INFORMATION ACQUISITION IN RUMOR BASED BANK RUNS

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ABSTRACT

We study information acquisition and dynamic withdrawal decisions when a spreading rumor exposes a solvent bank to a run. Uncertainty about the bank's liquidity and potential failure motivates depositors who hear the rumor to acquire additional noisy signals. Depositors with less informative signals may wait before gradually running on the bank, leading to an endogenous aggregate withdrawal speed and bank survival time. Private information acquisition about liquidity can subject solvent-but-illiquid banks to runs, and shorten the survival time of failing banks. Public provision of solvency information can mitigate runs by indirectly crowding-out individual depositors' effort to acquire liquidity information.

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1 Introduction

Bank runs are unfortunately still with us. Runs occurred recently on major traditional banks such as Northern Rock and IndyMac, and on non-traditional "shadow" banks. In Section 2, we document several intriguing features of the largest ever bank failure, which resulted from the 2008 run on Washington Mutual: (i) the bank survived a first run, followed by deposit inflows; (ii) rumors circulated, but there was no public information of the first run happening; (iii) uncertainty about bank liquidity played a key role in a second fatal run; (iv) both runs had an important time dimension; (v) worried depositors (even those covered by FDIC insurance) acquired additional information, then some withdrew immediately, while others decided to wait. The empirical pattern that runs have an important time dimension and that uncertainty and learning play a key role, has been documented as far back as the 1857 run on the Emigrant Industrial Savings Bank, and is especially important for the uninsured shadow banking system (e.g., money market funds behavior amid the European debt crisis discussed in Section 2). The dynamic effects of information acquisition and the incentives to wait, on the existence and duration of bank runs, have not been studied by existing bank run models. We aim to fill this gap.

We study dynamic rumor-based bank runs with endogenous information acquisition. Our model incorporates uncertainty about bank liquidity into the asynchronous awareness framework of Abreu and Brunnermeier (2003). A liquidity event occurs at some unobservable random time, at which point a solvent bank becomes illiquid with a known probability or remains liquid. An illiquid bank has only a limited amount of cash exposing it to a run, while a liquid bank has sufficient reserves to avoid a run. In the 07/08 financial crisis, the liquidity event can be thought of as banks with opaque exposure to mortgage-backed securities becoming weak and thus "illiquid" following adverse shocks in the housing market. The liquidity event triggers the spread of a *rumor* saying the liquidity event has occurred and the bank may be illiquid, exposing the bank to a run. Agents (depositors) can withdraw or deposit funds back at any time for a small transaction cost, but in the unique bank run equilibrium, upon hearing the rumor, agents wait, fully withdraw, and then redeposit if the bank survives. The optimal withdrawal time trades-off the marginal benefit of waiting inside the bank due to the yield on deposits, with the marginal cost of waiting due to risk of loss in a bank failure.

We first show that a straightforward application of Abreu and Brunnermeier (2003) to bank runs *without* information acquisition is rather uninteresting: in equilibrium, *either* bank runs never occur, *or* depositors run on the bank immediately upon hearing the rumor. Intuitively, if the conjectured (off-equilibrium) bank survival time is so long that every agent decides to wait a bit longer, then the illiquid bank would fail later. This longer survival time pushes agents to wait even longer. This force is so strong that, in equilibrium, agents never withdraw and the bank survives indefinitely, i.e. runs never occur. On the other hand, if every agent decides to withdraw a bit earlier, then the equilibrium force of a shorter bank survival time pushes all agents to withdraw immediately upon hearing the rumor. In this bank run equilibrium, the withdrawal speed and the implied bank survival time are exogenously determined by the rumor spreading speed.[1]

We then allow uncertainty-motivated informed agents who hear the rumor to acquire additional noisy signals. Our main result shows that private information acquisition about liquidity exposes otherwise safe banks to destructive runs with endogenous waiting. There are three potential signal realizations: *good*, *fair*, and *bad*, which are independent across agents conditional on the bank liquidity state. Information acquisition with noisy signals creates belief heterogeneity among depositors, which is key to our result. A good or bad signal reveals the liquidity state perfectly. Receiving a fair signal can be interpreted as seeing mixed evidences, leaving the agent still uncertain about bank liquidity as his prior. A higher quality signal is more likely to reveal the state perfectly. If the bank is indeed illiquid, then in aggregate a higher quality implies more agents receive bad signals thus knowing the bank is illiquid.

Fixing the signal quality, we show that in a setting where bank runs never exist otherwise, an interesting bank run equilibrium with endogenous waiting emerges once we allow agents to acquire information. Conditional on the bank being illiquid, agents with bad signals perceive a high bank failure hazard rate and withdraw immediately. Agents with uninformative fair signals maintain the same beliefs they had before acquiring the signals, but now also worry about withdrawals by bad-signal agents—hence fair-signal agents withdraw earlier than they would otherwise. This effect goes a long way in changing the qualitative nature of the bank run equilibrium. In equilibrium, instead of waiting indefinitely, agents with fair signals wait some endogenous time, earning a yield on

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[1]In deriving the endogenous bubble burst time, Abreu and Brunnermeier resolve this issue by assuming that the bubble component increases over time. This would be a hard assumption to swallow in a bank run setting: it would require that for a *perfectly liquid bank* the value of a dollar deposited declines over time.

deposits, but withdraw once the failure hazard rate becomes substantial, leading to an endogenous equilibrium (illiquid) bank survival time.

The mechanism is as follows. Without information acquisition, all agents effectively have fair signals. If all agents were to wait longer, the equilibrium bank failure time would be delayed by the same amount. This force pushes all agents to wait indefinitely and thus banks never fail. With information acquisition, there is belief heterogeneity among depositors; specifically, there are both fair-signal and bad-signal agents conditional on the bank being illiquid. When all fair-signal agents wait a bit longer, the delay in bank failure is limited, because more bad-signal agents have withdrawn. A "fear-of-bad-signal-agents" increases the equilibrium hazard rate perceived by fair-signal agents, and results in a unique endogenous withdrawal time that equates the marginal cost and benefit of waiting.

We then study the endogenous choice of signal quality with the bank run equilibrium. With this additional ingredient, our model features both strategic *complementarity* and *substitutability* in information acquisition among individual agents. Potential bank failure motivates agents to acquire higher quality signals, which in turn, shorten the equilibrium survival time of the failing bank. This positive spiral between bank runs and information acquisition makes bank runs more likely. We show that in an economy that without information acquisition is free of bank runs, there could be two equilibria once information acquisition is allowed: a *no-run-no-acquisition equilibrium* where agents do not acquire additional signals and do not run, and a *run-acquisition equilibrium* where agents acquire information aggressively and run on the illiquid bank. Socially inefficient information acquisition about bank illiquidity thus exhibits strategic complementarity regarding *whether to acquire information.*[2]

However, in our setting with smooth information acquisition costs, information acquisition exhibits strategic substitutability regarding *how much information to acquire* due to a learning effect. In our model there is a natural "public" signal conveyed by the fact that the bank has not yet failed. If others study more about the bank and run on the illiquid one, then the illiquid bank fails faster. As a result, this public signal becomes more informative. Because a more informative

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[2]This positive feedback effect is similar to Hellwig and Veldkamp (2009) who show that in their particular static setting, information acquisition exhibits complementarity if and only if ensuing actions (in our model, running on the bank) are complementary. In other economic settings, information acquisition complementarity arises even though ensuing actions are not complementary; one such example is the ex ante investment in high-frequency trading technology analyzed in Biais, Foucault, and Moinas (2013).

public signal lowers the value of private signals, individual agents hearing the rumor respond by acquiring less information. This learning effect also explains a counter-intuitive result that the wider the awareness window over which the rumor spreads, implying *more* potential withdrawers, the *longer* the illiquid bank survives. The intuition is that conditional on the bank surviving thus far, the bank is more likely to be liquid given that the rumor could have started spreading a long time ago, which discourages incentives to acquire information.

We extend our model along two dimensions. First, we consider fundamentally insolvent banks. When agents privately collect information about bank *solvency*, they inevitably learn about bank *liquidity*. Effort spent acquiring one reduces the effort needed to acquire the other. We show that public provision of solvency information curbs the private acquisition of bank liquidity information and delays a bank's failure, which, even for a short while, can help both regulators and industry organize an orderly resolution that minimizes damage to the banking system.[3] Thus our model implies that carefully constructed stress tests can help prevent or mitigate bank runs by crowding out information acquisition by individuals. Second, we show it is beneficial to avoid providing too much information that differentiates competing solvent but potentially illiquid banks, as small differences in liquidity can result in runs on slightly weaker banks. This justifies U.S. government actions that forced all "Big 9" banks to take government capital on October 13, 2008.

Our model is related to a vast literature on the role of information in *static* bank runs, and it is beyond the scope of our paper to have a thorough review on this topic.[4] The distinguishing feature is gradual withdrawal, which is an important aspect of modern bank runs (see Section 2). The dynamic nature of bank runs has been explored by studying the sequential service constraint in the Diamond and Dybvig setting. Green and Lin (2003) point out that no bank runs exist if depositors know their clock times of arrival and thus infer their relative positions in the queue, while Peck and Shell (2003) show that bank runs exist if the relative queue position information is

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[3] See March 16, 2008, *Financial Times* article titled "Bear races to forge deal with JPMorgan."

[4] See Gorton (1985), Bhattacharya and Gale (1987), Jacklin and Bhattacharya (1988), Chari and Jagannathan (1988), and recently, Ennis and Keister (2009). Our model provides an information-based theory of bank panics as opposed to a sunspot-based theory (Gorton and Winton, 2003, Ch. IV). On the topic of information acquisition and bank runs, Nikitin and Smith (2008) study how information acquisition about bank *solvency* affects the bank run equilibrium in a static Diamond and Dybvig setting. In contrast, we focus on information acquisition about bank *liquidity*. Based on the Morris and Shin (2002) global games technique, Goldstein and Pauzner (2005) study the optimal deposit contract by deriving a unique equilibrium when depositors in a Diamond and Dybvig type setting are endowed with private noisy signals about bank fundamentals. We allow for endogenous information acquisition, and show that excessive socially wasteful learning may lead to socially inefficient runs on solvent-but-illiquid banks.

unavailable.[5] In this regard, our paper is similar to Peck and Shell (2003) since each agent in our model assigns the same distribution to her relative position in the queue. Relative to these models, we emphasize the endogenous interaction between information acquisition and bank runs.

Gu (2011) studies depositors' withdrawal strategy sequentially for a potentially insolvent bank in a Diamond and Dybvig setting. Unlike our paper, (some) depositors in Gu (2011) observe previous withdrawals. Because withdrawals indicate bank insolvency, Gu (2011) shows that herding (ignoring private information which is endowed exogenously) after a long sequence of withdrawals may lead to inefficient bank runs. Our paper with endogenous information quality emphasizes individual depositors' uncertainty and forming of rational expectation over other depositors' withdrawal timing and strategies. This aspect is important given the modern age of electronic banking, the high frequency nature of repo in the context of shadow banking, or the large scale runs by money market funds that faced the European sovereign debt crisis. Thus our mechanism is complementary to Gu (2011) in understanding the dynamics of bank runs together with the role of information.

From a modeling perspective, our framework features more empirically appealing timing assumptions in that individual depositors can withdraw or redeposit their funds at any time; this represents a substantial improvement from the Diamond and Dybvig framework and the above-mentioned important extensions. If high frequency micro-level data is available, our analytically tractable model is particularly calibration-friendly thanks to its empirically-interpretable time dimension.[6] An alternative approach to bank run modeling is He and Xiong (2012a,b) who develop dynamic debt run models of a firm with a time-varying fundamental and a staggered debt structure.

Rumors are studied by Banerjee (1993) and van Bommel (2003). We build on the approach of Abreu and Brunnermeier (2002, 2003) who consider the asynchronous timing of awareness to study "rational" bubbles.[7] We add to their model uncertainty about the capacity of the bubble (bank), decouple the spreading rate from the length of the awareness window, and allow agents to acquire additional noisy information upon awareness. This ex post heterogeneity generates the

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[5]This literature focuses on optimal mechanism design; for related papers, see Wallace (1988, 1990), Andolfatto, Nosal, and Wallace (2007), and Andolfatto and Nosal (2008).

[6]By contrast, Green and Lin (2003) and Peck and Shell (2003) study finite-agent economy, and depositors can either withdraw at period 1, when called in line, or wait until period 2. Gu (2011) considers a relaxation that a finite number of depositors can withdraw at any time within an interim stage before period 2. A significant departure from the Diamond and Dybvig framework seems necessary for developing dynamic models that are both calibration-friendly and tractable. A sacrifice our model makes is to assume away the preference (early or late) type of depositors, an ingredient critical for Diamond and Dybvig, but inessential for our paper.

[7]Brunnermeier and Morgan (2010) generalize this idea to a class of "clock games" and test its main predictions.

fear-of-bad-signal-agent effect, which naturally leads fair-signal agents to withdraw earlier, giving rise to a bank run equilibrium in which the illiquid bank fails at an endogenous time.

The paper proceeds as follows. Section 2 summarizes empirical facts on bank run dynamics. Section 3 builds a rumor-based bank run setting. Section 4 shows that information acquisition with noisy signals leads to an interesting bank run equilibrium with an endogenous (aggregate) withdrawal speed. Section 5 discusses empirical predictions of our model. Section 6 provides extensions and discussion. We conclude in Section 7. Proofs are in the Appendix.

2 Empirical Bank Run Evidence

We begin by providing new evidence about the failure of Washington Mutual (WaMu), and synthesizing the empirical literature on bank run dynamics. Many of the stylized facts about runs on traditional regulated commercial banks also appear in shadow bank runs. In organizing these facts about bank runs, we highlight three aspects that are key contributions of our model: (i) bank runs are dynamic and gradual with rumors spreading among depositor-like agents; (ii) uncertainty and learning about the bank's liquidity drive agents' withdrawal/redeposit behavior; and (iii) individual agents are actively acquiring their own information before they run.

2.1 Traditional Banks

Figure 1 shows that the runs on WaMu were dynamic in nature. The bank suffered two separate runs: it experienced gradual withdrawals of $9 billion during a first run lasting 20 days in July 2008, and withdrawals of $15 billion during a second run lasting 15 days in September 2008, leading up to its takeover by the FDIC.

Information and uncertainty play an important role in both runs on WaMu. Following the failure of IndyMac on July 11, 2008, WaMu experienced massive withdrawals but managed to survive. Interestingly, consistent with our modeling of "rumor" as "information without discernible origin" in Section 3.1.3, although plenty of rumors about it circulated online, this run was never publicly reported before WaMu's failure.[8] After this first run, the bank regained much of its deposit

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[8]Some examples from answers.yahoo.com dated July 26, 2008: (1) "Anybody else hear rumors about WaMu bank potentially being taken over by the Federal Government? I heard that it is failing too. Who's next? Is it time to make that run yet?" (2) "I heard the same thing...I have a Wamu bank card....Yaaaaaaaaaaaahooooooo! This means I can tap it out and the Government will bail me out." (3) "I am so glad I took my money out." (4) "Actually I heard
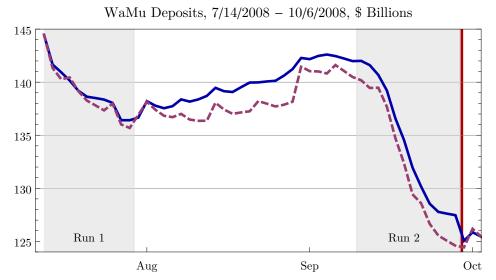
Figure 1: The Dynamic Nature of the Run on WaMu

WaMu Deposits, 7/14/2008 − 10/6/2008, $ Billions

Daily deposit balances as reported by Washington Mutual Bank to the Office of Thrift Supervision (OTS). Dashed line shows unadjusted balances which include consumer noninterest, consumer interest-bearing, and small business deposits. Solid line shows balances adjusted for Friday and end-of-month fixed effects, i.e. days with automatic payday deposits. The OTS appointed the FDIC as receiver of WaMu on the evening of September 25, 2008 (the vertical line in the figure) which then sold it to JPMorgan Chase.

base, suggesting that the mere survival of WaMu conveys positive information to its depositors. In September 2008 after Lehman Brothers failed, uncertainty regarding WaMu's liquidity and hence its eventual fate played a key role in a fatal second run. The bank's ability to secure funds from the Fed was questionable.[9] Depositors sought out additional information about its liquidity, then some depositors withdrew while others decided to wait.[10] Such noisy information acquisition and

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they had did better than expected in earnings. Only rumor I heard was that National City was going to be next."

[9]WaMu depended on borrowings from the FHLBs of San Francisco and Seattle and the San Francisco Fed acting as lenders of last resort. Given its composite CAMELS rating of 4, there was no assurance that funds would be available in the amounts and in the timing needed to meet its obligations. In fact, the bank lost its access to the Term Auction Facility program. See September 25, 2008 letter by Darrel W. Dochow, OTS Regional Director recommending FDIC receivership.

[10]"All across the country, customers streamed into hundreds of WaMu branches, on edge, brimming with questions. This run was nothing like July's, which had been tame by comparison. Now people were frenzied, acting purely out of fear. 'We just didn't have any control,' said one branch manager. The tellers' talking points no longer placated the customers—even the tellers didn't understand them. 'They kept telling us to say that we have $44 billion, whatever that meant,' said one employee, likely referencing WaMu's liquidity position. The customers demanded to talk to whoever was in charge. At one Seattle location, they lined up and waited all day to speak to the branch manager. 'Should I take my money out?' they asked her when they reached her, over and over again. 'Is WaMu going under?' And they asked the inevitable question: 'What would *you* do?' The branch manager, who knew most of the people's names and their kids, would say, 'I don't know any more than you do. But if you're uncomfortable, do what makes you sleep at night.' She wasn't sleeping, either. 'You went to work and you go hammered,' the branch manager said later. 'By the end of the day, you were exhausted. And you didn't know if you had a job, or what was going on.' WaMu headquarters told the branches repeatedly that everything was fine, but the news reports and the blogs and the stories posted on Facebook all said otherwise. 'You told the customers something, and then they watched the news, and then they would come back,' the branch manager said. Many took their money out anyway." (Grind,

the decision to wait before running are the main focus of our paper.

