $\hat{\sigma}_x = 8.2\%$ and $\hat{\sigma}_\xi = 11\%$ are estimated with considerable precision. Examining the GS sensitivity measure, we see that $\sigma_x$ is primarily identified by the volatility of consumption growth, and the correlation between consumption and investment. Conversely, $\sigma_\xi$ is mostly identified by the volatility of investment growth, and the correlation between investment and consumption. Recall from Figure 2 that both investment and consumption respond symmetrically to the disembodied shock $x$; by contrast, investment and consumption respond initially with opposite signs to $\xi$. Hence, the correlation between investment and consumption carries important information on the relative importance of these two shocks. In contrast, the parameters governing the mean growth rates of the two types of technology, $\mu_x$ and $\mu_\xi$, are much less precisely estimated. To estimate $\mu_x$, the model places considerable weight on the mean rate of consumption growth; by contrast, to estimate $\mu_\xi$ our estimation procedure leans on the the moments of the investment-to-output ratio; higher values of $\mu_\xi$ imply that the mean of the stationary distribution of $\omega$ is higher, which implies higher investment-to-output, but also a faster rate of mean reversion, and therefore a lower unconditional volatility for the investment-to-output ratio. However, since these moments also depend on other parameters (for instance, the mean growth rate of the economy depends on both $\mu_x$ and $\mu_\xi$) these two parameters are less precisely estimated.

The parameters governing the dynamics of $\lambda_{f,t}$ are identified by both the aggregate investment-to-output ratio, as well as the dispersion and persistence of firm-level investment, innovation, and valuation ratios. Specifically, the parameter $\lambda$, which corresponds to the cross-sectional mean of $\lambda_{f,t}$ is primarily determined by the unconditional volatility in the investment-to-output ratio; the aggregate ratio of the value of new projects to total wealth ($\nu_t/M_t$); and the persistence in innovation outcomes at the firm level. An increase in $\lambda$ implies that the economy responds faster to a positive shock to $x$ or $\xi$, which implies that the state variable $\omega$ exhibits a faster rate of mean reversion and hence lower unconditional volatility. In addition, increasing $\lambda$ implies an increase in the frequency of investment at the firm level, which implies higher firm-level persistence. The parameter $\lambda_D = (\lambda_H - \lambda_L)/\lambda$, which controls the relative differences in firm growth rates between the low- and high-growth regimes, is primarily identified by the cross-sectional dispersion in firm investment. The estimated value assigns considerable difference in the rate at which firms acquire projects in the high- and low-growth regimes. The parameters governing the transition across the low- and high-growth regimes (8) are identified by the persistence of investment growth, innovation, and valuation ratios at the firm level, as well as cross-sectional differences in innovation outcomes across firms. In particular, increasing either $\mu_H$ or $\mu_L$ lowers the persistence of investment and valuation ratios at the firm level. Our estimates imply that the high-growth rate is pretty transitory, while the low-growth rate is highly persistent. Hence, at any given time, most firms in the model are in the low-growth regime.

The remaining parameters governing the production side of the model are also relatively precisely estimated. The share of capital in the production function $\phi$ is primarily identified by the capital share. Recall that the capital share in the model, defined as the share of output that accrues to the owners of capital (shareholders) is equal to $\phi$ minus the share of output that accrues to inventors. The rate of mean reversion of firm-specific shocks $\kappa_u$ is identified by the persistence in profitability at the firm level. The volatility of firm-specific shocks $\sigma_u$ is primarily identified by the persistence and dispersion in profitability across firms. The depreciation rate $\delta$ is identified by the average investment-to-output ratio; similar to deterministic models, a higher depreciation rate of capital implies a higher mean investment-to-output ratio. Last, the degree of adjustment costs $\alpha$ is primarily identified by the volatility of investment growth, the cross-sectional dispersion in Tobin's $Q$, and the volatility of the market portfolio. Recall that a higher value of $\alpha$ lowers adjustment costs; similar to standard models, doing so makes investment more volatile but lowers the volatility of prices.

The parameter that governs the share of households that participate in the stock market $\psi$ is relatively well estimated, with a point estimate of 0.15 and a standard error of 0.05. Our estimate is largely in line with the facts reported in Poterba and Samwick (1995), who document that the households in the top 20% in terms of asset ownership consistently own more than 98% of all stocks. This parameter is primarily identified by the mean consumption share of stockholders.

The parameter that affects the division of surplus between shareholders and innovators is estimated at $\eta = 0.77$ with a standard error of 0.39. Hence, approximately one-fourth of the value of new investment opportunities in the economy accrues to the owners of the firm's public securities.[15] In general, this parameter has an ambiguous effect on the value premium. On the one hand, increasing $\eta$ increases the share of rents that go to innovators, and therefore increases the displacement risk that is faced by shareholders. On the other hand, however, it reduces the overall share of growth opportunities to firm value, which increases the volatility of the market

[15] A quantitative evaluation of the plausibility of this parameter is challenging due to the lack of available data on valuations of private firms. However, if we interpret the selling of ideas to firms as the inventors creating new startups that go public and are subsequently sold to large, publicly traded firms, this pattern is consistent with the empirical fact that most of the rents from acquisitions go to the owners of the target firm (Asquith and Kim, 1982). In addition, a high estimate of $\eta$ is consistent with the idea that ideas are a scarcer resource than capital, and since the 'innovators' own the ideas, it is conceivable that they extract most of the rents from the creation of new projects. An illustrative example is the case of the British advertising agency Saatchi and Saatchi, described in Rajan and Zingales (2000). In 1994, Maurice Saatchi, the chairman of Saatchi and Saatchi, proposed for himself a generous compensation package. The U.S. fund managers, who controlled 30 percent of the shares, voted down the proposal at the general shareholders' meeting. The Saatchi brothers, along with several key senior executives, subsequently left the firm and started a rival agency (M&C Saatchi), which in a short period of time captured several of the most important accounts of the original Saatchi & Saatchi firm. The original firm was severely damaged and subsequently changed its name. Another real world example of an inventor is a venture capitalist (VC). A large value of $\eta$ would be consistent with the evidence that VC firms add considerable value to startups, but investors in these funds effectively capture none of these rents (see, e.g. Korteweg and Nagel, 2016).

portfolio—since the market is now more vulnerable to displacement. Consequently, this parameter is mainly identified by the average returns of the market portfolio and the value factor (which are noisily estimated), and to a lesser extent by the long-run volatility of consumption. A higher long-run volatility of consumption growth implies that the model needs a smaller value of $\eta$ to match asset prices.

The remaining parameters correspond to household preferences, and therefore are not always well identified. For instance, the estimated utility curvature parameter is relatively high, ($\hat{\gamma} = 57$), but the large standard errors (41) suggest that the model output is not particularly sensitive to the precise value of $\gamma$. Examining the GS sensitivity measure, we see that the risk aversion coefficient is primarily identified by the mean of the market portfolio relative to its standard deviation and the mean of the value factor. Similarly, the elasticity of inter-temporal substitution has a point estimate of $\theta = 2.3$ but the large standard errors reveal that the quantitative implications of the model is robust to different values of $\theta$. Examining the GS sensitivity measure reveals that $\theta$ is primarily identified through the volatility of the interest rate in the model and the long-run volatility of consumption growth. A higher willingness to substitute across time implies a higher serial correlation for consumption growth, and hence a higher long-run variability in consumption growth. The parameter governing the household's effective discount rate $\rho$ is somewhat better identified, mainly through the mean of the investment-output ratio, as well as the mean interest rate relative to the growth rate of the economy.

Last, the parameter governing the share of relative consumption $h$ is identified primarily by the differences in risk premia between the market and the value factor. Recalling the discussion in Figure 4, higher values of $h$ imply that households are more averse to displacement risk—they resent being left behind—and hence increases the risk premium associated with displacement risk. However, higher values of $h$ also lower the household's effective risk aversion towards aggregate—that is, non-displacive—shocks, that is, the parameter $\gamma_1$ in the definition of the (30). Since the equity premium compensates investors partly for this aggregate risk, increasing $h$ lowers the equity premium while increasing the mean returns of the value factor. Our baseline estimates imply that households attach a weight approximately equal to 80% on relative consumption. The small standard error (0.06) implies that this parameter is quantitatively important for the implications of the model.

In sum, the two parameters that are key to our main mechanism, $\eta$ and $h$, are estimated with differing levels of precision, with $h$ much more precisely estimated that $\eta$. This suggests that the precise value of $\eta$ is much less important for the model's implications than $h$. However, these are local measures of sensitivity. In the next section, we explore this issue of robustness more broadly.

### 3.6 Sensitivity Analysis

Next, we further shed light on the model mechanism by estimating restricted versions of the model. Recall that our model has three relatively non-standard features. First, our model features technology shocks that are embodied in new capital. Second, markets are incomplete in that households cannot sell claims on their proceeds from innovation. Third, household preferences are affected by their consumption relative to the aggregate economy. In addition, when taking the model to the data, we assumed that a subset of households did not participate in financial markets. Here, we examine how important these features are for the quantitative performance of the model. Last, we also estimate a version of the model that imposes an upper bound on the risk aversion of the representative household.

We proceed by estimating the model imposing several parameter restrictions. To conserve space, in Table 3, we only report the key estimated parameters of the model, along with targeted moments; the full set of results is in the Online Appendix. To understand the impact of these parameter restrictions on asset prices, we also report the equilibrium risk prices for the two technology shocks $x$ and $\xi$, along with the risk exposures of the market portfolio and the value factor.[16]

### 3.6.1 Full participation

We first estimate a version of the model in which all households participate in financial markets. This is equivalent to restricting $\psi = 1$ in (37). Comparing columns two and three in Table 3, we see that the restricted model does almost as well as the full model. Both the level as well as the dispersion in risk premia are comparable to the baseline model and similar to the data; decomposing the risk premia to risk prices and risk exposures yields similar results as the baseline model. The overall measure of fit is also close (0.025 vs 0.021). However, the restricted model does require high curvature of the utility function ($\gamma$ of 105 vs 57), a higher surplus share to inventors (0.81 vs 0.77), and a larger preference weight on relative consumption (0.93 vs 0.84).

One reason why these estimated parameters are higher is that the restricted model underestimates the risks that financial markets participants face from innovation. As we saw in Figure 2, labor income rises in response to improvements in technology. This increase in labor income acts as a natural hedge for the displacement of households that occurs through financial markets. Restricting a fraction of the population to not participate effectively limits the share of the labor income that accrues to stock holders and therefore mitigates this hedging effect. Here, we should emphasize that this hedging benefit of labor income is an artifact of the stylized nature of our model. Specifically, we assume that technology has no displacive effect on labor, and that labor income is tradable without any frictions. A more realistic model that allows for endogenous displacement of human capital and possibly frictions, such as credit constraints, is outside the scope of this paper.

### 3.6.2 Restricting the surplus share to innovators

We next present the results when we restrict the surplus that accrues to innovators to be qual to zero, $\eta = 0$. Doing so effectively completes the markets for innovation outcomes, since in this case, all proceeds from new projects accrue to financial market participants. Examining column four of Table 3, we see that this restricted model performs relatively worse in fitting risk premia. The value

[16]In particular, risk premia can be written as $E[R_i] - r_f = \beta_{x,i}\lambda_x + \beta_{\xi,i}\lambda_\xi$, where $\beta$ are the risk exposures to the two technology shocks, and $\lambda_x \equiv -cov\left(dx_t, d\log\Lambda_t\right)/dt$ and $\lambda_\xi \equiv -cov\left(d\xi_t, d\log\Lambda_t\right)/dt$ the risk prices. This equation is an approximation that is useful for exposition: it only holds conditionally. If risk prices and risk exposures are correlated, the unconditional expression is more complicated. However, it is a relatively accurate and useful approximation in that it helps decompose the role of the two technology shocks in affecting risk premia.

premium is equal to 3.4% (versus 6.3% in the baseline model), and the equity premium is equal to 5.3% per year (versus 6.7% in the baseline model). The estimated parameters imply that the preference weight on relative consumption is quite high (92%), while the share of households that participate is 5%.

Examining the risk prices, we see that the price of the embodied technology shock is quite a bit smaller than in the baseline case (-4.3% vs -7.8%) which attenuates the dispersion in risk premia. Interestingly, even though markets for innovation are complete, the risk price of $\xi$ is still negative. This occurs due to an artifact of the stylized nature of our model. In particular, we assumed that households benchmark their consumption relative to the aggregate consumption in the economy, which includes the consumption of the non-participants. As we discussed in Section 3.6.1 above, labor income acts as a hedge because it is homogenous and not subject to displacement. Because aggregate consumption and labor income respond with opposite signs to an embodied shock $\xi$, the marginal utility of market participants rises on impact, leading to a negative risk premium. Further, the restricted model with $\eta = 0$ generates a counterfactually low stock market participation rate, reflected in the low consumption share of shareholders (0.32 vs 0.43 in the data).

We conclude that incomplete markets is a crucial feature of our model, since the restricted version with $\eta = 0$ is inadequate in matching the data. However, as long as markets are incomplete, the model output is rather robust to imposing an upper bound of $\eta$. Specifically, in columns five and six of Table 3, we present results when imposing an upper bound on $\eta$ of 0.3 and 0.6, respectively. Imposing an upper bound on $\eta$ has a minimal effect on the quantitative output of the model. In both cases, the model does a somewhat better job fitting both the level and the dispersion in risk premia—that is, the market portfolio and the value factor. The cost is somewhat higher utility curvature relative to the baseline model ($\gamma$ of 83 and 72, respectively, vs 57 in the baseline model).

### 3.6.3 Restricting the weight on relative preferences

We next consider versions of the model in which we impose restrictions on the preference share of relative consumption $h$. In column seven, we show results from estimating a version without relative consumption preferences, $h = 0$. We see that this restricted version can only fit the equity premium, but not the value premium. To understand why, recall that the embodied shock $\xi$ is primarily responsible in generating cross-sectional differences in risk premia across value and growth firms. The risk price on $\xi$ largely depends on households' aversion to displacement risk, and hence $h$. Hence, we see that the risk price associated with $\xi$ is about two-thirds smaller than in the baseline case.

In sum, preferences for relative consumption are an integral part of the model. However, the model can largely accommodate restrictions on the relative preference weight, $h$. To see this, in columns eight and nine, we present results using an upper bound of 0.3 and 0.6 on $h$. In this case, we see that imposing an upper bound on $h$ has only a moderate impact on the model's ability to fit the value premium. Imposing an upper bound of 0.3 and 0.6 leads to a value premium of 4.1% and 5.2%, respectively, compared to 0.063 in the baseline model. The cost of imposing these restrictions on $h$ is a somewhat higher equity premium than in the data (0.089 and 0.078, respectively).

