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QUIET BUBBLES

Harrison Hong
David Sraer

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ABSTRACT

Commentaries on the credit bubble of 2003-2007 routinely equate it with earlier episodes like the Internet boom. While credits were over-priced like Internet stocks a decade before, we show, using a model based on disagreement and short-sales constraints, that this is where the similarity ends. Equity bubbles are loud: price and volume go together as investors speculate on capital gains from reselling to more optimistic investors. But this resale option is limited for debt since its upside payoff is bounded. Debt bubbles then require an optimism bias among investors. But greater optimism leads to less speculative trading as investors view the debt as safe and having limited upside. Debt bubbles are hence quiet—high price comes with low volume. We find the predicted price-volume relationship of credits over the 2003-2007 credit boom.

Harrison Hong
Department of Economics
Princeton University
26 Prospect Avenue
Princeton, NJ 08540
and NBER
hhong@princeton.edu

David Sraer
Princeton University
Bendheim Center for Finance
26 Prospect Avenue
Princeton, NJ 08540
and NBER
dsraer@princeton.edu

1. Introduction

Many influential commentators point to a bubble in credit markets from 2003 to 2007, particularly in the AAA and AA tranches of the subprime mortgage collateralized default obligations (CDOs), as the culprit behind the Great Financial Crisis of 2008. These commentaries routinely lump together credit and equity bubbles, such as the Internet boom of the late nineties, as if they were one and the same. For instance, *The Economist*, in a number of opinion editorials following the Financial Crisis of 2008, argues for thinking about these bubble episodes over the past 40 years as the product of too lax monetary policy.[1] One reason perhaps is that the large literature on asset price bubbles often regards bubbles as being synonymous with price being greater than risk-adjusted fundamental value.[2] While there is indeed compelling evidence that investment-grade or highly-rated credit securities, especially in housing mortgage-back securities, were severely over-priced (see Coval, Jurek, and Stafford (2009), Greenwood and Hanson (2011)) in the same way Internet stocks were a decade earlier, we argue that this is where the similarity ends.

Classic equity bubbles are loud—high prices are accompanied by large trading volume as investors purchase in anticipation of capital gains (see Hong and Stein (2007) for a review of this evidence). For example, in the South Sea Bubble of 1720, transactions in the Bank of England stock, one of the three bubble stocks, were three times larger than in the prior three years (Carlos, Neal, and Wandschneider (2006)). Stock share turnover during the years before the Crash of 1929 in the United States were abnormally high by historical standards. In the dot-com bubble years of the late nineties, internet stocks accounted for nearly twenty percent of the trading volume in the stock market (Ofek and Richardson (2003)). The elevated trading volume, and in some of these instances elevated price volatility, associated with these episodes no doubt prompted the classic economists from Adam Smith to John

[1]See for instance *Bubble History* in their Buttonwood's Financial Markets section published on June 17th, 2010. This refrain is a common one in many other influential circles, going back to the works of Hyman Minsky.

[2]For instance, the earliest rational expectations bubble models a la Blanchard and Watson (1983) have no notion of trading volume.

Maynard Keynes to emphasize the role of speculation on anticipated capital gains in bubbles.

This signature of equity bubbles is captured parsimoniously in recent asset pricing models based on investor disagreement and short-sales constraints. The short-sales constraint imparts an upward bias in prices when there is sufficient disagreement among investors (Miller (1977), Chen, Hong, and Stein (2002)). In a dynamic framework, investors anticipate the potential to re-sell at a higher price to someone with a higher valuation due to binding short-sales constraints (Harrison and Kreps (1978), Morris (1996), Scheinkman and Xiong (2003) and Hong, Scheinkman, and Xiong (2006)).[3] This framework generates a bubble or overpricing in which the asset's price is above fundamental value. Importantly, the potential for disagreement tomorrow alone, as opposed to having to assume that all investors are optimistic today, is enough to get prices to be high today since price embeds this resale option. A distinguishing feature of this model is that the resale option is also associated with high share turnover, very much in line with anecdotes on classic equity bubbles.

Within this framework, we consider the pricing of a debt security.[4] Investors disagree over the underlying asset value. In the context of the subprime mortgage CDOs, the underlying asset values are real estate prices. For corporate credits, the underlying asset values are the assets of the companies. Whereas equity payoffs are linear in the investor beliefs regarding underlying asset value, debt up-side payoffs are capped at some constant and hence are concave in the investor beliefs about fundamental.

There is compelling evidence that short-sales constraints are at least as binding in debt markets as in equity ones. The theory only requires that *some* (not all) investors be short-sales constrained. Indeed, many mutual funds or insurance companies are required by charter to be long-only: that is they must simply own investment-grade debt or stocks and are prohibited by charter from shorting or trading derivatives (such as credit default swaps

[3]See Hong and Stein (2007) for a more extensive review of the disagreement approach to the modeling of bubbles.

[4]Earlier work on heterogeneous beliefs and bond pricing such as in Xiong and Yan (2010) assume that investors have disagreement about bond prices or interest rates and they do not model the nature of the concavity of debt pay-offs as a function of underlying asset value disagreement.

(CDSs)) (Almazan et al. (2004) and Koski and Pontiff (1999)). A significant portion of the \$28 trillion dollar of mutual fund money is in this long-only format. The short-sales constraints come not from the cost of shorting but institutional restrictions to shorting.[5] Moreover, Asquith et al. (2010) in their study of the cost of shorting corporate bonds point out that the rise of credit default swaps (CDS) did not influence the shorting activity or cost of shorting in debt. In other words, CDSs are not a substitute for shorting credit. And in the case of the mortgage CDOs, short-sales constraints on these CDOs were binding until the onset of the financial crisis (see Michael Lewis (2010)).

As a result, one might conclude that speculative credit bubbles would come as naturally in this setting as equity bubbles. But this is far from the case because debt upside payoffs are bounded in contrast to equity and hence the valuation of debt is less sensitive to disagreement about underlying asset value. This then limits the speculative resale option of credits.[6] As a result, a debt bubble has to be smaller and than an equity bubble if there is only this speculative disagreement motive at work. The safer is the debt claim, the more bounded is the upside, the less sensitive its valuation is to disagreement and therefore the lower the resale option and the smaller is the bubble.[7]

In other words, speculative resale alone is not enough to get a credit bubble, particularly for safe credits like AAA CDOs. Of course, we are not claiming that there cannot be bubbles in debt. Indeed, there is evidence from Greenwood and Hanson (2011) and Baker, Greenwood, and Wurgler (2003) that credit cycles in the U.S. over the past eighty years, including the eighties junk bond wave and the recent credit boom of 2003-2007, are associated

[5]There is evidence pointing to the impact of these institutional restrictions on over-pricing in equity (Chen, Hong, and Stein (2002)). These institutional restrictions to shorting are every bit as relevant for bonds as they are for stocks.

[6]Here we are implicitly comparing a debt claim with an equity claim that does not have limited liability, so that the only difference between the two claims is the upside of the payoff function. If one compares the debt claim with the complementary equity claim (i.e. with limited liability), then our results hold provided that the fundamental of the economy is good enough or that there is enough optimism among investors. Intuitively, when this is the case, the equity claim is mostly linear in the relevant range while the debt claim has upside bounded payoffs in the relevant range. The formal analysis of equity with limited liability is in Section 2.5.

[7]We provide a formal characterization that allows the ranking of two assets in terms of their disagreement sensitivity in Section 2.

with debt bubbles as issuance from poor credit quality firms forecast low future aggregate bond returns. Optimists told stories of how home prices had been too depressed historically and would keep rising (see Lereah (2005)) or rationalized the risk of the sub-prime mortgage CDOs by pointing to the fact that national home prices had never fallen in US history. Indeed, optimism over a lack of correlation among regional home prices seem to have played a role in the the credit rating agencies' models (Coval, Jurek, and Stafford (2009)).

To get debt overpricing, we allow for investor optimism in the disagreement framework. As investor optimism rises holding fixed fundamental value, debt prices naturally and unsurprisingly rise above fundamental value and a debt bubble emerges. But what is more interesting is that this optimism channel for debt bubbles makes them quieter in the process. When investors become more optimistic about the underlying fundamental of the economy, they view debt as being more risk-free with less upside and hence having a smaller resale option. Hence there is less trading of debt when there is greater investor optimism. In other words, debt bubbles are inherently quiet whereas equity bubbles are intrinsically loud.

We then provide some evidence for this prediction regarding the relationship between credit price and volume over the recent credit boom of 2003-2007 by looking at invesment-grade US corporates.[8] There was, of course, huge issuance of credits of all types during the credit boom of 2003-2007 in the same way that there was a lot of issuance of dot-com companies during the 1996-2000 (Greenwood and Hanson (2011) and Ofek and Richardson (2003)). Our focus, however, is on whether the trading of these credits went up during the 2003-2007 period in the same way that trading in stocks went up during the 1996-2000 period.

Figure 1 from Hong and Stein (2007) plots the monthly share turnover of Internet stocks and the cumulative returns to owning these stocks and those of the non-internet analogs.

[8]There is no comprehensive trading data on the investment-grade mortgage CDOs. But mortgage credits trade in a similar fashion to US corporates according to anecodotes we gathered from traders at Guggenheim Partners who trade both types of securities. Institutional investors such as insurance companies value them for their high credit rating and steady coupons in the same way that they value US corporate bonds. So we focus on the trading of investment-grade US corporates over the recent credit cycle to test our model.

The strong correlation of Internet stock share turnover and valuations can be seen in Figure 1. In Figures 2 and 3, we plot the monthly Bank of America/Merrill Lynch US Corporate 7-10 years Option-Adjusted Spread and trading activity measures (number of trades and dollar trading volume) of these US corporates by insurance companies. The trading data is obtained from Schedule D provided by the National Association of Insurance Commissioners (NAIC). These figures are the analogs to Figure 1 for dot-com stocks. The lack of a positive correlation between price and trading activity in credits in contrast to equity is apparent in Figures 2 and 3. It is also easy to see that trading activity fell over 2003-2007 when spreads also fell (i.e. when bond prices rose), consistent with the prediction of our model. We make this point more formally below.

Finally, we want to acknowledge that the quietness of the credit bubble did not mean that the excesses of the credit boom were not widely heard in other markets. Notably, there was indeed speculation in housing markets. Homes, especially ones bought with cheap leverage, have equity-like pay-offs. And consistent with our model, the housing bubble was loud in that there was speculation in the form of flipping of homes and buying in anticipation of capital gains. Our only point is that the bubble in credits, whose payoffs depended on these home values, was quiet.

Our paper proceeds as follows. The model and main results are discussed in Section 2. We discuss empirical evidence Section 3. We conclude in Section 4 by drawing out the implications of our work for policy and future research. In the Appendix, we collect proofs and derive an extension of our model to show that even in a setting with an unbiased average belief, an increase in the dispersion of priors can make bubbles larger and quieter at the same time, provided that trading costs and asset supply are sufficiently small.

2. Model

2.1. Set-up

Our model has three dates $t = 0$, 1, and 2. There are two assets in the economy. A risk-free asset offers a risk-free rate each period. A risky debt contract with a face value of D has the following payoff at time 2 given by:

$$\tilde{m_2} = \min\left(D, \tilde{G}_2\right), \tag{1}$$

where

$$\tilde{G}_2 = G + \tilde{\epsilon_2} \tag{2}$$

and G is a known constant and $\tilde{\epsilon_2}$ is a random variable drawn from a standard normal distribution $\Phi(\cdot)$. We think of $\tilde{G}_t$ as the underlying asset value which determines the payoff of the risky debt or the fundamental of this economy. There is an initial supply Q of this risky asset.

There are two groups of agents in the economy: group A and group B with a fraction 1/2 each in the population. Both groups share the same belief at date 0 about the value of the fundamental. More specifically, both types of agents believe at $t = 0$ that the underlying asset process is:

$$\tilde{V_2} = G + b + \tilde{\epsilon_2}, \tag{3}$$

where b is the agents' optimism bias. When $b = 0$, investor expectations are equal to the actual mean of the fundamental G and there is no aggregate bias. The larger is b, the greater the investor optimism.

