# INTERNATIONAL RESERVES AND ROLLOVER RISK

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**ABSTRACT**

We study the optimal accumulation of international reserves in a quantitative model of sovereign default with long-term debt and a risk-free asset. Keeping higher levels of reserves provides a hedge against rollover risk, but this is costly because using reserves to pay down debt allows the government to reduce sovereign spreads. Our model, parameterized to mimic salient features of a typical emerging economy, can account for significant holdings of international reserves, and the larger accumulation of both debt and reserves in periods of low spreads and high income. We also show that income windfalls, improved policy frameworks, and an increase in the importance of rollover risk imply increases in the optimal holdings of reserves that are consistent with the upward trend in reserves in emerging economies. It is essential for our results that debt maturity exceeds one period.

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# 1  Introduction

The large accumulation of international reserves in emerging economies has reinvigorated debates about the adequate levels of reserves.[1] A popular view suggests that emerging markets should build a large stock of reserves as a buffer against disruptions in international financial markets.[2] On the other hand, resources used to accumulate reserves could be used instead to lower debt levels, which could in turn help reduce sovereign risk and spreads. What constitutes an adequate level of reserves? Specifically, what is the optimal level of reserves for an indebted government facing default risk and the possibility of surges in borrowing costs? Furthermore, what explains the secular increase in reserves?

This paper studies the optimal accumulation of international reserves in a canonical model of sovereign default (Eaton and Gersovitz, 1981; Aguiar and Gopinath, 2006; Arellano, 2008) with long-term debt, a risk-free asset (i.e., reserves), and risk-averse foreign lenders.[3] In this setup, shocks to domestic fundamentals or to the risk premia required by bondholders exposes the government to rollover risk, that is, the risk of having to roll over its debt obligations at times when its borrowing opportunities deteriorate. We show that keeping higher levels of reserves provides a hedge against rollover risk, but this is costly because using reserves to pay down debt allows the government to reduce spreads. This is the key trade-off the government faces in our model.

In our quantitative investigation, we find that it is optimal for an indebted government paying a significant sovereign spread to hold large amounts of reserves. We calibrate the model by targeting salient features of a typical emerging economy paying a significant spread. Model simulations generate an optimal level of reserves of 6 percent of annual income on average, and reserve holdings can reach values as high as 40 percent in some simulation periods. We find that in the simulations, the government accumulates both reserves and debt in periods of low spread and high income (i.e., in good times), and conversely, the government reduces debt and reserves in periods of high spread and low income (i.e., in bad times). We show that this pattern is consistent with the behavior of debt and reserves in a majority of emerging economies.

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[1]International reserves reached $8 trillion in emerging markets in 2015. Section 2 presents empirical regularities of international reserves for emerging economies.

[2]Building a buffer for liquidity needs is the most frequently cited reason for reserve accumulation in the IMF Survey of Reserve Managers (80 percent of respondents; IMF, 2011). There is also extensive empirical evidence that supports the precautionary role of reserves (e.g., Aizenman and Lee, 2007, Calvo et al., 2012, Bussiere et al., 2013, Dominguez et al., 2012, Frankel and Saravelos, 2012, and Gourinchas and Obstfeld, 2012).

[3]In important early work, Alfaro and Kanczuk (2009) studied optimal reserve accumulation in a sovereign default model with *one-period debt*. They found that the optimal policy is not to hold reserves. As we explain below, the insurance motive for reserves that we study in this paper does not arise in their study.

We also show that the secular increase in reserves is consistent with three recent developments in emerging economies: (i) exceptional increases in real income arising, to a large extent, from the boom in commodity prices; (ii) improvements in policy frameworks that reduced political myopia; and (iii) increases in the severity of global disruptions to international financial markets. These developments also imply lower optimal debt levels, which are consistent with the data.

**Mechanism.** In our model, an indebted government facing default risk finds it optimal to accumulate reserves. When the government faces a negative shock, this leads to increases in spreads, which makes it costly for the government to roll over its debt. Having reserves in those states has the benefit of reducing borrowing needs and mitigating the drop in consumption, but so does having lower debt levels. Why, then, would the government choose to issue debt to finance the accumulation of reserves? Consider the next-period payoffs from issuing debt today to accumulate reserves. These are given by the stock of the reserves accumulated minus the next-period value of the debt issued. Because with long-term debt, the next-period value of the debt issued increases with the bond price, issuing debt to buy reserves provides higher payoffs in states in which the bond price is low. Issuing debt to buy reserves therefore allows the government to transfer resources from next-period states with low borrowing costs (high bond prices) to next-period states with high borrowing costs (low bond prices). Since consumption is lower in periods with high borrowing cost, such operation provides a hedging value to the government.[4] In a nutshell, issuing long-term debt to accumulate reserves provides insurance against rollover risk.

Issuing debt to accumulate reserves provides insurance against rollover risk only if debt maturity exceeds one period. With *one-period* debt, issuing one extra bond implies that the government has to allocate one extra unit of resources to service that bond in the next period, if it decides to repay. The benefit of accumulating one extra unit of reserves is that there is one extra unit of resources available in the next period, regardless of the cost of borrowing and the repayment decision. Thus, by issuing one-period debt to accumulate reserves, the government can only transfer resources from next-period repayment states to next-period default states (in contrast, issuing long-term debt allows the government to transfer resources *across* repayment states with different borrowing costs). We show that this channel is not quantitatively important. When we assume one-period bonds and recalibrate the model to match the same moments as in our benchmark calibration, we find that the level of reserves is close to zero, in line with

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[4]See equation (10) and the discussion therein for an analytical underpinning of the hedging benefit of accumulating debt and reserves and the role of debt maturity.

the results obtained by Alfaro and Kanczuk (2009).

Accumulating reserves is costly. The government pays a higher spread when choosing portfolios with higher debt and reserves positions. Although a higher spread reflects that the government defaults in more future states, this benefit from lower repayments is exactly offset by the extra default cost the government incurs. As a result, the higher spread implied by higher debt and reserves positions represents a cost for the government. When solving the optimal portfolio problem, the government trades off this cost against the insurance benefits of reserves. We illustrate this trade-off by tracing how the next-period sensitivity of consumption and the current level of spreads vary with the current portfolio decision.

We find that it is relatively more costly to accumulate reserves in bad times, when lenders are more risk averse or income is lower. In those states, the borrowing cost is more sensitive to increases in gross positions. Consequently, the government chooses lower debt and reserves in bad times. Conversely, in good times, the government buys more insurance against adverse future shocks by choosing portfolios with higher gross positions. This is consistent with the cyclical behavior of debt and reserves in the data, presented in Section 2.

**Related Literature.** We build on the quantitative sovereign default literature that follows Aguiar and Gopinath (2006) and Arellano (2008).[5] They show that predictions of the sovereign default model are consistent with several features of emerging markets, including countercyclical spreads and procyclical borrowing. We present a model with long-term debt and shocks to the lenders' risk aversion and we allow governments to simultaneously accumulate assets and liabilities. We show that the model's key predictions are still consistent with features of emerging markets. Our model is also able to account for the accumulation of reserves by indebted governments.

Alfaro and Kanczuk (2009) study the joint accumulation of international reserves and sovereign defaultable debt using a benchmark model with *one-period bonds*. Adding a risk-free asset to the sovereign debt model with one-period bonds increases spanning because by accumulating assets and a defaultable bond, the government shifts resources from repayment states to default states. They find, however, that the sovereign should not accumulate reserves at all.[6] In contrast, we study reserves as a hedge against rollover risk, an insurance role that, as we show, arises only when maturity exceeds one period. In our model, by accumulating assets and long-duration debt,

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[5]Aguiar and Amador (2013) and Aguiar et al. (2016b) analyze, respectively, theoretical and quantitative issues in this literature.

[6] In this sense, not allowing the government to accumulate reserves, as assumed in earlier studies mentioned above, is innocuous when the maturity of debt is one period.

the government transfers resources *across* repayment states. In particular, it transfers resources from states with low borrowing costs to states with high borrowing costs. Quantitatively, we find that this hedging motive against rollover risk makes the canonical sovereign debt model consistent with the pattern of reserves in the data.

Related studies on sovereign debt address debt maturity management. Arellano and Ramanarayanan (2012) study a quantitative model with default risk in which the government balances the incentive benefits of short-term debt and the hedging benefits of long-term debt. A similar trade-off is present in our model: reserves provide an insurance value for the government whereas paying down debt contributes to lower spreads by reducing future incentives to default. Trade-offs between short-term debt and long-term debt are also analyzed in an active growing literature, which includes Niepelt (2014), Hatchondo, Martinez, and Sosa Padilla (2015), Broner, Lorenzoni, and Schmukler (2013b), Dovis (2013), and Aguiar et al. (2016a). None of these studies allow the government to accumulate assets for insurance purposes.

Our work is also related to papers that study the optimal maturity structure of government debt in the presence of distortionary taxation. Angeletos (2002) and Buera and Nicolini (2004) study a closed-economy model in which the government can issue non-state-contingent bonds of different maturities under perfect commitment. They present examples in which the government can replicate complete market allocations by issuing nondefaultable long-term debt and accumulating short-term assets. In their model, changes in the term structure of interest rates, which contribute to offsetting shocks to the government budget constraint, arise as a result of fluctuations in the marginal rate of substitution of domestic consumers. Quantitatively, sustaining the complete market allocations requires gross positions that are on the order of a few hundred times output (Buera and Nicolini, 2004).[7] In contrast, in our model, gross positions affect incentives for debt repayment, and fluctuations in the interest rate reflect changes in the premium that foreign investors demand in order to be compensated for the possibility of a government default. This not only provides a different reason for the asset-spanning motive—without default risk, our model would have an indeterminate portfolio position—but also leads to empirically plausible levels of government assets and liabilities. Overall, our paper provides an alternative theory of debt management based on limited commitment, shifting the focus from managing deadweight losses of taxation to managing default risk.

Our paper is also related to the literature on precautionary savings, and sudden stops. Durdu,

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[7]Faraglia, Marcet, and Scott (2010) argue that the qualitative predictions of the complete market approach to debt management are sensitive to the type of shocks considered, and Debortoli, Nunes, and Yared (2016) argue that the predictions for gross positions are sensitive to the assumption that the government can commit to fiscal policy.

Mendoza, and Terrones (2009) study a dynamic precautionary savings model in which a higher *net* foreign asset position reduces the frequency and the severity of binding credit constraints. In contrast, we study a setup with endogenous borrowing constraints resulting from default risk and analyze *gross* portfolio positions. Jeanne and Ranciere (2011) present a simple analytical formula to quantify the optimal amount of reserves. They model reserves as an Arrow-Debreu security that pays off in a sudden stop. In a similar vein, Caballero and Panageas (2008) propose a quantitative setup in which the government issues nondefaultable debt that is indexed to the income-growth shock. They find that there are significant gains from introducing financial instruments that provide insurance against both the occurrence of sudden stops and changes in the sudden-stop probability. Aizenman and Lee (2007) and Hur and Kondo (2014) study reserve accumulation in open economy versions of Diamond and Dybvig (1983), generating endogenous sudden stops. In particular, Hur and Kondo consider a multicountry model with learning about the volatility of liquidity shocks to shed light on the upward trend in the reserves-to-debt ratio. Our paper complements this literature by considering endogenous borrowing costs that are due to default risk, by considering the role of debt maturity, and by focusing on non-state-contingent assets rather than insurance arrangements. Overall, a key contribution of our paper is to present a unified framework for studying the joint dynamics of reserves, debt, and sovereign spreads.

Other studies emphasize other benefits of reserve accumulation. Korinek and Servén (2011) and Benigno and Fornaro (2012) present models in which learning-by-doing externalities in the tradable sector lead the government to accumulate reserves to depreciate the real exchange rate. In Aguiar and Amador (2011), the accumulation of net foreign assets allows the government to credibly commit to not expropriating capital. These studies, however, do not present endogenous gross debt positions and hence do not address whether governments should accumulate reserves or lower their debt level.

Telyukova (2013) and Telyukova and Wright (2008) address the "credit card puzzle," that is, the fact that households pay high interest rates on credit cards while earning low rates on bank accounts. In their models, the demand for liquid assets arises because of a transaction motive, since credit cards cannot be used to buy some goods. Although we also study savings decisions by an indebted agent, we offer a distinct mechanism for the demand of liquid assets based on rollover risk. This mechanism could be relevant for understanding the financial decisions of households and corporate borrowers.

The rest of the article proceeds as follows. Section 2 documents stylized facts about reserves, public debt, and sovereign spreads. Section 3 presents the model. Section 4 presents the quantitative analysis. Section 5 concludes.

# 2  Facts on Reserves, Debt, and Spreads

This section presents empirical patterns of reserves, debt, and spreads for emerging markets. Our focus is on both the level and the comovement of these variables.

**Data description.**  Our data sample covers countries classified as emerging markets and developing economies by the IMF's World Economic Outlook and not classified as low-income countries by the World Bank. We exclude from our sample, data for sovereign default episodes (as listed in Asonuma and Trebesch, 2016).[8]

We consider reserves data from the International Financial Statistics. As defined by the IMF (2001), reserves are "official public sector foreign assets that are readily available." This definition includes foreign currencies and foreign-currency deposits and securities, special drawing rights (SDRs), and the reserve position in the IMF. Following the standard convention, we exclude gold.

We consider all forms of defaultable public debt, in line with our motivation. This includes debt denominated in domestic and foreign currency, and issued domestically or externally. Debt data are from the World Economic Outlook Database for the 2000-2014 period and the IMF Fiscal Affairs Department Historical Public Debt Database for the 1980-2014 period. Standard definitions of debt only include "all liabilities that are debt instruments" (Eurostat, 2014) and thus exclude currency as part of the government's debt.

For spreads, we use data from the Emerging Markets Bond Index Plus (EMBI+ blended).[9] We use GDP data from the World Economic Outlook Database.

**Three facts.**  We next present three basic facts on reserves, public debt, and sovereign spreads in emerging markets.

1 *Indebted governments that hold reserves pay significant and volatile spreads on their debt.* This fact, also highlighted by Rodrik (2006) and Alfaro and Kanczuk (2009) among others, is illustrated in Figure 1: panel (a) shows debt levels, panel (b) shows reserve levels, panel (c) shows spread levels, and panel (d) shows the spread volatility. In all panels, countries are sorted according to the reserve level for each country. Across countries, the median values for the average

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[8]Appendix B presents all available data for reserves, debt, and spread for all countries classified as emerging markets and developing economies by the IMF's World Economic Outlook.

[9]There are two EMBI series. The stripped EMBI, which strips out the effect of any credit enhancements such as collateral, and the blended EMBI, which does not.

levels of debt and reserves, are respectively 42 percent and 16 percent of GDP. For spreads, the median is 224 basis points, and the median standard deviation is 155 basis points.



Figure 1: International Reserves, Debt, and Spreads in Emerging Economies.
Note: The figure considers the 22 countries for which we can construct a balanced panel of data for reserves, debt, and spreads from 2000 (when data on sovereign spreads become more widely available) to 2014.

2  *There has been a secular increase in reserves.* Emerging markets have substantially increased their holdings of international reserves, a fact much noted in discussions on global imbalances. panel (a) of Figure 2 presents the trend for the median level of reserves, as well as the interquartile range, for the sample of countries considered. The figure also shows the evolution of the levels of debt (panel b) and spreads (panel c).

3  *Reserves and debt tend to increase when the sovereign spread is low or income is high.* Table 1 presents correlations between the growth rate of either reserves or debt (%$\Delta a$ and %$\Delta b$,

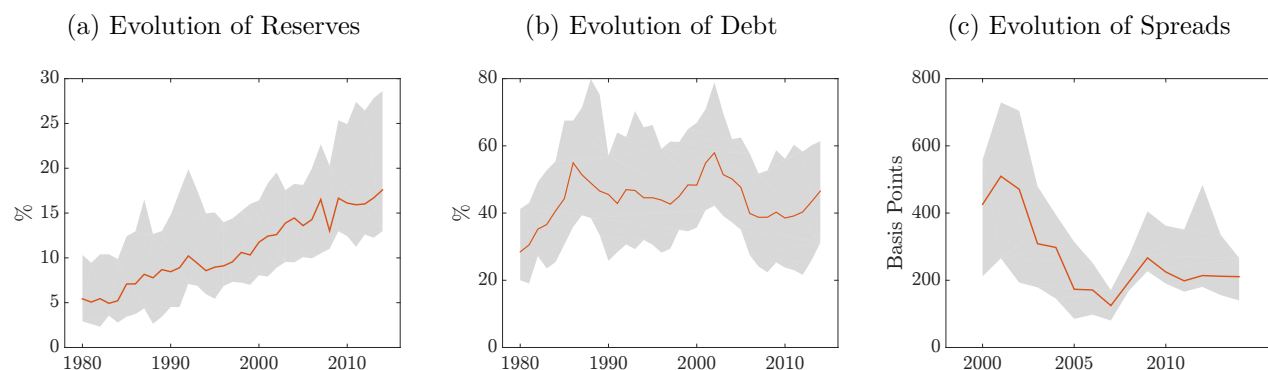| (a) Evolution of Reserves | (b) Evolution of Debt | (c) Evolution of Spreads |
|---|---|---|



Figure 2: Trends in Reserves, Debt, and Spreads.

Note: The figure presents the median level and interquartile range for reserves, public debt, and sovereign spreads in emerging economies. Panels (a) and (b) use the 51 emerging economies for which we have data on both public debt and reserves between 1980 and 2014. Panel (c) uses the 22 countries in Figure 1.

respectively) with either the spread or real GDP growth, all in annual terms.[10] This table shows that reserves and debt tend to increase when the sovereign spread is low or income is high. In other words, during good times, governments simultaneously borrow and save more.[11]

To summarize, these three facts present important regularities about the levels, trend, and cycle of debt and reserve positions. Next, we present a model of the optimal level of reserves as insurance against rollover risk that is consistent with these empirical regularities: (i) the government simultaneously holds large gross debt and asset positions while paying significant and volatile spreads for its debt, (ii) recent developments in emerging markets are consistent with significant increases of reserve holdings, and (iii) the government accumulates more reserves and debt when spreads are low and aggregate income is high.

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[10]Aguiar et al. (2016b) document a negative correlation between the growth rate of international debt and spreads for a majority of emerging economies. Because we use public debt, it is not surprising that we find a higher number (although still a minority) of countries in which the growth rate of public debt is positively correlated with the spread or negatively correlated with real GDP growth. An increase in public debt in bad times is consistent with emerging economies receiving loans from the official sector during crises (Boz, 2011).

[11]This pattern for public capital flows is also in line with the patterns of private and total capital flows in emerging markets. In particular, Broner et al. (2013a) document that broad measures of capital inflows and capital outflows are procyclical and positively correlated.