**Table 3:** Robustness: Comparison across restricted models

| Parameter | Symbol | BASE | FullPart | Restrict $\eta$ | | | Restrict $h$ | | | No $\xi$ | Restrict $\gamma$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\psi = 1$ | $\eta = 0$ | $\eta \leq 0.3$ | $\eta \leq 0.6$ | $h = 0$ | $h \leq 0.3$ | $h \leq 0.6$ | | $\gamma \leq 10$ |
| Risk aversion | $\gamma$ | 56.73 | 104.57 | 92.80 | 83.35 | 72.49 | 15.74 | 30.85 | 34.34 | 73.37 | 10.00 |
| Elasticity of intertemporal substitution | $\theta$ | 2.34 | 2.30 | 2.15 | 1.96 | 1.87 | 0.82 | 1.50 | 1.81 | 1.73 | 2.75 |
| Fraction of population that is a shareholder | $\psi$ | 0.15 | - | 0.05 | 0.06 | 0.13 | 0.05 | 0.05 | 0.05 | 0.16 | 0.05 |
| Preference weight on relative consumption | $h$ | 0.84 | 0.93 | 0.92 | 0.89 | 0.85 | 0.00 | 0.30 | 0.60 | 0.85 | 0.99 |
| Fraction of project NPV that goes to inventors | $\eta$ | 0.77 | 0.81 | 0.00 | 0.30 | 0.60 | 0.86 | 0.84 | 0.75 | 0.27 | 0.95 |
| Disembodied technology growth, volatility | $\sigma_x$ | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.07 | 0.08 | 0.08 | 0.08 |
| Embodied technology growth, volatility | $\sigma_\xi$ | 0.11 | 0.12 | 0.11 | 0.11 | 0.11 | 0.12 | 0.12 | 0.11 | - | 0.13 |

| Moment | DATA | BASE | FullPart | Restrict $\eta$ | | | Restrict $h$ | | | No $\xi$ | Restrict $\gamma$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\psi = 1$ | $\eta = 0$ | $\eta \leq 0.3$ | $\eta \leq 0.6$ | $h = 0$ | $h \leq 0.3$ | $h \leq 0.6$ | | $\gamma \leq 10$ |
| Shareholder consumption share, mean | 0.429 | 0.464 | - | 0.320 | 0.406 | 0.449 | 0.408 | 0.409 | 0.397 | 0.443 | 0.399 |
| Shareholder consumption growth, volatility | 0.037 | 0.039 | - | 0.044 | 0.039 | 0.039 | 0.039 | 0.036 | 0.039 | 0.036 | 0.042 |
| Investment-to-output ratio (log), volatility | 0.305 | 0.288 | 0.282 | 0.325 | 0.305 | 0.291 | 0.312 | 0.315 | 0.304 | 0.025 | 0.312 |
| Investment growth, volatility | 0.130 | 0.105 | 0.101 | 0.119 | 0.108 | 0.107 | 0.112 | 0.110 | 0.109 | 0.048 | 0.108 |
| Investment and consumption growth, correlation | 0.472 | 0.373 | 0.378 | 0.401 | 0.390 | 0.386 | 0.275 | 0.182 | 0.314 | 0.998 | 0.315 |
| Market portfolio, excess returns, mean | 0.063 | 0.067 | 0.068 | 0.053 | 0.066 | 0.071 | 0.081 | 0.089 | 0.078 | 0.053 | 0.039 |
| Market portfolio, excess returns, volatility | 0.185 | 0.131 | 0.138 | 0.132 | 0.125 | 0.129 | 0.139 | 0.119 | 0.128 | 0.116 | 0.161 |
| Value factor, mean | 0.065 | 0.063 | 0.051 | 0.034 | 0.056 | 0.059 | 0.014 | 0.041 | 0.052 | -0.010 | 0.050 |
| Distance (mean relative deviation) | | 0.014 | 0.020 | 0.024 | 0.015 | 0.015 | 0.061 | 0.050 | 0.022 | 0.240 | 0.028 |
| Additional Statistics | | | | | | | | | | | |
| Risk price for $x$ | | 0.027 | 0.026 | 0.021 | 0.031 | 0.032 | 0.053 | 0.053 | 0.039 | 0.041 | 0.000 |
| Market exposure to $x$ | | 1.313 | 1.347 | 1.375 | 1.245 | 1.293 | 1.324 | 1.265 | 1.221 | 1.313 | 1.263 |
| Value-minus-growth risk exposure to $x$ | | -0.184 | -0.154 | -0.303 | -0.237 | -0.194 | -0.096 | -0.194 | -0.252 | -0.327 | -0.200 |
| Risk price for $\xi$ | | -0.078 | -0.065 | -0.043 | -0.068 | -0.079 | -0.023 | -0.050 | -0.057 | | -0.049 |
| Market exposure to $\xi$ | | -0.653 | -0.687 | -0.657 | -0.468 | -0.464 | -0.743 | -0.610 | -0.647 | | -0.922 |
| Value-minus-growth risk exposure to $\xi$ | | -0.875 | -0.885 | -0.877 | -0.966 | -0.862 | -0.736 | -0.986 | -1.104 | | -1.066 |

This table results from estimating restricted versions of the model. To conserve space, we only report the key estimated parameters of the model, along with targeted moments; the full set of results are in the Online Appendix. In addition, we present the risk prices for the two technology shocks, defined as $\lambda_x \equiv -cov\,(dx_t, d\log\Lambda_t)\,/dt$ and $\lambda_\xi \equiv -cov\,(d\xi_t, d\log\Lambda_t)\,/dt$, as well as the risk exposures (beta) of the market portfolio and the value factor with respect to the two shocks.

### 3.6.4 Restricted model with no capital-embodied shocks

To further emphasize the role played by the embodied shock $\xi$, we next estimate a version of the model with only disembodied shocks. Column ten of Table 3 shows that this restricted model can generate the equity premium, but not the value premium. This pattern reinforces the discussion in Section 2.2, that the embodied shock is crucial in generating cross-sectional differences between value and growth firms.

### 3.6.5 Restricting the coefficient of relative risk aversion

Last, we also estimate a version of the model in which we impose an upper bound of 10 on the coefficient of relative risk aversion. As we see in the last column of Table 3, the restricted model generates an equity premium of 3.9% and a value premium of 5.0%. Examining the risk prices of the two technology shocks, we see that the price of $x$ is essentially zero; hence all risk premia are due to the embodied shock. To understand why this happens, notice that the parameter estimates imply that households place almost all of their preference weight on relative consumption, that is, $h = 0.99$. Doing so, magnifies the risk price of the embodied shock $\xi$, but at the cost of pushing the household's effective risk aversion towards purely aggregate shocks—the parameter $\gamma_1$ in the definition of the SDF in equation (30)— towards one. Hence, in this parametrization, households care almost exclusively about displacement risk, leading to a non-trivial risk price for $\xi$ but a zero risk price for $x$.

### 3.6.6 Summary

In sum, these results highlight the importance of the three relatively non-standard features of the model: incomplete markets, preference for relative consumption, and embodied technology shocks. Eliminating any of these three assumptions compromises the model's ability to simultaneously fit both the level as well as the cross-section in risk premia. Specifically, cross-sectional differences in risk premia arise because of displacement. To fit the data, the model requires that there is sufficient displacement risk, and that households care enough about displacement. The model can largely accommodate a strong prior belief about an upper bound on $\eta$ or $h$, as long as these features are not eliminated fully. Last, restricting the coefficient of relative risk aversion to lie below 10 still results in risk premia that are not too different than the data.

## 4 Additional Implications of the Model

Here, we examine the performance of the model in replicating some features of the data that we do not use as an explicit estimation targets. First, we show in Section 4.1 that the model can largely replicate the observed levels of income and wealth inequality, especially at the top of the distribution. Section 4.2 shows that the model can replicate the existence of certain 'asset pricing' anomalies related to the cross-section of value and growth firms. In Section 4.3, we consider the correlation between consumption growth and asset returns. In Section 4.4 we report the relation between consumption growth and corporate payout in the model. In Section 4.5 we show that the model implies an upward sloping term structure of real bond yields. Last, in Section 4.6 we show how the model implies a downward sloping term structure of risk premia for dividends claims on the market portfolio.

### 4.1 Consumption, Wealth, and Income Inequality: Model vs Data

One of the advantages of our incomplete markets model is that it has implications for inequality. In the model, inequality arises because households cannot share the rents to innovation. Lucky households that successfully innovate experience a temporary increase in their income due to the proceeds from innovation; in addition, their wealth (and consumption) also increases. Here, we compare the implications of the model for several measures of consumption, wealth and income inequality in the data. Since the model is largely silent regarding inequality between middle- and low-income households, whenever possible, we focus on the top end of the distribution.

We compute the following measures of inequality. Using the data of Piketty and Saez (2003) and Saez and Zucman (2016), we compute average top income and wealth shares at the 0.1%, 0.5% and 1% cutoff. In addition, we use the SCF, we report top percentile ratios in income and wealth (net worth) using the SCF weights. In order to be consistent with the model, we report these statistics only for the households that report direct or indirect stock ownership.We also remove cohort and age effects, though does not have a major impact on these estimates. For completeness, we report corresponding statistics using the CEX. However these should be interpreted with caution because wealthy households are largely absent from the CEX. By contrast, the SCF over-samples wealthy households, so the moments for wealth and income inequality are likely to be more reliable. We replicate the same statistics using a long simulation from the model. We try to be as close to the data as possible, so when computing top shares, we use the income, or wealth, of all households in the denominator, to make these estimates comparable with the data in Piketty and Saez (2003) and Saez and Zucman (2016). By contrast, when estimating the ratio of top percentiles, we only use the subset of households that participate in financial markets.

Table 4 presents the results. Recall that the parameter $\mu_I$ is chosen to match the income share of the top 0.1%. The rest of the statistics in Table 4 are not part of the estimation targets. Examining the table, we note that the model is largely successful in replicating the magnitude of income and wealth inequality, especially at the very top. Specifically, the top income and wealth shares are largely in line with the data. Further, focusing on the distribution of income and wealth among the stockholder sample, we can see that the model can largely replicate the empirical patterns.

The model differs from the data along some dimensions. First, the ratio of income between households in the 90-th to the 50-th percentile is much higher in the model than in the data. The reason is that in the model households are free to retrade claims on their future labor services $\mathrm{w}_t$; wealthy households therefore purchase a larger fraction of 'human wealth' in order to have the same ratio of financial to human wealth. This feature is essential in preserving the analytic tractability of the model, but also implies somewhat larger income inequality among stockholders in the middle of the distribution. Second, consumption inequality is somewhat higher in the model than in the CEX data. In the model, all shareholders consume the same fraction of their wealth, hence consumption inequality among stockholders is the same as wealth inequality. Top consumption shares differ from top wealth shares due to the presence of non-participating households, who simply consume their wage income. However, an important caveat in this comparison is that wealthy households are undersampled in the CEX, hence the empirical moments of consumption inequality should be interpreted with caution.

**Table 4:** Inequality moments

| | A. Consumption | | B. Wealth | | C. Income | |
|---|---|---|---|---|---|---|
| | Model | Data | Model | Data | Model | Data |
| top 0.1% share (%, all HH) | 5.3 | 0.7 | 12.1 | 13.2 | 5.6 | 5.6 |
| top 0.5% share | 11.2 | 2.5 | 26.2 | 26.2 | 11.5 | 10.8 |
| top 1.0% share | 16.7 | 4.2 | 36.5 | 32.6 | 15.6 | 14.2 |
| 99-90 ratio (stockholders) | 3.3 | 1.7 | 3.3 | 5.7 | 3.5 | 4.2 |
| 99-95 | 2.4 | 1.5 | 2.4 | 3.2 | 2.5 | 2.9 |
| 95-90 | 1.4 | 1.2 | 1.4 | 1.8 | 1.4 | 1.5 |
| 90-50 | 5.6 | 1.8 | 5.6 | 5.1 | 5.6 | 2.5 |

Table compares measures of inequality between the model and the data. Data sources are the Consumption Expenditure Survey (CEX), the Survey of Consumer Finances (SCF) and the data of Piketty and Saez (2003) and Saez and Zucman (2016). The top income and wealth shares are from Piketty and Saez (2003) and Saez and Zucman (2016). Top consumption shares are from CEX (1982-2010). Top shares are calculated relative to all households. The top percentile shares of income (total income) and wealth (net worth) are from the SCF (1989-2013); we report percentile ratios of the stock ownership sample (equity=1 in the SCF summary extracts) and after obtaining residuals from cohort and year dummies and cubic age effects. We use a similar procedure for the CEX data. The corresponding estimates in the model are computed from a long simulation of 10m households for 10,000 years. Percentile ratios are computed among the subset that participates in the stock market. In the model, income equals wages, payout and proceeds from innovations. In the data, we use the total income variable from the SCF, which includes salary, proceeds from owning a business and capital income. In the Piketty and Saez (2003) data, we use income shares inclusive of capital gains.

In sum, we see that the model generates realistic patterns of inequality, especially at the very top. This is important for several reasons. First, these moments indirectly rely on the parameters we estimated in Section 3, for instance, the share of surplus that accrues to innovators $\eta$. The fact that the calibrated value produces realistic moments for inequality is comforting, given those parameters, despite the fact that these were not explicit targets for calibration (with the exception of the income share of the top 1% that is used to calibrate $\mu_I$). Second, to the best of our knowledge, this is the first general equilibrium asset pricing model that can generate realistic patterns of inequality.

## 4.2 Cross-sectional 'asset pricing puzzles' and the failure of the CAPM

A stylized feature of the asset returns is that the CAPM does a poor job fitting the cross-section of risk premia (Fama and French, 1992, see, e.g.). Further, a closely related puzzle, is the existence of 'risk factors' in the cross-section of returns, for instance, the value factor. That is, spread portfolios formed by trading on the top and bottom deciles of characteristic sorted portfolios, not only have sizable CAPM alphas (they are mispriced by the CAPM) but they are substantially volatile, while also exhibiting low correlation with the market portfolio. As Cochrane (2005) writes, this 'comovement puzzle' is a challenge for existing general equilibrium models. These models largely imply that the market portfolio is a summary statistic for all aggregate risk in the economy.

**Table 5:** Cross-sectional 'asset pricing puzzles'

| | Mean | Std | CAPM $\alpha$ | CAPM $\beta$ | $R^2$ | |
|---|---|---|---|---|---|---|
| Value Spread | *0.063* | *0.152* | *0.046* | *0.226* | *0.038* | *Model* |
| | 0.064 | 0.184 | 0.050 | 0.263 | 0.051 | Data |
| | (0.022) | (0.004) | (0.021) | (0.038) | (0.015) | |
| I/K spread | *-0.027* | *0.092* | *-0.020* | *-0.050* | *0.011* | |
| | -0.056 | 0.112 | -0.067 | 0.190 | 0.068 | |
| | (0.017) | (0.003) | (0.016) | (0.029) | (0.023) | |
| E/P spread | *0.040* | *0.110* | *0.025* | *0.202* | *0.106* | |
| | 0.062 | 0.141 | 0.069 | -0.120 | 0.016 | |
| | (0.019) | (0.004) | (0.019) | (0.035) | (0.010) | |

Table reports the mean returns, volatilities, CAPM $\alpha$'s and $\beta$'s, and the regression $R^2$ from a market model for three spread portfolios: the value spread, defined as the difference in returns between the bottom and top decile in terms of market-to-book; the investment spread (I/K), defined as the return spread in decile portfolios of firms sorted based on their past investment rate; and the earnings-price spread (E/P), defined as the returns spread in decile portfolios based on earnings-to-price ratios. Numbers in italics denote results from a long simulation of the model (10,000 years). The moments in the data are based on portfolios available from Kenneth French's website. Data period for the value premium excludes data prior to the formation of the SEC (1936 to 2010); data period for the investment strategy (I/K) is 1964-2010; data period for the earnings-to-price strategy is 1952-2010. Standard errors for the empirical moments are included in parentheses. Standard errors for $R^2$ are computed using the delta method.