Surprisingly, more than half of running WaMu depositors were in fact covered by FDIC insurance. Withdrawals by insured depositors could make sense if they worry the FDIC might temporarily freeze their accounts, or that it could not afford to cover WaMu's insured deposits which were several times the Deposit Insurance Fund at the time, or simply as one depositor explained, they "just don't want to deal with it."

Iyer and Puri (2012) document the dynamic nature of an Indian bank run influenced by its depositors' social network. They also find that deposit insurance is only partially effective in preventing runs. Unlike in WaMu's first run in July 2008, deposit balances of most depositors who ran on the Indian bank, did not return to pre-run levels.[11]

O Grada and White (2003) find that the 1857 bank run on the Emigrant Industrial Savings Bank had an important time dimension: "*In neither 1854 nor 1857 did depositors respond to a single signal that led them to crowd into banks all at once. Instead, panics lasted a few weeks, building and sometimes ebbing in intensity, and only a fraction of all accounts were closed.*" They conclude that the 1857 run was driven by informational shocks in the face of asymmetric information about the true condition of bank portfolios. Kelly and O Grada (2000) provides further evidence on information transmission during this run.

2.2 Shadow Banks

Maturity transformation is no longer exclusively done in traditional commercial banks. Shadow banks, which include securitization vehicles and investment banks, provides short-term funding for long-term projects in a broad sense and has developed into an unregulated substitute to traditional commercial banks. With short-term funding contracts akin to demand deposits, these "shadow" banks are subject to runs with similar features.[12]

In September 2008 the Securities and Exchange Commission conducted an audit of the collapse

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2012, p. 277-278). See also July 24, 2008, *Bloomberg* article titled "WaMu Slumps as Gimme Credit Cites Liquidity Concern."

[11]In our main model with a single bank (which can be either liquid or illiquid), depositors return to the surviving bank once they endogenously learn it is liquid. In Section 6.2 we introduce competing banks. There, withdrawing depositors never return to their original bank, which offers a possible rationale for the findings of Iyer and Puri (2012).

[12]For instance, Gorton and Metrick (2011), Covitz, Liang, and Suarez (forthcoming), and Acharya, Schnabl, and Suarez (2013) document "shadow" bank runs on repo and asset-backed commercial paper (ABCP). Schroth, Suarez, and Taylor (2013) find that runs on ABCP are sensitive to leverage, asset values, and asset liquidity.

of Bear Stearns. Consistent with our modeling of dynamic nature of rumor, the executive summary starts with: "*During the week of March 10, 2008, rumors spread about liquidity problems at Bear Stearns. As the rumors spread, Bear Stearns was unable to obtain secured financing from counterparties.*"[13]

Given the sophisticated multi-layered structure of the modern financial system, "bank runs" can be interpreted broadly as funding withdrawals by one class of short-term lenders (i.e., not just household depositors) from other institutions. For instance, during the 2011 European sovereign debt crisis, money market funds (like depositors) divested from European banks. Fears of liquidity, or rollover risk, rather than solvency drove fund managers to cut their exposure to Spanish and Italian banks.[14] Consistent with our model, this run was dubbed the "slow-motion" bank run in the popular press, referring to its gradual nature: money market funds reduced their exposure to all Eurozone banks by 37% from $453 billion to $287 billion between May and August 2011 (Chernenko and Sunderam (2012)). More interestingly, individual information acquisition played a crucial role, as fund managers each did their own research and came up with different conclusions, prompting some to run and others to rollover their investment. [15] Eventually, U.S. money market funds returned to the European money market.[16] Their return to these positions is hard to generate using existing models, but is a feature of ours, which we turn to next.

3   The Base Model without Information Acquisition

We first build a rumor-based bank run model by introducing uncertainty into the asynchronous awareness setting of Abreu and Brunnermeier (2003), which is our base setting without information acquisition. Most of the analysis here, such as learning and optimal withdrawal strategies, applies to later situations with information acquisition. We show that, without information acquisition, *either* bank runs do not occur at all, *or* depositors run on the bank immediately upon hearing the rumor. Therefore, bank runs, if they occur, exhibit an exogenous withdrawal speed, which implies

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[13]"SEC's oversight of Bear Stearns and related entities: The consolidated supervised entity program," report no. 446-A published on September 25, 2008. Interestingly, Chairman Christopher Cox described Bear Stearns as a well-capitalized entity, which "*was done in by a cash shortage, because lending dried up and prime-brokerage clients moved their money elsewhere*" (April 3, 2008, Bloomberg article titled "Cox Defends SEC's Role in Regulating Bear Stearns (Update1)").

[14]See July 24, 2011, *Financial Times* article titled "Money market funds cut euro bank exposure."

[15]See July 5, 2011, *Reuters* article titled "U.S. money funds diverge on European bank paper."

[16]See July 29, 2013, *Financial Times* article titled "US money market funds return to EU banks."

the bank fails at some exogenous time. This result changes qualitatively once we allow agents to acquire additional noisy private signals.

3.1 The Setting

3.1.1 Bank and Depositors

Time is continuous on $t \in [0, \infty)$. There is a unit mass of infinitely-lived risk-neutral agents (depositors) with a zero discount rate. Bank deposits yield a constant rate of return $r > 0$ when the bank is operating, while holding cash outside the bank earns nothing. Broadly, one can interpret the bank as some investment vehicle in the shadow banking system or even the entire financial system, and the positive relative wedge reflects either a higher investment growth rate or a convenience yield for keeping funds in the institution. Our main analysis focuses on a single bank, but we consider competing banks offering similar returns as an extension in Section 6.2.

To avoid exploding values for a dollar inside the potentially safe bank, we assume that the bank's growth stops at some "maturing" event modeled as a Poisson shock with intensity $\delta > r$, and afterwards the game ends (e.g., each agent gets his deposit back for consumption).[17] Throughout, this maturing event is independent of any other random variables that we consider.

3.1.2 Uncertainty about Bank Liquidity

There are two potential types of banks that are fundamentally solvent (i.e., bank survival is the first-best allocation), with one type of bank being "illiquid," and a second "liquid" bank impervious to runs. Uncertainty about bank liquidity is crucial to our analysis, though its source could be fundamental shocks.[18]

Bank liquidity is defined as the mass of depositors it takes to run down the bank. For simplicity, we assume a binary structure for the state of bank liquidity $\widetilde{\kappa}$. When the bank is illiquid, $\widetilde{\kappa}$ takes a lower value $\kappa_L < 1$, and it fails when a κ_L measure of depositors have fully withdrawn their funds. One can literally interpret $\widetilde{\kappa}$ as the bank's cash reserves to meet withdrawals. We broadly interpret

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[17]A similar but more natural interpretation is that individual agent suffers liquidity shocks with immediate consumption need (therefore withdrawal). The analysis, which is available upon request, is similar but much more complicated (due to the replacement of agents).

[18]We consider uncertainty with known probabilities (risk). Generalizing our model to Knightian uncertainty might be interesting for future research.

$\widetilde{\kappa}$ as the liquidity of the bank, which could represent uncertainty about the willingness of a lender of last resort (e.g. central bank) to lend to bank (see footnote 9 for an example). For the liquid bank with a higher liquidity reserve, $\widetilde{\kappa} = \kappa_H > \kappa_L$. We focus on the case that there is no run on the liquid bank; hence for now it is useful to think of $\kappa_H > 1$, i.e. the sufficiently liquid bank can survive any severe run. However, in Section 6.3.1 we verify that our asynchronous awareness model can have $\kappa_H < 1$ but still rule out runs on the liquid bank.

In our economy, it is common knowledge that conditional on the liquidity event (to be introduced shortly), the probability of the illiquid state is $p \in (0,1)$. When the illiquid bank fails, its assets have a fire-sale price to recover $\gamma \in (0,1)$ dollar for each dollar of deposits. Afterwards, all agents go to autarky consuming their remaining wealth.

We focus on uncertainty about liquidity for solvent banks for better comparison to the literature.[19] Section 2.2 documents that during the 2007/08 financial crisis, concerns regarding liquidity of financial institutions are often highlighted among investors and regulators. No doubt, both solvency and liquidity are highly linked to bank fundamentals. In reality, banks are not just either solvent or insolvent; within solvent banks, there are strong ones and weak ones that differ in their fundamentals. Our modeling of liquid/illiquid banks can be equivalently interpreted as strong/weak banks. Indeed, as detailed in the next subsection, one can interpret the illiquid bank as an institution whose capital base is hit by adverse shocks to its mortgage-backed securities, making it weak but still solvent. Later in Section 6.1 we introduce fundamentally insolvent banks as an extension.

3.1.3 Liquidity Event, Spreading Rumors, and Informed Agents

At $t = 0$ the bank is liquid, i.e., $\widetilde{\kappa} = \kappa_H$. At some unobservable random time $\widetilde{t_0} > 0$ referred to as the liquidity event, the bank may become illiquid, and the uncertainty is as modeled in Section 3.1.2. Mapping to the 2007/08 crisis in which banks have opaque exposure to real estate, one may think of this event as the gradual deterioration of mortgage-backed securities which hurts banks' liquidity but keeps them solvent. Although the exact liquidity event time t_0 is publicly *unobservable*, it is common knowledge that $\widetilde{t_0}$ is exponentially distributed on $[0, \infty)$ with cumulative distribution $\Phi(t_0) \equiv 1 - e^{-\theta t_0}$, where θ is a positive constant.

Knowledge of this liquidity event starts spreading in the population after $\widetilde{t_0}$, i.e., some agents

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[19]For instance, Nikitin and Smith (2008) and Gu (2011) focus on bank solvency information.

hear a "*rumor*" that "the liquidity event $\widetilde{t_0}$ has occurred and thus the bank might be illiquid." Since the exact liquidity event time $\widetilde{t_0}$ is unknown, this spreading information captures the essence of an unverified rumor of uncertain origin that spreads gradually in the depositor population. Importantly, hearing the rumor informs the agent that "the liquidity event $\widetilde{t_0}$ has occurred and hence other agents in the population may have heard the rumor."

We call those agents who hear the rumor "informed," or simply "agents," and the rest "uninformed." Since at $t = 0$ the bank is liquid, throughout we assume that agents' beliefs are such that they stay inside the bank unless they hear the rumor. In Section 6.3.2 we show this is indeed the case under certain conditions, by analyzing the problem of an uninformed agent.

Given a realization of $\widetilde{t_0} = t_0$, the rumor begins to spread over an interval $[t_0, t_0 + \eta]$ with a positive constant (exogenous) length η. Following Abreu and Brunnermeier (2003) we refer to η as the "awareness window." At any interval $[t, t + dt]$ where $t \in (t_0, t_0 + \eta)$, uninformed agents become informed by hearing this rumor with probability βdt, where β is an exogenous positive constant. The rumor shock is i.i.d. across the population of uninformed agents. Then for $t \in [t_0, t_0 + \eta]$, the mass of uninformed and informed agents are $e^{-\beta(t-t_0)}$ and $1 - e^{-\beta(t-t_0)}$ respectively, and the mass of newly informed agents within $[t, t + dt]$ is $\beta e^{-\beta(t-t_0)} dt$. To make the problem interesting, we assume that at time $t_0 + \eta$ the fraction of informed agents, which is $1 - e^{-\beta\eta}$, is sufficient to take down the illiquid bank, i.e., $1 - e^{-\beta\eta} > \kappa_L$.

This spreading technology is different from that of Abreu and Brunnermeier (2003) in that we allow for separation between the spreading rate β and the awareness window η.[20] As shown later, the awareness window η has interesting effects opposite to common wisdom.

Call the informed agent who hears the rumor at $t_i \geq t_0$ simply agent t_i. Consistent with electronic age banking, he neither observes withdrawals before him, nor the potential queue in front of the bank. In the context of shadow banking, given the high frequency nature of repo financing, we assume that individual creditors do not observe, but do form rational expectations about, other creditors' rollover decisions. We share with Abreu and Brunnermeier (2003) the simplifying assumption that agents do not observe the market price of bank equity, which future work can introduce by adding noise to prices.[21]

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[20] Abreu and Brunnermeier assume a linear spreading technology with rate $\frac{1}{\eta}$ so that the entire population is informed at $t_0 + \eta$. This way, the awareness window η is artificially tied to the spreading rate $\frac{1}{\eta}$.

[21] If investors trading in the stock market have the same information as depositors, then bank stock prices convey

3.1.4 Transaction Costs and Agent's Problem

To eliminate strategies with infinite transactions, we assume a constant, yet arbitrarily small, transaction cost c per dollar when the agent receives cash from, or deposits cash back in the bank— including the maturing event (so the agent receives $1 - c$ per dollar of deposit) or bankruptcy (so he receives $\gamma(1 - c)$). This assumption is innocuous as our analysis carries through when $c \to 0$.

In sum, after hearing the rumor an agent can withdraw his deposits whenever he believes bank failure is imminent, and redeposit this cash in the future if the bank's survival sufficiently improves his posterior belief, both subject to a transaction cost c. Due to the linearity of this problem, a "bang-bang" strategy, i.e. keeping the entire wealth either in or out of the bank, is optimal.

Our model allows for the possibility of depositors redepositing back to the bank, unlike existing literature based on the Diamond and Dybvig model (e.g., Green and Lin (2003), Gu (2011), etc). Though it is not necessary for our qualitative results, the redepositing option is empirically motivated and theoretically appealing. Redepositing behavior is apparent in Figure 1, which shows that after the first run on WaMu, the bank regained much of its deposit base. From a theory perspective, we find it more coherent to allow agents to deposit or withdraw at any time, with some transaction costs. We take up this challenge and show that before the bank fails the class of threshold strategies is optimal. Otherwise, the optimality of threshold strategies becomes trivial given an assumption that prohibits redepositing.

3.2 Learning

3.2.1 Posterior Belief about t_0

Agent $t_i \in [t_0, t_0 + \eta)$ updates his posterior distribution of t_0 conditional on hearing the rumor at t_i. Given t_0, the likelihood of an individual uninformed agent hearing the rumor over $[t_i, t_i + dt]$ but not earlier, is $f(t_i|t_0) = \beta e^{-\beta(t_i - t_0)} dt$ for $t_i \in [t_0, t_0 + \eta)$.

Recall that the prior density of the liquidity event timing t_0 is $\phi(t_0) = \theta e^{-\theta t_0}$. We follow Abreu

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noisy signals of both the liquidity state of the bank and the time the rumor started spreading. These noisy signals would change the learning formula for depositors. The analysis of the bank run equilibrium might be challenging, as each agent's withdrawal decision could depend on the price history that she observes since hearing the rumor, and the entire price history enters the aggregate withdrawal condition. We conjecture this "learning effect" would only change the quantitative predictions of our model, because our model can be viewed as the limiting case where stock market prices are pure noise. Another important consequence of introducing noisy stock prices is that public signals can be coordination devices; see Abreu and Brunnermeier (2003) for a discussion of synchronizing events.

and Brunnermeier (2003) to focus on realizations of $t_0 \geq \eta$ such that the economy is already in a stationary phase; as shown shortly agent t_i's equilibrium strategy will be independent of the absolute timing of being informed.[22] With $t_0 \geq \eta$, the informed agent $t_i \geq t_0 \geq \eta$ updates his posterior belief about t_0 as

$$\phi(t_0|t_i) \equiv \frac{f(t_i|t_0)\phi(t_0)}{\int_{t_i-\eta}^{t_i} f(t_i|s)\phi(s)\,ds} = \frac{\theta-\beta}{e^{(\theta-\beta)\eta}-1}e^{(\theta-\beta)(t_i-t_0)} \equiv \frac{\lambda e^{\lambda(t_i-t_0)}}{e^{\lambda\eta}-1}, \tag{1}$$

where we define $\lambda \equiv \theta - \beta$. Without loss of generality, we assume that $\lambda > 0$.[23] Integrating (1) over t_0 we get the conditional cumulative distribution function for t_0 with support $t \in [t_i - \eta, t_i]$:

$$\Phi(t|t_i) \equiv \Pr\left(\widetilde{t_0} \leq t|t_i\right) = \int_{t_i-\eta}^t \phi(s|t_i)\,ds = \frac{e^{\lambda\eta} - e^{\lambda(t_i-t)}}{e^{\lambda\eta}-1}. \tag{2}$$

3.2.2 Bank Failure Hazard Rate

Suppose that agents believe the illiquid bank fails at $t_0 + \zeta$ where ζ is the survival time to be determined in equilibrium (potentially infinite), and the event of bank failure is $\{t_0 + \zeta < t, \kappa_L\}$.

For agent t_i at the absolute time $t > t_i$, denote by $\tau \equiv t - t_i$ the time elapsed since he heard the rumor. For this agent, the bank fails in the next instant if and only if t_0 occurs at $t_0 = t - \zeta = t_i + \tau - \zeta$, *and* the bank is illiquid. Since the bank being illiquid (with probability p) is independent of t_0, the bank failure hazard rate at $t = t_i + \tau$ is

$$h(t_i + \tau|t_i) = \frac{p\phi(t_i + \tau - \zeta|t_i)}{1 - p\Phi(t_i + \tau - \zeta|t_i)}.$$

Combining (1) and (2), we get $h(t_i + \tau|t_i)$ of the following proposition, which is independent of the absolute time agent t_i is informed. This stationarity allows us to denote $h(t_i + \tau|t_i)$ by $h(\tau)$.