Table 1: Correlations between the accumulation of reserves or debt with spreads or GDP growth.

| | Spread, $\%\Delta a$ | Spread, $\%\Delta b$ | Growth, $\%\Delta a$ | Growth, $\%\Delta b$ |
|---|---|---|---|---|
| Argentina | −0.67 | 0.52 | 0.36 | −0.16 |
| Brazil | −0.28 | −0.37 | 0.27 | 0.71 |
| Bulgaria | −0.20 | −0.35 | 0.52 | −0.28 |
| Chile | 0.14 | 0.41 | 0.05 | 0.02 |
| China | −0.17 | 0.14 | 0.65 | 0.36 |
| Colombia | −0.30 | −0.41 | 0.41 | 0.34 |
| Croatia | −0.27 | −0.44 | 0.42 | 0.40 |
| Ecuador | −0.36 | −0.40 | 0.03 | 0.08 |
| Hungary | −0.27 | −0.65 | −0.22 | 0.49 |
| Lebanon | 0.49 | 0.37 | 0.26 | 0.12 |
| Malaysia | −0.25 | 0.32 | 0.13 | 0.08 |
| Mexico | 0.31 | −0.32 | 0.09 | 0.54 |
| Morocco | 0.44 | −0.25 | 0.57 | −0.01 |
| Panama | −0.26 | −0.21 | −0.05 | 0.07 |
| Peru | −0.43 | 0.28 | 0.67 | 0.22 |
| Philippines | −0.40 | −0.17 | 0.18 | 0.34 |
| Poland | −0.13 | −0.45 | 0.38 | 0.25 |
| Russia | 0.04 | −0.07 | 0.63 | −0.15 |
| South Africa | −0.61 | −0.23 | 0.39 | −0.14 |
| Turkey | −0.12 | 0.52 | 0.47 | 0.15 |
| Ukraine | −0.04 | 0.09 | 0.65 | −0.19 |
| Venezuela | −0.53 | 0.29 | 0.09 | −0.15 |
| **Median** | **−0.26** | **−0.19** | **0.37** | **0.10** |

Note: The table uses the same sample used in Figure 1.

# 3 Model

This section presents a dynamic small open economy in which the government receives a stochastic endowment stream, issues non-state-contingent defaultable debt, and buys a reserve asset that pays the risk-free interest rate.[12]

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[12]In practice, reserves are often held by the monetary authority, while borrowing is conducted by the fiscal authority. Independence of the monetary authority, however, is often limited. As anecdotal evidence from Argentina, the *New York Times* reported that "President Cristina Fernandez fired Argentina's central bank chief Thursday after he refused to step down in a dispute over whether the country's international reserves should be used to pay debt." Similar issues are also present in developed economies. For instance, the Swedish National Debt Office and the Riksbank discussed whether the debt office should borrow to strengthen liquidity buffers and whether buffers should be at the fiscal authority or the Riksbank's disposal (Riksgalden, 2013). We treat the government as a consolidated entity and abstract from possible conflicts of interest between different branches of the government.

## 3.1 Environment

**Endowments.** Time is discrete and indexed by $t \in \{0, 1, ....\}$. The economy's endowment of the single tradable good is denoted by $y \in \mathbb{R}_{++}$. The endowment process follows:

$$\log(y_t) = (1 - \rho)\,\mu + \rho \log(y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$, and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

**Preferences.** Preferences of the government over private consumption are given by

$$\mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} u(c_j), \tag{1}$$

where $\mathbb{E}$ denotes the expectation operator, $\beta$ denotes the discount factor, and $c$ represents private consumption. The utility function is strictly increasing and strictly concave.

**Asset/Debt Structure.** As in Arellano and Ramanarayanan (2012) and Hatchondo and Martinez (2009), we assume that a bond issued in period $t$ promises a deterministic infinite stream of coupons that decreases at an exogenous constant rate $\delta$. In particular, a bond issued in period $t$ promises to pay $\delta(1 - \delta)^{j-1}$ units of the tradable good in period $t + j$, for all $j \geq 1$. Hence, debt dynamics can be represented by the following law of motion:

$$b_{t+1} = (1 - \delta)b_t + i_t, \tag{2}$$

where $b_t$ is the number of bonds due at the beginning of period $t$, and $i_t$ is the amount of bonds issued in period $t$. The government issues these bonds at a price $q_t$, which in equilibrium depends on the government's portfolio decisions and the exogenous shocks.

The government has access to a one-period risk-free asset (reserves) that pays one unit of the consumption good in the next period and is traded at a constant price $q_a$. We denote by $a_t \geq 0$ the government's reserve holdings at the beginning of period $t$.

**Budget constraint.** When the government has access to debt markets, it faces the following budget constraint:

$$c_t + g + \delta b_t + q_a a_{t+1} = y_t + a_t + i_t q_t. \tag{3}$$

The government finances consumption ($c_t$), a fixed government expenditure ($g$), coupon payments ($\delta b_t$), and reserve accumulation ($q_a a_{t+1}$) with income ($y_t$), the initial stock of reserves $a_t$, and the new issuances of debt ($i_t q_t$). The fixed government expenditure, $g$, captures rigidities in the

government budget constraint that are central to debt dynamics. For instance, in their debt sustainability analysis, the IMF (2013) assumes that the government cannot adjust spending for two years in response to negative shocks to GDP. Moreover, this feature lowers the speed at which the governments reduces borrowing after a negative shock, improving the quantitative performance of the model.[13] Overall, this fixed government expenditure makes it costlier for the government to adjust to adverse shocks and thus strengthens the incentives to accumulate assets for hedging against rollover risk.[14]

**Default.** When the government defaults, it does so on all current and future debt obligations. This is consistent with the observed behavior of defaulting governments, and it is a standard assumption in the literature.[15] As in most previous studies, we also assume that the recovery rate for debt in default (i.e., the fraction of the loan that lenders recover after a default) is zero.

In the default period, the government cannot borrow and suffers a one-time utility loss $U^D(y)$, which is increasing in income.[16] We think of this utility loss as capturing various default costs related to reputation, sanctions, or misallocation of resources. An alternative, which is more common in the literature, is to model the cost of defaulting as a loss of aggregate income instead of as a utility loss. One advantage of the utility loss specification is that the model that features only an income loss typically needs to overstate the income losses triggered by default to reproduce the debt levels in the data.[17] Overstating the income cost of defaulting is not innocuous because reserves could be used to smooth out those losses. By modeling a utility loss, we abstract from the role of reserves in smoothing out income losses triggered by default (which are difficult to identify in the data) and focus instead on their role in hedging against rollover risk. Section 4.7 shows that adding a plausible income cost of defaulting to the model does not substantially change our results.

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[13] In the conclusion of the handbook, Aguiar et al. (2016b) argue that "we need to rationalize a reduction in the speed with which the government chooses to undo the impact of negative shocks on the spread by borrowing less and yet not default on the debt." With a similar motivation, Bocola and Dovis (2015) introduce non-discretionary public spending in the government's utility function.

[14] Recalibrating our model to match the same targets with $g = 0$ generates half the average level of reserves in our benchmark simulations.

[15] Sovereign debt contracts often contain an acceleration clause and a cross-default clause. The first clause allows creditors to call the debt they hold in case the government defaults on a debt payment. The cross-default clause states that a default in any government obligation constitutes a default in the contract containing that clause. These clauses imply that after a default event, future debt obligations become current.

[16] In the calibration, a period in the model is a year and thus the exclusion from debt markets lasts for a year. A one-year exclusion period is at the short end of the range of empirical estimates. According to Cruces and Trebesch (2013), the consensus of empirical studies is that countries commonly regain access to credit after just one or two years. Section 4.7 shows that our findings are fairly insensitive to assuming longer exclusion periods.

[17] While GDP is typically low both before and after defaults, identifying the causal effect of default on GDP in the data is very difficult because low income is a trigger of defaults.

Upon default, the government retains control of its reserves and access to savings. Hence, the budget constraint becomes

$$c_t = y_t + a_t - q_a a_{t+1} - g. \tag{4}$$

**Foreign Lenders and Risk-Premium Shocks.** To capture dislocations to international credit markets that are exogenous to local conditions, we assume a global shock that increases the lenders' risk aversion. Several studies find that investors' risk aversion is an important driver of global liquidity (Rey, 2013) and that a significant fraction of the sovereign spread volatility in the data can be accounted for by the volatility of the risk premium (Borri and Verdelhan, 2015; Broner et al., 2013b; Longstaff et al., 2011; González-Rozada and Levy Yeyati, 2008). A vast empirical literature shows that extreme capital flow episodes are typically driven by global factors (Calvo et al., 1993; Uribe and Yue, 2006; Forbes and Warnock, 2012). Aguiar et al. (2016b) show that sovereign defaults are not tightly connected to poor fundamentals and that risk premia are an important component of sovereign spreads.

To introduce risk-premium shocks, we assume that foreign lenders price bonds' payoffs using the following stochastic discount factor:

$$m_{t,t+1} = e^{-r-(\kappa_t \varepsilon_{t+1} + 0.5\kappa_t^2 \sigma_\varepsilon^2)}, \quad \text{with} \quad \kappa_t \geq 0. \tag{5}$$

Here, $r$ is the discount rate, and $\kappa_t$ is the parameter governing the risk-premium shock. This formulation introduces a positive risk premium because bond payoffs are more valuable to lenders in states in which the government defaults (i.e., in states in which income shocks $\varepsilon$ are low). The risk-premium shock follows a two-state Markov process with values $\kappa_L = 0$ and $\kappa_H > 0$, and transition probabilities $\pi_{LH}$ and $\pi_{HL}$. Thus, in normal times, $\kappa_t = \kappa_L = 0$ and lenders are risk neutral. When $\kappa_t = \kappa_H$, lenders become risk averse and require a higher expected return to buy government bonds. A higher value of $\kappa_H$ can be seen as capturing how correlated the small open economy is with the lenders' income process, or alternatively, the degree of diversification of foreign lenders.[18]

This specification of the lenders' stochastic discount factor is a special case of the discrete-time version of the Vasicek (1977) one-factor model of the term structure, and it has been used in models of sovereign default (e.g., Arellano and Ramanarayanan, 2012). For our purpose, this specification is conveniently tractable and delivers a risk premium of bonds relative to reserves.

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[18]Aguiar et al. (2016b) explicitly model the lenders' portfolio problem featuring random finite wealth and limited investment opportunities. In their model, shocks to the foreign lenders' wealth shift the menu of borrowing opportunities.

Notice that this risk premium will be endogenous to the gross portfolio positions chosen by the government, which determine default risk. Without default, the risk premium would disappear, and the government's portfolio would become indeterminate. While not crucial for the core mechanism of the model, this shock plays an important role in our simulations (we study the importance of this shock in Section 4.7.2). In states in which lenders demand a higher premium for government bonds, the government uses the reserves accumulated in earlier periods to avoid rolling over debt at high rates.

**Discussion on Asset/Debt Management.** The government in our model has access to a saving instrument, a one-period risk-free asset, and a debt instrument, which is a long-duration bond. This asset/debt structure deserves some discussion. On the asset side, a key assumption for the mechanism in the paper is that reserves can be adjusted freely every period, which is consistent with reserves being liquid assets (e.g., US Treasury bills). Because reserves are a perfectly liquid risk-free asset that pays a constant interest rate each period, the assumed duration of reserves is irrelevant. We assume a duration of one period without loss of generality.

On the debt side, the fact that we take $\delta$ as a primitive of the model prevents us from addressing the management of the debt maturity structure. Notice that while choosing a longer maturity would mitigate the need of reserves to insure against rollover risk, a longer maturity is more costly in terms of debt dilution and risk premium. Given these costs from long-term debt, the government would remain exposed to rollover risk, even with endogenous maturity, and reserves would remain valuable in the government's portfolio.[19]

## 3.2 Recursive Government Problem

We now describe the recursive formulation of the government's optimization problem. The government cannot commit to future (default, borrowing, and saving) decisions. Thus, one may interpret this environment as a game in which the government making decisions in period $t$ is a player who takes as given the (default, borrowing, and saving) strategies of other players (governments) who decide after $t$. We focus on Markov perfect equilibria. That is, we assume that in each period, the government's equilibrium default, borrowing, and saving strategies depend only on payoff-relevant state variables.

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[19]Arellano and Ramanarayanan (2012), Hatchondo et al. (2015), and Aguiar et al. (2016a) analyze debt dilution, and Broner et al. (2013b) study the effect of the risk premium on the government's maturity choice. A joint analysis of reserve and maturity management would be interesting but is beyond the scope of this paper. Computationally, this would require introducing a third endogenous state variable (e.g., adding a short-term bond in addition to the long-duration bond).

Let $s = \{y, \kappa\}$ denote the current exogenous state of the world and $V$ denote the optimal value for the government. For any bond price function $q$, the function $V$ satisfies the following functional equation:

$$V(a, b, s) = \max \left\{ V^R(a, b, s), V^D(a, s) \right\}, \tag{V}$$

where the government's value of repaying is given by

$$V^R(a, b, s) = \max_{a' \geq 0, b' \geq 0} \left\{ u(c) + \beta \mathbb{E}_{s'|s} V(b', a', s') \right\}, \tag{VR}$$

subject to

$$c = y - \delta b + a + q(b', a', s)[b' - (1 - \delta)b] - q_a a' - g.$$

The value of defaulting is given by

$$V^D(a, s) = \max_{a' \geq 0} \left\{ u(c) - U^D(y) + \beta \mathbb{E}_{s'|s} V(0, a', s'), \right\} \tag{VD}$$

subject to

$$c = y + a - q_a a' - g.$$

The solution to the government's problem yields decision rules for default $\hat{d}(a, b, s)$, debt $\hat{b}(a, b, s)$, reserves in default $\hat{a}^D(a, s)$, reserves when not in default $\hat{a}^R(a, b, s)$, consumption in default $\hat{c}^D(a, s)$, and consumption when not in default $\hat{c}^R(a, b, s)$. The default rule $\hat{d}$ is equal to 1 if the government defaults and is equal to 0 otherwise. In a rational expectations equilibrium (defined below), lenders use these decision rules to price debt contracts.

**Equilibrium Bond Prices.**   To be consistent with lenders' portfolio conditions, the bond price schedule needs to satisfy

$$q(a', b', s) = \mathbb{E}_{s'|s} \left[ m(s', s) \left[ 1 - \hat{d}(a', b', s') \right] \left[ \delta + (1 - \delta)q(a'', b'', s') \right] \right], \tag{6}$$

where

$$b'' = \hat{b}(a', b', s')$$
$$a'' = \hat{a}^R(a', b', s').$$

Equation (6) indicates that, in equilibrium, an investor has to be indifferent between selling a government bond today and keeping the bond and selling it in the next period. If the investor keeps the bond and the government does not default in the next period, he first receives a coupon

payment of $\delta$ units and then sells the bond at the market price, which is equal to $(1-\delta)$ times the price of a bond issued in the next period. Using (5), lenders' portfolio condition for the risk-free assets yields

$$e^{-r} = q_a.$$

## 3.3 Recursive Equilibrium

**Definition 1** (Equilibrium)**.** *A Markov perfect equilibrium is defined by*

1. *a set of value functions $V$, $V^R$, and $V^D$,*

2. *rules for default $\hat{d}$, borrowing $\hat{b}$, reserves $\left\{\hat{a}^R, \hat{a}^D\right\}$, and consumption $\left\{\hat{c}^R, \hat{c}^D\right\}$,*

3. *and a bond price function $q$*

*such that*

i. *given a bond price function $q$, the policy functions $\hat{d}$, $\hat{b}$, $\hat{a}^R$, $\hat{c}^R$, $\hat{a}^D$, $\hat{c}^D$ and the value functions $V$, $V^R$, $V^D$ solve the Bellman equations (V), (VR), and (VD).*

ii. *given government policies, the bond price function $q$ satisfies condition (6).*

# 4 Quantitative Analysis

In this section we present the quantitative analysis of the model. Section 4.1 describes the computation of the model. Section 4.2 presents the calibration. Section 4.3 presents key statistics from the benchmark simulations. Sections 4.4 and 4.5 analyzes rollover risk, and portfolio choices. Section 4.6 inspects the key trade-offs of the model. Section 4.7 examines the importance of debt maturity and risk-premium shocks, and discusses the robustness of our results. Finally, Section 4.8 shows that the model can rationalize the upward trend in reserves observed in emerging economies.

## 4.1 Computation

The recursive problem is solved using value function iteration. As in Hatchondo et al. (2010), we solve for the equilibrium by computing the limit of the finite-horizon version of our economy. That is, the approximated value and bond price functions correspond to the ones in the first period of a finite-horizon economy with a number of periods large enough that the maximum deviation between the value and bond price functions in the first and second period is no larger than $10^{-6}$. We solve the optimal portfolio allocation in each state by searching over a grid of debt and reserve levels and then using the best portfolio on that grid as an initial guess

in a nonlinear optimization routine. The value functions $V^D$ and $V^R$ and the function that indicates the equilibrium bond price function conditional on repayment $q\left(\hat{b}(\cdot), \hat{a}^R(\cdot), \cdot, \cdot\right)$ are approximated using linear interpolation over $y$ and cubic spline interpolation over debt and reserves positions. We use 40 grid points for reserves, 40 grid points for debt, and 30 grid points for income realizations. Expectations are calculated using 50 quadrature points for the income shock.

## 4.2 Calibration

**Functional Forms.** The utility function displays a constant coefficient of relative risk aversion, that is,

$$u\left(c\right) = \frac{c^{1-\gamma} - 1}{1 - \gamma}, \text{ with } \gamma \neq 1.$$

The utility cost of defaulting is given by $U^D(y) = \alpha_0 + \alpha_1 log(y)$. As in Chatterjee and Eyigungor (2012), having two parameters in the cost of defaulting gives us the flexibility to match the levels of debt and spread in the data.

**Parameter Values.** We first choose a subset of parameters values that can be directly pinned down from the data, and then choose a second subset of parameter values so that model simulations match key aspects of the data. A first aspect of the calibration regards the economy we use as a reference. Following several quantitative models of sovereign default, we use Mexico for the calibration (of the endowment process, public consumption, and the parameters that govern the mean spread as well as the the level and duration of debt). Mexico is a common reference for studies on emerging economies because its business cycle displays the same properties that are observed in other emerging economies (Aguiar and Gopinath, 2007; Neumeyer and Perri, 2005). Moreover, Mexico also gives us calibration targets for the average levels of debt and spread that are close to the median value of these levels for emerging economies (Figure 1). Unless specified otherwise, we use data from 1993 to 2014.

Table 2 presents the benchmark values given to all parameters in the model. A period in the model refers to a year. The values of the risk-free interest rate and the domestic discount factor ($r = 0.04$ and $\beta = 0.92$) are standard in quantitative business cycle and sovereign default studies.