Our model can replicate these patterns, even though they are not part of the estimation targets. As we see in top two rows of Table 5, the model not only generates a realistic differences in risk premia between high-$Q$ (growth) and low-$Q$ (value) firms, but it also generates return comovement—the 'value factor' of Fama and French (1993). In both the model and in the data, the value minus growth portfolio is highly volatile but essentially uncorrelated with the market portfolio. In addition, it replicates the failure of the Capital Asset Pricing Model (CAPM) in accounting for these return differences. These patterns arise primarily from the fact that the two technology shocks in the model carry opposite risk prices (as we saw in the last column of Figure 4), and the fact that value and growth firms have differential exposures to the two technology shocks (as we saw in Figure 5). Specifically, as we can see in the bottom six rows of Table 3, the value factor and the market portfolio have different exposures to the disembodied shock $x$; since the disembodied shock $x$ has to be rather volatile to match the volatility of consumption — and the correlation between consumption and investment — this pushes the model to have a relatively low correlation between the market and the value factor, even when this correlation is not an explicit part of the estimation targets. As long as the model matches both the equity and the value premium, this low correlation between the market and value will result in a substantial CAPM alpha.

In the next set of rows, we show that the model can to a lesser degree, also replicate two other closely related 'anomalies' in the data, the high-minus-low investment strategy (Titman et al., 2004) and the high-minus-low earnings-to-price strategy (Rosenberg et al., 1985). The mechanism that delivers these patterns is essentially the same as that which delivers the value spread: firms with high investment and low earnings-to-price ratios are high growth firms (high $\lambda_f / K_f$) and are valued by investors because they help hedge displacement risk. However, at the current parametrization, the model cannot fully match the differences in risk premia observed in the data: the model-implied risk premia are approximately 1/3 to 1/2 their empirical values. We conjecture that extending the production side of the model along the lines of Kogan and Papanikolaou (2013) will deliver a much closer fit to the data. Last, we should mention the possibility that some of these empirical moments may be upward biased. In recent work, Linnainmaa and Roberts (2016) use a novel dataset collected from *Moody's Manuals* and show that the spread in the I/K pattern is much smaller in the pre-1963 period than in the post-1963 period. Linnainmaa and Roberts (2016) conclude that asset pricing models should be tested by whether the model is able to explain half of the in-sample alpha.

### 4.3 Consumption and Asset Returns

We next examine the implications of the model for the joint distribution of consumption and asset returns, specifically, returns to the market portfolio and the value factor. Since our baseline model features limited participation, we report results separately for stock holders, using the data of Malloy, Moskowitz, and Vissing-Jorgensen (2009). We compute correlations of asset returns and dividends from the market portfolio with the consumption of stockholders in absolute terms, but also relative to the consumption of non-participants. We follow standard practice and aggregate consumption growth over multiple horizons. We report results using 2-year growth rates, but the results are qualitatively similar using longer horizons.

**Table 6:** Consumption and asset returns

| Correlation with: | Market portfolio | Value factor | |
|---|---|---|---|
| Aggregate consumption | *0.66* | *0.00* | *Model* |
| | 0.39 | 0.16 | Data |
| | (0.15) | (0.13) | |
| Shareholder consumption | *0.71* | *0.14* | |
| | 0.21 | 0.35 | |
| | (0.12) | (0.18) | |

Table compares the empirical correlations between three consumption measures (aggregate consumption, consumption of stock holders, and relative consumption of stockholders) with the corresponding correlations in the model. Market returns are in excess of the risk-free rate. The value factor is the difference in returns between stocks in the top and bottom decile in terms of book-to-market. The empirical correlations with shareholder consumption are based on the data in (Malloy et al., 2009), which cover the 1982-2002 period. We use the convention of computing correlations of 1-yr asset returns with 2-yr consumption growth.

Table 6 shows that the baseline model generates empirically plausible levels of correlation between asset returns and aggregate consumption. Consumption and aggregate stock market returns are more highly correlated in the model, but the difference between the correlations in the model and in the data is not always statistically significant. The model also reproduces the low empirical correlation between aggregate consumption growth and the value factor, which implies the failure of the Consumption CAPM to capture the value premium in simulated data. Further, we see that the model replicates one of the main findings of Malloy et al. (2009): returns of value firms covary more with shareholder consumption than returns of growth firms. The model output does differ from the data in one respect, it generates a somewhat excessive correlation of shareholder consumption and the market portfolio (equal to 21% in the 1982-2002 sample versus 71% in the model).

## 4.4 Consumption and Corporate Payout

Here, we discuss the relation between consumption and net corporate payout in the model. In addition, we compute the risk and return characteristics of unlevered claims on both. We report the results in Table 7. Examining the table, there are several points worth discussing.

**Table 7:** Model: Consumption and Corporate Payout

| Model moments | Value |
|---|---|
| Consumption growth, volatility | 0.038 |
| Claim to aggregate consumption, return volatility | 0.047 |
| Claim to aggregate consumption, risk premium | 0.017 |
| Net corporate payout (‘dividend’) growth, volatility | 0.122 |
| Claim to aggregate corporate payout (financial wealth), return volatility | 0.061 |
| Claim to aggregate corporate payout (financial wealth), risk premium | 0.027 |
| Correlation, net payout and consumption growth | 0.367 |

The table reports the risk and return characteristics of aggregate consumption and corporate payout, defined in equation (23). In addition to the volatility of the underlying cashflows, we compute the volatility and risk premium on unlevered claims on these two components.

First, the volatility of payout growth is considerable in the model, much higher than consumption growth. As a result, an (unlevered) claim on corporate payout is considerably more risky than a claim to aggregate consumption, and therefore carries a higher risk premium. Further, consumption and net corporate payout (dividends) are positively correlated. These features are in contrast to most existing general equilibrium models, in which payout is often either negatively correlated with consumption or substantially less risky (see, e.g. Rouwenhorst, 1995; Kaltenbrunner and Lochstoer, 2010; Croce, 2014). In our model, dividends and consumption are positively correlated because they respond with the same sign to both technology shocks in the model. Recalling Figure 2, we see that a positive disembodied shock $x$ leads to an increase in both consumption and aggregate payout. By contrast, a positive embodied shock $\xi$ leads to a short-run drop in consumption and dividends as resources are diverted to investment. Second, the volatility of net payout growth is considerably larger than the volatility of the returns to the corporate sector. In the model, this happens because corporate payout has a mean-reverting component—following a positive shock to $\xi$, net corporate payout drops to finance investment but then rises as investment is transformed into productive capital.

These features of the model are consistent with the data. Specifically, Larrain and Yogo (2008) show that net corporate payout is substantially more volatile than returns. Lustig, Van Nieuwerburgh, and Verdelhan (2013) argue that a claim to aggregate consumption has a substantially lower risk premium—approximately one-third of the risk premium of equities.

### 4.5 Term structure of interest rates

Next, we examine the predictions of our model for the price of zero coupon bonds. The price of a zero coupon bond of maturity $T$ can be computed as

$$P_b(t,T) = E_t\left[\frac{\Lambda_T}{\Lambda_t}\right], \tag{42}$$

while zero coupon yields can be computed as

$$y(T,t) = -\frac{1}{T}\log P_b(t,T). \tag{43}$$

We solve for (43) numerically, and report the moments of bond yields in Table 8. We see that the model generates a real yield curve that is on average upward sloping. In addition, average excess returns on real bonds are positive, and rise with maturity. Real bonds are risky in the model because the risk-free interest rate is positively correlated with the stochastic discount factor. Specifically, recall that in Figure 3 interest rate is mostly sensitive to the embodied shock $\xi$, which as Figure 4 shows, is positively correlated with the SDF.

**Table 8:** Model: Term structure moments

| Maturity | 1y | 2y | 3y | 4y | 5y | 6y | 7y | 8y | 9y | 10y |
|---|---|---|---|---|---|---|---|---|---|---|
| Yield, average (%) | 2.0 | 2.0 | 2.1 | 2.2 | 2.2 | 2.3 | 2.4 | 2.4 | 2.5 | 2.6 |
| Yield, volatility (% ) | 0.7 | 0.7 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.9 |
| Excess return, mean (% ) | | 0.1 | 0.3 | 0.5 | 0.6 | 0.8 | 0.9 | 1.1 | 1.3 | 1.4 |
| Excess return, volatility ( %) | | 0.3 | 0.6 | 0.9 | 1.2 | 1.5 | 1.8 | 1.8 | 2.3 | 2.5 |

Table reports the moments for the term structure of real interest rates that is implied by the model under the baseline calibration. Bond excess returns are reported in excess of the risk-free rate. We report moments across a long simulation of the model (10,000 years).

The implications of our model for the yield curve are in stark contrast with most leading equilibrium asset pricing models, which typically predict a downward sloping real yield curve and negative yields on real bonds (Bansal and Yaron, 2004b; Piazzesi and Schneider, 2007; Bansal, Kiku, and Yaron, 2012; Wachter, 2013).[17] By contrast, the observed term structure of Treasury inflation protected securities (TIPS) has never had a quantitatively significant negative slope and the real yield on long-term TIPS has always been positive and is usually above 2% (Beeler and Campbell,

[17] A notable exception is Campbell and Cochrane (1999). The working paper version of Campbell and Cochrane (1999) shows that their endowment economy model also replicates an upward sloping real yield curve.

2012).

### 4.6 Term structure of risk and risk prices

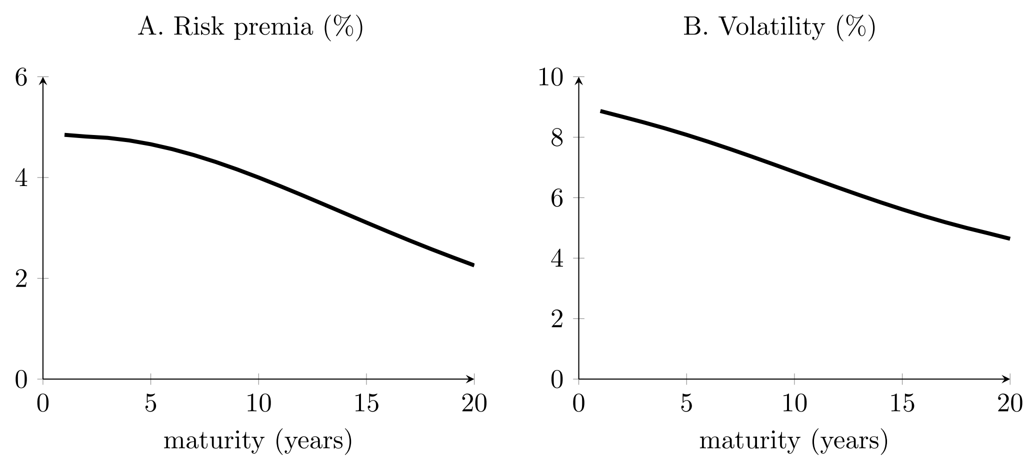
Our model generates a sizable equity premium due to joint movements in aggregate dividends and the stochastic discount factor. Here, we briefly examine how risk in the total payout of the market portfolio at different horizons contributes to asset prices. To do so, we compute prices of corporate payout 'strips', where each strip is a claim on the total payout of the corporate sector at a particular future date. The price of a payout strip is equal to

$$P_d(t,T) = E_t\left[\frac{\Lambda_T}{\Lambda_t} D_T\right], \tag{44}$$

where total payout $D_t$ is given by equation (23).

In Figure 7, we plot the risk premia, and volatilities, of corporate strips across maturities of one to twenty years. We report results for unlevered claims. In Panel A, we see that average returns of dividend strips are decreasing sharply across maturities. Specifically, risk premia range from 4.8% for the shortest maturity strips (1 year) to 2.1% for strips with maturity of 20 years. For comparison, the mean returns on an unlevered claim to corporate payout is 2.9%. Hence, the equity premium in the model is concentrated at shorter maturities. In Panel B, we also see that the prices of dividend strips at shorter maturities are more volatile than those of longer-maturity strips.

**Figure 7:** Model: Term structure of corporate payout strips




This figure shows the moments for the term structure of corporate strips that are implied by the model under the baseline calibration. We compute returns in excess of the risk-free rate and without leverage, using a long simulation of the model (10,000 years).

The above patterns are qualitatively consistent with recent empirical literature. Specifically, van Binsbergen, Brandt, and Koijen (2012) show that risk premia of dividend 'strips' on the market portfolio exhibit a downward-sloping term structure, with the short end of the curve accounting for most of the equity premium. In addition, they show that shorter maturity dividend claims are more volatile. van Binsbergen et al. (2012) argue that these patterns are in stark contrast with most existing equilibrium asset pricing models (e.g Campbell and Cochrane, 1999; Bansal and Yaron, 2004b; Bansal et al., 2012). However, we should emphasize that an important caveat in this comparison is that the empirical evidence pertains to claims on cash dividends, whereas the model results pertain to net payout.

In the model, these results are driven mainly by the dynamics of cashflows rather than the dynamics of risk prices. In particular, in the Online Appendix, we closely follow Borovička, Hansen, and Scheinkman (2014) and construct shock price elasticities, which are essentially risk prices across different horizons. In our model, these marginal risk prices are approximately flat across the horizons. This pattern implies that the model's implications for the term structure of risk premia stem mostly from the dynamics of cash flows – the impulse response of dividends in column four of Figure 2. Specifically, we saw in Panel A of Figure 2 that the contribution of the dividend dynamics induced by $x$ to the equity premium rises modestly with the horizon. Panel B implies the opposite pattern; the contribution of the dividend dynamics induced by $\xi$ to the equity premium is concentrated in the short and medium run, and the rise in long-run dividends contributes negatively to the equity premium. Thus, the term structure of dividend strip risk premia is downward sloping. To conserve space, we refer the reader to our Online Appendix and Borovička et al. (2014) for more details of these computations.

# 5 Examining the Mechanism

Our analysis thus far follows closely the existing literature and evaluates the success of the model based on the model-implied correlations between macroeconomic quantities and prices. Here, we use the estimated value of new innovations $\hat{\nu}_t$ that we constructed in Section 3.2 to examine more directly the predictions of the model's main mechanism. Specifically, we show in Section 5.1 that the innovation measure is related to aggregate quantities and prices in a manner that is consistent with the model. In Section 5.2, we show that growth firms, both in the data and in the model, are more likely to innovate than value firms; in Section 5.3 we show how these differences in innovation lead to differences in how the profitability of growth and value firms responds to an increase in the rate of innovation. Last, in Section 5.4 we provide evidence that increases in the rate of innovation are associated with rising inequality. Throughout this Section our focus is on comparing the empirical estimates to those implied by the model, both qualitatively as well as quantitatively.

## 5.1 Innovation and aggregate dynamics

We begin by documenting the correlation between our aggregate measure of the rate of innovation $\nu/M$ and aggregate quantities and prices. We then compare the empirical results with those in simulated data. One potential shortcoming of our empirical measure of $\hat{\nu}$ is that it identifies the timing of the innovation with the year when that patent is issued to the firm. While this timing is helpful in estimating the value of the patent based on the firm's stock market reaction, it is an imprecise measure of the actual timing of when the invention took place. To ensure that this potential timing mismatch does not affect our results, we report correlations across periods of one to three years.

In rows A and B of Table 9 we compare the correlations between (log differences) the rate of innovation $\nu/M$ and consumption growth—both aggregate consumption as well as the consumption of shareholders. We see that the rate of innovation has a weak negative relation with both measures of consumption growth. In our model, innovation is also weakly negatively correlated with the consumption growth of shareholders, and is essentially uncorrelated with the aggregate consumption growth. Importantly, the magnitudes are comparable, suggesting that the model mechanism does not rely on counter-factually large responses in aggregate consumption growth. Row C shows that both in the data and in the model, changes in the rate of innovation $\nu/M$ are positively correlated with investment growth. In the data, the estimated correlations are lower than in the model (27-30% vs 60-66%) and the difference is statistically significant. Though there are clearly other drivers of investment growth in the data than technological innovation, the empirical results suggest that the contribution of innovation to investment growth is not trivial.