At $t = 1$, agents' beliefs change stochastically: agents in group A believe the asset process is in fact

$$\tilde{V}_2 = G + b + \eta^A + \epsilon_2 \quad (4)$$

while agents in group B believe it is:

$$\tilde{V}_2 = G + b + \eta^B + \tilde{\epsilon_2}, \quad (5)$$

where η^A and η^B are drawn from a normal standard distribution with mean 0 and standard deviation 1. These revisions of beliefs are the main shocks that determine the price of the asset, its volatility and turnover at $t = 1$.

The expected payoff of an agent with belief $G + b + \eta$ regarding the mean of $\tilde{V}_2$ for this standard debt claim is given by:

$$\pi(\eta) = E[\tilde{V}_2|\eta] = \int_{-\infty}^{D-G-b-\eta} (G + b + \eta + \tilde{\epsilon_2})\phi(\tilde{\epsilon_2})d\tilde{\epsilon_2} + D\left(1 - \Phi\left(D - G - b - \eta\right)\right). \quad (6)$$

If the fundamental shock $\tilde{\epsilon}$ is sufficiently low such that the value of the asset underlying the credit is below its face value ($G + b + \eta + \tilde{\epsilon} < D$), then the firm defaults on its contract and investors become residual claimant (they receive $G + b + \eta + \tilde{\epsilon}$). If the fundamental shock $\tilde{\epsilon}$ is sufficiently good such that the value of the fundamental is above the face value of debt ($G + b + \eta + \tilde{\epsilon} \geq D$), then investors are entitled to a fixed payment D. Our analysis below applies more generally to any (weakly) concave expected payoff function, which would include equity as well standard debt claims. Note also that the unlimited liability assumption – the fact that debtholders may receive negative payoff – is not necessary for most of our analysis, but it allows us to compare our results with the rest of the literature.

Agents are risk-neutral and can borrow from a perfectly competitive credit market.[9] The

[9]This can be viewed as the limiting case of the following model with borrowing constraints. Agents are endowed with zero liquid wealth but large illiquid wealth W (which becomes liquid and is perfectly pledgeable at date 2). Credit markets are imperfectly competitive so that banks charge a positive interest rate, which we call $\frac{1}{\mu} - 1$, so that μ is the inverse of the gross rate charged by banks. μ is increasing with the efficiency of the credit market. We consider here the case where $\mu = 1$. The derivation of the model with $\mu < 1$ is available from the authors upon request. Results are qualitatively similar.

discount rate is normalized to 0 without loss of generality. Finally, the last ingredient of this model is that our risk-neutral investors face quadratic trading costs given by:

$$c(\Delta n_t) = \frac{(n_t - n_{t-1})^2}{2\gamma}, \tag{7}$$

where n_t is the shares held by an agent at time t. The parameter γ captures the severity of the trading costs – the higher is γ the lower the trading costs. These trading costs allow us to obtain a well-defined equilibrium in this risk-neutral setting. Note that $n_{-1} = 0$ for all agents, i.e. agents are not endowed with any risky asset. Investors are also short-sales constrained. This set-up is similar to the CARA-Gaussian platform in Hong, Scheinkman, and Xiong (2006) except that we consider non-linear payoff functions over disagreement about underlying asset value.

2.2. Date-1 equilibrium

Let P_1 be the price of the asset at $t = 1$. At $t = 1$, consider an investor with belief $G + b + \eta$ and date-0 holding n_0. Her optimization problem is given by:

$$J(n_0, \eta, P_1) = \left| \begin{array}{l} \max\limits_{n_1} \left\{ n_1 \pi(\eta) - \left((n_1 - n_0) P_1 + \dfrac{(n_1 - n_0)^2}{2\gamma} \right) \right\} \\ n_1 \geq 0 \end{array} \right. \tag{8}$$

where the constraint is the short-sales constraint.

Call $n_1^\star(\eta)$ the solution to the previous program. If $n_1^\star(\eta) - n_0$ is positive, an agent borrows $(n_1^\star(\eta) - n_0)P_1 + \frac{(n_1^\star(\eta)-n_0)^2}{2\gamma}$ to buy additional shares $n_1^\star(\eta) - n_0$. If $n_1^\star(\eta) - n_0$ is negative, the agent makes some profit on the sales but still has to pay the trading cost on the shares sold $(n_0 - n_1^\star(\eta))$. This is because the trading cost is symmetric (buying and selling costs are similar) and only affects the number of shares one purchases or sells, and not the entire position (i.e. $n_1 - n_0$ vs. n_1). In equation 8, $J(n_0, \eta, P_1)$ is the value function of an agent with belief $G + b + \eta$, initial holding n_0 and facing a price P_1. Clearly, $J(n_0, \eta, P_1)$ is driven in part by the possibility of the re-sale of the asset bought at $t = 0$ at a price P_1.

Our first theorem simply describes the date-1 equilibrium. At date 1, three cases arise, depending on the relative beliefs of agents in group A and B. If agents in group A are much more optimistic than agents in group B ($\pi(\eta^A) - \pi(\eta^B) > \frac{2Q}{\gamma}$), then the short-sales constraints binds for agents in group B. Only agents A are long and the price reflects the asset valuation of agents A ($\pi(\eta^A)$) minus a discount that arises from the effective supply of agents B who are re-selling their date-0 holdings to agents A.

Symmetrically, if agents in group B are much more optimistic than agents in group A ($\pi(\eta^B) - \pi(\eta^A) > \frac{2Q}{\gamma}$), then the short-sales constraints binds for agents in group A. Only agents B are long and the price reflects the valuation of agents B for the asset ($\pi(\eta^B)$) minus a discount that arises from the effective supply of agents A who are re-selling their date-0 holdings to agents B.

Finally, the last case arises when the beliefs of both groups are close (i.e $|\pi(\eta^A) - \pi(\eta^B)| < \frac{2Q}{\gamma}$). In this case, both agents are long at date 1 and the date-1 equilibrium price is simply an average of both groups' beliefs ($\frac{\pi(\eta^A)+\pi(\eta^B)}{2}$).

Theorem 1. *Date-1 equilibrium.*

At date 1, three cases arise.

1. *If $\pi(\eta^A) - \pi(\eta^B) > \frac{2Q}{\gamma}$, only agents in group A are long (i.e. the short-sales constraint is binding). The date-1 price is then:*

$$P_1 = \pi(\eta^A) - \frac{Q}{\gamma}.$$

2. *If $\pi(\eta^B) - \pi(\eta^A) > \frac{2Q}{\gamma}$, only agents in group B are long (i.e. the short-sales constraint is binding). The date-1 price is then:*

$$P_1 = \pi(\eta^B) - \frac{Q}{\gamma}.$$

3. *If* $|\pi(\eta^A) - \pi(\eta^B)| \leq \frac{2Q}{\gamma}$*, both agents are long. The date-1 price is then:*

$$P_1 = \frac{\pi(\eta^A) + \pi(\eta^A)}{2}.$$

Proof. Let (η^A, η^B) be the agents' beliefs at date 1. Agents in group i are solving the following problem:

$$\left| \begin{array}{l} \max\limits_{n_1} \left\{ n_1 \pi(\eta^i) - \left((n_1 - n_0) P_1 + \frac{(n_1 - n_0)^2}{2\gamma} \right) \right\} \\ n_1 \geq 0 \end{array} \right.$$

Consider first the case where both agents are long. Then, the date-0 holdings are given by the F.O.C. of the unconstrained problem and yield

$$n_1^A = n_0 + \gamma \left(\pi(\eta^A) - P_1 \right) \quad \text{and} \quad n_1^B = n_0 + \gamma \left(\pi(\eta^B) - P_1 \right)$$

The date-1 market-clearing condition ($n_1^A + n_1^B = 2Q$) combined with the date-0 market-clearing condition ($n_0^A + n_0^B = 2Q$) gives:

$$P_1 = \frac{\pi(\eta^A) + \pi(\eta^B)}{2},$$

and

$$n_1^A - n_0^A = \gamma \frac{\pi(\eta^A) - \pi(\eta^B)}{2} \quad \text{and} \quad n_1^B - n_0^B = \gamma \frac{\pi(\eta^B) - \pi(\eta^A)}{2}.$$

This can be an equilibrium provided that these date-1 holdings are indeed positive:

$$\frac{2n_0^A}{\gamma} > \pi(\eta^B) - \pi(\eta^A) \quad \text{and} \quad \frac{2n_0^B}{\gamma} > \pi(\eta^A) - \pi(\eta^B).$$

If this last condition is not verified, two cases may happen. Either agents in group B are short-sales constrained ($n_1^B = 0$). In this case, the date-1 market clearing condition imposes that:

$$P_1 = \pi(\eta^A) - \frac{Q}{\gamma}.$$

This can be an equilibrium if and only if group B agents' F.O.C. leads to a strictly negative holding or

$$\pi(\eta^A) - \pi(\eta^B) > \frac{2n_0^B}{\gamma}.$$

Or the agents in group A are short-sales constrained ($n_1^A = 0$). In this case, the date-1 market clearing condition imposes that

$$P_1 = \pi(\eta^B) - \frac{Q}{\gamma}.$$

This can be an equilibrium if and only if group B agents' F.O.C. leads to a strictly negative holding or

$$\pi(\eta^B) - \pi(\eta^A) > \frac{2n_0^A}{\gamma}$$

□

2.3. Date-0 equilibrium

We now turn to the equilibrium structure at date 0. Let P_0 be the price of the asset at $t = 0$. Then at $t = 0$, agents of group $i \in \{A, B\}$ have the following optimization program:

$$\left| \begin{array}{l} \max\limits_{n_0} \left\{ -\left(n_0 P_0 + \frac{n_0^2}{2\gamma} \right) + \mathbb{E}_\eta [J(n_0, \eta, P_1)] \right\} \\ n_0 \geq 0 \end{array} \right. \tag{9}$$

where the constraint is the short-sales constraint and the expectation is taken over the belief shocks (η^A, η^B).

The next theorem describes the date-0 equilibrium. In this symmetric setting, it is particularly simple. Both groups of agents are long and hold initial supply Q. The date-0 demand is driven by the anticipation of the date-1 equilibrium. When agents consider a large belief shock, they anticipate they will end up short-sales constrained. In this case, holding n_0 shares at date 0 allows the agents to receive $n_0 P_1$ at date 1 minus the trading cost associated with the reselling of the date-0 holding or $\frac{n_0^2}{2\gamma}$. Or agents consider a small belief shock, and thus anticipate to be long at date 1, i.e. that they will not become too pessimistic relative to the other group. In this case, it is easily shown that their utility from holding n_0 shares at date 0 is proportional to the expected payoff from the asset conditional on the date 1 belief $\pi(\eta^i)$.