The parameter values that govern the endowment process are chosen so as to mimic the behavior of logged and linearly detrended GDP in Mexico from 1980 to 2014. The estimation of the AR(1) process for the cyclical component of GDP yields $\rho = 0.66$ and $\sigma_\varepsilon = 0.034$. The level

of public goods $g$ is set to 12 percent to match the average level of public consumption to GDP in Mexico. We set $\delta = 0.28$. With this value and the targeted level of sovereign spread, sovereign debt has an average duration of 3 years in the simulations, which is roughly the average duration of public debt in Mexico.[20]

We use the average EMBI+ spread to parameterize the shock process to lenders' risk aversion. We assume that a period with high lenders' risk aversion is one in which the global EMBI+, excluding countries in default, is one standard deviation above the median over the sample period (we use quarterly data from 1993 to 2014). With this procedure, we obtain three episodes of a high risk premium every 20 years with an average duration of each episode equal to 1.25 years, which implies $\pi_{LH} = 0.15$ and $\pi_{HL} = 0.8$. The high risk-premium episodes are observed in 1994-1995 (Tequila crisis), 1998 (Russian default), and 2008 (global financial crisis). On average, the global EMBI+ was 2 percentage points higher in those episodes than in normal periods. Figure 3 shows that the average EMBI+ spread we use for calibrating high risk-premia shocks is highly correlated with the price of non-investment-grade corporate bonds, indicating that both series are strongly affected by global factors (which we model with risk-premium shocks). In particular, the high EMBI+ spread episodes we use to identify periods of adverse global factors also display low prices for corporate bonds, indicating that global factors played a role in these episodes.
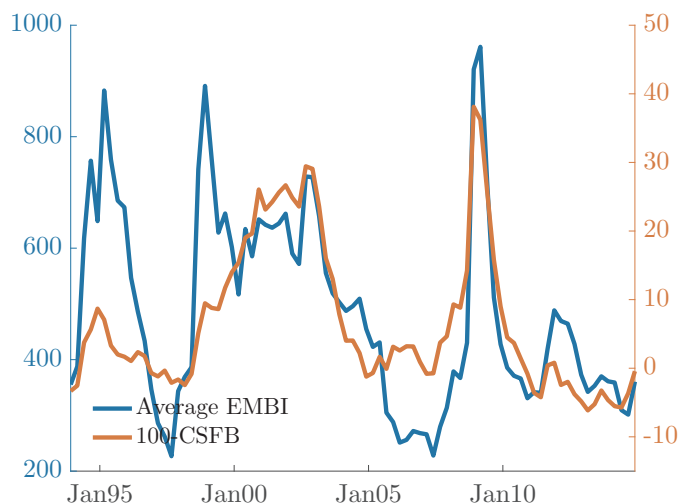


Figure 3: Sovereign spread and corporate bond prices.

Note: The figure presents the average EMBI+ spread and 100 minus the Credit Suisse First Boston High Yield Index, which measures the price of high-yield corporate securities denominated in U.S. dollars.

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[20]We use data from the central bank of Mexico for debt duration and the Macaulay definition of duration that, with the coupon structure in this paper, is given by $D = \frac{1+i_b}{\delta+i_b}$, where $i_b$ denotes the constant per-period yield delivered by the bond.

Table 2: Parameter Values

| Parameter | Description | Value |
|---|---|---|
| $r$ | Risk free rate | 0.04 |
| $\beta$ | Domestic discount factor | 0.92 |
| $\pi_{LH}$ | Probability of transiting to high risk-premium | 0.15 |
| $\pi_{HL}$ | Probability of transiting to low risk-premium | 0.8 |
| $\sigma_\varepsilon$ | Std. dev of innovation to $y$ | 0.034 |
| $\rho$ | Autocorrelation of $y$ | 0.66 |
| $g$ | Government consumption | 0.12 |
| $\delta$ | Coupon decaying rate | 0.2845 |
| | Parameters set by simulation | |
| $\alpha_0$ | Default cost parameter | 2.45 |
| $\alpha_1$ | Default cost parameter | 19 |
| $\kappa_H$ | Pricing kernel parameter | 23 |
| $\gamma$ | Coeff. of relative risk aversion | 3.3 |

**Targeted Moments.** We need to calibrate the value of four other parameters: the two parameters of the utility cost of defaulting $\alpha_0$ and $\alpha_1$, the parameter $\kappa_H$ determining the increase in lenders' risk aversion in periods of high risk premium, and the government's risk aversion $\gamma$. Our calibration strategy consists of choosing these four parameters $\{\alpha_0, \alpha_1, \kappa_H, \gamma\}$ to match four targets in the data: (i) a public debt-to-income ratio of 43.5 percent, (ii) a mean level of spreads of 240 basis points, (iii) an increase in the spread during high risk-premium periods of 200 basis points, which is the average increase in the sovereign spread observed in Mexico during the three high risk-premium periods we identify in the data, and (iv) a volatility of consumption relative to output equal to one. The values for the default cost $\alpha_0$ and $\alpha_1$, listed in the bottom panel of Table 2, mainly determine the average debt and spread levels, while $\kappa_H$ mainly determines the average increase in spreads in periods of high lenders' risk aversion. The choice of the value for the domestic risk aversion parameter is determined mainly by the consumption-volatility target.

We choose to make the domestic risk aversion part of the calibration because it is a key parameter determining the government's willingness to tolerate rollover risk. In particular, higher levels of risk aversion make the government more reluctant to tolerate sharp drops in consumption. Matching the volatility of consumption allows us to discipline the risk aversion parameter. A relative volatility of unity is consistent with the data. While durable consumption (and total

consumption) is more volatile than GDP both on average for emerging markets and for Mexico, this is not the case for non-durable consumption. Choosing a target for the volatility of consumption that is equal to the volatility of income brings us closer to the data for non-durable consumption without presenting a major departure from previous studies. The value of the risk aversion parameter that results from the calibration ($\gamma = 3.3$) is well within the range of values used for macro models.

To compute the sovereign spread that is implicit in a bond price, we first compute the yield $i_b$, defined as the return an investor would earn if he holds the bond to maturity (forever) and no default is declared. This yield satisfies

$$q_t = \sum_{j=1}^{\infty} \delta(1 - \delta)^{j-1} e^{-ji_b}.$$

The sovereign spread, $r_t^s$, is then computed as the difference between the yield $i_b$ and the risk-free rate $r$.

Debt levels in the simulations are calculated as the present value of future payment obligations discounted at the risk-free rate, that is, $\frac{\delta}{1-(1-\delta)e^{-r}}b_t$. To set our target of mean debt-to-income ratio of 43.5 percent, we use public debt data from the IMF Fiscal Affairs Department Historical Public Debt database. We should note that while these data include domestically held debt, in the model all debt is held by foreigners. We consider all public debt in the data because domestically held sovereign debt is also subject to rollover and default risk, and using only external debt would underestimate the debt burden faced by the government.[21] Section 4.7 presents a sensitivity analysis with respect to the level of debt.

## 4.3   Key Statistics: Model and Data

Table 3 reports long-run moments in the data and in the model simulations. The first panel of this table shows that the simulations match the calibration targets. The second panel shows that the model also does a good job in mimicking nontargeted moments. In particular, the simulations generate a volatile and countercyclical spread, and a high correlation between consumption and income.[22] This is in line with previous studies that have shown that the sovereign default model without reserve accumulation can account for these features of the data (Aguiar and Gopinath,

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[21] Our measure of public debt also includes debt denominated in domestic and foreign currency. Du and Schreger (2016) document that the credit spreads for local-currency and foreign-currency debt are positively correlated. This indicates that both forms of debt are subject to similar rollover risk.

[22]The spread volatility in the model is higher than in Mexico but is close to the median for emerging economies (Figure 1). The spread volatility in Mexico is also higher when computed using the stripped EMBI (throughout the paper we use the blended EMBI instead).

2006; Arellano, 2008). We show that this is still the case when we extend the baseline model to allow for the empirically relevant case in which indebted governments can hold reserves and choose to do so.

**Reserves.** Model simulations generate an average reserve-to-income ratio of 6 percent. Figure 4 shows that the simulations feature periods with reserve levels that are much higher than the average, of up to 40 percent of annual income.

Table 3: Basic Statistics: Model and Data

|  | Data | Model |
|---|---|---|
| **Targeted** | | |
| $\sigma(c)/\sigma(y)$ | 1.0 | 1.0 |
| mean (debt / y) | 43.0 | 43.5 |
| mean $r_s$ | 2.4 | 2.4 |
| Spread increase for $\kappa = \kappa_H$ | 2.0 | 2.0 |
| **Nontargeted** | | |
| $\sigma(r_s)$ | 0.9 | 2.0 |
| $\rho(r_s, y)$ | -0.5 | -0.7 |
| $\rho(y, c)$ | 0.8 | 0.9 |
| mean (reserves /y) | 8.5 | 6.0 |

Note: Moments are computed by generating 1,000 simulation samples of 300 periods each and taking the last 35 observations of samples in which the last default was observed at least 25 periods before the beginning of the sample. The standard deviation of $x$ is denoted by $\sigma(x)$. The coefficient of correlation between $x$ and $z$ is denoted by $\rho(x, z)$.
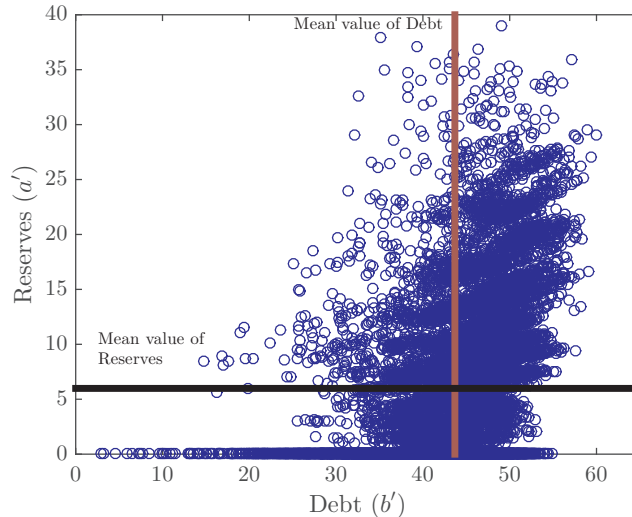
Figure 4: Portfolio of Debt and Reserves in the Simulations.

## 4.4 Rollover Risk

We now analyze the two sources of rollover risk in the model, which are crucial for understanding the optimal portfolio of the government. Figure 5 presents the spread the government is asked to pay as a function of its debt level for different income shocks (panel a) and risk-premium shocks (panel b) when the government chooses a level of reserves equal to the mean, $\bar{a}$. That is, we plot $r^s(b', \bar{a}, s)$ as a function of $b'$ for different values of $s$.

Panel (a) shows that for the same level of debt, investors demand higher spreads when income is low. This occurs because it is more attractive to default when income is low and income shocks are serially correlated. As emphasized by Aguiar and Gopinath (2006) and Arellano (2008), this feature enables the model to generate countercyclical spreads, as observed in the data.

Panel (b) shows that the government also faces higher spreads when lenders are more risk averse (and thus demand a larger compensation for default risk). Note that the effect of the exogenous risk-premium shock on the endogenous spread is an increasing function of the debt level. This illustrates how even though the risk-premium shock process is exogenous, the incidence of this shock on the domestic economy is a function of the government's portfolio choices (in particular, if the government were to commit to a portfolio that eliminates default risk, there would be no premium on government bonds).
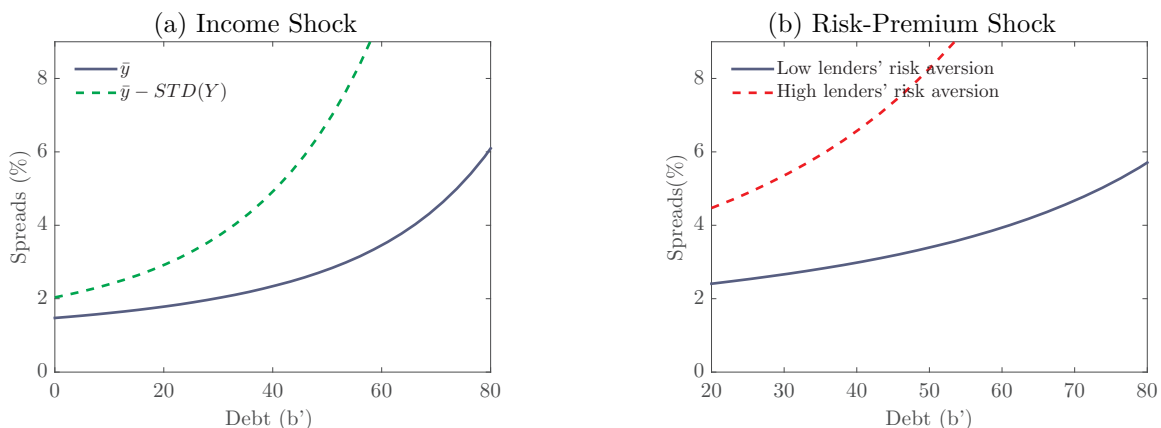
Figure 5: Spread Schedule.

Note: The figure presents the spread asked by lenders as a function of the debt level when the government chooses the average level of reserves in the simulations. In panel (a), lenders' risk aversion takes the low value. In panel (b), income takes the mean value. Debt levels are expressed as a percentage of average annual income.

## 4.5  Portfolio Policies

Figure 6 illustrates the government's optimal debt and reserves policies as functions of lenders' risk aversion and the level of income (the two sources of rollover risk in the model). Panels (a) and (b) show respectively the increase in reserves and debt, given initial values of debt and reserves equal to the mean, for the two values of lenders' risk aversion, and for income levels between 0.9 and 1.1. (Levels of income below 0.9 percent would lead the government to default.)

As panel (a) shows, when the risk aversion is low (blue straight line), the government increases reserve holdings for high income values and decreases reserve holdings for low income values. When the risk aversion is high (red broken line), the government depletes the initial stock of reserves (6 percent of income) in one year. This may look like a drastic response to the risk-premium shock but is consistent with governments sharply reducing reserve holdings during times of stress. In particular, following the 2008 financial crisis, there has been a significant decline in reserves held by emerging markets (Bussiere et al., 2013).[23]

Panel (b) shows that when lenders' risk aversion is low, the government accumulates debt when income is high and reduces debt when income is low. (For the initial states considered, the government reduces debt when income is below the mean value and vice versa.) Notice that the increase in debt slows down for high levels of income since consumption-smoothing motives

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[23]While it is natural in our model that governments hit the lower bound on the stock of reserves, in reality there may be reasons for governments to not fully deplete reserves. In addition, one would not expect a significant decline in reserves for the other two periods for which we identify a high risk premium in the data (1994-95 and 1998) because the stock of reserves was very low as central banks were starting to build up their buffers.

become stronger than the borrowing cost effect arising from high income improving credit market access. In fact, the ratio of debt to income actually decreases with income for levels of income larger than approximately 1.05.
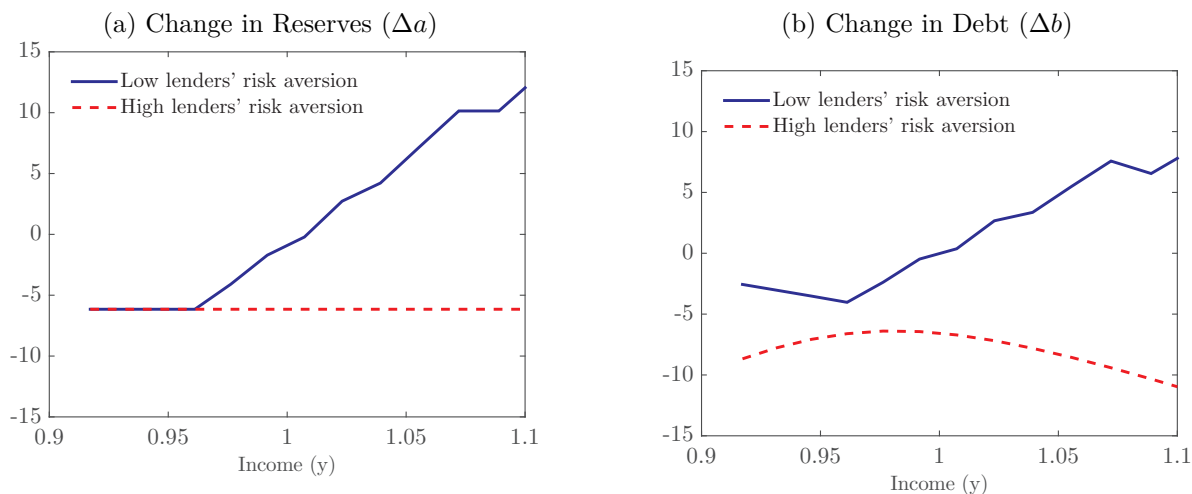


Figure 6: Optimal accumulation of reserves and debt.

Note: The figure shows the increase in reserves and debt for initial values of debt and reserves equal to the mean levels in the simulations. The figure displays income levels for which the government chooses to pay its debt. Changes in reserves and debt are expressed as a percentage of mean annual income.

Overall, Figure 6 shows that the government increases both reserves and debt when lenders' risk aversion is low and income is sufficiently high. In the simulations, the correlations of debt and reserves with income are 0.27 and 0.38, respectively, and the correlations of debt and reserves with spread are $-0.15$ and $-0.32$, respectively. This is consistent with the properties of the accumulation of reserves and debt in a majority of emerging economies, as illustrated in Table 1. This is also consistent with the broader movement of capital flows. In particular, capital inflows (sovereign debt accumulation) and capital outflows (reserve accumulation) are procyclical in both the model and the data, as documented by Broner et al. (2013a).

## 4.6 Inspecting the Mechanism

We now analyze the key forces that shape the optimal portfolio and the fundamental trade-off faced by the government. We show that reserves provide a hedge against adverse shocks that increase the cost of borrowing. On the other hand, using reserves to pay down debt reduces current spreads by reducing incentives to default in the future.

### 4.6.1 Issuing debt to accumulate reserves

To characterize the trade-off between the insurance benefits of reserves and the incentive costs of keeping larger gross debt positions, it will be useful to consider financial operations that allow for different portfolios that deliver the same level of current consumption. Below we will analyze these operations in detail, tracing their costs and benefits. For illustration purposes, we assume that the bond price and the value function are differentiable.[24] Details on the analytical derivations below can be found in Appendix A.