**Table 9:** Innovation, aggregate quantities and asset returns: Model vs Data

| Value of new inventions ($\hat{\omega}$) | Correlation $(t \to t+1)$ | Correlation $(t \to t+2)$ | Correlation $(t \to t+3)$ | |
|---|---|---|---|---|
| A. Consumption growth, aggregate | *0.04* | *0.00* | *0.00* | *Model* |
| | -0.02 | -0.08 | -0.14 | Data |
| | (0.16) | (0.16) | (0.15) | |
| B. Consumption growth, shareholders | *-0.13* | *-0.18* | *-0.18* | |
| | -0.22 | -0.13 | -0.13 | |
| | (0.11) | (0.11) | (0.13) | |
| C. Investment growth | *0.61* | *0.66* | *0.68* | |
| | 0.30 | 0.27 | 0.30 | |
| | (0.12) | (0.10) | (0.12) | |
| D. Market portfolio | *-0.34* | *-0.33* | *-0.32* | |
| | -0.50 | -0.30 | -0.09 | |
| | (0.11) | (0.13) | (0.13) | |
| E. Value factor | *-0.34* | *-0.45* | *-0.49* | |
| | -0.34 | -0.44 | -0.52 | |
| | (0.09) | (0.09) | (0.12) | |

This table reports the correlation between differences in the rate of innovation ($\hat{\omega} \equiv \log \hat{\nu}/M$) and returns to the market portfolio, the value factor (the return spread between the top and bottom decile portfolios of stocks sorted on book-to-market), aggregate consumption and investment growth (NIPA), and the consumption of shareholder using the data from Malloy et al. (2009). For portfolio returns, we compute the cumulative portfolio return between $t$ and $t+h$. For all other variables, including innovation, we use the log difference between $t$ and $t+h$. We compute correlations at horizons of 1 to 3 years. Numbers in parentheses are standard errors, computed using Newey-West with maximum lag length equal to three plus the number of overlapping observations. Data period is 1933 to 2008.

Last, rows D and E of Table 9 compare the correlation between changes in the rate of innovation and asset returns, specifically the market portfolio and the value factor. In the data, changes in innovation are negatively correlated with returns to the market portfolio, though the relation is stronger at the 1-year versus the 3-year frequency. In the model, the relation is also negative, and mostly comparable to the data. Last, we see that the value minus growth factor is negatively correlated with changes in the rate of innovation at all frequencies. Importantly, the magnitude of this correlation is comparable between the data and the model.

In sum, we see that the correlations between the aggregate rate of innovation and key quantities in the model are largely consistent with their empirical counterparts. We view the degree the model can quantitatively replicate these new facts—especially since they are not part of our estimation targets—as providing support for the main mechanism.

## 5.2 Firm valuation ratios and innovation

We next focus our attention at the more micro level, and specifically, on firm-level outcomes. The model implies that part of the reason why the value premium arises is that growth firms innovate more than value firms. This section examines this prediction in the data. Specifically, we are interested in whether a firm's valuation ratio (Tobin's $Q$) predicts the firm's innovative output. We therefore estimate

$$\frac{\nu_{f,t+1}}{M_{f,t+1}} = a + b \log Q_{f,t} + c\, Z_{f,t} + \varepsilon_{f,t}. \tag{45}$$

In addition to $Q$, the right-hand side of equation (45) includes controls for the firm's current size, time dummies, and lagged values of the dependent variable. When estimating (45), we also consider specifications with industry fixed effects, defined at the 3-digit SIC level.

**Table 10:** Valuation ratios and future innovation: Model vs Data

| $\nu_{f,t+1}/M_{f,t+1}$ | (1) | (2) | (3) | (4) | |
|---|---|---|---|---|---|
| $\log Q_{f,t}$ | *-0.038* | *0.244* | *0.244* | *0.125* | *Model* |
| | -0.007 | 0.211*** | 0.169*** | 0.080*** | Data |
| | (0.018) | (0.023) | (0.022) | (0.015) | |
| $\log K_{f,t}$ | | *0.160* | *0.160* | *0.045* | |
| | | 0.189*** | 0.243*** | 0.096*** | |
| | | (0.014) | (0.018) | (0.010) | |
| $\nu_{f,t}/M_{f,t}$ | | | | *0.569* | |
| | | | | 0.657*** | |
| | | | | (0.062) | |
| Observations | 70,987 | 70,987 | 70,986 | 65,823 | |
| $R^2$ | 0.055 | 0.194 | 0.309 | 0.571 | |
| Fixed Effects | T | T | T, I | T, I | |

This table reports the results of estimating equation (45). The first row (italics) reports results in simulated data based on a long sample of 10,000 years. The next two rows report results in Compustat data, along with standard errors clustered by firm. In addition to $Q$, we include controls for firm size ($K_{f,t}$ in the model, Compustat:ppegt in the data) and lagged values of firm innovation. Since firms that never patent may not be a valid control group, we restrict the sample to only include firms that have filed at least one patent.

Table 10 presents the results. We see that, both in the data as well as the model, Tobin's $Q$ is a strong predictor of whether the firm is likely to innovate in the future once we control for its current size. Importantly, the estimated elasticities are comparable between the model and the data. Furthermore, the positive relation survives controlling for whether the firm successfully innovated in the current period. Since the likelihood of successful innovation is somewhat random, Tobin's $Q$ carries additional information in the model about the firm's current state relative to whether the firm innovated in the past. The same is true in the data.

### 5.3 Technological innovation and firm displacement

Having established that growth firms are indeed more likely to innovate than value firms, we next examine the model's predictions about profitability. As we see in equations (35)-(36), the model has specific predictions about the response of firm profitability to changes in the technology frontier — summarized by changes in $\omega$ — as a function of the firm's current state (value or growth). Specifically, Figure 5 shows that a positive technology shock (either $x$ or $\xi$) increases the profitability of growth firms relative to value firms. To get sharper estimates of the impact of technological progress on firm outcomes, we take advantage of the substantial heterogeneity in innovation across industries documented in Kogan et al. (2016). Specifically, we construct a direct analogue of $\hat{\omega}$ at the industry level as

$$\hat{\omega}_{I\setminus f} = \frac{\sum_{f'\in I\setminus f} \hat{\nu}_{f,t}}{\sum_{f'\in I\setminus f} M_{f',t}}. \tag{46}$$

where industry $I$ is defined by the 3-digit SIC code and. $I \setminus f$ denotes the set of all firms in industry $I$ excluding firm $f$.

We estimate the impact of technology on log firm profitability using the following specification,

$$\pi_{f,t+T} - \pi_{f,t} = (a_0 + a_1\, q_{f,t})\, \hat{\omega}_{I\setminus f,t} + a_2\, q_{f,t} + c\, Z_{f,t} + \varepsilon_{t+T}. \tag{47}$$

The dependent variable is the change in the firm's log profits, $\pi_{f,t} = \log \Pi_{f,t}$, between time $t$ and $t+T$. We are interested in the impact of improvements in the technology frontier $\hat{\omega}_{I\setminus f}$ on firms with different levels of Tobin's $Q$. In particular, we classify firms as either value ($q_{f,t} = 0$) or growth ($q_{f,t} = 1$) depending on whether their Tobin's $Q$ falls below or above the industry median at time $t$. In addition, we include a vector of controls $Z_{f,t}$, which includes: the firm's own innovation outcome – the analogue of the $dN_{f,t}$ term in equation (36) – through $\hat{\nu}_{f,t}/M_{f,t}$; controls for lagged log profits $\pi_{f,t}$ and log size $K_{f,t}$ to be consistent with (35)-(36); and time and industry dummies. We cluster the standard errors at the firm level. In addition, we also estimate (47) in simulated data from the model (without the industry and time dummies). We scale $\hat{\omega}_{I\setminus f}$ to unit standard deviation to facilitate comparison between the model and the data. Table 11 compares the estimated coefficients $a_0$ and $a_1$ across horizons of 1 to 6 years in the model (italics) and in the data.

In sum, we find that, in the data, technological innovation—as measured by (47)—primarily hurts value (low $Q$) firms; in relative terms, growth firms benefit. These patterns are consistent with the model. In particular, the estimated coefficient $a_0$ in the data is negative and statistically different from zero, implying that the impact of technological progress on the expected profitability of low-$Q$ firms is negative. Further, the magnitudes are comparable between the model and the data. More importantly, consistent with the model's main mechanism, the estimated coefficient $a_1$ is positive and statistically significant across horizons, revealing substantial heterogeneity in how the profits of low-$Q$ and high-$Q$ firms respond to an increase in innovation at the industry level. Comparing the estimated coefficients $a_1$ between the model and the data, we see that magnitudes are somewhat comparable at shorter horizons (1-3 years), but they differ statistically at longer horizons. That is, the model implies somewhat larger heterogeneity between value and growth firms at longer horizons. This difference partly be driven by the fact that the smallest growth firms in the economy are not in the Compustat database, along with the fact that the less successful firms may exit the sample.

**Table 11:** Technology shocks and firm profitability

| $\Delta\pi_{f,t\to t+h}$ | Horizon | | | | | | |
|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | |
| $\hat{\omega}_{I\setminus ft}$ | *-0.026* | *-0.043* | *-0.054* | *-0.061* | *-0.065* | *-0.068* | *Model* |
| | -0.037*** | -0.036*** | -0.043*** | -0.040*** | -0.043*** | -0.050*** | Data |
| | (0.004) | (0.005) | (0.007) | (0.008) | (0.009) | (0.010) | |
| $\hat{\omega}_{I\setminus ft} \times G_{ft}$ | *0.010* | *0.030* | *0.046* | *0.055* | *0.062* | *0.069* | |
| | 0.024*** | 0.024*** | 0.030*** | 0.032*** | 0.031** | 0.038*** | |
| | (0.004) | (0.006) | (0.008) | (0.008) | (0.009) | (0.010) | |
| Observations | 68,818 | 63,618 | 59,278 | 55,321 | 51,551 | 48,053 | |
| $R^2$ | 0.118 | 0.137 | 0.150 | 0.149 | 0.157 | 0.165 | |

This table summarizes the estimated coefficients $a_0$ and $a_1$ from equation (47) in historical data and in simulated data from the model. We report standard errors in parenthesis clustered by firm. The model estimates (in italics) are from a long panel (10,000 years) comprised of 1,000 firms.

## 5.4 Innovation and inequality

A key feature of our model is that the benefits of innovation are asymmetrically distributed, even among households that participate in financial markets. Specifically, a small subset of households—those that successfully innovate—capture a substantial fraction of the rents from innovation. By contrast, the costs of innovation—the displacement of the installed capital stock—are shared by all market participants. As a result, our model has predictions about the relation between technological innovation and changes in inequality, especially at the very top. Our goal in this section is to examine this implication of the model in the data. Because our empirical analysis is constrained by the available empirical measures of inequality, we focus on the following three available measures of inequality that capture the main mechanism in the model.

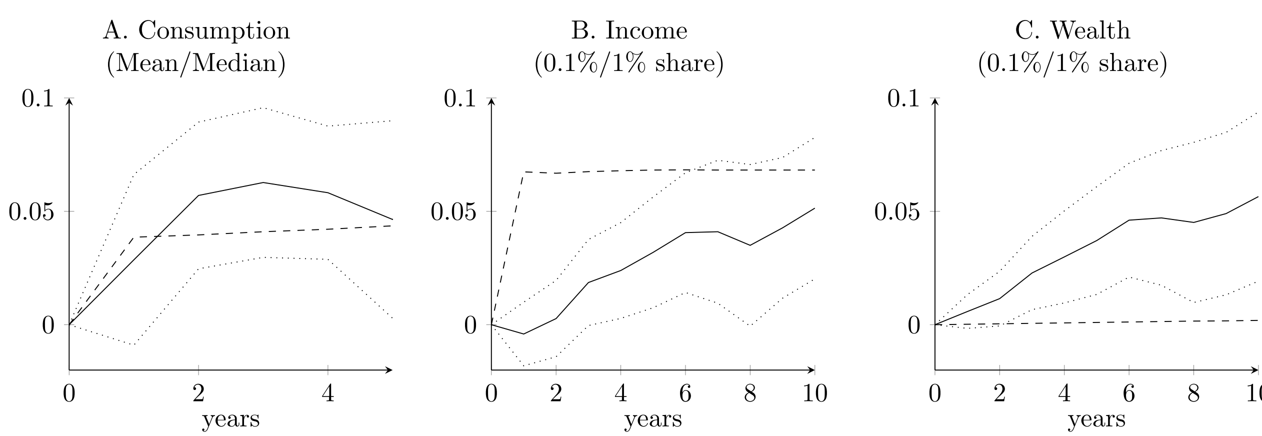
Our first measure of inequality is based on the difference between average and median consumption in the economy. Recalling Figure 4, which shows that improvements in technology have very different effects on the *average* versus the *median* future consumption path of an individual household, a natural measure of inequality is the difference between average consumption and the median in the cross-section of households.[18] Our next two measures of inequality aim to capture the idea that the benefits of innovation are shared by a handful of households. The first of these two measures, based on the data of Piketty and Saez (2003), is the ratio of the 0.1% top income share to the top 1%. This 'fractal' measure of inequality captures the share of the top 1% that accrues to the top 0.1%, and displays similar behavior as the top income shares (Jones and Kim, 2014; Gabaix, Lasry, Lions, and Moll, 2016). Similarly, the second measure uses the data in Saez and Zucman (2016), and is equal to the ratio of the top 0.1% wealth share to the top 1%. These two measures of inequality have the advantage that they are available over long periods of time, which is important for identifying low-frequency relations. To examine the quantitative implications of the model, we construct each of these three measures in simulated data from the model.

For each of the three inequality measures, we estimate the following specification:

$$\log INEQ_T - \log INEQ_t = a_0 + b_T\,(\hat{\omega}_T - \hat{\omega}_t) + c\,INEQ_t + d\,\hat{\omega}_t + u_t, \tag{48}$$

where $\hat{\omega}$ is equal to the log value of new projects (patents in the data) over the total market capitalization, defined in equation (40). To evaluate the magnitude of the empirical estimates, we also estimate (48) in a long simulation of the model (10,000 years).

**Figure 8:** Technology shocks and inequality




This figure plots the estimated coefficient $b_1$ from equation (48). For consumption inequality, we compute the difference between log aggregate per capita NIPA personal consumption expenditures and the logarithm of median consumption in the Consumer Expenditure Survey (CEX). To be consistent with the model, we estimate the median only using the set of households that are stockholders. For income, we use the series of top income shares of Piketty and Saez (2003) that include capital gains. Income in the model is defined as the sum of wage income, capital income ('payout') plus payments to innovators. For wealth, we use the series on top wealth shares by Saez and Zucman (2016). Wealth in the model includes financial wealth plus the present value of future wages (in the model, all market participants can sell claims against their future wage streams $w_t$. We plot empirical point estimates along with 90% confidence intervals. Model results are computed using a long panel of 10m households over 10,000 years.

[18]To estimate this measure in the data, we compute the difference between log aggregate per capita NIPA personal consumption expenditures and the logarithm of median consumption in the Consumer Expenditure Survey (CEX). To be consistent with the model, we estimate the median only using the set of households that are stockholders. The sample covers the 1982-2010 period.