Theorem 2. ***Date-0 equilibrium.***

At date-0, each group owns Q shares. The date-0 price is given by:

$$P_0 = \int_{-\infty}^{\infty} \left[\left(\pi(y) - \frac{2Q}{\gamma} \right) \Phi \left(\pi^{-1}[\pi(y) - \frac{2Q}{\gamma}] \right) + \int_{\pi^{-1}[\pi(y)-\frac{2Q}{\gamma}]}^{\infty} \pi(x)\phi(x)dx \right] \phi(y)dy - \frac{Q}{\gamma} \tag{10}$$

Proof. At date 0, group A's program can be written as:

$$\max_{n_0} \left\{ \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\pi^{-1}[\pi(y)-\frac{2n_0^A}{\gamma}]} \left(n_0 \left(\pi(y) - \frac{Q}{\gamma} \right) - \frac{n_0^2}{2\gamma} \right) \phi(x)dx \right. \right.$$
$$\left. \left. + \int_{\pi^{-1}[\pi(y)-\frac{2n_0^A}{\gamma}]}^{\infty} \left(n_1^\star(x)\pi(x) + (n_0 - n_1^\star(x))P_1(x,y) - \frac{(n_1^\star(x)-n_0)^2}{2\gamma} \right) \phi(x)dx \right] \phi(y)dy - \left(n_0 P_0 + \frac{n_0^2}{2\gamma} \right) \right\}$$

Let $G + b + x$ be the date-1 belief of group A agents and $G + b + y$ be the date-1 belief of group B agents. The first integral corresponds to the case where group A agents are short-sales constrained. This happens when $\pi(x) < \pi(y) - \frac{2Q}{\gamma} \Leftrightarrow x < \pi^{-1}\left(\pi(y) - \frac{2Q}{\gamma}\right)$. In this case, group A agents re-sell their date-0 holdings for a price $P_1 = \pi(y) - \frac{Q}{\gamma}$ and pay the trading cost $\frac{n_0^2}{2\gamma}$. The second integral corresponds to the case where group A agents are not short-sell constrained and their date-1 holding is given by the interior solution to the F.O.C., $n_1^\star(x)$. The corresponding payoff is the expected payoff from the date-1 holding with date-1 belief, i.e. $n_1^\star(x)\pi(x)$ plus the potential gains (resp. cost) of selling (resp. buying) some shares $((n_0 - n_1^\star(x))P_1(x,y))$ minus the trading costs $(\frac{(n_1^\star(x)-n_0)^2}{2\gamma})$ of adjusting the date-1 holding.

Note that the bounds defining the two integrals depend on the aggregate holding of group A, but group A agents have no impact individually on this aggregate holding n_0^A. Thus, they maximize only over n_0 in the previous expression and take n_0^A as given. Similarly, agents consider $P_1(x,y)$ as given (i.e. they do not take into account the dependence of P_1 on the aggregate holdings n_0^A and n_0^B).

To derive the F.O.C. of group A agents' program, use the envelope theorem to derive the second integral w.r.t. n_0. For this integral, the envelope theorem applies as $n_1^\star(x)$ is determined according to the date-1 interior F.O.C.. We thus have:

$$\frac{\partial \left(n_1^\star(x)\pi(x) + (n_0 - n_1^\star)P_1(x,y) - \frac{(n_1^\star - n_0)^2}{2\gamma} \right)}{\partial n_0} = P_1(x,y) + \frac{n_1^\star(x) - n_0}{\gamma} = \pi(x).$$

Thus, the overall F.O.C. writes:

$$\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y)-\frac{2n_0^A}{\gamma}]}\left(\pi(y)-\frac{Q}{\gamma}-\frac{n_0}{\gamma}\right)\phi(x)dx+\int_{\pi^{-1}[\pi(y)-\frac{2n_0^A}{\gamma}]}^{\infty}\pi(x)\phi(x)dx\right]\phi(y)dy-\left(P_0+\frac{n_0}{\gamma}\right)=0$$

The model is symmetric. Hence, it has to be that $n_0^A = n_0^B = Q$. Substituting in the previous F.O.C. gives the date-0 equilibrium price:

$$P_0 = \int_{-\infty}^{\infty}\left[\left(\pi(y)-\frac{2Q}{\gamma}\right)\Phi\left(\pi^{-1}[\pi(y)-\frac{2Q}{\gamma}]\right)+\int_{\pi^{-1}[\pi(y)-\frac{2Q}{\gamma}]}^{\infty}\pi(x)\phi(x)dx\right]\phi(y)dy-\frac{Q}{\gamma}$$

□

2.4. Comparative Statics

Now that we have solved for the dynamic equilibrium of this model, we are interested in how mispricing, share turnover and price volatility depend on the following parameters: the structure of the credit claim (D), the bias of the agents' prior (b) and the fundamental of the economy (G). We will relate the predictions derived from these comparative statics to the stylized facts gathered in Section 2.

To be more specific, we first define the bubble or mispricing, which we take to be P_0, the equilibrium price, minus $\bar{P}_0$, the price of the asset in the absence of short-sales constraints *and* with no aggregate bias ($b = 0$). This benchmark or unconstrained price can be written as:[10]

$$\bar{P}_0 = \int_{-\infty}^{\infty}\pi(\eta - b)\phi(\eta)d\eta - \frac{Q}{\gamma}. \tag{11}$$

Now define $\hat{P}_0$ as the date-0 price when there are no short-sales constraint but the aggregate bias is b. This price is given by

$$\hat{P}_0 = \int_{-\infty}^{\infty}\pi(\eta)\phi(\eta)d\eta - \frac{Q}{\gamma}. \tag{12}$$

[10]First, if there is no bias b, then the belief of an agent with belief shock η will be $G+\eta$. Thus, this agent will expect a payoff $\pi(\eta - b)$. Moreover, when there is no short-sales constraint, the formula for the price is similar to equation 10, except that the short-sales constraint region shrinks to 0.

The date-0 price can then be decomposed in the following way:

$$P_0 = \hat{P}_0 + \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\pi^{-1}[\pi(y) - \frac{2Q}{\gamma}]} \left(\pi(y) - \pi(x) - \frac{2Q}{\gamma} \right) \phi(x) dx \right) \phi(y) dy. \tag{13}$$

Then we can decompose the bubble into the following two terms:

$$\text{bubble} = \underbrace{\int_{-\infty}^{\infty} \left(\int_{-\infty}^{\pi^{-1}[\pi(y) - \frac{2Q}{\gamma}]} \left(\pi(y) - \pi(x) - \frac{2Q}{\gamma} \right) \phi(x) dx \right) \phi(y) dy}_{\text{resale option}} + \underbrace{\hat{P}_0 - \bar{P}_0}_{\text{optimism}} \tag{14}$$

In this simple model, the bubble emerges from two sources: (1) there is a resale option due to binding short-sales constraints in the future and (2) agents are optimistic about the asset payoff and thus drive its price up.

The second quantity we are interested in is expected share turnover. It is simply defined as the expectation of the number of shares exchanged at date 1. Formally:

$$\mathbb{T} = \mathbb{E}_{\{\eta^A, \eta^B\}} \left[|n_1^A - n_0^A| \right] \tag{15}$$

Share turnover can be expressed in our setting as:

$$\mathbb{T} = \int_{-\infty}^{\infty} \left(\underbrace{Q(\Phi(\underline{x}(y)) + (1 - \Phi(\bar{x}(y))))}_{\text{A,B short-sales constrained}} + \underbrace{\int_{\underline{x}(y)}^{\bar{x}(y)} \mu\gamma \frac{|\pi(y) - \pi(x)|}{2} d\Phi(x)}_{\text{no short-sales constraint}} \right) d\Phi(y),$$

where $\pi(\underline{x}(y)) = \pi(y) - \frac{2Q}{\gamma}$ and $\pi(\bar{(}x)(y)) = \pi(y) + \frac{2Q}{\gamma}$. Intuitively, if y is the interim belief shock of agents in group A, then when agents in group B have an interim belief shock x below $\underline{x}(y)$ (resp. above $\bar{x}(y)$) agents in group B (resp. agents in group A) are short-sales constrained. Conditional on one group of agent being short-sales constrained, share turnover is maximum and equal to Q. When neither group is short-sales constrained, turnover is just proportional to the difference in valuation between the optimistic and the pessimistic group.

The third object is price volatility between $t = 0$ and $t = 1$. Price volatility is defined simply by:

$$\sigma_P = \text{Var}_{\{\eta^A, \eta^B\}} \left[P_1(\eta^A, \eta^B) \right] \tag{16}$$

The following proposition shows how these three quantities depend on D.

Proposition 1. *A decrease in D (the riskiness of debt) leads to a decrease in (1) mispricing (2) share turnover and (3) price volatility.*

Proof. See Appendix. □

Proposition 1 offers a rationale for why debt bubbles are smaller and quieter than equity ones. The main intuition is that because the credit payoff is bounded by D, it is insensitive to beliefs on the distribution of payoffs above D. Thus, when D is low, there is very little scope for disagreement – the credit is almost risk-free and its expected payoff is close to its face value, and is in particular almost independent of the belief about the fundamental value. Short-sales constraint are thus not likely to bind (as short-sales constraints at date 1 arise from large differences in belief about the expected payoff). As a result, the resale option is low (i.e. the asset will most likely trade at its "fair" value at date 1) and mispricing is low. This, in turns, leads to low expected turnover as turnover is maximized when the agents' short-sales constraint binds. Similarly, volatility will be low as prices will be less extreme (intuitively, the date-1 price will be representative of the average of the two groups beliefs rather than of the maximum of the two groups' beliefs). This mechanism builds on the analysis in Hong, Scheinkman, and Xiong (2006) which relies on risk averse investors and a positive supply of the security so that there are regions in which both groups of investors are long.

Conversely, as D increases, agents' belief matters more for their valuation of the credit, both because of the recovery value conditional on default and because of the default threshold. In the extreme, when D grows to infinity, the credit becomes like an equity, beliefs become relevant for the entire payoff distribution of the asset and the scope for disagreement is maximum. This leads to more binding short-sales constraint at date 1, and hence more volatility and expected turnover.

A simple conclusion emerges from this analysis. For risk-less debt, that is D small, there is little scope for a bubble in credit to emanate from the resale option component. So it is very difficult for the speculative resale dynamics, which can easily drive equity bubbles, to create bubbles in safe credit. Perhaps such a mechanism can work for risky or junk bonds. But the bonds that were mispriced during the credit boom of 2003-2007 were investment-grade and in some cases AAA-rated.

For such safe credits, any bubble or mispricing has to emanate from the optimism component due to b—the optimism bias of investors. Indeed, as we discussed in the Introduction, there is compelling evidence for this explanation. Of course, when the aggregate bias increases (i.e. b increases), mispricing increases in our model. So a first take-away from our analysis is that the credit bubble had to emanate from the excess optimism of the average investor as opposed to disagreement or speculative resale among investors.

We can dig a bit deeper and ask what would happen to share turnover and volatility in our model as we increase b? In other words, high prices are only one symptom of asset price bubbles. In equity, high trading volume is another. We can see if credit and equity bubbles differ in these other dimensions as we increase b. That is, is there an auxiliary prediction associated with the higher prices coming from optimism.

It turns out that turnover and volatility also decrease as we increase b. We prove this in the following proposition.

Proposition 2. *Assume that $D < \infty$. An increase in aggregate optimism (i.e. b) leads to (1) higher mispricing (2) lower price turnover, and (3) lower volatility.*

Proof. We first look at mispricing. Note that $\bar{P}_0$ is independent of b. Thus:

$$\frac{\partial \text{mispricing}}{\partial b} = \frac{\partial P_0}{\partial b} = \int_{-\infty}^{\infty} \left(\Phi\left(\underline{x}(y)\right) \Phi(D - G - b - y) + \int_{\underline{x}(y)}^{\infty} \Phi(D - G - b - x)\phi(x)dx \right) \phi(y)dy > 0$$

Formally, the derivative of turnover and price volatility w.r.t. b is equal to the derivative of turnover and price volatility w.r.t. G. Thus, thanks to the proof of proposition 3 below:

$$\frac{\partial \mathbb{T}}{\partial b} < 0 \text{ and } \frac{\partial \mathcal{V}}{\partial b} < 0$$

□

When investors become more optimistic about the underlying fundamental of the economy, they view debt as being more risk-free with less upside and hence having a smaller resale option. Hence there is less trading of debt when there is greater investor optimism. In our model, provided that the payoff function is strictly concave (or equivalently that $D < \infty$), an increase in average optimism makes the bubble bigger and quieter at the same time. This can be contrasted with the case of a straight equity claim, where both volatility and turnover would be left unaffected by variations in the average optimism – even in the case of binding short-sales constraint. This is because differences in opinion about an asset with a linear payoff are invariant to a translation in initial beliefs. Thus while an increase in optimism would obviously inflate the price of an equity, it would not change its price volatility nor its turnover. In other words, debt bubbles are inherently quiet whereas equity bubbles are intrinsically loud.

In Proposition 2, we held fixed G and considered how a change in b influence properties of the credit bubble. In the next proposition, we hold fix b and consider the comparative static with respect to G.