Choose an initial bond, reserves, income, and risk-premium shock. In addition, consider a consumption target $\bar{c}$, and let $x = \{a, b, s, \bar{c}\}$ denote the vector of initial states and the consumption target. The combination of debt and reserves $(a', b')$ that delivers a level of consumption equal to $\bar{c}$ is then given by

$$q_a a' = y - \bar{c} - g - \delta b + a + q(b', a', s)[b' - (1 - \delta)b]. \tag{7}$$

How many reserves the government can accumulate with newly issued bonds depends on how the bond price varies with the portfolio. We define $\tilde{a}(b', x)$ as the amount of reserves that can be purchased when the government borrows $b'$, for a given value of of $x$ (i.e., $\tilde{a}(b', x)$ is the value of $a'$ consistent with equation 7). Applying the implicit function theorem to equation (7) implies that issuing an extra unit of debt in the current period enables the government to purchase

$$\frac{\partial \tilde{a}(b', x)}{\partial b'} = \frac{q(\tilde{a}(b', x), b', s) + \frac{\partial q(\tilde{a}(b', x), b', s)}{\partial b'}[b' - (1 - \delta)b]}{q_a - \frac{\partial q(\tilde{a}(b', x), b', s)}{\partial a'}[b' - (1 - \delta)b]} \tag{8}$$

additional reserves without changing current consumption. As (8) indicates, a government issuing debt can accumulate more reserves the more favorable are the changes in the bond price in response to the increase in debt and reserves (i.e., the higher are $\partial q / \partial a'$ and $\partial q / \partial b'$). The changes in the bond prices, in turn, depend on how future gross positions affect future incentives to default.

How does the government decide among combinations of debt and reserves that deliver the same level of current consumption? Because current utility is fixed by current consumption, the optimal portfolio needs to maximize the expected continuation value. Assuming that the government optimally chooses consumption and portfolios from next period onward, this implies

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[24] Clausen and Strub (2017) demonstrate that in the canonical default model, the objective function is continuously differentiable at the optimal choices and a version of the envelope theorem applies (see also Aguiar et al., 2016a for similar results with shocks to outside options to default). Figure 13 in Appendix A shows that the equilibrium accumulation of reserves obtained using Euler equations is almost identical to our numerical solution that uses a global solution method and does not rely on differentiability.

in turn that the continuation value is given by $V(\tilde{a}(b', x), b', s')$. The current optimal portfolio then solves

$$\max_{b', \tilde{a}(b', x) \geq 0} \mathbb{E}_{s'|s} V(\tilde{a}(b', x), b', s'). \tag{9}$$

Totally differentiating (9) with respect to $b'$ and using the envelope condition obtained from (VR) and (VD), we obtain

$$\frac{d\mathbb{E}_{s'|s} V(\tilde{a}, b', s')}{db'} = \overbrace{\underbrace{\frac{\partial \tilde{a}}{\partial b'}}_{\text{Reserves bought}}}^{\text{Benefit from buying reserves}} \mathbb{E}_{s'|s}\left[u'(c')\right] - \overbrace{\delta \mathbb{E}_{s'|s}\left[u'(c')(1-d')\right]}^{\text{Cost of coupon payments}} - \overbrace{(1-\delta)\mathbb{E}_{s'|s}\left[u'(c')q'(1-d')\right]}^{\text{Cost of carrying long-term debt}}.$$

$$\tag{10}$$

Equation (10) represents the expected net marginal benefits of issuing one more unit of debt and purchasing reserves with the proceeds. At an interior optimum (i.e., with $\tilde{a}(b', x) > 0$), the government equates this expression to zero. The first term presents the marginal utility benefit of carrying $\partial \tilde{a}/\partial b'$ additional reserves for next period. The remaining two terms represent the marginal costs from the issuance of an additional bond to finance the accumulation of reserves, which can be split into the cost of coupon payments and the cost of long-term debt.

In the next two subsections, we build on equation (10) to tease out the key forces in the model. The first studies the insurance benefits, and the second one addresses the incentive costs.

### 4.6.2 Insurance benefits

We now show that issuing debt to finance the accumulation of reserves allows the government to transfer resources to future states with low consumption—hence, the insurance role of reserves. For that, it is convenient to rearrange equation (10) by splitting next-period payoffs between the states in which the government repays and states in which the government defaults:

$$\frac{d\mathbb{E}_{s'|s} V(\tilde{a}, b', s')}{db'} = \mathbb{E}_{s'|s}\left[\overbrace{\underbrace{\overbrace{\frac{\partial \tilde{a}}{\partial b'}}^{\text{Net marginal utility}}_{\text{in default states}} d'u'(c')}_{\substack{\text{Payoffs in} \\ \text{default states}}} + \overbrace{\underbrace{\left[\overbrace{\frac{\partial \tilde{a}}{\partial b'} - \delta - (1-\delta)q'}^{\text{Net marginal utility in repayment states}}\right](1-d')u'(c')}_{\text{Payoffs in repayment states}}}\right]. \tag{11}$$

Equation (11) shows that by issuing debt to finance the accumulation of reserves, the government obtains $\partial \tilde{a}/\partial b'$ resources in default states, with an associated marginal utility value of $(\partial \tilde{a}/\partial b')u'(c')$ (first term), and $[\partial \tilde{a}/\partial b' - \delta - (1-\delta)q']$ resources in repayment states, with an associated marginal utility value of $[\partial \tilde{a}/\partial b' - \delta - (1-\delta)q']u'(c')$ (second term).

Maturity plays a crucial role in characterizing the second term in (11). If the bond maturity

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exceeds one period ($\delta < 1$), the payoffs obtained in repayment states are decreasing in the next-period bond price $q'$. States with lower next-period bond price $q'$, in turn, are states in which it is more expensive to issue debt, and hence those are states with higher marginal utility of consumption. Conversely, the government obtains lower payoffs in states with high next-period bond price $q'$ and low marginal utility of consumption. In equilibrium, the distribution of positive and negative payoffs must be such that investors obtain at least the risk-free return on average. This imply that issuing debt to accumulate reserves transfers resources from repayment states with high $q'$ to repayment states with low $q'$. Hence, reserves provide the government with insurance against rollover risk.

In contrast, if bond maturity is one period ($\delta = 1$), the payoffs in the second term are constant across next-period states. As a result, issuing one-period debt to purchase reserves does not transfer resources across repayment states. The payoffs are still different across states tomorrow, but they only vary between default states and repayment states: payoffs are $\partial \tilde{a} / \partial b'$ in default states and $\partial \tilde{a} / \partial b' - 1$ in repayment states. Section 4.7.1 shows that because of this lack of variation of payoffs across repayment states, the model with one-period debt produces a negligible amount of reserves.

**Numerical illustration of insurance properties.** Figures 7 and 8 illustrate the insurance properties of the optimal portfolio and demonstrate that reserves are indeed an instrument to hedge against rollover risk. Following the decomposition provided in equation (11), panels (a) and (b) of Figure 7 show, respectively, how next-period payoffs and consumption vary with next-period income and lenders' risk aversion.

Let us first describe panel (a) and focus on the blue straight line. One can distinguish three regions depending on the level of next-period income $y'$. For sufficiently low values of $y'$, the government does not repay its debt and therefore issuing debt and accumulating reserves deliver a positive (and constant) payoff. This payoff is given by $\frac{\partial \tilde{a}}{\partial b'}$, the reserves purchased with an additional unit of debt issued. As we move to the right, the government repays its debt. For the lower values of $y'$ such that the government repays ($0.89 \leq y' \leq 0.95$), the government still experiences a positive payoff from issuing debt to accumulate reserves (i.e., $\frac{\partial \tilde{a}}{\partial b'} - \delta - (1 - \delta)q' \geq 0$). As income increases, the payoffs for the government are reduced. Recall that default risk is countercyclical, as shown in Figure 5: higher income reduces spreads and increases bond prices. Eventually, for higher values of $y'$, the government experiences a negative payoff. This underscores how increasing gross positions at the margin allows the government to transfer resources from states with high income to states with low income. Likewise, the red

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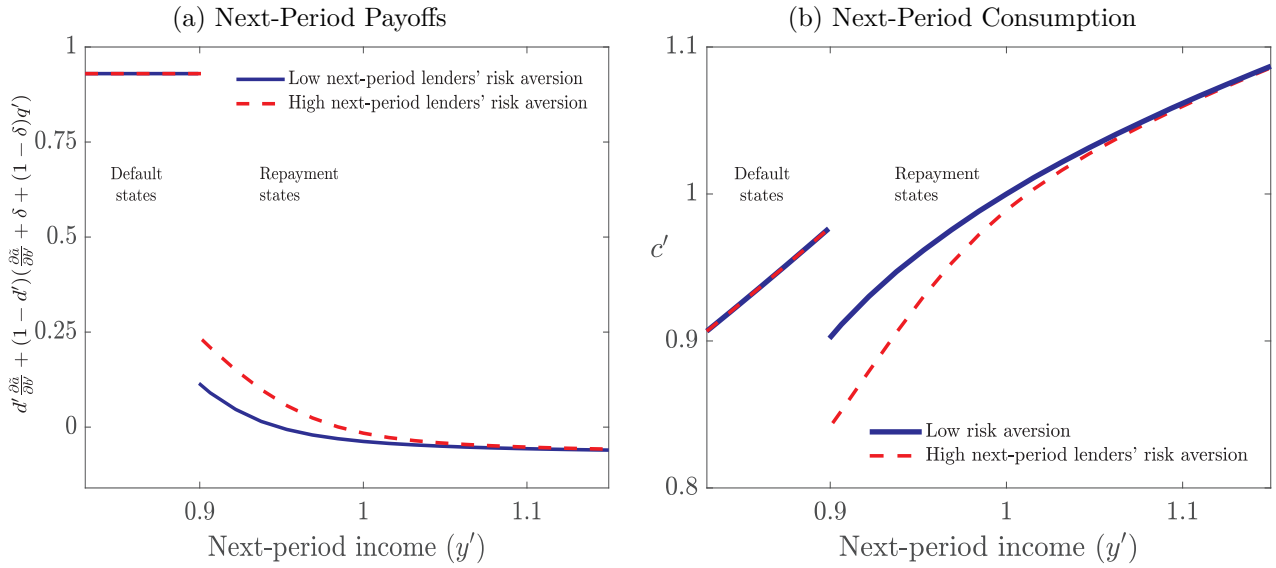(a) Next-Period Payoffs  (b) Next-Period Consumption

Figure 7: Insurance Benefits of Issuing Debt to Accumulate Reserves.

Note: Both panels assume that in the current period, the level of income and the initial levels of debt and reserves are equal to the mean level in the simulations, the lenders' risk aversion is low ($\kappa_t = 0$), and the government chooses the optimal portfolio.
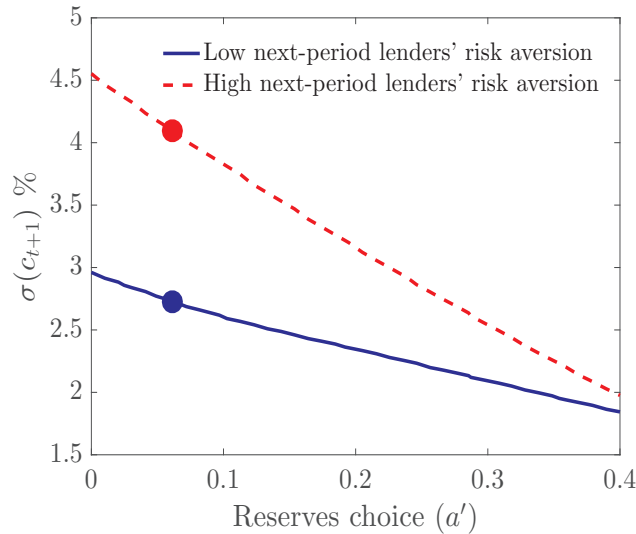


Figure 8: Volatility of Consumption

Note: The figure presents the standard deviation of next-period log consumption ($\log c'$) in repayment states. The initial states are those used in Figure 7.

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dashed line in Figure 7 shows how increasing gross positions deliver even higher payoffs when lenders's risk aversion is higher.

It is important to reiterate that it is long maturity that generates different payoffs across repayment states. As should be clear from equation (11), if $\delta = 1$, the transfer to repayment states would be constant and equal to $\partial \tilde{a} / \partial b' - 1$, and thus payoffs would be identical in repayment states (the government would not be able to transfer resources across states in which it repays its debt).

Panel (b) of Figure 7 shows that in repayment states, consumption is increasing in next-period income. Together, panels (a) and (b) show that issuing bonds to buy reserves transfers more resources to states with higher marginal utility. This is our key result regarding the insurance properties of issuing debt to accumulate reserves.

Figure 8 displays how an increase in gross positions today reduces the volatility of next-period consumption. To construct this plot, we assume that the government may deviate in the current period from the optimal portfolio while keeping current consumption at the optimal level. From next period onward, the government continues to choose according to the policy functions in the Markov equilibrium. The figure shows that when the government issues more debt to accumulate additional reserves, it reduces the standard deviation of next-period consumption in repayment states. (We focus on the volatility of consumption in repayment states to stress the importance of the insurance mechanism that we highlight.) By issuing debt and accumulating reserves today, the government transfers resources to more adverse states of nature and thus manages to improve consumption smoothing. In contrast, with one-period bonds the volatility of consumption conditional on repayments would be constant across reserve holdings.

### 4.6.3   Higher borrowing costs

We next establish that issuing debt to accumulate reserves is costly. In particular, we show that issuing debt to accumulate reserves can have negative effects on spreads, which imposes a cost on the government.

To allow for a sharper analytical characterization, let us assume that the government is risk neutral. In this way, we shut down the value from insurance described above, and isolate the effects of gross positions on incentives. Formally, we evaluate equation (10) but assuming a constant marginal utility normalized to one. Let us also define $\tilde{q}(b', x) = q(\tilde{a}(b', x), b', s)$, which expresses the bond price at which the government can issue debt for portfolios that satisfy (7). We denote by $\frac{d\tilde{q}}{db'}$ the total derivative of the bond price with respect to bond issuances, which includes both the direct effect of higher debt on the bond price, and the indirect effect through

the purchases of reserves with the newly issued bonds.

The following condition characterizes the net benefits from increasing gross positions (see Appendix A for a derivation):

$$\frac{d\mathbb{E}_{s'|s}V\left(\tilde{a}, b', s'\right)}{db'} \leq \frac{i}{q_a}\frac{d\tilde{q}}{db'}. \tag{12}$$

Equation (12) shows that the welfare effect of issuing debt to accumulate reserves depends on the effect of this operation on the bond price and on the level of debt issuances. If the bond price decreases when the government issues debt to buy reserves ($d\tilde{q}/db' < 0$), a risk-neutral government issuing debt ($i > 0$) prefers strictly lower reserves and chooses $a' = 0$. The intuition for this result is similar to the one of Aguiar et al. (2016a) in their analysis of maturity management. In the absence of a value from insurance, the sovereign's goal is to minimize borrowing costs, which is achieved in our model by holding zero reserves.[25]

Equation (12) was derived assuming the government is risk neutral, but it remains generally true that the effect of changing gross financial positions on the borrowing cost is a key component of the government's portfolio decision. When the government borrows more, it expands the set of states in which it will not repay next period. Together with these gains, however, there are higher costs from defaulting. In fact, the marginal benefits from increasing the set of states in which the government does not pay its debt are exactly compensated by the marginal cost of increasing the set of states in which the government pays the default cost. At the same time, if investors perceive a higher probability of default, they offer bonds to the government at a lower price. As a result, issuing debt to accumulate reserves imposes a cost if sovereign spreads deteriorate.

**Numerical illustration of gross positions and borrowing costs.** Figure 9 displays how changes in gross positions affect spreads. In particular, it shows how issuing debt to accumulate reserves indeed increases the cost of borrowing ($d\tilde{q}/db' < 0$) for the government.

Panel (a) presents the combination of debt and reserves that delivers the equilibrium level of consumption. The figure assumes initial portfolios equal to the mean, low lenders' risk aversion, and current income either equal to the mean or one standard deviation above the mean. The vertical axis presents possible debt levels $b'$, and the horizontal axis displays the level of reserves that can be purchased by issuing $b'$: $a' = \tilde{a}(b', x)$. These "isoquants" have a positive slope: the more debt issued, the more reserves the government can buy. The magnitude of the slope

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[25]Aguiar et al. (2016a) show that the sovereign issues only short-term debt because of the adverse price effects of managing the long-term bond position. Their model features i.i.d. shocks to the cost of defaulting and no income shocks. Somewhat analogously, we shut down the insurance motive by assuming risk neutrality.
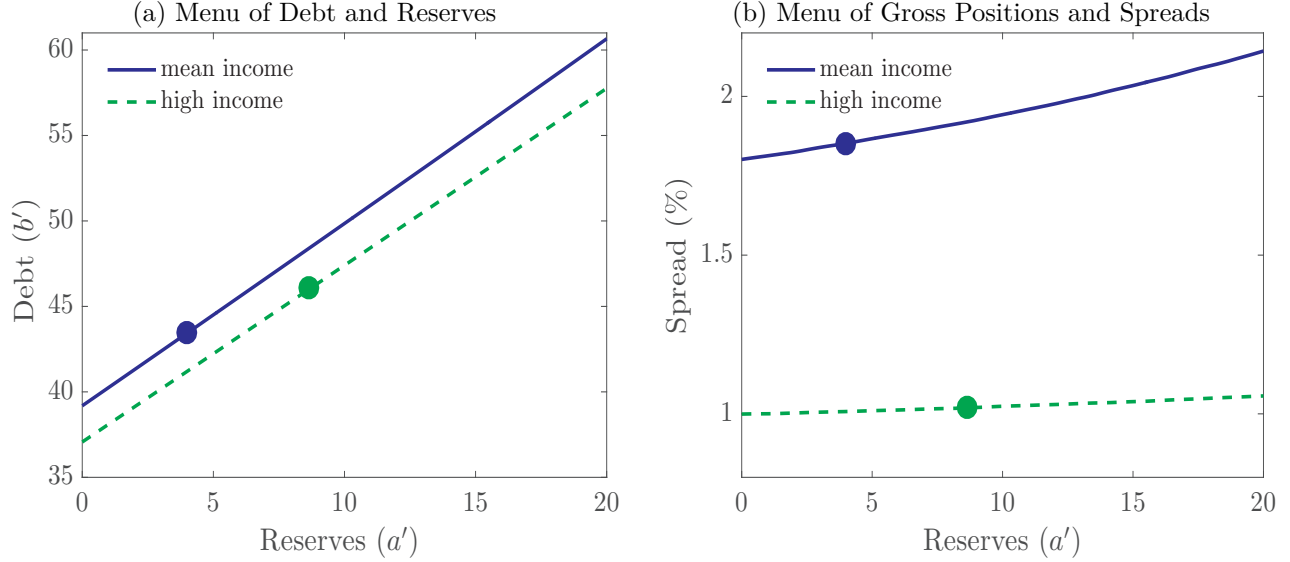
Figure 9: Issuing Debt to Accumulate Reserves.

