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ABSTRACT

This paper uses a New Keynesian model with banks and deposits to study the macroeconomic effects of policies that pay interest on reserves. While their effects on output and inflation are small, these policies require major adjustments in the way that the monetary authority manages the supply of reserves, as liquidity effects vanish in the short run. In the long run, however, the additional degree of freedom the monetary authority acquires by paying interest on reserves is best described as affecting the real quantity of reserves: policy actions that change prices must still change the nominal quantity of reserves proportionally.

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1 Introduction

Slowly but surely over the three decades that have passed since the Federal Reserve's monetarist experiment of 1979 through 1982, the role of the monetary aggregates in both the making and analysis of monetary policy has eroded. Bernanke's (2006) historical account explains how and why Federal Reserve officials gradually deemphasized measures of the money supply as targets and indicators for monetary policy over these years. Taylor's (1993) highly influential work shows that, instead, Federal Reserve policy beginning in the mid-1980s is described quite well by a strikingly parsimonious rule for adjusting the short-term interest rate in response to movements in output and inflation. Taylor's insight has since been embedded fully into theoretical analyses of monetary policy and its effects on the macroeconomy, which now depict central bank policy as a rule for managing the short-term interest rate. Indeed, textbook New Keynesian models such as Woodford's (2003) and Gali's (2008) typically make no reference at all to any measure of the money supply, yet succeed nonetheless in providing a complete and coherent description of the dynamics of output, inflation, and interest rates.

Still, as discussed by Ireland (2008) with reference to both practice and theory, the central bank's ability to manage short-term interest rates has rested, ultimately, on its ability to control, mainly through open market purchases and sales of government bonds, the quantity of reserves supplied to the banking system. Recently, however, Goodfriend (2002), Ennis and Weinberg (2007), and Keister, Martin, and McAndrews (2008) have all suggested that, to some extent, even this last remaining role for a measure of money in the monetary policymaking process can weaken when the central bank pays interest on reserves. In the United States, interest on reserves moved quickly from being a theoretical possibility to becoming an aspect of reality when, first, the Financial Services Regulatory Relief Act of 2006 promised to grant the Federal Reserve the power to pay interest on reserves starting on October 1, 2011, second, the Emergency Economic Stabilization Act of 2008 brought that starting date forward to October 1, 2008, and third, the Federal Reserve announced on October 6, 2008 that it would, in fact, begin paying interest on reserves.

Figure 1 begins to suggest how the mechanics of the Federal Reserve's federal funds rate targeting procedures can change with the introduction of interest payments on reserves. In both panels, the quantity of reserves gets measured along the horizontal axis and the federal funds rate along the vertical axis. The demand curve for reserves slopes downward, since as the federal funds rate falls, those banks that typically borrow reserves find that the cost of doing so has declined and those banks that typically lend reserves find that the benefit of doing so has declined: all banks, therefore, wish to hold more reserves. The notation in panel (a), $DR(FFR; RR = 0)$, makes clear that while the demand curve describes a relationship between banks' desired holdings of reserves and the federal funds rate FFR, this relationship also depends on the fact that, by assumption, the interest rate RR paid on reserves equals zero. In other words, a change in the federal funds rate leads to a movement along the downward-sloping demand curve, whereas a change in the interest rate paid on reserves shifts the demand curve. Panel (a) thereby shows that with $RR = 0$, the Federal Reserve hits its target FFR_0 for the federal funds rate by conducting open market operations that leave QR_0 dollars of reserves to circulate among banks in the system.

Panel (b) then shows how the payment of interest on reserves places a floor under the federal funds rate. For if the federal funds rate does fall below the rate $RR_0 > 0$ at which the Fed pays interest on reserves, any individual bank can earn profits by borrowing reserves from another bank and depositing them at the Fed; this excess demand for reserves then pushes the funds rate back to RR_0. In fact, the demand curve in panel (b) becomes horizontal when the federal funds rate reaches RR_0, assuming that then, banks become indifferent between lending reserves out and holding them on deposit at the Fed. Of course, these observations simply generalize those that could have been made when describing panel (a) for the case without interest on reserves: there, the lower bound for the federal funds rate equals zero, since no bank will lend reserves at a negative rate when those funds can be held without opportunity cost either as vault cash or as deposits at the Fed. However, the Federal Reserve Board (2008) cites this as one of the major rationales for its new interest-on-reserves policy:

that it can be used to set a positive lower bound for the federal funds rate.

When, in panel (b), the Federal Reserve's funds rate target FFR_0 remains above the interest rate RR_0 paid on reserves, the Fed must still conduct open market operations to make the quantity of reserves supplied, QR_1, equal to the quantity demanded. But with interest on reserves, the opportunity cost of holding funds on deposit at the Fed is measured by the spread $FFR_0 - RR_0$; since this spread is, of course, smaller in magnitude than the federal funds rate itself, the demand curve for reserves shifts to the right moving from panel (a) to panel (b). Thus, with interest on reserves, the level of reserves QR_1 required to support the funds rate target FFR_0 in panel (b) is generally larger than the level of reserves QR_0 required to support the same funds rate target shown in panel (a) for the case without interest on reserves. This is one of the points emphasized by Goodfriend (2002), Ennis and Weinberg (2007), and Keister, Martin, and McAndrews (2008): the ability to pay interest on reserves gives the Federal Reserve an additional tool of monetary policy that provides another degree of freedom in the policymaking process since, by adjusting the interest rate paid on reserves, the Fed can achieve different combinations of settings for both the federal funds rate and the quantity of reserves.

But panel (b) of figure 1 also highlights something more. Suppose that the Federal Reserve sets a funds rate target FFR_1 equal to the interest rate RR_0 it pays on reserves. Then a new equilibrium is reached in which the quantity of reserves supplied QR_2 can lie anywhere along the horizontal segment of the demand curve. In this case, to use Keister, Martin, and McAndrews' (2008) apt words, paying interest on reserves appears to "divorce money from monetary policy," since there is an entire continuum of values that can be chosen for the quantity of reserves, all of which remain consistent with the Federal Reserve's federal funds rate target.

Though highly suggestive, the two panels of figure 1 provide only a partial view of the changes that take place once a central bank begins paying interest on reserves. Most importantly, both panels hold the price level fixed, and while the Keynesian assumption of a fixed aggregate price level may be perfectly justified when looking at the effects of monetary policy over short horizons, measured in days or weeks, the question remains as to what will happen over longer intervals, as weeks blend into months and then quarter years and prices begin to change. Likewise, the graphs ignore the effects that changes in output, including those brought about in the short run by monetary policy actions themselves, may have on the demand for reserves. And to the extent that changes in the interest rate paid on reserves get passed along to consumers through changes in retail deposit rates, and to the extent that those changes in deposit rates then set off portfolio rebalancing by households, additional effects that feed back into banks' demand for reserves get ignored as well. One cannot tell from these graphs whether changes in the federal funds rate, holding the interest rate on reserves fixed either at zero or some positive rate, have different effects on output and inflation than changes in the federal funds rate that occur when the interest rate on reserves is moved in lockstep to maintain a constant spread between the two; if that spread between the federal funds rate and the interest rate on reserves acts as a tax on banking activity, those differences may be important too. All of these considerations underscore that assessing the full, dynamic effects of monetary policies that involve the payment of interest on reserves requires a fully dynamic and stochastic general equilibrium model. The purpose of this paper is to build and analyze such a model, so as to explore the macroeconomic effects of interest on reserves in more detail.

First and foremost, the dynamic model developed here provides a sharper view of the additional possibilities opened up by the extra degree of freedom a central bank obtains when it has the ability to pay interest on reserves. For a given setting of the short-term nominal interest rate – the model's analog to the federal funds rate – adjustments to the interest rate paid on reserves are best described as changing the *real* quantity of reserves demanded by banks. Thus, while the extra degree of freedom does allow the central bank to target simultaneously the short-term nominal interest rate and the real quantity of reserves, the model shows that monetary policy actions intended to bring about long-run changes in the aggregate price level must still be accompanied by proportional changes in the *nominal* supply of reserves. Quite strikingly, however, the model also reveals that households' shifts in and out of bank deposits change considerably when the central bank begins paying interest on reserves. As a consequence, the precise sequence of changes in the supply of reserves required to support a given series of movements in the short-term interest rate can also change considerably – not just in timing and magnitude but even in direction – depending on the central bank's policy for paying interest on reserves. When calibrated to match key features of the United States economy, the model implies that policies of paying interest on reserves have only small effects on output and inflation. On the other hand, the model suggests that these effects can be larger in other economies, where the banking system is less efficient and monetary policy is a larger source of instability. The paper's conclusion discusses the implications of these results for the Federal Reserve, as it begins to unwind the unprecedentedly large monetary policy actions taken during the financial crisis of 2008 and the severe recession that followed.

The basic features of the model used here can be described in relation to those that appear in previous work that explores the effects of policies that pay interest on reserves. Sargent and Wallace (1985) and Smith (1991) use overlapping generations models of money to see whether the payment of interest on reserves gives rise to problems of equilibrium indeterminacy; Hornstein (2010) does the same, using a model in which monetary assets appear in an infinitely-lived representative agent's utility function, output is exogenous, and the price level is perfectly flexible. Here, these issues of equilibrium determinacy are revisited, but with the help of a model with infinitely-lived agents, endogenous output, and sticky goods prices that resembles more closely the textbook New Keynesian frameworks of Woodford (2003) and Gali (2008).

Berentsen and Monnet (2008) also use a dynamic, general equilibrium model to investigate the workings of monetary policy systems that pay interest on reserves. In particular, Berentsen and Monnet employ a search-theoretic framework that highlights, in great detail, how schemes involving the paying of interest on reserves can help systems of payment operate more efficiently and thereby improve resource allocations supported by decentralized markets in which money serves as a medium of exchange. Here, as in Belongia and Ireland (2012) but in contrast to most other New Keynesian models, the medium of exchange role played by currency and bank deposits receives some attention. But, by generating a demand for money through a more stylized shopping-time specification as opposed to an explicit description of decentralized trade, the model used here can go beyond Berentsen and Monnet's in other ways, allowing for a more detailed analysis of the dynamics of macroeconomic variables including output, inflation, and interest rates that compares to similar analyses conducted with more conventional New Keynesian models.

Finally and most recently, Kashyap and Stein (2012) develop a detailed model of the financial sector, in which the spread between the between the federal funds rate and the interest rate paid on reserves acts as a time-varying tax, and show how a central bank might use this time-varying tax to stabilize a fractional reserves banking system. Here, the spread between the federal funds rate and the interest rate paid on reserves also appears as a tax on banks. Once again, however, the description of the banking system provided here remains more stylized so that, while some attention is paid below to shocks that disrupt the financial sector, issues relaing to the optimal design, structure, and regulation of the financial system cannot receive the extensive consideration they get in Kashyap and Stein (2012) and the three other studies mentioned previously: Goodfriend (2002), Ennis and Weinberg (2007), and Keister, Martin, and McAndrews (2008). Here, however, banks' activities get modeled together with those of households and all other firms in the economy, so that the broader focus can be on the macroeconomic effects of interest on reserves.

2 The Model

2.1 Overview

Belongia and Ireland (2012) extend the standard New Keynesian framework, exposited by Woodford (2003) and Gali (2008) and used by many others, to incorporate roles for currency and bank deposits in providing monetary services to households. There, the objective is to revisit issues first raised by Barnett (1980) concerning the ability of simple-sum versus Divisia monetary aggregates to track movements in the true quantity of monetary services provided by liquid assets supplied by both the government and the private banking system. Here, the same model gets extended still further to consider the macroeconomic effects of monetary policies that manage both a short-term market rate of interest, like the federal funds rate in the United States, and the rate of interest on reserves. This extended model allows the host of issues, raised above with the help of figure 1, to be addressed head on, directly and fully, with a dynamic, stochastic, general equilibrium model, but requires a more elaborate description of how banks optimally manage their holdings of reserves. In particular, the previous model in Belongia and Ireland (2012) simply posits an exogenously-varying reserve ratio that affects other aspects of bank behavior but is not itself an explicit choice variable as it is here. The extended model developed here also introduces the additional cost channel for monetary policy proposed and analyzed by Barth and Ramey (2001), Christiano, Eichenbaum, and Evans (2005), Ravenna and Walsh (2006), Rabanal (2007), and Surico (2008), both to provide a potentially important role for bank loans as well as deposits and to insure that the full effects of monetary policy on the dynamics of output and inflation are accounted for.

The model economy consists of a representative consumer, a representative finished goods-producing firm, a continuum of intermediate goods-producing firms indexed by $i \in [0, 1]$, a representative bank, and a monetary authority. During each period $t = 0, 1, 2, \ldots$, each intermediate goods-producing firm produces a distinct, perishable intermediate good.

Hence, intermediate goods may also be indexed by $i \in [0,1]$, where firm i produces good i. The model features enough symmetry, however, to allow the analysis to focus on the behavior of a representative intermediate goods-producing firm that produces the generic intermediate good i. The activities of each of these agents will now be described in turn.

2.2 The Representative Household

The representative household enters each period $t = 0, 1, 2, \ldots$ with M_{t-1} units of currency, B_{t-1} bonds, and $s_{t-1}(i)$ shares in each intermediate goods-producing firm $i \in [0,1]$. At the beginning of each period, the household receives T_t additional units of currency in the form of a lump-sum transfer from the monetary authority. Next, the household's bonds mature, providing B_{t-1} more units of currency. The household uses some of this currency to purchase B_t new bonds at the price of $1/r_t$ dollars per bond, where r_t denotes the gross nominal interest rate between t and $t+1$, and $s_t(i)$ new shares in each intermediate goods-producing firm $i \in [0,1]$ at the price of $Q_t(i)$ dollars per share.