Proposition 3. *A decrease in G (the riskiness of debt) leads to an increase in (1) mispricing (2) share turnover, and (3) price volatility.*

Proof. See Appendix. □

When fundamentals deteriorate, the credit claim becomes riskier and hence disagreement becomes more important for its valuation. This increase in disagreement sensitivity leads to an increase in the resale option (the speculative component of the date-0 price increases as short-sales constraints are more likely to bind at date 1) and hence higher mispricing. This triggers an increase in both price volatility and turnover as argued above and the bubble stops being quiet.

This prediction is very different from the prediction of the standard model of adverse selection (e.g., see the discussion by Holmstom (Forthcoming)). In this model, a deterioration in the fundamental of the economy destroys the information-insensitiveness of the credit, which reinforces adverse selection and potentially leads to a market breakdown. Thus a worsening of the economy leads to lower trading activity. In our model, however, when the economy worsens, agents realize that disagreement matters for the pricing of the credit in future periods – which drives up the resale option and subsequently increases volatility and trading volume.

2.5. Equity with Limited Liability

Our analysis so far has implicitly compared a credit claim (finite D) with an unlevered equity claim ($D = \infty$) on the same underlying asset. It is fairly direct to extend our results to the case where we compare the levered equity claim that complements the credit claim in the asset value space. Formally, we define the expected payoff function for the levered equity claim under fundamental G, aggregate optimism b, belief shock η and principal on the debt claim D as:

$$\pi^E(\eta) = \int_{D-G-b-\eta}^{\infty} (G + b + \eta + \epsilon - D)\, d\Phi(\epsilon)$$

The expected payoff of the debt claim is similar to our previous analysis and is easily defined by: $\pi^D(\eta) = G + b + \eta - \pi^E(\eta)$.

The following proposition compares the loudness of these two tranches:

Proposition 4. *There exists $\bar{G}$ such that if the fundamental is high enough ($G \geq \bar{G}$), the equity tranche has greater mispricing, share turnover and volatility than the debt tranche.*

Similarly, provided that aggregate optimism is high enough ($b \geq \bar{b}$) or that the principal on the loan is small enough $D \leq \bar{D}$, the equity tranche has greater mispricing, share turnover and volatility than the debt tranche.

Proof. See Appendix. □

Intuitively, when G increases, the debt tranche becomes less disagreement sensitive while the equity tranche becomes more disagreement sensitive. As a consequence, the relative mispricing of equity vs. debt increases, as well as their relative turnover and volatility. One purpose of this proposition is to show that provided that b is large, i.e. provided that the bubble is large enough, our results that credit bubbles are quieter than equity bubbles is robust to the consideration of shareholders' limited liability.

2.6. Characterizing Disagreement Sensitivity

In this section, we move away from the simple debt/equity dichotomy we have emphasized up to now. Our objective is to provide a characterization of the payoffs of various assets that allows us to rank them according to their disagreement sensitivity. We consider the following problem. Take two derivatives on the same underlying fundamental, with payoff function $\pi_1()$ and $\pi_2()$. To get rid of level effects, we make the assumption that π_1 and π_2 have the same expected fundamental value, i.e.:[11]

$$\int_0^\infty \pi_1(x)\phi(x)dx = \int_0^\infty \pi_2(x)\phi(x)dx$$

The next proposition proposes a sufficient condition under which π_1 will lead to a larger and louder bubble than π_2 for any distribution of the belief shocks (η^A, η^B):

Proposition 5. *Assume that for all $x \in \mathbb{R}$, $\pi_1'(x) \geq \pi_2'(x)$ and that the inequality holds strictly on a non-empty set. Then asset 1 has a strictly larger date-0 price, a strictly larger*

[11]Note that in our debt setup, an increase in D was increasing both the disagreement sensitivity of the debt *and* its expected payoff. In this sense, the exercise we consider in this section is more precise in that we only consider variations in the slope of the payoff function for a constant expected value.

expected turnover and a strictly larger expected volatility than asset 2.

Proof. Consider the function $\Delta(x) = \pi_1(x) - \pi_2(x)$. Thanks to our assumption on π_1 and π_2, Δ is increasing over $\mathbb{R}$. We thus have:

$$\forall\ y \geq x, \quad \Delta(y) \geq \Delta(x) \Leftrightarrow \pi_1(y) - \pi_1(x) - \frac{Q}{2\gamma} \geq \pi_2(y) - \pi_2(x) - \frac{Q}{2\gamma}$$

Moreover:

$$\begin{aligned} & \pi_1^{-1}(\pi_1(y) - \tfrac{2Q}{\gamma}) \geq \pi_2^{-1}(\pi_2(y) - \tfrac{2Q}{\gamma}) \\ \Leftrightarrow \quad & \pi_1(y) - \pi_1\left[\pi_2^{-1}(\pi_2(y) - \tfrac{2Q}{\gamma})\right] \geq \tfrac{2Q}{\gamma} \end{aligned}$$

But because π_2 is increasing, we know that $y \geq \pi_2^{-1}(\pi_2(y) - \frac{2Q}{\gamma})$. Moreover, because Δ is increasing, we know that: $\Delta(y) \geq \Delta(\pi_2^{-1}(\pi_2(y) - \frac{2Q}{\gamma}))$. This implies:

$$\pi_1(y) - \pi_2(y) \geq \pi_1\left[\pi_2^{-1}(\pi_2(y) - \frac{2Q}{\gamma})\right] - \pi_2(y) + \frac{2Q}{\gamma} \Leftrightarrow \pi_1(y) - \pi_1\left[\pi_2^{-1}(\pi_2(y) - \frac{2Q}{\gamma})\right] \geq \frac{2Q}{\gamma}$$

Thus, this proves that $P_0(\pi_1) \geq P_0(\pi_2)$. This is because (1) short sales constraint are binding more often under π_1 than π_2 and (2) when short-sales constraints are binding, the difference between the actual price and the no-short sales constraint price (which is proportional to the difference in beliefs between the two groups) is larger under π_1 than under π_2. Note that the inequality will be strict as soon as the derivatives of π_1 is strictly greater than the derivative of π_2 on a non-empty set of $\mathbb{R}$.

Similarly, it is direct to show that turnover and volatility will be greater under π_1 than under π_2. For instance, short-sales constraints bind more often with π_1 and turnover is then maximum and equal to Q. Moreover, when short-sales constraint do not bind, turnover is proportional to the difference in belief between the optimistic and the pessimistic group and we know that this difference will be larger under π_1 than under π_2. □

Intuitively, the payoff function with the largest slope will be such that differences in beliefs lead to larger differences in valuation for the asset. It will thus have the largest probability that short-sales constraints are binding and hence will have the largest price, expected turnover and expected volatility. Note that because of the constant expected value assumption, the condition in Proposition 5 is similar to a single crossing condition.[12]

Finally, note that to derive a necessary condition to rank the two assets, one needs to make assumptions on the p.d.f. of the beliefs shocks (η_1, η_2). In particular, if the condition has to hold for any distribution of the belief shocks, then the condition in Proposition 5 is also a necessary condition.

3. Price-Volume Relationship of Credits from 1998-2009

In this section, we provide evidence on the inverse relationship between the price and trading volume of credits over the recent credit cycle from 1998-2009. Our credit spread data is obtained from the St. Louis Federal Reserve Bank. We use the monthly Bank of America/Merrill Lynch US Corporate 7-10 years Option-Adjusted Spread (OASs). The Bank of America/Merrill Lynch OASs are the calculated spreads between a computed OAS index of all bonds in a given rating category and a spot Treasury curve. The US Corporate 7-10 Year OAS is a subset of the Bank of America Merrill Lynch US Corporate Master OAS, BAMLC0A0CM. This subset includes all securities with a remaining term to maturity of greater than or equal to 7 years and less than 10 years.

Our data source for bond trading volume is Schedule D provided by NAIC. Schedule D covers all the insurance companies in the US, including life and property insurance, and provides year-end holding and every trading record for bonds, stocks, mutual funds, private equity and short-term investments for each insurer. The sample includes 1097 life insurance companies and 2616 property ones or 3713 insurers in total. For the graphs here, we keep only corporate bond trades. We calculate two measures of bond trading activity. The first is number of trades by these insurance companies. The second is the dollar trading volume.

[12] We have $\lim_{x \to -\infty} \Delta(x) \leq 0$. If this was not the case, then $\Delta(x)$ would be strictly positive for all x and the two assets could not have the same expected value. Similarly, $\lim_{x \to -\infty} \Delta(x) \geq 0$.

These data are also monthly series. The latter data is comprehensive when it comes to the trades of insurance companies, which are the major owners of US corporates.

We plot in Figure 2(a) the monthly time series of the credit spread and the number of trades. Notice that the credit spreads are falling from the end of 2002 to the middle of 2007, when the financial crisis begins. It can also be easily seen that the number of trades are falling as the credit spreads are falling or prices are rising, consistent with our model. Notice moreover that from 1998 to 2002, credit spreads are rising and so is trading activity. We plot in Figure 2(b) the monthly series of the credit spread and the bond trading volume. We get a similar picture.

In Table 1, we conduct a regression of the bond spreads on either the log of the number of trades or of log trading volume. For the whole period, the coefficient is 0.1 but is statistically insignificant. We then break down the regression into sub-periods of 1998-2001 before the credit boom, the 2003-2007S1 years (S1 is the first six-months) of the credit boom, up to the start of the financial crisis in mid-2007, and the period of the financial crisis from 2007S2 to 2009. It is easy to see that there is a very strong correlation between spreads and log number of trades during the period of the credit boom. The coefficient of interest is 0.99, which attracts a t-statistic of 4. One standard deviation of our left-hand side variable, credit spreads, is 1.06. One standard deviation of our right-hand side variable, log number of trades is 0.323. So for the 2002-2007S1 period, a one standard deviation increase in number of trades (in logs) is associated with a 30% of a standard deviation increase in spreads, which is an economically meaningful fraction. The same holds true when we use dollar trading volume. The coefficient of interest is 0.73 with a t-statistic of 3.1 One standard deviation of log dollar trading volume is 0.283. So the economic significance is smaller compared to log number of trades but still sizeable. Notice that the same results holds for the 1998-2001 period.

The only period where the results are not significant is the 2007S2-2009 period of the financial crisis when credit spreads jumped but trading volume actually fell. Much of this is due to liquidity issues in credit markets. This liquidity dry-up has been prominently covered in other research. The interesting juxtaposition is that in 2003-2007 when there were no liquidity issues and the credit boom was taking place, trading in credits actually fell.

4. Conclusion

In this paper, we attempt to understand the dynamics of equity and credit bubbles within a unified framework built on investor disagreement and short-sales constraints. Our analysis is motivated by the observation that the classic speculative episodes such as the dot-com bubble usually come with high price, high price volatility and high turnover, while the recent credit bubble appears much quieter. We show that credit bubbles are quieter than equity ones because the up-side concavity of debt payoffs means debt instruments (especially higher rated ones) are less disagreement-sensitive than lower rated credit or equity. As a consequence, optimism which increases the size of credit and equity bubbles makes credit bubbles quiet but leaves the loudness of equity bubbles unchanged.

Our analysis drawing out the distinction that credit bubbles are quiet in contrast to equity bubbles adds to, or more accurately amplifies, a laundry list of potential rationales for the financial crisis, such as agency problems in banking, incentive problems of rating agencies, excess surplus of savings from China and a lack of price transparency in the mortgage credits until it was too late. The fact that credit bubbles are quiet might mean that it was difficult for banks and regulators to see or detect the speculative excesses in contrast to equities. It is interesting to ask whether this quietness might have contributed to why the crash of the credit bubble had more severe consequences than for the crash of the dot-com bubble?

Our analysis also suggests the potential usefulness of a taxonomy of bubbles. Here we offer a first attempt at a taxonomy of bubbles that distinguishes between loud equity bubbles and quiet credit bubbles. Future work elaborating on this taxonomy and providing other historical evidence would be very valuable.