Note: Panel (a) presents combinations of reserve and debt levels that would allow the government to finance the equilibrium level of consumption. Panel (b) presents the spread the government would pay for each combination in panel (a) (with each combination identified by the level of reserves). The figure assumes that the initial levels of debt and reserves are equal to the mean levels in the simulations, the lenders' risk aversion is low, and income is either equal to the mean level in the simulations or one standard deviation above this mean level. Solid dots represent the actual portfolios chosen $(\hat{b}(a, b, s), \hat{a}^R(a, b, s))$ and associated levels of spreads. The levels of reserves and debt are expressed as a percentage of mean annual income.

is determined by equation (8), which is a function of how sensitive bond prices are to gross positions. Solid dots in this panel represent the actual portfolio chosen by the government $(\hat{b}(a, b, s), \hat{a}^R(a, b, s))$ given the initial states considered. To complement these isoquants, panel (b) shows that issuing debt to finance the accumulation of reserves increases the spread that the government has to pay. This underscores how increasing gross positions is costly.

Figure 9 also shows that reserve accumulation is less costly when income is high. As panel (b) shows, the sensitivity of the spread to increases in gross positions decreases with income. When income is at the mean value, increasing reserves from 0 to 5 percent of mean income raises spreads from 1.7 percent to 1.8 percent. When income is higher, not only is the spread lower, but the response of the spread to an increase in gross positions is more modest, as reflected by the flatter dashed curve in panel (b). Thus, when income is higher, the lower sensitivity of spreads to gross positions induces the government to buy more insurance by increasing its debt and reserve

positions (as shown before in Figure 6).[26]

## 4.7 Sensitivity

We next study the quantitative importance of long-term debt and shocks to the lenders' risk aversion, and present the sensitivity of our results to other features of the model.

### 4.7.1 One-period bonds

We now show that having bonds with maturity exceeding one period is essential for obtaining high levels of reserves in the simulations. This enables us to quantify the use of reserves to insure against rollover risk and contrast our results with those presented by Alfaro and Kanczuk (2009).

As analyzed in Section 4.6, when debt is long term, issuing debt to accumulate reserves provides higher payoffs in repayment states with lower income and consumption. These insurance benefits from reserve accumulation do not arise with one-period debt. With $\delta = 1$, equation (10) is reduced to:

$$\frac{d\mathbb{E}_{s'|s}V\left(\tilde{a},b',s'\right)}{db'} = \overbrace{\frac{\partial\tilde{a}}{\partial b'}\mathbb{E}_{s'|s}\left[u'(c')d'\right]}^{\text{Net marginal utility in default states}} + \overbrace{\left(\frac{\partial\tilde{a}}{\partial b'} - 1\right)\mathbb{E}_{s'|s}\left[u'(c')(1-d')\right]}^{\text{Net marginal utility in repayment states}}. \tag{13}$$

In equation (13), payoffs in repayment states are constant and equal to $\frac{\partial\tilde{a}}{\partial b'} - 1$. Issuing debt to accumulate reserves does not provide the government with higher payoffs in states in which income is low and the government finds it optimal to repay the debt.

With one-period debt, reserves still provide some spanning. Higher gross positions still allow the government to transfer resources between repayment and default states.[27] As we show next, however, the net benefits from accumulating reserves, based on this alternative insurance role, are not quantitatively important. This finding is in line with the results presented by Alfaro and Kanczuk (2009).

To evaluate the quantitative role of reserves in the one-period-bond version of the model ($\delta = 1$), we change the value of four parameters to match the same targets that we match in our

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[26]The relatively modest increase of the spread when the government increases gross positions is not specific to the states considered in Figure 9. If the government deviates from the optimal portfolio by increasing its debt stock by 1 percent of aggregate income and allocates the extra proceeds to purchase reserves, the mean increase in the spread in the simulations would be equal to 3.4 basis points.

[27]Notice that if we were to assume that reserves are lost in case of default, issuing debt for accumulating reserves would serve no value when debt is one-period. This would not be the case, however, with long-term debt.

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Table 4: Sensitivity Analysis

|  | Benchmark | One-Period Bonds | No Risk Prem. Shock | 4-Year Exclusion | Income Cost | Lower Debt |
|---|---|---|---|---|---|---|
| std(cons) / std(y) | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| Mean (debt/y) | 43.5 | 42.6 | 44.5 | 44.2 | 46.0 | 26.6 |
| Mean spread | 2.4 | 2.3 | 2.4 | 2.3 | 2.2 | 2.4 |
| $\Delta$ Spread for $\kappa = \kappa_H$ | 2.0 | 2.0 | 0 | 1.9 | 1.7 | 2.1 |
| **Mean (reserves/y)** | **6.0** | **0.3** | **3.0** | **5.7** | **5.4** | **2.8** |

baseline calibration.[28] Table 4 shows that with one-period bonds, the average reserve ratio in the simulations falls to 0.3 percent. This is consistent with the zero reserve accumulation obtained by Alfaro and Kanczuk (2009). Again, the fact that reserves are close to zero with one-period bonds implies that the insurance value of transferring resources from repayment states to default states is not quantitatively important. In a nutshell, the insurance value of transferring resources from repayment states to default states is outweighed by the costs of facing higher spreads.

The differences in the equilibrium accumulation of reserves with one-period and long-duration bonds highlight the importance of debt maturity in understanding the role of reserves. Reserves provide insurance against rollover risk only if bonds' maturity exceed one period and the model with the debt maturity calibrated to match the data predicts a significant level of reserve accumulation.

However, the comparison between the benchmark economy and the one-period-bond economy not only reflects distinct insurance properties, but also differences in risk aversion and the equilibrium consumption, default, and bond price functions. With the goal of isolating the differences in the insurance properties, we present a counterfactual exercise that allows us to quantify the importance of incentives to hedge against rollover risk in the benchmark while abstracting from the other differences.

Consider first the Euler equation for debt accumulation (presented in equation A.8). We can decompose the marginal cost of issuing debt into an expected cost and a covariance term as

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[28]The recalibrated parameters are as follows: the parameters that affect the cost of defaulting are $\alpha_0 = 15.7$ and $\alpha_1 = 175.0$, the high level of lenders' risk aversion is $\kappa_H = 10.5$, and the risk aversion of domestic consumers is $\gamma = 4.5$. All other parameter values are the same as those assumed in the benchmark calibration. Table 4 shows that these parameter values allow the one-period-bond model to match the calibration targets well.

follows:

$$u'(c) \left[ q + \frac{\partial q(b', a', s)}{\partial b'} i \right] = \beta \mathbb{E}_{s'|s}[1 - d'] \mathbb{E}_{s'|s, d'=0} \left[ \delta + (1 - \delta) q' \right] \mathbb{E}_{s'|s, d'=0} \left[ u'(c') \right]$$

$$+ \beta \mathbb{E}_{s'|s}[1 - d'] \mathbb{COV}_{s'|s, d'=0} \left( \delta + (1 - \delta) q', u'(c') \right). \qquad (14)$$

The exercise consists of solving for the level of reserves consistent with: (i) equation (14) after setting the covariance term to *zero*, in addition to (ii) the Euler equation for reserve accumulation (presented in equation A.9) and (iii) the equilibrium bond price schedule and next-period default and consumption rules of the benchmark economy. Notice that after setting the covariance term to zero, equation (14) becomes analogous to the Euler equation for debt in the economy with one-period bonds. Recall that the positive covariance between the next-period bond price and next-period consumption (illustrated in Figure 7) makes issuing debt to accumulate reserves a useful hedge against rollover risk. By setting the covariance term in equation (14) equal to zero, the government in this exercise does not perceive the benefits of accumulating reserves to hedge against rollover risk.
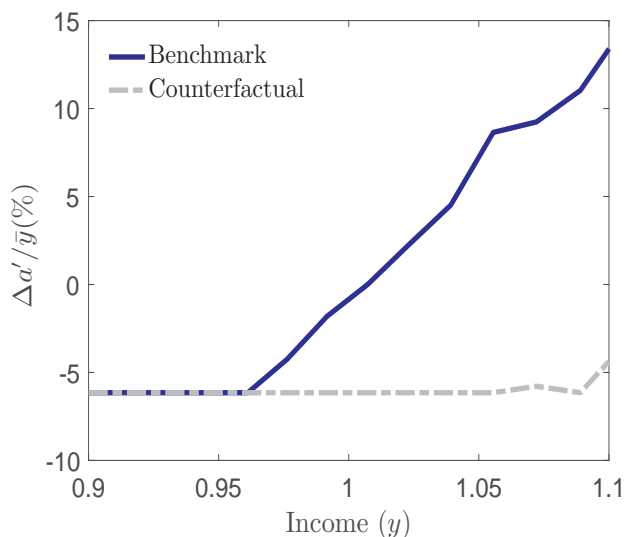


Figure 10: Counterfactual exercise.
Note: This figure compares the accumulation of reserves in the benchmark and counterfactual economies in which the government does not accumulate reserves for hedging against rollover risk.

Figure 10 shows that in the counterfactual economy, unless income is sufficiently high, the government chooses $a' = 0$. Moreover, when income is sufficiently high, the government chooses levels of reserves that are much lower than in the benchmark economy. Overall, this illustrates how virtually all the equilibrium accumulation of reserves in the benchmark is explained by

incentives to hedge against rollover risk.[29]

### 4.7.2 Role of Risk-Premium Shocks

The goal in this section is to assess the importance of the shock to lenders' risk aversion. To do this, we eliminate this shock by assuming that the pricing kernel is $m(s, s') = e^{-r}$ in every period. We recalibrate the parameters that affect the cost of defaulting ($\alpha_0 = 2.16$ and $\alpha_1 = 18.10$) to match the levels of sovereign debt and spread in the data matched in the benchmark calibration. All other parameter values are the same as those assumed in the benchmark calibration. Table 4 shows that the average holding of reserves in the simulations of the recalibrated model without shocks to the risk premium is half that of the benchmark simulations. This implies that the lack of a risk-premium shock significantly reduces the need for reserves to insure against rollover risk, but reserves continue to play an important role in the model without this shock.

### 4.7.3 Other Sensitivity Analysis

**Exclusion after Defaulting.** This section shows the robustness of the model predictions to assuming the government suffers a longer exclusion from debt markets after defaulting. As is standard in the sovereign default literature, we assume that in each period after the default, a government in default may regain access to debt markets with a constant probability. We calibrate this probability to obtain an expected duration of the exclusion from debt markets of four years. We keep all other parameter values as in the benchmark calibration.

Table 4 shows that assuming a defaulting government expects to be excluded from debt markets for four years (instead of for one year as in the benchmark) does not produce substantial changes in the simulations (since this is the case, we do not recalibrate the model to illustrate the effects of the post-default exclusion from debt markets). Since defaults occur after periods of low income, the government has typically depleted most of its reserve holdings before defaulting. The average reserve-to-income ratio when the government defaults is 0.6 percent. Thus, even though a longer exclusion increases the insurance value of defaulting with reserves, the fact that in equilibrium defaults occur in states with low reserves limits the effects of longer exclusions.[30]

**Income Cost of Defaulting.** This section shows the robustness of the model predictions to assuming that defaulting triggers a loss of income. Following Aguiar and Gopinath (2006), we

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[29]Since we rely on the Euler equations to solve for the portfolio in the counterfactual exercise, we also computed the optimal debt and reserve accumulation using the Euler equations for the benchmark. We found policies almost identical to the ones obtained using a global solution to the optimization problem (see Figure 13 in the appendix).

[30]In addition, notice that the sensitivity of the default cost to reserves is limited by the fact that reserves can be acquired during default (exclusion only applies to borrowing).

assume that defaulting triggers a loss of 5% of income.[31] We keep all other parameter values as in the benchmark calibration because, as Table 4 shows, introducing this income cost of defaulting has only modest effects on the statistics targeted in the calibration.

Table 4 shows that the level of reserve holdings is somewhat lower with the income cost of defaulting. Because reserves transfer resources to default periods, a lower income in default states implies that reserves are more effective in increasing utility in default periods, thus lowering the cost of defaulting. Therefore, issuing debt to accumulate reserves increases more the borrowing cost. As discussed in Section 4.6, this makes the accumulation of reserves more costly. Table 4 shows that, nevertheless, the optimal level of reserve holdings is still significant when we incorporate into the model a plausible income cost of defaulting.

**Debt Target.** This section discusses how the level of reserves in the simulations varies when we change the debt level targeted in the calibration. As we discussed above, choosing the debt target from the data requires some compromises, as there may be several valid ways to map the model to the data. To illustrate how equilibrium reserve holdings would change with a different debt target in the calibration, we recalibrate the model targeting a debt level of 26.6%, the average level of external debt in Mexico. We continue to target the same average spread, consumption volatility, and spread increase with high risk premium targeted in the benchmark calibration. The last column of Table 4 presents the simulation results obtained with the lower debt target ($\alpha_0 = 1.47$, $\alpha_1 = 10.77$, and $\kappa_H = 25.5$). As expected, with lower debt levels, the level of reserve holdings needed to insure against rollover risk is lower although reserves as a proportion of debt remains very close to the baseline.

## 4.8 Understanding the Upward Trend in Reserves

In this section, we analyze through the lens of our model how three developments in emerging markets can deliver the upward path in reserves observed in the data since the 1990s, as presented in Figure 2. We consider the following developments: (i) *income windfalls*, (ii) *a reduction in political myopia*, and (iii) *a increase in rollover risk*. Before providing an empirical background for these developments and describing how we map them into our model, we explain how we assess their effects.

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[31] Asonuma and Trebesch (2016) document that sovereign debt crises are associated with 2% lower annual GDP growth in the first two crisis years, consistently with the findings of previous studies.

**Trend Experiments.** To investigate possible sources of the upward trend in reserves, we feed into the model shocks or changes in structural parameters, associated with each development, and conduct transitional dynamics according to the following these four steps. First, we simulate the benchmark economy 10,000 times for 25 years, starting with low lenders' risk aversion, average levels of debt, and reserves, and initial values for income weighted by its stationary distribution. Second, we average the path of endogenous variables across the 10,000 samples.[32] Because we are starting with the mean level of debt and reserves, the average paths of these variables remain close to the initial values. Third, we simulate economies with each of the three developments mentioned above, following the same steps and starting at the same state used for the simulations of the benchmark economy. Finally, we report the differences between the average paths for the simulations of the economies with each of the three developments and the average paths for the benchmark economy.

### 4.8.1 Developments in Emerging Markets

We now describe the empirical background for the three developments and how we map them into our model.[33]

**(i) Income Windfalls.** As documented by Adler and Magud (2015), the boom in commodity prices led to large increases in real income in emerging markets. The median income windfall represented 60 percent of GDP in the last pre-boom year and lasted on average for seven years (see Table 1 in Adler and Magud, 2015).

To compute the effect of income windfalls in the accumulation of reserves, we select among the whole set of simulations those in which real income is on average around 60 percent (plus/minus 1%) above trend for the initial seven years of the simulations. We later present results for simulations in which real income is on average between 0 and 75 percent above trend for the initial seven years.

**(ii) Reduction in political myopia.** Various indicators, such as the reduction in the pro-cyclicality of fiscal and monetary policy (Vegh and Vuletin, 2012), point to an improvement in government policies in emerging markets. In fact, part of the apparent resilience of emerging markets during the Great Recession has been attributed to the increase in fiscal space that was

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[32]When averaging across samples, we exclude samples with default episodes, but results would be very similar without excluding these samples.

[33]Note that while we used Mexico for calibrating the model, we do not focus on the importance of each development for Mexico, and we discuss instead these developments for a broader set of emerging economies. As explained before, our calibration is broadly appropriate for analyzing other emerging economies.

generated because of less expansionary policies in the boom periods (Frankel et al., 2013). Furthermore, between 1991 and 2014, 78 countries adopted fiscal rules and 28 countries established independent fiscal councils (Debrun and Kinda, 2014).

We model a reduction in political myopia as an increase in the time discount factor of the government, which is a common way to account for political economy aspects in emerging economies (Aguiar et al., 2009). To illustrate the effects of a reduction in political myopia, we show simulation paths for $\beta = 0.94$ (Figure 11) and later present results for a range of discount factors between 0.92, the benchmark value, and 0.95 (Figure 12).

**(iii) Increases in rollover risk.** The final development we consider is an increase in the severity of global disruptions to financial markets. As documented by Chinn and Ito (2006), capital openness in emerging economies rose in the 1990s, which coincides with the beginning of the buildup of reserves. Especially following the East Asian crisis, there was a widespread view that emerging markets needed to self-insure against financial market turmoil by accumulating reserves (Feldstein, 1999), and empirical analysis (e.g., Aizenman and Lee, 2007, Calvo et al., 2012) tends to support that this motivation indeed played an important role in policy decisions.

To incorporate this development into the model, we consider a bigger risk premium shock. We first adjust $\kappa_H$ to obtain an average increase in spreads during high risk-premium episodes 2 percentage points higher than in the benchmark (this gives us an average increase in spreads that is comparable to the increase in the 'stripped' EMBI spread around the East Asian crisis). We later present results for spread increases during high risk-premium episodes between 0 and 2 percentage points higher than in the benchmark.

### 4.8.2 Results of Transition Experiments

Figures 11 and 12 present the results of the transition experiments. The figures present the difference between the levels of reserves, debt, and spreads in the simulations of the benchmark economy and the simulations of an economy with each of the three developments described above. Figure 11 presents the difference for each year of the simulations. Figure 12 presents the maximum difference throughout the simulations for a range of shock values.