Proposition 1. *Suppose that the illiquid bank fails at $t_0 + \zeta$. Then the failure hazard rate is positive*

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[22]The finite awareness window η over which the rumor spreads makes the cases of $t_0 < \eta$ and $t_0 \geq \eta$ different. In the event of $t_0 \geq \eta$, it always holds that $t_i \geq \eta$, and rational agents know that $t_0 \in [t_i - \eta, t_i]$. In Appendix C we consider the equilibrium behavior for $t_0 < \eta$.

[23]The assumption of $\lambda > 0$ is for exposition purpose, and the analysis goes though if $\lambda < 0$. To see this, if $\lambda < 0$, then the conditional density $\phi(t_0|t_i)$ can be written as $\frac{(-\lambda)e^{\lambda(t_i-t_0)}}{1-e^{\lambda\eta}}$ which is still positive.

14

for $\tau \in [\max(\zeta - \eta, 0), \zeta]$, with

$$h(\tau) \equiv h(t_i + \tau | t_i) = \frac{p\lambda e^{\lambda(\zeta - \tau)}}{(1 - p)(e^{\lambda \eta} - 1) + p(e^{\lambda(\zeta - \tau)} - 1)}. \tag{3}$$

The hazard rate is zero, if i) either $\zeta > \eta$ and $\tau < \zeta - \eta$, in which case the time $t_i + \tau$ is before the earliest possible bank failure at $t_i - \eta + \zeta$, or ii) $\tau > \zeta$, as the bank is revealed to be liquid.

We require the prior of the bank being illiquid, i.e. p, to be sufficiently high that

$$\frac{p}{1 - p} > e^{\lambda \eta} - 1. \tag{4}$$

Under this condition, the hazard rate in (3) increases with τ, the time elapsed since hearing the rumor. More time without failure lowers the posterior belief that the bank is illiquid and reduces the hazard rate. But, since the illiquid bank fails a fixed amount of time after the rumor starts to spread, every minute that passes since the agent heard the rumor brings him closer to potential failure, which increases the hazard rate. The condition (4) guarantees the latter effect dominates.[24]

3.3   Optimal Withdrawal Strategies

We present the key proposition that characterizes individual agents' optimal withdrawal policy, taking the equilibrium survival time ζ as given.

3.3.1   Value Functions

Due to stationarity, denote by $V_I(\tau)$ ($V_O(\tau)$) the optimal value of one dollar inside (outside) the bank at time $t = t_i + \tau$. Because withdrawal or redepositing costs c, the optimality condition implies $V_I(\tau) \geq (1 - c) V_O(\tau)$ and $V_O(\tau) \geq (1 - c) V_I(\tau)$ for all $\tau \geq 0$.

When $\tau \geq \zeta$, an inside dollar grows at rate r until the maturing event (with intensity δ), with a value of (recall that transaction costs also apply to the maturing payout)

$$V_I(\tau) = \int_0^\infty e^{rs}(1 - c)\delta e^{-\delta s} ds = \frac{(1 - c)\delta}{\delta - r} \text{ for } \tau \geq \zeta.$$

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[24]If (4) fails, the hazard rate decreases with τ, and agents, if they ever choose to withdraw, will withdraw upon hearing the rumor $\tau = 0$ and redeposit to the bank at some endogenous time. This case with endogenous entry could be interesting for future research on recovery after crises.

Given the transaction cost, the value of one dollar outside the bank at ζ is $V_O(\zeta) = (1-c)V_I(\zeta) = \frac{(1-c)^2\delta}{\delta-r}$. For sufficiently small c, $V_O(\zeta)$ is above 1, i.e. the agent's reservation value of one outside dollar. For notational convenience, define the additional value given the redepositing option at ζ:

$$v \equiv V_O(\zeta) - 1 = \frac{(1-c)^2\delta}{\delta-r} - 1 \geq 0. \tag{5}$$

When $\tau < \zeta$, consider a dollar outside the bank. If staying outside is optimal, the following Hamilton-Jacobi-Bellman (HJB) equation must hold:

$$0 = \underbrace{h(\tau)(1 - V_O(\tau))}_{\text{Bank failure}} + \underbrace{\delta(1 - V_O(\tau))}_{\text{Bank matures}} + \underbrace{V_O'(\tau)}_{\text{Time change}}. \tag{6}$$

The first term captures bank failure: the agent loses his value $V_O(\tau)$ but gets 1 if the bank fails with hazard rate $h(\tau)$. The second term captures the maturing event, and the third term is the change due to time elapsing. Because the agent can also deposit immediately (with transaction cost c), $V_O(\tau)$ satisfies:

$$0 = \max\left\{h(\tau)(1 - V_O(\tau)) + \delta(1 - V_O(\tau)) + V_O'(\tau), (1-c)V_I(\tau) - V_O(\tau)\right\}. \tag{7}$$

Similarly, for a dollar inside the bank, its value $V_I(\tau)$ satisfies:

$$0 = \max\left\{\underbrace{rV_I(\tau)}_{\text{Interest}} + \underbrace{h(\tau)(\gamma(1-c) - V_I(\tau))}_{\text{Bank failure}} + \underbrace{\delta((1-c) - V_I(\tau))}_{\text{Bank matures}} + \underbrace{V_I'(\tau)}_{\text{Time change}}, \underbrace{(1-c)V_O(\tau) - V_I(\tau)}_{\text{Withdrawal}}\right\} \tag{8}$$

3.3.2 Optimal Strategies

Let $\widehat{V}_O(\tau)$ denote the solution to (6). With boundary condition $\widehat{V}_O(\zeta) = V_O(\zeta) = 1 + v$ given in (5), the solution is

$$\widehat{V}_O(\tau) = \frac{e^{\lambda\eta}(1-p) - 1 + e^{\lambda(\zeta-\tau)}p + e^{-\delta(\zeta-\tau)}(1-p)\left(e^{\lambda\eta} - 1\right)v}{(1-p)(e^{\lambda\eta} - 1) + (e^{\lambda(\zeta-\tau)} - 1)p}. \tag{9}$$

Economically, $\widehat{V}_O(\tau)$ is the value of an outside dollar, following the *naive* strategy of staying outside the bank from τ to ζ, and redepositing if the bank survives. In general $\widehat{V}_O(\tau) \leq V_O(\tau)$ because the

16

naive strategy may be suboptimal, but we will show that analyzing this naive strategy is of great help in analyzing the optimal strategy.

The following function captures the marginal impact of postponing withdrawal on the value of the above naive strategy:

$$g(\tau) \equiv h(\tau)(1-\gamma) - r\widehat{V}_O(\tau). \tag{10}$$

Specifically, we compare two strategies: one is withdrawing at τ and staying outside until ζ, and the other is staying inside the bank at τ, withdrawing a bit later at $\tau + dt$ and staying outside until ζ. The marginal cost is the transaction-cost adjusted expected loss due to potential bank failure $h(\tau)(1-\gamma)(1-c)$. The marginal benefit is $r(1-c)\widehat{V}_O(\tau)$, because by waiting the deposited dollar grows at a rate of r, and under the naive strategy each deposited dollar is taken out immediately and worths $(1-c)\widehat{V}_O(\tau)$ after transaction cost. Factoring out $1-c$ we obtain $g(\tau)$.

The function $g(\tau)$ only applies to the naive strategy; however, Proposition 2 shows that the sign of $g(\tau)$ is the same as that of the first-order condition under the truly optimal strategy. What is behind this result is the optimality of threshold strategies, i.e., agents do not redeposit before ζ. This also implies $V_O(\tau) = \widehat{V}_O(\tau)$ for τ after the optimal withdrawal time τ_w.

At the optimal withdrawal time τ_w, we have $g(\tau_w) = 0$. Since $V_I(\tau_w) = (1-c)V_O(\tau_w) = (1-c)\widehat{V}_O(\tau_w)$, the optimality condition for τ_w can be rewritten as

$$\underbrace{h(\tau_w)(1-\gamma)(1-c)}_{\text{Staying Cost}} = \underbrace{rV_I(\tau_w)}_{\text{Staying Benefit}}. \tag{11}$$

Here, the left hand side is the transaction-cost adjusted marginal cost of staying (inside the bank) due to potential bank failure, while the right hand side is the marginal benefit of staying due to the yield on deposits.

We now show formally that a threshold strategy is optimal based on the function $g(\tau)$, under

the following sufficient parameter condition:[25]

$$\frac{\delta r \left(1 - p\right) \left(e^{\lambda \eta} - 1\right) \upsilon}{\lambda \left(r - \lambda \left(1 - \gamma\right)\right) p} \in [0, 1).$$ (12)

This requirement implies $r - \lambda \left(1 - \gamma\right) > 0$. Of course, if $\zeta = \infty$ so there is no bank run, then it is optimal to stay inside the bank always.

Proposition 2. *Given a finite equilibrium bank survival time $\zeta < \infty$, the optimal policy for the agent is as follows:*

1. If $g\left(\zeta\right) \leq 0$, then it is optimal to stay in the bank always.

2. If $g\left(0\right) \geq 0$, then it is optimal to withdraw at 0 and redeposit right after ζ.

3. Otherwise, $g\left(0\right) < 0$ and $g\left(\zeta\right) > 0$ must hold. There exists a unique waiting time $\tau_w \in \left(0, \zeta\right)$ so that $g\left(\tau_w\right) = 0$, and withdrawing at τ_w and redepositing at ζ is optimal.

3.4 Bank Run Equilibria

We define a bank run equilibrium and a no run equilibrium as follows:

Definition 1. A *bank run equilibrium* is a symmetric stationary Perfect Bayesian Nash equilibrium in which informed agents' strategies depend on the time since they heard the rumor, and the bank survival time is finite. A *no run equilibrium* is when the bank survival time is infinite.

We focus on symmetric equilibria where ex ante identical agents take the same strategy. Stationarity here means that agent t_i's strategy depends on $\tau = t - t_i$, not t; it follows from the hazard rate's independence of the absolute time t_i at which the agent becomes informed. A bank run means the illiquid bank fails at a finite $t_0 + \zeta < \infty$, while a no run equilibrium means the bank survives forever. Of course, a no run equilibrium always exists without further refinements on the behavior of agents, since ignoring the rumor is an equilibrium for solvent-but-illiquid banks. Throughout, our focus is on the existence and properties of bank run equilibria.

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[25]We emphasize that the sufficient parameter condition 12, which facilitates our analysis with detailed functional forms greatly, is far from the tight necessary condition. The optimality of threshold strategy is fairly general as long as the hazard rate is monotone.

3.4.1 Equilibrium conditions

The illiquid bank fails at $t_0 + \zeta$ when aggregate cumulative withdrawals deplete its capacity κ_L. Given the optimal waiting time τ_w, agents who contribute to the failure must hear the rumor at the window $[t_0, t_0 + \zeta - \tau_w]$. Recall the mass of newly informed agents within $[t, t + dt]$ is $\beta e^{-\beta(t-t_0)} dt$. At the failure time $t_0 + \zeta$ total withdrawals equal to the illiquid bank's capacity κ_L:

$$\int_{t_0}^{t_0+\zeta-\tau_w} \beta e^{-\beta(t_i-t_0)} dt_i = 1 - e^{-\beta(\zeta-\tau_w)} = \kappa_L. \tag{AW}$$

This is the *aggregate withdrawal* condition, which only depends on $\zeta - \tau_w$.

On the other hand, the *individual optimality* condition in Proposition 2 determines how long individual agents will wait (i.e. τ_w) given ζ. As the function $g(\tau)$ depends on ζ implicitly, the endogenous waiting in a bank run equilibrium (Case 3 in Proposition 2) requires

$$g(\tau_w; \zeta) = \frac{(\lambda(1-\gamma) - r) p e^{\lambda(\zeta-\tau_w)} - r\left[(1-p)\left(e^{\lambda\eta} - 1\right)\left(1 + ve^{-\delta(\zeta-\tau_w)}\right) - p\right]}{(1-p)(e^{\lambda\eta} - 1) + (e^{\lambda(\zeta-\tau_w)} - 1) p} = 0. \tag{IO}$$

We see that the individual optimality condition (IO) only depends on $\zeta - \tau_w$ as well!

That both the aggregate withdrawal condition (AW) and the individual optimality condition (IO) only depend on $\zeta - \tau_w$ is a result of stationarity, not the specific functional forms we chose. First, since in equilibrium withdrawing agents hear the rumor over $[t_0, t_0 + \zeta - \tau_w]$ (who then withdraw during $[t_0 + \tau_w, t_0 + \zeta]$), the aggregate withdrawal is a function of $\zeta - \tau_w$.

The individual optimality condition $g(\tau_w; \zeta)$ consists of $\widehat{V}_O$ in (9) as the benefit and the hazard rate h in (3) as the cost. Given stationarity, $\widehat{V}_O$ as the value of an outside dollar over the time interval $[\tau_w, \zeta]$ depends only on the difference $\zeta - \tau_w$. By Proposition 1 the hazard rate $h(\tau_w; \zeta)$ depends on $\zeta - \tau_w$ alone. In this economy, the underlying uncertainty is about the liquidity event t_0. At the withdrawal time $t_i + \tau_w$, the illiquid bank fails at the next instant if and only if t_0 occurs at $t_i + \tau_w$ minus the survival time ζ, which is $t_i - (\zeta - \tau_w)$. Again stationarity implies that the absolute value of t_i should not matter, and thus the hazard rate depends on $\zeta - \tau_w$ alone. The following lemma shows this general result formally:

Lemma 1. *Suppose that the liquidity event distribution $\phi(t_0)$ is memoryless, and the information*

spreading speed takes the form of $\beta (t - t_0)$, i.e. the mass of newly informed agents within $[t, t + dt]$ is $\beta (t - t_0) \, dt$ for $t > t_0$. Then the hazard rate of bank failure depends only on $\zeta - \tau$.

3.4.2 Characterizing the bank run equilibrium

In general, the two equilibrium conditions in (AW) and (IO) are inconsistent with each other. This implies that, except in knife-edge cases where primitive parameters are so that these two conditions coincide, there does not exist any bank run equilibrium with endogenous waiting: either there is no bank run, or a bank run occurs without waiting ($\tau_w = 0$).

Define $\tau_r \equiv \zeta - \tau_w$ as the bank's *remaining survival time* when agents start withdrawing. For ease of exposition define $G (\tau_r) \equiv g (\zeta - \tau_r; \zeta)$ in (10) so that

$$G (\tau_r) = \frac{(\lambda (1 - \gamma) - r) \, p e^{\lambda \tau_r} - r \left[(1 - p) \left(e^{\lambda \eta} - 1 \right) \left(1 + v e^{-\delta \tau_r} \right) - p \right]}{(1 - p) \left(e^{\lambda \eta} - 1 \right) + \left(e^{\lambda \tau_r} - 1 \right) p}, \tag{13}$$

which captures the same individual optimality of withdrawal as in Proposition 2.

The aggregate withdrawal condition in (AW) determines the equilibrium remaining survival time τ_r^u, where 'u' indicates upper bound as will shortly become clear

$$\tau_r^u \equiv - \frac{\ln (1 - \kappa_L)}{\beta}. \tag{14}$$

Once plugging this equilibrium τ_r^u into (13), Proposition 2 allows us to evaluate whether individual agents want to wait longer or withdraw earlier, depending on the sign of $G (\tau_r^u)$. Recall that a lower G is associated with a low marginal cost of staying (say γ close to one). If $G (\tau_r^u) < 0$, agents always want to wait, thus bank runs never occur. On the other hand, withdrawing earlier is optimal under $G (\tau_r^u) > 0$, implying immediate withdrawals $\tau_w = 0$ in equilibrium, with an exogenous (or at most trivially endogenous) survival time $\zeta = \tau_r^u = - \frac{\ln (1 - \kappa_L)}{\beta}$. Since the waiting time is cornered at zero, the equilibrium survival time mechanically depends only on the capacity of illiquid bank κ_L and the rumor spreading speed β. Finally, in a knife-edge case with $G (\tau_r^u) = 0$, agents are totally indifferent, thus equilibria are indeterminate. We have the following proposition.

Proposition 3. *The bank run equilibrium without information acquisition is as follows:*
1. If $G (\tau_r^u) > 0$, the unique bank run equilibrium is that every informed agent withdraws immedi-

ately upon hearing the rumor, so that $\tau_w = 0$, and $\zeta = \tau_r^u = -\frac{\ln(1-\kappa_L)}{\beta}$.

2. If $G(\tau_r^u) < 0$, a bank run equilibrium does not exist.

3. If the primitive parameters are such that $G(\tau_r^u) = 0$, then in this knife-edge case, any $\tau_w \geq 0$ and $\zeta = \tau_w + \tau_r^u$ constitute a bank run equilibrium.

Consider the second case in Proposition 3. Hypothetically, imagine that each agent believes others are running immediately with $\tau_w = 0$ hence $\zeta = \tau_r^u$. Because $G(\tau_r^u) < 0$, each agent postpones withdrawal, say to $\tau_w = \Delta > 0$.[26] But the aggregate withdrawal condition (AW) says that the survival time adjusts one-to-one to $\zeta = \tau_r^u + \Delta$. Then, given the new bank survival time, the individual optimality condition (IO) implies that the new waiting time responds one-to-one again with $\tau_w = \Delta + \Delta = 2\Delta$. Applying this thought experiment repeatedly implies that both waiting time and bank survival time are postponed indefinitely. Clearly, this logic reverses if $G(\tau_r^u) > 0$, pushing agents to withdraw as early as possible—hence in equilibrium they withdraw upon hearing the rumor.