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A. Appendix

A.1. Proof of Proposition 1 and 3

As shown in the text, mispricing can be written as:

$$\text{mispricing} = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\pi^{-1}[\pi(y)-\frac{2Q}{\gamma}]} \left(\pi(y) - \pi(x) - \frac{2Q}{\gamma} \right) \phi(x)dx \right) \phi(y)dy + \int_{-\infty}^{\infty} \left(\pi(y) - \pi(y-b) \right) \phi(y)dy$$

Note that $x < \pi^{-1}\left[\pi(y) - \frac{2Q}{\gamma}\right] \Rightarrow x < y$. Moreover, $\frac{\partial(\pi(y)-\pi(x))}{\partial D} = \Phi(D-G-b-x) - \Phi(D-G-b-y)$. Thus, for all $x < \pi^{-1}\left[\pi(y) - \frac{2Q}{\gamma}\right]$, $\frac{\partial(\pi(y)-\pi(x))}{\partial D} > 0$. Similarly, as $b > 0$, $\frac{\partial(\pi(y)-\pi(y-b))}{\partial D} = \Phi(D-G-y) - \Phi(D-G-b-y) > 0$. Thus, the derivative of mispricing w.r.t. D is strictly positive:

$$\frac{\partial(\text{mispricing})}{\partial D} = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\pi^{-1}[\pi(y)-\frac{2Q}{\gamma}]} \underbrace{\frac{\partial(\pi(y)-\pi(x))}{\partial D}}_{>0} \phi(x)dx \right) \phi(y)dy + \int_{-\infty}^{\infty} \underbrace{\frac{\partial(\pi(y)-\pi(y-b))}{\partial D}}_{>0} \phi(y)dy \tag{17}$$

Thus, as D increases, both the resale option and the mispricing due to aggregate optimism increases, so that overall mispricing increases.

We now turn to expected turnover. To save on notations, call $\bar{x}(y)$ the unique real number such that: $\pi(\bar{x}(y)) = \pi(y) + \frac{2Q}{\gamma}$. Similarly, call $\underline{x}(y)$ the unique real number such that: $\pi(\underline{x}(y)) = \pi(y) - \frac{2Q}{\gamma}$. Obviously, $\underline{x}(y) < y < \bar{x}(y)$. Expected turnover is:

$$\mathbb{T} = \int_{-\infty}^{\infty} \left(\underbrace{\int_{-\infty}^{\underline{x}(y)} Q\phi(x)dx}_{\text{A short-sales constrained}} + \underbrace{\int_{\underline{x}(y)}^{\bar{x}(y)} \gamma \frac{|\pi(y)-\pi(x)|}{2} \phi(x)dx}_{\text{no short-sales constraint}} + \underbrace{\int_{\bar{x}(y)}^{\infty} Q\phi(x)dx}_{\text{B short-sales constrained}} \right) \phi(y)dy$$

We can take the derivative of the previous expression w.r.t. D. Note that the derivative of the bounds in the various integrals cancel out, so that:

$$\frac{\partial \mathbb{T}}{\partial D} = \int_{-\infty}^{\infty} \left(\int_{\underline{x}(y)}^{\bar{x}(y)} \frac{\gamma}{2} \frac{\partial\, |\pi(y)-\pi(x)|}{\partial D} \phi(x)dx \right) \phi(y)dy \tag{18}$$

If $y \geq x$, $\frac{\partial|\pi(y)-\pi(x)|}{\partial D} = |\Phi(D-G-b-x) - \Phi(D-G-b-y)| > 0$. Thus turnover is strictly increasing with D.

We now turn to variance. Formally, note: $P_1(\tilde{x}, \tilde{y}, D)$ the date-1 price when one agent has belief shock $\tilde{x}$, the other $\tilde{y}$ and the face value of debt is D.

$$P_1(\tilde{x}, \tilde{y}, D) = \begin{cases} \pi(\tilde{x}) - \frac{Q}{\gamma} & \text{if } \pi(\tilde{x}) \geq \pi(\tilde{y}) + \frac{2Q}{\gamma} \\ \frac{\pi(\tilde{x}) + \pi(\tilde{y})}{2} & \text{if } |\pi(\tilde{x}) - \pi(\tilde{y})| \leq \frac{2Q}{\gamma} \\ \pi(\tilde{y}) - \frac{Q}{\gamma} & \text{if } \pi(\tilde{y}) \geq \pi(\tilde{x}) + \frac{2Q}{\gamma} \end{cases}$$

We have:

$$\mathbb{E}_{x,y}[(P_1(\tilde{x}, \tilde{y})^2] = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\underline{x}(y)} \left(\pi(y) - \frac{Q}{\gamma}\right)^2 \phi(x)dx + \int_{\underline{x}(y)}^{\bar{x}(y)} \left(\frac{\pi(x)+\pi(y)}{2}\right)^2 \phi(x)dx + \int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\gamma}\right)^2 \phi(x)dx \right) \phi(y)dy \tag{19}$$

We can take the derivative of the previous expression w.r.t. D. Call $K = D - G - b$:

$$\begin{aligned} \frac{1}{2}\frac{\partial \mathbb{E}[(P_1(\tilde{x},\tilde{y})^2]}{\partial D} \\ = \quad & \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\underline{x}(y)} (1 - \Phi(K-y)) \left(\pi(y) - \frac{Q}{\gamma}\right) \phi(x)dx + \int_{\underline{x}(y)}^{\bar{x}(y)} \left(1 - \frac{\Phi(K-x) + \Phi(K-y)}{2}\right) \left(\frac{\pi(x)+\pi(y)}{2}\right) \phi(x)dx \right] \phi(y)dy \\ & + \int_{-\infty}^{\infty} \left[\int_{\bar{x}(y)}^{\infty} (1 - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\gamma}\right) \phi(x)dx \right] \phi(y)dy \end{aligned}$$

Note first that:

$$\int_{-\infty}^{\infty} \left[\int_{-\infty}^{\underline{x}(y)} (1 - \Phi(K-y)) \left(\pi(y) - \frac{Q}{\gamma}\right) \phi(x)dx \right] \phi(y)dy = \int_{-\infty}^{\infty} (1 - \Phi(K-y)) \left[\int_{-\infty}^{\underline{x}(y)} \left(\pi(y) - \frac{Q}{\gamma}\right) \phi(x)dx \right] \phi(y)dy$$

Now the second term in equation 19 can be decomposed into:

$$\begin{aligned} & \int_{-\infty}^{\infty} \left[\int_{\underline{x}(y)}^{\bar{x}(y)} \left(1 - \frac{\Phi(K-x) + \Phi(K-y)}{2}\right) \left(\frac{\pi(x)+\pi(y)}{2}\right) \phi(x)dx \right] \phi(y)dy \\ = \quad & \int_{-\infty}^{\infty} \left[\int_{\underline{x}(y)}^{\bar{x}(y)} (1 - \Phi(K-y)) \left(\frac{\pi(x)+\pi(y)}{2}\right) \phi(x)dx \right] \phi(y)dy \\ & \quad + \underbrace{\int_{-\infty}^{\infty} \left[\int_{\underline{x}(y)}^{\bar{x}(y)} \frac{\Phi(K-y) - \Phi(K-x)}{2} \left(\frac{\pi(x)+\pi(y)}{2}\right) \phi(x)dx \right] \phi(y)dy}_{=0} \\ = \quad & \int_{-\infty}^{\infty} (1 - \Phi(K-y)) \left[\int_{\underline{x}(y)}^{\bar{x}(y)} \left(\frac{\pi(x)+\pi(y)}{2}\right) \phi(x)dx \right] \phi(y)dy \end{aligned}$$

Thus, eventually:

$$\frac{1}{2}\frac{\partial \mathbb{E}[(P_1(\tilde{x},\tilde{y})^2]}{\partial D} = \int_{-\infty}^{\infty}(1-\Phi(K-y))\left[\int_{-\infty}^{\underline{x}(y)}\left(\pi(y)-\frac{Q}{\gamma}\right)\phi(x)dx+\int_{\underline{x}(y)}^{\bar{x}(y)}\left(\frac{\pi(x)+\pi(y)}{2}\right)\phi(x)dx\right]\phi(y)dy$$
$$+\int_{-\infty}^{\infty}\left[\int_{\bar{x}(y)}^{\infty}(1-\Phi(K-x))\left(\pi(x)-\frac{Q}{\gamma}\right)\phi(x)dx\right]\phi(y)dy$$

Call $m=\mathbb{E}_{x,y}[P_1(\tilde{x},\tilde{y},D)]$. The derivative of m^2 w.r.t. to D is simply:

$$\frac{1}{2}\frac{\partial\left(\mathbb{E}[P(x,y)]\right)^2}{\partial D} = \int_{-\infty}^{\infty}(1-\Phi(K-y))\left[\int_{-\infty}^{\underline{x}(y)}m\phi(x)dx+\int_{\underline{x}(y)}^{\bar{x}(y)}m\phi(x)dx\right]\phi(y)dy$$
$$+\int_{-\infty}^{\infty}\left[\int_{\bar{x}(y)}^{\infty}(1-\Phi(K-x))\,m\phi(x)dx\right]\phi(y)dy$$

and $\mathcal{V}=Var(P_1(\tilde{x},\tilde{y},D))=\mathbb{E}_{x,y}[(P_1(\tilde{x},\tilde{y})^2]-m^2$. We have:

$$\frac{1}{2}\frac{\partial\mathcal{V}}{\partial D} = \int_{-\infty}^{\infty}(1-\Phi(K-y))\left[\int_{-\infty}^{\underline{x}(y)}\left(\pi(y)-\frac{Q}{\gamma}-m\right)\phi(x)dx+\int_{\underline{x}(y)}^{\bar{x}(y)}\left(\frac{\pi(x)+\pi(y)}{2}-m\right)\phi(x)dx\right]\phi(y)dy$$
$$+\int_{-\infty}^{\infty}\left[\int_{\bar{x}(y)}^{\infty}(1-\Phi(K-x))\left(\pi(x)-\frac{Q}{\gamma}-m\right)\phi(x)dx\right]\phi(y)dy$$
$$= \int_{-\infty}^{\infty}(1-\Phi(K-y))\left[\int_{-\infty}^{\underline{x}(y)}\left(\pi(y)-\frac{Q}{\gamma}-m\right)d\Phi(x)+\int_{\underline{x}(y)}^{\bar{x}(y)}\left(\frac{\pi(x)+\pi(y)}{2}-m\right)d\Phi(x)+\int_{\bar{x}(y)}^{\infty}\left(\pi(x)-\frac{Q}{\gamma}-m\right)d\Phi(x)\right]d\Phi(y)$$
$$+\int_{-\infty}^{\infty}\left[\int_{\bar{x}(y)}^{\infty}(\Phi(K-y)-\Phi(K-x))\left(\pi(x)-\frac{Q}{\gamma}-m\right)\phi(x)dx\right]\phi(y)dy$$

First note that $(1-\Phi(K-y))$ is an increasing function of y, as well as $\mathbb{E}[P(x,y)-m|y]$. Thus, these two random variables have a positive covariance and because $\mathbb{E}_y[\mathbb{E}_x[P(x,y)-m|y]]=0$, this implies that $\mathbb{E}_y[(1-\Phi(K-y))\times\mathbb{E}_x[P(x,y)-m|y]]\geq 0$, i.e. the first term in the previous equation is positive.