The first development we consider is the income windfall. As panel (a) of Figure 11 shows, the increase in reserves is quite significant throughout the windfall phase, reaching 12 percentage points. Once the windfall is over, the increase in reserves reverts back, somewhat slowly, to zero. In addition, the debt-to-income ratio fall in response to the windfall (panel d) and the same
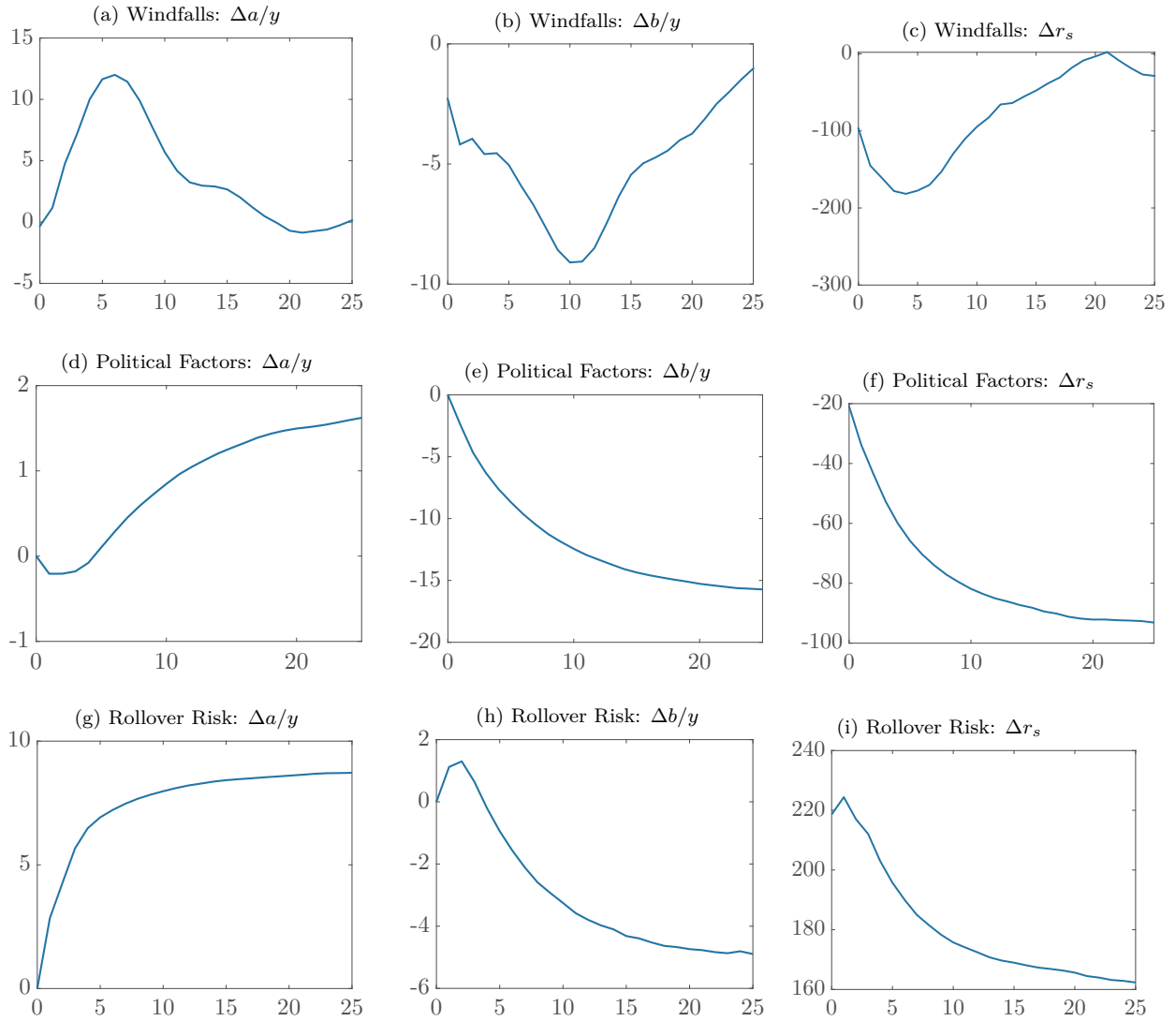
Figure 11: Transitional Dynamics

Note: The figure presents the difference between the average value of each variable in the simulations with each development minus the average value in the benchmark simulations. Differences in the reserves and debt to income ratios are expressed in percentage points. Differences in spreads are expressed in basis points.

occurs with spreads (panel g).[34]

The second development we address is the decrease in political myopia. The second column of Figure 11 shows that a more patient government accumulates more reserves, reduces debt levels, and faces lower spreads. Notice that in general the effect of the government's patience on the level of reserves is ambiguous. A rise in $\beta$ implies the government wants to transfer more resources from the present to the future, but this can be done by raising reserves or lowering

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[34]Notice that debt increases over the simulations but at a lower rate than income. Consistently with our discussion of optimal policy, when borrowing opportunities are more favorable, the government issues debt to finance the accumulation of reserves.

debt. Moreover, lower debt levels reduce rollover risk and, thus, the need for reserves. On the other hand, a higher $\beta$ also implies that the government becomes more concerned about future fluctuations in consumption and, therefore, has stronger incentives to self-insure by accumulating reserves. In our simulations, the second effect is dominant and, in spite of a large decline in debt levels, more patient governments choose to have more reserves. A notable difference of this development relative to the other two that we consider is that the increase in $\beta$ implies a change in debt positions that is much larger than the change in reserves.[35] This response is not specific to the change in $\beta$ considered in Figure 11 but applies to a wide range of values, as Figure 12 shows.

The third and final development we address is the increase in rollover risk. This development leads to a stronger precautionary demand for reserves. The emergence of higher rollover risk deteriorates the spread schedule as the future government's repayment incentives weakens, leading to an increase of spreads on impact (panel i). On impact, the government borrows more to finance the accumulation of reserves. Over time, the government ends up reducing the stock of debt, and keeping a larger buffer stock of reserves, which is close to 10 percentage points above the initial value.
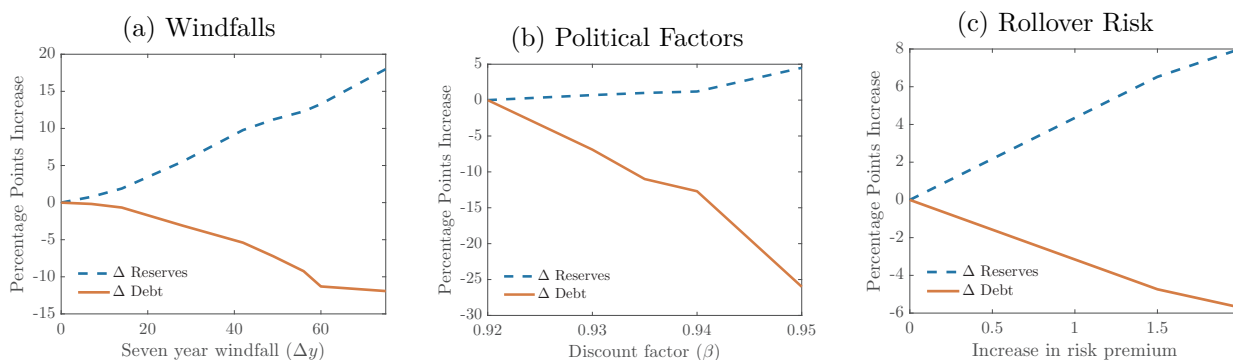


Figure 12: Sensitivity of Transitional Dynamics

Note: The figure presents the maximum difference across the simulation paths in the average levels of reserves and debt.

The relevance of each of these three developments in accounting for changes in the level of reserves would clearly depend on the country under consideration. For some countries, income windfalls were significant, for some others, they were not. Likewise, variations in institutions and political economy aspects were heterogeneous across countries. The increase in rollover risk was a global phenomenon, but still, riskier countries should have been more affected by this increase.

While the income windfall explanation is more directly quantifiable, measuring the reduction

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[35]In the data (Figure 2), governments have adjusted relatively more their asset than their liability positions.

in political myopia or the increase in rollover risk is more challenging. Nonetheless, one way to disentangle how plausible is each of these explanations is to examine their implications for the evolution of other variables in the model vis-à-vis the data. Namely, we can use the predictions of the model for debt and spreads to differentiate these two explanations. For example, as illustrated in Figures 11 and 12, our model predicts that countries that were more influenced by a decline in political myopia, should experience a larger decline in debt. Likewise, countries that were more affected by increases in rollover risk, should have experienced a larger increase in spreads. As Appendix B shows, there is significant heterogeneity in the time-series of reserves, debt, and spreads, which suggests that the importance of these developments vary across countries. Overall, a combination of the three developments presented in this section could easily account for the upward trend in reserve holdings in emerging economies.[36] A promising area for future research is to establish quantitatively the contribution of these empirical developments in the cross-section of countries.

# 5 Conclusions

We use a quantitative sovereign default model to study the optimal accumulation of international reserves. The model features long-term debt, risk-free assets, and shocks to the risk premium demanded by foreign creditors. We show that the optimal policy is to borrow and accumulate reserves in good times to hedge against future increases in borrowing costs. Allowing for the maturity of bonds to exceed one period is essential for uncovering the role of reserves as a hedge against rollover risk.

The model is calibrated to mimic salient features of a typical emerging economy, including the average public debt and spread levels. Model simulations display an average optimal reserve ratio that is equal to 6 percent of income. The reserve ratio reaches values close to 40 percent in some simulation periods. Furthermore, the government accumulates (decumulates) both reserves and debt in periods of low (high) spread and high (low) income, which we show is consistent with the behavior of debt and reserves in a majority of emerging economies.

The paper also shows that three recent developments in emerging economies (income windfalls, improved policy frameworks, and an increase in the severity of global shocks) imply significant increases in the optimal holding of reserves, which is consistent with the upward trend that has occurred since the 1990s. These developments also imply lower optimal debt levels, in line with what was observed in the data.

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[36]It should be noted that recently, developed economies have started accumulating more reserves, which appears to be related to the zero lower bound constraint on nominal interest rates (Amador et al., 2016).

Future research could explore how self-fulfilling crises, sovereign debt composition and private capital flows affect governments' management of international reserves. The mechanisms studied in this paper are not confined to sovereign debt markets and could be relevant for understanding the financial decisions of households and corporate borrowers facing rollover risk.

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# A   Further Details on Section 4.6

**Derivation of (10)**   To derive (10), we use the envelope theorem to obtain

$$\frac{\partial V^R(a,b,s)}{\partial b} = -u'(c)[\delta + q(1-\delta)] \tag{A.1}$$

$$\frac{\partial V^D(a,b,s)}{\partial b} = 0 \tag{A.2}$$

$$\frac{\partial V^R(a,b,s)}{\partial a} = u'(c) \tag{A.3}$$

$$\frac{\partial V^D(a,b,s)}{\partial a} = u'(c). \tag{A.4}$$

Differentiating (9), we obtain

$$\frac{d\mathbb{E}_{s'|s}V(\tilde{a},b',s')}{db'} = \frac{\partial \tilde{a}}{\partial b'}\frac{\partial \mathbb{E}_{s'|s}V(a',b',s')}{\partial a'} + \frac{\partial \mathbb{E}_{s'|s}V(a',b',s')}{\partial b'} \tag{A.5}$$

Using (A.1)-(A.4),(V) and A.5, we obtain (10) in the text.

  **Optimality conditions.** An alternative derivation of (10) uses the Euler equations. Taking first-order conditions with respect to $b'$ and $a'$, we first obtain

$$b' :: \qquad u'(c)\underbrace{\left[q + \frac{\partial q(b',a',s)}{\partial b'}i\right]}_{\text{Increase of this-period consumption}} = \beta\frac{\partial \mathbb{E}_{s'|s}[V(b',a',s')]}{\partial b'} \tag{A.6}$$

$$a' :: \qquad u'(c)\underbrace{\left[q_a - \frac{\partial q(b',a',s)}{\partial a'}i\right]}_{\text{Decline of this-period consumption}} \geq \beta\frac{\partial \mathbb{E}_{s'|s}[V_a(b',a',s')]}{\partial a'}. \tag{A.7}$$

where A.7 holds with strict inequality if $a' = 0$. Using (A.1)-(A.4), and (V), we obtain the following Euler equations for debt and reserves:

$$b' :: \qquad u'(c)\underbrace{\left[q + \frac{\partial q(b',a',s)}{\partial b'}i\right]}_{\text{Increase of this-period consumption}} = \beta\mathbb{E}_{s'|s}\left[u'(c')\underbrace{[\delta + q'(1-\delta)](1-d')}_{\text{Decline of next-period consumption}}\right] \tag{A.8}$$

$$a' :: \qquad u'(c)\underbrace{\left[q_a - \frac{\partial q(b',a',s)}{\partial a'}i\right]}_{\text{Decline of this-period consumption}} \geq \beta\mathbb{E}_{s'|s}u'(c')\underbrace{\mathbb{1}}_{\text{Increase of next-period consumption}} \tag{A.9}$$

Equation (A.8) equates the benefits of issuing one more bond in the current period to the expected cost of repaying it in the next period. The government issues a bond in exchange for $q$ units of consumption, but also lowers the price of new issuances, reducing proceeds by

$\frac{\partial q(b',a',s)}{\partial b'}i$. The marginal cost of borrowing is given by the cost of paying the coupon that matures in the next period and retiring the remaining $1 - \delta$ units of debt at the market price $q' = q\left(\hat{a}^R(a',b',s'), \hat{b}(a',b',s'), s'\right)$.

Equation (A.9) equates the cost of cutting consumption to buy one extra reserve asset to the benefits of consuming the proceeds from selling that extra reserve asset in the next period. The marginal cost of buying an extra reserve asset differs from $q_a$ because reserve purchases affect the current price at which the government issues debt.

Equations (A.8) and (A.9) can be combined to deliver the following expression:

$$\underbrace{\overbrace{\left(\frac{q + \frac{\partial q(b',a',s)}{\partial b'}i}{q_a - \frac{\partial q(b',a',s)}{\partial a'}i}\right)}^{\text{Mg utility benefit of issuing debt to buy reserves}} \mathbb{E}u'(c')}_{\text{Reserves bought}} \quad \leq \quad \overbrace{\mathbb{E}\left[u'(c')[\delta + q'(1-\delta)](1-\hat{d}')\right]}^{\text{Mg utility cost of repayment}} \tag{A.10}$$

which relates the marginal utility costs of starting next period with one more one unit of debt (right-hand side) to the marginal utility benefits of consuming the reserves bought with that unit of debt (left-hand side). At an interior optimum (i.e., with $a' > 0$), the government has to be indifferent between raising one unit of debt, collecting $q + \frac{\partial q(b',a',s)}{\partial b'}i$, and using the proceeds to buy reserves at a marginal cost of $q_a - \frac{\partial q(b',a',s)}{\partial a'}i$. Notice that rearranging equation (A.10) is equivalent to evaluating (10) at a strictly interior optimum.

**Derivation of (12).** To derive (12), we start from equation (10). Using constant marginal utility, we obtain

$$\frac{d\mathbb{E}_{s'|s}V(\tilde{a},b',s')}{db'} = \frac{\partial \tilde{a}}{\partial b'} - \mathbb{E}_{s'|s}\left[(\delta + (1-\delta)q')(1-d')\right], \tag{A.11}$$

$$\leq \frac{\partial \tilde{a}}{\partial b'} - \frac{q}{q_a}, \tag{A.12}$$

where the inequality in the second line follows from the no-arbitrage conditions and the non-negative risk premium required by foreign lenders.

Using (8), we have

$$\frac{\partial \tilde{a}}{\partial b'} - \frac{q}{q_a} = \frac{\left[q(\tilde{a},b',s) + \frac{\partial q(\tilde{a},b',s)}{\partial b'}i\right]q_a - \left[q_a - \frac{\partial q(\tilde{a},b',s)}{\partial a'}i\right]q}{q_a - \frac{\partial q(\tilde{a},b',s)}{\partial a'}i}$$

$$= \frac{i\left[\frac{\partial q(\tilde{a},b',s)}{\partial b'}q_a - \frac{\partial q(\tilde{a},b',s)}{\partial a'}q(\tilde{a},b',s)\right]}{q_a - \frac{\partial q(\tilde{a},b',s)}{\partial a'}i}. \tag{A.13}$$

We also have that

$$
\begin{aligned}
\frac{d\tilde{q}}{db'} &= \frac{\partial q\left(\tilde{a},b',s\right)}{\partial a'}\frac{\partial \tilde{a}}{\partial b'} + \frac{\partial q\left(\tilde{a},b',s\right)}{\partial b'} \\
&= \frac{q(\tilde{a},b',s) + \frac{\partial q(\tilde{a},b',s)}{\partial b'}i}{q_a - \frac{\partial q(\tilde{a},b',s)}{\partial a'}i}\frac{\partial q\left(\tilde{a},b',s\right)}{\partial a'} + \frac{\partial q\left(\tilde{a},b',s\right)}{\partial b'} \\
&= \frac{\left[q(\tilde{a},b',s) + \frac{\partial q(\tilde{a},b',s)}{\partial b'}i\right]\frac{\partial q(\tilde{a},b',s)}{\partial a'} + \frac{\partial q(\tilde{a},b',s)}{\partial b'}\left[q_a - \frac{\partial q(\tilde{a},b',s)}{\partial a'}i\right]}{q_a - \frac{\partial q(\tilde{a},b',s)}{\partial a'}i} \\
&= \frac{q(\tilde{a},b',s)\frac{\partial q(\tilde{a},b',s)}{\partial a'} - \frac{\partial q(\tilde{a},b',s)}{\partial b'}q_a}{q_a - \frac{\partial q(\tilde{a},b',s)}{\partial a'}i}.
\end{aligned}
\tag{A.14}
$$

Plugging (A.14) into (A.13) and using (A.12), we obtain

$$
\frac{d\mathbb{E}_{s'|s}V\left(\tilde{a},b',s'\right)}{db'} \leq \frac{i}{q_a}\frac{d\tilde{q}}{db'},
$$

which is equation (12) in the main text.

**Comparison of numerical solutions.** Figure 13 presents the equilibrium accumulation of reserves using our baseline algorithm and the one using Euler equations (A.7) and (A.6). The figure shows that the numerical results are extremely similar, a result consistent with the findings presented by Hatchondo et al. (2010) in a model with a singe long-term bond and no assets.
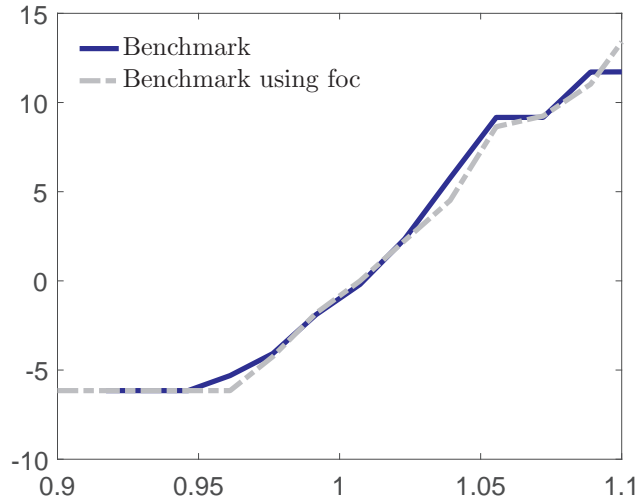


Figure 13: Comparison of Numerical Solutions

Note: The figure presents the equilibrium accumulation of reserves using our baseline algorithm and the one using the Euler equations. Initial values of debt and reserves are equal to the mean levels in the simulations and the lenders' risk aversion is assumed to take the low value. The figure displays income levels for which the government chooses to pay its debt. Changes in reserves are expressed as a percentage of mean annual income.

# Online Apppendix to
# International Reserves and Rollover Risk

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# B    Data on Reserves, Debt and Spreads

We collect here all sources of the data used in the paper.

- Reserves are from the International Financial Statistics.

   http://dsbb.imf.org/images/pdfs/opguide.pdf

- Debt data are from the World Economic Outlook Database for the 2000-2014 period (Figure 1) and from the IMF Fiscal Affairs Department Historical Public Debt Database for the 1980-2014 period (Figure 2)

- Spreads data is from the Emerging Markets Bond Index Plus (EMBI+ blended).