After this initial securities-trading session, the household is left with

$$M_{t-1} + T_t + B_{t-1} + \int_0^1 Q_t(i)s_{t-1}(i)\,\mathrm{d}i - B_t/r_t - \int_0^1 Q_t(i)s_t(i)\,\mathrm{d}i$$

units of currency. It keeps N_t units of this currency to purchase goods and deposits the rest in the representative bank. At the same time, the household also borrows L_t^h dollars from the bank, bringing the total nominal value of its deposits to

$$D_t^h = M_{t-1} + T_t + B_{t-1} + \int_0^1 Q_t(i)s_{t-1}(i)\,\mathrm{d}i - B_t/r_t - \int_0^1 Q_t(i)s_t(i)\,\mathrm{d}i - N_t + L_t^h. \tag{1}$$

During period t, the household supplies $h_t^g(i)$ units of labor to each intermediate goods-producing firm $i \in [0,1]$, for a total of

$$h_t^g = \int_0^1 h_t^g(i)\,\mathrm{d}i.$$

The household also supplies h_t^b units of labor to the representative bank. The household therefore receives $W_t h_t$ in labor income, where W_t denotes the nominal wage rate and

$$h_t = h_t^g + h_t^b \tag{2}$$

denotes total hours worked in goods production and banking.

Also during period t, the household purchases C_t units of the finished good at the nominal price P_t from the representative finished goods-producing firm. Making this transaction requires

$$h_t^s = \frac{1}{\chi}\left(\frac{v_t^a P_t C_t}{M_t^a}\right)^{\chi} \tag{3}$$

units of shopping time, where M_t^a is an aggregate of monetary services provided from currency N_t and deposits D_t^h according to

$$M_t^a = [(v^n)^{1/\omega} N_t^{(\omega-1)/\omega} + (1-v^n)^{1/\omega}(D_t^h)^{(\omega-1)/\omega}]^{\omega/(\omega-1)}. \tag{4}$$

In the shopping-time specification (3), the parameter $\chi > 1$ governs the rate at which the effort required to purchase goods and services increases as the household economizes on its holdings of monetary assets. The shock v_t^a impacts on the household's total demand for monetary services; it follows the autoregressive process

$$\ln(v_t^a) = (1-\rho_v^a)\ln(v^a) + \rho_v^a \ln(v_{t-1}^a) + \varepsilon_{vt}^a. \tag{5}$$

where $v^a > 0$ helps determine the steady-state level of real monetary services demanded relative to consumption, the persistence parameter satisfies $0 \leq \rho_v^a < 1$, and the serially uncorrelated innovation ε_{vt}^a has mean zero and standard deviation σ_v^a. In the monetary aggregation specification (4), the parameter $\omega > 0$ measures the elasticity of substitution between currency and deposits in creating liquidity services and the parameter v^n, satisfying $0 < v^n < 1$, helps determine the steady-state share of currency versus deposits in creating the monetary aggregate. Although (3)-(5) describe the liquidity services provided by currency and bank deposits in a highly stylized manner relative to more elaborate models of decentralized trade such as Berentsen and Monnet's (2008), they nevertheless allow for a considerably richer depiction of shifts in the household's portfolio of liquid assets that may arise when the monetary authority changes its policy of paying interest on reserves than those that appear in simpler cash-in-advance or money-in-the-utility function models. As shown below, these portfolio shifts have important implications for how the monetary authority must manage the supply of reserves when it adjusts its target for the short-term market rate of interest r_t while also paying interest on reserves. Jones, Asaftei, and Wang (2004) and Cysne and Turchick (2012) use similar models, but without interest on reserves, to estimate the welfare cost of inflation in economies where consumers also have opportunities to substitute between currency and deposits as alternative media of exchange.

At the end of period t, the household owes the bank $r_t^l L_t^h$ dollars, where r_t^l is the gross, competitively-determined interest rate on loans. At the same time, however, the bank owes the household $r_t^d D_t^h$ dollars, where r_t^d is the gross, competitively-determined interest rate on deposits. The household also receives a nominal dividend payment $F_t(i)$ for each share it owns in each intermediate goods-producing firm $i \in [0, 1]$. After all these payments get sent and received, the household carries M_t units of currency into period $t + 1$, where

$$M_t = N_t + W_t h_t + \int_0^1 F_t(i) s_t(i)\, \mathrm{d}i + r_t^d D_t^h - P_t C_t - r_t^l L_t^h. \tag{6}$$

The household, therefore, chooses sequences for B_t, $s_t(i)$ for all $i \in [0, 1]$, N_t, D_t^h, L_t^h, h_t, C_t, h_t^s, M_t^a, and M_t for all $t = 0, 1, 2, \ldots$ to maximize the expected utility function

$$E \sum_{t=0}^{\infty} \beta^t a_t [\ln(C_t) - \eta(h_t + h_t^s)], \tag{7}$$

where the discount factor satisfies $0 < \beta < 1$ and $\eta > 0$ measures the weight on leisure versus consumption. The preference shock a_t in (7) follows the autoregressive process

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_{at}, \tag{8}$$

where the persistence parameter satisfies $0 \leq \rho_a < 1$ and the serially uncorrelated innovation ε_{at} has mean zero and standard deviation σ_a. The household makes its optimal choices subject to the constraints (1), (3), (4), and (6), each of which must hold for all $t = 0, 1, 2, \ldots$, taking as given the behavior of the exogenous shocks described by (5) and (8) for all $t = 0, 1, 2, \ldots$.

A convenient way to characterize the solution to the household's problem is to substitute the shopping-time specification (3) into the utility function (7) and to express the remaining constraints (1), (4), and (6) in real terms by dividing through by the nominal price level P_t to obtain

$$\frac{M_{t-1} + T_t + B_{t-1} - B_t/r_t - N_t + L_t^h}{P_t} + \int_0^1 \left[\frac{Q_t(i)}{P_t}\right][s_{t-1}(i) - s_t(i)]\,\mathrm{d}i \geq \frac{D_t^h}{P_t}, \tag{9}$$

$$\left[(v^n)^{1/\omega}\left(\frac{N_t}{P_t}\right)^{(\omega-1)/\omega} + (1-v^n)^{1/\omega}\left(\frac{D_t^h}{P_t}\right)^{(\omega-1)/\omega}\right]^{\omega/(\omega-1)} \geq \frac{M_t^a}{P_t}, \tag{10}$$

and

$$\frac{N_t + W_t h_t + r_t^d D_t^h}{P_t} + \int_0^1 \left[\frac{F_t(i)}{P_t}\right] s_t(i)\,\mathrm{d}i \geq C_t + \frac{r_t^l L_t^h + M_t}{P_t}, \tag{11}$$

after allowing for free disposal. Letting Λ_t^1, Λ_t^2, and Λ_t^3 denote the nonnegative Lagrange multipliers on these three constraints, the first-order conditions for the household's problem can be written as

$$\frac{\Lambda_t^1}{r_t} = \beta E_t\left(\frac{\Lambda_{t+1}^1 P_t}{P_{t+1}}\right), \tag{12}$$

$$\Lambda_t^1\left[\frac{Q_t(i)}{P_t}\right] = \Lambda_t^3\left[\frac{F_t(i)}{P_t}\right] + \beta E_t\left\{\Lambda_{t+1}^1\left[\frac{Q_{t+1}(i)}{P_{t+1}}\right]\right\} \tag{13}$$

for all $i \in [0,1]$,

$$\frac{N_t}{P_t} = v^n \left(\frac{M_t^a}{P_t}\right)\left(\frac{\Lambda_t^2}{\Lambda_t^1 - \Lambda_t^3}\right)^{\omega}, \tag{14}$$

$$\frac{D_t^h}{P_t} = (1-v^n)\left(\frac{M_t^a}{P_t}\right)\left(\frac{\Lambda_t^2}{\Lambda_t^1 - r_t^d\Lambda_t^3}\right)^{\omega}, \tag{15}$$

$$\Lambda_t^1 = r_t^l \Lambda_t^3, \tag{16}$$

$$\eta a_t = \Lambda_t^3 \left(\frac{W_t}{P_t}\right), \tag{17}$$

$$\frac{a_t}{C_t}\left[1 - \eta\left(\frac{v_t^a P_t C_t}{M_t^a}\right)^{\chi}\right] = \Lambda_t^3, \tag{18}$$

$$\eta a_t \left(\frac{v_t^a P_t C_t}{M_t^a}\right)^{\chi} = \Lambda_t^2 \left(\frac{M_t^a}{P_t}\right), \tag{19}$$

and

$$\Lambda_t^3 = \beta E_t \left(\frac{\Lambda_{t+1}^1 P_t}{P_{t+1}}\right), \tag{20}$$

together with (3) and (9)-(11) with equality for all $t = 0, 1, 2, \ldots$. The implications of these optimality conditions for issues relating to the demand for monetary assets and services are discussed below.

2.3 The Representative Finished Goods-Producing Firm

During each period $t = 0, 1, 2, \ldots$, the representative finished goods-producing firm uses $Y_t(i)$ units of each intermediate good $i \in [0,1]$, purchased at the nominal price $P_t(i)$, to manufacture Y_t units of the finished good according to the constant-returns-to-scale technology described by

$$\left[\int_0^1 Y_t(i)^{(\theta-1)/\theta}\,\mathrm{d}i\right]^{\theta/(\theta-1)} \geq Y_t,$$

where $\theta > 1$ measures the elasticity of substitution for the various intermediate goods in producing the final good. Thus, the finished goods-producing firm chooses $Y_t(i)$ for all

$i \in [0,1]$ to maximize its profits, given by

$$P_t \left[\int_0^1 Y_t(i)^{(\theta-1)/\theta} \, \mathrm{d}i \right]^{\theta/(\theta-1)} - \int_0^1 P_t(i) Y_t(i) \, \mathrm{d}i,$$

for all $t = 0, 1, 2, \ldots$. The first-order conditions for this problem are

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta} Y_t \tag{21}$$

for all $i \in [0,1]$ and $t = 0, 1, 2, \ldots$.

Competition drives the finished goods-producing firm's profits to zero in equilibrium. This zero-profit condition implies that

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} \, \mathrm{d}i \right]^{1/(1-\theta)}$$

for all $t = 0, 1, 2 \ldots$.

2.4 The Representative Intermediate Goods-Producing Firm

During each period $t = 0, 1, 2, \ldots$, the representative intermediate goods-producing firm hires $h_t^g(i)$ units of labor from the representative household to manufacture $Y_t(i)$ units of intermediate good i according to the constant-returns-to-scale technology described by

$$Z_t h_t^g(i) \geq Y_t(i). \tag{22}$$

The aggregate technology shock follows a random walk with positive drift:

$$\ln(Z_t) = \ln(z) + \ln(Z_{t-1}) + \varepsilon_{zt}, \tag{23}$$

where $z > 1$ and the serially uncorrelated innovation ε_{zt} has mean zero and standard deviation σ_z.

To provide an additional role for bank lending and to introduce a cost channel of the kind described by Barth and Ramey (2001) into the model, it is assumed that the representative intermediate goods-producing firm must pay a fraction ϕ_d, with $0 \leq \phi_d \leq 1$, of its total wage bill $W_t h_t^g$ using bank deposits $D_t^f(i)$; the firm then pays the remaining fraction $1 - \phi_d$ of its wage bill out of revenues earned during the period. More specifically, the firm is assumed to borrow the amount $L_t^f(i)$ from the representative bank at the beginning of the period; since loans to households and firms have identical liquidity and risk characteristics, competitive banks will charge firms the same interest rate r_t^l on these loans to firms as they do on loans to households. The representative firm places the full amount of its loan on deposit, so that

$$D_t^f(i) = L_t^f(i), \tag{24}$$

thereby simultaneously earning interest on the funds at the deposit rate r_t^d. Then, during each period $t = 0, 1, 2, \ldots$, the firm faces the deposits-in-advance constraint,

$$D_t^f(i) \geq \phi_d W_t h_t^g(i) \tag{25}$$

and this constraint will bind so long as the interest rate on loans r_t^l exceeds the interest rate on deposits r_t^d. In a manner similar to Rabanal (2007) and Surico (2008), the importance of the cost channel can be varied here by adjusting the numerical value assigned to the parameter ϕ_d, with $\phi_d = 0$ corresponding to the case where this additional channel is absent and $\phi_d = 1$ to the case considered by Christiano, Eichenbaum, and Evans (2005) and Ravenna and Walsh (2006) where the cost channel reaches its maximum strength.

Since intermediate goods substitute imperfectly for one another in producing the finished good, the representative intermediate goods-producing firm sells its output in a monopolistically competitive market. Hence, during each period $t = 0, 1, 2, \ldots$, the intermediate goods-producing firm sets the nominal price $P_t(i)$ for its output, subject to the requirement that it satisfy the representative finished goods-producing firm's demand, described by (21). In addition, following Rotemberg (1982), the intermediate goods-producing firm faces a quadratic cost of adjusting its nominal price, measured in units of the finished good and given by

$$\frac{\phi_p}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right]^2 Y_t,$$

where the parameter $\phi_p > 0$ governs the magnitude of the price adjustment cost and where $\pi > 1$ denotes the gross, steady-state inflation rate.