Now, consider the function: $x\rightarrow\pi(x)-\frac{Q}{\gamma}-m$. It is strictly increasing with x over $[\bar{x},\infty($. Call $x^0=\pi^{-1}(\frac{Q}{\gamma}+m)$. Assume first that $\bar{x}(y)>x^0$ (i.e. $y>\pi^{-1}(m-\frac{Q}{\gamma})$).Then for all $x\in[\bar{x},\infty($:

$$(\Phi(K-y)-\Phi(K-x))\left(\pi(x)-\frac{Q}{\gamma}-m\right)>\left(\Phi(K-y)-\Phi(K-x^0)\right)\left(\pi(x)-\frac{Q}{\gamma}-m\right)$$

Now if $\bar{x} < x^0$, then:

$$\begin{aligned}
\int_{\bar{x}(y)}^{\infty} (\Phi(K-y) - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx &= \int_{\bar{x}(y)}^{x^0} (\Phi(K-y) - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx \\
&\quad + \int_{x^0}^{\infty} (\Phi(K-y) - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx \\
&\geq (\Phi(K-y) - \Phi(K-x^0)) \int_{\bar{x}(y)}^{x^0} \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx \\
&\quad + (\Phi(K-y) - \Phi(K-x^0)) \int_{x^0}^{\infty} \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx \\
&\geq (\Phi(K-y) - \Phi(K-x^0)) \int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx
\end{aligned}$$

Thus, for all $y \in \mathbb{R}$,

$$\int_{\bar{x}(y)}^{\infty} (\Phi(K-y) - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx \geq \left(\Phi(K-y) - \Phi(K-x^0)\right) \int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx$$

This leads to:

$$\begin{aligned}
&\int_{-\infty}^{\infty} \left[\int_{\bar{x}(y)}^{\infty} (\Phi(K-y) - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx\right] \phi(y)dy \\
\geq\ &\int_{-\infty}^{\infty} \left[(\Phi(K-y) - \Phi(K-x^0)) \int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx\right] \phi(y)dy
\end{aligned}$$

Now, $\Phi(K-y) - \Phi(K-x^0)$ is a decreasing function of y. $\int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx$ is also a decreasing function of y. Thus, the covariance of these two random variable is positive. But note that:

$$\int_{-\infty}^{\infty} \left[\int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx\right] \phi(y)dy = \mathbb{P}\left[\pi(x) \geq \pi(y) + \frac{2Q}{\gamma}\right] \times \left(\mathbb{E}\left[P(x,y) | \pi(x) \geq \pi(y) - \frac{2Q}{\gamma}\right] - \mathbb{E}[P(x,y)]\right)$$

Finally, note that the conditional expectation of prices, conditional on binding short-sales constraints has to be greater than the expected price, m. Thus, this last term is positive and finally the variance of date-1 prices is strictly increasing with D:

$$\frac{\partial Var(P_1(\tilde{x}, \tilde{y}, D)}{\partial D} \geq 0$$

We now turn to the comparative static w.r.t. G. First, note that $\frac{\partial \pi(y)}{\partial G} = \Phi(D - G - b - y) > 0$ and strictly decreasing with y. Now, the derivative of mispricing w.r.t. G is simply

$$\frac{\partial(\text{mispricing})}{\partial G} = \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\pi^{-1}[\pi(y)-\frac{2Q}{\gamma}]} \underbrace{\frac{\partial(\pi(y)-\pi(x))}{\partial G}}_{<0}\phi(x)dx\right)\phi(y)dy + \int_{-\infty}^{\infty}\underbrace{\frac{\partial(\pi(y)-\pi(y-b))}{\partial G}}_{<0}\phi(y)dy < 0$$

Similarly:

$$\frac{\partial \mathbb{T}}{\partial G} = \int_{-\infty}^{\infty}\left(\int_{\underline{x}(y)}^{\bar{x}(y)} \frac{\gamma}{2}\left|\frac{\partial\left(\pi(y)-\pi(x)\right)}{\partial G}\right|\phi(x)dx\right)\phi(y)dy < 0$$

Finally, note that:

$$\frac{\partial \mathcal{V}}{\partial G} = Cov(\frac{\partial P_1(x,y)}{\partial G}, P_1(x,y)) = Cov(\frac{1-\partial P_1(x,y)}{\partial D}, P_1(x,y)) = -\frac{\partial \mathcal{V}}{\partial D} < 0$$

QED.

A.2. Proof of Proposition 4

Consider first mispricing. The formula for the derivative of mispricing w.r.t. D (equation 17) holds irrespective of the nature of the claim, i.e. whether $\pi = \pi^E$ or $\pi = \pi^D$. We simply remark that π^E, as π^D, is increasing with x (the investor's belief) and $\frac{\partial \pi^E(y)-\pi^E(x)}{\partial D} = \Phi(D-G-b-y) - \Phi(D-G-b-x) = -\frac{\partial \pi^D(y)-\pi^D(x)}{\partial D}$. And similarly: $\frac{\partial \pi^E(y)-\pi^E(y-b)}{\partial D} = \Phi(D-G-b-y) - \Phi(D-G-y) = -\frac{\partial \pi^D(y)-\pi^D(y-b)}{\partial D}$. Thus:

$$\frac{\partial(\text{mispricing on } \pi^E)}{\partial D} = -\frac{\partial(\text{mispricing on } \pi^D)}{\partial D} < 0$$

Thus mispricing of the equity tranche decreases with D. The difference between the mispricing on the equity claim and the mispricing on the debt claim decreases with D as well. When D goes to infinity, there is no mispricing on the equity claim (which is worth 0) so that the difference between the mispricing on the equity claim and the mispricing on the debt claim is strictly negative. Similarly, when D goes to $-\infty$, there is no mispricing on the debt claim (which is worth 0) so that the difference between the mispricing and the equity claim on the mispricing on the debt claim is strictly positive. Thus, there exists a unique $\bar{D}^1 \in \mathbb{R}$ such that for $D \geq \bar{D}^1$, there is a larger mispricing on the equity claim than on the debt claim.

Consider now turnover. The formula for the derivative of turnover w.r.t. D (equation 18) holds irrespective of the nature of the claim, i.e. whether $\pi = \pi^E$ or $\pi = \pi^D$. We simply remark that π^E, as π^D, is increasing with x (the investor's belief) and $\frac{\partial \pi^E(y)-\pi^E(x)}{\partial D} = \Phi(D-G-b-y) - \Phi(D-G-b-x) = -\frac{\partial \pi^D(y)-\pi^D(x)}{\partial D}$.

Thus:

$$\frac{\partial \mathbb{T}(\pi^E)}{\partial D} = -\frac{\partial \mathbb{T}(\pi^D)}{\partial D} < 0$$

Thus, the turnover of the equity tranche decreases with D. The difference between the turnover on the equity claim and the turnover on the debt claim decreases with D as well. When D goes to infinity, there is no turnover on the equity claim (which is worth 0) so that the difference between the turnover on the equity claim and the turnover on the debt claim is strictly negative. Similarly, when D goes to $-\infty$, there is no turnover on the debt claim (which is worth 0) so that the difference between the turnover on the equity claim and the turnover on the debt claim is strictly positive. Thus, there exists a unique $\bar{D}^2 \in \mathbb{R}$ such that for $D \geq \bar{D}^2$, there is a larger mispricing on the equity claim than on the debt claim.

That the variance of $P_1^E(\tilde{x}, \tilde{y})$ is decreasing with D is direct from $\frac{\partial \pi^E}{\partial D}(x) = -\frac{\partial \pi^D}{\partial D}(x)$. Thus, we can apply the same argument as for mispricing and turnover and show the existence of a unique $\bar{D}^3 \in \mathbb{R}$ such that for $D \geq \bar{D}^3$, the variance of the equity tranche is larger than the variance of the debt tranche. To finish the proof, simply define $\bar{D} = \max(\bar{D}^1, \bar{D}^2, \bar{D}^3)$.

The proof for the existence of $\bar{G}$ and $\bar{b}$ follows exactly the same logic.

A.3. Extension: Interim Payoffs and Dispersed Priors

As we showed in the previous section, an increase in aggregate optimism leads to both larger *and* quieter bubbles while leaving unchanged the loudness of equity bubbles. In this section, we highlight another mechanism that makes credit bubbles both larger and quieter while still holding aggregate optimism fixed. In order to do so, we add two additional ingredients to our initial model. First, we introduce heterogenous priors. Group A agents start at date 0 with prior $G + b + \sigma$ and group B agents start with prior $G + b - \sigma$. Second, we introduce an interim payoff $\pi(G + \epsilon_1)$ that agents receive at date-1 from holding the asset at date 0. As a consequence, agents now hold the asset both for the utility they directly derive from it (consumption) and for the perspective of being able to resell it to more optimistic agents in the future (speculation). More precisely, the $t = 1$ interim cash-flow $\pi(G + \epsilon_1)$ occurs before the two groups of agents draw their date-1 beliefs. We also assume that the proceeds from this interim cash flow, as well as the payment of the date-0 and date-1 transaction costs all occur on the terminal date. This assumption is made purely for tractability reason so we do not have to keep track of the interim wealth of the investors.

Our next proposition shows that, provided that dispersion is large enough, an increase in the initial dispersion of belief, σ, leads to an increase in prices *and* simultaneously to a decrease in share turnover an price volatility. Thus, quiet bubbles emerge when there is sufficient heterogeneity among investors about the fundamental.

Proposition 6. *Provided the cost of trading are large enough/initial supply is low enough, there is $\bar{\sigma} > 0$ so that for $\sigma \geq \bar{\sigma}$ only group A agents are long at date 0. For $\sigma \geq \bar{\sigma}$, an increase in σ leads to (1) an increase in mispricing and (2) a decrease in trading volume.*

Proof. We now consider the case where group A has prior $G+b+\sigma$ and group B has prior $G+b-\sigma$. Thus, at date 1, beliefs are given by $(G+b+\sigma+\eta^A)$ for group A, with $\eta^A \sim \Phi()$ and $(F-\sigma+\eta^B)$ for group B, with $\eta^B \sim \Phi()$. Agents also receive at date 1 an interim payoff proportional to $\pi()$ from holding the asset at date-0. We first start by solving the date-1 equilibrium. At date 1, three cases arise:

1. Both groups are long. Thus demands are:

$$\begin{cases} n_1^A = n_0^A + \gamma\left(\pi(\sigma+\eta^A) - P_1\right) \\ n_1^B = n_0^B + \gamma\left(\pi(-\sigma+\eta^B) - P_1\right) \end{cases}$$

   The date-1 price in this case is: $P_1 = \frac{1}{2}\left(\pi(\sigma+\epsilon^A) + \pi(-\sigma+\epsilon^B)\right)$. This is an equilibrium if and only if: $\frac{2n_0^A}{\gamma} > \pi(-\sigma+\epsilon^B) - \pi(\sigma+\epsilon^A)$ and $\frac{2n_0^B}{\gamma} > \pi(\sigma+\epsilon^A) - \pi(-\sigma+\epsilon^B)$

2. Only A group is long. The date-1 equilibrium price is then simply: $P_1 = \pi(\sigma+\epsilon^A) - \frac{n_0^B}{\gamma}$

   This is an equilibrium if and only if $\pi(\sigma+\epsilon^A) - \pi(-\sigma+\epsilon^B) > \frac{2n_0^B}{\gamma}$.

3. Only B group is long. The date-1 equilibrium price is then simply: $P_1 = \pi(-\sigma+\epsilon^B) - \frac{n_0^A}{\gamma}$. This is an equilibrium if and only if $\pi(-\sigma+\epsilon^B) - \pi(\sigma+\epsilon^A) > \frac{2n_0^A}{\gamma}$

At date 0, group A program can be written as [13] :

$$\max_{n_0}\left\{n_0\pi(\sigma) + \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y-\sigma)-\frac{2n_0^A}{\gamma}]-\sigma}\left(n_0\left(\pi(y-\sigma) - \frac{n_0^A}{\gamma}\right) - \frac{n_0^2}{2\gamma}\right)\phi(x)dx + \int_{\pi^{-1}[\pi(y-\sigma)-\frac{2n_0^A}{\gamma}]-\sigma}^{\infty} n_0\pi(\sigma+x)\phi(x)dx\right]\phi(y)dy - \left(n_0P_0 + \frac{n_0^2}{2\gamma}\right)\right\}$$

The F.O.C. of group A's agents program is given by (substituting n_0^A for n_0 in the F.O.C.):

$$0 = \pi(\sigma) + \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y-\sigma)-\frac{2n_0^A}{\gamma}]-\sigma}\left(\pi(y-\sigma) - \frac{2n_0^A}{\gamma}\right)\phi(x)dx + \int_{\pi^{-1}[\pi(y-\sigma)-\frac{2n_0^A}{\gamma}]-\sigma}^{\infty}\pi(\sigma+x)\phi(x)dx\right]\phi(y)dy - \left(P_0 + \frac{n_0^A}{\gamma}\right)$$

[13] In group A agents' program, we note n_0^A group A agents aggregate holding – each agent in group A takes n_0^A as given.