- GDP data is from World Economic Outlook Database

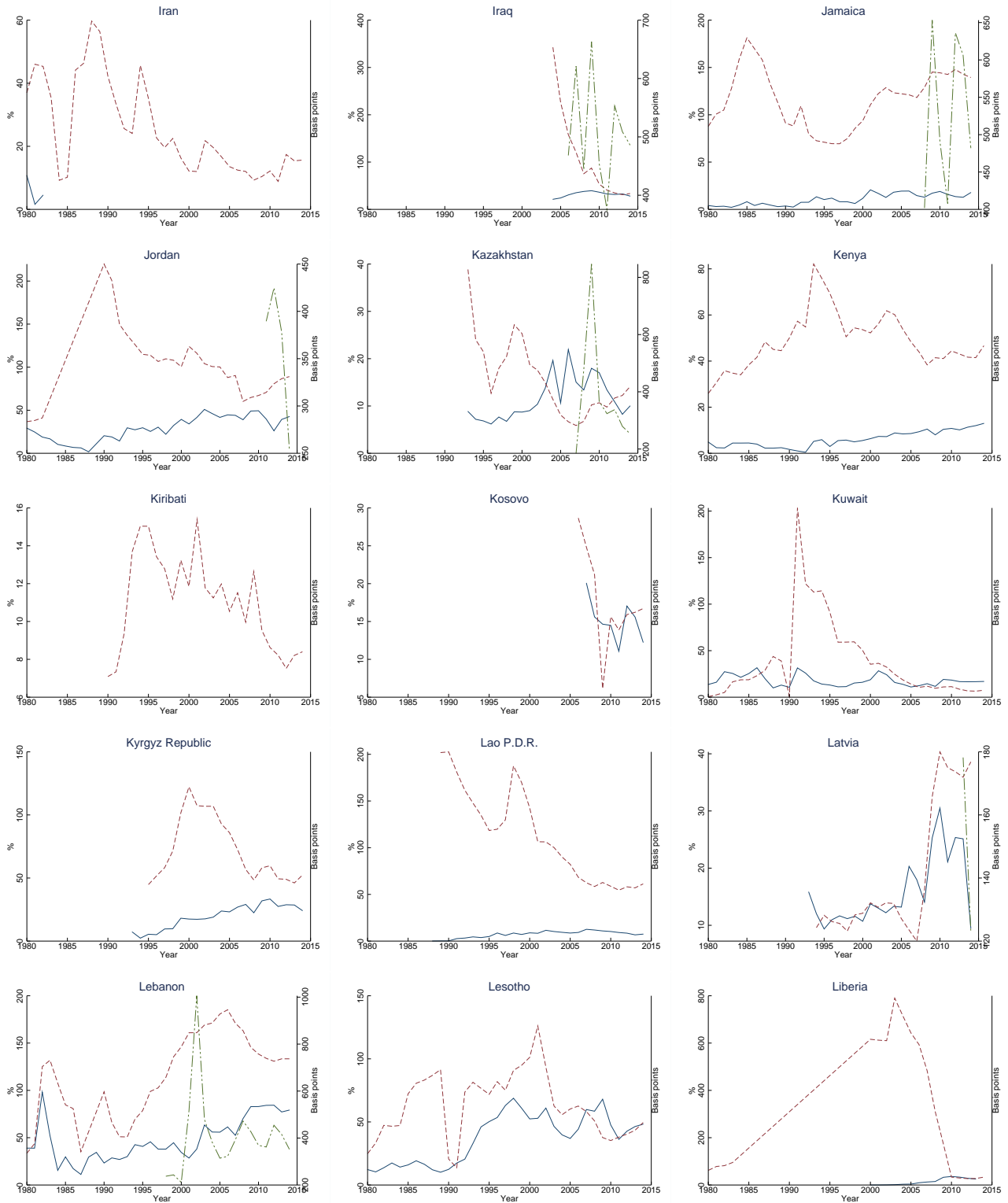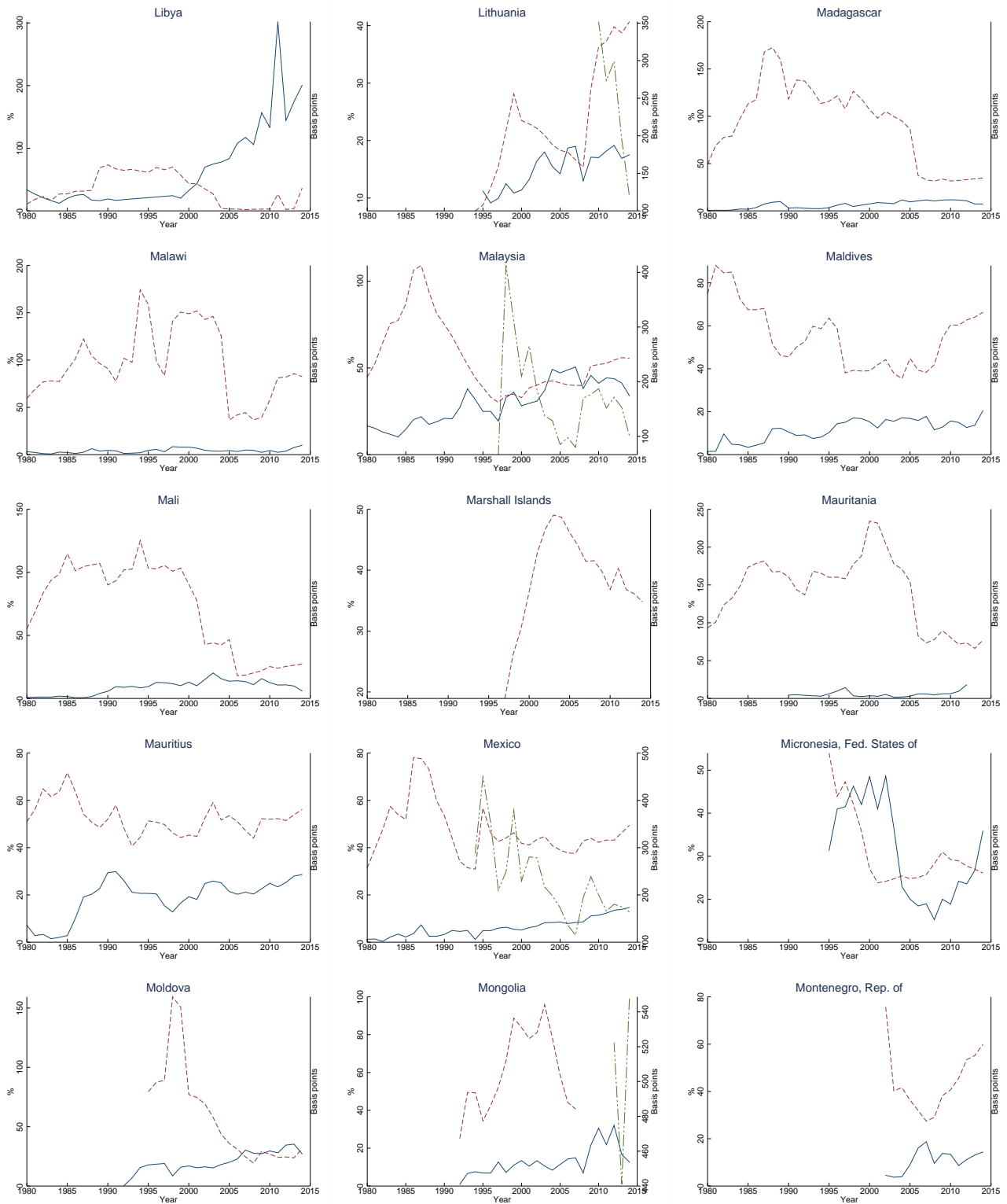   http://www.imf.org/en/Publications/WEO/
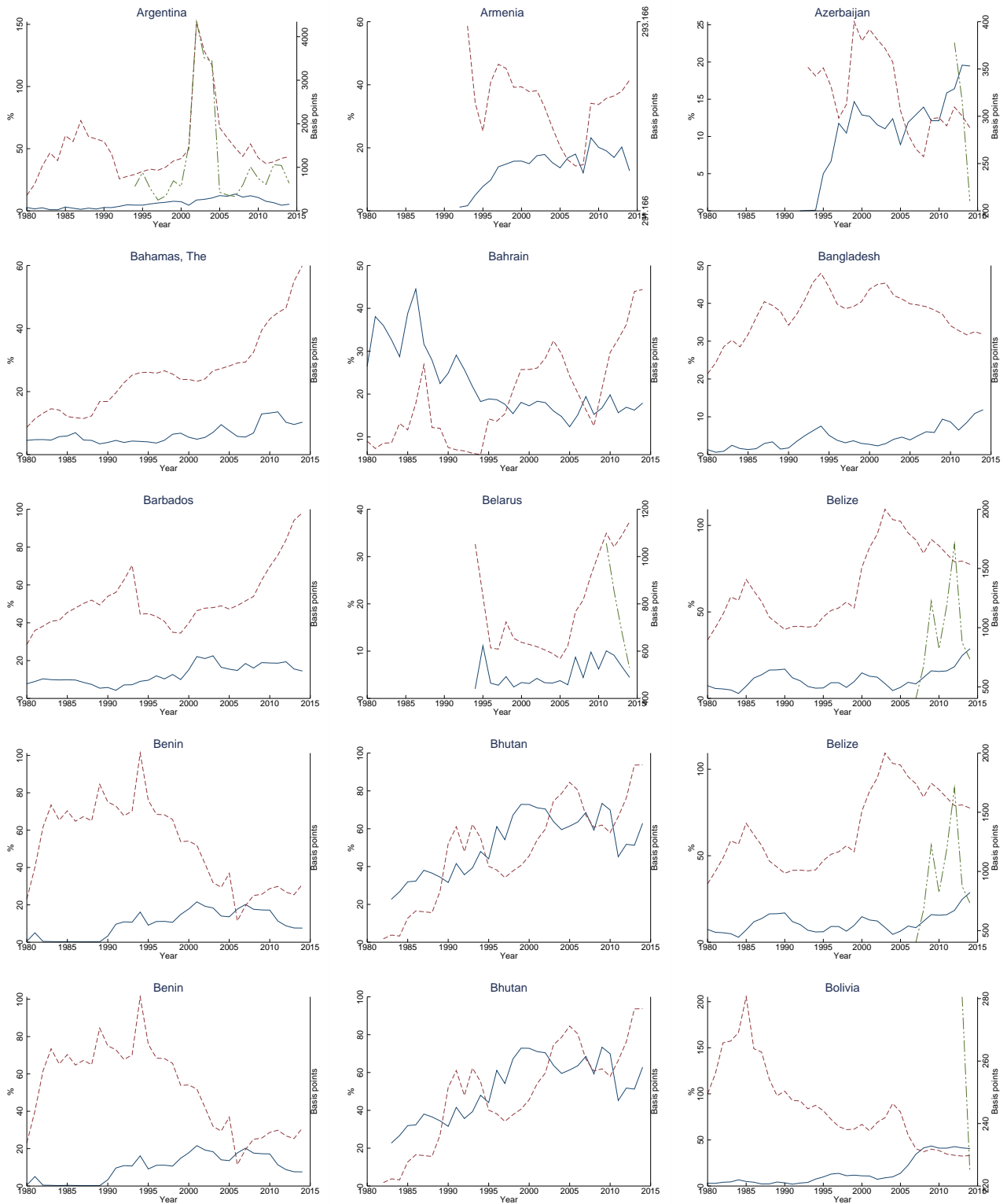
Figure 1: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios.
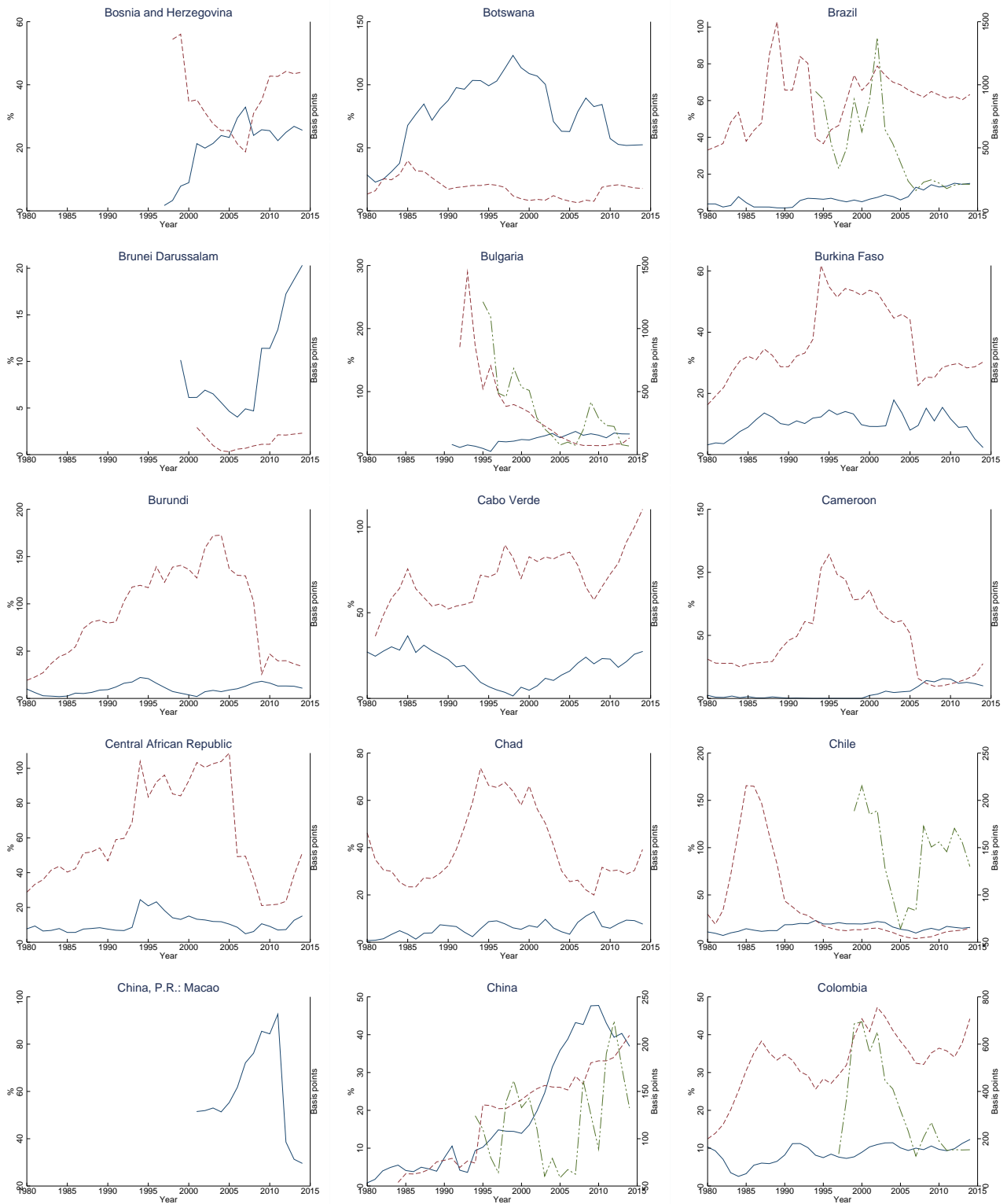
Figure 2: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios.
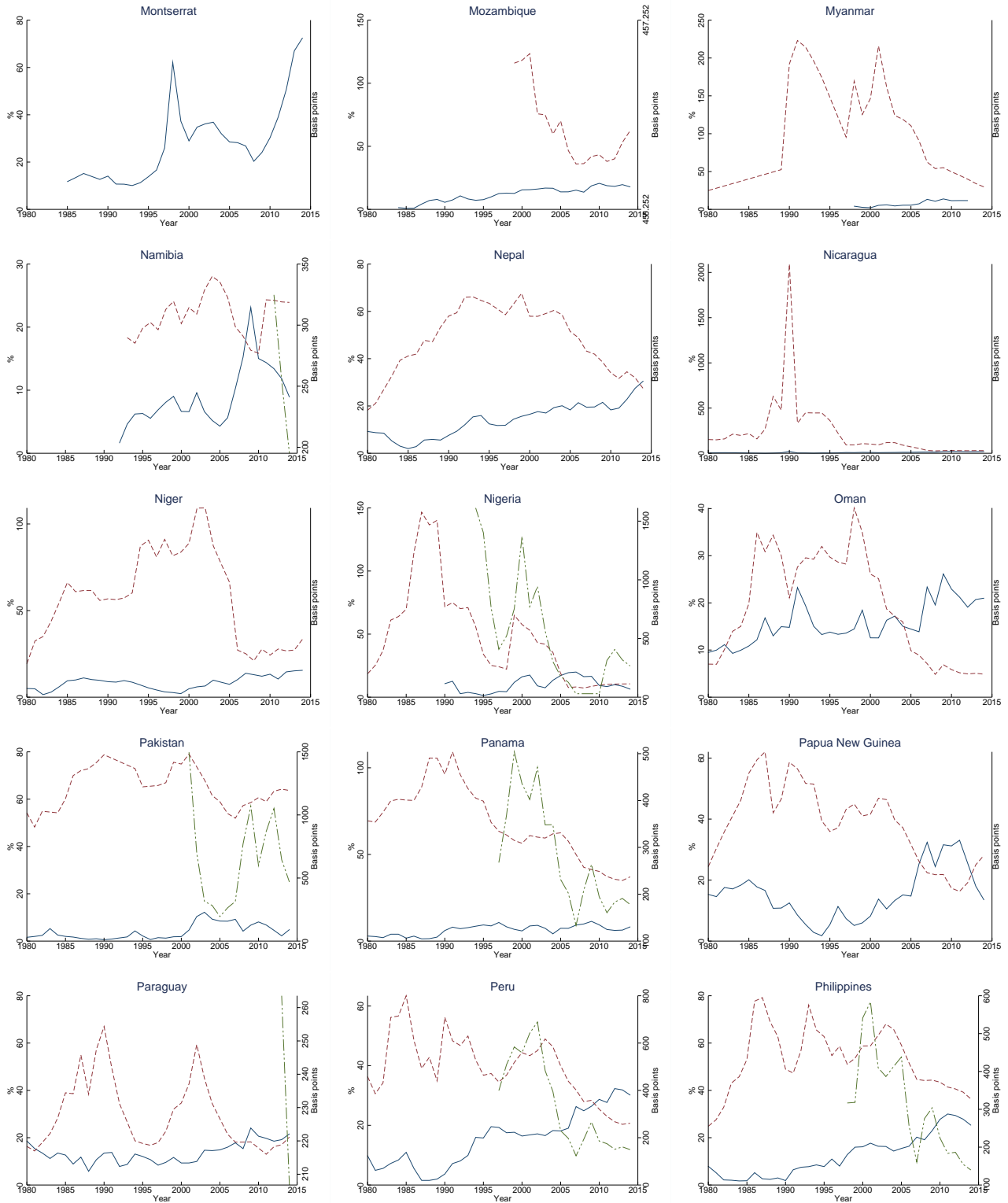
2
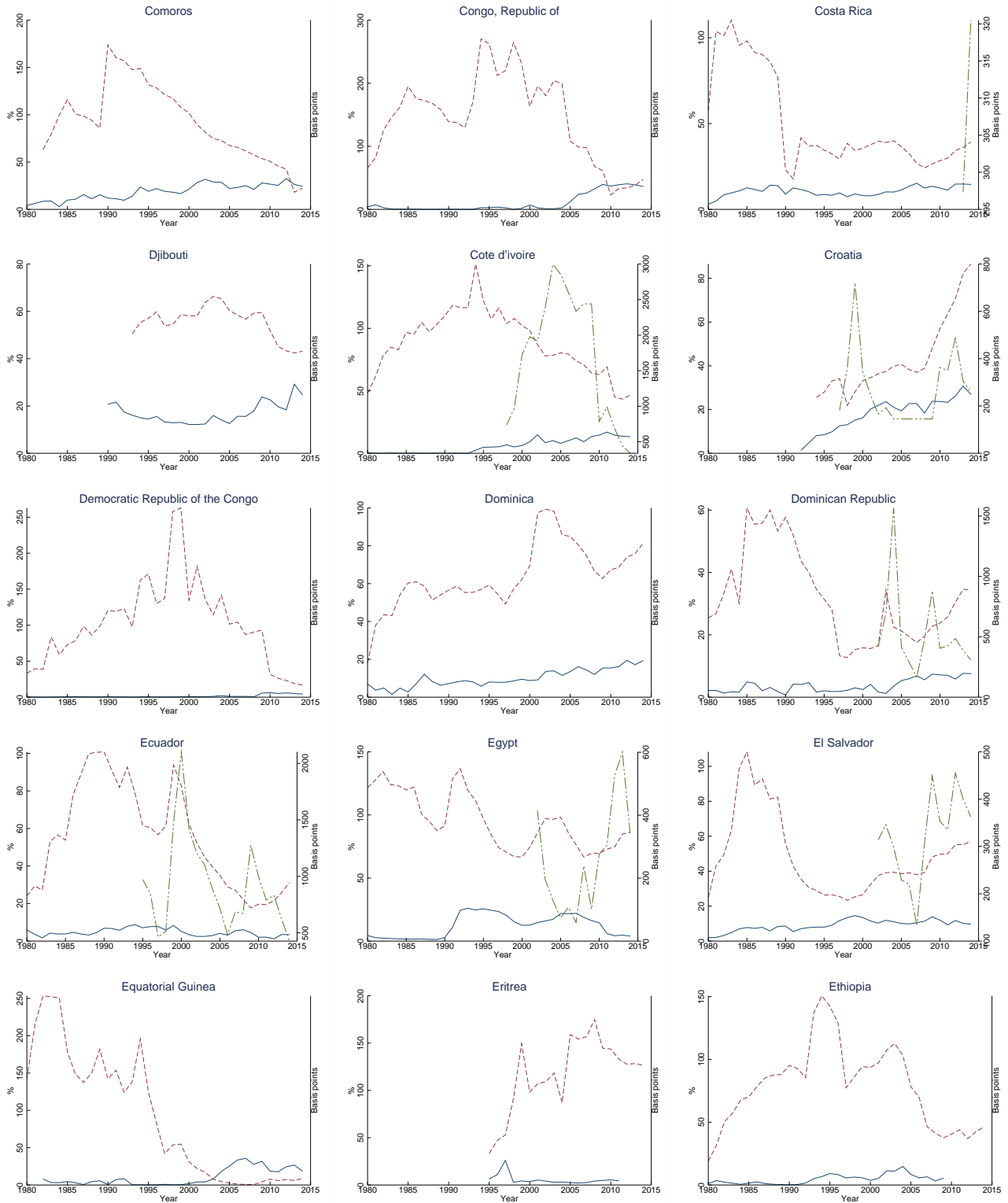
Figure 3: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios.
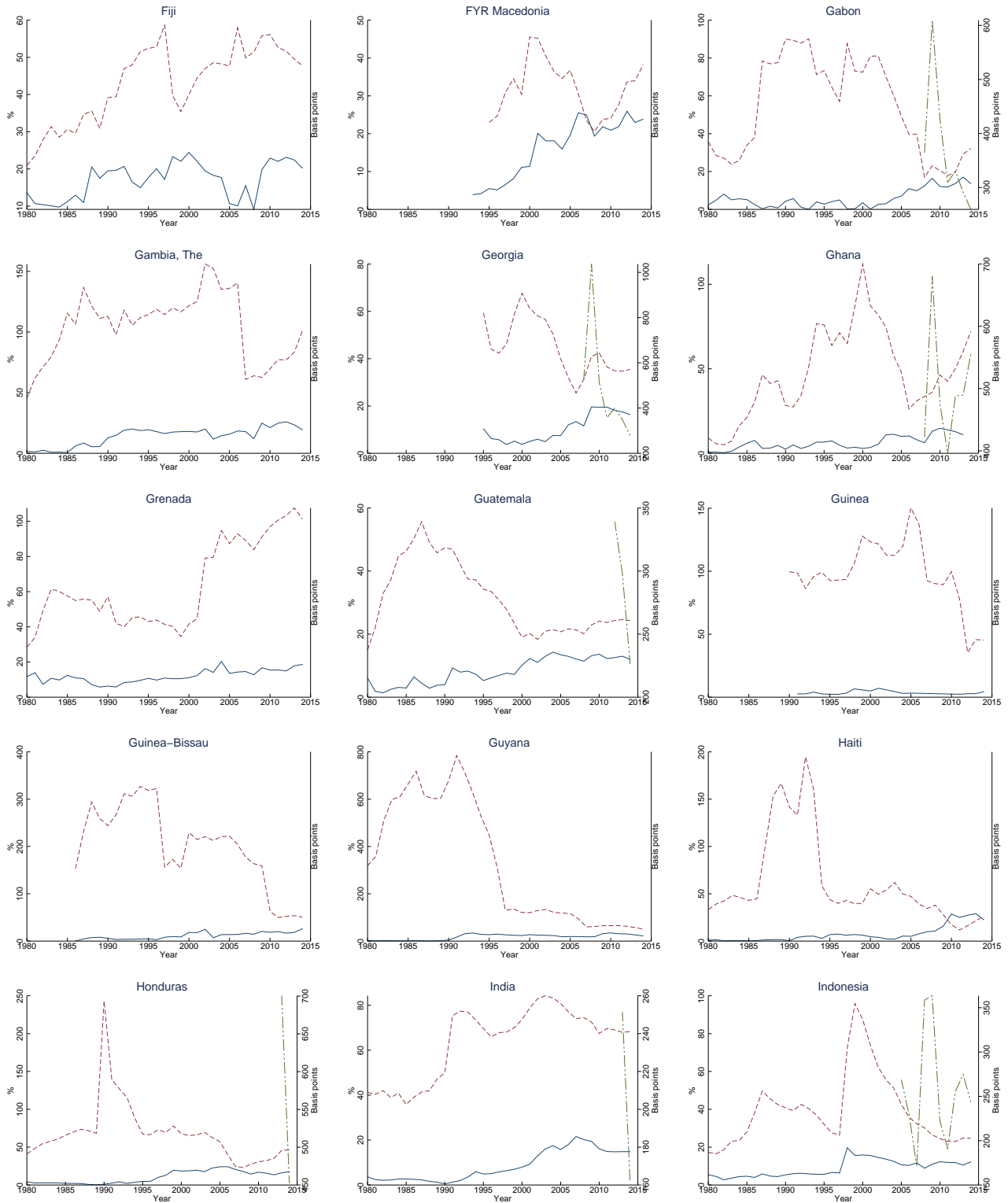
Figure 4: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios.
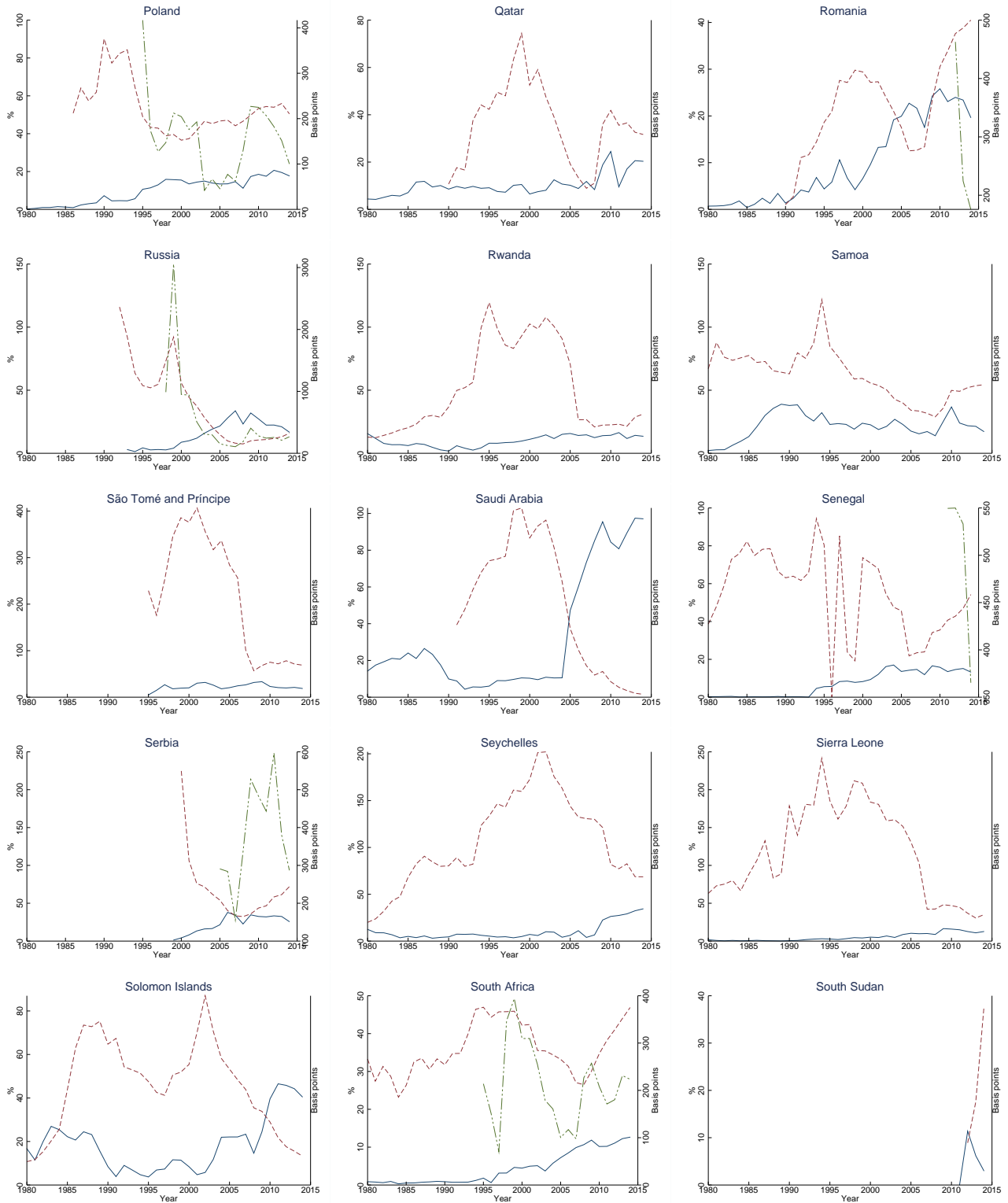
Figure 5: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios.