Figure 8 plots the estimated coefficients $b_T$ across horizons in the data (solid line) and in the model (dotted line). Panel A focuses on consumption inequality; given the short time dimension in the CEX, we focus on horizons $T$ of up to 5 years. We see that changes in $\hat{\omega}$ are associated with increases in consumption inequality in the data. Further, the estimated coefficients in the data are quantitatively close to those implied by the model. One concern with using CEX data is that it tends to undersample rich households. Even though using the full consumption distribution of the CEX is problematic for our purposes, the effect of undersampling on the median should be minor. However, we emphasize that, since the full extent of the sample selection biases in the CEX data is unknown, our results should be interpreted with caution.

In Panels B we examine the response of top 'fractile' income share over horizons $T$ of 1 to 10 years. We see that the estimated coefficients $b_T$ are mostly positive and statistically significant over horizons of more than 3 years; a one standard deviation increase in $\omega$ is associated with an increase in income inequality of about 5% over 10 years. In the model simulations, the increase in income inequality is quantitatively similar in the long run (approximately 6%), but it occurs much faster than in the data. In the model, increases in $\omega$ raise the income that accrues to innovators; since the measure of innovating households is very small, increases in $\omega$ increase income inequality at the very top of the distribution. However, unlike reality, this process of reallocation occurs instantaneously in the model, and hence the immediate increase in income inequality. That said, existing models have typically the opposite problem, that is, they generate an increase in income inequality that is too slow relative to the data (Jones and Kim, 2014; Gabaix et al., 2016). Hence, we view our model as also contributing to the literature focusing on the rapid increase in income inequality over the last few decades.

Last, in Panel C, we examine wealth inequality. In the data, we see a positive relation between $\hat{\omega}$ and wealth inequality. In the model, the relation between $\hat{\omega}$ and wealth inequality is also positive, but quantitatively very weak. The reason is that, in our model, the relation between innovation and top wealth shares is theoretically ambiguous. Specifically, increases in $\omega$ raise the income of innovating households and therefore their wealth shares. However, whether this process increases top *wealth* inequality largely depends on where exactly in the distribution of wealth these inventors lie. In the model, richer households are endowed with more ideas conditional on innovating—to keep the return on wealth independent of the level of wealth, which is important for model tractability. However, this feature is offset by the fact that all households have an equal likelihood of innovating, along with the fact that innovation leads to displacement, which erodes the share of wealth held by the richest households. The end result in the model is that these effects largely offset each other, leading to an essentially flat response in the top 0.1%/1% share.

We conclude that the model delivers realistic implications regarding the relation between innovation and the measures of income and consumption inequality that are available in the data. By contrast, even though the empirical results suggest that the correlation between our innovation measure and empirical measures of wealth inequality is positive, the relation in the model is substantially weaker. In the Online Appendix, we consider an extension of the model in which we allow the rate of innovation to vary among existing households. This extended model has similar implications for asset prices, but somewhat different implications for wealth inequality. Specifically, an increase in $\omega$ in the model leads to higher wealth inequality. However, this increase in wealth inequality happens much slower in the model than in the data. Given that the relation between wealth inequality and innovation is sufficiently important to merit a deeper analysis, we relegate this issue to future research.

## 6 Conclusion

We develop a general equilibrium model to study the effects of innovation on asset returns. The main feature of our model is that the benefits from technological progress are not shared symmetrically across all agents in the economy. Specifically, technological improvements partly benefit agents that are key in the creation and implementation of new ideas. As a result, technology shocks also lead to substantial reallocation of wealth among households. Embodied shocks have a large reallocative effects, whereas disembodied shocks have mostly a level effect on household consumption. In equilibrium, shareholders invest in growth firms despite their low average returns, as they provide insurance against increases in the probability of future wealth reallocation. Our model delivers rich cross-sectional implications about the effect of innovation on firms and households that are supported by the data.

Our work suggests several promising avenues for future research. First, labor income in our model is homogenous, and therefore workers benefit from both types of technological progress. In practice, however, technological advances are often complementary to only a subset of workers' skills. Recent evidence, for example, shows that the job market has become increasingly polarized (Autor, Katz, and Kearney, 2006). Thus, quantifying the role of technological progress as a determinant of the risk of human capital may be particularly important. Second, technological progress tends to disrupt traditional methods of production, leading to periods of increased uncertainty. If some agents have preferences for robust control, higher levels of uncertainty will likely increase the agents' demand for insurance against improvements in technology embodied in new vintages. Last, our model implies that claims on any factor of production that can be used across different technology vintages (as, for instance, land) can have an insurance role similar to the one played by growth firms in our current framework. Our model would therefore imply that claims on such factors should have lower equilibrium expected returns.

## Appendix A: Proofs and Derivations

Here, we provide proofs and detailed derivations. To conserve space, we provide the solution for the extended model with limited participation. We begin by describing the differences between the extended model and our baseline model. The solution to the baseline model is a special case in which $q_S = 1$.

### Limited Participation

There are two types of households in our economy, *workers* and *shareholders*. There is continuum of each type, with the total measure of households normalized to one. We denote the set of workers by $\mathcal{W}_t$ and the set of shareholders by $\mathcal{S}_t$

*Workers*
Workers in this economy are hand-to-mouth consumers. They do not participate in financial markets, supply labor inelastically and consume their labor income as it arrives. When a new household is born, it becomes a worker, independently of all other households and all other sources of randomness in the economy, with probability $1 - q_S$. Each worker receives a measure $\lambda(1-\psi)/\mu_I$ of blueprints upon innovating.

*Shareholders*
A newly born household becomes a shareholder with probability $q_S$. Workers that successfully innovate (see below) also become shareholders. Shareholders have access to financial markets, and optimize their life-time utility of consumption. Shareholders are not subject to liquidity constraints. In particular, shareholders sell their future labor income streams and invest the proceeds in financial claims. All shareholders have the same preferences, given by (13)-(14). Each shareholder $i$ receives a measure of projects in proportion to her wealth $W_{i,t}$ relative to shareholders as a group, specifically $\lambda\,\psi\,\mu_I^{-1}\,W_{i,t}\,\left(\int_{j\in\mathcal{S}_t} W_{j,t}\,dj\right)^{-1}$. Here,

$$\psi = \frac{\mu_I + q_S\,\delta^h}{\mu_I + \delta^h} \tag{A.1}$$

is the steady-state fraction of households that participate in financial markets. The case $q_S = 1$ (or equivalently, $\psi = 1$) corresponds to our baseline model.

## Proofs and Derivations

**Lemma 1 (Stationary distribution for $u$)** *The process $u$, defined as*

$$du_t = \kappa_u(1-u_t)\,dt + \sigma_u\,u_t\,dB_t^u \tag{A.2}$$

*has a stationary distribution given by*

$$f(u) = cu^{-2-\frac{2\,\kappa_u}{\sigma_u^2}} \exp\left(-\frac{2\theta}{u\sigma_u^2}\right), \tag{A.3}$$

*where $c$ is a constant that solves $\int_0^\infty f(u)du = 1$. Further, as long as $2\,\kappa_u \geq \sigma_u^2$, the cross-sectional variance of $u$ is finite.*

**Proof.** We follow (Karlin and Taylor, 1981, p. 221). The forward Kolmogorov equation for the stationary transition density $f(u)$ yields the ODE

$$0 = -\kappa_u \frac{\partial}{\partial u}\left[(1-u)f(u)\right] + \frac{1}{2}\sigma_u^2\,\frac{\partial^2}{\partial u^2}\left[u^2 f(u)\right] \tag{A.4}$$

Integrating the above with respect to $u$ yields

$$k = -\kappa_u\left[(1-u)f(u)\right] + \frac{1}{2}\sigma_u^2\,\frac{\partial}{\partial u}\left[u^2 f(u)\right] \tag{A.5}$$

where $k$ is a constant of integration. We set $k = 0$ and find

$$f(u) = cu^{-2-\frac{2\,\kappa_u}{\sigma_u^2}} \exp\left(-\frac{2\theta}{u\sigma_u^2}\right), \tag{A.6}$$

where $c$ is an unknown constant. By construction, the function $f$ is positive. Further, setting the constant $c$ to

$$\left(\int_0^\infty u^{-2-\frac{2\,\kappa_u}{\sigma_u^2}} \exp\left(-\frac{2\theta}{u\sigma_u^2}\right)\,du\right)^{-1} \tag{A.7}$$

(the above integral is finite as long as $\kappa_u > 0$) guarantees that $\int_0^\infty f(u)\,du = 1$, and therefore $f(u)$ is the stationary density of the diffusion process $u$.

The last part of the proof is to show that the variance of $u$ is finite and positive as long as $2\,\kappa_u - \sigma_u^2 > 0$. Given the solution for $c$,

$$\int_0^\infty (u-1)^2\, c\, u^{-2-\frac{2\,\kappa_u}{\sigma_u^2}} \exp\left(-\frac{2\theta}{u\sigma_u^2}\right) du = \frac{\sigma_u^2}{2\,\kappa_u - \sigma_u^2}, \tag{A.8}$$

which is finite as long as $2\,\kappa_u - \sigma_u^2 > 0$. ■

Before proving the Propositions in the main text, we establish some preliminary results. First, we show how to relate the stochastic discount factor (SDF) to the value function of an investor. This is a straightforward application of the results in Duffie and Skiadas (1994) on the relation between the utility gradient and the equilibrium SDFs.

We focus on a single household an omit the household index. To simplify exposition, we present the result in a slightly more general form, not limiting it to the exact structure of our economy. As in our model, the household is solving a consumption-portfolio choice problem with one non-standard element: it receives a stochastic stream of gains from innovation in proportion to its financial wealth. Let $W_t$ denote the household's wealth.

The market consists of $I$ financial assets that pay no dividends. Let $S_t$ denote the vector of prices of the financial assets. $S_t$ is an Ito process

$$dS_t = \mu_t\, dt + \sigma_t\, dB_t. \tag{A.9}$$

The first asset is risk-free, its price growth at the equilibrium rate of interest $r_t$. Let $\mathcal{F}$ denote the natural filtration generated by the Brownian motion vector $B_t$.

The investor receives a flow of income from innovation projects according to an exogenous Poisson process $N$ with the arrival rate $\lambda$. The process $N$ is independent of the Brownian motion $B$. We assume that conditional on innovating, household's wealth increases by a factor of $\exp(\varrho_t)$, where the process $\varrho$ is adapted to the filtration $\mathcal{F}$. The consumption process of the household, $C$, and its portfolio vector $\theta$, are adapted to the filtration generated jointly by the exogenous processes $N$ and $B$.

As in our model, the investor maximizes the stochastic differential utility function given by equations (13-14) in the main text, where we take the process $\overline{C}_t$ to be a general Ito process adapted to the filtration $\mathcal{F}$, subject to the dynamic budget constraint

$$dW_t = \delta W_t\, dt - C_t\, dt + \left(e^{\varrho_t} - 1\right) W_t\, dN_t + \theta_t dS_t, \quad W_t = \theta_t S_t, \tag{A.10}$$

and a credit constraint, which rules out doubling strategies and asymptotic Ponzi schemes:

$$W_t \geq 0. \tag{A.11}$$

Note that the first term in (A.10) captures the flow of income from annuities that the household collects conditional on its continued survival. The death process is a Poisson process with arrival rate $\delta$, which is independent of $N$ and $B$. We are now ready to define an SDF in relation to the value function of the household. In particular, we construct an SDF process that is adapted to the filtration $\mathcal{F}$, and hence does not depend on the household-specific innovation arrival process $N$.

**Lemma 2 (SDF)** *Let $C_t^\star$, $\theta_t^\star$, and $W_t^\star$ denote the optimal consumption strategy, portfolio policy, and the wealth process of the household respectively. Let $J_t^\star$ denote the value function under the optimal policy. Define the process $\Lambda_t$ as*

$$\Lambda_t = \exp\left(\int_0^t \delta + \frac{\partial\phi(C_s^\star, J_s^\star; \overline{C}_s)}{\partial J_s^\star} + \lambda\left(e^{(1-\gamma)\varrho_s} - 1\right) ds\right) A_t, \tag{A.12}$$

*where*

$$A_t = \frac{\partial\phi(C_t^\star, J_t^\star; \overline{C}_t)}{\partial C_t^\star} \exp\left(\int_0^t \gamma\varrho_s\, dN_s\right). \tag{A.13}$$

*Then $\Lambda_t$ is a stochastic discount factor consistent with the price process $S$ and adapted to filtration $\mathcal{F}$.*

**Proof.**

Let $\mathcal{M}$ denote the market under consideration, and define a fictitious market $\hat{\mathcal{M}}$ as follows. $\hat{\mathcal{M}}$ has the same information structure as $\mathcal{M}$, with modified price processes for financial assets. Specifically, let

$$R_t = \exp\left(\int_0^t \delta\, ds + \varrho_s\, dN_s\right) \tag{A.14}$$

and define price processes in the market $\hat{\mathcal{M}}$ as

$$\hat{S}_t = R_t S_t. \tag{A.15}$$

The budget constraint in the market $\hat{\mathcal{M}}$ is standard,

$$d\hat{W}_t = -C_t\, dt + \hat{\theta}_t d\hat{S}_t, \quad \hat{W}_t = \hat{\theta}_t \hat{S}_t. \tag{A.16}$$

If a consumption process $\{C\}$ can be financed by a portfolio policy $\theta$ in the original market $\mathcal{M}$, it can be financed by the policy $R^{-1}\theta$ in the fictitious market $\hat{\theta} = R^{-1}\theta$, and vice versa. Thus, the set of feasible consumption processes is the same in the two markets, and therefore the optimal consumption processes are also the same. Since the consumption-portfolio choice problem in the fictitious market is standard, according to (Duffie and Skiadas, 1994, Theorem 2), the utility gradient of the agent at the optimal consumption policy defines a valid SDF process $\hat{\Lambda}_t$,

$$\hat{\Lambda}_t = \exp\left(\int_0^t \frac{\partial\phi(C_s^\star, J_s^\star; \overline{C}_s)}{\partial J_s^\star}\, ds\right) \frac{\partial\phi(C_t^\star, J_t^\star; \overline{C}_t)}{\partial C_t^\star}. \tag{A.17}$$

Thus, for all $t < T$,

$$\hat{\Lambda}_t R_t S_t = \hat{\Lambda}_t \hat{S}_t = \mathrm{E}_t\left[\hat{\Lambda}_T \hat{S}_T\right] = \mathrm{E}_t\left[\hat{\Lambda}_T R_T S_T\right] \tag{A.18}$$

and therefore $\Lambda_t' = \hat{\Lambda}_t R_t$ is a valid SDF in the original market $\mathcal{M}$. Note that $\Lambda_t'$ is not adapted to the filtration $\mathcal{F}$, since it depends on the agent's innovation process $N$. In other words, $\Lambda_t'$ is an agent-specific SDF process.