The third knife-edge case is uninteresting, not only because of its indeterminacy of bank run equilibria but also because it holds only on a parameter space with zero measure. For the latter reason, the knife-edge case is not "*generic*," and we only focus on generic equilibria throughout the rest of the paper.
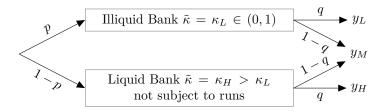
4   The Model with Information Acquisition

In the base model above, the aggregate withdrawal condition (AW) implies that the survival time ζ responds one-to-one to changes in the waiting time τ_w, and vice versa according to the individual optimality condition (IO). This somewhat interlocked stationarity is behind the tension between conditions (AW) and (IO) that precluded bank runs with waiting except in a knife-edge case.

In deriving the endogenous bubble burst time, Abreu and Brunnermeier (2003) resolves this issue by assuming that the bubble component of prices increases over time. As a result, the net benefit of attacking increases, so that in the *individual optimality condition* the waiting time τ_w responds less than one-to-one to the survival time ζ. In a bank run setting, this would be a hard assumption to swallow, which would require that the value of a dollar in a perfectly safe bank

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[26]Implicitly, it says that $G(\tau_r^* - \Delta) = 0$ which is the condition for the optimal withdrawal time.

Figure 2: Probability Distribution of the Additional Signal $\tilde{y}$ with Quality q

declines exogenously over time.

In this paper we take a novel route. We introduce noisy information acquisition, and thus ex-post *heterogeneity* among agents with different perceptions about the likelihood of bank failure. Different from Abreu and Brunnermeier (2003), this naturally changes the *aggregate withdrawal condition* so that the survival time ζ responds less than one-to-one to the waiting time τ_w. As we show, this translates to a realistic "fear-of-bad-signal-agents" effect for those agents with median signal realizations, and a unique bank run equilibrium with endogenous waiting emerges.

We find that individual information acquisition regarding bank liquidity makes solvent banks that are free from runs prone to runs, and shortens the survival time of failing banks. The point is that, although bank runs never exist if information acquisition is not possible, inefficient bank runs with endogenous waiting reemerge once we allow for the possibility of acquiring additional signals, because individual information acquisition imposes negative externalities on each other.

4.1 Information Acquisition and Signal Structure

At time $t_i > t_0$ when agent i hears the rumor, he can acquire an additional signal about the bank's type at some cost. For example, agents could spend time sifting through call reports, calling their branch manager, etc. To emphasize that it is the noisiness of the signal, rather than its precision, that plays the crucial role, we first assume a binary information acquisition structure: either no acquisition, or acquiring an additional signal with a fixed precision $q > 0$. We endogenize q in Section 4.4 by assuming a quadratic acquisition cost.

Agents pay $\chi > 0$ per dollar of deposits for the additional signal. Suppose χ is realtively small that when facing a bank run equilibrium it is optimal to acquire the signal. For tractability, the

signal takes three possible values $\tilde{y} \in \{y_L, y_M, y_H\}$ with conditional probabilities as in Figure 2. With probability $q > 0$, the bank's type is perfectly revealed by the signal y_H (y_L); while with probability $1 - q$, the agent receives an uninformative "medium" signal y_M (e.g., seeing mixed evidences).

Conditional on the bank's state, the realizations of these signals are i.i.d. across agents. Unconditionally, the signals are correlated across agents, as signals tend to be low (high) in the illiquid (liquid) state. Hence there are more agents with unfavorable signals given the bank being illiquid, a feature captured by the simple signal structure in Figure 2.[27] We emphasize that this signal structure is consistent with the fact documented in Section 2.2, that during the Euro zone crisis, money market fund managers each did their own research and came up with different conclusions.

The analysis of optimal withdrawal strategies in Section 3.3 applies once we condition on the signal realization. For y_H (y_L) agents, we can simply replace the prior p with 0 (1) in Proposition 2. Agents with y_H know the bank is not subject to runs and thus stay inside the bank always. For agents with y_L, it is optimal to withdraw immediately if

$$g(0; p = 1) = \frac{(\lambda(1 - \gamma) - r) e^{\lambda \zeta} + r}{e^{\lambda \zeta} - 1} > 0, \tag{15}$$

We will show later in Lemma 2 that in bank run equilibrium $\zeta < \eta$. Recall $\lambda(1 - \gamma) < r$ given condition (12), the condition (15) is implied by the following imposed parameter condition:

$$(\lambda(1 - \gamma) - r) e^{\lambda \eta} + r > 0,$$

which requires the loss $1 - \gamma$ to be significant.[28] Finally, Proposition 2 directly applies to y_M agents, and our later analysis focuses on their endogenous waiting time τ_w, from which we omit the index M from now on without risk of confusion.

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[27]More formally, we need the signal to be informative on average, i.e., $Cov[\tilde{y}, \tilde{\kappa}] > 0$. Our results are robust to the introduction of richer signal structures with potentially multiple levels of medium signals.

[28]In fact, (15) has to hold in any bank run equilibrium. As mentioned in Lemma 2 later, generically (i.e. except for some zero measure sets of parameters), for bank run equilibrium to exist y_L agents must withdraw immediately upon hearing the rumor; otherwise bank runs do not exist.

4.2 Bank Run Equilibrium with Acquired Information

We now solve for the bank run equilibrium given that agents are acquiring signals upon hearing the rumor. We focus on the case where both y_M and y_L agents are driving the bank failure. Conditional on the bank being illiquid, there could be at most $q\left(1 - e^{-\beta\eta}\right)$ measure of y_L agents. To ensure that y_L agents alone are not enough to take down the illiquid bank, the signal precision q satisfies:

$$q < \frac{\kappa_L}{1 - e^{-\beta\eta}}. \tag{16}$$

4.2.1 Equilibrium Conditions

As in Section 3.4.1, the individual optimality and aggregate withdrawal conditions pin down the bank run equilibrium. We know that y_L agents withdraw immediately under (15). For y_M agents, the individual optimality condition $G(\tau_r) = 0$ is unchanged.

The ex post heterogeneity due to acquired noisy signals gives rise to an interesting twist in the aggregate withdrawal condition. In contrast to the base model with homogeneous informed agents, now there are two groups of agents running on the illiquid bank. The first group is y_L agents who withdraw immediately, and at $t_0 + \zeta$ the total withdrawals by them are $q\int_{t_0}^{t_0+\zeta} \beta e^{-\beta(t_i-t_0)}dt_i = q\left(1 - e^{-\beta\zeta}\right).$[29] Agents with y_M signals wait for τ_w, and at $t_0 + \zeta$ their total withdrawals are $(1-q)\int_{t_0}^{t_0+\zeta-\tau_w} \beta e^{-\beta(t_i-t_0)}dt_i = (1-q)\left(1 - e^{-\beta(\zeta-\tau_w)}\right)$. Let τ_r denote the y_M agents' remaining survival time, and the illiquid bank fails if

$$q\left(1 - e^{-\beta\zeta}\right) + (1-q)\left(1 - e^{-\beta\tau_r}\right) = \kappa_L. \tag{AW'}$$

Both the survival time ζ and the y_M agent's remaining survival time τ_r enter the new aggregate withdrawal condition (AW'), in contrast to (AW) where only τ_r enters.

For better understanding, Figure 3 depicts the cumulative withdrawal patterns for both banks. For illiquid banks in the left panel, y_L agents begin to withdraw right after t_0 as the rumor starts spreading. At $t_0 + \tau_w$, y_M agents join the force of withdrawals, until eventually cumulative withdrawals reach κ_L at $t_0 + \zeta$. For liquid banks in the right panel, no agent withdraws immediately

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[29]We implicitly assume that at the survival time ζ there are still newly informed y_L agents withdrawing, which requires that $\zeta < \eta$, as we prove in Lemma 2.

Cumulative withdrawal patterns for an illiquid bank with capacity κ_L and a liquid bank. At $\tau = 0$ the rumor starts to spread. In equilibrium y_M agents set the waiting interval τ_w, and the illiquid bank fails while the liquid bank starts experiencing redeposits at ζ.

after t_0, but y_M agents start withdrawing at $t_0 + \tau_w$. At $t_0 + \zeta$ those early informed y_M agents realize that the bank is liquid and start redepositing, which makes net aggregate withdrawals decrease over time.

Because (AW') involves both τ_r and ζ, we can solve for the bank run equilibrium as follows. First, the individual optimality condition for y_M agents determines the equilibrium remaining survival time τ_r:

$$0 = G\left(\tau_r\right) = \frac{\left(\lambda\left(1-\gamma\right)-r\right)e^{\lambda\tau_r}p - r\left(1-p\right)\left(e^{\lambda\eta}-1\right)ve^{-\delta\tau_r} + r\left(1-e^{\lambda\eta}\left(1-p\right)\right)}{\left(1-p\right)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda\tau_r}-1\right)p}. \qquad (17)$$

Then, the aggregate withdrawal condition (AW') determines the equilibrium survival time ζ:

$$\zeta = -\frac{1}{\beta}\ln\left[1 - \frac{\kappa_L - \left(1-q\right)\left(1-e^{-\beta\tau_r}\right)}{q}\right]. \qquad (18)$$

4.2.2 Characterizing the Bank Run Equilibrium with Acquired Signals

There are natural bounds for the remaining survival time τ_r in equilibrium. When $\tau_w = 0$ so that y_M agents withdraw immediately upon hearing the rumor, $\tau_r = \zeta - \tau_w$ assumes its upper bound value $\tau_r^u = -\frac{\ln(1-\kappa_L)}{\beta}$, which is independent of signal quality q and identical to (14) of the base model without information acquisition. Intuitively, in this extreme case, all agents withdraw from

25

the illiquid bank immediately, regardless of their signals, leading to the same bank survival time $\zeta = \tau_r^u$ found without information acquisition.

On the other hand, the following lemma establishes that in any bank run equilibrium $\zeta \leq \eta$ holds generically. Recall that the rumor spreading window is $[t_0, t_0 + \eta]$. Lemma 2 implies that the illiquid bank fails before the rumor stops spreading. The intuition is similar to the two-equation-one-unknown situation encountered in the base model in Section 3.4.1.[30]

Lemma 2. *Generically, in any bank run equilibrium we have $\zeta \leq \eta$, and y_L agents must withdraw immediately upon hearing the rumor.*

Combining (18) and Lemma 2 gives a lower bound for τ_r, which depends on signal quality q:

$$\tau_r^l(q) \equiv \frac{1}{\beta} \ln \left(\frac{1-q}{1 - \kappa_L - q e^{-\beta \eta}} \right).$$

Parameter condition (16) implies $\tau_r^l(q) > 0$, so that y_M withdrawals contribute to the bank's failure.

Proposition 4. *Given that agents acquire the additional signals with precision q, the bank run equilibrium is characterized as follows:*

1. *If $G(\tau_r^u) \geq 0$, the unique bank run equilibrium is $\tau_r = \zeta = \tau_r^u$ and $\tau_w = 0$, i.e. y_M agents withdraw immediately upon hearing the rumor, coinciding with Case 1 in Proposition 3.*

2. *If $G\left(\tau_r^l(q)\right) \leq 0$, a bank run equilibrium does not exist.*

3. *Otherwise, we have $G(\tau_r^u) < 0$ and $G\left(\tau_r^l(q)\right) > 0$. In the unique bank run equilibrium $\tau_r \in \left(\tau_r^l, \tau_r^u\right)$ with $G(\tau_r) = 0$, we have $\zeta = \frac{1}{\beta} \ln \left[\frac{q}{1 - \kappa_L - (1-q)e^{-\beta \tau_r^*}} \right]$ and $\tau_w = \zeta - \tau_r > 0$, i.e. y_M agents wait to withdraw.*

When $q = 0$, i.e., additional signals have no information value, then $\tau_r^l(q = 0) = \tau_r^u$, and Proposition 4 collapses to Proposition 3 of the base model.

Suppose that we have $G(\tau_r^u) < 0$ and $G\left(\tau_r^l\right) > 0$ (Case 3). Following the thought experiment discussed after Proposition 3, imagine that each y_M agent believes that other y_M agents withdraw

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[30]If y_L agents have all withdrawn when the bank fails ($\zeta > \eta$), then the aggregate withdrawal condition is (just replacing ζ by η in (AW')):
$$\kappa_L = q\left(1 - e^{-\beta \eta}\right) + (1-q)\left(1 - e^{-\beta \tau_r}\right).$$
Again, as in Section 3.4.1, this aggregate withdrawal condition only depends on y_M's remaining survival time τ_r. Then, τ_r that solves the above equation will violate y_M agents' individual optimality condition (17) generically. As a result, the discussion in Section 3.4.1 implies either y_M agents want to wait so that bank runs do not exist, or y_M agents withdraw earlier which pushes ζ all the way to fall below η.

immediately ($\tau_w = 0$) so that the remaining survival time τ_r takes its upper bound τ_r^u (graphically, setting $\tau_w = 0$ in the left panel of Figure 3). $G(\tau_r^u) < 0$ implies that the marginal cost of waiting is outweighed by the marginal benefit, and each y_M agent postpones his withdrawal, say to $\tau_w = \Delta > 0$.

Once y_M agents decide to wait longer by $\Delta > 0$, importantly they know there are more y_L agents withdrawing before them—this is the key difference from the base model without signal acquisition where all agents wait longer by Δ. Bank failure now requires less withdrawing y_M agents, and hence a shorter $\tau_r < \tau_r^u$ in (AW'). A shorter remaining survival time τ_r implies bank failure is more imminent, giving rise to a higher failure hazard rate in (3) and a greater marginal cost of waiting. This captures the realistic "*fear-of-bad-signal-agents*" effect in the bank run environment: agents unsure about bank liquidity (i.e. y_M agents who receive mixed evidences) worry that there are agents receiving much worse signals and thus running before them, which increases their incentives to run as well.[31] Finally, if y_M agents wait sufficiently long that the remaining survival time attains its lower bound $\tau_r = \tau_r^l$, then $G\left(\tau_r^l\right) > 0$ implies that they want to withdraw a bit earlier. Then, by the intermediate value theorem, a bank run equilibrium exists for some intermediate τ_r with $G(\tau_r) = 0$, even though a bank run equilibrium does not exist if no acquisition of liquidity signals is allowed.

4.3 Bank Run Equilibrium with Endogenous Information Acquisition

We have derived the bank run equilibrium, given that every agent acquires the signal upon hearing the rumor. We now study the agent's incentive to acquire the signal, given the bank run equilibrium. Recall Lemma 2 states that $\zeta \leq \eta$ holds generically in any bank run equilibrium. As we will discuss in detail in Section 4.5, this implies that hearing the rumor and observing the bank is still alive at t_i is informative, and agent t_i's posterior probability of the bank being illiquid, denoted by p_{t_i}, drops from p to

$$p_{t_i} = \frac{\left(e^{\lambda\zeta} - 1\right)p}{(1-p)\left(e^{\lambda\eta} - 1\right) + \left(e^{\lambda\zeta} - 1\right)p}.$$

Agents with y_H stay inside the bank always with a payoff of $\frac{(1-c)\delta}{\delta-r}$, while agents with y_L

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[31]This effect of "fear-of-bad-signal-agents" shares a similar spirit with global games (Morris and Shin (2002), Goldstein and Pauzner (2005)), where private signals are not only informative about the bank fundamental but also informative about other agents' strategies. The key difference is that we do not rely on this effect to derive the unique equilibrium: in our model, there is a unique "cornered" equilibrium even without information acquisition.

withdraw immediately with a payoff of $1 - c$ facing a bank run. For the y_M signal, we can calculate the agent's value using Proposition 2. Given the cost χ, agent t_i acquires the additional signal if and only if

$$q \left[p_{t_i} (1 - c) + (1 - p_{t_i}) \frac{(1 - c) \delta}{\delta - r} - V_I (0|y_M) \right] \geq \chi. \tag{19}$$

Suppose that the information cost χ is sufficiently small and signal quality q is sufficiently large so that (19) holds in a bank run equilibrium. Proposition (5) summarizes our main result.

Proposition 5. *Suppose that $G(\tau_r^u) < 0$ so that bank runs never occur without information acquisition. Then,*

1. If $G\left(\tau_r^l(q)\right) < 0$, a bank run equilibrium does not exist and no information is acquired.

2. If $G\left(\tau_r^l(q)\right) \geq 0$, there are two equilibria: one where agents do not acquire information and the bank does not fail, and the bank run equilibrium with information acquisition of Proposition 4.

Recall that $G(\tau_r^u) < 0$ implies that bank runs do not exist if agents cannot acquire further information. As a result, no-run-no-acquisition must be an equilibrium under that condition: if nobody acquires additional information, then bank runs never occur; on the other hand, nobody wants to acquire information either if nobody runs. This is the unique equilibrium under $G\left(\tau_r^l(q)\right) < 0$, shown by the first statement in Proposition 5.

However, information acquisition opens the possibility of bank runs, as run-acquisition might also be an equilibrium if the marginal cost of staying is sufficiently low that $G\left(\tau_r^l(q)\right) \geq 0$ holds. The multiplicity of equilibria reflects the strategic complementarity of information acquisition combined with bank runs: either nobody acquires information without worries about bank runs, which rules out bank runs given nobody knows exactly whether the bank is illiquid; or everybody acquires information and runs on the bank according to their private signal, which justifies the acquisition of information in the first place.