The cost of price adjustment makes the intermediate goods-producing firm's problem dynamic: the firm chooses a sequence for $P_t(i)$ for all $t = 0, 1, 2, \ldots$ to maximize its total, real market value, which from the equity-pricing relation (13) implied by the household's optimizing behavior is proportional to

$$E\sum_{t=0}^{\infty}\beta^t\Lambda_t^3\left[\frac{F_t(i)}{P_t}\right]$$

where real profits during each period $t = 0, 1, 2, \ldots,$

$$\frac{F_t(i)}{P_t}=\left[\frac{P_t(i)}{P_t}\right]^{1-\theta}Y_t-[1+\phi_d(r_t^l-r_t^d)]\left[\frac{P_t(i)}{P_t}\right]^{-\theta}\left(\frac{W_tY_t}{P_tZ_t}\right)-\frac{\phi_p}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right]^2Y_t, \quad (26)$$

equal revenue from sales minus the cost of hiring labor, inclusive of the net interest cost that depends on $\phi_d(r_t^l - r_t^d)$, and the cost of price adjustment. The first-order conditions for this problem are

$$\begin{aligned}0 =&(1-\theta)\Lambda_t^3\left[\frac{P_t(i)}{P_t}\right]^{-\theta}Y_t+\theta\Lambda_t^3[1+\phi_d(r_t^l-r_t^d)]\left[\frac{P_t(i)}{P_t}\right]^{-\theta-1}\left(\frac{W_tY_t}{P_tZ_t}\right)\\&-\phi_p\Lambda_t^3\left[\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right]\left[\frac{Y_tP_t}{\pi P_{t-1}(i)}\right]\\&+\beta\phi_pE_t\left\{\Lambda_{t+1}^3\left[\frac{P_{t+1}(i)}{\pi P_t(i)}-1\right]\left[\frac{Y_{t+1}P_{t+1}(i)P_t}{\pi P_t(i)^2}\right]\right\}\end{aligned} \quad (27)$$

for all $t = 0, 1, 2, \ldots$. When log-linearized, (27) takes the form of a New Keynesian Phillips curve, like Ravenna and Walsh's (2006), that allows for the additional cost channel when

$\phi_d > 0$ so that the firm's borrowing costs affect its total marginal costs of production.

2.5 The Representative Bank

During each period $t = 0, 1, 2, \ldots$, the representative bank issues deposits worth a total of

$$D_t = D_t^h + D_t^f, \tag{28}$$

where

$$D_t^f = \int_0^1 D_t^f(i)\,\mathrm{d}i$$

measures deposits issued to all of the various intermediate goods-producing firms $i \in [0, 1]$. Creating and maintaining these deposits requires N_t^v dollars in reserves and h_t^d units of labor, where

$$x_t^a \left[(x^n)^{1/\nu} \left(\frac{N_t^v}{P_t} \right)^{(\nu-1)/\nu} + (1 - x^n)^{1/\nu} (Z_t h_t^d)^{(\nu-1)/\nu} \right]^{\nu/(\nu-1)} \geq \frac{D_t}{P_t}. \tag{29}$$

In (29), the parameter $\nu > 0$ measures the elasticity of substitution between reserves and labor in the deposit creation process and the parameter x^n, satisfying $0 < x^n < 1$, helps determine the share of reserves relative to labor in producing deposits. The shock x_t^a to productivity in the banking sector follows the autoregressive process

$$\ln(x_t^a) = (1 - \rho_x^a)\ln(x^a) + \rho_x^a \ln(x_{t-1}^a) + \varepsilon_{xt}^a, \tag{30}$$

where $x^a > 0$, $0 \leq \rho_x^a < 1$, and the serially correlated innovation ε_{xt}^a has mean zero and standard deviation σ_x^a.

Similar in spirit to the shopping-time specification in (3)-(5), the banking technology described by (29) and (30) is intended to account – once again in a stylized way – not only for the costs of maintaining bank branches and automated teller machines for the convenience of deposit holders, but also for the costs of managing assets and liabilities with different characteristics within the bank. Thus, for instance, while there is no literal maturity mismatch between bank loans and bank deposits in the model, the specification in (29) reflects the idea that the representative bank can cope more easily with deposit inflows and outflows when it holds a larger stock of reserves.

In fact, (29) by itself implies that the representative bank can always reduce its labor input at the margin by holding additional reserves. In particular, the bank's demand for reserves grows without bound, rather than approaching a finite satiation point as shown in figure 1, as the opportunity cost of holding reserves shrinks towards zero. Without further modification, therefore, a well-defined equilibrium for this model will fail to exist when the monetary authority pays interest on reserves at the market rate r_t offered by bonds. Suppose, however, that in order to manage its stock of reserves worth N_t^v/P_t in real terms, the bank must hire h_t^v additional units of labor, where

$$Z_t h_t^v \geq \phi_v \left(\frac{N_t^v}{P_t} \right). \tag{31}$$

When the parameter $\phi_v > 0$ is very small but strictly positive, the additional labor required by (31) has little effect on aggregate resource allocations, but ensures that banks' holdings of reserves remain finite and uniquely-determined even when the monetary authority pays interest on those reserves at the market rate. With the addition of this feature, the total amount of labor employed by the bank is given by

$$h_t^b = h_t^d + h_t^v, \tag{32}$$

and the assumption, reflected in both (29) and (31), that labor productivity across all banking activities grows at the same stochastic rate as it does in the production of intermediate goods described by (22) and (23), ensures that the model remains consistent with balanced growth.

After deciding on its optimal holdings of reserves N_t^v, the bank lends out its remaining funds L_t; its balance sheet constraint requires that

$$\frac{D_t}{P_t} \geq \frac{N_t^v + L_t}{P_t}, \tag{33}$$

where

$$L_t = L_t^h + L_t^f \tag{34}$$

and

$$L_t^f = \int_0^1 L_t^f(i)\,\mathrm{d}i$$

describe the breakdown of total loans L_t into those channeled to households, L_t^h, and those channeled to all of the intermediate goods-producing firms, L_t^f. As noted above, all deposits pay interest at the gross rate r_t^d and all loans earn interest at the gross rate r_t^l. Hence, during each period t, the bank chooses D_t, N_t^v, L_t, h_t^d, and h_t^v to maximize its profits, given in nominal terms by

$$(r_t^l - 1)L_t + (r_t^v - 1)N_t^v - (r_t^d - 1)D_t - W_t(h_t^d + h_t^v), \tag{35}$$

subject to the constraints (29), (31), and (33) and taking as given the behavior of the exogenous shocks described by (23) and (30) for all $t = 0, 1, 2, \ldots$.

A convenient way to characterize the solution to the bank's problem is to substitute the constraints (29), (31), and (33) into the expression (35) for profits, which can be rewritten in real terms as

$$\begin{aligned} &(r_t^l - r_t^d)x_t^a \left[(x^n)^{1/\nu} \left(\frac{N_t^v}{P_t} \right)^{(\nu-1)/\nu} + (1-x^n)^{1/\nu} (Z_t h_t^d)^{(\nu-1)/\nu} \right]^{\nu/(\nu-1)} \\ &- \left[r_t^l + \phi_v \left(\frac{W_t}{P_t Z_t} \right) - r_t^v \right] \left(\frac{N_t^v}{P_t} \right) - \left(\frac{W_t}{P_t} \right) h_t^d. \end{aligned} \tag{36}$$

Equation (36) serves to highlight that the bank earns revenue from charging a higher interest rate on its loans than it must pay on its deposits, but also incurs both an opportunity and a real resource cost of holding reserves and must compensate its workers for their efforts in creating deposits. Note that (12), (16), and (20) imply that $r_t = r_t^l$ for all $t = 0, 1, 2, \ldots$: since households can obtain funds either by selling bonds or borrowing from banks, the interest rate on bonds must equal the interest rate on loans. In light of this no-arbitrage condition, the first-order conditions for the bank's problem can be written as

$$\frac{N_t^v}{P_t} = (r_t - r_t^d)^\nu (x_t^a)^{\nu-1} x^n \left(\frac{D_t}{P_t}\right) \left[r_t + \phi_v \left(\frac{W_t}{P_t Z_t}\right) - r_t^v\right]^{-\nu}, \tag{37}$$

$$h_t^d = (r_t - r_t^d)^\nu (x_t^a)^{\nu-1} (1 - x^n) \left(\frac{D_t}{P_t}\right) Z_t^{\nu-1} \left(\frac{W_t}{P_t}\right)^{-\nu}, \tag{38}$$

and (29), (31), and (33) with equality for all $t = 0, 1, 2, \ldots$.

Equation (37) confirms that in the absence of the additional labor requirement imposed through (31), the representative bank's demand for reserves grows without bound as the spread $r_t - r_t^v$ between the market rate and the interest rate on reserves shrinks to zero. Equation (37) also helps foreshadow many of the quantitative results that follow. Suppose, in particular, that the monetary authority uses a Taylor (1993) rule to manage the short-term interest rate r_t. Equation (37) then shows how, as anticipated in figure 1 and by Goodfriend (2002), Ennis and Weinberg (2007), and Keister, Martin, and McAndrews (2008), the ability to pay interest on reserves at the positive rate r_t^v gives the monetary authority an extra degree of freedom that it can use to target the supply of reserves to the banking system independently from the short-term market rate. Equation (37) suggests, however, that in this dynamic model, the additional degree of freedom is best described as one that determines the real quantity of reserves N_t^v/P_t as opposed to the nominal quantity of reserves N_t^v, so that a set of monetary policy actions intended to increase or decrease the aggregate price level P_t will still have to involve a proportional change in the nominal supply of reserves.

Finally, (37) can be combined with (29) and (38) to obtain

$$r_t - r_t^d = \left(\frac{1}{x_t^a}\right)\left\{x^n\left[r_t + \phi_v\left(\frac{W_t}{P_tZ_t}\right) - r_t^v\right]^{1-\nu} + (1-x^n)\left(\frac{W_t}{P_tZ_t}\right)^{1-\nu}\right\}^{1/(1-\nu)}, \tag{39}$$

which shows how the cost of deposit creation, which in turn depends on the opportunity cost of holding reserves and the cost of labor, drives a spread between the competitively-determined interest rate on bonds and loans and the competitively-determined interest rate on deposits.

2.6 The Monetary Authority

As usual in New Keynesian models like this one, the monetary authority will be assumed to conduct monetary policy by adjusting the short-term market rate of interest r_t in response to movements in inflation $\pi_t = P_t/P_{t-1}$ and a stationary measure of real economic activity, in this case the rate of output growth

$$g_t = Y_t/Y_{t-1}, \tag{40}$$

since the level of output inherits a random walk from the nonstationary technology shock (23), The modified Taylor (1993) rule

$$\ln(r_t/r) = \rho_r \ln(r_{t-1}/r) + \rho_\pi \ln(\pi_{t-1}/\pi) + \rho_g \ln(g_{t-1}/g) + \varepsilon_{rt} \tag{41}$$

allows, in addition, for interest rate smoothing through the lagged interest rate term on the right-hand side. In (41), the constants r, π, and g denote the steady-state values of the short-term nominal interest rate, the inflation rate, and the output growth rate, the Taylor rule coefficients $\rho_r \geq 0$, $\rho_\pi \geq 0$, and $\rho_g \geq 0$ are chosen by the monetary authority, and the serially uncorrelated innovation ε_{rt} has mean zero and standard deviation σ_r.

In addition, here, the monetary authority must choose a rule for determining the interest rate r_t^v it pays on reserves. By specifying a general rule of the form $r_t^v = \tau_t r_t^\alpha$ or, in logs,

$$\ln(r_t^v) = \ln(\tau_t) + \alpha \ln(r_t), \tag{42}$$

where $\alpha \geq 0$ is a parameter, the new variable τ_t follows the autoregressive process

$$\ln(\tau_t) = (1 - \rho_\tau) \ln(\tau) + \rho_\tau \ln(\tau_{t-1}) + \varepsilon_{\tau t} \tag{43}$$

with $\tau \geq 1$ and $0 \leq \rho_\tau < 1$, and the serially uncorrelated innovation $\varepsilon_{\tau t}$ has mean zero and standard deviation σ_τ, the general model allows flexibly for a number of special cases, including: (i) the standard case with $\alpha = 0$, $\tau = 1$, $\rho_\tau = 0$, and $\sigma_\tau = 0$, in which no interest is paid on reserves, (ii) the case with $\alpha = 0$, $\tau > 1$, $\rho_\tau = 0$, and $\sigma_\tau = 0$, in which interest is paid on reserves at the constant, gross rate τ, (iii) the case with $\alpha = 1$, $0 < \tau < 1$, $\rho_\tau = 0$, and $\sigma_\tau = 0$, in which the monetary authority maintains a constant, $100(1 - \tau)$ percentage-point spread between the market rate and the interest rate on reserves, (iv) the case with $\alpha = 1$, $\tau = 1$, $\rho_\tau = 0$, and $\sigma_\tau = 0$, in which interest is paid on reserves at the market rate, and (v) a variety of cases with $0 \leq \rho_\tau < 1$ and $\sigma_\tau > 0$, in which there is independent, stochastic variation in the rate of interest on reserves, giving rise to a time-varying spread between the market rate and the rate of interest on reserves.

2.7 The Demand for Monetary Assets and Services

In this model with currency and deposits, the variable M_t^a represents the true aggregate of monetary services demanded by the representative household during each period $t = 0, 1, 2, \ldots$. Note that (12) and (20), describing the representative household's optimizing behavior, imply that

$$\Lambda_t^1 = r_t \Lambda_t^3 \tag{44}$$

for all $t = 0, 1, 2, \ldots$. Substituting (44), together with (14) and (15), into (10) then yields

$$\frac{\Lambda_t^2}{\Lambda_t^3} = [v^n(r_t - 1)^{1-\omega} + (1 - v^n)(r_t - r_t^d)^{1-\omega}]^{1/(1-\omega)}. \tag{45}$$

Define the own rate of return r_t^a on the monetary aggregate M_t^a with reference to the right-hand side of (45):

$$r_t - r_t^a = [v^n(r_t - 1)^{1-\omega} + (1 - v^n)(r_t - r_t^d)^{1-\omega}]^{1/(1-\omega)}. \tag{46}$$

Equations (44)-(46) then allow (17) and (19) to be combined to obtain

$$\ln\left(\frac{M_t^a}{P_t}\right) = \frac{\chi}{1+\chi}\ln(C_t) + \frac{1}{1+\chi}\ln\left(\frac{W_t}{P_t}\right) - \frac{1}{1+\chi}\ln(r_t - r_t^a) + \frac{\chi}{1+\chi}\ln(v_t^a), \tag{47}$$

and (14) and (15) to be rewritten as

$$\frac{N_t}{P_t} = v^n\left(\frac{u_t^a}{u_t^n}\right)^{\omega}\left(\frac{M_t^a}{P_t}\right) \tag{48}$$

and

$$\frac{D_t^h}{P_t} = (1 - v^n)\left(\frac{u_t^a}{u_t^d}\right)^{\omega}\left(\frac{M_t^a}{P_t}\right), \tag{49}$$

where

$$u_t^a = \frac{r_t - r_t^a}{r_t}, \tag{50}$$

$$u_t^n = \frac{r_t - 1}{r_t}, \tag{51}$$

and

$$u_t^d = \frac{r_t - r_t^d}{r_t} \tag{52}$$

use Barnett's (1978) formula to define the user costs u_t^a, u_t^n and u_t^d of the monetary aggregate M_t^a, currency N_t, and deposits D_t.