The last remaining step is to show that the process $\Lambda_t$ is adapted to the filtration $\mathcal{F}$ and a valid SDF. First, we show that the process $\exp\left(\int_0^t \partial\phi(C_s^\star, J_s^\star; \overline{C}_s)/\partial J_s^\star\right)$ is adapted to $\mathcal{F}$, and the process $\partial\phi(C_t^\star, J_t^\star; \overline{C}_t)/\partial C_t^\star$ can be decomposed as $A_t R_t^{-\gamma}$, where $A_t$ is also adapted to $\mathcal{F}$. Given the homotheticity of the stochastic differential utility function and the budget constraint (A.10), standard arguments show that the agent's value function and the optimal consumption policy can be expressed as

$$J_t^\star = (W_t^\star)^{1-\gamma}\, \varrho_{J,t}, \tag{A.19}$$

where $\varrho_{J,t}$ is a stochastic process adapted to $\mathcal{F}$. The optimal wealth and consumption processes then take form

$$W_t^\star = R_t \varrho_{W,t}, \quad C_t^\star = R_t \varrho_{C,t}, \tag{A.20}$$

where $\varrho_{W,t}$ and $\varrho_{C,t}$ are adapted to $\mathcal{F}$, and therefore

$$J_t^\star = R_t^{1-\gamma} \varrho_{W,t}^{1-\gamma} \varrho_{J,t}. \tag{A.21}$$

We next use these expressions to evaluate the partial derivatives of the aggregator $\phi$:

$$\frac{\partial\phi(C_t^\star, J_t^\star; \overline{C}_t)}{\partial J_t^\star} = -\frac{\rho(1-\gamma)}{1-\theta^{-1}} - \frac{\rho(1-\gamma)^{\frac{1-\theta^{-1}}{\gamma-1}}(\gamma - \theta^{-1})}{1-\theta^{-1}} \left(\varrho_{C,t}^{1-h}\left(\varrho_{C,t}/\overline{C}_t\right)^h\right)^{1-\theta^{-1}} \left(\varrho_{W,t}^{1-\gamma}\varrho_{J,t}\right)^{\frac{1-\theta^{-1}}{\gamma-1}}, \tag{A.22}$$

which is adapted to $\mathcal{F}$; and

$$\frac{\partial\phi(C_t^\star, J_t^\star; \overline{C}_t)}{\partial C_t^\star} = \rho(1-\gamma)^{\frac{1-\theta^{-1}}{\gamma-1}} \overline{C}_t^{h(1-\theta^{-1})} \varrho_{C,t}^{-\theta^{-1}} \left(\varrho_{W,t}^{1-\gamma}\varrho_{J,t}\right)^{\frac{1-\theta^{-1}}{\gamma-1}} R_t^{-\gamma}. \tag{A.23}$$

Thus, the process

$$A_t = \frac{\partial \phi(C_t^\star, J_t^\star; \overline{C}_t)}{\partial C_t^\star} e^{-\gamma\delta t} R_t^\gamma \tag{A.24}$$

is adapted to $\mathcal{F}$. Based on the above results, we express $\Lambda_t'$ as

$$\Lambda_t' = \exp\left( \int_0^t \frac{\partial \phi(C_s, J_s; \overline{C}_s)}{\partial J_s} + \gamma\delta \, ds \right) A_t R_t^{1-\gamma} \tag{A.25}$$

Define $\Lambda_t = \mathrm{E}[\Lambda_t' | \mathcal{F}_t]$. Since all asset price processes in the original market are adapted to $\mathcal{F}$, $\Lambda_t$ is also a valid SDF process. Using the equality (see below)

$$\mathrm{E}\left[ R_t^{1-\gamma} | \mathcal{F}_t \right] = \exp\left( \int_0^t \delta(1-\gamma) + \lambda\left( e^{(1-\gamma)\varrho_s} - 1 \right) \, ds \right), \tag{A.26}$$

we find

$$\Lambda_t = \exp\left( \int_0^t \frac{\partial \phi(C_s^\star, J_s^\star; \overline{C}_s)}{\partial J_s^\star} + \delta + \lambda\left( e^{(1-\gamma)\varrho_s} - 1 \right) \, ds \right) A_t. \tag{A.27}$$

To complete the proof, we show that

$$\mathrm{E}\left[ (R_t)^{1-\gamma} | \mathcal{F}_t \right] = \exp\left( \int_0^t \delta(1-\gamma) + \lambda\left( e^{(1-\gamma)\varrho_s} - 1 \right) \, ds \right). \tag{A.28}$$

Fix the path of $\varrho_s$ and consider only the uncertainty associated with Poisson process $N$. Define

$$M_t = \exp\left( \int_0^t \varrho_s (1-\gamma) \, dN_s - \int_0^t \lambda \left( e^{(1-\gamma)\varrho_s} - 1 \right) \, ds \right). \tag{A.29}$$

Then

$$dM_t = -M_t \lambda \left( e^{(1-\gamma)\varrho_s} - 1 \right) \, dt + \left( e^{(1-\gamma)\varrho_s} - 1 \right) M_t \, dN_t, \tag{A.30}$$

and therefore

$$\mathrm{E}\left[ dM_t | N_s, \, s \le t \right] = 0 \tag{A.31}$$

and $M_t$ is a martingale. So, $\mathrm{E}\left[ M_t | \mathcal{F}_t \right] = 1$. ■

Next, we consider some of the equilibrium relations in order to gain intuition for the overall structure of the solution. Define

$$\zeta_{j,t} = u_{j,t} \, e^{\xi_{\tau(j)}} \, K_{j,t}. \tag{A.32}$$

and

$$Z_t = \int_{J_t} e^{\xi_{s(j)}} \, u_{j,t} \, K_{j,t} \, dj. \tag{A.33}$$

The labor hiring decision is static. The firm managing project $j$ chooses $L_{jt}$ as the solution to

$$\Pi_{jt} = \sup_{L_{jt}} \left[ \zeta_{jt}^\phi \, (e^{x_t} L_{jt})^{1-\phi} - \mathrm{w}_t \, L_{j,t} \right] \tag{A.34}$$

The firm's choice

$$L_{j,t}^\star = \zeta_{j,t} \left( \frac{(1-\phi) \, e^{(1-\phi)x_t}}{\mathrm{w}_t} \right)^{\frac{1}{\phi}}. \tag{A.35}$$

After clearing the labor market, $\int_{\mathcal{J}_t} L_{j,t} dj = 1$, the equilibrium wage is given by

$$\mathrm{w}_t = (1-\phi) \, e^{(1-\phi) \, x_t} \, Z_t^\phi, \tag{A.36}$$

and the choice of labor allocated to project $j$ is

$$L^{\star}_{j,t} = \zeta_{j,t}\, Z_t^{-1}. \tag{A.37}$$

Aggregate output of all projects equals

$$Y_t = \int_{J_t} \zeta_{j,t}\, e^{(1-\phi)\,x_t}\, Z_t^{\phi-1}\, dj = e^{(1-\phi)\,x_t}\, Z_t^{\phi}. \tag{A.38}$$

Here, note that the definition of aggregate output, along with other aggregate quantities in the model, requires aggregating firm-level quantities. Aggregation over the continuum of firms should satisfy a law of large numbers, canceling out firm-specific randomness. Several aggregation procedures with such property have been developed in the literature, and the exact choice of the aggregation procedure is not important for our purposes. Specifically, we follow Uhlig (1996) and define the aggregate as the Pettis integral. We denote the aggregate over firms by an integral over the set of firms, $\in_0^1 \cdot df$. For alternative constructions that deliver the law of large numbers in the cross-section, see for instance Sun (2006) and Podczeck (2010), as well as the discussion of similar issues in Constantinides and Duffie (1996).

The project's flow profits are

$$\Pi_{j,t} = \sup_{L_{j,t}} \left[ \zeta_{jt}^{\phi}\; \left(e^{x_t} L_{j,t}\right)^{1-\phi} - \mathrm{w}_t\, l_{j,t} \right] = p_t\, \zeta_{jt} \tag{A.39}$$

where

$$p_t = \phi\, Y_t\, Z_t^{-1} \tag{A.40}$$

Because firms' investment decisions do not affect its own future investment opportunities, each investment maximizes the net present value of cash flows from the new project. Thus, the optimal investment in a new project $j$ at time $t$ is the solution to

$$\sup_{K_{j,t}} \mathrm{E}_t \left[ \int_t^{\infty} \frac{\Lambda_s}{\Lambda_t}\, \Pi_{j,s}\, ds \right] - K_{j,t}^{1/\alpha} = \sup_{K_{j,t}} \left[ P_t\, K_{j,t} e^{\xi_t} - K_{j,t}^{1/\alpha} \right], \tag{A.41}$$

where $P_t$ is the time-$t$ price of the asset with the cash flow stream $\exp(-\delta(s-t))p_s$:

$$P_t = \mathrm{E}_t \left[ \int_t^{\infty} \frac{\Lambda_s}{\Lambda_t} e^{-\delta(s-t)}\, p_s\, ds \right]. \tag{A.42}$$

The optimal scale of each new project is then given by

$$k_t^{\star} = \left(\alpha\, e^{\xi_t}\, P_t\right)^{\frac{\alpha}{1-\alpha}}. \tag{A.43}$$

Note that the solution does not depend on the identity of the firm, i.e., all firms, faced with an investment decision at time $t$, choose the same scale for the new projects. The optimal investment scale depends on the current market conditions, specifically, on the current level of the embodied productivity process $\xi_t$, and the current price level $P_t$.

We thus find that the aggregate stock of quality-adjusted installed capital in the intermediate good sector, defined by (7), evolves according to

$$dK_t = \left(-\delta\, K_t + \lambda\, e^{\xi_t}\, k_t^{\star}\right)\, dt = \left(-\delta\, K_t + \lambda\, e^{\xi_t}\, \left(\alpha\, e^{\xi_t}\, P_t\right)^{\frac{\alpha}{1-\alpha}}\right)\, dt. \tag{A.44}$$

An important aspect of (A.44) is that the growth rate of the capital stock $K_t$ depends only on its current level, the productivity level $\xi_t$, and the price process $P_t$. Furthermore, as we show below, we can clear markets with the price process $P_t$ expressed as a function of the state vector $X_t = (x_t, \xi_t, K_t)$. Thus, $X_t$ follows a Markov process in equilibrium.

We express equilibrium processes for aggregate quantities and prices as functions of $X_t$. For instance, the fact that investment decisions are independent of $u$ implies that $Z_t = K_t$. Aggregate investment $I_t$ is given by

$$I_t = \lambda \left(k_t^{\star}\right)^{1/\alpha}, \tag{A.45}$$

The aggregate consumption process satisfies

$$C_t = Y_t - I_t = K_t^{\phi} e^{(1-\phi)x_t} - \lambda \left(k_t^{\star}\right)^{1/\alpha}. \tag{A.46}$$

Prices of long-lived financial assets, such as the aggregate stock market, depend on the behavior of the stochastic discount factor. In equilibrium, the SDF is determined jointly with the value function of the households, as shown in Lemma 2. Below we fully characterize the equilibrium dynamics and express $\Lambda_t$ as a function of $X_t$.

Define the two variables

$$\chi_t = \frac{1-\phi}{1-\alpha\,\phi}\, x_t + \frac{\phi}{1-\alpha\,\phi}\xi_t. \tag{A.47}$$

and

$$\omega_t = \Big(\xi_t + \alpha\,\chi_t - \log K_t\Big) \tag{A.48}$$

$\omega_t$ and $\chi_t$ are linear functions of the state vector $X_t$. In Lemma 3 below, we characterize the SDF and aggregate equilibrium quantities as functions of $\omega_t$ and $\chi_t$.

In the formulation of the lemma, we characterize the value function of a household, as well as prices of financial assets, such as $P_t$ in (A.42), using differential equations. Verification results, such as (Duffie and Lions, 1992, Sec. 4), show that a classical solution to the corresponding differential equation, subject to the suitable growth and integrability constraints, characterizes the value function. Similarly, the Feynman-Kac Theorem (Karatzas and Shreve, 1991, e.g, Theorem 7.6) provides an analogous result for the prices of various financial assets. Because we solve for equilibrium numerically, we cannot show that the classical solutions to our differential equations exist and satisfy the sufficient regularity conditions. With this caveat in mind, in the following lemma we characterize the equilibrium processes using the requisite differential equations.

**Lemma 3 (Equilibrium)** *Let the seven functions, $f(\omega)$, $s(\omega)$, $\kappa(\omega)$, $i(\omega)$, $v(\omega)$, $g(\omega)$, $h(\omega)$ solve the following system of four ordinary differential equations,*

$$\begin{aligned} 0 =& A_1(\omega) f(\omega)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} + f(\omega)\left\{c_0^f - (1-\gamma)\, s(\omega) + \left[\left(1+\frac{\psi}{\mu_I}\, s(\omega)\right)^{1-\gamma} - 1\right]\mu_I\right\} \\ &+ f'(\omega)\left\{c_1^f - (1-\alpha\,\phi)\,\kappa(\omega)\right\} + f''(\omega)\, c_2^f, \end{aligned} \tag{A.49}$$

$$\begin{aligned} 0 =& \phi\, e^{-\phi\omega} B(\omega) + v'(\omega)\left\{c_1^f - (1-\alpha\,\phi)\,\kappa(\omega)\right\} + v''(\omega)\, c_2^f, \\ &+ v(\omega)\left\{c_0^f - \frac{\gamma-\theta^{-1}}{1-\gamma} A_1(\omega) f(\omega)^{\frac{1-\theta^{-1}}{\gamma-1}} + \mu_I\left(\left(1+\frac{\psi}{\mu_I}s(\omega)\right)^{1-\gamma} - 1\right) + \gamma\, s(\omega) - \kappa(\omega)\right\}, \end{aligned} \tag{A.50}$$

$$\begin{aligned} 0 =& (1-\eta)\,(1-\alpha)\, v(\omega)\,\kappa(\omega) + g'(\omega)\left\{c_1^f - (1-\alpha\,\phi)\,\kappa(\omega)\right\} + g''(\omega)\, c_2^f \\ &+ g(\omega)\left\{c_0^f - \frac{\gamma-\theta^{-1}}{1-\gamma} A_1(\omega) f(\omega)^{\frac{1-\theta^{-1}}{\gamma-1}} + \mu_I\left(\left(1+\frac{\psi}{\mu_I}s(\omega)\right)^{1-\gamma} - 1\right) + \gamma\, s(\omega)\right\}, \end{aligned} \tag{A.51}$$

$$\begin{aligned} 0 =& (1-\phi)\, e^{-\phi\omega} B(\omega) + h'(\omega)\left\{c_1^f - (1-\alpha\,\phi)\,\kappa(\omega)\right\} + h''(\omega)\, c_2^f \\ &+ h(\omega)\left\{c_0^f - \delta^h - \frac{\gamma-\theta^{-1}}{1-\gamma} A_1(\omega) f(\omega_t)^{\frac{1-\theta^{-1}}{\gamma-1}} + \mu_I\left(\left(1+\frac{\psi}{\mu_I}s(\omega_t)\right)^{1-\gamma} - 1\right) + \gamma\, s(\omega)\right\}, \end{aligned} \tag{A.52}$$

*and three algebraic equations,*

$$s(\omega) = \frac{\eta\,(1-\alpha)\,v(\omega)\,\kappa(\omega)}{v(\omega)+g(\omega)+\psi\,h(\omega)}, \tag{A.53}$$

$$\kappa(\omega) = \lambda^{1-\alpha}\,e^{(1-\alpha\phi)\,\omega}[i(\omega)]^{\alpha}, \tag{A.54}$$

$$\left(\frac{i(\omega_t)}{\lambda}\right)^{1-\alpha} = \alpha e^{(1-\alpha\phi+\phi(1-\theta_1^{-1}))\omega}\,v(\omega)\,f(\omega)^{\frac{\gamma-\theta^{-1}}{1-\gamma}}\;[(1-i(\omega))]^{\theta_1^{-1}}\left(1-\frac{(1-\psi)(1-\phi)}{1-i(\omega)}\right)^{1/\theta}. \tag{A.55}$$