Proposition 5 supposes $G(\tau_r^u) < 0$. Suppose instead $G(\tau_r^u) \geq 0$, i.e., bank runs occur even if agents cannot acquire liquidity information and they are already withdrawing upon hearing the rumor. The next corollary shows that in equilibrium agents acquire information, then run immediately on the illiquid bank that generates equal (y_M) or worse (y_L) signals only. Hence in our model, in this case information acquisition has no effect on the existence of the bank run equilibrium, nor on the survival time of the illiquid bank:

Corollary 1. *Suppose that $G(\tau_r^u) \geq 0$, so that bank runs occur even without information acquisition. Then with information acquisition, in the unique bank run equilibrium agents acquire information, withdraw immediately given y_M and y_L signals, and the illiquid bank fails at $\zeta = \tau_r^u$.*

In general, collecting information about the bank's liquidity status is individually beneficial but socially wasteful, in the situation where without the additional information runs do not occur. However, if without the additional information bank runs did occur, information acquisition could mitigate runs on stronger banks.[32] Moreover, collecting information about bank solvency could be socially beneficial, a topic we revisit in Section 6.1.

Belief heterogeneity is key to generating the interesting bank run equilibrium with endogenous waiting, which feeds back to agents' ex ante information choice. Endogenous information acquisition offers perhaps the most natural way to introduce belief heterogeneity into the model. Exogenously endowed noisy signals, i.e. $\chi = 0$, would suffice to generate the bank run equilibrium with endogenous waiting. However, if $\chi = 0$, then there is another uninteresting equilibrium where agents ignore their signals (so there is no bank run), in addition to the equilibria characterized in Proposition 5. A strictly positive signal cost eliminates such an uninteresting equilibrium, because agents do not acquire signals that they plan to ignore.

4.4 Smooth Information Acquisition Cost

The qualitative result in Proposition 5 remains when we endogenize the signal precision q in a setting with quadratic acquisition cost $\chi(q) = \frac{\alpha}{2}q^2$, where $\alpha > 0$ is constant and $q \in \left[0, \frac{\kappa_L}{1-e^{-\beta\eta}}\right)$. Agent t_i solves:

$$\max_{q \in \left[0, \frac{\kappa_L}{1-e^{-\beta\eta}}\right)} q p_{t_i}(1-c) + q(1-p_{t_i})\frac{(1-c)\delta}{\delta - r} + (1-q)V_I(0|y_M) - \chi(q).$$

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[32]But whether information acquisition can prevent failures of stronger banks depends on the setting. As Corollary 1 shows, in our model because the liquid bank is sufficiently strong, it survives independently of possible information acquisition.

The first order condition for optimal q satisfies:[33]

$$\underbrace{(1-c)\left[p_{t_i} + (1-p_{t_i})\frac{\delta}{\delta-r}\right]}_{\mathbb{E}[V_I(0)|\text{informative signals}]} - \underbrace{V_I(0|y_M)}_{\mathbb{E}[V_I(0)|\text{uninformative signal}]} \geqslant \alpha q, \text{ with equality if } q < \frac{\kappa_L}{1 - e^{-\beta\eta}} \quad (20)$$

Combining Proposition 4 with condition (20), we can solve for the signal precision q, the waiting time τ_w, and the survival time ζ simultaneously. In general, multiplicity may occur even among the class of bank run equilibria with positive information acquisition. The next lemma states that under a sufficient conditions provided in the Appendix A.8, there exists at most one such equilibrium.
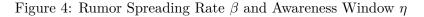
Lemma 3. *Under condition (26) provided in the Appendix A.8, the bank-run equilibrium with positive precision, if one exists, is unique.*
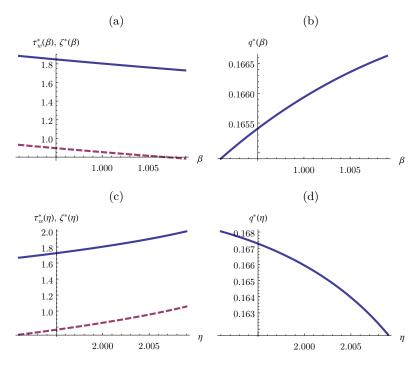
Let $q^*|_{\zeta=\eta}$ be the optimal signal precision given $\zeta = \eta$ in (20); whether there is a bank run equilibrium depends on whether $G\left(\tau_r^l\left(q^*|_{\zeta=\eta}\right)\right) > 0$. The next proposition is a generalization of Proposition 5 to the quadratic information acquisition cost setting.

Proposition 6. *Suppose that condition (26) in Lemma 3 holds.*

1. *If $G\left(\tau_r^u\right) < 0$ so that bank runs never occur without information acquisition. Then,*

   a. If $G\left(\tau_r^l\left(q^*|_{\zeta=\eta}\right)\right) < 0$, a bank run equilibrium does not exist and no information is acquired.

   b. If $G\left(\tau_r^l\left(q^*|_{\zeta=\eta}\right)\right) \geq 0$, there are two equilibria: one where agents do not acquire information and the bank does not fail, and the bank run equilibrium with information acquisition of Proposition 4.

2. *If $G\left(\tau_r^u\right) \geq 0$ so that bank runs occur without information acquisition. Then with information acquisition, in the unique bank run equilibrium agents acquire information, withdraw immediately given y_M and y_L signals, and the illiquid bank fails at $\zeta = \tau_r^u$.*

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[33]The fact that we are focusing on bank run equilibria and that the agent with y_L finds immediate withdrawal optimal imply that in (20) the optimal q cannot bind at zero, as $(1-c)\left[p(t_i|t_i) + (1 - p(t_i|t_i))\frac{\delta}{\delta-r}\right] - V_I(0|y_M) > 0$ always. In words, information has positive value as it improves the agent's decision, but the marginal cost of information acquisition at $q = 0$ is zero given a quadratic cost. On the other hand, the no run equilibrium must have $q = 0$ and is analyzed in Section 4.3.

Figure 4: Rumor Spreading Rate β and Awareness Window η

Panels (a) and (c) show the equilibrium survival time of the illiquid bank ζ (solid line) and the waiting time τ_w of medium signal agents (dashed line), while Panels (b) and (d) show the equilibrium quality of the additional signal q, as a function of the rumor spreading rate β and the length of the awareness window η. Parameters are $r = 0.09$, $\beta = 1$, $\theta = 1.03$, $\eta = 2$, $p = 0.8$, $\delta = 0.12$, $c = 10^{-6}$, $\kappa_L = 0.65$, $\alpha = 0.7$, $\gamma = 0.75$.

4.5 Strategic Complementarity vs. Substitution in Information Acquisition

Our model features both strategic *complementarity* and *substitutability* in information acquisition among individual agents. For better understating, we first take the model in Section 4.4 and study the comparative statics of the rumor spreading rate β and the awareness window η. Recall that relative to Abreu and Brunnermeier (2003), our model decouples the rumor spreading rate from the awareness window.

The comparison between these two comparative static results are intriguing. The top panels of Figure 4 shows that, intuitively, the greater the rumor spreading rate β, the faster the illiquid bank fails (Panel 4a), and the greater precision the acquired signals (Panel 4b). A surprising result emerges when we turn to the awareness window η in the bottom panels. When η increases so that everybody knows that potentially there will be more informed agents attacking the bank, each individual agent acquires *less* information (Panel 4d) and the illiquid bank survives *longer* (Panel

31

4c).

Dynamic learning when faced with uncertainty plays a key role here. The posterior probability that the bank is illiquid upon hearing the rumor and *observing that the bank is still alive*, recalling that $\Phi(t|t_i)$ is given by (2), is:

$$p_{t_i} = \frac{\Pr\{\text{bank survives at } t_i|\kappa_L, t_i\}\Pr\{\kappa_L\}}{\Pr\{\text{bank survives at } t_i|\kappa_L, t_i\}\Pr\{\kappa_L\} + \Pr\{\kappa_H\}} = \frac{[1 - \Phi(t_i - \zeta|t_i)]p}{[1 - \Phi(t_i - \zeta|t_i)]p + 1 - p} \quad (21)$$

The posterior probability that the illiquid bank survives at t_i is decreasing in η:

$$\Pr\{\text{illiquid bank survives at } t_i|\kappa_L, t_i\} = 1 - \Phi(t_i - \zeta|t_i) = 1 - \frac{e^{\lambda\eta} - e^{\lambda(t_i - t_i + \zeta)}}{e^{\lambda\eta} - 1} = \frac{e^{\lambda\zeta} - 1}{e^{\lambda\eta} - 1}.$$

Intuitively, from the point of view of agent t_i, a greater η implies that $t_0 \in [t_i - \eta, t_i]$ could occur earlier, and other depositors may have run on the bank for a longer time. As a result, conditional on the bank being alive at t_i, the bank is more likely to be liquid.

We have seen that in our bank run setting, information acquisition exhibits strategic complementarity regarding *whether to acquire information*. This is evident from the existence of multiple equilibria in Proposition 5 with binary information choices: the information acquisition of other agents makes a bank run possible, which prompts this agent to acquire his information as well. In this way, acquisition of liquidity information can subject solvent but illiquid banks to runs. This positive feedback effect shares the same spirit to Hellwig and Veldkamp (2009) who show that, in a static model, information acquisition exhibits complementarity if actions (in our model, running on bank) are complementary.

In the setting with smooth information acquisition cost, (21) implies that individual information acquisition exhibits strategic substitutability regarding *how much information to acquire*. This is because there is a natural public signal conveyed by the fact that the bank has not yet failed. If others study more about the bank and run on the illiquid one, then the illiquid bank fails sooner. As a result, this public signal becomes more informative—(21) suggests that the mere survival of the bank, upon hearing the rumor, leads the agent to perceive the bank to be stronger. Because a more informative public signal lowers the value of private signals, individual agents hearing the rumor respond to the public signal of "bank survival till t_i" by acquiring less information.[34]

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[34]This is similar to the effect of price information in a noisy Rational Expectation Equilibrium (REE) asset pricing

The result that η mitigates bank runs goes against the casual intuition that runs are more severe with more prone-to-run agents, which stems from the premise that bank failure requires a sufficient mass of running depositors. By contrast, we show that in a dynamic learning setting, when cumulative informed agents are enough to run down the bank, artificially shortening the awareness window gives rise to a novel information effect with the exact opposite direction.[35]

5 Empirical Predictions

Beyond the empirical bank run evidence presented in Section 2, our model offers additional insights for future empirical research, especially for high-frequency deposits and withdrawals data.
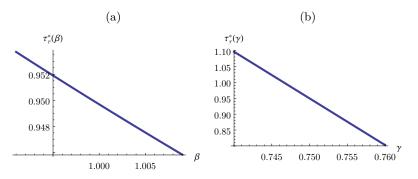
First, our model makes several qualitative testable predictions about the nature of bank runs. Specifically, bank runs would empirically manifest as follows. There are two broad types of depositors, with one type better informed than the other. These better informed agents start withdrawing first, then joined by less informed agents. Importantly, a unique prediction of our model is that for banks that survive a run, we should observe some agents withdrawing at the same time other agents are redepositing back into the bank. This cannot occur in a model where a homogeneous depositor population responds to a common signal as in He and Xiong (2012a).

Second, our comparative statics generate testable predictions about how bank run dynamics would respond to variation in key model parameters, such as those shown in Figure 4. Since rumors are often not made public, it could be challenging to empirically measure the endogenous withdrawal time and bank survival time. However, the remaining survival time τ_r, which is the difference between the endogenous withdrawal time and bank survival time $\zeta - \tau_w$, is free of this issue. Given high-frequency depositor transaction data, τ_r can be measured as the time between when withdrawals begin and when the bank fails, or until redeposits begin for a surviving bank. In

models (e.g. Grossman and Stiglitz, 1980). We thank an anonymous referee for pointing out the similarity between "bank failure" in our model and "price" in a noisy REE model as public signals.

[35]One real world example of this point involves the subprime mortgage crisis. On May 17, 2007 Fed Chairman Bernanke indicated in a speech about the subprime mortgage market that looser lending standards were pervasive especially in loans originated in 2006 (Bernanke, 2007). The speech took place at a time when low teaser rates on many adjustable-rate mortgages were set to expire, suggesting that the rise in defaults was just the tip of an iceberg. Subsequently, asset-backed commercial paper (ABCP) outstanding dropped from \$1.3 trillion in July 2007 to \$833 billion in December 2007 (Acharya, Schnabl, and Suarez, 2013). One interpretation within our model of this speech is that it signaled a relatively short awareness window, and there was not much to learn from the survival of ABCP conduits up to that point in time. Suppose that instead that looser lending standards were pervasive in loans originated since 2003. Having observed that the ABCP market kept growing despite this fact, investors would perceive a high probability of the system being liquid and conclude that a run might not occur.

Figure 5: Rumor Spreading Rate β and Recovery Value γ

Panels (a) and (b) show the equilibrium remaining survival time τ_r as a function of the rumor spreading rate β and the length of the recovery value γ. Parameters are $r = 0.09$, $\beta = 1$, $\theta = 1.03$, $\eta = 2$, $p = 0.8$, $\delta = 0.12$, $c = 10^{-6}$, $\kappa_L = 0.65$, $\alpha = 0.7$, $\gamma = 0.75$.

our model, the higher the rumor spreading speed β, the faster the (illiquid) bank fails, the smaller the remaining survival time. The effect of bankruptcy recovery γ on the remaining survival time τ_r is also negative: a lower recovery γ prompts depositors to withdraw earlier, leading to a longer remaining survival time $\tau_r = \zeta - \tau_w$. Figure 5 shows these two comparative static results.

Third, our model has testable implications regarding the probability of bank runs. Proposition 6 characterizes different parameter regions in which a bank run can constitute an equilibrium. By assigning a probability measure on the parameter space,[36] one can relate model parameters to the likelihood of bank runs, which has a straightforward empirical counterpart. Consider, for example, the rumor spreading speed. One can show that all else equal, a larger β makes the condition $G(\tau_r^u) \geq 0$ easier to satisfy; thus Proposition 6 implies that bank runs are more likely. In the working paper version of this article, which is listed as NBER Working Paper 18513, we also show that the lower the information acquisition cost α, the more likely is the bank run equilibrium. These are testable predictions given proxies for the spreading speed or the cost of information. Approaches for their measurement using social media and press coverage have been recently developed in the asset pricing literature (e.g. Manela, 2014).

Finally, if our model withstands the above tests, one could use our structural framework to estimate key model parameters. For example, using the moment condition (17) of remaining survival time τ_r, one could estimate the expected deposit recovery rate γ given bankruptcy as perceived by depositors, a highly interesting quantity.

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[36]In those parameter regions where multiple equilibria are possible, one need some exogenous sunspots with certain likelihood to predict the equilibrium outcome.

6 Extensions and Discussions

We study two extensions of our model with quadratic information acquisition cost. In Section 6.1 we introduce fundamentally insolvent banks, so that it is also *socially* efficient for individual agents to acquire information and run on insolvent banks. Section 6.2 considers a two-bank economy where competition amplifies the individual agent's socially wasteful information production. Finally we provide discussion on several theoretical issues.

6.1 Insolvent Banks and Stress Tests

6.1.1 Solvency information versus liquidity information

Suppose that the bank might become *insolvent* at the time t_0, which randomly fails (rather than matures) with intensity $\xi > r$ and recovers nothing. Of course the bank can also be solvent, and if so it can be either liquid or illiquid as we modeled before.

When agent t_i hears the rumor, the possibility of insolvency motivates him to spend a fixed amount of effort $\underline{e} > 0$ to obtain a signal $\mathbf{1}_s \in \{0, 1\}$; for simplicity, this signal $\mathbf{1}_s$ perfectly reveals whether the bank is solvent or not. We focus on situations where the agent acquires this solvency signal $\mathbf{1}_s$ (which is optimal if the default intensity ξ is sufficiently high). Hence after hearing the rumor, all agents will figure out the bank's insolvency and run on the insolvent bank immediately.

The information regarding bank solvency is both socially and individually valuable. In contrast, the information regarding bank liquidity is individually valuable but socially destructive when agents realize that runs on the illiquid bank become a concern. We aim to capture the fact that bank solvency information and liquidity information are naturally linked when individual agents are acquiring them.

We assume that a by-product of the agent's private learning about bank solvency, i.e., collecting the solvency signal $\mathbf{1}_s$, is that he also learns something about the liquidity of the bank. To model this, we assume that given the effort $\underline{e}$ of acquiring $\mathbf{1}_s$, if the bank turns out to be solvent, then the baseline quality q of the bank's liquidity signal y—which is the signal we modeled in Section 4.1—is just $\underline{e}$. As a result, the agent's additional liquidity information precision choice is $q \geq \underline{e}$ with cost $\frac{\alpha}{2}(q - \underline{e})^2$. The interpretation is that the process of collecting insolvency information inevitably teaches the agent something about bank liquidity. The more effort that the agent spends in figuring

out whether the bank is insolvent, the more he knows about the bank's liquidity. Our modeling that the effort of collecting insolvency information (i.e., $\underline{e}$) becomes the baseline quality of liquidity information captures this idea in the simplest way.
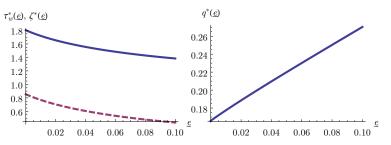
6.1.2 Policy Implication: Stress Tests

The above setting has important implications for stress tests in revealing the fundamentally problematic banks. By *providing insolvency information alone*, the government can use stress tests to reduce $\underline{e}$ to eliminate runs on solvent-but-illiquid banks. According to our model, if a great effort is required to learn about whether the bank is insolvent (say institutions like Lehman), i.e., a higher $\underline{e}$, then each agent will be automatically endowed with significant information about the liquidity of solvent banks (say institutions like Citibank with weak liquidity condition). As a result, runs on solvent banks may start, which pushes each agent to acquire even more liquidity information.