Equation (47) takes the form of a demand curve for the monetary aggregate M_t^a, which having been derived from a shopping-time specification has the real wage as well as consumption as its scale variables, a result that echoes Karni's (1973), and has $r_t - r_t^a$ as its opportunity cost term. Meanwhile, (48) and (49) show how the household's optimal choices of currency and deposits in creating the monetary aggregate depend on the share parameter v^n as well as the user cost of each monetary asset relative to the whole. These equations also help foreshadow many of the quantitative results to follow and provide yet another perspective on the possibilities opened up by the extra degree of freedom the monetary authority obtains when it can pay interest on reserves. From (51), movements in the market rate of interest r_t, brought about as the monetary authority follows the Taylor rule (41), translate into variations in the user cost of currency. Equation (52) shows that the user cost of deposits depends, as well, on the opportunity cost $r_t - r_t^d$, which by (39) can be manipulated separately by the monetary authority through variations in the interest rate r_t^v it pays on reserves. Thus, this general equilibrium model reveals how the extra degree of freedom identified by Goodfriend (2002), Ennis and Weinberg (2007), and Keister, Martin, and McAndrews (2008) allows the central bank to vary the relative prices of currency and deposits as competing media of exchange, triggering shifts in households' portfolios of liquid assets.

While the true monetary aggregate M_t^a and its user cost u_t^a are well-defined and observable within the model, their magnitudes depend not only on the quantities of currency and deposits but also on functional forms and parameters that may not be known to outside agents, including analysts at the monetary authority and applied econometricians more generally. Belongia and Ireland (2012) show, however, that in a model like this one, but without interest on reserves, movements in both the true aggregate and its user cost are approximated very closely by movements in Divisia price and quantity indices for monetary services like those proposed by Barnett (1980); and the advantage of these Divisia aggregates is that they can be constructed without reference to unknown functional forms and parameters. Until 2006, the Federal Reserve Bank of St. Louis compiled and released data on these monetary services indices, building closely on Barnett's work as described by Anderson, Jones, and Nesmith (1997*a*, 1997*b*); below, therefore, data on the St. Louis Fed's monetary services price and quantity indices will be used as proxies for M_t^a and u_t^a. The more familiar, simple-sum aggregate

$$M_t^s = N_t + D_t \tag{53}$$

is, of course, constructed quite easily both in the model and the data; it, too, does not depend on unknown parameters, and in the extended model developed here it can be used to keep track of total deposits D_t and not just deposits D_t^h owned by households.

Two other monetary variables considered below are the reserve ratio and the monetary base. The former can be measured in the usual way, dividing bank reserves by deposits:

$$rr_t = N_t^v / D_t. \tag{54}$$

The latter is measured most easily by observing that since, for simplicity, households in this model do not carry deposits across periods and banks do not carry reserves across periods either, the variable M_t that keeps track of the currency possessed by the representative household at the end of each period $t = 0, 1, 2, \dots$ also equals the monetary base. And since, within each period, the monetary base gets increased both through the lump-sum transfer made by the monetary authority to households and the interest payments on reserves made by the monetary authority to banks, it evolves according to

$$M_t = M_{t-1} + T_t + (r_t^v - 1)N_t^v \tag{55}$$

for all $t = 0, 1, 2, \dots$.

2.8 Symmetric Equilibrium

In a symmetric equilibrium, all intermediate goods-producing firms make identical decisions, so that $Y_t(i) = Y_t$, $h_t^g(i) = h_t^g$, $L_t^f(i) = L_t^f$, $D_t^f(i) = D_t^f$, $P_t(i) = P_t$, $F_t(i) = F_t$, and $Q_t(i) = Q_t$ for all $i \in [0,1]$ and $t = 0, 1, 2, \ldots$. In addition, the market-clearing conditions $B_t = 0$ and $s_t(i) = 1$ for all $i \in [0,1]$ must hold for all $t = 0, 1, 2, \ldots$. After imposing these conditions (2), (3), (5), (8)-(20), (22)-(34), (37), (38), (40)-(43), (46), (50)-(55) can be collected together to form a system of 42 equations determining the equilibrium behavior of the 42 variables C_t, Y_t, g_t, h_t^s, h_t, h_t^g, h_t^b, h_t^d, h_t^v, F_t, Λ_t^1, Λ_t^2, Λ_t^3, M_t, T_t, N_t, D_t, D_t^h, D_t^f, L_t, L_t^h, L_t^f, M_t^a, N_t^v, M_t^s, P_t, W_t, Q_t, r_t^l, r_t^d, r_t, r_t^v, r_t^a, u_t^a, u_t^n, u_t^d, rr_t, v_t^a, a_t, Z_t, x_t^a, and τ_t.

This system implies that many of these variables will be nonstationary, with real variables inheriting a unit root from the random walk in the technology shock (23) and nominal variables inheriting a unit root from the conduct of monetary policy as described by the Taylor rule (41). However, the real variables become stationary when scaled by the lagged technology shock Z_{t-1} and the nominal variables become stationary when expressed in growth rates. When the 42-equation system is rewritten in terms of these appropriately-transformed variables, it implies that the economy has a balanced growth path, along which all of the stationary variables remain constant at steady-state values in the absence of shocks. The transformed system can therefore be log-linearized around its steady state to form a set of linear expectational difference equations that can be solved using methods outlined by Blanchard and Kahn (1980) and Klein (2000).

3 Results

3.1 Calibration

Numerical implementation of the solution procedure just described requires that specific values be assigned to each of the model's 30 parameters: χ, β, η, θ, ω, ν, ϕ_d, ϕ_p, ϕ_v, π, ρ_r,

ρ_π, ρ_g, α, τ, υ^a, υ^n, z, x^a, x^n, ρ_υ^a, ρ_a, ρ_x^a, ρ_τ, σ_υ^a, σ_a, σ_z, σ_x^a, σ_τ, and σ_r. Hence, the exercise continues by calibrating a version of the model without interest on reserves to match various statistics from the United States economy, mainly during the period extending from the fourth quarter of 1987 through the third quarter to 2008. This sample period starts with the appointment of Alan Greenspan as Federal Reserve Chairman and continues through Ben Bernanke's term until the onset of the financial crisis; throughout this period, likewise, the Federal Reserve did not pay interest on reserves.

Although the cost channel will be reintroduced later, when assessing the robustness of the results, the benchmark calibration sets $\phi_d = 0$, to keep the model as close to the standard New Keynesian framework as possible. This initial choice implies that (47) describes the aggregate demand for monetary services, so that $-1/(1+\chi)$ measures the elasticity of money demand with respect to the opportunity cost variable $r_t - r_t^a$, and this coefficient on the opportunity cost term is equal in absolute value to the coefficient on the real wage, which in turn equals one minus the coefficient on consumption. Using data on the M2 monetary services quantity index and the associated price index compiled by the Federal Reserve Bank of St. Louis to measure M_t^a and $r_t - r_t^a$, as well as data on real personal consumption expenditures and the associated chain-type price index from the National Income and Product Accounts to measure C_t and P_t and the index of compensation per hour in the nonfarm business sector assembled by the Bureau of Labor Statistics for its report on Productivity and Costs to measure W_t, an ordinary least squares regression of this form, with all of the parameter constraints imposed, yields

$$\ln(M_t^a/P_t) = -4.4 + 0.83\ln(C_t) + 0.17\ln(W_t/P_t) - 0.17\ln(r_t - r_t^a),$$

suggesting a calibrated value of $\chi = 5$. Since the St. Louis Fed discontinued its monetary services series in 2005:4, the data used to estimate this equation run from 1987:4 through 2005:4. Likewise, (48) indicates that the parameter ω measuring the elasticity of substitution between currency and deposits in creating the monetary aggregate M_t^a can be calibrated based on a regression of the ratio of currency to the M2 monetary services index on the ratio of the user cost associated with M2 monetary services to the user cost of currency. Again, these data are available from the St. Louis Fed from 1987:4 through 2005:4 and yield the estimated equation

$$\ln(N_t/M_t^a) = -6.7 + 0.53\ln(u_t^a/u_t^n),$$

suggesting the calibrated value $\omega = 0.50$. Note that this setting for ω makes the elasticity of substitution between currency and deposits smaller than that implied by the Cobb-Douglas specification that represents the special case of (4) with $\omega = 1$. In principle, a setting for ν, measuring the elasticity of substitution between reserves and labor in deposit creation, might be pinned down from a detailed study of bank productivity, but a search of the literature yielded no estimate covering the 1987-2008 period. Introspection suggests that there is likely to be very little substitutability between these two quite different inputs, however, and the calibrated value $\nu = 0.25$ reflects this idea.

Data on the federal funds rate and the growth rates of the GDP deflator and real GDP, 1987:4-2008:3, yield ordinary least squares estimates of the coefficient of the Taylor rule (41):

$$\ln(r_t) = -0.0018 + 0.95\ln(r_{t-1}) + 0.20\ln(\pi_{t-1}) + 0.13\ln(g_{t-1}),$$

suggesting the settings $\rho_r = 0.95$, $\rho_\pi = 0.20$, and $\rho_g = 0.15$. Of course, under the benchmark regime where interest is not paid on reserves, (42) and (43) get specialized by setting $\alpha = 0$ and $\tau = 1$. The analysis below considers two alternative policy regimes under which interest does get paid on reserves. In the first alternative, $\alpha = 1$ and $\tau = 1 - 0.000625$; under this policy, the monetary authority maintains an average spread of 25 basis points between the market rate of interest r_t and the interest rate on reserves r_t^v when both rates are expressed in annualized terms. In the second alternative regime, $\alpha = 1$ and $\tau = 1$, so that interest gets paid on reserves at the market rate.

Interpreting each period in the model as a quarter year in real time, the settings $z = 1.005$ and $\pi = 1.005$ imply an annualized, steady-state growth rate for real, per-capita variables of 2 percent and an annualized, steady-state inflation rate of 2 percent as well. Given these choices, the setting $\beta = 0.995$ then implies a steady-state market rate of interest of about 6 percent per year. The settings $\theta = 6$ and $\phi_p = 50$, drawn from previous work by Ireland (2000, 2004*a*, 2004*b*), make the steady-state markup of price over marginal cost equal to 20 percent and, as explained in Ireland (2004*a*), imply a speed of price adjustment in this model with quadratic price adjustment costs that is the same as the speed of price adjustment in a model with staggered price setting following Calvo's (1983) specification in which individual goods' prices are adjusted, on average, every 3.75 quarters, that is, just slightly more often than once per year.

Values for the next six parameters, η, v^a, v^n, x^a, x^n, and ϕ_v, get selected to match six facts. First, the steady-state value of hours worked equals 0.33, meaning that the representative household allocates 1/3 of its time to labor. Second, the steady-state ratio of the simple-sum monetary aggregate M_t^s to nominal consumption P_tC_t equals 3, matching the fact that during the 1987:4-2008:3 period, the average ratio of simple-sum M2 to quarterly nominal personal consumption expenditures equals 3.04; since the St. Louis Fed's monetary services indices are just that, namely index numbers for monetary services, they track growth rates not levels and therefore cannot be used to match the ratio of M_t^a to P_tC_t in the model. Third, the steady-state ratio of currency to deposits equals 0.10, approximating the fact that the average ratio of currency to deposits in simple-sum M2 equals 0.1077. Fourth, the steady-state ratio of reserves to deposits equals 0.02, matching the fact that the average ratio of Federal Reserve Bank of St. Louis adjusted reserves to simple-sum deposits in M2 equals almost exactly 0.02. Fifth, the steady-state ratio of employment in banking to total employment equals 0.007, or seven-tenths of one percent. In data from the Bureau of Labor Statistics' Current Employment Survey, 1.4 percent of all workers on total nonfarm payrolls were employed in depository credit intermediation on average over the period from 1990 through 2011. Of course, those employees engaged in a range of banking activities that extends beyond deposit creation; hence, the smaller, 0.7 percent figure is taken as the one to be matched by the model. Sixth, in data covering 2008:4 through 2011:4, the ratio of Federal Reserve Bank of St. Louis adjusted reserves to simple-sum deposits in M2 averaged 15 percent. During most of that period, the Federal Reserve paid interest on reserves at a rate intended to match the federal funds rate. Based on these observations, ϕ_v is chosen so that in the steady state of the model in which $\alpha = 1$ and $\tau = 1$, so that interest is paid on reserves at the market rate, the ratio of reserves to deposits equals 15 percent.

Searching over various parameter combinations with these targets in mind leads to the settings $\eta = 2.5$, $v^a = 0.90$, $v^n = 0.20$, $x^a = 65$, $x^n = 0.75$, and $\phi_v = 0.000005$. These parameter values also imply, through the relationship shown in (39), a steady-state spread between the market rate of interest r_t and the deposit rate r_t^d equal in annualized terms to 0.97 percent, just below one percentage point. In United States data, 1987:4-2008:3, the average spread between the three-month Treasury bill rate and the own rate of return on the deposit component of M2 equals 1.22 percent. Hence, the model and the data are not too far out of line along this added dimension; indeed, that the spread between the bond and deposit rates in the data exceeds the same spread in the model provides reassurances that the real resource cost of deposit creation in the model is based on a conservative estimate.