*The constants $c_0^f$, $c_1^f$, $c_2^f$ and $\phi^d$ are*

$$c_0^f = \Bigg\{\delta^h(1-\gamma) - \frac{\rho(1-\gamma)}{1-\theta^{-1}} + (1-\gamma_1)(1-\phi)\mu_x + \frac{1}{2}(1-\phi)^2\,\sigma_x^2\,(1-\gamma_1)^2 + \frac{1}{2}\left(\frac{\phi(1-\gamma_1)}{1-\alpha\,\phi}\right)^2\left(\sigma_\xi^2+\alpha^2(1-\phi)^2\sigma_x^2\right)$$
$$+\frac{\phi(1-\gamma_1)}{1-\alpha\,\phi}\left(\mu_\xi+\alpha\,(1-\phi)\,\mu_x+(1-\gamma_1)\,\alpha(1-\phi)^2\,\sigma_x^2\right)\Bigg\}, \tag{A.56}$$

$$c_1^f = \left\{\mu_\xi+\alpha\,(1-\phi)\,\mu_x+(1-\alpha\,\phi)\delta+(1-\gamma_1)\,\alpha(1-\phi)^2\,\sigma_x^2+\frac{\phi(1-\gamma_1)}{1-\alpha\,\phi}\left(\sigma_\xi^2+\alpha^2(1-\phi)^2\sigma_x^2\right)\right\}, \tag{A.57}$$

$$c_2^f = \frac{1}{2}\left(\sigma_\xi^2+\alpha^2(1-\phi)^2\sigma_x^2\right) \tag{A.58}$$

*and the functions $A_1(\omega)$ and $B(\omega)$ are defined as*

$$A_1(\omega) = \frac{\rho(1-\gamma)}{1-\theta^{-1}}\;[(1-i(\omega))]^{1-\theta_1^{-1}}\left(1-\frac{(1-\psi)(1-\phi)}{1-i(\omega)}\right)^{1-\theta^{-1}}\;e^{-\phi(1-\theta_1^{-1})\,\omega}, \tag{A.59}$$

$$B(\omega) = [(1-i(\omega))]^{-\theta_1^{-1}}\left(1-\frac{(1-\psi)(1-\phi)}{1-i(\omega)}\right)^{-1/\theta}\;f(\omega)^{\frac{\gamma-\theta^{-1}}{\gamma-1}}\;e^{\phi\,\theta_1^{-1}\,\omega}. \tag{A.60}$$

*Then we can construct price processes and individual policies that satisfy the definition 1, so that the value function of a shareholder household $n$ with relative wealth $W_n/W = w_n$ is given by*

$$J(w_n,\chi,\omega) = \frac{1}{1-\gamma}w_n^{(1-\gamma)}e^{(1-\gamma_1)\chi}\,f(\omega), \tag{A.61}$$

*where $\gamma_1 = 1-(1-\gamma)(1-h)$, and $K_t$ follows*

$$\frac{dK_t}{K_t} = -\delta\,dt + \kappa(\omega_t)\,dt. \tag{A.62}$$

**Proof.** We start with a conjecture, which we confirm below, that the equilibrium price process $P_t$ satisfies

$$P_t = K_t^{-1}e^{\chi_t}\,v(\omega_t)B(\omega_t)^{-1}, \tag{A.63}$$

the equilibrium aggregate value of assets in place is

$$V_t = e^{\chi_t}\,v(\omega_t)B(\omega_t)^{-1}, \tag{A.64}$$

the value of growth opportunities for the average firm ($\lambda_f = \lambda$) is

$$G_t = e^{\chi_t}g(\omega_t)\,(B(\omega_t))^{-1}, \tag{A.65}$$

and the aggregate value of human capital is

$$H_t = e^{\chi_t}h(\omega_t)\,(B(\omega_t))^{-1}. \tag{A.66}$$

We then characterize the equilibrium SDF and the optimal policies of the firms and households, and show that all markets clear and the above conjectures are consistent with the equilibrium processes for cash flows and the SDF.

We denote the time-$t$ net present value of the new projects (the maximum value in (A.41)) by $\nu_t$. According to equations (A.43, A.45) above,

$$\nu_t = \left(\alpha^{\alpha/(1-\alpha)} - \alpha^{1/(1-\alpha)}\right) \left(P_t e^{\xi_t}\right)^{1/(1-\alpha)} = \alpha^{\frac{1}{1-\alpha}} \left(\frac{1}{\alpha} - 1\right) \left(e^{\xi_t} P_t)\right)^{\frac{1}{1-\alpha}}. \tag{A.67}$$

The aggregate investment process, according to (A.43, A.45), is given by

$$I_t = \lambda \left(\alpha \, e^{\xi_t} \, P_t\right)^{\frac{1}{1-\alpha}} = \lambda \, \frac{\alpha}{1-\alpha} \nu_t. \tag{A.68}$$

Using (A.68) and market clearing (A.45), $K_t$ follows

$$\frac{dK_t}{K_t} = \left(-\delta \, K_t + \lambda \, e^{\xi_t} \, \left(\alpha \, e^{\xi_t} \, P_t\right)^{\frac{\alpha}{1-\alpha}}\right) dt = -\delta \, dt + \kappa(\omega_t) \, dt,$$

where we have used (A.63), (A.55), and (A.60) for the last equality. The equilibrium dynamics of the aggregate quality-adjusted capital stock thus agrees with (A.62).

Next, we establish the dynamics of the SDF $\Lambda$. Consider the evolution of household's wealth share. All shareholders solve the same consumption-portfolio choice problem, different only in the level of household wealth, and households have homothetic preferences. Thus, the evolution of a shareholder household's wealth share (defined as the ratio of household wealth to the total wealth of all shareholders) is given by an equation similar to (29) but taking into account the presence of households that do not participate in financial markets:

$$\frac{dw_{n,t}}{w_{n,t}} = \delta^h \, dt - \frac{\lambda \eta \nu_t}{V_t + G_t + \psi \, H_t} \, dt + \psi \, \frac{\lambda \eta \nu_t}{V_t + G_t + \psi \, H_t} \, \mu_I^{-1} dN_{it}^I. \tag{A.69}$$

Equation (A.69) takes into account that the total measure of new blueprints that accrue to shareholders is equal to $\psi$. The benchmark model in the paper corresponds to the case where $\psi = 1$. Based on the asset prices in (A.64–A.66) and (A.53), we find that

$$\frac{\lambda \eta \nu_t}{V_t + G_t + \psi \, H_t} = s(\omega_t), \tag{A.70}$$

and therefore wealth shares follow

$$\frac{dw_{n,t}}{w_{n,t}} = (\delta^h - s(\omega_t)) \, dt + \psi \mu_I^{-1} s(\omega_t) \, dN_{it}^I. \tag{A.71}$$

Next, we derive the the consumption process of households from the market clearing conditions. Then, optimality of this process follows from asset prices being consistent the SDF implied by this process. Based on the aggregate consumption process (A.46) and equilibrium wage process (A.36), along with the definition of $\omega$ and $\chi$, the consumption of shareholders as a group is

$$C_t^S \equiv \int_{i \in \mathcal{S}_t} C_{i,t} \, di = C_t - (1-\psi) \, \mathrm{w}_t = e^{\chi_t} e^{-\phi \, \omega_t} \left(1 - i(\omega_t) - (1-\psi)(1-\phi)\right). \tag{A.72}$$

Preference homotheticity implies that the consumption of a each shareholder household is proportional to its wealth share, so

$$C_{n,t} = w_{n,t} \, C_t^S. \tag{A.73}$$

Optimality of household consumption and portfolio choices implies that the SDF in Lemma 2 above, defined using a shareholder households' consumption process, is a valid equilibrium SDF in this economy. In particular, we obtain

$$\Lambda_t = \Lambda_0 \, \exp\left(\int_0^t \delta^h + \frac{\partial \phi(C_{n,s}, J_{n,s}; \overline{C}_s)}{\partial J_{n,s}} + \mu_I \left(\left(1 + \frac{\psi}{\mu_I} s(\omega_s)\right)^{1-\gamma} - 1\right) ds\right) A_t, \tag{A.74}$$

where $\Lambda_0$ is a constant and

$$A_t \equiv \frac{\partial \phi(C_{n,t}, J_{n,t}; \overline{C}_t)}{\partial C_{n,t}} \exp\left( \int_0^t \gamma \log\left(1 + \frac{\psi}{\mu_I} s(\omega_s)\right) dN_{n,s} \right). \tag{A.75}$$

In the formulation of Lemma 2, we set the gain from innovation to its equilibrium value,

$$\varrho_t \equiv \log\left(1 + \frac{\psi}{\mu_I} s(\omega_t)\right). \tag{A.76}$$

Note also that the SDF is defined only up to a multiplicative positive constant.

Equation (A.49) is the Hamilton-Jacobi-Bellman equation for the value function, if the latter is expressed in the form of (A.61). Using this expression for the value function and the process for shareholder consumption (A.73), we find that

$$A_t = A_0\, e^{-\gamma_1 \chi_t} \hat{w}_t^{-\gamma}\, B(\omega_t), \tag{A.77}$$

where $A_0$ is a constant and $B(\omega)$ satisfies (A.60) and

$$d\hat{w}_t = \left(\delta^h - s(\omega_t)\right) dt. \tag{A.78}$$

The process $\hat{w}_t$ is the same as the process for the household wealth shares, conditional on no innovation shocks, i.e., setting $N_{n,t}$ to be constant. Further,

$$\frac{\partial \phi(C_t^\star, J_t^\star; \overline{C}_t)}{\partial J_t^\star} = -\frac{\rho}{1-\theta^{-1}} \left( (\gamma - \theta^{-1})\, l(\omega)^{1-\theta^{-1}} \, [f(\omega)]^{\frac{1-\theta^{-1}}{\gamma-1}} + (1-\gamma) \right) \tag{A.79}$$

where

$$l(\omega) \equiv (1 - i(\omega) - (1-\psi)(1-\phi)) \, ((1 - i(\omega)))^{-h}. \tag{A.80}$$

We are now in a position to complete the proof by verify that the conjectured price processes in (A.63–A.66) are consistent with the equilibrium SDF above. Note that equations (A.50–A.52) are the valuation equations for $V_t$, $G_t$, and $H_t$ respectively, based on the Feynman-Kac Theorem (Karatzas and Shreve, 1991, e.g, Theorem 7.6), given the equilibrium SDF above and the conjectured expressions in (A.64–A.66). By definition of $V_t$ and $K_t$, $P_t = K_t^{-1} V_t$, which establishes the consistency of A.63. ■

**Proof of Proposition 1.** Proposition follows directly from (A.74) and (A.77) in the proof of the Lemma 3 above. The process $b(\omega_t)$ is given by

$$b(\omega) = (1-\gamma)\delta^h - \rho\,\kappa - \rho\,(1-\kappa)\,(1 - i(\omega_t))^{(1-\theta_1^{-1})}\,(f(\omega))^{-\kappa^{-1}} + \gamma\, s(\omega) + \mu_I \left( \left(1 + \frac{\psi}{\mu_I} s(\omega)\right)^{1-\gamma} - 1 \right). \tag{A.81}$$

where the functions $i(\omega)$, $f(\omega)$, and $s(\omega)$ are defined in Lemma 3 above. ■

**Lemma 4 (Market value of a firm)** *The market value of a firm equals*

$$S_{f,t} = e^{\chi_t} \left[ v(\omega_t)\, \frac{z_{f,t}}{\bar{u}_{ft}} \left(1 + \frac{v_1(\omega_t)}{v(\omega_t)}\, (\bar{u}_{f,t} - 1)\right) + g(\omega_t) + \left(\frac{\lambda_{f,t}}{\lambda} - 1\right) g_1(\omega_t) \right] (B(\omega_t))^{-1} \tag{A.82}$$

*where $v(\omega)$ and $g(\omega)$ are defined above and the functions $v_1$ and $g_1$ solve the ODEs*

$$\begin{aligned} 0 = {} & \phi\, e^{-\phi\omega}\, B(\omega) + v_1'(\omega) \left\{ c_1^f - (1 - \alpha\,\phi)\,\kappa(\omega) \right\} + v_1''(\omega)\, c_2^f \\ & + v_1(\omega) \left\{ c_0^f - \kappa_u - \frac{\gamma - \theta^{-1}}{1-\gamma} A_1(\omega) f(\omega)^{\frac{1-\theta^{-1}}{\gamma-1}} + \mu_I \left( \left(1 + \frac{\psi}{\mu_I} s(\omega)\right)^{1-\gamma} - 1 \right) + \gamma\, s(\omega) - \kappa(\omega) \right\} \end{aligned} \tag{A.83}$$

$$
\begin{aligned}
0 = &\ (1-\eta)\,(1-\alpha)\,v(\omega)\,\kappa(\omega) + g_1'(\omega)\left\{c_1^f - (1-\alpha\,\phi)\,\kappa(\omega)\right\} + g_1''(\omega)\,c_2^f \\
&+ g_1(\omega)\left\{c_0^f - \mu_L - \mu_H - \frac{\gamma-\theta^{-1}}{1-\gamma}A_1(\omega)f(\omega)^{\frac{1-\theta^{-1}}{\gamma-1}} + \mu_I\left(\left(1+\frac{\psi}{\mu_I}s(\omega)\right)^{1-\gamma} - 1\right) + \gamma\,s(\omega)\right\} \qquad \text{(A.84)}
\end{aligned}
$$

**Proof.** The proof follows closely the derivations of equations (A.63) and (A.65) above. We have that the value of assets in place for an existing firm with capital stock $K_{f,t}$ and profitability $Z_{f,t}$ are given by

$$
VAP_{f,t} = P(X_t)K_{f,t} + P_1(X_t)\,(Z_{f,t} - K_{f,t})\,, \qquad \text{(A.85)}
$$

where

$$
P_1(X_t) \equiv \mathrm{E}_t\left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t}e^{-(\delta+\kappa_u)(s-t)}\,p_s\,ds\right] = K_t^{-1}e^{\chi_t}\,v_1(\omega_t)B(\omega_t)^{-1} \qquad \text{(A.86)}
$$

where $v_1(\omega)$ satisfies the ODE (A.94). As above, we have used the SDF (A.74), equation (A.40), the definition of $\chi$ and $\omega$ and the Feynman-Kac theorem. Similarly, the present value of growth opportunities for a firm equals

$$
\begin{aligned}
PVGO_{f,t} &\equiv (1-\eta)\,\mathrm{E}_t\left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t}\lambda_{f,s}\,\nu_s\,ds\right] \\
&= PVGO_t + \lambda\,(1-\eta)\,\left(\frac{\lambda_{f,t}}{\lambda} - 1\right)E_t\int_t^\infty \frac{\Lambda_s}{\Lambda_t}e^{(\mu_L+\mu_H)(s-t)}\,\nu_s\,ds \qquad \text{(A.87)} \\
&= e^{\chi_t}g(\omega_t)\,(B(\omega_t))^{-1} + e^{\chi_t}g_1(\omega_t)\,(B(\omega_t))^{-1} \qquad \text{(A.88)}
\end{aligned}
$$

where $g_1(\omega)$ satisfies the ODE (A.94). As above, we have used the SDF (A.74), the definition of $z$ and $\omega$ and the Feynman-Kac theorem , and the fact that

$$
\mathrm{E}[\lambda_{fs}|\lambda_{f,t}] = \lambda + \lambda\left(\frac{\lambda_{f,t}}{\lambda} - 1\right)e^{(\mu_L+\mu_H)(s-t)}. \qquad \text{(A.89)}
$$