Similarly, at date 0, group B agents' program can be written as:

$$\max_{n_0}\left\{\pi(-\sigma)+\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y+\sigma)-\frac{2n_0^B}{\gamma}]+\sigma}\left(n_0\left(\pi(y+\sigma)-\frac{n_0^B}{\gamma}\right)-\frac{n_0^2}{2\gamma}\right)\phi(x)dx+\int_{\pi^{-1}[\pi(y+\sigma)-\frac{2n_0^B}{\gamma}]+\sigma}^{\infty}n_0\pi(-\sigma+x)\phi(x)dx\right]\phi(y)dy \\ -\left(n_0P_0+\frac{n_0^2}{2\gamma}\right)\right\}$$

Group B agents' F.O.C.:

$$0=\pi(-\sigma)+\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y+\sigma)-\frac{2n_0^B}{\gamma}]+\sigma}\left(\pi(y+\sigma)-\frac{2n_0^B}{\gamma}\right)\phi(x)dx+\int_{\pi^{-1}[\pi(y+\sigma)-\frac{2n_0^B}{\gamma}]+\sigma}^{\infty}\pi(-\sigma+x)\phi(x)dx\right]\phi(y)dy-\left(P_0+\frac{n_0^B}{\gamma}\right)$$

Consider now an equilibrium where only group A is long, i.e. $n_0^A=2Q$ and n_0^B=0. In this case, the date-0 price is given by:

$$P_0=\pi(\sigma)+\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y-\sigma)-\frac{4Q}{\gamma}]-\sigma}\left(\pi(y-\sigma)-\frac{4Q}{\gamma}\right)\phi(x)dx+\int_{\pi^{-1}[\pi(y-\sigma)-\frac{4Q}{\gamma}]-\sigma}^{\infty}\pi(\sigma+x)\phi(x)dx\right]\phi(y)dy-\frac{2Q}{\gamma}$$

This is an equilibrium if and only if:

$$P_0>\pi(-\sigma)+\int_{-\infty}^{\infty}\left[\int_{-\infty}^{y+2\sigma}\pi(y+\sigma)\phi(x)dx+\int_{y+2\sigma}^{\infty}\pi(-\sigma+x)\phi(x)dx\right]\phi(y)dy$$

We now show that P_0 is increasing with σ (noting $K=D-G-b$):

$$\begin{aligned}\frac{\partial P_0}{\partial\sigma} &= \pi'(\sigma)+\int_{-\infty}^{\infty}\left[\int_{\pi^{-1}[\pi(y-\sigma)-\frac{4Q}{\gamma}]-\sigma}^{\infty}\Phi(K-\sigma-x)\phi(x)dx-\int_{-\infty}^{\pi^{-1}[\pi(y-\sigma)-\frac{4Q}{\gamma}]-\sigma}\Phi(K-y+\sigma)\phi(x)dx\right]\phi(y)dy \\ &= \pi'(\sigma)+\int_{-\infty}^{\infty}\phi\left(\pi^{-1}[\pi(y+\sigma)+\frac{4Q}{\gamma}]+\sigma\right)\Phi(K-\sigma-y)\phi(y)dy-\int_{-\infty}^{\infty}\phi\left(\pi^{-1}[\pi(y-\sigma)-\frac{4Q}{\gamma}]-\sigma\right)\Phi(K-y+\sigma)\phi(y)dy \\ &\geq \pi'(\sigma)+\int_{-\infty}^{\infty}\phi\left(y+2\sigma\right)\Phi(K-\sigma-y)\phi(y)dy-\int_{-\infty}^{\infty}\phi\left(y-2\sigma\right)\Phi(K-y+\sigma)\phi(y)dy \\ &\geq \pi'(\sigma)+\int_{-\infty}^{\infty}\phi\left(y+2\sigma\right)\Phi(K-\sigma-y)\phi(y)dy-\int_{-\infty}^{\infty}\phi\left(y\right)\Phi(K-y-\sigma)\phi(y+2\sigma)dy \\ &\geq \pi'(\sigma)+\int_{-\infty}^{\infty}\Phi(K-\sigma-y)\left[\phi\left(y+2\sigma\right)\phi(y)-\phi\left(y\right)\phi(y+2\sigma)\right]dy\end{aligned}$$

Call $\psi(\sigma)=\phi\left(y+2\sigma\right)\phi(y)-\phi\left(y\right)\phi(y+2\sigma)$. $\psi'(\sigma)=2\phi(y+2\sigma)\left(\phi(y)+(y+2\sigma)\phi(y)\right)$ Thus, ψ is increasing if and only if: $2\sigma>-y-\frac{\phi(y)}{\phi(y)}$. Now consider the function $\kappa:y\in\mathbb{R}\to y\phi(y)+\phi(y)$. $\kappa'(y)=\phi(y)>0$. Thus, κ is increasing strictly with y. But $\lim_{y\to-\infty}\kappa'(y)=0$. Thus: $\forall\ y$, $\kappa(y)>0$. Thus, for all $\sigma>0$, $-y-\frac{\phi(y)}{\phi(y)}=-\frac{\kappa(y)}{\phi(y)}<0<2\sigma$ so that ψ is strictly increasing with σ, for all $\sigma>0$ and $y\in\mathbb{R}$. Now $\psi(0)=0$.

Thus, $\psi(y) > 0$ for all $\sigma > 0$. As a consequence:

$$\frac{\partial P_0}{\partial \sigma} \quad \geq \quad \pi'(\sigma) + \int_{-\infty}^{\infty} \Phi(K-\sigma-y)\psi(y)dy > 0$$

Thus, when the equilibrium features only group A long at date 0, the price is strictly increasing with dispersion σ.

We now simply show that in this equilibrium, turnover is strictly increasing. Turnover is $2Q$ when group B only is long at date 1 (i.e. $\pi(-\sigma+\eta^B) > \pi(\sigma+\eta^A) + \frac{4Q}{\gamma}$), it is given by: $\gamma\frac{\pi(y-\sigma)-\pi(x+\sigma)}{2}$ when group B and group A are long at date 1 ((i.e. $\pi(-\sigma+\eta^B) < \pi(\sigma+\eta^A) + \frac{4Q}{\gamma}$ and $\pi(-\sigma+\eta^B) > \pi(\sigma+\eta^A)$ and it is 0 if only group A is long at date 1 (i.e. $\pi(-\sigma+\eta^B) < \pi(\sigma+\eta^A)$). Thus, conditioning over η^B, expected turnover can be written as:

$$\mathbb{T} = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\pi^{-1}(\pi(-\sigma+y)-\frac{4Q}{\gamma})-\sigma} 2Q\phi(x)dx + \int_{\pi^{-1}(\pi(-\sigma+y)-\frac{4Q}{\gamma})-\sigma}^{y-2\sigma} \gamma\frac{\pi(y-\sigma)-\pi(x+\sigma)}{2}\phi(x)dx \right] \phi(y)dy$$

Again, the derivative of the bounds in the integrals cancel out and the derivative is simply:

$$\frac{\partial \mathbb{T}}{\partial \sigma} = \int_{-\infty}^{\infty} \int_{\pi^{-1}(\pi(-\sigma+y)-\frac{4Q}{\gamma})-\sigma}^{y-2\sigma} -\gamma\frac{\Phi(K-y+\sigma)+\Phi(K-x-\sigma)}{2}\phi(x)dx\phi(y)dy < 0$$

Now consider the equation defining the equilibrium where only group A is long at date 0. This condition is:

$$\delta(\sigma) = P_0(\sigma) - \left(\pi(-\sigma) + \int_{-\infty}^{\infty} \left[\int_{-\infty}^{y+2\sigma} \pi(y+\sigma)\phi(x)dx + \int_{y+2\sigma}^{\infty} \pi(-\sigma+x)\phi(x)dx \right] \phi(y)dy \right) > 0$$

Notice that the derivative of the second term in the parenthesis can be written as:

$$\begin{aligned}
& \int_{-\infty}^{\infty} \left[\int_{-\infty}^{y+2\sigma} \pi'(y+\sigma)\phi(x)dx - \int_{y+2\sigma}^{\infty} \pi'(-\sigma+x)\phi(x)dx \right] \phi(y)dy \\
= & \int_{-\infty}^{\infty} \phi(y+2\sigma)\pi'(y+\sigma)\phi(y)dy - \int_{-\infty}^{\infty} \phi(y-2\sigma)\pi'(y-\sigma)\phi(y)dy \\
= & \int_{-\infty}^{\infty} \Phi(K-y-\sigma)\left(\phi(y+2\sigma)\phi(y) - \phi(y)\phi(y+2\sigma)\right)dy
\end{aligned}$$

We thus have:

$$\frac{\partial \delta}{\partial \sigma} \geq (\pi'(\sigma) + \pi'(-\sigma)) > 0$$

Thus, there is $\bar{\sigma} > 0$ such that for $\sigma \geq \bar{\sigma}$, the equilibrium has only group A long at date 0, the price increases with dispersion and turnover decreases with dispersion. QED.

□

The intuition for this result is the following. The condition on trading costs/initial supply allow the optimists to have enough buying power to lead to binding short-sales constraints at date 0. In the benchmark setting, low trading costs/low supply were associated with a louder credit bubble. But it turns out that when there is dispersed priors they can lead to large but quiet mispricings.

To see why, first consider the effect of dispersed priors on mispricing. Mispricing (i.e. the spread between the date-0 price and the no-short-sales constraint/ no bias price) increases with dispersion for two reasons. First, group A agents' valuation for the interim payoff increases. This is the familiar Miller (1977) effect in which the part of price regarding the interim payoff reflects the valuations of the optimists as short-sales constraints bind when disagreement increases. Second, as dispersion increases, so does the valuation of the marginal buyers (or the optimists) at date 1 – which leads to an increase in the resale option and hence of the date-0 price.

As dispersion increases, group A agents – who are more optimistic about the interim payoff than group B agents – own more and more shares until they hold all the supply at date 0 (which happens for $\sigma > \bar{\sigma}$).[14] As σ increases, the probability that group B become the optimistic group at date 1 also becomes smaller. As a consequence, an increase in dispersion leads to an increase in the probability of the states of nature where turnover is zero or equivalently where the group A agents hold all the shares at date 0 and 1. So overall expected turnover decreases.

Price volatility also decreases as σ increases. This is because in these states of nature where only group A agents are long, the expected payoff becomes more concave as a function of their belief shock, so that the price volatility conditional on these states decreases. When σ becomes sufficiently large, group A agents are long most of the time and the asset resembles a risk-free asset and price volatility goes down to zero.

[14] Note that this occurs despite the fact that both types share the same valuation for the date-1 resale option – this is entirely driven by the interim payoff: in a pure resale option setting (i.e. without the interim payoff), all agents would end up long at date 0 as, in the margin, there would be no disagreement about the value of the resale option.

Table 1: The Relationship between Trading Activity and Investment-Grade Credit Spreads, 1998-2009

	Spreads							
	Whole period		1998-2001		2002-2007S1		2007S2-2009	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log(Number of Trades)	.1		.5***		.99***		-.92	
	(.41)		(3.3)		(4)		(-.72)	
Log(Volume of Trades)		-.034		.39***		.73***		-1.7
		(-.16)		(3.1)		(3.1)		(-1.6)
Constant	.74	2.6	-3.2**	-8**	-8.4***	-17***	12	45
	(.31)	(.49)	(-2.2)	(-2.6)	(-3.6)	(-2.9)	(.98)	(1.7)
Observations	144	144	48	48	66	66	30	30
R^2	.00074	.0001	.14	.13	.18	.14	.018	.077

This table reports the contemporaneous relationship between bond spreads and volume measured either as the log of number of trades during the month or the log of $ transacted during the month. Robust T-stats are in parenthesis. *, **, and *** means statistically different from zero at 10, 5 and 1% level of significance.