Intriguingly, the strictly positive setting for ϕ_v in the cost specification (31) for managing reserves allows, as revealed by (37), for the existence of equilibria not only when the interest rate on reserves r_t^v equals the market rate r_t but also when r_t^v rises slightly above r_t. The model is therefore consistent, at least to an extent, with the puzzling fact that contrary to the intentions specified in the Federal Reserve Board's (2008) press release, the federal funds rate has actually fallen below the interest rate on reserves for much of the period between 2008:4 and 2011:4. It should be noted, however, that the very small value assigned to ϕ_v implies that the model can account for a negative spread between the market rate and the rate on reserves of at most 0.0017 basis points. Hence, more likely, the institutional factors described, modeled, and analyzed by Bech and Klee (2011) explain why the interest rate on reserves has not worked perfectly in placing a hard floor beneath the federal funds rate.

Since the technology shock in (23) follows a random walk with drift, it is highly persistent by assumption. The settings $\rho_v^a = 0.95$ and $\rho_a = 0.95$ make the shocks to the demand for monetary services and preferences highly persistent as well. The additional settings $\rho_x^a = 0.50$ and $\rho_\tau = 0.50$ introduce a more modest amount of persistence in the shocks to productivity in the banking system and to the spread between the market interest rate and the interest rate paid on reserves. Since most of the analysis that follows focuses on impulse responses from the log-linearized model, the settings $\sigma_v^a = 0.01$, $\sigma_a = 0.01$, and $\sigma_z = 0.01$ are really just normalizations that make one-standard-deviation money demand, preference, and technology shocks, equivalently, into one-percentage-point shocks. The setting $\sigma_r = 000625$, however, means that the monetary policy shock leads to a 25-basis-point change in the annualized, short-term interest rate. The setting $\sigma_\tau = 0.0003125$ is half that size, so that the rate of interest paid on reserves remains below the market rate of interest after a one-standard-deviation shock to the interest rate spread, even under the alternative policy considered below in which $\alpha = 1$ and $\tau = 1 - 0.000625$, so that the monetary authority maintains an average 25-basis-point spread between the annualized market rate of interest and the annualized interest rate on reserves. Finally, the setting $\sigma_x^a = \ln(10)$ is used below to capture some of the effects of a financial crisis, in which an adverse shock reduces the productivity of reserves and labor in producing bank deposits by an entire order of magnitude.

3.2 Equilibrium Determinacy

The larger size of this model with currency, deposits, and banks precludes the derivation of analytic results like those obtained by Woodford (2003) and Bullard and Mitra (2005), identifying conditions on the coefficients of Taylor rules like (41) that ensure the determinacy of rational expectations equilibria in smaller-scale New Keynesian models. Numerical analysis indicates, however, that for this model, familiar conditions for determinacy apply, both with and without interest on reserves. Specifically, a grid search over 2001 evenly-spaced values for ρ_r between 0 and 2, 2001 evenly-spaced values for ρ_π between 0 and 2, and 11 evenly-spaced values for ρ_g between 0 and 1, making a total of more than 44 million cases in all, reveals that for all three policy regimes described above, without interest on reserves ($\alpha = 0$ and $\tau = 1$), with interest paid on reserves at an annualized rate that is, on average, 25 basis points below the market rate ($\alpha = 0$ and $\tau = 1 - 0.000625$), and with interest paid on reserves at the market rate ($\alpha = 1$ and $\tau = 1$), the exact same condition

$$\rho_r + \rho_\pi > 1 \tag{56}$$

is both necessary and sufficient on the grid for the system to have a unique dynamically stable rational expectations equilibrium according to the criteria of Blanchard and Kahn (1980). Condition (56), of course, requires the monetary authority to satisfy what Woodford (2003) calls the "Taylor principle," increasing the short-term market rate of interest more than proportionally in response to any change in inflation. This result – that the Taylor principle continues to hold even when interest is paid on reserves – reinforces the interpretation suggested above, in reference to (37) depicting the representative bank's demand for reserves. In this dynamic model, the additional degree of freedom that the monetary authority obtains from the ability to pay interest on reserves is most appropriately described as one that allows the central bank to target the real quantity of reserves separately from the market rate of interest. The dynamic properties of the short-term interest rate, therefore, remain essential for determining uniquely the dynamic paths for prices and other nominal variables.

Thus, the payment of interest on reserves, at a rate below or equal to the market rate, does not give rise to special problems of equilibrium determinacy in this New Keynesian model as it appears to in the overlapping generations models studied by Sargent and Wallace (1985) and Smith (1991). Hornstein's (2010) more detailed analysis of equilibrium determinacy in a flexible-price, money-in-the-utility function model indicates that these differences can be traced back to the specification of fiscal as well as monetary policies in Sargent and Wallace's and Smith's earlier models. Borrowing Leeper's (1991) terminology, Hornstein points out that indeterminacy prevails in Sargent and Wallace's and Smith's models in cases where both monetary and fiscal policies are "passive" in that monetary policy fails to respond vigorously to changes in prices and fiscal policy fails to respond vigorously to changes in government debt. Hornstein goes on to demonstrate that a unique, stable equilibrium, with or without interest on reserves, is pinned down in his model, just as in the model with sticky prices used here, when an "active" monetary policy rule that satisfies the Taylor principle is combined with a passive fiscal policy that leaves government taxes and transfers to be determined, endogenously, so as to support the given interest rate path.

It should be noted, again, that the small but positive labor cost (31) of managing larger stocks of reserves plays a key role in this model, to allow an equilibrium in which interest on reserves is paid at the market rate to exist in the first place. Without this additional cost, (37) confirms that banks' demand for reserves will be unboundedly large, exactly as anticipated by Sargent and Wallace (1985). On the other hand, one version of Hornstein's (2010) specification resembles the case illustrated in panel (b) of figure 1, in which there is a satiation point beyond which banks are indifferent between holding any level of reserves once the opportunity cost of doing so equals zero. In this case, Hornstein shows that a unique equilibrium can still be determined if the monetary authority adopts, in addition to the rule for managing the market rate of interest and, in lockstep, the interest rate on reserves, a rule for managing the real stock of reserves or, equivalently, the monetary base. This additional policy rule plays the same role in Hornstein's model that the small cost of managing reserves does here: it makes the real quantity of reserves uniquely-determined when interest is paid on reserves at the market rate, while having little or no impact on the behavior of other equilibrium values. Further, in both this model and in Hornstein's, these extra mechanisms for ensuring determinacy could be eliminated if the monetary authority lowers the rate of interest it pays on reserves ever so slightly below the market rate; here, an arbitrarily small but still positive interest rate spread would work, through (37), in exactly the same way as the arbitrarily small but positive labor requirement measured by the parameter ϕ_v, to keep the demand for real reserves finite and well-defined.

3.3 The Steady-State Effects of Paying Interest on Reserves

Table 1 compares the steady-state values of a range of variables under the benchmark policy that does not pay interest on reserves to the steady-state values of the same variables under the alternative policies of paying interest on reserves, either at a rate that, in annualized terms, lies 25 basis points below the market rate or that coincides with the market rate. In the model, the steady-state rate of output growth gets pinned down by the rate of technological change, as measured by the parameter z in (23), describing the process for the technology shock. The steady-state rate of inflation gets chosen by the monetary authority at the same it fixes the coefficients of the Taylor rule (41). The steady-state market rate of interest then gets determined by the Fisher relationship: it gross terms, it equals the product of the inflation rate π and the real interest rate z/β. Hence, the first three rows of table 1 confirm that none of those steady-state values depends on whether or not interest is paid on reserves.

Instead, a decision by the monetary authority to pay interest on reserves has its steady-state effects on banks' demand for real reserves as described by (37) and, through the pricing relationship shown in (39), the interest rate that banks pay on deposits. Changes in the deposit rate then set off portfolio adjustments by households, which given the shopping-time specification (3)-(5) also have implications for the levels of output and hours worked. Not surprisingly, table 1 reveals that the biggest effects in percentage terms are on banks' holdings of real reserves, which more than double moving from the steady state without interest on reserves to the steady state in which interest is paid at a 25-basis-point spread and rise by nearly a factor of eight when interest is paid at the market rate. The technological specification (31) implies that the amount of labor that banks use to manage reserves rises proportionally to changes in the stock of reserves; hence, table 1 shows that large percentage changes in h_t^v also appear across steady states. In all cases, however, the very small value for the parameter ϕ_v selected above implies that the management of reserves requires nonzero but extremely small resource costs.

Competitive pressures in the banking system imply, through (39), that reductions in banks' opportunity cost of holding reserves brought about by the payment of interest on reserves get passed along to households in the form of higher deposit rates. Table 1 shows that, in particular, the annualized interest rate on deposits rises by 15 or 16 basis points, depending on whether the interest rate on reserves is held 25 basis points below or set equal to the market rate of interest. These changes seem modest when quoted by themselves, but imply sizable reductions in the user cost of deposits, which according to Barnett's (1978) formula (52) depends not on the level of the deposit rate but rather on the spread between the market and deposit rates. Hence, according to the relationships (47)-(49), households shift out of currency and into deposits when interest gets paid on reserves, and their overall demand for monetary services as reflected in the real value of the true monetary aggregate M_t^a increases as well. These effects serve to reinforce the second interpretation, suggested above with reference to the user cost formulas (51) and (52), that by adjusting the interest rate r_t^v it pays on reserves, the central bank can use its extra degree of freedom to influence the relative prices that household's face in choosing between currency and deposits as competing media of exchange. Table 1 also shows that shopping time, while always small relative to the household's other time commitments, falls by 8 or 9 percent across steady states when interest gets paid on reserves.

In this shopping-time model as in Cooley and Hansen's (1989) cash-in-advance model and Belongia and Ireland's (2006) real business cycle model with currency and deposits, inflation acts as a tax on market activity, since households must use monetary assets that pay interest at below-market rates to purchase consumption but do not receive nominal wages payments in exchange for their labor until the end of each period. But while both of those previous studies examine how changes in the inflation rate π affect the magnitude of the inflation tax as a whole, here, instead, the focus lies on how a single component of the inflation tax gets reduced through the payment of interest on reserves. Under the benchmark policy of no interest on reserves, reserves are small when compared to both the monetary base and the level of deposits. Hence, the incremental inflation-tax effects of paying interest on reserves are small as well: as shown in table 1, total hours worked and aggregate output rise by only one-tenth of a percentage point when interest gets paid on reserves.

These small changes still imply, however, that there are welfare gains from paying interest on reserves. The last row of table 1 quantifies these gains, by showing how the representative household's steady-state utility rises moving from the benchmark policy of no interest on reserves to the alternatives in which interest is paid on reserves. To place these numbers in some perspective, the table also reports the permanent, percentage changes in consumption that generate equivalent changes in utility. In particular, the table shows that moving from the benchmark of no interest on reserves to the regime where interest is paid at a rate that is 25 basis points below the market rate yields a welfare gain equivalent to that provided by a permanent 0.0275 percent increase in consumption; moving from the benchmark to the alternative where interest is paid at the market rate yields a gain equivalent to a permanent 0.0300 percent increase in consumption. Thus, while Goodfriend (2002), Ennis and Weinberg (2007), and Keister, Martin, and McAndrews (2008) identify a variety of other, potentially more important, ways through which the payment of interest on reserves can help banks operate more efficiently, the model used here does provide an additional, albeit modest, rationale for the Federal Reserve's decision to pay interest on reserves.

3.4 The Dynamic Effects of Macroeconomic Shocks

Figure 2 plots the impulse responses of output Y_t, the aggregate price level P_t, the market rate of interest r_t, and reserves N_t^v to one-standard-deviation innovations to the preference shock a_t, the technology shock Z_t, and the monetary policy shock ε_{rt}, both under the benchmark policy without interest on reserves (solid lines) and the alternative policy of paying interest on reserves at a rate that lies 25 basis points below the market rate (dashed lines). Since the impulse response for the case where the central bank pays interest on reserves at the market rate resemble so closely those for the case with the 25-basis-point spread, results for this third case are not shown. The panels express output, the price level, and nominal reserves in logs and the interest rate in annualized percentage-point terms.

The preference shock acts as an exogenous, non-monetary, demand-side disturbance, increasing both output and prices and, under the Taylor rule (41) calling forth a tightening of monetary policy in the form of higher interest rates. The technology shock increases output and decreases the price level. The random walk specification (23) implies that the technology shock's effect on the level of output is permanent, and the implied short-run increase in output growth dominates the decrease in inflation so that, under the Taylor rule, the monetary authority responds with a modest, 8-basis-point increase in the market rate of interest. Finally, the monetary policy shock generates a 25-basis-point increase in the market rate of interest that, in this purely forward-looking model, begins to reduce output and prices immediately. The implied movements in output growth and inflation then lead the interest rate, through the Taylor rule, back quite quickly to its steady-state value, despite the large setting $\rho_r = 0.95$ for the interest rate smoothing coefficient in (41). These are the variables and shocks that hold center stage in most New Keynesian analysis, and here they display their usual behavior.

The new results shown in figure 2 can be summarized by observing that, in each of the first three rows, the solid and dashed lines overlap, so much so that they are indistinguishable. While, in fact, the changes in the market rate of interest shown in the third row do give rise to changes in banks' opportunity cost of holding reserves under the benchmark policy without interest on reserves but not under the alternative in which the positive interest rate on reserves tracks changes in the market rate to maintain the 25-basis-point spread, and while, in principle, these differences in the cost of holding reserves might translate into variable inflation tax effects that then impact differently on output and inflation as well, these effects turn out, quantitatively, to be very small. These results for the model's output and price dynamics echo those for the steady states described earlier in table 1.

Again as in table 1, however, measures of money, particularly bank reserves, behave quite differently across policy regimes. Although, in the fourth row of figure 2, important differences appear in the aftermath of all three macroeconomic shocks, they are most striking in the case of a monetary policy shock. In the traditional case in which interest is not paid on reserves, the monetary authority must drain reserves from the banking system in order to bring about the desired liquidity effect: the 25-basis-point increase in the market rate of interest. When interest is paid on reserves, however, the change in reserves required to generate the 25-basis-point increase in the market rate differs not just in magnitude but in sign: the monetary authority must initially expand the nominal supply of reserves to prevent the market interest rate from rising even more.