■

**Lemma 5 (Bond prices)** *The price at time $t$ of a zero coupon bond of maturity $T$, is given by*

$$
P_b(\omega_t, T-t) = \frac{b(\omega_t, T-t)}{B(\omega_t)} \qquad \text{(A.90)}
$$

*where $b(\omega_t, T-t)$ solves the following PDE*

$$
\begin{aligned}
0 = &\ \frac{\partial}{\partial t}b(\omega,T-t) + \frac{\partial}{\partial\omega}b(\omega,T-t)\left\{c_1^f - (1-\alpha\,\phi)\,\kappa(\omega)\right\} + \frac{\partial^2}{\partial\omega^2}b(\omega,T-t)\,c_2^f \\
&+ b(\omega,T-t)\left\{c_0^b - \frac{\gamma-\theta^{-1}}{1-\gamma}A_1(\omega)f(\omega)^{\frac{1-\theta^{-1}}{\gamma-1}} + \mu_I\left(\left(1+\frac{\psi}{\mu_I}s(\omega)\right)^{1-\gamma} - 1\right) + \gamma\,s(\omega)\right\} \qquad \text{(A.91)}
\end{aligned}
$$

*where $c_1^f$ and $c_2^f$ are constants defined above,*

$$
\begin{aligned}
c_0^b = &\left\{\delta^h(1-\gamma) - \frac{\rho(1-\gamma)}{1-\theta^{-1}} - \gamma_1\,(1-\phi)\mu_x + \frac{1}{2}(1-\phi)^2\,\sigma_x^2\,\gamma_1^2 + \frac{1}{2}\left(\frac{\phi\,\gamma_1}{1-\alpha\,\phi}\right)^2(\sigma_\xi^2 + \alpha^2(1-\phi)^2\sigma_x^2)\right. \\
&\left. - \frac{\phi\,\gamma_1}{1-\alpha\,\phi}\left(\mu_\xi + \alpha\,(1-\phi)\,\mu_x - \gamma_1\,\alpha(1-\phi)^2\,\sigma_x^2\right)\right\}, \qquad \text{(A.92)}
\end{aligned}
$$

*along with the terminal condition $b(\omega, 0) = B(\omega)$.*

**Proof.** Proof follows from the definition of the zero coupon bond and the Feynman-Kac theorem. ■

**Lemma 6 (Dividend Strip prices)** *The price at time $t$ of a dividend strip of maturity $T$, is given by*

$$P_d(\omega_t, T-t) = e^{\chi_t} \frac{d(\omega_t, T-t)}{B(\omega_t)} \tag{A.93}$$

*where $d(\omega_t, T-t)$ solves the following PDE*

$$0 = \frac{\partial}{\partial t} d(\omega, T-t) + \frac{\partial}{\partial \omega} d(\omega, T-t) \left\{ c_1^f - (1-\alpha\,\phi)\,\kappa(\omega) \right\} + \frac{\partial^2}{\partial \omega^2} d(\omega, T-t)\, c_2^f$$
$$+ d(\omega, T-t) \left\{ c_0^f - \frac{\gamma - \theta^{-1}}{1-\gamma} A_1(\omega) f(\omega_t)^{\frac{1-\theta^{-1}}{\gamma-1}} + \mu_I \left( \left(1 + \frac{\psi}{\mu_I} s(\omega_t)\right)^{1-\gamma} - 1 \right) + \gamma\, s(\omega) \right\} \tag{A.94}$$

*where $c_1^f$ and $c_2^f$ are constants defined above, along with the terminal condition $d(\omega, 0) = B(\omega) e^{-\phi\omega} \left(1 - i(\omega) - \eta\, i(\omega) \frac{1-\alpha}{\alpha}\right)$.*

**Proof.** Proof follows from the definition of the dividend strip and the Feynman-Kac theorem. ■

# Appendix B: Data and Estimation Methodology

Here, we briefly discuss the construction of the estimation targets and our empirical methodology.

## B1. Construction of Estimation Targets

**Aggregate consumption**: We use the Barro and Ursua (2008) consumption data for the United States, which covers the 1834-2008 period. We compute the estimate of long-run risk using the estimator in Dew-Becker (2014). We thank Ian Dew-Becker for sharing his code.

**Volatility of shareholder consumption growth**: The volatility of shareholder consumption growth is from the unpublished working paper version of Malloy et al. (2009) and includes their adjustment for measurement error. Data period is 1980-2002. We are grateful to Annette Vissing-Jorgensen for suggesting this.

**Aggregate investment and output**: Investment is non-residential private domestic investment. Output is gross domestic product. Both series are deflated by population and the CPI. Data on the CPI are from the BLS. Population is from the Census Bureau. Data range is 1927-2010.

**Firm Investment rate, Tobin's Q and profitability**: Firm investment is defined as the change in log gross PPE. Tobin's Q equals the market value of equity (CRSP December market cap) plus book value of preferred shares plus long term debt minus inventories and deferred taxes over book assets. Firm profitability equals gross profitability (sales minus costs of goods sold) scaled by book capital (PPE). When computing correlation coefficients, we winsorize the data by year at the 1% level to minimize the effect of outliers. We simulate the model at a weekly frequency, $dt = 1/50$ and time aggregate the data at the annual level. In the model, we construct Tobin's Q as the ratio of the market value of the firm divided by the replacement cost of capital using end of year values. Replacement cost is defined as the current capital stock adjusted for quality, $\hat{K}_{ft} = e^{-\xi_t} K_{ft}$ . The investment rate is computed as $I_{ft}/\hat{K}_{ft-1}$, where $I_{ft}$ is the sum of firm investment expenditures in year $t$ and $\hat{K}_{f,t-1}$ is capital at the end of year $t-1$. Similarly, we compute firm profitability as $p_{Z,t} Z_{f,t}/\hat{K}_{f,t-1}$, where $p_{Z,t} Z_{f,t}$ is the accumulated profits in year $t$ and $\hat{K}_{f,t-1}$ is capital at the end of year $t-1$. Data range is 1950-2010. Following standard practice, we exclude financials and utilities.

**Market portfolio and risk-free rate moments:** We use the reported estimate from the long sample of Barro and Ursua (2008) for the United States and cover the 1870-2008 sample (see Table 5 in their paper). In the data, the risk-free rate is the return on treasury bills of maturity of three months or less. The reported volatility of the interest rate in Barro and Ursua (2008), which equals 4.8%, is the volatility of the realized rate. Hence it is contaminated with unexpected inflation. We therefore target a risk-free rate volatility of 0.7% based on the standard deviation of the annualized yield of a 5-year Treasury Inflation Protected Security (the shortest maturity available) in the 2003-2010 sample. In the model, $r_f$ is the instantaneous short rate; and $R_M$ is the return on the value-weighted market portfolio.

**Value factor, I/K, and E/P moments:** We use the 10 value-weighted portfolios sorted on each of these characteristics from Kenneth French's Data Library. The value factor is the 10 minus 1 portfolio of firms sorted on book-to-market. The I/K and E/P spread are defined analogously. Data period for the value premium excludes data prior to the formation of the SEC (1936 to 2010); data period for the investment strategy (I/K) is 1964-2010; data period for the earnings-to-price strategy is 1952-2010. Standard errors for the empirical moments are included in parentheses. Standard errors for $R^2$ are computed using the delta method.

**Consumption share of stockholders:** Consumption share of stock holders is from Table 2 of Guvenen (2006). This number is also consistent with Heaton and Lucas (2000): using their data on Table AII we obtain an income share for stockholders of approximately 43%.

**Consumption growth of shareholders:** We use the series constructed in Malloy et al. (2009), which covers the 1980-2002 period. We follow Jagannathan and Wang (2007) and construct annual consumption growth rates by using end-of-period consumption. In particular, we focus on the sample of households that are interviewed in December of every year, and use the average 8 quarter consumption growth rate of non-stockholders and stockholders, defined as in Malloy et al. (2009).

**Value of new blueprints:** We create $\hat{\omega}$ using a non-parametric variant of the Kogan et al. (2016) procedure. First, we create idiosyncratic stock returns for firm $f$ around the day that patent $j$ is granted to equal the 3-day return of the firm minus the return on the CRSP value-weighted index around the same window,

$$r^e_{fj} = r_{fj} - r_{mj}. \tag{B.1}$$

Patents are issued every Tuesday. Hence, $r_{fj}$ are the accumulated return over Tuesday, Wednesday and Thursday following the patent issue. Second, we compute an estimate of the value of patent $j$ as the firm's market capitalization on the day prior the patent announcement $V_{fj}$ times the idiosyncratic return to the firm truncated at zero,

$$\hat{\nu}_j = \frac{1}{N_j} \max(r^e_{fj}, 0)\, V_{fj}. \tag{B.2}$$

If multiple patents were granted in the same day to the same firm, we divide by the number of patents $N$. Relative to Kogan et al. (2016), we replace the filtered value of the patent $E[x_{fd}|r_{fd}]$ with $\max(r^e_{fj}, 0)$. Our construction is an approximation to the measure in Kogan et al. (2016) that can be easily implemented in simulated data without the additional estimation of parameters. See the earlier working paper version of Kogan et al. (2016) for a comparison between the two measures. We follow a similar approach when constructing $\hat{\nu}_j$ in simulated data. We compute the excess return of the firm as in equation (B.1) around the times that the firm acquires a new project, and then construct $\hat{\nu}_j$ as in equation (B.2).

Given the estimated patent values $\hat{\nu}_j$, we construct an estimate of the aggregate value of new blueprints at time $t$ as

$$\hat{\nu}_t = \sum_{j \in P_t} \hat{\nu}_j, \tag{B.3}$$

where where $P_t$ denotes the set of patents granted to firms in our sample in year $t$. Similarly, we measure the total dollar value of innovation produced by a given firm $f$ in year $t$ by summing the estimated values for all patents $\nu_j$ that were granted to the firm during that year $t$,

$$\hat{\nu}_{f,t} = \sum_{j \in P_{f,t}} \nu_j, \tag{B.4}$$

where now $P_{f,t}$ denotes the set of patents issued to firm $f$ in year $t$. In the context of our model, (39) can be interpreted as the sum of the net present values of all projects acquired by firm $f$ in the interval $s \in [t-1, t]$,

$$\nu_{f,t} = \int_{t-1}^{t} \nu_s \, dN_{f,s}. \tag{B.5}$$

To avoid scale effects, we normalize (B.3), and (B.4) by market capitalization in both the data and in the model.

The dollar reaction around the issue date is an understatement of the dollar value of a patent. The market value of the firm is expected to change by an amount equal to the NPV of the patent times the probability that the patent application is unsuccessful. This probability is not small; in the data less than half of the patent applications are successful. Consequently, when comparing the dispersion in firm innovation between the data and the model, we scale the firm innovation measures $\nu_{f,t}/M_{f,t}$ in both the data and the model such that they have the same mean (equal to 1) conditional on non-zero values.

**Inequality Data** Data sources are the Consumption Expenditure Survey (CEX), the Survey of Consumer Finances (SCF) and the data of Piketty and Saez (2003) and Saez and Zucman (2016). The top income and wealth shares are from Piketty and Saez (2003) and Saez and Zucman (2016). Top consumption shares are from CEX (1982-2010). Top shares are calculated relative to all households. The top percentile shares of income (total income) and wealth (net worth) are from the SCF (1989-2013); we report percentile ratios of the stock ownership sample (equity=1 in the SCF summary extracts) and after obtaining residuals from cohort and year dummies and cubic age effects. We use a similar procedure for the CEX data. The corresponding estimates in the model are computed from a long simulation of 10m households for 10,000 years. Percentile ratios are computed among the subset that participates in the stock market. In the model, income equals wages, payout and proceeds from innovations. In the data, we use the total income variable from the SCF, which includes salary, proceeds from owning a business and capital income. In the Piketty and Saez (2003) data, we use income shares inclusive of capital gains.

## B2. Estimation Methodology

We estimate the parameter vector $p$ using the simulated minimum distance method (Ingram and Lee, 1991). Denote by $X$ the vector of target statistics in the data and by $\mathcal{X}(p)$ the corresponding statistics generated by the model given parameters $p$, computed as

$$\mathcal{X}(p) = \frac{1}{S}\sum_{i=1}^{S}\hat{X}_i(p), \tag{B.6}$$

where $\hat{X}_i(p)$ is the $21 \times 1$ vector of statistics computed in one simulation of the model. We simulate the model at a weekly frequency, and time-aggregate the data to form annual observations. Each simulation has 1,000 firms. For each simulation $i$ we first simulate 100 years of data as 'burn-in' to remove the dependence on initial values. We then use the remaining part of that sample, which is chosen to match the longest sample over which the target statistics are computed. Each of these statistic is computed using the same part of the sample as its empirical counterpart. In each iteration we simulate $S = 100$ samples, and simulate pseudo-random variables using the same seed in each iteration.

Our estimate of the parameter vector is given by

$$\hat{p} = \arg\min_{p\in\mathcal{P}}(X - \mathcal{X}(p))'W(X - \mathcal{X}(p)), \tag{B.7}$$

where $W = diag(XX')^{-1}$ is our choice of weighting matrix that ensures that the estimation method penalizes proportional deviations of the model statistics from their empirical counterparts.

We compute standard errors for the vector of parameter estimates $\hat{p}$ as

$$V(\hat{p}) = \left(1+\frac{1}{S}\right)\left(\frac{\partial}{\partial p}\mathcal{X}(p)'W\frac{\partial}{\partial p}\mathcal{X}(p)\right)^{-1}\frac{\partial}{\partial p}\mathcal{X}(p)'W'V_X(\hat{p})W\frac{\partial}{\partial p}\mathcal{X}(p)\left(\frac{\partial}{\partial p}\mathcal{X}(p)'W\frac{\partial}{\partial p}\mathcal{X}(p)\right)^{-1}, \tag{B.8}$$

where

$$V_X(\hat{p}) = \frac{1}{S}\sum_{i=1}^{S}(\hat{X}_i(\hat{p}) - \mathcal{X}(\hat{p}))(\hat{X}_i(\hat{p}) - \mathcal{X}(\hat{p}))' \tag{B.9}$$

is the estimate of the sampling variation of the statistics in $X$ computed across simulations.

The standard errors in (B.8) are computed using the sampling variation of the target statistics across simulations (B.9). We use (B.9), rather than the sample covariance matrix, because the statistics that we target are obtained from different datasets (e.g. cross-sectional versus time-series), and we often do not have access to the underlying data. Since not all of these statistics are moments, computing the covariance matrix of these estimates would be challenging even with access to the underlying data. Under the null of the model, the estimate in (B.9) would coincide with the empirical estimate. If the model is misspecified, (B.9) does not need to be a good estimate of the true covariance matrix of $X$. Partly for these reasons, we specify the weighting matrix as $W = diag(XX')^{-1}$, rather than scaling by the inverse of the sample covariance matrix of $X$. In principle we could weigh moments by the inverse of (B.9). However, doing so forces the model to match moments that are precisely estimated but economically less interesting, such as the dispersion in firm profitability or Tobin's $Q$.

Solving each iteration of the model is computationally costly, and thus computing the minimum (B.7) using standard methods is infeasible. We therefore use the Radial Basis Function (RBF) algorithm in Björkman and Holmström (2000). The Björkman and Holmström (2000) algorithm first fits a response surface to data by evaluating the objective function at a few points. Then, it searches for a minimum by balancing between local and global search in an iterative fashion. We use a commercial implementation of the RBF algorithm that is available through the TOMLAB optimization package.

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