Stress tests provide transparency on potentially insolvent banks (Bernanke, 2009), and therefore reduce $\underline{e}$. Government, by providing higher quality information about banks' insolvency, can crowd out private acquisition of insolvency information. Because public information can be better targeted at insolvency alone, while the process of private acquisition of solvency information inevitably reveals liquidity information, public provision of solvency information helps all agents know that other agents do not have superior information regarding banks liquidity situation. Therefore, our model suggests that the public provision of insolvency information indirectly reduces the socially wasteful information acquisition regarding liquidity, and therefore make runs on illiquid banks less likely. Figure 6 plots the equilibrium signal precision when we vary $\underline{e}$ which measures the extent of public provision of solvency information.

This view is consistent with the Federal Reserve Board's recent break from the traditional supervisory view of opaqueness in favor of more public disclosure to restore the confidence of investors. However, our rationale is different than the one described by Bernanke (2010) who argues that transparency allows for scrutiny by outside analysts, which enhances the credibility of the tests. Instead, we argue that by providing more information, the government crowds out the information collection effort by individuals about the solvency of the banks, and the perfect revelation of insolvent bank clarifies the channel that we are emphasizing.[37] Since liquidity and

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[37]Releasing better solvency information helps illiquid banks, but in our model it is not through a higher average

Figure 6: The effect of public provision of solvency information

Bank run equilibrium with endogenous information acquisition when we vary the individual solvency infor-
mation collection effort $\underline{e}$, which is *decreasing* in the public provision of solvency information. Parameter
values are $r = 0.09$, $\beta = 1$, $\theta = 1.03$, $\eta = 2$, $p = 0.8$, $\delta = 0.12$, $c = 10^{-6}$, $\alpha = 0.7$, $\gamma = 0.75$.

solvency are tightly linked, this government policy has the useful by-product of reducing information

collection about bank liquidity and therefore reducing the incidence of runs on illiquid banks.

6.2 Multiple Solvent Banks

We now investigate the model with competing solvent banks. Instead of holding cash, a bank

run in this setting involves the transfer of deposited funds from one illiquid bank to another more

liquid one. Information is privately more valuable in this setup since the outside option is a nearly

identical bank rather then holding cash. Here, the two banks' difference might be minuscule,

and transfers between the two institutions only involve social losses (transaction fees and bank

failure). Apparently, if only one of the multiple competing banks could be potentially illiquid, then

redepositing back to the surviving bank becomes off-equilibrium, as it is optimal to stay inside the

liquid competing bank. This might be the underlying reason for the finding of Iyer and Puri (2012)

that only 10% of running depositors' bank balances return to pre-crisis levels.

Suppose there are two banks, ex-ante identical except half the population deposits in bank A

and half deposits in bank B. Both banks promise the same rate of return r. However, transferring

funds between banks requires a transaction cost c. A liquidity event occurs at a random time t_0,

and a rumor starts that exactly one of the banks is illiquid (with κ_L) while the other is liquid (with

κ_H). The prior probability that each bank is illiquid is $p = 0.5$ since they are ex-ante identical. The

bank quality once the stress test isolates those insolvent ones. When the planner varies $\underline{e}$, agents can always perfectly
spot insolvent banks, therefore the channel of insolvency information is shut down. Rather, the channel is through
the strategic interaction of individual agents, as now everybody knows that everybody will wait to see the stress test
instead of scrambling to search the insolvent banks. As a result, everybody will have less precise information on
which solvent bank is less liquid and susceptible to a run.

learning process in the two bank set-up is simpler, because the passage of time without a failure teaches agents nothing about the relative viability of their bank.

Agents can acquire a costly signal $\widetilde{y} \in \{y_L, y_M, y_H\}$ about their bank with probability distribution as before. Agents who receive the y_H signal know their bank is liquid and therefore never withdraw; agents who receive the y_M signal gain no useful information and staying in their original bank is optimal; and agents with y_L signals run on their bank immediately.[38] Proposition 7 in the Appendix A.10 characterizes the equilibrium in this setting with two banks.

In this setting, information acquisition to figure out the relative liquidity position of two solvent banks is socially undesirable, as it shortens the survival time of the illiquid bank by inducing more agents to realize that one bank strictly dominates the other. Thus it is socially beneficial to blur the differences between competing solvent banks. Consider the Capital Purchase Program commonly known as the bailout of the nine largest U.S. financial institutions on October 13, 2008. When presenting the program to the CEOs of the 9 banks, Secretary of Treasury Henry M. Paulson was concerned the strongest banks (e.g. JPMorgan) would not participate. To make sure they do, government officials suggested that if a bank refused the funds, its regulator would later force it to raise capital anyway and under worse terms.[39] The government was in fact injecting noise about the liquidity of competing solvent banks into the economy. By pooling banks together the incentive to transfer funds between them was kept low enough so that none of the nine banks suffered a run.

6.3 Other Theoretic Issues

In this section we tie some theoretic loose ends and provide further discussions.

6.3.1 Minimum threshold liquidity capacity κ to eliminate runs

The dynamic rumor-based run equilibrium derived in our setting has a qualitatively different nature relative to the one in the static setting (e.g., Diamond and Dybvig (1983), Goldstein and Pauzner

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[38]From the perspective of y_L agents, the value of a dollar in their bank falls and the value of a dollar in the competing bank increases to a riskless $\frac{\delta}{\delta-r}$; as long as the transaction cost c is small enough so that $V_I(0|y_M) < (1-c)V_{-I}(0|y_H) = (1-c)\frac{\delta}{\delta-r}$, immediate withdraw is optimal. Here, information is (privately) more valuable in this setup since the outside option is a nearly identical bank rather then holding cash.

[39]"I was concerned about Jamie Dimon, because JPMorgan appeared to be in the best shape of the group, and I wanted to be sure he would accept the capital." "Look, we're making you an offer," I said, jumping in. "If you don't take it and sometime later your regulator tells you that you are undercapitalized and you have to raise private-sector capital but you are unable to do so, you may not like the terms if you have to come back to me." - Paulson (2010)

(2005)). Unlike the typical static setting where runs occur if bank reserves are below all potential withdrawals, our rich dynamic setting gives a non-trivial minimum reserve threshold that eliminates the run equilibrium. In other words, even for the illiquid bank, the minimum reserve requirement to fence off bank runs, κ_L, is below 1 which is the level that is sufficient to cover all potential withdrawals. As fully analyzed in the working paper version NBER Working Paper 18513 and explained briefly in Appendix B.1, this result is rooted in the uncertainty over the timing of liquidity event and other depositors' withdrawal timings in our asynchronous awareness setting.

Following the same logic, there exists a minimum reserve threshold for κ_H, so that rumor-based bank runs never occur for the liquid bank as long as κ_H is above this threshold. This result justifies the assumption that staying in the bank is optimal given a y_H signal (and hence the bank is liquid), which we have taken as given in Section 4.1. For detailed argument, see Appendix B.1.

6.3.2 What about uninformed agents?

Uninformed agents play no role in our analysis of rumor-based bank runs. So far, we have assumed that uninformed agents remain fully deposited before they hear the rumor. We now provide a justification for the optimality of this strategy. Uninformed learning is more complicated because, in addition to the failure hazard rate, they keep updating the rumor arrival rate, i.e. the probability of hearing the rumor in the next instant given no bank failure and no rumor. Nevertheless, because it does not matter much whether an agent hears the rumor inside or outside the bank (up to small transaction costs), the rumor arrival rate plays a minor role in the withdrawal decision.

The failure hazard rate from the perspective of uninformed agents, on the other hand, is important even when transaction costs vanish. Importantly, the failure hazard rate converges to zero as $t \to \infty$; a formal analysis is available upon request. Intuitively, as time passes and uninformed agents observe no failure and no rumor, they will attach a higher and higher probability to the event that $\widetilde{t}_0$ already happened (and they missed the rumor), but the bank is liquid so no failure occurs. Similar to the analysis for informed agents in (11), an uninformed agent will stay inside the bank from time t going forward if the following holds:

$$h^U(s)(1-\gamma)(1-c) \le rU_I(s) \text{ for } s \ge t, \tag{22}$$

where $h^U(s)$ and $U_I(s)$ are the failure hazard rate and the value of an inside dollar perceived by uninformed agents. Since when $t \to \infty$ the failure hazard rate decreases to zero, while the right hand side $U_I(s) > p\gamma(1-c) + (1-p)\frac{(1-c)\delta}{\delta-r}$,[40] condition (22) must hold for any t such that $0 \le t^f \le t < \infty$, for some finite t^f. In fact, for the parameters chosen to illustrate our results, the maximum hazard rate is sufficiently low that $t^f = 0$, i.e. it is always optimal for uninformed agents to remain fully deposited.

7   Conclusion

We study above new dimensions of bank runs, previously unexplored, by focusing on information acquisition in rumor-based runs. Our model incorporates uncertainty and information acquisition about bank liquidity into the asynchronous awareness framework of Abreu and Brunnermeier (2003). Information acquisition about liquidity exposes otherwise safe banks to destructive runs with endogenous waiting through a "*fear-of-bad-signal-agents*" effect: agents unsure about bank liquidity worry that agents who receive worse signals withdraw before them, which increases their own incentives to run. Even in the situation where runs never occur without information acquisition, this effect of information acquisition results in a unique and interior bank run equilibrium with an endogenous bank survival time.

We use our model to analyze the role that government information policy can play in bank run prevention, when solvency is uncertain, and in a multiple bank setting. Empirically, our model sheds new light on both traditional bank runs and debt runs more broadly. Our model makes new predictions linking the likelihood of a run to the spreading rate of the rumor, the cost of acquiring information, and the recovery value in case of bankruptcy. It also generates the unique prediction that for banks that survive a run, we should observe agents withdraw at the same time earlier informed agents redeposit. Our model is flexible and rich enough for structural estimation or calibration of its parameters given detailed withdrawals data, which can result in meaningful policy implications. In the Appendix, we discuss the possibility of depositors choosing the optimal time to acquire signals (Section B.2), and the possibility of endogenizing the rumor spreading speed

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[40]The lower bound for the value $U_I(s)$ is achieved under the simple strategy of never withdrawing from the bank, and assuming that the illiquid bank fails immediately. This is a lower bound because i) the illiquid bank survives for a while; and ii) at any later time, the posterior probability of an illiquid bank conditional on no failure is weakly lower than their prior p (similar to the effect in (21); the formal proof is available upon request).

(Section B.3).

More broadly, the dynamic bank run model we provide above can shed new light on other economic settings such as arbitrageur behavior, currency attacks, and R&D investment games.

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A Omitted Proofs

A.1 Proof of Proposition 1

Since the bank failure occurs at $t_0 + \zeta(\kappa)$ where $\zeta(\kappa_L) = \zeta$ and $\zeta(\kappa_H) = \infty$, then from the perspective of agent t_i, the cumulative distribution of the failure time $t_0 + \zeta(\kappa)$, witch is denoted by $\Pi(t|t_i)$, is given by:

$$\Pi(t|t_i) = \int_{t_0 + \zeta(\kappa) < t} d\Phi(t_0|t_i) = \Phi(t - \zeta(\kappa)|t_i).$$

For $t = t_i + \tau$, $\Pi(t_i + \tau|t_i) = \Phi(t_i + \tau - \zeta(\kappa)|t_i)$. The hazard rate is therefore

$$h(t_i + \tau|t_i) = \frac{d\Pi(t_i + \tau|t_i)}{1 - \Pi(t_i + \tau|t_i)} = \frac{p\phi(t_i + \tau - \zeta|t_i)}{1 - p\Phi(t_i + \tau - \zeta|t_i)},$$

since in the κ_H state an infinite survival time makes both the density and distribution of t_0 zero. Plugging in the expression of $\phi(t_i + \tau - \zeta|t_i)$ in (1) and $\Phi(t_i + \tau - \zeta|t_i)$ in (2) we get the expression for h. Finally, when $\zeta > \eta$ we have $h(\tau) = 0$ for $\tau < \zeta - \eta$ because $\phi(t|t_i)$ takes non-zero value only for $t \in [t_i - \eta, t_i]$.

A.2 Proof of Proposition 2

It is without loss of generality to focus on the strictly positive part of $h(\cdot)$. We first establish the following lemma.

Lemma 4. *The function*

$$g(\tau) = \frac{(\lambda(1-\gamma) - r) e^{\lambda(\zeta-\tau)}p - r(1-p)\left(e^{\lambda\eta} - 1\right)ve^{-\delta(\zeta-\tau)} + r\left(1 - e^{\lambda\eta}(1-p)\right)}{(1-p)(e^{\lambda\eta} - 1) + (e^{\lambda(\zeta-\tau)} - 1)p},$$

crosses zero from below at most once in the interval $[0, \zeta]$.

Proof: Since it is the change of the numerator that dominates around $g(\tau) = 0$, it suffices to show that the numerator of $g(\tau)$ (ignoring the constant)

$$[\lambda(1-\gamma) - r] e^{\lambda(\zeta-\tau)}p - r(1-p)\left(e^{\lambda\eta} - 1\right)ve^{-\delta(\zeta-\tau)} \tag{23}$$

is increasing over the interval $[0, \zeta]$. To show this, note that $\lambda(1-\gamma) - r < 0$ from (12) implies that (23) is concave in τ. Let $\bar{\tau}$ be the unique maximizer. At the maximum

$$\zeta - \bar{\tau} = \frac{1}{\lambda + \delta} \ln \frac{\delta r(1-p)\left(e^{\lambda\eta} - 1\right)v}{\lambda(r - \lambda(1-\gamma))p} < 0$$

due to (12). Thus, the function in (23) attains its maximum to the right of ζ and is therefore increasing over $[0, \zeta]$.

Lemma 4 implies that if $g(\zeta) \leq 0$ $(g(0) \geq 0)$ then $g(\tau) < 0$ $(g(\tau) \geq 0)$ always for $\tau \in [0, \zeta]$. We next consider the optimal strategy for the three cases of the proposition:

Case 1. If $g(\zeta) \leq 0$ so that $g(\tau_w) \leq 0$ always, then it is optimal to stay in the bank always. To prove our claim, it suffices to show that $V_I(\tau) > (1-c)V_O(\tau)$ for $\tau \in [0, \zeta]$. Suppose not, then there must exist a largest τ_w so that $V_I(\tau_w) = (1-c)V_O(\tau_w)$ and $V_I'(\tau_w) > (1-c)V_O'(\tau_w)$ because $V_I(\zeta) > (1-c)V_O(\zeta)$. From HJB equations, (7) implies $h(\tau_w)(1 - V_O(\tau_w)) + \delta(1 - V_O(\tau_w)) + V_O'(\tau_w) = 0$ (because $(1-c)V_I(\tau_w) < V_O(\tau_w)$), and (8) implies $rV_I(\tau_w) + h(\tau_w)(\gamma(1-c) - V_I(\tau_w)) + \delta(1 - c - V_I(\tau_w)) + V_I'(\tau_w) = 0$ (this holds for all $\tau > \tau_w$ and the continuity gives this equality). Multiplying the first equation by $1-c$ and using the second equation, we have

$$h(\tau_w)(1-c)(1-\gamma) - r(1-c)V_O(\tau_w) = V_I'(\tau_w) - (1-c)V_O'(\tau_w) > 0.$$

However, since $V_I(\tau_w) = V_O(\tau_w) \geq \widehat{V}_O(\tau_w)$ (the second inequality is because $\widehat{V}_O(\tau_w)$ may be derived under suboptimal policy), we have

$$h(\tau_w)(1-\gamma) - rV_O(\tau_w) \leq h(\tau_w)(1-\gamma) - r\widehat{V}_O(\tau_w) = g(\tau_w) \leq 0,$$

a contradiction.

Case 2. If $g(0) \geq 0$, then it is optimal to withdraw at 0 and redeposit right after ζ. If $g(0) \geq 0$, then $g(\tau) \geq 0$ always for $\tau \in [0, \zeta]$, and $g(\zeta) > 0$. Using $g(\zeta) > 0$, we first show that since c is arbitrarily small, there exists some $\widehat{\tau}$ close to ζ so that $V_O(\widehat{\tau}) = V_I(\widehat{\tau})$. To show this, we show that $(1-c)V_O'(\zeta) - V_I'(\zeta)$ is strictly below zero when c is arbitrarily small. To see this, from the HJB equations we know that

$$(1-c)V_O'(\zeta) - V_I'(\zeta) = h(\zeta)((1-c)V_O(\zeta) - V_I(\zeta) + (1-c)(\gamma - 1)) + \delta(V_I(\zeta) - (1-c)V_O(\zeta)) + rV_I(\zeta)$$
$$= -(1-c)g(\zeta) + (h(\zeta) + \delta - r)((1-c)V_O(\zeta) - V_I(\zeta))$$

The first term is strictly negative while the second and third terms converge to zero as $c \to 0$. Therefore, when c is arbitrary small, $V_I(\zeta) - (1-c)V_O(\zeta) = c(2-c)\frac{(1-c)\delta}{\delta - r}$ is close to zero, and thus there exists some ε so that $V_I(\zeta - \varepsilon) < (1-c)V_O(\zeta - \varepsilon)$. Due to continuity there exists some $\widehat{\tau}$ close to ζ so that $V_I(\widehat{\tau}) = (1-c)V_O(\widehat{\tau})$. Note that $V_O(\widehat{\tau}) = \widehat{V}_O(\widehat{\tau})$.