Equation (39) helps, once again, in explaining this surprising result. In the model, banks create deposits with a combination of reserves and labor. Hence, as shown in (39), the wedge between the market rate and the competitively-determined interest rate on deposits depends both on the opportunity cost of holding reserves $r_t - r_t^v$ and the real wage relative to productivity $(W_t/P_t)/Z_t$. Without interest on reserves, the rise in the market rate increases the opportunity cost term, more than offsetting the decline in the real wage brought about by the contractionary macroeconomic effects of the monetary policy shock. When the monetary authority increases r_t^v in lockstep with r_t, so as to maintain a constant spread between the two, the opportunity cost of holding reserves gets held fixed and the only effect that remains works through the decline in wages, so that the spread $r_t - r_t^d$ declines, as does the user cost of deposits given by (52). Figure 3 displays these differences in the response of the user cost of deposits and also shows how, as a consequence, households substitute more strongly into deposits after a monetary policy shock in the case where interest is paid on reserves. Hence, the liquidity effect vanishes entirely when interest is paid on reserves: because the increase in the market interest rate has general equilibrium effects on households' demand for deposits and banks' demand for reserves, the market interest rate and the quantity of reserves move together, in the short run, when interest is paid on reserves.

Still, both with and without interest on reserves, the Taylor rule (41) associates a contractionary monetary policy shock with a transitory fall in inflation but a permanent decline in the aggregate price level. Hence, moving back to figure 2, the bottom, right-hand panel shows that even when the monetary authority pays interest on reserves, it must in the long-run contract the supply of reserves after a monetary policy shock. So while the dynamic behavior of reserves differs dramatically depending on whether or not interest is paid on reserves, the long-run effects coincide: a monetary policy action that decreases the price level always requires a proportional reduction in the nominal supply of reserves. These results suggest, once again, that the additional degree of freedom that the monetary authority obtains when it can pay interest on reserves is best interpreted as one that, in the long run, influences the real, as opposed to the nominal, quantity of reserves. At the same time, however, these results highlight how moving to a policy of paying interest on reserves requires sometimes dramatic changes in short-run operating procedures that go well beyond those that can be illustrated in simple, static diagrams like those from figure 1.

3.5 The Effects of Financial Shocks

Figure 4 repeats the impulse response analysis from figure 2, but for the remaining three shocks to money demand v_t^a, bank productivity x_t^a, and the spread τ_t between the market rate and the interest rate paid on reserves. The left-hand column confirms that Poole's (1970) classic result, showing that by holding the market rate of interest fixed in the face of shocks to money demand, the monetary authority automatically accommodates the shifts in demand with appropriate shifts in the supply of liquid assets and thereby insulates the macroeconomic from effects of those disturbances, carries over to this setting just as it does to the simpler New Keynesian model studied by Ireland (2000). In particular, the figure shows how, under the Taylor rule (41), the supply of reserves increases to meet the additional demand generated by an increase in v_t^a, leaving output, inflation, and the market rate of interest virtually unchanged.

The calibrated value σ_x^a is selected above to make the banking productivity shock very large and thereby simulate the effects of a severe disruption to the financial system that makes it much more difficult for private financial institutions to supply households with highly liquid assets like bank deposits. Hence, the middle column of figure 4 traces out the effects of an adverse shock of this kind, that is, a negative one-standard-deviation innovation to x_t^a. The bottom panel shows how the monetary authority floods the economy with reserves to partially offset the negative effects of this shock. Nevertheless, in the top row, output still falls by one percent. These effects appear much the same, regardless of whether or not the monetary authority pays interest on reserves. Evidently, what matters most in shaping the effects of this financial-sector shock is how the monetary authority expands the supply of reserves, while (37) indicates, to the contrary, that changes in the rate of interest on reserves work mainly to change the demand for reserves.

Finally, the right-hand column of figure 4 shows the effects of a 12.5-basis-point increase in the interest rate that the monetary authority pays on reserves. The solid line in the bottom panel shows that starting from the benchmark case in which no interest gets paid on reserves, this small and temporary increase in the interest rate on reserves has only modest effects on the demand for reserves and can therefore be supported by a small increase in reserve supply. Starting from the alternative case in which there is a 25-basic-point spread between the market rate of interest and the interest rate on reserves, however, this same shock cuts banks' opportunity cost of holding reserves in half, and therefore sets off much larger changes in monetary variables, including the supply of reserves. Figure 4 also confirms that once again, changes in the interest rate paid on reserves have very small effects on output and inflation.

3.6 Robustness

Running through all of the results displayed in table 1 and figures 2-4 is the basic finding that while the monetary authority's decision to pay interest on reserves can have very important effects on both the steady-state levels and dynamic behavior of reserves and other monetary variables, the effects on output and inflation are by contrast quite small. Behind these results lie some very basic features of the contemporary United States economy, which are reflected in the model's calibration. In the United States prior to 2008, when the Federal Reserve paid no interest on reserves, the stock of reserves was small, relative to both the monetary base and the level of deposits. Moreover, inflation and market rates of interest remained low and stable. Putting these two sets of facts together: with a small tax base as measured by the stock of reserves and a low tax rate as measured by the market rate of interest relative to the zero rate of interest paid on reserves, the distortionary effects on the macroeconomic stemming from banks' demand for reserves were modest and, therefore, the effects of changes in the opportunity cost of holding reserves more modest still.

The robustness of these findings is confirmed, for example, by the results shown in figure 5, which reintroduces the cost channel into the model by resetting the parameter ϕ_d from the representative intermediate goods-producing firms deposits-in-advance constraint (25) equal to one, while holding all other parameters fixed at their benchmark values. Under this alternative parameterization, the ratio of deposits held by firms to deposits held by households is just slightly less than 0.30 in the steady state without interest on reserves. Over the period from 1987:3 through 2008:3, the Federal Reserve's flow of funds data show that the ratio of deposits held by nonfinancial businesses to deposits held by households and nonprofit organizations was just slightly less than 0.21. Hence, the adjusted value $\phi_d = 1$ leads the model to overstate the importance of firms' holdings of liquid assets. Moreover, since with all the other parameter values held fixed, households continue to hold the same level of deposits as under the benchmark parameterization, this example overstates the importance of deposits economywide as well: the steady-state ratio of the simple-sum monetary aggregate M_t^s to nominal consumption P_tC_t, for instance, rises to 3.9, above the comparable figure of 3 used to guide the benchmark calibration. Nevertheless, figure 5 reveals that the effects of macroeconomic shocks on output, prices, and interest rates depend little on whether or not the monetary authority pays interest on reserves.

To show, however, that these basic results reflect, not the inevitable workings of the model itself, but rather the way in which the model gets calibrated to match the most relevant aspects of the United States economy, figure 6 displays impulse responses generated after ϕ_d is reset to its benchmark value of zero but two, more important, sets of changes are made to the model's parameter values. First, new values $v^a = 3.75$, $v^n = 0.225$, $x^a = 11$, and $x^n = 0.98$ increase the steady-state ratio of the simple-sum monetary aggregate M_t^s to nominal consumption P_tC_t from 3 to 10 and the steady-state ratio of reserves N_t^v to deposits D_t from 0.02 to 0.10, increasing greatly the importance of reserves in creating deposits and deposits in providing transaction services, while holding constant the steady-state ratio of currency to deposits at 0.10 and the steady-state ratio of employment in banking to total employment at 0.007, the values used for the benchmark calibration. This first set of changes, therefore, has the effect of enlarging dramatically the tax base that gets hit when the monetary authority does not pay interest on reserves. Second, new values $\rho_r = 0.50$, $\rho_\pi = 0.75$, and $\rho_g = 0$ for the coefficients of the Taylor rule (41) make monetary policy shocks more persistent. Since, when the monetary authority does not pay interest on reserves, movements in the market rate translate directly into movements in the opportunity cost of holding reserves, this second set of changes makes swings in the distortionary tax rate on reserves more persistent as well.

In figure 6, therefore, output responds quite differently to shocks, depending on whether or not the monetary authority pays interest on reserves. The additional inflation tax effects, for instance, make the decline in output that follows the monetary policy shock depicted in the figure's far right column significantly larger when interest is not paid on reserves. Likewise, the rise in the market rate of interest that follows a preference shock turns the initial output expansion into a subsequent contraction when interest is not paid on reserves. These results suggest that, in other economies with different basic features, the monetary authority's decision to pay or not to pay interest on reserves might have larger macroeconomic consequences. But with this alternative calibration just as before, the dynamic behavior of reserves themselves shifts most dramatically when the monetary authority decides to pay interest on reserves.

4 Conclusion

The analysis performed here, with the help of a dynamic, stochastic, general equilibrium New Keynesian model, shows that the Federal Reserve's recent decision to begin paying interest on reserves is unlikely to have large effects on the behavior of macroeconomic variables, such as output and inflation, once normal times return. These results serve also to confirm the basic thrust of arguments advanced, using a variety of quite different models, by Goodfriend (2002), Ennis and Weinberg (2007), Keister, Martin, and McAndrews (2008), and Kashyap and Stein (2012), all suggesting that the ability to manipulate the spread between the federal funds rate and the interest rate it pays on reserves provides the Fed with an additional degree of freedom that it can use to expand its policymaking strategies and objectives. Specifically, the results obtained here show how the Fed can continue to adjust its target for the federal funds rate to achieve its goals for macroeconomic stabilization, while independently varying the interest rate on reserves, as necessary, to help enhance the efficiency of and reinforce the stability of private financial institutions and the financial sector as a whole.

Re-examining these issues using a fully dynamic and stochastic macroeconomic model, however, sharpens and extends the insights gleaned from this previous work. Results obtained here, for instance, suggest that in the long run, the additional degree of freedom provided by the ability to pay interest on reserves is best described as one that gives the Federal Reserve the ability to target the real quantity of reserves separately from the federal funds rate. Even when it pays interest on reserves, the Fed must continue to use open market operations to adjust the nominal quantity of reserves proportionally, following any policy action intended to bring about a long-run change in the aggregate price level. On the other hand, the results obtained here also show that the decision to pay interest on reserves does require rather dramatic changes in the way in which the Fed manages the supply of reserves in the short run. In particular, with interest on reserves, the traditional short-run liquidity effect, associating a monetary policy tightening with higher interest rates brought about through a reduction in the supply of reserves, vanishes. Instead, as discussed above, portfolio reallocations by households may actually require the monetary authority to initially increase the supply of reserves when raising its target for the short-term interest rate.

These results have implications, too, for the Fed as its acts to unwind the unprecedentedly large policy actions it took during the financial crises of 2008 and the severe recession that followed. Most obviously, yet perhaps most importantly, the model shows how the steady-state level of reserves held willingly by the banking system can increase enormously when the monetary authority reduces the opportunity cost of doing so by paying interest on reserves. Thus, to the extent that the Federal Reserve continues to pay interest on reserves at a rate that equals or falls just slightly below its target for the federal funds rate, even as it moves to normalize that target for the federal funds rate, banks' demand for reserves will remain permanently higher. It follows that the Fed might leave most or perhaps even all of the additional dollars in reserves it has injected since mid-2008 to continue circulating within the banking system without creating any inflation. In fact, the model's dynamics referred to above imply that if the Fed begins to raise its target for the federal funds rate while holding the spread between the funds rate and the interest rate it pays on reserves fixed, private agents' portfolio shifts may actually increase banks' demand for reserves still further in the economy's general equilibrium, requiring the Fed to expand the supply of reserves by even more to prevent interest rates from rising too quickly.

As discussed by Goodfriend (2002, 2011), a decision to continue paying interest on reserves at rates close to the federal funds rate would necessarily reduce the magnitude of the Fed's transfers to the United States Treasury as those interest rates rise and, under certain conditions, might even require the Fed to approach the Treasury for additional funding for its day-to-day operations, potentially threatening the central bank's independence. Plosser (2010) highlights a less direct, but possibly more dangerous channel, through which a policy of paying interest on reserves could weaken the Fed's independence if its greatly enlarged balance sheet was taken as a signal of an ability and willingness to intervene significantly in other markets if asked to by Congress or the Treasury. Of course, these public finance and political economy considerations are absent from the model developed here. To the extent that they become important in reality, however, the model clearly implies that convergence back to a steady state with a wider spread between the federal funds rate and the interest rate on reserves would require large open market operations to re-absorb the immense stocks of excess reserves now held within the banking system.

Finally, the analysis shows how these results, particularly those having to do with the small macroeconomic effects of paying interest on reserves, depend on some specific features of the United States economy. Before 2008, in the years leading up to the Fed's decision to begin paying interest on reserves, banks operated successfully with fairly small stocks of reserves. Moreover, inflation, interest rates, and by extension banks' opportunity cost of holding reserves, remained low as well. With a relatively small base taxed at a relatively low rate to begin with, incremental changes in the opportunity cost of holding reserves, brought about through independent variations in the policy rate paid on reserves, have only modest effects outside the banking system. Building on this same intuition, however, the model also demonstrates how, in other economies where banks operate less efficiently and market rates of interest are higher on average, paying interest on reserves may have more profound macroeconomic consequences.

Although the model developed here reveals and highlights various aspects of the macroeconomic effects of these new policies, it abstracts from many important features of the banking and payments systems that might justify further the Federal Reserve's recent decision to begin paying interest on reserves. Elaborating on the model in future research, so as to incorporate more of these institutional features, would surely yield additional insights and would serve, as well, to bring the analysis into closer contact with the complementary efforts by Goodfriend (2002), Ennis and Weinberg (2007), Berentsen and Monnet (2008), Keister, Martin, and McAndrews (2008), Bech and Klee (2011), and Kashyap and Stein (2012).