Figure 1: ABX Prices

The figure plots the ABX 7-1 Prices for various credit tranches including AAA, AA, A, BBB, and BBB-.

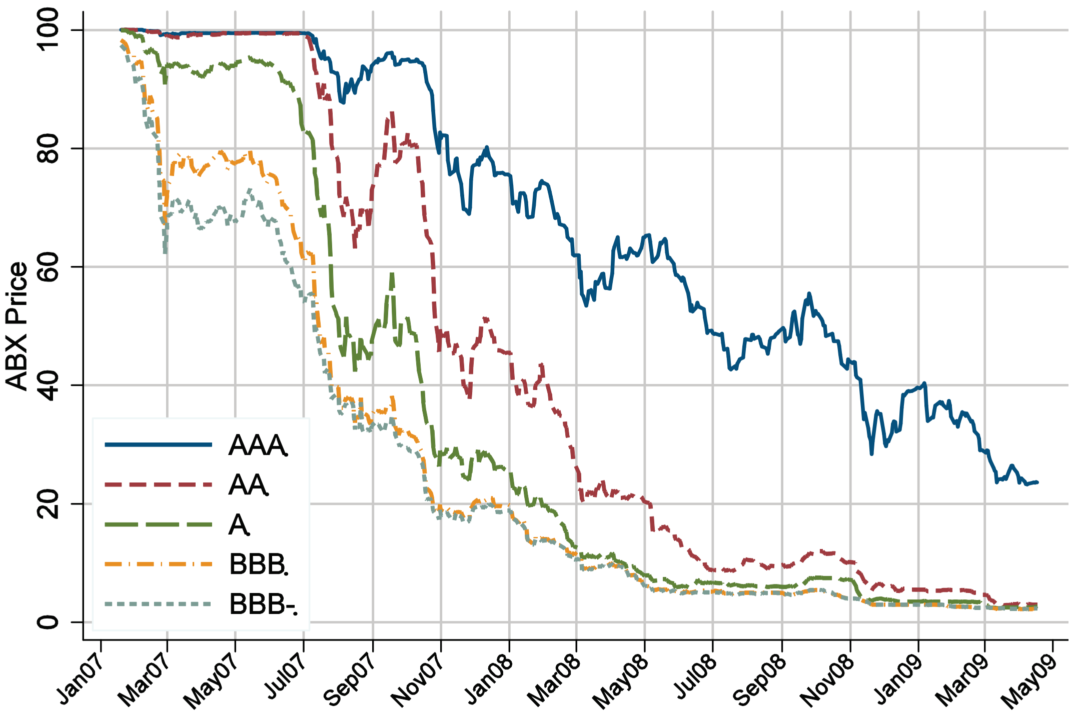

Figure 2: CDS Prices of Basket of Finance Companies

Source: Moodys KMV, FSA Calculations. The figure plots the average CDS prices for a basket of large finance companies between December 2002 and December 2008. Firms included: Ambac, Aviva, BancoSantander, Barclays, Berkshire Hathaway, Bradford & Bingley, Citigroup, Deutsche Bank, Fortis, HBOS, Lehman Brothers, Merrill Lynch, Morgan Stanley, National Australia Bank, Royal Bank of Scotland and UBS.

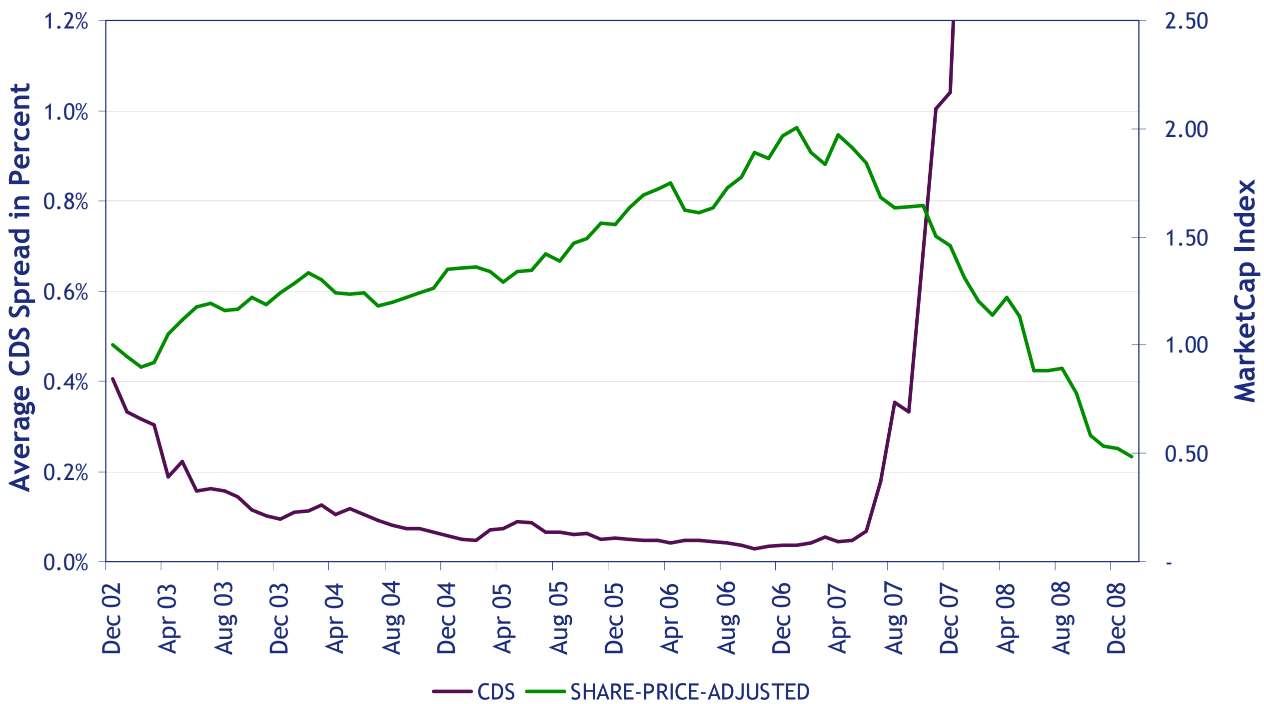

Figure 3: Monthly Share Turnover of Financial Stocks

The figure plots the average monthly share turnover of financial stocks.

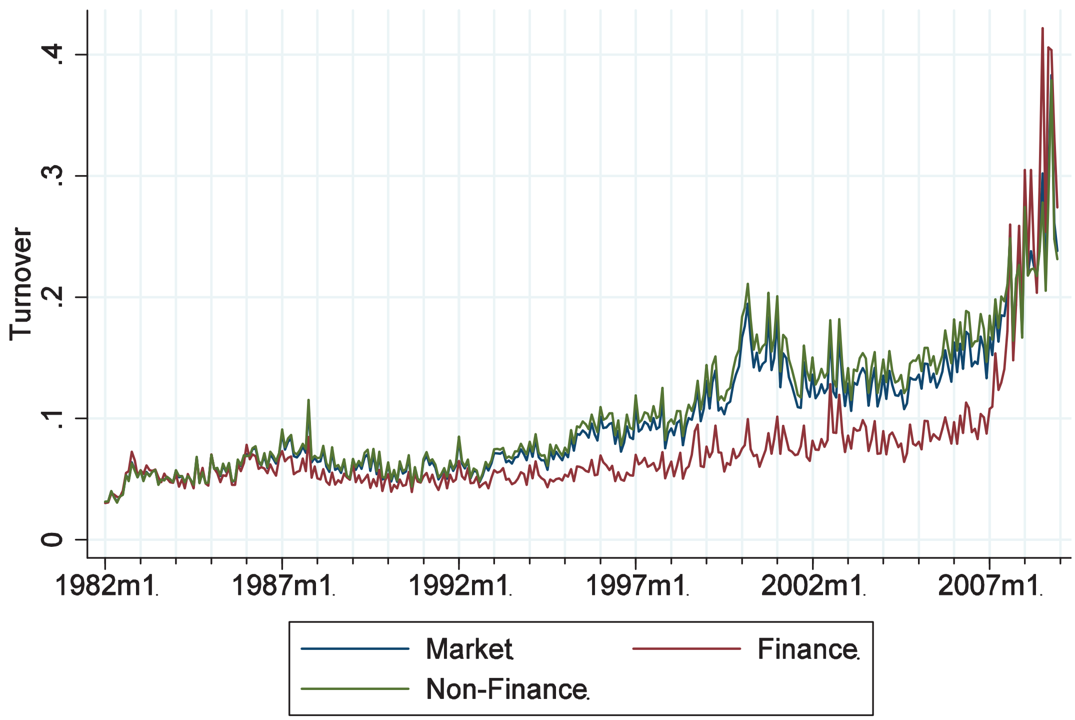

Figure 4: Monthly Share Turnover of Internet Stocks

Source: Hong and Stein (2007). The figure plots the average monthly share turnover of Internet stocks and non-Internet stocks from 1997 to 2002.

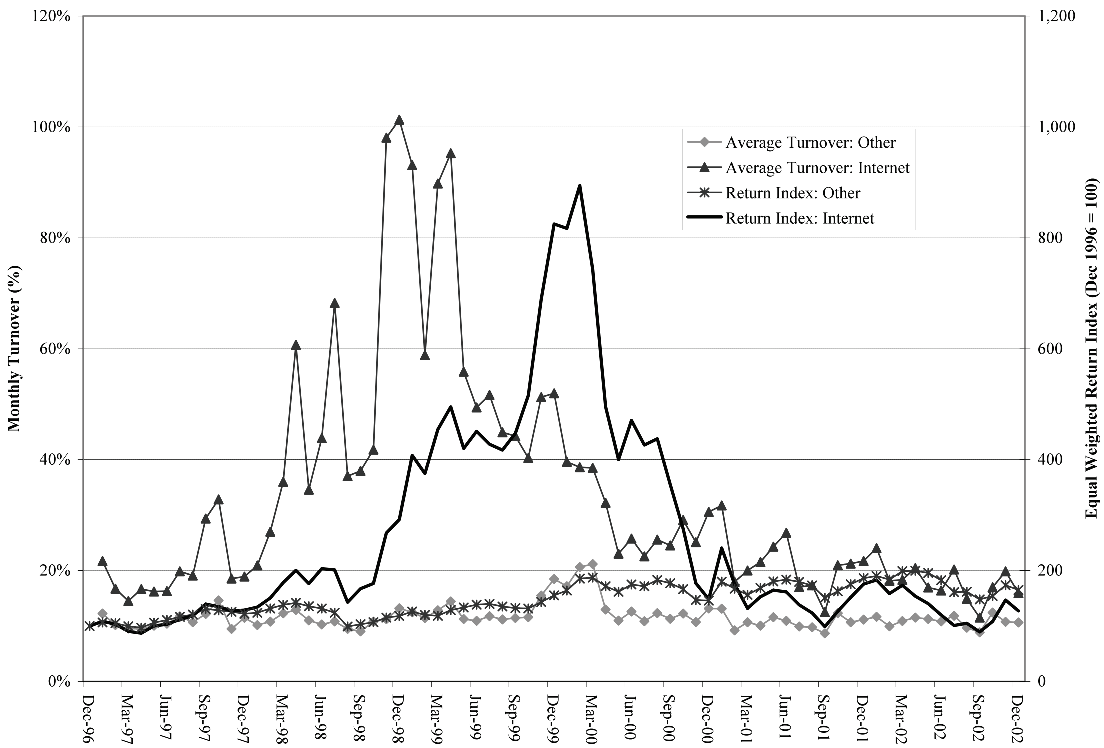

Figure 5: Issuance of IG synthetic CDOs (Monthly)

Source: Citygroup. The figure plots the monthly issuance of investment grade synthetic CDOs.