Now to prove that "it is optimal to withdraw at 0 and redeposit right after ζ," we only need to show that $V_I(\tau) = (1-c)V_O(\tau)$ holds for all $\tau \in [0, \widehat{\tau}]$ (intuitively, at any point of time a dollar inside the bank has the value of taking outside). Suppose that this does not hold; since $V_I(\tau) \geq (1-c)V_O(\tau)$ in general, there must exits some point $\tau_w \in [0, \widehat{\tau}]$ so that $V_I(\tau_w) = (1-c)V_O(\tau_w)$ and $V_I'(\tau_w) < (1-c)V_O'(\tau_w)$. Choosing the largest value τ_w, so that $V_O(\tau_w) = \widehat{V}_O(\tau_w)$ holds (i.e., the optimal continuation strategy is wait outside the bank until ζ). Similar to the argument before, we have

$$h(\tau_w)(1-c)(1-\gamma) - r(1-c)\widehat{V}_O(\tau_w) = h(\tau_w)(1-c)(1-\gamma) - r(1-c)V_O(\tau_w) = V_I'(\tau_w) - (1-c)V_O'(\tau_w) < 0,$$

but this contradicts with $g(\tau) \geq 0$ always.

Case 3. It follows from the Lemma 4 that $g(\zeta) > 0$ and $g(0) < 0$ imply that there exists a unique $\tau_w \in (0, \zeta)$ so that $g(\tau_w) = 0$, $g(\tau) > 0$ for $\tau \in (\tau_w, \zeta)$ and $g(\tau) < 0$ for $\tau \in (0, \tau_w)$. Following the same argument in the second part by replacing 0 with τ_w, we know that it is optimal to withdraw at τ_w and redeposit at $\zeta+$, and $V_I(\tau_w) = (1-c)V_O(\tau_w) = (1-c)\widehat{V}_O(\tau_w)$. Then to prove our claim we only need to show that $V_I(\tau) > (1-c)V_O(\tau)$ for $\tau \in (0, \tau_w)$. Let $H(\tau) \equiv V_I(\tau) - (1-c)V_O(\tau)$ with $H(\tau_w) = 0$, then we need to show that $H(\tau) > 0$ for

45

$\tau \in (0, \tau_w)$. Recall $H(\tau) = V_I(\tau) - (1-c) V_O(\tau) \geqslant 0$ in general. We first rule out $H(\tau) = 0$ uniformly on any interval $(\tau_w - \Delta, \tau_w)$ where $\Delta > 0$; if it is true then it must be that $V_I(\tau) = (1-c) V_O(\tau) = (1-c) \widehat{V}_O(\tau)$ on that interval so that (as $(1-c) V_I < V_O$):

$$
\begin{aligned}
0 &= r V_I(\tau) + h(\tau) (\gamma(1-c) - V_I(\tau)) + \delta(1 - c - V_I(\tau)) + V_I'(\tau) \\
&= r(1-c) \widehat{V}_O(\tau) + h(\tau)(1-c)\left(\gamma - \widehat{V}_O(\tau)\right) + \delta(1-c)\left(1 - \widehat{V}_O(\tau)\right) + (1-c) \widehat{V}_O'(\tau) \\
&= (1-c)\left[r\widehat{V}_O(\tau) - h(\tau)(1-\gamma)\right] = -(1-c) g(\tau) > 0
\end{aligned}
$$

where the first equality is (8) and third equality is using the ODE for $\widehat{V}_O(\tau)$ with $0 = h(\tau)\left(1 - \widehat{V}_O(\tau)\right) + \delta\left(1 - \widehat{V}_O(\tau)\right) + \widehat{V}_O'(\tau)$. This contradiction implies that we must have $V_I(\tau) > (1-c) V_O(\tau)$ for some τ close to τ_w. Now suppose that there exists another point $\tau_w^1 < \tau_w$ so that $V_I(\tau_w^1) = (1-c) V_O(\tau_w^1)$. Take τ_w^1 that is the closet to τ_w so that $V_I'(\tau_w^1) \geq (1-c) V_O'(\tau_w^1)$. At τ_w^1, $V_I(\tau_w^1)$ must satisfy the HJB in (8) (because τ_w^1 is in the inaction region around the neighborhood, i.e., $(1-c) V_I(\tau) < V_O(\tau)$ for τ close to τ_w). Then

$$
\begin{aligned}
0 &= r V_I\left(\tau_w^1\right) + h\left(\tau_w^1\right)\left(\gamma(1-c) - V_I\left(\tau_w^1\right)\right) + \delta\left(1 - c - V_I\left(\tau_w^1\right)\right) + V_I'\left(\tau_w^1\right) \\
&\geq r(1-c) V_O\left(\tau_w^1\right) + h\left(\tau_w^1\right)(1-c)\left(\gamma - V_O\left(\tau_w^1\right)\right) + \delta(1-c)\left(1 - V_O\left(\tau_w^1\right)\right) + (1-c) V_O'\left(\tau_w^1\right) \\
&= (1-c)\left[r V_O\left(\tau_w^1\right) - h\left(\tau_w^1\right)(1-\gamma)\right] \geq -(1-c) g\left(\tau_w^1\right) > 0
\end{aligned}
$$

where we have used the HJB for $V_O(\tau_w^1)$ (since $(1-c) V_O(\tau_w^1) = V_O(\tau_w^1)$) and the fact that $V_O(\tau_w^1) \geq \widehat{V}_O(\tau_w^1)$ in general. Again we get a contradiction with (8).

A.3 Proof of Lemma 1

The distribution of t_0, $\phi(t_0)$ is memoryless implying ϕ follows an exponential distribution. Given that information spreading $\beta(t_i - t_0)$ depending on $t_i - t_0$ only, the following derivation shows that the conditional distribution of t_0, $\phi(t_0|t_i)$, takes the form of $\phi(t_i - t_0)$, i.e. it only depends on time elapsed $t_i - t_0$:

$$
\begin{aligned}
\phi(t_0|t_i) &\equiv \frac{\beta(t_i - t_0)\phi(t_0)}{\int_{t_i-\eta}^{t_i} \beta(t_i - s)\phi(s)\, ds} \overset{\phi \text{ is exponential}}{=} \frac{\beta(t_i - t_0)}{\int_{t_i-\eta}^{t_i} \beta(t_i - s)\phi(s - t_0)\, ds} \\
&\overset{\text{let } y = s - t_i + \eta}{=} \frac{\beta(t_i - t_0)}{\int_0^\eta \beta(\eta - y)\phi(y - \eta + t_i - t_0)\, dy} \equiv \phi(t_i - t_0).
\end{aligned}
$$

Suppose the function Σ is an indefinite integral of ϕ so that $\Sigma' = \phi$. This implies that the conditional cumulative distribution of t_0 also depends on $t_i - t_0$ only:

$$
\Phi(t|t_i) \equiv \Pr\left(\tilde{t}_0 \leq t|t_i\right) = \int_{t_i-\eta}^t \phi(t_i - s)\, ds \overset{\text{let } y = s - t_i + \eta}{=} \int_0^{t - t_i + \eta} \sigma(\eta - y)\, dy = \Sigma(\eta) - \Sigma(t_i - t).
$$

This allows us to write $\Phi(t|t_i)$ as $\Phi(t - t_i)$. Hence, the conditional hazard rate depends on $\zeta - \tau$ only:

$$
h(t_i + \tau|t_i; \zeta) = \frac{\phi(t_i + \tau - \zeta|t_i)\, p}{1 - p\Phi(t_i + \tau - \zeta|t_i)} = \frac{\phi(\zeta - \tau)\, p}{1 - p\Phi(\zeta - \tau)}.
$$

A.4 Proof of Proposition 3

In any bank run equilibrium, the equilibrium remaining survival time is $\tau_r^* = -\frac{\ln(1-\kappa_L)}{\beta}$. Then depending on the sign of $G\left(-\frac{\ln(1-\kappa_L)}{\beta}\right)$, the agent in equilibrium is either willing to wait a bit longer or withdraw a bit earlier. "Waiting" contradicts to the bank run equilibrium, while "withdrawing a bit earlier" implies that in equilibrium the waiting time binds at zero, i.e. every one withdraw their deposits upon hearing the rumor. The knife-edge case is obvious.

A.5 Proof of Lemma 2

Suppose that $\zeta > \eta$, so that at ζ the cumulative withdrawal from y_L agents is $q\left(1 - e^{-\beta\eta}\right)$. Then using the aggregate condition (AW'), we can back out the equilibrium τ_r for y_M agents as

$$\tau_r = -\frac{1}{\beta} \ln\left(1 - \frac{\kappa_L - q\left(1 - e^{-\beta\eta}\right)}{1 - q}\right).$$

However, unless parameters are such that the above τ_r happens to satisfy $G\left(\tau_r\right) = 0$ which is the y_M agents' optimal waiting decision, generically this cannot occur.

Now we rule out the case that y_L agents wait a positive time $\tau_w^L > 0$. Suppose that y_L agents wait to withdraw upon hearing the rumor; it is easy to show that y_M agents must wait as well. Denote $\tau_r^L > 0$ and $\tau_r^M > 0$ as the remaining survival times optimally chosen by y_L and y_M agents, respectively; then the two individual optimal conditions in the spirit of (17) determine τ_r^L and τ_r^M. However, generically they will be inconsistent with the aggregate withdrawal condition $\kappa_L = q\left(1 - e^{-\beta\tau_r^L}\right) + (1-q)\left(1 - e^{-\beta\tau_r^M}\right)$; or at ζ there are no y_L agents withdrawing then the aggregate withdrawal condition is $\kappa_L = (1-q)\left(1 - e^{-\beta\tau_r^M}\right) + q\left(1 - e^{-\beta\eta}\right)$.

A.6 Proof of Proposition 4

First note that the G function is the mirror image of individual FOC condition function, i.e., $G\left(\tau_r\right) = g\left(\zeta - \tau_r\right)$, and it shares the same (but opposite) property of $g\left(\cdot\right)$ shown in Lemma 4:

Corollary 2. $G\left(\tau_r\right)$ *crosses zero from above at most once on* $\tau_r \in [0, \zeta]$.

This result implies that the following holds for the three cases of the proposition:

Case 1. If $G\left(\tau_r^l\right) \leq 0$, then $G\left(\tau_r\right) \leq 0$ for all $\tau_r \geq \tau_r^l$. Thus if all other agents strategy is to redeposit after any $\tau_r \geq \tau_r^l$, it is optimal for the individual agent to deviate and wait a bit longer. Therefore, $\zeta^* \to \infty$ and no run equilibrium exists.

Case 2. If $G\left(\tau_r^u\right) \geq 0$, then $G\left(\tau_r\right) \geq 0$ for all $\tau_r \leq \tau_r^u$. Thus, if all other agents' strategy is to withdraw at some interior $\tau_r \leq \tau_r^u$, it is optimal for the individual agent to deviate and withdraw earlier. Therefore, agents withdraw immediately in the only symmetric equilibrium.

Case 3. Finally, if $G\left(\tau_r^u\right) < 0$ and $G\left(\tau_r^l\right) > 0$ then by continuity of G and Corollary 2, there exists a unique bank run equilibrium $\tau_r^* \in \left(\tau_r^l, \tau_r^u\right)$ so that $G\left(\tau_r^*\right) = 0$. Plugging into (AW') we get

the equilibrium survival time ζ^* and waiting time τ_w^*. A second implication of Corollary 2 is that $G'(\tau_r^*) < 0$. Therefore the equilibrium is stable.

A.7 Proof of Proposition 5

It is easy to show that $\tau_r^l(q) = \frac{1}{\beta} \ln \left[\frac{1-q}{1-\kappa_L - qe^{-\beta\eta}} \right] < \frac{1}{\beta} \ln \left[\frac{1}{1-\kappa_L} \right]$, and we aim to show that $G\left(\frac{1}{\beta} \ln \left[\frac{1}{1-\kappa_L} \right] \right) < 0$ cannot rule out the case of $G\left(\tau_r^l(q) \right)$ being negative or positive. Corollary 2 tells us that $G(\cdot)$ to cross zero at most once from above; if anything, this implies that $G(\cdot)$ is decreasing, which means that $G\left(\tau_r^l(q) \right) > G\left(\frac{1}{\beta} \ln \left[\frac{1}{1-\kappa_L} \right] \right)$; this implies that $G\left(\tau_r^l(q) \right)$ might take both signs. The rest of result is a direct application of Proposition 4.

A.8 Proof of Lemma 3

First, when the bank run equilibrium occurs with corner solution $\zeta = \tau_r^u = \frac{1}{\beta} \ln \frac{1}{1-\kappa_L}$, then the marginal benefit of information $MB = p(t_i|t_i)(1-c) + (1 - p(t_i|t_i)) \frac{(1-c)\delta}{\delta-r} - V_I(0|y_M)$ is independent of q, and the equilibrium q^* equates $MC = \alpha q^* = MB$. Therefore the equilibrium is unique and stable (MB is constant while MC increases with q). Also if q^* takes the upper bound corner value, the associated run equilibrium is unique as well. So the rest of proof focus on the case where both the information quality of equilibrium survival time take interior solutions.

From now on we focus on interior bank run equilibrium. Importantly, this implies that τ_r is determined in (17) which only depends on primitives. Therefore we treat τ_r as a primitive parameter.

Given τ_w simple integration yields

$$V_I(0|y_M) = \frac{(1-c) \left[\begin{array}{c} \frac{\delta(e^{\lambda\eta}(1-p)-1)}{\delta-r} \left(1 - e^{-(\delta-r)\tau_w} \right) + \frac{\delta+\lambda\gamma}{\lambda+\delta-r} e^{\lambda\zeta} p \left(1 - e^{-(\lambda+\delta-r)\tau_w} \right) \\ + e^{-(\delta-r)\tau_w} \left(e^{\lambda\eta} - 1 - \left(e^{\lambda\eta} - e^{\lambda(\zeta-\tau_w)} \right) p \right) V_O(\tau_w|y_M) \end{array} \right]}{e^{\lambda\eta} - 1 - (e^{\lambda\eta} - e^{\lambda\zeta}) p}.$$

The FOC (20) when the agent sets q^* is

$$\frac{\left[\begin{array}{c} (e^{\lambda\zeta} - 1) p + (1-p) (e^{\lambda\eta} - 1) \frac{\delta}{\delta-r} - \frac{\delta(e^{\lambda\eta}(1-p)-1)}{\delta-r} \left(1 - e^{-(\delta-r)\tau_w} \right) \\ -\frac{\delta+\lambda\gamma}{\lambda+\delta-r} e^{\lambda\zeta} p - e^{-(\delta-r)\tau_w} e^{\lambda\tau_r} \left[\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)} \right] p - \frac{\alpha}{1-c} q \left[(1-p) (e^{\lambda\eta} - 1) + (e^{\lambda\zeta} - 1) p \right] \end{array} \right]}{(1-p) (e^{\lambda\eta} - 1) + (e^{\lambda\zeta} - 1) p} = 0 \quad (24)$$

where ζ and $\tau_w = \zeta - \tau_r$ depend on q through (18). Note that $h(\tau_w)(1-\gamma) = rV_O(\tau_w)$ implies

$$\left(e^{\lambda\eta} - 1 - \left(e^{\lambda\eta} - e^{\lambda(\zeta-\tau_w)} \right) p \right) V_O(\tau_w) = \frac{\lambda(1-\gamma) e^{\lambda(\zeta-\tau_w)} p}{r}$$

where we used the expression for h in (3). Then note that

$$\frac{\lambda(1-\gamma)}{r} - \frac{\delta + \lambda\gamma}{\lambda+\delta-r} = \frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)}$$

48

which gives expression 24. We have

$$\zeta'(q) = \tau_w'(q) = \frac{e^{-\beta\zeta} - e^{-\beta\tau_r}}{q\beta e^{-\beta\zeta}} = \frac{1 - e^{\beta\tau_w}}{q\beta} < 0.$$

The derivative at the point where (24) takes zero value yields:

$$\lambda\zeta'e^{\lambda\zeta}p_0 - \delta\left(e^{\lambda\eta}(1-p_0) - 1\right)e^{-(\delta-r)\tau_w}\tau_w' - \frac{\delta+\lambda\gamma}{\lambda+\delta-r}p_0\lambda e^{\lambda\zeta}\zeta' + (\delta-r)e^{-(\delta-r)\tau_w}e^{\lambda\tau_r}p_0\left(\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)}\right)\tau_w'$$

$$-\frac{\alpha}{1-c}\left((1-p_0)\left(e^{\lambda\eta}-1\right) + \left(e^{\lambda\zeta}-1\right)p_0\right) - \frac{\alpha}{1-c}qe^{\lambda\zeta}\lambda\zeta'p_0$$

$$= \lambda\zeta'e^{\lambda\zeta}p_0\left[\frac{\lambda(1-\gamma)-r}{\lambda+\delta-r} - \frac{\alpha}{1-c}q\right] + e^{-(\delta-r)(\zeta-\tau_r)}\tau_w'\left[e^{\lambda\tau_r}p_0(\delta-r)\left(\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)}\right) - \delta\left(e^{\lambda\eta}(1-p_0)-1\right)\right]$$

$$-\frac{\alpha}{1-c}\left((1-p_0)\left(e^{\lambda\eta}-1\right) + \left(e^{\lambda\zeta}-1\right)p_0\right)$$

$$= \zeta'(q)e^{-(\delta-r)\zeta}\left[e^{(\delta-r)\tau_r}\left[e^{\lambda\tau_r}p_0(\delta-r)\left(\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)}\right) - \delta\left(e^{\lambda\eta}(1-p_0)-1\right)\right] + \lambda e^{(\lambda+\delta-r)\zeta}p_0\left[\frac{\lambda(1-\gamma)-r}{\lambda+\delta-r} - \frac{\alpha}{1-c}q\right]\right]$$

$$-\frac{\alpha}{1-c}\left((1-p_0)\left(e^{\lambda\eta}-1\right) + \left(e^{\lambda\zeta}-1\right)p_0\right)$$

The second line is clearly negative. If we can ensure that

$$e^{(\lambda+\delta-r)\tau_r}p_0(\delta-r)\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)} + e^{(\delta-r)\tau_r}\delta\left(1 - e^{\lambda\eta}(1-p_0)\right) + \lambda e^{(\lambda+\delta-r)\zeta}p_0\left[\frac{\lambda(1-\gamma)-r}{\lambda+\delta-r} - \frac{\alpha}{1-c}q\right] > 0, \quad (25)$$

then since $\zeta'(q) < 0$, the first line is negative, and as a result the derivative of (24) is always negative. Therefore, when (24) equals zero, it must go down. Combined with differentiability of (24), this result rules out multiple solutions, because if there exist, then there must have one solution with the local slope being nonnegative. Therefore, (24) crosses zero at most once from above, which implies that the bank run equilibrium, if exists, is unique and stable.