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Table 1. Steady-State Effects of Paying Interest on Reserves

	No Interest Paid On Reserves $\alpha = 0$, $\tau = 1$	Interest Paid At 25 Basis Point Spread $\alpha = 1$, $\tau = 1 - 0.000625$		Interest Paid At The Market Rate $\alpha = 1$, $\tau = 1$	
Variable	Steady-State Value	Steady-State Value	Percentage Change	Steady-State Value	Percentage Change
Output Growth Y_t/Y_{t-1}	1.0050	1.0050	0.00	1.0050	0.00
Inflation P_t/P_{t-1}	1.0050	1.0050	0.00*	1.0050	0.00*
Market Interest Rate r_t	1.0151	1.0151	0.00*	1.0151	0.00*
Output Y_t/Z_{t-1}	0.3314	0.3317	0.09	0.3317	0.10
Shopping Time h_t^s	0.0009	0.0008	−7.97	0.0008	−8.78
Hours Worked h_t	0.3320	0.3324	0.09	0.3324	0.10
Hours in Goods Production h_t^g	0.3297	0.3300	0.09	0.3300	0.10
Hours in Banking h_t^b	0.0023	0.0023	0.88	0.0023	1.00
Hours in Deposit Creation h_t^d	0.0023	0.0023	0.87	0.0023	0.97
Hours in Reserves Management h_t^v	0.0000	0.0000	122.45	0.0000	683.50
Real Reserves $(N_t^v/P_t)/Z_{t-1}$	0.0191	0.0425	122.45	0.1496	683.50
Real Monetary Base $(M_t/P_t)/Z_{t-1}$	0.1113	0.1323	18.88	0.2408	116.27
Real Currency $(N_t/P_t)/Z_{t-1}$	0.0922	0.0893	−3.23	0.0890	−3.56
Real Deposits $(D_t/P_t)/Z_{t-1}$	0.9202	0.9675	5.14	0.9729	5.72
Real True Monetary Aggregate $(M_t^a/P_t)/Z_{t-1}$	0.8856	0.9013	1.76	0.9029	1.95
Real Simple-Sum Monetary Aggregate $(M_t^s/P_t)/Z_{t-1}$	1.0125	1.0568	4.38	1.0618	4.88
Real Wage $(W_t/P_t)/Z_{t-1}$	0.8375	0.8375	0.00	0.8375	0.00
Interest Rate on Reserves r_t^v	1.0000	1.0145	5.79*	1.0151	6.04*
Interest Rate on Deposits r_t^d	1.0127	1.0130	0.15*	1.0131	0.16*
Own Rate on True Monetary Aggregate r_t^a	1.0110	1.0114	0.16*	1.0114	0.17*
User Cost of Currency u_t^n	0.0149	0.0149	0.00	0.0149	0.00
User Cost of Deposits u_t^d	0.0024	0.0020	−15.29	0.0020	−16.79
User Cost of True Monetary Aggregate u_t^a	0.0040	0.0036	−9.57	0.0036	−10.53
Reserve Ratio rr_t	0.0207	0.0439	111.57	0.1537	641.09
Utility	−387.3589	−387.3040	0.0275**	−387.2989	0.0300**

Notes: Each row shows the steady-state value of the variable indicated under the benchmark policy of no interest on reserves and the alternative policies of paying interest on reserves either at an annualized rate that is 25 basis points below the annualized market rate or at the market rate. "Percentage Change" refers to the percentage change in the steady-state value of each variable under each alternative policy with interest on reserves compared to the value of the same variable under the benchmark policy of no interest on reserves, except starred (*) entries that show percentage-point changes in annualized inflation and interest rates and double-starred (**) entries that convert differences in utility into equivalent, permanent percentage changes in consumption.

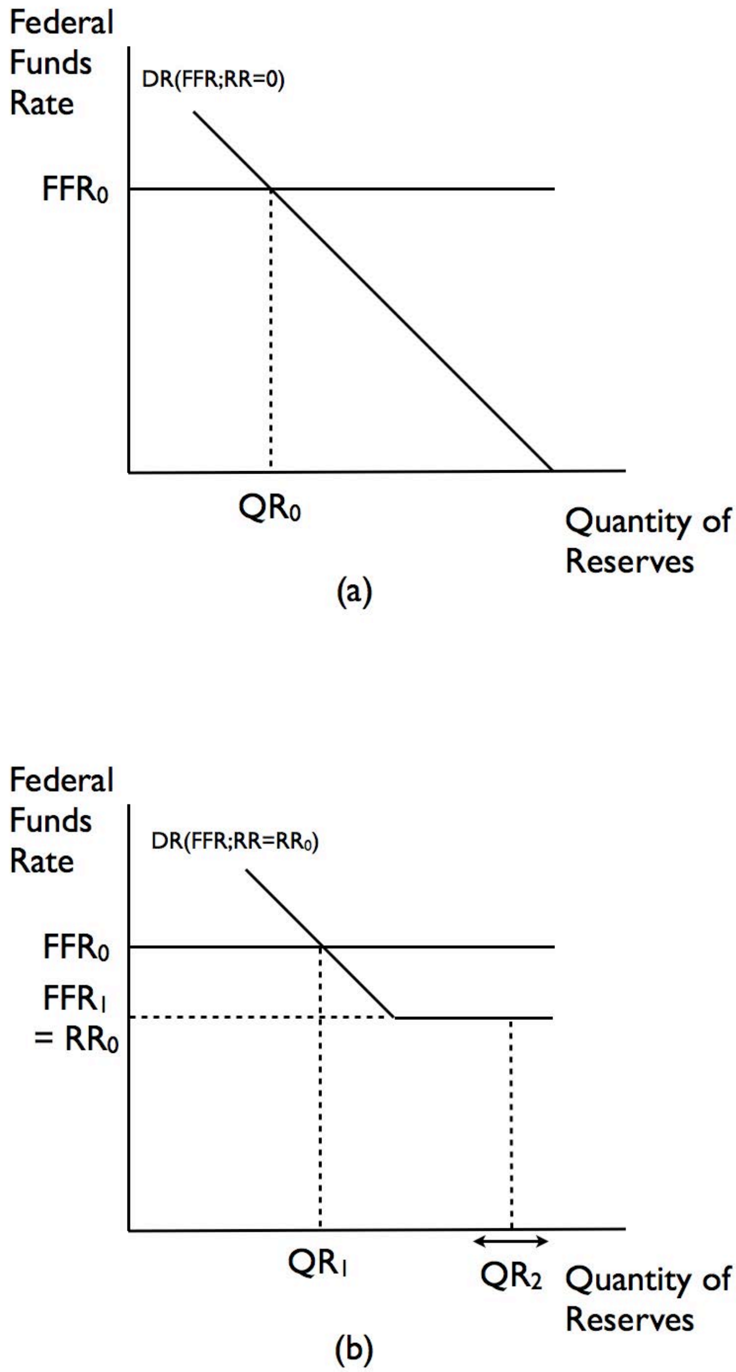

Figure 1. Federal Funds Rate Targeting, With and Without Interest on Reserves. In panel (a), the central bank does not pay interest on reserves and supplies QR_0 dollars in reserves to support its federal funds rate target FFR_0. In panel (b), the central bank pays interest on reserves at the rate RR_0. It can supply QR_1 dollars in reserves to support the funds rate target $FFR_0 > RR_0$ or it can lower its federal funds rate target to $FFR_1 = RR_0$, in which case the quantity of reserves supplied, QR_2, can lie anywhere along the horizontal segment of the demand curve.

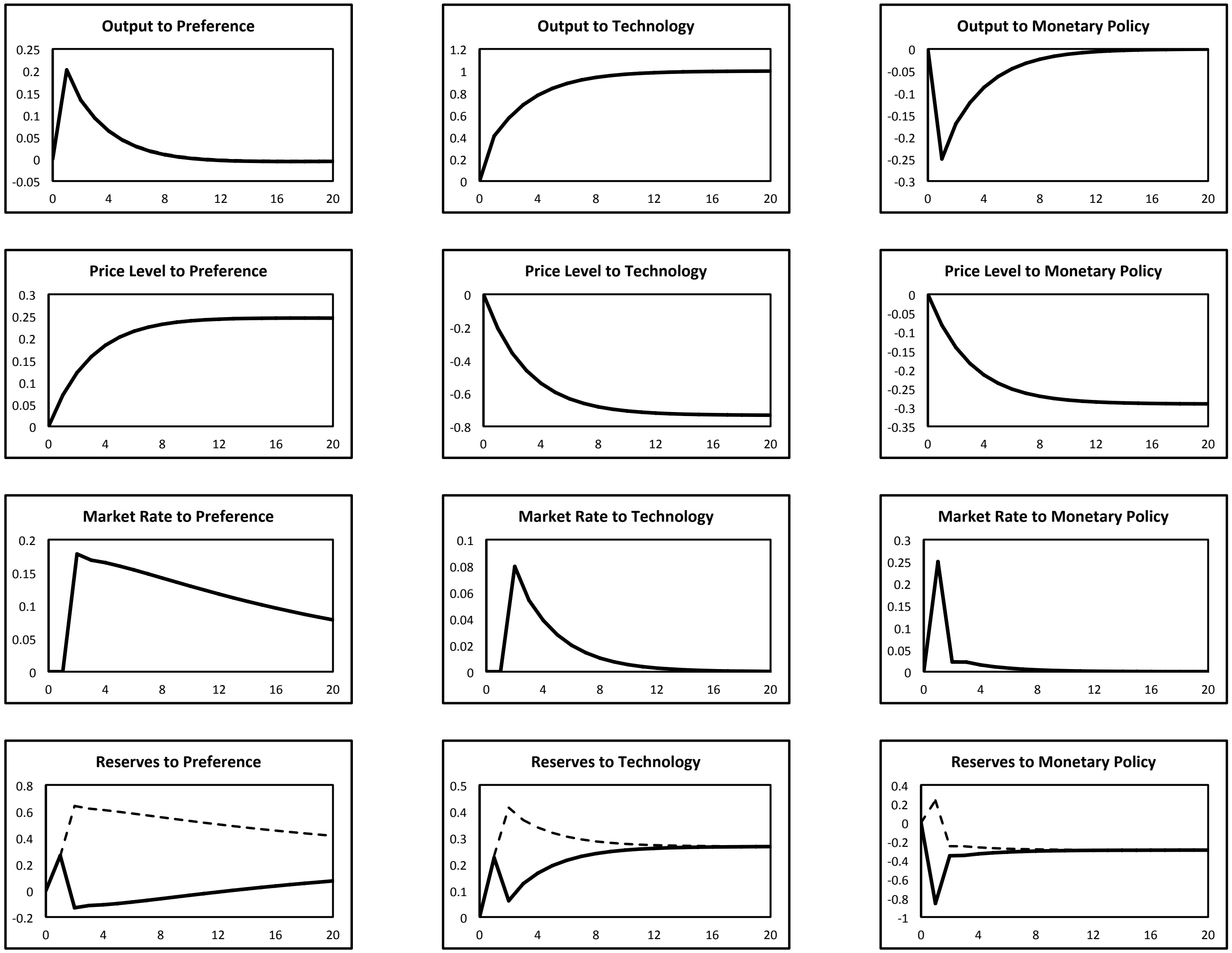

Figure 2. Impulse Responses to Macroeconomic Shocks: Benchmark Parameterization. Output, the price level, and reserves are in logs; the market rate of interest is annualized. Solid lines track the responses under the benchmark policy without interest on reserves; dashed lines track the responses under the alternative policy when interest is paid on reserves at a rate that is 25 basis points below the market rate.

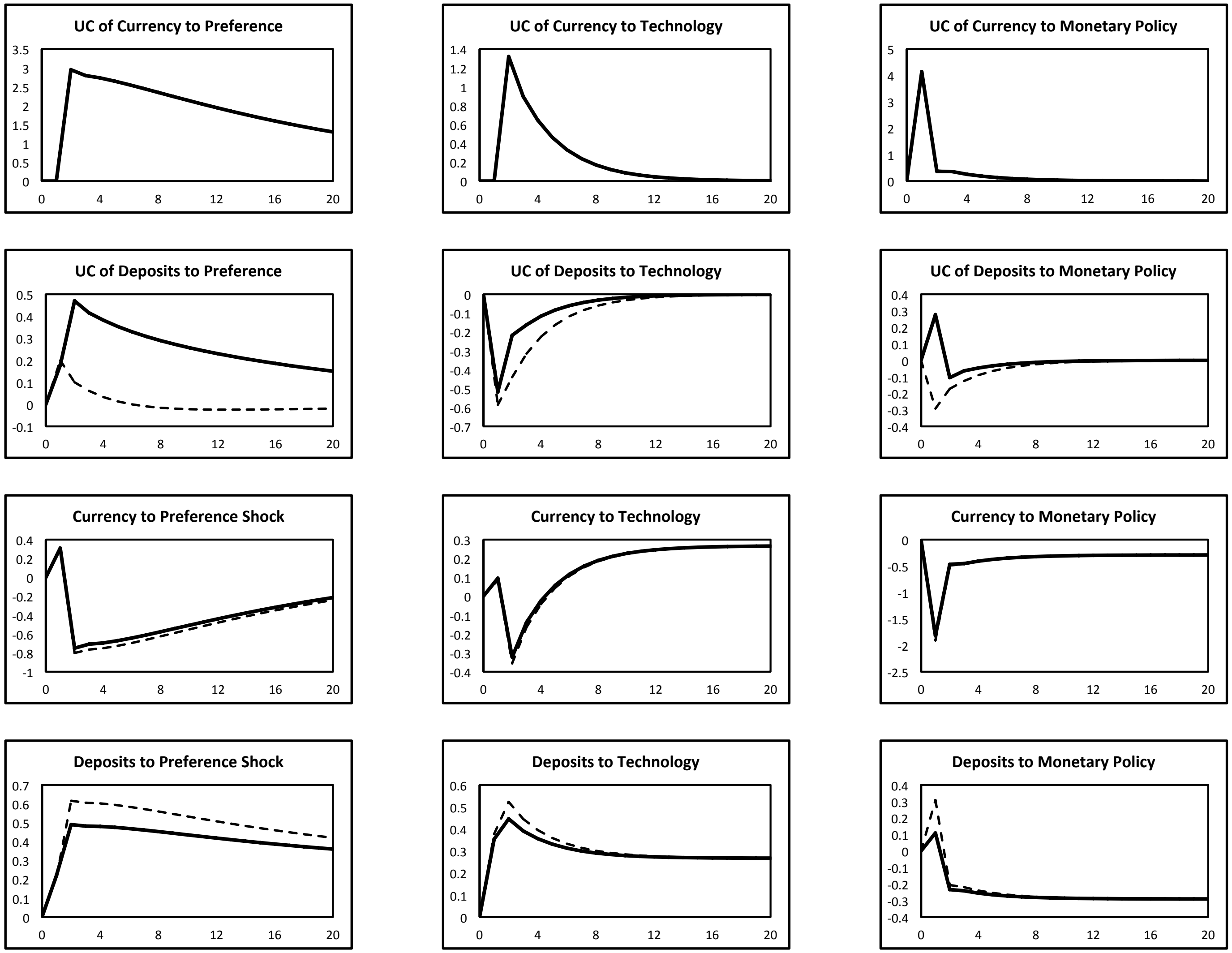

Figure 3. Impulse Responses to Macroeconomic Shocks: Benchmark Parameterization. Currency, deposits, and their user costs are all in logs. Solid lines track the responses under the benchmark policy without interest on reserves; dashed lines track the responses under the alternative policy when interest is paid on reserves at a rate that is 25 basis points below the market rate.