Figure 6: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios.

Figure 7: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios.

Figure 8: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios.
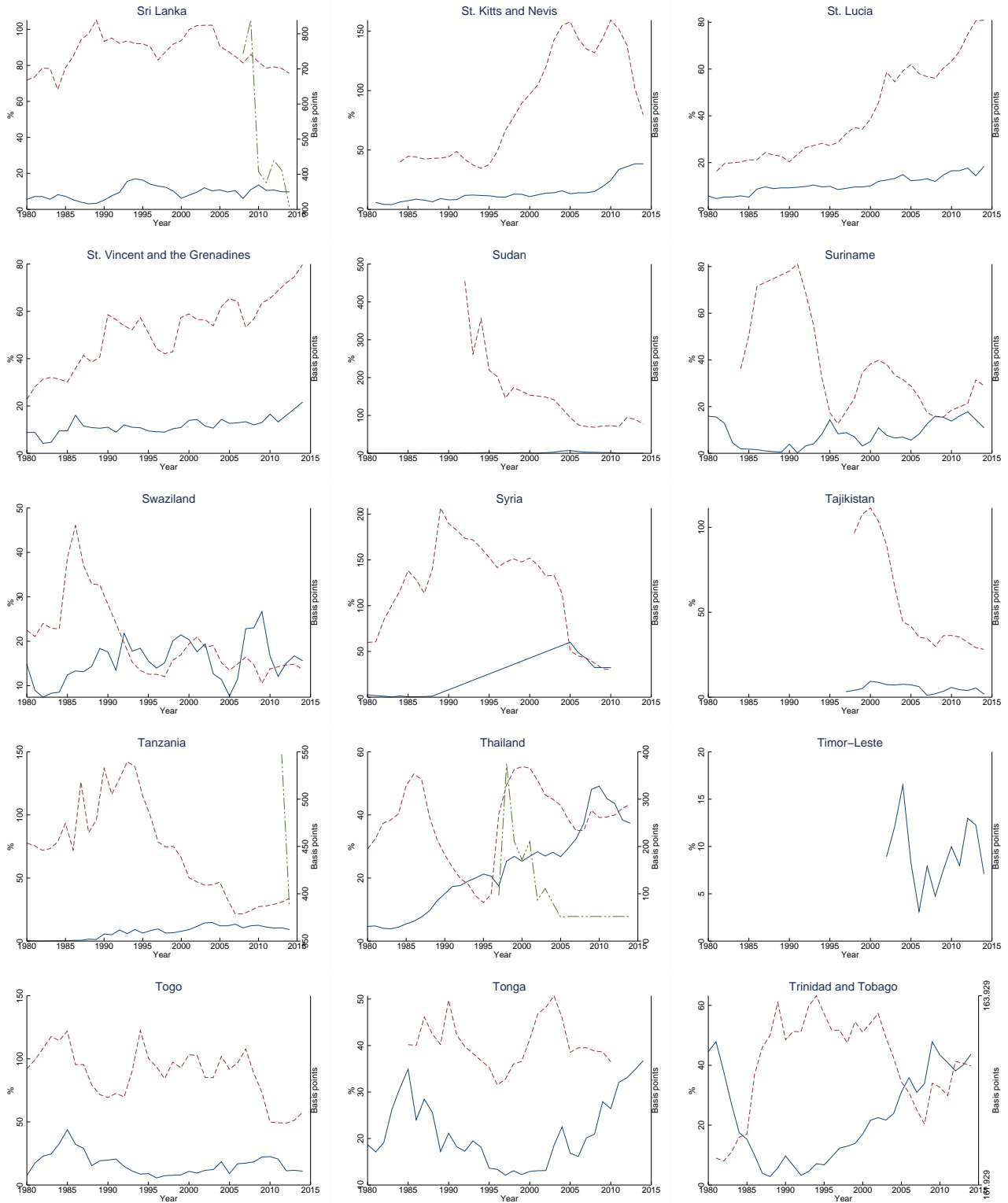
Figure 9: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios.
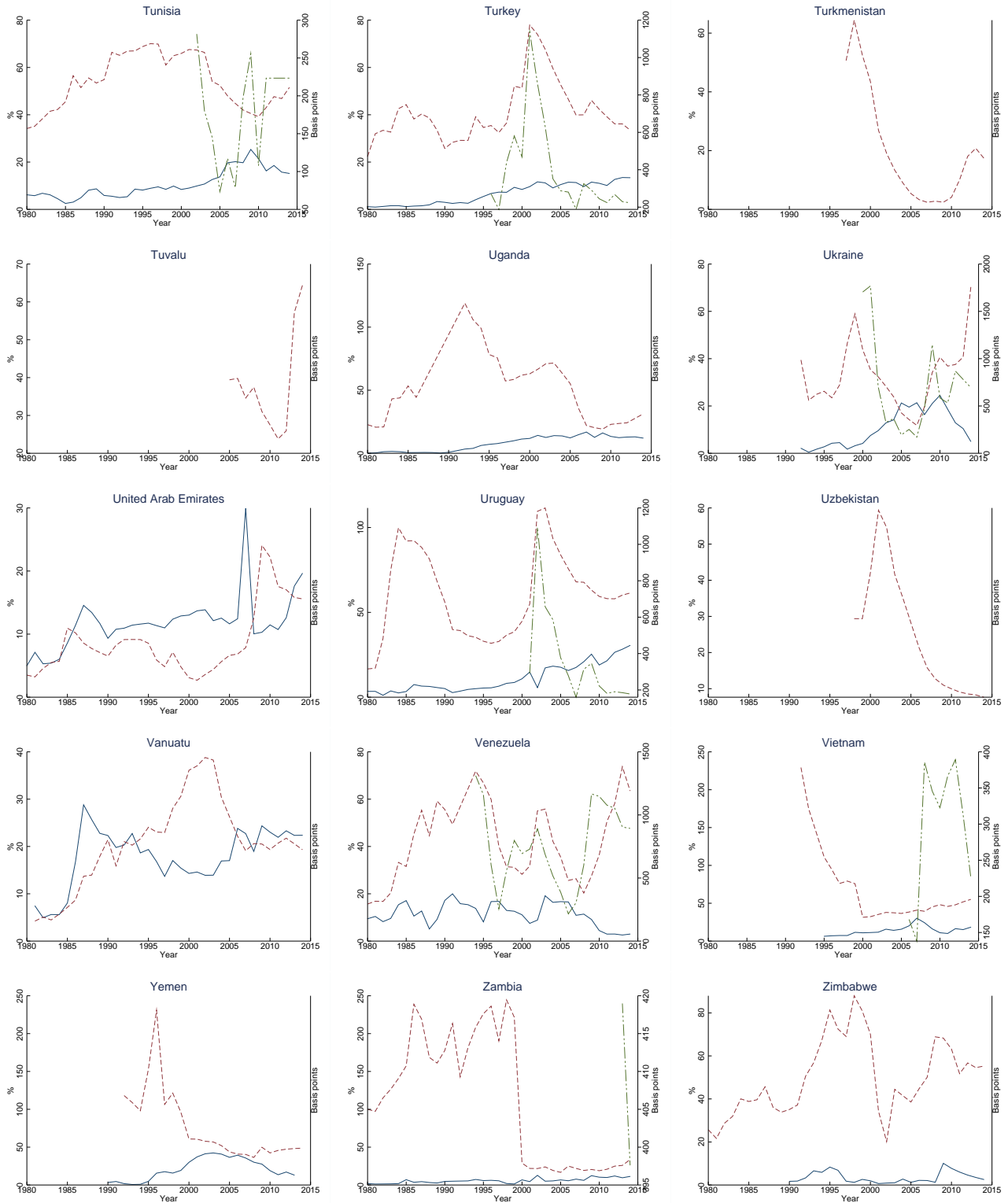
Figure 10: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios.

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