To finish the argument, we show that the sufficient condition for condition (25) is (note that $e^{\lambda\eta}(1-p_0) < 1$)

$$e^{(\lambda+\delta-r)\tau_r^u}p_0(\delta-r)\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)} + e^{(\delta-r)\tau_r^d}\delta\left(1 - e^{\lambda\eta}(1-p_0)\right) \quad (26)$$

$$+\lambda e^{(\lambda+\delta-r)\eta}p_0\left[\frac{\lambda(1-\gamma)-r}{\lambda+\delta-r} - \frac{\alpha}{1-c}\frac{\kappa_L}{1-e^{-\beta\eta}}\right] > 0.$$

We only need to verify that (25) dominates (26) term by term. The first term is because $\lambda(1-\gamma) - r < 0$ (recall (12)) and $\tau_r \leq \tau_r^u$; the second term is because $\tau_r \geq \tau_r^d$, and the third term is because $\lambda(1-\gamma) - r < 0$, $\zeta < \eta$, and $q^* < \frac{\kappa_L}{1-e^{-\beta\eta}}$.

A.9   Proof of Proposition 6

Follows immediately from Lemma 2 and Lemma 3.

A.10   Equilibrium with Multiple Solvent Banks

At the time of hearing the rumor, the value of a dollar in the bank for agents with y_L and y_H signals are respectively $V_I(0|y_L) = \frac{(1-c)^2\delta}{\delta-r}$ and $V_I(0|y_H) = \frac{(1-c)\delta}{\delta-r}$. In order to calculate the value

for agents with the y_M signal, note that with probability $1/2$ the original bank is the illiquid one, but the agent can deposit his funds (after the liquidation cost $1 - \gamma$) to the liquid one in case of failure while deposited. As a result, the value with y_M signal is

$$
\begin{aligned}
V_I\left(0|y_M\right) &= \frac{1}{2}\frac{1-c}{1-\Pi\left(0|\kappa_L\right)}\int_0^\zeta\left[\delta e^{-(\delta-r)s}\left(1-\Pi\left(s|\kappa_L\right)\right)+e^{-(\delta-r)s}\pi\left(s|\kappa_L\right)\gamma\left(1-c\right)\frac{\delta}{\delta-r}\right]ds+\frac{1}{2}\frac{(1-c)\delta}{\delta-r} \\
&= \frac{1-c}{2}\left\{\frac{1}{1-e^{-\lambda\zeta}}\left[-e^{-\lambda\zeta}\frac{\delta}{\delta-r}\left(1-e^{-(\delta-r)\zeta}\right)+\frac{\delta}{\delta-r+\lambda}\left(1+\frac{\lambda\gamma\left(1-c\right)}{\delta-r}\right)\left(1-e^{-(\delta-r+\lambda)\zeta}\right)\right]+\frac{\delta}{\delta-r}\right\}.
\end{aligned}
$$

Proposition 7. *Under the two banks setup, the bank run equilibrium $\{\zeta^*, q^*\}$ is determined by the following two equations:*

$$
\zeta^* = -\frac{1}{\beta}\ln\left(1-\frac{\kappa_L}{q^*}\right),\ \text{ and } \frac{1}{2}\left(1-c\right)\frac{\delta}{\delta-r}+\frac{1}{2}\frac{\delta}{\delta-r}-V_I\left(0|y_M;\zeta^*,q^*\right)=\frac{\alpha}{1-c}q^*.
$$

A bank run equilibrium requires that withdrawals by the y_L agents alone can destroy a bank, i.e. $\kappa_L < q\left(1-e^{-\beta\eta}\right)$.[41] The threshold $\bar{q}$ so that no run would occur is $\bar{q} \equiv \frac{\kappa_L}{1-e^{-\beta\eta}}$. If the planner raises α so that the marginal benefit of acquiring information is below its marginal cost, i.e.,

$$
\frac{1}{2}\left(1-c\right)\frac{\delta}{\delta-r}+\frac{1}{2}\frac{\delta}{\delta-r}-V_I\left(0|y_M\right)\le\frac{\alpha}{1-c}\bar{q},
$$

then the illiquid bank is always sufficiently liquid to sustain a run.

B   Additional Results and Extensions 6.3

B.1   Minimum Liquidity Capacity Threshold that Eliminate Runs

The threshold liquidity capacity κ_L that eliminates bank runs on the illiquid bank can be found numerically as the κ_L so that $G\left(\tau_r^l\left(q\right);\kappa_L\right)=0$ in Proposition 5 with binary information choices, or $G\left(\tau_r^l\left(q^*|_{\zeta^*=\eta}\right)\right)=0$ in Proposition 6 with convex acquisition costs. In the baseline parameters in Figure 4, the threshold liquidity capacity κ_L is 0.6537.

In general, due to the asynchronous nature of our rumor-based setting, even without information acquisition, immediate withdrawal after hearing the rumor might not be an equilibrium. Similar to the insight of Abreu and Brunnermeier (2003), this is because the depositor knows that the bank survives for a while, a feature absent in any static bank run setting. Thus, eliminating dynamic rumor-based runs requires less liquidity reserve than the one suggested by the static perspective.

Here is an outline on how to determine the threshold threshold κ_H so that there never exist runs on the liquid bank. Our information structure in Figure 2 implies that the agent with y_H knows the bank is liquid for sure. More importantly, the agent knows that he is among the q fraction of y_H agents, and there are $1-q$ fraction of informed agents with medium signals y_M. We are interested in the equilibrium behavior of y_H agents who endogenously wait a while to withdraw, by holding the belief that $1-q$ fraction of y_M agents withdraw immediately upon hearing the rumor. Thus, y_H agents face a tradeoff similar to (11). This harshest belief regarding y_M agents allows us to

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[41] In this two-bank setting we no longer impose the restriction of $q \le \frac{\kappa_L}{1-e^{-\beta\eta}}$ as in condition (16), because now only y_L agents are withdrawing to potentially take down the illiquid bank.

determine the upper bound of the κ_H threshold that rules out runs by y_H agents. Interestingly, this is in the same nature as the equilibrium for the illiquid bank studied in the main model, where y_L agents withdraw immediately while y_M agents wait for a while; and hence the same algorithm that pins down threshold κ_L applies.[42] Under our baseline parameters given in Figure 4, the threshold κ_H required to eliminate runs on the liquid bank is 0.8241.

B.2 Optimal Time to Acquire the Additional Signal(s)

We have so far assumed agents acquire the additional signal y immediately upon hearing the rumor. Although early information allows for a superior action, there is an option value derived from waiting to acquire the signal later: the agent learns from the passage of time that the bank is more likely to be liquid, and the bank might fail (thus making the additional signal unnecessary).

The following Proposition provides a sufficient condition for the optimality of acquiring information immediately. Intuitively, it suffices that the information acquisition cost is low enough relative to the net benefit of delaying this expense.

Proposition 8. *The optimal time to acquire the additional signal $\tilde{y}$ is immediately upon hearing the rumor if*

$$[\delta + h(\zeta)] \frac{\alpha}{2(1-c)} \frac{\kappa_L}{1 - e^{-\beta\eta}} - \frac{r - c\delta}{\delta - r} \delta (1 - p) < 0, \tag{27}$$

$$[\delta - r + h(\zeta)] \frac{\alpha}{2(1-c)} \frac{\kappa_L}{1 - e^{-\beta\eta}} - ((1-\gamma) h(0) - rp) < 0, \tag{28}$$

and

$$(1 - p) \frac{(1-c)\delta}{\delta - r} \geq \alpha. \tag{29}$$

Proof. Let $\tau_y \equiv t_y - t_i$ denote the time an agent waits between hearing the rumor and acquiring the additional signal y. We first show that under condition (27), if $0 \leq \tau_y \leq \tau_w$ then the optimal $\tau_y^* = 0$. We then show that under condition (28), if $\tau_w \leq \tau_y \leq \zeta$ then $\tau_y^* = \tau_w$. Finally, we show that $\tau_y = \tau_w$ is dominated by $\tau_y = \tau_w - \epsilon$, which implies setting $\tau_y^* = 0$ is everywhere optimal.

First, suppose $0 \leq \tau_y \leq \tau_w$. Informed agents maximize the value of a deposited dollar at t_i:

$$\max_{q,\tau_y} v(0; q, \tau_y) = (1-c) \left[\int_0^{\tau_y} \delta e^{-(\delta-r)s} (1 - \Pi(s)) \, ds + \int_0^{\tau_y} \gamma e^{-(\delta-r)s} \pi(s) \, ds \right] $$
$$+ e^{-(\delta-r)\tau_y} (1 - \Pi(\tau_y)) v(\tau_y; q, \tau_y).$$

The marginal benefit of earlier information acquisition time τ_y dominates the marginal cost if

$$[\delta - r + h(\tau_y)] \frac{\chi(q)}{1 - c} < q((1-\gamma) h(\tau_y) - rp(\tau_y)) \text{ for all } \tau_y \in [0, \tau_w].$$

Condition (27) guarantees this is the case.

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[42]More specifically, when the liquid bank reserve κ_H increases, the waiting benefit of y_H agents increases and they wish to wait longer. Once the resulting survival time ζ exceeds the awareness window η, the similar logic as in Lemma 2 and Proposition 4 implies that y_H agents will not run in equilibrium. Here, the implicit assumption is that $1 - q$ measure of y_M agents are not enough to take down the bank.

Second, suppose $\tau_w < \tau_y \leq \zeta$. The agent now maximizes:

$$\max_{q,\tau_y} v\left(\tau_w; q, \tau_y\right) = \int_{\tau_w}^{\tau_y} \delta e^{-\delta s} \left(1 - \Pi\left(s\right)\right) ds + \int_{\tau_w}^{\tau_y} e^{-\delta s} \pi\left(s\right) ds + e^{-\delta \tau_y} \left(1 - \Pi\left(\tau_y\right)\right) v\left(\tau_y; q, \tau_y\right).$$

For this case, the marginal benefit of earlier information acquisition time τ_y dominates if

$$\left[\delta + h\left(\tau_y\right)\right] \frac{\chi\left(q\right)}{1 - c} < q \frac{r - c\delta}{\delta - r} \delta \left[1 - p\left(\tau_y\right)\right] \text{ for all } \tau_y \in \left[\tau_w, \zeta\right],$$

which is guaranteed by condition (28).

The earlier region dominates if $v\left(\tau_w^-; q\right) \geq \left(1 - c\right) v\left(\tau_w^+; q\right)$, i.e. if acquiring the signal just before withdrawing dominates acquiring it just after. Using the fact that $V_I\left(\tau_w | y_M\right) = \left(1 - c\right) V_O\left(\tau_w | y_M\right)$, the earlier region dominates if and only if $q\left(1 - p\left(\tau_w\right)\right) \frac{\left(1-c\right)\delta}{\delta-r} c \geq \chi\left(q\right) c$, which is guaranteed under condition (29). The intuition is that since the money is inside the bank just before τ_w, an agent who withdraws, acquires information, and then redeposits would waste the transaction cost, but gain on the proportional cost of information by delaying it until after he pays transaction cost to withdraw.

We could also allow agents to acquire signals more than once. But if we impose, as one should, a small fixed cost of acquiring information, we get in equilibrium a finite number of such acquisitions. After these, the population is fully described by the same three types $\left(y_L, y_M, y_H\right)$ as in the main text. Thus, the bank run equilibrium structure remains qualitatively the same.

B.3 Endogenous Rumor Spreading Speed

One can extend our model by linking the spreading speed to the fraction already informed, which leads the equilibrium fraction of informed agents to be an S-shaped function in the spreading time. Such a modification only makes the conditional distribution for $\tilde{t}_0$ in (2) more complicated, but would not affect our main results.

A more ambitious/interesting extension would be to follow the spirit of Banerjee (1993) by allowing the spreading speed to depend on the fraction of depositors who have run on the bank, say $\beta R\left(t | t_0, \tilde{\kappa}\right) dt$ where $R\left(t | t_0, \tilde{\kappa}\right) = \left(1 - q\right)\left(1 - e^{-\beta\left(t - \tau_w\right)}\right) + q\left(1 - e^{-\beta t}\right) \times \mathbf{1}_{\{\kappa_L\}}$ is the cumulative withdrawal shown in Figure 3. As a result, the rumor spreading speed depends on not only the underlying bank liquidity $\tilde{\kappa}$, but also the endogenous equilibrium waiting time τ_w of y_M agents. Both can be viewed as amplification forces. First, since y_L agents immediately withdraw only when the bank is illiquid, relative to the case of $\tilde{\kappa} = \kappa_H$ the rumor spreads faster, giving rise of a more severe run on the illiquid bank. Second, if y_M agents wait a bit longer to withdraw, the rumor spreading speed is slower, and thus the illiquid bank will survive longer, feeding back to later withdrawal of y_M agents. Although mathematically challenging, the critical feature for our analysis, that the hazard rates depend only on time elapsed since hearing the rumor, would be preserved. As a result, with the aid of numerical methods one can analyze such settings with endogenous rumors, and we leave this possibility for future research.

C Non-stationary Equilibria

We consider the non-stationary part of the model here. If $t_0 < \eta$, then some early informed agents with $t_i < \eta$ knows that $t_0 \in [0, t_i]$, and this truncation implies strategy may be t_i-dependent. However, as shown in Abreu and Brunnermeier (2003), those early agents will be bunching together to eliminate the non-stationarity. We modify their results to our setting.

Focus on the bank being illiquid. To be precise, follow Abreu and Brunnermeier (2003) in our model the agents who hears the rumor before $\zeta - \tau_w = \tau_r$ will behave as if the agent who hears the rumor exactly at τ_r. The strategy of the agent who hears rumor at τ_r is that, independent of signal (y_L or y_M) he will withdraw at $\tau_r + \tau_w = \zeta$. Moreover, for agents who hear rumor at $t_i \in [\tau_r, \zeta]$, they take the following strategy. If they receive y_L signal then he will withdraw at ζ, while if they get y_M signal then they withdraw at $t_i + \tau_w$. This additional modification is because relative to Abreu and Brunnermeier (2003) agents may have different signals in our model.

For illustration, suppose that $t_0 = 0$ so the bank should fail at ζ. Recall that there are q measure of y_L signals and $1 - q$ measure of y_M signals. Since the information keeps spreading at ζ (recall that $\eta < \zeta$) and all agents hears the rumor before ζ will withdraw at ζ, there are $q\left(1 - e^{-\beta\zeta}\right)$ measure of y_L agents withdrawing. On the other hand, y_M agents who hear the rumor in the interval $[0, \tau_r]$ are withdrawing at ζ, with a total mass of $(1 - q)\left(1 - e^{-\beta\tau_r}\right)$. Therefore, we have

$$(1 - q)\left(1 - e^{-\beta\tau_r}\right) + q\left(1 - e^{-\beta\zeta}\right) = \kappa_L,$$

which is exactly (AW'). A similar argument can be applied to the case of $t_0 > 0$ so that the bank failure time is $t_0 + \zeta$: this is because endogenously there are less agents bunching at the physical time ζ, so the bank failure time is postponed to $t_0 + \zeta > \zeta$.

There is one issue that our richer (than Abreu and Brunnermeier (2003)) setting leads to potential non-stationarity. Although withdraw behavior can be stationary, the endogenous learning about bank liquidity is non-stationary when $\eta < \zeta$. In fact, initially when $t_0 = 0$, agents have no other information so $p(t_i|t_i) = p$ must holds. In stationary state, $p(t_i|t_i) = \widehat{p}_0 \equiv \frac{(e^{\lambda\zeta}-1)p}{(1-p)(e^{\lambda\eta}-1)+(e^{\lambda\zeta}-1)p} < p$. This difference potentially alters the optimal withdraw strategies for agents with different timings. To resolve this issue, we simply assume that for $t_i < \eta$, the prior is time-varying

$$p(t_i) = \frac{\left(e^{\lambda t_i} - 1\right)p}{[(1-p)(e^{\lambda\eta} - 1) + p(e^{\lambda t_i} - 1)]},$$

and one can show that with this specification, the resulting posterior upon hearing the rumor, $p(t_i|t_i)$, is always $\widehat{p}_0$. One can presumably achieve this by a more structural way; for instance, introduce other shocks so that, conditional on survival and hearing the rumor, the posterior of the bank being illiquid is always $\widehat{p}_0$. Also, we have to fix the signal quality structure q to be the same as in the stationary phase. We deem these technical issue non-essential for the economic questions that we are after in this paper.