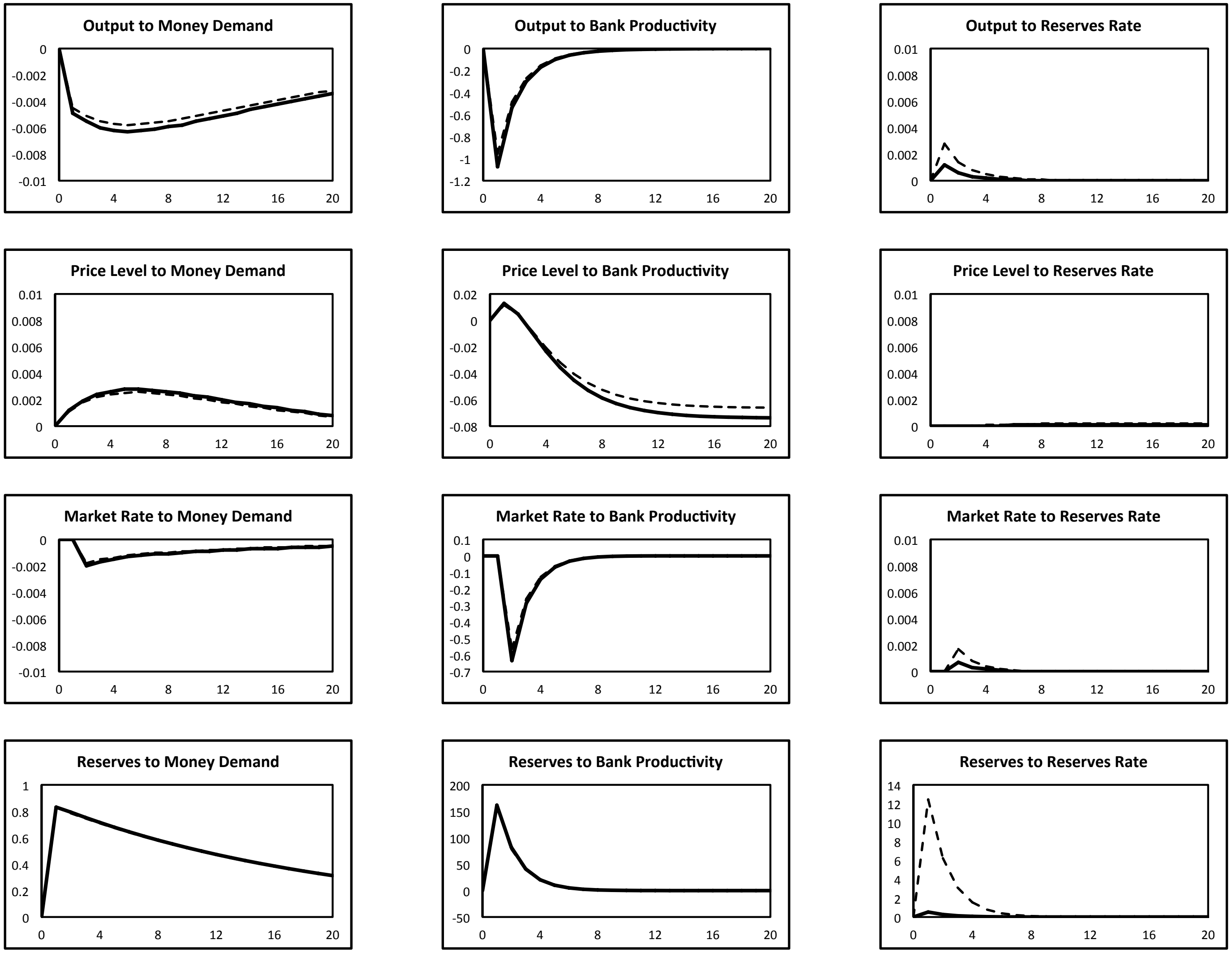

Figure 4. Impulse Responses to Financial Shocks: Benchmark Parameterization. Output, the price level, and reserves are in logs; the market rate of interest is annualized. Solid lines track the responses under the benchmark policy without interest on reserves; dashed lines track the responses under the alternative policy when interest is paid on reserves at a rate that is 25 basis points below the market rate.

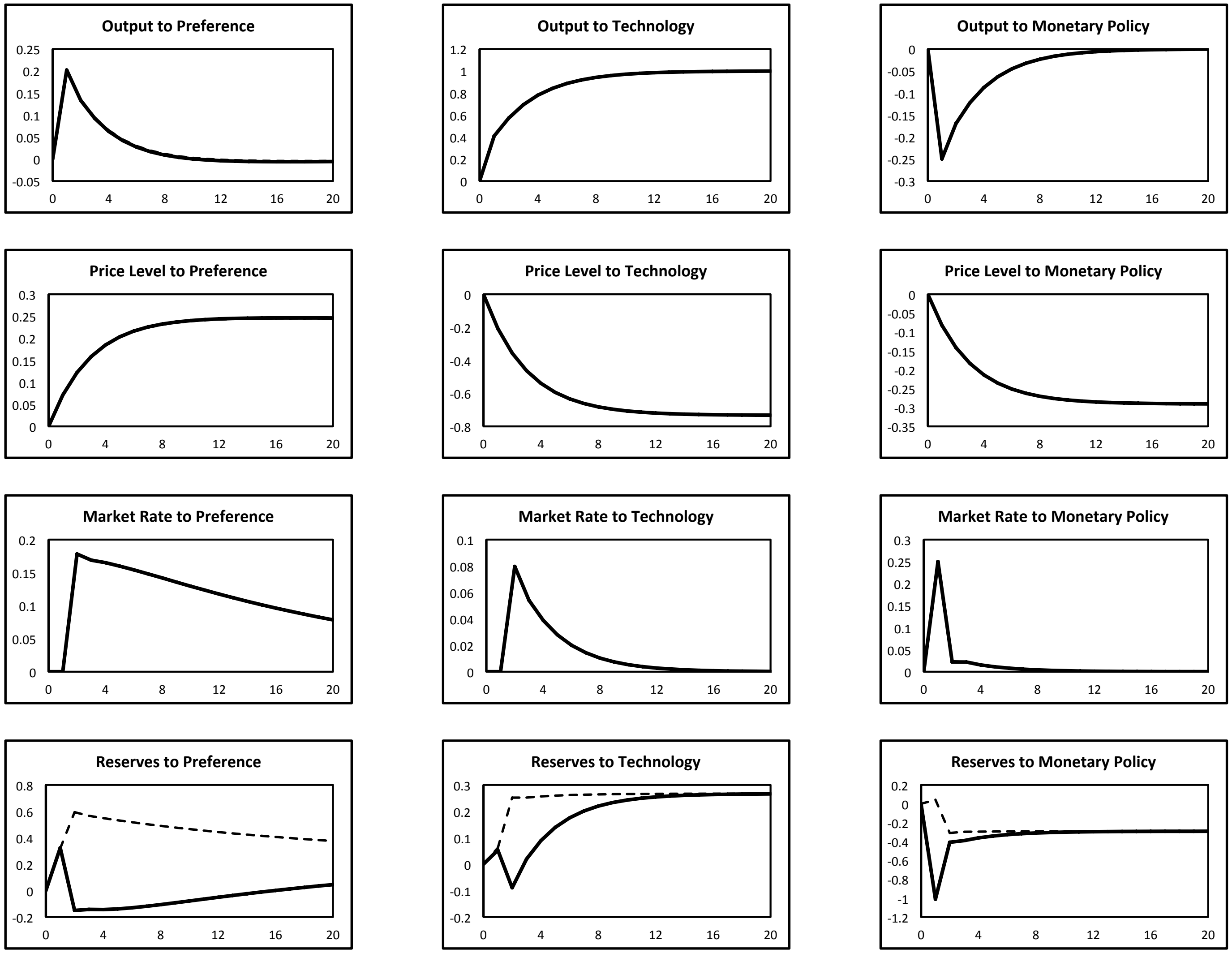

Figure 5. Impulse Responses to Macroeconomic Shocks: Alternative Parameterization with Cost Channel. Output, the price level, and reserves are in logs; the market rate of interest is annualized. Solid lines track the responses under the benchmark policy without interest on reserves; dashed lines track the responses under the alternative policy when interest is paid on reserves at a rate that is 25 basis points below the market rate.

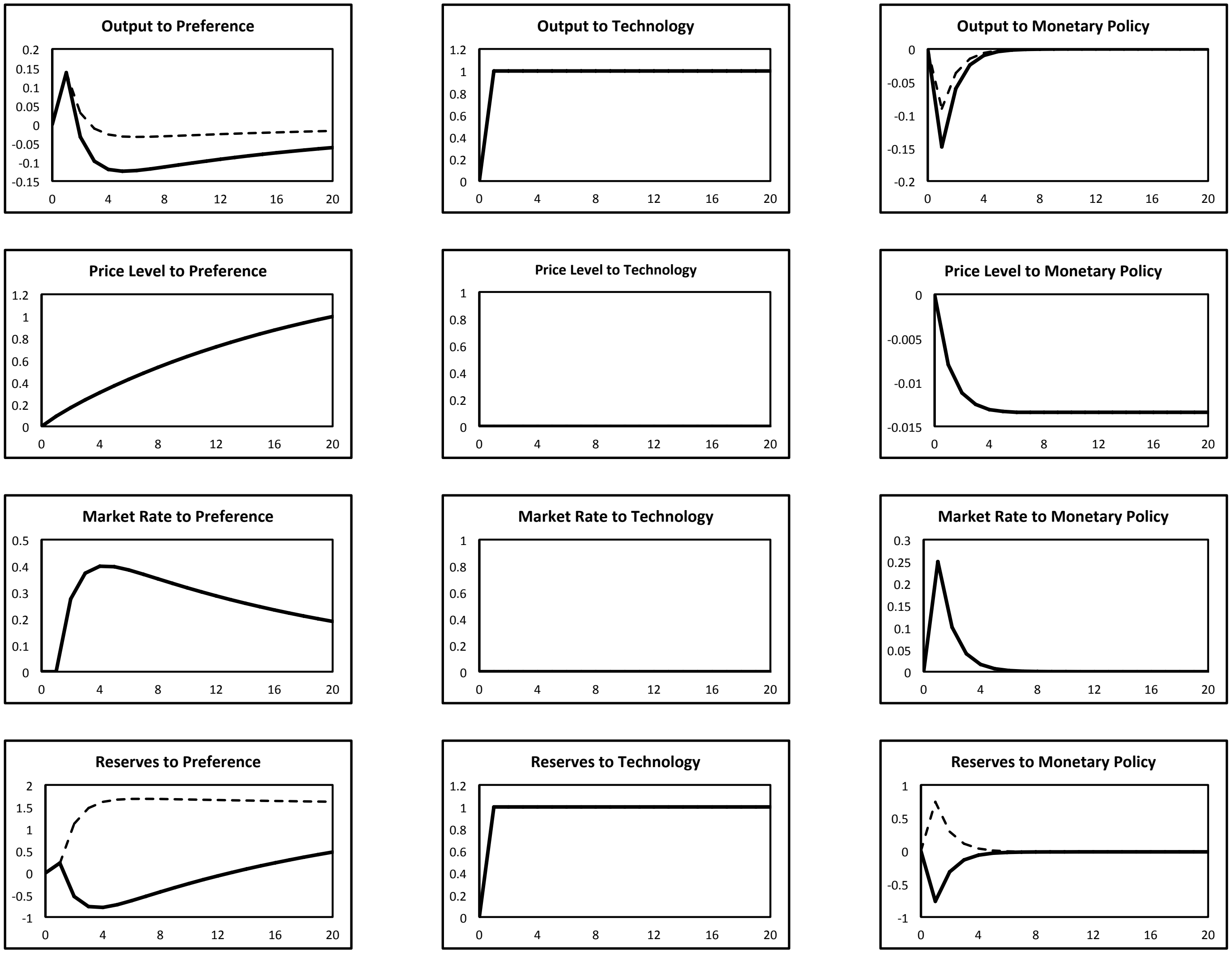

Figure 6. Impulse Responses to Macroeconomic Shocks: Alternative Parameterization with Larger Inflation Tax Effects. Output, the price level, and reserves are in logs; the market rate of interest is annualized. Solid lines track the responses under the benchmark policy without interest on reserves; dashed lines track the responses under the alternative policy when interest is paid on reserves at a rate that is 25 basis points below the market rate.